Investigation of Compliant Space Mechanisms with Application to the Design of a Large-Displacement Monolithic Compliant Rotational Hinge

Robert McIntyre Fowler

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ABSTRACT

Investigation of Compliant Space Mechanisms with Application to the Design of a Large-Displacement Monolithic Compliant Rotational Hinge

Robert M. Fowler
Department of Mechanical Engineering, BYU
Master of Science

The purpose of this research is to investigate the use of compliant mechanisms in space applications and design, analyze, and test a compliant space mechanism. Current space mechanisms are already highly refined and it is unclear if significant improvements in performance can be made by continuing to refine current designs. Compliant mechanisms offer a promising opportunity to change the fundamental approach to achieving controlled motion in space systems and have potential for dramatic increases in mechanism performance given the constraints of the space environment. A compliant deployment hinge was selected for development after industry input was gathered. Concepts for large-displacement compliant hinges are investigated. A design process was developed that links the performance requirements of deployment to the design parameters of a deployment hinge. A large-displacement monolithic compliant rotational hinge, the Flex-16, is designed, analyzed, and tested. It was developed for possible application as a spacecraft deployment hinge and designs were developed using three different materials (polypropylene, titanium, and carbon nanotubes) and manufacturing processes (CNC milling, electron beam manufacturing metal rapid prototyping, and a carbon nanotube framework) on two size scales (macro and micro). A parametric finite element model allowed for prediction of prototype behavior before fabrication. The Flex-16 hinge is capable of 90° of deflection without failure or contact and can be designed to meet industry requirements for space.

Keywords: compliant space mechanisms, compliant mechanisms, space mechanisms, deployment hinge, large displacement, monolithic, rotational joint
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CHAPTER 1. INTRODUCTION

1.1 Research Motivation

Satellites, rovers, the international space station, and other space vehicles require mechanisms to perform mechanical tasks. These space mechanisms have been designed to perform in the demanding environments of space and launch. Efforts are continually being made to improve their performance and reliability while considering cost [1]. Many current mechanisms are already highly optimized and it is unclear if significant improvements in performance can be made by continuing to refine current designs. Compliant mechanisms offer a promising opportunity to change the fundamental approach to achieving controlled motion in space systems and have potential for dramatic increases in mechanism performance given the constraints of the space environment.

Compliant mechanisms gain motion from the elastic deflection of flexible components. Advantages of compliant mechanisms over traditional mechanisms include increased performance through reduced weight, increased precision, reduced friction, elimination of lubricants, ease of miniaturization, and integration of functions into fewer components. Figure 1.1 shows a few examples of compliant mechanisms designed by the Compliant Mechanisms Research Group (CMR) at Brigham Young University. The field of compliant mechanisms has matured to the point that design and analysis methods, and the increase in commercial compliant mechanisms, makes it possible to apply them in critical applications.

1.2 Thesis Objective & Approach

The purpose of this thesis is to:

1. Introduce the field of compliant space mechanisms, a merger of compliant mechanism and space mechanism technologies. The feasibility of this technological merger will be examined by identifying and discussing the crucial parameters, namely by
Figure 1.1: Examples of compliant mechanisms designed by the CMR: (a) scanning electron micrographs of compliant micro mechanism next to a white blood cell, a micro compliant joint for high g-loads, and a micro bistable mechanism, (b) a high precision device for nanoscribing, (c) an exercise device with specified force-displacement characteristics, (d) a compliant overrunning (one-way) clutch, (e) a lamina emergent mechanism (LEM), (f) a custom prototyping kit, compatible with Legos, to facilitate quick design and testing of compliant mechanisms, and (g) a compliant prosthetic knee.

• examining the current state of both compliant and space mechanisms
• identifying the advantages compliant mechanism could offer in light of the documented challenges and failures faced by current space mechanisms.
• examining the challenges associated with the space and launch environments
• identifying design guidelines and performance requirements on the mechanism and system level for space mechanisms
• aligning the research with the future direction of the aerospace industry’s technological advancements

• defining the avenues of research in this field

2. Select a space mechanism to design using compliant mechanism technology. This will be done by:

• reviewing the different types of space mechanisms and identifying application opportunities for compliant space mechanisms

• selecting a specific mechanism type to design - deployment hinge was chosen

• developing and identifying possible concepts for compliant hinges

• developing and identifying possible concepts for compliant dampers

• investigating the current design of a deployment hinge and its components and determining opportunities for compliant mechanism design

3. Develop a deployment hinge design process. The design of the deployment hinge is vital in dictating the dynamics of the deployment. This step-by-step design process links the deployment performance requirements with the hinge parameters and allows a hinge design to be selected that meets the requirements levied by the mission or program. This will be done by

• summarizing the dynamic model of a deployment hinge

• explaining the links between the requirements and hinge parameters

• creating specific steps that allow for a hinge design that meets the performance requirements

• showing an example of using the deployment hinge design process

4. Design, analyze, and test a compliant space mechanism. This will be done by:

• setting the desired capabilities and possible space application of the mechanism

• investigating research already done in this area
• inventing the design
• creating a parametric finite element model to analyze designs
• creating a parametric CAD model to facilitate prototyping
• performing a configuration study to select ideal design after analyzing many different geometric arrangements of the design
• analyzing the selected designs
• validating the analytical model
• prototyping the selected designs in three different materials and on two different size scales
• testing the prototypes and analyzing results
• creating CAD models of the hinge integrated with spacecraft components in its intended application
• discussing strengths and challenges of the design

1.3 Thesis Outline

Chapter 1 introduces the motivation, objectives, and approach of the research. Chapter 2 provides the motivation for research in compliant space mechanisms. This work was published in the Journal of Mechanical Sciences after being presented at the International Symposium on Compliant Mechanisms in Delft, Netherlands. The mathematical foundation for deployment performance is summarized and linked to hinge design parameters by a deployment hinge design process developed in Chapter 3. Chapter 4 investigates concepts for compliant hinges in light of current deployment hinge design. The Flex-16, a monolithic large-displacement compliant rotational hinge was developed for possible application on a spacecraft deployment hinge and designs were developed using three different materials and manufacturing processes. Its design, analysis, and testing is developed in detail in Chapter 5. It also aligned the hinge design with industry requirements for space. The work in Chapter 5 will be published as a journal article in Mechanism and Machine Theory. Chapter 6 presents the conclusions from the research and describes recommendations for future work.
CHAPTER 2. COMPLIANT SPACE MECHANISMS

2.1 Introduction

This chapter proposes Compliant Space Mechanisms as a new research direction in compliant mechanisms, provides the motivation to do so, shows the application and benefits of this research, and discusses the crucial factors in beginning to understand this field. 

2.2 Proposed Research Direction

A compliant space mechanism is a moveable mechanical assembly that achieves its desired motion, force, or displacement by means of the deflection of flexible members and that can perform a necessary function in the environments of launch and space.

The field of compliant space mechanisms has the potential for significant impact on the performance of space mechanisms because compliant mechanisms offer distinct advantages that can address many of the issues encountered in current rigid-link mechanisms. The advent of design and analysis methods for compliant mechanisms allows for design engineers to accurately model flexible segments and to design precision devices for specific tasks. The pseudo-rigid-body model [2] enables flexures to be modeled as rigid-link assemblies with representative link lengths and torsional spring constants. It also enables the use of well-known rigid-link kinematic and dynamic analysis software packages in analyzing compliant mechanisms. The advent of this approach has brought about many advances in numerous fields and lends itself particularly well to the space industry. Topology optimization [3–6], another method of compliant mechanism design, can be used in the conceptual phase of design to arrive at the optimal geometry for specific loading and boundary conditions. These and other tools, further discussed in Section 2.4.2, provide the

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1This chapter has also been published as a journal article entitled “Compliant Space Mechanisms: A New Frontier for Compliant Mechanisms” in the Journal of Mechanical Sciences after being presented at the International Symposium on Compliant Mechanisms in Delft, Netherlands.
ability to merge compliant mechanism technology and space mechanisms so it can be readily used and implemented by designers and analysts in the space industry.

2.3 Motivation

This section addresses the motivation for creating compliant space mechanisms, including the challenges faced by current space mechanisms, the potential advantages of compliant space mechanisms, and limiting factors. The known challenges inherent in compliant mechanisms are discussed, as are the lessons learned from space mechanism failures. The relationship between compliant mechanism research and NASA’s Technology Roadmaps is summarized.

2.3.1 State of Current Space Mechanisms

Current space mechanisms are almost entirely composed of traditional rigid-link assemblies. These mechanisms perform a variety of functions. However, the harsh environments of space impose demanding requirements and rigid-link mechanisms can experience a variety of issues, including lubrication outgassing, friction and binding of joints, and inadequate force or torque margin of safety. The mission objectives or desired mechanism functionality also have demanding requirements and rigid-link mechanisms are naturally prone to issues concerning size, weight, and accuracy of motion. Continual effort is put into improving the performance and reliability of space mechanisms, yet many of the inherent challenges still remain and lead to compromises in performance and reliability. Since mechanisms often perform functions that are singularly vital for mission success, a failure could be catastrophic to the mission. Many of the failures of space mechanisms have been documented [7, 8] and occur because of the design tradeoffs and inherent challenges.

2.3.2 Possible Advantages of Compliant Mechanisms in Space Applications

The application of compliant mechanism technology could prove vital in overcoming some of the difficult challenges that current space mechanisms face when put in the space environment. Tables 2.1 and 2.2 show key challenges of space mechanisms and which advantages of compliant mechanisms could possibly address each challenge.
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<td>Outgassing of lubrication, friction, wear and binding of joints</td>
<td>Elimination of joints requiring lubrication or friction</td>
</tr>
<tr>
<td>Large mass/weight (and accompanying cost)</td>
<td>Significant part count reduction; miniaturization possible</td>
</tr>
<tr>
<td>Large size/volume</td>
<td>Significant part count reduction, increased number of possible joints and design configurations, integration of multiple functions into one mechanism, simpler geometries can lead to a reduction in material and assemblies needed to achieve the required motion</td>
</tr>
<tr>
<td>Incorrect mechanism stiffness (components assumed rigid are not entirely rigid)</td>
<td>Ability to accurately model and predict flexible joint and mechanism stiffnesses; distributed compliance better represents real-world applications than lumped compliance; flexures with distributed compliance can exhibit lower stress concentrations than those with lumped compliance [9]</td>
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<tr>
<td>Complex or costly to manufacture and integrate</td>
<td>Less expensive manufacturing methods possible (e.g. planar); simpler, more integrated geometries can achieve the required motion; significant part count reduction; little or no assembly required</td>
</tr>
<tr>
<td>Challenges of Current Space Mechanisms</td>
<td>Advantages of Compliant Space Mechanisms</td>
</tr>
<tr>
<td>---------------------------------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>Feedback noise in attitude control system due to mechanism dynamics</td>
<td>Precision motion provides increased predictability and control over mechanism mode shapes, natural frequencies, and component stiffnesses; can improve isolation from deployment dynamics with distributed compliance</td>
</tr>
<tr>
<td>Single point failure modes</td>
<td>Redundancy in actuation and motion possible; elimination of lubrication and friction dependent joints</td>
</tr>
<tr>
<td>Inadequate force or torque margin of safety</td>
<td>Accurate analysis methods; redundancy in actuation and motion possible</td>
</tr>
<tr>
<td>Lack of accurate modeling and analysis methods for flexible and large-displacement segments</td>
<td>Proven design and analysis methods</td>
</tr>
<tr>
<td>Reduced reliability in off-nominal conditions</td>
<td>Analysis methods provide increased predictability of behavior in off-nominal conditions; reduced susceptibility to foreign objects during testing and operation</td>
</tr>
<tr>
<td>Thermal gradients cause joint binding or misalignment</td>
<td>Mechanisms constructed of a single continuous material</td>
</tr>
<tr>
<td>Backlash, hysteresis, and joint misalignment</td>
<td>Monolithic (single piece) nature of compliant joints eliminates backlash, makes hysteresis predictable, and reduces the need for assembly, thus reducing the possibility of joint misalignment</td>
</tr>
</tbody>
</table>
These advantages eliminate or reduce many of the disadvantages inherent in rigid-link space mechanisms. Compliant mechanisms also offer an increased number of mechanism designs, joints, and configurations. This provides more options in finding an optimized, low-cost mechanism design. The distributed compliance of some compliant mechanisms could be particularly useful in robotic grasping, sample collection, landing platforms, and rover suspensions. Resulting from reduced part count and simpler topologies is the possible effect on a program level of fewer schedule delays during design, procurement, or manufacturing. The possible advantages of compliant space mechanisms are summarized here:

- Elimination of lubrication, friction, binding, backlash, rigid joints
- Reduced weight, part count, volume, complexity, and cost
- Increased precision, ease of manufacture, integration, and redundancy
- Large variety of innovative joints and flexures; accurate analysis of mechanisms undergoing large deflection
- Increased designer control over stiffness, deflection, natural frequencies, deployment dynamics, mode shapes, and stresses

The advantages of compliant space mechanisms provide the opportunity to change the fundamental approach to achieving controlled motion in space systems and to design simpler, more reliable, better performing, and more cost-effective solutions for many space applications.

2.3.3 Limiting Factors

The Space Environment

Much is known about the environment of space and the challenges it presents in designing mechanisms [10–12]. The basics of understanding the space environment include the effects of the vacuum, thermal radiation, charged particle radiation, atomic particles, micrometeoroids and debris. The degree to which each of these has an effect on spacecraft is a function of orbit altitude.
The vacuum of space creates several potential problems:

- **Outgassing**: the release of gasses from spacecraft materials, often seen in composites and joint lubrication. This can weaken materials and cause failure. It can also cause material property degradation from emission condensation on neighboring components.

- **Pressure in sealed structures**

- **Water desorption and material contraction**

- **Cold welding**: fusing together of metal components since the vacuum eliminates small air gaps

- **Limited means of heat transfer**: heat can only be transferred by radiation between components and space, and by conduction between components in contact

Outgassing and cold welding can be avoided by designing joints requiring neither lubrication nor contact surfaces. The often monolithic nature of compliant mechanisms lends itself well to designing joints that overcome these challenges.

The limited means of heat transfer is a problem inherent to the space environment. Compliant space mechanisms need to meet the thermal requirements of their specific mission and must be able to handle the thermal loading or be properly isolated from it. The reduction in number of parts and contact surfaces of compliant mechanisms provides increased simplicity in predicting the temperatures and thermal paths in the mechanism. The thermal properties of space vehicles are important and are carefully analyzed and monitored. The thermal environment is determined by the temperature and radiation interactions of all the components of the space vehicle and the sun or planets. Convection is not a means of heat transfer in space and excess heat must be properly dissipated from critical components using radiation, or conduction into a neighboring component. A monolithic compliant joint has the potential for significantly better heat transport capability than a multi-element ball bearing.

Thermal radiation causes thermoelastic loading, distortions, and stresses. It could also result in interface contact and jamming, complex thermal analyses, and a the necessity of a thermal control system. Electromagnetic radiation from the sun heats up surfaces exposed to it. Whether the mechanism is exposed to the sun or in its shadow can significantly change its temperature and
thus its material properties. The resulting thermal extremes can reduce the accuracy and reliability of mechanism performance and can cause damage to surfaces and electronic components. The performance of compliant mechanisms designed for use in space applications must not be adversely affected by the extreme range of temperature. The temperature dependent performance of flexures will need to be better understood.

Charged particles are encountered in the space environment and come from different sources such as the solar wind and flares on the sun, galactic cosmic rays from outside the solar system, or charged particles from the Van Allen radiation belts. They can cause spacecraft charging and possible discharges, sputtering, and single event phenomenon [10]. Charged particle radiation can necessitate protective metal enclosures for electronics which adds extra mass.

Atomic particles can cause material erosion on exposed surfaces as well as degrading thermal control coatings.

Micrometeoroids and debris can cause catastrophic damage from collisions.

The effects of these problems could be mitigated by employing compliant mechanisms. Research will be vital in evaluating how these challenges affect compliant space mechanisms and how those differ from the challenges affecting current space mechanisms.

The Launch Environment

The launch environment subjects mechanisms to some of the most intense loads and vibrations experienced on a mission. The launch loads often range from 25 to 100 g’s. The vibration profile is random and can excite many modes. Understanding these loads and the vibrational response of the mechanisms requires extensive analysis and testing. The stresses, deflections, natural frequencies, mode shapes, and other crucial characteristics will need to be found for any compliant space mechanism.

Compliant mechanisms lend themselves well to vibration isolation applications. A satellite and all its appendages have many needs for vibration isolation. Compliant mechanisms offer the potential to design custom vibration isolation management systems. The design and analysis of large-displacement flexures for space applications is a matter of energy management. The understanding and control of strain energy in compliant mechanisms can be applied to address
this critical need. Compliant mechanisms provide the ability to safely manage the vibrations from launch, separation, or deployment events.

**Performance Requirements**

The NASA Space Mechanisms Handbook [8] identifies distinct measurable quantities as mechanism performance metrics. They are:

- Range of Motion
- Torque (or Force) Margin
- Operating Speed
- Operating Life
- Pointing Accuracy
- Slew (or Scan) Rate
- Deployment Time
- Restow Capability
- Stowage Duration

A listing of similar metrics is shown in [11].

**System Requirements**

Aside from mechanism performance, a mechanism must be able to be integrated into the other subsystems and become part of a whole space vehicle. These considerations [8] affect not only the mechanism but the entire space vehicle.

