PREDICTING DRAG POLARS FOR MICRO AIR VEHICLES

by

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ABSTRACT

PREDICTING DRAG POLARS FOR MICRO AIR VEHICLES

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Drag polars for three Micro Air Vehicles (MAVs) were measured at Reynolds numbers of 70,000, 50,000, 30,000, and 10,000 and compared to predictions generated using the classical approach. The MAVs tested had different configurations and aspect ratios varying from 1.2 to 1.6 and ratios of wetted surface area to planform area from 2.6 to 3.9.

A force balance was used to measure the lift and drag on the MAVs at angles of attack ranging from -5° (or -10°) to 10°. The force balance allowed the MAVs to rotate in the pitching axis. The MAV angle of attack was set using an elevator installed on the MAV and controlled using a standard radio control used by RC plane enthusiasts. Uncertainty analysis performed on the data showed the uncertainty for high Reynolds numbers was dominated by velocity uncertainty, and uncertainty for the lower Reynolds numbers was dominated by uncertainty in the force measurements.
Agreement between the measured and predicted drag polars was good with the measured drag never being more than two times the predicted drag. For the majority of the tests, the drag coefficients followed the expected Reynolds number trend: increasing with decreasing Reynolds number.
ACKNOWLEDGEMENTS

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<td>Maximum Lift-To-Drag Ratio</td>
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<td>Micro Air Vehicle</td>
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<tr>
<td>mV</td>
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<td>Number of Data Points in a Set of Data</td>
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<td>$P_{\text{amb}}$</td>
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<td>$W$</td>
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<td>$W/S$</td>
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<td>Arbitrary variable</td>
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Chapter 1: Introduction

1.1 BACKGROUND

Micro Air Vehicles, or MAVs, have recently experienced a surge in interest and research. A MAV is generally defined as an aircraft with a wingspan of less than 15 cm and a mass of less than 100 grams\(^1\). An example of a MAV built at Brigham Young University is shown in Figure 1.1.

![An example of a Micro Air Vehicle](image)

**Figure 1.1** An example of a Micro Air Vehicle

Various military and commercial applications have been proposed, ranging from surveillance and visual reconnaissance to air sampling and sensing of chemical or biological agents. However, the development of MAV technology is in its infancy and is in a stage of development similar to the first decade of flight in the early 20\(^{th}\) century.
Currently there are no formalized methods for predicting the drag polar for a MAV, which is needed to predict thrust and power requirements. This information is critical to the design of any aircraft. The purpose of this research is, therefore, to develop a method for predicting the drag polar for a MAV concept design. As much as possible, the classical approach of estimating the drag polar for conventional (larger scale) air vehicles will be extended to the domain of MAVs with modifications to compensate for the reduced Reynolds numbers characterizing MAV flight. It should be noted that this research is confined to predicting the drag polar in the *conceptual stage* of a MAV design, because the drag polar is determined empirically (and at greater cost) in the later stages of a MAV design.

Figure 1.2 shows a generic aircraft design process that illustrates how and in what stages the drag polar is used in the design process. It also delimits the area of study of this research.

![Diagram of a typical aircraft design process showing when the drag polar is predicted](image_url)
Of course, this is a greatly simplified design process, but this depiction suffices as a description of when and how estimating the drag polar for an aircraft occurs during the design of an aircraft. As one might deduce, the more accurately the drag polar is predicted in the early stages of the design, the greater will be the reduction in resources and time needed to revise and finalize an aircraft design.

Unfortunately, two problems could potentially arise in predicting the drag polar for MAVs. One, some of the theory upon which the classical method is based is extracted from experimental data taken in high Reynolds number regimes and may not be applicable to MAV flight which occurs in a very low Reynolds number regime (ie. Re < 200,000). Second, gathering useful empirical data for low Reynolds number regimes and MAV flight is plagued with a high degree of difficulty and uncertainty. The purpose of this research is to overcome these problems and present a comparison between predicted and measured drag polars for MAVs along with relevant conclusions.

The material that follows in this chapter will cover the following related to predicting and measuring drag polars for MAVs:

- Explain how the drag polar is predicted for traditional air vehicles
- Discuss the importance of estimating the drag polar
- Investigate aspects of the classical drag polar prediction methodology which might produce erroneous results for MAVs
- Discuss considerations relating to measuring drag polars for MAVs

1.2 PREDICTING DRAG POLARS FOR CONVENTIONAL AIRCRAFT
Since this thesis investigates the application of the classical methodology for predicting drag polars to MAVs, it is important to understand the classical methodology and how it is derived. This section explains the drag polar and how it is predicted.

The drag polar is a plot of lift coefficient, $C_L$, vs. drag coefficient, $C_D$, for a given aircraft. The true drag polar is empirically measured. However, in the linear range of $C_L$ vs. angle of attack, $\alpha$, the drag polar can be approximated by a parabolic equation expressed in terms of $C_L$ and $C_D$ as the independent and dependent variables\(^2\). To determine the drag of an aircraft at any value of lift, a designer must be concerned with the two major components of drag: “Parasite” drag (or drag at zero lift – the component of drag that is independent of lift) and drag due to lift (which varies approximately quadratically with lift). An example of a drag polar is depicted below.

![Sample drag polar](image)

Figure 1.3 Sample drag polar

The equation for the drag polar is given by Brandt, et al.\(^3\) as follows:

$$C_D = C_{D,0} + k_1 C_L^2 + k_2 C_L$$  \hspace{1cm} (1.1)
$C_{D,0}$ is the drag at zero-lift (the X-intercept) and is called the parasite drag coefficient. The $k_1C_L^2$ and $k_2C_L$ terms represent drag induced by lift, or drag due to lift.

In reality, the parabola is only a good approximation while $C_L$ is a linear function of angle of attack, $\alpha$. At high $\alpha$, the relationship between $C_L$ and $\alpha$ becomes nonlinear, especially near the maximum value of $C_L$ at stall. In the nonlinear region, the parabola no longer approximates the real drag polar. Fortunately, most airplanes fly at angles of attack in the linear $C_L(\alpha)$ range below stall.

The challenge for the MAV designer is to estimate parasite drag coefficient $C_{D,0}$ and the coefficients $k_1$ and $k_2$. Once these have been estimated, the MAV designer can predict the MAV’s performance using the predicted drag polar. Methods of estimating the two components of the drag polar for conventional air vehicles are described as follows.

1.2.1 **Parasite Drag Coefficient**

The parasite drag coefficient is the first component of the drag polar and is represented by the term $C_{D,0}$. There are two methods by which the parasite drag coefficient of an aircraft can be estimated. The easiest method is the Equivalent Skin-Friction Coefficient method. The other is the Component Build-up method, which sums drags for individual components like wings and fuselage and also accounts for interaction between them. However, the Component Build-up method will not be covered by this thesis because of the additional complexity involved in validating this method for MAVs.

Brandt, et al. give an expression for $C_{D,0}$ for the Equivalent Skin-Friction Coefficient method as shown below:

$$C_{D,0} = C_{D,\text{min}} + k_1(C_{L,\text{min}D})^2$$

(1.2)
where $C_{D,\text{min}}$ is the minimum drag coefficient, $k_1$ is the same coefficient as shown in Eq. (1.1), and $C_{L,\text{minD}}$ is the coefficient of lift when drag is at a minimum (at $C_{D,\text{min}}$).

$C_{D,\text{min}}$ is found using the following expression:

$$C_{D,\text{min}} = \frac{S_{\text{wet}}}{S} C_{fe}$$  \hspace{1cm} (1.3)

$S_{\text{wet}}/S$ is the ratio of the aircraft’s wetted surface area $S_{\text{wet}}$ (the plane’s total external surface area exposed to the air) to its wing planform area $S$ (the 2-D area of the wings if viewed from the top or bottom). $C_{fe}$ is the equivalent skin friction drag coefficient (including skin friction and pressure drag). It is derived based on the assumptions that a well-designed aircraft exhibits mostly skin-friction drag, and that separation drag is a consistent percentage of total drag for a specific class of aircraft.
Typically, a designer can look up from charts values for $S_{wet}/S$ for the aircraft class most similar to the design concept (see Figure 1.4 below$^2$).

![Figure 1.4](image)

**Figure 1.4** $S_{wet}/S$ vs. wing loading $W/S$

Values for $C_{f_e}$ can also be found graphically by aircraft class for a range of Reynolds numbers between $10^7$ and $10^9$ (see Figure 1.5$^2$).
For example, if a new design configuration mostly resembles a commercial airliner, the designer would use values for $S_{wet}/S$ and $C_{fe}$ from a similar existing airliner. $C_{fe}$ can also be approximated from theory using flat plate skin friction equations for one side of a flat plate shown below. Equation (1.4) is for laminar flow ($Re < 350,000$). Equations (1.5 through (1.7) can be used for turbulent flow ($Re > 1,000,000$)$^{2,5}$.

$$c_f = \frac{1.328}{\sqrt{Re}} \quad \text{(Blasius)}$$  \hspace{1cm} (1.4)

$$\left(c_f\right)^{-1/2} = 4.13 \log(Re \ c_f) \quad \text{(Karman-Schoenherr)}$$  \hspace{1cm} (1.5)

$$c_f = \frac{0.42}{\ln^2(0.056 Re)} \quad \text{(White and Christoph)}$$  \hspace{1cm} (1.6)
According to Anderson, $c_{fe}$ can then be approximated by the following equation:

$$c_{fe} = 1.5 \times c_f$$ \hfill (1.8)

Once $C_{D,max}$ has been estimated, values for $k_1$ and $C_{L,minD}$ must be determined in order to complete the prediction of $C_{D,0}$. The method for calculating $k_1$ will be described in section 1.2.2.1. The method for determining $C_{L,minD}$ is described in section 1.2.2.2.

Finally, it is worth noting that in the special case in which the wing has no or little camber, Eq. (1.2) collapses into a simplified expression for $C_{D,0}$ given in most texts where $C_{D,0}$ is equal to $C_{D,min}$:

$$C_{D,0} = \frac{S_{wet}}{S} C_{fe}$$ \hfill (1.9)

1.2.2 Drag Due To Lift

The other component of the drag polar is the drag due to lift, which is represented by the last two terms of the drag polar: $k_1 C_L^2 + k_2 C_L$. $C_L$ is simply the lift coefficient and is the independent variable in the drag polar. The $k_2 C_L$ term equals zero for uncambered wings. How the coefficients $k_1$ and $k_2$ (which account for induced drag) are determined classically is discussed below.

