2016-10

Polynomial Chaos for the Computation of Annual Energy Production in Wind Farm Layout Optimization

Santiago Padrón
Stanford University

Andrew P.J. Stanley
Brigham Young University, stanley_andrewpj@yahoo.com

See next page for additional authors

Original Publication Citation
Padrón, S., Stanley, A. P. J., Thomas, J., Alonso, J. J., and Ning, A., "Polynomial Chaos for the Computation of Annual Energy Production in Wind Farm Layout Optimization," Journal of Physics: Conference Series, Vol. 753, No. 032021, The Science of Making Torque from Wind, Oct. 2016. doi:10.1088/1742-6596/753/3/032021

Authors
Santiago Padrón, Andrew P.J. Stanley, Jared Thomas, Juan Alonso, and Andrew Ning

Polynomial chaos for the computation of annual energy production in wind farm layout optimization

A S Padrón[1], A P J Stanley[2], J J Thomas[2], J J Alonso[1] and A Ning[2]

[1] Department of Aeronautics & Astronautics, Stanford University, Stanford, CA, 94305, USA
[2] Department of Mechanical Engineering, Brigham Young University, Provo, UT, 84602, USA

E-mail: padronas@stanford.edu, aning@byu.edu

Abstract. Careful management of wake interference is essential to further improve Annual Energy Production (AEP) of wind farms. Wake effects can be minimized through optimization of turbine layout, wind farm control, and turbine design. Realistic wind farm optimization is challenging because it has numerous design degrees of freedom and must account for the stochastic nature of wind. In this paper we provide a framework for calculating AEP for any relevant uncertain (stochastic) variable of interest. We use Polynomial Chaos (PC) to efficiently quantify the effect of the stochastic variables—wind direction and wind speed—on the statistical outputs of interest (AEP) for wind farm layout optimization. When the stochastic variable includes the wind direction, polynomial chaos is one order of magnitude more accurate in computing the AEP when compared to commonly used simplistic integration techniques (rectangle rule), especially for non grid-like wind farm layouts. Furthermore, PC requires less simulations for the same accuracy. This allows for more efficient optimization and uncertainty quantification of wind farm energy production.

1. Introduction

Wind farm optimization is multidisciplinary, the nature of wind is inherently stochastic, and the design of wind farms contains numerous potential design degrees of freedom. Because of this complexity, industry practice generally relies on sequential design processes, simplistic assessments of uncertainty, and a relatively small number of design variables for designing wind farms. The results are simplistic layout optimization formulations, often grid-like, with significantly reduced energy capture than could be realized. Wind farm installations typically face energy production losses of around 10–20% because of wake interference [1, 2]. This loss in energy capture results in millions of dollars of loss for operators and investors and increased economic uncertainty for new installations. Methods to better design and optimize wind farms to minimize these wake impacts would have a substantial impact on the industry.

In addition to optimizing the wind farm layout, optimizing control at the level of the wind farm is a promising approach to minimizing wake impacts [3, 4]. Control parameters generally include turbine yaw angles or blade pitch angles. If turbine control and layout are optimized simultaneously, the control parameters become coupled to the stochastic wind directions, and the total number of design variables across the wind farm grows rapidly. Our past research has focused on enabling large-scale coupled layout-yaw optimization [5, 6] by reformulating wake models to provide continuously differentiable output, using automatic differentiation to provide numerically exact gradients, and taking advantage of scalable, gradient-based optimization methods [7]. Another approach to enable large-scale optimizations is to reduce the number of stochastic wind directions needed to accurately compute the statistical quantities of interest, which is the approach explored in this paper.

For wind farm optimization, many stochastic variables exist in addition to wind direction, namely the wind speed, and wake model parameters. Past studies have computed statistical outputs, like the annual energy production, using simple integration methods like the rectangle rule and trapezoid rule. These quadrature methods are inefficient and generally require a large number of function evaluations to quantify wind farm performance for the stochastic wind conditions. This is especially true if multiple stochastic inputs are considered simultaneously (e.g., uncertainty in wind direction and speed).

The field of uncertainty quantification [8] moves beyond these simple integration methods to compute the statistical outputs. An efficient uncertainty quantification method is polynomial chaos [9] and it's starting to find its use in computting statistics for wind farms [10].

In this paper we explore the use of polynomial chaos (PC)—a scalable stochastic expansion method—to efficiently predict the statistical outputs of the wind farm and use those outputs in wind farm optimization. In section 2 we describe the current integration methods and our proposed method, polynomial chaos, to compute statistics of the wind farm. In section 3 we provide the definition of the problem of interest and describe the wake model used to determine the wake effects. Finally, sections 4 and 5 contain the results and conclusions.

