



Theses and Dissertations

---

2020-12-14

## Instructional Case Studies in the Field of Windfarm Optimization

N. Francesco Baker  
*Brigham Young University*

Follow this and additional works at: <https://scholarsarchive.byu.edu/etd>



Part of the [Engineering Commons](#)

---

### BYU ScholarsArchive Citation

Baker, N. Francesco, "Instructional Case Studies in the Field of Windfarm Optimization" (2020). *Theses and Dissertations*. 8770.

<https://scholarsarchive.byu.edu/etd/8770>

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact [ellen\\_amatangelo@byu.edu](mailto:ellen_amatangelo@byu.edu).

Instructional Case Studies in the Field of Windfarm Optimization

N. Francesco Baker

A thesis submitted to the faculty of  
Brigham Young University  
in partial fulfillment of the requirements for the degree of  
Master of Science

Andrew Ning, Chair  
Julie Crockett  
John Salmon

Department of Mechanical Engineering  
Brigham Young University

Copyright © 2020 N. Francesco Baker

All Rights Reserved

## ABSTRACT

### Instructional Case Studies in the Field of Windfarm Optimization

N. Francesco Baker  
Department of Mechanical Engineering, BYU  
Master of Science

Wind farm layout optimization is a multidisciplinary undertaking, requiring students and researchers to integrate many skillsets in order to optimize turbine placement. There is currently a lack of useful benchmarking exercises for participants in the field to compare the efficacy of their methods. This work details the construction and completion of a set of four case studies meant to satisfy this need, with the hope of providing some insight into useful layout optimization approaches. These case studies are intended to also serve as instructive introductory exercises with which newcomers researching wind energy may incrementally practice and increase their abilities. The first two case studies were released globally and attracted participants from around the world who attempted the optimization problems. A detailed analysis of their results is presented herein. The second two case studies are currently being worked on by researchers in the field, with initial feedback regarding the formulations also included.

Keywords: wind energy, wind turbines, optimization, wind farm optimization, case studies

## ACKNOWLEDGMENTS

I would like to, first and foremost, express the utmost appreciation for my graduate advisor, Dr. Andrew Ning. His guidance, assistance, patience, and support have empowered me to complete this work and improve myself as an engineer in ways I could not have done on my own. This is a debt I will never be able to repay.

Dr. Julie Crockett and Dr. John Salmon have also been incredibly helpful along my journey, providing me educational opportunities both as a student and as a teacher's assistant.

A few members of the BYU FLOW Lab have also gone above and beyond on my behalf, proofreading, problem solving, providing thought guidance, and more than I could ever catalogue. Jared Thomas has been there for me from my first day at BYU and done much to support me during my transition from the military to the world of graduate studies, from coding the most basic of wake models to conceptualizing complex aerodynamic phenomena. P.J. Stanley has been a consistent model of work ethic and personal efficiency and has read countless drafts of the academic papers I wrote during my time at BYU.

Eduardo Alvarez and Judd Mehr were both always available to bounce ideas off of, patiently watch my marker board drawings, help me see holes in my logic, and improve the science behind my thought processes.

Hayley Marta spent many hours proofreading and editing this work for clarity, and it is an infinitely better document for it.

Finally, no list would be complete without mention of my mother and father. They both have supported me in my dream to attend BYU as a mechanical engineer and done nothing but continuously enable me to do my best as a student.

## TABLE OF CONTENTS

<b>LIST OF TABLES</b> . . . . .	<b>vi</b>
<b>LIST OF FIGURES</b> . . . . .	<b>vii</b>
<b>Chapter 1 Introduction</b> . . . . .	<b>1</b>
1.1 Wind . . . . .	2
1.1.1 Wind Resource . . . . .	4
1.2 Wind Power . . . . .	6
1.2.1 AEP . . . . .	7
1.3 Wind Turbines . . . . .	9
1.3.1 Functional Attributes . . . . .	11
1.3.2 Power Curve . . . . .	12
1.4 Wind Turbine Wake . . . . .	13
1.5 Engineering Wake Models . . . . .	14
1.6 Optimization . . . . .	17
1.7 Computational Derivatives . . . . .	17
1.7.1 Symbolic Differentiation . . . . .	18
1.7.2 Numerical Differentiation . . . . .	19
1.7.3 Algorithmic Differentiation . . . . .	20
1.8 Wind Farm Layout Optimization . . . . .	22
1.9 IEA Task37 . . . . .	23
1.10 Need . . . . .	23
<b>Chapter 2 Case Study 1 &amp; Case Study 2</b> . . . . .	<b>26</b>
2.1 Problem Formulation . . . . .	26
2.1.1 Case Study 1: Optimization Only . . . . .	33
2.1.2 Case Study 2: Combined Case . . . . .	35
2.2 Results . . . . .	36
2.2.1 Case Study 1: Optimization Only . . . . .	37
2.2.2 Case Study 2: Combined Physics Model/Optimization Algorithm . . . . .	41
<b>Chapter 3 Case Study 3 &amp; Case Study 4</b> . . . . .	<b>45</b>
3.1 Problem Formulation . . . . .	45
3.1.1 Case Study 3: Concave Boundary . . . . .	53
3.1.2 Case Study 4: Discrete Boundary Regions . . . . .	54
3.2 Results . . . . .	55
3.2.1 Interesting Methods . . . . .	56
3.2.2 Author’s Attempt . . . . .	57
3.2.3 Lessons Learned . . . . .	60
<b>Chapter 4 Conclusion and Future Work Recommendations</b> . . . . .	<b>62</b>

