Preliminary Design Approach for Prosthetic Ankle Joints Using Compliant Mechanisms

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PRELIMINARY DESIGN APPROACH FOR PROSTHETIC ANKLE JOINTS USING COMPLIANT MECHANISMS

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ABSTRACT

PRELIMINARY DESIGN APPROACH FOR PROSTHETIC ANKLE JOINTS USING COMPLIANT MECHANISMS

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The objective of this thesis is to develop design approaches and models for prosthetic ankle joints using kinematic models of the human ankle and compliant mechanisms technology. Compliant mechanisms offer several potential design advantages over traditional rigid-body designs including high reliability and low cost. These design advantages are ideal for use in prosthetics. Some prosthetic ankle/foot systems currently on the market have multiple degrees of freedom yet are expensive. Additionally, even though these systems have multiple degrees of freedom, none of them are designed after the actual movements of the biological ankle. In this thesis a two, single degree-of-freedom hinge joint model, which is a kinematic model based on the biological ankle during walking, is used to develop compliant prosthetic ankle joints. The use of the model together with compliant mechanisms may provide the ability to develop highly functional prosthetic ankle joints at a lower cost than current high-performance prosthetic systems. Finally, a design approach for ankles may facilitate future development for knees, hips or other biological joints.
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Chapter 1  Overview

1.1 Background

The objective of this thesis is to develop design approaches and models for prosthetic ankle joints using kinematic models of the human ankle and compliant mechanisms technology. With the positive characteristics of compliant mechanisms the approaches and models will lower the cost and improve performance of prosthetic ankles.

Three factors come together to drive the development of the compliant ankle: compliant mechanism research, prosthetic technology, and the needs of individuals in developing countries (Figure 1-1). The following sections highlight each of these factors.

Figure 1-1. Factors driving the development of the compliant ankle
1.1.1 Background – Compliant Mechanism Research

Compliant mechanisms use the compliant (deflecting) properties of a material to transfer motion or energy. Primary advantages of compliant mechanisms are their low cost to manufacture and assemble, and their minimal maintenance requirements. An example of a compliant mechanism is the compliant gripping device shown in Figure 1-2. This device can be machined or injection molded in one piece or several modular pieces as shown in the figure. The joints and springs of a traditional rigid body gripping device are replaced by a few compliant members.

Similar to the motion and energy transformation of the compliant gripping device, we can use a compliant mechanism to mimic the motion and support of an ankle joint. The complex system of bones, muscles, ligaments and tendons in the human ankle allow it to withstand high compression impacts, to move in 3-degrees, and to transfer energy from the leg to the foot while walking or running.

Compliant mechanisms exploit the elastic deformation of materials to create motion. Normally, these motions are accomplished with rigid-body mechanisms, rigid bars connected by pins and springs.

Compliant mechanisms can store and release energy. For instance, energy stored in the flexible member of the compliant crimping device is similar to the energy stored in the spring of its rigid body counterpart.

Figure 1-2. Compliant gripping device developed at Brigham Young University
Compliant mechanisms are often more reliable, lighter, require less maintenance and may cost less to manufacture than their rigid body counterparts.

Some compliant mechanisms that have been developed at BYU include the figures above.

Traditionally, compliant mechanisms have been applied in situations where the flexible members are in tension, rather than compression. Attributes that give compliant mechanisms their favorable characteristics, such as thin walls to promote bending, make it difficult to apply them in high compression. Groundwork for designing high compression compliant mechanism (HCCMs) has been laid through the research of Alexandre Guérinot who studied development of a compliant prosthetic knee [1] (Figure 1-5). In his thesis, Guérinot conducted a detailed classification of compliant joints and
identified buckling as the major obstacle to producing compliant joints capable of bearing high compressive loads. He found that compliant mechanisms can be designed to carry high compression forces using at least two design principles: isolation and inversion. These two principles, and how they can be applied to an ankle joint, will be discussed later on.

1.1.2 Background – Developing Country Needs

In the United States there are estimated to be hundreds of thousands of lower leg amputees. There is a well-developed prosthetic market, offering prosthetic models of varying price and performance, with the highest performing models generally costing the most. Some knee models with complex electronic feedback systems can cost over $25,000.

While the exact need of prosthetics in developing countries is hard to determine, it does far exceed the need in the United States. Common causes of amputations include birth defects, Buerger's disease, diabetes, gangrene, infections, necrotizing fasciitis, and war. Looking just at the effect of war gives us an idea of the need for prosthetics in developing countries:

“Over 110 million land-mines of various types — plus millions more unexploded bombs, shells and grenades — remain hidden around the world, waiting to be triggered by the innocent and unsuspecting. Angola alone has an estimated 10 million land-mines and an amputee population of 70,000, of whom 8,000 are children.”\(^a\)

The boy in Figure 1-7 is a common example of the need for prosthetics in developing countries. “Because a child's bones grow faster than the surrounding tissue, a wound may require repeated amputation and a new artificial limb as often as every six months.”\(^a\) Unfortunately, the high cost of prosthetics makes it difficult for people in developing countries to receive one, let alone one every year for children.

\(^a\) Land-mines: A deadly inheritance, UNICEF
1.1.3 Background – Prosthetic Research

The Prosthetic Outreach Foundation\(^a\) (POF) is one organization that is currently helping people like the boy in Figure 1-7. The mission of the POF is “to restore mobility and independence to disadvantaged amputees worldwide by providing the best available practical solutions for prosthetic care and rehabilitation.” Since 1989, the foundation has restored mobility to more than 12,000 people. The POF has expressed interest in contributing to the success of this research project. They have designed a prosthetic leg with no ankle (see Figure 1-8), and would like to see the development of a reliable low-cost ankle joint.

Prosthetic foot-ankle systems can be placed into two categories: articulated and non-articulated systems. An articulated system prosthetic is composed of rigid or flexible links connected by joints. A non-articulated prosthetic is made of one piece or several pieces making up one segment, but has no articulating joint.

Common non-articulated feet are the Solid Ankle Cushioned Heel (SACH), and the Solid Ankle Flexible Endoskeleton (SAFE). These basic prosthetic feet consist of neoprene or urethane molded over a wood or plastic keel. They have good energy absorption capabilities for impact loading, but have poor energy release capabilities for

\(^a\) http://www.pofsea.org/
propulsion. The largest advantages of the SAFE and the SACH feet are their low manufacturing cost and relatively low maintenance.

Articulated feet may be single or multi-axial. A single-axis foot is generally lighter and easier to maintain than a multi-axis foot, but a multi-axis foot, with added degrees of freedom, is more suited for walking, running, or navigating uneven terrain. Essentially, a multi-axis biological foot can do more of the things a biological foot can do.

Today there are many high performance articulated and non-articulated lower limb prosthetics on the market. Dynamic response prosthetics typically use the compliance in carbon fiber beams to store and release energy to help in the push-off of the toe during walking. Most of these dynamic response products attempt to simulate the ankle function by combining it with the foot piece. One of the most popular models is the Freedom Innovations FS1000 (Figure 1-10).

![Figure 1-9. The non-articulated SACH (Solid Ankle Cushioned Heel) foot](image)

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\(^a\) Scheck & Siress
Figure 1-10. Popular prosthetic foot (Freedom Innovations FS1000) that uses carbon fiber beams to store and release energy while walking

Figure 1-11. Split-foot, multi-axial prosthetic (College Park TruStep)

Other more complex prosthetic feet allow for movement in several planes. The TruStep by College Park (Figure 1-11) attempts to mimic the human foot in structure and function. Multiple axes allow for mult-planar movement and bumpers add shock absorption.

One of the few lower leg prosthetics on the market that is specifically designed as an ankle joint is Ohio Willow Wood’s Impulse™ Ankle. The Impulse™ Ankle can move in all three planes of the ankle (see Figure 2-1 for planes of the ankle). However in the sagittal plane, which splits the body from nose to back, this prosthetic ankle inhibits
plantarflexion (the movement of the toe pushing down) in order to allow greater loading of a connected energy-storing foot.

Figure 1-13 below shows a prosthetic ankle concept machined from Nylon 6. The water jet cut shown through the middle of the ankle allows for a small amount of movement in the plane of progression. This design has the potential to be manufactured at a very low price.

In summary, there are many different prosthetic ankle/feet models with varying performance and costs. The SACH foot is a common model that offers good shock absorption, but poor energy release during push-off. The dynamic response feet offer good energy release during walking, however, only the expensive models have multi-planar movement.

Figure 1-12. The Impulse™ Ankle, with 3 degrees of motion, inhibits plantarflexion to increase loading in energy storing feet

Figure 1-13. Nylon 6 prosthetic ankle -- water jet cut allows for movement in plane of progression
1.2 Limitations of Current Prosthetics

Having introduced the driving factors (compliant mechanisms, prosthetic technology & developing country needs) for development of the compliant ankle, it is appropriate to outline particular problems that need to be overcome.

1.2.1 Lack of Affordable Prosthetics

The largest need for affordable prosthetics is in developing countries. Unfortunately, few can afford any kind of prosthetic in these countries. In his paper, *Orthopedic appliances for developing countries*, Huckstep writes:

“In developing countries lower limb amputations are common. In South East Asia and Africa a considerable number are due to landmines. They are also the result of motor vehicle accidents and badly treated compound fractures, as well as gunshot wounds and other causes. In many patients, the lack of an artificial limb may lead to loss of livelihood, and even death due to starvation of the patient and sometimes his family in times of famine and war. It is therefore important that simple artificial limbs be made available” [2].

Of course the need for affordable prosthetics is not limited to developing countries. Countries such as the United Kingdom, Russia, China, and Israel are a great market opportunity for a compliant ankle. These countries’ medical needs are defined by their socialized health care systems. Socialized health care offers coverage to everyone; however, there are often long waiting lists where patients are prioritized based on gender, age, and overall contribution to society. This common problem that stems from socialized health care opens a market opportunity for private companies offering affordable medical equipment and care.

1.2.2 Limited Degrees of Freedom

Previous research done by Guérinot focused on improving compliant mechanisms intended to function in a single plane. This was a logical first step for a prosthetic knee, which although it has a large range of motion, it has only one degree-of-freedom in the sagittal plane (Figure 2-1). The ankle joint, in contrast, is a three degree-of-freedom joint. It has a small range of motion compared to the knee, however it can rotate on the transverse, frontal and sagittal planes (see Figure 2-1). There is a problem with current compliant mechanisms being able to mimic the tri-planar motion of the biological ankle.
1.2.3 Compression with Deflection

The work by Guérinot focused on methods to increase the allowable magnitude of the compressive force for compliant joints in their undeflected position. When talking about a knee joint during the gait\(^a\) cycle this focus is reasonable because we can assume the knee, while under compression, can remain collinear\(^b\) (i.e. the knee is unbent) while the ankle and hip joints are responsible for deflection.

These assumptions cannot be applied to the ankle, since the ankle joint is under compression as it deflects through the heel strike and push-off of the gait cycle (see section 2.1.3 for details on the gait cycle). Therefore, consideration must be given to the design of compliant mechanisms that can accommodate high compressive loads while deflected.

1.3 Thesis Statement

The objective of this thesis is the development of preliminary design approaches and models for prosthetic ankle joints using kinematic models of the human ankle and compliant mechanism technology. This will include design for deflection in multiple planes, modeled after the human ankle, as well as design for high compression. The design approaches and models will take advantage of the positive characteristics of compliant mechanisms while simultaneously considering current problems such as buckling and off-plane deflection.

1.3.1 Delimitations

Related areas of research that will not be included in this thesis are the following:

- Internal ankle replacements – Internal ankle replacements are used to replace damaged ankle joints, usually as a result of arthritis. This thesis will only focus on full prosthetics for those without an ankle. However, learning from design solutions of internal replacements is definitely within the bounds of the thesis.

- Bilateral joints – Bilateral joints are joints that are subject to both compression and tension. The wrist is an example of a bilateral joint. There are circumstances

\(^a\) Heel strike to heel strike of the same foot while walking (see section 2.1.3 for details)

\(^b\) In reality there is loading in the knee joint when the knee is bent. This is how we can climb up or down stairs.
when the ankle is under tension, however since they are rare, this thesis will not explore solutions for an ankle joint in tension.

- Material analysis – Materials are obviously an important aspect of compliant mechanism design since varying the materials varies important compliant mechanism properties such as weight and elasticity. The affect of material characteristics on compliant mechanism performance will be considered, however detailed analysis of material selection is not included in this thesis.

### 1.3.2 Potential Impact of Thesis

Results of this thesis will advance the understanding of compliant mechanisms used for prosthetic joints. As work on the compliant knee has led to work on this thesis, this thesis may lead to continued improvement to the knee as well as to research in other compliant prosthetics. Additionally, the specific design advancements achieved through this research can be used in other non-prosthetic high compression compliant mechanism applications.

More important will be the possible social impact of the compliant prosthetic ankle. A prosthetic with good functionality for a fraction of the cost of a standard prosthetic is what developing countries require. The boy in Figure 1-7 is a common example of the need for these prosthetics. Since the boy has a below-knee amputated leg, he is a perfect candidate for a compliant ankle. He will be growing fast and will require several prosthetics before he is an adult. Unfortunately, due to high costs of current prosthetics he probably can’t afford even one. Development of the compliant ankle may provide mobility for many people in developing countries who currently cannot afford prosthetics.
Chapter 2    Literature Review

This chapter includes summaries of existing literature relevant to this thesis. The purpose of the review is to summarize the existing boundaries of understanding. The chapter is divided into three sections that will focus on current research on biomechanics of the ankle, compliant mechanisms, and lower leg prosthetics.

2.1 Biomechanics of the Ankle

This section introduces biomechanical literature on the human ankle. The purpose of this literature is to provide general background on biomechanical terms and definitions that will be useful in following chapters or future research. Specific subjects include: anatomy of the ankle, its joints and movements and the gait cycle (walking cycle).

2.1.1 Anatomy Defined

For the purposes of this chapter, the foot can be divided into three planes of movement and three bone sections. The anatomical planes allow discussion of the ankle in any one of its three degrees of freedom. The planes are the frontal, transverse and sagittal planes (Figure 2-1). The three bone sections are the hindfoot, midfoot, and forefoot (Figure 2-2). The hindfoot is made up of the calcaneus and the talus bones. The midfoot, which forms the arch, is made up of the cuboid, navicular and three cuneiforms. The forefoot includes the cuneiforms, metatarsals and phalanges.
Robert Donatelli describes the interrelationships of the three bone sections of the foot: “The [hindfoot] converts the torque of the lower limb...The [hindfoot] also influences the function and movement of the midfoot and forefoot. The midfoot transmits movement from the [hindfoot] to the forefoot, and promotes stability. The movements and function of the midfoot are dependent upon the mechanics of the

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a www.footmax.com
[hindfoot]. The forefoot adapts to the ground as the terrain changes, adjusting to the uneven surface.\textsuperscript{a}

\subsection*{2.1.2 Joints and Movement}

The major joints of interest are the talocalcaneal, or subtalar joint and the talocrural, also known as the tibiotalar or the true ankle (Figure 2-3). The talocrural joint is made up of the tibia, or the medial (inside) bone, the fibula, or the lateral (outside) bone, and the talus, which is distal or the bone underneath. The talocrural joint is responsible for the up (dorsiflexion) and down (plantar flexion) motion of the foot in the sagittal plane (Figure 2-4).