- Weight
- Stiffness
- Envelope
• Clearance

• Alignment

• Interface

• Environments (temperature, vibration, shock, vacuum, transportation, and storage)

These are often vital factors that influence the design of mechanisms and will become avenues of opportunity for innovation as compliant space mechanism design research advances.

Along with the performance requirements, it is important to remember the key analyses performed on space mechanisms. They are:

• Stress
  – Strength
  – Fatigue

• Stiffness

• Thermal and thermoelastic

• Mass properties

• Modal

• Structural response

• Mass properties

• Failure mode identification

• Dynamics

Awareness of the different requirements and analyses that will be performed on compliant mechanisms will aid designers in developing viable compliant space mechanisms.
Possible Challenges of Compliant Mechanisms in Space Applications

The numerous advantages of compliant mechanisms also come with some challenges. Two distinct challenges are:

• The coupling of motion and forces in compliant mechanisms creates a more complex design situation

• Off-axis stiffness and motion are possible

Typically, a mechanism is designed so it can achieve a certain motion, then, the forces in the joints and links are determined. In compliant mechanisms, the kinematics and dynamics are coupled.

In current space mechanisms, off-axis stiffnesses are significantly larger than the stiffness in the desired direction/axis and are often neglected. In compliant mechanisms, off-axis stiffness can be lower and could create undesirable parasitic motion.

Other known challenges associated with compliant mechanisms are:

• Designing joints or flexures capable of large deflection without failure due to stress, fatigue, or vibration

• Stress relaxation or creep (e.g. under preload, particularly at elevated temperatures or extended stowage duration)

• Designing thin flexible segments to withstand the vacuum and thermal extremes of space [10]

• Continuously rotating joints often require a hybrid, or partially compliant design

• Designing complex compliant mechanisms that can be manufactured from a planar (or similar) state

• Stored strain energy (can be an advantage or drawback depending on the application)

Fortunately, these challenges are surmountable if proper attention is given to already established compliant mechanism design and analysis guidelines. It will remain the mechanism designer’s responsibility to be vigilant in accounting for these challenges. Some aspects, such as manufacturing and testing, will involve a certain level of difficulty due to the stringent requirements for space applications.
2.3.4 Documented Failures: Space Mechanisms Lessons Learned Study

The Space Mechanisms Lessons Learned Study performed by NASA [7] is an extensive collection of knowledge learned from past space mechanism failures, and is evidence that many of the failures in space mechanisms are due to problems for which compliant space mechanisms may provide solutions. The Lessons Learned Study investigated available literature on mechanism failures as well as the research being performed to eliminate those failures. It included a review of the first 28 Annual Proceedings of the Aerospace Mechanism Symposium, documents on deployable appendages from NASA Goddard, an industrial survey that yielded meaningful anomaly reports from companies in the space industry, and a review of the European literature contributed by the European Space Tribology Lab. The “Needs Analysis” section lists future needs for space mechanisms and will be useful in guiding compliant space mechanism research. It begins by stating, “A review of the information compiled for the Lessons Learned study reveals that bearing and lubrication problems are the most prevalent and, thus, improved technologies are most needed in these areas.” It then details the specific needs for each of the three categories: Deployable Appendages, Rotating Systems, and Oscillating Systems.

Compliant mechanisms show potential for overcoming lubrication and friction issues caused by traditional mechanisms with lubricated contact surfaces, e.g. ball bearings, and present an opportunity to meet the needs defined by past experience and summarized in the NASA Lessons Learned Study.

2.3.5 A Call for Future Direction: NASA Technology Roadmaps

The NASA Technology Roadmaps ([13] and [14]) provide specific areas where compliant mechanisms may make an immediate impact and help to show how research in this field contributes to technological goals on a national level. Technology Roadmaps 12 and 9 are most applicable to compliant space mechanisms.

Technology Roadmap 12, “Materials, Structures, Mechanical Systems, and Manufacturing” [13] provides detailed technologies that are priority for research and development in the areas most related to compliant mechanisms. Several of the identified technologies are listed in Table 2.3. Compliant mechanisms show great promise in these areas.
<table>
<thead>
<tr>
<th>Roadmap Section</th>
<th>Subsection</th>
<th>Specific Technologies Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.1 Materials</td>
<td>Flexible Material Systems</td>
<td>Expandable Habitat; Flexible EDL Materials; Solar Sail; Shape Morphing Materials; Advanced Expandable Materials</td>
</tr>
<tr>
<td>12.2 Structures</td>
<td>Lightweight Concepts</td>
<td>Composite/Inflatable Habitats; Expandable Structures (Precision Mirrors &amp; Solar/Antenna Arrays); Landers</td>
</tr>
<tr>
<td>12.2 Structures</td>
<td>Innovative, Multifunctional Concepts</td>
<td>Reusable Modular Components</td>
</tr>
<tr>
<td>12.3 Mechanical Systems</td>
<td>Deployables, Docking and Interfaces</td>
<td>Common Universal Interchangeable Interfaces; Restraint/Release Devices; Deployment of Flex Materials; Large Lightweight Stiff Deployable; Precision Structure Deploy Mechanism</td>
</tr>
<tr>
<td>12.3 Mechanical Systems</td>
<td>Electro-mechanical, Mechanical and Micromechanisms</td>
<td>Active Landing Attenuation System; New Concepts</td>
</tr>
<tr>
<td>12.3 Mechanical Systems</td>
<td>Design and Analysis Tools and Methods</td>
<td>Kinematics &amp; Rotor Dynamics Analysis</td>
</tr>
</tbody>
</table>

Technology Roadmap 9 “Entry, Descent, and Landing” [14] also provides areas for application of compliant space mechanisms, such as: flexible thermal protection systems for entry; mechanical deployments for attached deployable decelerators for descent; and anchoring, touchdown, and extreme terrain suspension systems for landing.

Some of the areas listed are large scope goals for the future and it is understood that compliant mechanisms would be an integral part of the solutions.
2.3.6 Building upon, not Replacing, Heritage

From the Russian’s Sputnik I (Figure 2.1), to present day, an impressive amount has been learned about space mechanisms. Many mechanisms have performed vital functions, such as the sample collection mechanism on the Phoenix Mars Lander (Figure 2.2) that uncovered ice on Mars.

Figure 2.1: Sputnik I. Courtesy of NASA.

This rich heritage is not to be replaced or forgotten; it is to be continued and built upon to take space mechanisms to the next level of performance and reliability.

Figure 2.2: Example of a current space mechanism. Phoenix Mars Lander sample collection shovel. Courtesy of NASA/JPL-CalTech-University of Arizona.
2.4 Merging Compliant Mechanisms and Space Mechanisms

The merger of compliant mechanism technology to space mechanisms is addressed in this section by discussing the current state of space mechanisms, the current state of compliant mechanism research, and the proposed merger of these two fields. Possible technologies to develop and suggested areas to begin research are summarized.

2.4.1 Current Space Mechanisms

Many earth-sensing satellites, planetary rovers and orbiters, and manned spacecraft have been developed that require mechanisms to perform specific tasks, such as deployments, instrument pointing, stage separations, dockings, sample return, landings, retention and release, attitude stability, etc.

Space Mechanism Design Rules

Since the general paradigm for designing space mechanisms has remained fairly consistent over the years, design rules exist for space mechanisms and have been generally well established by industry. A good example of a list of design rules is shown on page 499 in Space Vehicle Mechanisms - Elements of Successful Design [12]. The NASA Space Mechanisms Handbook [8] is the space industry’s authoritative document on space mechanism design and contains guidelines and details for space mechanisms of all types. It is available for free to approved U.S. citizens at www.grc.nasa.gov/WWW/spacemech/ (with updates through 2009). The AIAA has a similar document that standardizes how to, for example, calculate margins of safety for a space mechanism [15] and is commonly used in the space industry. Research needs to be performed in collaboration with these knowledge standards.

In addition to design rules, NASA’s Space Mechanisms Handbook [8] cites general design objectives for space mechanisms:

- Identify and eliminate failure modes

- Increase mechanism robustness (performance in off-nominal conditions

- Provide adequate force or torque margin
Table 2.4: Space mechanism types

<table>
<thead>
<tr>
<th>Space Mechanism Types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployable</td>
</tr>
<tr>
<td>Rotating</td>
</tr>
<tr>
<td>Suspension</td>
</tr>
<tr>
<td>Restraint &amp; Release</td>
</tr>
<tr>
<td>Latches &amp; Stops</td>
</tr>
<tr>
<td>Drive</td>
</tr>
<tr>
<td>Vibration Isolation</td>
</tr>
<tr>
<td>Separation</td>
</tr>
<tr>
<td>Landing</td>
</tr>
<tr>
<td>Sample Return</td>
</tr>
<tr>
<td>Docking</td>
</tr>
<tr>
<td>Shape Control</td>
</tr>
<tr>
<td>Pointing/Oscillating</td>
</tr>
</tbody>
</table>

- Provide redundancy
- Design for producibility/manufacturability (without compromising performance)
- Design mechanisms for test and analysis
- Provide access for repair
- Provide adequate instrumentation
- Design for storage (stress relaxation, lubrication evaporation)

Many of these design objectives can be met by the advantages of compliant space mechanisms (Tables 2.1 and 2.2).

**Space Mechanism Types & Mechanism Components**

Table 2.4 lists different types of mechanisms needed in space, most of which are identified in the NASA Space Mechanisms Handbook [8].

Common deployable mechanisms include: hinges, booms, antennas, covers, solar arrays, trusses, and dampers. Common rotating mechanisms include control moment gyroscopes, momentum wheels, reaction wheels, slip rings, and solar array drives. Common pointing/oscillating
mechanisms include gimbals, harmonic drives, bearings, swash plates, pivot joints, antennas, telescopes, and scanning or pointing mirrors. Precision pointing mirrors are common in space astronomy and can often have tight requirements for pointing, even needing to have a resolution of 1 microradian. A common example of a restraint & release mechanism is a pyrotechnic separation nut used to disconnect the solar arrays from the structure at deployment. A common example of a separation mechanism is known as a Lightband, which releases the payload from the launch vehicle at the separation stage. An example of a mobility suspension mechanism is the Mars Rover rocker-bogie suspension or the ATHLETE rover multi-DOF legs.

2.4.2 Current State of Compliant Mechanisms

Significant work has already been done in the field of compliant mechanisms. Many compliant mechanisms have become commercially available and research is ongoing in many areas of application. A complete review of the research and publications in compliant mechanisms is not feasible to present here. Compliant mechanisms symposia are held annually as part of the ASME Mechanisms & Robotics Conference, and the International Symposium on Compliant Mechanisms has been held every four years. Significant advances have been made in compliant mechanisms:

- analysis and design methods (e.g. the pseudo-rigid-body model [2], topology optimization [3–6], and FACT [16–18])

- advanced applications (e.g. lamina emergent mechanisms [19–22], bistable mechanisms [2, 23–25], constant-force mechanisms [2, 26], metamorphic mechanisms [27–29], large-displacement mechanisms [2,30], contact-aided mechanisms [31–35], spherical mechanisms [36,37], embedded actuators and sensors [38], and statically balanced mechanisms [39,40]).

A Handbook of Compliant Mechanisms will be published by John Wiley & Sons in 2012 and will include a library to illustrate a few hundred examples.

The research in compliant mechanisms and their resulting components and systems are available to address many of the issues related to space mechanisms. An active compliant mechanisms research community exists with applications ranging from microelectromechanical systems to shape morphing wings to spinal implants.
2.4.3 Applying Compliant Mechanism Technology to Space Mechanisms

The types of space mechanisms listed in Table 2.4 can benefit from the application of compliant mechanism technology. Design and analysis methods, such as the pseudo-rigid-body model, topology optimization, and FACT, allow for the design of joints or linkages using flexible segments of all types of boundary conditions. Simple flexures, four bar mechanisms, spherical linkages, and many other types of kinematic chains can be modeled and designed. Revolute, torsional, spherical, planar, and many other types of joints can be designed. Examples could include a fixed-guided flexure for attaching a thermally expanding-contracting telecom waveguide to the structural panels or a one-piece bistable compliant four-bar deployment hinge.

The different types of current space mechanisms [8, 11, 12] (also shown in Table 2.4) can be evaluated for their potential for being replaced by compliant mechanisms. Some types will be better suited for the merger than others. In doing this, the distinct motions and functions [8, 41] that are commonly required by space mechanisms need to be defined. Working from the understanding of current space mechanism functions and requirements, compliant space mechanisms can be created that are either compliant redesigns of current space mechanisms or are novel mechanism designs that fulfill a required function or motion. This can be done using a toolbox of compliant mechanism joint types [2, 20, 30, 42] and design techniques [2, 19, 30, 41, 43, 44]. Design guidelines and rules need to be developed on how to apply compliant mechanism technology to space mechanisms so that the process is standardized and widely usable. The breadth of possible applications and configurations of compliant mechanisms in space applications should also be explored.

Candidate designs can be evaluated against the design criteria that has been developed. Promising candidate designs will be selected for detailed analysis. Analysis of the mechanisms will be performed to better understand the behavior of the mechanisms, especially in terms of their desired functionality in the space environments. The performance of the mechanisms should be quantified in several categories, which were outlined earlier as the Performance and System Requirements of the NASA Space Mechanisms Handbook or similarly in [11, 12].

Testing will be performed following the same standards that are used to qualify flight hardware. The testing results will be compared to the analytical results and models can be verified or corrected, and added to the pool of design knowledge. Efforts should be taken to not only evaluate the performance of the mechanism but to optimize its design and performance. The lessons learned
through this process of design, analysis, and testing will be documented as initial guidelines for this field and act as a stepping stone for continued advancements.

**Example of a Compliant Space Mechanism**

Although not yet widespread, flexible links have been successfully used on space mechanisms [12] and the field is just beginning to be explored [45]. Compliant vibration isolation systems such as SoftRide are produced by CSA Engineering (a division of Moog) and have been used on numerous satellite missions. Figures 2.3 and 2.4 show these compliant mechanisms being effectively used to isolate a satellite from launch vehicle vibrations. Similar mechanisms have been used on the Hubble Space Telescope for on-orbit jitter reduction and solar array vibration damping. Propellant tank tab flexures and compliant universal joints are other common examples.

![Figure 2.3: Example of a compliant space mechanism. SoftRide vibration isolation mechanism. Compliant segments are labeled. Courtesy of U.C. Berkeley.](image)

**Possible Concepts to Develop for Compliant Space Mechanisms**

In addition to converting current space mechanisms to compliant mechanisms, innovative compliant mechanism designs may be developed. This could include the development of the concepts listed here:

- Compliant joints capable of 90 and even 180 degree deflection; multiple segment joints
Figure 2.4: Example of a compliant space mechanism. SoftRide vibration isolation mechanism. The compliant segments separate the launch vehicle adapter and the satellite WISE that was launched on a Delta II in December 2009. Courtesy of U.C. Berkeley.

- Statically balanced hingeline joints
- Compliant joints with improved off-axis properties
- Temperature insensitive joints
- Modular vibration isolation mechanisms
- Compliant bi/tri-stable hinges
- Creasing joints for deployable arrays and masts
- Integrated deployment springs
• Redundant compliant latches
• Multi-Layer lamina emergent thermal radiators
• Pointing array mechanisms
• Two/three axis compliant gimbal platforms
• Compliant joints with integrated damping

2.4.4 Areas of Research & Questions to Answer

Merging the fields of compliant and space mechanisms opens up many possible areas of research. Research can be started in a variety of ways. Each of the space mechanism types should be looked at for their compliant potential. A desired mechanism function and its requirements could be the starting point. Replacing a current mechanism component, such as a ball bearing, would yield valuable results. Designing to overcome a reoccurring issue or failure in a current mechanism could guide the research. Guidelines, methods, and tools for the design, analysis, and testing of compliant space mechanisms need to be developed and established for use in industry.

This research hopes to yield ground-breaking answers to detailed questions such as this one: How do compliant segments, especially large-displacement, behave in the thermal extremes and the vacuum of space? There are many fascinating questions and there will be even more as this field unfolds.

Compliant space mechanism research will help fulfill the needs expressed in the NASA Technology Roadmaps and Lessons Learned Study, while also creating next generation space mechanisms.

2.5 Conclusions

Failures in space can be costly, dangerous, and can hinder progress of technology development and space exploration. The field of compliant space mechanisms has the potential for significant impact on the performance of future space mechanisms. Research in compliant space mechanisms will create innovative new space technologies. It could initiate critical advances that
would lead to more affordable and more capable space mechanism solutions. This research cross-cuts several technologies and applies to a large range of different space mechanism types.

Compliant space mechanism research hopes to inspire collaboration between government space agencies (such as NASA), companies in the space industry, and academia. Overall, it would present opportunities for game-changing technologies and lead to enhanced capabilities in space exploration.
CHAPTER 3. CONCEPTUAL DESIGNS FOR COMPLIANT HINGES

3.1 Introduction

Chapter 2 identified numerous types of space mechanisms that have the potential for development as compliant space mechanisms. Research is viable in all those areas. The purpose of this chapter is to determine one space mechanism type to pursue through detailed research.

Industry input was sought to determine the mechanism to pursue. A seminar was conducted at Lockheed Martin Space Systems in Littleton, CO. All the mechanisms experts throughout Space Systems were invited to attend, resulting in over 30 mechanisms experts attending. A brainstorming session was held to answer the questions: “What applications of compliant mechanisms do you envision in space applications?” and “What space mechanism type is the most common and would benefit most from the application of compliant mechanism technology?”

The final consensus among everyone was that deployment hinges are one of the most common and most important space mechanisms and they would benefit most from being designed as compliant mechanisms.

