Meaning in the Numberline

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William Blake said "if the doorways to perception were cleansed, the world would appear to man as it really is--infinite." It is our good fortune that the world does not appear to us in its infinite state; we should be overwhelmed by its vastness if we did not have, as Blake put it "doorways to perception" by which the information present in the world "out there" is selected and classified, as it enters our inner world of experience. Blake's 19th Century poetic statement finds modern scientific expression in Russell DeValois' (1966) recent experiments, which explicitly show the neurological mechanisms by which an infinite visual spectrum is categorized into separate colors. Indeed, such neuro-perceptual mechanisms are doorways to perception in providing structure to an otherwise infinite world.

Take, for example, the following design, analyzable from any number of different perspectives. The two perspectives I shall describe, however, demonstrate that the difference between the mathematical operations addition and multiplication is based on a simple difference in perception. For example, if the box in the upper left-hand corner is viewed as empty, gradually being filled with lines in the succeeding boxes to the right and down (the second box has one line, the third, two etc.) we have the basis
for addition. Hence, if we take the third box to the right, which has two lines and the fourth box down, which has three lines, and look where they intersect, we see a box containing five lines: \(2 + 3 = 5\). If, on the other hand we view the box in the upper left-hand corner as a plane, subject to successive division by the lines in the succeeding boxes to the right and down, we have the basis for multiplication. Hence, if we take the third box to the right to be divided into three parts by two lines, and the fourth box down to be divided into four parts by three lines, and look where the boxes intersect, we see a box divided into twelve parts: \(3 \times 4 = 12\).

Thus, the difference between addition and multiplication is here a mere difference in perception. Nothing in the referential schema changes; only the perceptual starting point shifts.

Jakobson describes a similar perceptual shift in terms of every day language (1973:52):

About the same bottle one can say that it is half empty or half full. Half empty and half full are not the same, they are not synonyms because they have a different frame of reference, a different starting point. Taking the empty bottle as unmarked is different from taking the full bottle as unmarked.

We thus take language to be a system of signs through which the speaker asks the hearer to view the referential world. If I the speaker choose to say that a bottle is half full, I am asking the hearer to view it differently than if I had said the same bottle was half empty. If I ask the hearer to view the upper left-hand corner of the design as a divisible plane, the mathematical consequences are ultimately different than if I had asked him to view it as an empty box.

We therefore define language as a device by which the speaker conveys information to the hearer; more specifically, it is a tool by which the speaker triggers the hearer's perceptual attitude; it is a means by which the speaker cues the hearer to view the referential world. These perceptual categories, both as the phonological distinctive features of the signans, and as the semantic conceptual features of the signatum ("interpretants" in Pierce's terms), work to "readjust...extrinsic matter, selecting, dissecting and classifying along their own lines."

Of paramount linguistic interest, therefore, is the nature of the above-mentioned distinctive and conceptual features, which significantly are central to Jakobson's enormous body of work on linguistics. Much has been said regarding the phonological, distinctive features, but the semantic conceptual features remain relatively obscure. They were first described in impressive detail in Jakobson's pioneering "Beitrag zur Allgemeinen Kasuslehre..." and later in his "Morphological Inquiry into Slavic Declension" which in my opinion are among the most original and important contributions to linguistic theory.

Their application to linguistic study has yielded significant statements about the semantic nature of the Russian case system, as well as other grammatical and lexical systems of Russian, the French tense system, the French prepositional system, to name a few.
In his discussion of the Russian case system, Jakobson demonstrates that the eight cases, taken together, form a perfectly organized system, as defined by their formal and concomitant conceptual markers. He speaks of 1) the unmarked case: the nominative, which has no associated conceptual feature, 2) the singly marked cases: accusative, marked with the conceptual feature "directionality," instrumental, marked with the "marginality" and the so-called genitive II, marked with the conceptual feature "quantification," 3) the doubly marked cases: dative marked with both "directionality" and "marginality," genitive I marked with both "quantification" and "directionality," locative II, marked with "quantification" and "marginality," and 4) the triply marked case: locative I, marked with "directionality," "marginality" and "quantification." Structurally, the paradigm may be presented as follows:

The three conceptual markers—"directionality," marginality and quantification—indissolubly linked to their formal markers, combine to form the logical system schematized above. These conceptual features, in their paradigmatic, syntagmatic and extralinguistic contexts, exemplify a portion of the invariant cues, the "interpretants," referred to above, by which the Russian speaker conveys information to his hearer.