The talocalcaneal joint consists of the talus on top and calcaneus on the bottom. The talocalcaneal joint creates the twisting motion (inversion and eversion) of the foot in the frontal plane (Figure 2-5).

The ankle gets its transverse motion (abduction and adduction in Figure 2-6) from the transverse taral joint \textsuperscript{4}. The back side of the joint is composed of the talus and calcaneus bones, and the front side of the joint is composed of the navicular and cuboid bones.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2-3.png}
\caption{Major two ankle joints–talocalcaneal joint and talocrural joint – in the (A) lateral view and the (B) posterior view [3].}
\end{figure}

\textsuperscript{a} Robert Dontelli, \textit{The Biomechanics of the Foot and Ankle}, 1990, pg 4
Figure 2-4. Sagittal plane movement – dorsiflexion and plantar flexion

Figure 2-5. Frontal plane movement – eversion and inversion

Figure 2-6. Transverse plane movement – abduction and adduction

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*a* www.footmax.com
2.1.3 The Gait Cycle

The three foot sections act as a unit during the stance phase of gait (see section 2.1.3 for details on the gait cycle). Interdependency of the hindfoot, midfoot and forefoot are made possible by a complex system of muscle and connective tissues. As a result, the ankle and foot joints move in a way that together creates an oblique plane of movement to the three planes of Figure 2-1. This movement is referred to as tri-planar motion.® The tri-planar movements of the ankle are pronation (dorsiflexion, eversion and abduction) and supination (plantarflexion, inversion and adduction), shown in Figure 2-7.

Five main functions of the gait cycle outlined by David A. Winter in *The Biomechanics and Motor Control of Human Gait* include:
1. Maintenance of support of the upper body during stance
2. Maintenance of upright posture and balance of the total body
3. Control of foot trajectory to achieve safe ground clearance and a gentle heel or toe landing

® Robert Dontelli, *The Biomechanics of the Foot and Ankle*, 1990, pg 9
4. Generation of mechanical energy to maintain the present forward velocity or to increase the forward velocity

5. Absorption of mechanical energy for shock absorption and stability or to decrease the forward velocity of the body

These five functions will be important in forming some of the design metrics of the compliant ankle.

2.1.3.1 Stance Phase

The stance phase is divided into three sub-phases: contact, midstance and propulsion. Footmaxx, a company specializing in gait analysis and foot orthotics describes the three sub-phases of stance:

Contact phase - (heel strike until the first sign of forefoot loading)

The heel hits the ground slightly lateral of center. The calcaneus is inverted about 2 degrees. At this point the foot aids in shock absorption and accepts leg rotation from above. The calcaneus begins to pronate at heel strike and continues until about 22% of the stance phase when a position of almost 4 degrees of pronation (see Figure 2-7) is reached (total pronation =almost 6 degrees). Forefoot loading terminates contact phase.

Midstance phase - (first sign of forefoot loading until heel lift)

Midstance begins with forefoot loading. Motion at the subtalar joint is continuous supination (see Figure 2-7) from 22% to 100% of the stance phase. The end of midstance is heel-lift of the support limb. This occurs at about 50% of the stance. Stability of the limb is required at this point and to achieve this, the foot must be in a position to lock the midtarsal joint. Near the end of mid-stance, at about 55 per cent of stance phase the subtalar joint should be in neutral position (which means the midtarsal joint is locked).

Propulsive phase - (heel lift until toe-off)

This is the final 50% of the stance phase. The foot continues to supinate (for a final total of 6 degrees) and attains about a 2 degree supinated position. The midtarsal joint is locked and maximum forefoot loading takes place at about 75-80% of the stance phase. Toe-off is at a 2 degree supinated position. The leg moves into swing phase.

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a http://www.footmaxx.com/clinicians/principles.html#normbio
2.1.3.2 Swing Phase

Footmax description of the swing phase:
The swing phase consists of 40% of normal gait cycle and occurs from toe-off to heel strike. During this phase the foot pronates first and then supinates. Pronation shortens the foot, which helps it to clear the ground. Pronation also minimizes the energy expenditure necessary for ground clearance as the non-weight bearing limb passes the weight bearing limb. Supination stabilizes the bony architecture of the foot thus preparing it for heel strike, when the foot must absorb the shock of striking the ground.a

2.1.4 Kinematic Models of the Human Ankle

The previous two sections have outlined general ankle anatomy and the gait cycle. With that understanding, attention is now turned to kinematic models of the human ankle. The purpose of reviewing such models is to 1) understand ankle characteristics such as joint rotation angles and moments and 2) aid in the selection of a design model for the compliant prosthetic ankle.

2.1.4.1 Talocrural and Talocalcaneal Joint Kinematics and Kinetics

Kinematic and kinetic profiles provide valuable information that can be used to understand the motion patterns of the ankle as well as moments at specific joints within the ankle. A particular model that has been shown to be satisfactory in it’s approximation of the human ankle is the two monocentric single degree-of-freedom hinge joint model. The talocrural and talocalcaneal joints (see Figure 2-3) are approximated as hinge joints and have axis of location as described in Figure 2-8.

The talocrural joint is principally responsible for movement in the sagittal plane, however, the combined movements of the talocrural and talocalcaneal joints with axes oblique to the anatomical planes provide pronation and supination.
Procter and Paul [5] used the two axis model to create a three-dimensional model for normal gait. The model included the effects of forces from ligaments and muscles acting on two segments, the talus alone and the talus plus the hindfoot (see Figure 2-2). Model parameters were gathered using anatomical dimensions based on cadavers. Forces were divided into medial and lateral components acting through points along the joint axes.

Since considering ligament and muscle forces are beyond the scope of this thesis, experimental details and results are not listed in this thesis, but instead introduced as a reference for future work.

Dul and Johnson [9] modeled the spatial gross motion of the foot with respect to the shank, or the combined tibula and fibula, using the two hinge joint model. Transformation matrices are used to go from the shank to the talocrural axis, to the talus, to the talocalcaneal axis and then to the foot (see Figure 2-9). Model parameters are

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(a) Superior view

(b) Lateral view

Figure 2-8. Relative location of talocrural (Tc) and talocalcaneal (Tcn) axes [5] according to (a) Manter, 1941 [6] and (b) Isman and Inman [7] (c) Inman (1976) [8]

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*medial means toward the sagittal plane; lateral means away from sagittal plane*
gathered using externally visible bony landmarks. The Denavit and Hartenberg method is used to assign coordinate systems and define parameters. When given the rotation angles of the talocrural and talocalcaneal axes, the resulting transformation matrix can be used to relate points in the foot to points in the shank.

Scott and Winter [10] used the two hinge joint model to find the rotation and moments of the talocrural and talocalcaneal joints during the stance phase of normal gait. Ten markers on the lower leg were identified using bony landmarks and manual manipulation of each joint. A video system collected visual movements of the markers and two-dimensional data was converted to three-dimensional coordinates using the DLT technique.

Figure 2-9. Dul and Johnson’s kinematic model using Denavit /Hartenberg method for Transformations. With this model, given talocrural and talocalcaneal rotation angles, any point on the foot can be described with respect to the shank.
Results of Scott and Winters investigation for one subject are found in Figure 2-10 and Figure 2-11. Moments acting about the talocrural and talocalcaneal joints were calculated using standard inverse dynamics. Ground reaction and gravitational forces were transformed to the talocrural and talocalcaneal joints using standard inverse dynamics and moments calculated using the moment arms or mechanical advantage.
2.1.5 Kinetics of the Human Ankle

With kinematics outlined above, it is appropriate to now discuss kinetics, or the forces that cause movement. Internal forces in a biological foot are created from muscles. Prosthetic limbs do not have internal forces that biological limbs do. Instead, prosthetics must rely on available external forces to function.

External forces affecting the prosthetic ankle are ground reaction forces (the ground’s opposite reaction to the force the foot exerts on the ground). Ground reaction forces can be calculated using force plates such as the one found in Figure 2-12. The forces recorded by force transducers in the plate include four vertical forces. From these forces an equivalent force and torque at a center-of-pressure (Y, Z) can be calculated. The center-of-pressure location is the point on the bottom of the foot where the force is applied. This point location changes along the foot during gait, as shown in Figure 2-13. With ground reaction forces and center-of-pressure locations, the kinematic relationships can be used to find moments exerted on the talocrural and talocalcaneal joints mentioned in the previous section.

Figure 2-12. Example of a force plate used to record ground reaction forces, where \( F + M_x \) at \((Y, Z)\) is equivalent to \( F_1 + F_2 + F_3 + F_4 \)


2.2 Compliant Mechanisms

With the biomechanical background of human anatomy, human gait, and existing kinematic and kinetic models, attention is now turned to compliant mechanisms. After introducing current methods and procedures for the design of relevant compliant mechanisms a discussion of lower leg prosthetics will follow.

In his thesis *Compliant Mechanisms Subjected to Compressive Loads*, Alexandre Guérinot [11] analyzed several existing compliant joint types subjected to direct compression. The joints are organized into categories and then evaluated based on four criteria. The purpose of the evaluation is to provide a framework for predicting the adequacy of one joint relative to another.

Guérinot uses four metrics in his evaluation: (1) maximum allowable compressive loads, (2) maximum infinite fatigue life deflection angle (3) required actuation forces at maximum infinite life deflection angle, and (4) directional flexibility.

Generally, Guérinot found that compliant joints are unfit for direct, high compressive loads because the joints, in order to remain flexible, must be thin, which results in smaller cross-sectional areas, which in turn are prone to buckling failure.

The following sections will review Guérinot’s joint evaluation and two design techniques, isolation and inversion, developed for high compression compliant mechanisms.
2.2.1 Joint Evaluation

Guérinot first establishes three assumptions that allow the consistent comparison between the compliant joints in compression. They are:

1. Compliant joints will be subjected to compressive forces only when the compression forces are aligned, as in Figure 2-14 (a) where joints 1, 2, and 3 are collinear. Also, the pin joints in positions 1 and 3 of Figure 2-14 (a) imply that no moment will be applied to the compliant joint.

2. The compressive load is removed when the joint is actuated by an external moment (see Figure 2-14 (b)).

3. For materials with an endurance limit the threshold between infinite and finite life will be used as a reference in analyses and evaluations. For materials without endurance limits, the assumed standard will be $5.0 \times 10^8$ cycles in conformity with generally-accepted design practices proposed by most scholars.

![Figure 2-14. Assumed loading environment. (a) Joint 1, 2 and 3 are collinear when under compression. (b) The joint is not under compression during actuation.](image)
Guérinot separates joint types into two categories, basic joints and complex joints. The categories and the joints belonging to each category are summarized in Figure 2-15. These are a group of joints most commonly utilized in compliant mechanisms.

**Figure 2-15. Compliant joint categorization**
<table>
<thead>
<tr>
<th>Table 2-1. Basic compliant joints</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Living Hinges</strong></td>
</tr>
<tr>
<td><img src="Image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Small-Length Flexural Pivots</strong></td>
</tr>
<tr>
<td><img src="Image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Long Flexible Segments</strong></td>
</tr>
<tr>
<td><img src="Image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Initially Curved Segments</strong></td>
</tr>
<tr>
<td><img src="Image" alt="Diagram" /></td>
</tr>
</tbody>
</table>
### Table 2-2. Complex compliant joints

<table>
<thead>
<tr>
<th></th>
<th>Cross Axis Flexural Pivots</th>
<th>Torsional Hinges</th>
<th>Q-Joints</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Link A</strong></td>
<td>![Diagram of Link A and B]</td>
<td>![Diagram of Link A and B]</td>
<td>![Diagram of Living hinges]</td>
</tr>
<tr>
<td><strong>Link B</strong></td>
<td>![Diagram of Link A and B]</td>
<td>![Diagram of Link A and B]</td>
<td>![Diagram of Living hinges]</td>
</tr>
<tr>
<td><strong>Living hinges</strong></td>
<td>![Diagram of Living hinges]</td>
<td>![Diagram of Living hinges]</td>
<td>![Diagram of Living hinges]</td>
</tr>
</tbody>
</table>
For the purposes of this literature review, the detailed analysis of the joint categories will not be shown. However, as mentioned above, the joints were analyzed against four characteristics: maximum allowable compressive loads ($P_{cr}$), maximum infinite fatigue life deflection angle ($\theta_{\text{max}}$), required actuation forces at maximum infinite life deflection angle ($M_{\text{actuation}}$), and off-plane stiffness, or the joints stiffness in planes that resist deflection. These characteristics are shown in Figure 2-16. The purpose of these criteria is to establish a norm by which the various joint categories can be compared. The results of these comparisons are summarized in Table 2-3 and Table 2-4.

Figure 2-16. Evaluation criteria for joint types under compression: (a) Maximum allowable load $P_{cr}$, (b) maximum infinite fatigue life bending angle $\theta_{\text{max}}$ (infinite life), (c) actuation moment $M_{\text{actuation}}$, and (d) off-plane stiffness
Table 2-3. Analysis summary of basic compliant joints in compression

<table>
<thead>
<tr>
<th>JOINTS</th>
<th>( P_{cr} )</th>
<th>( \theta_{max} ) ( \text{(infinite life)} )</th>
<th>( M_{actuation} )</th>
<th>Mobility Characteristics &amp; Other Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Living Hinges</td>
<td>Very Low</td>
<td>High</td>
<td>Negligible</td>
<td>Not suitable in current form</td>
</tr>
<tr>
<td>Small-Length Flexural Pivots</td>
<td>Medium</td>
<td>Very Low</td>
<td>Low</td>
<td>Medium rigidity in torsion Medium rigidity in lateral bending</td>
</tr>
<tr>
<td><strong>Long Flexible Segments:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed-Free</td>
<td>Low</td>
<td>Medium</td>
<td>Low</td>
<td>Not suitable for application</td>
</tr>
<tr>
<td>Pinned-Pinned</td>
<td>Low</td>
<td>N/A</td>
<td>Low</td>
<td>Not suitable for application</td>
</tr>
<tr>
<td>Fixed-Pinned</td>
<td>Low</td>
<td>Medium to High</td>
<td>Low</td>
<td>Must be used as part of a system</td>
</tr>
<tr>
<td>Fixed-Fixed</td>
<td>Medium</td>
<td>Medium to High</td>
<td>Low</td>
<td>Low rigidity in torsion Low rigidity in lateral bending</td>
</tr>
<tr>
<td><strong>Initially Curved Segments:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pinned-Pinned</td>
<td>Negligible</td>
<td>N/A</td>
<td>Low</td>
<td>Not suitable for application</td>
</tr>
<tr>
<td>Fixed-Pinned</td>
<td>Negligible</td>
<td>N/A</td>
<td>Low</td>
<td>Not suitable for application</td>
</tr>
<tr>
<td>Fixed-Fixed</td>
<td>Negligible</td>
<td>Medium to High</td>
<td>Low</td>
<td>Not suitable for application</td>
</tr>
</tbody>
</table>
Table 2-4. Analysis summary of complex compliant joints in compression

<table>
<thead>
<tr>
<th>JOINTS</th>
<th>P_{cr}</th>
<th>(\theta_{\text{max}}) (infinite life)</th>
<th>M_{actuation}</th>
<th>Mobility Characteristics &amp; Other Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-Axis Flexural Pivots</td>
<td>Medium</td>
<td>Medium</td>
<td>Low</td>
<td>High rigidity in torsion High rigidity in lateral bending</td>
</tr>
<tr>
<td>Torsional Hinges</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
<td>High rigidity in torsion High rigidity in lateral bending</td>
</tr>
<tr>
<td>Q-joints</td>
<td>None</td>
<td>High</td>
<td>Negligible</td>
<td>Not suitable in current form</td>
</tr>
</tbody>
</table>

As the tables show, none of the basic or complex joints analyzed are able to carry high compressive loads, P_{cr}. It is possible to increase P_{cr} by manipulating the geometry of the joint (increasing the width or decreasing the length), however the trade off is decreasing the maximum infinite fatigue life deflection angle, \(\theta_{\text{max}}\).

