Reconstruction of the Temperature Profile Along a Blackbody Optical Fiber Thermometer

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RECONSTRUCTION OF THE TEMPERATURE PROFILE
ALONG A BLACKBODY OPTICAL FIBER THERMOMETER

By
David G. Barker

A thesis submitted to the faculty of
Brigham Young University
In partial fulfillment of the requirements for the degree of

Master of Science

Department of Mechanical Engineering
Brigham Young University
April 2003
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ABSTRACT

RECONSTRUCTION OF THE TEMPERATURE PROFILE ALONG A BLACKBODY OPTICAL FIBER THERMOMETER

David G. Barker
Department of Mechanical Engineering
Master of Science

A blackbody optical fiber thermometer consists of an optical fiber whose sensing tip is given a metallic coating. The sensing tip of the fiber forms an isothermal cavity, and the emission from this cavity is approximately equal to the emission from a blackbody. Standard two-color optical fiber thermometry involves measuring the spectral intensity at the end of the fiber at two wavelengths. The temperature at the sensing tip of the fiber can then be inferred using Planck’s law and the ratio of the spectral intensities. If, however, the length of the optical fiber is exposed to elevated temperatures, erroneous temperature measurements will occur due to emission by the fiber. This thesis presents a method to account for emission by the fiber and accurately infer the temperature at the tip of the optical fiber. Additionally, an estimate of the temperature profile along the fiber may be obtained.
A mathematical relation for radiation transfer down the optical fiber is developed. The radiation exiting the fiber and the temperature profile along the fiber are related to the detector signal by a signal measurement equation. Since the temperature profile cannot be solved for directly using the signal measurement equation, two inverse minimization techniques are developed to find the temperature profile. Simulated temperature profile reconstructions show the techniques produce valid and unique results. Tip temperatures are reconstructed to within 1.0%.

Experimental results are also presented. Due to the limitations of the detection system and the optical fiber probe, the uncertainty in the signal measurement equation is high. Also, due to the limitations of the laboratory furnace and the optical detector, the measurement uncertainty is also high. This leads to reconstructions that are not always accurate. Even though the temperature profiles are not completely accurate, the tip-temperatures are reconstructed to within 1%—a significant improvement over the standard two-color technique under the same conditions. Improvements are recommended that will lead to decreased measurement and signal measurement equation uncertainty. This decreased uncertainty will lead to the development of a reliable and accurate temperature measurement device.
The pursuit of knowledge is in vain without the guidance of those who have gone before. Dr. Matthew Jones had enough faith in me as an undergraduate to ask me to take on the task of solving this problem. Had he not, I would not be graduating for another year, and I would have missed the many opportunities I have had because of this research. I wish to thank him for his guidance through the theory, the math, the writing and the editing. I appreciate his patience while working with a not-so-humble graduate student, and will be always grateful for the many professional lessons he has taught me. I especially appreciate his insights into learning that have forever affected my opinion of education and its purpose. I wish to acknowledge funding for this research from the College of Engineering and Technology at Brigham Young University, and from an undergraduate research grant from the Office of Research and Creative Activities at Brigham Young University.

My desire for learning how things work is undoubtedly a product of my heritage. My grandfather, Dee H. Barker, is a chemical engineer. My father, Gary T. Barker, is a mechanical engineer. I thank both of them for their example of successful engineering and of hard work. I wish to also thank my mother, Susan E. Barker, for teaching me faith and patience.
My wife, Robyn, is my motivation. Her love for me and our interdependence on each other have propelled me through the sometimes rough waters of this research. I wish to thank her for her willingness to listen to my complaints and problems and for trying to understand. Most importantly, I would like to thank her for being so excited for my accomplishments throughout the research—that excitement continually pushed me forward.

Finally, I would like to acknowledge the hand of my Heavenly Father in this work. While the results of the research might not be of highest importance, the lessons of faith I have been taught are. Without Him I am nothing, and without Him this work would not have been completed in the manner it has.
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\( \eta_g \)  Monochromator grating efficiency
\( \eta_m \)  Mirror reflection efficiency
\( \theta_c \)  Critical angle for total internal reflection
\( \kappa \)  Spectral calibration constant
\( \lambda \)  Wavelength
\( \nu \)  Frequency
\( \sigma \)  Standard deviation
\( \psi \)  Detector signal
\( \omega \)  Solid angle
\( \nabla \)  Gradient operator

Subscripts
\( b \)  Blackbody
\( c \)  Calibration
\( d \)  Detector
\( i, j, k \)  Indices
\( L \)  Length of the optical fiber
\( m \)  Measured
\( S \)  Light path length
\( \lambda \)  Wavelength
\( \nu \)  Frequency
\( 1, 2, 3 \)  Surfaces

Superscripts
\( k \)  Iteration
CHAPTER 1 INTRODUCTION

Optical fiber thermometers (OFT) are devices that use radiative emission to infer temperature. OFT have several advantages over other temperature measurement methods such as thermocouples. OFT are more stable, have a wider dynamic range, and are more capable of withstanding harsh environments (Fang et. al., 1994). This chapter will give a brief background of the development of optical fiber temperature sensors and show the main objectives for this research.

1.1 Background

This section will briefly discuss the evolution of optical fibers as temperature sensors, concluding with the findings most relevant to this thesis.

In the late 1970’s, Gottlieb et. al. (1980) documented research that explored the use of optical fibers as temperature sensors. They showed that propagation and loss characteristics of optical fibers could be altered sufficiently by temperature, making detection of these variations possible. By measuring these variations it was possible to infer the temperature that caused them. Gottlieb et. al. addressed specifically two characteristics: interaction between core and cladding refractive indices, and losses due to bending.

For an optical fiber to act as a light guide, the core must have a higher refractive index than that of the cladding—a material that surrounds and protects the glass core or fiber. Gottlieb et. al. proposed that a fiber and cladding could be chosen such that at low
temperatures the core would have a higher refractive index than the cladding, but as the surrounding temperature became greater, the refractive index of the core would decrease while the index of the cladding would increase. At a certain temperature the refractive index values would cross and the fiber would no longer allow light transmission. This temperature sensor would not be able to detect different temperatures, only a single temperature, thereby acting as a shut-off monitor. This was very restrictive because few core and cladding combinations existed that could provide the desired characteristics, and each was only useful at one temperature.

As an optical fiber bends, the transmission properties are affected until the point when the critical bend radius is reached and no transmission occurs. According to Gottlieb et. al. (1980), if either the core or cladding refractive index were dependent on temperature, the loss due to bending would also be a function of temperature. In this manner, the light transmission in a bended fiber with known core and cladding refractive indices could be monitored, and the temperature inferred. No experimental results were shown for either of the proposed temperature sensors.

Gottlieb and Brandt (1981) observed that temperature sensors of this kind were somewhat impractical because it was necessary to control the optical properties of the fiber very precisely. Accordingly, they developed further methods of using optical fibers as temperature sensors, but this time the fibers were actually the source of the thermal radiation and not merely light pipes.

They proposed that a heated fiber would emit as a blackbody whose temperature could be inferred using Planck’s Law. Specifically, this technique could be used to find hot spots in the domain where the fiber was being used. Theoretically, the hot spot would
be much hotter than the rest of the fiber, therefore dominating the emission. They proposed a method for finding the temperature and location of the hot spot.

A hotspot would cause temperature gradients in the fiber. Since no data were available on the temperature dependence of the fiber material, it was assumed by Gottlieb and Brandt (1981) that the absorption coefficient of the optical fiber did not vary with temperature. This absorption constant was used to approximate the emission in the calculations. In the actual experiments, the absorption of the fiber was a known parameter of the fiber, but again it was treated as a constant. This is critical to the proposed thesis because the temperatures to which the blackbody OFT will be exposed will be extreme, and the temperature dependence of the absorption coefficient on temperature will need to be known.

Experiments to test this theory were conducted using Pyrex and quartz fibers of different diameters in conjunction with silicon, germanium and lead-sulfide detectors. Results are shown in Gottlieb and Brandt for the temperature range 200-700 C and plotted as detector signal strength vs. temperature. The results presented are in good agreement with blackbody theory and the predicted signal.

Building further on using an optical fiber as a thermal radiation source, Dils (1983) proposed an optical fiber thermometer (OFT) that would function at temperatures far above those for which a standard OFT would function. An OFT was constructed using a single crystal sapphire optical fiber with an optically opaque coating at the tip. If the thermal conductivity of the optically opaque coating is high, the cavity created is isothermal, so emission from this cavity can be approximated as emission from a blackbody. The Planck equation can be used to determine the spectral radiative flux
exiting the cavity. The high temperature fiber can be coupled to a standard glass fiber in order to transmit the emission to a detector. Any number of detectors can be used depending on the spectral band and emissive power to be measured.

The approximations made by Dils were:

- Apparent emissivity of the blackbody cavity is not a strong function of temperature
- Transmission losses in the low temperature region of the high temperature fiber as well as in the entire low temperature fiber are negligible

For the experiments, coated and uncoated 1.25 mm diameter 30 cm long sapphire fibers were used with a 10 µm thick platinum coating with a L/D ratio of 2 on the tip.

Following Dils, Krieder noted that an important part of the blackbody OFT is the construction of the blackbody cavity at the sensing tip of the fiber (1985). He stated, “the source of signal generation on an open light pipe can be either the target area viewed by the tip or self-radiation. To use the light pipe in measuring the temperature of gases, it is far more preferable to have the signal generation from a well-characterized source in order to gain spatial resolution and known emissivity factors” (p.152).

Advantages of this type of optical fiber thermometry are well-characterized signal generation, well-characterized transmissions of the light guides, material stability in oxidizing environments up to the melting point of the optical fiber (approximately 2300 K for sapphire), and excellent sensitivity to temperature changes as described by Planck’s equation.

A major problem deals with the calibration of the device since the emission incident on the detector can be affected by “changes in the emitter, the scattering
properties of the sapphire surface, and the alignment to the optical fiber, lens, filters and diode” (pp. 152-53).

In the experiments conducted by Kreider to test OFT theory, the blackbody cavity at the tip was created by vacuum sputtering a 2 \( \mu \text{m} \) thick platinum coating. Kreider concluded that temperature measurements using this type of OFT were accurate within 5\(^\circ\text{C} \) up to 300\(^\circ\text{C} \) from the temperature used for calibration. Several geometries were used for the blackbody cavity, but due to variations in sapphire properties and coupling losses, no conclusions were drawn about the effects of the different geometries.

This type of blackbody OFT—a sapphire optical fiber with an optically opaque coating on the tip—described by Dils and Kreider was used for the results presented here. The following section describes the method that was used to reconstruct the temperature profile along the blackbody OFT.

1.2 Objectives

Blackbody OFT are created by coating the tip of an optical fiber with a material that is optically opaque (see Fig. 1.1). This creates an isothermal cavity at the tip of the fiber that emits like a blackbody. The emission can then be measured at the opposite end of the fiber and the temperature can be inferred using Planck’s law (Dils, 1983). This type of OFT can measure a cavity temperature, or the temperature of the tip’s surroundings.

In most applications where blackbody OFT are used, only the sensing tip and a small length of the fiber are exposed to the high temperature environment. In these situations, it is acceptable to assume that all radiation detected has been emitted by the blackbody cavity (Dils, 1983). However, it has been shown that when a significant
portion of the fiber is exposed to elevated temperatures, the temperature measurements are corrupted by emission by the fiber (Jones et. al., 1999).

Figure 1.1 Diagram of a typical blackbody optical fiber thermometer.

A widely used application of OFT is lightpipe radiation thermometers (LPRT) which are used in temperature control of silicon wafer processing. LPRT are similar to blackbody OFT without the blackbody coating on the tip. They infer the temperature of the target—whatever fills the field of view of the optical fiber. Temperature is inferred from the radiative emission exiting the fiber based on the known emissivity of the target (i.e. the silicon wafer). Tests performed at the National Institute of Standards and Technology (NIST) have shown that commercial LPRT temperature measurements are also corrupted when a portion of the lightpipe is exposed to elevated temperatures (Tsai et. al., 2000). Currently, calibration is performed, on a case-by-case basis, to account for different temperature measurement environments, but these calibrations require operating conditions of the LPRT to be nearly exactly the same as the calibration conditions. This time-intensive process causes the price of LPRT to be higher than it would if a more robust, scientific approach were used to infer the target temperature.
The objective of this research is threefold: (1) To develop a numerical optimization routine that is capable of finding the temperature profile along a heated optical fiber. (2) To use this numerical algorithm to account for fiber self-emission and obtain an inferred tip-temperature that is more accurate than the standard two-color method. An estimate of the temperature profile along the optical fiber will also be obtained using this algorithm. (3) To design and perform preliminary experiments to validate the numerical simulations for both the tip-temperature and the temperature profile. The results presented in this thesis show the fulfillment of these objectives and provide a basis for further development.
CHAPTER 2 MATHEMATICAL FORMULATION

2.1 Radiation Transfer in the Optical Fiber

A schematic of a typical blackbody OFT with a heated portion of the optical fiber is shown in Figure 2.1.

Figure 2.1 Schematic of an OFT with a heated portion of the optical fiber.

Usually, the optical fiber is a single crystal sapphire (Al₂O₃) rod, and the cavity is created by sputtering platinum or iridium (Dils, 1983) on the tip of the fiber. The spectral irradiation is equal to the product of the spectral intensity exiting the fiber and the solid angle subtended by the fiber as seen from the detector.

\[
G_{\lambda} = I_{\lambda}(L) \omega_d
\]  

(2.1)

Initially frequency is used as the spectral variable. Light is transferred down the fiber, then through air with a different refractive index than sapphire, until it is incident
on the detector. Since wavelength changes at the boundary of two different refractive indices, frequency, which remains constant as the refractive index varies, has been used.

Modeling the cavity as an isothermal enclosure and the fiber as an emitting, absorbing and non-scattering medium, yields the following differential equation for the spectral intensity in the fiber (Eq. (2.2)) with the boundary condition specified by Eq. (2.3) (Brewster, 1992).

\[
\frac{dI_v}{ds} = -K_{av}I_v + K_{av}I_{bw}(T(s)) \tag{2.2}
\]

\[
I_v(0) = \varepsilon I_{bw}(T_o) \tag{2.3}
\]

where the emissivity of the isothermal cavity is included in the boundary condition since it is less than unity. The path length, \(s\), is the actual path traveled by the light. As illustrated in Fig. 2.2, the light path is longer than the length of the heated section of the optical fiber, and the effect of the difference in the lengths of the various possible light paths are included in the uncertainty analysis.

Solving for \(I_v(t_{vs})\) from Eqs. (2.2) and (2.3) using an integrating factor gives

\[
I_v(t_{vs}) = \varepsilon I_{bw}(T_o) \exp\{\int_{-t_{vs}}^{t_{vs}} I_{bw}(T(t_v)) \exp\{t_v - t_{vs}\}dt_v \tag{2.4}
\]

where the independent variable has been transformed from \(s\) to the optical depth, \(t_v = K_{av}s\). The upper limit of the integral in Eq. (2.4) is the optical depth for the average of the possible light paths. The effect of the uncertainty in the absorption coefficient has also been included in the uncertainty analysis. The term on the left of the equal sign in Eq. (2.4) represents intensity exiting the isothermal cavity. The first exponential term on the right accounts for attenuation of the blackbody emission by the fiber. The integral
term represents an increase in the intensity exiting the fiber due to emission by the fiber. If the temperature along the OFT is low, the integral term can be neglected since emission by the fiber is insignificant compared to emission by the cavity.

Figure 2.2 Schematic of the light path due to internal reflection.

If the optical properties of the fiber and the temperature profile are known, Eq.(2.4) can be solved directly for the isothermal cavity intensity. In the current problem, however, the spectral intensity measurements exiting the fiber are available and the temperature profile is unknown, making the problem an inverse problem. An inverse problem can be described in the following manner. A temperature profile along the optical fiber is guessed. Equation (2.4) is used with the assumed temperature profile to calculate predicted spectral measurements. These predicted measurements are compared with the actual measurements and then the temperature profile guess is updated. This process continues until convergence.

