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On the Use of Size Premiums,
Arithmetic or Geometric Average Returns,
and Liquidity Premiums in Determining Discount Rates

BY HAL HEATON, PH.D.

In recent court cases, a number of technical issues have arisen in determining appropriate discount rates for use in the discounted cash flow approach to valuation. This article examines three issues, reviews the literature about the issues, and summarizes the key considerations.

The first issue is whether a size premium is appropriate. The size premium refers to an adjustment to the discount rate (also referred to here as the cost of capital) to reflect a higher required return for smaller companies. Return data often are obtained from large, publicly traded companies, but then are used to value small companies or properties. Should a premium be added to the required return to reflect the fact that the subject company or property is much smaller than the companies used to estimate the required return?

The second issue is the use of historical arithmetic or geometric averages to determine the equity risk premium in the capital asset-pricing model (CAPM). Most appraisers use historical return data from the stock market and compare those returns to the historical returns from Treasury bonds to estimate the additional return that investors require to invest in the stock market rather than in Treasuries. However, in comparing the historical returns, should the arithmetic or geometric average returns be used to calculate the difference in returns?

The third issue is whether a liquidity premium is appropriate when illiquid investments or properties are being valued. Often appraisers obtain return or cost-of-capital data from liquid, publicly traded securities. Liquidity refers to the ease or cost of buying or selling an investment. Should a premium be added to data from traded securities to reflect the fact that tangible, physical properties take much longer to sell, are much more expensive to sell than publicly traded stocks or bonds, and require much larger discounts in price to sell quickly?

Each of these questions is addressed in this article.

The Size Premium

Appraisers are often asked to value properties that are small compared to

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publicly traded companies. Return and cost-of-capital data often are available only from large, publicly traded companies. A large number of researchers have documented market evidence that small companies have higher returns than the CAPM indicates. The CAPM estimates the required return on equity with the equation

\[ \text{Required return} = R_f + \beta (R_m - R_f) \]

where \( R_f \) is the risk-free rate, \( R_m \) is the market expected return, and \( \beta \) (beta) is the measure of systematic risk and incorporates both the volatility of the investment and the correlation of the investment with the market. (For a sample of research articles that document this size premium, see the Suggested Reading section at the end of this article.) In each of its Stocks, Bonds, Bills, and Inflation Yearbooks (2007a; 2007b), Ibbotson Associates devotes an entire chapter to the historical risk premiums above the risk premiums suggested by the CAPM for companies of various sizes. The evidence is overwhelming that the smaller the company, the higher the required return.

There is speculation about why the size premium exists. Some believe that small companies are subject to higher risk since single events such as the death of a key executive, the loss of a key customer, the change of a single technology, or the introduction of a new competitor has a much larger impact on a small company than on a large company. Smaller companies often are in weaker competitive positions, have fewer facilities and product diversification, and can be more vulnerable to regulatory risk or labor disruptions.

Others believe that the premium can be largely a liquidity premium. It is much harder and more expensive to sell a position in a small company than in a large company. The bid-ask spreads of the traded shares for small companies are usually a much larger percentage of price than for larger companies. Fewer analysts issue reports on small companies primarily since there are fewer investors. Small companies can have a handful of investors that control a large percentage of outstanding shares. All these factors can make it more difficult to buy or sell a position in a small company.

Another line of research indicates that because of the difficulty in obtaining information on small companies, it may take longer for the share prices to react to new information. As a result, the usual method for estimating CAPM betas for a company (ordinary least squares regression) leads to a beta estimate that understates the risk. In a Journal of Portfolio Management article, researchers found that using a multiple regression technique on returns, and also the lagged returns, corrects for much of this bias; the correct beta must be estimated by adding the beta on returns and the beta on the lagged returns together (Ibbotson, Kaplan, and Peterson 1997).

