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Simulation of Guozheng Lake Temperature by Using a New Water Temperature Model

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Abstract: Water temperature in lakes plays an important role in aquatic ecosystem. Wind sheltering effect has an important influence on water temperature distribution. An improved wind sheltering model, which was developed to quantify wind sheltering effect, was coupled to vertical one dimensional heat conduction model to create a new water temperature model. Based on the relationship between the wind direction and obstacle location, as well as the characteristics of wind variation downwind from the obstacle, the wind sheltering model could calculate a time-dependent wind sheltering coefficient. For numerical solution, a new operator-splitting method was adopted. To compensate for the deviation caused by poor treatment of the source-sink term, the source-sink term was solved by Crank-Nicolson scheme which has second order accuracy. The numerical tests show that the proposed numerical method has higher accuracy than the traditional. It also proved that: Even the diffusion term solving by scheme has high-accuracy and good-stability, but if the source-sink term was not treated appropriately, there might still exist a large deviation. Then, the proposed model and method were applied to simulation for Guozheng Lake. The results show that: the enhanced water temperature model is an effective tool for temperature simulation.

Keywords: Water temperature simulation; wind sheltering model; splitting method; source-sink term

1 INTRODUCTION

Optimum lake temperature is critical to aquatic ecosystem. Wind is one of the most important factors that influences the process of lake temperature change and thermal stratification for its close relationship with heat flux of water-air interface. Due to wind sheltering effect (Hansen 1979) caused by complex surroundings around the lakes, wind speed on the lake surface is often lower than that in flat and open terrain. Literatures prove that the wind sheltering effect is important for estimating the lake-integrated evaporation heat flux (Venäläinen et al. 1998). In order to compensate for the deviation caused by the wind sheltering effect, the input wind speed in the water environmental model is usually multiplied with a wind sheltering coefficient \( W_{sr} \) (Ford and Stefan 1980).

It is difficult to determine \( W_{sr} \) because the physical mechanism of wind sheltering effect is very complex, causing present researches to regard \( W_{sr} \) as a simple calibrated constant (Branco and Torgersen 2009). Though easy and practical, because of a lack of physical foundations, this manner requires great subjective experience. On the other hand, though certain improvements have been made, some methods on aerodynamics (Taylor and Lee 1984) have many limitations in complex calculating procedures and have difficulty in coupling with a water environment model. Therefore, a wind sheltering model that is practical and has sufficient predictive capabilities could effectively solve those problems.

For the solution of water temperature model, the source-sink term is very important because it includes heat budget process like solar radiation, which has a great influence on water temperature distribution. Unfortunately comparing with convection term and diffusion term, little research has been focus on the source-sink term. However, researchers gradually realized that an accurate discretization of source-sink term has the same significance with convection term and diffusion term (Toro and
Garcia 2007). Therefore it is necessary to find an improved numerical solution with the reasonable treatment of the source-sink term.

To summarize, this paper aims to provide a simple but sufficiently accurate method to calculate the value of $W_{str}$ in order to perform a more accurate water temperature simulation. Based on the relationship between the wind direction and the location of the obstacle along with the variation in wind characteristics downwind from the obstacle, an improved wind sheltering model was developed to calculate a time-dependent $W_{str}$ which serves to dynamically adjust the wind data. The wind sheltering model was coupled to vertical one dimensional heat conduction model to create a new water temperature model. Another contribution of this paper is applying a new operator-splitting method to the solution of water temperature model to compensate for the deviation caused by poor treatment of the source-sink term. Lastly, application of the new water temperature model was suggested for the Guozheng Lake, and the validity of proposed methods was discussed.

2 REGION OVERVIEW

Guozheng Lake (114°21’ E, 30°33’ N) was chosen as the case study (Figure 1). It lies in the East Lake basin, Wuhan city, Hubei Province (Central China). It has a gross area of 11.3 km$^2$ with the water level of 20m, with the average depth of 3.81m and the maximum depth of 4.75m. There are considerable amount of trees in some parts of regions along the lake which will generate obvious wind sheltering effect, such as S1 (Segment AB), S2 (Segment CD), S3 (Segment EF), and S4 (Segment GH).