Eq. (2.4) may be simplified as follows. Wien’s limit can be used to approximate the spectral intensity of a blackbody for the frequencies and temperatures of interest.

\[
I_{\text{be}}(T_o) = \frac{2h^3n_e^2}{c_o^2 \left[ \exp \left\{ \frac{hv}{kT_o} \right\} - 1 \right]} \approx \frac{2h^3n_e^2}{c_o^2 \left[ \exp \left\{ \frac{hv}{kT_o} \right\} \right]} \tag{2.5}
\]
Since the spectral intensity is measured as a function of wavelength, it is convenient to convert the spectral variable from frequency to wavelength. After exiting the fiber, the radiation propagates through air until reaching the detector, so the appropriate conversion is (Brewster 1992)

$$I_{b\lambda}(T_o) = \frac{c_o}{\lambda^2} I_{b\nu}(T_o)$$  \hspace{1cm} (2.6)

With $\omega_\lambda = c_o/\nu$, $C_1 = 2\pi hc_o^2$, $C_2 = hc_o/k$, and Eq. (2.1), Eq. (2.4) becomes

$$G_\lambda = \frac{C_in^2_\lambda \omega_\lambda}{\pi \lambda^5} \left[ \varepsilon_\lambda \exp\left(-t_{\lambda S}-\frac{C_2}{\lambda T_o}\right)+\int_0^{t_{\lambda S}} \exp\left(t_{\lambda}-t_{\lambda S}-\frac{C_2}{\lambda T(t_{\lambda})}\right) dt_{\lambda} \right]$$  \hspace{1cm} (2.7)

Equation (2.7) provides the desired relationship between the temperature profile along the fiber, $T(t_{\lambda S})$, and the irradiance incident on the detector, $G_\lambda$. Irradiance describes the amount of light intensity exiting the fiber that is incident on the detector. This equation will be used in the signal measurement equation which describes aspects of the light attenuation.

### 2.2 Assumptions in the Two-Color Method

When an OFT is held at a low temperature over the length of the fiber, the tip being the only part exposed to elevated temperatures, the integral term in Eq. (2.7) can be neglected (Dils, 1983). The emissivity is also assumed to be constant over a narrow wavelength band. To develop the standard two-color approach—measuring emission at two wavelengths, at the same temperature, and using those measurements to infer the tip temperature—absorption in the fiber is also neglected and $T_o$ can be found in the following manner. Taking measurements at $\lambda_1$ and $\lambda_2$, and taking their ratio gives
\[
\frac{G_{\lambda_1}}{G_{\lambda_2}} = \frac{\kappa_{\lambda_1}}{\kappa_{\lambda_2}} \exp\left\{\frac{C_2}{\lambda_2 T_o}\right\}
\]

(2.8)

Where \( \kappa \) is a calibration constant that will be defined later. Solving for \( T_o \) gives

\[
T_o = T_{2c} = \frac{C_2 - C_2}{\frac{\lambda_2}{\lambda_1} \ln \left( \frac{\kappa_{\lambda_1} G_{\lambda_1}}{\kappa_{\lambda_2} G_{\lambda_2}} \right)}
\]

(2.9)

Eq. (2.9) is most accurate (Dils, 1983; Kreider, 1985) using two wavelengths narrowly spaced to the left of the wavelength of peak emission where the sensitivity to temperature change is greatest. This, however, is only the case if the tip of the fiber is the only part of the fiber exposed to elevated temperatures. If the length of the fiber is exposed to elevated temperatures, it has been shown that the two-color temperature is biased towards higher temperatures (Jones et. al., 1999; Tsai et. al., 2000). This is because at elevated temperatures, the emission is significant and the integral term in Eq. (2.7) becomes large enough to be non-negligible.

This thesis presents methods of accounting for the emission by the fiber when the fiber is exposed to elevated temperatures. Taking into account the emission by the fiber, measuring spectral irradiance at several wavelengths, and using an optimization algorithm, will make it possible to reconstruct the tip temperature and obtain an estimate of the temperature profile along the optical fiber thermometer.
CHAPTER 3 OPTICAL DETECTION SYSTEM

This chapter describes the optical detection system used in the experiments. The measurements from this system are used in the signal measurement equation which is used by the minimization algorithms to reconstruct the tip-temperature and the temperature profile along the optical fiber. Since the uncertainty in the optical measurements has a significant impact on the reconstructions, an uncertainty analysis on the optical measurements is addressed.

3.1 Experimental Equipment

Figure 3.1 shows a diagram of the apparatus used in the experiments (See Appendix B for photographs of the actual setup).

![Diagram of the apparatus used in the experiments]

Figure 3.1 Schematic representation of the equipment used to measure the intensity exiting the blackbody OFT.
The OFT is similar to the schematic in Fig. 1.1. It is made of single crystal sapphire (AL₂O₃), is 0.5 mm in diameter and 1.0 m in length. The blackbody cavity at the tip was created with a thin-walled platinum tube with an inside diameter (1.2 mm) slightly larger than the optical fiber and a length to diameter ratio of 15. One end was crimped to form a cavity and the optical fiber was inserted in the other end. An analysis performed on a cylindrical cavity with a length to diameter ratio of 15 showed that the effective emissivity of the cavity is 0.92 ±0.03. This calculation will be discussed in Section 3.3, Calibration.

The furnace is a cylindrical resistance furnace capable of temperatures up to 1366 K. Characterization of the furnace temperature profile at several temperature settings was performed with a multipoint thermocouple inserted through the furnace port. The temperature was measured at 5 points spaced 5 cm apart in the heated zone of the furnace with the first point’s location corresponding to the location of the tip of the OFT (Fig 3.2).

Figure 3.2 Laboratory furnace with multipoint thermocouple inserted for temperature profile characterization
The multipoint thermocouple actually had six measurement locations, but the fiber was only inserted into the furnace so that the tip of the fiber was at the second measurement location from the tip. This was done due to size constraints of the fiber and detection system.

Observing the change in temperature of the temperature points over time with the furnace at steady state, it was calculated that the standard deviation of the temperature at each point was less than 0.5K for each temperature profile. These temperature points will be used to assess the accuracy of the temperature reconstructions.

In order to measure the radiant flux in portions of the spectrum, the light from the OFT was passed through a monochromator. To maximize throughput, an F/# matcher was used to match the lower F/# of the optical fiber with the higher F/# of the monochromator. This made the output solid angle of the fiber more closely match the acceptance cone of the monochromator. Reflection losses in the F/# matcher were specified by the manufacturer and are less than would be experienced by the difference in F/#’s of the fiber and monochromator.

The monochromator was an Oriel 1/8m monochromator with a micrometer-driven entrance slit and a fixed-width exit slit. The manufacturer specified mirror reflection efficiency for each of the four mirrors in the monochromator. A single grating was used with the blaze wavelength at 2.0 $\mu m$ and a line density of 300 lines/mm. This yielded a grating bandwidth of 22 nm. The usable wavelength region was from 1.0 $\mu m$ to 4.0 $\mu m$. An approximate spectral efficiency plot of the grating was provided, but the grating was not calibrated. The grating efficiency was accounted for in the calibration procedure.
After the light was separated into wavelength bands, a miniature thermopile detector was used to measure the radiant flux exiting the monochromator. A miniature thermopile uses an array of thermocouples to measure the temperature rise associated with incident radiant flux. The thermopile outputs a current that corresponds with the temperature rise. This detector was calibrated, and the responsivity was given as 0.765 nA/(mW/cm$^2$). Since a thermopile measures temperature rise, the response is assumed to be constant over all wavelengths. The detector window was made of sapphire and the spectral transmissivity was given by the manufacturer. The transmissivity was given as a general plot, not as tabulated data, and not specific to the window used. It was therefore also treated as an unknown and accounted for in the calibration.

The signal from the thermopile detector was read by an Oriel Optical Power Meter capable of reading a signal of 1.0 pA. This is slightly less than the noise level of the thermopile detector so the limiting factor on the signal is the detector. This signal was read into a PC by a LabVIEW® program. To take a signal measurement at one wavelength, 50 measurements were taken with the monochromator shutter closed to provide a dark reading, 50 measurements were taken with the shutter open and then 50 were again taken with the shutter closed. With these measurements, the offset between light and dark signals was calculated along with the variance between each of the measurements. This gave the actual signal along with the measurement uncertainty.

### 3.2 Signal Measurement Equation

With knowledge of the apparatus used to measure the radiant flux exiting the OFT it is possible to develop the signal measurement equation (SME). The SME relates an amperage output of the detector to the radiant flux exiting the heated portion of the
optical fiber. Eq. (3.1) shows this relationship. The information on the SME presented here is specific to the current problem, but more general information can be found in the literature (DeWitt and Nutter, 1988; McCluney, 1994; NIST, 2002).

\[
\psi_\lambda = A \alpha R \eta F/\# T_m^4 \eta_{G_\lambda} \tau_\lambda \Delta \lambda G_\lambda 
\]  

(3.1)

The radiant flux exiting the heated portion of the optical fiber, \( G_\lambda \), is attenuated by the components of the optical detection system before it is incident on the detector. The following is a brief description of these components as addressed in Section 3.1.

- \( A \) Amplifier gain of the optical power meter
- \( \alpha \) Absorption in the low-temperature portion of the optical fiber
- \( R \) Responsivity of the optical detector
- \( \eta_{F/\#} \) Efficiency of the F/\# matcher
- \( \eta_m \) Mirror reflection efficiency (four mirrors in the monochromator)
- \( \eta_{G_\lambda} \) Spectral grating efficiency
- \( \tau_\lambda \) Spectral transmissivity of the sapphire detector window
- \( \Delta \lambda \) Monochromator bandwidth

Using the SME in conjunction with Eq. (2.7) it is then possible to relate the detector amperage output to the temperature profile along the OFT, or to predict the amperage output with a given temperature profile. Eq. (3.2) shows this relationship.

\[
\psi_\lambda = A \alpha R \eta F/\# T_m^4 \eta_{G_\lambda} \tau_\lambda \Delta \lambda \int_0^{t_{JS}} \frac{C_1 n_\lambda^2 T_0}{\lambda^5} \left[ \varepsilon_\lambda \exp \left( -t_{JS} \frac{C_2}{\lambda T_0} \right) + \exp \left( t_{JS} \frac{C_2}{\lambda T(t_\lambda)} \right) \right] dt_\lambda 
\]  

(3.2)
Since many of the parameters before the brackets are poorly or incompletely known, a calibration coefficient is defined that will be discussed in the following section. Equation (3.3) shows this calibration coefficient.

$$\kappa_\lambda = A_\alpha R_\eta F_{\lambda m} \eta_{\lambda_i} \tau_\lambda A_\lambda \frac{C_n \eta^2_{\lambda d}}{\pi \lambda^5} \quad (3.3)$$

Using this definition, Eq. (3.2) becomes

$$\psi_\lambda = \kappa_\lambda \left[ \varepsilon_\lambda \exp \left\{ -t_{\lambda S} - \frac{C_2}{\lambda T_o} \right\} + \int_0^{t_{\lambda S}} \exp \left\{ t_{\lambda} - t_{\lambda S} - \frac{C_2}{\lambda T(t_{\lambda})} \right\} \, dt_{\lambda} \right] \quad (3.4)$$

This is the final form of the equation that will be used by the minimization algorithms to estimate the tip-temperature of the OFT and to reconstruct the temperature profile along it.

3.3 Calibration

Note that all of the parameters included in the definition of the spectral calibration coefficient (Eq. (3.3)) are associated with the detection system, and none of these parameters depend on the temperature profile along the fiber. Therefore, the spectral calibration coefficients may be used when the fiber is exposed to any arbitrary temperature profile if the detection system is well characterized and all the parameters in Eq. (3.3) are known. However, many of these parameters are poorly characterized for the detection system used in these experiments, so it is necessary to determine the spectral calibration coefficients using Eq. (3.5)

$$\kappa_\lambda = \frac{\psi_\lambda \left( T_{\lambda} \left( t_{\lambda S} \right) \right)}{\varepsilon_\lambda \exp \left\{ -t_{\lambda S} - \frac{C_2}{\lambda T_o} \right\} + \int_0^{t_{\lambda S}} \exp \left\{ t_{\lambda} - t_{\lambda S} - \frac{C_2}{\lambda T_{\lambda} \left( t_{\lambda} \right)} \right\} \, dt_{\lambda} \quad (3.5)$$
where $T_c(t_d)$ is the temperature profile existing along the heated portion of the fiber during the calibration procedure and $\psi_m(\lambda, T_c(t_d))$ are the spectral measurements obtained during the calibration procedure.

The use of Eq. (3.5) requires knowledge of the spectral emissivity of the cavity and the spectral optical depth. The emissivity of the cavity is unknown and will be accounted for in the calibration. An estimated value must however be used to calculate the spectral calibration coefficient. The following describes the estimation of the cavity emissivity.

Figure 3.3 shows a schematic of the isothermal cavity. The cavity is actually crimped at the bottom, but analyzing it as a flat surface makes the problem more tractable and provides for a conservative estimate of the emissivity. Surfaces one and two are isothermal at the temperature of the tip of the fiber, and surface three is chosen to be room temperature. This will provide the lowest estimate of the effective emissivity of the cavity.

![Schematic of the isothermal platinum cavity.](image)
Since this is only an estimate—the inaccuracy is accounted for in the calibration—it is not necessary to know the exact temperature the end of the cavity sees. Since the cavity is coupled to one end of the fiber, the other end of the fiber being at room temperature, assuming total internal reflection, the temperature it actually sees is room temperature. This estimated analysis is also necessary to establish an uncertainty in the emissivity for use in the uncertainty analysis.

First it is necessary to find the view factors for the three surfaces. The formulas for the view factors can be found in (Howell, 2003). The view factor from surface one to three, $F_{1-3}$, is the same as from three to one and is given below.

\[
F_{1-3} = \frac{1}{2} \left[ X - \left( X^2 - 4 \right)^{1/2} \right] \tag{3.6}
\]

with

\[
X = \frac{2R^2 + 1}{R^2} \quad \text{and} \quad R = \frac{D_c}{2L_c} \tag{3.7}
\]

\[
F_{1-3} = F_{3-1} = 1 + 2 \left( \frac{L_c}{D_c} \right)^2 - 2 \left\{ 4 \left( \frac{L_c}{D_c} \right)^4 \left[ \frac{1}{2} \left( \frac{D_c}{L_c} \right)^2 + 1 \right] - 1 \right\}^{1/2} \tag{3.8}
\]

Since $F_{1-1} = F_{3-3} = 0$, by summation, $F_{1-2} = 1 - F_{1-3}$ and $F_{3-2} = 1 - F_{3-1}$. Then, by reciprocity, $F_{2-3} = \frac{A_2}{A_3}F_{3-2}$ and $F_{2-1} = \frac{A_2}{A_3}F_{1-2}$. Finally, by summation,

\[
F_{2-2} = 1 - F_{2-1} - F_{2-3}. \]

Using these view factors and a length-to-diameter ratio of 15 for the platinum cavity gives the following view factor matrix

\[
F_{L_c/D_c=15} = \begin{bmatrix}
0 & 0.999 & 0.001 \\
0.017 & 0.967 & 0.017 \\
0.001 & 0.999 & 0
\end{bmatrix} \tag{3.9}
\]
Approximating the cavity as a diffuse gray enclosure, the radiosity of each surface may be calculated by the following (Incropera and DeWitt, 1996)

\[ J_i = \varepsilon_i E_{bi} + \left(1 - \varepsilon_i\right) \sum_{j=1}^{N} J_j F_{ij} \quad (3.10) \]

The surface temperatures of each of the surfaces are as given above, and the emissivity of platinum is 0.15 (Incropera and DeWitt). With the known radiosities, the radiosity exiting the cavity can be found by calculating the irradiance incident on surface three

\[ G_3 = J_1 F_{3-1} + J_2 F_{3-2} \quad (3.11) \]

The effective emissivity can then be found by taking the ratio of Eq. (3.11) to the blackbody emissivity of a surface with the temperature of surfaces one and two. This yields and effective emissivity for the cavity of 0.92 ± 0.03. This is a conservative estimate, and again used only to allow calculation of the spectral calibration coefficient—its inaccuracy is accounted for in the calibration.

