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You Do Math Like a Girl: How Women Reason Mathematically Outside of Formal and School Mathematics Contexts

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ABSTRACT

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Outside of Formal and School Mathematics Contexts

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Females continue to have negative dispositions towards mathematics even though the performance gap between females and males has all but disappeared. While there are many hypotheses for why these negative dispositions exist among females towards mathematics, this paper explores the possibility that the field of mathematics could favor more masculine ways of reasoning at the exclusion of valid, non-masculine mathematical thought. To research this idea, the day-to-day, non-formal, non-school mathematical activities of two women were identified and analyzed. The analysis uncovered complex mathematical processes among both women that were fundamentally different from the mathematical processes common in the mathematics field. Such results seem to affirm the idea that females' ways of doing mathematics are not acknowledged or validated by the mathematics community and therefore suggest the development of more inclusive mathematics research and instruction.

Keywords: women in math, female perspective, everyday mathematics, context, quantities, quantitative process

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FOREWORD

It is important that I provide clarification on how I refer to gender throughout this thesis. In this paper, I adopt the perspective that gender is binary in order to explore the idea that the mathematics field might favor more masculine ways of doing mathematics at the expense of non-masculine mathematical activity. I therefore seek to give voice to non-masculine perspectives by collecting data from female participants. It is possible that female-centric perspectives would also resonate with those who see gender as a spectrum or in other non-binary ways. It is also possible that those who identify as men do not reason about mathematics in traditionally masculine ways. By adopting a gender-binary perspective, I am not meaning to suggest that these other perspectives or experiences are invalid or inaccurate in any way. It is my hope that the field of mathematics will continue to open up mathematical spaces for all peoples, regardless of how they identify.

In this thesis, I use the word "females" to refer to female humans. I use "girls" and "women" to refer to female humans who are under the age of 18 and over the age of 18, respectively. I use "boys" and "men" similarly. I do not seek to limit who is categorized as girls and women or boys and men, as I believe gender is partially socially constructed. For this study I chose two humans who identify as women to be participants. Both women also happened to be assigned the sex of female at birth.

CHAPTER 1: INTRODUCTION

Although the achievement gap has diminished between boys and girls in secondary mathematics (Mullis, Martin, Foy, & Hooper, 2016; Mullis, Martin, Foy & Arora, 2012), the disposition boys and girls have towards mathematics is very different. Research shows that middle school and high school females have a more negative disposition towards mathematics than their male counterparts, generally feeling more dislike towards the subject, believing that mathematics is not as relevant for their future (Samuelsson & Samuelsson, 2016; Meyer & Kohler, 1990), experiencing mathematics as a chilly environment where they do not belong (Lubienski & Ganley, 2017; Herzig, 2004), and feeling insecure and anxious about their mathematical ability (Else-Quest, Hyde, & Linn, 2010; Hill, Mammarella, Devine, Caviola, Passolunghi, & Szucs, 2016; Lacampagne, Campbell, Herzig, Damarin, & Vogt, 2007; Devine, Fawcett, Szucs, & Dowker, 2012). Researchers suggest several reasons for this gendered difference in mathematical dispositions, particularly teacher beliefs and actions that marginalize females in the mathematics classroom, as well as mathematics classroom activity that is not inclusive of female Discourse (Gee, 1989).

Marginalization of Females in Mathematics

Teacher Beliefs and Actions

Past studies have shown that mathematics educators believe boys to be naturally better at mathematics than girls. Specifically, mathematics educators subscribe to the idea that boys are mathematically gifted, and girls must work harder than their male counterparts in mathematics to mirror the males' mathematical achievement (Li, 1999; Fennema, Peterson, Carpenter, Lubinski, 1990). For example, Fennema et. al (1990) asked mathematics educators to name the most successful and least successful students in their classes. When naming the most successful students, 79% of teachers named a boy as the top student, and 56% named a boy as the second highest performing student, overestimating boys' mathematics performance about 20% of the time (Fennema et. al, 1990). Teachers in this study also attributed boys' high mathematical performance more frequently to ability and girls' high mathematical performance more frequently to effort. While it is possible that mathematics teachers' beliefs about girls' and boys' achievement have changed over the past 30 years, no follow up studies have been conducted, so it is possible that mathematics teachers' beliefs may still be inequitable.

But even if teacher beliefs about males' and females' abilities to do mathematics have changed since the 1990s, research shows that teacher actions are still biased in favor of males in the mathematics classroom. Particularly, teachers respond more to male students than female students (Leyva, 2017; Oakes, 1990); they make more encouraging comments to males about their mathematical potential (Becker, 1981); and they ask more open-ended questions to males and more procedural questions to females, creating for males the expectation and opportunity to think more conceptually about mathematical concepts (Duffy, Warren, & Walsh, 2001). Teachers also interact more colloquially with male students, joking and speaking about extra-curricular topics more frequently with them than female students (Solomon, Lawson, & Croft, 2011; Becker, 1981). Thus, teachers seem to be socializing boys into the mathematics community more often than they do with their female students, which could contribute to female students' marginalization in mathematics.

Mathematics as a Masculine Discourse

Other studies have shown that some teachers equate mathematical success with more masculine behaviors like speaking up frequently in class, talking authoritatively, working on tasks independently, challenging mathematical ideas and procedures, and playing the mathematics game of getting to the right answer the fastest (Solomon et. al, 2011; Lacampagne et. al, 2007). These masculine ways of acting and speaking are what I mean by a masculine Discourse (Gee, 1989). However, a masculine Discourse conflicts with more feminine patterns of mathematical activity that favor collaborating with other students, qualifying original mathematics ideas with uncertainty, and needing a deep conceptual understanding before feeling comfortable with a procedure (Solomon et. al, 2011; Lacampagne et. al, 2007) Unfortunately, female students are not only experiencing mathematics in a masculine Discourse, they are also discouraged from even adopting this masculine Discourse. Teachers prefer female students to follow directions, not question authority, and be “good girls” (Walkerdine, 1998), which means that the authoritativeness, risk-taking, and outspokenness that mathematics teachers seem to value cannot be achieved “appropriately” by females. Females are therefore stuck in a paradoxical situation: if they are to be good at mathematics, then they cannot be feminine; and if they are to be feminine, they cannot be good at mathematics (Walkerdine, 1998).

This paradox exists not only in patterns of valued behavior but also in the definition and pedagogy of mathematics itself. Mathematics is often abstract, taken out of contexts that make it more relatable and meaningful. In fact, abstraction is valued because mathematics is taken out of specific situations and generalized to become applicable to many scenarios. Unfortunately, focusing on abstraction discourages students from joining the mathematics field, since they see little real-world relevance in the subject (Lacampagne et. al, 2007). Focusing on abstraction also contributes to the overlooking of complex mathematics accomplished in everyday life, especially in contexts that generally attract women. Quantitative reasoning, maximization, spatial awareness, and other mathematical skills are all necessary in quilting, sewing, basket-weaving, and interior designing; however, these skills are deemed *feminine,* not *mathematical* (Walls, 2009).

Why Problematic?

This bias towards males over females in mathematics is problematic for several reasons. First, we see that inequitable education exists in a land touted to be the "Land of Equal Opportunity." If public education is supposed to provide all students with the same opportunities for learning, then the aforementioned studies show that this is not always taking place.

Second, this marginalization of females in mathematics correlates with low female participation in STEM fields. As of 2015, 47% of all jobs in the United States were filled by women. However, in that same year, only 24% of STEM jobs were filled by women (Noonan, 2017). Since most STEM jobs have higher earning potential than non-STEM jobs (Noonan, 2017), we are essentially dissuading women from participating in careers that would offer them better access to health care, better education, and better opportunities for social capital. Additionally, the dearth of women in STEM fields means we are limiting potentially powerful contributions to the field of mathematics, since we have a fraction of the qualified minds we could have in the field.

Some might argue that females' lack of participation in STEM fields is a personal choice. Research suggests that women are more inclined to choose professions that work with people (e.g., teacher, nurse, caregiver) instead of objects (e.g., engineer, electrician, plumber, architect) (Eccles & Wang, 2016). Women are also not as motivated by money in their career choices as men, being more swayed by enjoyment of the work they would be doing and how easily they can balance work and family life in their careers (Eccles & Wang, 2016; Zafar, 2013). Using these findings, one could conclude that the lack of female participation in STEM fields is not due to marginalization; rather, women have the freedom to choose careers they enjoy without the societal pressure of finding high-paying jobs in STEM fields to support their families. Though this may be the case, the fact remains that girls are reporting higher levels of mathematics anxiety, insecurity, and exclusion than boys (Else-Quest, Hyde, & Linn, 2010; Hill, Mammarella, Devine, Caviola, Passolunghi, & Szucs, 2016; Lacampagne, Campbell, Herzig, Damarin, & Vogt, 2007; Devine, Fawcett, Szucs, & Dowker, 2012). These adverse experiences do not seem symptomatic of a group that simply chooses to do something else with their career.