This chapter describes the components of a current deployment hinge and presents possible concepts (and building blocks) of how a compliant deployment hinge could be designed.

3.2 Space Deployment Hinge Components

A satellite deployment hinge is comprised of

1. a component that provides the torque.

2. a component that allows for the range or motion.

3. a component that provides damping.

4. a component that provides latching at the end of the range of motion.
5. brackets to attach hinge components to the spacecraft and to the deployable appendage.

On current satellites, these components are commonly

1. a constant torque spring or a torsional spring.
2. a ball bearing or lubricated pin in slot.
3. a viscous fluid or eddy current damper.
4. an assembly of a rigid link, pin joint, and a torsional spring.
5. metallic brackets bolted the spacecraft and deployable appendage with the above components bolted to the brackets

For the hinge to benefit from the advantages of compliant mechanisms, each component is evaluated for its potential for being replaced by a compliant space mechanism. Component 1 is already a compliant mechanism. Components 2 and 3 can be replaced by compliant space mechanisms. Specifically, designs for the following two components should be investigated and developed:

- compliant hinge that replaces ball bearings and allows for the range of motion
- compliant damper that replaces conventional dampers and provides damping to the deploying appendage

Component 4 can be designed as a compliant mechanism with existing compliant latch designs. Component 5 does not require motion. It may be possible for components 2 through 5 to be integrated into one component.

This chapter identifies and investigates several possible concepts for the design of compliant hinges. One of these concepts is developed in more detail in later chapters.

3.3 Concepts for Large-Displacement Rotational Compliant Hinges

The objective for the compliant hinge is to provide the required range of motion. It is to do so without the use of a traditional pin joint or lubricated joint. The avoidance of contact
between moving parts is desired. The design of a monolithic compliant hinge is also sought. The hinge will be designed to minimize weight, complexity, backlash, size, misalignment, off-axis motion, friction, and contact surfaces. Stored strain energy for long durations should be avoided. Designs with two equilibrium positions can apply well to deployments with stowed and deployed configurations. Designs with redundancy or the ability to be used in parallel are preferred over those without.

Designing flexures for large displacement (for most boundary conditions) is possible thanks to the design methods discussed in Chapter 2. However, it is difficult to design flexures that can individually achieve rotations of 90 or even 180° as is often required in space applications. Geometries comprised of flexures in series, parallel, or other combined arrangements could help achieve these large deflections. For complex geometries of this nature, analysis using closed-form solutions is limited and finite element models are used for analysis. Although not many designs exist that can achieve angular deflection from 90 to 180°, research is being performed to develop compliant hinges capable of this motion. Some of the concepts discussed here are geometries being designed and analyzed to achieve this large displacement objective and the others are building blocks that have potential to be modified or combined to achieve the desired deflection.

### 3.3.1 Large Deformation Torsion Rods

Torsional loading can be applied to rods or beams of various cross-sections to produce rotational displacement. The large deformation of torsion rods is analyzed here for several different cross-sectional shapes, including: circular, rectangular, hollow circular, and hollow open circular or split tube. An optimization is performed for each cross-sectional shape. The optimization problem solved is

\[
\begin{align*}
\text{Optimize } & \theta = 180 \text{ degrees} \\
\text{Subject to } & \tau_{\text{max}} < \frac{S_y}{2}
\end{align*}
\]

where \( \theta \) is the angular displacement, \( \tau_{\text{max}} \) is the maximum shear stress, and \( S_y \) is the yield strength of the material used. The optimization problem is set up not to minimize or maximize any quantities but to drive angular displacement to 180° keeping the stress below an allowable limit. The
The approach taken for this optimization problem is more of an attempt to find a feasible solution for torsion rods capable of 180° of rotation than a classical optimization problem. The optimization constraint uses the more conservative maximum shear stress failure theory to predict failure.

The design variables are similar but different for each case. The discrete variables are the material properties $E$ (Young’s Modulus), $\nu$ (Poisson’s ratio), and $S_y$ (yield strength), as well as $T$ (the applied torque). Each cross-section type was optimized for both polypropylene (PP) and Titanium Ti6Al4V (Ti). The applied torque is dependent on the mission requirements and was set at 100 in-lb for this optimization. The analysis variables are $\theta$, $\tau_{\text{max}}$, and $k$ (the stiffness), and are defined separately for each case according to the following equations [46].

The angular displacement of a torsion rod is

$$\theta = \frac{TL}{KG}$$

where $T$ is the applied torque, $L$ is the length of the rod or beam, $K$ is a function of cross-sectional geometry, and $G$ is the shear modulus. $G$ is defined as

$$G = \frac{E}{2(1+\nu)}$$

The optimization drives the angular displacement to 180° for each torsion rod.

The maximum shear stress of a torsion rod is

$$\tau_{\text{max}} = \frac{T}{Q}$$

where $Q$ is a function of the cross-section geometry. According to the optimization problem, the result for each torsion rod will have a maximum shear stress that is less than half the tensile yield strength of the material and will not fail from the applied torque.

The torsional stiffness of a torsion rod is

$$k = \frac{KG}{L} = \frac{T}{\theta}$$
Since each rod or beam was optimized to reach 180° of deflection with an applied torque of 100 in-lb, the stiffness of all the torsion rods will be 31.83 in-lb/rad.

**Circular**

For the circular cross-section, the design variables are $L$, and $r$ (radius of the rod).  

For a circular cross-section $K$ is replaced by $J$, the polar moment of inertia, and is

$$J = \frac{\pi r^2}{2} \quad (3.6)$$

and $Q$ is defined as

$$Q = \frac{J}{r} \quad (3.7)$$

The optimized geometries for each material are

**Table 3.1: Optimized geometry of a circular torsion rod**

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>$L$ (in)</th>
<th>$r$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimized Value PP</td>
<td>26.02</td>
<td>0.289</td>
</tr>
<tr>
<td>Optimized Value Ti</td>
<td>47.97</td>
<td>0.112</td>
</tr>
</tbody>
</table>

**Rectangular**

The rectangular cross-section has design variables $L$, $b$ (width of the beam), and $h$ (height of the beam).

For a rectangular cross-section

$$K = ac^3 \left( \frac{16}{3} - 3.36 \frac{b}{a} \left( 1 - \frac{b^4}{12a^4} \right) \right) \quad (3.8)$$

where $a = b/2$ and $c = h/2$ and $Q$ is defined as

$$Q = \frac{8a^2b^2}{3a + 1.8b} \quad (3.9)$$
The optimized geometries for each material are

Table 3.2: Optimized geometry of a rectangular torsion rod

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>$L$ (in)</th>
<th>$b$ (in)</th>
<th>$h$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimized Value PP</td>
<td>28.51</td>
<td>1.707</td>
<td>0.288</td>
</tr>
<tr>
<td>Optimized Value Ti</td>
<td>47.95</td>
<td>0.800</td>
<td>0.100</td>
</tr>
</tbody>
</table>

Hollow Circular

The design variables for a hollow circular cross-section are $L$, $d_o$ (outer diameter of the rod), and $d_i$ (inner diameter of the rod). For a hollow circular cross-section $K$ is replaced by $J$ and is

$$ J = \frac{\pi (d_o^4 - d_i^4)}{32} \quad (3.10) $$

and $Q$ is defined as

$$ Q = \frac{J}{r} \quad (3.11) $$

The optimized geometries for each material are

Table 3.3: Optimized geometry of a hollow circular torsion rod

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>$L$ (in)</th>
<th>$d_o$ (in)</th>
<th>$d_i$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimized Value PP</td>
<td>50.29</td>
<td>0.772</td>
<td>0.606</td>
</tr>
<tr>
<td>Optimized Value Ti</td>
<td>51.29</td>
<td>0.227</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Split Tube

The design variables for a split tube flexure are $L$, $r$, and $t$ (thickness of the hollow rod). For a split tube cross-section

$$ K = \frac{2\pi rt^3}{3} \quad (3.12) $$
and $Q$ is defined as

$$Q = \frac{4\pi^2 r^2 t^2}{6\pi r + 1.8t} \quad (3.13)$$

Split tube flexures were developed and analyzed by Goldfarb and Speich [47]. Detailed equations for stiffness and stresses can be found there.

The optimized geometries for each material are

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>$L$ (in)</th>
<th>$r$ (in)</th>
<th>$t$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimized Value PP</td>
<td>12.12</td>
<td>0.989</td>
<td>0.135</td>
</tr>
<tr>
<td>Optimized Value Ti</td>
<td>23.98</td>
<td>0.500</td>
<td>0.049</td>
</tr>
</tbody>
</table>

The designs all achieve $180^\circ$ of rotation. The rotational displacement is sensitive to the design variables $r$ and $h$. To meet the stress constraint, the geometries are too long and skinny for practical application. If the applied torque is increased, the length could be reduced and the geometry thickened, however, doing so would also increase the maximum shear stress and possibly make the design infeasible. The split-tube torsion rods have the shortest lengths yet could be susceptible to torsional buckling and warping. The hollow circular torsion rods are longer than the circular and rectangular torsion rods. If the required angular deflection was decreased, the designs would be more practical. For example, when optimized to reach only $90^\circ$, the Titanium circular torsion rod would require a length of only 23.98 inches in length instead of 47.97 inches.

The following are some key relations between torsion rod parameters. An increase in $T$ will result in a linear increase in $\theta$ and will increase $\tau_{max}$. An increase in $L$ will result in a linear increase in $\theta$ and but will not affect $\tau_{max}$. An increase in $b$ or $h$ will result in a decrease in $\theta$ and $\tau_{max}$. You want to design $b$ and $h$ to be small to achieve greater $\theta$ but you want them larger for a smaller $\tau_{max}$. You want a lower $E$, $G$, and $J$ and a higher $\nu$ if that is possible through material selection.

In summary, large deformation is possible with the correct geometry and material.
3.3.2 Large Deflection of a Long Flexure with Pure Moment Loading

If a pure moment is applied to a long flexure, the large deflection has a closed-form solution that can be used to design the flexure. The goal of the following analysis is to determine at what thickness a flexure fails due to stress and is unable to deflect 180°. Slope is proportional to curvature

\[
\frac{d\theta}{ds} = C \frac{d^2 y}{dx^2}
\]  

(3.14)

where

\[
C = \frac{1}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}
\]  

(3.15)

The Bernoulli-Euler moment curvature relationship states

\[
\frac{d\theta}{ds} = \frac{M}{EI}
\]  

(3.16)

separating variables and solving yields

\[
\int_0^{\theta_0} d\theta = \int_0^L \frac{M}{EI} ds
\]

\[
\theta_0 = \frac{ML}{EI}
\]  

(3.17)

(3.18)

Solving for the pure moment load that achieves the desired displacement \(\theta_0\) results in

\[
M = \frac{\theta_0 EI}{L}
\]  

(3.19)

The stress in the flexure is

\[
\sigma = \frac{Mc}{I} = \frac{6M}{bh^2}
\]  

(3.20)

substituting Equation (3.19) into (3.20) gives

\[
\sigma = \frac{\theta_0 Eh}{2L}
\]  

(3.21)

This represents the maximum stress in the flexure when a pure moment load is applied. The stress is in terms of the desired angle \(\theta_0\), Young’s Modulus of the material \(E\), and the thickness \(h\) and
length \( L \) of the flexure. The thickness of the flexure, \( h \), can be adjusted until the stress magnitude results in the desired safety factor.

Additionally, the vertical \( (b) \) and horizontal \( (a) \) location of the end of the displaced flexure can be found by

\[
b = \frac{L(1 - \cos \theta_0)}{\theta_0} \quad (3.22)
\]

\[
a = \frac{L \sin \theta_0}{\theta_0} \quad (3.23)
\]

Figure 3.1 is an example of designing a flexure from Titanium Ti4Al6V using the MathCad worksheet that has been developed for this purpose.

---

**Figure 3.1: Example long flexure design with pure moment load.**
3.3.3 Butterfly Pivot

Eight flexures are used in series in the design of the butterfly pivot for aerospace application developed by Henein et al. [48]. The butterfly pivot was designed for precision pointing purposes and had an angular stroke of approximately 15°. It is mainly used for small loads and small angular deflections. If designed for large displacements, the butterfly pivot could act as a basis for designing a compliant mechanism capable of large angular deflection. The advantages of this design are that the individual flexures act in series to share the angular deflection among themselves so that the total rotation of the hinge is much larger than the angular displacement of any one flexure. It is also planar and monolithic, which allows for increased ease and lower costs of prototyping and manufacturing. It was used as a building block for the Flex-16 design of Chapter 5.

3.3.4 Tape Springs

Tape springs are flexures with a curved cross-section that are capable of large deflection. Watt and Pellegrino used tape springs in conjunction with rolling contact elements to create a deployment hinge for space applications [49].

Mallikarachchi and Pellegrino have developed a tape springs that can be used as a hinge and can self-deploy by releasing the stored strain energy [50]. The simplicity of this design allows it to be made from carbon-fiber reinforced plastic. This system is capable of large deflections and is compact and light weight. Thorough analysis has been completed on this tape spring hinge and optimization methods have been used in their designs [51]. One drawback of this design is the need for a device to pinch the two carpenter tape members to allow the device to bend without necessitating the members to buckle.

Three tape springs are used by Pellerin et al. to create a hinge [52]. This hinge removes the need for a device to pinch the tape spring to allow deflection. One drawback of this device is the need to construct the hinges to be strong and the base plates to be lightweight. Due to this requirement, it is usually constructed of multiple parts. It can however, achieve 180° of rotation.
3.3.5 Statically Balanced Pivots

Statically balanced pivots have preloaded elements that combine with other compliant elements to balance the input load for quasi-static loads. Statically balanced hinges require zero force to deflect and have zero stiffness. They are at equilibrium in all positions by balancing the stored strain energy between two or more pairs of flexures. Research in exploring different balancing configurations and geometries would be beneficial.

A generic zero stiffness statically balanced hinge was developed and tested by Morsch et al. [53]. The design could be adapted for use in large displacement applications.

In one possible concept, a bistable pre-stressed curved plate is deflected and clamped to a flat plate to result in a statically balanced plate. The bistable plate could also disconnect from flat plate right before the second equilibrium position so it becomes bistable and no longer statically balanced and can latch in its second equilibrium position.

3.3.6 Spherical Nuremburg Scissors

The Nuremburg scissor mechanism is used in many common applications and is an effective means of getting a large output displacement from a small input displacement. They are pairs of rigid links attached at their midpoints and connected at their ends to another pair in series. They function as scissor to achieve large linear motion. If the joint at the midpoint is offset from the center, curved planar motion results.

If the links are connected such that their axes connect at a point and are attached in the same manner, the resulting motion is spherical and out-of-plane and all the links revolve around the center of a sphere. The geometry of the links and attachment points can be adjusted to achieve certain motion and deflections. The following is a description of a few of the adjustments that can be made to spherical nuremburg scissors and how it affects the resulting motion:

- If the center joints are offset the resulting motion is a curved spherical trajectory.

- If the center joints are not offset and only one type of link arc length is used, it reduces the distance traveled and reduces the coverage of the sphere.
• If two different sizes of link arc lengths are used in the same mechanism, it deploys and retracts all with the same input motion direction instead of continuing to extend. The shorter arc links end up lining up and moving through a change point, after which the mechanism retracts. The maximum extension is defined by the sum of the arc lengths of the shorter links for that case.

• If the center joints are offset and there are different arc lengths, it results in some complex and interesting motion.

• If planar and spherical components are combined, the resulting motion is helical. The geometry and kinematics of this type of mechanism are more complex. The mechanism center of rotation is not longer a point but an axis. A conceptual model of one set of links for a non-compliant helical nuremburg scissor mechanism is depicted in Figure 3.2.

Many different configurations of spherical nuremburg scissors exist to be explored for the possible use on large-displacement applications. An advantage of spherical nuremburg scissors is that the many pairs of links spreads the deflection out over the many joints. A hinge of this nature would be more stable than flexures in series and off-axis stiffness would not be a large problem. The 3D nature of the motion and geometry makes design and analysis more difficult. A guide or housing may be needed for this type of hinge. The rotation of the mechanism is much larger than the deflection of the individual joints and shows promise for large-displacement compliant designs.

Figure 3.2: Conceptual model of one set of scissor links for a (non-compliant) helical nuremburg scissor mechanism.
3.3.7 Other Preliminary Compliant Hinge Concepts

A number of other concepts show promise for possible application as large-displacement rotational hinges. These are documented here to facilitate future development.

- Serpentine Rotational Flexures. Radial serpentine rotational flexures were used in the design of a micro-clasp gripper device [54]. These serpentine flexures were used as flexible torsional springs for knife-edge cam joints. The knife-edge cam joints allow for large displacement but are passive joints [2]. The serpentine flexures also allow for this rotation and have low operating stresses. The serpentine flexures demonstrated the ability to undergo rotation greater than 20°. If the serpentine flexures were used alone as a joint, they would need to be adapted to handle the loading conditions while taking into consideration concerns with stresses.

- Large Displacement Joints. Trease et al. [30] completed a benchmarking survey of large-displacement linear and rotational compliant joints. They found the most promising rotational design to be the compliant revolute (CR) joint which is a large deformation torsion rod and they provide detailed analysis on its behavior.

