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In-fiber Optical Devices Based on D-fiber

Kevin H. Smith
Brigham Young University - Provo

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IN-FIBER OPTICAL DEVICES BASED ON D-FIBER

by

Kevin H. Smith

A dissertation submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Electrical and Computer Engineering

Brigham Young University

April 2005
This dissertation has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

Date
Richard H. Selfridge, Chair

Date
Stephen M. Schultz

Date
Michael A. Jensen

Date
Karl F. Warnick

Date
Justin B. Peatross
As chair of the candidate’s graduate committee, I have read the dissertation of Kevin H. Smith in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

Date

Richard H. Selfridge
Chair, Graduate Committee

Accepted for the Department

Michael A. Jensen
Graduate Coordinator

Accepted for the College

Douglas M. Chabries
Dean, Ira A. Fulton
College of Engineering and Technology
This dissertation presents the fabrication and analysis of in-fiber devices based on elliptical core D-shaped optical fiber. Devices created inside optical fibers are attractive for a variety of reasons including low loss, high efficiency, self-alignment, light weight, multiplexibility, and resistance to electromagnetic interference. This work details how D-fiber can be used as a platform for a variety of devices and describes the creation and performance of two of these devices: an in-fiber polymer waveguide and a surface relief fiber Bragg grating.

In D-fiber the core is very close to the flat side of the ‘D’ shape. This proximity allows access to the fields in the fiber core by removal of the cladding above the core. The D-fiber we use also has an elliptical core, allowing for the creation of polarimetric devices. This work describes two different etch processes using hydrofluoric acid (HF) to remove the fiber cladding and core. For the creation of devices in the fiber core, the core is partially removed and replaced with another material possessing the required optical properties. For devices which interact with the evanescent field, cladding removal is terminated before acid breaches the core.
Etching fibers prepares them for use in the creation of in-fiber devices. Materials are placed into the groove left when the core of a fiber is partially removed to form a hybrid waveguide in which light is guided by both the leftover core and the inserted material. These in-fiber polymer waveguides have insertion loss less than 2 dB and can potentially be the basis for a number of electro-optic devices or sensors. A polarimetric temperature sensor demonstrates the feasibility of the core replacement method.

This work also describes the creation of a surface relief fiber Bragg gratings (SR-FBGs) in the cladding above the core of the fiber. Because it is etched into the surface topography of the fiber, a SR-FBG can operate at much higher temperatures than a standard FBG, up to at least 1100 °C. The performance of a SR-FBG is demonstrated in temperature sensing at high temperatures, and as a strain sensor.
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Chapter 1

Introduction

1.1 Optical communication and sensing

Optical fibers have revolutionized the way we communicate with each other. Their low loss, low dispersion, and high bandwidth have allowed for the complete overhaul of telecommunications networks in the last 25 years. The bandwidth-hungry demands of data, voice, and video transmissions have fueled rapid growth in the optical fiber industry. The ever-expanding data transmission capabilities of fiber were demonstrated in so-called ‘hero experiments’ in February 2004 in which 149 channels were transmitted over a distance of 6,120 km at 42.7 Gb/s \[1\].

Optical fibers have not only revolutionized communications; they have begun to make inroads into other areas. “Fibers go beyond communications to deliver laser energy inside the body for delicate orthoscopic [sic] surgery on joints. Optical fibers molded into concrete or plastic composites can monitor the strengths of structures from bridges to aircraft. Subtle changes in light waves passing through fibers looped many times around a spool can measure rotation, serving as gyroscopes with no moving parts. The wonders go on and on \[2\].”

The main advantages of optical fibers in communication and sensing are that they have very low loss, have very large bandwidth, are immune to electromagnetic interference, are electrically inert, are very light weight, and the signals they carry can have very little power \[3\]. In addition, many signals or sensing points can be multiplexed onto a single fiber. These advantages have resulted in optical fibers supplanting metal wires in many communications and sensing applications. Some of
the drawbacks of optical fibers include the difficulty in splicing and aligning fibers and the high cost of transmitters and receivers. However, continuing research continues to magnify the benefits of optical fibers and diminish their drawbacks. This dissertation will discuss several new types of optical devices based on optical fibers.

1.2 Optical devices

Optical devices now play an important role in communications and sensing, and in the future that role will only expand with the increasing prominence of optical signals in communication and sensing. All-optical networks remain a goal of the research community which promise “increased bandwidth, faster service provisioning, easier management, reduced costs, and greater reliability” [4].” Fiber-optic sensors are also making progress in many different applications including sensing of “rotation, acceleration, electric- and magnetic-field measurement, temperature, pressure, acoustics, vibration, linear and angular position, strain, humidity, viscosity, chemical measurements, and a host of other sensor applications [5].”

Three general methods exist for creating optical devices. The first method involves the use of bulk optical components such as lenses, mirrors, and gratings. A second method involves the integration of optical components into or onto the guiding medium of planar optical waveguides. The integration of optical devices onto planar substrates streamlines and simplifies the fabrication process, improves optical alignment, and allows for much more compact optical systems. These advantages mirror those of integrated electronic circuits.

A third method involves the integration of devices into optical fibers – in-fiber optical devices. Devices created with bulk or integrated optical components do not interface well with single-mode fibers, which form the backbone of optical communication systems and sensors. This results in high insertion loss, power reflection, and poor mechanical integrity. In addition, alignment of optical fibers with bulk optics will always be a difficult and time-consuming process. While the integration of optical components into planar waveguides overcomes many of these problems, planar waveguides still suffer from the difficulty of pigtailing optical fibers to them.
1.3 In-fiber devices

In-fiber devices solve many of the problems listed above because the optical signal never leaves the fiber. In-fiber devices enable the labor-intensive fiber pigtailing process to be replaced with automated fusion splicing, which results in good mechanical integrity and low insertion loss. Two prime examples of the benefits of in-fiber devices are erbium-doped fiber amplifiers \[6\] and fiber Bragg gratings \[7\].

The efficiency of in-fiber devices is determined by the strength of the interaction between the optical field and the interacting medium. In order to create the most efficient in-fiber devices it is therefore necessary to interact directly with light in the core instead of with the evanescent field in the cladding of the fiber. However, for some applications, weaker interaction with the light in the fiber is required to produce the desired response of the device.

1.4 In-fiber devices based on D-shaped optical fiber

This dissertation investigates the fabrication and analysis of several different types of in-fiber devices. These devices are an in-fiber electro-optic waveguide, an in-fiber polymer waveguide used for sensing, and a surface relief fiber Bragg grating (SR-FBG). All of these devices are based on D-shaped optical fiber supplied by KVH Industries, Inc. Figure 1.1 shows the cross section of a D-fiber. The major axis of the elliptical, germania-doped core of the D-fiber can be oriented either perpendicular (vertical core) or parallel (horizontal core) to the flat surface of the D shape. Most of the optical power propagating through this type of fiber travels in the germania-doped core. The small undoped region at the center of the core is an artifact of the fabrication process.

This work begins by describing two different methods used to etch fibers. The first method involves the partial removal of the core of the fiber, and is used for creating devices in which the core of the fiber is partially replaced with another material. The second method involves the removal of a predetermined amount of the cladding material above the core, and is used in devices in which light interacts evanescently with structures or materials on the flat side of the fiber. This work then
discusses the application of polymer to a fiber whose core has been partially removed. Polymer application creates a hybrid waveguide in which much of the power in the fiber is guided within the polymer and the rest is guided in the cladding and what remains of the core. The polymer waveguide forms the basis for two of the in-fiber devices described in this work: an in-fiber polymer waveguide sensor and an electro-optic modulator. The last device described in this dissertation is the surface relief fiber Bragg grating.

1.4.1 Etching of D-fibers

D-fiber is advantageous in the formation of in-fiber devices because of the proximity of the core of the fiber to the flat side of the fiber. This proximity allows access to either light in the core of the fiber or to the evanescent field of light in the core by the removal of the core itself, or by removal of some of the cladding above the core of the fiber. Hydrofluoric acid (HF) removes the cladding and core of the fiber so that in-fiber devices can be created. Figure 1.2 shows illustrations of the cross sections of three different vertical core fibers. From left to right they are an unetched fiber, a fiber in which the cladding above the core has been mostly removed, and a fiber in which the core has been partially removed.
Figure 1.2: Illustrations of cross sections of three fibers. From left to right, the first fiber is unetched, the second has much of the cladding removed, and the third has the core partially removed.

The second chapter of this dissertation details the steps performed to etch into the cladding or the core of a D-fiber. The more a device overlaps with the high-intensity portion of the optical field in the fiber, the stronger the interaction between the light and that device. Therefore, devices needing strong interaction may require the partial removal and replacement of the core of the fiber, while devices requiring weaker interaction with light in the fiber may need to be placed in the cladding above the core.

### 1.4.2 Creation of in-fiber polymer waveguides

The basis for several of the devices described in this dissertation is the in-fiber polymer waveguide. This waveguide is formed by the partial removal and replacement of a section of the core of the fiber with a polymer. Chapter 3 describes how polymer is applied to an etched fiber to create an in-fiber polymer waveguide. It details the loss mechanisms of polymer waveguides and describes how loss can be minimized. It also addresses the tradeoffs between loss and efficiency that affect this type of device. Figure 1.3 shows cross sections of two in-fiber polymer waveguides,
one based on vertical core fiber and the other on horizontal core fiber. A thin layer of polymer coats the fiber, and the polymer is thickest in the groove left by partially removing the fiber core.

Figure 1.3: Cross-sectional images showing polymer partially replacing the core of a D-fiber for both vertical and horizontal core orientations.

As an example of a device that could potentially be fabricated from an in-fiber polymer waveguide, Appendix B discusses the design of an electro-optic modulator. Poling an in-fiber polymer waveguide makes the polymer electro-optic, allowing for the creation of devices based on the interaction of an applied electric field with light in the fiber. Figure 1.4 shows an illustration of a conceptual electro-optic device fabricated on a D-fiber. In the figure, electrodes are placed to the sides of the polymer waveguide. The application of an electric field to the polymer changes its optical characteristics, enabling the creation of devices such as modulators or electric field sensors.
1.4.3 Sensors based on in-fiber polymer waveguides

Replacing the core of a D-fiber with a material that responds to some environmental stimulus allows the fiber to function as a sensor. The core can be replaced with materials sensitive to temperature, pressure, humidity, strain, or the presence of a chemical or biological substance. Chapter 4 discusses how the replacement of the core of the fiber with polymer results in a sensor. The feasibility of this approach is demonstrated by using core-replaced D-fiber for temperature sensing. Figure 1.5 shows a diagram of how a sensor is formed by partially replacing the core of a D-fiber with a sensing material.
1.4.4 Surface relief fiber Bragg gratings

Etching gratings into the surface of D-fibers is an alternative to standard index modulation fiber Bragg gratings (FBGs). Surface relief fiber Bragg gratings (SR-FBGs) differ in several important aspects from standard FBGs. While standard photoinduced FBGs wash out quickly at high temperatures, SR-FBGs are actually etched into the surface topography of the fiber, and so are able to perform up to at least 1100 °C. SR-FBGs may ultimately be limited by the melting point of the glass of which the fiber is made. SR-FBGs also are asymmetrical with respect to the fiber core, the result being that different polarizations of light in the fiber respond differently to the same grating. This effect could be exploited to sense multiple stimuli simultaneously with a single SR-FBG. Figure 1.6 shows an illustration of a SR-FBG, with the grating etched into the flat side of the D-shaped optical fiber. Chapter 5 describes how SR-FBGs are modeled and fabricated, and demonstrates the fabrication of a temperature sensor and a strain sensor from a SR-FBG.

![Surface relief grating](image)

Figure 1.6: A surface relief fiber Bragg grating (SR-FBG) showing the grating etched into the flat side of the fiber.

1.5 Numerical simulations

In this work I use the commercial software package BeamPROP™ for numerical simulations in which I find modes in waveguides and propagate beams through three-dimensional structures. The numerical foundation of BeamPROP™ is the beam propagation method (BPM), is based on the paraxial approximation to the
Helmholtz equation. BPM assumes that the field varies slowly along the propagation axis and has small angular divergence from this axis. These assumptions are well suited to problems dealing with light propagation in optical fibers and waveguides [8].

BeamPROP™ finds supported modes in optical fibers via the imaginary distance beam propagation method (IDBPM) by launching a Gaussian mode into a two-dimensional refractive index profile with no variance in the propagation direction [8]. Once the field profile has converged to a supported mode, the effective refractive index, $N$, of that mode is calculated.

BeamPROP™ can also propagate light in three-dimensional structures. Three-dimensional propagation is initiated by specifying a launch field at the input to the 3-D structure. BeamPROP™ then steps through the structure using BPM, calculating the electric field at each successive transverse plane.

This research also involves the use of other numerical tools. I have developed three numerical tools which are described in Appendices A, C, and D. The first is an etch simulator, which helps to visualize how materials are etched in acid or a reactive ion etcher (RIE). The second is a two-dimensional eigenvalue mode solver which finds fields and effective indices of modes in waveguiding structures. The third is a Runge-Kutta solver for the coupled-mode equations, which predicts power reflection and transmission for FBGs.

1.6 Contributions

This work contributes to the field of fiber optics by demonstrating the fabrication and analysis of new types of devices based on D-shaped optical fiber. Optical fibers are making inroads into many different applications in communications and sensing, and D-fiber has unique properties that allow it to form the basis of several different devices. This dissertation shows the potential of D-fiber to impact the growing field of fiber-optic devices. The research contained herein has produced four publications in peer-reviewed journals, other papers which are in submission, and several conference papers.
Chapter 2

D-fiber as a Platform for In-fiber Optical Devices

2.1 D-fiber geometry

D-shaped optical fiber provides an alternative to circular fiber for creating in-fiber optical devices because of the proximity of the core to the flat side of the fiber. The primary benefit of this proximity is that it allows access to the fiber core via chemical etching while leaving most of the cladding intact. Etch selectivity of the different doping regions in the fiber allows for partial removal of the core of the fiber and its replacement with other materials. The properties of these materials determine which kinds of devices can be created with them. The first advantage of this technique is that it maintains the mechanical integrity of the fiber by leaving the cladding mostly intact. The second advantage is that replacing the fiber core allows the interaction between the materials and the optical field to be very strong. Fibers can also be etched so that some of the cladding above the core remains, enabling the creation of devices in which the evanescent field of the light guided in the fiber interacts with materials and/or structures on the flat side of the fiber.

D-fibers used in this research are made of silica with a germania-doped core surrounded by a fluorine-doped cladding, which is surrounded by a pure silica supercladding. The diameter of the D-fiber is \( \sim 125 \mu m \), and the core is \( \sim 12-13 \mu m \) from the flat side of the fiber. The dimensions of the elliptical germania-doped core are approximately \( 2 \mu m \times 4 \mu m \). The elliptical core can be oriented with the major axis of the ellipse either perpendicular to the flat side of the fiber (vertical core fiber) or parallel to it (horizontal core fiber). In the preform fabrication process, germania
diffuses out of the center of the core, leaving undoped silica there. Figure 2.1 shows a diagram of the cross section of a D-fiber. In this work I use elliptical core D-fiber supplied by KVH Industries, Inc.

![Figure 2.1: Cross section of a D-fiber showing the doping of the different regions of the fiber.](image)

Figure 2.1 shows a backscatter scanning electron microscope (SEM) image of a cleaved D-shaped fiber. The lighter ellipse toward the bottom of the image is the fiber core. This image shows that the core is elliptical in shape and is located approximately 13 µm from the flat side of the fiber. The small undoped region at the center of the core can also be seen in this image.

2.1.1 Fabrication of elliptical core D-fiber

The following brief description of the fabrication process for elliptical core D-fiber summarizes that of Dyott [9]. Hollow circular tubes made of pure silica are the starting point in the fabrication of D-fiber. Opposite sides of the silica tube are first ground flat, then the tube is cleaned. Burning different mixtures of gases in the tube deposits materials onto its inner surface. This modified chemical vapor deposition (MCVD) process creates the fluorine-doped cladding and the germania-doped core.
The tube is then collapsed at high temperature and slight vacuum to form a solid preform, in which process it again assumes a circular cross-section. However, because opposing sides had previously been ground flat, the core and cladding take on an elliptical shape. This circular preform is then ground again on one side to give it a D-shaped cross-section, placing the core of the fiber close to the flat side of the D. Figure 2.3 shows the steps in the fabrication of a D-fiber preform. Finally, the preform is drawn in a drawing tower and coated with a plastic jacket to complete the process.

The end result of the fabrication process is the D-fiber shown in Fig. 2.4. The figure is taken from the book by Dyott [9] and is used by permission. It shows a photograph of the end of a D-fiber in which the D shape is clearly visible, as well as the elliptical core and the fluorine-doped cladding.

2.1.2 Properties of elliptical core fiber

The single-mode elliptical core fiber used in this research allows light to propagate in two orthogonal modes at a wavelength of 1550 nm, one quasi-polarized along the major axis of the ellipse with the other quasi-polarized along the minor
Figure 2.3: Steps in the fabrication of a D-fiber preform. The process starts with a silica tube (upper left) and ends up with a D-shaped preform with an elliptical core (lower right).

axis. This dissertation adopts the convention that the $x$-axis is defined parallel to the flat surface of the D-fiber and the $y$-axis is defined perpendicular to the flat surface. Hence, the primary component of the electric field of an $x$-polarized mode is parallel to flat surface. For a $y$-polarized mode, the electric field is primarily perpendicular to the flat surface.

Because of the relatively large difference in effective index between the two polarization modes, there is little coupling between them and the fiber is said to be polarization maintaining, whereas the polarization state in a circular core fiber fluctuates almost randomly as light propagates through it. This polarization maintaining property makes elliptical core fibers ideal for applications in which polarization is important, such as couplers, polarization analyzers, polarimetric modulators, Mach-Zehnder interferometers, etc. However, elliptical core fibers are not used in long-haul telecommunications because they have higher loss than circular core fibers and have smaller cores than telecom fiber (making alignment more difficult).

2.2 Partial removal of the core of a D-fiber

The proximity of the core of a D-fiber to the flat side of the fiber allows access to light in the fiber via chemical etching in hydrofluoric (HF) acid or buffered
hydrofluoric acid (BHF). The evanescent field of the fiber can be accessed by etching the cladding of the fiber above the core. Structures or materials can then be placed on the flat side of the fiber just above the core.

For stronger interaction with light in the fiber, the core of the fiber can be completely or partially removed. The doping of optical fibers results in differential etch rates of the core, cladding, and supercladding materials, allowing the core to be removed while leaving the cladding intact. Partial or complete removal of the core of the fiber allows for the creation of high-efficiency devices by allowing for the replacement of the removed core with an active material. This process allows light to be guided within the active material. The greater the amount of power guided in the active material, the more efficient the device becomes.

One of the benefits of wet-etching fibers is that there is no need for precise alignment because the HF acid etch is selective – it removes much more of the core of the fiber than the cladding. The groove left by the removal of the core is exactly aligned with the path of the optical power in the fiber. This allows for the creation of optical devices with low insertion loss by replacing the fiber core with other materials.
In contrast, a photolithographic process would involve the creation of a mask and would necessitate the alignment of that mask with an optical fiber to submicron accuracy in order to etch only the core of the fiber.

2.2.1 Monitored etching of D-fibers in hydrofluoric (HF) acid

In HF acid, the germania-doped core etches approximately 8 times faster than the fluorine-doped cladding and 11.5 times faster than the silica supercladding \[10\]. This differential etch rate allows a desired amount of the core to be removed while maintaining the cladding of the fiber mostly intact. With the cladding intact, the fiber maintains most of its structural integrity.

To begin the fabrication of core-replaced devices, we first cut a length of fiber approximately 1 m long from a spool of D-fiber. A 3 cm long section of the fiber is soaked in dichloromethane to soften the jacket. The jacket is then stripped from the fiber to expose the silica cladding so that it can be etched in HF acid. 2 cm of the stripped section of the fiber is placed into a container of 25% HF acid in a dip-etch configuration as shown in Fig. \[2.5\]. The dip-etch setup allows for gradual transitions between the unetched section of the fiber and the etched section. Gradual transitions have lower loss than abrupt transitions because the modes are allowed to slowly transition from the unetched to the etched section and vice versa. Factors such as the meniscus height, a decreasing acid level due to evaporation, the angle at which the fiber enters the etch bath, and etching by acid vapor all contribute to gradual transitions \[11\].