Past Reform Efforts

Past reforms have tweaked elements of mathematics instruction to make mathematics instruction more equitable for women by adding in mathematical tasks that are more collaborative or conceptual (Hyde & Lindberg, 2007; Boaler & Irving, 2007). Because the performance gap has closed, it is possible that these reforms might have contributed to more equitable instruction. However, few studies have questioned the core mathematical values and practices that have existed for centuries. It is my hypothesis that as we open up the definition of what counts as doing mathematics to better include females, females will have a more positive disposition towards mathematics.

CHAPTER 2: BACKGROUND

Historically, the field of mathematics has been dominated by men. We see evidence of this in the volume of publications written by men in mathematics (Mihaljevic-Brandt, Santamaria, Tullney, 2016), the number of men recognized for mathematical achievement or discoveries, and especially the laws and culture that have historically excluded women from mathematics publication and even mathematics learning (Osen, 1974). It follows, then, that a subject historically developed by, practiced by, and taught to men would reflect a masculine Discourse. I use Gee's (1989) definition of Discourse throughout this paper, specifically that Discourse means the "words, acts, values, beliefs, attitudes, and social identities as well as gestures, glances, body positions, and clothes" a person must take on in order to be recognized as part of a given group (p. 7). A masculine Discourse is one in which the ways of speaking, acting, and believing that are associated with being male are privileged. It is my hypothesis that women feel insecurity and anxiety about mathematics, much more so than men, because mathematics reflects a masculine Discourse, which makes mathematics more comfortable and "natural" to men than to women.

As was mentioned in the chapter above, mathematicians and mathematics teachers typically favor masculine behaviors (e.g., speaking out, challenging ideas, working independently). However, the maleness of mathematics can actually go deeper than just preferred ways of acting to the very foundation of mathematical activity. From my review of the literature, three common themes of mathematical activity appeared: abstraction, deduction and formalisms, and the widespread acceptance of instrumental understanding. In this chapter, I will describe each of these aspects of mathematics more fully, specifically addressing how each aspect might contribute to the marginalization of females in mathematics.

Abstraction

The abstraction process starts with looking beyond the superficial characteristics of two or more situations and recognizing sameness in the situations' underlying structures (Mitchelmore & White, 2004). These structures can contain objects, properties of those objects, and operations performed on those objects. For example, our experience with manipulating real world objects has led to the construction of whole numbers, their properties, and the arithmetic operations we perform on them—all of which make up a structure that seems to underlie many real world situations. Abstraction is complete when we no longer need to refer to the original situations the structure came from in order to make sense of and talk about the structure. In other words, the structure takes on its own entity (Mitchelmore & White, 2004). Once we have multiple abstract structures, we may consider looking across those for underlying sameness in structure, just like we did before, and then create another level of abstraction. Theoretically, the number of times we perform the abstraction process has no limit. Abstraction is useful in mathematics because general, decontextualized structures can be applied to many different contexts.

Though abstraction is useful, there is pushback from some mathematics educators about favoring abstraction at the expense of context. Research shows that students can better understand symbolic manipulations (i.e., operating on abstract structures) when the quantities are first given in context (Walkington, 2013; Mitchelmore & White, 1995), especially a context that is meaningful to them. Walkington (2013) found that students who were given mathematics problems in contexts that were personalized to the students' individual interests were much better at solving those problems than students given traditional story problems from their algebra textbook. In fact, the personalized story problems correlated with increased performance in writing symbolic equations about the relationships given in context. And even after these contexts were removed, students continued to be successful in abstracting relationships to symbolic equations—particularly students who had *not* been successful with abstraction previously in the course. Thus, contexts do not seem to be limiting students' mathematical understanding or adaptability; rather, familiar and interesting contexts can act as access points for students to understand mathematical relationships. And once this stronger foundation of understanding is in place, students show increased success in doing symbolic manipulations.

Contexts also give students purpose. In a study completed by Lacampagne et. al. (2007), graduate students dropped out of their respective mathematics graduate programs because the mathematics they were spending all their time on was devoid of context. They felt that they were not contributing meaningfully to the world.

Contexts also seem to be a preferred element of mathematical problem-solving for women. Heffler (2001) found that women were much more likely to prefer concrete, experiential learning over abstract conceptualization than men. Specifically, women self-reported feeling more partiality towards learning that required feeling-based judgements and concrete role-play and was more people-oriented and ambiguous. Boaler's (1994) research shows girls' aptitude in contextual, experience-based mathematics. In her study, girls and boys were given problems about different mathematical concepts, each concept having both an abstract and contextualized question. Her findings showed that the boys were better able to focus only on the relevant numbers in the contextual task, whereas the girls reasoned through the contextual task using their mathematical knowledge as well as their knowledge gained from personal experience in the given context. Unfortunately, the girls performed worse on those questions with familiar contexts because they accounted for real-world variables that would naturally be in those contexts but were not included in the question. These results suggest that while boys may play the game of mathematics (i.e., get to the right answer the fastest), girls actually think more carefully about real-world contexts than boys do. In other words, if the girls in this study were actually solving problems in the real world, they would apply mathematics more sensibly to the situation than their male counterparts. However, because even the contextual tasks were more oriented towards dismissing contextual variables for the sake of abstraction, girls were penalized.

Additionally, emphasizing abstraction delegitimizes authentic mathematical activity that is only ever embedded in context. For example, seamstresses, construction workers, indigneous basket-weavers, and interior decorators all engage in complex spatial reasoning, pattern-building, and optimizing--all legitimate mathematical practices But their work is not deemed mathematical (Walls, 2009), as it is context-dependent.

With these converse arguments in mind, we see how context is actually helpful in students' understanding of relationships and that favoring abstraction actually excludes a large portion of what makes mathematical activity seem natural and meaningful, particularly to women.

Deductive Reasoning

In addition to abstraction, mathematics favors logic and reason. From the literature, we see logic and reason in mathematics approached in two different ways: deduction and induction (Lange, 2009; Hanna & de Villiers, 2008; Knuth & Elliott, 1998). Deductive arguments are often given inside an axiomatic system, where results are built upon a foundation of axioms and their consequences until the result of interest is proved as a natural and indisputable consequence of the set of axioms upon which it was predicated (Christiansen, 1969). Simply, deduction is proving a result based on theorems and axioms that are either taken as given or already proved. Conversely, induction involves recognizing patterns in one's own experience and forming conjectures about whether those patterns will continue in the same situation or apply to another situation (Christiansen, 1969).

Deduction is the reasoning of choice in mathematics, and is the only accepted method of proving knowledge in the field. Theorems and definitions are presented in their final, deductive form in journal articles, textbooks, etc., and mathematics teachers expect deduction from their students in secondary mathematics (e.g., two-column proofs in high school geometry) and college mathematics (e.g., formal proofs in an introduction to proofs course). Though induction is acknowledged in mathematics and mathematics education literature, it is mostly recognized as a natural but immature, preliminary step to deductive reasoning (Lange, 2009; Knuth, 2002; Herbst, 2000; Sowder & Harel, 1998; Knuth & Elliott, 1998). However, inductive reasoning has more merit than mathematicians might allocate to it.

Though deduction is the standard of finished work, mathematicians still rely on induction to conjecture about potential truths they have yet to deductively prove. Polya (1954) explains the need for induction when explaining his own problem-solving process:

> "...having verified the theorem in several particular cases, we gathered strong inductive evidence for it. The inductive phase overcame our initial suspicion and gave us a strong confidence in the theorem. Without such confidence we would have scarcely found the courage to undertake the proof which did not look at all a routine job. When you have satisfied yourself that the theorem is true, you start proving it" (pp. 83-84).

Polya claims that after seeing a certain pattern hold for several cases, he and his colleagues inductively assumed that the pattern would continue enough to be a theorem. Though they had yet to prove the theorem deductively, their empirical evidence convinced themselves that the theorem could be proven true. Polya acknowledges that it was the inductive process that gave himself and his colleagues the confidence to continue on to prove the theorem deductively. Thus, induction was absolutely necessary in accomplishing their proof by deduction.

Induction also correlates with ways that women may reason naturally. The intuition that exists in induction is similar to the "inner voice" talked about in Belenky, Clinchy, Goldberger, and Tarule's book *Women's Ways of Knowing* (1986). Belenky et. al, describe a path that women take in their knowledge and moral development throughout their lives. A crucial stage of knowledge development is transitioning from received knowledge—where women take as given the ideas and positions told them by others in authority, commonly men—to subjective knowledge, where women give authority to and trust their inner voice or intuition. This inner voice is described as a gut feeling or intuitive reaction to an idea or situation, "something experienced, not thought out, something felt rather than actively pursued or constructed" (p. 69). According to Belenky et. al, giving authority to their inner voice is fundamental to the intellectual development of women. At this stage, women are able to see how truth can be subjective and how one person's idea of truth is localized to his or her own set of experiences. Once women acknowledge the power of their inner voice, they are able to progress to reasoning that integrates both the voices of others and their inner voice to create a more balanced, robust knowledge.

This path of knowing for women seems to mirror the ways students—and as Polya suggested, mathematicians—naturally use induction and deduction in mathematics. For students to move past the blind faith of relying on received knowledge from their teachers and textbooks, they must start exploring and making sense of the mathematics for themselves. Through exploration and sense making, students develop intuition about mathematics, similar to the "inner voice." It is this intuition that helps them recognize and form conjectures, and provides them support to move forward deductively. Thus, induction is imperative for students, and maybe especially girls and women, in mathematics, as it enables them to engage in the activity of doing mathematics through recognizing, forming, and confirming conjectures.