- Cross-Axis Flexure Pivot. Cross-axis flexure pivots are formed by crossing two flexures in different planes at their midpoints and attaching one pair of ends to a fixed base and the other pair to a rotating base. The load-deflection relationship has been found through an analytical model and validated through testing [55]. The geometry is fairly simple but the analysis is not. Manufacturing may also be challenging. These pivots can deflect through a large angular range and have potential for application in large-displacement hinges. Tubular cross-axis flexural pivots are a variation on the cross-axis flexure pivots with the top and bottom rigid blocks replaced by the walls of a cylinder that can rotate relative to each other. Friction is a problem of this design if the tubes are allowed to contact. An advantage is that the design is self-contained and compact, but they are difficult to manufacture. The analysis used is the same as for cross-axis flexure pivots.

- CORE Joint. The CORE joint [56] uses curved contact surfaces rolling on each other with contact-aided flexures connecting them. They function well in compression, but not in ten-
sion. They can achieve 180° of rotation, could work in series, and could easily incorporate a latch. The drawbacks lie in off-axis stiffness, contact surfaces, functionality during vibration, and possible loss of surface connection.

• CORE Bearing. The core bearing offers the ability to achieve significant rotation by unwinding the curved, contact-aided flexures [57]. The design is planar and monolithic. The stability out of plane is poor and the center of rotation of the center shaft could be unstable. This is a promising concept that should be explored further.

• Rotary Bistable. Rotary bistable hinges are an axis within an axis similar to the core bearing concept but display a bistable behavior. They are compact and could incorporate multiple layers. Several rotary bistable concepts are shown in Figure 3.3. Other geometries are possible. These designs have potential for large displacement and research is recommended in this area.

• Compliant fan. The compliant fan concept comes from the fans people used to cool them-
selves on hot days. There are many segments to decrease individual joint deflection and can fold up into a small volume. The challenges are that there are too many unconstrained DOF and controlling the deployment motion would be difficult. A simple sketch depicts the concept in Figure 3.4. One side could be connected to the spacecraft and the other to the deploying appendage.

- Spherical LEMs in series. Spherical lamina emergent mechanisms allow rotational motion by the out-of-plane deflection of spherical links [36]. This allows for planar manufacturing. One drawbacks is that several would be required in series to achieve a large deflection. Another drawback is that the individual joints undergo an excessive amount of torsion (approx. 90°). It would need to be redesigned to achieve more overall rotation with less individual joint rotation.

- Bistable four-bar linkage. A bistable four-bar compliant mechanism has the potential to allow large displacement. Kinematic analysis would yield linkage designs but complex analysis would need to be performed to ensure the flexible joints do not fail to stress and to ensure bistability. Energy methods could help in this situation [23].

- Flexures in series. Small-length flexure pivots are a basic joint type [2]. If put in series they can achieve large deflection but also suffer from many unconstrained degrees of freedom, vibration issues, and inaccurate motion. They can be manufactured in a planar state and are easily adapted to different ranges of motion. Overall, their challenges appear to outweigh their advantages for application in large-displacement joints. Putting longer flexures in series would result in the same problems as small-length flexure pivots in series.

- Switch-back joints in series. A switch-back joint is a planar rectangle with slits that divide it into 3 links that can rotate. This concept is simple and monolithic yet will have problems with unconstrained DOF and vibrations. They would require a lot of space if put in series and off-axis stiffness is not ideal. Link interference is possible. They may require a housing to constrain their motion. Figure 3.5 shows the number of switch-back joints required for a range of deflections. Using switch-back joints in series presents a way to decrease the required deflection of individual joints while achieving large displacement.
• LET joints in series. Lamina emergent torsional (LET) joints are capable of large rotational displacement [58] and present a promising joint type for use in large-displacement applications. They are monolithic, planar, simple and there are new LET joints available that allows stability in compression and tension [59]. Several of the challenges are twisting when placed in series, center of rotation shift, too many degrees of freedom, and vibration issues. LET joints with improved off-axis stiffness and large rotational capabilities are desired.

• Telescoping flexures. Telescoping flexures are staged flexures that exit a containment housing and deflect sequentially, as shown in Figure 3.6. This type of staged deflection is limited by the friction that would exist on the contact surfaces, the need to actuate each individual flexure, and creating a means of allowing them to slide relative to each other. It is really just a more complex version of the concept of having long flexures in series.

Figure 3.5: Number of switch-back joints vs overall deflection.

Figure 3.6: Telescoping flexures concept.
3.4 Conclusions

Several concepts have been presented that could be used to create a large-displacement rotational hinges. Each idea was evaluated and one concept, the butterfly pivot, was selected as a building block for the design of a compliant deployment hinge that is developed in Chapter 5.
CHAPTER 4. DEPLOYMENT DYNAMICS MATHEMATICAL MODEL & DESIGN PROCESS

4.1 Introduction

Rotational joints or hinges are commonly used mechanisms in the aerospace industry. A deployment hinge is a specific type of rotational hinge and finds frequent application on spacecraft and satellites. The designer is typically left to review the dynamics that govern deployment and then is responsible for designing the hinge to meet the specific requirements given by the mission objectives.

The design of the hinge has a direct effect on the performance of the deployment. The objective of this chapter is to develop a design process that designers in the aerospace industry can easily use to determine the parameters of a deployment hinge that result in the required deployment performance. This design process allows the designer to create a hinge that meets the requirements set by the objectives of the mission while also letting the designer quickly understand the fundamental mathematical model describing the dynamics of a deployment hinge. The effect of varying each parameter can be readily observed. A Mathcad interactive worksheet and a Matlab script were developed and use the input parameters dictated by the mission requirements and calculate the parameters necessary to define the performance of the deployment and the hinge design that will result in that performance. The worksheet and script are aimed at becoming an understandable, basic, standardized design tool that allows designers the ability to model the dynamics of a deployment hinge and to provide key parameters for the design of the hinge itself.

The mathematical model and design process are described next, followed by an example of using the design process.
4.2 Description of System for which the Design Process is Used

The deployment system design process is mathematically rooted in the dynamics of deployment. The deployment system modeled is a single-degree-of-freedom rotational mass-damper system. Since the design of a deployment hinge for space applications is the focus of this research, the specific example used is a satellite deployment hinge. The system is composed of a spacecraft, deployment hinges, and a solar array panel, as shown in Figure 4.1.

![Figure 4.1: System diagram.](image)

The solar array deploys, rotates $180^\circ$, then latches. In this 1 DOF system, the properties of the hinge are crucial to the performance of the solar array. The spacecraft is considered to be fixed because any large rotational accelerations from the deployment are considered disturbances to the spacecraft dynamics and attitude control and need to be quantified and taken into account into the spacecraft control system. The solar array has an inherent mass moment of inertia. The hinge has a spring constant, damping, and an applied torque. According to Newton’s second law of motion in rotational form, the free body diagram can be developed as shown in Figure 4.2.

This is a second-order damped system with forced response. The second order differential equation that describes the motion of the solar array is

$$I\ddot{\theta} + b\dot{\theta} + k\theta = \sum T$$  \hspace{1cm} (4.1)
where $\theta$ is the angular displacement, $\dot{\theta}$ is the angular velocity, and $\ddot{\theta}$ is the angular acceleration of the solar array. $I$ is the mass moment of inertia, $b$ is the damping rate, and $k$ is the torsional spring constant. The right hand side is the sum of the torques externally applied to the system.

Two of these quantities are specified and two are determined by the design process. Typically, $I$ is dictated by the mass of the deploying appendage and has limits set by the mission requirements. The magnitude of the externally applied torques is dictated by the source of deployment torque, commonly a constant torque spring, where its magnitude is calculated by multiplying the sum of the torque losses across the hingeline with the factors of safety applied to the hingeline. This results in allowing the source of deployment torque to have adequate torque margin. Both the stiffness of the hinge, $k$, and the damping rate of the hinge, $b$, are unknown design variables and will be determined by the design process.

$I$ is the mass moment of inertia about the hingeline axis and can be found by first finding the mass moment of inertia of a beam or rectangular panel then shifting it to the hingeline axis using the parallel axis theorem. The mass moment of inertia about the center of a rectangular plate is

$$I_c = \frac{1}{12}mL^2 \tag{4.2}$$

The parallel axis theorem states that the mass moment of inertia about a shifted axis is

$$I = I_c + md^2 \tag{4.3}$$

where $d = L/2$ for a symmetric plate. This makes the mass moment of inertia about the hingeline

$$I = \frac{1}{3}mL^2 \tag{4.4}$$
In current, non-compliant mechanism designs, $k$, the torsional spring constant, is typically achieved by a torsional spring or constant torque spring. In compliant mechanism designs, $k$ of the hinge will be defined by the stiffness of the compliant joint. In current designs, $b$, the damping rate, is achieved commonly by either viscous fluid or eddy current dampers. In compliant mechanism designs, it will be achieved by energy absorbing flexure designs.

The coefficients $I$, $k$, $b$, and $T$ are typically assumed to be constants. However, in this system the damping rate, spring constant, and applied torque may vary with time. The solution shown here is for constant coefficients. Time dependent coefficients require solving a nonlinear differential equation. This can be difficult and the ability of the solution to converge depends highly on the curves of the coefficients and becomes increasingly less likely to converge as the desired range of motion increases.

The solution to Equation 4.1 is shown in Appendix A. The solution to Equation 4.1 will be summarized and is shown for underdamped, critically damped, and overdamped cases. This solution will be the starting point for the design process.

### 4.3 Summary of Deployment Dynamics Mathematical Model

Equations (4.5), (4.6), and (4.7) show the solution for rotational displacement of a deploying appendage for an overdamped, critically damped, and underdamped case, respectively.

**Overdamped ($\zeta > 1$)**

$$\theta = -\frac{T}{k(1 - \frac{r_1}{r_2})} e^{r_1 t} + \frac{r_1 T}{k(r_2 - r_1)} e^{r_2 t} + \frac{T}{k} \quad (4.5)$$

**Critically Damped ($\zeta = 1$)**

$$\theta = -\frac{T}{k} e^{r_1 t} + \frac{T r_1}{k} t e^{r_2 t} + \frac{T}{k} \quad (4.6)$$

**Underdamped ($\zeta < 1$)**

$$\theta = -\frac{T}{k} e^{\lambda t} cos(\mu t) + \frac{T \lambda_1}{k \mu_2} e^{\lambda t} sin(\mu t) + \frac{T}{k} \quad (4.7)$$

48
where $r_1$ and $r_2$ are the characteristic roots and are

\[
\begin{align*}
    r_1 &= \frac{-b + \sqrt{b^2 - 4Ik}}{2I} \\
    r_2 &= \frac{-b - \sqrt{b^2 - 4Ik}}{2I}
\end{align*}
\]  \hspace{1cm} (4.8)

For the underdamped case, the roots are complex and are

\[
\begin{align*}
    r_1 &= \lambda + i\mu \\
    r_2 &= \lambda - i\mu
\end{align*}
\]  \hspace{1cm} (4.9)

The three solutions in Equations (4.5), (4.6), and (4.7) represent the fundamental mathematical model of the dynamics of a deployment hinge. This model assumes the solar array is at rest in a stowed position as the initial conditions. The first and second derivatives of these equations provide the angular velocity and angular acceleration of the solar array and complete the description of its motion. Note that the roots will both be negative and will provide the exponentially decaying transient behavior.

With the mathematics in place, it is necessary to understand how the performance requirements and the hinge design relate to it. The design process is now detailed with an example of its use presented afterwards.

4.4 Design Process

4.4.1 Inputs & Outputs

The required deployment angle, $\theta_{des}$, and minimum deployment time, $t_{depl}$, are typically specified by the mission and are givens in the design process. The mathematical model and design process are created to find the following performance parameters of the deploying appendage and its hinge:

- $\theta$ is the angular displacement
- $\dot{\theta}$ is the angular velocity
- $\ddot{\theta}$ is the angular acceleration
• $KE$ is the kinetic energy

• $k$ is the stiffness of the compliant hinge to be designed

• $b$ is the damping rate of the compliant damper to be designed

4.4.2 Six Step Design Process

The design process is defined in six steps here.

1. Understand the mathematical model and enter givens. Gather the following inputs:

   (a) $T$ is the externally applied torque with a minimum specified by the hingeline torque losses and Factors of Safety. Common sources are a constant torque spring or a torsional spring.

   (b) $I$ is the Mass Moment of Inertia about the hingeline defined by the size of the solar array or appendage

   (c) $\theta_{des}$ is the required deployment angle defined by the mission

   (d) $t_{depl}$ is the required deployment time defined by the mission

2. Determine $k$. The necessary hinge stiffness can be calculated

   $$k = \frac{T}{\theta_{des}}$$  \hspace{1cm} (4.10)

   This will be the needed stiffness of the compliant hinge to achieve the desired steady-state response value.

3. Set $b$. Use the mathematical model to set the damping rate at the value that makes the system critically damped. This is a starting value. The damping value that makes the system critically damped is

   $$b = 2\sqrt{Ik}$$  \hspace{1cm} (4.11)

4. Check $\theta_{des}$ and $t_{depl}$ and adjust $b$. Assure that the damping rate allows the appendage to reach the desired deployment angle within the required deployment time. Decrease $b$ to
make the system more underdamped because if \( b \) is too high it will not reach the required deployment angle or will not deploy within the required time. Adjust \( b \) until the system meets the deployment time requirements.

5. Verify other performance parameters

(a) Verify angular velocity. Plot the angular velocity (Equations (4.12), (4.13), and (4.14)) for the range of motion and look at the maximum and final values. The final value of angular velocity will dictate the latching loads and should be minimized while still allowing deployment within \( t_{depl} \).

\[
\dot{\theta} = r_1 c_1 e^{r_1 t} + r_2 c_2 e^{r_2 t} \tag{4.12}
\]

Critically Damped

\[
\dot{\theta} = r_1 c_1 e^{r_1 t} + c_2 e^{r_2 t} + r_2 c_2 e^{r_2 t} \tag{4.13}
\]

Underdamped

\[
\dot{\theta} = c_1 \lambda_1 e^{\lambda_1 t} \cos(\mu_1 t) + c_2 \lambda_2 e^{\lambda_2 t} \cos(\mu_2 t) - c_1 \mu_1 e^{\lambda_1 t} \sin(\mu_1 t) + c_2 \lambda_2 e^{\lambda_2 t} \sin(\mu_2 t) \tag{4.14}
\]

where the coefficients for each case are

Overdamped

\[
c_1 = -\frac{T}{k(1 - \frac{r_1}{r_2})} \tag{4.15}
\]
\[
c_2 = \frac{r_1 T}{k(r_2 - r_1)}
\]

Critically Damped

\[
c_1 = -\frac{T}{k} \tag{4.16}
\]
\[
c_2 = -c_1 r_1 = \frac{T r_1}{k}
\]
Underdamped

\[
c_1 = -\frac{T}{k} \\
c_2 = -\frac{c_1 \lambda_1}{\mu_2} = \frac{T \lambda_1}{k \mu_2}
\]

(4.17)

(b) Verify angular acceleration. Plot the angular acceleration (Equations (4.18), (4.19), and (4.20)) for the range of motion and verify that it begins to approach zero before \(\theta_{des}\). Also verify that its maximum magnitude does not adversely affect the attitude dynamics of the spacecraft. The angular accelerations for the overdamped, critically damped, and underdamped cases are

Overdamped

\[
\ddot{\theta} = r_1^2 c_1 e^{r_1 t} + r_2^2 c_2 e^{r_2 t}
\]

(4.18)

Critically Damped

\[
\ddot{\theta} = r_1^2 c_1 e^{r_1 t} + r_2^2 c_2 e^{r_2 t} + 2 r_2 c_2 e^{r_2 t}
\]

(4.19)

Underdamped

\[
\ddot{\theta} = c_1 \lambda_1^2 e^{\lambda_1 t} \cos(\mu_1 t) - c_1 \mu_1^2 e^{\lambda_1 t} \cos(\mu_1 t) \\
+ c_2 \lambda_2^2 e^{\lambda_2 t} \sin(\mu_2 t) - c_2 \mu_2^2 e^{\lambda_2 t} \sin(\mu_2 t) \\
+ 2 c_2 \lambda_2 \mu_2 e^{\lambda_2 t} \cos(\mu_2 t) - 2 c_1 \lambda_1 \mu_1 e^{\lambda_1 t} \sin(\mu_1 t)
\]

(4.20)

(c) Verify kinetic energy. Plot the kinetic energy over the range of motion and determine its maximum value. This will be used in the damper design as the amount of energy the compliant damper will need to absorb to achieve the specified damping rate. The kinetic energy of the deploying appendage is

\[
KE = \frac{1}{2} I \dot{\theta}^2
\]

(4.21)
(d) Verify natural frequency. It must be larger than the minimum requirements from the mission. The natural frequency of the deploying appendage is

\[ \omega_n = \sqrt{\frac{k}{I}} \]  

(4.22)

The damping ratio is

\[ \zeta = \frac{b}{2\sqrt{Ik}} \]  

(4.23)

The damped natural frequency is

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]  

(4.24)

6. Use \( k \) and \( b \) as specifications to design the compliant hinge and compliant damper.

This design process allows the designer to understand the performance of the deployment and define parameters necessary for the compliant hinge and damper designs.

The following are a few notes on the dynamics model and design process.

- Damping causes the angular acceleration to decrease and at a certain point come to zero (where the angular velocity reaches a max), then the angular acceleration becomes negative and the angular velocity decreases. This is how damping is used to reduce latching loads at the end of deployment travel.

- If the compliant hinge design involves stored strain energy that is released at deployment, it can be considered either an externally applied torque or a stiffness. In this model we will consider it a stiffness.

- This model assumes a space environment where there are no gravity loads or gravitational potential energy stored in the system.

- Different geometries have different possible ranges of torque, stiffness, and damping. Be aware of physical limitations when designing to match the parameters output from math model that give desired performance.
Given mission requirements, a designer can predict the deployment performance and solve for the corresponding design parameters of the hingeline components.