Once the cladding above the core has been removed and the core has been breached by the acid, the core is very quickly removed because of the high etch rate of the germania-doped silica. Because the etch rate of the germania-doped core is so fast, it is necessary to remove the fiber from the 25% HF solution as soon as the core is breached. It is then placed in a much more dilute 5% HF solution. The much slower etch rate of the more dilute acid gives greater control over the depth of the etch in the core of the fiber. The 25% HF solution can remove all of the cladding above the core in 23-27 min, and the 5% HF solution removes half of the core in
about 4-6 min. More detailed descriptions of this core removal process, including a step by step outline of the procedure, are provided in Refs. 12, 13.

Figure 2.6 shows SEM images of the progression of the etch from the point where the etch has just reached the core (Fig. 2.6(a)) to the point where the core has been almost completely removed (Fig. 2.6(f)). The figure shows that the germania-doped core of the fiber etches much more quickly than the cladding. Figure 2.6(f) shows that the core can be completely removed while leaving the cladding mostly intact. Also, the undoped region in the center of the core etches much more slowly than the rest of the core, leaving a hump if the etch is terminated when about half of the core has been removed, as in Fig. 2.6(e). Figure 2.7 shows the region of the fiber that we magnify to get images of the core such as those shown in Fig. 2.6.

The etch rate of silica fibers in HF acid depends strongly on temperature, humidity, glass doping, and acid concentration. Also, the geometry of the fibers can vary slightly from spool to the next. This sensitivity makes it very difficult to simply use a timed etch to remove the core of the fiber. Initial experiments involved a simple timed etch, but the results were not very repeatable. The etch depth must be controlled precisely to achieve the desired optical characteristics, so we decided to monitor the etch process in situ by transmitting laser light through the fiber as it was
etched. The \textit{in situ} technique is accomplished by monitoring the power transmitted through the fiber as it is etched, similar to a process used to monitor the fabrication of side-polished fibers [14, 15]. Accurate correlation of the depth of the etch in the core with the amount of power transmitted through the fiber would yield a highly reproducible process.

### 2.2.2 Analysis of the \textit{in situ} power monitoring technique

Initial experiments involved the use of light of wavelength 1550 nm. We launched the laser light into the fiber and measured the transmitted power at the output with a detector as in Fig. 2.5. The amount of transmission loss through a fiber while it is being etched depends strongly on the modal structure of the etched section of the fiber. Figure 2.1 shows the different regions of the fiber and how they are doped. At a wavelength of 1550 nm, the undoped silica supercladding and the undoped center of the fiber core have a refractive index of $n_{\text{SiO}_2} = 1.444$, the fluorine-doped cladding has an index of $n_{\text{clad}} = 1.441$, and the germania-doped core has an index of $n_{\text{core}} = 1.4756$. The fluorine-doped cladding index is depressed below that...
Figure 2.7: SEM image showing the region of the fiber that is magnified in order to see the how the core has been etched.

of undoped silica in order to increase the birefringence between the two polarization modes [16].

A mode supported by a waveguide is characterized by its propagation constant $\beta_z = k_0 N$, where $N$ is the effective index of refraction, $k_0 = 2\pi/\lambda$, and $\lambda$ is the free space wavelength. When $n_{\text{core}} > N > n_{\text{clad}}$ for a given mode the mode is said to be a core mode, and the intensity of the guided light is highest in the region of the core. The electric field of a core mode is evanescent outside of the core and decays exponentially with distance from the core. As the cladding is removed by etching, the light is influenced more and more by the lower index of the HF acid bath $n_{HF} = 1.33$, because it begins to take on the role of a cladding material. The greater the amount of field in the acid bath, the lower the effective indices of the modes in the fiber. When the fiber has been etched far enough, the effective indices of modes begin to drop below the cladding index. At this point, the modes become cladding modes and are no longer evanescent in the cladding. The power in a cladding mode is not concentrated in the core region, but is dispersed throughout the cladding material. When the effective index of a mode drops below the cladding index, it is said to be cut off.

Etched fibers have two transition regions where the fiber meets the surface of the HF acid bath. When light traveling through an etched fiber reaches the first
transition, some of it is coupled into radiation modes, some into the modes supported by the etched section of the fiber, and some is reflected. Light that couples into radiation modes does not couple back into core modes at the second transition. Similarly, the light that couples into cladding modes in the etched region also does not couple back into core modes at the second transition. When light couples into different cladding modes at the first transition, the combined fields at the second transition bear very little resemblance to the field that was launched into the etched section at the first transition region. In other words, the overlap between the fields of cladding modes in the etched section and the fields in the unetched section is very low. The overlap integral between two modes is defined by

\[
\eta = \frac{\int \int E_1 \cdot E_2^* dS}{\sqrt{\int \int |E_1|^2 \cdot dS \int \int |E_2|^2 \cdot dS}},
\]

where \(E_1\) and \(E_2\) correspond to the electric fields of a mode in the etched region and the unetched region, respectively, and \(S\) is the entire transverse plane. The overlap integral predicts the fraction of light that is coupled from one mode into another at a discontinuity in a waveguide. While etched fibers do not have a sharp discontinuity, the overlap integral provides an intuitive method of visualizing the amount of coupling between modes. The more a fiber is etched, the more power couples into radiation and cladding modes at the transitions because of the smaller overlap between modes in the unetched and etched sections of the fiber.

BeamPROP\textsuperscript{TM} simulations show that the fiber is single-mode at 1550 nm, and the two linearly polarized (LP) modes it supports are cut off when about half of the core has been removed. These modes are in reality quasi-LP modes because each mode has some electric field component orthogonal to the polarization axis, but this component is negligible. After this cutoff point is reached, the power transmitted through the fiber should be close to zero, because the remaining core no longer guides light. Simulations also show that unetched fiber supports 8 modes (4 of each polarization) at a wavelength of 670 nm and some of these modes cut off at a later point in the etch process than the modes at 1550 nm. The etched section of the fiber should therefore continue to support guided modes at a wavelength of 670 nm even
after the modes at 1550 nm are cut off. This allows more control over the etch depth using light at a wavelength of 670 nm because a significant amount of power continues to be transmitted through the fiber until the core is almost completely removed.

Figure 2.8 shows a model of the cross-sectional profile of a D-fiber during the etch process. When the fiber has been etched to the point shown in this figure, the simulated effective index, $N$, of the fundamental mode (for both polarizations) at a wavelength of 1550 nm drops below the cladding index, $n_{clad}$, and the core mode transitions to a cladding mode. As the profile of a fiber being etched approaches that shown in Fig. 2.8, the power transmitted through the fiber should drop very rapidly. Because of this, the etch depth cannot be monitored with 1550 nm light unless only a small fraction of the core is removed.

Figure 2.8: Model of the etch profile at which the simulated $N$ of the fundamental mode at 1550 nm drops below $n_{clad}$.

Figure 2.9 shows cross-sectional profiles of a D-fiber having about half of the core removed. The profiles show the point in the etch process when the simulated $N$ of each mode at a wavelength of 670 nm drops below $n_{clad}$. Each of the four images is labeled with a mode number, with 1 corresponding to the fundamental
mode. Successively higher supported modes are labeled with numbers 2 through 4. There are four profiles instead of eight because each mode has two polarizations which are cut off at almost precisely the same point in the etch process. The profiles show that the second and third modes at 670 nm cut off at nearly the same point, which is also at nearly the same point as the fundamental mode at 1550 nm. The cutoff profile for the fundamental mode shows that almost the entire core can be removed before the fundamental mode cuts off. This means that red light can be used to monitor etches in which a very significant portion of the core is to be removed.

Figure 2.9: Model of the etch profile at which the simulated $N$ of the supported modes at 670 nm drop below $n_{clad}$. 
2.2.3 Experimental results of the in situ monitoring technique

The power transmitted through a fiber at a wavelength of 1550 nm quickly drops into the noise floor once the core has been breached by the acid. For this reason and because of the results of numerical simulations we modified the in situ monitoring technique by using light of wavelength 670 nm to monitor the etch. Simulations show that the fiber should continue to guide light at this wavelength until the core is almost completely removed. We conducted an experiment in which we simultaneously sent both wavelengths of light, 1550 nm and 670 nm, into a switch. The switch alternated which wavelength was launched into the fiber. We then measured the output at each wavelength as a fiber was etched in HF acid. Figure 2.10 shows the amount of power transmitted through the fiber versus time as the fiber is etched. Plots of the normalized power for both 1550 nm and 670 nm light are shown. The plot shows that the 670 nm light drops into the noise floor later than the 1550 nm light. Since the 670 nm light drops out later, it can be used to monitor the etch depth in the core for greater etch depths than light of wavelength 1550 nm. Point (a) in the figure shows the point at which the core is breached by the acid. Once the core is breached, the etch proceeds to remove the core very rapidly, and the transmitted power begins to decrease sharply. Point (b) shows the point at which the two modes at $\lambda = 1550$ nm are cut off. Once they are cut off, the transmitted power quickly falls into the noise floor. However, red light continues to guide for a period of time after the transmitted 1550 nm light is no longer measurable.

Through SEM imaging we correlated the amount of core and cladding removed with the amount of power the fiber transmitted at the time it was removed from the etch. By removing the fiber from the etch bath when the transmitted power drops to a predetermined level, we were able to achieve very consistent etch depths. We determined that the fiber should be extracted from the etch when the transmitted power (at $\lambda = 670$ nm) reaches a point 6.8 dB below the initial power reading taken before the fiber is etched. Later in this work I describe how extracting the fiber from the etch at this point allows for the formation of a low loss single-mode waveguide in the core region through the application of polymer. Figure 2.11 shows how the power
transmitted through a fiber at a wavelength of 670 nm correlates to the etch depth in the core region. The SEM image with the label 3 shows the profile of a fiber that has been removed at the transmission loss point of 6.8 dB.

When analyzing the mode cutoffs for 670 nm light we expected the plot of power transmission to show individual modes dropping out sharply as their effective index went below $n_{clad}$. We believe that this is not demonstrated in the plots because the gradual transition is able to couple some power from higher order modes in the unetched region to lower order modes in the etched region even after the higher order modes are cut off in the etched region. Modes also become more leaky before they cut off. Both of these factors smooth out the expected sharp drops in the transmitted power when modes cut off.

Figure 2.10 shows a cross-sectional view of a vertical core fiber which we removed when the transmission loss dropped to 6.8 dB. We etch horizontal core fiber with the same process as that for vertical core fiber. These fibers are also withdrawn from the etch at the 6.8 dB loss point. Figure 2.12 shows a SEM image of
Figure 2.11: Plot showing the correlation between the normalized power transmitted through a fiber (at 670 nm) during the etch and the cross-sectional profile of the fiber.

A horizontal core fiber withdrawn from the HF acid etch bath when the transmission reaches 6.8 dB.

2.3 Polarimetric monitoring of etches

Some applications require access to the evanescent field of the fiber while leaving the core intact. For these applications, etching must be terminated before acid breaches the fiber core. The resulting proximity of the core of the fiber to the flat surface allows materials or structures on the flat surface of the fiber to interact with the evanescent field of the guided light. The \textit{in situ} power monitoring technique described above does not work well for etching only the cladding because the transmitted power begins to drop only when the core of the fiber has been breached.

However, the birefringence between the modes in the fiber changes significantly even before the core has been breached by acid. For this reason it is beneficial to polarimetrically monitor the birefringence between the two modes in the fiber to determine how close to the core the flat surface of the fiber is.
2.3.1 Birefringence

Because of the elliptical geometry of the core, the $x$- and $y$-polarized modes have different propagation constants, $\beta_z$, where $\beta_z = k_0 N = 2\pi/\lambda N$. In this equation, $k_0$ is the free space wavenumber, $\lambda$ is the free space wavelength, and $N$ is the effective index of refraction. Birefringence is defined as the difference between the effective indices of the $x$- and $y$-polarized modes, $B = N_x - N_y$.

As the cladding of the fiber is etched, the effective index of each polarization mode changes, but they change by different amounts. This causes the birefringence to change as the fiber is etched. Figures 2.13 and 2.14 show how the birefringence of each of the two types of fiber (horizontal and vertical core, respectively) changes as the distance from the core to the flat side of the fiber decreases.

The simulation results shown in Fig. 2.13, computed using BeamPROP™, are very similar to those of Jensen and Selfridge which were found using an analytic approximation [18]. Simulations show that in horizontal core fiber the birefringence
remains nearly constant until the etch brings the surface of the fiber to within 4 \( \mu \text{m} \) of the core and the birefringence begins to increase dramatically once the cladding has been etched to within 2 \( \mu \text{m} \) of the core. For vertical core fiber, Fig. 2.14 shows that the birefringence does not significantly change until the core-to-flat distance is less than 2 \( \mu \text{m} \), and it only changes rapidly when the core is within 1 \( \mu \text{m} \) of the flat side. These simulations assume that the fiber is immersed in water.

By placing a polarizer at the input to the fiber and an analyzer at the output, the change in birefringence can be measured by monitoring the output power, which oscillates as the core-to-flat distance decreases during the etch. The input polarizer orientation ensures that equal amounts of power are launched into each polarization mode, and the analyzer either has the same orientation as the input polarizer or is orthogonal to it. By closely monitoring the oscillations in output power, we can tightly control the distance from the core to the flat, enabling the placement of structures at any predetermined distance from the core.
Because they have different effective indices, the two modes accumulate a phase difference as they propagate through the fiber. Since the birefringence changes as the acid etches the fiber, the accumulated phase difference between the modes changes as well. The change in the phase difference between the two polarizations is directly proportional to the change in the birefringence and is given by

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta B(d)L, \]  

(2.2)

where \( \lambda \) is the free space wavelength, \( B \) is the birefringence, \( d \) is the distance from the top of the core to the flat side of the fiber, and \( L \) is the length of the etched section of the fiber. In this birefringence monitoring process we have used \( L = 1.5 \) cm. The optical power of the light at the output is then given by

\[ P(d) = P_0 \sin^2 \left( \frac{\Delta \phi + \phi_0}{2} \right), \]  

(2.3)

where \( P_0 \) is the peak power at the output and \( \phi_0 \) is the phase difference between the two modes before etching. Figures 2.15 and 2.16 show plots of the simulated
power received at the detector as a function of $d$ for horizontal and vertical core fiber, respectively. In these plots we have set $\phi_0 = 0$. Different values of $\phi_0$ would change the phase of the plot, but not the number of oscillations. As the distance from the surface of the cladding to the fiber core decreases, the oscillations in the received power increase in frequency because the birefringence begins to change more rapidly as the core-to-flat distance decreases.

![Figure 2.15](image)

Figure 2.15: Plot of the simulated output power through the analyzer as a function of the distance from the top of the elliptical core to the flat for a horizontal core fiber.

Figures 2.15 and 2.16 show that by closely monitoring the oscillations in output power, the distance from the core to the flat can be tightly controlled. We use the plots to determine when to extract fibers from the etch. For a horizontal core fiber, if we extract the fiber when 3 full oscillations have occurred in the output power, we expect the core-to-flat distance to be 0.4 µm. Similarly, for vertical core fiber the core-to-flat distance should be just greater than 0.1 µm if the fiber is extracted from the etch after one full oscillation.
2.3.2 Experimental results

Figure 2.17 shows the etch setup used to monitor the birefringence. A Teflon etch frame holds the fiber so that a 1.5 cm section of the fiber is immersed in HF. Light of wavelength 1550 nm is launched into the fiber through a polarizer at a 45° angle relative to the major axis of the elliptical core of the fiber. This allows equal amounts of light to be launched into each of the two polarization modes. One mode is polarized along the major axis of the ellipse, and the other along the minor axis. An analyzer is placed at the output of the fiber, at a −45° angle with respect to the major axis of the ellipse, and a detector measures the transmitted power.

Figure 2.18 shows a plot of the output power as a function of time as the fiber is etched in BHF. The BHF used in these experiments is supplied by Transene Company, Inc. For the first 90 minutes of the etch, there is essentially no change in the output power, so this data has not been included. The fiber was etched for a total of 170 minutes. The oscillations in output power increase in frequency as the core-to-flat distance decreases. We believe that the signal becomes noisy after 3 1/2
Figure 2.17: Setup to monitor the birefringence of the fiber as it is etched, with a polarizer at the input and an analyzer at the output.

Oscillations have occurred because of high scattering and radiation loss when the fiber has been etched to this degree. This is also the reason for the decreasing amplitude of the oscillation peaks.

Figure 2.19 shows a SEM image of a fiber removed from the acid bath after three complete oscillations in the output power, corresponding to point (a) in Fig. 2.18. The core-to-flat distance measured with the SEM is 0.41 µm, as shown in the figure. This result shows that the numerical model is in good agreement with experimental results. Good agreement between the two means that we can accurately achieve predetermined core-to-flat distances, enabling the repeatable placement of structures within a certain distance of the fiber core.

2.4 Summary

This chapter has demonstrated the etching of D-fibers as the first step in creating in-fiber devices. The first etch method described in this chapter uses an in situ power monitoring technique which allows for tight control in removing the core of the fiber. This method provides a platform for the partial replacement of the core with other materials. Chapters 3 and 4 will discuss the partial replacement of the core with a polymer, and analyze devices that can be created in this manner.
Figure 2.18: Plot of the output power through the analyzer vs. time as a fiber is etched in BHF.

The second etch method described achieves tight control over etches which decrease the core-to-flat distance without etching the core of the fiber. This kind of etch involves monitoring the birefringence between the two modes in the fiber as it is etched. It allows for the creation of structures in or the placement of materials on the cladding just above the core, in the evanescent field of fiber modes. This method is the first step in the creation of SR-FBGs, which are described in Chapter 5.
Figure 2.19: SEM image of a fiber removed from the acid bath after three full oscillations in output power. The core-to-flat distance agrees well with numerical models.
Chapter 3

Polymer Waveguide Fabrication

In order to create an in-fiber device, light in the fiber must interact with the material that forms the basis of the device. For applications in which strong interaction is needed, we etch the fiber until the core is partially removed. When an active medium is then deposited into the region of the core that has been removed, light can be guided in this medium, resulting in strong interaction. For applications in which weaker interaction is desired, the fiber is etched until a predetermined amount of cladding remains above the core. The interaction then takes place in the evanescent field of the light in the core.

Creating devices within optical fibers by replacing the core yields several advantages. In-fiber devices are self-aligned, meaning the device is directly in the path of the optical field without any extra alignment steps. This decreases the loss and increases the efficiency of the device since much of the light guides in the active material. In-fiber devices can also be fusion spliced into optical networks, making the expensive pigtailling process unnecessary. Having the device inside the fiber also reduces the transmission loss through the device since the light does not have to be taken out of the fiber system and then coupled back in.

This chapter details the process of forming low-loss polymer waveguides within optical fibers. For maximum coupling between the unetched fiber and polymer waveguide sections, the polymer that replaces the core would have the same index of refraction and shape as the core. This would also ensure that a large fraction of the light would be guided in the polymer, resulting in a high-efficiency device. The greater the fraction of the light that is guided in the polymer, the greater the efficiency of
the device. The new waveguide must also support a single mode for efficient coupling into and out of the waveguide and for proper operation of polarimetric devices. To support a single mode, the waveguide dimensions must be small enough to cut off higher modes.

Most active polymers have higher indices of refraction than germania-doped glass, and the shape of the polymer waveguide is severely restricted by current etching and polymer deposition techniques. This gives rise to tradeoffs in polymer waveguide fabrication. The following sections detail how we confronted tradeoffs and fabricated a low-loss polymer waveguide that guides a significant fraction of the optical power within the polymer. Also presented are the numerical analyses that guided the polymer waveguide design process to create a low loss and high efficiency device.

3.1 Polymer application

To partially remove the core of D-fibers, the fiber is etched in HF acid as described in Chapter 2. This process leaves a 2 cm long groove along the flat side of the fiber into which polymer is deposited. The fiber is taped to a silicon wafer with the flat side of the fiber facing up. Polymer in solution is then applied to the fiber using a pipet. The wafer upon which the fiber is mounted is then spun in a standard commercial spinner. The spinning process coupled with surface forces spreads a uniform layer of polymer onto the flat surface of the fiber and forces a thicker layer into the groove left by the etch. The polymer in the groove, in combination with the remaining core and cladding, acts as a waveguide. Waveguide dimensions are dependent on many factors, such as polymer viscosity, spin speed, spin ramp time, and the profile of the removed core. We selected vertical core fiber for this type of waveguide. In vertical core fiber, the major axis of the elliptical core is perpendicular to the flat surface of the fiber, and therefore has a much higher aspect ratio when etched than does horizontal core fiber. The higher aspect ratio allows a greater thickness of polymer to pool in the groove left by the etch.