Instrumental Understanding

Though there are many definitions of understanding in the mathematics education literature, Skemp's (1976) definition has proven useful in distinguishing between two general approaches to mathematics. Skemp argues that there are two types of mathematical understanding: instrumental and relational understanding. Instrumental understanding is the memorization of mathematical rules and algorithms *without* understanding the reasoning behind them. Whereas, relational understanding is "knowing what to do [mathematically] and why" (p. 89).

While mathematics is certainly not instrumental in nature, traditional mathematics instruction is (Boaler, 2002; Stipek, Givvin, Salmon, MacGyvers, 2001; Skemp, 1976). Students are able to succeed in traditional mathematics classes, even some advanced mathematics classes, with only an instrumental understanding of the curriculum. This lack of relational understanding becomes problematic, however, when the memorization of so many rules and procedures gets to be too much to retain without some relational connections. Solomon, Lawson, and Croft (2011) found that women especially lose confidence in their mathematical abilities in upper-level mathematics classes when they are taught mathematics instrumentally. They describe the experience of a mathematics undergraduate student, Debbie, who sought relational understanding about multiplying matrices. Though she could accurately perform the procedures of matrix multiplication, she felt insecure about her understanding since she did not understand *why* matrix multiplication was done that way. Similarly, another mathematics major in the study, Diane, feared that she would do poorly in her undergraduate mathematics classes, even when getting questions right, because she did not understand why she was getting the correct answer. She knew how to do the procedure, but because she did not understand why the procedure worked, she felt insecure about her mathematical understanding. Interestingly, males in the same year as Debbie and Diane did not feel the same insecurity as their female counterparts. They recognized they should probably care more about relational understanding, but since they were performing well in their courses, they did not find relational understanding necessary. Solomon et. al. (2011) suggest that the lack of relational understanding in mathematics courses could be contributing to females' negative disposition towards mathematics, specifically their insecurity in their mathematics understanding and performance.

Boaler (2002) found that females perform better in classes that teach for relational understanding. She describes the results of two schools in England, Amber Hill and Phoenix Park, that although demographically similar, taught mathematics in contrasting ways. Amber Hill took a procedural, skills-based approach to mathematics teaching, whereas Phoenix Park taught mathematics in an open-ended, concept-based way. At the end of her three year study, Boaler found that there was a statistically significant gap in achievement between boys and girls at Amber Hill, with boys outscoring girls on nationalized tests and course grades. Conversely, there was no gender disparity in achievement at Phoenix Park, with both boys and girls at Phoenix Park performing significantly better than the Amber Hill students. This suggests that a conceptual based approach to mathematics might create a more equitable learning environment, especially for female students who seem to be disadvantaged by a procedural approach.

Boaler (2002) used “procedural” and “conceptual” to describe the teaching of Amber Hill and Phoenix Park respectively, not “instrumental” and “relational.” I recognize that the terms “procedural” and “instrumental” are not synonyms in mathematics education literature, nor are “conceptual” and “relational.” However, these terms have significant overlap with each other: procedural and instrumental approaches focus heavily on methods and formulas, and relational and conceptual approaches emphasize mathematical meaning behind the methods and formulas. These similarities between terms show that Boaler’s data generally supports the idea that women perform better when learning meaning behind rules and procedures in mathematics, not just the rules and procedures alone.

Unfortunately, not many papers have been written about the gendered effects or gendered preference of Skemp’s (1976) two types of understanding. Even papers that mention procedural and conceptual approaches to mathematics have not looked at those approaches through the lens of gender. Thus, I use Solomon et. al. (2011) and Boaler’s (2002) papers to represent the literature published on the gendered effects of teaching for instrumental understanding compared to relational understanding. From their papers we see that it is likely that female students prefer and actually perform better in classes that teach for relational understanding.

Theoretical Framework

Abstraction, deductive reasoning, and instrumental understanding are often defining characteristics of mathematical activity, particularly in formal and/or school mathematics. In order to find a more inclusive mathematics Discourse, we need to look outside of formal and school mathematics contexts, particularly where women engage with mathematical activity that is comfortable to them. This raises the question, then, about what counts as mathematical activity, outside of formal and school mathematics contexts.

There are at least two camps about what defines mathematical activity—pattern recognition or pattern building (Mulligan & Mitchelmore, 2009; Mulligan, Prescott, Papic, Mitchelmore, 2006; Mumford, 2002) and quantitative reasoning (Thompson, 2011; Castillo-Garsow, 2012). Some mathematicians describe mathematics in terms of pattern building because there is a lot of pattern recognition and creation of patterns in mathematics. However, the pattern building approach to defining mathematics seems to encompass a lot of non-mathematical activity as well. For instance, realizing that an unreliable friend will likely forget to come to a social event is recognizing your friend's behavior and inferring future behavior based on old patterns. This qualifies as pattern recognition and pattern building; however, there does not seem to be mathematics present in the analysis of the situation. In contrast to the pattern building approach, quantitative reasoning excludes some activities that do not seem mathematical, while still being general enough to encompass a lot of mathematical activity outside of what is normally done in school mathematics. Therefore, I choose to use quantitative reasoning as the defining feature of mathematical activity. I am choosing to use Thompson's (1990) framework of quantitative reasoning because he provides a specific, detailed theoretical model of quantity-based reasoning in mathematics, and his framework is well-known.

I recognize that I am using a framework that was developed by a man and then accepted in a generally male-dominated field. One could say that choosing such a framework to analyze female mathematical activity seems to be pigeon-holing female mathematical activity to what fits in a male lens. In a sense, I could be perpetuating the exclusion that I am advocating to eliminate. Though I acknowledge the validity of these claims, I am not arguing that the results of this study will define a feminist mathematics. Rather, I am hoping to expand the current definition of mathematics to better include feminine mathematical activity.

Quantitative Reasoning

Thompson (1990) defines quantitative reasoning as "the analysis of a situation into a quantitative structure—a network of quantities and quantitative relationships" (p. 12). Though seemingly broad, Thompson fleshes out his definition of quantitative reasoning by also defining quantity and quantitative relationship. According to Thompson (1990), a quantity is a measurable quality of an object that is then given an appropriate unit of measurement. It follows that quantification is the process by which someone assigns numerical values to qualities. A quantitative relationship, therefore, is " the conception of three quantities, two of which determine the third by a quantitative operation [e.g., addition, subtraction, multiplication, division, composition]" (p. 11). For example, consider the quantitative relationship of doubling flour in a recipe. One must first have the quantity of the original flour. Then in order for the flour to increase in size, we know we must perform some quantitative operation. In order to know how much the original flour amount should increase, we have the quantity associated with doubling, which is 2. The third quantity, the amount of new flour, is therefore determined by multiplying the first quantity (original flour amount) by the second quantity (the number of times as large the quantity of new flour amount is as the original quantity of flour). The "doubling quantity" could also be thought of as the amount of times the original flour amount must be scooped out and put in the mixing bowl to achieve the new amount of flour. In this case, the quantitative operation would be repeated addition instead of the scaling operation described above. Thompson (1990) emphasizes the importance of distinguishing exactly how the person is quantifying and relating the quantities in a quantitative relationship, as this is the essence of their quantitative reasoning.

It is the goal of this study to better understand the types of mathematical activity women engage in outside of formal and school mathematics in order to identify characteristics of mathematical activity that could be added to mathematics education so that mathematics is more inclusive of women. Therefore my research question is: How do some women reason quantitatively in the non-formal, non-school contexts they choose to participate in?

CHAPTER 3: METHODS

Introduction

I used interviews and general qualitative methods to answer my research question. The interviews helped elicit the quantitative thinking of the participants, and general qualitative methods, like coding data and finding patterns within those codes, allowed me to find underlying quantitative processes the participants engaged in. Because I was investigating the qualitative question of how women reason quantitatively in their daily lives, it followed that I should be engaging in qualitative research methods.

I first explain the setting, participants, treatment, and data type of my data collection. Then I explain the specific process I went through to analyze the data I collected.

Data Collection

Setting

I collected data from women who live in Utah County. The people of this county are predominantly white, with about 10% of the population being Hispanic and less than 6% of the population identifying as nonwhite, non-Hispanic (Statistical Atlas, 2018). Utah County is also home to one college and two universities. Therefore, the young adult population is inflated beyond the national average, with 24.8 being the median age of the county's population. This county also has a strong religious presence, as 30,000 of its residents attend a Christian university that requires faithful attendance of worship services and living a strict code of conduct. The median household income in this county is about $70,000 a year, and the poverty rate is about 12% (Data USA, 2017). Because the population from which I sampled is a generally homogenous group of 606,000 people, and I only collected data on two people from that population, the results of my study are not meant to be extrapolated to the Utah County population or to women in general.