In summary, deployment performance is vital and dictates hinge design. This mathematical model and design process allow for valuable system modeling and parameter determination before significant resources are invested in prototyping and testing. An example using this design process is now presented.

4.5 Design Process Example

Suppose we want to design a hinge based on performance requirements. Using the design process, the given requirements are used to calculate the performance and then adjusted to meet the requirements. The requirements of this example are arbitrary but realistic. Matlab code was written to calculate and plot the data used in the design process for this example. The code is found in Appendix B. A Mathcad worksheet was also developed and is available upon request.

1. Define inputs:

   (a) \( T = 90 \text{ in-lb} \) as specified by deployment torque margin requirements.

   (b) \( I = 45,720 \text{ lbm-in}^2 \). The mass of the deploying solar array panel is 26.46 lbm (12 kg). The length of the solar panel from the hingeline to the end of the panel is 72 in. It was calculated using Equation 4.4.

   (c) \( \theta_{des} = 180^\circ \)

   (d) \( t_{depl} = 150 \text{ seconds} \)

2. Calculate the rotational stiffness. \( k = 28.648 \text{ in-lb/rad} \) using Equation 4.10.

3. Set damping at critically damped value using Equation 4.11. \( b = 2289 \text{ lb-s/in} \).

4. Check performance and adjust \( b \).

   (a) Figure 4.3 shows the deployment angle vs time (Equations 4.5, 4.6, and 4.7) for critically damped conditions. The required deployment angle was not reached within the minimum deployment time. It reached 2.79 rad at 150 seconds, lacking 0.35 rad. The
red circle in the figure indicates the required angle and time with the red line extended to show the required angle. The green line indicates what angle it reaches by the required deployment time. The distance between the red and green lines indicates the angle remaining to reach full deployment.

(b) Decreasing the damping makes the system underdamped and increases the angle achieved in the same amount of time. It also increases the angular velocity at the end of the deployment time; when latching occurs. If $b$ is decreased to 1620 lb-s/in, the deployment behaves as shown in Figure 4.4. With this value of $b$, the deployment reaches
the desired angle within the required time. The red line extends to the point where the required deployment angle is reached; in this case at 133.3 seconds. The green line indicates the angle achieved within the minimum required deployment time. The resulting angular velocity and acceleration which will be calculated in the next step.

5. Verify performance parameters

(a) Angular Velocity. Figure 4.5 shows the angular velocity (Equations (4.12), (4.13), and (4.14)) of the deployment with the red circle indicating the magnitude of the angular velocity at the time the deployment reaches the required angle. The angular velocity is 0.0074 rad/s at 133.3 seconds. This will be reacted by the latch and efforts should be made to minimize it while still reaching the required angle in time.

![Angular velocity graph](image)

Figure 4.5: Angular velocity.

Suppose $b$ was decreased even further to 1,200 lb-s/in. The deployment angle and angular velocity are shown in Figures 4.6(a) and 4.6(b). The required angle was reached in 99.6 seconds but the angular velocity at that time has increased to 0.0213 rad/s.

(b) Angular Acceleration. Returning to the value of $b$ that gave us desirable performance, Figure 4.7 shows the angular acceleration (Equations (4.18), (4.19), and (4.20)) during deployment with the red circle indicating the magnitude at the time it reaches the required deployment angle. The angular acceleration is -0.00026 rad/s$^2$ at 133.3 seconds.
with a maximum deceleration of 0.002 rad/s\(^2\). The maximum acceleration is at release, when deployment begins, and is typically a function of the pyrotechnic actuation.

(c) Kinetic Energy. Figure 4.8 shows the kinetic energy (Equation 4.21) of the deploying appendage with the red circle indicating the location of reaching the required angle. The total energy to be absorbed by the damper would be the area under the graph up until the time indicated by the red circle. The kinetic energy at latching is 1.26 J with a maximum of 29.36 J.
(d) Natural Frequency. Using Equations 4.22, 4.23, and 4.24, the natural frequency, damping ratio, and damped natural frequency of the deployment hinge are 0.025 Hz, 0.708, and 0.018 Hz, respectively.

6. Use the values selected for $k$ and $b$ to design the hinge. If any of the performance parameters are undesirable or infeasible, modify the design until they are feasible.

4.6 Conclusion

The design process for a deployment hinge has been developed and demonstrated with an example. The design process can be used to develop compliant deployment hinges that meet design requirements in the space industry.
CHAPTER 5. FLEX-16: A LARGE-DISPLACEMENT MONOLITHIC COMPLIANT ROTATIONAL HINGE

5.1 Introduction

5.1.1 Objective

The objective of this chapter is to describe the design, prototyping, and testing of a monolithic compliant mechanism capable of large rotational displacement for potential use as a compliant spacecraft deployment hinge. It aims to provide a replacement for ball bearings and other lubricated joints in space mechanisms. The mechanism is desired to be capable of at least 90 degrees of angular displacement and benefit from planar manufacturing methods.

5.1.2 Motivation

Of the many types of space mechanisms, deployment hinges are among the most common and their performance is vital to the objectives of a spacecraft mission. Current deployment hinges exhibit numerous possible failure modes. The NASA Space Mechanisms Handbook Lessons Learned document [7] details and summarizes the failures that have occurred in deployment mechanisms. Many of the failure modes are lubrication and tribology related. The number of failure modes can be reduced by application of compliant mechanism technology to the design of deployment hinges. A large-displacement monolithic compliant deployment hinge would not be susceptible to lubrication outgassing, cold welding, friction, binding, and backlash [60]. Among other things, it has the potential to eliminate lubrication and rigid-link joints while reducing part count, complexity, and cost. It would increase the ease of manufacturing and integration with the other spacecraft components. It would also increase the ability of the hinge designer to control the stiffness, stresses, and natural frequencies of the hinge and tailor the dynamics of the deploying appendage to meet the mission deployment performance requirements.
A common current space hinge solution is to use ball bearings. They are challenged by the complexities associated with quantifying reliability and performance of a joint whose rotation is dependent on lubrication and contact surfaces in the harsh environments of space. A large-displacement monolithic compliant deployment hinge could be immune to those challenges.

The Flex-16, as depicted in Figure 5.1, is proposed as a solution. Its design, analysis, and testing are detailed in this paper.

5.2 Background

Compliant mechanism hinges [2] have been developed for use in precision devices [61–65], spherical mechanisms [36, 66], lamina emergent mechanisms [28, 59], and other applications [67–69] including some for large deflections [30, 54]. Several tape spring designs have been presented that can offer large displacement [49–52].

The butterfly pivot developed by Henein et al. [48], as shown in Figure 5.2, was designed for precision pointing and had an angular stroke of approximately 15°. It was comprised of 8 flexures, 2 intermediate rigid shuttles, one central X-shaped rigid block, and two rigid ends. This design offered a starting point for the design of a large-displacement compliant rotational hinge.

Significant challenges in creating a monolithic device capable of the desired large displacement included ensuring that stresses at the maximum deflection did not cause failure, and that the
device elements did not collide at any point during the large deflection. Many possible design configurations were evaluated and the Flex-16 design was selected. The following sections describe the Flex-16 and the analysis, prototyping, and testing undergone to arrive at the current design.

5.3 Flex-16 Large-Displacement Compliant Rotational Hinge

5.3.1 Hinge Description

The Flex-16 (Figure 5.1) is a large-displacement compliant rotational hinge that is capable of the desired 90 degrees of rotation and is monolithic. It is comprised of 16 flexures that generally radiate outward from the center of the joint, 4 intermediate rigid shuttles (arc shaped), 1 vertical flexure between the two innermost shuttles, and two rigid ends for fixing the hinge to both the spacecraft and the deploying appendage, as shown in Figure 5.3.

The Flex-16 is actuated by fixing one rigid end and applying a moment on the other rigid end. The radial flexures allow for large angular displacement by acting in series to divide up the stress and displacement among the individual flexures. The lengths of the flexures are designed to be as long as possible to reduce stresses while still fitting within the size envelope of the hinge and not causing contact between flexures as the hinge displaces. The two innermost intermediate rigid shuttles are connected by a vertical flexure to provide axial stability yet still allow rotation. The outer intermediate rigid shuttles increase stability during rotation and geometrically allow for 4 flexures per quadrant.
Figure 5.3: Basic components of Flex-16: 16 radial flexures, 4 intermediate rigid shuttles, 1 vertical flexure, and 2 rigid ends.

5.3.2 Hinge Features

The following list identifies the unique features of the Flex-16 that combine to create a novel mechanism:

- Capable of large-displacement; at least 90° of rotation
- Monolithic design; only one material needed
- Can be manufactured from planar materials which increases the number of applicable manufacturing processes; planar manufacturing processes often easier and less costly; reduces complexity
- Low mass; small volume; reduced material cost
- Compact design; smaller size envelope required; hinge capability maximized within its size envelope
• Analysis tools developed to allow development of the hinge on different size scales or using different materials

• Eliminates many current space application failure modes; increases reliability

5.4 Parametric Finite Element Model

A parametric nonlinear finite element model was created in ANSYS for the analysis of the Flex-16 and the elements are shown in Figure 4. BEAM3 elements were used to allow nonlinear effects and the model contained approximately 1000 elements. The boundary conditions were fixed on one rigid end and a rotational displacement of $\pi/2$ radians applied on the other end. The parametric model allowed for iterative design capability to efficiently change and analyze the geometry to improve performance. The finite element model results were evaluated in terms of

• achieving the desired rotational displacement (90°)

• ensuring a maximum von Mises stress lower than the material tensile yield strength

• having a displaced shape that does not self-intersect or contact
Knowing the stiffness is important for matching a hinge design to the proper torque loading on the hinge. A Matlab script was created to read in output data from ANSYS and plot the torque-displacement behavior of the hinge designs. This allowed the nonlinear stiffness of the hinge to be quantified. All the results from the parametric finite element model are shown in Section 5.6.

![Keypoints](image)

**Figure 5.5: Keypoints used in parametric finite element models.**

The parametric finite element model geometry is defined by 47 keypoints, as labeled in Figure 5.5. Each keypoint is defined in polar coordinates by an angle from the vertical and a fraction of the radius, called the length factor. The angles and length factors are shown for one quadrant of the hinge in Figures 5.6(a) and 5.6(b), respectively.

The equations for the Cartesian coordinate locations of each keypoint are listed in Table 5.1. This allows the user to easily use the model by specifying the locations of all 47 keypoints using only the radius, eight angles, and eight length factors. The model inputs the keypoints into ANSYS
Figure 5.6: The geometry of the Flex-16 is comprised of keypoints defined by (a) angles and (b) length factors.

and CAD software in Cartesian coordinates as required. The model facilitates rapid translation of design concepts into finite element models.

These parameters and equations constitute the parametric design and allow the geometry to be changed quickly to iterate on the design. This allows the designer to identify which parameter changes will reduce stress and give the desired displaced shape. These equations and parameters were coded into ANSYS batch files that were used to run the analyses.

Figure 5.7: The parametric CAD model allows for rapid creation of CAD models of different geometries.
A parametric CAD model was also created in SolidWorks that takes the same inputs as the ANSYS parametric model. It creates a 3D model with the same geometry that the ANSYS model represents by accounting for thicknesses of beams. This allowed for time-efficient modeling and prototyping of designs. Figure 5.7 shows two different CAD models that were created using the parametric CAD model.

The model was verified with prototypes constructed in three different materials, as described later.

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5.5 Configuration Study

The parametric finite element model was used to conduct a configuration study to explore and analyze different combinations of parameters. Figure 5.8 shows the different geometric variations that were analyzed. Visual depictions of several of the configurations are shown with their descriptions in this section. The configuration study provided a better understanding of how each of the parameters affects the stress and displaced shape and decreased the design time required for prototypes on different size scales or of different materials.

Figure 5.8: Design configuration cluster.
5.5.1 Radial Configurations

To begin the exploration of different configurations by changing the design parameters, it was proposed that the most natural design would be one where each of the flexures pointed directly to the center of rotation, similar to radial serpentine flexures [54]. The loading would benefit from a transmission angle closest to 90°, making the load perpendicular to the flexure and the flexure obtaining maximum deflection from the load. Designs were explored and analyzed that changed the angles of Figure 5.6(a) and length factors of Figure 5.6(b) to make three out of the four flexures per quadrant point to the center and then another to have all the flexures point to the center. These designs are depicted in the line drawings of Figure 5.9. Four additional keypoints, 42, 43, 44, and 45, were added to accommodate the fully radial design of Figure 5.9(b) and are only needed for fully radial designs.

The addition of keypoints to obtain a fully radial design increased the arc length of the two inner intermediate blocks and created self-interference issues. The fully radial design was unable to achieve 90° of rotation. The design with three radial flexures achieved 90° but predicted higher stresses than the horizontal configuration, shown in Figure 5.10. Having the centerline between the second and third flexure per quadrant pass through the center of rotation showed a decrease in stress and was adopted into future design configurations.
5.5.2 Flexure Length Configurations

The differences between designs with different relative lengths were compared, including where

1. the four flexures per quadrant are all different length

2. the first and second flexures per quadrant are the same length and the third and fourth flexures per quadrant are the same length

3. all four flexures per quadrant are the same length

The second and third concepts were tested against the first and are shown in line drawings in Figure 5.11.

In the cases evaluated, the longer the flexure length, the lower the stress in that flexure. It is advantageous to make every flexure as long as possible and still have them all fit within the hinge and not interfere with each other.

5.5.3 End Shape Configurations

After investigating squared, diamond, hemispherical and other end cap shapes, the results suggested that it would be advantageous to maximize the length of each flexure and that the end shapes reduced the effective flexure length and increased stresses. Figure 5.12 shows a line drawing of the hemispherical configuration.
To maximize the length of each flexure and still avoid self-interference, the length factors and angles were adjusted until the flexures were as long as possible while the predicted displaced shape did not self-intersect. This design resulted in a lower maximum stress, going from above 24 MPa to just below 17 MPa.

5.5.4 Vertical Configuration

The designs without a flexure in the center had poor axial stiffness. To address this problem, a vertical flexure was added between the inner intermediate blocks, as shown in Figure 5.13. Two
additional keypoints, 46 and 47, were added to accommodate a vertical flexure design. The design with the addition of the vertical flexure did not self-intersect, had a much lower maximum stress, and deflected $90^\circ$.

![Vertical configuration](image)

Figure 5.13: Vertical configuration.

The vertical flexure proved effective in increasing the axial stiffness of the design while allowing for the desired displacement and a similar displaced shape. The addition of a vertical flexure helped to improve distribution of the stress and slightly lowered the maximum stress.

### 5.5.5 Flexure Thickness

The in-plane thickness of the flexures, $t_1$, is independent of the other design variables and can be used to lower or raise stresses. The lower limit is a function of manufacturing feasibility. During design, the thickness of the flexures was decreased to reduce stress, but this also results in reduced stiffness. The stiffness must also match the applied torque across the hingeline defined by the design requirements. If the hinge is not stiff enough, it would be necessary to use several of them in parallel and the cost and production time would increase. Increasing flexure thickness also provides increased off-axis stiffness and increases the stability of the hinge.
5.5.6 Intermediate Block Placement and Thickness

The intermediate blocks provide stability for the hinge by helping to guide the 16 flexures through the range of motion. The in-plane thickness of the intermediate blocks, \( t_2 \), has a large effect on self-interference problems. If made too thick, they will come into undesirable contact during deflection. Friction in contact surfaces is a possible failure mode for the hinge in space applications and must be avoided. If the intermediate blocks are too thin, they become flexures and cease to provide stability and symmetry to the hinge.

The distance of the two pairs of intermediate blocks from the center are defined by length factors \( f_2 \) and \( f_7 \). For the conditions evaluated, if the radii were increased, the length of the flexures was decreased and the stresses increased. It was also found that if the radii were decreased past a certain value, the location of the maximum stress would change, the maximum stress would increase, and self-interference would also occur. Two length factor values were chosen that balanced these competing outcomes.

5.5.7 Overall Size of the Hinge

The radius of the overall hinge, \( R \), determines the size of the hinge since every keypoint parametrically scales according to it. This parameter allows the designer to design the hinge on any scale. When the radius is decreased, the length of the flexures decrease and the stress increases. When sizing down, the flexure thickness needs to be decreased to keep within stress limits. The design is bounded by stress as the size decreases since there are physical boundaries, related to manufacturing processes, on how thin the flexures can be.

5.5.8 Summary of Lessons Learned from the Configuration Study

Different configurations were combined and the Flex-16 was a composite of the best qualities of each of the different branches of the cluster. The design configuration found by exploring and analyzing these different configurations maximizes the hinge capability and results in the lowest stress for any size of the hinge. This means that when scaling up or down, without changing materials, the only parameter necessary to adjust is the flexure thickness. Some of the resulting design characteristics are
• the first and third flexure of each quadrant are radial
• the end shape is squared (with fillets to reduce stress concentrations)
• none of the flexures are the same length
• the length of each flexure is maximized without self-interference
• the horizontal beam was removed and a vertical flexure was added
• the intermediate blocks were positioned at an optimal radii
• the intermediate block thickness is adequate for stability and avoids self-interference
• the flexure thickness was set to reduce stress and still be manufacturable

If the material used for the hinge changes, the designer needs only to change the material properties in the parametric ANSYS model. When the material and scale changes, adjustment of all the parameters is required.

5.6 Results

5.6.1 Overview of Prototypes

After the configuration study was completed, and the Flex-16 characteristics determined, five prototypes were fabricated using three different materials and three different manufacturing processes. Table 5.2 describes the five prototypes.