2.2.2 High Compression Design Solutions

Guérinot offers two design solutions to overcome buckling failure in high compression compliant joints. They are isolation and inversion.

2.2.2.1 Isolation

The isolation principle involves isolating the flexible segment that is susceptible to buckling so that it is removed or isolated from being subject to any compressive loads. Instead the compressive loads are diverted through a passive rest, which is a load bearing contact between two rigid-body links. Figure 2-17 shows how the load runs through the joint at the passive rest of links B and A. This principle is effective when used with assumptions 1 and 2 of section 2.2.1, which says compliant joints are under compression only when the compression forces are aligned and the compressive forces are removed when the joint is actuated by an external moment. Therefore the passive joint is used only when the compressive forces are aligned and is not needed otherwise, because there are no compressive forces when the joint is actuated.

In practice a compliant joint using the isolation principle is only as effective as its passive rest. The compliant joint will not fail under compression, so long as the passive rest does not fail.
Limitations of the passive rest include inherent friction where the rigid links meet, and its inability to be used in tension.

2.2.2.2 Inversion

A compliant segment’s flexibility is largely due to its geometry. Compliant segments become more flexible by increasing the length or by decreasing the width or thickness. However, making the segment more flexible also decreases its tolerance to buckling while under compression. The second principle proposed by Guérinot, inversion, comes from the fact that compliant segments, designed for flexibility, can withstand higher forces in tension than in compression. A compliant joint under compression can be inverted so that its flexible segment is turned into tensural pivots.

Figure 2-17. Isolation principle – compliant joint with passive rest

Generally the kinematic properties of the device are preserved while the flexible segment loading is inverted from compression to tension. This inversion can be achieved by

---

“rigidly coupling the top link to the bottom force and the bottom link to the top force” as shown in Figure 2-18.

Inverted compliant joints are limited by the tensile yield strength of the joint material. Additionally the joints can only be used when the rigid joints (links A & B in Figure 2-18) are under compression. The compliant joint will not function when the links are in tension.

In summary, compliant mechanisms are traditionally designed for tension loads and not compressive loads. The thin flexible members of compliant mechanisms are susceptible to buckling failure when in compression. Two methods that allow for high compression, while retaining flexibility, are isolation and inversion.

The remainder of the chapter will be devoted to reviewing current lower leg prosthetics.

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Figure 2-18. Inversion Design Principle

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2.3 Lower Leg Prosthetics

As mentioned in section 1.1.3, one of the most common lower leg prosthetics is the SACH foot. The SACH foot is a simple, non-articulate prosthetic composed of an inner wooden core and an outer layer of urethane. Due to its simplicity and high effectiveness, the SACH is used as a standard by which other prosthetic models are compared.

The SACH foot is effective at shock absorption, however it does not offer any energy return for the propulsive stage of gait (see section 2.1.3.22.1.3.1). Most of the recent prosthetic technology is based on an energy returning model called dynamic elastic response (DER). DER prosthetic feet are designed to store energy in the stance phase of gait and return a portion of that energy at toe-off to assist in limb propulsion.

DER prosthetics store energy in the form of a deformed leaf spring during the midstance phase and release that energy through plantarflexion during the propulsive phase. Many models have a foam heel to dissipate energy at the contact phase.

Often it is difficult for prosthetists to select the best DER foot because any technical specifications from the manufacturers are not standardized – even fundamental properties such as stiffness. Extensive material testing has been done on DER feet and some reports suggest that the properties of the DER feet naturally fall into categories that would make standardization process simple to accomplish [12].

---

*Scheck & Siress*
In addition to the lack of standardization, there is a lack of designs for features, other than energy return during the late stance phase of gait. Little attention, for example, has been paid to how the increase in energy return effects the transmitting of loads from the prosthesis to residual limb. A current prosthetic ankle concept, called the rolling joint foot/ankle, RJFA, has been designed to address this issue. The RJFA focuses on “middle and late stance phase relationship between sagittal plane ankle joint moment and ankle joint rotation” [13].

DER and SACH feet typically have a moment profile similar to Figure 2-20. This means the prosthetics have a very stiff initial dorsiflexion. A typical profile seen in more complicated feet with rotational axes and shock absorption is found in Figure 2-21. The RJFA attempts to mimic the concave pattern of resistance which is characteristic of a human ankle, where for the first half of the dorsiflexion period there is very little resistance (see Figure 2-22).

![Figure 2-20. Convex pattern of resistance typical of SACH and DER prosthetics [14]](image)

![Figure 2-21. Single axel foot moment pattern: Resistance is initially low before foot encounters convex resistance pattern](image)
Figure 2-22. Concave pattern of resistance in a biological ankle

Figure 2-23 is a schematic diagram of an RJFA concept. The elastic bands shown in the diagram are used to generate moments to oppose relative rotation between foot and shank as well as to keep the proximal and distal portion of the ankle in contact during swing phase when compressive loads are absent. As the ankle undergoes dorsiflexion or plantar flexion, the center of rotation of the proximal portion of the RJFA moves away from the neutral position. As a result, the moment arms associated with the resistance in the elastic bands increase. Therefore, ankle joint stiffness (local slope of joint moment versus joint rotation) is lower at smaller displacement angles.

The ultimate goal of decreasing ankle stiffness during midstance is to increase dorsiflexion during midstance, leading to forward tibial rotation, which leads to reduced pain and discomfort at the stump/socket interface [13].
Chapter 3  Research Methodology

The purpose of this chapter is to outline the research methodology that is used in the development of compliant prosthetic ankle joints. The chapter will discuss two methods (Process and Product) associated with the development of compliant prosthetic ankles. The Process method will be used for the purposes of this research. The outcome of this method is a preliminary set of design approaches and models for compliant prosthetic ankle joints. The design approaches and models can then be used in conjunction with the Product method to create low-cost, high-performance prosthetic ankles.

3.1 Process: creating design approaches and models

The methodology to turn the function of a biological ankle during walking into a compliant mechanism concept is summarized in Figure 3-1. This method may be applied not only to ankles during walking, but also knees during running, etc. Details of the steps will not be explained in this section; rather the actual realization of steps one through three will be documented in Chapters 4, 5 and 6. Step four, behavior check and testing, will be reserved for further research. This step includes testing and comparing experimental results with the kinematic model of step one.
3.2 Product: using design approaches to create a product

This section outlines the methodology to turn design approaches and models from the Process method into working prosthetic ankles. Details of the Product method for the compliant ankle are out of the scope of this thesis; however the steps are explained here for the purpose of future work. The Product methodology is summarized in Figure 3-2.a

---

a This research process is partially based on Ulrich and Eppinger’s design methodology, Product Design and Development [21].
The research is mapped into five phases: Planning, Concept Development, Modeling, Detailed Design, and Testing and Refinement. The combined phases form a research and development process focusing on both internal and external product design factors. Internal factors include research done in both compliant mechanisms and prosthetics (Figure 3-3). Internal factors are important not only in designing a prosthetic ankle, but are also important to establish a technology for the design of HCCMs that can be used in various joint applications. External factors are found in the developing country needs.
3.3 Planning

The foundation of the planning phase is the literature review. This helps determine existing technology that can be applied to HCCM design. The outcome of the planning stage includes core research questions, key assumptions, constraints and research goals.

3.4 Concept Development

In this phase, specific needs of the prosthetic ankle users will be identified and then used in conjunction with output from the planning stage to develop HCCM joint concepts. One or more concepts will be chosen and developed further. Other outputs include cost analysis and development feasibility.

3.5 Modeling

The modeling stage is used to verify that concepts, particularly product architecture, will fulfill the function that it is designed to fulfill. This phase may use
finite element analysis or other analysis techniques to uncover weaknesses in product architecture.

### 3.6 Detailed Design

With modeling complete, design details can then be considered. In this phase, plans are made to drive the concepts to the point where they can be tested and proved. Details include geometry specifications, materials, and prototypes.

This phase, along with testing and refinement, are used to validate the design model and illustrate its use.

### 3.7 Testing and Refinement

This phase includes building and testing the compliant joint concepts using the details from the previous stage. According to the results of preliminary testing, it may be necessary to iterate back to the Detailed Design (denoted by the dotted line of Figure 3-2). Testing of the prototypes should not only answer questions about performance and reliability, but should also answer some of the core research questions.
Chapter 4  Kinematic/Kinetic Model

Several kinematic models have been developed for the human ankle. Section 2.1.4.1 discusses research that has modeled the human ankle as two monocentric single hinge joints. In researching the current prosthetic market (section 1.1.3), no prosthetics were found to follow this model. Instead, the majority of prosthetics allow for a single degree-of-freedom, and if they allow more, the motion does not closely simulate that of a biological ankle. Therefore the two single hinge joint model is a good choice to improve existing prosthetic functionality. The resulting prosthetic will more closely mimic a biological ankle and will do so with relative simplicity. That is, tri-planar motion of supination and pronation (see Section 2.1.2) can be achieved with only two simple hinge joints.

A prototyped ankle can be useful to help visualize the two hinge joint model. The ankle shown in Figure 4-1 was used to keep track of locations and positions of reference frames in the development of the kinematic model.
4.1 Kinematics

The two monocentric, single degree-of-freedom hinge joint model is formulated using two methods. The first method (matrix transformation) is commonly used in robotics. Matrices holding translation and rotation information are used to describe reference frames relative to each other. The second method (Vector Rotation) is developed for two reasons, first to validate the matrix transformation method, and second to simplify the analytical model. To describe a point on the foot relative to a point on the shank, the first method requires multiplication of seven transformation matrices, while the second method after a few dot and cross products only requires the sum of three vectors.

4.1.1 Kinematics Method I (Matrix Transformation)

Matrix transformation uses matrix multiplication to describe reference frames relative to each other. The first step in constructing the model is assigning reference frames to relevant features on the ankle and defining the parameters that describe the
locations of the reference frames. Frames are described in Table 4-1 and parameters are described in Table 4-2.

Table 4-1. Reference frames used to create left foot kinematic model

<table>
<thead>
<tr>
<th>Frame</th>
<th>Description</th>
</tr>
</thead>
</table>
| {0}   | Left leg shin with the following axes:\(^a\)  
        + x pointing distal (away from the knee)  
        + y pointing anterior (toward the front)  
        + z pointing medial (toward inside of leg) |
| {1}   | Oriented as {0} but with origin at talocrural joint (see S of Table 4-2) |
| {2}   | Origin same as {1} but now in proper talocrural orientation (see \(Tb_x\) and \(Tb_y\) of Table 4-2) |
| {3}   | Origin same as {2} but now in rotated position (see \(\theta\) of Table 4-2) |
| {4}   | Origin of talocalcaneal, but in {0} orientation (when \(\theta = 0\)) (see \(L\) of Table 4-2) |
| {5}   | Origin same as {4} but now in proper talocalcaneal orientation (see \(Tc_x\) and \(Tc_y\) of Table 4-2) |
| {6}   | Origin same as {5} but now in rotated position (see \(\phi\) of Table 4-2) |
| {7}   | Oriented as {0} (when \(\theta = \phi = 0\)), with origin on some point on the bottom of the foot (see \(Px\), \(Py\), and \(Pz\) of Table 4-2) |

Table 4-2. Parameters used in left foot kinematic model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S)</td>
<td>Distance of {0} up the shank from the talocrural</td>
</tr>
<tr>
<td>(L)</td>
<td>Distance between talocrural and talocalcaneal joints in {0} when (\theta = 0)</td>
</tr>
<tr>
<td>(Tb_x)</td>
<td>x rotation to orient talocrural joint (see Figure 2-8)</td>
</tr>
<tr>
<td>(Tb_y)</td>
<td>y rotation to orient talocrural joint (see Figure 2-8)</td>
</tr>
<tr>
<td>(Tc_x)</td>
<td>x rotation to orient talocalcaneal joint (see Figure 2-8)</td>
</tr>
<tr>
<td>(Tc_y)</td>
<td>y rotation to orient talocalcaneal joint (see Figure 2-8)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Talocrural rotation</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Talocalcaneal rotation</td>
</tr>
<tr>
<td>(Px)</td>
<td>Some x distance from talocalcaneal to a point on the foot, measured in {0} when (\theta = \phi = 0)</td>
</tr>
<tr>
<td>(Py)</td>
<td>Some y distance from talocalcaneal to a point on the foot, measured in {0} when (\theta = \phi = 0)</td>
</tr>
<tr>
<td>(Pz)</td>
<td>Some z distance from talocalcaneal to a point on the foot, measured in {0} when (\theta = \phi = 0)</td>
</tr>
</tbody>
</table>

\(^a\) Conventional human reference frames have + x pointing anterior and + y pointing proximal.
Describing vectors relative to different reference frames requires transformations or matrix multiplication. Each transformation is a 4x4 matrix (see Equation (4-1)) holding information that can be used to transform the description of vectors between different frames. The top left 3x3 of Equation (4-1), \( R^o_b \), is a stack of unit vectors describing the \( x \), \( y \) and \( z \) axes of frame \( b \) relative to frame \( a \). It is also rotation information that can be used to transform a vector described in frame \( b \) into a vector described in frame \( a \). The top right 3x1 column of (4-1), \( P^o_b \), holds the translation information describing the position of the origin of frame \( b \) relative to frame \( a \).

\[
T^o_b = \begin{bmatrix}
R^o_b & P^o_b \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(4-1)

Since \{0\} is some distance \( S \) up the shank, then the transformation of \{0\} to \{1\} is:

\[
T^0_1 = \begin{bmatrix}
1 & 0 & 0 & S \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(4-2)

Between \{1\} and \{2\} there are two rotations, one about \( x \), and one about \( y \). The order of rotation is of course important to achieve the desired position.

\[
R_{xb} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(Tc_x) & -\sin(Tc_x) & 0 \\
0 & \sin(Tc_x) & \cos(Tc_x) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(4-3)

\[
R_{yb} = \begin{bmatrix}
\cos(Tc_y) & 0 & \sin(Tc_y) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(Tc_y) & 0 & \cos(Tc_y) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(4-4)