2. Uncertainty quantification: Calculating the statistics

We propose a novel way of computing the statistics of interests of a wind farm. A common statistic of interest of a wind farm is the Annual Energy Production (AEP), which can be formulated as a function of expected power

$$AEP = \frac{hr}{yr}\mathrm{E}[P(\boldsymbol{\xi})] \tag{1}$$

where hr/yr is the number of hours in a year (i.e., 8760). The expected power, $\mathrm{E}[P(\boldsymbol{\xi})]$, is defined as

$$\mathrm{E}[P(\boldsymbol{\xi})] = \int_{\Omega} P(\boldsymbol{\xi})\rho(\boldsymbol{\xi})\, d\boldsymbol{\xi} \tag{2}$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_n)$ is a vector of random (stochastic) variables, which we will refer to as the uncertain variables, $\rho(\boldsymbol{\xi}) = \prod_{k=1}^{n} \rho_k(\xi_k)$ is the joint probability density function of the uncertain variables, Ω is the domain of the uncertain variables, and P is the power produced by the wind farm. Common uncertain variables are the wind direction β and the freestream wind speed U_∞.

Below we describe the method commonly used to calculate the expected value (rectangle rule) for the 1-dimensional case when the uncertain variable is the wind direction ($\xi = \beta$), and then describe our proposed method (polynomial chaos) for the same case.

2.1. Rectangle rule

The rectangle rule is the simplest and most straightforward quadrature method, and the most commonly used approach in computing AEP. It consists of dividing the domain for wind direction $\Omega = [a = 0, b = 2\pi]$ into m equal subintervals of length $\Delta\xi = (b - a)/m$. Next, we construct rectangles with base $b = \Delta\xi$ and height equal to the power and the density evaluated at the mid-point of the subinterval $h = P(\xi_j)\rho(\xi_j)$. Finally, the rectangle rule approximates the expected power by adding up the areas $(b \cdot h)$ of the m rectangles

$$\mathrm{E}[P(\xi)] = \int_0^{2\pi} P(\xi)\rho(\xi)\, d\xi \approx \sum_{j=1}^{m} P(\xi_j)\rho(\xi_j)\Delta\xi. \tag{3}$$

A simple improvement is to integrate the density exactly within each subinterval

$$\mathrm{E}[P(\xi)] = \int_0^{2\pi} P(\xi)\rho(\xi)\,d\xi \approx \sum_{j=1}^{m} P(\xi_j) \int_{\xi_{j-1/2}}^{\xi_{j+1/2}} \rho(\xi)\,d\xi. \tag{4}$$

This is easily done as the density is known and usually given by an interpolation of a wind rose. This modification is helpful for a small number of evaluations m. We will use this modified rectangle rule and simply refer to it as the rectangle rule.

2.2. Polynomial Chaos

Polynomial chaos is an uncertainty quantification method that aims to quantify the effect of uncertain inputs ξ on our output of interest $P(\xi)$. PC approximates our function by a polynomial expansion

$$P(\xi) \approx \hat{P}(\xi) = \sum_{i=0}^{p} \alpha_i \phi_i(\xi), \tag{5}$$

where the larger the polynomial order p is, the closer the approximation $\hat{P}(\xi)$ is to the true response $P(\xi)$. The orthogonal polynomials ϕ_i are chosen such that their orthogonality weighting functions match the probability density function of the uncertain input up to a constant factor. From PC (eq. (5)) we can obtain the mean and the variance analytically as

$$\mu_P = \mathrm{E}[P] = \alpha_0 \qquad \text{and} \qquad \sigma_P^2 = \sum\nolimits_{i=1}^{p} \alpha_i^2 \langle \phi_i^2(\xi) \rangle. \tag{6}$$

The coefficients α_i can be computed by linear regression [9] or by spectral projection which gives

$$\alpha_i = \frac{1}{\langle \phi_i^2(\xi) \rangle} \int_0^{2\pi} P(\xi)\phi_i(\xi)\rho(\xi)\,d\xi. \tag{7}$$

This integral is then evaluated with Gaussian quadrature.

Note the equation for the zeroth coefficient is the same as the expected power ($\phi_0 \equiv 1$)

$$\alpha_0 = \mathrm{E}[P] = \int_0^{2\pi} P(\xi)\rho(\xi)\,d\xi \approx \sum_{j=1}^{m} P(\xi_j) w_j. \tag{8}$$

Thus, for the computation of the expected value for this 1-dimensional case, the difference between the currently used rectangle rule (eq. (4)) and the proposed PC is that the expected value (eq. (8)) is computed by Gaussian quadrature as opposed to the rectangle rule.

However, PC also provides a functional representation eq. (5) from which other statistics such as probabilities can be estimated. In addition, extension to multiple dimensions or multiple uncertain variables (wind direction, wind speed, wake model parameteres, etc.) can easily be incorporated in the PC framework [11].