<b>REFERENCES . . . . .</b>	<b>65</b>
<b>Appendix A Case Study 1-2 Startup Files . . . . .</b>	<b>69</b>
<b>Appendix B Case Study 3-4 Startup Files . . . . .</b>	<b>91</b>
<b>Appendix C Case Study 1-2 Participant Results . . . . .</b>	<b>151</b>
<b>Appendix D Relevant Code . . . . .</b>	<b>156</b>

## LIST OF TABLES

2.1	Wake Deficit Equation Variable Definitions . . . . .	34
2.2	Case Study 1, 16 Turbine Farm Results . . . . .	38
2.3	Case Study 1, 36 Turbine Farm Results . . . . .	38
2.4	Case Study 1, 64 Turbine Farm Results . . . . .	38
2.5	Case Study 2, Participant 1's Cross-Comparison Results . . . . .	42
2.6	Case Study 2, Participant 2's Cross-Comparison Results . . . . .	42
2.7	Case Study 2, Participant 3's Cross-Comparison Results . . . . .	42
2.8	Case Study 2, Participant 4's Cross-Comparison Results . . . . .	42
2.9	Case Study 2, Participant 5's Cross-Comparison Results . . . . .	42
3.1	Case Study 4's Example Layout Turbine Apportionment . . . . .	49

## LIST OF FIGURES

1.1	Diurnal Heating Cycle . . . . .	4
1.2	Example Wind Rose (tri-directional) . . . . .	5
1.3	Horizontal Axis Wind Turbine (HAWT) Cog and Wheel . . . . .	9
1.4	Turbine Nacelle Cutaway . . . . .	10
1.5	Turbine Gearbox Cutaway . . . . .	10
1.6	Turbine Blade Cross-Sectional . . . . .	10
1.7	15-MW Offshore Reference Turbine Attributes . . . . .	12
1.8	Wind Turbine as Streamtube . . . . .	14
1.9	Wind Turbine as Actuator Disk . . . . .	14
1.10	Jensen Wake Model . . . . .	15
1.11	FLORIS Wake Model . . . . .	16
1.12	Dynamic Wake Meandering (DWM) in 3-Dimensions . . . . .	16
1.13	DWM in Trefftz Plane . . . . .	16
1.14	Simple Derivative Optimization . . . . .	18
1.15	Rosenbrock Design Space . . . . .	19
1.16	Central Difference Example . . . . .	20
1.17	IEA Wind Task 37 Holistic Approach . . . . .	24
2.1	3.35-MW Onshore Reference Turbine Attributes . . . . .	27
2.2	Case Study 1 Example Layouts . . . . .	29
2.3	Case Studies 1 & 2 Test Wind Roses and Optimization Performance . . . . .	32
2.4	Wake Expansion Continuation (WEC) Example Wind Farm Layout . . . . .	39
2.5	WEC Variable Perturbation Effects on Design Space . . . . .	39
2.6	Case Study 1 Participant-Reported Wall Times . . . . .	43
3.1	Borssele Offshore Wind Farm Location . . . . .	46
3.2	Borssele Wind Farm UTC Coordinates . . . . .	46
3.3	Preliminary Example Layout for Case Study 4 . . . . .	47
3.4	Published Example Layout for Case Study 4 . . . . .	47
3.5	10-MW Offshore Reference Turbine Attributes . . . . .	48
3.6	Wind Resource Bin Convergence Study for Case Studies 3 & 4 . . . . .	51
3.7	Demonstration of Weibull Function Variability . . . . .	52
3.8	3-Dimensional Wind Speed Distribution for Case Studies 3 & 4 . . . . .	53
3.9	Single Direction Wind Speed Distribution for Case Studies 3 & 4 . . . . .	53
3.10	Boundary Released for Case Study 3 . . . . .	54
3.11	Boundary Released for Case Study 4 . . . . .	55
3.12	Depiction of BPM . . . . .	57
3.13	Simple Case Study 4 Optimized Turbine Layout with AEP Calculation . . . . .	59



## CHAPTER 1. INTRODUCTION

Wind energy is one of the most plentiful resources on the Earth. Multiple wind turbines arrayed in *wind farms* harness this energy mechanically. Altering the placement and attributes of a wind farm's turbines in order to maximize the power they extract from the wind is a profitable and useful focus area. Strategically altering wind turbine placement can increase the wind farm's annual energy production (AEP), which in turn generally decreases the farm's cost of energy (COE). Many factors effect calculation of a wind farm's AEP and COE, including windfarm geographic location, turbine height, blade geometry, generator mechanics, attributes of construction material, turbine rotor radius, and local wind resource. The interplay of these factors (and others not here enumerated) creates complicated problems that require strategic solutions in order to be solved efficiently.

The costs in labor, time, and materials to physically construct and adjust turbine attributes for optimal output in the field without prior calculations tend to be prohibitively inefficient. Though non-physical computer simulations can be more easily manipulated than actual turbines, these simulations only consider finite facets of physical reality, and introduce complications of their own. Computing power, model realism, algorithm efficiency, and computational precision all factor into the efficacy of a simulation's results when translated into physical wind farms.