For the polymer we used poly(methyl methacrylate), or PMMA, as a host polymer with DR1 azo dye as guest chromophore. The chromophore is added the host
polymer for two reasons. First, it increases the polymer index of refraction, allowing a greater fraction of the light to be guided in the polymer, resulting in greater device efficiency. Second, the chromophore can be poled to make it optically active so that it can be used in electro-optic devices. We chose the mixture of PMMA and DR1 azo dye because it is inexpensive, readily available, and offers proof of concept. More sophisticated polymers would be needed for commercially viable devices because of the small electro-optic coefficient of the PMMA-DR1 combination.

Our basic solution consists of 0.6 g of DR1 azo dye and 8 g of PMMA dissolved in a solvent consisting of methyl ethyl ketone and chlorobenzene. We adjust the viscosity of the solution by controlling the ratio between the volume of the solvents and the mass of the solids. Once it is mixed we place the solution in a closed container on a stirring plate for 24 h to produce a uniform mixture. The solution is then filtered with a 0.2-µm filter. After application of the solution to a fiber by spin casting, the wafer and the fiber are placed in an oven at 90 °C to evaporate the remaining solvent. References [12, 13] give a more detailed procedure for creating and applying the polymer to a fiber.

### 3.2 Experimental results

The thickness of the polymer layer on the fiber is directly related to the viscosity of the polymer as well as inversely and more weakly related to the speed at which the polymer is spun onto the fiber. Since polymer thickness is more sensitive to viscosity, we chose to vary only the viscosity by increasing the solvent volume from 60 to 150 mL while holding the other variables constant: the spin rate at \( \sim 2000 \) rpm, the ramp time at a few seconds, and the fiber etch profile as shown in Fig. 3.1.

Table 3.1 shows the properties of three in-fiber polymer waveguides fabricated with different polymer solution viscosities. These properties include the amount of solvent used in creating the polymer solution, the measured insertion loss of the polymer waveguide after deposition in the etched fiber, and the thickness of the polymer on the flat surface of the fiber. We measured transmission loss by comparing the power transmitted through a fiber with a 2 cm in-fiber polymer waveguide section to
Figure 3.1: SEM image of the cross-section of an etched fiber showing about half of the core removed.

Table 3.1: Representative fabrication and loss parameters.

<table>
<thead>
<tr>
<th>Figure Label</th>
<th>Side Polymer Thickness (nm)</th>
<th>Loss (dB)</th>
<th>MEK (mL)</th>
<th>Chlorobenzene (mL)</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>177</td>
<td>1.6</td>
<td>37.5</td>
<td>112.5</td>
</tr>
<tr>
<td>b</td>
<td>695</td>
<td>36</td>
<td>30</td>
<td>90</td>
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<td>c</td>
<td>1131</td>
<td>$\infty$</td>
<td>15</td>
<td>45</td>
</tr>
</tbody>
</table>

the power transmitted through an unetched reference fiber. We measured polymer thickness using a SEM.

Figure 3.2 shows cross-sectional SEM images of the polymer waveguides listed in Table 3.1. Figure 3.2(a) shows a polymer waveguide produced from a low-viscosity polymer solution. The polymer thickness on the flat surface of the fiber is $\sim$200 nm. However, the aspect ratio of the etched fiber results in a thicker polymer layer in the groove ($\sim$700 nm) than on the flat surface of the fiber. Figures 3.2(b) and 3.2(c) show that once the polymer viscosity is above a certain value, any further increase in viscosity increases the polymer thickness in the groove and on the flat surface by the same amount. Thus, a thick layer of polymer in the groove is accompanied
by a thick layer of polymer on the flat surface. Of the three samples shown, only the sample in Fig. 3.2(a) has low enough loss for most applications. Once characterized, our fabrication process consistently produced low-loss samples similar to that shown in Fig. 3.2(a) with \( \sim 1.6 \text{ dB} \) of loss.

Figure 3.2: SEM images of the cross-section of a three etched fibers with polymer filling the core. The fabrication and loss parameters for these three in-fiber waveguides are listed in Table 3.1.

Figure 3.3 shows the electric field distribution of the fundamental \( x \)-polarized mode for an in-fiber polymer waveguide similar to the one shown in Fig. 3.2(a). The polymer index used in this simulation is 1.54, and the wavelength in 1550 nm. The contour lines that specify the magnitude of the electric field are overlaid onto an illustration of the in-fiber polymer waveguide. The field strength is high in the polymer material, although a significant portion of the power carried by the mode is outside of the polymer material. The greater the fraction of the field confined by the polymer, the higher the efficiency of the device based on the polymer waveguide.
Figure 3.3: Cross section of a polymer waveguide with an overlaid contour plot of the field intensity of the fundamental TE mode.

3.3 Sources of loss

There are a variety of factors that contribute to transmission loss through the polymer waveguide section. First, the electro-optic polymer in which the light is guided is lossy. It has a higher bulk absorption coefficient than the germania-doped glass core of the fiber. DR1 azo dye-doped PMMA has a bulk absorption coefficient of less than 1 dB/cm at a wavelength of 1300 nm [20].

Light scattering along the length of the polymer waveguide is another source of loss. Any large particles or rough surfaces incorporated into the light guiding region will cause light to scatter out of the waveguide. We can minimize this source of loss by keeping fibers free of contamination between the core removal step and the polymer application step.

In addition to these loss factors, Table 3.1 shows that there is a strong relationship between the polymer’s thickness on the flat surface of the fiber and the transmission loss. This loss has been attributed to the excitation of slab modes, as explained in more detail below. There is also transmission loss associated with the mismatch between the mode supported by the unetched fiber section and that
supported by the polymer waveguide. This loss can be reduced by ensuring that the transition to and from the polymer waveguide section is gradual. A discussion of the transition is also given below.

### 3.3.1 Loss from coupling to slab modes

Experiments consistently demonstrate that waveguides with thick polymer layers on the flat surface of the fiber have high insertion loss. These thick polymer layers result in high loss because they are thick enough to guide light in slab modes. These modes carry light away from the core region of the fiber.

We used BeamPROP\textsuperscript{TM}'s three-dimensional simulation capabilities to model propagation of light through the transition from the unetched fiber core to the polymer waveguide and then let the light propagate in the polymer waveguide section to see how it behaved. We performed two different simulations in which we butt-coupled an unetched section of fiber to a section which had had the core partially replaced with polymer. The first simulation models coupling from unetched fiber to core-replaced fiber having the cross-sectional profile shown in Fig. 3.4(a), with a very thin layer of polymer. The second models coupling from unetched fiber to core-replaced fiber having the cross-sectional profile shown in Fig. 3.4(b), with a very thick layer of polymer. Figure 3.4(a) is very similar to the cross-sectional SEM image shown in Fig. 3.2(b), and Fig. 3.4(a) is similar to the SEM image shown in Fig. 3.2(c).

Figure 3.5 shows a top view of the results of the three-dimensional simulation. An abrupt transition was made between the unetched section of optical fiber and the core-replaced sections illustrated in Fig. 3.4. The transitions occurred at the longitudinal position of $z = 10 \ \mu m$.

Figure 3.5 shows that for the thin polymer waveguide, much of the light guided in the unetched section couples out of the fiber at the transition between the two sections. This is because the mode overlap of the unetched section with the core-replaced section is very poor. Any light not coupled into the waveguiding core at the transition is quickly radiated away because the polymer on the flat side of the fiber is not thick enough to act as a slab waveguide. However, after the initial loss at
the transition, the light for the most part remains in the core region of the fiber, and there is very little loss along the core-replaced section.

Reference [21] contains a link to a video showing the full three-dimensional simulation of light propagating through a transition region into the polymer waveguide section for both thin and thick polymer waveguides. This video shows how light continuously radiates out of the thick polymer waveguide.

The simulation using a thick polymer layer shows much better coupling between the fiber core and the polymer waveguide. Figure 3.4(b) shows that in addition to the thicker polymer in the core region there is also a thicker layer on the flat surface of the fiber. Most of the light at the transition is coupled into the core-replaced waveguide because of the greater overlap between modes in the two sections. However, Figure 3.5 shows that this thick layer of polymer also results in continuous radiation away from the central region of the fiber along the length of the core-replaced section. This is because the thick polymer layer on the flat surface of the fiber supports slab waveguide modes. The thick polymer layer allows light to couple out of the core region of the polymer waveguide and into slab modes on the flat side of the fiber. Once coupled out of the core, light simply radiates away, increasing the loss of the device.
Figure 3.5: Simulations of light being coupled from an unetched section of fiber to a core-replaced section of fiber for thin (a) and thick (b) polymer layers. Light continuously radiates out of the central guiding region of the thick polymer waveguide giving it very high loss.

Figure 3.6 plots the power remaining in the thin and thick waveguides as a function of position. It demonstrates the high initial loss of the thin polymer waveguide, as well as the settling of power in the waveguide to a constant value. The plot shows that the thick polymer continues to radiate power all along its length. Note that the waveguides we have fabricated are 50 times as long as the simulated waveguide, so the continual radiation from the thick polymer waveguide would have very low total transmitted power after the full 2 cm.

The tradeoff here is that a thick polymer layer allows for good coupling from the unetched section to the polymer waveguide, but it also results in continual loss along the length of the polymer waveguide. The optimal solution therefore, is to create a polymer waveguide that is as thick as possible without supporting slab modes on the flat surface of the fiber. The ideal solution would be to deposit polymer only in the core of the fiber, but we have not found a straightforward method for accomplishing this. We therefore must keep the thickness of the polymer on the flat surface below the minimum thickness that supports slab modes.
We model the polymer layer on the flat surface of the fiber as an infinite slab waveguide with air as the cover medium, the polymer as the guiding medium, and glass as the substrate. Since the slab waveguide is asymmetric, it only supports guided modes if the guiding layer is thicker than a minimum value.

The cutoff thicknesses for various polymer indices give an estimate of the upper bound on the thickness of the polymer on the flat side of the optical fiber. The cutoff thickness is defined as the thickness at which the effective index of refraction of the fundamental mode falls below the index of refraction of the substrate.

Figure 3.7 shows a plot of the cutoff slab thickness versus polymer refractive index for 1550 nm light in an infinite slab waveguide. It shows that for a polymer index of 1.54 the TE mode cutoff thickness is 0.5 µm. To ensure that no slab modes were supported (neither TE nor TM) we kept the thickness of the polymer on the flat surface below 0.4 µm. This resulted in very low insertion loss.
Figure 3.7: Slab mode cutoff thicknesses versus polymer index for TE and TM modes. Air is the cover medium and glass is the substrate.

3.3.2 Loss at the transition

The field profile of the modes supported by the unetched optical fiber is different from the mode supported by the polymer waveguide. This mode mismatch contributes to the transmission loss of the device. One can reduce the loss associated with the mode mismatch by producing a gradual transition between the two waveguides [22].

The process used to chemically remove part of the core of the optical fiber results in a gradual transition between the unetched portion of the optical fiber and the partially removed core [11]. This gradual transition ensures that when the polymer is deposited onto the fiber, it too will change shape gradually in the transition regions.

The shortest possible transition is simple butt coupling of the polymer waveguide to the unetched fiber. Butt coupling analysis shows that with a thin polymer layer on the flat surface and with $\sim1$ µm of polymer in the core region, we can expect a total insertion loss of $\sim3$ dB [23]. Any transitions longer than this should result in decreased loss.
Butt coupling analysis was performed using the overlap integral, which is defined by \[ \eta = \frac{|\int \int E_1 \cdot E_2^* dS|^2}{\int\int |E_1|^2 \cdot dS \int\int |E_2|^2 \cdot dS} \] \[ (3.1) \]

$E_1$ and $E_2$ correspond to the electric fields in an unetched fiber and a fiber whose core has been partially replaced with polymer and $S$ represents the entire transverse plane. The overlap integral predicts the fraction of light that is coupled from the unetched fiber to the polymer waveguide at the discontinuity between the two.

Light propagation through transitions from fiber core to polymer waveguide and back into the fiber core was modeled with BeamPROP™'s three-dimensional simulation capability. We simulated the propagation of light through transitions of different lengths as an aid to understanding the effect of transition length on insertion loss. In this model we neglected the bulk absorption loss of the polymer. The model consists of a short unetched length of fiber followed by a transition region, a 2500 µm section of polymer waveguide, another transition, and a final length of unetched fiber 2500 µm long. A link to a movie showing a cross-sectional view of the fiber as one travels in the longitudinal direction can be found in Ref. [21].

Figure 3.8 shows the relationship between loss and propagation distance for waveguides with different transition lengths. The fiber with the longest transition (1100 µm in length) performs best, having only 0.5 dB of loss. The plot shows that the overall loss is approximately evenly divided between the transitions to and from the polymer waveguide.

3.4 Summary

This Chapter has described the creation of a low-loss polymer waveguide which partially replaces the core of a D-fiber. These waveguides are 2 cm long and the fabrication process consistently yields an insertion loss of less than 2 dB. Analysis has explained the sources of loss in polymer waveguides. This chapter also addresses some of the tradeoffs between waveguide efficiency and loss.

To demonstrate the feasibility of these waveguides in the creation of in-fiber devices, Chapter 4 discusses the fabrication and testing of a core-replaced temperature...
Figure 3.8: Plot of loss as light propagates through transitions of different lengths.

Sensor. It shows how partially replacing the core of a D-fiber significantly improves upon the sensitivity of elliptical core fiber to changes in temperature. I have also simulated the performance of an in-fiber polymer waveguide as a polarimetric electro-optic modulator in Appendix B.
Chapter 4

Core-replaced Fiber Sensors

Optical fiber sensors are playing an increasingly important role in many applications. They are used to monitor environmental conditions such as temperature, strain, pressure, and the presence or absence of chemicals [24]. They are having a tremendous impact on civil structures such as roads and buildings, on the aviation and navigation industries, and are beginning to play a more prominent role in national security.

The optical characteristics of fiber sensors change in response to changes in the environmental variables to be measured. Changes in the optical response of the fibers are then correlated to changes in the environmental variables. Figure 4.1 shows a schematic of a fiber sensor. The optical response may be a change in reflection/transmission spectrum, amplitude, phase, frequency or polarization. The majority of fiber sensors currently in use rely on the response of the intrinsic characteristics of the optical fiber. Fiber Bragg gratings (FBGs) are the most prevalent example of intrinsic fiber sensors [5], wherein the period of the grating and/or the effective indices of fiber modes are a function of environmental variables. Alternative methods of sensing involve coating a fiber with a sensing material and using evanescent field interaction [25] or long period gratings (LPGs) [26] to couple light from the fiber core to the coated cladding. This extrinsic method allows the detection of changes in index of refraction, the presence of chemicals, etc. which cannot be detected with FBGs.

This chapter describes a method for creating sensors in optical fiber by replacing a section of the core of a D-shaped fiber with a sensing material [27]. This
places the sensing material directly in the path of the beam propagating in the fiber, resulting in strong interaction between the light and the sensing material. Figure 4.2 shows a D-fiber with a sensing material replacing a section of the core to form an in-fiber sensor. This type of sensor allows a wide variety of sensing materials to be incorporated into a short section of the fiber without changing the rest of the fiber.

4.1 In-fiber sensor formation via core removal and replacement

The first step in the fabrication process of core-replaced fiber sensors is to partially remove the core of the fiber as outlined in Chapter 2. Figure 4.3 shows the cross section of a horizontal core fiber that has been etched in HF acid. The core has been partially removed and the fiber is ready for polymer deposition.
Depositing sensing material into the groove creates a new waveguide which interacts strongly with the light traveling through it. For best results, Chapter 3 points out that this new waveguide should be single-mode, have low loss, and have as much power guided in the sensing material as possible to maximize interaction strength. The efficiency of the waveguide is highest when the amount of sensing material in the core region is maximized while still maintaining single-mode operation. Chapter 3 details the process of depositing a polymer into the partially removed core of the fiber. Figure 4.4 shows a scanning electron microscope (SEM) image of the profile of a low-loss core-replaced fiber fabricated with a polymer as the sensing material.

4.2 In-fiber temperature sensing

In order to test the performance of this core-replaced in-fiber sensor technique we deposited poly(methyl methacrylate) (PMMA) doped with DR1 azo dye into the 2 cm long groove left by the HF acid etch. PMMA was chosen as the sensing material for these experiments because it was readily available and because the change in its bulk index of refraction with temperature ($\frac{dn}{dT}$) is much greater than that in germania-doped silica, allowing for the creation of a simple temperature sensor. The PMMA is doped with DR1 azo dye to increase its index of refraction so that more optical power is confined within the polymer when it is deposited into the core region. The efficiency of the sensing device is then governed by two effects. It is
directly related to the amount of power guided in the sensing material and also to the geometry of the waveguide since the birefringence is purely a geometrical effect.

The simplest method of correlating changes in bulk index of refraction with temperature is to place the temperature sensor in a polarimetric configuration. The only pieces of equipment needed to create a polarimetric sensor are two bulk polarizers which are placed at the input and output of the fiber. We compare the performance of a so-called polarimetric thermal sensor made from an unetched D-fiber to one made of a D-fiber whose core has been partially removed and replaced with polymer. While not ideal for temperature sensing because of the sinusoidal temperature response, the polarimetric configuration offers a simple and easily implementable method to measure the performance of an in-fiber waveguide sensor and gauge its performance relative to that of an unetched fiber [28], offering proof of concept for core-replaced sensors.
4.2.1 Polarimetric thermal sensing

Elliptical-core fiber supports light in two orthogonal modes, linearly polarized along the major and minor axes of the core. The two polarization modes have different effective indices and propagation constants. The fiber is therefore birefringent and a relative phase difference accumulates between the two modes as they propagate. This phase difference is fixed for a given length of fiber at a given temperature. Depositing PMMA into the groove of an etched fiber as in Fig. 4.4 maintains the birefringent nature of the waveguide because the asymmetry of the fiber is preserved.

The bulk indices of refraction of the different materials composing the fiber sensor respond differently to changes in temperature. Changes in bulk indices of refraction cause changes in the propagation constant of each of the two modes traveling through the fiber. The propagation constants of the two modes change at different rates with temperature so the birefringence also changes with temperature. This change in birefringence can be detected by placing the fiber between a polarizer and an analyzer. The optical power detected through the analyzer varies according to the change in the phase difference. This property can be used to create a temperature sensitive device in elliptical core D-fiber.

Figure 4.5 shows how polarizers are placed at the input and output of the fiber. The input polarizer is placed at the input of the fiber at a 45° angle with respect to the major axis of the core. This angle allows equal amounts of power to be launched into both polarization modes. The analyzer is placed at the output of the fiber at a −45° angle with respect to the major axis of the fiber, perpendicular to the input polarizer. These angles of orientation allow for the greatest extinction ratio in the output power.

As the fiber is heated, the effective indices of the two modes change, resulting in a change in the birefringence given by

\[
\Delta B = (N_{x,T} - N_{y,T}) - (N_{x,0} - N_{y,0}),
\]  

(4.1)

where \(N_{x,T}\) and \(N_{y,T}\) are the effective indices of the two polarization modes at temperature \(T\) and \(N_{x,0}\) and \(N_{y,0}\) are the effective indices of the modes at room temperature.
Figure 4.5: Illustration of a polarimetric in-fiber polymer waveguide sensor showing the waveguide placed between a polarizer and an analyzer.

The change in relative phase difference between the two polarizations of light in the fiber is directly proportional to the change in birefringence and is given by

\[ \Delta \phi = \frac{2\pi}{\lambda} \Delta BL, \]  

(4.2)

where \( L \) is the length of the fiber being heated and \( \lambda \) is the free space wavelength. Note that thermal expansion of the fiber has been ignored because the change in birefringence contributes much more than the change in fiber length \([9]\). The optical power of the light at the output is then given by \([19]\)

\[ P = P_0 \sin^2 \left( \frac{\Delta \phi + \phi_0}{2} \right), \]  

(4.3)

where \( P_0 \) is the peak power at the output and \( \phi_0 \) is the phase difference between the two modes at room temperature. The output power can be correlated with temperature so that the system can then be used as a temperature sensor.