3. Problem definition

We will consider the power of the wind farm to be a function of three classes of variables: uncertain variables $\boldsymbol{\xi}$, design variables $\mathbf{x}$, and parameters $\boldsymbol{\theta}$

$$P = P(\boldsymbol{\xi}, \mathbf{x}, \boldsymbol{\theta}). \tag{9}$$

We consider uncertain variables to be variables that follow a probability distribution, design variables to be those variables that an optimizer can vary, and parameters to be constants. The classification of the variables is problem dependent. For instance, the height of a turbine could be considered as a design variable if we allow the optimizer to vary it, as an uncertain variable if we assume that due to terrain or manufacturing variability the exact height is unknown but follows a particular probability distribution, or as a parameter if we set the turbine height in the model at a fixed value.

For the problems considered in this work, table 1 lists in which category we put each variable that affects the power computation. The uncertain variables we consider are the wind direction

Table 1. The variables used for calculating the power.

Uncertain $\boldsymbol{\xi}$	β - wind direction. Polynomial distribution (see footnote 1) U_∞ - freestream wind speed. Weibull distribution.
Design $\mathbf{x}$	x - The x location of each turbine. y - The y location of each turbine.
Parameters $\boldsymbol{\theta}$	yaw angles, turbine characteristics, and wake parameters (see figure 1).

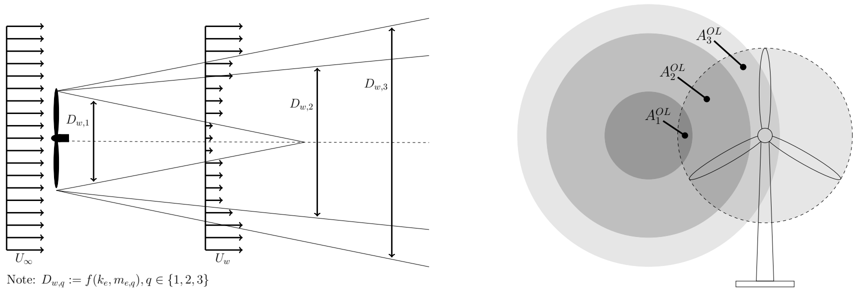

Figure 1. The FLORIS wake model. The model has three zones with varying diameters, $D_{w,q}$, that depend on tuning parameters k_e and $m_{e,q}$. The effective hub velocity is computed using the overlap ratio, A_q^{OL}, of the total rotor-swept area to the part of the rotor-swept area overlapping each wake zone respectively.

β which follows a distribution specified by a polynomial[1] and the wind speed U_∞ which follows a Weibull distribution. The wake model we use to compute the power of the wind farm is described below, after which we describe the optimization problem.

3.1. Wake Model: FLORIS

We can estimate the power conversion of a wind farm for a single wind direction by summing the power captured by each wind turbine as follows:

$$P = \sum_{i=1}^{n_{turb.}} P_i \tag{10}$$

The power capture of individual turbines, P_i, is defined as

$$P_i = \frac{1}{2}\rho C_P A_i U_i^3, \tag{11}$$

where ρ is the air density, A_i is the rotor swept area, C_P is the power coefficient, and U_i is the effective hub velocity for each turbine. The effective hub velocity is determined with the aid of a wake model. We used the FLow Redirection and Induction in Stead-state (FLORIS) wake model [13] to estimate the wake effects, using recent changes described in [7] that reduce the required number of function calls and improve convergence during optimization. The FLORIS model is a derivative of the Jensen wake deficit model [14] and the wake deflection model presented in [15]. The FLORIS model builds on the Jensen model by defining three separate wake zones with differing expansion and decay rates to more accurately describe the velocity deficit across the wake region. A simple overlap ratio of the area of the rotor in each zone of each shadowing wake

[1] The polynomial was formed by solving a constrained least squares fit to the wind rose of the Princess Amalia wind farm [12], removing regions of (near) zero probability, and redefining the zero location to be the max probability direction.

to the full rotor area is used to determine the effective hub velocity of a given turbine. A simple overview of the FLORIS model, showing the zones and overlap areas, is shown in figure 1. The FLORIS model can be described in the terms outlined above as follows:

$$U_i = f(\underbrace{\beta, U_\infty}_{\boldsymbol{\xi}}, \underbrace{\mathbf{x}, \mathbf{y}}_{\mathbf{x}}, \underbrace{\boldsymbol{\gamma}, \mathbf{T}, \boldsymbol{\Theta}}_{\boldsymbol{\theta}}) \tag{12}$$

where β is the wind direction, U_∞ is the freestream wind speed, $(\mathbf{x}, \mathbf{y})$ is the set of all turbine locations, $\boldsymbol{\gamma}$ is the set of individual yaw-offset angles for each turbine with respect to the wind direction, $\mathbf{T}$ is a set of turbine characteristics (e.g., rotor diameter), and $\boldsymbol{\Theta}$ is a set of constant parameters used to tune the FLORIS model. In this work, we set the yaw-offset angle of each turbine to zero and used the parameter values recommended in [13] and [7].