In my investigation of the numberline as a semiotic system, I have found, predictably, that the numbers themselves are also signs by which information is communicated, and that they too, form a logical system, different from, but similar in logical rigor, to the case system described above. Most strikingly, close analysis of the different types of numbers (whole, negative, fractions, irrationals, and complex) has revealed conceptual features starkly similar to those defined by Jakobson in his description of the Russian case system. This is not to minimize the profound differences that exist between the two linguistic and mathematical systems, based on their form, their paradigmatic structure, their use, and other semantic differences. But, significantly, because both natural language and the language of mathematics are products of the human mind, it should not be surprising to discover that ultimately they share the same conceptual constitution. It is this conceptual constitution which is the subject of this investigation—not with the end of proving that mathematics is ultimately the basis of linguistic organization, or the reverse, but rather that both natural language and the language of mathematics have the same conceptual core.
Quantification

Mathematics, founded on the simple act of counting, is often called the language of quantification. Similarly, Jakobson named the genitive and the locative cases of Russian the cases of quantification. This term, quantification, is wholly appropriate both because the whole numbers (0 1 2 3 ...), which answer the question "how many X's are there?", express quantity, and because genitive/locative also quantify, as in the examples, stakan vody 'a glass of water' iast' doma 'a part of the house' or polnyj mysliej 'full of thoughts.'

This particular use of the genitive is called the Genitivus Partitivus, the partitive genitive, where "the use of the genitive signifies a definite or indefinite degree of involvement and thus establishes a spatial or temporal boundary."²

A second variant use of the genitive might be called contextual absence, which in Jakobson's terms signals a referent "which remains outside the prediction." Such is the meaning of zero. For example, if I asked the question "how many cotton wood trees are there in this room?" an appropriate answer would be zero because any existing cotton wood trees are absent from the given context of this room. Zero makes it possible to speak of things not present.

From a mathematical point of view the zero in 10 means that all counting numbers are absent in the context of the one column; 100 means that no counting numbers exist either in the one column or in the ten column, and so on.

The semantic notion given by zero--absence from a particular context--is consistent with certain uses of the genitive, as suggested above.

For example, limončika by! "Oh for a little lemon!" signals that the lemon has its existence independent of the speaker--the lemon is only imagined, not real. In the sentence vody, vody[G]...no ja naprasno stradal'cu vodu[A] podaval (Puškin) 'Water, water!...but in vain I offered the sufferer water' the first two instances of water are perceived absolutely outside the sufferer's consumption--we know that he never tastes of the water. In Jakobson's terms, the referent "remains outside the prediction."

Normally in Russian the accusative marks the so-called direct object, but it is usual for the direct object to take the genitive, if the sentence is negated:

ne našel kvartiry 'found no apartment'
našel kvartiru 'found an apartment'

The co-occurrence of the genitive and negative makes perfect semantic sense because the verbal action (including the direct object) remain outside the predication.
The above rule is not iron-clad, however. The accusative can be found marking the direct object in negative propositions, but with semantic consequences:

ja ne slychal etoj sonaty[G]  
'I have not heard the sonata'  

ja ne slychal etu sonatu[A]  
'I have not heard the sonata'  

In the first sentence, according to Jakobson "the emphasis is on the unknown-ness of the sonata on the part of the speaker"; the sonata is predicationally absent, whereas in the sentence with the accusative "this emphasis is lacking and the fact that I have not heard it becomes mere accident, which is unable to eliminate the sonata from the predication--the presence of the sonata takes precedence: this nuance requires the A in contrast to the G." (p.27)

To summarize, quantification is found both in the whole numbers and in the genitive (and locative) cases of Russian, in both the contextual variants described by Jakobson:

1) the counting numbers are analogous to the genitival use that signals partitive.

2) zero is analogous to the gentival use that signals absence from predication.

Marginality

Up to this point the discussion has included only the whole numbers, which can be represented on the numberline as follows:

0 1 2 3 4 5...

The system of whole numbers is infinite in that a new segment can always be added to the right, thus increasing the length and number. But such changes in the numberline are only quantitative. The numberline can be changed qualitatively (with semantic consequences) in its infinite extension to the left, with the addition of the negative numbers:

... -3 -2 -1 0 1 2 3 ...

The perceptual attitude triggered by the negative numbers is comparable to the perceptual cue given by the so-called peripheral cases, the instrumental, the dative, the genitive I and the locative I.