The size premium affects not only the cost of equity but also the cost of debt. Small companies with the same credit ratios simply pay higher rates than larger companies with the same ratios. Standard and Poor’s, an agency that evaluates the risk of the debt of issuers, mentions a number of factors that can cause a small company to have a higher cost of credit than a large company:

For example, the fact that a company may only have one major production facility normally is regarded as an area of vulnerability. Similarly, reliance on one product creates risk, even if the product is highly successful. …size turns out to be significantly correlated to ratings. The reason: size often provides a measure of diversification, and/or affects competitive position. (Standard and Poor’s 2006)

A Wall Street Journal article also cited this phenomenon:

…But all of this is a double-whammy for smaller companies, which are now penalized on both the equity and debt sides of their capital. It is well known
that small companies’ stocks don’t get the analyst coverage or the market valuations awarded to bigger concerns. But investors have also become notably more cautious about bonds and loans from smaller or less credit-worthy companies.

“Several years ago we were much more willing to look at deals that were below $100 million,” says Kevin Mathews, senior portfolio manager at Pilgrim America High Yield Fund in Phoenix. “To look at anything below that size now it has to be super-compelling.” While a year ago the minimum was $100 million, now “we tend to look at anything $150 million or bigger.”

The change in demand can be seen clearly in the extra yields investors are demanding on smaller deals. Since 1994, bond issues smaller than $100 million have paid an average 0.48 of a percentage point more yield than issues larger than $300 million, says Steven Ruggiero, head of high-yield securities research at Chase Securities Inc. But the spread has soared since last fall’s credit crunch, rising from a slim 0.07 percentage point during the flush times of May 1998, peaking at 1.87 percentage points this April and currently at 1.44 percentage points, nearly a full percentage point higher than the long-term average …. (Scherer 1999, C1)

Some appraisers have cited an article indicating that the size premium can be an illusion caused by a delisting bias (Shumway 1997, 327–340). When a company gets in trouble, the value of its debt or equity falls, and as a result, it is delisted from the exchange that trades the security. Since the data from these securities are then eliminated from some databases, they are not included in the average returns. If the negative returns from these companies were included, the returns would be lower. However, the data in this article show that the bias is trivial for all but the very smallest companies, and yet the evidence is documented in firms much larger than those for which the bias may exist.

A PricewaterhouseCoopers study showed that when companies were grouped by size into 25 subcategories, even in the 25th group of companies the delisting bias was only 22 basis points (Grabowski and King 1997). This bias was trivial compared to the size premiums demonstrated in the data. The delisting bias certainly does not explain the size premium.

In short, size affects cost of capital and must be adjusted for in a cost of capital determination.

Arithmetic Versus Geometric Means for the Market Risk Premium

Another issue stems from the use of historical arithmetic versus geometric means. The Stocks, Bonds, Bills, and Inflation Classic Edition Yearbook (Ibbotson Associates 2007b) indicated that the arithmetic average return of common stocks from 1926 through 2006 was 12.3 percent per year, whereas the geometric average return was 10.4 percent per year. This represents almost a 2 percent difference in average return.

The key issue for appraisers is which average represents the correct average to use in determining the equity risk premium for the CAPM. The equity risk premium is the portion of the CAPM equation in parentheses:

\[ \text{Required return} = R_f + \beta (R_m - R_f). \]

Ibbotson Associates makes it clear that the arithmetic average is the appropriate average:

The equity risk premium data presented in this book are arithmetic average risk premia as opposed to geometric average risk premia. The arithmetic average equity risk premium can be demonstrated to be most appropriate when discounting future cash flows. For use as the expected equity risk premium in either the CAPM or the building block approach, the arithmetic mean or the simple difference of the arithmetic
means of stock market returns and riskless rates is the relevant number. This is because both the CAPM and the building block approach are additive models, in which the cost of capital is the sum of its parts. The geometric average is more appropriate for reporting past performance …. (Ibbotson Associates 2007b)

Financial textbooks that cite the use of historical data to estimate the equity risk premium for the future indicate that, if historical data are used, the arithmetic average is the appropriate average:

*If the cost of capital is estimated from historical returns or risk premiums, use arithmetic averages, not compound annual rates of return.* (Brealey and Myers 2003)

The issue is muddied by a few authors who choose to use the geometric average:

Conventional wisdom argues for the use of the arithmetic average. In fact, if annual returns are uncorrelated over time and our objective was to estimate the risk premium for the next year, the arithmetic average is the best unbiased estimate of the premium. In reality, however, there are strong arguments that can be made for the use of geometric averages. First, empirical studies seem to indicate that returns on stocks are negatively correlated over time. Consequently, the arithmetic average return is likely to overstate the premium. (Damodaran 2006, 98)

The arithmetic average is the best estimate of future expected returns because all possible paths are given equal weighting. … Empirical research … indicates that a significant long-term negative autocorrelation exists in stock returns. The implication is that the true market risk premium lies between the arithmetic and geometric averages. (Copeland, Koller, and Murrin 2000, 218–221)

The last two citations are correct that, historically, there has been negative autocorrelation in stock returns. The problem is that if this were true going forward, investors could earn excess returns in the market by buying after periods in which the market has fallen and selling after periods in which the market has risen. If it were this easy to beat the market, all investors would do it and prices would move in such a way that the opportunity would disappear. Even if it were true in the past, it cannot be true for future expected returns.

To show that the arithmetic average is the correct average for both short-term and long-term returns, consider the following example.

Toss a coin three times. If it comes up heads, you double your money. If it comes up tails, you lose 0 percent of your money. Start with $1. Table 1 illustrates the possible outcomes.

What is your expected return on the next toss?

With 50 percent probability, you will have $0.50, or a –50 percent return.

**Table 1. Arithmetic mean as outcome predictor—Coin-toss example**

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Returns</th>
<th>Ending Amount</th>
<th>Arithmetic Mean</th>
<th>Geometric Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>T T T</td>
<td>–50%, –50%, –50%</td>
<td>$0.125</td>
<td>–50%</td>
<td>–50%</td>
</tr>
<tr>
<td>T T H</td>
<td>–50%, –50%, 100%</td>
<td>$0.50</td>
<td>0%</td>
<td>–21%</td>
</tr>
<tr>
<td>T H T</td>
<td>–50%, 100%, –50%</td>
<td>$0.50</td>
<td>0%</td>
<td>–21%</td>
</tr>
<tr>
<td>T H H</td>
<td>–50%, 100%, 100%</td>
<td>$2.00</td>
<td>50%</td>
<td>26%</td>
</tr>
<tr>
<td>H H H</td>
<td>100%, 100%, 100%</td>
<td>$8.00</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>H H T</td>
<td>100%, 100%, –50%</td>
<td>$2.00</td>
<td>50%</td>
<td>26%</td>
</tr>
<tr>
<td>H T H</td>
<td>100%, –50%, 100%</td>
<td>$2.00</td>
<td>50%</td>
<td>26%</td>
</tr>
<tr>
<td>H T T</td>
<td>100%, –50%, –50%</td>
<td>$0.50</td>
<td>0%</td>
<td>–21%</td>
</tr>
<tr>
<td>Long-run average</td>
<td>$1.95</td>
<td>25%</td>
<td>8.1%</td>
<td></td>
</tr>
</tbody>
</table>
With 50 percent probability, you will have $2.00, or a 100% return. Therefore the “expected” outcome is ($2.00 + $0.50)/2 = $1.25, which is a 25 percent expected return, the arithmetic mean. Hence the arithmetic—not the geometric mean—best estimates the expected return in the short term.

Note too that the long run expected payoff to the above game is $1 x (1.25^3) = $1.95, not $1 x (1.081^3). Again, the arithmetic mean, not the geometric mean, is the best estimate of the future outcome.

The arithmetic mean is the appropriate average for both long-term and short-term expected returns.