3.2. Optimization problem

The objective of the wind farm design optimization is to maximize the AEP. The optimization problem can be summarized as:

$$\begin{aligned} \underset{x,y}{\text{maximize}} \quad & AEP(x, y) \\ \text{subject to} \quad & S_{i,j} \geq 2D \quad i, j = 1...60 \quad i \neq j \\ & N_{i,b} \geq 0 \quad i = 1...60 \quad b = 1...14 \end{aligned} \tag{13}$$

where $S_{i,j}$ is the distance between each pair of turbines i and j. The normal distance, $N_{i,b}$, from each turbine i to each boundary b was defined as positive when a turbine was inside the boundary, and negative when it was outside of the boundary. The boundary is the convex hull of the baseline layout.

The baseline for the optimization is the Princess Amalia wind farm layout. The Princess Amalia wind farm has 60 turbines, which results in 120 design variables for position-only optimization. While the Princess Amalia wind farm is composed of Vestas V80 turbines, we used the NREL 5MW reference turbine [16] since we needed an open-access turbine model.

For the optimization problem, the uncertain variable over which the expected value—in the AEP—is taken is the wind direction. The wind speed is taken as a constant at 8 m/s. And the optimization problem is solved with the gradient-based optimizer SNOPT [17].

4. Results

For the uncertainty quantification—calculating the AEP—we focus on 4 different layouts: grid, Amalia, optimized, and random. All of the layouts have 60 turbines. The grid layout fits in a box of equivalent area to that of the Amalia layout. The Amalia layout (which is grid-like) is that of the Princess Amalia wind farm. The optimized layout was obtained by running the optimization problem, eq. (13), with PC using 30 wind directions and using the Amalia layout as the starting point. The optimized layout is not necessarily the global optimum, but it is a representative optimal layout. The random layout was generated by random sampling and keeping the turbines whose location are contained within the convex hull of the Amalia wind farm.

The power production as a function of wind direction and wind speed for each of the four layouts is shown in figure 2. The power vs. direction results are calculated for a wind speed of 8 m/s and the power vs. speed results for the most probable wind direction, 225 deg CW from North. The more grid-like the layout, the larger the variability in the power for the wind direction. An optimal layout not only has a higher mean power but also has a much smaller variance than the grid-like layouts. The response with speed is as expected (approximately cubic) until it reaches rated speed, with more random layouts showing a smoother transition to the maximum power because individual turbines reach their rated speed at different freestream speeds.