### 4.2.2 Numerical simulations

PMMA has much greater temperature sensitivity than the fiber core with bulk values of \( \frac{dn}{dT} \) being \(-2.6 \times 10^{-6}(^\circ C)^{-1}\) for the germania-doped core, \(1 \times 10^{-5}(^\circ C)^{-1}\) for fused silica \([30]\), and \(-1.2 \times 10^{-4}(^\circ C)^{-1}\) for PMMA. Because of the greater temperature sensitivity of the PMMA, core-replaced fiber sensors have a much greater change in birefringence for a given temperature change than does unetched fiber.
In order to simulate the performance of the temperature sensor relative to an unetched fiber we used the mode solver in the software package BeamPROP™ to compute the propagation constants of each of the two modes at different temperatures. Figure 4.6 shows an image of the cross-sectional models used to predict the birefringence of in-fiber polymer waveguides as a function of temperature. The birefringence of this type of polymer waveguide is due to the geometry of the waveguide and not to any intrinsic material birefringence. Because of this, the shape of the deposited polymer is important. In order to have significant geometric birefringence, the guiding region must be asymmetric.

![Cross-sectional models](image)

(a) Vertical core  
(b) Horizontal core

Figure 4.6: Cross-sectional models used to predict the birefringence of polymer waveguides.

Figure 4.7 shows a plot of how the birefringence changes from the room-temperature value for four different types of fiber. The change in birefringence for the polymer waveguides is denoted by the letter P, whereas the unetched fibers are denoted by the letter U. The cross-sectional geometries of the polymer waveguide models is shown in Fig. 4.6. The change in birefringence, $\Delta B$, is greatest for the polymer waveguide based on vertical core fiber. Unetched fibers have a much smaller
\( \Delta B \) than the polymer waveguides, and \( \Delta B \) is even negative for unetched vertical core fiber. The reason for the positive values of \( \Delta B \) in the other three fibers is that the long axis of the guiding material in each of them is in the \( x \)-direction.

Figure 4.7: Plot of the change in birefringence as a function of temperature for core-replaced fibers (P) and unetched fibers (U) with cores oriented vertically and horizontally.

To match the experiments outlined later in this chapter, the simulations modeled sensors which were 12 cm long. The reason for the 12 cm length is that the fibers were taped to 4 inch silicon wafers and they were heated by placing the wafer on a hot plate. For the core-replaced sensor, 2 cm of this length had the core partially replaced with PMMA, while the remaining 10 cm were composed of unetched fiber. The change birefringence from the 10 cm of unetched length adds with the change from the polymer waveguide section. Because of the negative change in birefringence in unetched vertical core fiber, the net \( \Delta B \) is smaller in vertical core polymer waveguides than in horizontal core polymer waveguides because the positive and negative values of \( \Delta B \) partially cancel each other out. The result is that
horizontal core polymer waveguides as we have fabricated them are more sensitive to temperature than vertical core polymer waveguides. However, if only the core-replaced section is heated, Fig. 4.7 indicates that vertical core fiber could be more sensitive because of the greater sensitivity of the birefringence to temperature.

Figure 4.8: Plot of the expected output power of a polarimetric thermal sensor based on a polymer waveguide (solid line) and an unetched fiber (dashed line).

Figure 4.8 shows the expected output power of polarimetric thermal sensors based on a horizontal core polymer waveguide and an unetched horizontal core fiber. The simulation shows that core-replaced sensors were about 3 1/2 times more sensitive than unetched D-fiber sensors. Simulations of a core-replaced sensor having the entire 12 cm length filled with polymer showed an improvement in sensitivity by a factor of 21 over a completely unetched 12 cm sensor. In the following section we show experimentally that replacing the partially removed core of a D-fiber with PMMA does improve significantly upon the sensitivity of an unetched fiber.
4.3 Experimental verification of core-replacement technique

4.3.1 Experimental setup

To create a polarimetric temperature sensor, a groove approximately 2 cm long is etched into a fiber as described in Chapter 2. The etch depth was chosen such that about half of the core was removed as shown in Fig. 4.4. The fiber is then taped to a silicon wafer and PMMA is applied according to the process described in Chapter 3. This process leaves a thicker layer of PMMA in the core region of the fiber than on the flat surface of the fiber because the PMMA pools somewhat in the etched-out groove. This process results in a polymer thickness of \( \sim 700 \) nm in the core of the fiber. With the core etched to the proper depth and the desired amount of polymer applied, a low-loss waveguide is formed.

To heat the fiber, the wafer with the fiber attached is placed on a hot plate such that only the section of fiber on the wafer plus about 1 cm on each side of the wafer is heated. About 12 cm of fiber is heated, including the 2 cm region with polymer. The rest of the fiber is not in contact with the hot plate or wafer, and so it stays essentially at room temperature. A reference fiber is mounted on the wafer next to the fiber with the polymer waveguide. The reference fiber is left unetched, but a 2 cm section of the jacket is removed to ensure similar heating conditions for both the reference fiber and the fiber with the polymer waveguide.

A 1550 nm wavelength laser is focused into one end of the fiber, and a detector is placed at the output. A polarizer is placed at the input and an analyzer at the output of the fiber as described in the previous section. A LabVIEW program interfaces with the detector and a thermocouple to monitor output power as a function of temperature. Changes in the birefringence of the light through the fiber cause the output power to vary according to Eq. (4.3) as the hot plate heats the fiber from 20°C to about 95°C.
4.3.2 Experimental results

Figure 4.9 shows a plot of the output power from the reference fiber as a function of temperature. The output power from this fiber does not go through a full period over this temperature range. For the temperature range used, the change in the phase difference is linear \[31\], which means that the measured optical power varies periodically with temperature.

![Plot of output power as a function of temperature for an unetched fiber.](image)

Figure 4.9: Output power as a function of temperature for an unetched fiber.

Figure 4.10 shows a plot of the output power using a fiber with polymer replacing a 2 cm section of the core. A SEM image of the cross-section of the fiber used to generate this data is shown in Fig. 4.4. For the given temperature range, the power curve from the core-replaced fiber goes through about three and a half cycles. This is about five times greater than in the reference fiber, showing a large increase in temperature sensitivity due to the core-replacement technique. This factor of improvement is roughly in agreement with the numerical simulations presented earlier in this chapter. The oscillations become more frequent at higher temperatures. This
indicates that the change in birefringence in the polymer waveguide does not remain linear with temperature. This could mean that the change in index of refraction with temperature of the waveguide materials is nonlinear.

![Graph showing output power as a function of temperature for a core-replaced fiber.](image)

Figure 4.10: Output power as a function of temperature for a core-replaced fiber.

### 4.4 Summary

This chapter describes how the partial replacement of the core of a D-fiber greatly enhances its sensitivity to temperature. This is demonstrated by a polarimetric temperature sensor. It shows how core replacement can serve as the basis for a number of different sensors because the technique is amenable to a variety of sensing materials. For example, the response of other materials to environmental stimuli could be changes in index of refraction, absorption, or fluorescence. Core-replaced fiber sensors are also amenable to many different detection schemes, depending on the response of the sensing material to the measurand. For more discussion of core-replaced sensors, see Ref. [13].
Chapter 5 also addresses sensing with D-fiber, but these sensors are based on the etching of surface relief Bragg gratings into the flat side of the D-fiber. Placing a grating on a fiber results in a narrowband filter/mirror, whose reflection wavelength changes in response to environmental stimuli.
Chapter 5

Surface-relief Fiber Bragg Gratings on D-fibers

The most widely deployed type of fiber sensor is the fiber Bragg grating (FBG) sensor [5]. FBGs reflect light in a narrow wavelength band about the Bragg wavelength, and transmit light of other wavelengths. The reflection band shifts with changes in temperature, strain, etc. Shifts in the reflection peak are then correlated with the environmental variable to be measured to create a sensor. Standard FBGs are fabricated by exposing optical fibers to an intense ultraviolet (UV) interference pattern. This exposure induces a periodic refractive index modulation in the fiber by modifying the structure of the core material [32]. This chapter presents an alternative method for fabricating Bragg gratings in fibers by etching the gratings into the surface of a D-shaped optical fiber [33].

Figure 5.1 shows typical reflection and transmission spectra of a fiber Bragg grating with a sinusoidal index modulation using an analytical formula from Ref. [34]. In this formula, I used an index modulation of $5 \times 10^{-4}$ and a grating length of 10 cm. The $x$-axis shows deviation from a central wavelength of 1550 nm, and the $y$-axis shows normalized power.

One of the drawbacks of standard FBG sensors is that their performance decays when they are exposed to temperatures above $\sim 200$ °C [35] because the core modifications caused by UV exposure begin to anneal out. Loading fibers with hydrogen makes gratings decay at even lower temperatures [36]. The thermal stability of FBGs has been improved upon by various methods, such as doping fibers with tin [37] or writing gratings with a single excimer laser pulse [38], but these types of FBGs also decay quickly above 800 °C.
Surface relief fiber Bragg gratings (SR-FBGs) can be used in the same applications as conventional FBGs for temperature, strain, and pressure sensing. The asymmetry of the grating structure (it is on one side of the core) along with the elliptical shape of the core could also possibly allow etched gratings to be used in applications not possible with standard FBGs, such as multiaxis strain measurement \[39\]. Additionally, because SR-FBGs are written into the surface topography of the fiber, they are able to function at high temperatures, possibly up to the melting point of the glass of which the fiber is made.

Previous research has demonstrated the fabrication of etched gratings on the flat side of D-fibers \[40\], but these gratings were used to couple light out of fibers. This dissertation extends previous work by demonstrating the fabrication of etched gratings with a shorter period, short enough to create Bragg gratings with a reflection peak centered at a wavelength of 1550 nm.
5.1 Surface relief Bragg gratings

The reflectivity of a fiber Bragg grating depends on the length of the grating, the amplitude of the periodic index modulation, and the amount of overlap between the grating and the field of the guided modes in the fiber. The overlap is quantified as the fraction of the modal field that falls in the physical area where the index modulation occurs. Increasing the grating length, the index modulation amplitude, or the overlap each serve to increase the reflectivity of a FBG. The details of calculating the reflectivity of a grating are given in Appendix D.

The germania-doped core of standard optical fibers is sensitive to high-intensity ultraviolet (UV) light. Various techniques have been developed to increase the photosensitivity of optical fiber cores [41, 42]. Exposure of fibers with a high-intensity periodic pattern of UV light changes the refractive index of the exposed areas of the core by a small amount, anywhere from $10^{-5}$ to $10^{-2}$ [43]. The periodic modulation of the core index of refraction results in the creation of a fiber Bragg grating. Because the index modulation occurs over the entire cross-section of the fiber core, the overlap between the grating and the fields in a standard FBG is large. Figure 5.2(a) shows an illustration of a standard FBG fabricated in a circular fiber, while Fig. 5.2(b) shows a surface relief FBG fabricated on D-fiber.

SR-FBGs are fabricated in the cladding above the fiber core or just into the top of the core and are shallow compared to the core dimensions. The fields of guided modes in a fiber are evanescent outside of the core, meaning that they decay exponentially in the cladding region. The gratings we fabricate are small and are primarily in the evanescent field of modes in the fiber, so their overlap with the fields is small. SR-FBGs therefore need to be fabricated close to the core in order to have high reflectivity. The large index modulation ($\sim 0.45$) of SR-FBGs can overcome their small overlap with the fields in the fiber if the grating is close to or within the core. This index modulation is much greater than in standard FBGs.

In order to determine how close to the core we should place etched gratings, we numerically simulated their performance. The coupled-mode equations describe how a forward traveling mode couples to a backward traveling mode in a fiber Bragg
To model the performance of SR-FBGs I solved the coupled-mode equations numerically with a Runge-Kutta solver. One of the key parameters in the coupled-mode equations is the coupling coefficient, which determines how strongly light is coupled from the forward traveling mode into the reverse traveling mode. It is proportional to the fraction of the power of a given mode that falls in the region of the grating. Figure 5.3 shows a cross section of a D-fiber as well as a side cutaway of a grating. By integrating the modal power over the area shown from the peak of the grating to its trough, one can compute the coupling coefficient.

Appendix D discusses the solution of the coupled-mode equations in much more detail. The numerical solver predicts the amount of power an etched Bragg grating reflects as a function of several variables, including the distance from the top of the core to the center of the grating, the height of the grating, the grating period, polarization state, and the wavelength of light. The solver can also accommodate non-sinusoidal grating profiles, chirp, and apodization.
Figure 5.3: Image of a D-fiber with a grating above the core. The image indicates the region over which the modal power is integrated to find the coupling coefficient.

Figure 5.4 shows the results of a computation using the coupled-mode equations solver. It plots the power reflected at the Bragg wavelength as a function of the core-to-flat distance for the two orthogonal polarizations and two core orientations. The overall length of the gratings in these simulations is 1.5 cm. The peak-to-trough depth of the surface relief gratings simulated in this figure is 100 nm. Vertical core fiber has a much shallower notch in the transmission spectrum because the field of the fundamental mode in vertical core fiber has a smaller overlap with the grating than does the fundamental mode in horizontal core fiber.

Figure 5.4 also shows that $y$-polarized light is reflected less strongly than $x$-polarized light. Figure 5.5 shows a plot of the difference between the magnitudes of the transverse electric fields of the $x$- and $y$-polarized modes for a horizontal core fiber with a core-to-flat distance of 80 nm. The fields are normalized so that the maximum field value over the cross section of the fiber is 1. Positive values on the plot represent areas where the field of the $x$-polarized mode is more intense than the $y$-polarized mode and vice versa for negative values. Because it has less field concentrated in the area of the grating, the $y$-polarized mode reflects less strongly than the $x$-polarized mode.

Because of its greater efficiency, we decided to use horizontal core fiber to create SR-FBGs. Figure 5.4 shows that a 1.5 cm-long SR-FBG with a grating depth
of 100 nm on a horizontal core fiber needs the core-to-flat distance to be less than \( \sim 0.7 \ \mu m \) to achieve a transmission notch greater than 30 dB deep.

### 5.2 Grating fabrication

To create SR-FBGs on D-fiber, we first remove the cladding above the core in a bath of HF or BHF to give access to the light guided in the fiber. We then form a surface relief grating in photoresist on the flat side of the fiber. A reactive ion etcher (RIE) directionally etches the photoresist and glass, transferring the grating into the fiber, which allows it to be used as a sensor. Figure 5.6 shows a diagram of a D-fiber in five successive stages of the fabrication process. The top image is of an unetched
Figure 5.5: Plot of the difference, $|E_{t,x}| - |E_{t,y}|$, between the transverse electric fields of the $x$- and $y$-polarized modes.

fiber, then of a fiber etched to remove the cladding above the core. Photoresist has been applied to the third fiber and the fourth shows a grating on the fiber surface after photoresist development. The bottom image is of a fiber having a surface relief grating on the flat surface above the core after a RIE transfers the grating from the photoresist into the fiber.

The simulations described above show that in order to create gratings with high reflectivity, they must be placed very close to the core or even into the core. We initially tried to place gratings into the top of the core for high reflectivity. However, we found experimentally that photoresist did not adhere well to fibers that had been etched long enough to expose the core. We therefore decided to terminate the etch before it reaches the fiber core, which necessitated the change from the \textit{in situ} power monitoring technique to the birefringence monitoring etch technique. Monitoring the birefringence between the two modes in the fiber allows us to controllably monitor the etch and terminate it when a predetermined amount of cladding is left above the core. The birefringence monitoring technique is described in detail in Chapter 2. In
order to create high efficiency SR-FBGs, we remove fibers from the etch when the core is \( \sim 0.4 \mu m \) from the flat side of the fiber.

### 5.2.1 Photoresist application, exposure, and development

After the cladding has been etched so that the core is near the surface of the fiber, the fiber is mounted on a silicon wafer with the flat surface of the fiber facing up. The fiber is placed in the RIE for 30 seconds using CF\(_4\) gas at a power of 250 Watts to passivate the fiber surface for the application of photoresist. We added
this RIE step to the process in order to remove the very thin layer of the fiber surface which had been exposed to BHF. Fibers that had not been etched demonstrated much better photoresist adhesion than fibers etched in BHF, so we concluded that the BHF was altering the fiber’s surface properties. The added 30 second RIE step improved photoresist adhesion. To further prepare the fiber surface it is soaked in the adhesion promoter SurPass 4000 for 1 minute, rinsed in deionized water, and blown dry with nitrogen.

With the fiber surface prepared we apply Shipley 1.2L photoresist over the length of the etched section of the fiber. The resist is thinned by mixing 2 parts photoresist to 1 part thinner. Thinning is necessary to reduce the thickness of the photoresist grating. If the photoresist layer is too thick, the gratings are nearly washed out by the RIE before they can be transferred into the fiber. This happens because the RIE etches somewhat isotropically. After applying photoresist, the wafer is spun in a commercial spinner for 1 minute at 6000 rpm. After spinning we soft bake the fiber for 1 minute at 90 °C.

The next step is to expose the photoresist with two interfering beams of light to create a surface relief grating in the photoresist on the flat side of the fiber. Figure 5.7 shows the Lloyd’s Mirror arrangement that we use to expose the photoresist [40], in which the mirror and the fiber meet at a right angle. Half of the incident beam falls on the fiber. The other half falls on the mirror, which reflects it onto the fiber. The two halves of the beam combine to create an interference pattern on the surface of the fiber. We use a Lloyd’s mirror configuration because it overcomes the problem of small vibrations of optical components, which can wash out a two-beam interference pattern, by having the mirror and the fiber firmly connected to the same base.

We design the gratings on the fibers so that the Bragg wavelength, \( \lambda_B \), is 1550 nm. The resulting grating period, \( \Lambda \), is given by

\[
\Lambda = \frac{\lambda_B}{2N},
\]

where \( N \) is the effective index of the mode in the fiber. The effective indices of both
polarizations are very close to 1.45, so the grating period should be 534 nm in order to achieve a reflection peak at 1550 nm.

For the incident beam we use the 363.8 nm line of an argon ion laser. The recorded grating period is given by

$$\Lambda = \frac{\lambda}{2 \sin \theta},$$

where $\theta$ is the angle between the normal to the fiber and the interfering beams. To achieve a grating period of 534 nm, the two beams need to be incident onto the fiber at an angle of $\theta = 19.9^\circ$. The photoresist on the fiber is exposed so that the total dose in the most intense portion of the beam is about 20 mJ/cm$^2$.

Figure 5.7: Top view of a Lloyd’s mirror arrangement for creating an optical interference pattern on the fiber.
After exposure, we develop the resist on the fiber in Shipley 1.2L developer for 10 seconds then rinse it thoroughly with deionized water. At this point, there is a sinusoidal photoresist grating on the fiber with a peak-to-trough distance of ∼300 nm. Drying with nitrogen and then hard baking the fiber for 5 min increase the stability of the photoresist grating.

### 5.2.2 Transfer of the grating into the fiber

To transfer the grating from the photoresist into the flat side of the fiber we place the fiber in the RIE for 60 seconds using just CF$_4$ gas with 250 W of RF power. This step directionally etches both the resist and the glass, transferring the sinusoidal surface relief grating into the flat side of the fiber. To complete the process, any photoresist remaining on the fiber is removed with acetone and isopropyl alcohol.

### 5.3 Experimental results

Once SR-FBGs are fabricated, we measure both their physical dimensions and transmission spectra. To measure their physical dimensions we use both scanning electron microscopy (SEM) and atomic force microscopy (AFM). SEM images show a top view of the grating structure over large areas and AFM images quantitatively show the depth and structure of individual grating ridges. We measure the transmission spectrum of each grating with an optical spectrum analyzer (OSA).

Figure 5.8 shows a SEM image of a grating etched into the surface of a D-fiber using the procedure outlined above. While there are some irregularities in the grating, its structure is very distinct. The gratings cover a large portion of the flat surface of the fiber, but there are intermittent areas where they are nearly washed out. Figure 5.9 shows surface height data taken with an AFM as it scanned over the surface of the same etched grating shown in Fig. 5.8. The data was low-pass filtered to get rid of high-frequency noise. The plot shows that the grating is ∼80 nm deep from peak to trough.

To test the performance of these gratings we launch a broad band amplified stimulated emission (ASE) source into the fibers and measure the transmission
spectrum using an OSA. At the Bragg wavelength, $\lambda_B$, there is a sharp dip in the transmission spectrum. This occurs because the grating back-reflects light in a narrow band about $\lambda_B$. The depth of the transmission notch varied from fiber to fiber, but the maximum notch depth we were able to see was 14 dB below the power value just to the left or right of the notch. Figure 5.10 shows the transmission spectrum of a SR-FBG. The notch is $\sim 8$ dB below the off-Bragg wavelength value and the notch linewidth is $\sim 0.3$ nm when measured 3 dB below the off-Bragg power.