![Figure 1. Overview of Guozheng Lake.](image)

The geometric shape of Guozheng Lake is almost round. Suppose Guozheng Lake to be an equivalent circle with a diameter of D=3.74 km. S1, S2, S3, and S4 are proportional to arcs AB, CD, EF, and GH respectively. For the sake of convenience, the authors also adopted a special manner to describe the location of one point on the circle i.e., the location of one point on the circle is the clockwise angle from due north on the circle to this point. For example, Point A is 20°, Point B is 67°, S1 (Segment AB) ranges from 20°-67°. The statistical information of obstacles can be seen in Table 1.

<table>
<thead>
<tr>
<th>Tree height $h_c$ (m)</th>
<th>Embankment height $h_e$ (m)</th>
<th>Length (km)</th>
<th>the starting and ending</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 9.8±2.25</td>
<td>2.1±0.4</td>
<td>1.56</td>
<td>AB (20°-67°)</td>
</tr>
<tr>
<td>S2 10.9±2.15</td>
<td>3.8±0.5</td>
<td>1.85</td>
<td>CD (102°-158°)</td>
</tr>
<tr>
<td>S3 13.5±1.5</td>
<td>4.0±1.4</td>
<td>1.95</td>
<td>EF (197°-256°)</td>
</tr>
<tr>
<td>S4 14.2±2.5</td>
<td>2.5±0.45</td>
<td>2.42</td>
<td>GH (279°-352°)</td>
</tr>
</tbody>
</table>

3 NEW WATER TEMPERATURE MODEL

An improved wind sheltering model was coupled to vertical one dimensional heat conduction model to create a new water temperature model.
3.1 Improved Wind Sheltering Model

On the whole, the new wind sheltering model developed by this paper is an improvement on the framework of Markfort’s wind sheltering model (Markfort et al. 2010). And some improvements were made.

Researchers have conducted a lot of physical experiments or actual field observations to explore the variation in wind speed behind the obstacle as a function of downwind distance (Counihan 1969; Mons and Sforza 1970; Jaster et al. 2007; Markfort et al. 2010). The characteristics of wind variation within the range of wind deficit length can be described by an exponential function:

\[ \omega(x) = a \exp[b \times (x / h_c)] + c \exp[d \times (x / h_c)] \]  

(1)

\( \omega(x) \) is the wind speed deficit coefficient which is defined as the ratio of wind speed decreased by wind sheltering effect to the original wind speed. Where \( x \) represents the distance; \( h_c \) represents the obstacle. \( a, b, c, \) and \( d \) are constants. After several iterations, it is found that when the values of \( a, b, c, \) and \( d \) are equal to 0.8475, 0.0013, -0.8001, and -0.1063 respectively, the curve of the exponential function is generally identical to the data from research above. So Equation (1) is believed to be effective at generally reflecting the characteristics of wind speed variation from behind the obstacle.

In an actual case study [Figure 2(a)], based on the wind deficit length \( x \) (the range in which the wind sheltering effect is applicable), the lake can be divided into two parts: (1) square area AA’B’B’, where the wind speed decreases as a function of fetch; (2) area of wind access [area under shadow in Figure 2(a)], where the wind speed is not completely influenced by the wind sheltering effect and thus is equal to the background wind speed \( w \). The lake is denoted as Circle \( O \) with diameter \( D \). The coordinate system setting is shown in Figure 2(b), where the positive direction of \( x \)-axis is wind direction. The equation for Circle \( O \) is given by: \((x-D/2)^2+y^2=D^2/4\). The imaginary Circle \( O' \) can be obtained by moving a distance \( x \), towards the negative direction of \( x \)-axis. The equation for Circle \( O' \) is given by: \((x-D/2-x')^2+y'^2=D^2/4\). \( B \) and \( y \) are the angles between \( x \)-axis and the lines \( AB \) and \( CD \) respectively. The extended lines \( A'A \) and extended line \( B'B \) cross the \( y \)-axis at \( y2 (y2=D/2*\sin\beta) \) and \( y1 (y1=-D/2*\sin\gamma) \) respectively.