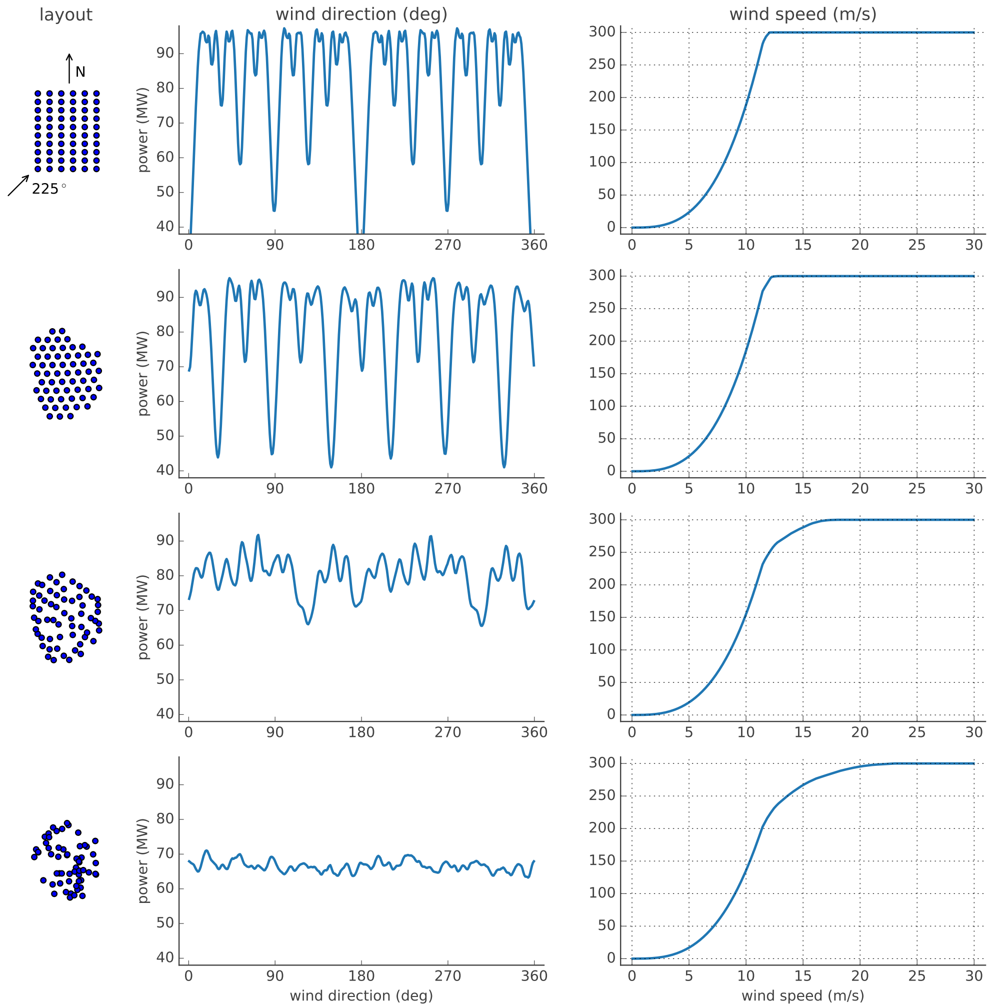

Figure 2. Power response of the wind farm for grid, Amalia, optimized, and random layouts as a function of wind direction and wind speed. The power vs. direction results are calculated for a wind speed of 8 m/s and the power vs. speed results for the most probable wind direction 225 deg. The more grid-like the layout the larger the variability in the power for the wind direction. An optimal layout has much smaller variance than the grid-like layouts. The response with speed is as expected (cubic in nature) until it reaches rated speed, the more random the layout the smoother the transition to the maximum power.

The convergence of the AEP (weighted integral of the power response eq. (1)) as a function of the number of power simulations needed is shown in figure 3. Overall we see that as the layouts become less grid-like (which is the case during an optimization), PC performs better than the rectangle rule. For the optimal layout, PC can estimate the AEP within 1%[2] by considering only 4 wind directions, whereas the rectangle rule requires the computation of the power at around 20 wind directions for the same accuracy. Furthermore, PC is usually one order of magnitude more accurate in computing the AEP for the same number of simulations.

When the uncertain variable is the speed, the rectangle rule and PC perform similarly for computing the integrated quantity of the AEP. However, the good performance of the rectangle

[2] The reference AEP used to compute the error for each of the different uncertain variable cases are the AEP values obtained by the rectangle rule with 100 wind directions, or 100 wind speeds, or for the combined case 1600 wind directions and speeds.