Thus,

\[
T^1_2 = R_{xb} R_{yb}
\]  
(4-5)

and
This $T^0_2$ is the matrix that describes the talocrural (see Figure 4-1) joint {2} relative to {0}. It is important at this point to realize the vector from {0} to {2} is the transformation information in $T^0_2$, that is $P^0_2$. If the talocrural is rotated $\theta$ during walking then the transformation is a rotation of $\theta$ about $z$ of {2}:

$$
T^2_3 = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & 0 & 0 \\
\sin(\theta) & \cos(\theta) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(4-7)

If {3} needs to be expressed relative to {0} then:

$$
T^0_3 = T^0_2 T^2_3
$$

(4-8)

Next is a transformation from the talocural, {3}, to the origin of the talocalcaneal joint, {4}. The distance of the talocalcaneal distal to the talocrural is $L$, measured in {0} when $\theta = 0$. The transformation is:

$$
T^3_4 = (T^0_3)^{-1} T^0_4
$$

(4-9)

where

$$
(T^0_3)^{-1} = (T^4_2)^{-1} (T^0_1)^{-1}
$$

(4-10)

And when $\theta = 0$, $T^0_4$ is simply the $T^i_2$ rotations plus a translation:

$$
T^0_4 = T^i_2 + \begin{bmatrix}
0 & 0 & 0 & S + L \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

(4-11)

Now expressing {4} relative to {0} yields:

$$
T^0_4 = T^0_3 T^3_4
$$

(4-12)

Continuing, {4} has the proper origin, but is not yet in the proper orientation. To transform into the proper talocalcaneal position a simple rotation about $x$ and $y$ will not suffice, because the talocalcaneal orientation is dependent upon the talocrural rotation, or $\theta$. For this transformation:
\[ T_5^4 = \left( T_4^0 \right)^{-1} T_5^0 \]  

(4-13)

where \( T_4^0 \) is defined in Equation (4-11) and

\[ T_5^0 = R_{x2} R_{y2} + \begin{bmatrix} 0 & 0 & 0 & S + L \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  

(4-14)

and

\[ R_{xc} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(Tc_x) & \sin(Tc_x) & 0 \\ 0 & \sin(Tc_x) & \cos(Tc_x) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(4-15)

\[ R_{yc} = \begin{bmatrix} \cos(Tc_y) & 0 & \sin(Tc_y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(Tc_y) & 0 & \cos(Tc_y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(4-16)

Expressing \{5\} relative to \{0\} is:

\[ T_5^0 = T_4^0 T_5^4 \]  

(4-17)

The transformation for the rotated talocalcaneal is:

\[ T_6^5 = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(4-18)

If \{6\} needs to be expressed relative to \{0\} then:

\[ T_6^0 = T_5^0 T_6^5 \]  

(4-12)

Next is a transformation from the rotated talocalcaneal, \{6\}, to some point on the bottom of the foot, \{7\}. Ground reaction data (see Table A-2 of the appendix) are given with respect to \{0\}, when \( \theta = \phi = 0 \), therefore it is convenient to express \{7\} in the same orientation as \{0\}, when \( \theta = \phi = 0 \). The transformation between the talocalcaneal and a point on the bottom of the foot (\( P_x, P_y \) and \( P_z \) of \{0\}) is:
where $T_{\theta=0}^{0}$ is used instead of $T_{\phi=0}^{0}$ because it excludes the $\phi$ and $\theta$ rotation, and

$$
T_{\theta=0}^{0} = \begin{bmatrix}
1 & 0 & 0 & S + L + Px \\
0 & 1 & 0 & Py \\
0 & 0 & 1 & Pz \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Finally the transform needed to express a point on the foot relative to the universal frame is:

$$
T_{7}^{0} = T_{\phi=0}^{0} T_{7}^{6} 
$$

The vector describing a point on the foot relative to the shin (universal frame) is then the translation information in $T_{\phi=0}^{6}$, or $P_{\phi}^{0}$. This vector, or other vectors such as $P_{z}^{0}$ and $P_{x}^{0}$, can be used to translate ground reaction forces and moments on the bottom of the foot to the talocrural or talocalcaneal joints.

4.1.2 Kinematics Method II (Vector Rotation)

In vector rotation finding the relationship of a point on the foot relative to the shank is the sum of three vectors $\vec{r}_{1}$ – the vector from the shank to the talocrural, $\vec{r}_{2}$ – the vector from the talocrural to the talocalcaneal, and $\vec{r}_{3}$ – the vector from the talocrural to a point on the bottom of the foot. This method makes use of the Equation (4-21) describing the rotation of a vector $\vec{v}$ about a unit vector $\hat{m}$ by an angle $\psi$ [15].

$$
\vec{r} = (1 - \cos(\psi))(\hat{m} \cdot \vec{v})\hat{m} + \cos(\psi)\vec{v} + \sin(\psi)(\hat{m} \times \vec{v})
$$

where $(\hat{m} \cdot \vec{v})$ is a dot product and $(\hat{m} \times \vec{v})$ is a cross product.

The first vector, $\vec{r}_{1}$, is

$$
\vec{r}_{1} = \begin{bmatrix}
S \\
0 \\
0
\end{bmatrix}
$$

It should be noted that the same information can be derived from the first method (matrix transformation), that is $\vec{r}_{1} = P_{z}^{0}$.  

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Finding $\vec{r}_2$ requires Equation (4-21), where $\hat{m}_{r_2}$ is the rotational matrix of $T^o_2$, or $R^o_2$, and

$$\vec{v}_{r_2} = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}$$

and $\psi_{r_2} = \theta$.

$$\vec{r}_2 = (1 - \cos(\theta)) \left( R^o_2 \cdot \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \right) R^o_2 + \cos(\theta) \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} + \sin(\theta) \left( R^o_2 \times \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \right) \right)$$

(4-23)

Again it should be noted that the same information can be derived from the first method (matrix transformation), that is $\vec{r}_2 = P^o_4 - P^o_2$.

Finding $\vec{r}_3$ is similar, however $\hat{m}_{r_3} = R^o_5$, the rotational 3x3 of $T^o_5(\theta = 0)$, and $\vec{v}_{r_3} = \begin{bmatrix} Px \\ Py \\ Pz \end{bmatrix}$ must first be rotated about $\hat{m}_{r_2}$ by $\theta$. The result is:

$$\hat{m'}_{r_3} = (1 - \cos(\theta)) \left( R^o_2 \cdot \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \right) R^o_2 + \cos(\theta) R^o_5 + \sin(\theta) \left( R^o_2 \times R^o_5 \right)$$

(4-24)

$$\vec{v'}_{r_3} = (1 - \cos(\theta)) \left( R^o_2 \cdot \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \right) R^o_2 + \cos(\theta) \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} + \sin(\theta) \left( R^o_2 \times \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \right)$$

(4-25)

Vector $\vec{v'}_{r_3}$ is then rotated about $\hat{m'}_{r_3}$ by $\phi$.

$$\vec{r}_3 = (1 - \cos(\phi))(\hat{m'}_{r_3} \cdot \vec{v'}_{r_3})\hat{m'}_{r_3} + \cos(\phi)\vec{v'}_{r_3} + \sin(\phi)(\hat{m'}_{r_3} \times \vec{v'}_{r_3})$$

(4-26)

Once again the same information can be derived from the first method (matrix transformation), that is $\vec{r}_3 = P^o_7 - P^o_2 - (P^o_4 - P^o_2)$.

The vector describing a point on the foot relative to the shank is then:

$$\vec{r}_{\text{Shank to Foot}} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$$

(4-27)

This vector, $\vec{r}_{\text{Shank to Foot}}$ is equivalent to $P^o_7$. 

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4.2 Kinetics

With the kinematics defined and validated through two different methods (matrix transformation and vector rotation) the kinetics or forces that cause the motion can be discussed. The relevant forces for this model will be ground reaction forces obtained from a force plate like the one in Figure 2-12. The variables associated with ground reactions include a force, $F_x$, a moment about the x axis, $M_x$, and three center-of-pressure locators (measured in $\{0\}$), $P_x$, $P_y$ and $P_z$ (see Figure 4-2). $P_z$ is a fixed amount to locate the pressure on the bottom of the foot, while $P_x$ and $P_y$ vary during gait (see Figure 2-13). Mass of the foot, which is only about 1.5% of the total body weight [16], and associated dynamic forces during acceleration and deceleration of gait will be neglected.

4.2.1 Joint Moments and Forces

This section demonstrates how to find the moments around x, y and z of the talocrural and talocalcaneal axes. The moments can then be used to design joints that have desirable deflection associated with the z moment ($z$ is the axis of rotation) and withstand moments associated with the x and y axes ($x$ and $y$ are the off-plane axes – off-plane stiffness was introduced in section 2.2.1).

Joint moments are calculated by “locking” the joints so that the ankle becomes a structure. Then forces and moments are balanced for each link in terms of individual link frames. Craig [17] uses $f_i$, the force exerted on link $i$ by link $i-1$, and $n_i$, the torque exerted on link $i$ by link $i-1$, to define static force propagation as:
\[
\mathbf{f}_i = R_{i+1}^i \mathbf{f}_{i+1}
\]  \hspace{1cm} (4-28)

where \(R_{i+1}^i\) is the transform of \(i+1\) relative to \(i\); and

\[
n_i = R_{i+1}^i n_{i+1} + P_{i+1}^i \times \mathbf{f}_i
\]  \hspace{1cm} (4-29)

where \(P_{i+1}^i\) is the translation array described in 4.1.1.

Applying Equations (4-28) and (4-29) and starting with the foot frame (the three frames are the foot, the talocalcaneal, Tc, and the talocrural, Tb) yields:

\[
\mathbf{f}_{\text{foot}} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}
\]  \hspace{1cm} (4-30)

\[
n_{\text{foot}} = \begin{bmatrix} 0 \\ 0 \\ M_z \end{bmatrix}
\]  \hspace{1cm} (4-31)

Moving to the forces and moments on the talocalcaneal:

\[
f_{\text{Tc}} = \left(R_6^6 R_5^c\right) \mathbf{f}_{\text{foot}}
\]  \hspace{1cm} (4-32)

\[
n_{\text{Tc}} = \left(R_6^6 R_5^c\right) n_{\text{foot}} + \left(\mathbf{f}_T \times f_{\text{Tc}}\right)
\]  \hspace{1cm} (4-33)

Finally the forces and moments of the talocrural:

\[
f_{\text{Tb}} = \left(R_3^2 R_4^3 R_5^4\right) f_{\text{Tc}}
\]  \hspace{1cm} (4-34)

\[
n_{\text{Tb}} = \left(R_3^2 R_4^3 R_5^4\right) n_{\text{Tc}} + \left(\mathbf{f}_T \times f_{\text{Tb}}\right)
\]  \hspace{1cm} (4-35)

\(n_{\text{Tc}}\) and \(n_{\text{Tb}}\) are 3x1 arrays describing the moment about the x, y and z axes of their respective frames. For instance the moment about the z axis or the axis of rotation of the talocrural joint is

\[
n_{\text{Tb}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

while the moments in the off-plane x and y axes are \(n_{\text{Tb}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\) and \(n_{\text{Tb}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\) respectively.
4.2.2 Static Axis

Although not important to this thesis but mentioned for use in future research, is the existence of a “static axis”, STA, which is the axis where any forces acting along this line do not, or at least should not, contribute to motion. The axis, a unit vector, is easily found from the cross product of the talocrural and talocalcaneal axes.

\[
STA = \left( R_3^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)
\]

(4-36)

where \( R_3^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \) (from Equation (4-8)) is the rotational transform of talocrural axis, the z axis of \{3\}, relative to \{0\} and \( R_6^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \) (from Equation (4-12)) is the rotational transform of the talocalcaneal axis, the z axis of \{6\}, relative to \{0\}.

4.3 Model Validation

To validate the model developed in the preceding sections (Wiersdorf Model) it is compared to results found by Scott [10]. Scott showed that the biological ankle during walking can be modeled as two monocentric single degree-of-freedom hinge joints and published rotation and moment profiles for both the talocrural and the talocalcaneal. If the Wiersdorf model produces moment profiles that are generally similar to Scott’s results then we are confident that the Wiersdorf model is valid, or more specifically the Wiersdorf model is valid for design of a general ankle prosthesis. Anthropometrics of the Scott study are not available, nor are the ground reaction forces that were used. To compare moment results the Wiersdorf model will use rotation data from Scott (see Figure 2-10), anthropometrics estimated from the author’s foot, and ground reaction data from the International Society of Biomechanics. This combination of different data sources for validation will suffice since the compliant prosthetic ankle concept is not
being designed for a specific person. Ideally, each prosthetic would be designed after the user’s specific limb measurements and body weight (i.e., use the left foot if the person is a right foot amputee).

Additionally, the Wiersdorf model neglects the weight of the foot. These model differences will cause some discrepancies in comparison; however, the important element is to see a similar trend. A list of the data used for both models is found in Table A-2 and Table A-3 of the appendix.

![Talocrural joint moment (z axis) for Wiersdorf and Scott kinematic/kinetic models](image)

**Figure 4-3.** Talocrural joint moment (z axis) for Wiersdorf and Scott kinematic/kinetic models
Figure 4-4. Talocalcaneal moment (z axis) for Wiersdorf and Scott kinematic/kinetic models

Figure 4-3 is a graph of the moment about the talocrural joint for Wiersdorf and Scott models. The shapes of both moment profiles are similar which gives us confidence that the Wiersdorf model mimics the biological ankle with sufficient accuracy for prosthetic design. The talocalcaneal moment profile of Wiersdorf and Scott is very similar as shown in Figure 4-4. One difference that is apparent between the two models is the maximum moment. The Wiersdorf model has a maximum moment that extends about 5% of stance longer than the Scott model. Studies such as Scott’s [10] clearly show the unique moment profile of individuals. Body weight, ankle anatomy and gait characteristics all have an effect on the moment and forces that are produced at the talocrural and talocalcaneal joints. The results in Figure 4-3 and Figure 4-4, as well as the two methods of kinematic calculations (matrix transformation and vector rotation) in sections 4.1.1 and 4.1.2 give sufficient reason to move forward confidently with the Wiersdorf model.

The advantages of the Wiersdorf model over other available kinematic and kinetic models (see Section 2.1.4.1) are:

1. The Wiersdorf model calculates the moments and forces about all three axes of the talocrural and talocalcaneal joints, not just the z axis or axis of rotation. This is important for minimizing off-plane deflection.
The Wiersdorf model can be used with any ground reaction force data, such as data for running or stair climbing, to get the associated forces and moments about the talocrural and talocalcaneal joints\(^a\). This is valuable for future research.

\(^a\) Granted \(\theta\) and \(\phi\) must also be known for a particular activity.
Chapter 5  Compliant Joint Design Principles

With the kinematic and kinetic models established and validated, moments and forces about the rotational plane (desirable deflection about the $z$ axis) and off-planes (undesirable deflection) of the talocrural and talocalcaneal joints are known. This information is the key element for design of compliant joint concepts for the two joints.

The purpose of this chapter is to show how the force-deflection information from Chapter 4 can be used to determine the desired joint characteristics. Choosing appropriate joint concepts and parameter values, such as length, width, and material will result in the desired deflection or resistance to deflection about each axis.