Table 5.2: Prototype materials and processes

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<td>CNT-1</td>
<td>Carbon Nanotubes</td>
<td>CNT Framework Growth &amp; Infiltration</td>
</tr>
<tr>
<td>CNT-2</td>
<td>Carbon Nanotubes</td>
<td>CNT Framework Growth &amp; Infiltration</td>
</tr>
</tbody>
</table>
Prototypes were fabricated at the micro and macro size scales. Table 5.3 lists the parameters that describe the design for each of the five prototypes.

Table 5.3: Prototype design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PP-1</th>
<th>PP-2</th>
<th>Ti-1</th>
<th>CNT-1</th>
<th>CNT-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>15.24 cm</td>
<td>15.24 cm</td>
<td>6.66 cm</td>
<td>4 mm</td>
<td>2 mm</td>
</tr>
<tr>
<td>t1</td>
<td>2.5 mm</td>
<td>2.5 mm</td>
<td>0.7 mm</td>
<td>30 μm</td>
<td>20 μm</td>
</tr>
<tr>
<td>t2</td>
<td>8.0 mm</td>
<td>8.0 mm</td>
<td>3.0 mm</td>
<td>100 μm</td>
<td>80 μm</td>
</tr>
<tr>
<td>w</td>
<td>1.27 cm</td>
<td>1.27 cm</td>
<td>1.27 cm</td>
<td>250 μm</td>
<td>250 μm</td>
</tr>
<tr>
<td>a1</td>
<td>10°</td>
<td>7°</td>
<td>7°</td>
<td>10°</td>
<td>10°</td>
</tr>
<tr>
<td>a2</td>
<td>25°</td>
<td>22°</td>
<td>22°</td>
<td>25°</td>
<td>25°</td>
</tr>
<tr>
<td>a3</td>
<td>39°</td>
<td>30°</td>
<td>34°</td>
<td>39°</td>
<td>39°</td>
</tr>
<tr>
<td>a4</td>
<td>46°</td>
<td>41°</td>
<td>45°</td>
<td>46°</td>
<td>46°</td>
</tr>
<tr>
<td>a5</td>
<td>49°</td>
<td>52°</td>
<td>56°</td>
<td>49°</td>
<td>49°</td>
</tr>
<tr>
<td>a6</td>
<td>55°</td>
<td>52°</td>
<td>57°</td>
<td>55°</td>
<td>55°</td>
</tr>
<tr>
<td>a7</td>
<td>84°</td>
<td>85°</td>
<td>85°</td>
<td>84°</td>
<td>84°</td>
</tr>
<tr>
<td>a8</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
<td>90°</td>
</tr>
<tr>
<td>f1</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>f2</td>
<td>0.35</td>
<td>0.31</td>
<td>0.29</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>f3</td>
<td>0.88</td>
<td>0.94</td>
<td>0.94</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>f4</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>f5</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>f6</td>
<td>0.86</td>
<td>1.0</td>
<td>1.0</td>
<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>f7</td>
<td>0.20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>f8</td>
<td>0.90</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In the upper section of the table, the radius (R), flexure thickness (t1), the intermediate shuttle thickness (t2), and the out-of-plane hinge width (w) for each of the prototypes are listed. This can be used for a quick comparison of the sizes of the prototypes. In the middle section of the table, the 8 angles (a1-a8) used to define the 47 keypoints are listed for each prototype. In the lower section of the table, the 8 length factors (f1-f8) used to define the 47 keypoints are listed for each prototype. The middle and lower sections can be used for a quick comparison of the designs of the prototypes.
5.6.2 Overview and Summary of Analyses Performed on Prototypes

The parametric finite element model in ANSYS was used to calculate the following for each prototype:

- Displaced Shape
- Stiffness
- Stresses

A stiffness plot and a stress/displaced shape plot were created for each of the five prototypes. The two plots are shown for one prototype, PP-2; the plots for the other prototypes are similar but with different magnitudes. The stress and stiffness results for the five prototypes are summarized in Table 5.4.

<table>
<thead>
<tr>
<th>Name</th>
<th>Maximum Stress at 90°</th>
<th>Yield Factor of Safety</th>
<th>Rotational Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-1</td>
<td>20.6 MPa</td>
<td>1.50</td>
<td>0.4497 N-m/rad</td>
</tr>
<tr>
<td>PP-2</td>
<td>15.3 MPa</td>
<td>2.02</td>
<td>0.3113 N-m/rad</td>
</tr>
<tr>
<td>Ti-1</td>
<td>805 MPa</td>
<td>1.03</td>
<td>1.2719 N-m/rad</td>
</tr>
<tr>
<td>CNT-1</td>
<td>30.0 MPa</td>
<td>3.50</td>
<td>1.7405 N-m/rad</td>
</tr>
<tr>
<td>CNT-2</td>
<td>40.1 MPa</td>
<td>2.62</td>
<td>1.0327 N-m/rad</td>
</tr>
</tbody>
</table>

The ANSYS predicted stiffness plot for PP-2 is shown in Figure 5.14. The predicted stiffness is nearly linear. The ANSYS predicted stress and displaced shape plot for PP-2 is shown in Figure 5.15. The maximum von Mises stress is predicted to be 15.3 MPa and is located at the junction of the flexures and the intermediate rigid shuttles.

Table 5.5 lists the material properties used for the analysis of the prototypes. The tensile yield strengths for polypropylene and titanium are on the low end of the commonly given ranges; adding conservatism to the designs. If a Young’s Modulus of 41 MPa (higher end of the range) is used instead, the predicted rotational stiffness is 0.5057 N-m/rad and 0.03502 N-m/rad for PP-1 and PP-2, respectively. The tensile yield strength and Young’s modulus for the carbon nanotube
Figure 5.14: Predicted stiffness of PP-2.

Figure 5.15: Predicted stress of PP-2. Maximum stress is located in the flexures where they meet the intermediate shuttles.
framework are estimated values as this is an active area of research and the properties are still being verified by testing. When further material properties data is available for in-plane loading conditions, the material properties will provide a more accurate prediction of the displaced shape and stress.

Table 5.5: Prototype material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile Strength</th>
<th>Yield Strength</th>
<th>Young’s Modulus</th>
<th>Poisson’s Ratio</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polypropylene</td>
<td>31 MPa</td>
<td>1.379 GPa</td>
<td>0.3</td>
<td>—</td>
<td>900 kg/m³</td>
</tr>
<tr>
<td>Titanium</td>
<td>827 MPa</td>
<td>113.8 GPa</td>
<td>0.34</td>
<td>2360 kg/m³</td>
<td></td>
</tr>
<tr>
<td>Carbon Nanotubes</td>
<td>105 MPa</td>
<td>6 GPa</td>
<td>0.28</td>
<td>—</td>
<td></td>
</tr>
</tbody>
</table>

The following additional analyses were performed on specified prototypes and are shown in later sections:

- Parasitic Center Shift
- Modal Analysis
- Thermal Stresses
- Off-Axis Stiffness

5.7 Polypropylene Prototypes PP-1 & PP-2

5.7.1 Model Verification

Two polypropylene prototypes were created to test the performance of the Flex-16 design and validate the analytical model. They were fabricated on a CNC mill and are shown in Figures 5.16(a) and 5.16(b).

PP-1 was deflected from 0 to 90 degrees and Figure 5.17 shows the prototype at 0, 45, and 90 degrees. The same is shown for PP-2 in Figure 5.18.
Both polypropylene prototypes were able to rotate 90° without failure, as predicted by the analytical model. PP-1 showed contact of the horizontal flexures at 90°. PP-2 demonstrated a more desirable displaced shape with only minimal contact with the fixed base.

The force required to hold the prototype in a position where it was deflected to 90° was measured and compared to the analytical prediction from ANSYS. The comparison is shown in Table 5.6. The correlation factor describes the degree to which ANSYS is under-predicting the force required to hold deflected at 90°. The differences in the values originate primarily from the variability of the material properties of polypropylene. The Young’s Modulus used for the analysis
Figure 5.18: PP-2 deflected from 0° to 45° and 90°.

Table 5.6: Model validation - comparison of Y direction force required to hold deflection at 90°

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Predicted Y Force</th>
<th>Tested Y Force</th>
<th>Correlation Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-1</td>
<td>5.12 N</td>
<td>6.76 N</td>
<td>1.32</td>
</tr>
<tr>
<td>PP-2</td>
<td>3.42 N</td>
<td>4.89 N</td>
<td>1.43</td>
</tr>
</tbody>
</table>

was at the low end of the range of common values which adds conservatism to the design but increases the error in the predicted stiffness.

5.7.2 Parasitic Center Shift

Table 5.7: Magnitude of parasitic center shift for PP-2 at 90°

<table>
<thead>
<tr>
<th>X Displacement</th>
<th>Y Displacement</th>
<th>Rotation in Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>-19.7 mm</td>
<td>-7.0 mm</td>
<td>0.798 rad</td>
</tr>
</tbody>
</table>

The center of the hinge shifts during rotation and can be a challenge if used in precision applications. The maximum parasitic center shift occurs when the hinge is deflected to the point
where the rigid shuttles come into contact. The measured parasitic center shift at 90° for PP-2 is listed in Table 5.7.

![Image of center shift](image1.png)

**Figure 5.19: Visual depiction of center shift.**

To put the center shift in context of the size of the hinge, the x-direction shift is 0.129 times the radius and the y-direction shift is 0.046 times the radius. The rotation of the center point is approximately 45° as expected.

![Image of center shift motion](image2.png)

**Figure 5.20: Motion of center shift predicted for PP-2.**

The motion of the center point between the two indicated locations in Figure 5.19 can be seen in Figure 5.20.

The center shift on a compliant deployment hinge is a problem if it is not quantified properly. If it is quantified and taken into account, it can be part of the predictable behavior of the system.
5.7.3 Additional Testing

(a) PP-1 test setup  
(b) PP-2 test setup

Figure 5.21: Test setup for PP-1 and PP-2.

PP-1 and PP-2 were deflected to $90^\circ$ then undeflected, to experimentally measure the rotational stiffness. The testing was performed using a force load cell, a potentiometer, and a LabView program calibrated to output and plot the force-displacement data, as shown in Figure 5.21 for both prototypes.

Figure 5.22: Deflection test results for PP-1.

A load cell measured force normal to the moment arm, at a distance specified by the radius (R) of the design, while a calibrated potentiometer measured rotational displacement. The data
from the load cell and potentiometer were brought into LabView. The force data from LabView was multiplied by the moment arm and plotted against the rotation data. The data for PP-1 and PP-2 is plotted in Figures 5.22 and 5.23, respectively.

Using polypropylene leads to the hysteresis that can be seen in the plots as the difference in the deflecting and undeflecting stiffnesses. Table 5.8 reports the tested rotational stiffness of PP-1 and PP-2 along with the linear approximation correlation coefficient. The tested stiffnesses are nearly linear.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Tested Stiffness</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-1</td>
<td>0.5841 N-m/rad</td>
<td>0.9981</td>
</tr>
<tr>
<td>PP-2</td>
<td>0.4376 N-m/rad</td>
<td>0.9959</td>
</tr>
</tbody>
</table>

5.8 Titanium Prototype Ti-1

A titanium hinge was designed, analyzed, and fabricated. It was prototyped at NASA Marshall Space Flight Center using an electron beam manufacturing metal rapid prototyping process [70]. Ti-1 is shown in its undeflected state in Figure 5.24.

The predicted displaced shape and stress plot is shown in Figure 5.25(b) for comparison with the deflected prototype in Figure 5.25(a).
Figure 5.24: Prototype Ti-1 as fabricated at NASA Marshall Space Flight Center.

(a) Ti-1 deflected to 90°.

(b) Ti-1 predicted deflection and stress from ANSYS.

Figure 5.25: Tested and predicted displaced shape for prototype Ti-1.

There is no self-interference or contact during deflection and it achieves 90° of rotational displacement without failure. The prototype deflected shape is predicted by the finite element model.
5.8.1 Modal Analysis

Modal analysis was performed for PP-2 and Ti-1 designs using ANSYS. The first 10 mode shapes and natural frequencies were found and the first two mode shapes are shown in Figures 5.26(a) and 5.26(b). Table 5.9 lists the first five natural frequencies for both prototypes. The first natural frequency excites a simple rotational mode shape. The second natural frequency excites a simple axial (up and down) mode shape. The third and higher natural frequencies are complex mode shapes. The first natural frequency for Ti-1 is high enough to avoid most low frequency launch vibrations for space applications.

<table>
<thead>
<tr>
<th>Natural Frequency</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-2</td>
<td>2.99</td>
<td>8.47</td>
<td>11.76</td>
<td>12.92</td>
<td>19.26</td>
</tr>
<tr>
<td>Ti-1</td>
<td>21.51</td>
<td>58.73</td>
<td>86.99</td>
<td>96.79</td>
<td>137.63</td>
</tr>
</tbody>
</table>

5.8.2 Thermal Stresses

A thermal analysis was performed in ANSYS for prototype Ti-1 to simulate typical space environment temperatures. It was analyzed for both -35 and 75 °C, using a reference temperature
of 20 °C. A thermal expansion coefficient of $8.6 \times 10^{-6} m/mK$ was used in the analysis. The thermal stress plot is shown in Figure 5.27 with a predicted maximum von Mises stress due to thermal expansion/contraction of 1.6 MPa. The location of maximum stress is at the end of the third flexure per quadrant and is different from the location of maximum stress from rotational displacement. The thermal stresses are low but could be added as a preload to the displacement model.

![Figure 5.27: Plot of thermal stresses for Ti-1 at 75 and -35 °C.](image)

### 5.8.3 Off-Axis Stiffness

The off-axis stiffness in the axial direction of prototype Ti-1 was found to be 3107.4 N/m. This means it would require 37.28 N of vertical force to deflect the hinge vertically 12 mm until the rigid intermediate shuttles contact. The displaced shape and stresses at this point are shown in Figure 5.28. The maximum von Mises stress is predicted to be 327 MPa and is at the same location as for thermal stresses. With the maximum stress from axial loading being nearly 40 percent of yield, it is important to avoid axial forces when rotating the hinge to very large displacements. The vertical flexure increases off-axis axial stiffness, but it could be further increased to alleviate concerns with launch vibrations.
5.8.4 Additional Testing

Ti-1 was tested to experimentally measure the rotational stiffness. The force data from LabView was multiplied by the moment arm and plotted against the rotation data. The data for Ti-1 is plotted in Figure 5.29.

![Graph of Ti-1 tested rotational stiffness](image)

Figure 5.29: Ti-1 tested rotational stiffness.

<table>
<thead>
<tr>
<th>Prototype</th>
<th>Tested Stiffness</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-1</td>
<td>0.7885 N-m/rad</td>
<td>0.9974</td>
</tr>
</tbody>
</table>

Table 5.10: Ti-1 tested rotational stiffness
Table 5.10 reports the tested rotational stiffness of Ti-1 along with the linear approximation correlation coefficient.

### 5.9 Carbon Nanotube-Framework Prototypes CNT-1 & CNT-2

Two designs were created for micro-scale applications and were fabricated from a carbon nanotubes framework process [71]. This served as a preliminary investigation of the feasibility of rotational joints on the micro scale as well as aiding in gathering more data to refine our understanding of the material properties of carbon nanotubes undergoing in-plane loading. It also served to demonstrate the scalability of the Flex-16 hinge.

The carbon nanotubes framework process used is summarized as follows. Both designs (CNT-1 & CNT-2) were modeled in CAD software. 2D outlines were created from the geometry in another program where the profile of the designs were augmented with drawings of necessary support structures. The finished drawings were used to make the mask for performing photolithography. The mask is used to generate the pattern in the photoresist on an alumina coated silicon wafer. The exposed photoresist is chemically dissolved. A thin layer of iron is deposited on the wafer. Another chemical is used to remove the remaining photoresist, which also removed the iron that sits on top of photoresist. This leaves the pattern in iron on the alumina coated silicon wafer. Figure 5.30 shows the silicon wafer at this point.

![Figure 5.30: CNT-1 & CNT-2 during the carbon nanotube fabrication process.](image-url)
The carbon nanotubes are grown on the wafer by passing ethylene and hydrogen gases over the wafer at 750 °C. Carbon infiltration is performed by passing argon and ethylene gases over the wafer at 900 °C. Growth and infiltration must take place for the proper amount of time to grow the mechanisms to the desired out-of-plane width of the hinge. Once completed, the hinges are removed from in between the support structures and are isolated for deflection testing and imaging. Prototypes CNT-1 and CNT-2 are shown in Figure 5.31. CNT-1 was capable of 90° of deflection.

![Figure 5.31: Prototypes CNT-1 and CNT-2.](image)

A scanning electron micrograph shows CNT-1 deflected to 90° in Figure 5.32. CNT-2 experienced brittle failure during deflection. While deflecting CNT-2, axial loading was inadvertently induced on the hinge and failure was possibly due to off-axis stresses combined with the stresses predicted during rotation. The material properties used to predict the stresses were shown in Table 5.5 but are only estimates and could help account for the failure.