5.3.1 Temperature sensing

The key to using FBGs as sensors is to correlate changes in the reflection/transmission spectrum with the environmental variable being measured. Different stimuli affect $\lambda_B$ in different ways. For example, $\lambda_B$ shifts with increases in
temperature because of two different effects. First, changes in the bulk indices of refraction of the fiber materials change the effective indices of modes in the fiber, and second, thermal expansion causes the period of the Bragg grating to increase. Taking the partial derivative of the Bragg wavelength with respect to temperature gives

$$\frac{\partial \lambda_B}{\partial T} = 2 \left( \frac{\partial N}{\partial T} \Lambda + N \frac{\partial \Lambda}{\partial T} \right)$$

$$\frac{\partial \lambda_B}{\partial T} = 2 \Lambda (\xi + N \alpha),$$

(5.3)

where $\Lambda$ is the grating period, $\xi$ is the temperature sensitivity of the effective index of refraction, $N$, of the guided mode, and $\alpha$ is the coefficient of thermal expansion.

To test the performance of the SR-FBG as a sensor, we measured the shift in $\lambda_B$ as a function of temperature. Figure 5.11 shows a plot of the shift of $\lambda_B$ for a SR-FBG with temperature. All shifts are referenced to $\lambda_{B,20} = 1552.6$ nm, the Bragg wavelength at a temperature of 20 $^\circ$C. The asterisks represent data points and the line on the plot shows the linear fit to those data points. The slope of the line is 15.9 pm/$^\circ$C. The data show that $\lambda_B$ monotonically increases with temperature.

The insets in Fig. 5.11 show the transmission spectrum of the fiber at three different temperatures, 50 $^\circ$C, 80 $^\circ$C, and 110 $^\circ$C. The depth of the notches in the inset transmission spectra is $\sim$8 dB, and the wavelength range of the horizontal axis
on the three insets is from 1550 nm to 1555 nm. Once they are calibrated, SR-FBGs can be used as temperature sensors by detecting the location of either the null in the transmission spectrum or the peak in the reflection spectrum.

### 5.3.2 High temperature sensing

To perform high temperature tests of SR-FBGs, instead of using the hot plate we threaded a fiber through an 18 inch-long steel tube and placed the tube in a tube furnace, making sure that the grating was at the center of the furnace to ensure maximum heating. Figure 5.12 shows the experimental setup for high temperature testing. To measure the temperature of the fiber during heating we placed a long thermocouple probe within the tube. For the heating process, the fiber was heated from room temperature up to 1100 °C. For each increase in temperature we dialed in the new temperature on the furnace, waited for 30 minutes for everything to reach equilibrium, then measured the temperature and Bragg wavelength. Data were also taken as we let the oven cool down. Figure 5.13 shows the resulting data. The Bragg wavelength seems to increase quite linearly over the temperature range. The black
asterisks show temperature data taken while the fiber was heating up and the red circles show the data as the fiber was returning to room temperature. The line on the plot is the linear fit to the black asterisks. The slope of the line is 16.2 pm/°C. This value is close to the value obtained from Fig. 5.11 and is somewhat larger than the value of 10.3 pm/°C for commercial FBGs [46], possibly because the core of a D-fiber has greater germania doping than commercial fiber.

Figure 5.12: Setup for testing SR-FBGs at high temperature.
5.3.3 Strain sensing

Other factors, such as stress and strain also cause changes in the effective indices of modes, causing changes in $\lambda_B$. The Bragg wavelength shifts with strain according to [46]

$$\Delta \lambda_B = 2n\Lambda \left(1 - \left(\frac{n^2}{2}\right) \left[P_{12} - \nu(P_{11} + P_{12})\right]\right) \epsilon,$$

(5.4)

where $\epsilon$ is the applied strain, $P_{i,j}$ are the Pockel’s coefficients of the stress-optic tensor, and $\nu$ is Poisson’s ration.

To test a SR-FBG as a strain sensor, we glued it to a thin sheet of aluminum, 60 cm x 12 cm. We also placed four strain gauges on the aluminum sheet.
in a Wheatstone bridge configuration to have a calibration standard for the sensor. An ASE source launched broadband light into the fiber, and an OSA measured the transmission spectrum. To apply strain to the fiber we fixed one end of the aluminum sheet and deflected the other end. Figure 5.14 shows a diagram of the setup for applying strain to the fiber.

![Diagram of setup for applying strain to a fiber and measuring the transmission spectrum.](image)

Figure 5.14: Diagram of setup for applying strain to a fiber and measuring the transmission spectrum.

Figure 5.15 shows a plot of the shift in the Bragg wavelength of the fiber as a function of applied strain. Strain is defined as the change in the length of the fiber divided by the length of the fiber and is measured in microstrain (µϵ). Transmission spectra were obtained by deflecting one end of the aluminum sheet down from the horizontal in 1 cm increments, while keeping the other end fixed. Later, once the strain gauges were calibrated, we measured the amount of strain applied at each of these 1 cm increments. We did this by setting the zero point of strain to occur when the sheet was horizontal and then deflecting the sheet in 1 cm increments. The plot shows the resulting data, along with three insets showing transmission spectra for three different amounts of applied strain. All shifts in $\lambda_B$ are relative to a wavelength of 1542.56 nm. The slope of the linear fit line in Fig. 5.15 is 1.57 pm/µϵ, whereas
the strain sensitivity of commercial FBGs is listed in Ref. [46] as 1.2 pm/µε at a wavelength of 1550 nm.

![Plot of the shift in the Bragg wavelength as a function of applied strain.](image)

The strain sensitivity of commercial FBGs is listed in Ref. [46] as 1.2 pm/µε at a wavelength of 1550 nm.

**Figure 5.15:** Plot of the shift in the Bragg wavelength as a function of applied strain.

One of the benefits of using polarization maintaining fiber in the creation of FBGs is that the fiber is sensitive to multiple strain axes. Because of their different effective indices of refraction, the two polarization modes in a fiber have a different Bragg wavelength. The separation between the two peaks can be found from

\[
\lambda_{B,x} - \lambda_{B,y} = 2(N_x - N_y)\Lambda \\
\Delta \lambda_B = 2B\Lambda,
\]

where \( B \) is the birefringence. For a 535 nm grating written onto a horizontal core fiber etched so that the core-to-flat distance is 0.5 µm, Fig. 2.13 shows that the birefringence is \( \sim 4.5 \times 10^{-4} \), yielding a peak separation of 0.48 nm. When longitudinal strain is applied to a fiber, the peaks shift in the same direction. However, when transverse strain is applied, the peaks shift in opposite directions [39], allowing for the detection of multiple strain axes simultaneously. Because its low efficiency, we
have not yet been able detect a notch in the transmission spectrum of the $y$-polarized mode. Multiaxis strain measurement remains a goal of this research.

5.4 Summary

This chapter has introduced the surface relief fiber Bragg grating (SR-FBG), providing details of the analysis and fabrication procedure. This type of sensor has two properties which set it apart from standard FBGs – it is etched into the surface of the fiber, and it has an elliptical core. The fact that the grating is etched into the surface topography of the fiber allows the grating to survive up to very high temperatures, at least 1100 °C. This is much higher than other existing FBG sensors. The elliptical core of the fiber will also possibly allow it to be used in sensing multiple measurands or multiple components of strain simultaneously.
Chapter 6

Conclusion

The advantages of optical fibers in communications and sensing are many. They have low loss, high bandwidth, immunity to electromagnetic interference, many signals can be multiplexed on a single fiber, and they are lightweight. These advantages make optical fibers attractive in the creation of optical devices for many applications.

6.1 Contributions

This dissertation describes several different optical devices based on a unique specialty fiber – elliptical core D-fiber. The principal benefit of D-fiber is that the light in the core of the fiber can be easily accessed by removing the cladding above the core. This work shows how D-fiber can be a platform for a wide variety of in-fiber optical devices, outlines fabrication steps, and provides analysis of D-fiber based devices.

Creating devices on D-fibers represents a shift away from standard integrated optical fabrication techniques. This shift brings new flexibility in that structures or materials can be placed very close to the fiber core or can even replace it. The flexibility to incorporate a myriad of structures or materials into a fiber is where D-fiber has its great advantage.

The first contribution of this work is the analysis and refinement of the D-fiber etch process, which enables structures or materials to be placed near the fiber core. The ability to repeatably etch a predetermined amount of the fiber is very important in the creation of devices. Two etch procedures are analyzed and
demonstrated. One partially removes the core so that a material can be inserted into the groove left by the etch and light can then guide in that material. The other controllably removes the cladding above the core to reduce the core-to-flat distance by a predetermined amount. This technique allows evanescent interaction with light in the fiber core.

The second contribution of this work is the analysis and demonstration of a low-loss in-fiber polymer waveguide. This waveguide replaces some of the partially removed core of the fiber. The analysis has explained how to achieve low loss in this waveguide while making its efficiency as high as possible. The viability of this core replacement technique is demonstrated in the creation of a temperature sensor, which demonstrates much greater temperature sensitivity than an unetched fiber.

The last contribution is the analysis and fabrication of SR-FBGs on D-fiber. These gratings are fabricated in the cladding above the fiber core, which makes differentiates them from standard FBGs, which rely on a small photoinduced index modulation of the core material. Because they are etched into the surface of the fiber, they can function up to very high temperatures, whereas commercial FBGs wash out at relatively low temperatures. The elliptical core of the fiber also presents the possibility of the simultaneous detection of multiple measurands with a single fiber. The SR-FBG is demonstrated to function as a temperature sensor up to a temperature of 1100 °C and is also demonstrated as a strain sensor.

6.2 Future work

There are many avenues for the research presented in this dissertation to take in the future. One common denominator among all of the devices listed is that in order to create commercial devices, packaging issues must be addressed. Below I list how the ideas presented in this dissertation could be extended and refined.

The in-fiber polymer waveguide has the potential to be the basis for many different types of sensing and communication devices. In order to create an electro-optic modulator, electrodes must be placed on the fiber and the polymer must be poled to become electro-optic. While polymer films have been poled and electrodes have
been deposited onto fibers in our lab, these elements have not yet been integrated onto the same fiber. Polymer modulators hold the promise of very high photonic switching speeds. A poled polymer waveguide without electrodes on the flat side of the fiber could also be used as a sensor to detect electric field strength.

Different materials also need to be tested in the creation of in-fiber polymer waveguides in order to create sensors that can detect a wider range of measurands. One direction that core-replaced sensors could take is for the detection of chemical or biological agents by the incorporation into the fiber of specialized sensing materials. Another possibility is using this type of sensor in the creation of smart structures. The fiber sensor would be embedded into a structure and would be able to monitor conditions such as corrosion or humidity.

There is much work to be done to more fully develop the potential of the SR-FBG. The exploitation of the elliptical core of the fiber to measure strain fields simultaneously in both the longitudinal and transverse directions remains a goal of the research. Two ideas which could possibly further increase the number of variables which a single SR-FBG can measure are the use of polarimetric sensing in conjunction with wavelength peak detection, and the combination of photoinduced gratings with SR-FBGs on the same fiber. Additionally, the application of another material on top of the SR-FBG might yield some new possibilities in sensing.

In order to improve the ability of a SR-FBG to measure multiple strain axes simultaneously, the efficiency of the grating in reflecting $y$-polarized light must be improved. This could be accomplished by etching gratings into the core of the fiber instead of just above it. A deep RIE machine would also help this effort. As well there is the possibility of researching fiber with the major axis of the elliptical core aligned at an angle somewhere between 0° and 90° in order to balance the efficiency of the two polarization modes.

The applications listed above highlight the flexibility of D-fiber in the creation of in-fiber devices. I look forward to the time when some of these ideas will come to fruition and become viable devices.
Appendix
Appendix A

Etch Simulations

In order to help visualize how fibers etch in HF, BHF, or in the RIE, I created a probabilistic model of the etch process. Simulating etches can give insight into the profile of a fiber at a specific point in the etch process without having to resort to imaging tools such as a SEM or an AFM. Simulations can also help to spot problems that may occur in etching, such as undercutting.

The etch simulator takes as input an arbitrary cross-sectional profile of the structure being etched along with the selectivities of the different materials of which it is composed. The profile is discretized so that it consists of ‘molecules’, which represent either the materials which are being etched, or the actual etchant itself. The etch process is carried out in discrete time steps, each complete execution of the for loop representing one time step. Within each time step, each molecule has a certain probability of being etched. This probability is directly related to the number of ‘etchant’ molecules which surround it.

To create the cross-sectional grid of molecules, values which represent the etch rate of each type of material are assigned to each point on the grid. Each molecule of the material with the highest etch rate is assigned the etch index of 0.125 \( \left( \frac{1}{8} \right) \). To determine the etch indices of each of the other materials, 0.125 is divided by the relative etch rate. For example, if material A etches twice as fast as material B, material A is assigned the value of 0.125 and material B is assigned a value of 0.0625. In addition, an etch index of 0 is assigned to all molecules of acid.

To determine which molecules will be etched in a given time step, the code looks at all eight neighbors of each molecule and counts how many of those
neighbors are acid. It then multiplies the etch index of that molecule by the number of neighbors that are acid. The resulting number is compared with a random number in the inequality

\[ X < n_{\text{etch}} \times N_{\text{acid}}, \]

where \( X \) is a uniformly distributed random number between 0 and 1, \( n_{\text{etch}} \) is the etch index of the molecule, and \( N_{\text{acid}} \) is the number of neighbors that are acid. If the molecule has an etch index of 0.125 and all eight neighbors are acid, it has a 100% chance of being etched. This is the highest probability allowed, and all other etch probabilities are lower than this. This is why the maximum etch index is set to 0.125.

As an example, if a molecule has an etch index of 0.025 (one fifth of etch rate of the highest etch rate), and two of its eight neighbors are acid molecules, the probability that it will be etched in the current time step is 0.05 or 1 in 20.

This probabilistic model does not take into account the amount of time it will take some structure to etch to a certain depth. It only determines what the structure looks like during and after the etch. To determine what the structure looks like as a function of time, temporal etch rates must be correlated with the number of time steps it takes to etch a structure to a given depth.

### A.1 Simulations

I simulated the etching of a D-fiber in hydrofluoric acid to try to help understand what the fiber looked like at different stages of the etch process. Figure A.1 shows profiles of a horizontal core D-fiber in various stages of the etch process. The top left image is of the core of the fiber before the etch begins, and the bottom right image shows the core entirely removed. The faster etch rate of the germania-doped core is very evident. The etch simulator has also been used to simulate the etching of fibers in BHF and in the RIE. As an example of an etch simulation, Ref. [21] contains a link to a movie of a vertical core fiber being etched in HF acid.
A.2 Code

Below I have included the Matlab code for the etch simulator, whose filename is `etchsim.m`. This code generates a two-dimensional profile, assigns etch indices to each molecule, and then iterates in a for loop for each time step of the etch process. Each iteration of the for loop calls the function `etchmatr.m` to determine which molecules to etch. The code also generates a movie of the etch simulation. A code very similar to this one generated the images in Fig. A.1.

% This file sets up a grid and calls the etchmatr.m program to etch a
% 2-D matrix of molecules which represents a 2-D cross section of some
% structure whose materials etch selectively. The 'index' or value
% assigned to the fastest etching material is .128 or 1/8. To get the
% values for the other materials, divide .125 by the relative etch rate.
% The 'index' of the etchant is always 0.

clear all; close all;
% Set up grid of molecules
N=1001;
xx=linspace(-2.5,2.5,N);
yy=xx;
[x,y]=meshgrid(xx,yy);

% Set up indices of different materials
na=0;
ge=.125;
nsf=nge/11.5;
nsi=nge/8;

% Define geometry of structure
eps=nsf*ones(size(x));
eps((x/1).^2 + (y/2).^2 < 1)=nge;
eps(x.^2+y.^2<.12)=nsi;

% Perform a convolution because there is some diffusion of materials
num=round((N-1)/40)
A=ones(2*num+1);
c=conv2(A,eps);
eps=c(num+1:end-num,num+1:end-num)/length(A)^2;
size(eps);
eps(1:num,:)=nsf;
eps(end-num:end,:)=nsf;
eps(:,end-num:end)=nsf;
eps(:,1:num)=nsf;
eps(y>2.2)=na;

% Get image of object before etch
pcolor(xx,yy,eps); shading interp; axis image; colorbar;
v=colormap;
v(1,:)=[1 1 1];
colormap(v);

% Initialize movie and two variables used in loop
mov = avifile('Diss_etchsim.avi','fps',5);
mov.Quality = 100
epsnew=eps;
n1=1;

for nn=1:round(1.8*N) %1800 when 1001 grid points
    epsnew=etchmatr(eps,epsnew,N,N);
    eps=epsnew;
    if mod(nn,15)==0 % Don’t display every frame
        pcolor(x,y,eps); shading interp;
        axis([min(xx) max(xx) min(yy) max(yy) 0 nge 0 nge]);
        axis image;
        colormap(v);
        pause(.001); title(num2str(nn));
        F(n1) = getframe(gca);
        n1=n1+1;
        mov = addframe(mov,F);
    end
end
mov = close(mov);
map=colormap;
options=[1 0 1 1 10 8 10 25];
mpgwrite(F, map, 'testmov.mpg',options)
Below is the engine of the `etchsim.m` code, the `etchmatr.m` function. It checks each molecule in the grid to see how many neighbors are acid and then determines whether or not to etch them.

% This function 'etches' molecules according to how many nearest neighbors % are acid molecules (with a value of 0). It loops through each molecule % in the 2-D grid, and uses probability to determine whether to etch it or % not.

function epsnew=etchmatr(eps,epsnew,Nx,Ny)
for n=2:Ny-1
    for m=2:Nx-1
        if eps(n,m)~=1
            b=rand;
            % See how many nearest neighbors are acid
            switch (eps(n+1,m+1)==0) + (eps(n-1,m-1)==0) + (eps(n-1,m+1)==0)
                + (eps(n+1,m-1)==0) + (eps(n+1,m)==0) + (eps(n-1,m)==0)
                    + (eps(n,m+1)==0) + (eps(n,m-1)==0)
            case 8
                if b<eps(n,m)*8*5
                    epsnew(n,m)=0;
                end
            case 7
                if b<eps(n,m)*7*4
                    epsnew(n,m)=0;
                end
            case 6
                if b<eps(n,m)*6*3
                    epsnew(n,m)=0;
                end
        end
    end
end
case 5
    if b<\text{eps}(n,m) \times 5/2
        \text{epsnew}(n,m)=0;
    end

case 4
    if b<\text{eps}(n,m) \times 4
        \text{epsnew}(n,m)=0;
    end

case 3
    if b<\text{eps}(n,m) \times 3
        \text{epsnew}(n,m)=0;
    end

case 2
    if b<\text{eps}(n,m) \times 2
        \text{epsnew}(n,m)=0;
    end

case 1
    if b<\text{eps}(n,m) \times 1
        \text{epsnew}(n,m)=0;
    end

end
end
end
end
Appendix B

Polarimetric Modulation Using In-fiber Polymer Waveguides

B.1 Introduction

The in-fiber polymer waveguide discussed in Chapter 3 can form the basis of a number of optical devices. To gauge the possible performance of the waveguide, we characterize its efficiency by simulating its theoretical performance as a conceptual polarimetric modulator. To create a device such as this, the polymer must be made electro-optic.

To make the polymer electro-optic, it must be poled to permanently orient the guest chromophore molecules, introducing asymmetry into the material [49]. In this research we use DR1 azo dye as the guest chromophore because of its availability. Once a polymer is poled, applied electric fields change its index of refraction, and this change is dependent on the direction of the applied field. The changes in index of refraction retard or advance the propagation of different polarizations of light, so interferometric devices can be constructed. A poled polymer can serve as part an active optical device, such as a modulator [50], or as a passive device such as an electric field transducer [51].