Participants

Due to the Covid-19 Pandemic, I was limited in my ability to interact with women outside my own social circle. Thus, both the women I interviewed are women I know personally. I desired to recruit women in their early to mid twenties because women of this age range are generally in a period of transition; they have usually moved away from home and are exploring who they are separate from the identities they formed while with their family of origin. This period of exploration is particularly helpful for my study because women of this age group often involve themselves in a variety of activities, which creates a rich data mine of potential QR situations. I recognize that not all women are as involved in their community as others,so I selected two women who seemed especially involved.

Kailey is a white, middle class 22 year-old woman. She has limited mathematics exposure, taking up to 10th grade math in high school and then taking a life skills mathematics course while she was in college. She admitted that her disposition towards mathematics is teacher-dependent; when she had a good teacher who worked with her, Kailey was very confident in math and enjoyed it. However, she stated that the majority of her mathematics teachers were not good teachers and left her to fend for herself with the subject. Because of that, she grew frustrated with mathematics and was not confident in her abilities.

She went to college for two semesters but left academia to work full-time as a wife and mother. She has been married for three years and has a one year-old daughter. I recruited Kailey for the study because her involvement with homemaking and child-rearing could result in many quantitative activities like cooking, budgeting, and scheduling. From my experiences with

Kailey, I also knew she enjoyed art and photography, and I anticipated that there could be instances of QR with camera angles, shutter speeds, or scaling elements in her projects to give depth or appear a certain way. Kailey was also in the process of moving when I began my data collection, so I was aware that she was selling most of her furniture before she moved. I anticipated that QR would be present in those exchanges as well.

Lucy is a single, 25 year-old, white, middle class woman. She recently graduated from university with a degree in Elementary Education and is teaching fourth grade. In high school, Lucy earned a 5 on the BC Calculus exam in her junior year, and then took two mathematics education courses for her major in college. She loved math in high school and excelled at it. But she expressed that she felt some anxiety towards advanced mathematical thinking at the time I interviewed her because she had not had exposure to advanced mathematics since high school.

I chose Lucy as a participant because I knew she excelled in school and might feel comfortable performing mathematical operations in her everyday activities. I also knew she was a self-aware, metacognitive person who could give me a lot of detail about her own thinking. Her daily activities also had potential for QR. From my interactions with Lucy, I knew she was concerned about diet, exercise, and losing weight. I expected there would be instances of QR as she determined how much she ate, how many calories she burned from exercise, and the fluctuations of her weight over time. I also was aware that she worked as a student teacher at an elementary school, so I supposed there would be quantitative thinking about scheduling, making copies, and even teaching mathematics to her students.

This pair of participants is particularly viable because several aspects of their lives contrast each other. While Kailey is a wife and mother, Lucy is single. While Lucy loved mathematics and completed advanced education, Kailey had a negative disposition towards mathematics and left college to raise a family. Such differences provide opportunities for more diverse data.

Also, I purposely did not choose women that had STEM occupations or were particularly immersed in mathematics through their schooling. I did this for three reasons. The first is that women in STEM fields might engage in different QR than women not involved in those fields. This is related to my second reason. As was stated in Chapter 1, STEM occupations did not seem to be very representative of women's careers. Thus, choosing participants with non-STEM careers like homemaking and teaching seemed to be more consistent with the trend of female career choices (Eccles & Wang, 2016). Therefore, the thinking of a homemaker or teacher might be more representative of the female population than the thinking of a mathematician or engineer. Third, the intent of my research was to begin to understand why women are choosing *not* to go into STEM fields, so choosing women who *did* pursue STEM related careers seemed unproductive.

Treatment

I went through the entire treatment process with one participant at a time, first with Lucy and then with Kailey. I did this so I could use the findings of my first participant to better inform the interview questions I would ask to my second participant. With Lucy, I had to rely on my own experience with QR outside of formal and school contexts to infer which of her activities might have involved QR. However, after performing the treatment process with Lucy, I could use mine *and* Lucy's experiences to better inform which activities Kailey likely reasoned about quantitatively and then ask Kailey about those activities in her interview.

Log. I requested that the participants of this study log their activities for one week for the purpose of identifying possible activities where they engaged in QR. This log was completed before I interviewed the participant. I chose to have each participant log her activities because a log would likely give a more accurate and detailed description of her week than the participant's recall a week later; the women might forget or misremember activities they engaged in during the week by the time I interviewed them.

I asked each participant to make a note on their cell phone every time they completed an activity, as well as give a short description of the activity and when it occurred during their day. Figure 1 shows a portion of Kailey's log that illustrates the desired specificity of the log entries.

Figure 1

Kailey's Log

- 9 am → woke up and fed [baby] a bottle, played on bed and cuddled
- 10am → Mom took [baby] and I slept till 12
- 12pm → ate oatmeal and played Animal Crossing
- 1pm → fed [baby] and got her dressed, played, ate my lunch while [baby] napped
- 2pm → went to Target and Sally's Beauty for supplies, researched what I needed beforehand, had a budget of $60 and only spent $50

Asking the participants to log every time they changed an activity might seem unnecessarily arduous, but I anticipated that my understanding of what they did during their logged days would be more accurate the more activities they logged.

Following their one week of activity logging, I looked over the participant's log and did some preliminary analysis, identifying which activities seemed to have the most potential for QR and then creating questions about those activities to elicit the participant's quantitative thinking, if it occurred, in the interview. I explain more about this analysis in my data analysis section.

Interview. After receiving and reviewing the participant's log, I then interviewed the participant for approximately two hours. The purpose of the interview was to determine which activities in their logs actually resulted in QR and what characteristics of that QR were present.

I began each interview with a few broad questions in the hopes of being led by the participants themselves to activities where they reasoned quantitatively. I then asked more specific questions about their thinking as I learned more about their participation in each activity. If there was ever a lull in the conversation, I used the questions I had created from my preliminary log analysis to talk about activities we had not yet mentioned in the interview that had potential for QR.

This small sample of Lucy's interview questions (see Figure 2) show the progression of questions from broad to specific during her interview. Refer to Appendix A for the full preliminary question list. To be clear, I simply used the question list as a guide. Many of the questions I asked in the interview were not on this list but were developed in the moment during the interview in response to the participant's answers.

Figure 2

Interview Questions

Introduction Questions

1. When did you think about numbers, amounts, shapes, measurements, speed, or anything else number-related this week?
2. Was there a time this week where you performed calculations in your head?

Time Management/Scheduling

1. How did you decide *when* to do this activity?
2. How did you realize you were running late?

Data Types

As was stated above, the data I collected was each participant's activity log and interview. Lucy emailed me her log in the form of a spreadsheet, and Kailey sent her log in the body of an email. I converted Kailey's log into a spreadsheet for ease of organization. Lucy's interview was conducted in person and recorded with a video camera so I could see and analyze Lucy's facial expressions and hand gestures during our conversation. Because Kailey was out of town at the time of our interview, Kailey's interview occurred via Zoom (Banyai, 1995), and the camera showed mainly her face with only a few hand gestures dramatic enough to appear in the frame of the camera. This Zoom call also was recorded. I transcribed both interviews completely, and once transcribed, I generally referred to the transcript during my data analysis rather than the video footage.

Data Analysis

Preliminary Analysis with Logs

Once I received the activity log, I identified categories of activities where QR seemed likely. For example, Lucy noted that she weighed herself each morning and restricted her sugar intake to lose weight. Since weight and caloric intake are both quantities, I anticipated there would be QR as she determined her weight loss progress. Lucy also frequently referenced time, and not just the hour something occurred, but whether she was running late, how much time she thought she had to do an activity, etc. Thus, time and scheduling became a category. I continued this analysis of the participant's activities until I felt I had identified every type of activity in her log that likely contained QR. As was said above, for Lucy's activity log analysis, I was only using my own experience to identify QR activities. But for Kailey's activity log analysis, I used both my own experience and the information I received from Lucy's data to direct me towards QR activities.

Following the identification of QR activities for the participant, I wrote a list of questions for each category that would elicit quantitative thinking and QR, if either were actually present.

I then referenced these questions during the participant's interview, using them to guide, but not necessarily dictate, the questions I asked the participant.

Interview Analysis

I began analyzing Lucy's interview by rewatching the video footage and marking instances of QR on Vosaic (Plicanic, 2016), an online video coding application. After rewatching the two-hour interview multiple times, I found only three instances of actual QR because Lucy rarely performed quantitative operations. I was surprised to only find three instances, though, since Lucy demonstrated that she was constantly engaging with quantities. I figured that perhaps I was not asking specific enough questions to get at the actual QR, so I scheduled a second interview to ask questions focused on the quantitative operations Lucy used in each of the activities related to quantities. But even in the second interview there was little evidence of quantitative operations taking place in Lucy's reasoning.

I then recruited Kailey, hoping that Lucy was an anomaly and that Kailey would show more evidence of QR. I went through the treatment process with Kailey, and then rewatched Kailey's interview video several times, coding where there were instances of QR. There were only two instances where Kailey engaged in QR. And again, this was due to Kailey rarely performing quantitative operations. But I knew from the data that Kailey was constantly engaging with quantities, just like Lucy was.