1.2.2.1 $k_1$

The coefficient $k_1$ is calculated using the following equation:

$$k_1 = \frac{1}{\pi e_0 AR}$$ \hfill (1.10)

where $e_0$ is the Oswald efficiency factor and $AR$ is the wing aspect ratio.
The Oswald efficiency factor accounts for extra drag due to 3-D effects and a non-elliptical lift distribution. It was originally presented by W. B. Oswald in his doctoral dissertation at the California Institute of Technology\textsuperscript{4}.

Curve-fit equations for the Oswald efficiency factor, $e_0$, have been determined from empirical data for straight-wing and swept-wing aircraft\textsuperscript{3,4}. For straight-wing aircraft, the empirical correlation for $e_0$ is shown in Eq. (1.11):

$$e_0 = 1.78(1 - 0.045AR^{0.68}) - 0.64$$  \hspace{1cm} (1.11)

For swept-wing aircraft (with sweep angle > 30 degrees), $e_0$ is estimated with Eq. (1.12):

$$e_0 = 4.61(1 - 0.045AR^{0.68})(\cos \Lambda_{LE})^{0.15} - 3.1$$  \hspace{1cm} (1.12)

where $\Lambda_{LE}$ is the sweep angle of the wing’s leading edge. Whether or not these equations can be used to approximate $e_0$ for MAVs is discussed in Chapter 5 of this thesis.

The aspect ratio $AR$ is defined as:

$$AR = \frac{b^2}{S}$$  \hspace{1cm} (1.13)

where $b$ is the wingspan and $S$ is the planform area of the wing discussed earlier\textsuperscript{2}.

1.2.2.2 $k_2$

The coefficient $k_2$ is a function of $k_1$ and the lift coefficient at minimum drag, or $C_{L,minD}$. The expression for $k_2$ is:

$$k_2 = -2k_1C_{L,minD}$$  \hspace{1cm} (1.14)

For uncambered wings, $C_{L,minD}$ equals 0, and thus the $k_2C_L$ term in the drag polar vanishes for uncambered wings. For cambered wings, $C_{L,minD}$ can be estimated using the following approximation:
\[ C_{L,\text{min}D} = \frac{C_{L,\alpha=0}}{2} \]  

where \( C_{L,\alpha=0} \) is the lift coefficient at zero angle of attack\(^3\). Therefore, for cambered wings, the designer will have to know the lift coefficient at zero angle of attack.

1.3 **USEFULNESS OF THE DRAG POLAR**

The drag polar describes the lift and drag characteristics of an aircraft. From the drag polar the designer can infer the minimum drag coefficient and the maximum lift-to-drag ratio \((L/D)_{\text{max}}\) for a design. Other flight performance parameters like maximum velocity, endurance, and rate of climb also include the drag polar in their calculations\(^2\).

This information provided by the drag polar is crucial to the designer of an airplane. Having this information helps the designer predict the airplane’s performance and size the airplane’s propulsion system.

The limitation on the drag polar, of course, is that it does not provide information on the stability and control of the aircraft. In order to design an aircraft, other information must be available to the designer to deal with stability issues, but that is not within the scope of this research. However, it is important to remember that the influence of stabilizers on drag and lift is taken into account by the drag polar.

1.4 **PREDICTING DRAG POLARS FOR MICRO AIR VEHICLES**

1.4.1 **Extending Classical Approaches**

Much of the classical approach for estimating the drag polar is still applicable to MAVs. However, at the beginning of this research, it was believed that various aspects of the classical approach needed to be verified for MAVs because of their basis on empirical data gathered from conventional air vehicles.
1.4.2 Aspects of the Classical Approach To Be Verified

1.4.2.1 Parasite Drag Coefficient

1.4.2.1.1 $S_{we}/S$

Typical MAV’s have configurations (i.e. shape of the wings and fuselage) and weights that are significantly different from those of large-scale airplanes. These differences translate into $S_{we}/S$ ratios that correspond with much lower values of wing loading $W/S$ (where $W$ is the weight of the aircraft). Designers of conventional airplanes can look on charts or tables of $S_{we}/S$ vs. $W/S$ and estimate an average $S_{we}/S$ value for the class of aircraft to which the new airplane would belong because of its configuration. However, such data for MAVs are not available to MAV designers. Therefore, this research will provide $S_{we}/S$ vs. $W/S$ data for the MAVs studied.

1.4.2.1.2 $C_{fe}$

In order to estimate the drag polar using the Equivalent Skin-Friction method, a value for $C_{fe}$ must be known. $C_{fe}$ can be predicted using flat plate theory, but as can be seen in Figure 1.5, there is a difference between empirical values and those based on the flat plate assumption. For conventional aircraft, the difference appears to be about a 50% increase over the value of $c_f$ given by flat plate theory, but what this difference would be for MAVs is unpredictable. Unfortunately, there are no empirical values of $C_{fe}$ available in the reference literature for the lower range of Reynolds numbers (between $10^3$ and $10^5$) experienced by MAVs. Therefore, this research will provide empirical $C_{fe}$ data for low Reynolds numbers and quantify correlation with flat plate theory.
1.4.2.2 Drag Due to Lift

1.4.2.2.1 $e_0$ – Oswald Efficiency Factor

The equations for the Oswald efficiency factor, or $e_0$, given in section 1.2.2.1 are curve-fits of wind tunnel data. Since MAVs experience much lower Reynolds numbers and have lower wing aspect ratios, the curve-fit equations given by Raymer may not accurately predict $e_0$ for MAVs. Agreement of these equations with experimental data will be checked.

1.4.2.2 $C_{L_{\text{minD}}}$

The approximation for $C_{L_{\text{minD}}}$ mentioned in section 1.2.2.2 may hold true for the MAV flight regime. It will be validated or modified as part of this research.

1.4.3 Prior Work

There is not yet a large body of research related to MAVs. Numerous MAVs have been built, and a drag polar has been measured for at least one of them empirically. Research projects have also been conducted to study the effects of various aerodynamic characteristics of MAVs including airfoil camber, airfoil planform shape, etc. One example is research completed at the U.S. Air Force Academy in 1991 to construct predictive equations for $k_1$ and $C_{D_{\text{0}}}$ for student-made balsa gliders for $Re$ less than 36,000.

Another example is research completed by Kevin Paulson at Brigham Young University. In a master’s thesis, he published his finding that parasite drag for flat plate wings in the Reynolds number range of 10,000 to 40,000 was double the flat plate predictions. There have been no comprehensive studies, however, that address the topic
of how designers can estimate (with reasonable accuracy) a full drag polar for a design in the conceptual stage.

1.5 MEASURING DRAG POLARS FOR MAVS

Much time and effort was invested into developing the equipment and procedures used to measure drag polars for the MAVs studied in this research. Various issues dealt with are described here.

1.5.1 Force Balance Considerations

At the time this research was started, BYU did not have a force balance sensitive enough to measure the minute aerodynamic forces that MAVs experience, which are on the order of tenths of a newton (or hundredths of a pound of force). Thus a force balance needed to be developed that was sensitive enough to measure these forces in both the lift and drag directions. Once a force balance was designed or modified, it would also require calibration. The calibration is described in Chapter 3 of this thesis.

In reality, in forward flight an aircraft may experience a pitching moment in addition to the lift and drag forces. In order to fully quantify the aerodynamic forces experienced during forward flight, the pitching moment should also be measured. To reduce the complication of the setup involved, it was decided that instead of developing a force balance that could also measure pitching moment, each MAV would be mounted at its center of gravity and allowed to rotate freely in the pitching axis. Thus no moment could be exerted by the force balance to counteract any pitching moment produced by the airflow about the MAV, and the MAV, if statically stable, would settle on its own to an equilibrium angle of attack (or trim). Using this testing scheme therefore obviated the need to measure the pitching moment, since it was constrained to always be zero.
1.5.2 Testing Considerations

In conjunction with the MAV mounting scheme, each MAV would need a functioning horizontal control surface (or elevator) in order to experiment with different angles of attack. In flight, the angle of attack is controlled by the elevator setting. If an aircraft is statically stable, it should trim to an equilibrium angle of attack based on its elevator setting, and the pitching moment about the center of gravity would go to zero. Including control surface deflections not only allowed measurements at different angles of attack, but it also provided the added benefit of more precisely approximating the drag polar for a MAV in actual flight, since control surface deflections also have an effect on the lift and drag forces experienced. In actual flight, different angles of attack are desirable in order to change velocity or altitude\textsuperscript{11}.

Because of the scheme developed to control the angle of attack, a way to measure the angle of attack also needed to be devised. The device used to accomplish this task is described in Chapter 2.

Finally, it should be noted that tests for this research were conducted unpropelled. Thus, any prop wash effects on the MAV aerodynamics are not taken into account in this research.
Chapter 2: Research Facility

2.1 WIND TUNNEL

Testing was done in BYU’s largest wind tunnel, located in B-38, next to the Clyde Building. The pictures of the wind tunnel are shown in Figures 2.1 through 2.5:

![Inlet Section](image1)
![Test Section](image2)

Figure 2.1 Picture of the wind tunnel showing the inlet and test sections. Also shown is the data acquisition system, workstation, and pressure-measuring equipment
Figure 2.2 The front of the inlet section is shown on the left. The first of the metal screens used as flow straighteners is visible. On the right, a close-up of the test section shows the test section window.

Figure 2.3 The contraction from the inlet to test sections. The author is standing inside the inlet section to show the scale.

It is an open circuit wind tunnel, which circulates air at the ambient conditions within the building. The test section is 2 feet high and 4 feet wide. The inlet is 6 feet tall and 8 feet wide. The flow straighteners in the wind tunnel are 3 fine metal screens separated with 2 inches of spacing between them and fastened on the front of the inlet.
Figure 2.4 Wind tunnel outlet along with the motor and its controller.

Figure 2.5 Louver control on the top of the wind tunnel for finer speed control

A 60-Hz Westinghouse motor and controller with 9 forward and 9 backward settings powers the wind tunnel at the outlet. Finer speed control is accomplished by turning louvers near the outlet.

2.2 FORCE BALANCE

The force balance is an external balance, which is fastened to the bottom of the test section. The beam of the force balance extends into the wind tunnel, along with wiring to power the servo that controls the horizontal control surface, or elevator, on each MAV. The signal acquired from the force balance is the output of two strain gages that produce voltages corresponding with deflection in the lift and drag directions. Pictures of the force balance are shown in Figures 2.6 and 2.7.
Figure 2.6 Picture showing the force balance mounted to the bottom of the wind tunnel test section. Not shown is the beam of the force balance which extends from the drag platform into the interior of the wind tunnel.