Finally, to calculate the calibration coefficient it is necessary to know the spectral optical depth. The spectral optical depth is equal to the product of the spectral absorption coefficient and the light path length. The light path length can be characterized using the average path length for the fiber, but determining the spectral absorption coefficient is more problematic. Data on the absorption coefficient of sapphire available in the literature as a function of temperature are inconsistent. Figure 3.4 shows the data available for the spectral absorption coefficient at an unspecified temperature (Brewster, 1992), 1200ºC (Gryvnak and Burch, 1965) and 1000ºC (Oppenheimer and Even, 1962). Brewster actually tabulated the imaginary part of the refractive index for sapphire, so the
absorption coefficients shown in Fig. 3.4 are calculated using this data and Eq. (3.12), where the units for $K_{a\lambda}$ are $\text{mm}^{-1}$ when $\lambda$ is given in $\mu\text{m}$.

$$K_{a\lambda} = \frac{4000\pi k}{\lambda}$$  \hspace{1cm} (3.12)

![Figure 3.4 Absorption coefficient for sapphire.](image)

Due to the uncertainty regarding the variation in the spectral absorption coefficient with temperature, it was necessary to perform an in-situ calibration similar to the procedure used to calibrate blackbody OFT for the two-color approach (Dils, 1983). The temperature profile measured with the furnace set at 1344 K was taken as the calibration temperature profile. The spectral calibration coefficients were calculated using Eq. (3.5), the estimated spectral emissivity value of the cavity, and the spectral absorption coefficients given by Brewster (1992). This approach neglects the dependence of the spectral absorption coefficient on temperature, so the resulting calibration coefficients are only valid at temperatures close to the calibration temperature profile. This restriction
could be eliminated if the dependence of the spectral absorption coefficient on temperature were known.

With the light path characterized, it is now possible to perform an uncertainty analysis on the predicted or measured signal.

3.4 Uncertainty Analysis

Since the error in the tip-temperature is a function of the error in the measured signal, normally an uncertainty analysis would be performed on the tip temperature as a function of the other parameters in Eq (3.4). However, no closed form solution is available for the temperature profile along the optical fiber—hence the necessity for solving it by an inverse minimization algorithm. Since a closed form solution is not available, partial derivatives cannot be calculated and an uncertainty analysis cannot be performed. The uncertainty in the tip-temperature will be ascertained from the different reconstructions that are performed.

It is possible, however to perform an uncertainty analysis on the signals predicted by Eq (3.4). This is necessary because the convergence criteria of the minimization algorithms depend on the degree of uncertainty in the predicted signals. Also, the more uncertainty in the predicted signals, the more error will exist in the reconstructed tip-temperature and temperature profile. Finally, an uncertainty analysis on the predicted signals is necessary to make simulated signal measurements and reconstructions possible.

The uncertainty of the signals predicted by Eq. (3.4) are a function of the following parameters

- Isothermal cavity emissivity, $\varepsilon_{\lambda}$
- Fiber tip temperature, $T_o$
• Fiber temperature profile, $T(t_\lambda)$

• Optical depth, $t_\lambda$

• Spectral calibration coefficient, $\kappa_\lambda$

The first step in calculating the uncertainty in the SME is to take the partial derivative of the signal, $\psi$ (Eq. (3.4)), with respect to each source of uncertainty. This is done analytically and the resulting partial derivatives are shown here.

$$\frac{\partial \psi_\lambda}{\partial \epsilon_\lambda} = \kappa_\lambda \exp \left\{ -t_{\lambda S} - \frac{C_2}{\lambda T_o} \right\}$$

(3.13)

$$\frac{\partial \psi_\lambda}{\partial T_o} = -\kappa_\lambda \epsilon_\lambda C_2 \frac{t_{\lambda S}}{\lambda T_o^2} \exp \left\{ -t_{\lambda S} - \frac{C_2}{\lambda T_o} \right\}$$

(3.14)

$$\frac{\partial \psi_\lambda}{\partial T(t_\lambda)} = -\frac{C_2 \kappa_\lambda}{\lambda} \int_0^{t_{iS}} \frac{\exp \left\{ t_\lambda - t_{\lambda S} - \frac{C_2}{\lambda T(t_\lambda)} \right\}}{T(t_\lambda)^2} \, dt_\lambda$$

(3.15)

$$\frac{\partial \psi_\lambda}{\partial t_\lambda} = \kappa_\lambda \left[ \epsilon_\lambda \exp \left\{ -\frac{C_2}{\lambda T_o} - t_{\lambda S} \right\} \exp \left\{ -\frac{C_2}{\lambda T(t_\lambda)} \right\} \right]$$

$$+ \int_0^{t_{iS}} \exp \left\{ t_\lambda - t_{\lambda S} - \frac{C_2}{\lambda T(t_\lambda)} \right\} \, dt_\lambda$$

(3.16)

$$\frac{\partial \psi_\lambda}{\partial \kappa_\lambda} = \epsilon_\lambda \exp \left\{ -t_{\lambda S} - \frac{C_2}{\lambda T_o} \right\} + \int_0^{t_{iS}} \exp \left\{ t_\lambda - t_{\lambda S} - \frac{C_2}{\lambda T(t_\lambda)} \right\} \, dt_\lambda$$

(3.17)

The partial derivatives are then multiplied by the uncertainty of each source of uncertainty. The squares of these products are then summed and the square root of the sum gives the error in the predicted signals. This is shown by Eq. (3.18)

$$u_{\psi, p} (\lambda) = \left[ \sum_{i=1}^{N} \left( \frac{\partial \psi_\lambda}{\partial x_i (\lambda)} \right)^2 u^2 (x_i (\lambda)) \right]^{1/2}$$

(3.18)
where the $x_i$ represent each of the unknown parameters and $u$ represents the uncertainty associated with each unknown parameter.

The following is a list of the estimated uncertainty in each of the unknown parameters

- $u_{x_i} = 0.03$ Calculated using a method similar to the one used to calculate the uncertainty in the predicted signals, but using the Eqs. (3.6)-(3.11) for the isothermal cavity emissivity
- $u_{T_o} = 0.5 K$ From steady-state thermocouple readings
- $u_{T_{tuK}} = 0.5 K$ From steady-state thermocouple readings
- $u_{t_{ij}}$ varies with wavelength and is discussed below
- $u_{K_{ij}}$ varies with wavelength. Calculated by dividing the standard deviation of the measurements obtained during the calibration procedure at each wavelength by the square-root of the number of measurements at each wavelength

The uncertainty in the optical depth is calculated by the following expression

$$u_{t_{ij}} = \left[ \left( \frac{\partial t_{ij}}{\partial K_{ij}} u_{K_{ij}} \right)^2 + \left( \frac{\partial t_{ij}}{\partial s} u_s \right)^2 \right]^{1/2}$$  \hspace{1cm} (3.19)

The uncertainty in the spectral absorption coefficient as given in Gryvnak and Burch is 30%, and the uncertainty in the length of the light path at the end of the heated portion of the fiber is 0.175 m.
Figure 5 shows the normalized product of the partial derivative and the uncertainty squared for each of the sources of uncertainty at several wavelengths.

Figure 3.5 Normalized uncertainty parameters at several wavelengths.

This uncertainty analysis provides upper and lower bounds for the signals predicted by Eq. (3.4). These bounds will be used in simulating signal measurements and simulating temperature profile reconstructions using the minimization algorithms. With these simulations, it will be possible to ascertain the error in the tip-temperature measurement as a function of the error in the measured signals.
CHAPTER 4 INVERSE OPTIMIZATION TECHNIQUES

This chapter will describe two possible methods for inverting Eq. (3.4). Since Eq. (3.4) is non-linear and ill-posed, certain difficulties are present in inverting the equation. The following techniques arrive at accurate solutions despite the difficulties. The development of these techniques fulfill the first part of the objective of this research.

4.1 Genetic Algorithm

Genetic algorithms are based on the principle of natural selection or survival of the fittest. The structure of a typical genetic algorithm is shown in Fig. 4.1 (Goldberg, 1989; Pham and Karaboga, 2000).

![Figure 4.1 Simple flowchart for a genetic algorithm](image-url)
The following paragraphs briefly describe the aspects of the implementation unique to this study (Jones and Barker, 2002). Detailed discussions of the fundamentals of genetic algorithms and descriptions of the wide variety of optimization problems successfully treated using genetic algorithms are available in the literature (Goldberg, 1989; Jones et. al., 1996; Pham and Karaboga, 2000).

4.1.1 Selection of an Initial Population

An initial population of 100 possible temperature profiles was created by randomly perturbing an initial estimate of the temperature profile along the optical fiber. The initial estimate consisted of 147 temperature points equally spaced along the OFT.

4.1.2 The Fitness Function

The error for the jth possible temperature profile in the population is the rms value of difference between the measurements and the value of \( G_{\lambda} \) obtained from Eq. (3.4) using the jth temperature profile.

\[
e_j = \frac{1}{N} \sum_{i=1}^{N} \sqrt{\left( \frac{G_{\lambda}^{\text{measured}} - G_{\lambda}^{j}}{G_{\lambda}^{\text{measured}}} \right)^2}
\]  

(4.1)

The smoothness of each temperature profile was quantified using the second finite differences.

\[
s_j = \frac{2}{N_p - 2} \sum_{k=2}^{N-1} \sqrt{\frac{(T(z_{k+1}) - T(z_k))(z_k - z_{k-1}) - (T(z_k) - T(z_{k-1}))(z_{k+1} - z_k)}{(z_{k+1} - z_{k-1})(z_k - z_{k-1})}}
\]  

(4.2)

The fitness of the jth possible temperature profile was defined as

\[
f_j = \frac{e_{\text{max}} - e_j}{e_{\text{avg}}} + \frac{s_{\text{max}} - s_j}{s_{\text{avg}}}
\]  

(4.3)
where $e_{\text{max}}$ and $e_{\text{avg}}$ are the maximum and average values for the current generation and $s_{\text{max}}$ and $s_{\text{avg}}$ are the maximum and average values of the smoothness criterion for the current generation.

4.1.3 Creation of a New Population

The possible temperature profile with the greatest fitness value and the possible temperature profile with the minimum error from the previous generation were copied directly into the new generation. The next twenty possible temperature profiles in the new population were created by repeatedly applying a smoothing operator to the possible temperature profile with the minimum error. Ten of the twenty new possible temperature profiles were created using the average of the temperature at a point with its two nearest neighbors. The next ten new possible temperature profiles were created using the average of the temperature at a point with its four nearest neighbors. The remaining 78 possible temperature profiles in the new generation were created using the crossover and mutation operators (Goldberg, 1989; Jones et. al., 1996; Pham and Karaboga, 2000). The crossover operation consists of selecting two possible temperature profiles according to their fitness values. A crossover site for each point in the temperature profile was randomly selected, and the portion of the temperature profile above the crossover site in the first possible profile was combined with the remaining portion of the second possible profile to create a new possible temperature profile. The mutation operator was applied after the new generation had been filled. The mutation operator consists of randomly selecting 1% of all the temperature values and then randomly perturbing these values within a range of $\pm 400$ K.
The process of selection, crossover and mutation continued until the following criteria were met. First, the profile with the lowest rms error for the current generation had to reach a specified minimum at the same time the average of the rms errors for the entire population reached a specified value. If these two criteria were satisfied and the maximum actual error of the radiant flux compared to the measured flux was less than a specified tolerance, the algorithm would terminate and yield the most-fit solution. The algorithm was also terminated if significant improvement was not made over a specified number of generations. This was determined by the variation of the rms average error; if it did not change more than a specified amount over twelve generations, the algorithm was terminated and the results returned.

4.2 Conjugate Gradient Algorithm

Another method for inverting Eq. (3.4) is to assume the temperature profile can be parameterized as

$$T(z) = \sum_{j=1}^{N} P_j C_j (z)$$

(4.4)

where $P_j$ are the unknown parameters and $C_j$ are trial functions. The trial functions may be any type of function: polynomial, sinusoidal, exponential or a combination of these functions. If $a priori$ knowledge of the temperature profile exists, it is most efficient to choose trial functions that will accurately model the temperature profile with the fewest number of parameters. In this study, fourth-order polynomials will be used to reconstruct the temperature profiles, and the parameters will be the coefficients of the polynomials.
A conjugate gradient algorithm (CGA) will be used to solve for the vector of parameters that characterize the unknown temperature profile. The implementation of the CGA used in this study is based on the algorithm developed by Özişik and Orlande (2000). More detailed developments of the CGA are also available in the literature (Polak, 1971; Stoer and Burlisch, 1980).

The function to be minimized, the objective function, is the least squares difference between the actual and predicted signal measurement based on the current estimate of the temperature profile.

\[
S = \sum_{i=1}^{\lambda} (\psi_{\lambda,\text{actual}} - \psi_{\lambda,\text{predicted}}(P_i))^2
\]  \hspace{1cm} (4.5)

This can also be expressed in matrix form

\[
S(P) = (\psi_{a\lambda} - \psi_{p\lambda}(P))^T (\psi_{a\lambda} - \psi_{p\lambda}(P))
\]  \hspace{1cm} (4.6)

The minimum of this function is found by calculating the gradient and setting it equal to zero. The gradient is found by taking the derivative of the least squares norm with respect to each parameter

\[
\frac{\partial S(P)}{\partial P_1} = \frac{\partial S(P)}{\partial P_2} = \cdots = \frac{\partial S(P)}{\partial P_N} = 0
\]  \hspace{1cm} (4.7)

where \(N\) is the number of unknown parameters. Eq. (4.7) can also be expressed in matrix form

\[
\nabla S(P) = \frac{\partial S(P)}{\partial P} = -2 \left[ \frac{\partial \psi_{p\lambda}^T(P)}{\partial P} \right] [\psi_{a\lambda} - \psi_{p\lambda}(P)] = 0
\]  \hspace{1cm} (4.8)

An important concept in parameter estimation problems is that of the sensitivity coefficient. The sensitivity coefficient can be defined as the sensitivity of the measured
quantity, such as spectral intensity, to a change in the parameter $P_i$. This definition of sensitivity coefficients can be seen in Eq. (4.8) and is defined by Eq. (4.9).

$$\frac{\partial \psi_{p_{\lambda}}}{\partial P} = \left[ \frac{\partial}{\partial P_1} \psi_{p_{\lambda_1}} \psi_{p_{\lambda_2}} \cdots \psi_{p_{\lambda_N}} \right]$$ (4.9)

The sensitivity matrix, also known as the Jacobian matrix, is defined as the transpose of Eq. (4.9) and is shown below

$$J(P) = \left[ \frac{\partial \psi_{p_{\lambda}}}{\partial P} \right]^T = \begin{bmatrix}
\frac{\partial \psi_{p_{\lambda_1}}}{\partial P_1} & \frac{\partial \psi_{p_{\lambda_1}}}{\partial P_2} & \cdots & \frac{\partial \psi_{p_{\lambda}}}{\partial P_N} \\
\frac{\partial \psi_{p_{\lambda_2}}}{\partial P_1} & \frac{\partial \psi_{p_{\lambda_2}}}{\partial P_2} & \cdots & \frac{\partial \psi_{p_{\lambda_2}}}{\partial P_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \psi_{p_{\lambda_N}}}{\partial P_1} & \frac{\partial \psi_{p_{\lambda_N}}}{\partial P_2} & \cdots & \frac{\partial \psi_{p_{\lambda_N}}}{\partial P_N}
\end{bmatrix}$$ (4.10)

Rewriting Eq. (4.8) with this definition yields

$$\nabla S(P) = -2J^T(P)\left[ \psi_{a_{\lambda}} - \psi_{p_{\lambda}}(P) \right]$$ (4.11)

This becomes the function that is to be minimized for the vector of unknown parameters.

Sensitivity coefficients are important because they determine how sensitive the measured quantity is to changes in the unknown parameters. It is desirable for sensitivity coefficients to be large so that small changes in the parameters result in large changes in the measurements. If the sensitivity coefficients are small, the problem is considered ill-conditioned (Özişik and Orlande, 2000), and a wide range of parameter values will yield
nearly the same measurement. For ill-conditioned problems, measurement errors have a much more significant effect than for problems that have large sensitivity coefficients.

Several methods for computing the sensitivity matrix exist, and in this study, they will be calculated using the forward difference approximation (Özişik and Orlande, 2000) given by Eq. (4.12)

\[ J_{ij} = \frac{\psi(p_i, p_2, \ldots, p_j + \epsilon p_j, \ldots, p_N) - \psi(p_i, p_2, \ldots, p_j, \ldots, p_N)}{\epsilon p_j} \]  

(4.12)

where \( \epsilon \) is an arbitrary perturbation on the order of \( 10^{-5} \) or \( 10^{-6} \).

In non-linear optimization problems such as the current problem, an iterative approach must be used. The iterative approach (Özişik and Orlande, 2000) to this method updates the vector of unknown parameters using the following expression

\[ P^{k+1} = P^k - \beta_k d^k \]  

(4.13)

The calculation of \( \beta_k \) follows. Eq. (4.14) shows the calculation of \( d \) as a conjugation of the least squares gradient direction (Eq. (4.8)) and the previous direction of descent.

\[ d^k = \nabla S(P^k) + \gamma^k d^{k-1} \]  

(4.14)

where \( \gamma^k \) is an estimate of the change in gradient, or second derivative, from one iteration to the next.