![Figure 2. Geometric schematic of wind sheltering effect.](image)

By definition, \( W_{se} \) is the ratio of \( w_{total}' \) to \( w_{total} \). Where \( w_{total}' \) is the average wind speed of lake in the presence of wind sheltering effect; \( w_{total} \) is the average wind speed of lake in the absence of wind sheltering effect. \( w_{total} \) is equal to the background wind speed \( w \). Now, the crucial point is the expression of \( w_{total}' \), which can be regarded as the ratio of the whole lake "wind speed summation" \( S_{lake} \) (\( S \) represents the summation of wind speed at each element in one region) to the whole lake area \( A_{lake} \) namely, \( w_{total}'=S_{lake}/A_{lake} \). The wind speed summation of the whole lake is given by: \( S_{lake}=S_{shadow}+S_{AA'B'B} \). Wind speed at each element is uniformly quantified to ensure a more significant comparison. \( \omega \) is introduced in Equation (1) to represent the ratio of the wind speed at one element to the original wind speed. Detailed mathematical expression of \( S_{shadow} \) and \( S_{AA'B'B} \) isn’t presented here. The redefined obstacle height is denoted as \( h_c' \) and \( h_c=h_c+h_o \). The final expression of \( W_{se} \) is written as:
\[ W_{str} = \frac{w_{total}}{w_{total}} \times \frac{\left(ah_b^* \cdot [\exp(b/b^* g) - 1] + \frac{ch_d^*}{d} \cdot [\exp(d/d^* g) - 1] - D(\sin\beta + \sin\gamma) / 2 \right)}{\pi D^2 / 4} + 1 \]  

With \( W_{str} \) calculated by Equation (2), the time series of the observed wind speed can be modified to the maximum possible extent, enabling it to be used as the input for the temperature model based on the actual wind condition.

### 3.2 Heat Conduction Model and Numerical Solution

Usually heat conduction model in lakes and rivers is vertical one-dimensional model:

\[ \frac{\partial T}{\partial t} = \frac{1}{A(z)} \frac{\partial}{\partial z} \left[ A(z) K \frac{\partial T}{\partial z} \right] + \frac{1}{\rho c_p} \frac{\partial [A(z) I(z)]}{\partial z} + H_R \]

\[ I(z) = (1 - \beta)I_0 e^{-k_z z} \]

Where \( z \) denotes vertical coordinate and \( z=0 \) is the surface; \( t \) is time; \( T \) is water temperature \(^\circ\text{C}\); \( A(z) \) is horizontal area (function of depth \( z \)); \( K \) is the vertical diffusion coefficient \((\text{m}^2/\text{s})\); \( I(z) \) is solar radiation at depth \( z \) \((\text{W/m}^2)\); \( \rho \) is water density \((\text{kg/m}^3)\); \( c_p \) is water specific heat capacity \([\text{J/(kg}^\circ\text{C})]\); \( H_R \) is the source-sink term \((\text{W/m}^2)\); \( \beta \) is the proportion of solar radiation absorbed at the water surface and \( k_z \) is the extinction coefficient \((\text{m}^{-1})\).

A splitting method was adopted for solution of Equation (3). Layout of variables should be established in advance. Suppose water body is divided into \( K \) layers in vertical direction. \( k \) denotes the cell centre in the \( z \) directions with space step of \( \Delta z \). \( k=1 \) is the bottom layer, and \( k=K \) is the surface layer, water temperature \( T \) locates in the middle of the layer, index is \( T_k \). Horizontal area \( A_0 \) locates in the upper and lower surface of layer, index is \( A_{k-1/2} \); \( H \) is total depth of water; \( z_i \) is the displacement from water surface to the center of layer \( k \). Superscript \( n+1 \) and \( n \) represent next time step and the current time step respectively. Superscript \( \ast \) represents the average value of variables in upper and lower layer. \( T_1 \) is the intermediate solution between \( T^n \) and \( T^{n+1} \).