The definition of the negative numbers, as well as those grammatical morphemes that have the conceptual feature marginality, often includes the notion of cancellation. Notice how the notion cancellation figures in the following typical mathematical definition of the negative numbers (Wheeler, p. 122):

If a is a natural number, then -a will be defined to be a unique number such that a + (-a) = (-a) + a = 0
That is, $5 - 5 = 0$. In other words, in the context of addition, the negative tells the mathematician to perform an operation of cancellation. Children learning to subtract are often given such concrete examples of cancellation as: if I had five apples and ate two, how many would be left? Later they learn that the minus sign, $-$, in the context of addition means cancellation.

Jakobson's conceptual feature "marginality" is similarly described. C. H. Van Schooneveld (1977:4) says that "the Russian instrumental and such prepositions or preverbs as vy-, ot, and iz have in common the cancellation of the initial narrated situation and the break between this situation and the ensuing situation."

Cancellation, however, is only one contextual perspective. In further describing the Russian cases that share the feature marginality, Jakobson uses the term "peripheral": The instrumental "indicates that its referent occupies a peripheral status in the overall semantic content of the utterance." Similarly on the numberline the negative numbers are peripheral in the sense that they presuppose the counting numbers, which are perceptually central to the numberline. Jakobson is careful to point out that "what is specific to the peripheral cases is not that they indicate the presence of two points in the semantic content of the utterance, but only that they render one peripheral with respect to the other." (Pp. 33, 34) Likewise, on the numberline 2 and 4 represent random points, as $-1$ and 1 might. But $-1$ is peripheral with respect to 1, whereas 2 has no such relationship to 4. Each, peripheral, negative number has associated with it a central, counting number, but the reverse is not true: one can imagine counting without negative numbers, but it is difficult to imagine a negative priority over the positive numbers. Thus, in counting, the negative numbers are marginal or peripheral, while the whole numbers are central.

A similar marginal/central distinction emerges in the study of the instrumental. For example, in the sentence *oxotnik[N] ranil olenja streloj[I]* 'the hunter wounded the deer with an arrow' the focus is on the agentive hunter, while the instrumental arrow is peripheral. On the other hand, in the sentence *olenja ranila strela[N] 'the arrow wounded the deer' the focus in on the arrow—no comment is made regarding peripherality.

In sentences as on *zdes' sud'ej 'he functions here as judge,' budet sud'ej 'he has been elected judge,' stal sud'ej 'become a judge,' on isbran sud'ej 'he has been elected judge,' sud'ej on posetil nas 'he visited us as judge,' one sees the centrality of the individual referred to by the full cases, and the marginality of the individual as given by his role as judge. Here the instrumental "refers to the same entity as the corresponding (expressed or implied) full case in the same sentence, and signifies that a special function of that entity—a passing, occasional property—is involved." (p. 37)

When we pass from the negative numbers of mathematics to the marginal cases of Russian, we at first glance appear to have changed worlds; all the landmarks seem so dissimilar. But a broader view reveals compelling parallelisms. The negative numbers can be described in terms of
Cancellation in the context of addition, just as van Schooneveld similarly described the semantic effects of the instrumental. In the context of the numberline the negative numbers are peripheral to the central counting numbers, just as the referent of the instrumental is marginal with respect to a centrally implied referent.

Directionality

With the addition of the negative numbers, we have a numberline which extends infinitely to the right (counting numbers) and infinitely to the left (negative numbers), with zero as the origin.

The next qualitative change is given by subdividing, or in other words by providing for fractions in the numberline, as represented in the following:

\[
\begin{array}{cccccccccccc}
\ldots & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \ldots \\
0 & 1 & 2 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\end{array}
\]

In this case each unit is subdivided into aliquot parts. For example, the unit from 7 to 8 is divided into eight units: \( \frac{1}{8} \) or the unit from 1 to 2 is divided into two units: \( \frac{1}{2} \). This systematic subdivision now permits us to understand what fractions mean. We will look at the fractions from two points of view, each of which is equally valid; each of which figures prominently in Jakobson's description of the accusative case.

The first perspective is given in these terms: a fraction, for example \( \frac{1}{8} \) has a numerator of one and a denominator of eight. The denominator, eight, tells us where on the numberline to start counting (on the subdivided unit segment between 7 and 8) and the numerator, one, tells us how many subdivided segments to count (one): \( \frac{1}{8} \).