At this point in my analysis, I realized that "quantitative reasoning" as defined by Thompson (1990) was too restrictive of a definition for identifying and understanding the participants' thinking about quantities. I could see that both participants were constantly thinking about numbers and doing mental work that seemed mathematical. But the quantitative reasoning framework with which I had started the study was not an adequate tool to describe or evaluate the mental work of the participants. Therefore, I turned to general qualitative methods of analysis to help me accurately describe the participants' thinking.

Coding

To begin my new analysis, I first transcribed Kailey's interview. I started with Kailey because her interview was the most recent and fresh in my mind. I then broke up her transcript into sections based on quantitative topics. By quantitative topics I mean general topics, like Kailey's baby napping schedule or Kailey's cooking, that involve numbers or amounts and also address how the participant reasons about the numbers. I found that each quantitative topic generally had a quantitative process associated with it. By quantitative process I mean the actual numerical activity that took place within a certain context. For example, while Kailey's baby napping schedule was the quantitative topic, how Kailey actually determined that schedule was the quantitative process. I then studied each quantitative process, determining what the process accomplished, where it started and ended, and how the quantities changed within it to accurately define the process.

In conjunction with my macro analysis of general quantitative processes, I looked closer at the individual quantities in each section to see how the quantities informed the quantitative processes I was finding. I began to form codes for quantities by attending to their characteristics. While quantities might share a plethora of similarities to each other, the characteristics I ultimately found most relevant to distinguish were the quantity's purpose, origin (external or internal), place in time (past, present, or future), breadth of existence (multiple occurrences or just once), and specificity. These characteristics of quantities were deemed the most relevant because they were the most helpful in distinguishing the quantitative process in which the quantities were being used. I then developed codes for specific combinations of these characteristics of quantities. For example, an external, specific quantity that the participant obeyed or adhered to (like a bill or item in a recipe) was originally coded as a *command*. After revisiting the data multiple times and making many revisions, I developed a descriptive list of internal codes containing descriptions of Kailey's quantitative processes as well as the types of quantities she used.

I then transcribed Lucy's interview. I knew I would find different processes at the macro level of analysis with Lucy's data, but I expected the codes I made for the individual quantities to generally apply to Lucy's data as well. However, I was mistaken. Because Lucy's quantitative processes were completely different from Kailey's, these processes brought about new types of quantities that were not present in Kailey's data. Unfortunately, I only accepted this fact after months of trying to develop a coding scheme that assimilated the thinking of the two women into a common, shared list of codes for quantities and processes. During those months, I knew the revisions of my codes were not helping me to interpret the data correctly because I struggled to portray Lucy's thinking as reasonable and sensible. I personally identified more with Kailey's thinking, so her quantitative processes naturally made more sense to me, and from my Kailey-biased perspective, Lucy's quantitative processes did not.

I worked to let go of that bias and reviewed Lucy's data without trying to assimilate it into the coding scheme I developed for Kailey's data. I followed the same process of analysis as I did with Kailey's, breaking the sections up into quantitative topics and defining the quantitative process(es) and quantities within each section. Again, after multiple revisions, I created a list of internal codes containing descriptions of Lucy's quantitative processes and the types of quantities she used.

I knew I had reconciled the issues I had before with Lucy's data with this new list of codes because the descriptions contained therein were true to the data and presented Lucy's thinking as reasonable and sensible. Because I only achieved this reconciliation after separating the women's data from each other in my analysis, I present the women as two distinct case studies in the next chapter.

CHAPTER 4: RESULTS

I present my findings as two case studies. For each case study, I describe the values each participant possessed, the types of quantities each participant used, and how those quantities worked within a quantitative process. I then discuss the similarities and differences within the participants' quantitative thinking.

Case Study 1: Kailey

Context and Values

Recall that Kailey is a 21 year-old woman, wife, and new mother working in the private sphere. Her life is very busy and riddled with quantitative thinking. In Kailey's week-long log of activities, she engaged with quantities when determining her 7 month-old baby's nap time and food schedule, budgeting, cooking for herself and family, dieting and exercising, selling old furniture, playing games on her phone, and generating public interest on her Instagram account.

Kailey consistently demonstrated a personal value of structure and predictability when describing her daily routines to me during our interview. In fact, the majority of her engagement with quantities centered around creating structure and predictability in her life. She adhered to specific nap times for her baby; she closely followed a specific, monthly budget; she set a specific diet and exercise plan to lose weight. There were very few instances in Kailey's interview where she talked about doing something impulsively or spontaneously.

The presence of structure and predictability in Kailey's life provided her a sense of security as well as some autonomy. With set budgeting, Kailey was able to allocate spending money for her personal use, with which she could buy what she pleased without worrying about finances. Similarly, since much of her time was spent attending to the needs of her baby, set patterns allowed her to set aside uninterrupted personal time while her baby was sleeping.

Quantities

Two main categories of quantities surfaced in Kailey's interview data: specific descriptors and practical references. *Specific descriptors* are numeric quantities used to describe past event(s). They are often used as data to be considered when making a decision. Consider the statements below that Kailey made in her interview:

1. I knew I needed a lot of bleach, so I got three packets.
2. We tried to stay in a $300 limit for gas and travel and such.
3. I worked out an extra 15 minutes to burn that off.

The specific descriptors in these statements are "three packets," "$300," and "15 minutes" respectively since they are specific quantities that describe past events. In contrast, had Kailey used phrases such as "a few packets of bleach," "a couple hundred dollars," or "enough minutes to get my heart rate up," she would not be using specific descriptors because those phrases do not indicate a specific amount. Had Kailey said something like, "I typically try to exercise 15 minutes a day," this would not be a specific descriptor because it is describing current and future events rather than those in the past. Although it is likely that these numerals are not always perfectly accurate (e.g., Kailey likely did not work out for exactly 15 minutes and 0 seconds), they give us a very close approximation to the actual quantity in the situation.

Specific descriptors were then used to create *practical references*, which are specific, numeric quantities that act as predictors or targets in current or future activity. In the statement "I exercise for 30 minutes every day," the quantity "30 minutes" is a practical reference because the 30 minutes is numeric and informs future events. Had Kailey said "I exercise for a few minutes every day," this would not be considered a practical reference because it does not refer to a specific quantity. Likewise, the statement,, "I exercised for 30 minutes today," would not be considered a practical reference because it refers to a past event.

Again, I make the distinction between specific descriptors and practical references by their place in time because their place in time changed the type of reasoning in which Kailey engaged about the quantity. Specific descriptors were data and practical references were targets or predictors.

Quantitative Process

Specific descriptors and practical references are the building blocks of Kailey's primary quantitative process, *pattern recognition and implementation* (PRI). This is the process by which Kailey draws upon specific descriptors to construct reasonable practical references. Through PRI, Kailey noticed the same specific descriptors occurring naturally in her daily activities (i.e., pattern recognition). She then used those specific descriptors as reference points to plan her other related activities (implementation), henceforth shifting the quantity from specific descriptor to practical reference. More specifically, once Kailey recognized repeated specific descriptors, she would use this quantity as a predictor for events that would happen outside of her control or as a target for events that she could control.

For example, Kailey noticed regularities in the time that her baby got fussy and needed a nap each day. The baby typically became fussy and tired at around 10am; Kailey then put her down for a nap; the baby woke up at around 12pm and then got fussy again at around 2pm; Kailey put her down for a nap; and then the baby woke up at around 4pm. At this point, the times listed above would be specific descriptors because they are describing past, numeric quantities of time. (Recall that while specific descriptors are numeric, they are not necessarily exact, just a close approximation.) After noticing the general consistency of these times, Kailey said, “We implemented it by just being stricter about it, so when it was 10, we would lay [our baby] down instead of waiting for her to show signs of being tired.” Thus, she would put her baby down at 10am and 2pm (targets for which to aim) and then anticipate that the baby would wake up at around 12pm and 4pm (predictors). It is this aiming and/or anticipating that shifts the specific descriptors into practical references. At first, the 10am, 12pm, 2pm, and 4pm, simply described past events. But after Kailey’s implementation of those times into her schedule, those quantities became practical references. They now informed current and future nap times either because Kailey aimed to put her baby down at a certain time or because she knew when to anticipate that her baby would wake up.

To be clear, PRI is an ongoing process. In the example above, Kailey still continually monitored her baby’s fussiness, even after the nap schedule was decided on, to determine whether the nap schedule was meeting her baby’s needs. Several months after the interview, Kailey has implemented a new nap schedule based on the recent changes in her baby’s fussiness: she now puts her baby down from 12pm till 2pm. Thus, practical references can change as new data is collected.

PRI also occurred when quantitative constraints were imposed by an outside source (e.g., income, rent, cost of items when shopping, etc.) and Kailey acknowledged and implemented patterns within those constraints. For example, when determining her family’s finances, Kailey kept track of the trends of their monthly expenditures and built the family budget around those trends. She said, “I knew how much things would cost just based on the meals we typically make and what I usually buy. So, we determined around $250 for food each month.” Kailey surveyed the family’s groceries each month (specific descriptors) and then built her budget around those needs, making what was simply a past occurrence now a target amount to aim for (practical reference). Also, Kailey continued to monitor her family's expenses after the budget was determined, in case the current budget was not consistently meeting her family's needs and a new budget needed to be put in place. As evidence of this, Kailey and her husband bought a house 10 months after her interview. Thus, their budget has changed dramatically after adding a mortgage payment to their expenses. In summary, PRI is a continuous cycle of recognition, implementation, recognition, and implementation.