1) First step: Crank-Nicolson Method for the source-sink term

Only considering the source-sink term of the heat conduction equation (solar radiation), Crank-Nicolson Method which has second-order accuracy in space and time was used:

\[ \frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{1}{A(z)} \frac{\partial [A(z) I(z)]}{\partial z} \]

\[ T_k^{n+1} = T_k^n + \frac{\Delta t}{4 \rho c_p A_k} \frac{\Delta z}{\Delta z} \left[ (A_{k+1}^{n+1} T_{k+1}^{n+1} - A_{k-1}^{n+1} T_{k-1}^{n+1}) + (A_{k+1}^n T_{k+1}^n - A_{k-1}^n T_{k-1}^n) \right] \]  

2) Second step: implicit discretization for the diffusion term

After calculating the intermediate solution \( T_1 \), only considering the diffusion term of the heat conduction equation, Equation (7) was solved implicitly:

\[ \frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{1}{A(z)} \frac{\partial [A(z) K \frac{\partial T}{\partial z}]}{\partial z} \]

\[ \frac{T_k^{n+1} - T_k^n}{\Delta t} = \left. \frac{K_k}{A_k} \right| \frac{\Delta z}{\Delta z} \left( A_{k+1}^{n+1} T_{k+1}^{n+1} - A_{k-1}^{n+1} T_{k-1}^{n+1} - A_{k+1}^n T_{k+1}^n + A_{k-1}^n T_{k-1}^n \right) \]

The final finite difference result was written as a matrix form:

\[ B_k T_{k+1}^{n+1} + C_k T_k^{n+1} + D_k T_{k-1}^{n+1} = F_k \]

Where \( B_k, C_k, T_k \) and \( F_k \) are parameters involving the known variables, they form a system of linear equations [Equation (9)] about water temperature \( T^{n+1} \) of all layer. The matrix is a tri-diagonal matrix, TMDA method was adopted to solve the matrix equation.

### 4 NUMERICAL TESTS

To test capability, the proposed numerical method will be applied in the solution of the numerical test model (Equation 10).
\[
\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial z^2} + a_0 \cos(\omega_p t) T
\]  
(10)

Where \( T \) is a specific factor in convection-dispersion substance (here refers to water temperature); \( D \) is diffusion coefficient; \( a_0 \) is amplitude; \( \omega_p = 2\pi / T_p \); \( T_p \) is period of oscillation. The second term on the right of the equation is the source-sink term. Suppose water depth \( L = 4000 \), setting the initial condition as:

\[
T(z,0) = \begin{cases} 
T_0 = 10 & z_L < z < z_R \\
T_0 = 0 & z > z_R \text{ or } z < z_L 
\end{cases}
\]  
(11)

Where \( z_L \) and \( z_R \) are 1500m and 1800m respectively. A final result \( (t=9600s) \) was chosen.

Suppose \( a_0 = 0.005, \ T_p = 1500s \). Time step is \( \Delta t = 10s \). Space step is \( \Delta z = 10 \). Set different diffusion value \( D (D=2, D=8) \) to perform numerical simulations. Two methods were adopted to treat source-sink term: the improved method proposed in this paper and the conventional pointwise method \([\text{the source-sink term treated simply: } R = R(T^n)]\). The simulated results were shown in Figure 3. From the figure we could see that the calculated curve of the improved method was almost overlapped with analytic curve. However, although the results calculated by the conventional method could generally reflect the trend of temperature change, the simulated curve deviated from the analytic curve distinctly, and the predicted results of wave form and temperature peak value both presented obvious deviation comparing with the analytical results. So it could be concluded that: the improved method has more accuracy comparing with conventional method. It also prove that: Even the diffusion term solving by scheme has high-accuracy and good-stability, but if the source-sink term was not treated appropriately, there might still exist a large deviation.