This chapter will model the talocrural and talocalcaneal as a cantilever beam (Figure 5-1). The free end of the beam is subject to the forces and moments calculated in the previous chapter. This is a generalized loading representation that works well for many joint types discussed in section 2.2.1. More complex joints, such as hybrid systems, require specialized methods for calculating deflection. The pseudo-rigid-body model [18] was developed for this purpose. Numerical methods such as finite element analysis will work well for refining initial designs or analyzing mechanisms with varying cross sections along their lengths.

5.1 Design Principle Definitions

As demonstrated in section 4.2.1 each joint is subject to reaction forces and moments transformed from ground reactions on the bottom of the foot. Figure 5-1 is a free body diagram of the reaction forces and moments found on the talocrural and talocalcaneal joints, the joint represented as a cantilever beam. The grounded portion of Figure 5-1 is considered proximal (grounded side of the joint is toward the knee).
Table 5-1. Summary of significant design issues on talocrural and talocalcaneal joints (* indicates desirable deflection)

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Factors Involved</th>
<th>Deflection Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending</td>
<td>$F_y, M_z$</td>
<td>Angular deflection about $Z^*$</td>
</tr>
<tr>
<td></td>
<td>$F_z, M_y$</td>
<td>Angular deflection about $Y$</td>
</tr>
<tr>
<td>Transverse Shear</td>
<td>$F_y$</td>
<td>Linear deflection along $Y$</td>
</tr>
<tr>
<td></td>
<td>$F_z$</td>
<td>Linear deflection along $Z$</td>
</tr>
<tr>
<td>Torsion</td>
<td>$M_x$</td>
<td>Angular deflection about $X$</td>
</tr>
<tr>
<td>Bending and Transverse Shear</td>
<td>$F_y, M_z$</td>
<td>Maximum Linear deflection along $Y$</td>
</tr>
<tr>
<td></td>
<td>$F_z, M_y$</td>
<td>Maximum Linear deflection along $Z$</td>
</tr>
<tr>
<td>Axial</td>
<td>$F_x$</td>
<td>Buckling along $X$</td>
</tr>
</tbody>
</table>

The $z$ axis is the axis of rotation, and the design goal is to achieve a specific angular deflection for a given force and moment. In the off-plane (any deflection other than angular deflection about $z$) the design goal is to minimize deflection. Table 5-1 summarizes the different reaction forces and moments and how they affect a given axis.
The next three sections will outline the basic design principles for dealing with desired deflection about the $z$ axis, undesired off-plane deflections and buckling. After that, several sections will be devoted to applying those basic design principles to the specific load types in Table 5-1 for both the talocrural and talocalcaneal joints.

### 5.1.1 Desirable, Large Deflection

The $z$ axis is the axis of desirable angular deflection. Angular deflection about the $z$ axis is relatively large and therefore introduces geometric nonlinearities. Designing a compliant mechanism with large deflections and nonlinearities can be accomplished through various means, such as elliptical integration or the pseudo-rigid-body method [18]. Current elliptical integral and pseudo-rigid-body solutions involve knowing the final linear displacements in order to solve for the required input forces or moments. However from Chapter 4, angular deflections ($\theta$ and $\phi$) are known, but linear displacements (required for elliptical or pseudo-rigid-body models) are not known. For this reason, this thesis will use an approximation of the Bernoulli-Euler equation. The Bernoulli-Euler equation (for isotropic materials of constant cross sectional area) states that the bending moment of a beam is proportional to its curvature. That is,

$$\frac{M}{EI} = \frac{d\phi}{ds} = K_0$$

where, $\phi$ is the angular deflection of the beam end, $s$ is the arc length of the deflected beam and $K_0$ is the curvature. Since the curvature at each point along the beam is unknown, it can be approximated by an average curvature:

Approximation

$$\frac{\bar{M}}{EI} = \frac{\Delta \phi_0}{L} = \bar{K}_0$$

where $\phi_0$ is the beam’s end angle ($\theta$ or $\phi$), and $\frac{\Delta \phi_0}{L}$ is the change in beam curvature over the entire beam length. Using $\frac{\Delta \phi_0}{L}$ instead of $\frac{d\phi}{ds}$ results in an average curvature corresponding to the average moment along the beam instead of true curvature at the beam’s end (see Figure 5-2).
Another important design issue related to angular deflection about the $z$ axis is stress. Thin flexible beams are required to achieve a specific deflection; however, thicker beams are required to oppose a specific moment and force ($M_z$ and $F_y$). A thicker beam deflected the same amount as a thinner beam will have higher stresses. Solving Equation (5-2) for a given moment and rotation could result in a beam that will fail due to high stresses at the fixed end of the beam.

Determining if a beam will fail due to stress at the fixed-end (highest stress location) can be done by comparing the required linear deflection with the maximum linear deflection of the beam tip. The linear deflection, $\delta$, of the flexural pivot tip and the maximum allowable deflection before failure, $\delta_{\text{max}}$, are found by Howell [18]:

$$\delta = L \frac{1 - \cos(\varphi)}{\varphi}$$  \hspace{1cm} (5-3)

---

a Assumes failure of the beam will occur when the maximum stress equals the yield strength.
Pure Moment Maximum Deflection \[ \delta_{\text{max}} = \frac{S_y}{E} \frac{L^2}{h} \quad (5-4) \]

Force at Beam Tip Maximum Deflection \[ \delta_{\text{max}} = \frac{2}{3} \frac{S_y}{E} \frac{L^2}{h} \quad (5-5) \]

where \( S_y \) is the yield strength.

Solutions to stress failure at the fixed-end of a beam include using appropriate materials, or creating a hybrid compliant joint that uses multiple thinner beams. Both these solutions can lower the stresses at the fixed end of the beam.

### 5.1.2 Off-plane, Small Deflection

Off-plane deflection is undesirable deflection occurring along or about every axis except angular deflection about the \( z \) axis. Unlike deflection about the \( z \) axis, off-plane deflections will be small and therefore geometric linearity is assumed. For joint design, the energy method, together with Castigliano’s theorem can be used. The relevant energy equations include [19]:

- **Bending** \[ U = \int_0^l \frac{M^2}{2EI} \, dx \quad (5-6) \]
- **Transverse Shear** \[ U = \int_0^l \frac{3V^2}{5GA} \, dx \quad (5-7) \]
- **Torsion** \[ U = \int_0^l \frac{T^2}{2GI} \, dx \quad (5-8) \]

where \( I \) is the moment of inertia, \( E \) is the modulus of elasticity, \( G \) is the shear modulus, \( A \) is the beam cross sectional area, and \( K \) is a property of cross sectional area (polar moment of inertia for round cross sections). \( M, V \) and \( T \) are all functions of \( x \). Castigliano’s theorem [19] is then used to calculate the deflection.

- **Angular deflection** \[ \Delta = \frac{\partial U}{\partial M} \quad (5-9) \]
- **Linear deflection** \[ \delta = \frac{\partial U}{\partial F} \quad (5-10) \]
Bending contributes to both angular deflection and linear deflection, while transverse shear only contributes to linear deflection and torsion only contributes to angular deflection\(^a\).

Equations (5-9) and (5-10) can be used to find the off-plane linear or angular deflections of both the talocrural and talocalcaneal joints. By choosing the proper geometry \((I, A\) and \(K)\) and material properties \((E\) and \(G)\) the joint can be designed to minimize off-plane deflection.

### 5.1.3 Buckling

Buckling is joint failure due to compressive loads along the \(x\) axis. The critical load, \(P_{cr}\), is the critical axial load, or the axial force required to cause buckling of a beam. The critical load can be calculated by \(20\).

Fixed-pinned ends

\[
P_{cr} = \frac{2.046 \pi^2 EI}{L^2}
\]  

Equation (5-9) assumes a beam with fixed-pinned end conditions. Fixed-pinned end conditions are used because the compliant prosthetic ankle has neither true fixed-fixed nor true fixed-free end conditions\(^b\). Instead a middle value is achieved with fixed-pinned. The critical load of a beam can be maximized be using the proper geometric \((L\) and \(I)\) or material \((E)\) properties.

It should be noted that Equation (5-11) only considers axial loading. However, the ankle is subject to forces and moments that will cause the beam to deflect at the onset of axial loading. A beam subject to an axial load and a bending moment is equivalent to an eccentrically loaded beam. The maximum deflection produced by an eccentrically loaded beam is found by Gere and Timoshenko \(20\).

Sections 5.2 and 5.3 below will apply the design equations introduced above to specific load types on the talocrural and talocalcaneal joints.

---

\(^a\) Partial differentiation of Equation 6-2 relative to \(M\) is zero and partial differentiation of Equation 6-3 relative to \(F\) is zero.

\(^b\) For fixed-fixed ends, \(P_{cr} = \frac{4 \pi^2 EI}{L^2}\). For fixed-free ends, \(P_{cr} = \frac{\pi^2 EI}{4L^2}\).
5.2 Talocrural Joint

The first step when designing for deflection (or minimizing deflection in off-plans) is to examine the forces and moments the joint will experience. These forces and moments come from Equation (4-34) and Equation (4-35) and represent the reaction forces and moments at the talocrural transformed from ground reaction forces and moment at the bottom of the foot. Using the Wiersdorf model (data from Table A-4 and Table A-5 of the appendix) results in the following force and moment profiles over the stance phase of gait:

![Figure 5-3. Reaction forces at x, y and z axes of the talocrural joint (Wiersdorf model)](image-url)
Figure 5-4. Moments about x, y and z axes of the talocrural joint (Wiersdorf model)

The largest forces on the talocrural joint are related to body weight, which is force in the x direction. Buckling will be the major issue related to this vertical force. The small force in y will give a small contribution to a desired deflection about the z axis. The small force in the z direction contributes to an undesirable deflection about the y axis. The large moment about the z axis is the major contributor to the desired deflection about the z axis, and makes sense since it is in the plane of progression during gait. The two other small moments contribute to undesirable deflections.

5.2.1 Bending – Angular Deflection about Z

Angular deflection is desired about the z axis. The design goal is to produce a specific rotational pattern for the talocrural joint given reaction forces and moments (z axis of Figure 5-3 and Figure 5-4). Plotting the talocrural moment (z axis) with respect to the rotation results in Figure 5-5.
Figure 5-5. Talocrural moment (z axis) with respect to degrees of rotation (Wiersdorf model)

Figure 5-5 shows the relationship between moment and rotation is not a function; that is, there are multiple moments for a single ankle position. The complex system of muscles and ligaments in a biological ankle makes this profile possible. Duplicating the entire stance phase moment profile would require a complex joint that has one force deflection relationship for heel strike, a separate force deflection relationship for the propulsion phase, and still another for the toe-off phase. The best starting point to design a joint that has a similar moment profile, without the multiple force-deflection complexities, is to use a linear approximation of the propulsion phase as shown in Figure 5-5. This phase, which has a single moment value for a single rotation value, is the portion between 10% to 80% of stance (see section 2.1.3.1 for a summary of the stance phase of gait). A linear approximation of the 10% to 80% stance phase is shown in Figure 5-6.
Assuming the linear approximation of Figure 5-6 can be modeled as a cantilever beam then Equation (5-2) can be used for its design. Note that in order for the joint to have the linear profile shown in Figure 5-6 the joint must be oriented so that negative ten degrees ($\theta = -10\text{ }^\circ$) corresponds with $M_z = 0$. Therefore the joint must have an initial deflection of negative 10 degrees at heel strike, after which joint will rotate 20 degrees reaching positive 10 degrees.

The forces influencing the bending of the beam about the z axis are $F_y$ and $M_z$. The average moment along the beam is:

$$\bar{M} = M_z + \frac{F_y L \cos(\theta)}{2} \tag{5-12}$$

Substituting Equation (5-12) into Equation (5-2) yields:

$$\frac{2M_z + F_y L \cos(\theta)}{2EI} = \frac{\theta}{L} \tag{5-13}$$

$F_y$, $M_z$ and $\theta$ of Equation (5-13) are known. Solving for $EI$:

$$EI = \frac{2LM_z + F_y L^2 \cos(\theta)}{2\theta} \tag{5-14}$$
In this case $I = \frac{bh^3}{12}$. Choosing $b$, $h$, $L$ and $E$ to satisfy Equation (5-14) should be done so that buckling is avoided and off-plane deflection is kept to a minimum. Ideally, $L$ would be on the left side of Equation (5-14) so that all geometric and material variables made up a single constant while the right side was made up of the input parameters ($M_z$, $F_y$ and $\theta$). This however cannot be solved analytically and instead requires the use of numerical methods.

Equations (5-3), (5-4) and (5-5) should be used to determine if the beam will fail due to stress at the fixed-end.

5.2.2 Bending – Angular Deflection about Y

The loads related to angular deflection about the $y$ axis are $F_z$ and $M_y$. $M_y$ is a pure moment across the entire beam while $F_z$ is a force causing a moment that changes with $x$ (hence $M = M_y + F_zx$). From Equation (5-3), the strain energy from bending is:

$$U = \int_0^L \frac{(M_y + F_zx)^2}{2EI} \, dx$$

(5-15)

After integration,

$$U = \frac{F_z^2 L^3}{6EI} + \frac{F_z M_y L^2}{2EI} + \frac{M_y^2 L}{2EI}$$

(5-16)

Then by applying Castigliano’s theorem:

$$\Delta = \frac{\partial U}{\partial M} = \frac{F_z L^2}{2EI} + \frac{M_y L}{EI}$$

(5-17)

In this case $I = \frac{hb^3}{12}$. Note the difference between this moment of inertia and the moment of inertia for deflection about the $z$ axis ($h$ and $b$ are switched). The design goal is to minimize the deflection about the $y$ axis. This can be done by increasing $I$ and $E$ and decreasing $L$.

5.2.3 Transverse Shear – Linear Deflection along Y

Normally transverse shear (in longer beams) is minimal and is ignored; however in this case the joint length is not much greater than its width and transverse shear may be
significant. The factor causing transverse shear resulting in a linear deflection along the \( y \) axis is \( F_y \). Energy due to transverse shear caused by \( F_y \) is:

\[
U = \int_0^L \frac{3F_y^2}{5GA} \, dx \tag{5-18}
\]

After integration:

\[
U = \frac{3F_y^2 L}{5GA} \tag{5-19}
\]

Applying Castigliano’s theorem:

\[
\delta = \frac{\partial U}{\partial F} = \frac{6F_y L}{5GA} \tag{5-20}
\]

This linear deflection from transverse shear does not contribute to the angular deflection about the \( z \) axis and should be minimized without compromising the desirable deflection.

### 5.2.4 Transverse Shear – Linear Deflection along \( Z \)

The factor causing transverse shear resulting in a linear deflection along the \( z \) axis is \( F_z \). Energy due to transverse shear caused by \( F_z \) is:

\[
U = \int_0^L \frac{3F_z^2}{5GA} \, dx \tag{5-21}
\]

After integration:

\[
U = \frac{3F_z^2 L}{5GA} \tag{5-22}
\]

Applying Castigliano’s theorem:

\[
\delta = \frac{\partial U}{\partial F} = \frac{6F_z L}{5GA} \tag{5-23}
\]

### 5.2.5 Torsion – Angular deflection about \( X \)

Torsion about the \( x \) axis is undesirable deflection and should be minimized. The moment responsible for the \( x \) axis torsion is \( M_x \).