Case Study 2: Lucy

Context and Values

Recall that Lucy is a single, 25 year-old woman who recently graduated from university with a degree in Elementary Education. In Lucy's week-long list of activities, she engaged with quantities when determining her daily schedule, the amount of food she would buy and eat, and her weight loss and exercise progress.

In stark contrast to Kailey, Lucy found patterns and schedules restrictive rather than empowering. In fact, the only time Lucy engaged with specific descriptors was when they were put in place by someone or something else (e.g., start times of events, prices of items, and income). Otherwise, Lucy preferred to make many of her day-to-day decisions in the moment instead of in advance, which contributed to those decisions changing often.

Lucy's most pervasive personal values were autonomy and reliability. She wanted the freedom to determine her own schedule, but she also wanted others to be able to rely on her to show up to events she promised to attend; she wanted to eat good tasting food but she also wanted to rely on her own self-discipline to stick to her diet. The fact that autonomy and reliability often conflicted with each other explains why she made her day-to-day decisions in the moment; some days autonomy was her foremost desire and other days it was reliability. Most often Lucy made choices that struck a balance between the two values so that they were both adhered to in some way. For example, Lucy constantly arrived at social gatherings, work, or school about 5 to 10 minutes late. Her tardiness exhibited her resistance to event times determined by someone else, but the fact that she was only 5 to 10 minutes late showed her desire to be reliable.

Quantities

As was suggested above, the quantities Lucy engaged with were less specific, contrasting with the more definite nature of Kailey's specific descriptors and practical references The main types of quantities used by Lucy were *loose descriptors* and *ranges*. Lucy also used *levels* and *thresholds*, which were not always quantitative, but were so intertwined with her quantitative thinking that to remove them from the results would create an incomplete picture of Lucy's thinking. I explain loose descriptors, ranges, levels, and thresholds below.

Loose descriptors are numerically unspecified quantities that can describe past, present, or future quantities. Consider these three statements Lucy made in her interview:

1. I feel comfortable going faster than the flow of traffic.
2. I'm almost never late to church.
3. I'm just gonna eat some cookies.

The loose descriptors in these examples would be "faster," "late," and "some" respectively. While subtle, quantities are implied in these words: one has to compare speeds in order to have a greater speed than the flow of traffic; if someone is late, then there must be a specific hour at which they arrived some place; and if a person has "some" of something, then they have a certain amount of it - the actual amount just remains unknown or unsaid. Counterexamples of loose descriptors would be phrases like “15 minutes” since 15 minutes is a specific quantity or “how full I am” since that characteristic is not quantified in the participant’s mind.

The unspecific nature of loose descriptors is also present in *ranges*. Ranges are clusters of quantities that fall near each other on a spectrum. I use spectrum to mean a general characteristic of an activity that describes an array of possible quantities or states concerning that characteristic. So, in other words, ranges parse out a spectrum into relevant categories of quantities on that spectrum. For example, Lucy was finishing up her student-teaching experience when I interviewed her, and she stated she arrived at the school where she was teaching around 8:05 or 8:10 each morning. In this instance, the spectrum is the general characteristic of “arrival time,” while the range Lucy describes is her typical arrival time, which was between 8:05 and 8:10. (As is evidenced in this example, Lucy typically referred to ranges by their endpoints.) In a different context, Lucy said she typically drove between 80 and 85 miles per hour on the highway. The spectrum in this context is the general characteristic of “driving speed,” the range is the speeds in between 80 to 85 miles per hour, and the endpoints of the range are 80 and 85 miles per hour. Ranges differ from specific descriptors because, although their endpoints are numerical, they represent a spectrum of quantities rather than one specific quantity. Thus ranges are less specific because they do not point to a single quantity. However, they are still more specific than loose descriptors because they limit the amount of possible quantities to a range of values.

Levels are the qualitative equivalent of ranges, and loose descriptors typically describe the state on the spectrum that I refer to as a level. Using the driving example, Lucy considered the likelihood she would get a ticket on her commute to student teaching. While likelihood could be characterized by numeric values, she did not think about the likelihood of her getting a ticket this way. Instead, she thought of it in levels: not at all likely, a little likely, probably going to happen, and definitely going to happen. Just like with ranges, levels described sections on a spectrum. But unlike ranges, Lucy described levels by their state on the spectrum rather than their endpoints.

It is important to note that ranges could become levels if quantities are no longer present in the participant's thinking. For example, Lucy originally thought of the food she ate in ranges of their caloric value. She knew fruits and vegetables contained a certain range of calories, meats and grains contained calories of a higher range, and so on. She stated that she once knew what the numeric values of the endpoints of those ranges were. However, after a while, she simply started thinking of foods in levels of caloric value: low-calorie, medium-calorie, and high-calorie. There was no longer any quantitative thinking, but simply the memorization that certain foods were in certain levels of caloric value. Thus, the ranges became levels.

Finally, *thresholds* describe the tipping point where a qualitative state was reached in a given situation. Referencing the driving example again, Lucy considered her personal, moral commitment to being a good citizen and obeying the state's speed limit laws. The threshold in this example is the place on the spectrum of Lucy's moral commitment where she goes from feeling no commitment to obeying the laws to feeling committed, or vice versa. This spectrum of commitment only contains a threshold, rather than a series of levels, because according to Lucy, one is either obedient to God's laws or one is not. There is no gray area. Thus, unlike ranges and levels, there are not multiple descriptors along the relevant spectrum - just the line where something happens or does not happen.

Quantitative Process

Lucy's primary quantitative process was the *negotiation of ranges, levels, and thresholds* (NRLT), which encompassed each of the four terms just described.

I described loose descriptors, ranges, levels, and thresholds using the example of Lucy's commute to student teaching, so I use this same context to describe Lucy's thinking with NRLT. As was mentioned above, the spectrums referenced in this situation were Lucy's arrival time, her driving speed, the likelihood she would get a speeding ticket, and her commitment to obey the speed limit. Ranges were used to parse out the first two spectrums, and levels and thresholds were used for the last two spectrums respectively.

Thus, the first step of NRLT is the establishment of the ranges, levels, and thresholds in a given situation. While Lucy likely did not consciously think about their establishment, nor did she use the terminology of spectrums, ranges, levels, and thresholds, she had to at least subconsciously recognize the spectrums that influenced her decision-making in a certain context. Then she used her past experiences to determine for what ranges, levels, or thresholds along each of the spectrums a desirable outcome might occur. We see evidence of this parsing out of spectrums when Lucy determined what ranges of time it would be appropriate for her to arrive at the school where she was student teaching. Though her contracted time began at 8:00am, Lucy knew her mentor teacher typically did not arrive till 8:15-8:20am, and the students would not arrive until 8:30am. So, Lucy created ranges of arrival times based on her knowledge of these events. Lucy knew that arriving anytime before 8:10 would mean no negative consequences and an adequate amount of preparation time before school started. If she arrived between 8:10 and 8:15, she would likely arrive before her mentor teacher but it was not a guarantee. If she arrived between 8:15 and 8:30, she would likely arrive after her mentor teacher, her mentor teacher would know that she was late, and she would have little preparation time before the students arrived. And if she arrived after 8:30, then her mentor teacher *and* the students would know she was late, and she would have no preparation for the work day. Lucy used this same process of noticing trends and parsing out spectrums to establish all ranges, levels, and thresholds.

It is important to note that some of the spectrums in a situation are positively correlated, and others are negatively correlated. For instance, Lucy's driving speed positively correlated with the likelihood of her getting a ticket; the more she sped, the more likely it was that she would get pulled over. However, other spectrums were negatively correlated, like her commitment to follow the speed limit and her driving speed; if she was committed to drive the speed limit, then her driving speed would decrease.

The second and final step of NRLT is the navigation of these spectrums by negotiating the ranges, levels, and thresholds to produce a desired outcome. Lucy knew that the more miles per hour she drove over the speed limit the more likely she was to get pulled over. So, she could not drive too fast. But if she drove too slow, she would be really tardy for student teaching and have a late arrival time. Thus, she had to negotiate between these ranges, levels, and thresholds to find a speed that maximized positive consequences but minimized negative ones. She described a portion of the NRLT she engages in about her commute each morning in the quote below.

> "I'm supposed to be there at 8 ...and if I speed a lot, then I can make it there in like 40 minutes. So if I leave right at 7:20, and I speed, then I know I can make it there on time. And that's why I'm always thinking, 'Okay, well it's okay if I leave a little bit later, because I know I can speed and get there on time. If I get there at 8:05 or 8:10, there's no one that's holding me accountable. So it's okay if I get there at 8:05 or 8:10, or even

> 8:15 sometimes.' My mentor teacher, she doesn't get there till 8:15 or 8:20 most days, so she's always late. So as long as I'm there before her, I feel like I'm okay."