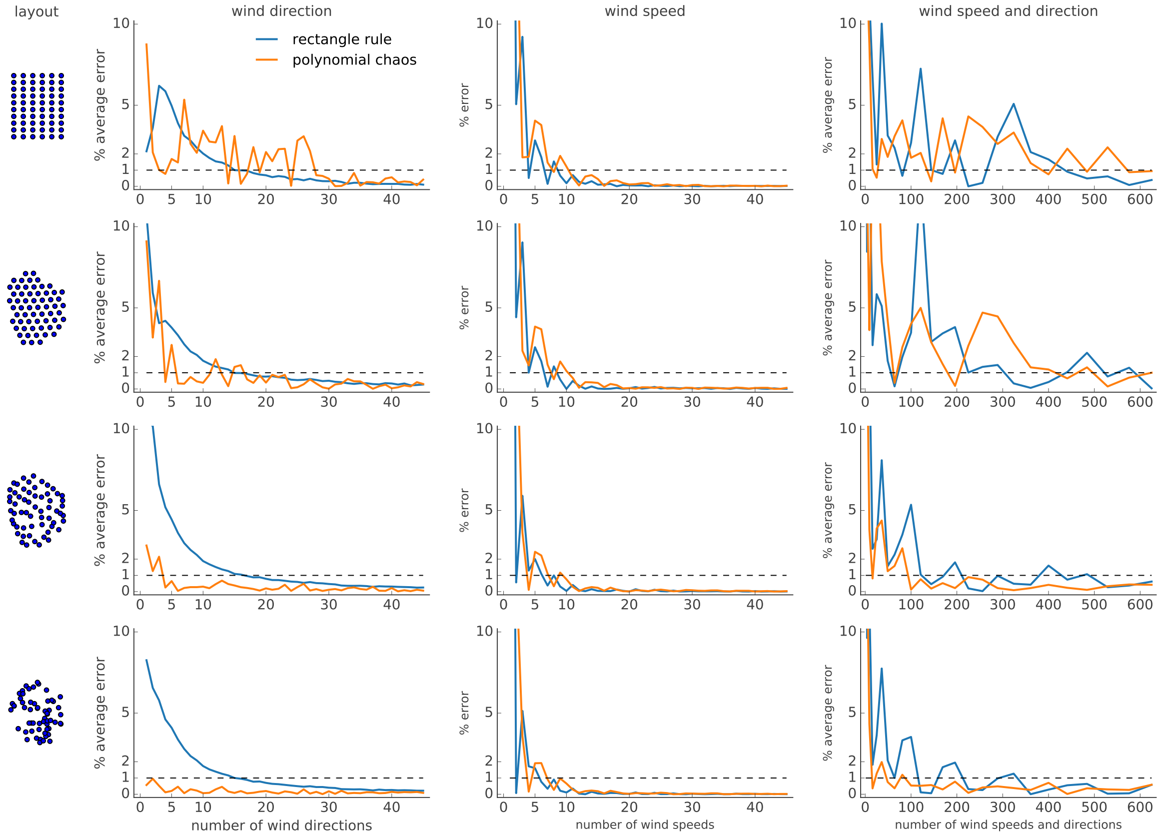

Figure 3. The error in computing the AEP as a function of the number of power simulations by both methods rectangle rule and polynomial chaos (PC). The results in each column are for different cases of the uncertain variable: wind direction, wind speed, and the 2d case (wind direction and speed). Overall we see that as the layouts become less grid-like PC performs better than the rectangle rule, especially for the wind direction cases.

rule for small number of wind speeds is more caused by luck in approximating the integral, where over-predictions and under-predictions almost entirely cancel, rather than in correctly approximating the function (power response) and thus the integral. The speed response is less complex than the direction response (figure 2) and more easily approximated by a polynomial, although the convergence of the AEP with PC is similar for both uncertain variables.

For the combined case, PC again outperforms the rectangle rule while the number of evaluations is small for non grid-like layouts. The combined case is a more realistic case where both the wind direction and wind speed can vary. The evaluation points are the tensorial product of the direction and speed quadrature points.

The convergence results in figure 3 that include the wind direction as the uncertain variable show the average error, which more clearly ilustrates the differences between the PC and rectangle rule. The average error is computed from 10 sets of wind directions.[3]

The variability in the AEP from these 10 sets of directions for the 1d case (wind direction)

[3] For the rectangle rule for each number of wind directions, 10 different sets of those directions were used. For example for 36 wind directions the 10 sets are $\{[0, 10, 20, \ldots, 340, 350\,\text{degrees}];\ [1, 11, 21, \ldots, 341, 351]; \ldots;\ [9, 19, 29, 349, 359]\}$. For the polynomial chaos the quadrature points are the numerically generated gaussian quadrature points for the interval. Thus to create 10 different sets we pick 10 different intervals. When considering 36 wind directions the chosen intervals are $\{[0, 360]; [10, 370]; [20, 380]; \ldots;\ [340, 700]; [350, 710]\}$.

and for the 2d case (wind direction and speed) for the optimal layout is illustrated in figure 4. The line in the figure represents the average of these 10 sets and the shaded region represents the max and min value calculated from these 10 sets for each number of wind directions. The variability is similar for both the rectangle rule and PC and it is more pronounced for the 1d case. The variability decreases as we increase the number of wind directions considered. Also, the variability in these 10 sets is smaller for the less grid-like layouts (this result is not shown).