The strain energy, \( U \), due to \( M_x \) about the \( x \) axis can be calculated from Equation (5-8). After substituting \( M_x \) for \( T \) and integrating:
\[ U = \frac{M_z^2 L}{2GK} \]  \hspace{1cm} (5-24)

Then by applying Castigliano’s theorem the torsion around the \( x \) axis is:

\[ \Delta = \frac{\partial U}{\partial M_x} = \frac{M_x L}{GK} \]  \hspace{1cm} (5-25)

In this case \( K = \frac{bh^3}{16} \left( \frac{16}{3} - 3.36 \frac{h}{b} \left( 1 - \frac{h^4}{12b^4} \right) \right) \) \cite{19}. The goal is to minimize the angular deflection of Equation (5-25), which can be accomplished by decreasing \( L \) and increasing \( G \) and \( K \).

**5.2.6 Bending and Transverse Shear – Maximum Linear Deflection along \( Y \)**

There is a desired amount of angular deflection about the \( z \) axis, and thus necessary linear deflection along the \( y \) axis that can be calculated with Equations (5-6) and (5-7). These equations are derived from a beam with stresses that remain in the elastic range. Past the elastic range yielding (failure) occurs. Therefore it is important to insure that the desired deflection remains within the elastic range. The total linear deflection along \( y \) is a sum of the linear deflection due to bending and the linear deflection due to transverse shear.

From Equation (5-6) the strain energy due to bending is

\[ U = \frac{F_y^2 L^3}{6EI} + \frac{F_y M_z L^2}{2EI} + \frac{M_z^2 L}{2EI} \].

Applying Castigliano’s theorem the linear deflection due to bending is:

\[ \delta = \frac{2F_y L^3}{6EI} + \frac{M_z L^2}{2EI} \]  \hspace{1cm} (5-26)

Adding Equation (5-20) and Equation (5-26) yields:

\[ \delta = \frac{2F_y L^3}{6EI} + \frac{M_z L^2}{2EI} + \frac{6F_y L}{5GA} \]  \hspace{1cm} (5-27)

The calculated linear deflection of Equation (5-27) should be less than the beam’s maximum allowable deflection. Otherwise the beam will fail before the desired angular deflection is reached.
5.2.7 Axial – Buckling along X

The axial load $F_x$ is the force of interest regarding buckling failure. To calculate the critical axial load (maximum axial load before buckling failure), Equation (5-11) is used and the smaller of two moment of inertia options (\( \frac{bh^3}{12} \) or \( \frac{hb^3}{12} \)) is chosen. In this case $I = \frac{bh^3}{12}$ is smaller. This is the same as the moment of inertia used to calculate angular deflection about the $z$ axis. Decreasing the risk of buckling can be achieved by decreasing $L$ and increasing $E$ and $I$. Adjusting these parameters should be done carefully since doing so affects the desired deflection about the $z$ axis.

5.3 Talocalcaneal Compliant Joint

As done for the talocrural joint, the first thing to examine is the forces and moments the talocalcaneal joint will experience. These forces and moments come from Equation (4-32) and Equation (4-33) and represent the reaction forces and moments at the talocalcaneal. Using the Wiersdorf model (data from Table A-4 and Table A-5 of the appendix) results in the following moment profiles over the stance phase:

![Figure 5-7. Reaction forces at x, y and z axes of the talocalcaneal joint (Wiersdorf model)](image)
Figure 5-8. Moments about x, y and z axes of the talocalcaneal joint (Wiersdorf model)

Unlike the talocrural joint with high forces only along the x axis, the talocalcaneal joint has high forces occurring along both the x axis and the z axis (as shown in Figure 5-7). Since the talocalcaneal x axis is oriented 42 degrees relative to horizontal (see Figure 2-8) the x and y axes share the vertical force of body weigh. The large moment about the y axis shown in Figure 5-8 is related to the large moment about the z axis of the talocrural joint. This y axis moment is essentially in the sagittal plane or the plane of progression during gait. Unlike the z axis of the talocrural, the y axis of the talocalcaneal must resist deflection. Resisting this deflection and maximizing the critical load along the x axis are the key design issues for the talocalcaneal joint. Although less critical because the joint along the z axis of the talocalcaneal joint is relatively long, the transverse shear due to high $F_z$ should be examined. The following sections will outline design details of the talocalcaneal joint.

5.3.1 Bending – Angular Deflection about Z

The z axis is the axis where deflection is desired. The design goal is to achieve a specific rotational pattern for the talocalcaneal joint given reaction forces and moments (z axis of Figure 5-7 and Figure 5-8). Plotting the talocrural moment (z axis) with respect to the rotation results in Figure 5-9.
Like the talocrural, the relationship between the rotation and moment is not a function; that is, there are multiple moments for a single rotation value. Again, a linear
approximation of the propulsion phase (approximately 10% to 80% of stance) is calculated (Figure 5-10).

With the approximation of Figure 5-9, the same methods of design can now be used for the talocalcaneal as the talocrural. See section 5.2.1 for details.

5.3.2 All other Talocalcaneal Deflections

The calculations for undesirable deflections and buckling are the same as those done for the talocrural joint. Refer to sections 5.2.2 through 5.2.7 for details.

5.4 Summary

The main issues for designing talocrural and talocalcaneal joints are (1) achieving proper rotation for given forces and moments (2) minimizing off-plane deflections and (3) maximizing the critical axial load. Geometric properties \((L, I, A, \text{ and } K)\) and material properties \((E \text{ and } G)\) are the variables of design. The \(z\) axis is the axis of rotation, and trade-offs between stiffness and rotation exist. Designing a beam to deflect to a specific rotation given a maximum moment may cause inelastic deformation if stress analysis is not included in the design.
Chapter 6 Prototyping and Simulation

The prototyping process is a concurrent process, meaning prototyping began as early as the choice of a kinematic model. During the design development process, a prototype can be used for learning, communication, integration and for milestones [21]. The purpose of this chapter is to summarize the learning that has occurred through use of prototyping. The prototypes include (1) a rapid-prototyped model, (2) a modular prototype with basic compliant joint inserts, (3) a modified talocrural modular insert and (4) a modified talocalcaneal modular insert.

6.1 Rapid Prototype

The first prototype produced was a rapid-prototyped model of two simple hinges representing the talocrural and talocalcaneal joints. This first prototype aided in constructing the kinematic model, making transformation of the reference frames easier to visualize, as well as provided a visual basis on which future prototypes could be made.
Figure 6-1. Rapid-prototyped concept of two single hinge joint model. This prototype aided in developing the kinematic model by making reference frame transformation easier to visualize.

6.2 Modular Prototype

The next prototype constructed was the modular experimental ankle, MERA. The purpose of MERA is to allow for the examination of joint motion and load capacity of various compliant joint types, and ultimately to aid in the simulation of desired rotation and moment profiles shown in Figure 5-6 and Figure 5-10.
Figure 6-2. Modular Experimental Research Ankle (MERA). Shown in dark gray are three rigid aluminum parts wherein various compliant joints (light grey) can be inserted.

Figure 6-3. MERA and the Ohio Willow Wood Earthwalk foot and pylon.
As shown in Figure 6-2 and Figure 6-3 MERA is a three piece aluminum module that is mountable with the Ohio Willow Wood Earthwalk prosthetic foot and pylon. The lower rigid segment of the ankle attaches to the Earthwalk foot using a standard 10mm bolt connection and the upper rigid ankle segment attaches to the pylon using a standard set screw interface. The middle ankle piece is the rigid segment connecting the talocrural and talocalcaneal joints. Set screws in the rigid members are used for securing modular compliant pieces acting as the talocrural and talocalcaneal joints.

The MERA design benefits are that it follows the two-axis kinematic model of the human ankle and simultaneously lends itself to use with multiple types of compliant joints. MERA provides the ability to quickly turn an idea into something tangible that can then be subject to testing and evaluation. Initial compliant joints that fit into MERA were designed during early stages of this thesis before design methods of Chapter 5 were developed. Early compliant joint concepts used in MERA are found in Figure 6-4.

Figure 6-4. Early compliant joint concepts used with MERA.
The next step in prototyping, while concurrently developing the design methods of Chapter 5, was to improve the joints in Figure 6-4, adding higher compressive and stiffness capabilities. The following section outlines the prototyping of the improved compliant joint concepts for both the talocrural and talocalcaneal joints.

6.3 Modified Talocrural

As discussed in section 5.2 the talocrural joint is subject to a large force along the $x$ axis and a large moment about the $z$ axis. The rolling contact joint is an appropriate choice for the talocrural joint because of its ability to handle high axial or compressive loads along the $x$ axis (for rolling contact joint details see [22]). In fact, the significant improvement in compression virtually eliminates buckling as a possible failure. The rolling contact of Figure 6-4 was redesigned to increase the stiffness about the $z$ axis and to decrease its deflection in torsion (angular deflection about $x$ axis). As can be seen in Figure 6-5 the improved rolling joint has two external protrusions on each side. One side is a rigid stop and is oriented so that the $\theta = -15$ degrees, when $M_z = 0$. The rigid stop is simply a safety feature to keep the ankle from twisting farther than is desired for walking. The other protrusion is oriented so that a flexural pivot will deflect for a positive rotation and will reach a rigid stop at $\theta = -15$ degrees, corresponding to the maximum moment. The flexural pivot is responsible for the stiffness while bending about the $z$ axis.
Figure 6-5. Improved rolling joint for talocrural concept
($\zeta$ is the moment arm of $F_y$ on the flexural pivot)

Figure 6-6 is a picture of a polypropylene prototype inserted into MERA. A steel link (master link) connects the foci of the top and bottom portion of the rolling joint. The master link is necessary to keep the joint together since the flexural pivot protruding out the side of the joint acts as a lever, forcing the joint apart. The master link also decreases the $y$ axis linear and angular deflection.

Finally, an important design improvement to mention is the decreased torsional deflection about the $x$ axis due to the protruding flexural pivot and rigid stop. Since the rolling joint of Figure 6-6 is made up of four pieces like the one in Figure 6-5, the protrusions on each piece overlap, restricting angular deflection about the $x$ axis.
The talocrural rolling joint is made up of four compliant pieces like the ones shown in Figure 6-5. The pieces are connected by 4 rigid rods and the master link. Because of the multiple elements that make up this joint, off-plane deflection calculations of Chapter 5 cannot be used by themselves. Additional analysis, for future research, must be performed. However, the desirable deflection about the \( z \) axis can be calculated with the Bernoulli-Euler of Equation (5-1). After separating variables, integrating and solving Equation (5-1) for \( \varphi \) (\( \theta \) for talocrural joint):

\[
\theta = \frac{ML}{EI}
\]  

(6-1)

As shown in sagittal view of Figure 6-5 the flexural pivot of the rolling joint is subject to a sliding contact force from the rigid link of MERA (shown in Figure 6-6). Therefore \( M \) of Equation (6-1) is the sum of \( M_z \) and the pure moment produced by \( F_y \) and the moment arm \( \zeta \) (see the sagittal view of Figure 6-5). Equation (6-1) becomes:

\[
\theta = \frac{(M_z + F_y \zeta)L}{EI}
\]  

(6-2)
Table 6-1 is a summary of design results for various flexible pivots of the rolling
contact joint in Figure 6-5. In order to maximize flexibility and stability, $L$ and $b$ were
given a maximum value based on the size constraints of the ankle (i.e. any larger values
of $L$ and $b$ would create an awkward, bulky prosthetic).

The information in Table 6-1 is by no means optimized. Rather it is a simple
comparison of the relationship between materials, pivot thickness and linear deflection of
the pivot tip. The maximum rotation of the talocrural was kept to 18 degrees and the
resulting beam thickness, $h$, was the only varying geometric parameter.

From Table 6-1, the linear deflection of the beam tip, $\delta$, is higher than the
maximum allowable beam deflection, $\delta_{\text{max}}$, for all three materials. In other words the
flexible pivot will fail due to stress in the fixed end before it reaches its maximum 20
degrees of rotation.

The findings of the table suggest at least three options: (1) a more extensive
optimization needs to be performed to find the appropriate geometric and material
parameters and/or (2) a hybrid solution such as multiple thinner beams may be used so
that deflections remain under the maximum allowable deflection and/or (3) scale down
the design requirements so that the compliant beam is designed for a smaller deflection.

Table 6-1. Basic design results for the flexible pivot concept on the rolling contact joint of Figure 6-5
(load is a pure moment, thus Equation (5-4) for $\delta_{\text{max}}$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Polypropylene</th>
<th>Steel (4140)</th>
<th>Titanium (Ti-13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>mm</td>
<td>1.98 $(\theta = 20^\circ)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_z$</td>
<td>Nm</td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>mm</td>
<td>4.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>mm</td>
<td>12.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>mm</td>
<td>50.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>GPa</td>
<td>1.4</td>
<td>207</td>
<td>114</td>
</tr>
<tr>
<td>$S_y$</td>
<td>MPa</td>
<td>34</td>
<td>1641</td>
<td>1170</td>
</tr>
<tr>
<td>$h$</td>
<td>mm</td>
<td>0.045</td>
<td>0.008</td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta_{\text{max}}$</td>
<td>mm</td>
<td>0.09</td>
<td>0.15</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Table 6-2. A hybrid compliant beam design for the improved talocrural rolling joint. The design uses multiple thinner beams to satisfy stiffness and deflection requirements. The required deflection is decreased from 20 to 10 degrees. The effect of $F_y$ is small and neglected.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Polypropylene</th>
<th>Steel (4140)</th>
<th>Titanium (Ti-13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>mm</td>
<td>1.10 $(\theta = 10^o)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_z$</td>
<td>Nm</td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>mm</td>
<td>12.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>mm</td>
<td>50.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>GPa</td>
<td>1.4</td>
<td>207</td>
<td>114</td>
</tr>
<tr>
<td>$S_y$</td>
<td>MPa</td>
<td>34</td>
<td>1641</td>
<td>1170</td>
</tr>
</tbody>
</table>

| Independent Variables | | | | |
|-----------------------|---|---|---|
| $h$                   | mm | 3.5 | 1.2 | 1.5 |
| # of beams            |    | 27  | 5   | 4   |
| Dependent Variables   | | | | |
| Reaction Moment of single beam (z-axis) | Nm | 3.6 | 18.6 | 22.2 |

An example of combining options 2 and 3 is found in Table 6-2. With the independent variables listed in the table, the resulting beams for the given materials should be used to satisfy both the stiffness and flexibility issues. As an example, if the flexible beam were made out of steel, and had maximum allowable deflection of 10 degrees then four beams must be used together to oppose a moment of 96 Nm. Through limiting the amount of deflection and using multiple beams the stiffness and deflection requirements can be satisfied.