It should be noted that when \( \gamma^k = 0 \), the conjugate gradient method becomes the method of steepest descent with the direction of descent equal to the negative gradient direction. The method of steepest descent, while the simplest approach, is not the most efficient. It requires that each new direction of descent be perpendicular to the previous direction of descent, and may require many iterations to find the minimum. Figure 4.2
shows a visualization of the steepest descent method versus the conjugate gradient method (Press et al., 1999).

The direction specified by Eq. (4.14) is a combination of steepest descent and the previous direction of descent. This is a more efficient minimization routine, allowing minimization in fewer iterations (Özişik and Orlande, 2000), than the steepest descent method.

\[
\gamma^k = \frac{\sum_{j=1}^{N} \left[\nabla S(P^k)\right]_j \left[\nabla S(P^k) - \nabla S(P^{k-1})\right]_j}{\sum_{j=1}^{N} \left[\nabla S(P^k)\right]_j} \quad \text{and} \quad \gamma^0 = 0 \quad (4.15)
\]

Several methods are available for determining the conjugation coefficient, \(\gamma\), but according to (Özişik and Orlande, 2000; Press et al., 1999), the Polak-Ribiere expression (Eq. (4.15)) provides the quickest convergence for non-linear estimation problems.

Figure 4.2 Visual comparison of the steepest descent method and the conjugate gradient algorithm. The steepest descent requires a new search direction that is exactly perpendicular to the previous search direction, while the conjugate gradient moves more efficiently.
With the sensitivity coefficients (Eq. (4.12)) and the gradient direction (Eq. (4.8)) known, the conjugation coefficient can be calculated and then the direction of descent.

The search step-size can be calculated by minimizing \( S(P^{k+1}) \) with respect to \( \beta \).

\[
\min_{\beta} S(P^{k+1}) = \min_{\beta} \left[ \psi_{a\lambda} - \psi_{p\lambda}(P^{k+1}) \right]^T \left[ \psi_{a\lambda} - \psi_{p\lambda}(P^{k+1}) \right] \tag{4.16}
\]

substituting from Eq.(4.13),

\[
\min_{\beta} S(P^{k+1}) = \min_{\beta} \left[ \psi_{a\lambda} - \psi_{p\lambda}(P^k - \beta^k d^k) \right]^T \left[ \psi_{a\lambda} - \psi_{p\lambda}(P^k - \beta^k d^k) \right] \tag{4.17}
\]

The \( \psi_{p\lambda}(P^k - \beta^k d^k) \) are linearized using a Taylor series expansion (Özisik and Orlande, 2000) and then the minimization is performed, yielding the following for \( \beta^k \) in matrix form

\[
\beta^k = \frac{\left[ J^k d^k \right]^T \left[ \psi_{p\lambda}(P^k) - \psi_{a\lambda} \right]}{\left[ J^k d^k \right]^T \left[ J^k d^k \right]} \tag{4.18}
\]

The iterations continue until one of two stopping criteria have been met. The first criterion is the discrepancy principle. The discrepancy principle is based on the premise that the predicted measurements cannot have lower least squares error than the magnitude of the standard deviation of the actual measurement error. The actual measurement error is quantified by the variance of sequential measurements at a given wavelength and temperature. This is shown by Eq. (4.19)

\[
S(P^{k+1}) < \sum_{i=1}^{I} \sigma_i^2 \tag{4.19}
\]

The second criterion is met if a certain number of iterations have occurred without a significant change in the least squares error. If this occurs, the solution with the least error to that point is taken as the solution. In practice, the second stopping criterion is
only used when the standard deviation of the measurement errors is assumed to be very small \( \sigma \ll 1 \). In cases when the standard deviation is larger, the discrepancy principle is satisfied before the second stopping criterion.

In summary, the computational algorithm (Özişik and Orlande, 2000) proceeds as follows.

1. Set \( k = 0 \) and provide an initial guess \( \mathbf{P}^0 \)
2. Solve the direct problem (Eq. Signal measurement equation) using the current estimate of \( \mathbf{P}^k \) and obtain the predicted measured emissions \( \psi_{\tilde{P}} \left( \mathbf{P}^k \right) = \left[ \psi_{\tilde{P}_1} \; \psi_{\tilde{P}_2} \; \cdots \; \psi_{\tilde{P}_\lambda} \right] \).
3. Check whether the stopping criteria is satisfied; stop if satisfied
4. Compute the sensitivity matrix \( \mathbf{J}^k \) as defined by Eq. (4.12)
5. Compute the gradient direction \( \nabla \mathbf{S} \left( \mathbf{P}^k \right) \) given by Eq. (4.11) and then the conjugation coefficient \( \gamma^k \) given by Eq. (4.15)
6. Compute the direction of descent \( \mathbf{d}^k \) using Eq. (4.14)
7. Compute the search step size \( \beta^k \) using Eq. (4.18)
8. Using \( \beta^k \) and \( \mathbf{d}^k \) compute the new estimate \( \mathbf{P}^{k+1} \) using Eq. (4.13)
9. Replace \( \mathbf{P}^k \) with \( \mathbf{P}^{k+1} \) and repeat from step 2 until convergence is obtained
This chapter shows results which fulfill the second part of the objective for this thesis—to accurately infer the tip-temperature of the optical fiber using simulated measurements and the optimization algorithms. Estimates of the temperature profiles are also shown.

5.1 Reconstruction Using the Genetic Algorithm

The genetic algorithm was the initial approach to reconstructing the temperature profile along the OFT since it was suspected that standard inverse techniques such as a conjugate gradient algorithm would be likely to become trapped in local minima and fail to converge. The genetic algorithm was very effective at reconstructing an estimate of the temperature profile, but the computation time was extreme. Since derivatives can be calculated from the signal measurement equation, derivative-based methods are more appropriate. The results presented below (Jones and Barker, 2002) show that the genetic algorithm is indeed robust, but the computational time makes it impractical. It should be noted that no simulated error was introduced for the following results.

Three typical axial temperature profiles for the microgravity furnace illustrated in Fig. 5.1 were obtained from a detailed thermal model (NASA-MSFC, 1999) and used for simulated temperature reconstructions.
Simulated measurements of the dimensionless emission were calculated using these temperature profiles and Eq. (2.7) for a number of wavelengths. For these simulated measurements, as well as those for the conjugate gradient algorithm, the absorption coefficient is calculated using the data given in Brewster (1992).

The reconstructed temperature profiles obtained using the genetic algorithm are compared with the actual temperature profiles in Fig. 5.2. The initial estimates for the population used to obtain these results are also shown in these figures. Table 2 lists the average and maximum deviations for the high and low temperature regions of the reconstructed temperature profiles for each case.
Figure 5.2  Reconstructed temperature for three different temperature profiles using the genetic algorithm. (a) Case 1, (b) Case 2, (c) Case 3
In order to assess the effect of the initial guess on the results, Case 1 was run using the initial temperature profile shown in Fig. 12. After 466 generations, the tip temperature had 0.14% error, and the entire profile had an average of 13.4% error.

Figure 5.3  Temperature reconstruction using the genetic algorithm with a poor initial guess.

Comparison of these results with the results given in Fig. 5.8 and Table 5.1, shows that a poor initial guess greatly increases the time required to find a solution. However, this case and the results of other cases not presented indicate that the algorithm

| Case | High Temperature Region | | Low Temperature Region | | |
|------|-------------------------|-------------------------|-------------------------|-------------------------|
| | Average Temperature Error (%) | Maximum Temperature Error (%) | Average Temperature Error (%) | Maximum Temperature Error (%) |
| | | | | |
| 1 | 1.4 | 3.7 | 2.4 | 8.2 |
| 2 | 1.5 | 6.9 | 5.1 | 9.4 |
| 3 | 3.5 | 11.1 | 15.0 | 35.5 |
will eventually find a reasonably accurate solution independent of the choice made for the initial temperature profile.

It can be seen that lower temperature profiles produce less accurate reconstructions. This is because emission by the fiber is a function of temperature. If the temperature profile is low, the integral in Eq. (3.4) becomes negligible (as in the assumptions for the two color method). This makes the emission due to the actual temperature profile insignificant compared to the emission from the tip. It can be seen in Figs. 5.2 and 5.3 that even though the reconstructions were not completely accurate, the tip-temperatures were.

5.2 Furnace Temperature Profile Characterization

The following describes the temperature profiles used for the conjugate gradient algorithm and for comparison between the CGA and the genetic algorithm.

The temperature profile in a laboratory furnace was characterized as shown in Fig. 3.2. For the simulated temperature profile reconstructions, the furnace was set at 1366K and the temperature points were measured using the multipoint thermocouple. In order to simulate spectral signal measurements, it was necessary to have a continuous profile along the OFT. Figure 5.4 shows this approximated temperature profile along with the measured points.

This approximated temperature profile was arrived at by curve-fitting a fourth-order polynomial to the five temperature points and a sixth point where the fiber temperature had approximately reached room temperature. The shape of this profile is not significant for the simulated measurements since they are just simulations. Any temperature profile could have been assumed for the simulated reconstructions, but it was
desirable to have a physically-based and realistic temperature profile. Results will also be shown using two additional profiles (shown in Fig. 5.2) to demonstrate the capabilities of the algorithms.

Figure 5.4  Measured temperature points and approximated temperature profile at with the furnace at 1366K.

Figure 5.5  Simulated temperature profiles for use with the reconstruction algorithms.
The temperature profile shown in Fig. 5.4 was also used to simulate reconstructions using the genetic algorithm. This is the temperature profile that was used to compare the genetic algorithm and the conjugate gradient algorithm and to develop the hybrid method. The following results are from (Barker and Jones, 2002). Figure 5.6 shows the simulated temperature reconstruction and Table 5.2 shows the statistics for the run.

![Graph](image)

**Figure 5.6** Simulated temperature profile reconstruction for the furnace temperature profile using the genetic algorithm.

It can be seen that the computation time is large, and that the temperature profile exhibits oscillations that are not physically realistic for the given system. This is due to the nature of the temperature profile—it consists of 147 individual temperature points. The genetic algorithm randomly reproduces temperature profiles to fill new generations and the smoothest ones are chosen, but this does not prevent the oscillations that are seen. For these two reasons, the computation time and the characterization of the temperature profile, the conjugate gradient algorithm was developed. The following section shows
how simulated measurement error was introduced to the simulated signal measurements, and the section after that shows reconstructions using the CGA.

<table>
<thead>
<tr>
<th>Table 5.2 Convergence statistics for the genetic algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Iterations</td>
</tr>
<tr>
<td>Actual Iterations</td>
</tr>
<tr>
<td>σ (%)</td>
</tr>
<tr>
<td>Computation Time (s)</td>
</tr>
<tr>
<td>Tip Temperature Error (%)</td>
</tr>
<tr>
<td>RMS Temperature Error (%)</td>
</tr>
</tbody>
</table>

5.3 Simulated Error

Error was introduced to the predicted signal by considering the confidence level on the uncertainty. The confidence level describes the percentage of measurements that fall within the calculated uncertainty. For example, if the confidence level on the uncertainty is 68.3% (see Fig. 5.7), 68.3% of the measurements are within the predicted uncertainty and 31.7% are greater.

If the confidence level is 99.7%, then 99.7% and 0.3% of the measurements are respectively within and greater than the calculated uncertainty. In this study, three confidence levels were used: 86.6%, 95.4% and 99.7%.

Confidence levels may also be expressed in the number of standard deviations a measurement lies from the mean. For example, if the predicted uncertainty in a measurement is ±1%, and the confidence level for the predicted uncertainty is 95.4%, then 1% measurement error occurs at exactly two standard deviations above and below the actual value (see Fig. 5.8).
Figure 5.7  For a 68.3% confidence level for the uncertainty, 31.7% of the time the uncertainty would be greater than the maximum predicted uncertainty.

Figure 5.8  With a confidence level of 95.4% on the uncertainty and the maximum uncertainty being ±1%, the standard deviation for the distribution is 0.5%.
This means that $2\sigma = 1\%$ and $\sigma = 1\%/2 = 0.5\%$ for this particular distribution. If the confidence interval were 99.7%, three standard deviations would separate the maximum predicted uncertainty from the mean with $3\sigma = 1\%$ and $\sigma = 1\%/3 = 0.33\%$. Figure 5.9 shows that a higher confidence level generates a distribution that is more tightly centered around the mean. Error is introduced to the measurements based on this discussion.

![Figure 5.9](image-url)  
**Figure 5.9** A higher confidence level generates a distribution that is more tightly centered about the mean.

To simulate measurement error, a normally distributed random number, $\omega$, with a zero mean is generated and then multiplied by the maximum uncertainty ($\omega \cdot 1\%$ for the situation discussed above). This product, which specifies the measurement error for a particular wavelength, is then added to the actual calculated measurement and the new value is taken as the spectral measurement. The standard deviation used to calculate the random number is calculated as discussed in the previous paragraph. For example, if the
maximum predicted uncertainty is ±3% and the confidence level on the uncertainty is 95.4% (±2σ from the mean), then 2σ = 3% and σ = 3%/2 = 1.5%. This value is then normalized by the maximum predicted uncertainty, making the standard deviation used to calculate ω equal to 1.5%/3.0% = 0.5 (see Fig. 5.10).

![Normalized Uncertainty](image)

**Figure 5.10** For a predicted uncertainty of 3% with a 95.4% confidence level, the standard deviation for the distribution is 1.5% (σ = 0.5 normalized).

This makes the maximum predicted uncertainty occur when the normalized uncertainty equals unity, and 95.4% (the desired confidence level) of the time the calculated error will be less than or equal to this maximum. In this way the confidence level determines how narrowly the simulated measurements are distributed around the actual spectral measurements, resulting in spectral measurements with normally distributed error centered about the predicted signal with the same confidence level as the uncertainty—the higher the confidence level, the lower the overall error.
Another way to introduce error into the signal is by considering the strength of the signal compared to the noise level of the detector. The detector itself has a certain standard deviation associated with the measurements it is capable of making. The closer the measured signal is to this noise level, the more significant effect the detector variance will have on the signal. This is important when considering the error for two reasons: the measurement uncertainty determines how closely the measurements are grouped about the actual spectral distribution, and this measurement standard deviation is the standard deviation used by the discrepancy principle shown in Eq. (4.19).

If the measured signal is close to the noise of the signal, the stopping criterion will be large relative to the actual error of the signal causing the CGA to terminate prematurely—most likely before any significant progress has been made. If the signal is strong (several orders of magnitude) compared to the noise, the stopping criterion will be small and the algorithm will converge to an accurate solution.

This measurement standard deviation can be simulated considering the relationship between the signal and the fiber geometry. The signal is directly proportional to the area of the fiber

\[ \psi \propto d^2 \]

(5.1)

The signal strength increases as the square of the fiber diameter. For a small change in diameter, the signal increase will be substantial and the difference in the measured signal and the noise of the detector will increase likewise. To simulate this signal-to-noise ratio, either the simulated signal may be increased or the standard deviation of the detector noise may be decreased. The latter was done in the simulated reconstructions.
Observing the detector used in the actual experiments, it was seen that the standard deviation of the detector readings was approximately 0.003 nA regardless of wavelength or temperature. To simulate a stronger signal, this standard deviation was divided by the square of a simulated diameter increase. In this manner, the combined uncertainty in the signal could be varied by using both the confidence level and the signal-to-noise ratio. More will be discussed about this in a following section.

5.4 Reconstruction Using the Conjugate Gradient Algorithm (CGA)

Initially, the trial functions, as discussed in Chapter 4, for the CGA were cubic splines. The number of parameters used in the c-spline were specified and the predicted temperature profile then consisted of a cubic interpolation between these points. This provided a much smoother curve than the temperature profile predicted by the genetic algorithm and with many fewer parameters. Figure 5.11 shows a temperature profile reconstruction using the CGA with the same temperature profile and initial guess as given in Fig. 5.6. Table 5.3 shows the statistics for this run.

<table>
<thead>
<tr>
<th>Table 5.3. Convergence statistics for the gradient-based method.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Parameters</td>
</tr>
<tr>
<td>Maximum Iterations</td>
</tr>
<tr>
<td>$\sigma$ (%)</td>
</tr>
<tr>
<td>Actual Iterations</td>
</tr>
<tr>
<td>Computation Time (s)</td>
</tr>
<tr>
<td>Tip Temperature Error (%)</td>
</tr>
<tr>
<td>RMS Temperature Error (%)</td>
</tr>
<tr>
<td>RMS Emission Error (%)</td>
</tr>
</tbody>
</table>
Figure 5.11  Temperature profile reconstruction with the CGA using the same initial guess and actual temperature profile as for the genetic algorithm in Fig. 5.6.