Newcomers to the field of wind farm optimization have countless variables to consider and may be overwhelmed with the magnitude of the task. Optimization algorithms can be difficult to compare; not only is a comparison of computing resources between different systems multivariate, but if different algorithms use different physics models, results between them would be incomparable.

To better study the efficiencies and draw-backs that various computing algorithms have under different wind farm scenarios, we, in the Brigham Young University (BYU) Flight, Optimization, and Wind Laboratory (FLOW) Lab, created a set of four case studies, increasing in

complexity and realism, to be used by the wind farm optimization community at large. The case studies are intended to serve as well-thought benchmarks, through which established researchers can compare their methods, but also as a series of instructional problems that newcomers to the field can use to build their understanding of wind farm layout optimization.

These case studies (numbered one through four) were created generally under the supervision of the International Energy Association (IEA) and specifically for their Task 37 [1], which deals with wind farms and wind energy. Involvement in Task 37 provided these studies with global recognition, including subsequent participation from individuals in Italy, Germany, France, Denmark, and the United States.

To better understand the case studies we created, some foundational knowledge is required. The basics of wind, wind turbines, the manner with which they draw power from the wind, and the way computer optimization can help increase the power they generate will be explained herein.

## 1.1 Wind

The composition of the lowermost layer of the Earth's atmosphere where most weather phenomena occur, called the *troposphere*, and can generally be considered of uniform molecular composition. A given volume of tropospheric gas consists, on average, of 78% nitrogen and 21% oxygen [2]. Air density, as a trend, increases as observed altitude decreases. As such, we can model the characteristics of the Earth's atmosphere as an ideal gas adhering to the ideal gas law (IGL). Specifically, the IGL states that,

$$P = \frac{n}{V}RT \quad (1.1)$$

where  $P$  is pressure,  $n$  is the number of molecules of the gas present,  $V$  is volume,  $R$  is a gas constant, and  $T$  is the absolute temperature. Note that the quantity  $\left(\frac{n}{V}\right)$  is equivalent to density, denoted  $\rho$ .

While the IGL illustrates many relationships, for the purpose of exploring the major causes of wind, we must isolate and examine the relationship between volume and temperature. Equation (1.1) demonstrates that, if all else remains constant, as temperature increases, the gas' volume will also increase (or expand). Scenarios in which  $P$  and  $n$  are fixed can be illustrated best in

small systems, such as the one exhibited in fig. 1.1. Here, the system is closed in regard to mass and pressure changes due to altitude are negligible. Symbolically, this proportional relationship between volume and temperature is represented as:

$$V \propto T \tag{1.2}$$

Alternatively, some scenarios don't exhibit this trend, including atmospheric temperature inversions or at the boundaries of disparate pressure systems, since those scenarios alter different variables in the IGL. However, knowing this general correlation allows us to draw conclusions from broader patterns.

Many factors beyond those examined here are involved in the constant motion of the Earth's atmosphere on a macro level, but one major contributor worth discussing is the uneven heating of the Earth's surface [3]. Radiative solar heating causes the majority of the Earth's surface temperature imbalance. This imbalance varies in time as the Sun follows its path through the Earth's sky during each 24-hour day. Especially in coastal regions, the drastically different heat-absorbing properties of water and soil create temperature disparities that warm or cool the air above each medium, as depicted in fig. 1.1.

During the day, warm (less dense) air above the ground rises and is laterally replaced by cool (more dense) air that was previously above the water. The isolated IGL relationship demonstrated in eq. (1.2) illustrates that, if all else remains constant, as  $T$  of the air over land increases, the air in that moving volume will become less dense and lighter, rising as it warms. The cooler air over the relatively temperature-stable water will flow landward to replace the rising warmer air, creating a circular motion. At night, this cycle is inverted, as the rapidly cooling soil creates a dense, falling mass of air above it, while the temperature-stable water is relatively warm, creating a light, rising mass of air that subsequently pulls the cooler neighboring air across the shoreline to replace it.

On an even larger, global scale, areas of low pressure tend to exist over equatorial regions and areas of high pressure tend to exist over polar regions. This pressure differential causes global circulation that is further propelled by the constant rotation of the Earth and the Coriolis forces that result from its spin. All of these natural and unavoidable atmospheric phenomena contribute