Figure 2.7 The picture on the left shows the under side of the force balance. Here, the drag platform and the beam extending into the wind tunnel can be seen. On the right, the beam protrudes into the interior of the test section.

The force balance is comprised of several components. The components are shown in Figure 2.8.
The beam extending into the wind tunnel interior is attached directly to the drag platform. The drag platform is attached to the lift platform using two shims. The lift platform is also attached to the base of the balance using two other shims. Strain gages are attached between the drag and lift platforms and between the lift platform and the base, as shown.

The purpose of the shims is to allow the platforms to move in only one direction relative to each other. The shims attached to the drag platform allow it to move back and forth along the drag direction, but prevent motion in the lift direction. The shims attached from the base to the lift platform allow it to move in the lift direction but prevent motion in the drag direction. Thus, the lift and drag platforms move independently of each other. These movements are translated by the strain gages into electrical voltages, which can be measured using the data acquisition system. Also, with the selection of the

---

**Figure 2.8** Picture of force balance showing various components.
proper shim thickness, the force balance can measure forces as small as 0.01 newtons (about 0.002 lbs.), which is the force exerted on 1 gram of mass by gravity.

In order to minimize the aerodynamic force on the force balance beam and its effect on the flow field, a shroud in the shape of an airfoil was used to shield the beam from the flow in the wind tunnel. This shroud did not contact the beam of the force balance but was fixed to the wind tunnel. The shroud is visible in the figure below:

![Figure 2.9 Final setup showing airfoil shroud](image)

Two holes were drilled and tapped in the end of the force balance beam inside the wind tunnel. This formed an interface which was universal with all the mounts which attached the MAVs to the force balance. Each MAV had an integrated mount which fastened to the beam of the force balance.
2.3 DATA ACQUISITION SYSTEM

Signals from the strain gages, thermocouple, and pressure transducer were acquired using a National Instruments SCXI-1000 module. The module was connected to a PC, and the data was processed with Labview software, also by National Instruments (Figure 2.11).

![Data acquisition system](image)

**Figure 2.10** Data acquisition system. The SCXI-1000 module is at left, and Labview is run on the PC at right.

Two Omega LCL-113G strain gages were used in the force balance. They had a rated capacity of 113 grams with a combined error of .25% of full scale (Figure 2.11).

![Omega LCL-113G strain gage](image)

**Figure 2.11** Omega LCL-113g strain gage
A T-type thermocouple was used to measure ambient temperature. It was placed outside the wind tunnel near the pressure transducer, so that the same air was sampled for both readings. The computer CPU and monitor were placed away from the pressure transducer and thermocouple so as to not affect the readings from either instrument.

Speed in the wind tunnel was measured by obtaining the pressure difference between the inside of the test section and outside the wind tunnel. Assuming negligible pressure losses through the flow straightening screens, the velocity in the wind tunnel was given by Bernoulli’s principle:

$$U = \sqrt{\frac{2(P_{\text{amb}} - P_{\text{tunnel}})}{\rho}}$$  \hspace{1cm} (2.1)

The pressure transducer used to measure this pressure difference was a Gems Sensors 865 Series Differential Pressure Transducer (shown in Figure 2.12). It was an electronic transducer, with a range of 0 to 0.5 inches of water. Readings from the transducer could be recorded directly by the Labview software package. The transducer measured the pressure differential between the inside of the test section and the surroundings. Four pressure taps (one on each side of the test section) about 19 inches upstream of the force balance were connected to the low pressure side of the transducer, and the high pressure side was left open to the surroundings outside the wind tunnel. The transducer was factory-calibrated, but the calibration was verified by comparing its full-scale reading to the corresponding reading from a water manometer connected to the same...
pressure taps. Agreement between the two readings was within 0.01 inches of water, and thus it was deemed satisfactory to use the transducer without an additional calibration.

![Figure 2.13 Water manometer (the electronic pressure transducer is also shown in the lower right corner)](image)

2.4 MAVS TESTED

Replicas of MAVs that have been successfully flown rather than fanciful or conceptual MAV designs were used for this research. There are two reasons for this. First, a significant amount of time and resources can be required to develop a single MAV, which is beyond the scope of this thesis. Second, research on fanciful but unproven designs may be of little practical use to researchers and designers because of control and stability issues, which are also beyond the scope of this thesis.

The following MAVs were tested in this research:

- BYU MAV
- Replica of the Black Widow by Aerovironment, called the “Blue Widow”
- MAV donated by KIST of Korea
All the MAVs were donated except for the Blue Widow, which was constructed from scratch based on dimensions and pictures provided by a paper on the Black Widow\(^1\). All of the MAVs, with exception of the KIST MAV, were constructed of foam. Each MAV required assembly as well as modifications so that it could be mounted in the wind tunnel. These modifications consisted of skewering each MAV along an axis parallel to the span of the wing with a rod at the best-estimated location of its center of gravity. The rod was allowed to rotate inside a mount connected to the beam of the force balance by means of two screws. Each MAV had the same style of mount, because the mounting interface on the force balance was intended to be universal for all the MAVs, thus expediting changeovers from one MAV to another.
Each MAV was also outfitted with a functioning horizontal control surface, or elevator, so that the angle of attack could be controlled. The elevator on each MAV was actuated by a servo embedded inside (the servos in each MAV are by Hitec). The receiver/controller and power source for the servo were located externally (outside the wind tunnel) and were hardwired to the servo by running wires along the beam of the force balance. (Due to this configuration, part of the wire was exposed in the flow field, and a subtraction had to made for the lift and drag on the wiring.)

Inputs to the servos were provided using a standard radio control used by RC plane enthusiasts (see Figure 2.15). During the course of testing, it was found that radio emissions from the radio control created severe noise in the strain gage data, thus necessitating that the radio transmitter not be used during testing. The solution for this dilemma was to use a Direct Servo Control, or DSC cable, which hardwires the radio control to the receiver/controller and bypasses the radio transmitter. The controller module, radio control, and DSC cable used are all by JR.

Overall, this system allowed the angle of attack for each MAV to be controlled simply by using the remote control outside the wind tunnel during wind tunnel operation.

2.5 MEASURING ANGLE OF ATTACK

One of the considerations of the testing procedure was to be able to change and measure the angle of attack without needing to interrupt the operation of the wind tunnel.
Since the remote control used to adjust the angle of attack did not provide a reading on the angle, a way to measure the angle of attack had to be devised.

A device (shown in Figure 2.16) was therefore constructed for measuring the angle of attack.

*Figure 2.16 Device for measuring angle of attack*

It consists simply of a straight edge mounted on a platform which could be adjusted vertically. A protractor was mounted on the platform, and its center coincides with the point about which the straight edge rotates. The angle of attack could be measured visually with this device by lining up the wing chord with the top (or bottom) of the straight edge, and the angle was read with the protractor.
Chapter 3: Force Balance Calibration

The force balance used for this research was new and required calibration in order to relate strain gage voltage outputs to the actual forces being measured. The setup, procedure, and results of the calibration are detailed as follows.

3.1 CALIBRATION SETUP

Calibration of the force balance was accomplished by a system of strings and pulleys in order to apply the desired load in the desired direction. A load was applied in the drag (or free stream) direction by placing a mass in a bowl attached to one end of a string, which was draped over a pulley and attached at the other end to the sting of the force balance. A load was applied to the negative-lift direction (the direction of gravity) by placing a mass in a bowl hanging from a string, whose end was attached directly to the lift platform. A load was applied in the positive-lift direction by tying a mass to a string draped over a pulley, extending through the top of the wind tunnel, and attached to the tip of the force balance beam. The system used to calibrate the force balance is shown in the Figures 3.1 and 3.2.
Figure 3.1  Pictures of the string, pulley, and bowl used to calibrate the drag directions. A string is attached to the tip of the force balance beam at left. At right, a weight is shown in the bowl.

Figure 3.2  Pictures of the strings, pulley, and bowl used to calibrate the lift directions. The picture at the left shows the bowl attached by string to the base of the force balance to apply loads in direction of gravity. The top right shows a weight permanently attached to a string which extends through the top of the wind tunnel to the tip of the force balance beam. The bottom right shows the string that connects the weight shown (at upper right) to the tip of the force balance beam.
In order to maximize repeatability, the positions of these pulleys and strings were fixed and never changed throughout the calibrations.

### 3.2 CALIBRATION PROCEDURE

Six series of calibrations were performed on the force balance. Each series consisted of the following four calibrations:

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Loading in lift direction</th>
<th>Loading in drag direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Held constant at 0</td>
<td>Started at 0 and applied various loadings sequentially, and then unloaded following the reverse sequence</td>
</tr>
<tr>
<td>2</td>
<td>Held constant at -100 or 100 grams (alternated order by series so that 3 calibration series used -100 grams and the other 3 series used 100 grams)</td>
<td>Same as Calibration 1</td>
</tr>
<tr>
<td>3</td>
<td>Started at -100 g (or 100 g) and applied loadings sequentially up to 100 g (or down to -100 g), and then unloaded (or loaded) following the reverse sequence</td>
<td>Held constant at 0</td>
</tr>
<tr>
<td>4</td>
<td>Same as Calibration 3</td>
<td>Held constant at 100 g</td>
</tr>
</tbody>
</table>

The reason for the four different calibrations in each series was to capture possible interactions between the lift and drag cell readings. Ideally, the lift and drag platforms (and the attached strain gages) would be orthogonal to each other, and a reading on one strain gage would not be accompanied by a correlated reading from the other. However, this is difficult to achieve in practice, thus creating the possibility that the lift cell would, in fact, produce a small reading as a pure drag force was applied, and vice versa as a pure lift force was applied.

In order to check the resolution of the force balance, the first three calibration series were performed with most of the loadings applied between 0 and 5 grams for the
drag direction and from -10 to 10 grams in the lift direction. The latter three calibration series were performed more evenly dispersed in 10- or 20-gram increments in order to estimate voltage and calibration uncertainty in the gaps left by the first three series.

Also, as mentioned in Table 3.1, a loading sequence was followed by an unloading sequence for each calibration, thus completing a “loop.” The purpose of this was to check for hysteresis. Hysteresis occurs as components of the calibration system (ie. the strings or pulleys) or the lift and drag platforms themselves do not return to their initial position, thus creating an offset in the readings. This was indeed observed as an offset in the loading and unloading trend lines in every calibration performed, although the effect was small compared to other uncertainties measured (ie. voltage and curve-fit).