Another possible cause for failure of the smaller design (CNT-2) is that the flexure thickness after fabrication for both prototypes was thicker than designed due to inaccuracies in the photolithography exposure time. The flexures for the larger design (CNT-1) were measured to be approximately 43 µm thick instead of 30 µm as designed. The increase in thickness for CNT-2 would be the same since photolithography, growth, and infiltration occurred at the same time for both. Figure 5.33 shows a scanning electron micrograph of a CNT-1 flexure with the fabricated thickness measured.
The fabricated thickness of CNT-1 was input into the ANSYS model and solved. It predicted a maximum von Mises stress of 42.6 MPa (up from 30.0 MPa as predicted in Table 5.4 using a thickness of 30\(\mu m\)), which is still below the yield stress, and this is supported by the fact that it did not break when deflected. The brittle failure of CNT-2 was likely due to a combination of increased flexure thickness and off-axis stresses.
5.10 Flex-16 Fabrication and Application

5.10.1 Manufacturability

The Flex-16 benefits from the ability to be fabricated using planar manufacturing processes including:

- CNC milling (used for PP-1 & PP-2)
- Wire EDM
- Electron Beam Manufacturing (used for Ti-1)
- Direct Metal Laser Sintering Rapid Prototyping
- MEMS Fabrication Processes (used for CNT-1 & CNT-2)

It could also be stamped from thin metal sheets which could be subsequently bolted together. These options are lower cost alternatives compared to the manufacturing methods often used for more complex space mechanisms.

5.10.2 Material Selection

Table 5.11 identifies four possible candidate materials for compliant space mechanisms and shows a relative comparison of their strength, temperature resistance, fatigue resistance, and strength-to-modulus ratio. The temperature resistance is based on their coefficient of thermal expansion.

<table>
<thead>
<tr>
<th>Material</th>
<th>Strength</th>
<th>Temperature Resistance</th>
<th>Fatigue Resistance</th>
<th>Strength-to-Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inconel</td>
<td>Low</td>
<td>High</td>
<td>Poor</td>
<td>Poor</td>
</tr>
<tr>
<td>Titanium</td>
<td>High</td>
<td>Fair</td>
<td>Poor</td>
<td>Fair</td>
</tr>
<tr>
<td>Nitinol</td>
<td>Fair</td>
<td>Fair</td>
<td>High</td>
<td>Fair</td>
</tr>
<tr>
<td>Elgiloy</td>
<td>High</td>
<td>High</td>
<td>Fair</td>
<td>Fair</td>
</tr>
</tbody>
</table>

Table 5.11: Candidate materials for compliant space mechanisms
Inconel is a primarily nickel, chromium, and molybdenum alloy. The titanium alloy commonly used is primarily titanium, aluminum, and vanadium. Nitinol is a primarily nickel and titanium alloy. Elgiloy is a primarily cobalt, chromium, and nickel alloy. Each material has its own unique advantages and challenges. The specific application should be considered carefully when selecting a material.

5.10.3 Flex-16 Ti-2: A Compliant Space Deployment Hinge Example

A Flex-16 hinge is designed to meet typical requirements from the space industry and is shown conceptually integrated into a spacecraft deployment hingeline in Section 5.10.4.

Suppose we want to design a deployment hinge to allow for 90° of rotation using two Flex-16s in parallel on the hingeline and a constant torque spring as the source of torque. To do this we sum the torque losses across the hingeline that come from various sources and determine the amount of torque required at 90° in the Flex-16s and the torque required in the constant torque spring. This will then allow us to design the Flex-16 to have the proper stiffness. For this example, non-SI units will be used to provide results for which that we can more readily understand the physical interpretations.

The typical torque loss across a hingeline comes from several sources. The following are typical estimated torque losses in in-lb for a typical current hingeline components: Harness 23, Damper 2, Latch 3, and Bearings 2. This sums to around 30 in-lb torque loss across the hingeline. The factor of safety design requirement for most conceptual design reviews is 2.75 [15]. This brings the required torque on the hingeline to 82.5 in-lb. This is the torque that the constant torque spring or torsional spring will be required to produce. There will be two hinges on the hingeline which must allow for a 90° deployment. This means the hinges combined must produce 30 in-lb of torque at 90°. Each hinge needs to produce 15 in-lb at 90° since they are used in parallel.

A Flex-16 design was created out of titanium to meet this requirement by using the same analysis methods as the other designs and can be described by the following parameters The design has the torque-displacement relationship shown in Figure 5.35 and can be seen to have a torque of approximately 15 in-lb at 90 degrees. The stiffness of the hinge is 9.535 in-lb/rad (1.0773 Nm/rad).

The predicted stress and displaced shape are shown in Figure 5.35. The maximum von
Figure 5.34: Ti-2 predicted stiffness.

Figure 5.35: Ti-2 predicted stress and displaced shape.
Table 5.12: Ti-2 example design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.0787 m</td>
</tr>
<tr>
<td>t1</td>
<td>0.7 mm</td>
</tr>
<tr>
<td>t2</td>
<td>3.0 mm</td>
</tr>
<tr>
<td>w</td>
<td>1.27 cm</td>
</tr>
<tr>
<td>a1</td>
<td>7 deg</td>
</tr>
<tr>
<td>a2</td>
<td>22 deg</td>
</tr>
<tr>
<td>a3</td>
<td>34 deg</td>
</tr>
<tr>
<td>a4</td>
<td>45 deg</td>
</tr>
<tr>
<td>a5</td>
<td>56 deg</td>
</tr>
<tr>
<td>a6</td>
<td>57 deg</td>
</tr>
<tr>
<td>a7</td>
<td>85 deg</td>
</tr>
<tr>
<td>a8</td>
<td>90 deg</td>
</tr>
<tr>
<td>f1</td>
<td>1.05</td>
</tr>
<tr>
<td>f2</td>
<td>0.29</td>
</tr>
<tr>
<td>f3</td>
<td>0.94</td>
</tr>
<tr>
<td>f4</td>
<td>1.25</td>
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<tr>
<td>f5</td>
<td>0.95</td>
</tr>
<tr>
<td>f6</td>
<td>1.0</td>
</tr>
<tr>
<td>f7</td>
<td>0.15</td>
</tr>
<tr>
<td>f8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Mises stress is predicted to be 682 MPa, which is less than the yield strength of 827 MPa used in this analysis. The Flex-16 design meets the requirements.

5.10.4 Spacecraft Integration

The Flex-16 is intended for future application as a compliant spacecraft deployment hinge where specific torque requirements are applied to the mechanism. Different configurations of the Flex-16 have been developed that provide options when designing for this application. The hinge can be used in different configurations in a modular manner for application in space. Figure 5.36(a) shows two Flex-16s in parallel. This may be used to increase the stiffness for higher torque applications while still allowing for planar manufacturing and reasonable material thicknesses. Figure 5.36(b) shows two hinges in parallel, offset from each other by 90°. These configurations could be used to improve off-axis stiffness and minimize undesirable vibrations.
Figure 5.36: Two configurations of combined Flex-16s. These configurations can be used to tailor the stiffness to meet specific torque requirements or to alleviate off-axis stiffness problems.

Figure 5.37: Flex-16 integrated with brackets.

The hinge can be integrated into the bracketry that attaches it to the spacecraft as depicted in Figure 5.37. The hinge can be integrated with a constant torque spring to allow actuation, as depicted in Figure 5.38. Two hinges with integrated bracketry would attach to a deploying panel in a manner similar to Figure 5.39.
5.11 Conclusions

5.11.1 Strengths of the Flex-16

The Flex-16 demonstrates desirable performance and has potential for application in space and other applications. While its intended application is as a compliant spacecraft deployment hinge, it is well-suited for non-space applications as a compliant revolute joint and future work is desirable in this area. The strengths and challenges associated with the Flex-16 are listed here.

The strengths of the Flex-16 design are:

- Large-displacement hinge capable of 90° of rotation
• Monolithic, simplified manufacturing
• Compact and lightweight
• Nearly linear stiffness over range of motion
• Customizable stiffness possible for various torque applications
• Elimination of lubrication, friction, backlash, and contact surfaces
• Can be integrated with spacecraft and current components
• Analytically predictable torque vs angular displacement behavior

The Flex-16 presents large displacement capability from a monolithic compliant mechanism. The ability to parametrically model and predict the nonlinear stiffness, displaced shape, and stress over the full range of motion will allow the designer to more readily design a deployment hinge that meets requirements. The nearly linear stiffness of the Flex-16 makes it easy to approximate a linear stiffness in the deployment dynamics model. The benefit of monolithic construction is the ability to manufacture the hinge from a planar material with a simple milling or similar process. This can help to reduce manufacturing time and costs. The hinge can be made compact to fit within the small size envelopes of launch. The hinge is able to be reset, be installed, replaced, or removed, used modularly, used in series or parallel for redundancy or axis alignment, fitted with a latch or brackets or torque application devices, etc. This allows for flexibility in the assembly, integration, and testing phase of a spacecraft lifecycle.

5.11.2 Challenges of the Flex-16

The challenges of the Flex-16 design are:

• Low off-axis stiffnesses (axial and out-of-plane bending) leading to vibration problems
• Parasitic center shift if not taken into account
• Possible creep and stress relaxation if stowed in deflected position
The off-axis stiffness, axially and out-of-plane bending, are both a problem but can be alleviated by having two or more hinges in parallel on a hingeline or by widening the out of plane dimension of the hinge. The off-axis stiffness will also result in challenges with launch vibrations and further work may need to be done to stiffen the hinge so the fundamental natural frequency is above the design requirement. If the hinge is designed to be stowed in the deflected position, creep can occur. Proper design of the hinge and bracketry allow the hinge to be in the undeflected state during stow and launch configurations. The parasitic center shift will affect pointing or deployment accuracy and should be quantified.

The Flex-16 design was demonstrated in five prototypes and three different materials and manufacturing processes and presents a promising design as a monolithic large-displacement compliant deployment hinge.
CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

In Chapter 1, the research motivation is introduced and the thesis objectives and approaches are enumerated.

In Chapter 2, the advantages of compliant mechanisms were defined and shown in the context of designing for space applications. The challenges of designing mechanisms for the space environment were outlined. Current industry performance requirements were addressed and possible challenges for compliant space mechanisms were identified. Identification of historical evidence and future industry goals validating the motivation for this research was presented. The role of heritage design was discussed as the different space mechanism types were identified. Merging compliant and space mechanism technology was discussed in terms of current space mechanism design rules and the current state of research in compliant mechanisms. Tools for developing compliant space mechanisms were identified, the research and design approach was discussed, and an example compliant space mechanism was shown. Several possible new mechanism concepts were listed and several questions this research hopes to answer were asked.

In Chapter 3, the composition of a current deployment hinge was described. Each component was evaluated for its feasibility for being replaced by compliant mechanisms. Two of the components were identified as showing many possible advantages over current designs if replaced by compliant counterpart designs. A survey of concepts for developing a large-displacement compliant rotation hinge were discussed. Several concepts were identified and investigated.

In Chapter 4, the fundamental mathematical model for a spacecraft deployment hinge was summarized. The performance of the deployable appendage was quantified for each of the three solution cases for a forced second-order dynamics model. To make this deployment model useful, a design process was created to take mission requirements as inputs and obtain deployment per-
formance and specification of vital hinge design parameters as outputs. The design process was outlined and an example of its use was shown.

In Chapter 5, one of the concepts identified in Chapter 4 was selected and in-depth design, analysis, prototyping, and testing was performed to investigate the feasibility of the design as a compliant space mechanism. The Flex-16, a large-displacement compliant rotational hinge, was designed that can achieve 90° of rotation, not fail to stress, and not self-interfere. The hinge parameters were defined and a configuration study using a parametric non-linear finite element model was used to investigate a myriad of potential design configurations. Designs were analyzed for desired displaced shape and maximum von Mises stress. Two different designs were selected and the analysis results were presented. Prototypes were fabricated out of polypropylene on a CNC mill and were tested. The torque-rotation test data was compared to the analytical model results. A titanium design was developed, analyzed, and submitted for fabrication using electron beam manufacturing metal rapid prototyping process at NASA Marshall Space Flight Center. Two micro-scale designs were developed for fabrication out of a carbon nanotube framework. The prototypes were deflected and imaged using a scanning electron microscope. The manufacturability of the Flex-16 is discussed along with an overview of candidate materials for compliant space mechanisms. An design example of a Flex-16 meeting typical industry requirements is presented. Integration of the Flex-16 with spacecraft components is depicted. Strengths and challenges of the Flex-16 are summarized.

In Chapter 6, the research was summarized, conclusions were drawn, and recommendations for future work were outlined.

6.2 Conclusions

Here are several outcomes of this research:

- This research has provided a meaningful beginning for research and development in the field of compliant space mechanisms.
- The investigation into the possible merger of compliant mechanism and space mechanism technologies provided a positive affirmation that compliant mechanisms could offer distinct advantages when designed in space applications.
• A deployment hinge design process has been developed that will aid future designers of compliant hinges and compliant dampers to determine necessary design parameters that provide the desired deployment performance while also meeting mission requirements.

• A preliminary investigation of concepts for compliant hinges yielded some promising concepts that merit further research efforts.

• A novel large-displacement compliant rotation hinge capable of 90 degrees of rotation was created. The analytical model was validated to an appropriate extent by prototype test results. The design was proven in five prototypes, out of three different materials and three different manufacturing processes. After detailed design, analysis, prototyping, and testing, the design shows promise in further refinement and possible application on a spacecraft deployment hinge.

6.3 Recommendations

I recommend that future work be performed in the following areas:

• Investigation of feasibility for application of compliant mechanism technology to the other types of space mechanisms listed in Chapter 2.

• Research into the behavior of compliant mechanisms, especially large-displacement compliant mechanisms, in thermal environments similar to space.

• Development of a compliant “ball and socket” hinge.

• Investigation into friction-free and contact-surface-free compliant joints.

• Further development and standardization of the deployment hinge design process developed in Chapter 3 for widespread industry adoption and increased capability to handle nonlinear coefficients.

• Development of a detailed mathematical model of the dynamics of deployment latching. Development of related compliant latch designs that improve the response to latch loads.
• In-depth investigation and development of concepts presented in Chapter 4 for compliant hinges.

• Development of designs that integrate hinge and damping functions into one compliant space mechanism.

• Further development of the Flex-16 developed in Chapter 5 by improving off-axis stiffnesses, increasing natural frequencies, increasing torque capability, improving the design to be more compact without failing to stress, improving for increased fatigue life, improving for increased motion accuracy and repeatability.

• Collaboration with industry for environmental testing of the Flex-16. Possible environments are thermal-vacuum deployment testing, modal survey testing, random or sine vibration testing, ambient deployment testing, and acoustic testing.

• Study and design work on how to design compliant deployment hinges for improved integration into adjacent spacecraft components.
REFERENCES


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APPENDIX A. DERIVATION OF THE DEPLOYMENT DYNAMICS MATHEMATICAL MODEL

To understand the behavior of the deploying appendage, we solve a second order differential equation. We must find the homogeneous and particular solutions. We begin by putting the differential equation in the following form

\[ \ddot{\theta} + \frac{b}{I} \dot{\theta} + \frac{k}{I} \theta = \frac{\sum T}{I} \]  

(A.1)

Several important quantities can be found with the equation in this form. The natural frequency of the deploying appendage is

\[ \omega_n = \sqrt{\frac{k}{I}} \]  

(A.2)

The damping ratio is

\[ \zeta = \frac{b}{2\sqrt{Ik}} \]  

(A.3)

The damped natural frequency is

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]  

(A.4)

The characteristic equation is

\[ r^2 + \frac{b}{I} r + \frac{k}{I} = 0 \]  

(A.5)

Solving for the two roots gives

\[ r_1 = \frac{-b + \sqrt{b^2 - 4Ik}}{2I} \]  

(A.6)

\[ r_2 = \frac{-b - \sqrt{b^2 - 4Ik}}{2I} \]  

(A.7)
Note that the roots will both be negative and will provide the exponentially decaying transient behavior.

The homogeneous solution will be of the form

\[ \theta = c_1 e^{r_1 t} + c_2 e^{r_2 t} \quad (A.8) \]

where \( c_1 \) and \( c_2 \) are coefficients found later using the initial conditions.

To solve the particular solution, we assume a solution of

\[ \theta = A + Bt \]
\[ \dot{\theta} = B \]
\[ \ddot{\theta} = 0 \quad (A.9) \]

\( \theta \) and its derivatives are substituted into Eqn. (A.1) and values for \( A \) and \( B \) are solved by equating terms with \( t \), then equating terms without \( t \), as shown below.

\[ 0 + \frac{b}{I} B + \frac{k}{I} A + \frac{k}{I} Bt = \frac{T}{I} \]
\[ kB = 0 \]
\[ B = 0 \quad (A.10) \]
\[ kA = T \]
\[ A = \frac{T}{k} \]

Knowing \( A \) and \( B \), the particular solution reduces to

\[ \theta_p = \frac{T}{k} \quad (A.11) \]

This represents the steady state value of the response and is added to the homogeneous solution.

The full solution to the differential equation is thus

\[ \theta = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \frac{T}{k} \quad (A.12) \]
We can solve for $c_1$ and $c_2$ by knowing that the solar array is at rest in a stowed position before deployment. The initial conditions are

$$\theta(0) = 0$$
$$\dot{\theta}(0) = 0$$  \hspace{1cm} (A.13)

However, the solution depends on the damping. There are three solution cases.

1. Overdamped
2. Critically Damped
3. Underdamped

The form of the solution and the calculation of the coefficients depends on the case. Equation (A.12) shows the full solution to the overdamped case. Each of these cases is described next.