**Liquidity Premium**

Most return data are obtained from markets in which investments trade easily, quickly, and at low cost. This presents a problem when the subject property is physical operating property, real estate, or a privately held company that is not actively traded on an exchange. It also is a problem for a small company that is traded but has larger bid-ask spreads for its shares or so few investors that a large premium or discount to the last trade must be offered to find a new buyer or seller.

The lack of an active market leads to a substantially higher required rate of return by investors, because of the time and cost involved for investors to sell their investment and the difficulty and expense of getting cash, which may be unexpectedly needed.

This is sometimes called a risk premium, but it technically is not a risk premium in the classical sense. Risk refers to the inability to forecast a price. In general, the more difficult it is to estimate a future price, the higher the risk. Risk also depends on the correlation of the error with the overall market or economy. Two investments can be equally difficult to forecast and hence have the same risk. However, if the first investment takes a long time to sell or has high transaction costs and the second can be converted to cash quickly at low cost, investors require a much higher return on the first.

The higher required return for the lack of an active market is referred to as a liquidity premium. This premium must be added to return estimates based on data from actively traded stocks when the subject property is not actively traded. For example, betas and risk-free rates used in the CAPM almost always are based on data from actively traded securities.

The difference in price of investments without active markets when compared to investments with active markets is referred to as a liquidity discount. Numerous studies give overwhelming evidence of discounts of 20–40 percent for stocks that are not actively traded compared to equities that are actively traded (see Pratt [1989] for a more detailed discussion of this point). Such studies have looked at sales of restricted, or letter, stocks. These stocks have all the rights and privileges of common stock but are not traded on an exchange. Owners of this stock can sell this stock only in privately arranged transactions, not on an exchange.

One study (Silber 1991) found discounts averaging 33.75 percent for transactions involving restricted stock when compared to the price at which the common stock was trading at the same time on an exchange. The Internal Revenue Service has allowed a 35 percent discount from the estimated value for equity not traded on an exchange to reflect the lack of liquidity (Pratt 1989).

Financial research has clearly determined the value of liquidity:

*Liquidity (or marketability) is a key attribute of capital assets, and it strongly affects their pricing. ... investors prefer to commit capital to liquid investments, which can be traded quickly and at low cost whenever the need arises. Investments with less liquidity must offer higher expected returns to attract investors.*

(Amihud and Mendelson 1991)
liquidity-increasing financial policies may increase the value of the firm. This was demonstrated for our numerical example. If the spread is reduced to 0.486% (from 3.2%) (as in our low-spread portfolio group), our estimates imply that the value of the asset would increase to $75.8, about a 50% increase. (Amihud and Mendelson 1986)

Our study contributes to the academic literature since we believe we offer the cleanest and most precise measures of the value of liquidity. Due to the unique experimental design inherent in REITs, especially the precision of underlying asset values, we are able to not only verify a link between liquidity and required returns but we are able to accurately quantify these gains. Specifically, we find that exchange trading increases shareholder wealth by around 10–15% at the margin compared to the relatively illiquid real estate market. However, our estimates of wealth creation jump to around 23% when comparing exchange traded claims to nontrading ones. (Benveniste, Cappozza, and Seguin 2001)

Securities that can be converted quickly and cheaply into cash offer relatively low yields. (Brealey and Myers 2003, 891)

The rate of return on investment combines a safe rate with a premium to compensate the investor for risk and the illiquidity of invested capital. (Appraisal Institute 2001, 492)

The research is overwhelming in indicating higher rates of return and corresponding lower prices for less-liquid investments.

Summary
In recent litigation, the issues of size and liquidity premiums have been a central issue. The question of whether to use a risk premium in the CAPM based on the difference between arithmetic or geometric historical average returns also has been intensely debated. The research is clear that a size premium and a liquidity premium are warranted. The logic and evidence also are compelling that if historical returns are used, the arithmetic mean returns should be used to determine the equity risk premium for use in the CAPM.

References
Scherer, P.M. 1999. For small business
credit, the squeeze is on. *The Wall Street Journal*, September 30.


**Suggested Reading**