Electro-optic polymers have been shown to be a viable candidate for high-speed modulators [50]. While we have yet to realize an in-fiber polarimetric modulator, Fig. B.1 shows how one could be constructed by placing an electro-optic polymer waveguide section between electrodes.
B.2 Theory of operation

The application of voltage to the electrodes, as shown in Fig. B.1, causes an electric field to be applied transverse to the propagation axis of the waveguide. The applied field causes a change in the birefringence between the $x$- and $y$-polarized modes. Changing the birefringence varies the output polarization state of the waveguide. With crossed linear polarizers placed at the input and output of the modulator and oriented at $\pm 45^\circ$ with respect to the flat side of the fiber, the power transmission is given by

$$T = \frac{P_{\text{out}}}{P_{\text{in}}} = \sin^2 \left( \frac{\pi L \Delta B + \phi_0}{2} \right),$$  

(B.1)

where $\lambda$ is the free-space wavelength, $L$ is the modulator length, and $\phi_0$ is the phase difference between the two modes with no field applied. The change in birefringence, $\Delta B$, is given by

$$\Delta B = (N_{x,V} - N_{y,V}) - (N_{x,0} - N_{y,0}),$$  

(B.2)

where $N_{x,V}$ and $N_{y,V}$ are the effective indices of the modes with a voltage applied and $N_{x,0}$ and $N_{y,0}$ are the effective indices of the modes with no voltage applied.

Equation (B.1) shows that the power transmission is sinusoidal and it changes from a minimum to a maximum when the argument of the sine function changes by $\frac{\pi}{2}$. This phase change also corresponds to a $\pi$ phase shift between the $x$- and $y$-polarized modes. The modulator is characterized by determining the length
$L_\pi$ required to achieve a phase shift of $\pi$ radians between the two polarization states for a given applied voltage and polymer waveguide cross-section. $L_\pi$ is given by

$$L_\pi = \frac{\lambda}{2\Delta B}$$  \hspace{1cm} (B.3)

Changes in the bulk indices of refraction of the materials composing the fiber cause changes in the effective indices of the fiber modes. The change in birefringence $\Delta B$ depends on the change in effective index of the guided modes. The change in the bulk index of refraction of the polymer depends on the applied electric field, the zero-field polymer index $n_{p0}$, and the polymer electro-optic coefficients $r_{33}$ and $r_{13}$ through [53]

$$\Delta n_{p,x} = \frac{1}{2} n_{p0}^3 r_{33} E$$  \hspace{1cm} (B.4)

$$\Delta n_{p,y} = \frac{1}{2} n_{p0}^3 r_{13} E$$  \hspace{1cm} (B.5)

where the value of $r_{13}$ is typically taken to be $\frac{1}{3}$ of the value of $r_{33}$. The PMMA/DR1 azo dye combination that we use to create the in-fiber polymer waveguide can probably yield an electro-optic coefficient of almost 10 pm/V, so a more efficient polymer would need to be used in a practical device. Other polymers have been developed with $r_{33}$ coefficients as high as 97 pm/V [54].

### B.3 Simulations

In numerical simulations of a polarimetric modulator, we assume an electro-optic coefficient of $r_{33} = 50$ pm/V. The applied electric field $E$ is estimated using HFSS simulations which showed that electrodes with a 10 volt drive voltage and a separation of 10 $\mu$m yield a field in the polymer region of approximately 600 kV/m that is primarily in the horizontal direction. Reference [55] provides additional analysis of modulators based on electro-optic polymer core waveguides.

The effective indices of refraction for the various modes were calculated based on the polymer waveguide cross-section shown in Fig. [B.2] using the two-dimensional mode solving capability of BeamPROP$^\text{TM}$. These calculations were performed with a polymer index of $n_p = n_{p0}$, to determine $N_{x,0}$ and $N_{y,0}$, and with
\[ n_p = n_{p0} + \Delta n_{p,x} \text{ and } n_p = n_{p0} + \Delta n_{p,y} \] to determine \( N_{x,V} \) and \( N_{y,V} \) respectively. Equation (B.3) is then used to calculate \( L_{\pi} \).

![Figure B.2: Refractive index profile used to model the efficiency of the electro-optic waveguide.](image)

Figure B.2 plots the \( L_{\pi} \) length for a polarimetric modulator using a free-space wavelength of 1550 nm. The primary simulation parameters are the polymer thickness in the core, \( t \), and the zero field polymer index \( n_{p0} \). The polymer waveguide becomes more weakly guiding as the polymer thickness \( t \) gets smaller. A decrease in the polymer index of refraction also causes the polymer waveguide to become more weakly guiding. This results in less optical power being concentrated in the polymer region. The reduction in the fraction of power guided in the polymer region decreases the efficiency of the modulator. Figure B.3 shows this increase in \( L_{\pi} \) (or decrease in efficiency) as the polymer thickness and index decrease. These simulations show that in order to obtain a reasonable value for \( L_{\pi} \) the polymer thickness in the core of the fiber needs to be greater than 1.3 µm. The tradeoff is to produce a thick enough polymer layer in the fiber core while still not exciting slab modes on the flat side of the fiber, which contribute to high loss.
Figure B.3: Modulator length for a phase shift between polarizations as a function of polymer thickness and index.
Appendix C

Finite-difference Waveguide Mode Solver

As a check on BeamPROP\textsuperscript{TM}\textsuperscript{\textregistered}’s mode solving capabilities and to give another representation of the fields of a two-dimensional waveguiding structure I developed a finite-difference eigenvalue mode solver. This solver was developed using the derivation in Ref. [56]. To solve for modes in a waveguide one needs to solve the Helmholtz wave equation

\[ \nabla^2 \Phi + k_c^2 \Phi = 0, \]  

(C.1)

where \( \Phi \) represents \( E_z \) or \( H_z \) for TM or TE modes, respectively, and \( k_c \) is the cutoff wavenumber. Appropriate boundary conditions must also be applied. The cutoff wavenumber is defined by

\[ k_c^2 = \omega^2 \mu \epsilon - \beta^2, \]  

(C.2)

where \( \omega \) is the angular frequency of the wave, \( \mu \) and \( \epsilon \) are, respectively, the permeability and permittivity of the material inside the waveguide, and \( \beta \) is the propagation constant of the guided mode.

In deriving the Helmholtz wave equation for a dielectric waveguide, I have omitted some terms which complicate the solution of the equation. By applying the curl operator to Faraday’s and Ampere’s laws, we find the full wave equations for \( \mathbf{E} \) and \( \mathbf{H} \) in a dielectric waveguide:

\[ \nabla^2 \mathbf{E} + \nabla (\mathbf{E} \cdot \nabla \ln \epsilon) = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \]  

(C.3)

\[ \nabla^2 \mathbf{H} = \mu \epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} + \mathbf{E} \times \nabla \epsilon. \]

By making the approximation that the index change between materials is not a sharp step function, and that the overall index difference between materials is small, we
can consider as negligible the terms $\nabla \ln \epsilon$ and $\nabla \epsilon$. It is desirable to neglect these terms in numerical solutions because they either couple transverse components with longitudinal components, or couple $\mathbf{E}$ and $\mathbf{H}$, making numerical solution much more time-intensive. Neglecting these terms, assuming harmonic time dependence, assuming $\exp(-j\beta z)$ propagation, and solving for the longitudinal fields, we are able to derive the Helmholtz wave equation from Eq. (C.3).

Discretizing the Helmholtz wave equation over the cross section of the waveguide yields the finite-difference Helmholtz wave equation

$$\Phi(i + 1, j) + \Phi(i - 1, j) + \Phi(i, j + 1) + \Phi(i, j - 1) - \left(4 - h^2k_c^2\right)\Phi(i, j) = 0,$$  

(C.4)

where $i$ and $j$ label the grid points and $h$ is the grid spacing. See Ref. [56] for more details on the setting up of the eigenvalue equation and the numbering of grid points.

One of the computational bottlenecks in the numerical solution of this kind of waveguide problem can be the setting up of the sparse matrix whose eigenvalues need to be found. If the grid representing the waveguide is made up of a $m \times n$ grid, the number of elements in the matrix is $m^2n^2$, so problems do not scale well. Sparse matrix eigenvalue solvers perform well in finding eigenvalues, especially if only a few are needed, but generating the sparse matrix can take a long time if it is not done efficiently.

I have created a sparse matrix generator in Matlab to assist in the solution of waveguide modes. Figure C.1 shows the grid used to set up the numerical solver, where point (a) corresponds to point $(i, j)$, and points (b), (c), (d), and (e) label the points above, below, to the right, and to the left of point (a). Each row of the matrix to be solved represents each grid point in the entire domain. Using the grid shown in Fig. C.1 we can rewrite Eq. (C.4) using Eq. (C.2) as

$$\Phi_b + \Phi_c + \Phi_d + \Phi_e - \left(4 - h^2\omega^2\mu\epsilon_a\right)\Phi_a = \beta^2\Phi_a.$$  

(C.5)

This form of the equation is necessary to solve for dielectric waveguides, whose permittivity, $\epsilon$ is not a constant. The eigenvalue on the right-hand side is the square of the propagation constant, $\beta$.  

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C.1 Boundary conditions

Dielectric waveguides do not have Dirichlet or Neumann boundary conditions at the edges of the simulation domain as do conducting waveguides so I implemented a linear extrapolation boundary condition. Figure C.2 shows how the linear extrapolation is performed. If a point on the right boundary of the simulation domain, point (a), is the point we are solving for, and point (d) is outside the simulation domain, then $\Phi_d$ can be estimated through linear extrapolation by assuming that the line connecting $\Phi_e$ and $\Phi_a$ also passes through $\Phi_d$. More elaborate and accurate boundary conditions can be developed, but linear extrapolation is sufficient for our purposes.

In order to estimate the value of $\Phi_d$, we set up a coordinate system in which the origin is at point (a), and the line that goes through $\Phi_d$ is given by the equation

$$\Phi_d = mx + b,$$

where $m$ is the slope of the line connecting $\Phi_e$ and $\Phi_a$, and $b$ is the point where that
Figure C.2: Diagram showing how linear extrapolation is performed.

The line intercepts the $x$-axis. The equation then becomes

\[ \Phi_d = \frac{\Phi_a - \Phi_e}{h} h + \Phi_a \]

\[ \Phi_d = 2\Phi_a - \Phi_e. \]  \hfill (C.7)

Making this substitution for $\Phi_d$ in Eq. (C.5) then gives the discrete equation for points on the right boundary of the domain,

\[ \Phi_b + \Phi_e - (2 - h^2 \omega^2 \mu \epsilon_a) \Phi_a = \beta^2 \Phi_a. \]  \hfill (C.8)

Similar steps are performed to find the discrete equations for all points on the edges of the domain. However, for corner points the extrapolation step must be applied twice, in both the $x$- and $y$-directions, yielding

\[ h^2 \omega^2 \mu \epsilon_a \Phi_a = \beta^2 \Phi_a. \]  \hfill (C.9)

### C.2 Simulation results

Figure [C.3] shows the results of finding the fundamental mode of an un-etched elliptical core waveguide at a wavelength of 1550 nm. The simulation region is $16 \times 16 \mu m$ with 401 grid points in each direction. The fiber is a vertical core fiber and the dimensions of the core and the refractive indices of the different materials are given in Chapter 2. The mode on the left is found using BeamPROP™ while the
mode on the right is found using the eigenvalue solver. Technically, BeamPROP™ finds the transverse field, while the eigenvalue solver finds the longitudinal field, but the structure of the modes can still be easily compared. The fields look very similar and the effective indices are also close.

\[ N = 1.44986 \quad N = 1.45021 \]

Figure C.3: Simulated fundamental mode of an elliptical core fiber using BeamPROP™ (left) and an eigenvalue solver (right).

C.3 Code

The codes included below create a two-dimensional grid representing the dielectric waveguide, generate the matrix to be solved, and then solve matrix and plot the fields. The 400 × 400 grid used to find the fundamental mode in Fig. C.3 was generated in 0.75 seconds, whereas the eigenvalue solver eigs.m took over an hour to find the first two modes. The speed of the matrix generator means that it is no longer a bottleneck in this type of problem.

% This code generates a two-dimensional dielectric waveguide, calls a % function to generate the matrix to be solved, and then finds the % effective index of the modes and unpacks the electric fields.
clear all; close all;

% Set up grid and define constants
l=16e-6;
a=1e-6; % x-dimension of core in meters
b=2e-6; % y-dimension
Nx=400; % Number of cells in the x-direction
Ny=Nx; % Number of cells in y
dx=l/Nx; % Step size
N=(Nx-1)*(Ny-1); % Number of points to solve for
lam=1.55e-6; % Wavelength of light

% Define grid and permittivity profile
x=linspace(-l/2,l/2,Nx+1);
y=x;
[xx,yy]=meshgrid(x,y);
eps=1.441^2*ones(Nx+1,Ny+1);
eps((xx/a).^2 + (yy/b).^2 <=1)=1.4756^2;
eps(xx.^2 + yy.^2 <= .15*a^2)=1.444^2;

% Find sparse matrix with user-defined function
A=Lin_BC(N,Nx,Ny,dx,lam,eps);

% Set options, do eigenvalue solve, and find cutoff freqs.
opts.disp=0;

% Find first nmod modes
nmod=2;
[v,d]=eigs(A,nmod,'lr',opts);

% Find effective index and Fields
\[ \text{neff} = \frac{\sqrt{\text{diag}(d)}}{2\pi/d\lambda} \]

\[ E = \text{reshape}(v(:,1), N_x-1, N_y-1); \]

\[ E = \text{padarray}(E, [1 1], 0, 'both'); \]

\[ E = E / \text{max}(\text{max}(\text{abs}(E))); \quad \% \text{Normalize E-field} \]

\[ \text{if } \min(\min(E)) < 0 \]
\[ \quad E = -E; \]
\[ \text{end} \]

\% Plot fields

\text{for } n = 1:n_{\text{mod}} \]
\[ \quad \text{figure} \]
\[ \quad E = \text{reshape}(v(:,n), N_x-1, N_y-1); \]
\[ \quad E = \text{padarray}(E, [1 1], 0, 'both'); \]
\[ \quad \text{pcolor}(x, y, E); \text{shading interp; axis image;} \]
\[ \quad \text{text}(8/9*\min(x), 8/9*\max(y), ['n_{\text{eff}} = ' \text{num2str}(\text{neff}(n), 5)]); \]
\[ \quad \text{pause}(0.1); \]
\[ \text{end} \]

\% Overlay contour map of fields

\text{figure;} \]
\[ \text{pcolor}(x, y, \text{eps}); \text{axis image; shading interp;} \]
\[ \text{hold on;} \]
\[ \text{contour}(xx, yy, E, 10) \]
\[ \text{hold off;} \]
function A=Lin_BC(N,Nx,Ny,dx,lam,eps);

% Define constants
mu=4e-7*pi;
eps0=8.854e-12;
c=1/sqrt(mu*eps0);
w=2*pi*c/lam;
nr=Nx-1; %number of rows/columns in 2-D grid

% Unpack the permittivity matrix into a vector
eps2=eps0*eps(2:end-1,2:end-1);
eps2=reshape(eps2,1,N);

% Find indices of nonzero matrix elements
% Main diagonal (point a in grid)
i=1:N;

% Far-off diagonal (points b and c in grid). Points off of grid are ignored
i1=Nx:N;
j1=1:N-nr;
s1=ones(1,N-nr);

% One-off diagonal (points e and d in grid)
i2=2:N;
j2=1:N-1;
s2=ones(1,N-1);
s2(nr:nr:end)=0; % Set lateral points off of grid to zero

% Row indices of points on top, left, right, bottom of grid (for BC).
% Per Sadiku, a represents the grid point in the eigenvector, b the point
% above, c the point below, d the point to the right, and e the point to
% the left of a.
\[
\begin{align*}
  b &= 1: nr; \\
  c &= nr \times (nr - 1) + 1: N; \\
  d &= nr: nr: N; \\
  e &= 1: nr: N; \\
  i3 &= [b \ c \ d \ e]; \\
  s3 &= 2 * \text{ones}(\text{size}(i3));
\end{align*}
\]

% For the linear BC, extrapolation negates the points opposite the
% extrapolated points
\[
\begin{align*}
  ib &= b; \\
  jb &= b + nr; \\
  sb &= -\text{ones}(\text{size}(ib)); \\
  ic &= c; \\
  jc &= c - nr; \\
  sc &= -\text{ones}(\text{size}(ic)); \\
  id &= d; \\
  jd &= d - 1; \\
  sd &= -\text{ones}(\text{size}(id)); \\
  ie &= e; \\
  je &= e + 1; \\
  se &= -\text{ones}(\text{size}(ie));
\end{align*}
\]

% Indices and vector of -4s and 1s
\[
\begin{align*}
  is &= [i, i1, j1, i2, j2, i3, ib, ic, id, ie]; \\
  js &= [i, j1, i1, j2, i3, jb, jc, jd, je]; \\
  s &= [-4 * \text{ones}(1, \text{length}(i)) + w^2 * \mu * \text{eps2} * dx^2, s1, s1, s2, s2, s3, sb, sc, sd, se];
\end{align*}
\]
% Build output matrix for eigenvalue equation
A=sparse(is,js,s,N,N);
Appendix D

Coupled-mode Theory

D.1 Introduction

There are two fundamental approaches for analyzing the behavior of fields in perturbed waveguide structures. The first is to solve Maxwell’s equations in that structure. The second method is to express the fields in the perturbed waveguide as a sum of the normal modes of the unperturbed structure.

The first method of solution should give correct results, but only the simplest cases yield analytical solutions. The second method yields only an approximate answer, but is often the only way a solution can be found. The purpose of this tutorial is to explain coupled mode theory in some brief detail and then apply it to the problem of finding reflection spectra of gratings placed onto the flat surface of a D-shaped optical fiber. The following mathematical analysis follows to some degree that of Nishihara [44] and also draws on elements from Lee [45].

D.2 Modes in a waveguide

D.2.1 Lorentz reciprocity theorem

In order to develop the coupled-mode equations, we first derive the Lorentz reciprocity theorem, starting with some arbitrary waveguiding structure in which the time harmonic fields can be expressed as a sum over orthogonal modes

\[ E_0 = \sum_\nu a_\nu(z)E_\nu(x, y)e^{-j\beta_\nu z}, \quad H_0 = \sum_\nu a_\nu(z)H_\nu(x, y)e^{-j\beta_\nu z}, \]  

(D.1)
where the modal fields \((E_\nu, H_\nu)\) are normalized and the amplitude \(a_\nu\) is a function of \(z\). These fields are assumed to be time-harmonic, with \(e^{j\omega t}\) time dependence.

The fields propagate in the \(+z\) direction and satisfy the Maxwell equations

\[
\nabla \times \mathbf{E}_0 = -j\omega \mu \mathbf{H}_0, \quad \nabla \times \mathbf{H}_0 = j\omega \epsilon_0 \epsilon_r \mathbf{E}_0, \quad (D.2)
\]

where \(\epsilon_r\) is the relative dielectric permittivity of the waveguide and generally is a function only of the transverse components. To perturb the structure of the waveguide, we let the permittivity change in the longitudinal direction: \(\epsilon' = \epsilon_r(x, y) + \Delta \epsilon_r(x, y, z)\).