3.3 CALIBRATION RESULTS

The calibration data were checked for linearity, hysteresis, drift, and sensitivity. Plots of the data show linear relationships between voltages and applied loads, an example of which is given in Figure 3.3. As proof, regressions were performed on the data with an $R^2$ greater than 0.994 in every case, thus showing that a linear fit correlates well with the calibration data, which is a highly desirable situation.
Figure 3.3 Results from Calibration 5 for lift (drag held at 0). The data exhibit very linear behavior. Hysteresis is also shown in this graph as a slight offset between the loading and unloading trend lines.

The sensitivity of the force balance is shown by graphing the voltage means of a calibration performed with error bars corresponding to one sample standard deviation above and below the mean. As can be seen in Figure 3.4, the force balance can detect a difference of one gram, which is the smallest mass increment available for the calibrations.
Initially, a multi-variable or multiple regression was performed on the data to account for the interactions between the readings from the two strain gages and reduce the uncertainty in lift and drag force data. Since the data observed exhibited linear behavior, the curve-fit equations used for the multiple regression were as follows:

\[ L = a_{m1}v_1 + a_{m2}v_2 + a_{m3} \]  \hspace{1cm} (3.1)

\[ D = b_{m1}v_1 + b_{m2}v_2 + b_{m3} \]  \hspace{1cm} (3.2)

The least squares method was used to solve for the coefficients in Eqs. (3.1) and (3.2) for each calibration set.

Single regressions were also performed on the data, again using the least squares method. In this case, lift was assumed to be a function of the lift cell voltage only, and
drag was assumed to be a function of the drag cell voltage only. The equations fitted are shown in Eqs. 3.3 and 3.4.

\[ L = a_1 v_1 + a_2 \quad (3.3) \]

\[ D = b_1 v_2 + b_2 \quad (3.4) \]

As it turned out, the single regressions had an \( R^2 \) and standard error very similar to the multiple regressions. For both the single and multiple regressions, the \( R^2 \) values exceeded .994 in every case and the \( R^2 \) values of the single regression nearly matched those of the multiple regressions. The single regression standard errors (on the order of 1 to 3 grams for the lift curve and 0.6 to 0.8 grams for the drag curve) nearly matched those of the multiple regressions as well. Overall, there seemed to be very little added precision provided by the multiple regression. Thus, a single regression was deemed sufficient for the calibrations.

Furthermore, since the regression for each calibration set produced a slightly differing slope, an analysis of variance, or ANOVA, using a general linear model was performed on the data to account for the variance induced by each individual calibration. The ANOVAs were performed with the lift cell or drag cell voltage and calibration set number as the factors and the lift or drag force respectively as the response. This operation succeeded in separating out variance components due to the individual calibrations and providing more accurate calibration coefficients with lower standard errors.

The ANOVA results provided the coefficients \( a_1 \) and \( b_1 \) with their associated standard errors. The ANOVAs also provided values for the intercepts \( a_2 \) and \( b_2 \) with their associated standard errors. Only the standard errors for the coefficients \( a_2 \) and \( b_2 \) were
used during the experimental phase, however, because significant drift in the computed intercepts occurred between calibrations. Only a small amount of drift (occurring as the observed hysteresis) was observed within a calibration run, though, so it was concluded that the intercepts $a_2$ and $b_2$ could be measured with zero-velocity readings taken just prior to and after a data run. Based on behavior observed during the calibrations, it was assumed that no drift would occur during a data acquisition run, and that the intercept standard errors computed by the ANOVAs could reasonably be used to represent the error in the intercepts measured for each data run.
Chapter 4: Uncertainty Analysis

4.1 METHODOLOGY

As in any experiment, sources of uncertainty were present and needed to be quantified. An uncertainty analysis was also crucial in order to identify the primary sources of uncertainty.

4.1.1 Measurement Uncertainty

On the data acquisition level, the following sources of uncertainty were present:

- Voltage sampling variation for the pressure transducer, the thermocouple, and for each strain gage
- Barometer sampling variation
- Resolution limits of all the above instruments or of the data acquisition system itself

These variations have a number of possible causes, including hysteresis, fluctuations in ambient conditions, unsteady flow in the wind tunnel, instrumentation system inaccuracy, and/or experimental drift.

In addition to the sources of variation listed above, nonlinearities or other inaccuracies in the instrument calibrations provided additional sources of error. These were expressed as standard errors. For example, each calibration coefficient for the force balance readings had a standard error associated with it due to nonlinearities (ie. each data point that did not coincide with the straight-line relationship provided by a
calibration equation described in Chapter 3 increased the standard error of the computed coefficient).

4.1.2 Resulting Drag Polar Uncertainty

All of these sources of variation propagate into the uncertainty of the final results (the aerodynamic coefficients comprising the drag polar), as illustrated in Figure 4.1.

The Kline-McClintock method of error propagation was chosen to estimate the uncertainty in the final results\(^ {12} \). This method relates measured uncertainties at the measurement level to uncertainties in the computed intermediate and final results. Simply explained, the Kline-McClintock method sums together the measured variances, each of which is appropriately scaled by its effect on the calculated result. The scaling of each variance is accomplished by computing the partial derivative of each defining relation with respect to the variable whose variance is expressed. For example, drag force uncertainty is a function of the voltage uncertainty measured but is also a function of the calibration coefficients’ standard errors. The voltage component of variance is therefore scaled by the partial derivative of the calibration equation (given by Eq. 3.2) with
respect to voltage, and the coefficient component of variance is scaled by the partial derivative of Eq. 3.2 with respect to the coefficient. Once the component variances are scaled by the partial derivatives, they are comparable because they are now in terms of the same units and are also both expressed in the same units as the computed result (in this case: drag force). General equations used in the uncertainty analysis are provided in Appendix A.

Figure 4.2 illustrates the use of the Kline-McClintock method to compute force, density, velocity, and aerodynamic coefficient ($C_L$ and $C_D$) uncertainties from the measured voltage, pressure, and temperature uncertainties and the calibration uncertainties for the strain gages and pressure transducer. This same method was applied to estimate the Reynolds number uncertainty. Figure 4.3 shows the Kline-McClintock method applied to compute the Reynolds number uncertainty.
Figure 4.2 Uncertainty analysis for $C_L$ and $C_D$. The procedure progresses from bottom to top.

Figure 4.3 Uncertainty analysis for Reynolds number
4.2 CONCLUSIONS

The results of the uncertainty analysis showed that, generally speaking, at higher Reynolds numbers, strain gage voltage and calibration uncertainties (and thus lift and drag force uncertainties) tend to dominate the uncertainty in the measured drag polars. At lower Reynolds numbers, however, the pressure transducer’s calibration uncertainty (and thus, velocity uncertainty) dominates, thus increasing both the drag polar and the Reynolds number uncertainties.

Uncertainty due to changes in ambient conditions was much smaller in comparison for both the high and the low Reynolds number cases. Thus, ambient pressure, temperature, and density uncertainties were considered to be negligible.

Other sources of uncertainty include the measurements in angle of attack. At high Reynolds numbers, it was relatively easy to set and measure the angle of attack for the BYU and Blue Widow MAVs with an uncertainty no greater than a 0.5° because of friction in the mount system. The KIST MAV, on the other hand, exhibited irregular fluctuations on the order to 1° to 2° in either direction, possibly denoting wind tunnel turbulence or poor aerodynamic performance of the MAV itself. The difference in uncertainties is probably due to the fact that the mount for the KIST MAV had less friction than the other mounts and thus allowed the KIST MAV to adjust instantaneously to unsteadiness in the flow. The flow unsteadiness seemed to decrease for the KIST MAV tests at Re of 50,000 and 30,000. However, at Re = 10,000, the KIST MAV would sometimes not remain trimmed for very long at the same angle of attack, indicating flow unsteadiness in this situation as well. These fluctuations could be caused by any of a number of possible sources:
• Vortex shedding from the MAV itself
• Unsteady drag characteristics of the mounting scheme, force balance shroud, or the servo wiring exposed in the airstream
• Flow unsteadiness due to obstructions in front of the wind tunnel or because of the wind tunnel itself

At lower Reynolds numbers, it was more difficult to measure the angle of attack. For example, at a Reynolds number of 10,000, velocity inside the wind tunnel was on the order of 2 or 3 mph – the speed at which a human being can walk. There was not enough aerodynamic force for the BYU and Blue Widow MAVs to trim to the desired angle of attack, so they needed to be adjusted manually to the desired angle. Due to the difficulty of this task, the uncertainty in angle measurement was increased by approximately another 1°.
Chapter 5: Experimental Results

5.1 PROCEDURE

Drag polars were measured for the MAVs described in Chapter 2 at Reynolds numbers of 70,000, 50,000, 30,000, and 10,000. Each drag polar was measured twice in order to check repeatability.

It is important to note that these Reynolds numbers are effective Reynolds numbers because they were adjusted for the wind tunnel turbulence factor of 1.123 set forth by Paulson\textsuperscript{10}. In other words, although the wind tunnel velocity was set to achieve the effective Reynolds numbers stated above, the actual Reynolds numbers based on wind tunnel velocity were slightly lower.

The methodology employed for measuring each drag polar was as follows.

1. A zero-velocity reading (lasting 20 seconds at a sampling rate of 10 Hz) was taken from the strain gages, pressure transducer, thermocouple, and barometer prior to operating the wind tunnel.

2. The wind tunnel was turned on and was set at the desired wind speed corresponding with the Reynolds number to be tested (adjusted appropriately for wind tunnel turbulence). The wind tunnel velocity was calculated based on the ambient conditions measured in step 1.

3. The MAV wing angle of attack was set and strain gage readings were taken. First the straight edge of the measuring device described in Chapter
2 was set to the desired angle. Then the MAV elevator was actuated with
the remote control until the wing chord was visually aligned with the
straight edge. Once this was completed, the strain gages, pressure
transducer, and thermocouple were sampled at 10 Hz for 20 seconds.

4. The process in step 3 proceeded for angles of attack from –5° to 10° in 1°
   increments. Then, an additional reading was taken at the initial angle of
   attack in order to check for hysteresis.

5. The wind tunnel was turned off, and one more zero-velocity reading was
taken from the strain gages, pressure transducer, and thermocouple as an
additional check for hysteresis.