6.4 Modified Talocalcaneal

As discussed in section 5.3 the talocalcaneal joint is subject to high forces along the x and z axes and a high moment about the y axis. The short flexural pivot is an appropriate choice for the talocalcaneal joint because of its off-plane stiffness, resisting forces along the z axis, and its large moment of inertia about the y axis, opposing $M_y$. The short flexural pivot in Figure 6-4 was redesigned to withstand the large $F_x$, improving both the critical axial load. The zero-moment position of the compliant
member of the short flexural pivot was designed to be at positive eight degrees rotation. The resulting concept is found in Figure 6-7.

Figure 6-8 is a picture of a polypropylene concept inserted into MERA. A rigid stop may be connected to the rigid aluminum pieces of MERA as a safety precaution to keep the joint from rotating past negative eight degrees. Pictures of the combined improved talocalcaneal joint prototype and the improved talocrural joint prototype inside of MERA can be found in Figures A-1, A-2 and A-3 of the appendix.

![Figure 6-7. Improved talocalcaneal compliant joint concept](image-url)
Figure 6-8. Improved polypropylene talocalcaneal short flexural pivot concept inserted into MERA.

Table 6-3. Talocalcaneal polypropylene short flexural pivot – off-plane deflections and critical load values. \((E=1.4\times10^9 \text{ Pa}, \sigma_y=34\times10^6 \text{ Pa}, L=6.4 \text{ mm}, b=36.8 \text{ mm}, h=3.2 \text{ mm})\)

<table>
<thead>
<tr>
<th>Deflection Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desirable angular deflection about Z ((\phi))</td>
<td>16.00 degrees</td>
</tr>
<tr>
<td>Off-plane angular deflection about Y</td>
<td>-1.07 degrees</td>
</tr>
<tr>
<td>Off-plane linear deflection along Y</td>
<td>-0.01 mm</td>
</tr>
<tr>
<td>Off-plane linear deflection along Z</td>
<td>0.04 mm</td>
</tr>
<tr>
<td>Off-plane angular deflection about X</td>
<td>-2.73 degrees</td>
</tr>
<tr>
<td>Buckling force along X (Critical Axial Load)</td>
<td>327,000 N</td>
</tr>
</tbody>
</table>

Using Equation (5-12) to design the beam in Figure 6-7 so it can rotate through the positive eight degrees to negative eight degrees, results in the off-plane deflections and critical axial load shown in Table 6-3. The data of Table 6-3 shows small off-plane deflections except for torsional deflection about the \(x\) axis. Running optimizations on the design may improve the torsional deflection.
Table 6-3 is just half of the analysis. The maximum allowable deflection must now be considered. Table 6-4 is a simple numerical experiment comparing the maximum allowable deflection of several materials. The thickness, $h$, was chosen as the variable to keep the beam rotation to 16 degrees.

As for the talocural concept joint, the table above indicates that the talocalcaneal joint will need an optimization process to find the appropriate material and geometric combination, a hybrid solution should be found. Since the difference between the desired deflection and the maximum allowable deflection of Table 6-4 is small ($\delta - \delta_{\text{max}} \approx 4\text{mm}$), adding the necessary stiffness using a hybrid mechanism should be feasible. In order to demonstrate the feasibility, the analysis for a multi-beamed talocalcaneal joint is shown in Table 6-5. Table 6-5 shows that when limiting the talocalcaneal z axis motion to five degrees, then a single titanium beam can be used or 2 steel beams can be used. The off-plane deflection about the x axis, torsion, is fairly high (7 degrees for steel and 3 degrees for titanium), therefore methods to increase the torsional stiffness should be investigate.

**Table 6-4. Summary of maximum allowable deflection by varying materials and thickness, $h$.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Polypropylene</th>
<th>Steel (4140)</th>
<th>Titanium (Ti-13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>mm</td>
<td>0.826 ($\theta = 15^\circ$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_z$</td>
<td>Nm</td>
<td>31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>mm</td>
<td>6.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>mm</td>
<td>36.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>GPa</td>
<td>1.4</td>
<td>207</td>
<td>114</td>
</tr>
<tr>
<td>$S_y$</td>
<td>MPa</td>
<td>34</td>
<td>1641</td>
<td>1170</td>
</tr>
<tr>
<td>$h$</td>
<td>mm</td>
<td>3.77</td>
<td>0.71</td>
<td>0.87</td>
</tr>
<tr>
<td>$\delta_{\text{max}}$</td>
<td>mm</td>
<td>0.26</td>
<td>0.45</td>
<td>0.48</td>
</tr>
</tbody>
</table>
Table 6-5. A hybrid compliant beam design for the improved talocalcaneal joint. The design uses multiple thinner beams to satisfy stiffness and deflection requirements. The required deflection is decreased from 15 to 5 degrees. The effect of $F_y$ is small and neglected to simplify the analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Polypropylene (4140)</th>
<th>Steel (4140)</th>
<th>Titanium (Ti-13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>mm</td>
<td>0.826 ($\theta = 5^\circ$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_z$</td>
<td>Nm</td>
<td>31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_x$</td>
<td>Nm</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>mm</td>
<td>6.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>mm</td>
<td>50.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>GPa</td>
<td>1.4</td>
<td>207</td>
<td>114</td>
</tr>
<tr>
<td>$G$</td>
<td>GPa</td>
<td>-</td>
<td>45</td>
<td>42</td>
</tr>
<tr>
<td>$S_y$</td>
<td>MPa</td>
<td>34</td>
<td>1641</td>
<td>1170</td>
</tr>
</tbody>
</table>

Independent Variables

| $h$                        | mm    | 3.5                   | 1.15         | 1.49             |
| # of beams                 |       | 9                     | 2            | 1                |
| Reaction Moment of single beam (z-axis) | Nm | 3.6 | 18.5 | 22.1 |
| Deflection about x axis (Torsion) | degrees | - | 7 | 3 |

Dependent Variables

6.5 Summary

The prototyping process is a concurrent process, meaning prototyping began as early as the choice of a kinematic model. During the design process, a prototype can be used for learning, communication, integration and for milestones [21].

The first prototype produced was a rapid-prototype model of two simple hinges. This prototype aided in the development of the kinematic model as well as provided a visual basis on which future prototypes could be made.

The next prototype constructed was the modular experimental ankle, MERA. The purpose of MERA is to allow for the examination of joint motion and load capacity of various compliant joint types. Multiple compliant joint concepts were developed to fit
into MERA’s rigid connections. Improved concepts for the talocrural (a rolling contact joint) and for the talocalcaneal (small flexural pivot) were designed based off of the design principles of Chapter 5.

Optimization of Chapter 5 design equations, hybrid solutions or decreasing the required $z$ axis rotation must be used to overcome the trade-off problem between stiffness (opposing reaction moments and forces) and flexibility (rotating to the appropriate $\theta$ and $\phi$). Limiting the amount of rotation to less than that of the biological ankle may limit the functionality but at this point in the development of a design process it is necessary to achieve the required stiffness and deflection. Since the current prosthetics on the market have very limited motion compared to the biological ankle, designs similar to those in Table 6-2 and Table 6-5 are an improvement.
Chapter 7 Conclusion

The purpose of this thesis has been to establish the beginnings of a development process for the design of prosthetic ankles using compliant mechanism technology and kinematic models based on the biological ankle. The driving factors for this development process are:

1. Compliant mechanism research – making mechanisms more reliable and more affordable.
2. Prosthetic technology – making prosthetics with higher functionality and performance.
3. Developing country needs – making prosthetics that are reliable at an affordable price.

This chapter is divided into two sections which will discuss contributions of the thesis to the compliant prosthetic ankle development process, and recommendations for future research.

7.1 Contributions

The objective of this thesis was to lay the groundwork for a comprehensive compliant mechanism prosthetic development process. Chapter 2 is a literature review of the knowledge boundaries of kinematic models of the ankle, related compliant mechanism research, and lower leg prosthetics. It was shown that many high end prosthetic ankle/systems have trip-planar motion; however, none of them simulate the motion of an actual biological ankle during normal gait. To this end, one of the more significant contributions of this chapter is the review of the two single hinge joint model for compliant prosthetic ankles.
Chapter 3 is an outline of the development process and includes a section devoted to product development, which uses the development process summarized in Figure 7-1. This thesis provides significant preliminary work for steps 1, 2 and 3 of the development process. Step 4 is reserved for future research.

Chapter 4 outlines step 1, the development of the kinematic and kinetic models. The complex analysis of human gait can be simplified by modeling the ankle as two single-degree-of-freedom hinge joints. With these kinematic and kinetic models the reaction forces and moments on the talocrural and talocalcaneal joints can be found when ground reaction forces on the bottom of the foot are given as input.

Chapter 5 is a compilation of design principles that can be used in conjunction with the kinematic model to design simple compliant mechanisms that will withstand the loads found on a biological ankle. The chapter includes design issues for bending, transverse shear, torsion, buckling and stress analysis. The Bernoulli-Euler equation can be used about the axis of rotation to calculate large deflections. Small angular and linear deflections about the off-plane axis can be calculated using Castigliano’s theorem.

Chapter 6 contributes preliminary work on prototyping and design simulation. The modular experimental research ankle, MERA, was introduced as a prototyping tool that allows examination of different compliant mechanism joints within a modular set-up. The flexibility of MERA saves valuable design time through modularizing the
prototyping process. Additionally Chapter 6 provided a basic look at numerical results, using Chapter 5 principles, of two compliant joint concepts. A rolling contact/flexural pivot hybrid was used for the talocrural and a short flexural pivot for the talocalcaneal.

Optimization of Chapter 5 design equations, hybrid solutions or decreasing the required z axis rotation must be used to overcome the trade-off problem between stiffness (opposing reaction moments and forces) and flexibility (rotating to the appropriate $\theta$ and $\phi$). Limiting the amount of rotation to less than that of the biological ankle may limit the functionality but for now is necessary to achieve the required stiffness and deflection. Since the current prosthetics on the market have very limited motion compared to the biological ankle, designs similar to those in Table 6-2 and Table 6-5 are an improvement.

To summarize, the major contributions of this thesis include a (1) kinematic/kinetic model that calculates forces and moments about and along the axis of rotation as well as in the off-planes (2) a set of joint design principles whose input are the forces and moments calculated by the kinematic/kinetic model and (3) prototypes and analytical concepts which show promising design solutions for development of an improved commercial prosthetic ankle joint.

7.2 Future Research

The remainder of this chapter will be devoted to possible future research areas. Since this thesis is preliminary work on the development process for prosthetics using compliant mechanisms there are still a few key issues to address.

7.2.1 Numerical Methods and Optimization

Chapter 5 of this thesis established the groundwork for detailed compliant joint design. For large deflections, the Bernoulli-Euler equation was used. Using this method, as well as using elliptical integral analysis or the pseudo-rigid-body method, is more convenient when the input variables are linear deflection requirements rather than angular deflection requirements. From Equation (5-11), geometric and material variables ($E$, $I$, and $L$) cannot be analytically separated (moved to one side of the equation) from the parameters ($F_y$, $M_z$ and $\theta$ or $\phi$) without the use of numerical methods. Establishment of a numerical method would speed up the design process.
Optimization techniques should be used to find best geometric and material variable values to satisfy rotation and stiffness requirements. The simple calculations performed in Chapter 6 only examined the changes in thickness, while holding other geometric variables constant. An optimization method may find a solution that will satisfy both rotation and stiffness requirements.

7.2.2 Hybrid Compliant Mechanisms

Hybrid compliant mechanism research may be the most important area of future research for a few reasons:

(1) A hybrid compliant mechanism may be necessary to satisfy rotation and stiffness requirements. A hybrid may use several thinner beams, which can have the appropriate rotation, whose sum total stiffness is sufficient to oppose the input forces.

(2) A hybrid compliant mechanism may be used to create non-linear moment profiles. In Chapter 5, linear estimates of the talocrural and talocalcaneal moment profiles were formed to simplify design requirements. Future working prototypes may combine multiple compliant beams with a CAM surface to produce a desired, non-linear moment profile. This idea is similar to the RJFA of Section 2.3.

(3) Finally, a hybrid compliant mechanism may be used to create a multi-function moment profile, duplicating the entire stance phase moment profile with one force deflection relationship for heel strike, a separate force deflection relationship for the propulsion phase, and still another for the toe-off phase.

7.2.3 Behavior Check and Testing

Step 4 in the development process of Figure 7-1 involves making sure the calculated behavior of a compliant ankle concept behaves as predicted. This requires both static testing to determine strength and cyclic testing to determine fatigue durability. Both axial and torsional forces should be applied to the prosthetic. Future work may include designing prosthetic testing aparati to perform tests in-house or partnering with prosthetic manufactures to do the testing. Ultimately, clinical testing will determine if the kinematic model and design process produce an ankle which increases performance and/or reduces discomfort.
7.2.4 Failure Modes

The only failures considered in this thesis were off-plane deflections, buckling and failure at the beam’s fixed-end due to stress (the maximum allowable deflection of Equation (5-4) was used for this purpose). Eccentrically loaded beams (equivalent to a beam with an axial force and a bending moment) were mentioned in 5.1.3, however no analytical tests in Chapter 6 considered eccentric loading. Failure due to eccentric loads and many other failure types exist and should be considered.

7.2.5 Market and Commercial Research

Although a few commercial and non-profit prosthetic development groups were consulted in the early stages of this thesis, there is much to be gained by a focused commercial research or commercial partnering. If functionality and comfort can be improved by mimicking the biological ankle, then why are current prosthetics so rigid? Why are prosthetics so expensive? What do prosthetic users say about improving prosthetics? Interviews with commercial prosthetic companies and prosthetic users could provide the answers to these questions.

7.2.6 Other model applications

This thesis has focused on walking for the ankle joint. Other functions such as running or stair climbing could also be used. If it could be shown that the two, single-degree-of-freedom hinge joint model can be used in running, then the kinematic and kinetic models in Chapter 4 can be used. The only difference would be new ground reaction forces and new rotational positions for the model input. If the ankle cannot be modeled as two, single-degree-of-freedom hinge joints then the kinematic model of Chapter 4 must be modified for running. Other functions of interest include standing, stair climbing and hiking.

Once a full process for prosthetic ankle development has been completed, there is promising applications to other biological joints. If simple kinematic models are available for a particular joint, as well as force input data, the same development process can be used for that joint as is used for the ankle. In fact, the knee and hip would simply be an extension of the ankle process. The kinematic transformation used for mapping the
ankle could be extended to map the knee and hip. The ground reaction forces as model input would be the same for the knee and hip as for the ankle.

### 7.2.7 Inflection Points

An inflection point in a beam is caused when a pure moment promotes bending in one direction while a force at the beam’s end promotes bending in the opposite direction. The result is a beam with both positive and negative curvature along its length. This may not be an issue when using relatively short beams, however, over the entire stance phase of gait the ankle is subject to various force moment combinations as can be seen in Figure 5-3 and Figure 5-4 for the talocrural and Figure 5-7 and Figure 5-8 for the talocalcaneal. Investigating the significance of inflection points in the compliant ankle may be of some worth.