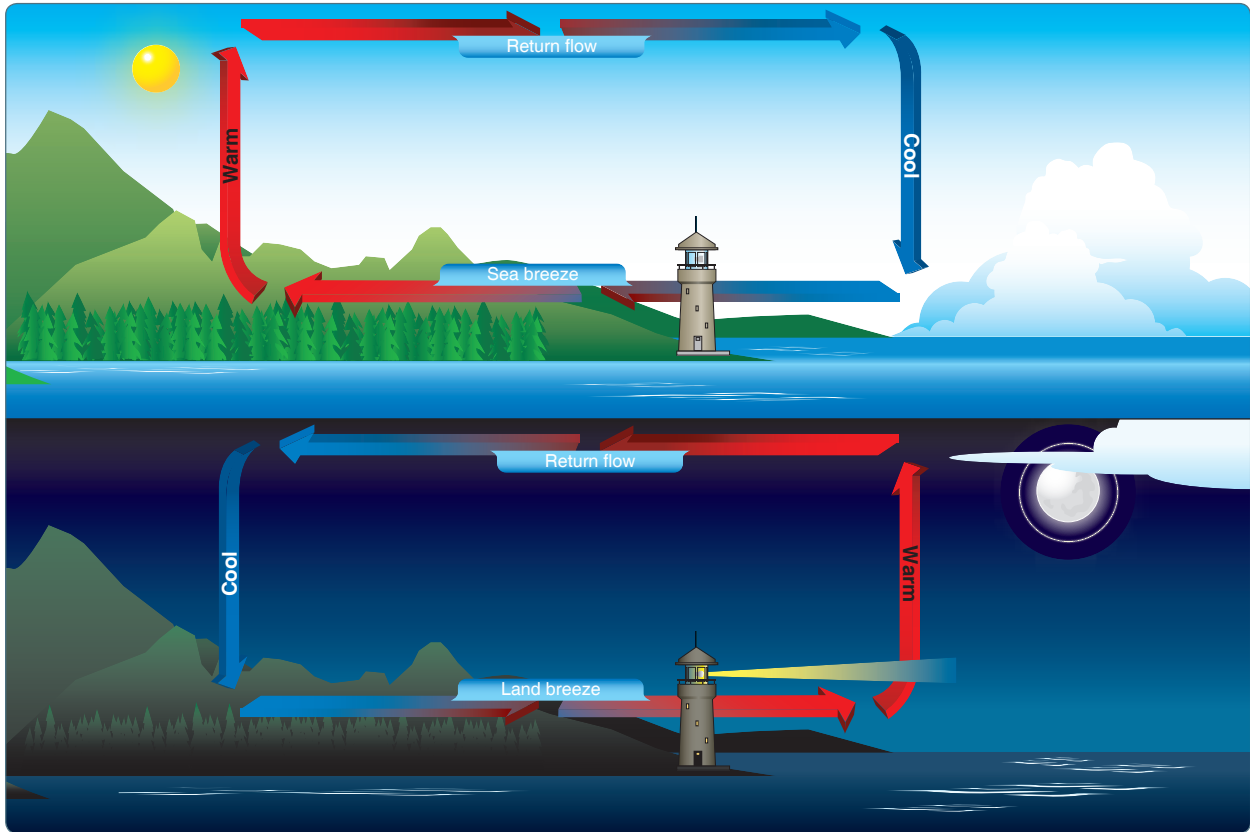


Figure 1.1: The diurnal heating cycle. From the Federal Aviation Administration’s Pilots Handbook of Aeronautical Knowledge [3] in the public domain.

to constant and non-uniform motion of the atmosphere. So long as these physical laws exist, there will be wind flowing across the Earth’s surface.

### 1.1.1 Wind Resource

The geographic attributes of a wind farm’s location affect the strength, frequency, and directionality of experienced wind on a micro scale throughout the day and on a macro scale over the course of each year. Those factors: wind strength, wind frequency, and wind directionality (for a given period of time), are three fundamental ways in which a location’s *wind resource* can be quantified. For example, the venturi effect caused by the geography of a bottleneck canyon will result in strong winds up and down the draw, the lower part of the canyon, exhibiting a much different wind resource than would be observed across an unobstructed shoreline.

This directional wind strength and frequency is often codified in a graphical format termed a *wind rose*, an example of which is depicted in fig. 1.2. Around the polar wind rose plot in fig. 1.2,

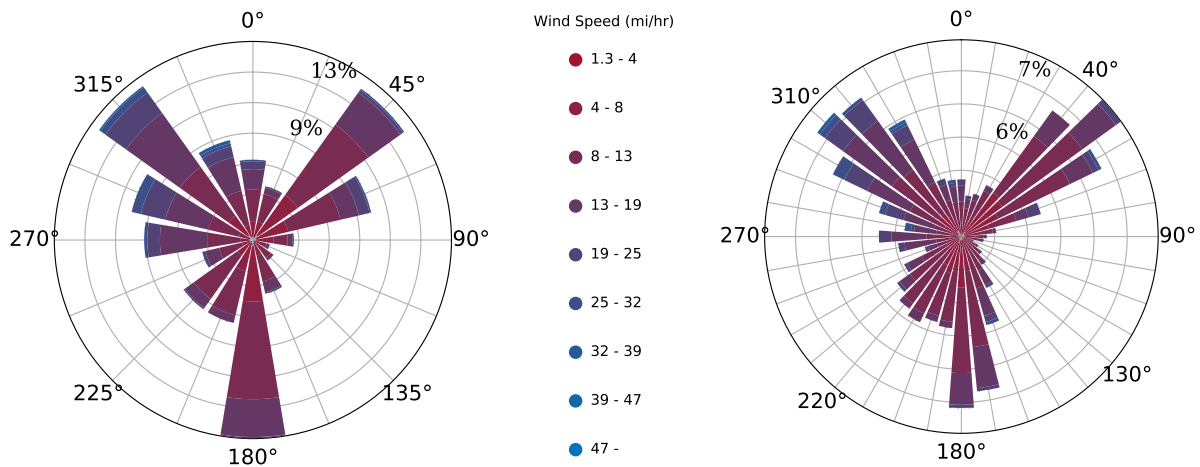


Figure 1.2: A wind rose with three major directions, radial magnitude indicating directional frequency, windspeeds are depicted by colored echelons. The same data has been divided into 16 bins (left) and 36 bins (right).

each radial corresponds to a direction on the compass (i.e.,  $0^\circ$  corresponds to North,  $180^\circ$  to South, and so on). Magnitude in the radial direction corresponds to the frequency for that given direction, here ranging from roughly 2% to 12%. In other words, for the slice corresponding to the North East, or  $45^\circ$ , the wind comes from that direction for roughly 11% of the time period of which this rose depicts.