6. The strain gage measurements were then translated into lift and drag force
measurements (using the calibration equation developed in Chapter 3),
which were then used to calculate the lift and drag coefficients $C_L$ and $C_D$
using the following definitions\textsuperscript{2,5}:

\begin{align}
C_L &= \frac{2L}{\rho V^2 S} \quad (5.1) \\
C_D &= \frac{2D}{\rho V^2 S} \quad (5.2)
\end{align}

where $L$ and $D$ are the measured lift and drag forces, $\rho$ is the density of the
ambient air, $V$ is the free stream velocity measured in the wind tunnel, and
$S$ is the wing planform area discussed in Chapter 1. The uncertainties for
each variable of interest was also calculated according to the procedure
describe in Chapter 4. The result was that for each MAV, different values
of lift and drag (and thus $C_L$ and $C_D$), along with their associated uncertainties, were produced for each angle of attack measured.

7. Once the testing on the MAV was completed for all Reynolds numbers, drag was measured for the mount and exposed servo wire (without the MAV) at each Reynolds number. Each drag measurement was translated into a drag coefficient using Eqs. (5.1) and (5.2) and was subsequently subtracted from the drag coefficient measured for the MAV. Drag coefficient subtractions and their uncertainties are found in Appendix G.

8. Finally, the revised drag coefficient $C_D$ (excluding drag on the mount and wire) was plotted versus lift coefficient $C_L$ with uncertainties for each MAV to produce a drag polar. $C_L$ was also plotted vs. $\alpha$ in order to find the range over which $C_L$ varies linearly with $\alpha$ (or, in other words, the bounds in which the predicted drag polar applies).

9. Each drag polar was curve-fit with a 2$^{\text{nd}}$-order polynomial using the least-squares method. The coefficients from the resulting polynomial provided values of $k_1$ and $k_2$ and an estimate of $C_{D,0}$ (in addition to the $C_{D,0}$ which could be either interpolated or extrapolated from the raw data). From these coefficients, $e_0$, $C_{fe}$, and $C_{L,minD}$ could be ascertained from the experimental data.

5.2 EXPERIMENTAL RESULTS AND DISCUSSION OF RESULTS

5.2.1 Measured vs. Predicted Drag Polars

Data used for predicting the values of $C_{D,0}$, $k_1$, and $k_2$ for the MAVs is contained in Appendices E and F. Appendix E contains physical characteristics measured for each
MAV. Appendix F contains the values assumed for $e_0$, $C_{fs}$, and $C_{L, minD}$ for each MAV and how they were derived. The full listing of all drag polars measured is contained in Appendices B, C, and D.

Measured and predicted drag polars for all 3 MAVs at $Re = 70,000$ are shown below. The drag polars in Figures 5.1 to 5.3 show trends representative of all drag polars measured.

**Figure 5.1** Predicted vs. measured drag polars for the BYU MAV at $Re = 70,000$
The agreement between predicted and measured drag polars is very good for the Blue Widow (Figure 5.2). For the BYU and KIST MAVs, (Figures 5.1 and 5.3) the
classical approach tended to under-predict drag by as much as a factor of approximately two.

It should be mentioned that for all of the predicted drag polars, the highest value of the laminar or turbulent $c_f$ (multiplied by 1.0) was used for the predicted $C_{f_e}$. Therefore, for all $Re$ except for 10,000, the turbulent value of $c_f$ was used in the drag polar prediction. For $Re = 10,000$, the laminar $c_f$ is greater than the turbulent $c_f$.

One reason which should be mentioned as to why agreement is good is that data for $C_{L,\alpha=0}$ from each MAV (instead of the author’s guess of $C_{L,\alpha=0}$ made prior to testing) was used to estimate $C_{L,minD}$. The original guess for $C_{L,\alpha=0}$ resulted in drag polar predictions positioned too high vertically (with respect to the axes), and agreement was poor. In reality, a MAV designer may not have this data available in order to predict a drag polar. Thus, the experimental $C_{L,\alpha=0}$ (averaged from the data for $Re = 70,000$ and 50,000) used for the final drag polar predictions are listed in Appendices B, C, and D for each MAV, so they can provide a benchmark for future use.

One notable anomaly in the data occurred for the Blue Widow at $Re = 50,000$ (Figure 5.4). In this case, the classical approach *over-predicted* drag for $Re = 50,000$, as shown below.
In fact, for Re = 50,000, the actual $C_{fe}$ for the Blue Widow was measured to be less than the theoretical turbulent and laminar $c_f$ ‘s. This difference is nearly resolved if the laminar value of $c_f$ is used for $C_{fe}$. The new predicted drag polar in this case is shown in Figure 5.5.
This anomaly is investigated further in the discussion of the experimentally measured $C_{fe}$ for all MAVs.

The last trend to be discussed in regards to the measured drag polars is the experimental uncertainty. The following four figures (Figures 5.6 to 5.10) show the predicted and measured drag polars with uncertainty bars (denoting the uncertainty in the individual measurements) for the KIST MAV for all Re. These drag polars represent the same trend shown for each MAV.
Figure 5.6 Predicted vs. measured drag polars for the KIST MAV at Re = 70,000, showing uncertainty bars.

Figure 5.7 Predicted vs. measured drag polars for the KIST MAV at Re = 50,000, showing uncertainty bars.
As shown, the experimental uncertainty increases with decreasing Reynolds number. Repeatability also decreases with decreasing Re, as shown by the differences
between measured drag polars for the same Re. In the case of Re = 10,000, the uncertainty rules out making any sort of conclusions from the data. Repeatability is also very poor for Re = 10,000. For Re = 30,000, the data look okay but are inconclusive when attempts are made to extract data for $e_0$ and $C_{L_{\text{min}}}D$.

5.2.2 $S_{\text{wet}}/S$ vs. $W/S$

Figure 5.10 shows the chart of $S_{\text{wet}}/S$ vs. $W/S$ produced in this research.

![Wetted Surface Area Ratio vs. Wing Loading](image)

Figure 5.10 $S_{\text{wet}}/S$ vs. $W/S$ for the MAVs studied

None of the MAVs had a wing loading greater than 1 lb/ft$^2$. The chart shown in Figure 1.3 is reproduced below, showing the relative location of the region encompassing the data shown in Figure 5.11.
From this chart, it is apparent that MAVs experience wing loadings 1 or 2 orders of magnitude lower than those of conventional aircraft, although their wetted surface area ratios are similar to those of many conventional aircraft.

5.2.3 Equivalent Skin Friction Coefficient, $C_{fe}$

Two values of $C_{D_{min}}$ were derived from each test: one from the actual data and another from the least-squares fit. For each value of $C_{D_{min}}$, the associated $C_{fe}$ was calculated. Figure 5.12 shows the experimentally measured $C_{fe}$’s for each MAV compared with the flat-plate values for $c_f$ provided by Eqs. (1.4) through (1.7).
Almost all of the $C_{fe}$ data shown are under-predicted by flat plate theory, with the exception of the Blue Widow. The Blue Widow, in one case (Re = 50,000), exhibited $C_{fe}$ even below the flat plate predictions.

The $C_{fe}$ data from all MAVs were also averaged by Reynolds number, and correlation was checked between the average $C_{fe}$ for all MAVs and the theoretical $c_f$ for each Reynolds number tested. The results are shown in Figure 5.13.
These results show that the data are best correlated with laminar theory by a factor of 1.8 and with turbulent theory (Schlichting and Karman-Schoenherr only) by a factor of 1.6. This is close to the correlation factor of 1.5 (for Karman-Schoenherr) given by Anderson for conventional aircraft\(^2\).

### 5.2.4 Oswald Efficiency, \(e_0\)

During the course of predicting drag polars for the MAVs tested, it was discovered that the curve-fit equations given in Eq. (1.11) gives results greater than unity for the low aspect ratios of the MAVs tested. Table 5.1 shows the aspect ratios measured for each of the MAVs studied in this research.

<table>
<thead>
<tr>
<th>MAV</th>
<th>(AR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BYU</td>
<td>1.46</td>
</tr>
<tr>
<td>Blue Widow</td>
<td>1.18</td>
</tr>
<tr>
<td>KIST</td>
<td>1.59</td>
</tr>
</tbody>
</table>

---

**Figure 5.13** Graph showing experimentally measured \(C_{fe}\)'s and their trends related to the theoretical \(C_f\)'s
A plot of $e_0$ vs. $AR$ (Figure 5.14) shows that Eq. (1.11) is only valid for aspect ratios of at least 2.2.

![Predicted $e_0$ vs AR for Straight Wing Aircraft](image)

**Figure 5.14** Oswald efficiency vs. aspect ratio

Since the aspect ratios of all four MAVs tested are less than 2, Oswald efficiency cannot be predicted for them using Eq. (1.11). Since no swept-wing MAVs were tested in this research, conclusions cannot be drawn about Eq. (1.12).

The following table shows the experimentally measured Oswald efficiency factor $e_0$ for each MAV used in the research.

<table>
<thead>
<tr>
<th>MAV</th>
<th>Wing Shape</th>
<th>$e_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BYU</td>
<td>Circular</td>
<td>0.7</td>
</tr>
<tr>
<td>Blue Widow</td>
<td>Elongated octagon</td>
<td>1.0</td>
</tr>
<tr>
<td>KIST</td>
<td>Zimmerman</td>
<td>0.6</td>
</tr>
</tbody>
</table>

In the table, each $e_0$ value is the average for each MAV from the tests done at Reynolds numbers of 50,000 and 70,000. Tests done at Reynolds numbers of 10,000 and 30,000 yielded widely varying and unrealistic values of $e_0$ which were rejected. Apart from this, no other Reynolds number trends were observed.
5.2.5 $C_{L,\text{min}D}$

The only definitive trend discovered for $C_{L,\text{min}D}$ was greater dispersion of the data with lower Reynolds number. Figure 5.15 shows this trend.

![CL, minD vs Re](image)

**Figure 5.15 $C_{L,\text{min}D}$ vs. Re**

Figure 5.16 shows experimental $C_{L,\text{min}D}$ vs. $C_{L,\alpha=0}$ for all MAVs studied. The data for this graph exclude measurements for Reynolds numbers of 30,000 or less because of the higher spread and uncertainty.
Linear regressions performed on the data had an $R^2$ of less than 0.5. It appears that $C_{L_{\text{minD}}}$ could be roughly equal to $C_{L,a=0}$ (rather than $\frac{1}{2}$ of $C_{L,a=0}$, which is a standard first approximation).