The problems of computational time and smoothness were resolved with the conjugate gradient algorithm, but one limitation of the CGA was the initial guess. The genetic algorithm could eventually converge with any initial guess, but the computation time increased drastically depending on how good the initial guess was.

Figure 5.12  Converged temperature profile with a flat initial guess at 600 K.
Figure 5.12 shows the lowest flat initial guess for which the conjugate gradient algorithm converged. The genetic algorithm converged for temperature profiles with a lower initial guess, but the computation time was extreme.

5.4.1 Hybrid Method

In order to make the CGA more robust, a hybrid algorithm was developed that would use the genetic algorithm to narrow down the search space. It was found that the genetic algorithm would reach a better approximation of the tip temperature soon in the iterations. This new tip-temperature was used to create a straight-line initial guess from the tip temperature down to room temperature. The CGA used this initial guess to converge the temperature profile. Figure 5.13 and Table 5.4 show the results of the hybrid algorithm.

![Figure 5.13](image-url)  
**Figure 5.13** Temperature profile reconstruction using the hybrid algorithm.
Table 5.4 Convergence Statistics for the hybrid algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generations for Genetic Algorithm</td>
<td>572</td>
</tr>
<tr>
<td>$\sigma$ (%)</td>
<td>5</td>
</tr>
<tr>
<td>Computation Time (s)</td>
<td>12669</td>
</tr>
<tr>
<td>Tip Temperature Error (%)</td>
<td>0.7</td>
</tr>
<tr>
<td>Parameters for Gradient-Based Method</td>
<td>5</td>
</tr>
<tr>
<td>Maximum Iterations</td>
<td>2000</td>
</tr>
<tr>
<td>Actual Iterations</td>
<td>17</td>
</tr>
<tr>
<td>Computation Time (s)</td>
<td>63</td>
</tr>
<tr>
<td>Tip Temperature Error (%)</td>
<td>0.52</td>
</tr>
<tr>
<td>RMS Temperature Error (%)</td>
<td>8.1</td>
</tr>
<tr>
<td>Total Convergence Time (s)</td>
<td>12732</td>
</tr>
</tbody>
</table>

5.4.2 Improvements to the CGA

With development of the CGA resulting in much faster computational times, it was easier to explore the strengths and weaknesses of the algorithm. The improvements explored dealt with different possible objective functions.

5.4.2.1 Fourth-order polynomial trial function

A fourth-order polynomial trial function was explored to reduce the number of parameters the CGA would use. Reconstructions are shown below for the three different temperature profiles given in Fig. 5.2 (Barker and Jones 2003a). These results are shown for confidence levels from 86.6% to 99.7% in Figs. 5.14-5.16 and Table 5.5.

Computation time for each of these runs was an average of 100 seconds. This is faster than the CGA took to converge in the hybrid method using the cubic spline trial function and a much better initial guess.
Figure 5.14  Reconstructed temperature profiles for profile A with confidence intervals on the uncertainty ranging from 86.6% to 99.7%.

Figure 5.15  Reconstructed temperature profiles for profile B with confidence intervals on the uncertainty ranging from 86.6% to 99.7%.
Figure 5.16 Reconstructed temperature profiles for profile C with confidence intervals on the uncertainty ranging from 86.6% to 99.7%.

Table 5.5 Results for three simulated temperature profiles using the CGA

<table>
<thead>
<tr>
<th>Profile</th>
<th>CI (%)</th>
<th>Tip Error (%)</th>
<th>High Temperature</th>
<th>Low Temperature</th>
<th>Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>95.5</td>
<td>0.02</td>
<td>1.41</td>
<td>7.5</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>99.7</td>
<td>0.57</td>
<td>1.41</td>
<td>8.75</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td>86.6</td>
<td>0.02</td>
<td>4.73</td>
<td>19.7</td>
<td>0.68</td>
</tr>
<tr>
<td>B</td>
<td>95.5</td>
<td>0.57</td>
<td>3.58</td>
<td>31.0</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>99.7</td>
<td>0.28</td>
<td>3.58</td>
<td>19.7</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>86.6</td>
<td>0.22</td>
<td>0.75</td>
<td>4.41</td>
<td>2.22</td>
</tr>
<tr>
<td>C</td>
<td>95.5</td>
<td>0.04</td>
<td>1.19</td>
<td>7.69</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>99.7</td>
<td>0.10</td>
<td>1.19</td>
<td>5.41</td>
<td>0.43</td>
</tr>
</tbody>
</table>

An eighth-order polynomial was also used to see if it would increase the accuracy. Figure 5.17 shows the results for profile A.

It can be seen that the higher-order polynomial did not increase the accuracy of the reconstruction obtained by the fourth-order trial function. Additionally, the
computation time was greater since more parameters were optimized in this case. Similar results were obtained for the other two profiles.

Figure 5.17  Simulated temperature profile reconstruction for profile A using an eighth-order polynomial for the trial function. A confidence interval for the simulated error of 99.7% was used.

5.4.2.2 Bezier curve trial function

Bezier curves are frequently used in shape optimization. Since the temperature profile reconstruction is somewhat of a shape optimization problem, a Bezier curve was used for the trial function for the CGA. A Bezier curve uses a pre-determined number of control points to define the shape of the curve. The curve is defined by the following

\[
P(t) = \sum_{i=0}^{n} \left[ \frac{n!}{i!(n-i)!}(1-t)^{n-i} t^i \right] P_i
\]

(5.2)

where \( t \) varies from 0 to 1 (the non-dimensional length between the control points), \( P(t) \) defines a continuous curve, the \( P_i \) are the control points and \( n \) is the number of control
points minus one. Several Bezier curves can be used together with the end control points being common. Using this definition with three Bezier curves, four control points for each curve and Eq. (3.4), the following result was obtained from the CGA.

![Graph showing actual versus reconstructed temperature profiles.](image)

**Figure 5.18** Reconstruction using three Bezier curves with four control points each as the trial function.

While the tip-temperature was reconstructed with 1.3% error, the rest of the temperature profile is flawed. Also, the reconstruction time using the more-complicated trial function resulted in a computation time of 1000 seconds—a tenfold increase over the polynomial trial function.

### 5.5 Sensitivity Coefficients

The Bezier curve trial function reconstruction brings up a critical aspect of optimizing Eq. (3.4) for the temperature profile. It is observed from each of the
reconstruction methods that the tip-temperature was always accurately reconstructed while the rest of the temperature profile left much to be desired in some instances. This is due to the magnitude of the sensitivity of the optimization parameters to changes in optical detector signal. For the genetic algorithm, the parameters are individual temperature points along the fiber. For the CGA the parameters are determined by the trial function—coefficients for the polynomial or control points for the Bezier curves. In the CGA, the sensitivity coefficients control the minimization as discussed previously. Figure 5.19 shows the sensitivity of the five coefficients in a fourth-order polynomial with respect to signal change.

Figure 5.19  Sensitivity coefficients for the polynomial trial function coefficients.

It can be seen from Fig. 5.19 that the zeroth-order coefficient sensitivity is at least one order of magnitude greater than any of the other coefficients at every wavelength. Because of this, the zeroth-order coefficient dominates the direction of the minimization. Once an accurate tip-temperature (represented by the zeroth-order coefficient) is reached,
very large changes in the other coefficients would have to be made to affect small changes in the signal. Because of this, the low-temperature portion of the temperature profile is not as accurate as the high temperature portion.

This is the same reason why the Bezier curve trial function was so ineffective. Where the fourth-order polynomial dictates a fourth-order curve, each of the Bezier curves is nearly independent of the others—only the common control point is dependent. This makes wavy configurations (as seen in Fig. 5.18) possible. Because of the dominance of the high-temperature portion of the fiber and overwhelming sensitivity of the tip-temperature, choosing the lowest-order trial function possible is desirable.

5.6 Comparison With the Two-Color Method

For comparison purposes, simulated tip temperatures were also obtained using the standard two-color method. The two-color approach is arrived at by neglecting absorption and emission by the fiber (Dils, 1983; Kreider, 1985). These approximations give

\[ \psi_\lambda = \kappa_\lambda \epsilon_\lambda \exp \left\{ - \frac{C_2}{\lambda T_{2C}} \right\} \]  \hspace{1cm} (5.3)

Taking the ratio of measurements at two wavelengths and solving Eq. (5.3) for \( T_{2C} \) gives

\[ T_{2C} = \frac{C_2}{\frac{\lambda_2}{\lambda_2} - \frac{\lambda_2}{\lambda_1}} \ln \left( \frac{\psi_\lambda \kappa_\lambda \epsilon_\lambda}{\psi_\lambda \kappa_\lambda \epsilon_\lambda} \right) \] \hspace{1cm} (5.4)

As previously discussed, it is desirable to choose closely spaced wavelengths to the left of the peak signal where the sensitivity to changes in the measurements is greatest when using the two-color approach (Dils, 1983; Kreider, 1985). The spectral
measurements used to obtain $T_{2C}$ were selected from the set of spectral measurements used for the reconstructions with the conjugate gradient algorithm (see Figs. 5.14-5.16).

Table 5.6 shows the resulting tip-temperature measurements for each of the three temperature profiles. In each case, the wavelengths used in the two-color approach were carefully chosen to give the best possible results. These results show that the tip temperatures obtained from reconstruction of the temperature profile is much more accurate than the two-color method.

<table>
<thead>
<tr>
<th>Profile</th>
<th>CI (%)</th>
<th>$\lambda_1$ ($\mu$m)</th>
<th>$\lambda_2$ ($\mu$m)</th>
<th>$T_o$ (K)</th>
<th>$T_{2C}$ (K)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>86.6</td>
<td>1.8</td>
<td>1.9</td>
<td>1342</td>
<td>1098</td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td>95.5</td>
<td>1.8</td>
<td>1.9</td>
<td>1342</td>
<td>1522</td>
<td>11.8</td>
</tr>
<tr>
<td></td>
<td>99.7</td>
<td>1.8</td>
<td>1.9</td>
<td>1342</td>
<td>1202</td>
<td>11.7</td>
</tr>
<tr>
<td>B</td>
<td>86.6</td>
<td>1.9</td>
<td>2.0</td>
<td>1200</td>
<td>1907</td>
<td>37.1</td>
</tr>
<tr>
<td></td>
<td>95.5</td>
<td>1.9</td>
<td>2.0</td>
<td>1200</td>
<td>1050</td>
<td>14.3</td>
</tr>
<tr>
<td></td>
<td>99.7</td>
<td>1.9</td>
<td>2.0</td>
<td>1200</td>
<td>1249</td>
<td>3.93</td>
</tr>
<tr>
<td>C</td>
<td>86.6</td>
<td>1.6</td>
<td>1.7</td>
<td>1573</td>
<td>1426</td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>95.5</td>
<td>1.6</td>
<td>1.7</td>
<td>1573</td>
<td>1467</td>
<td>7.19</td>
</tr>
<tr>
<td></td>
<td>99.7</td>
<td>1.6</td>
<td>1.7</td>
<td>1573</td>
<td>1481</td>
<td>6.24</td>
</tr>
</tbody>
</table>

This comparison proves the fulfillment of the second part of the objective for this thesis—to use the minimization algorithm to accurately infer the tip-temperature of the optical fiber and to obtain an estimate of the temperature profile. It can be seen from Table 5.6 that the CGA produces tip-temperatures that are significantly better than the two-color approach even in the presence of error.
5.7 Unique Solutions

While the simulated results are good, it is necessary to establish a few more points to solidify the usefulness of the CGA in this particular application. For any optimization routine it is necessary to explore the search space to determine if a global optimum has been reached. It is also important in problems as the one presented here to determine the uniqueness of the solution. If the same temperature profile is not arrived at for a given set of spectral measurements, regardless of the initial guess, the solution is not unique. For a temperature measurement method to be valid and useful, the measurement must be independent of the initial guess.

To test this, one set of spectral measurements was used. Several different initial guesses were used and the CGA was run to convergence. Figure 5.20 shows the different initial guesses (dashed lines) along with the simulated temperature profile (solid dark line) used in the reconstructions.

While the temperature profile reconstructions were unnoticeably different—any small differences due to the direction at which the optimum was approached for each initial guess—the reconstructed tip temperature errors were all within 0.01% of each other. Table 5.7 shows these results referenced from the highest-temperature initial guess to the lowest.

<table>
<thead>
<tr>
<th>Initial Guess</th>
<th>Tip-temperature</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1319.1</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>1318.1</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>1318.7</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>1318.9</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>1319.1</td>
<td>0.02</td>
</tr>
<tr>
<td>6</td>
<td>1318.7</td>
<td>0.01</td>
</tr>
</tbody>
</table>
According to these results, the reconstruction method using the CGA provides a unique solution to Eq. (3.4), within the value of the stopping criteria.

This can also be seen by visualizing the search space. Figure 5.21 shows a surface plot of the search space generated by holding all the coefficients of the fourth-order polynomial trial function constant except for the zeroth and first-order coefficients.

The fourth-order curvature is expected from the fourth-order polynomial trial function. The greater dependence on the zeroth-order coefficient than the first-order coefficient is also predicted from the discussion on the sensitivity coefficients (see Fig. 5.19). It can be seen that the optimal region of the search space is a long, narrow valley with an obvious minimum at one point. This corresponds to the values of the zeroth and first-order coefficients at the optimum. Similar plots are also obtained for the other coefficients with greater dependence on the lower order coefficients.
While in the 2-D search space it looks like many values could possibly satisfy the stopping criterion, when combined with the other coefficients, the optimal region of the search space is significantly narrowed. The fourth-order coefficient has the least sensitivity and therefore has the widest range of possible solutions. This justifies the use of a fourth-order polynomial trial function. Increasing the order of the trial function would only add computation time without adding accuracy.

Figure 5.21 Search-space plot for the zeroth and first-order coefficients.
CHAPTER 6 ACTUAL TEMPERATURE MEASUREMENTS

The final part of the thesis objective is to obtain successful experimental reconstructions of the tip-temperature. This chapter shows that tip-temperature measurements can be obtained to within 1% of the actual temperature using physical apparatus. Even though the suitability of the equipment was relatively low, the resulting tip-temperature reconstructions improve significantly on the two-color method.

6.1 Measurement Procedure

Kreider (1985) stated that the calibration values calculated in his experiments were valid for a temperature range of ±300 K, so it was assumed for these experiments that temperature measurements could be obtained with the OFT at temperature settings of 1353K (+9K from the calibration temperature) and 1331K (-13K).

To take the measurements in the furnace, the OFT was inserted through the furnace port (see Figs. 3.1, 3.2 and Appendix B) the desired distance. The platinum tube was then placed on the tip of the optical fiber and the tip was rested on a wire stand. The furnace door was closed and sufficient time was allowed for the furnace to reach steady state.

When sufficient time had elapsed, spectral measurements were obtained in the same way as described previously (see Chapter 3.1). Two temperature profiles (1366K and 1331K) were used, and multiple sets of measurements were taken at each temperature setting.
6.2 Reconstructions

The CGA was used to reconstruct the temperature profiles from the sets of spectral signal measurements. Figure 6.1 shows Run 1 at a furnace setting of 1366 K and Fig. 6.2 shows Run 1 at a furnace setting of 1331 K.

While the reconstructed temperature profile shown in Fig. 6.1 is in good agreement with the thermocouple measurements, the reconstructed temperature profile shown in Fig. 6.2 only matches the thermocouple measurement at the tip of the fiber.

These results highlight the fact that the emission from the cavity dominates the measured signal, and the measurements are much more sensitive to changes in the tip temperature than to changes in the temperature profile along the fiber. Due to the relatively low sensitivity of the measurements to variations in the temperature profile, a high level of noise in the measurements prevents consistently accurate reconstruction of the entire temperature profile. Other measurements were made, but these reconstructions shown highlight the best and the worst of the reconstructions. Possible methods of improving the reconstructions will be discussed in the final chapter.

6.3 Comparison to the Two-Color Method

The tip-temperature reconstructions are a considerable improvement over the standard two-color method. Table 6.1 shows this comparison.