Also of note are the number of *bins*, or divisions around the wind rose. The rose depicted on the left side of fig. 1.2 has 16 bins, meaning each slice represents an average of the  $22.5^\circ$  section it estimates, with  $11.25^\circ$  on either side of each bin's central direction. The rose depicted on the right has 36 bins, each bin representing  $10^\circ$  of the total. It is also worth noting that the radial values for the two figures are not equivalent. This is due to the fact that as the number of bins increases, their frequency percentages decrease because each bin accounts for less and less of the wind resource.

A wind rose can be further complicated by the graphical inclusion of wind speed probability. Depicted in fig. 1.2, each bin is itself echeloned into probabilities of a given wind speed along that radial. So, while the given graphics show wind direction and frequency, the same wind roses also depict the probability of seeing a range of wind speeds within each binned direction.

Roses with  $360 \times 1^\circ$  bins or even smaller can be utilized though the need for so many data points depends largely on their intended application. For a snapshot of the data, more granularity,

like that of the wind rose on the right of fig. 1.2, paints a better picture for the observer. On the other hand, for large scale optimizations where each data point needs to be iterated through multiple times, fewer bins results in a quicker process. Yet, with fewer data points, calculations lose some granularity necessitating convergence studies to ensure that results using fewer bins still come within some acceptable computational margin ( $\sim 1\%$  is often used).

## 1.2 Wind Power

As wind moves through the atmosphere, the air molecules push with a force  $\left(\frac{kg \cdot m}{s^2}\right)$ , in the direction the wind flows, over a distance ( $m$ ) and through time ( $1/s$ ). These factors combine into a Watt (W), the International System of Units' (SI) unit for power, symbolically represented as:

$$1W = 1 \frac{kg \cdot m^2}{s^3} \quad (1.3)$$

This power conveyed by the wind can be harnessed using wind turbines. As the wind passes over the turbine's blades, the wind's force works on the blades to induce lift, in turn spinning the rotor. This spin caused by the wind's power is converted into a rotational kinetic energy by the turbine. Rotational kinetic energy is then converted into electrical energy by the turbine's generator, which can be stored for external use.

Many factors contribute to an individual turbine's ability to harvest the wind's power. To optimize the rotational (and hence electrical) energy a given turbine can produce, we must maximize the air flow working on the turbine's blades and minimize the resistance any mechanics may have on the blade's rotation.

Despite the fact that blade geometry and turbine construction have and will continue to be extensively studied, they are outside the scope of this work. Instead of experimenting with and optimizing the best blade geometry for each problem set, we chose to use reference turbines created by the IEA. After selecting two idealized reference turbines for these case studies and considering their attributes fixed, the wind resource experienced by each turbine was the sole element influencing maximization of the wind farm's AEP.

Site selection plays a major role in the wind resource available to a farm carrying several potential trade-offs with it. Placement in a venturi-like canyon, for example, will increase

windspeeds but will also reduce the area available for turbine placement due to the limitations associated with sloping and mountainous terrain. Offshore locations offer constant and repetitious air flow fluctuations but introduce a slew of complications including irregular tide levels, saltwater corrosion, and turbine anchoring requirements.

### 1.2.1 AEP

Annual energy production is a common metric for comparing wind farm layout efficacy. AEP, the total amount of electrical energy produced by a wind farm per year, is typically measured in kilowatt hours or megawatt hours (*kWh* or *MWh*). Its value is dependent on various factors, including the number of turbines in the farm, the power rating of said turbines, how the turbines are physically arranged, and the geographic wind resource the farm experiences.

To calculate a wind farm's AEP, we must calculate the power produced by each turbine over each wind directional bin and for every wind speed bin, and multiply it by the frequency each wind speed and direction occurs in the wind resource. The following algorithm describes the steps in AEP calculation, one by one:

1. For each binned direction ( $\theta$ ), rotate the turbine locations  $(x, y)$  into the wind frame of reference  $(x_w, y_w)$ :

$$\Psi = -\left(\frac{\pi}{2} + \theta\right)$$

$$x_w = x \cos(\Psi) + y \sin(\Psi)$$

$$y_w = -x \sin(\Psi) + y \cos(\Psi)$$

2. For each wind speed bin, ( $V_\infty$ ), iterate through each turbine in the field to compute its power:
  - Compute the wake deficit between each turbine pair (there is no wake effect of a turbine on itself).
  - Compute effective wind speed ( $V_e$ ) at each turbine:

$$V_e = V_\infty \left[ 1 - \left( \frac{\Delta V}{V_\infty} \right)_{total} \right]$$