5.2.6 $L/D$

$(L/D)_{\text{max}}$ was recorded in each experiment in order to observe perhaps the most relevant Reynolds number trends (see Figure 5.17). Surprisingly, $(L/D)_{\text{max}}$ increases for Reynolds numbers of 10,000 and 50,000, which is an unexpected phenomenon. The data for $\text{Re} = 10,000$ has already been deemed inconclusive, but the slight increase in $(L/D)_{\text{max}}$ for $\text{Re} = 50,000$ is unexpected.

The range of Reynolds numbers covered was not broad enough to capture the large changes in $(L/D)_{\text{max}}$ caused by transition from laminar to turbulent flow. Because the values for $(L/D)_{\text{max}}$ are small, the data must fall in the range of Reynolds numbers below the laminar-to-turbulent transition point.
COMMENTS ON EXPERIMENTAL PROCEDURE

5.3.1 Challenges

Various challenges had to be overcome in devising and implementing the strategy used for this research. The greatest challenge discovered and overcome was the interference caused by radio waves emitted from the radio control. Fortunately, the RF transmitter could be bypassed through the use of a DSC cable (this is only available for certain radio control models), thereby remedying the problem.

Other challenges of this methodology include:

- Overcoming the force balance uncertainty which is larger than the measured forces for low Re tests
- Measuring and subtracting a large amount of drag for the servo wire exposed in the free stream, which may in fact change during a drag polar test, thus adding to the uncertainty of the measured $C_{D,0}$, $C_{L,minD}$, and $e_0$
• Using MAVs which are statically stable (this is essentially a requirement for flight anyway)
• Estimating the MAV’s center of gravity and using ballast to manipulate the center of gravity
• Measuring the uncertainty in angle of attack

5.3.2 Advantages

Advantages of the methodology are as follows:

• Elevator deflections are taken into account, thus more accurately reflecting on the drag and lift experienced by a MAV in flight
• An existing 2-component force balance could be used instead of developing and calibrating a 3-component force balance (with the ability to also control the angle of attack), which saved a lot of time and effort and possibly reduced the size of intrusions in the flow immediately around the MAV
• Angle of attack could be controlled using a simple radio control used by radio plane enthusiasts and measured with a simple visual device outside the wind tunnel
• The force balance used was sensitive enough to measure very small aerodynamic forces down with resolution down to 1 gram

5.3.3 Recommendations

Various recommendations can be made on improvements to the research methodology or areas to focus further research in the future. Although the testing
methodology used was very useful and solved a number of problems, improvements still can be made in the following areas:

- Reduce, eliminate, or somehow cover the length of servo wire so that no wire is exposed in the airstream to affect the drag and lift on the MAV
- Do not use radio frequencies to control MAV elevators
- Devise a way to adjust angle of attack for low-velocity flow without requiring adjustments by hand (i.e., reduce the friction in the mounting scheme, match the mounting position with the MAV center of gravity with greater precision, or use some other method to fix the angle of attack)
- Devise a way to measure angle of attack in real-time in order to relate lift and drag uncertainty to angle uncertainty, possibly using a position sensor embedded in the force balance mount
- Reduce hysteresis on the force balance and increase force balance sensitivity even further for Reynolds numbers 30,000 and below
- Quantify and/or reduce flow unsteadiness in the wind tunnel (flow unsteadiness is probably the largest source of hysteresis due to occasional spikes in the forces measured on the force balance)
- Cushion the force balance from wind tunnel vibrations which may also be exacerbating the hysteresis (the natural frequency of the force balance is around 84 Hz, and the wind tunnel motor operates at 60 Hz)
Chapter 6: Conclusions

6.1 PREDICTED VS. MEASURED DRAG POLARS

This research may be the first attempt to validate the application of the classical methodology for predicting drag polars to MAVs, and it has produced data that can be potentially of great benefit to MAV designers in the future. Agreement between predicted and measured drag polars was quantified. Data for $S_{we}/S$, $C_{fe}$, $e_0$, and $C_{L,minD}$ were measured, and conclusions were drawn about the drag polar prediction methodology itself.

The most consistent relationship that could be observed for $C_{fe}$, $e_0$, or $C_{L,minD}$ (other than Reynolds number trends) was between the $C_{fe}$’s measured for all the MAVs studied and the theoretical values of $c_f$ for laminar and turbulent flow. $C_{fe}$ was measured to be a factor of 1.8 larger than the $c_f$ from laminar skin friction theory (and 1.6 larger than $c_f$ given by Schlichting and Karman-Schoenherr turbulent skin friction theory).

It was found that $C_{L,minD}$ was closer to being equal to $C_{L,a=0}$. This is in contrast with one half of $C_{L,a=0}$, as dictated by the classical approach.

No defining relationship was found between $e_0$ and $AR$. The Blue Widow was shown to have the highest $e_0$ (near unity) while the KIST MAV exhibited the lowest $e_0$ measured (around 0.6).
Finally, it was also discovered that although $S_{wet}/S$ values for MAVs are similar to those of conventional aircraft, the corresponding wing loadings $W/S$ are much lower for MAVs.

Generally speaking, it appears that the classical drag polar prediction methodology is usable for designing MAVs as long as the designer realizes that it appears to typically under-predict drag by up to 100%.

6.2 REYNOLDS NUMBER TRENDS

The methodology used was successful in measuring drag polars at low Reynolds numbers. Various trends with Reynolds number were observed. First, the quality of the data decreased with decreasing Reynolds number. Uncertainty increased with decreasing Re. At higher Re (ie. Re of 70,000 and 50,000), voltage and calibration uncertainty dominated the overall uncertainty. At lower Re, pressure transducer and velocity uncertainty dominated the overall uncertainty. At Re = 10,000, the experimental uncertainty ballooned to a point where the data were inconclusive and exceeded the sensitivity of the force balance.

Data for $C_{fe}$ were conclusive for Re between 30,000 and 70,000. $C_{fe}$, for the most part, increased with decreasing with Re.

No trends for $e_0$ and $C_{L,minD}$ with Re were observed. Data for both $e_0$ and $C_{L,minD}$ were inconclusive for Re below 50,000.

$(L/D)_{max}$ showed an expected trend downward with decreasing Re. As discussed previously, the data for Re = 10,000 are inconclusive due to large experimental uncertainty. The low values of $(L/D)_{max}$ measured indicate the Reynolds numbers studied were below the laminar-to-turbulent transition point.
6.3 RESEARCH METHODOLOGY

The approach conceived and adopted in order to measure drag polars for MAVs proved to be extremely useful. It consisted of mounting each MAV on a strain gage force balance, controlling the angle of attack using actuated control surfaces (during wind tunnel operation), and acquiring the lift and drag strain gage data. Measurement uncertainty was measured and used to estimate the uncertainty of the final result.

Significant advantages were obtained by using this approach. First and foremost, it decreased the complexity of the force balance design. This method also proved to be a convenient and rapid way to measure lift and drag at various angles of attack. Another advantage was the fact that control surface deflections were included in the drag and lift measurements, which more closely reflects the true conditions during flight.

Due to the electrical sensitivity of the force balance, the biggest challenge initially was interference between the radio control and the strain gage readings. Fortunately this was discovered and overcome by using a DSC cable to bypass the radio transmitter on the remote control. Afterward, the biggest remaining challenges of the methodology were the sensitivity and hysteresis of the strain gage force balance.

6.4 SUMMARY OF RECOMMENDATIONS

Areas for further improvement include somehow improving the accuracy of the force measuring system so that data for Reynolds number of 10,000 and 30,000 can be gathered and analyzed. Another important area for improvement is either to eliminate the need to have a servo wire exposed in the airstream or to measure the drag on the exposed servo wire with greater accuracy and precision.
Improvement is also needed in setting the angle of attack at Reynolds numbers of 10,000 or even 30,000. A way to measure the angle of attack uncertainty and correlate it with the force uncertainty would also be very beneficial and would further reduce the overall uncertainty.
Bibliography


Appendix
### Appendix A: Uncertainty Equations

**Table A.1** Equations used in the Kline-McClintock method of uncertainty analysis

<table>
<thead>
<tr>
<th>Equation</th>
<th>Eqn.</th>
<th>Designation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{x_i} = \frac{\partial X}{\partial x_i} )</td>
<td>(A.1)</td>
<td>Sensitivity index</td>
<td>Amount of influence that ( x ) has on the calculation of ( X )</td>
</tr>
<tr>
<td>( R_{x_i} = \frac{1}{2} \text{leastcount} )</td>
<td>(A.2)</td>
<td>Resolution or bias error</td>
<td>Obtained from the smallest increment of the instrument</td>
</tr>
<tr>
<td>( R_x = \sqrt{\sum_{i=1}^{k} (\theta_{x_i} R_{x_i})^2} )</td>
<td>(A.3)</td>
<td>Resolution or bias limit</td>
<td>Amount that the calculation of ( x ) will vary due to resolution error</td>
</tr>
<tr>
<td>( S_{x_i} = \sqrt{\frac{\sum_{j=1}^{N} (x_{ij} - \bar{x}_i)^2}{N - 1}} )</td>
<td>(A.4)</td>
<td>Sample standard deviation</td>
<td>Measure of the precision of a measurement</td>
</tr>
<tr>
<td>( S_{x_i} = \frac{S_{x_i}}{\sqrt{N}} )</td>
<td>(A.5)</td>
<td>Standard deviation of the means</td>
<td>Estimates the dispersion of the sample means about the true mean</td>
</tr>
<tr>
<td>( P_x = \sqrt{\sum_{i=1}^{k} (\theta_{x_i} S_{x_i})^2} )</td>
<td>(A.6)</td>
<td>Precision index</td>
<td>Amount the calculation of ( x ) will vary due to precision error</td>
</tr>
<tr>
<td>( \nu = \left[ \frac{\sum_{i=1}^{k} (\theta_{x_i} S_{x_i})^2}{\sqrt{\sum_{i=1}^{k} (N-1)x_i}} \right]^2 )</td>
<td>(A.7)</td>
<td>Degrees of freedom (Welch-Satterthwaite eqn.)</td>
<td>Used to determine the student t value to used in Eqn. (A.8)</td>
</tr>
<tr>
<td>( u_x = \sqrt{R_x^2 + (t_{\nu,95} P_x)^2} )</td>
<td>(A.8)</td>
<td>Uncertainty</td>
<td>Used to give the 95% confidence interval for any value of ( x )</td>
</tr>
<tr>
<td>( u_X = \sqrt{\sum_{m=1}^{q} (\theta_m u_{x_m})^2} )</td>
<td>(A.9)</td>
<td>Higher-level uncertainty</td>
<td>Gives the 95% confidence interval for ( X ), based on the uncertainty components of ( x_m )</td>
</tr>
</tbody>
</table>
Appendix B: Experimental Results for the BYU MAV