<table>
<thead>
<tr>
<th>Furnace Temperature Setting (K)</th>
<th>$T_{TC}$ (K)</th>
<th>$T_R$ (K)</th>
<th>$T_{2C}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1366 (Run 1)</td>
<td>1340.6</td>
<td>1337.3</td>
<td>1221.0</td>
</tr>
<tr>
<td>1366 (Run 2)</td>
<td>1340.6</td>
<td>1347.1</td>
<td>1600.6</td>
</tr>
<tr>
<td>1331 (Run 1)</td>
<td>1318.8</td>
<td>1327.6</td>
<td>1354.5</td>
</tr>
<tr>
<td>1331 (Run 2)</td>
<td>1318.8</td>
<td>1323.8</td>
<td>878.0</td>
</tr>
</tbody>
</table>
Figure 6.1  (a) Temperature profile reconstruction for Run 1 at a furnace setting of 1366 K.  (b) Measured and predicted signals with corresponding uncertainty.
Figure 6.2  (a) Temperature profile reconstruction for Run 1 at a furnace setting of 1331 K.  (b) Measured and predicted signals with corresponding uncertainty.
The spectral measurements used to obtain $T_{2C}$ were selected from the set of spectral measurements used for the corresponding reconstructions. Using the data from Run 1 at each furnace temperature setting, the combination of wavelengths that resulted in a value for $T_{2C}$ that most closely matched the thermocouple reading was determined. For the 1366 K furnace setting, $\lambda_1 = 1.6$ and $\lambda_2 = 1.7$. For the 1331 K furnace setting, $\lambda_1 = 2.0$ and $\lambda_2 = 2.1$. The same two wavelengths were then used to obtain the two-color temperature based on the measurements for Run 2, and the results are listed in Table 1. In each case, the reconstructed tip-temperature was significantly more accurate than the two-color temperature. Despite the high level of noise in the measurements and the large uncertainty in the predicted signal, the reconstructed tip-temperature agreed with the thermocouple readings to within 0.7%. This is because the reconstruction algorithm uses several wavelengths, and the error in the measurement at one wavelength is frequently offset by the error in the other measurements. Since the two-color method only uses two measurements, it is much more unstable in the presence of error.

The reconstruction using the CGA accomplishes the final part of the thesis objective by accurately inferring the tip-temperature to within 0.7% —a considerable improvement over the standard two-color method.
CHAPTER 7 IMPROVEMENTS TO THE TEMPERATURE PROFILE RECONSTRUCTIONS

While the thesis objective has been fulfilled, a few suggestions are given here to improve the temperature profile reconstructions.

Based on the presented material, it is clear that significant improvement can be made in the experimental method of reconstructing the temperature profile along the optical fiber thermometer. Figures 6.1 and 6.2 show the uncertainty in the measured signals and the uncertainty in the signal measurement equation. This degree of uncertainty directly affected the accuracy of the reconstructions. To improve the accuracy of the reconstructions, it is necessary to reduce the error in the spectral signal measurements and reduce the uncertainty in the SME. This can be done in the following ways.

7.1 Improvements to the Optical Fiber Probe

For this research, an optical fiber with a 0.5 mm diameter was used because it was the only one available in the length that was needed (1.0 m). It was assumed that this length was necessary in order to heat a sufficient length of the fiber and to provide enough distance for the fiber to cool before reaching the detection system. The main problems with this type of optical fiber are crystal purity and size. While a single fiber prevents coupling losses, the small diameter and considerable length required contribute to an increased likelihood that the fiber has internal and surface defects that adversely
affect the light transmission down the fiber. The small diameter also drastically limits the total amount of light that travels down the fiber.

Equation (5.1) shows that the signal increases as the diameter of the fiber squared. For the experiments presented here, the signal-to-noise ratio was an average of approximately 9. An improved signal-to-noise ratio can be simulated by simulating an increased signal with an increased fiber diameter.

In the experiments by Dils (1983) and Kreider (1985) larger diameter sapphire rods that were also shorter in length were coupled to a low temperature fiber that carried the light from the rod to the detection system. Coupling losses are introduced when using multiple fibers, but these losses are worth the extra light obtained with a larger fiber diameter. Sapphire optical rods are available commercially in diameters from 1.0 to 4.0 mm at lengths from 2 cm to 30 cm. Table 7.1 shows an example of increased throughput with a larger diameter fiber, taking into account a 10% coupling loss for diameters greater than 0.5mm.

It can be seen that increasing the fiber diameter greatly increases the signal strength even when taking into account a conservative 10% coupling loss (Dils 1983).

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Signal-to-Noise Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>119</td>
</tr>
<tr>
<td>4</td>
<td>477</td>
</tr>
</tbody>
</table>

Another problem with the optical fiber probe used in these experiments was the platinum cavity used at the tip. In order to accurately predict the signal at the other end
of the fiber it is necessary to know the emissivity of the blackbody cavity or target. Since the optical fiber used in the experiments was long, it would not fit in most sputtering chambers. It was attempted to coil the fiber and sputter a coating in this manner, but that proved ineffective. To create a blackbody cavity at the tip of the fiber, a thin walled, platinum tube with an inside diameter slightly larger than the fiber was used. One of the ends was crimped to create an enclosure and the fiber was inserted in the opened end. The analysis discussed earlier showed that the emissivity of the cavity was approximately 0.92, but it was necessary to account for it in the calibration.

To obtain a more accurate cavity emittance and reduce calibration dependence, it is necessary to sputter a coating on the tip of the fiber. Dils characterized the effective emittance of a sputtered cavity for different length to diameter ratios (1983), so a better estimate of the emissivity would be available. Using a shorter fiber would improve the suitability for sputtering since it would be small enough to fit easily into most sputtering chambers.

In summary, to create a better optical fiber probe, it is necessary to use a sapphire rod with a diameter of between 1 and 4 mm. This fiber would be coupled to a low temperature fiber that would transfer the light to the detection system. The sapphire rod would have a sputtered coating at the detection end that would act as a blackbody cavity whose effective emittance is well characterized. This type of probe will increase throughput and predictability of the light incident on the detector.

7.2 Improvements to the Optical Detection System

The optical components used in this research were purchased from Oriel because of the ease of assembly and incorporation into the whole detection system. One of the
main problems with the optical system was the detector that was used. Since it was initially anticipated that a wider spectral range would be necessary than actually was, a thermopile detector was used. This type of a detector is generally not sensitive to changes in wavelength, but it has a very high noise equivalent power (NEP). NEP describes the amount of incident intensity required to produce a signal that is above the noise of the detector. Detector responsivity is frequently referred to by the detectivity (D) of the detector or the normalized detectivity (D*) which are basically reciprocals of the NEP—the higher the D*, the less intensity is required to produce a signal above the noise. The D* rating for the thermopile is about $2 \cdot 10^8$ whereas the D* for photoelectric or photon detectors can be as high as $2 \cdot 10^{14}$—an increase of six orders of magnitude.

An appropriate detector for this application would be a lead-sulfide (PbS) detector. This detector has a D* of between $5 \cdot 10^9$ and $5 \cdot 10^{11}$ which is not as high as other photon detectors but that is from one to three orders of magnitude higher than the thermopile. This detector also covers a wide enough spectral range (about 1.5 to 4.0 µm) that only one detector would be necessary. It would be possible to use another detector with a higher D*, such as an InGaAs, for the shorter wavelengths, but the best choice for longer wavelengths would be the PbS detector.

Whatever detector is used, it is necessary to have it calibrated. The thermopile was calibrated which made prediction and interpretation of the signal possible. For a photon detector, calibration is even more important since the responsivity of the detector is a function of wavelength, whereas the thermopile responsivity was constant over all wavelengths. It was possible to make measurements with the thermopile and the small-diameter fiber, but the temperature range was severely limited, and many of the
measurements were near the noise. With a more sensitive, calibrated detector and a larger fiber, the uncertainty in the measurements will decrease significantly.

Another major problem with the detection system was the lack of accurate knowledge of the efficiency of the monochromator. Specific losses for some parts of the monochromator were stated by the manufacturer, but the part that was unclear, and perhaps most significant was the grating efficiency. One grating was used with a spectral range from 1.3 to 5.0 µm. This was sufficient to cover the desired spectral range. A plot of the efficiency for this type of grating was available from the manufacturer, but it is necessary to calibrate for the grating efficiency specifically since it is likely that every grating is not the same.

It is possible to quantify the grating efficiency in the following way. A calibrated radiation source, like a quartz tungsten halogen lamp, in conjunction with an integrating sphere are necessary to know exactly the radiation incident on the monochromator slit. With the knowledge of the incident intensity on the monochromator from the calibrated radiation source, and using a calibrated detector, it will be possible to quantify the losses due to grating inefficiency. Knowing the grating efficiency will decrease the uncertainty in the measurements even further.

7.3 Other Improvements

The furnace used for the discussed experiment is old and not capable of extreme temperatures. Since the temperature range was limited, both by the furnace and the capabilities of the detection system, it was not possible to study in-depth the temperature dependence of the measurements. It is desirable to have a wide temperature range which
would provide a stronger signal to be detected and to be able to take measurements at several different temperature profiles.

With a better furnace and optical detection system, it would also be possible to measure the absorption coefficient which is probably the largest hindrance to successful, low-error measurements. As discussed in the previous chapters, data on the absorption coefficient for sapphire is limited and then only tabulated at room temperature. It had been observed in the experiments that the absorption coefficient is a function of temperature, and it is necessary to know this temperature dependence. It is also possible that different fibers might have different absorption characteristics, and it is also necessary to determine this in order to make more accurate measurements. Figure 7.1 shows a possible experimental setup to measure the absorption as a function of temperature.

Measurements of the radiative flux exiting the fiber are obtained with the furnace operating at steady state. Thermocouples are used to measure the temperature of the cavity, and multipoint thermocouple probes are used to measure the temperature profile along the axis of the furnace.

In this configuration, all the parameters in Eq. (3.4) except the spectral absorption coefficients are known. Based on assumed values for the spectral absorption coefficients, the output from the detector is predicted using Eq. (3.4). An objective function is defined as the absolute value of the difference between the measured and predicted signals.

\[ F = |\psi_{\lambda,\text{measured}} - \psi_{\lambda,\text{predicted}}| \]  \hspace{1cm} (6.1)

Using an inverse approach similar to the method used to reconstruct the temperature profile, the values for the spectral absorption coefficient will be obtained at
points along the fiber. Since the temperature of the fiber is known at each of these points, the spectral absorption coefficient will be determined as a function of temperature. It would also be possible to use this furnace to perform temperature profile reconstructions.

![Diagram of laboratory setup](image)

**Figure 7.1** Possible laboratory setup for measuring the absorption of the sapphire fiber.

### 7.4 Recommended Procedure

With the appropriate components as discussed previously, the following procedure will lead to accurate reconstructions of the temperature profile along the blackbody OFT. Firstly, the grating efficiency needs to be established. The radiation source will be coupled to the monochromator and the calibrated detector will be used at the exit slit. Adjustments are then made to the grating position to diffract the light at different wavelengths. Stepping through the wavelength range of the monochromator, recording the signal output of the detector, the measured signals are then compared with
the power of the radiation source. This will provide data for the spectral grating efficiency which will allow signals obtained from the optical fiber to be accurately translated to intensity exiting the fiber.

Following the determination of the grating efficiency, it is necessary to determine the absorption properties of the optical fiber. The high temperature fiber is coupled to the low-temperature fiber which is attached to the detection system. The fiber assembly is then positioned in the furnace (see Fig. 7.1) in front of the blackbody cavity. The temperature profile in the furnace will be measured with the multipoint thermocouple.

With the high temperature fiber in the furnace, measurements can be taken with the calibrated optical system. Using Eq. (3.4), the known temperature profile in the furnace, the blackbody cavity temperature and room temperature values for the absorption coefficient, predicted signal values can be calculated. These values are then compared with the measured values, and using a minimization routine, corrected values for the absorption coefficient can be calculated. This should be done at several temperatures and with several fibers to determine the temperature dependence and individual fiber dependence of the absorption.

With the known grating efficiency and absorption coefficient for the sapphire fiber, a coated fiber can then be calibrated using a blackbody calibration furnace. Calibration is necessary to account for anomalies on a fiber-to-fiber basis. It is not expected that this calibration factor will deviate significantly from unity since the prior steps have been taken to characterize the detection system and the absorption properties of the fiber. This calibration step is carried out by measuring the signal at a given temperature for all wavelengths of interest and comparing the measurements to the

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expected signal given by Eq. (3.4). This gives a spectral calibration factor that can be used to accurately predicted the temperature at the tip of the fiber from a given spectral measurement.

7.5 Possible results

With the improvements discussed in this chapter, the following improved results can be expected. Simulated measurements were generated using a fourth-order polynomial fit to the temperature points measured with the thermocouple with the furnace set at 1331K. By implementing the improvements to the optical system discussed above, it is estimated that the signal to noise ratio in the measurements can be conservatively increased by a factor of 10 (increasing by a factor of 100 is realistic) using a more suitable detector, and the uncertainty in the predicted signals can be reduced by 30%. This is accomplished by reducing the uncertainty in the absorption coefficient to 10% and reducing the uncertainty in the cavity emissivity to 0.02. The reconstructed temperature profile obtained with this level of noise in the measurements and uncertainty in the predicted signals is shown in Fig. 7.2 The improved CGA discussed earlier was used for this simulation.

Further reductions in the uncertainty will lead to increased accuracy in the reconstructions for the tip-temperature and the temperature profile. The increased accuracy will occur due to a smaller stopping criterion which reduction is caused by lower measurement uncertainty. This will allow the conjugate gradient algorithm to explore the search space further.
Figure 7.2  Improved reconstruction by increasing the signal-to-noise ratio and decreasing signal measurement uncertainty.

Not only will accuracy be improved, but so will computation time. With less uncertainty, the CGA will find a converged solution faster. This is because the measured signals will more closely match the signals predicted by the signal measurement...
equation. Implementing these improvements will lead to consistently accurate and repeatable results that could be feasibly used as a dependable temperature measurement device.
CHAPTER 8 CONCLUSIONS

The three-fold objective of this thesis has been fulfilled. Two numerical inversion techniques have been developed to minimize the root-sum-squares error between the measured and predicted signals and reconstruct the temperature at the tip of the optical fiber. These techniques also provide an estimate of the temperature profile along the fiber. Simulated measurements have shown that it is possible to obtain a reconstructed tip-temperature that has an error of within 1%. Finally, actual experiments have shown that this method is feasible and reliable even in the presence of significant measurement uncertainty. These actual measurements improve significantly over the standard-two color method. Additionally, this new method of optical fiber thermometry (OFT) can be used in environments where standard OFT cannot be used.

In order to use the numerical inversion techniques, it was necessary to know the relationship between the temperature profile along the optical fiber and the signal output of the optical detector. This relationship is called the signal measurement equation (SME) and accounts for attenuation from the end of the fiber until the light exiting the fiber is incident on the detector. A detailed description of the optical detection system is necessary to accurately develop the SME. The description of the optical components used in the experiments was given to the degree of accuracy they were known.

Since many of the components of the optical detection system were poorly or incompletely known, it was necessary to calibrate the optical fiber thermometer in-situ.
The temperature profile in a laboratory furnace was characterized at a calibration temperature and this temperature profile was used to calibrate the optical fiber thermometer.

The equation describing radiation transfer down the fiber is non-linear and ill-posed. Hence there are difficulties in extracting the temperature profile from the equation. At a single wavelength it is possible that many temperature profiles could satisfy Eq. (3.4), but by using several wavelengths it has been shown that the solution is unique. It has been shown that the CGA is stable to the degree that is has been tested and will arrive consistently at the same solution regardless of initial guess.

Simulated temperature measurements were performed using an estimate of the temperature profile in a laboratory furnace. Error was introduced randomly using a standard uncertainty analysis and expressing a confidence level on the uncertainty. A higher confidence level will yield normally distributed error that is more tightly distributed around the mean. The confidence levels used in this research were 86.6, 95.5 and 99.7%. An improved method of introducing error is to simulate an increased fiber diameter and decreased noise.

For the simulated reconstructions with error introduced randomly as discussed, it is possible to retrieve accurate tip temperatures (within 1.0%) and to reconstruct an estimate of the temperature profile along the OFT.

For the actual experiments, two temperature profiles on either side of a calibration temperature profile were used. In these actual experiments the tip-temperature was measured to within 0.7%, but the temperature profiles had significant error in most of the cases. This is due to large measurement uncertainty and the large uncertainty in the
signal measurement equation. While the temperature profiles were not completely accurate, the tip-temperature reconstructions improved considerably on the standard two-color method.