A.1 Overdamped

The solution is overdamped ($\zeta > 1$) when

$$b^2 - 4Ik > 0$$  \hspace{1cm} (A.14)

and the homogeneous part of the solution takes the form

$$\theta = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$  \hspace{1cm} (A.15)

The coefficients are found by taking the first derivative of the full solution (Equation (A.12)) then substituting in the initial conditions. The derivative is

$$\dot{\theta} = r_1 c_1 e^{r_1 t} + r_2 c_2 e^{r_2 t}$$  \hspace{1cm} (A.16)
Substituting in the initial conditions (Equation (A.13)) and solving for the coefficients $c_1$ and $c_2$ results in

\[
c_1 = -\frac{T}{k\left(1 - \frac{r_1}{r_2}\right)} \quad \text{and} \quad c_2 = \frac{r_1 T}{k(r_2 - r_1)} \tag{A.17}
\]

Substituting the coefficients into the full solution ((A.12)) results in

\[
\theta = -\frac{T}{k\left(1 - \frac{r_1}{r_2}\right)} e^{r_1 t} + \frac{r_1 T}{k(r_2 - r_1)} e^{r_2 t} + \frac{T}{k} \tag{A.18}
\]

This is plotted to show the deployment of the solar array with respect to time.

### A.2 Critically Damped

The solution is critically damped ($\zeta = 1$) when

\[
b^2 - 4Ik = 0 \tag{A.19}
\]

and the homogeneous part of the solution takes the form

\[
\theta = c_1 e^{r_1 t} + c_2 t e^{r_2 t} \tag{A.20}
\]

The full solution of the critically damped case for this system is

\[
\theta = c_1 e^{r_1 t} + c_2 t e^{r_2 t} + \frac{T}{k} \tag{A.21}
\]

The coefficients are found by taking the first derivative of the full solution (Equation (A.21)) then substituting in the initial conditions. The derivative is

\[
\dot{\theta} = r_1 c_1 e^{r_1 t} + r_2 c_2 t e^{r_2 t} \tag{A.22}
\]
Substituting in the initial conditions (Equation (A.13)) gives coefficients \(c_1\) and \(c_2\)

\[
c_1 = -\frac{T}{k} \\
c_2 = -c_1 r_1 = \frac{Tr_1}{k}
\] (A.23)

Substituting these coefficients into the full solution (Equation (A.21)) results in

\[
\theta = -\frac{T}{k} e^{r_1 t} + \frac{Tr_1}{k} t e^{r_2 t} + \frac{T}{k}
\] (A.24)

### A.3 Underdamped

The solution is underdamped (\(\zeta < 1\)) when

\[
b^2 - 4Ik < 0
\] (A.25)

and the homogeneous part of the solution takes the form

\[
\theta = c_1 e^{\lambda_1 t} \cos(\mu_1 t) + c_2 e^{\lambda_2 t} \sin(\mu_1 t)
\] (A.26)

where

\[
s_1 = \lambda + i\mu \\
s_2 = \lambda - i\mu
\] (A.27)

The full solution of the underdamped case for this system is

\[
\theta = c_1 e^{\lambda_1 t} \cos(\mu_1 t) + c_2 e^{\lambda_2 t} \sin(\mu_1 t) + \frac{T}{k}
\] (A.28)

The coefficients are found by taking the first derivative of the full solution (Equation (A.28)) then substituting in the initial conditions (Equation (A.13)). The derivative is

\[
\dot{\theta} = c_1 \lambda_1 e^{\lambda_1 t} \cos(\mu_1 t) + c_2 \mu_2 e^{\lambda_2 t} \cos(\mu_2 t) - c_1 \mu_1 e^{\lambda_1 t} \sin(\mu_1 t) + c_2 \lambda_2 e^{\lambda_2 t} \sin(\mu_2 t)
\] (A.29)
Substituting in the initial conditions and solving gives

\begin{align*}
c_1 &= -\frac{T}{k} \\
c_2 &= -c_1 \frac{\lambda_1}{\mu_2} = \frac{T \lambda_1}{k \mu_2}
\end{align*}

(A.30)

Substituting these coefficients into the full solution (Equation (A.28)) results in

\[ \theta = -\frac{T}{k} e^{\lambda t} \cos(\mu t) + \frac{T \lambda_1}{k \mu_2} e^{\lambda t} \sin(\mu t) + \frac{T}{k} \]

(A.31)

The three solutions in Equations (A.18), (A.24), and (A.31) represent the fundamental mathematical model of the dynamics of a deployment hinge.
clear;clc;clf(figure(1));clf(figure(2));clf(figure(3));clf(figure(4));
format compact
%Enter givens and desired parameter values
T=90; %10.17
b=1620; %18.6134
I=45720; %13.162
tdepl=150
thetades=pi
div=10;
tf=200
%Calculate rotational stiffness
k=T/thetades
%Calculate characteristic roots
r1=(-b+sqrt(b^2-4*I*k))/(2*I)
r2=(-b-sqrt(b^2-4*I*k))/(2*I)
%Calculate value resulting in critically damped system
cd=2*sqrt(I*k)
%Determine state of solution
u=b^2-4*I*k
%overdamped
if u>0
  %Create time vector
t=[0:1/div:tf];
  %Calculate coefficients
  c1=-T/(k*(1-(r1/r2)));
  c2=(r1*T)/(k*(r2-r1));
  %Calculate theta
  theta=c1*exp(r1*t)+c2*exp(r2*t)+T/k;
  %Calculate angular velocity
  thetadot=c1*r1*exp(r1*t)+r2*c2*exp(r2*t);
  %Calculate deployment angle reached in minimum deployment time
tdepl2=tdepl*div+1
deplang=theta(tdepl2)
%Verify whether deployment angle reached in required time
if deplang>=thetades
  a='Yes, reaches desired deployment angle in required time'
  ttd=find(theta'>=thetades);
  %Calculate time to reach required deployment angle
td=t(ttd(1))
%Calculate the angular velocity at end of deployment time
angu=thetadot(round(td*div))
else
    a='No, does not reach desired deployment angle in required time'
%Calculate the angle remaining to achieve full deployment
lack=thetades-deplang
%Calculate what angle is reached in minimum deployment time
ttd2=find(theta'>=deplang);
td=t(ttd2(1))
angu=thetadot(td*div)
end
%Calculate how long it takes to reach theta desired
end
%critically damped
if u==0
    t=[0:1/div:tf];
c1=-T/k;
c2=T*lam1/(k*mu2);
theta=c1*exp(lam1.*t).*cos(mu1.*t)+c2*exp(lam2.*t).*sin(mu2.*t)+T/k;
    thetadot=c1*lam1.*exp(lam1.*t).*cos(mu1.*t)+c2*mu2.*exp(lam2.*t).*cos(mu2.*t)...
        -c1*mu1.*exp(lam1.*t).*sin(mu1.*t)+c2*lam2.*exp(lam2.*t).*sin(mu2.*t);
\[ \theta_{\cdot\cdot} = c_1 \lambda_1^2 \exp(\lambda_1 t) \cdot \cos(\mu_1 t) - c_1 \mu_1^2 \exp(\lambda_1 t) \cdot \cos(\mu_1 t) + c_2 \lambda_2^2 \exp(\lambda_2 t) \cdot \sin(\mu_2 t) - c_2 \mu_2^2 \exp(\lambda_2 t) \cdot \sin(\mu_2 t) + 2c_2 \lambda_2 \mu_2 \exp(\lambda_2 t) \cdot \cos(\mu_2 t) - 2c_1 \lambda_1 \mu_1 \exp(\lambda_1 t) \cdot \sin(\mu_1 t); \]

t_{\text{depl2}} = t_{\text{depl}} \ast \text{div} + 1

degl = \theta(t_{\text{depl2}})

if degl >= \theta_{\text{des}}
  a = 'Yes, reaches desired deployment angle in required time'
  t_{\text{td}} = \text{find}(\theta > = \theta_{\text{des}});
  td = t(t_{\text{td}}(1))
  angu = \theta_{\cdot}(\text{round}(td \ast \text{div}))
  accel = \theta_{\cdot\cdot}(\text{round}(td \ast \text{div}))
else
  a = 'No, does not reach desired deployment angle in required time'
  lack = \theta_{\text{des}} - degl
  t_{\text{td2}} = \text{find}(\theta > \text{degl});
  td = t(t_{\text{td2}}(1))
  angu = \theta_{\cdot}(\text{round}(td \ast \text{div}))
  accel = \theta_{\cdot\cdot}(\text{round}(td \ast \text{div}))
end
end

%Plot Rotational Displacement
tend = t(end);
figure(1)
plot(t, \theta)
xlabel('Time (s)')
ylabel('\theta (rad)')
grid
hold on
plot(t_{\text{depl}}, \theta_{\text{des}}, 'ro')
hold on
plot([t_{\text{depl}} tend], [\text{degl} \text{degl}],'g')
hold on
plot([td tend], [\theta_{\text{des}} \theta_{\text{des}}],'r')

%Plot Angular Velocity
figure(2)
plot(t, \theta_{\cdot}, 'b')
grid
xlabel('Time (s)')
ylabel('\omega (rad/s)')
hold on
plot(td, angu, 'ro')

%Plot Angular Acceleration
thetaddmax = \text{max}(\theta_{\cdot\cdot})
figure(3)
plot(t,thetadotdot,'b')
grid
xlabel('Time (s)')
ylabel(r'$\alpha (\text{rad/s}^2)$')
hold on
plot(td,accel,'ro')

%Plot Kinetic Energy
KE=0.5*I*thetadot.^2;
KE2=KE(round(td*div))
KEmax=max(KE)
figure(4)
plot(t,KE,'b')
grid
xlabel('Time (s)')
ylabel('Kinetic Energy (J)')
hold on
plot(td,KE2,'ro')

%Calculate natural frequency, damping ratio, and damped natural frequency
wn=sqrt(k/I)
zeta=b/(2*sqrt(I*k))
wd=wn*sqrt(1-zeta^2)
APPENDIX C. FLEX-16 ANSYS BATCH FILES FOR PROTOTYPES

This appendix contains the ANSYS batch files that create the parametric finite element model for Ti-1. The batch files for the other prototypes are the same except for the adjustable parameters section which can be updated for the desired prototype by plugging in parameter values from Table 5.3. The ANSYS batch files for modal, off-axis, and thermal analysis of Ti-1 are available upon request.

C.1 ANSYS Batch File for Ti-1

FINISH
/CLEAR

! ADJUSTABLE parameters-------------

*SET,E,113.8e9 ![N/m^2] ,E-modulus
*SET,v,0.34 ![ ] ,Poisson Ratio
*SET,w1,1.27e-2 ![m] ,Width beams
*SET,t1,.7e-3 ![m] ,Thickness beams
*SET,w2,1.27e-2 ![m] ,Width beams
*SET,t2,3e-3 ![m] ,Thickness beams
*SET,a1,7*3.14156/180 ![deg] ,Angle of flexure
*SET,a2,22*3.14156/180 ![deg] 
*SET,a3,34*3.14156/180 ![deg] 
*SET,a4,45*3.14156/180 ![deg] 
*SET,a5,56*3.14156/180 ![deg] 
*SET,a6,57*3.14156/180 ![deg] 
*SET,a7,85*3.14156/180 ![deg] 
*SET,a8,90*3.14156/180 ![deg] 
*SET,f1,1.05
*SET,f2,.29 !.31
*SET,f3,.94
*SET,f4,1.25
*SET,f5,.95
*SET,f6,1
*SET,f7,.15 !.15
*SET,f8,1
*SET,R,.0666 ![m] ,Radius 2.6 in

! Define element
/PREP7
ET,1,BEAM3
! Real constant
R,1,w1*t1,w1*t1**3/12,t1, , , ,
R,2,w2*t2,w2*t2**3/12,t2, , , ,

! Material properties
MPTEMP,1,0
MPDATA,EX,1,,E
MPDATA,PRXY,1,,v

! Define keypoints
K,1,0,0,,
K,2,0,R,,
K,3,-f1*R*sin(a1),R,,
K,4,-f2*R*sin(a1),f2*R*cos(a1),,
K,5,-f2*R*sin(a2),f2*R*cos(a2),,
K,6,-f3*R*sin(a3),f3*R*cos(a3),
K,7,-f4*R*sin(a3),R,,
K,8,-f5*R*sin(a4),f5*R*cos(a4),
K,9,-f6*R*sin(a5),f6*R*cos(a5),
K,10,-f7*R*sin(a6),f7*R*cos(a6),
K,11,-f8*R*sin(a7),f8*R*cos(a7),
K,12,-f8*R,0,,
K,13,0,-R,,
K,14,-f1*R*sin(a1),-R,,
K,15,-f2*R*sin(a1),-f2*R*cos(a1),
K,16,-f2*R*sin(a2),-f2*R*cos(a2),
K,17,-f3*R*sin(a3),-f3*R*cos(a3),
K,18,-f4*R*sin(a3),-R,,
K,19,-f5*R*sin(a4),-f5*R*cos(a4),
K,20,-f6*R*sin(a5),-f6*R*cos(a5),
K,21,-f7*R*sin(a6),-f7*R*cos(a6),
K,22,-f8*R*sin(a7),-f8*R*cos(a7),
K,23,f1*R*sin(a1),R,,
K,24,f2*R*sin(a1),f2*R*cos(a1),
K,25,f2*R*sin(a2),f2*R*cos(a2),
K,26,f3*R*sin(a3),f3*R*cos(a3),
K,27,f4*R*sin(a3),R,,
K,28,f5*R*sin(a4),f5*R*cos(a4),
K,29,f6*R*sin(a5),f6*R*cos(a5),
K,30,f7*R*sin(a6),f7*R*cos(a6),
K,31,f8*R*sin(a7),f8*R*cos(a7),
K,32,f8*R,0,,
K,33,f1*R*sin(a1),-R,,
K,34,f2*R*sin(a1),-f2*R*cos(a1),
K,35,f2*R*sin(a2),-f2*R*cos(a2),
K,36,f3*R*sin(a3),-f3*R*cos(a3),,
K,37,f4*R*sin(a3),-R,,
K,38,f5*R*sin(a4),-f5*R*cos(a4),,
K,39,f6*R*sin(a5),-f6*R*cos(a5),,
K,40,f7*R*sin(a6),-f7*R*cos(a6),,
K,41,f8*R*sin(a7),-f8*R*cos(a7),,
K,46,0,f7*R,,
K,47,0,-f7*R,,

! Draw lines between keypoint
LSTR,3,4 !Start of real constant set 1
LSTR,5,6
LSTR,6,8
LSTR,8,9
LSTR,9,10
LSTR,10,11
LSTR,14,15
LSTR,16,17
LSTR,17,19
LSTR,19,20
LSTR,20,21
LSTR,21,22
LSTR,23,24
LSTR,25,26
LSTR,26,28
LSTR,28,29
LSTR,29,30
LSTR,30,31
LSTR,33,34
LSTR,35,36
LSTR,36,38
LSTR,38,39
LSTR,39,40
LSTR,40,41
LSTR,1,46
LSTR,1,47
LSTR,2,3 !Start of real constant set 2
LSTR,3,7
! LSTR,4,5
LSTR,11,12
! LSTR,12,1
LSTR,13,14
LSTR,14,18
! LSTR,15,16
LSTR,22,12
LSTR,2,23
LSTR,23,27
! LSTR,24,25
LSTR,31,32
LSTR,13,33
LSTR,33,37
! LSTR,34,35
LSTR,41,32
! LSTR,1,32
! Draw arc between keypoint
LARC,4,24,1,f2*R,
LARC,4,5,1,f2*R,
LARC,24,25,1,f2*R,
LARC,15,34,1,f2*R,
LARC,15,16,1,f2*R,
LARC,34,35,1,f2*R,
LARC,10,46,1,f7*R,
LARC,21,47,1,f7*R,
LARC,46,30,1,f7*R,
LARC,47,40,1,f7*R,

! Glue the lines to one beam
LGLUE, ALL,

! Mesh, create elements
LESIZE,ALL, ,20, ,1, , ,1,
TYPE, 1
REAL, 1
LMESH,1,26

TYPE, 1
REAL, 2
LMESH,27,48

! Define constraints
DK,13,ALL, , , , , , , ,
DK,14,ALL, , , , , , , ,
DK,18,ALL, , , , , , , ,
DK,33,ALL, , , , , , , ,
DK,37,ALL, , , , , , , ,

! Define Rotational Displacement
DK,2, ,1.7, ,0,ROTZ, , , , ,

! Find the node at keypoint 2
ksel,s,kp,,2
nslk,s
*get,nkp2,node,0,num,max
nsel,all
ksel,all
! Defining Analysis specifications
NLGEOM,1
AUTOTS,0
NSUBST,50,0,0
OUTRES,ALL,1

! Solve the analysis
bcsoption, ,maximum
/SOL
SOLVE
FINISH

! Plot deformed shape
/POST1
PLDISP,1

/SHRINK,0
/ESHAPE,1.0
/EFACET,1
/RATIO,1,1,1
/CFORMAT,32,0
/REPLOT

! Plot graph
/POST26
NSOL,2,nkp2,ROT,Z,rotz !Displacements node 2
RFORCE,3,nkp2,M,Z,mzz !Forces node 2
XVAR,2
PLVAR,3

*CREATE,scratch,gui
*DEL,_,P26_EXPORT
*DIM,_,P26_EXPORT,-table,50,2
VGET,_,P26_EXPORT(1,0),2
VGET,_,P26_EXPORT(1,1),3
/output,'moment','txt','J:\Grad School\Research'
*WRITE,'ROTZ','MZ'
*WRITE,_,P26_EXPORT(1,0),_,P26_EXPORT(1,1)
/OUTPUT,TERM
*END
/INPUT,scratch,gui

/AXLAB,X,DEFLECTION [rad] !Renaming axis labels
/AXLAB,Y,TORQUE [Nm]
/REPLOT