Figure B.1 Drag polars for BYU MAV, Re = 70,000

Figure B.2 Drag polars for BYU MAV, Re = 70,000, showing uncertainty bars
Figure B.3 Drag polars for BYU MAV, $Re = 50,000$

Figure B.4 Drag polars for BYU MAV, $Re = 50,000$, showing uncertainty bars
Figure B.5 Drag polars for BYU MAV, Re = 30,000

Figure B.6 Drag polars for BYU MAV, Re = 30,000, showing uncertainty bars
Figure B.7  Drag polars for BYU MAV, $Re = 10,000$ (shown on a larger scale than for $Re$ 30,000 through 70,000)

Figure B.8  Drag polars for BYU MAV, $Re = 10,000$, showing uncertainty bars (shown on a larger scale than for $Re$ 30,000 through 70,000)
Figure B.9  $C_L$ vs. $\alpha$ for BYU MAV, $Re = 70,000$

Figure B.10  $C_L$ vs. $\alpha$ for BYU MAV, $Re = 50,000$
Figure B.11 $C_L$ vs. $\alpha$ for BYU MAV, Re = 30,000

Figure B.12 $C_L$ vs. $\alpha$ for BYU MAV, Re = 10,000 (shown on a larger scale than for Re 30,000 through 70,000)
Figure B.13  $C_D$ vs. $\alpha$ for BYU MAV, $Re = 70,000$

Figure B.14  $C_D$ vs. $\alpha$ for BYU MAV, $Re = 50,000$
Figure B.15 $C_D$ vs. $\alpha$ for BYU MAV, $Re = 30,000$

Figure B.16 $C_D$ vs. $\alpha$ for BYU MAV, $Re = 10,000$
Figure B.17 $L/D$ vs. $\alpha$ for BYU MAV, $Re = 70,000$

Figure B.18 $L/D$ vs. $\alpha$ for BYU MAV, $Re = 70,000$, showing uncertainty bars
Figure B.19 $L/D$ vs. $\alpha$ for BYU MAV, $Re = 50,000$

Figure B.20 $L/D$ vs. $\alpha$ for BYU MAV, $Re = 50,000$, showing uncertainty bars
Figure B.21 $L/D$ vs. $\alpha$ for BYU MAV, Re = 30,000

Figure B.22 $L/D$ vs. $\alpha$ for BYU MAV, Re = 30,000, showing uncertainty bars
Figure B.23  \(L/D\) vs. \(\alpha\) for BYU MAV, \(Re = 10,000\) (shown on a larger scale than for \(Re\) 30,000 through 70,000)

Figure B.24  \(L/D\) vs. \(\alpha\) for BYU MAV, \(Re = 10,000\), showing uncertainty bars (shown on a larger scale than for \(Re\) 30,000 through 70,000)
Table B.1 Table of experimental data for all tests for BYU MAV

<table>
<thead>
<tr>
<th>Nominal Re:</th>
<th>70,000</th>
<th>50,000</th>
<th>30,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 (up)</td>
<td>2 (down)</td>
<td>1 (up)</td>
<td>2 (down)</td>
</tr>
<tr>
<td>L/D MAX</td>
<td>5.07</td>
<td>5.04</td>
<td>6.09</td>
<td>5.40</td>
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<tr>
<td>Avg Re</td>
<td>70290</td>
<td>70490</td>
<td>49869</td>
<td>49837</td>
</tr>
<tr>
<td>k1</td>
<td>0.313</td>
<td>0.351</td>
<td>0.2377</td>
<td>0.4338</td>
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<tr>
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<td>-0.0918</td>
<td>-0.0560</td>
<td>-0.1063</td>
</tr>
<tr>
<td>CD,0 (data)</td>
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<td>0.0610</td>
<td>0.0510</td>
<td>0.0550</td>
</tr>
<tr>
<td>CD,0 (LS)</td>
<td>0.0512</td>
<td>0.0608</td>
<td>0.0521</td>
<td>0.0537</td>
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<tr>
<td>e0</td>
<td>0.70</td>
<td>0.62</td>
<td>0.92</td>
<td>0.50</td>
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<td>CL, minD (data)</td>
<td>0.1506</td>
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<td>CL, minD (LS)</td>
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<td>0.1178</td>
<td>0.1225</td>
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<td>CD, min (data)</td>
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<td>CD, min (LS)</td>
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<tr>
<td>Cfe (Data)</td>
<td>0.0137</td>
<td>0.0148</td>
<td>0.0133</td>
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<tr>
<td>Cfe (LS)</td>
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<td>0.0150</td>
<td>0.0134</td>
<td>0.0129</td>
</tr>
<tr>
<td>CL, alpha=0</td>
<td>0.1025</td>
<td>0.110</td>
<td>0.0989</td>
<td>0.1223</td>
</tr>
<tr>
<td>Lift</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hysteresis (% of max force)</td>
<td>2.9%</td>
<td>27.4%</td>
<td>22.6%</td>
<td>38.6%</td>
</tr>
<tr>
<td>Max force measured (g)</td>
<td>29.3</td>
<td>25.9</td>
<td>14.9</td>
<td>12.3</td>
</tr>
<tr>
<td>Drag</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hysteresis (% of max force)</td>
<td>9.5%</td>
<td>22.2%</td>
<td>3.0%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Max force measured (g)</td>
<td>7.4</td>
<td>6.7</td>
<td>3.4</td>
<td>3.3</td>
</tr>
</tbody>
</table>
Appendix C: Experimental Results for the Blue Widow MAV

Drag Polars for Blue Widow MAV, RE = 70,000

Figure C.1 Drag polars for Blue Widow MAV, Re = 70,000

Figure C.2 Drag polars for Blue Widow MAV, Re = 70,000, showing uncertainty bars
Figure C.3  Drag polars for Blue Widow MAV, $Re = 50,000$

Figure C.4  Drag polars for Blue Widow MAV, $Re = 50,000$, showing uncertainty bars
Figure C.5 Drag polars for Blue Widow MAV, Re = 30,000

Figure C.6 Drag polars for Blue Widow MAV, Re = 30,000, showing uncertainty bars
Figure C.7 Drag polars for Blue Widow MAV, Re = 10,000 (shown on a larger scale than for Re 30,000 through 70,000)

Figure C.8 Drag polars for Blue Widow MAV, Re = 10,000, showing uncertainty bars (shown on a larger scale than for Re 30,000 through 70,000)
Figure C.9 $C_L$ vs. $\alpha$ for Blue Widow MAV, Re = 70,000

Figure C.10 $C_L$ vs. $\alpha$ for Blue Widow MAV, Re = 50,000
Figure C.11  $C_L$ vs. $\alpha$ for Blue Widow MAV, $Re = 30,000$

Figure C.12  $C_L$ vs. $\alpha$ for Blue Widow MAV, $Re = 10,000$ (shown on a larger scale than for $Re = 30,000$ through 70,000)
Figure C.13  $C_D$ vs. $\alpha$ for Blue Widow MAV, $Re = 70,000$

Figure C.14  $C_D$ vs. $\alpha$ for Blue Widow MAV, $Re = 50,000$
Figure C.15 \( C_D \) vs. \( \alpha \) for Blue Widow MAV, \( Re = 30,000 \)

Figure C.16 \( C_D \) vs. \( \alpha \) for Blue Widow MAV, \( Re = 10,000 \)
Figure C.17  $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 70,000$

Figure C.18  $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 70,000$, showing uncertainty bars
Figure C.19  $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 50,000$

Figure C.20  $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 50,000$, showing uncertainty bars
Figure C.21 $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 30,000$

Figure C.22 $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 30,000$, showing uncertainty bars
Figure C.23 $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 10,000$ (shown on a larger scale than for $Re$ 30,000 through 70,000)

Figure C.24 $L/D$ vs. $\alpha$ for Blue Widow MAV, $Re = 10,000$, showing uncertainty bars (shown on a larger scale than for $Re$ 30,000 through 70,000)
Table C.1 Table of experimental data for all tests for Blue Widow MAV

<table>
<thead>
<tr>
<th>Nominal Re:</th>
<th>70,000</th>
<th>50,000</th>
<th>30,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>1 (up)</td>
<td>2 (down)</td>
<td>1 (up)</td>
<td>2 (down)</td>
</tr>
<tr>
<td>L/D MAX</td>
<td>8.42</td>
<td>8.74</td>
<td>10.53</td>
<td>10.88</td>
</tr>
<tr>
<td>Avg Re</td>
<td>69661</td>
<td>69285</td>
<td>49892</td>
<td>49955</td>
</tr>
<tr>
<td>k1</td>
<td>0.2456</td>
<td>0.2744</td>
<td>0.2166</td>
<td>0.3318</td>
</tr>
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<td>k2</td>
<td>-0.0409</td>
<td>-0.0430</td>
<td>-0.0215</td>
<td>-0.0348</td>
</tr>
<tr>
<td>CD,0 (data)</td>
<td>0.0270</td>
<td>0.0230</td>
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<td>0.0120</td>
</tr>
<tr>
<td>CD,0 (LS)</td>
<td>0.0273</td>
<td>0.0235</td>
<td>0.0155</td>
<td>0.0121</td>
</tr>
<tr>
<td>e0</td>
<td>1.10</td>
<td>0.99</td>
<td>1.25</td>
<td>0.82</td>
</tr>
<tr>
<td>CL,minD (data)</td>
<td>0.0904</td>
<td>0.0781</td>
<td>0.0158</td>
<td>0.0671</td>
</tr>
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<td>CL,minD (LS)</td>
<td>0.0833</td>
<td>0.0784</td>
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<td>CD,min (data)</td>
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<td>0.0199</td>
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<td>CD,min (LS)</td>
<td>0.0256</td>
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<td>0.0112</td>
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<tr>
<td>Cfe (Data)</td>
<td>0.0102</td>
<td>0.0078</td>
<td>0.0059</td>
<td>0.0041</td>
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<tr>
<td>Cfe (LS)</td>
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<td>0.0086</td>
<td>0.0059</td>
<td>0.0044</td>
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<td>CL,alpha=0</td>
<td>0.0579</td>
<td>0.0781</td>
<td>0.0568</td>
<td>0.0671</td>
</tr>
<tr>
<td>Lift</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hysteresis (% of max force)</td>
<td>11.7%</td>
<td>11.5%</td>
<td>16.7%</td>
<td>23.7%</td>
</tr>
<tr>
<td>Max force measured (g)</td>
<td>51.7</td>
<td>47.7</td>
<td>25.4</td>
<td>20.2</td>
</tr>
<tr>
<td>Drag</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hysteresis (% of max force)</td>
<td>18.6%</td>
<td>11.8%</td>
<td>10.7%</td>
<td>10.4%</td>
</tr>
<tr>
<td>Max force measured (g)</td>
<td>10.7</td>
<td>9.6</td>
<td>4.7</td>
<td>4.3</td>
</tr>
</tbody>
</table>
Appendix D: Experimental Results for the KIST MAV