The new fields supported by this perturbed structure also satisfy the Maxwell equations,

\[
\nabla \times \mathbf{E} = -j\omega \mu \mathbf{H}, \quad \nabla \times \mathbf{H} = j\omega \epsilon_0 (\epsilon_r + \Delta \epsilon_r) \mathbf{E}. \quad (D.3)
\]

We now cross multiply Eqs. (D.3) and the conjugate of Eqs. (D.2),

\[
\mathbf{H}_0^* \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H}_0^* + \mathbf{H} \cdot \nabla \times \mathbf{E}_0^* - \mathbf{E}_0^* \cdot \nabla \times \mathbf{H} = \\
- j\omega \mu \mathbf{H}_0^* \cdot \mathbf{H} + j\omega \epsilon_0 \epsilon_r \mathbf{E} \cdot \mathbf{E}_0^* + j\omega \mu \mathbf{H} \cdot \mathbf{H}_0^* - j\omega \epsilon_0 (\epsilon_r + \Delta \epsilon_r) \mathbf{E}_0^* \cdot \mathbf{E}. \quad (D.4)
\]

The terms on the right hand side of this equation cancel each other with the exception of the term with \(\Delta \epsilon_r\) in it. Using the vector identity

\[
\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \quad (D.5)
\]

twice on the left hand side yields the result

\[
\nabla \cdot (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) = - j\omega \epsilon_0 \Delta \epsilon_r \mathbf{E}_0^* \cdot \mathbf{E}. \quad (D.6)
\]

We now integrate Eq. (D.6) over a volume extending infinitely in the transverse plane but infinitesimally thin in the \(z\) direction. We also make use of the Gauss’ theorem

\[
\int_V \nabla \cdot \mathbf{A} dV = \oint_S \mathbf{A} \cdot d\mathbf{S}
\]

to come up with

\[
\oint_S (\mathbf{E} \times \mathbf{H}_0^* + \mathbf{E}_0^* \times \mathbf{H}) \cdot d\mathbf{S} = - j\omega \epsilon_0 \int_V \Delta \epsilon_r \mathbf{E}_0^* \cdot \mathbf{E} dV. \quad (D.7)
\]
In the limit as the thickness $\Delta z \to 0$, the integrand on the right is constant with respect to $z$. This allows us to pull everything out of the $z$-integral and integrate $dz$ from $z_0$ to $z_0 + \Delta z$, giving just $\Delta z$. Since the surface $dS$ is oriented in the $\pm z$ direction, the dot product on the left picks off only those components parallel to the $z$ direction. Any cross products involving the $z$ components then vanish when dotted into $\hat{z}$, so only the transverse fields contribute to the integral. The integral over the surface at $z = z_0$ is subtracted from that at $z = z_0 + \Delta z$, so we get a differential. Moving the $\Delta z$ from the right to the left hand side we get the Lorentz reciprocity theorem

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial z} \left[ E_t \times H_{0,t}^* + E_{0,t}^* \times H_t \right]_z \text{d}x \text{d}y = -j\omega \epsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta \epsilon_r E_0^* \cdot E \text{d}x \text{d}y. \quad (D.8)$$

### D.2.2 Orthogonality

We now discuss the orthogonality of the modes of an arbitrary lossless dielectric waveguiding structure, which will prove useful in the next section. This structure is uniform in the $z$ direction and is defined by the cross-sectional dielectric permittivity profile $\epsilon = \epsilon(x, y) = \epsilon_0 \epsilon_r(x, y)$ where $\epsilon_r$ is the relative permittivity. The time-harmonic electric and magnetic fields in this structure satisfy the Maxwell equations:

$$\nabla \times E = -j\omega \mu H, \quad \nabla \times H = j\omega \epsilon_0 \epsilon_r E. \quad (D.9)$$

Modes of this structure will propagate in the $z$ direction and have $z$ dependence of $e^{-j\beta z}$. Any two modes of the structure with fields $E_1, H_1$ and $E_2, H_2$ will also satisfy the Maxwell equations as well as the following relationship (same derivation as for Eq. (D.6) but with $\Delta \epsilon_r = 0$):

$$\nabla \cdot (E_1 \times H_2^* + E_2^* \times H_1) = 0. \quad (D.10)$$

The fields of the modes will have the functional form

$$A_n(x, y, z) = A_n(x, y)e^{-j\beta_n z}. \quad (D.11)$$
Inserting two modes with labels \( \mu \) and \( \nu \) into Eq. (D.10) yields
\[
\nabla_t \cdot \left[ \mathbf{E}_\mu \times \mathbf{H}^*_\nu + \mathbf{E}^*_\nu \times \mathbf{H}_\mu \right]_t e^{-j(\beta_\mu - \beta_\nu)z} \\
- j (\beta_\mu - \beta_\nu) \left[ \mathbf{E}_{t\mu} \times \mathbf{H}^*_{t\nu} + \mathbf{E}^*_{t\nu} \times \mathbf{H}_{t\mu} \right]_z e^{-j(\beta_\mu - \beta_\nu)z} = 0, \quad (D.12)
\]
where the subscripts \( t \) and \( z \) refer to the transverse and longitudinal components, respectively. Notice that the cross-product of transverse components produces longitudinal components. Simplifying, rearranging, and integrating Eq. (D.12) over the entire transverse plane gives
\[
\iint \nabla_t \cdot \left[ \mathbf{E}_\mu \times \mathbf{H}^*_\nu + \mathbf{E}^*_\nu \times \mathbf{H}_\mu \right]_t \, dx \, dy \\
= j (\beta_\mu - \beta_\nu) \iint [\mathbf{E}_{t\mu} \times \mathbf{H}^*_{t\nu} + \mathbf{E}^*_{t\nu} \times \mathbf{H}_{t\mu}]_z \, dx \, dy. \quad (D.13)
\]
We can now use a two dimensional version of Gauss’ theorem
\[
\int_S \nabla \cdot \mathbf{F} \, dS = \oint \mathbf{F} \cdot d\mathbf{s} \quad (D.14)
\]
to recast the left hand side of the integral as a line integral along the rim of the flat surface \( S \) where \( d\mathbf{s} \) is the vector differential element along the surface with the vector pointing normal to the rim of the surface. All modes guided in the structure will exponentially decay as distance from the \( z \) axis goes to infinity, so the integral on the left is zero if one or both of the modes is guided. Nishihara also states without proof that the integral goes to zero if both modes are radiation modes by the application of periodic boundary conditions. Another way of showing that the right hand side goes to zero is to let the waveguide be contained within a circular perfect conductor. Then let the radius of the cylinder go to infinity. It then contains the same modes as the structure without the conductor, but boundary conditions on the conductor make the integral zero. The final result is the orthogonality relationship
\[
j (\beta_\mu - \beta_\nu) \iint [\mathbf{E}_{t\mu} \times \mathbf{H}^*_{t\nu} + \mathbf{E}^*_{t\nu} \times \mathbf{H}_{t\mu}]_z \, dx \, dy = 0. \quad (D.15)
\]
A final method of demonstrating the above orthogonality relationship is to recognize that it can be derived from the left hand side of the Lorentz reciprocity theorem (Eq. (D.8)) by assuming \( e^{-j\beta z} \) \( z \)-dependence and setting \( \Delta \epsilon_r = 0 \).
Equation (D.15) is obviously satisfied if $\beta_\mu = \beta_\nu$, but in the case that they are not equal, i.e. $\mu$ and $\nu$ label modes with different propagation constants, the fields of those modes are orthogonal.

D.2.3 Normalization

Power flow density for electromagnetic waves is described by the Poynting vector, $\mathbf{S} = \mathbf{E} \times \mathbf{H}^*$ and the time averaged total power carried by a waveguide can be written

$$P = \frac{1}{2} \Re \left( \iint \mathbf{S} \cdot \hat{z} \, dx \, dy \right)$$

$$= \frac{1}{2} \Re \left( \iint (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{z} \, dx \, dy \right)$$

$$= \frac{1}{4} \iint [\mathbf{E} \times \mathbf{H}^* + \mathbf{E}^* \times \mathbf{H}] \cdot \hat{z} \, dx \, dy. \tag{D.16}$$

The power is normalized by scaling the fields such that the total power is unity.

We generally deal only with the electric field components, because we only solve for the electric field in our mode solver. Thus, in order to normalize the field, we use the Maxwell equations to express $\mathbf{H}$ in terms of $\mathbf{E}$. We only need to deal with the transverse components of $\mathbf{E}$ and $\mathbf{H}$ because they are the only ones that contribute to power (because of the dot product with $\hat{z}$) so we have

$$\mathbf{H}_t = \frac{1}{-j\omega \mu} (\nabla \times \mathbf{E})_t$$

$$= \frac{1}{-j\omega \mu} \left[ \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{x} + \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{y} \right]. \tag{D.17}$$

The longitudinal component of the field is much smaller than the transverse components, and for the fundamental mode in structures that we are interested in, it also varies somewhat slowly in the transverse direction, so we throw out all $z$-components. The $z$-derivatives of the transverse components just bring down a factor of $-j\beta_z$ giving

$$\mathbf{H}_t \approx \frac{\beta}{\omega \mu} (-E_y \hat{x} + E_x \hat{y}). \tag{D.18}$$

Taking the cross product we get

$$\mathbf{E}_t \times \mathbf{H}_t^* \approx \frac{\beta}{\omega \mu} (E_x \hat{x} + E_y \hat{y}) \times (-E_y^* \hat{x} + E_x^* \hat{y})$$

$$\approx \frac{\beta}{\omega \mu} |\mathbf{E}_t|^2. \tag{D.19}$$
Inserting this into the expression for the power, we get

\[ P \approx \beta^2 \omega \mu \left| E_t \right|^2 \text{d}x \text{d}y. \]  \hspace{1cm} (D.20)

To normalize the electric field then all that is necessary is to scale it so that the above expression is equal to 1.

Assuming that the fields in Eq. \((D.16)\) are normalized, we can combine it with Eq. \((D.15)\) to express the orthonormality of the modes in one equation:

\[ \frac{1}{4} \int \left[ \mathbf{E} \times \mathbf{H}^* + \mathbf{E}^* \times \mathbf{H} \right] \cdot \hat{z} \text{d}x \text{d}y = \pm \delta_{\mu,\nu} \]  \hspace{1cm} (D.21)

for discrete guided modes, and

\[ \frac{1}{4} \int \left[ \mathbf{E} \times \mathbf{H}^* + \mathbf{E}^* \times \mathbf{H} \right] \cdot \hat{z} \text{d}x \text{d}y = \pm \delta \left( \beta_\mu - \beta_\nu \right) \]  \hspace{1cm} (D.22)

for radiation modes with a continuous distribution of propagation constants.

**D.3 Coupled-mode equations**

Now for the fields in the unperturbed waveguide we insert the normalized fields for the \(\mu\)th mode,

\[ \mathbf{E}_{0,t} = \mathbf{E}_{t,\mu}(x,y)e^{-j\beta_\mu z}, \quad \mathbf{H}_{0,t} = \mathbf{H}_{t,\mu}(x,y)e^{-j\beta_\mu z}. \]  \hspace{1cm} (D.23)

We construct the fields in the perturbed structure as a sum over the normal modes of the unperturbed structure

\[ \mathbf{E}_t = \sum_\nu a_\nu(z)\mathbf{E}_{t,\nu}(x,y)e^{-j\beta_\nu z}, \quad \mathbf{H}_t = \sum_\nu a_\nu(z)\mathbf{H}_{t,\nu}(x,y)e^{-j\beta_\nu z}. \]  \hspace{1cm} (D.24)

We then substitute these into the Lorentz reciprocity theorem (Eq. \((D.8)\)). For the left-hand side of the equation we take the sum over \(\nu\) and all \(z\)-dependent terms outside of the integral, to get

\[ \text{LHS} = \frac{\partial}{\partial z} \sum_\nu \left\{ e^{-j(\beta_\nu - \beta_\mu)z}a_\nu(z) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\mathbf{E}_{t,\nu} \times \mathbf{H}_{t,\mu}^* + \mathbf{E}_{t,\nu}^* \times \mathbf{H}_{t,\mu}] \text{d}x \text{d}y \right\}. \]  \hspace{1cm} (D.25)
We now apply the partial derivative with respect to \( z \), along with the orthonormal relationship of Eq. (D.21) to get

\[
LHS = \sum_{\nu} \left\{ \left( \frac{d}{dz} a_{\nu}(z) - j (\beta_{\nu} - \beta_{\mu}) a_{\nu}(z) \right) e^{-j(\beta_{\nu} - \beta_{\mu})z} \right\} (\pm 4\delta_{\mu,\nu})
\]

\[
LHS = \pm 4 \frac{d}{dz} a_{\nu}(z), \quad (\beta_{\mu} \geq 0).
\]

(D.26)

Using eqs. (D.3) and (D.24) we can express the electric field in the perturbed structure as

\[
E = E_t + E_z = E_t + \frac{\nabla_t \times H_t}{j \omega \epsilon_0 (\epsilon_r + \Delta \epsilon_r)}
\]

\[
= \sum_{\nu} a_{\nu}(z) \left( E_{\nu t} + \frac{\epsilon_r}{\epsilon_r + \Delta \epsilon_r} E_{\nu z} \right) e^{-j\beta_{\nu}z}.
\]

(D.27)

Now we can substitute in the relation

\[
E_{z\nu} = \frac{\nabla_t \times H_t}{j \omega \epsilon_0 \epsilon_r},
\]

(D.28)

which allows us to write

\[
E = \sum_{\nu} a_{\nu}(z) \left( E_{\nu t} + \frac{\epsilon_r}{\epsilon_r + \Delta \epsilon_r} E_{\nu z} \right) e^{-j\beta_{\nu}z}.
\]

(D.29)

Note that the in the preceding equations, the sums are replaced by integrals if we allow for radiation modes.

With the above expression for the electric field we can now write right-hand side of Eq. (D.8) as

\[
RHS = -j \omega \epsilon_0 \sum_{\nu} a_{\nu}(z) e^{-j(\beta_{\nu} - \beta_{\mu})z} \int \int \Delta \epsilon_r E_{\mu t}^* \left( E_{\nu t} + \frac{\epsilon_r}{\epsilon_r + \Delta \epsilon_r} E_{\nu z} \right) dxdy.
\]

(D.30)

We can now finally write Eq. (D.8) in the form of the coupled-mode equation:

\[
\pm \frac{d}{dz} a_{\mu}(z) = - j \sum_{\nu} \kappa_{\mu\nu}(z) a_{\nu}(z)e^{-j(\beta_{\nu} - \beta_{\mu})z}, \quad (\beta_{\mu} \geq 0)
\]

\[
\kappa_{\mu\nu}(z) = \kappa_{\mu\nu}^t(z) + \kappa_{\mu\nu}^z(z),
\]

(D.31)

\[
\kappa_{\mu\nu}^t(z) = \frac{\omega \epsilon_0}{4} \int \int \Delta \epsilon_r(x, y, z) E_{\mu t}(x, y) \cdot E_{\nu t}(x, y) dxdy,
\]

\[
\kappa_{\mu\nu}^z(z) = \frac{\omega \epsilon_0}{4} \int \int \frac{\epsilon_r \Delta \epsilon_r(x, y, z)}{\epsilon_r + \Delta \epsilon_r(x, y, z)} E_{\mu z}^*(x, y) \cdot E_{\nu z}(x, y) dxdy.
\]
where $\kappa^t$ and $\kappa^z$ are, respectively, the coupling coefficients due to the transverse and longitudinal fields. They describe how strongly power from the $\nu^{th}$ mode is coupled into the $\mu^{th}$ mode. Equation (D.31) relates the complex amplitude of the $\mu^{th}$ mode to the complex amplitude of the $\nu^{th}$ mode and is the starting point for all of the analysis in the following sections.

### D.4 Coupled-mode theory for Bragg gratings on D-fibers

The analysis of Bragg gratings on fibers begins with the coupled mode equation (Eq. (D.31)). This equation describes how power is transferred between modes in a fiber. It looks like a simple linear, first order differential equation, but in order to solve for field amplitudes of two coupled modes, we must simultaneously solve two coupled, linear, first-order differential equations, for $a_\mu(z)$ and $a_\nu(z)$. The term that governs the coupling between the modes, $\kappa$, is in general a complicated function of $z$. For example, if the Bragg grating is purely sinusoidal, $\kappa$ is not, in general, sinusoidal. This is because $\Delta \epsilon_r$ is only nonzero over a small area of the cross-section of the fiber and the electric field is not constant in this region.

In finding the modes in a fiber, BeamPROP™ computes and saves only the transverse fields. Because of this, we use only the transverse component of $\kappa$, the coupling coefficient. This is not a bad approximation because the $z$-components of the fields are much smaller than the transverse components. The index of the germania-doped core of the fiber is assumed to be 1.4756, while that of the cladding is assumed to be 1.441. Based on a simple bouncing ray analysis we can approximate the fraction of the total field that is in the $z$-direction by analyzing the critical angle for a core-cladding interface,

$$\theta_c = \sin^{-1} \left( \frac{n_{clad}}{n_{core}} \right) = \sin^{-1} \left( \frac{1.441}{1.4756} \right) = 77.6^\circ. \quad (D.32)$$

With this angle of propagation, the normalized field amplitudes are

$$E_t = \sin (\theta_c) = 0.977$$

$$E_z = \cos (\theta_c) = 0.215. \quad (D.33)$$
Because the $z$-components of the field are so small compared to the transverse components, we simply ignore $\kappa^z$, because it is formed as the product of $z$-components.

For Bragg reflection, the grating period is designed so that it will couple the forward-traveling fundamental mode to the backward-traveling fundamental mode. The coupling from the fundamental to other modes, radiation or otherwise, should be negligible, so we ignore it. For our research, we desire the reflection to be centered at a wavelength of 1550 nm. Therefore, the grating vector must therefore be twice the propagation constant of the fundamental mode,

$$k_r = k_i - K_G$$

$$-2k_i = -K_G$$

$$\frac{4\pi n_{eff}}{\lambda_B} = \frac{2\pi}{\Lambda}$$

$$\Lambda = \frac{\lambda_B}{2n_{eff}},$$  \hspace{1cm} (D.34)

where $K_G$, $\lambda_B$, and $\Lambda$ are the grating vector, the Bragg reflection wavelength, and the grating period, respectively.

Because the interaction is only between the forward- and reverse-traveling fundamental modes (which have the same normalized electric field profile and propagation constant), we can simplify the coupled-mode equation even further. The coupling coefficient is the same for all pairs of modes, i.e. $\kappa_{ff} = \kappa_{rr} = \kappa_{fr} = \kappa_{rf}$, where $f$ and $r$ label the forward- and reverse-traveling modes respectively. We can therefore write the coupled-mode equation as

$$\frac{d}{dz}a_f(z) = -j\kappa(z)(a_f(z) + a_r(z)e^{j2\beta z})$$

$$\frac{d}{dz}a_r(z) = +j\kappa(z)(a_r(z) + a_f(z)e^{-j2\beta z})$$  \hspace{1cm} (D.35)

$$\kappa(z) = \frac{\omega\epsilon_0}{4} \int \int \Delta\epsilon_r(x,y,z)|E_{\mu}(x,y)|^2 dx dy.$$

To find $\kappa(z)$ numerically we first solve for the modes in the unperturbed waveguide – without a grating. To construct the unperturbed waveguide we assume that it extends to the center line of the grating. The resulting fields are then used in Eq. (D.35) to find the coupling coefficient. The integration is performed from the
center line of the grating to the surface of the grating at the given point in $z$. For example, if at a certain point in $z$ there is a trough in the grating, the electric field is integrated from the center of the grating down to the bottom of the grating. Figure D.1 shows a diagram of a D-fiber with a grating on the top surface. At the right in the figure is a vertical cut of the fiber showing how the topography varies in the $z$-direction.

![Diagram showing the profile of a grating on a D-fiber from the front and side.](image)

Figure D.1: Diagram showing the profile of a grating on a D-fiber from the front and side.

In practice, once the field of the mode is found, it is interpolated on a finer grid, and $\kappa$ is determined as a function of the grating height for a given number of points between a trough and a peak. In a trough, the height is $-\frac{h}{2}$ and at a grating peak the height is $\frac{h}{2}$, where $h$ is the peak-to-trough grating height. Once $\kappa(h)$ is determined, it is mapped onto the structure of the grating (whether sinusoidal, square, apodized, chirped, etc.) to get $\kappa(z)$.

### D.5 Numerical solution of the coupled-mode equations

The solution of the coupled-mode equations, Eq. (D.35), is greatly complicated by the presence of $\kappa(z)$. While $\kappa(z)$ is periodic (because of the presence of the grating), it is not a simple function. Also, if the grating is apodized or chirped,
it becomes even more complicated. Because of this, we solve the coupled differential equations numerically using the Runge-Kutta method instead of analytically.