It has been shown that a small reduction in uncertainty in the SME and the uncertainty in the measurements will lead to significant improvements in the temperature profile reconstructions. Recommendations for implementing these improvements have been given. Application of these recommendations will lead to the development of a more-dependable optical fiber temperature sensor. This temperature sensor will have the added benefit of providing an accurate reconstruction of the temperature profile along the optical fiber.
REFERENCES


National Institute of Standards and Technology (NIST), 2002, *Radiation Thermometry Short Course*, Gaithersburg, MD.


Conjugate Gradient Algorithm using the Signal Measurement Equation to Reconstruct the Temperature Profile Along a Sapphire Optical Fiber Thermometer

David G. Barker
Last Modified: 19 February 2003
Mathcad Version 11

User Inputs

Fiber Properties
Fiber Diameter

\[ d_f := 0.000425 \text{ [m]} \]

\[ A_f := \pi \left( \frac{d_f}{2} \right)^2 \]

\[ A_f = 1.419 \times 10^{-7} \text{ [m]} \]

Fiber area

Numerical Aperture

\[ NA := \phi \]

\[ a := \arcsin(NA) \]

\[ a = 0.201 \]

\[ \omega := \pi \left( \tan(a) \right)^2 \]

\[ \omega = 0.131 \]

Acceptance solid angle

Fiber Attenuation

\[ \text{Atten} := 0 \text{ [dB/m]} \]

Fiber Length

\[ L_c := \frac{1.35}{2} \]

Any units may be used here, the conversion factor converts to millimeters

\[ uc := 1000 \]

Conversion factor to millimeters
Fiber F/#

\[ F_{\text{fib}} := \frac{1}{2 \cdot \sin(\text{NA})} \quad F_{\text{fib}} = 2.517 \]

Monochromator Properties

Monochromator F/#

\[ F_{\text{mono}} := 3.7 \quad \text{Coup} := 1 \]

F/# Matcher efficiency

\[ \eta_{\text{match}} := .75 \]

Monochromator mirror reflection efficiency

\[ \eta_{\text{mirrors}} := .88 \]

Vignetting factor (1 if source image is smaller than grating size)

\[ \text{Vign} := 1 \]

Slit width calculation

\[ \text{slitopen} := 2 \]

\[ \text{slit} := \left[ 1 \cdot (\text{slitopen}) \cdot d_1^2 \right] \]

\[ \text{slit} = 8.5 \times 10^{-4} \]

Grating Properties

\[ G_1 = 74025 \quad G_2 = 74029 \]

Grating reciprocal dispersion

\[ \text{RDG1} := 6.2 \quad \text{RDG2} := 26 \quad \text{[nm/mm]} \]

Grating blaze wavelength

\[ \text{BWG1} := 750 \quad \text{BWG2} := 2000 \quad \text{[nm]} \]

Minimum grating wavelength

\[ \text{MINWG1} := 450 \quad \text{MINWG2} := 1100 \quad \text{[nm]} \]
Maximum grating wavelength

\[
\begin{align*}
\text{MaxWG1} & := 1600 \\
\text{MaxWG2} & := 5000
\end{align*}
\quad [\text{nm}]
\]

Grating bandwidth calculation

\[
\begin{align*}
\text{BandG1} & := \text{RDG1}\cdot\text{slit}\cdot10^0 \\
\text{BandG2} & := \text{RDG2}\cdot\text{slit}\cdot10^0
\end{align*}
\quad [\text{mm}]
\]

Detector Properties

\[
\begin{align*}
\text{Gain} & := 1 \\
\text{Responsivity} & := 76.5 \quad [\text{pA/(W/m}^2\text{)]}
\end{align*}
\]

\[
\text{Resp} := \text{Responsivity}\cdot10^{-12}
\]

\[
\text{Resp} = 7.65 \times 10^{-11} \quad [\text{A/(W/m}^2\text{)]}
\]

\[
\text{NEP} := 1.2\cdot10^{-9} \quad [\text{W/Hz}^{1/2}]
\]

Calibrated solid angle subtended by the detector

\[
\omega_d := .0049878
\]

Emissivity of Tip Cavity

\[
\varepsilon(\lambda) := .92
\]

User Inputs

Other Inputs and Calculations

Radiation Constants

\[
\begin{align*}
\text{c1} & := 119000000 \quad [\text{W mm}^4 / \text{m}^2 \text{ sr}] \\
\text{c2} & := 14388 \quad [\text{mm K}]
\end{align*}
\]

\[
\text{kf} := \kappa\text{tx}
\]

Calibration coefficient data
\[
\text{sk} := \text{cspline}\left(\text{kf}^{(0)}, \text{kf}^{(1)}\right)
\]
\[
\text{k}(\lambda) := \text{interp}\left(\text{sk}, \text{kf}^{(0)}, \text{kf}^{(1)}, \lambda\right)
\]

\[
\text{kin} := \ldots \text{kbrews.bdl}
\]

Imaginary refractive index from brewster

\[
\text{sk} := \text{cspline}\left(\text{kin}^{(0)}, \text{kin}^{(1)}\right)
\]
\[
\text{k}(\lambda) := \text{interp}\left(\text{sk}, \text{kin}^{(0)}, \text{kin}^{(1)}, \lambda\right)
\]

\[
\text{Ka}(\lambda) := \frac{4000 \cdot \pi \cdot \text{k}(\lambda)}{\lambda} \cdot (\text{Lc} \cdot \text{uc})
\]
\[
\text{Kal}(\lambda) := \frac{4000 \cdot \pi \cdot \text{k}(\lambda)}{\lambda} \cdot \text{Lc} \cdot \text{uc}
\]

Define optical depth function

modified Planck's function

\[
\text{mplank}(T, \lambda) := \frac{1}{\left(\frac{c^2}{e^{\lambda \cdot T}}\right)}
\]

Dimensionless length start and end

\[
\text{Zo} := 0 \quad \text{Zf} := 1
\]

Wavelength range, resolution and number of wavelengths calculation

\[
\lambda_{\text{initial}} := 1.5 \quad \lambda_{\text{final}} := 3.3
\]
\[
\lambda_{\text{num}} := 19
\]
\[
\lambda_{\text{res}} := \frac{\lambda_{\text{final}} - \lambda_{\text{initial}}}{\lambda_{\text{num}} - 1}
\]
\[
\lambda_{\text{res}} = 0.1
\]
Fahrenheit to Kelvin conversion function

\[ FK(T_f) := (T_f - 32) \cdot \frac{5}{9} + 273.15 \]

Furnace temperature profile (measured)

\[ TACTin := \ldots\backslash 2000F\.txt \]

Temperature profile discretization

\[
TACT := \begin{cases} 
\text{for } j \in 0..5 \\
\quad \text{for } i \in 0..19 \\
\quad \quad t_{i,j} \leftarrow FK(TACT_{in,i,j}) \\
\end{cases}
\]

\[
Tact := \begin{cases} 
\text{for } i \in 0..5 \\
\quad t_i \leftarrow \text{mean}(TACT^{(i+1)}) \text{ if } i < 5 \\
\quad t_i \leftarrow 300 \text{ otherwise} \\
\end{cases}
\]

\[
Xact := \begin{cases} 
\text{for } i \in 0..5 \\
\quad x_i \leftarrow \frac{i}{10} \text{ if } i < 5 \\
\quad x_i \leftarrow \frac{i + 5}{10} \text{ otherwise} \\
\end{cases}
\]

\[
Tact = \begin{pmatrix} 
1340.589 \\
1348.378 \\
1351.522 \\
1349.033 \\
1338.589 \\
300 \\
\end{pmatrix} 
\]

\[
Xact = \begin{pmatrix} 
0 \\
0.1 \\
0.2 \\
0.3 \\
0.4 \\
1 \\
\end{pmatrix} 
\]
output order of amperage $10^{-9} = \text{nanoamps}$

\[
\text{ORDER} := -9
\]

**Enter actual temperature profile**

\[
s := \text{cspline}(\text{Xact}, \text{Tact})
\]

C spline interpolation for actual temperature profile

\[
\text{Tz}(x) := \text{interp}(s, \text{Xact}, \text{Tact}, x)
\]

\[
\text{Tz}(x) := 1300 \cdot e^{-\frac{1}{1} \cdot x} + 273
\]

Alternative exponential profile

\[
\text{Tz}(x) := 1200 + 2000 \cdot x - 2500 \cdot x^2
\]

Alternative parabolic profile

**Enter coefficients for guessed temperature profile (add Cg's if a higher order desired)**

<table>
<thead>
<tr>
<th>Constant Term</th>
<th>First-order Term</th>
<th>Second-order Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{g0} := 900 )</td>
<td>( C_{g1} := -600 )</td>
<td>( C_{g2} := .001 )</td>
</tr>
</tbody>
</table>

CGA constants

\( P_{\text{Order}} := 4 \) Order of reconstructed temperature profile

\( \text{maxiterations} := 200 \) Maximum number of iterations

\( \text{stoptol} := 1 \cdot 10^{-4} \) Orders of magnitude less than discrepancy principle for second stopping criterion

\( \text{stopct} := 40 \) Number of iterations below stoptol for convergence

\( \text{SigCt} := 3 \) Number of standard deviations for maximum uncertainty used in error simulation

\( \text{RoomTemp} := 2 \) Minimum valid temperature

\( \text{MaxT} := 1800 \) Maximum valid temperature

\( N := P_{\text{Order}} + 1 \) Number of trial function parameters

\( \text{Actual} := "\text{ys}" \) Enter "yes" for actual measurements or anything else for simulated

\( \text{Jfreq} := 1 \) Not used

\( \text{kreset} := 6000 \) Reset to steepest descent after this many iterations
\begin{align*}
\text{smooth} & := .2 & \text{Not used} \\
\text{UseSmooth} & := \text{"Ys"} & \text{Not used} \\
\text{Tstoptol} & := .0005 & \text{Not used} \\
\text{Tstopct} & := 2000 & \text{Not used} \\
\end{align*}

\section*{Temperature Discretization}

Polynomial temperature function

\[ T(x, u) := u_0 + u_1 x + u_2 x^2 + u_3 x^3 + u_4 x^4 + u_5 x^5 + u_6 x^6 + u_7 x^7 + u_8 x^8 \]

Guessed profile discretization

\begin{align*}
\text{polynomial}(x, U) := & \quad p \leftarrow U_0 \\
& \quad \text{for } i \in 1..\text{rows}(U) - 1 \\
& \quad \quad p \leftarrow p + U_i x^i \\
& \quad p
\end{align*}

\begin{align*}
\text{CgN} := & \quad \text{for } i \in 0..N - 1 \\
& \quad \quad \text{cgn}_i \leftarrow \text{Cg}_i \quad \text{if } i < \text{rows}(\text{Cg}) \\
& \quad \quad \text{cgn}_i \leftarrow 10^{-3} \quad \text{otherwise}
\end{align*}

TgN(x) := polynomial(x, CgN)

TactN(x) := Tz(x)

r := 0.01..1
\[ TN(x, u) := \text{polynomial}(x, u) \]

\[ T_{actN}(x) := T_z(x) \]

\[ T_{gN}(x) := TN(x, CgN) \]

\[ \textbf{Temperature Discretization} \]

Measured Signal File

\[ Y_{input} := \ldots \text{2000}_072302.prn \]

\[ \textbf{Emission Equation} \]

Signal measurement equation

Equation for predicted signal

\[ Smel(\lambda, z, M) := \kappa(\lambda) \left[ \left( m\text{planck}(TN(0, M), \lambda) \cdot e(\lambda) \ldots \right) \cdot e^{- Ka(\lambda)} \right] + Ka(\lambda) \int_{0}^{1} e^{Ka(\lambda) \cdot e - m\text{planck}(TN(z, M), \lambda)} \cdot dz \]

100
Equation for actual signal

\[ Smelact(\lambda, z) := \kappa(\lambda) \left[ mplanck(Tz(0), \lambda) e(\lambda) \ldots \right] e^{-KaL(\lambda)} + Ka(\lambda) \int_0^1 e^{Ka(\lambda)-z} mplanck(Tz(z), \lambda) \, dz \]

\[
\begin{align*}
Y_{\text{meas}} := & \quad \text{for } i \in 0 \ldots \lambda_{\text{num}} - 1 \\
& \quad \text{istart } \leftarrow i \text{ if } \left( \text{Yinput}^{(0)} \right)_i = \lambda_{\text{initial}} \\
& \quad \text{for } i \in 0 \ldots \lambda_{\text{num}} - 1 \\
& \quad y_i \leftarrow \left( \text{Yinput}^{(2)} \right)_{i+\text{istart}} \\
& \quad y
\end{align*}
\]

Puts the information from the signal data file into a vector

\[
\begin{align*}
Y_{\text{unc}} := & \quad \text{for } i \in 0 \ldots \lambda_{\text{num}} - 1 \\
& \quad \text{istart } \leftarrow i \text{ if } \left( \text{Yinput}^{(0)} \right)_i = \lambda_{\text{initial}} \\
& \quad \text{for } i \in 0 \ldots \lambda_{\text{num}} - 1 \\
& \quad \begin{bmatrix}
 y_{i,0} \leftarrow \left( \text{Yinput}^{(1)} \right)_{i+\text{istart}} \\
 y_{i,1} \leftarrow \left( \text{Yinput}^{(3)} \right)_{i+\text{istart}} \\
 y
\end{bmatrix}
\end{align*}
\]

Puts the uncertainty data from the signal file into a vector

\[
\begin{align*}
\Lambda := & \quad \text{for } i \in 0 \ldots \lambda_{\text{num}} - 1 \\
& \quad L_i \leftarrow \lambda_{\text{initial}} + \lambda_{\text{res}} \cdot i \\
& \quad L
\end{align*}
\]

Wavelength Vector

Predicted and Measured Signal Values
Signal discretization function

Calculates the spectral signal distribution for a given set of trial function parameters

\[ ME(M, z) := \begin{align*}
\text{for } i &\in 0..\lambda_{\text{num}} - 1 \\
\lambda_i &\leftarrow \lambda_{\text{initial}} + \lambda_{\text{res}}^i \\
y_i &\leftarrow Smel(\lambda_i, z, M) \\
y &\leftarrow
\end{align*} \]

Calculates the spectral signal distribution for the actual temperature profile (simulated)

\[ ME_{\text{act}}(z) := \begin{align*}
\text{for } i &\in 0..\lambda_{\text{num}} - 1 \\
\lambda_i &\leftarrow \lambda_{\text{initial}} + \lambda_{\text{res}}^i \\
y_i &\leftarrow Smel_{\text{act}}(\lambda_i, z) \\
y &\leftarrow
\end{align*} \]

\[ Y := ME_{\text{act}}(Zf) \quad \text{Smel}_{\text{act}}(2, Zf) = 0.04 \]

\textbf{Emission Equation}

\textbf{Sensitivity Matrix}

Define the sensitivity matrix

\[ \varepsilon_{\text{sens}} := 10^{-3} \quad \text{Perturbation} \]

\[ N = 5 \]
\[ JI(M, z) := \begin{array}{l}
J \leftarrow 0 \\
n \text{for } j \in 0..N - 1 \\
p \leftarrow M \\
p_j \leftarrow M_j \cdot \text{esens} + M_j \\
n \text{for } i \in 0..\lambda_{\text{num}} - 1 \\
\lambda \leftarrow \lambda_{\text{initial}} + \lambda_{\text{res}} \cdot i \\
J, j \leftarrow \frac{\text{Sdef}(\lambda, z, p) - \text{Sdef}(\lambda, z, M)}{M_j \cdot \text{esens}} \\
p \leftarrow M
\end{array} \]

\[ \text{rr4} := 0..\lambda_{\text{num}} - 1 \]

\[ J := JI(CgN, Zf) \]

\[ \text{Sensitivity Matrix} \]

\[ d := 10 \] Diameter Multiple
\[ \sigma_s := \frac{.003}{d^2} \] Simulated measurement standard deviation

\[ u_{\text{red}} := \frac{\sigma_s}{.003} \] Reduced uncertainty in calibration coefficient

\[ u_{\text{red}} := .01 \] Reduced Uncertainty

### Uncertainty

Uncertainty values for the different contributors

\[ u_{\text{Resp}} := \text{Resp}.036 \]
\[ u_{\text{Resp}} = 2.754 \times 10^{-12} \] [A/W]

\[ u_{\text{L}} := 175 \] [mm]
\[ u_{\text{ka}} := .05 \]

\[ u_{\text{Ka}}(\lambda) := \sqrt{(Ka(\lambda) \cdot u_{\text{ka}})^2 + \left(\frac{Ka(\lambda)}{uc \cdot Lc} \cdot u_L\right)^2} \]

\[ u_{\text{ust}} := 1 - \frac{.85}{9} \]
\[ u_{\text{ust}} = 0.056 \] [\text{]}\n
\[ u_{\text{udf}} := \frac{.00045 - .0004}{3} \]
\[ u_{\text{udf}} = 1.667 \times 10^{-5} \] [m²]

\[ u_\omega := \pi \left[\frac{(\tan(\text{asin}.201))^2 - (\tan(\text{asin}.199))^2}{6}\right] \]
\[ u_\omega = 4.545 \times 10^{-4} \] [sr]

\[ u_{\epsilon} := .01 \]

\[ u_{\text{Tip}} := \sqrt{.25 + .25} \] [K]

\[ \text{pert} := 10^{-6} \]
\[ u_T := u_{\text{Tip}} \]
\[ u_T = 0.707 \]

\[ u_{\text{xin}} := \ldots/\text{unc\_kappa.xl} \]
Read in uncertainty in the calibration coefficient
uxs := cspline(uxi, uxi)
ux(\lambda) := interp(uxs, uxi, \lambda)