- Use  $V_e$  and the turbine's idealized power curve formulated from the cut-in wind speed ( $V_{cut-in}$ ), rated wind speed ( $V_{rated}$ ), and cut-out wind speed ( $V_{cut-out}$ ) explained in section 1.3.2, to calculate each turbine's power:

$$P_{turb}(V_e) = \begin{cases} 0 & V_e < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V_e - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V_e < V_{rated} \\ P_{rated} & V_{rated} \leq V_e < V_{cut-out} \\ 0 & V_e \geq V_{cut-out} \end{cases} \quad (1.4)$$

3. Sum powers ( $P$ ) from all  $t$  turbines at each wind speed.

$$P_{speed} = \sum_{k=1}^t P_{turb,k} \quad (1.5)$$

4. Sum powers for all  $m$  wind speed bins for that wind direction, multiplied by the probability ( $w$ ) that each speed will occur.

$$P_{dir} = \sum_{j=1}^m w_j P_{speed,j} \quad (1.6)$$

5. Compute AEP using farm power for all  $n$  directions over an idealized 24 hours a day for 365 days a year, where  $P$  is the wind farm power for direction  $i$ , and  $f$  is the corresponding frequency for direction  $i$ .

$$AEP = \left( \sum_{i=1}^n f_i P_{dir,i} \right) (365 \times 24) \frac{\text{hrs}}{\text{yr}} \quad (1.7)$$

Equation (1.7) gives us the calculated AEP from a given wind farm, assuming that every turbine is running 24 hours every day for 365 days. For a small, theoretical farm of 16 turbines (described in case study 1) with each turbine producing up to 10 MW of power, participants averaged approximately 400 kW in overall farm AEP. This farm was intentionally small in order to ensure that the turbines were close enough to each other for their wakes to interact frequently. Based on these calculations, one can see that several elements go into determining the overall energy production of a given farm.



### 1.3 Wind Turbines

Some of the earliest records of horizontal axis wind turbines (HAWTs) in western culture can be traced as far back as 1137 [4]. These ancient structures translated the wind's power into rotational kinetic energy used to grind flour, pump water, and power sawmills. Mechanisms like the cog and ring depicted in fig. 1.3 translated the horizontal rotation of the rotor to a vertical rotation down the shaft, powering whichever structure was attached below.

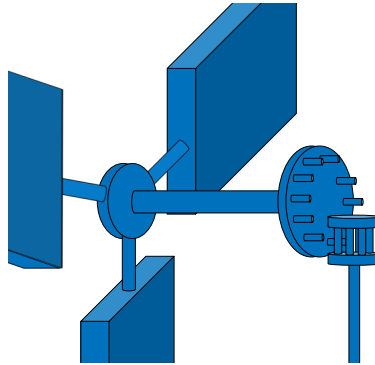


Figure 1.3: Early HAWT cog and wheel.

The descendants of these early windmills are wind turbines, which convert the wind's power into storable energy. Instead of mechanically translating rotation down a vertical shaft, most modern turbines house a generator directly behind the spinning rotors. Simple depictions of the inner workings of these turbines are shown in figs. 1.4 and 1.5.

Encased in the housing behind the spinning blades are, in broad terms, the turbine's *governor*, *gearbox*, and *generator*. In addition to maximizing power output, the governor adjusts blade pitch and controls rotational speed in order to prevent the damage an unregulated rotation would cause. The governor connects to the main shaft, which in turn connects to the gearbox (fig. 1.4). Like gears on a bicycle, a gearbox deterministically amplifies or dampens the main shaft's rotational speed in order to maintain a constant rotational speed for the generator (fig. 1.5). The generator creates power through a combination of magnets and copper wire, employing the principle of magnetic induction. The copper wire spinning along the generator's main shaft shifts electrons in the magnetic field around it, producing an electrical current [5].

This simplified design can be scaled from small science experiments, measuring a few centimeters tall and producing only 10 W of power, to the largest commercial turbine currently in

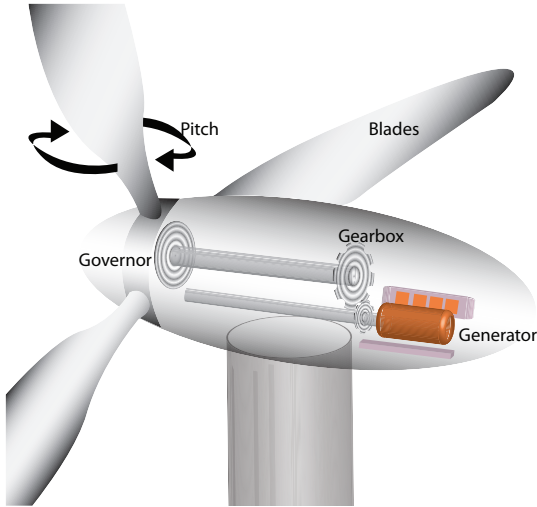


Figure 1.4: Turbine Nacelle Cutaway.  
NASA instructional image, public domain [5].