The conclusion to be drawn is simply this: the geometric value of the numerator is determined by the denominator. The numerator one of \( \frac{1}{8} \) is this long \( \_ \), whereas the numerator one of \( \frac{1}{2} \) is this long \( \_ \). The numerator in and of itself has no specific geometric value. One, for example, could be geometrically equivalent to two given a denominator of four and eight respectively: \( \frac{1}{4} = \frac{2}{8} \).

It might be said that a fraction specifies two things: a) how much and b) in which denomination. Twenty has quite a different value in the denomination of dollars than in the denomination of cents. The denomination ascribes the value.

From this perspective it is now obvious why \( \frac{0}{8} \) is a possible fraction, whereas its inverse \( \frac{8}{0} \) is not. \( \frac{0}{8} \) says to count nothing in denominations of 8, whereas \( \frac{8}{0} \) requires something that is impossible: to count to eight in a nonexistent denomination. Simply put, a fraction with a zero denominator has no meaning in the logic of mathematics: nothing exists by which the value of the numerator can be ascribed.
This first view of fractions we shall call "ascription" because the denominator ascribes a relative value to the number given by the numerator.

The second perspective is given in terms of an imaginary trip. Suppose that one were to take a trip on the unit segment from 7 to 8, which has been subdivided so, 7 8, where zero represents the point of departure and eight, the goal. Thus, 1/8 means that on the trip from zero to eight, the traveler has gone the distance of one toward the goal of eight: 12345678. 2/8 would mean of course that he has traveled two towards eight: 7 8. 0/8 would mean that he stayed home, whereas 8/0 is impossible because no goal is given.

This second view we shall call "goal" because the denominator is the goal with respect to the numerator.

The notion of goal, as suggested above, coincides with Jakobson's description of the accusative, which in its primary contextual variant of marking the direct object, "denotes an object upon which is directed the action of the verb." (p. 6 Morphological). Thus, čit’ knigu 'to read a book' the action of reading is directed on the book. In the phrase na stol 'onto the table' the notion of goal is present, whereas in na stol'ë 'on the table' no such goal is apparent.

The notion of ascription (recall that the denominator ascribes a specific value to the numerator) emerges in the so-called weakly governed accusative "where a segment of space or time...is entirely filled by the action," as žit' god 'to live a year' idt'i v'erstu 'to go one verst'. Here one's living or one's going is given a specific value as determined by "a year" or "one mile" respectively.

The remaining untouched portions of the numberline are those segments bounded by the fractional numbers. These bounded segments contain the so-called irrational numbers, which can be defined in terms of van Schooneveld's conceptual feature dimensionality. A full description, however, exceeds the limits of this paper.

The final number type is the complex numbers, which in effect, turn the numberline into a Cartesian plane, the points of which are codetermined by the x and y axes. Their conceptualization can be defined in van Schooneveld's conceptual feature "duplication," the elaboration of which is reserved for another paper.

The expansion of the numberline can be described in these structural terms:
The right-to-left numberline becomes more and more dense, until it is completely filled with the real numbers. It is finally extended upward and downward, giving the imaginary numbers, which together with the real numbers constitute the whole numbers.

Conclusion

The proposition of this paper is that the language used by Jakobson in describing the conceptual features of the Russian cases is starkly similar to the language needed to describe the information conveyed by the whole numbers, negative numbers and fractions; that is, the conceptual features isolated in describing the cases are those features necessary to the description of the number types above.

Such a proposition is based on the assumption that language is a system of signs, paradigmatically given and ordered, by which information is encoded and decoded. The nature of such information, which is invariantly linked to its signans, and mediated by its paradigmatic, syntagmatic and referential context, is given at least in part by the conceptual features quantification, marginality and directionality.

We emphasize that these features do not derive from the referential world, but are rather perceptually given, and as such are doorways to perception, as it were, or interpretants by which the in-coming information of the referential world is classified and categorized.

Of course, we are only beginning to understand such semantic similarities as are proposed in this paper, but in Sapir's terms (p. 144) "some day, it may be, we shall be able to read [from our further research] the great underlying ground plans." If indeed the proposition of this paper has validity, then we are that much closer to understanding the essence of man's greatest intellectual endowment--natural language, and in a broader sense, language in all its aspects.
FOOTNOTES

1 This is quoted from Waugh (1975) who takes it from a source unavailable to me: Jakobson (1949).

2 My quotations of Jakobson's "Beitrag..." and "Morphological Inquiry..." are taken from xerox copies of English translations done by Kenneth L. Miner and Rodney Sangster respectively.

BIBLIOGRAPHY