Analytic partial derivatives for the uncertainty parameters

pSp(\lambda) := \kappa(\lambda) \cdot mplanck(Tz(0), \lambda) \cdot e^{-KaL(\lambda)}

pSp(2) = 0.038

pSpTip(\lambda) := \left| -\kappa(\lambda) e^{-KaL(\lambda)} \cdot \frac{\lambda^{2}}{Tz(0)^{2}} \right|

pSpTip(2) = 1.382 \times 10^{-4}

pSpT(\lambda) := \left| -\kappa(\lambda) e^{-KaL(\lambda)} \cdot \frac{\lambda^{2}}{Tz(0)^{2}} \right|

pSpT(1.7) = 2.469 \times 10^{-5}

pSpKa(\lambda) := \left| -\kappa(\lambda) e^{-KaL(\lambda)} \cdot mplanck(Tz(0), \lambda) \right|

pSpKa(2) = 4.01335 \times 10^{-2}

pSpx(\lambda) := mplanck(Tz(0), \lambda) \cdot e(\lambda) + Ka(\lambda) \cdot \int_{0}^{1} e^{Ka(\lambda) \cdot z \cdot mplanck(Tz(z), \lambda)} dz \cdot e^{-KaL(\lambda)}
\[ \text{pSpx2}(2) = 4.042 \times 10^{-3} \]

\[ r_{\lambda} := \lambda_{\text{initial}} + \lambda_{\text{res}} + \lambda_{\text{final}} \]

**Total Uncertainty**

\[ u_{S2}(\lambda) := \sqrt{\left( pSp\epsilon^2(\lambda)^2 \cdot u_{\epsilon}^2 + pSpT^2(\lambda)^2 \cdot u_T^2 + pSpT\text{tip2}(\lambda)^2 \cdot u\text{tip2}^2 \right) + \left( pSp\kappa^2(\lambda)^2 \cdot u\kappa(\lambda)^2 + pSp\kappa^2(\lambda)^2 \cdot u\kappa(\lambda)^2 \right)} \]
The equation in the text is:

$$uS2(2) = 2.28 \times 10^{-3}$$
\[ \text{ERRmel} := \text{for } i \in 0..\lambda_{\text{num}} - 1 \]
\[ i2 \leftarrow \lambda_{\text{initial}} + \lambda_{\text{res}}^i \]
\[ \text{unc} \leftarrow \text{uS2}(i2) \]
\[ \text{em}_{i,0} \leftarrow Y_i - \text{unc} \]
\[ \text{em}_{i,0} \leftarrow Y_i \text{ if } \text{em}_{i,0} \leq 0 \]
\[ \text{em}_{i,1} \leftarrow Y_i \]
\[ \text{em}_{i,2} \leftarrow Y_i + \text{unc} \]

Uncertainty in the signal measurement equation

\[ \text{ERRout} := \text{augment}(\Lambda, \text{ERRmel}) \]

\textbf{Uncertainty}

\textbf{OUTPUTsp := "c:\mca\out\sp\whole.prm"}

\textbf{Least Squares Norm Calculation
Calculation for randomly introduced error to simulated measurements including stopping criterion

\[ Y_{\text{err}}(Y) := \begin{align*}
\text{unc} & \leftarrow 0 \\
\text{for } i & \in 0..\lambda_{\text{num}} - 1 \\
\lambda_i & \leftarrow \lambda_{\text{initial}} + \lambda_{\text{res}} \cdot i \\
\omega & \leftarrow \text{norm}(1, 0, \sigma) \\
\sigma_{\lambda_i} & \leftarrow \sigma_s \\
\text{unc}_i & \leftarrow \sqrt{\left(\sigma_{\lambda_i}\right)^2 + \left(uS^2(\lambda_i) \cdot u_{\text{red}}\right)^2} \\
\varphi_{\lambda_i} & \leftarrow \text{unc}_i \cdot \omega_0 + Y_i \\
\Lambda_i & \leftarrow \lambda_i \\
u_{s_i} & \leftarrow uS^2(\lambda_i) \cdot u_{\text{red}} \\
\varepsilon & \leftarrow \sum_{j=0}^{\lambda_{\text{num}} - 1} \left(\sigma_{\lambda_j}\right)^2 \\
\varepsilon^2 & \leftarrow \sum_{j=0}^{\lambda_{\text{num}} - 1} \left(\varphi_{\lambda_j} - Y_j\right)^2 \\
\left(\begin{array}{c}
\varphi_{\lambda_i} \\
\Lambda \\
\varepsilon \\
0 \\
\varepsilon^2 \\
\text{unc} \\
u_{s_i}
\end{array}\right)
\end{align*} \]

\[ Y_1 := Y_{\text{err}}(Y) \]
\[ u_s := Y_{1.6} \quad \text{Uncertainty in the measured signal} \]
\[ Y_{\text{err}} := Y_{1.0} \quad \text{Simulated measurements with error} \]
\[ \Lambda_1 := Y_{1.1} \quad \text{Wavelength Vector} \]
\( \epsilon := Y_{1,2} \)  
Stopping Criterion

\( \epsilon_{2,\text{test}} := Y_{1,4} \)  
Test variable for stopping criterion

\[
Y_{\text{avg}} := \frac{1}{\text{rows}(Y_{\text{err}}) - 1} \sum_{i = 0}^{\text{rows}(Y_{\text{err}}) - 1} Y_{\text{err},i}
\]
Average signal strength

\[
\text{SNR} := \frac{Y_{\text{avg}}}{\sigma_s}
\]
Signal-to-noise ratio

\( u_c := Y_{1,5} \)  
Uncertainty in the signal measurement equation

\( \text{sc} := \epsilon \)  
Stopping criterion redefined

Calculation of stopping criterion for actual measurements

\[
Y_{\text{Err}} :=
\begin{align*}
y & \leftarrow Y_{\text{meas}} \\
st & \leftarrow 0 \\
& \text{for } i = 0 \ldots \lambda_{\text{num}} - 1 \\
& \quad y_{h,i} \leftarrow y_i + \sqrt{\left[ (\text{Input}^{(3)}_i)^- y_i \right]^2} + 0 \\
& \quad y_{l,i} \leftarrow y_i - \sqrt{\left[ y_i - (\text{Input}^{(1)}_i) \right]^2} + 0 \\
& \quad st \leftarrow st + \left[ (\text{Input}^{(3)}_i)^- y_i \right]^2 \\
& \quad st \leftarrow st \\
& \quad y \leftarrow \text{augment}(y, y_l, y_h) \\
& \quad (y, st)
\end{align*}
\]

\( Y_{\text{Err}N} := Y_{\text{Err}}_0 \)  
Input signal

\( \epsilon_{2} := Y_{\text{Err}} \cdot 1 \)  
Stopping criterion for actual measurements

\( \text{sc} := \text{if}(\text{Actual} = \text{"yes", } \epsilon_{2}, \epsilon) \cdot 1 \)  
Determines which stopping criterion to use

\( \text{stoptol} := \text{stoptol} \cdot \text{sc} \)  
Defines the stopping tolerance for the second stopping criterion
\[ stoptol = 1.71 \times 10^{-12} \]

\[ se = 1.71 \times 10^{-8} \]
Actual = "ys"

\[ Y := \text{if}(\text{Actual} = \text{"Yes"}, Y_{\text{meas}}, Y_{\text{err}}) \]

Chooses which set of measurements to use

\[ Y_{\text{end}} := Y \]

Used for comparison after algorithm
Write the ordinary least squares norm

\[
SPM := Y - \hat{E}_{est}
\]

\[
SP := |SPM^T \cdot SPM|
\]

\[
SP = 1.2601883 \times 10^{-2}
\]

Write to output file

---

**Conjugate Gradient Algorithm**
DKM1 := 0  
Initialize direction of descent vector

Technique_2 := 
"Time counter"
  to ← time(1)
"while-loop control variable"
  stop ← 0
"Initialize trial function parameters"
  pk ← CgN
"Initialize minimum error solution"
  minpk ← pk
"Output to trial function file"
  pkout ← stack(0,CgN)
"Previous iteration trial function parameters"
  pkm1 ← CgN
"Previous iteration direction of descent"
  dkm1 ← DKM1
"Current sensitivity coefficients"
  Jm ← Juse
"Other structure variable initialization"
  ct ← 0
  jct ← 0
  kset ← 0
  other ← 0
  rmsct ← 0
  why ← "Null"
"Write intial parameters to files"
  WRITEPRN(OUTPUTtip,CgN_0)
  WRITEPRN(OUTPUTpk,(pkout)^T)
  Jold ← Juse
"Main loop"
  while stop ≠ 1
      "Calculate current and previous error"
      spmk ← Y − ME(pk,Zf)
      spkm1 ← Y − ME(pkm1,Zf)
      ""
      spkl ← (spmk)^T.spmk
spk ← |(spmk)ᵀ·spmk|
spkstore. ← spk

"Check if current iteration error is the minimum error"
minspk ← spk if ct = 0
if spk < minspk if ct > 0
  minspk ← spk
  minpk ← pk
  minct ← ct

spkm1 ← |(spmkm1)ᵀ·spmkm1|
kset ← 0 if kset = kreset ∨ ct = 0

"Arrange for output and output current iteration error"
spk1_a0 ← spk1
APPENDPRN(OUTPUTsp, spk1_a)

spkct 0, ← spk

"Check to see if the first stopping criterion is satisfied"
if spk < sc
  stop ← 1
  pf ← pk
  maxit ← ct – 1
  spkf ← spk
  why ← "Converged"
  break

"Calculate the sensitivity matrix for current and previous iterations"
Jm ← J/J(pk1, Zf)
J ← J/J(pk, Zf)

"Calculate current and previous gradients"
Dspk ← −2·(J)ᵀ·spmk
Dspkm1 ← −2·(Jm)ᵀ·spmkm1

"Calculate the conjugation coefficient"

\[
gk ← \sum_{i=0}^{N-1} \frac{[Dspk_i (Dspk_i - Dspkm1_i)]}{\sum_{i=0}^{N-1} (Dspkm1_i)^2}
\]

""
gk ← 0 if kset ≠ 0
kset ← kset + 1
ksetout_{ct} ← kset
pspk_{ct,1} ← gk

"Calculate the direction of descent"
dk ← Dspk + gk·dkm1

"Calculate the search step-size"
bkt ← ((J·dk))^T·(ME(pk,Zf) − Y)
bkb ← ((J·dk))^T·(J·dk)
bkv ← \frac{bkt}{bkb}
bk ← bkv
pspk_{ct,2} ← bk

"Set current values to be previous iteration values"
pk1 ← pk − bk·dk
pkm1 ← pk
pk ← pk1

"Check temperature criterion"
pk_0 ← pkm1·2 if pk_0 ≤ RoomTemp

for i ∈ 0..100
    ix ← i/100
    tPk_i ← TN(ix,pk)
    if tPk_i ≤ RoomTemp − 0
        for j ∈ 1..N − 1
            jp ← if pk_j < 0,−.5,1
            pk_j ← pk_{j,jp}
        kset ← kreset
    pk_{ct} ← pk
dkm1 ← dk
tiperr_{0} ← 1 − \frac{pk_0}{Tact_{0}}
pkout ← stack(ct,pk)

"Output data"

"***
APPENDPRN(OUTPUTpk,(pkout)^T)"
APPENDPRN(OUTPUTtip, tiperr)

"Check second stopping criterion"
if ct ≥ stopct + 1 ∧ (|spkstore_{ct} - spkstore_{ct-stopct}| ≤ stoptol)
  if spk < minspk
    minspk ← spk
    minpk ← pk
    minct ← ct
    pf ← minpk
    maxit ← ct - 1
    spkf ← minspk
    stop ← 1
    other ← |spkstore_{ct} - spkstore_{ct-stopct}|
    why ← "Unchanged SP"
rmstp_{ct} ← 0

"Stop if iterations are greater than maximum iteration value"
ct ← ct + 1
if ct ≥ maxiterations + 1 ∧ stop ≠ 0
  stop ← 1
  pf ← minpk
  maxit ← ct - 1
  spkf ← minspk
  why ← "Maximum Iterations"

"Output variables to be used further"
(time(1) − to)
  pf
  maxit
  spkf
  pks
  why
  other
  pspk
  tpk
  minct
  ksetout
OUTunits := o ← ORDER
ou ← "nA" if o = 9
ou ← "pA" if o = 12
ou ← "microA" if o = 6
ou ← "Unknown" if o ≠ 12 ∧ (o ≠ 9 ∧ o ≠ 6)
ou

OUTunits = "nA"

\[
\begin{align*}
\text{TIME} \\
\text{Cpred} \\
\text{Iterations} \\
\text{SPKf} \\
\text{PkS} \\
\text{Why} := \text{Technique_2} \\
\text{other} \\
\text{Values} \\
\text{tpk} \\
\text{minct} \\
\text{Kset}
\end{align*}
\]

Assigns variables from Conjugate Gradient Algorithm

\[
\text{Yfinal} := ME(Cpred,Zf)
\]

Predicted signals

\[
\text{Tpred}(z) := TN(z,Cpred)
\]

Predicted temperature profile

\[
\text{ErrorConfidence} := \text{pnorm} (\text{SigCt} \cdot \sigma, 0, \sigma) - \text{pnorm} (-\text{SigCt} \cdot \sigma, 0, \sigma)
\]

Error confidence interval

\[
\text{TipError} := \left| 1 - \frac{Tpred(0)}{TactN(0)} \right|
\]

\[
rz := 0, \frac{Zf}{20} .. Zf
\]

Conjugate Gradient Algorithm
Results

\( T_{\text{pred}}(0) = 1.342 \times 10^3 \)

Predicted tip-temperature

\( \text{TipError} = 0.126 \% \)

\( \text{SPKf} = 1.70306734211515 \times 10^{-8} \)

Root-sum-squares error

\( \text{sc} = 1.71 \times 10^{-8} \)

Stopping criterion

Why = "Converged"

Why the program terminated

\[ \text{C}_{\text{pred}} = \begin{pmatrix} 
1.342 \times 10^3 \\
86.087 \\
-119.261 \\
-361.397 \\
-438.529 
\end{pmatrix} \]

Predicted trial function parameters

\( \text{minct} = 55 \)

Solution with minimum error

\( \text{ErrorConfidence} = 99.73 \% \)

Confidence level for the uncertainty

\( \text{C}_{\text{pred}}F := \text{stack}(\text{Tact}_0, \text{C}_{\text{pred}}) \)

\[ \text{RMS2} := \int_0^1 \sqrt{\left( \frac{T_{\text{act}}(z) - T_{\text{pred}}(z)}{T_{\text{act}}(z)} \right)^2} \, dz \]

RMS temperature error

\[ \text{RMS} = \frac{1}{\lambda_{\text{num}}} \left[ \sum_{i=0}^{\lambda_{\text{num}}-1} \sqrt{\left( \frac{Y_i - Y_{\text{final},i}}{Y_i} \right)^2} \right] \]

RMS signal error
APPENDIX B PHOTOGRAPHS OF THE EXPERIMENTAL EQUIPMENT
Figure B.1  Optical detection system

Figure B.2  Set-up for calibrating the optical detection system with a blackbody calibration source.
Figure B.3  Location of furnace port in the laboratory furnace.

Figure B.4  Laboratory furnace used in the experiments.