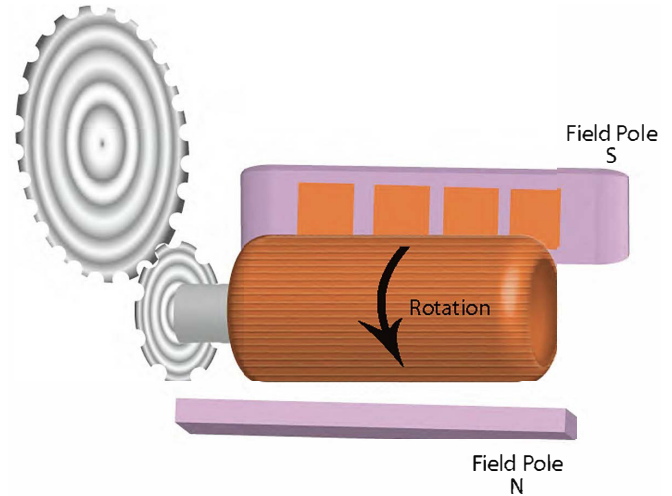


Figure 1.5: Blown-up Turbine Gearbox.  
NASA instructional image, public domain [5].

production, the Vestas V164 [6], with a rotor swept area alone measuring  $164\text{ m}$  in diameter and a production capacity of  $10,000,000\text{ W}$  (10 MW).

Modern turbine blades are composed primarily of fibreglass or carbon fibre composites [7]. They are much like propeller blades, in that they rotate about a central axis and have cross-sections that are simple airfoils decreasing in size towards the tip (fig. 1.6). This geometry enables wind flowing into the turbine face to create lift on the individual blades, generating a force that ultimately spins the rotor.

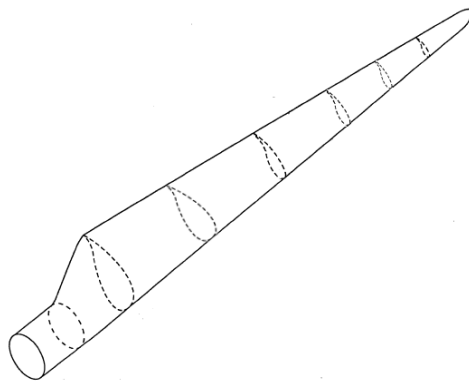


Figure 1.6: Turbine Blade with airfoil cutaways [8].

### 1.3.1 Functional Attributes

While turbines have several functional attributes, four are particularly important for the purpose of defining its power curve: *power rating*, *cut-in speed*, *rated wind speed*, and *cut-out speed*. Power rating, simply put, is the maximum amount of power a turbine can produce in an ideal set of conditions, usually given in kW or MW. A 10 MW turbine, if operating at maximum efficiency, will produce its rated 10 MW of power. While factors such as decreased wind speed, wind turbulence, partial wake, or worn components, can cause a turbine to produce less than its rated power, the number of turbines in a wind farm multiplied by their individual power rating generally serves as an upper limit for a farm’s energy production capacity.

Because turbines are large structures, mechanical assistance is often needed to initiate rotation of the blades around the main shaft. The blade geometry will generate lift sufficient to rotate the entire shaft above a certain wind speed, called the cut-in speed. Below a turbine’s cut-in speed, however, the wind is not blowing fast enough to overcome internal mechanical resistance and make the turbine produce more power than it would itself use, so turbine rotation isn’t merited.

Once the cut-in speed is exceeded, though the turbine has a net positive power production, it is not producing its rated power until the rated wind speed is reached. At speeds between the cut-in speed (where power production begins) and rated wind speed (where maximum power is produced), turbine power can be modelled in an idealized way after a cubic power curve:

$$P_{urb}(V_e) = P_{rated} \cdot \left( \frac{V_e - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 \quad (1.8)$$

Figure 1.7 graphically depicts eq. (1.8) in the velocity region between the cut-in and rated wind speeds of the example turbine.

On the other end of the speed spectrum, a turbine functioning in high winds poses a potential danger to itself for a few reasons. In the HAWT configuration (depicted in figs. 1.3 and 1.4), each blade passes between the incoming wind and the vertical support structure every rotation. Though blades are built to slightly flex and give, wind that is too powerful can bend the turbine’s blade tips to the point of causing a “blade-strike,” where the spinning rotor impacts the vertical support structure. Similarly, the internal governors and gearboxes are themselves mechanically

limited in terms of rotation speed. If spun too fast, they can be structurally dismantled, or incur unsustainable mechanical friction and heating which could result in serious damage.

Because of the dangers that exist, the “cut-out” speed serves as the maximum operable wind speed, above which the turbine is not safe to operate. In wind conditions above the cut-out speed, turbine rotation is halted and each blade is usually “feathered,” or turned into the wind so that they produce minimal lift even when stationary.

### 1.3.2 Power Curve

Using these four attributes, power rating, cut-in speed, rated wind speed, and cut-out speed, we can construct a graph of windspeed versus power produced, called a *power curve*. A power curve provides a better understanding of a given turbine’s power production capacity, an example of which is given in fig. 1.7.