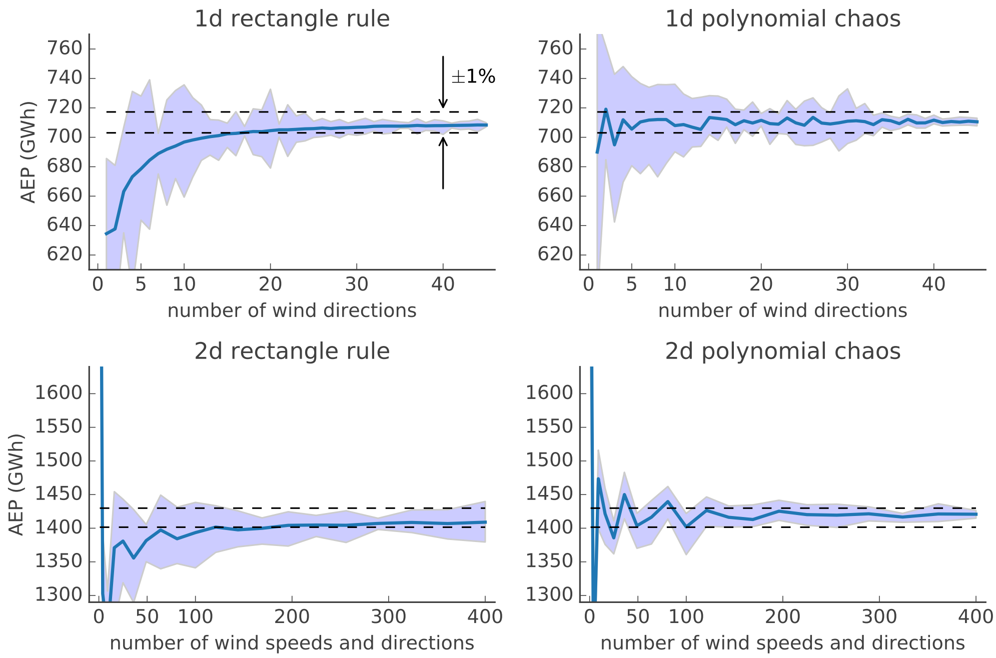

Figure 4. Variability in the convergence of the AEP for both the rectangle rule and polynomial chaos (PC) for the 1-dimensional and 2-dimensional uncertain variable case. The results shown are for the optimal layout. The variability is calculated from 10 sets of wind directions (see footnote 3). The line represents the average of these 10 sets and the shaded region represents the max and min value calculated from these 10 sets for each number of wind directions. The dashed lines show ±1% of the converged value (see footnote 2). The variability is similar for both the rectangle rule and PC and it is more pronounced for the 1d case.

Next, we perform an optimization eq. (13) with both methods (rectangle rule and PC) considering different numbers of wind directions to evaluate the AEP. The optimal layouts for each method and number of directions, along with the AEP for each optimized layout, is shown in figure 5. We observe that there are many local optima—the optimization layouts are significantly different for each method and for the number of wind directions considered. Furthermore, the different local optima can produce essentially the same amount of AEP (PC 30 and 40). The optimal AEP for the different number of directions considered are visualized in figure 6. The AEP for both methods is similar, with PC performing better for small number of wind directions. It is interesting to see that increasing the number of directions continuously improves the optimization result even after we are well beyond the number of directions needed for a converged AEP. To better quantify the performance of the optimization, multiple simulations should be run with different sets of a particular number of wind directions, and then average the results as was done previously. Nevertheless, these initial optimization results are promising.

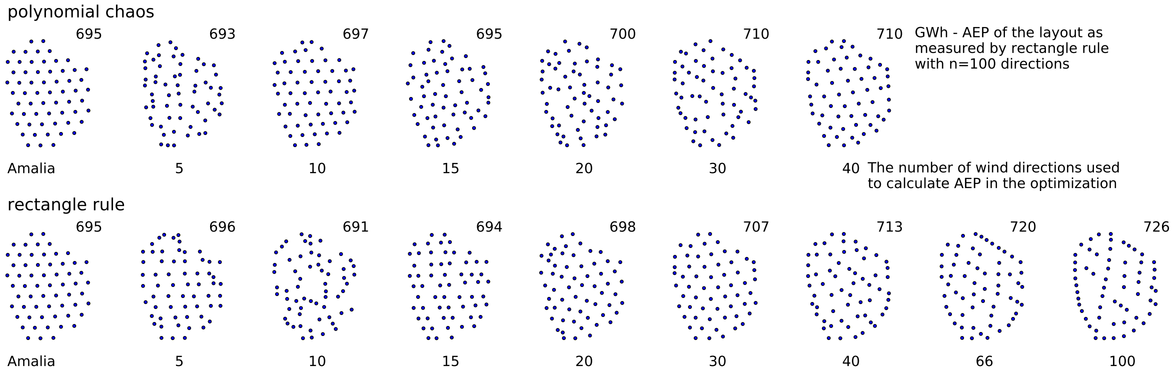

Figure 5. Diagram showing the different optimal layouts optimized with different methods (polynomial chaos top row, rectangle rule bottom row) and considering different number of directions to evaluate the AEP in the optimization problem.

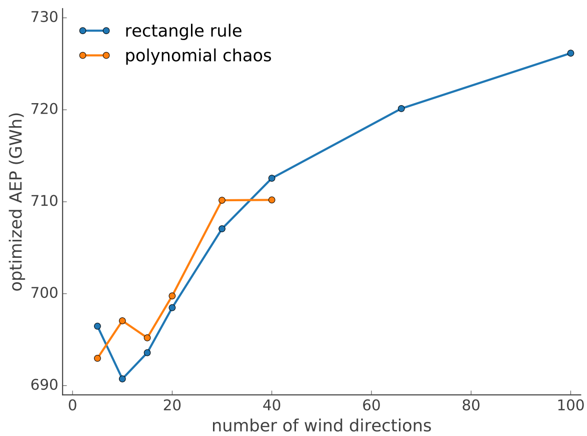

Figure 6. The AEP after running an optimization with various numbers of wind directions. The reported AEP is calculated by the rectangle rule with 100 directions for each optimal layout.

5. Conclusion

This paper has focused on improving uncertainty quantification with application in wind farm design. In particular, we have provided a framework—in polynomial chaos—for addressing AEP calculations for any relevant uncertain variable of interest and applied it for uncertain wind direction and wind speed. The application of polynomial chaos can allow AEP calculations to be performed with a desired accuracy using fewer (20%–500%) simulations than is possible with the typical rectangle rule approach, especially when the turbines are not in a grid-like layout. Because the majority of wind farm layouts during optimization iterations are unlikely to be grid-like, PC is a promising method for reducing the computational cost of wind farm design optimization.