Figure 3. Comparison of the simulated value and the analytical value: (a) \( D=2 \); (b) \( D=8 \).

5 APPLICATION FOR GUOZHENG LAKE

The temperature observed from July 1 to August 31, 1978 in Guozheng Lake was used to verify the proposed model and method. The lake was divided into 10 vertical layers with space step \( \Delta z = 0.4 \) m and time step \( \Delta t = 10 \) s.

Figure 4 shows the comparison between the \( W_{str} \) time series calculated using water temperature model with the improved wind sheltering model and Markfort model \([\text{Markfort et al. 2010}]\). According to Figure 4, \( W_{str} \) calculated via WST is time-dependent with an average value of 0.96. However, the value of \( W_{str} \) calculated via Markfort model is fixed constant \((0.86)\). Additionally, the \( W_{str} \) value calculated by the Markfort model was generally smaller than that from the improved wind sheltering model. This is because the Markfort model considers \( W_{str} \) to be the ratio of area of wind access \( A_{windaccess} \) (where the wind is not completely influenced by wind sheltering effect) to the whole lake area (Figure 3). However, the wind in non-shallow areas of the lake still contributes to the lake-averaged wind even though its speed decreases because of wind sheltering effect.
The comparison between the results of the daily average simulated value and the observed value are shown in Figure 5. The results clearly show that the improved wind sheltering model was very effective. Markfort model has also been able to generally reproduce the variation in actual water temperature. However, the simulated results of these two models for the initial days all showed an obvious deviation. This could be because the simulated results were heavily influenced by initial conditions in the early simulation stage. Additionally, for the same input, the simulated value of Markfort model was apparently higher than that of the improved wind sheltering model. The reason for explaining the above phenomenon is that the $W_{str}$ value calculated using the Markfort model was relatively small, causing the wind speed as the input of water temperature model to become smaller.

The simulated values and the observed values of the vertical distribution of water temperature at 8:00 am and 5:00 pm on July 14, 1978 are shown in Figure 6. It clearly shows that the major features of the negative temperature distribution in the morning and positive temperature distribution in the afternoon were accurately reproduced when the improved wind sheltering model was applied. Another interesting phenomenon is that if the wind sheltering effect is ignored (namely $W_{str}=1$), the simulated value at 8:00 am was consistent with the observed value. However, at 5:00 pm the difference of the simulated water temperature between the upper and middle layers was not obvious, which meant the lake was still being overturned and the appearance of positive temperature distribution was put off. The result coincided with Ahsan’s statement (Ahsan 1999), i.e., if the wind speed observed by the meteorological station differs from the actual value, it could easily result in an earlier or later overturn.
Figure 6. The comparison of the simulated and the observed vertical water temperature on July 14, (a)8:00am; (b)5:00pm.

6 CONCLUSION

To compensate for the failure of previous studies to take wind sheltering effect into account or the low accuracy of the method used to calculate the wind sheltering coefficient, an improved wind sheltering model was developed and coupled with a one-dimensional vertical heat conduction model to create a new water temperature model. Based on the relationship between wind directions, the location of the obstacle, and the characteristics of wind variation downwind from the obstacle, the improved wind sheltering model could calculate a time-dependent wind sheltering coefficient $W_{st}$ that would dynamically adjust the wind data to be used as input for the water temperature model. This computational methodology enhances the representation of heat balance for temperature simulation. Another contribution of this paper is applying a new operator-splitting method to the solution of water temperature model to compensate for the deviation caused by poor treatment of the source-sink term. The numerical tests show that the proposed numerical method has higher accuracy than the traditional. It also proved that: Even the diffusion term solving by scheme has high-accuracy and good-stability, but if the source-sink term was not treated appropriately, there might still exist a large deviation. The proposed model was verified using measurements from Guozheng Lake. The results were found to be consistent, thus proving the validity of the model and the method.

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