### 7.2.8 Mass Customization

Mass customization is the process of creating custom products at high volumes. The process development model in this thesis has the potential to be mass customized. After further work has been done to refine the design tools and procedures, the process may be linked together becoming parametric. Then a few customer specifications such as body weight, height, and other anthropometrics are used as input to automatically create (through the linked process development) a custom compliant prosthetic ankle.
References


Table A-1. Talocrural rotation (theta) and talocalcaneal rotation (phi) during stance (From Scott [11])

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<th>theta (rad.)</th>
<th>phi (rad.)</th>
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Table A-2. Ground reactions used in Wiersdorf model (Fx = force along x axis, Fy = force along y axis, Tx = torque about x axis, Ycp = y axis location of center of pressure, Zcp = z axis location of center of pressure). Data from International Society of Biomechanics Data Resources.

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Table A-3. Moments calculated about the $z$ axis of the talocrural (Tb) and the talocalcaneal (Tc) joints. Scott model used 65.1 kg male (ground reaction data not available), Wiersdorf model used 64.9 kg male with ground reaction data in Table A-2.

| % Stance | Scott Model | | | Wiersdorf Model |
|----------|-------------|-------------------|-------------------|
|          | Tb$_z$ (Nm) | Tc$_z$ (Nm)       | Tb$_z$ (Nm)       | Tc$_z$ (Nm)       |
| 2%       | -2          | -1                | -10               | -4                |
| 5%       | -5          | -2                | -14               | -5                |
| 7%       | -3          | -1                | -11               | -5                |
| 10%      | 1           | 1                 | -6                | -2                |
| 12%      | 5           | 2.5               | 1                 | 3                 |
| 15%      | 10          | 3                 | 5                 | 5                 |
| 17%      | 15          | 6                 | 6                 | 6                 |
| 20%      | 18          | 7.5               | 10                | 7                 |
| 22%      | 25          | 8                 | 14                | 9                 |
| 24%      | 30          | 12                | 17                | 11                |
| 27%      | 33          | 13                | 23                | 14                |
| 29%      | 35          | 14                | 26                | 15                |
| 32%      | 38          | 15                | 30                | 17                |
| 34%      | 40          | 15.5              | 33                | 18                |
| 37%      | 41          | 16                | 35                | 18                |
| 39%      | 42          | 16.5              | 36                | 18                |
| 41%      | 45          | 17.5              | 35                | 15                |
| 44%      | 46          | 18                | 37                | 15                |
| 46%      | 48          | 18.5              | 39                | 15                |
| 49%      | 50          | 19                | 43                | 16                |
| 51%      | 52          | 19.5              | 47                | 19                |
| 54%      | 55          | 20                | 49                | 19                |
| 56%      | 58          | 21                | 52                | 19                |
| 59%      | 60          | 22                | 57                | 20                |
| 61%      | 65          | 23                | 64                | 22                |
| 63%      | 70          | 24                | 72                | 24                |
| 66%      | 75          | 25                | 77                | 26                |
| 68%      | 79          | 26                | 83                | 28                |
| 71%      | 85          | 27.5              | 88                | 30                |
| 73%      | 90          | 28                | 92                | 30                |
| 76%      | 92          | 28                | 96                | 31                |
| 78%      | 93          | 27.5              | 97                | 31                |
| 80%      | 90          | 25                | 100               | 31                |
| 83%      | 83          | 22.5              | 100               | 31                |
| 85%      | 75          | 20                | 100               | 30                |
| 88%      | 65          | 15                | 98                | 29                |
| 90%      | 50          | 10                | 87                | 26                |
| 93%      | 35          | 6                 | 71                | 21                |
| 95%      | 15          | 2.5               | 46                | 13                |
| 98%      | 5           | 1                 | 20                | 5                 |
| 100%     | 0           | 0                 | 9                 | 2                 |
Table A-4. Calculated joint moments for Wiersdorf model (Nm)
(Tb = talocrural, Tc = talocalcaneal)

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Table A-5. Calculated joint forces for Wiersdorf model (N)
(Tb = talocrural, Tc = talocalcaneal)

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</table>
function T = myMoments(theta, phi, Fin, Ycp, Zcp, M)

% myMoments.m computes the kinematic transformation for shin to foot system

% Jason Wiersdorf
% April 5, 2005
% MS Thesis

% Frame 0 is in the left leg shin with the following axes:
% +x pointing distal (away from the knee)
% +y pointing anterior (toward the front)
% +z pointing medial (toward inside of leg)
% ______________________________________________________________________

% KINEMATIC METHOD 1

% Parameters (lengths in meters, angles in radians)

% theta % Angle of talocrural joint; 0 is standing; positive rotation
% corresponds to dorsiflexion (toe up) negative rotation corresponds to
% plantarflexion (toe down) For proper right hand rule use, considered
% the shin fixed and the foot rotating.
% phi % Angle of talocalcaneal joint (0 is standing) positive rotation
% corresponds to eversion (toe moves lateral) negative rotation
% corresponds to inversion (toe moves medial)
% Together positive theta and phi rotation corresponds to pronation and
% negative rotation corresponds to supination

S  = 0; % distance from frame 0 to frame 1 (i.e. length of shin)
L  = .00889; % distance of talocalcaneal below talocrural in the 0 coordinate
            % system (L is not the link length)
Tb_x = -6*(pi/180); % Procter and Paul "Ankle Joint Biomechanics"
Tb_y = -10*(pi/180); % Procter and Paul "Ankle Joint Biomechanics"
Tc_x = (23-90)*(pi/180); % Procter and Paul "Ankle Joint Biomechanics"
Tc_y = -42*(pi/180); % Procter and Paul "Ankle Joint Biomechanics"
Px  = 0.06; % Some distance on x axis (measured on frame 0's x axis when
            % theta=phi=0, 0.06 comes from ISB)
Py  = Ycp; % Some distance on y axis (measured on frame 0's y axis when
            % theta=phi=0)
Pz  = Zcp; % Some distance on z axis (measured on frame 0's z axis when
            % theta=phi=0)

% Translate down shaft
T01 = [1 0 0 S; 0 1 0 0; 0 0 1 0; 0 0 0 1];

% Rotate into Tb position => order Rx*Ry since it is a moving frame rotation

Rx =[1 0 0 0; 0 cos(Tb_x) -sin(Tb_x) 0; 0 sin(Tb_x) cos(Tb_x) 0; 0 0 0 1];
\[
R_y = \begin{bmatrix}
\cos(T_b_y) & 0 & \sin(T_b_y) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(T_b_y) & 0 & \cos(T_b_y) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix};
\]

\[
T_{12} = R_x R_y;
\]

\[
T_{02} = T_{01} T_{12};
\]

% Rotate according to theta

\[
T_{23} = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & 0 & 0 \\
\sin(\theta) & \cos(\theta) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix};
\]

\[
T_{03} = T_{02} T_{23};
\]

% Translate down link

\[
T_{04_0} = T_{12} + \begin{bmatrix}0 & 0 & 0 & (S+L) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}; % For \theta = 0
\]

\[
T_{34} = (T_{12}^{-1}) (T_{01}^{-1}) T_{04_0};
\]

\[
T_{04} = T_{03} T_{34};
\]

% Rotate into Tc position => order Rx*Ry since it is a moving frame rotation

\[
R_{x2} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(T_c_x) & -\sin(T_c_x) & 0 \\
0 & \sin(T_c_x) & \cos(T_c_x) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix};
\]

\[
R_{y2} = \begin{bmatrix}
\cos(T_c_y) & 0 & \sin(T_c_y) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(T_c_y) & 0 & \cos(T_c_y) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix};
\]

\[
T_{05_0} = (R_{x2} R_{y2}) + \begin{bmatrix}0 & 0 & 0 & (S+L) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}; % for \theta = 0
\]

\[
T_{45} = (T_{04_0}^{-1}) T_{05_0};
\]

\[
T_{05} = T_{04} T_{45};
\]

% Rotate according to phi

\[
T_{56} = \begin{bmatrix}
\cos(\phi) & -\sin(\phi) & 0 & 0 \\
\sin(\phi) & \cos(\phi) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix};
\]

\[
T_{06} = T_{05} T_{56};
\]

% Translate down foot

\[
T_{07_0} = \begin{bmatrix}1 & 0 & 0 & (S+L+P_x) \\
0 & 0 & 1 & \text{Py} \\
0 & 0 & 0 & \text{Pz} \\
0 & 0 & 0 & 1
\end{bmatrix}; % For \phi = 0
\]

\[
T_{67} = (T_{05_0}^{-1}) (T_{07_0});
\]

\[
T_{07} = T_{06} T_{67};
\]

% ______________________________________________________________________

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% Static Coordinate system. Any forces along the z axis do not contribute to motion

STF_temp = cross(T03(1:3,1:3),T06(1:3,1:3)); % Z axis is perpendicular to Tc and Tcn axes

% Convert to unit vector (can't use A/norm(A) with 'syms'

STFz = STF_temp(:,3)/sqrt(STF_temp(1,3)^2+STF_temp(2,3)^2+STF_temp(3,3)^2);
STFy = STF_temp(:,2)/sqrt(STF_temp(1,2)^2+STF_temp(2,2)^2+STF_temp(3,2)^2);
STFx = STF_temp(:,1)/sqrt(STF_temp(1,1)^2+STF_temp(2,1)^2+STF_temp(3,1)^2);
STF = [STFx, STFy, STFz]; % Unit vectors of static coordinate system

% KINEMATIC METHOD 2

% r1 = vector down shin to talocrural joint
% r2 = vector from talocrural to talocalcaneal
% r3 = vector from talocalcaneal to point on foot
% a = coordinate system aligned with universe
% b = coordinate system aligned with talocrural
% c = coordinate system aligned with talocalcaneal

% To find vectors (r1, r2, r3) we use the following equation to rotate a
% vector, v, about a unit vector m by an angle t:
% R = (1-cos(t))(m.v)m+cos(t)v+sin(t)(mxv)
% Specialized Cross_this is used instead of default matlab cross product
% function to avoid imaginary numbers

% r1
r1 = [S;0;0];
z1 = T02(1:3,4);

% r2
bz = T02(1:3,3); % axis of rotation (unit vector)
v2 = [L;0;0]; % vector to be rotated
r2 = (1-cos(theta))*DOT_this(bz,v2)*bz+cos(theta)*v2+sin(theta)*CROSS_this(bz,v2);
z2 = T04(1:3,4)-z1;

% r3
cza = T05_0(1:3,3); % axis of rotation (unit vector)
czb = (1-cos(theta))*DOT_this(bz,cza)*bz+cos(theta)*cza+sin(theta)*CROSS_this(bz,cza);
v3a = [Px;Py;Pz]; % vector to be rotated
v3b = (1-cos(theta))*DOT_this(bz,v3a)*bz+cos(theta)*v3a+sin(theta)*CROSS_this(bz,v3a);
r3 = (1-cos(phi))*DOT_this(czb,v3b)*czb+cos(phi)*v3b+sin(phi)*CROSS_this(czb,v3b);
z3 = T07(1:3,4)-z1-z2;

r = r1+r2+r3;

%
% FORCE DEFLECTION RELATIONSHIPS (pg. 175 of Craig)

% a  coordinate system aligned with universe
% b  coordinate system aligned with talocrural
% c  coordinate system aligned with talocalcaneal
% fb reaction forces at talocrural
% fc reaction forces at talocalcaneal
% nb torque at talocrural
% nc torque at talocalcaneal
% Finx Ground reaction force in x direction, frame 0
% Finy Ground reaction force in x direction, frame 0

T57 = T56*T67;
fcc = T57(1:3,1:3)*Fin;
nc = T57(1:3,1:3)*M+cross(T57(1:3,4),fc);
T(:,4:6) = transpose(nc);  % Talocalcaneal torque
T(:,10:12) = transpose(fc);

T25 = T23*T34*T45;
fc = T25(1:3,1:3)*fc;
nc = T25(1:3,1:3)*nc+cross(T25(1:3,4),fb);
T(:,1:3) = transpose(nb);  % Talocrural torque
T(:,7:9) = transpose(fb);
Matlab Code A-2. The code (Ankle_loop.m) uses myMoments.m to create a matrix storing the forces and moments about the x, y and z axes of the talocrural and talocalcaneal joints during stance

% Ankle_loop.m uses myMoments.m to create the matrix T

% Where T is 41x12 matrix with the following columns:
% Column 1: talocrural moment about x
% Column 2: talocrural moment about y
% Column 3: talocrural moment about z
% Column 4: talocalcaneal moment about x
% Column 5: talocalcaneal moment about y
% Column 6: talocalcaneal moment about z
% Column 7: talocrural force along x
% Column 8: talocrural force along y
% Column 9: talocrural force along z
% Column 10: talocalcaneal force along x
% Column 11: talocalcaneal force along y
% Column 12: talocalcaneal force along z

Data = xlsread('ISB_65kg');  % Ground reaction forces from ISB (64.9kg)
% Moment data from Scott (Subject WK06 -- 52.4kg (115 lb))
for i = 1:41  % start loop
    theta = Data(i,2);
    phi = Data(i,3);
    Fin = [Data(i,4);Data(i,5);0];
    Ycp = Data(i,6);
    Zcp = Data(i,7);
    M = [Data(i,8);0;0];
    T(i,:) = myMoments(theta, phi, Fin, Ycp, Zcp, M);
end

% Plots

figure(1)
plot(Data(:,1),T(:,1),Data(:,1),T(:,2),Data(:,1),T(:,3))
grid on;
legend('X','Y','Z',2);
xlabel('% Stance');
ylabel('Moment (Nm)');
title('Talocrural Moments (Wiersdorf)');

figure(2)
plot(Data(:,1),T(:,4),Data(:,1),T(:,5),Data(:,1),T(:,6))
grid on;
legend('X','Y','Z',3);
xlabel('% Stance');
ylabel('Moment (Nm)');
title('Talocalcaneal Moments (Wiersdorf)');
figure(3)
plot(Data(:,1),T(:,3),Data(:,1),T(:,6))
grid on;
legend('Talocrural Z','Talocalcaneal Z',4);
xlabel('% Stance');
ylabel('Moment (Nm)');
title('Ankle Moments (Wiersdorf)');

figure(4)
plot(Data(:,1),T(:,3),Data(:,1),Data(:,9),Data(:,1),T(:,6),Data(:,1),Data(:,10))
grid on;
legend('Tb Wiersdorf','Tb Scott','Tc Wiersdorf', 'Tc Scott',2)
xlabel('% Stance');
ylabel('Moment (Nm)');
title('Scott/Wiersdorf Comparison');

figure(5)
plot(Data(:,1),T(:,7),Data(:,1),T(:,8),Data(:,1),T(:,9))
grid on;
legend('X','Y','Z',2);
xlabel('% Stance');
ylabel('Force (N)');
title('Talocrural Forces (Wiersdorf)');

figure(6)
plot(Data(:,1),T(:,10),Data(:,1),T(:,11),Data(:,1),T(:,12))
grid on;
legend('X','Y','Z',3);
xlabel('% Stance');
ylabel('Force (N)');
title('Talocalcaneal Forces (Wiersdorf)');
Figure A-1. Improved talocrural and talocalcaneal joint concepts connected by MERA's rigid link.

Figure A-2. MERA with improved talocrural and talocalcaneal joint concepts.
Figure A-3. Hybrid talocrural and talocalcaneal joints. Flexible beams connected to the outside of MERA add z axis stiffness to polypropylene concept joints.