First, we note that the derivative is defined as:

\[
\frac{df}{dx} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.
\]

The \(z\)-derivative of the complex amplitude function \(a(z)\) can therefore be written as:

\[
\frac{da}{dz} = \lim_{\Delta z \to 0} \frac{a(z + \Delta z) - a(z)}{\Delta z}.
\]

If we now let \(a(z)\) be defined only at discrete points, \(z_n\), with a spacing of \(\Delta z\) between them, we can approximate the derivative numerically as:

\[
\frac{da_n}{dz} \approx \frac{a_{n+1} - a_n}{\Delta z}.
\]

This derivative is only exact in the limit as \(\Delta z \to 0\). Notice that the derivative is not centered at the point \(z_n\), but at \(z_n + \frac{1}{2}\). In other words, this form of the derivative is a better approximation at the midpoint between the points labeled \(n\) and \(n + 1\), which leads us to the centered derivative

\[
\frac{da_{n+\frac{1}{2}}}{dz} \approx \frac{a_{n+1} - a_n}{\Delta z}.
\]

We can now put the above definition of the derivative into Eq. (D.35) and rearrange to approximate the differential equation:

\[
\begin{align*}
a_{f,n} &= a_{f,n+1} + j\Delta z\kappa(z_{n+\frac{1}{2}}) \left( a_{f,n+\frac{1}{2}} + a_{r,n+\frac{1}{2}} e^{2j\beta(z_{n+\frac{1}{2}} - \frac{\Delta z}{2})} \right) \\
a_{r,n} &= a_{r,n+1} - j\Delta z\kappa(z_{n+\frac{1}{2}}) \left( a_{r,n+\frac{1}{2}} + a_{f,n+\frac{1}{2}} e^{-2j\beta(z_{n+\frac{1}{2}} - \frac{\Delta z}{2})} \right).
\end{align*}
\]

The boundary conditions for this solution must be given at the end of the grating, after the incident light has passed through it. The boundary condition is that at the end of the simulation domain, there is no power in the reverse traveling wave. This is reasonable because the only contribution to the reverse traveling wave comes when it is reflected by the grating, but at the end of the simulation region there is no more grating to reflect any power. The boundary condition on the forward traveling wave is that it be any value that is nonzero. If it is identically zero, there will be no
light in the entire simulation region. To find how long a grating would need to be to reflect 99% of the incident power, for example, the boundary condition on the forward traveling wave could be that it have 0.01 units of power at the end of the simulation domain. Then, iterate back to the beginning of the grating until the forward traveling wave has 1 unit of power. This would then give the grating length to achieve a 99% reflection.

D.6 Chirp and apodization

Chirping and apodization of gratings gives more control over the transmission and reflection spectra of gratings, allowing for dispersion compensation, bandwidth augmentation, sidelobe suppression, etc. A chirped grating is one whose grating period changes along the length of the grating. An apodized grating is one whose coupling coefficient changes in strength along the length of the grating.

The period of a linearly chirped grating as a function of $z$ can be written

$$\Lambda(z) = (\Lambda_f - \Lambda_i) \frac{z}{L} + \Lambda_i, \quad (D.41)$$

where $\Lambda_f$ and $\Lambda_i$ are the final and initial grating periods, and $L$ is the length of the grating. Alternately, we can define a chirp ratio

$$R_{ch} = \frac{(\Lambda_f - \Lambda_i)}{\Lambda_c}, \quad (D.42)$$

where $\Lambda_c$ is the central grating period. This lets us rewrite the initial and final grating periods as

$$\Lambda_i = \Lambda_c \left(1 - \frac{R_{ch}}{2}\right) \quad (D.43)$$
$$\Lambda_f = \Lambda_c \left(1 + \frac{R_{ch}}{2}\right).$$

Finally, we can write the overall grating period as

$$\Lambda(z) = \Lambda_c \left[1 + R_{ch} \left(\frac{z}{L} - \frac{1}{2}\right)\right]. \quad (D.44)$$

To incorporate chirp into the coupled-mode code, I start by mapping $\kappa(h)$ onto a sinusoidal grating. This is done by simply taking the formula for the height of
the grating \( h(z) \), and inverting it to find \( z(h) \),

\[
h(z) = h_{\text{max}} \cos \left( \frac{2\pi}{\Lambda} z \right),
\]

\[
z(h) = \frac{\Lambda}{2\pi} \cos^{-1} \left( \frac{h}{h_{\text{max}}} \right).
\]

(D.45)

This function of \( h \) is only computed for half of one period, the result is mirrored and replicated many times, adding one full grating period to each successive replication until the end of the grating is reached at \( z = L \). Note that this produces non-uniform spacing in \( z \). Successive grid points in \( z \) are only placed where the discretized height of the grating changes.

The next step in incorporating chirp is to relate the phase of the sinusoidal grating to that of the chirped grating to do a new mapping of the grid points in \( z \). The phase of a non-chirped grating can be written

\[
\phi = \frac{2\pi}{\Lambda} z,
\]

(D.46)

but the phase of a chirped grating must be written

\[
\phi = \int \frac{2\pi}{\Lambda(z)} dz.
\]

(D.47)

The phase accumulated (by a chirped grating) in going from the origin to a point \( z_0 \) can be written

\[
\phi = \int_0^{z_0} \frac{2\pi}{\Lambda(z)} dz
\]

\[
\phi = \frac{2\pi L}{\Lambda c R_{ch}} \int_0^{z_0} \frac{dz}{\alpha + z}, \quad \alpha = L \left( \frac{1}{R_{ch}} - \frac{1}{2} \right).
\]

(D.48)

This equation can be integrated by performing a \( u \) substitution, with \( u = \alpha + z \) and \( du = dz \), giving

\[
\phi = \frac{2\pi L}{\Lambda c R_{ch}} \int_{\alpha}^{\alpha + z_0} \frac{du}{u}.
\]

(D.49)

This integral is easily evaluated as

\[
\phi = \frac{2\pi L}{\Lambda c R_{ch}} \ln \left( \frac{\alpha + z_0}{\alpha} \right).
\]

(D.50)
Now, by equating the phase accumulated by a chirped grating with the phase accumulated by a non-chirped grating, we can figure out how to map grid points from the non-chirped to those of the chirped grid.

\[
\frac{2\pi L}{\Lambda_c} z' = \frac{2\pi L}{\Lambda_c R_{ch}} \ln \left( \frac{\alpha + z}{\alpha} \right),
\]

(D.51)

where the left-hand side represents the phase accumulated at point \(z'\) for a non-chirped grating with a grating period of \(\Lambda_c\). Solving for \(z\) we get

\[
z = \alpha \left( e^{\frac{R_{ch} L}{z}} - 1 \right),
\]

(D.52)

where \(\alpha\) is defined above. This mapping allows for simple mapping of the grid used for a sinusoidal grating into the grid used for a chirped grating.

Apodization of gratings is accomplished in a somewhat simpler manner than chirping. First, the non-apodized grating shape is determined over the length of the grating, then this function is multiplied by the apodizing function, which is a gaussian in my code. The grating heights are then sorted from smallest to largest (keeping track of the change in order), and they are binned to the nearest point in the height grid using Matlab’s hist.m command. The function \(\kappa(h)\) is then mapped and unpacked back into the correct order, as well as the grid points in \(z\), to get \(\kappa(z)\).

**D.7 Simulation results**

In order to compare the results of this numerical code with analytical results for standard FBGs, I constructed a numerical model of an index modulation grating instead of a surface relief grating. The results of the simulation are shown below in Fig. D.2. The analytical solution was found using Ref. [34]. The simulation parameters were a design wavelength of 1550 nm, a confinement factor of 0.7113, effective index of 1.45446, \(\delta n_{co}\) of \(2 \times 10^{-4}\), a length of 1 cm, and no chirp. The plot shows that the results are almost identical. The difference came in that the ‘DC’ effective index of the numerical solution was slightly lower than that of the analytical solution, which caused the reflection peaks to be separated by 76 pm.

Figure D.3 shows the results of chirping and apodizing a grating. The grating simulated in this figure is 0.5 \(\mu\)m from peak-to-trough, and the center line of
the grating is 0.3 µm from the top of the horizontal core of the fiber. The figure shows how chirping broadens the reflection peak of the grating and apodizing suppress the sidelobes.

### D.8 Code

The first code listed below takes the transverse electric field profile of a mode, the grating depth, and the center line of the grating and finds the coupling coefficient, $\kappa$ as a function of height. The second code maps from $\kappa(h)$ to $\kappa(z)$ so that the coupling coefficient is ready to be put into the solver for the coupled-mode equations. The third code calls the mapping code and sets up different constants needed for the Runge-Kutta solver, and the fourth code is the solver.

% This function computes kappa, the coupling coefficient for different
% points in height on the grating. It divides the peak to trough height of
% the grating into 81 segments and computes kappa at each of those
% "altitudes." In order to do this, the electric field of the given mode
% is first interpolated at a finer grid spacing and then integrated to find
Figure D.3: Plots of the reflection spectrum of a purely sinusoidal grating compared with a chirped and an apodized grating.

% kappa(h)

% Load Ex and Ey, along with effective index of mode
[Ex,neff]=loadE([name '.m00']);
[Ey,neff]=loadE([name '.n00']);

% Define constants
k0=2*pi/mean(lambda); % mean is used in case we are doing a spectral run
mu=4e-7*pi;
w=k0*c;
eps0=8.854e-12;
beta=neff*k0;

% Find grating period
if Lam==1
    Lam=mean(lambda)/2/neff % unetched fundamental neff is 1.45006101
% Convert everything from microns to meters; define dx, dy; define grid
if max(x)>1
    x=x*1e-6;
    y=y*1e-6;
    d=d*1e-6;
    hgrat2=hgrat2*1e-6
end

dx=x(2)-x(1);
dy=y(2)-y(1);
[xx,yy]=meshgrid(x,y);

% Find field magnitude squared (proportional to power)
Esq=Ex.^2+Ey.^2;

% Normalize the field so the total power in the mode is 1W
int=beta/2/w/mu*trapz(trapz(Esq))*dx*dy;
Esq=Esq/int;

maxE=max(max(Esq))

% Define the number of points to define on the height grid
% and define a new interpolation grid
Nhz=81;
hz=linspace(-hgrat2,hgrat2,Nhz);
yk=hz+d;
dyk=yk(2)-yk(1);
xk=linspace(-4e-6,4e-6,321); % Narrow the integration window
dxk=xk(2)-xk(1);
[xx,yk]=meshgrid(xk,yk);
% Interpolate in the region of the grating to get more precise
% values for the E-field squared and delta_epsilon in this region
Esqi=interp2(xx,yy,Esq,xk,yk,'spline');

% Multiply interpolated E_sq by delta_epsilon -> the u is for the
% part of the grating above the center point, and d for below it.
% Note that this approximation is not entirely correct. It assumes
% that the index of the core is 1.441 while it is 1.4756.
Esqid=Esqi*(1-1.441^2);
Esqiu=Esqi*(1.441^2-1);

% Define handy factors and initialize kap_h (kappa as a function of
% height relative to the grating center
fac=w*eps0/4*dyk*dxk;
nmid=(Nhz+1)/2;
kap_h=zeros(size(hz));

% Compute the coupling coefficient as a function of height (kap_h)
% by integrating
for n=1:Nhz
    if n<nmid
        kap_h(n)=fac*trapz(trapz(Esqid(n:nmid,:)));%Optional
    elseif n>nmid
        kap_h(n)=fac*trapz(trapz(Esqiu(nmid:n,:)));%Optional
    else
        kap_h(n)=0;
    end
end
function [kappa, z]=kappa_map(kap,hz,Lam,L,sq_sin, ...  
chirp_ratio,apod,apod_sigma)

% This function computes kappa(z), the coupling coefficient, given
% different parameters. As input, it needs kappa as a function of depth.
% h should be a vector of heights at which kappa is defined, then this
% function maps those heights onto different grating types. The extrema
% of h are the lowest and highest points in the grating. Lam is the
% initial grating period and L is the overall length of the grating. If
% the grating is sinusoidal with no chirp or apodization, sq_sin,
% chirp_ratio, and apod should all be zero. To make a square grating,
% sq_sin is set equal to the fraction of one period that the grating is
% high. To chirp the grating, chirp_ratio is set to the ratio of the
% initial period to the final period. At this time only linear chirp is
% allowed. If the grating is apodized, apod is set to the ratio of the
% deepest region of the grating to the shallowest region, and the
% apodization is approximately gaussian with sigma given by apod_sigma
% where the gaussian envelope is defined by
% max(h)/apod+max(h)*(1-1/apod)*exp(-(z-L/2).^2/2/apod_sigma^2)
% See Kevin Smith’s dissertation for more details.

% Define slope of square sidewalls if grating is square
rise=Lam/20;
pt1=sq_sin*Lam-rise;
pt2=sq_sin*Lam;
pt3=Lam-rise;
pt4=Lam;
npts=80;

if chirp_ratio~=0 % chirped gratings
    % approximately find the number of periods

nper=ceil(L/Lam);
alpha=L*(1/chirp_ratio-1/2);  % Convenient factor
zcent=L/2;

if sq_sin~0 % square gratings
  % Z-points where kappa is defined (sidewalls of square)
  z1=[linspace(pt1,pt2,npts), linspace(pt3,pt4,npts)];
  Len=length(z1);
  z=zeros(1,nper*Len);
  for nn=1:nper
    z(1+(nn-1)*Len:nn*Len)=z1+Lam*nn;
  end
  h1=linspace(max(hz),min(hz),npts);

  if apod==0 | apod==1 % non-apodized gratings
    kap1=interp1(hz,kap,h1);
    kappa=repmat([kap1 fliplr(kap1)],1,nper);
  else % apodized gratings
    htemp=repmat([h1 -h1],1,nper);
    h_env=1/apod+(1-1/apod)*exp(-(z-zcent).^2/2/apod_sigma^2);
    htemp=htemp.*h_env;
    [ht,ind]=sort(htemp);
    numbin=hist(ht,hz);
    ii=1;
    for nn=1:length(numbin)
      kappa(ii:ii+numbin(nn)-1)=kap(nn);
      ii=ii+numbin(nn);
      nn;
    end
    kappa2=kappa;
    for nn=1:length(z)
      kappa(ind(nn))=kappa2(nn);
    end

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else % sinusoidal gratings
    zz1 = acos(hz/max(abs(hz)))*Lam/2/pi;
    zz2 = fliplr(zz1);
    zz3 = Lam - fliplr(zz2(zz2<Lam/2));
    z = [zz2 zz3(1:end-1)];
    ztemp = z;
    Len = length(ztemp);
    z = zeros(1,nper*Len);
    for nn = 1:nper
        z(1+(nn-1)*Len:nn*Len) = ztemp + Lam*nn;
    end
if apod == 0 | apod == 1 % non-apodized gratings
    kap2 = fliplr(kap(2:end));
    kappa = [kap2 kap(1:end-1)];
    kappa = repmat(kappa,1,nper);
else % apodized gratings
    htemp = max(hz)*cos(2*pi/Lam*z);
    h_env = 1/apod + (1-1/apod)*exp(-((z-zcent/2).^2/2/apod_sigma^2);
    htemp = htemp.*h_env;
    [ht,ind] = sort(htemp);
    numbin = hist(ht,hz);
    ii = 1;
    for nn = 1:length(numbin)
        kappa(ii:ii+numbin(nn)-1) = kap(nn);
        ii = ii + numbin(nn);
        nn;
    end
    kappa2 = kappa;
end
end

end % sinusoidal gratings

else % sinusoidal gratings
    zz1 = acos(hz/max(abs(hz)))*Lam/2/pi;
    zz2 = fliplr(zz1);
    zz3 = Lam - fliplr(zz2(zz2<Lam/2));
    z = [zz2 zz3(1:end-1)];
    ztemp = z;
    Len = length(ztemp);
    z = zeros(1,nper*Len);
    for nn = 1:nper
        z(1+(nn-1)*Len:nn*Len) = ztemp + Lam*nn;
    end
if apod == 0 | apod == 1 % non-apodized gratings
    kap2 = fliplr(kap(2:end));
    kappa = [kap2 kap(1:end-1)];
    kappa = repmat(kappa,1,nper);
else % apodized gratings
    htemp = max(hz)*cos(2*pi/Lam*z);
    h_env = 1/apod + (1-1/apod)*exp(-((z-zcent/2).^2/2/apod_sigma^2);
    htemp = htemp.*h_env;
    [ht,ind] = sort(htemp);
    numbin = hist(ht,hz);
    ii = 1;
    for nn = 1:length(numbin)
        kappa(ii:ii+numbin(nn)-1) = kap(nn);
        ii = ii + numbin(nn);
        nn;
    end
    kappa2 = kappa;
end
end
for nn=1:length(z)
    kappa(ind(nn))=kappa2(nn);
end

dv=diff(kappa);
iii=find(dv);
kappa=[kappa(1) kappa(iii+1)];
z=[z(1) z(iii+1)];
end

else % non-chirped gratings

    nper=ceil(L/Lam);
    if sq_sin~=0 % square gratings
        % Z-points where kappa is defined (sidewalls of square)
        z1=[linspace(pt1,pt2,npts), linspace(pt3,pt4,npts)];
        Len=length(z1);
        z=zeros(1,nper*Len);
        for nn=1:nper
            z(1+(nn-1)*Len:nn*Len)=z1+Lam*nn;
        end
        h1=linspace(max(hz),min(hz),npts);
    end
    if apod==0 | apod==1 % non-apodized gratings
        kap1=interp1(hz,kap,h1);
        kappa=repmat([kap1 fliplr(kap1)],1,nper);
    else % apodized gratings
        htemp=repmat([h1 -h1],1,nper);
        h_env=1/apod+(1-1/apod)*exp(-((z-L/2).^2/2/apod_sigma^2);
        htemp=htemp.*h_env;
    end

end

z=alpha*(exp(z*chirp_ratio/L)-1);
[ht,ind]=sort(htemp);
numbin=hist(ht,hz);
ii=1;
for nn=1:length(numbin)
    kappa(ii:ii+numbin(nn)-1)=kap(nn);
    ii=ii+numbin(nn);
end
kappa2=kappa;
for nn=1:length(z)
    kappa(ind(nn))=kappa2(nn);
end

else % sinusoidal gratings
    zz1=acos(hz/max(abs(hz)))*Lam/2/pi;
    zz2=fliplr(zz1);
    zz3=Lam-fliplr(zz2(zz2<Lam/2));
    z=[zz2 zz3(1:end-1)];
    ztemp=z;
    Len=length(ztemp);
    z=zeros(1,nper*Len);
    for nn=1:nper
        z(1+(nn-1)*Len:nn*Len)=ztemp+Lam*nn;
    end
    if apod==0 | apod==1 % non-apodized gratings
        kap2=fliplr(kap(2:end));
        kappa=[kap2 kap(1:end-1)];
        kappa=repmat(kappa,1,nper);
    else % apodized gratings
        htemp=max(hz)*cos(2*pi/Lam*z);
        h_env=1/apod+(1-1/apod)*exp(-(z-L/2).^2/2/apod_sigma^2);

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htemp=htemp.*h_env;
[ht,ind]=sort(htemp);
numbin=hist(ht,hz);
ii=1;
for nn=1:length(numbin)
    kappa(ii:ii+numbin(nn)-1)=kap(nn);
    ii=ii+numbin(nn);
    nn;
end
kappa2=kappa;
for nn=1:length(z)
    kappa(ind(nn))=kappa2(nn);
end
dv=diff(kappa);
iii=find(dv);
kappa=[kappa(1) kappa(iii+1)];
z=[z(1) z(iii+1)];
end
kappa=kappa(z<L);
z=z(z<L);
The following code calls the mapping function and sets up values for use in the solver.

```matlab
if ~exist('sq_sin')
    sq_sin=0;
end
if ~exist('apod')
    apod=0;
end
if ~exist('chirp_ratio')
    chirp_ratio=0;
end
apod_sigma=L/6;
[kappa, z]=kappa_map2(kap_h, hz, Lam, L, sq_sin, chirp_ratio, apod, apod_sigma);
Nz=length(z);

% Set up handy values of kappa (take multiplications out of the for loop)
% at the points in z and halfway between points in z
dz=[0 diff(z)];
kaphalf=j*dz.*[0 (kappa(2:end)+kappa(1:end-1))/2];
kap=j*.5*dz.*kappa;
```
The following code solves the coupled-mode equations numerically.

```matlab
% Get rid of unnecessary stuff
clear kappa indprofun indprof depsd depsu Esq Esqi
Esqid Esqiu Ex Ey x xk xx y yk yy

% Set up more constants for fast execution of the for loop
exp1=exp(j*2*beta*z);
exp3=exp(j*2*beta*(z-dz/2));

% Set up coefficients for forward and backward traveling waves
af=zeros(1,Nz);
ab=af;
% Set boundary conditions at point z=L (af=.1 and ab=0)
af(Nz)=sqrt(.01);
% af(Nz)=.1;
ab(Nz)=0;

% Run the iterative loop to compute af and ab
for n=Nz:-1:2
    afhalf=af(n)+kap(n)*(af(n)+ab(n)*exp1(n));
    abhalf=ab(n)-kap(n)*(ab(n)+af(n)*conj(exp1(n)));
    af(n-1)=af(n)+kaphalf(n)*(afhalf+abhalf*exp3(n));
    ab(n-1)=ab(n)-kaphalf(n)*(abhalf+afhalf*conj(exp3(n)));
end
```

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Bibliography