Drag Polars for KIST MAV, RE = 70,000

Figure D.1 Drag polars for KIST MAV, Re = 70,000

Drag Polars for KIST MAV, RE = 70,000

Figure D.2 Drag polars for KIST MAV, Re = 70,000, showing uncertainty bars
Figure D.3  Drag polars for KIST MAV, Re = 50,000

Figure D.4  Drag polars for KIST MAV, Re = 50,000, showing uncertainty bars
Figure D.5 Drag polars for KIST MAV, Re = 30,000

Figure D.6 Drag polars for KIST MAV, Re = 30,000, showing uncertainty bars
Figure D.7 Drag polars for KIST MAV, Re = 10,000 (shown on a larger scale than for Re 30,000 through 70,000)

Figure D.8 Drag polars for KIST MAV, Re = 10,000, showing uncertainty bars (shown on a larger scale than for Re 30,000 through 70,000)
Figure D.9  $C_L$ vs. $\alpha$ for KIST MAV, $Re = 70,000$

Figure D.10  $C_L$ vs. $\alpha$ for KIST MAV, $Re = 50,000$
Figure D.11 $C_L$ vs. $\alpha$ for KIST MAV, $Re = 30,000$

Figure D.12 $C_L$ vs. $\alpha$ for KIST MAV, $Re = 10,000$ (shown on a larger scale than for $Re 30,000$ through $70,000$)
Figure D.13 \( C_D \) vs. \( \alpha \) for KIST MAV, \( Re = 70,000 \)

Figure D.14 \( C_D \) vs. \( \alpha \) for KIST MAV, \( Re = 50,000 \)
Figure D.15 $C_D$ vs. $\alpha$ for KIST MAV, $Re = 30,000$

Figure D.16 $C_D$ vs. $\alpha$ for KIST MAV, $Re = 10,000$
Figure D.17 $L/D$ vs. $\alpha$ for KIST MAV, $Re = 70,000$

Figure D.18 $L/D$ vs. $\alpha$ for KIST MAV, $Re = 70,000$, showing uncertainty bars
Figure D.19  $L/D$ vs. $\alpha$ for KIST MAV, $Re = 50,000$

Figure D.20  $L/D$ vs. $\alpha$ for KIST MAV, $Re = 50,000$, showing uncertainty bars
Figure D.21 $L/D$ vs. $\alpha$ for KIST MAV, $Re = 30,000$

Figure D.22 $L/D$ vs. $\alpha$ for KIST MAV, $Re = 30,000$, showing uncertainty bars
Figure D.23  $L/D$ vs. $\alpha$ for KIST MAV, $Re = 10,000$ (shown on a larger scale than for $Re$ 30,000 through 70,000)

Figure D.24  $L/D$ vs. $\alpha$ for KIST MAV, $Re = 10,000$, showing uncertainty bars (shown on a larger scale than for $Re$ 30,000 through 70,000)
### Table D.1 Table of experimental data for all tests for KIST MAV

<table>
<thead>
<tr>
<th>Nominal Re:</th>
<th>70,000</th>
<th>50,000</th>
<th>30,000</th>
<th>10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>1 (up)</td>
<td>2 (down)</td>
<td>1 (up)</td>
<td>2 (down)</td>
</tr>
<tr>
<td><strong><strong>L/D MAX</strong></strong></td>
<td>3.02</td>
<td>3.10</td>
<td>2.95</td>
<td>2.87</td>
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<tr>
<td>Avg Re</td>
<td>69673</td>
<td>69882</td>
<td>50145</td>
<td>49978</td>
</tr>
<tr>
<td>k1</td>
<td>0.6236</td>
<td>0.6176</td>
<td>0.6647</td>
<td>0.7269</td>
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<td>k2</td>
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<td>-0.0999</td>
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<tr>
<td>CD,0 (data)</td>
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<td>0.0720</td>
<td>0.0620</td>
<td>0.0800</td>
</tr>
<tr>
<td>CD,0 (LS)</td>
<td>0.0762</td>
<td>0.0716</td>
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<td>0.0824</td>
</tr>
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<td>e0</td>
<td>0.32</td>
<td>0.32</td>
<td>0.30</td>
<td>0.28</td>
</tr>
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<td>CL, minD (data)</td>
<td>0.1012</td>
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<td>0.1040</td>
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<td>CL, minD (LS)</td>
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<td>0.0809</td>
<td>0.0495</td>
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<td>CD, min (data)</td>
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<td>0.0663</td>
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<td>0.0757</td>
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<tr>
<td>Cfe (Data)</td>
<td>0.0183</td>
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<td>0.0190</td>
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<tr>
<td>Cfe (LS)</td>
<td>0.0185</td>
<td>0.0175</td>
<td>0.0158</td>
<td>0.0196</td>
</tr>
<tr>
<td>CL, alpha=0</td>
<td>0.1180</td>
<td>0.1134</td>
<td>-0.0147</td>
<td>0.1382</td>
</tr>
</tbody>
</table>

**Lift**

| Hysteresis (% of max force) | 3.0% | 9.1% | -28.0% | 12.6% | -2.1% | 59.1% | 127.3% | 242.1% |
| Max force measured (g)      | 63.5 | 61.4 | 27.8 | 29.1 | 10.7 | 9.2 | 2.9 | 3.3 |

**Drag**

| Hysteresis (% of max force) | 7.7% | 13.4% | -9.2% | 9.9% | -4.0% | -0.7% | -12.4% | -19.8% |
| Max force measured (g)      | 24.3 | 23.4 | 11.2 | 11.6 | 4.0 | 4.2 | 0.5 | 0.7 |
## Appendix E: Characteristic Data for Each MAV

Table E.1 Data used for drag polar predictions and Re calculations for each MAV. Masses are published values where available; otherwise they are educated guesses.

<table>
<thead>
<tr>
<th>MAV</th>
<th>AR</th>
<th>$S_{web}/S$</th>
<th>$W/S$ (lb/ft$^2$)</th>
<th>$b$ (in)</th>
<th>$S$ (in$^2$)</th>
<th>$c$ (used for Re calculation)</th>
<th>$c$ (root)</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BYU</td>
<td>1.46</td>
<td>3.65</td>
<td>0.246</td>
<td>7.5</td>
<td>38.6</td>
<td>6.0</td>
<td>6.0</td>
<td>30</td>
</tr>
<tr>
<td>Blue Widow</td>
<td>1.18</td>
<td>2.55</td>
<td>0.830</td>
<td>6.0</td>
<td>30.6</td>
<td>4.0 (tip)</td>
<td>5.5</td>
<td>80</td>
</tr>
<tr>
<td>KIST</td>
<td>1.59</td>
<td>3.86</td>
<td>0.733</td>
<td>8.3</td>
<td>43.3</td>
<td>4.5 (tip)</td>
<td>5.5</td>
<td>100</td>
</tr>
</tbody>
</table>

## Appendix F: Misc. Data Used for Drag Polar Predictions

Table F.1 Additional data used for drag polar predictions for each MAV

<table>
<thead>
<tr>
<th>MAV</th>
<th>$C_{f,\text{fe}}$ (The greater of the laminar or turbulent [Schlichting] solutions was used for $c_f$)</th>
<th>Best Guess $\varepsilon_0$ (based on data from Re = 50,000 and 70,000)</th>
<th>$C_{L,\alpha=0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BYU</td>
<td>$= 1.0 \times c_f$</td>
<td>0.8</td>
<td>0.11</td>
</tr>
<tr>
<td>Blue Widow</td>
<td>$= 1.0 \times c_f$</td>
<td>0.7</td>
<td>0.07</td>
</tr>
<tr>
<td>KIST</td>
<td>$= 1.0 \times c_f$</td>
<td>0.5</td>
<td>0.13</td>
</tr>
</tbody>
</table>

## Appendix G: Table of Lift and Drag Subtractions for Mounts and Wires With Uncertainties

Table G.1 Measured corrections of $C_L$ and $C_D$ (along with their uncertainties $u_{CL}$ and $u_{CD}$) for the mounts and wires exposed in the wind tunnel during the tests.

<table>
<thead>
<tr>
<th>MAV</th>
<th>$C_L$</th>
<th>$u_{CL}$</th>
<th>$C_D$</th>
<th>$u_{CD}$</th>
<th>$C_L$</th>
<th>$u_{CL}$</th>
<th>$C_D$</th>
<th>$u_{CD}$</th>
<th>$C_L$</th>
<th>$u_{CL}$</th>
<th>$C_D$</th>
<th>$u_{CD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BYU MAV</td>
<td>0.0140</td>
<td>0.00941</td>
<td>0.0237</td>
<td>0.00058</td>
<td>0.0310</td>
<td>0.00517</td>
<td>0.0361</td>
<td>0.00036</td>
<td>0.0221</td>
<td>0.00466</td>
<td>0.0308</td>
<td>0.00039</td>
</tr>
<tr>
<td>Blue Widow MAV</td>
<td>0.0019</td>
<td>0.01814</td>
<td>0.0240</td>
<td>0.00115</td>
<td>0.0343</td>
<td>0.01045</td>
<td>0.0393</td>
<td>0.00080</td>
<td>0.0254</td>
<td>0.00934</td>
<td>0.0297</td>
<td>0.00076</td>
</tr>
<tr>
<td>KIST MAV</td>
<td>0.0131</td>
<td>0.00560</td>
<td>0.0250</td>
<td>0.00319</td>
<td>0.0164</td>
<td>0.02787</td>
<td>0.0337</td>
<td>0.00190</td>
<td>0.0275</td>
<td>0.02467</td>
<td>0.0295</td>
<td>0.00200</td>
</tr>
</tbody>
</table>