In this excerpt, we see the spectrums of driving speed, departing time, and arrival time at play with each other. If Lucy leaves too late, then she must speed in order to arrive at school between 8:05 and 8:10. But, as Lucy describes in a different portion of her interview, if she speeds too much, it is likely she will get a ticket, which seems to be more undesirable than arriving late.

Each day Lucy performed this negotiation of ranges, levels, and thresholds on her way to the school, since each day brought about different locations on the spectrums that the ranges, levels, and thresholds were describing. For example, while she might not feel morally obligated to follow the speed limit one day, her commitment to the law might be present by the next. This would then influence her to drive at the speed limit, which might then change her arrival time, etc. Thus, Lucy constantly balanced the ranges and levels and thresholds to bring about that day's desired outcome.

Lucy also often engaged in NRLT when reasoning about her diet. In one instance, she explained her thought process in determining how much cheese she should put on her salad.

> "I used to eat so much cheese, because I love cheese. And then when I started logging my calories, I realized cheese is pretty high-calorie, so I'm just wasting all of those calories. If I'm trying to lose weight, then I should be eating a lot of other stuff instead of that cheese. So when I do put cheese, I just put as little as I can, that I know I'll still be able to taste it and enjoy it, but not any more than that."

Within this quote, Lucy acknowledges both steps of NRLT. She first established the spectrums that are important to her in the situation: the amount of cheese in her salad, the calories cheese contains, her ability to taste the cheese, and her ability to enjoy the cheese. She then parses out these spectrums once she logs her calories and notices the high level of calories contained in cheese and once she reflects on her ability to taste and enjoy the cheese at different amounts. Finally, she references her negotiation of each of the ranges, levels, and thresholds by stating that she puts as little as she can to still taste and enjoy the cheese, but not any more than that.

In this situation, ranges, levels, and thresholds are all present. The amount of cheese Lucy adds to the salad would be a range of quantities. The calorie status of the cheese would be a level, with high, medium and low being the endpoints of each level. Then, there are two thresholds: whether Lucy can taste the cheese and whether Lucy can enjoy the cheese. These are thresholds because they either happen or they do not, there are not levels or ranges. Lucy engages with NRLT by taking into account the range, level, and thresholds listed above and then determining how much cheese she should actually put on her salad based on those ranges, levels, and thresholds. As she stated in the excerpt, cheese is high-calorie, so she must compensate for the high amount of calories by only putting a small amount of cheese on her salad. However, if she puts too small of an amount, she will not be able to taste or enjoy the cheese. So she must negotiate between the high-calorie status of the cheese and her desire to taste and enjoy the cheese to determine the appropriate amount of cheese to put on her salad. This negotiation is an example of NRLT.

Discussion

After seeing the complex quantitative processes in which the participants of this study engaged, I believe the definition of quantitative reasoning accepted by the mathematics education community is too restrictive as a definition for mathematical activity. Recall Thompson's (1990) definition of quantitative reasoning: "the analysis of a situation into a quantitative structure—a network of quantities and quantitative relationships" (p. 12). We established in Chapter 2 that quantities were measurable attributes of an object, and quantitative relationships were formed with three quantities, two of which formed the third through a quantitative operation.

However, after reviewing the data, there were few instances when the two participants actually performed quantitative operations in their daily activities - at least the ones mentioned in their activity logs or interviews. I acknowledge that it is certainly possible they performed quantitative operations in activities that were not captured in the data. But I can only analyze the data I collected, and within those data, there were few instances of quantitative reasoning because the participants rarely performed quantitative operations.

Although students seldom engaged in quantitative reasoning as defined by Thompson (1990), that does not mean that the participants rarely engaged in thinking about quantities. In fact, the women were *constantly* engaging with quantities. As was described in the case studies above, raising children, losing weight, budgeting, scheduling, balancing conflicting values, etc. are all activities that require quantitative predictions, the creation of quantitative targets, the formation of relevant ranges of quantities, or the coordination of multiple quantities to achieve particular goals. Thus, saying that these ways of thinking are not mathematical because they do not meet Thompson's (1990) definition of quantitative reasoning overlooks ways of thinking about quantities that are authentic in these women's everyday activity. The only other feasible argument against classifying this thinking as mathematical is that perhaps the type of reasoning the women engaged in is not complex enough. However, each of the activities listed above required complex problem-solving and the consideration of many real-world variables. So the "too simplistic" argument does not hold either. I therefore refer to the quantitative thinking the participants (and those who think similarly to the participants) of this study engaged in for the rest of the paper as “the participants’ mathematics.”

There are several dangers in not acknowledging the participants’ mathematics as being mathematical. The first is that mathematics educators could be making mathematics more difficult to learn because they fail to connect, in either traditional or reform instruction, the mathematics these types of thinkers perform in their day-to-day lives with the school mathematics being taught. The participants' mathematics is missing from traditional instruction because there is a dearth of quantitative thinking in traditional instruction. There is little to no connection to the real-world contexts people actually encounter in their day-to-day lives because students use abstract numbers and variables devoid of context. So, the scaffolding to real world contexts present in the participants' mathematics is missing from traditional instruction. The participants' mathematics is missing from reform instruction as well. For while quantities and context abound, the mathematics performed in reform instruction almost always uses quantitative operations. But as the results of this study suggest, the participants engaged in mathematical activity and rarely used quantitative operations. Thus, students like the participants of this study are missing out on the advantage of connecting personal life experiences and their natural ways of sense-making to school mathematics because school mathematics is introduced as something separate from, rather than a perpetuation of, these students’ natural thinking. Again, this is because there is either no everyday context present in the curriculum (traditional instruction) or when everyday context is presented, there is no acknowledgement of mathematical thinking that does not use quantitative operations (reform instruction).

Second, the lack of connection between school mathematics and the participants' mathematics could not only make school mathematics more difficult to learn but also less relatable and meaningful to students. Mathematics educators are often asked the question, "When am I ever going to use this?" By failing to validate these types of students' mathematical thinking in their day-to-day lives, mathematics could be needlessly portrayed as separate from what those students care about. In other words, mathematics educators could be alienating some students from the subject when simply validating the mathematics in which students engage in their day-to-day lives could help students see relevance to school mathematics and maybe improve their dispositions towards mathematics altogether.

There are other benefits to incorporating the participants' ways of thinking into mathematics. One benefit is the development of tools and processes in mathematics education that could help students reason about quantities in their own lives. Once we acknowledge the mathematics students accomplish day-to-day outside of school, we can expand the instruction we give students to encompass their daily activities and their ways of thinking mathematically about those activities. This inclusion of the quantitative thinking demonstrated in the participants' mathematics would make explicit the type of mathematical reasoning that is useful in many everyday activities and help students be better equipped to handle daily, real-world mathematics problems.

Another benefit in incorporating the participants' mathematics into mathematics instruction is the field of mathematics can become richer with the additional types of quantities the participants introduced in their thinking. Practical references and specific and loose descriptors are not explicitly present in any mathematics curriculum, and yet those types of quantities are relevant and functional in the participants' mathematics. They could be relevant and functional to other mathematical thinkers or areas of mathematics as well.

Additionally, there are types of factors present in the participants' mathematics that are not quantities but still affect the outcome of a quantitative situation. Both participants attended to non-quantitative contextual factors in their mathematical thinking, like Kailey's desire to create her baby's eating schedule around Kailey's own eating schedule, or Lucy's moral commitment to obey the laws of the road while driving to work. While often not quantified, these types of factors weigh heavily in everyday decision making. Thus, mathematics could be made all the richer and more relevant by including and attending to these factors in mathematical activity.

Validating the Literature

The results of this study also validate the findings in the literature regarding how women prefer to reason contextually, relationally, and inductively. I explain how each of these traits manifested in the data below.

Context was omnipresent throughout the data. While the importance of context might be seen as a direct result of asking the participants to identify and discuss how they use quantities in the context of their day-to-day lives, the interview questions would have also prompted the women to talk about instances of abstract mathematics or recreational mathematics present in their daily activities. However, no such instances were present in the logs or interviews. Rather, context motivated each of the quantities present in the women's activity log and interview transcript because every situation required a context-dependent renegotiation of how the women thought about the quantities. For example, while Kailey might have developed a set schedule for her baby's nap time, she was still attune to her baby's fussiness and made changes to her schedule if her baby woke up early or became fussy before it was nap time. Similarly, Lucy's process of NRLT required the analysis of each situation to determine her course of action. For example, how much of each food item she would put in a salad depended on her present commitment to eating healthy, her present cravings of the food item, how much she currently weighed, etc. So, again, while these women had general strategies for approaching the changing dynamics of their day-to-day decisions, the quantities they identified and reasoned about in each situation were unique to the contexts. They did not have specific algorithms that they regularly followed. Instead, they used their general strategies and the present context of each activity to reason about and with quantities.

Similarly, both of the women only used strategies and made decisions that made sense to them, showing a preference for relational understanding in their decision-making. Their proposed solution for each of the problems they faced appeared to be only as complex as it needed to be, which demonstrated a deep level of understanding about each problem and their choices in responding to it. Just as a taxi driver demonstrates their expertise of the layout of a city by sifting through the many routes they know are possible to take and choosing the most efficient one, so too did Kailey and Lucy demonstrate their relational understanding when making their own decisions. For Kailey, the most efficient way to make decisions was by implementing patterns in her schedule, budget, and purchases. For Lucy, the most efficient way to take into account the oftentimes conflicting variables of each situation was to negotiate the ranges, levels, and thresholds as each decision came along. Each woman's general strategy made sense to her, and there was no evidence of either woman blindly using algorithms or procedures in their activities that they did not understand.