An initial optimization study showed that PC and the rectangle rule provide results with comparable AEP for a given number of wind directions, but with significantly different wind farm layouts. The final optimized wind farm layouts differed considerably when we varied the number of wind directions used in the optimization. Also, increasing the number of wind directions—beyond what was necessary to compute the AEP accurately—continued to produced better optimization results. This sensitivity of the optimization needs to be further explored.

Also, for the optimization other future work may include application of PC with gradients, and further investigating methods of performing wind farm design optimization under uncertainty. For the uncertainty quantification, future work would include considering additional uncertain variables (especially parameters of wake models) and exploring the use of regression within PC as opposed to quadrature.

Acknowledgements
Funding for this research was provided by the National Science Foundation under a collaborative research grant No. 1539384 and 1539388.

References

[1] Barthelmie R J, Frandsen S T, Nielsen M N, Pryor S C, Rethore P E and Jørgensen H E 2007 Modelling and measurements of power losses and turbulence intensity in wind turbine wakes at middelgrunden offshore wind farm *Wind Energy* **10** 517–528 URL http://dx.doi.org/10.1002/we.238

[2] Briggs K 2013 Navigating the complexities of wake losses *North American Windpower* **10**

[3] Pao L Y and Johnson K E 2009 A tutorial on the dynamics and control of wind turbines and wind farms *2009 American Control Conference* (Institute of Electrical & Electronics Engineers (IEEE)) URL http://dx.doi.org/10.1109/acc.2009.5160195

[4] Fleming P A, Gebraad P M, Lee S, van Wingerden J W, Johnson K, Churchfield M, Michalakes J, Spalart P and Moriarty P 2014 Evaluating techniques for redirecting turbine wakes using SOWFA *Renewable Energy* **70** 211–218 URL http://dx.doi.org/10.1016/j.renene.2014.02.015

[5] Fleming P, Ning A, Gebraad P and Dykes K 2016 Wind plant system engineering through optimization of layout and yaw control *Wind Energy* **19** 329–344

[6] Gebraad P, Thomas J J, Ning A, Fleming P and Dykes K 2015 Maximization of the annual energy production of wind power plants by optimization of layout and yaw-based wake control *Wind Energy* (in press)

[7] Thomas J, Gebraad P and Ning A 2016 Improving the floris wind plant model for compatibility with gradient-based optimization (in review)

[8] Smith R C 2014 *Uncertainty Quantification Theory, Implementation, and Applications* (SIAM) ISBN 9781611973211 URL http://bookstore.siam.org/cs12/

[9] Eldred M S and Burkardt J 2009 Comparison of Non-Intrusive Polynomial Chaos and Stochastic Collocation Methods for Uncertainty Quantification *47th AIAA Aerospace Sciences Meeting including The New Horizons Forum and Aerospace Exposition* 1–20 URL http://arc.aiaa.org/doi/pdf/10.2514/6.2009-976

[10] Murcia J P, Réthoré P E, Natarajan A and Sørensen J D 2015 How Many Model Evaluations Are Required To Predict The AEP Of A Wind Power Plant? *Journal of Physics: Conference Series (Online)* vol 625 pp 1–11

[11] Padrón A S, Alonso J J and Eldred M S 2016 Multi-fidelity Methods in Aerodynamic Robust Optimization *AIAA SciTECH 2016* 1–14

[12] Brand A J, Wagenaar J W, Eecen P J and Holtslag M C 2012 Database of measurements on the Offshore Wind Farm Egmond aan Zee *Proceedings of the EWEA Annual Meeting* (Copenhagen, Denmark)

[13] Gebraad P M O, Teeuwisse F W, van Wingerden J W, Fleming P A, Ruben S D, Marden J R and Pao L Y 2014 Wind plant power optimization through yaw control using a parametric model for wake effects—a CFD simulation study *Wind Energy* **19** 95–114

[14] Jensen N O 1983 A note on wind generator interaction Tech. rep. Risø National Laboratory DK-4000 Roskilde, Denmark

[15] Jiménez Á, Crespo A and Migoya E 2010 Application of a LES technique to chracterize the wake deflection of a wind turbine in yaw *Wind Energy* **13** 559–572

[16] Jonkman J, Butterfield S, Musial W and Scott G 2009 Definition of a 5-MW reference wind turbine for offshore system development Tech. Rep. NREL/TP-500-38060 National Renewable Energy Laboratory Golden, CO

[17] Gill P, Murray W and Saunders M 2005 SNOPT: An SQP algorithm for large-scale constrained optimization *SIAM review* **47** 99–131