Lastly, both women demonstrated a preference for induction in their day-to-day activities. I previously defined induction as the use of data-based patterns to make conjectures about a situation, whereas deduction was defined as the use of logic and axiomatic systems to make conclusions. Both Kailey and Lucy used patterns to influence their future decisions, and there was no evidence that they used solely abstract reasoning or formal logic at all. Through PRI, Kailey used the patterns of her baby's fussiness and her spending habits to then make decisions about her schedule and budget. Instead of deducting why the pattern exists in the first place or whether there are confounding variables in the situation she is not acknowledging, Kailey simply recognized a pattern and ran with it. Similarly, Lucy used patterns to build the levels and thresholds present in NRLT. For example, she noticed at around what speed she would get a ticket on her commute to work, and then used that data to inform the different levels of speed she was willing to drive each day. Again, there were no instances of deductive reasoning present in either participant's thinking.

Commonality Beyond the Literature

In addition to the context, relational thinking, and induction that was present in the data for both women, there was also a common characteristic of the participants' thinking that was not included in the literature. Specifically, both women were relatively flexible with the measurements of the quantities they used. With both Kailey's specific descriptors and Lucy's loose descriptors and ranges, precision was not particularly important. For instance, when Kailey said she exercised for 15 minutes, it made no difference to her if she actually exercised for some seconds or minutes under or above the 15 minutes she stated. For Lucy, precise quantities held even less significance, since she never used numeric quantities with loose descriptors, and when she did use numerals with her ranges, they were described as a range of quantities, not a precise number. As was noted in the results, Lucy only ever used an exact quantity if it was imposed by an outside source (e.g., the price imposed by the grocery store or the time her boss set her shift at work to start).

While this flexibility with quantities might seem to be a natural byproduct of living in a world where exact measurements are impossible, or at the very least inconvenient, it is interesting to note that there is rarely such flexibility with quantities in school mathematics. For example, the responses of 19 and 21 would both be considered incorrect for a problem whose correct answer was 20. In school mathematics, there is often one right answer, and therefore there is little room for flexibility with numbers. But the participants in this study demonstrated a lot of flexibility in the precision of the quantities present in their day-to-day activities.

Relating Back to the Problem

From the results of this study, it is evident that the participants' mathematics contains the following characteristics: it is contextual and it contains quantitative thinking without using quantitative operations. Due to its contextual nature, the participants' mathematics is not included in traditional mathematics instruction. Again, this is because traditional mathematics instruction treats numbers and variables as mainly abstract entities with little contextual basis. And because the participants' mathematics did not include quantitative operations, the participants' mathematics is likely not present in reform mathematics instruction, either. Again, while reform instruction does attend to context, the mathematics problems almost always require the use of quantitative operations. Thus, the mathematics the two participants engaged in is likely not included in mathematics instruction at all.

If females are not seeing their day-to-day mathematical thinking acknowledged or included in mathematics instruction, their negative dispositions towards mathematics make sense. It follows that they would deem mathematics as irrelevant for their future (Samuelsson & Samuelsson, 2016; Meyer & Kohler, 1990) because it is irrelevant to their life now. It follows that they would view the field of mathematics as a chilly environment that does not care to include them (Lubienski & Ganley, 2017; Herzig, 2004) because it does not include them now. And it follows that they would feel insecure about their mathematical ability (Else-Quest, Hyde, & Linn, 2010; Hill, Mammarella, Devine, Caviola, Passolunghi, & Szucs, 2016; Lacampagne, Campbell, Herzig, Damarin, & Vogt, 2007; Devine, Fawcett, Szucs, & Dowker, 2012) because the mathematics they perform naturally and easily in their day-to-day activities is not mathematical enough to be included in the curriculum now.

CHAPTER 5: CONCLUSION

Contributions

The results of this study contributed to the mathematics and mathematics education fields in four ways. First, we now know that using QR as the definition of mathematics is too restrictive to explore women's mathematical thinking. As I explained in Chapter 4, both of the participants in this study reasoned mathematically in ways that rarely involved quantitative operations, so it is possible that other women also engage in mathematical thinking without using mathematical operations. Therefore, using a definition of mathematics with mathematical operations as one of its requirements does not allow us to explore the mathematics that occurs in women's thinking before quantitative operations ever take place.

Second, because QR was not a sufficient framework, I developed a new process for identifying mathematical thinking among my participants. Specifically, I identified the values of each participant, then I determined how those values related to the quantitative processes in which the participants engaged, and finally I discovered categories of quantities that acted as building blocks for those quantitative processes. Future researchers that seek to understand females' mathematical thinking can replicate this process in their own studies or use this process as a starting point for their analysis. Additionally, the quantitative processes I discovered in this thesis could be used as frameworks for future studies.

Third, the results of this study found a common characteristic of mathematical thinking among the two participants, namely the flexibility in quantitative measurements. As was concluded from the results, the exact amount of something—be it time, money, volume, etc.—was often unnecessary to determine in the participants' day-to-day lives. A close estimation was just as useful as the exact measurement. In fact, a close estimation was perhaps more useful since it fit the level of precision with which numbers were used in the daily contexts. It is possible that flexibility in quantitative measurements is present in other women's mathematical thinking as well and may be a common characteristic of female mathematical thought.

Fourth, the data showed that school mathematics was rarely, if ever, present in the day-to-day lives of the participants. Conversely, the mathematical thinking the participants did engage in is not present in school mathematics, either. It is likely that this disconnect from school mathematics and everyday life is experienced by many women and could contribute to their negative dispositions towards mathematics.

Implications

Because QR was too restrictive of a definition of mathematics, and general pattern finding was not restrictive enough (as explained in Chapter 2), it seems using a pre-existing, external theoretical framework is insufficient to identify and categorize women's mathematical thinking outside the classroom. Instead, researchers should develop a framework about women's mathematical thinking from actual data collected from women about their mathematical thinking. In other words, it would be advantageous for future researchers who seek to explore women's mathematical thinking to use a framework developed from internal codes based on women's own thinking.

In addition to using women's ways of mathematical thinking to build mathematics frameworks, women's ways of mathematical thinking should also be used to build curricula for school mathematics. Curriculum should incorporate real-world data collection from contexts females care about and teach pattern-finding in the data. Additionally, these real-world contexts should include the consideration of non-quantitative variables. For example, a task exploring how to determine the profit of running a small business might also include the ethical concerns of whether the business' employees are earning a fair and living wage. Including the qualitative variables with the quantitative is more authentic to the actual mathematical work the female (and possibly male) students will accomplish outside of school. Including their day-to-day mathematical thinking in school mathematics allows women to see connections between mathematics and their day-to-day lives. As was said in the Discussion section, this inclusion could contribute to women seeing the relevance of studying mathematics and feeling represented in the field, as well as enrich the subject of mathematics with their diverse ways of thinking.

Limitations and Directions for Future Research

There were three main limitations with this study. The first is I had a very small, homogeneous sample. I only studied two participants; they are both women I know personally; and they both are of the same race and have similar socioeconomic backgrounds. Because of this, I was not able to extrapolate my research to any larger population. This study could therefore be replicated with a much larger sample of a more diverse group of women. These larger studies would likely provide information that could be inferred to a larger population of women.

Second, my data consisted only of each woman's interpretation of a week's worth of events in their lives. In other words, I had no observational data to triangularize the results of the logs and interviews. It is possible the participants engaged in more mathematical activities than I was aware of. It is also possible they misremembered the quantitative thinking they reported. Thus, future studies could extend my methods to include observational data of the women in their various activities, and possibly in the moment questions about how the women were thinking about the quantities in a given activity. Data could also be collected for more than a week of activities.

The third limitation is the possibility that the results of this study are not only representative of women but of men also. It is possible that many men do not perform quantitative operations in their day-to-day lives but still engage in mathematical thinking in other ways. It is also possible that precise quantities do not play an important role in men's day-to-day activities. Thus, this study could be extended to include both men and women to see if there is a gendered distinction in ways women reason quantitatively.

There is more work to be done to include women more fully in the field of mathematics. As we recognize and legitimize women's ways of quantitative thinking in our research and curriculum, we make space for the intelligent, innovative contributions women can make to the field.

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APPENDIX A

Interview Questions

Introduction Questions

1. When did you think about numbers, amounts, shapes, measurements, speed, or anything else number-related this week?
2. Was there a time this week where you performed calculations in your head?

Time Management/Scheduling

1. How did you decide *when* to do this activity?
2. How did you realize you were running late?

Cooking

1. Did you follow a recipe for this meal? If not, how did you determine how much of each ingredient you should use when you were cooking?
2. When doubling the recipe, how did you make sure that each of the quantities was doubled?

Shopping

1. How did you choose which product to buy at the store?
2. Why did you decide to buy that specific product when you did?