Turbine Power Rating	15	MW
Cut-In Wind Speed	3	m/s
Rated Wind Speed	10.59	m/s
Cut-Out Wind Speed	25	m/s

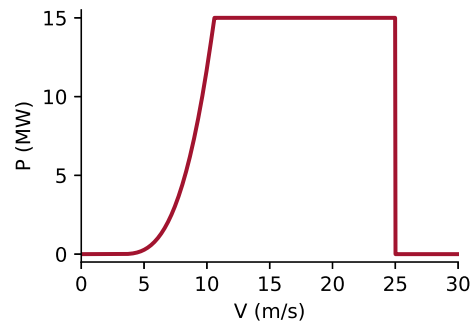


Figure 1.7: Attributes (left) and power curve (right) for the IEA Wind 15-MW reference turbine [9].

The IEA Wind 15-MW offshore reference turbine was used in fig. 1.7 as an example to understand plotted power curves. When analyzing the curve, note that at wind velocities below the cut-in speed of  $3\text{ m/s}$ , power production is zero because the rotor is not spinning. Once the cut-in speed is reached, power produced by the rotor increases cubically following the formula explained in eq. (1.8), reaching the maximum turbine rated power of  $15\text{ MW}$  at the turbine’s rated wind speed of  $10.59\text{ m/s}$ . The turbine produces its rated power for all wind speeds above the rated wind speed and below the cut-out speed which, in this case, is  $25\text{ m/s}$ . Beyond the cut-out speed, the turbine drops power production to zero since rotation above that speed is deemed unsafe for the turbine.

## 1.4 Wind Turbine Wake

The rotating blades of a horizontal wind turbine sweep out a circular geometry in space, as depicted by the gray region in fig. 1.8. This area is modelled as an infinitely thin disk, called an *actuator disk*, that uniformly affects the incoming free stream air. When examining the free-stream air flowing perpendicular to the face of a turbine, a few trends can be observed in both the upstream and downstream wind areas. The relationship between the properties of the closed volume of wind before, throughout, and beyond the actuator disk is described in part by the formula for *mass flow rate*,

$$\dot{m} = \rho VA \quad (1.9)$$

where  $\dot{m}$  is *mass flow rate*,  $\rho$  is *density*,  $V$  is air stream *velocity*, and  $A$  is cross-sectional *area*.

In accordance with the law of *conservation of mass*, the mass flow upstream ( $\dot{m}_\infty$ ) must be equivalent to both the mass flow at the actuator disk ( $\dot{m}_d$ ) and the mass flow at the downstream waked area ( $\dot{m}_w$ ). Assuming a uniform  $\rho$  in a closed system, any variability in a velocity term must be counteracted by movement of the same magnitude and opposite direction in the corresponding cross-sectional area term. This relationship is demonstrated in eq. (1.11) below.

$$\dot{m}_\infty = \dot{m}_d = \dot{m}_w \quad (1.10)$$

$$\rho V_\infty A_\infty = \rho V_d A_d = \rho V_w A_w$$

$$\rho V_\infty A_\infty = \rho V_d A_d = \rho V_w A_w$$

$$V_\infty A_\infty = V_d A_d = V_w A_w \quad (1.11)$$

The downstream region with decreased freestream velocity and increased cross-sectional area is the turbine's *wake* [10], a word generally defined as a disturbance in the air caused by a solid object. Turbine wake is characterized by a slowed freestream velocity that has passed through the turbine's face, and an increased cross-sectional diameter of the affected wind volume (figs. 1.8 and 1.9). Downstream rotors in a wake-affected area experience reduced power production due to the decreased experienced wind velocity and a shortened lifespan as a result of the increased turbulence intensity [11].

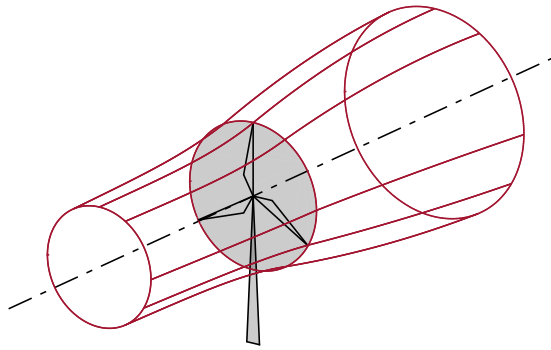


Figure 1.8: Three-dimensional visualization of the streamtube of air passing through a HAWT modelled as an actuator disk.

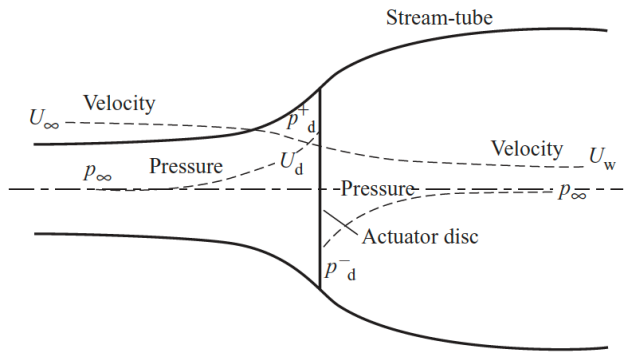


Figure 1.9: Velocity and pressure gradients along the streamtube [10]. The subscript  $\infty$ ,  $d$ , and  $w$  represent the freestream, actuator disk, and downstream (waked) locations, respectively.

In this example (depicted in figs. 1.8 and 1.9), many simplifications have been made, including assumptions that: (1) flow is only in the  $x$ -direction, fully-developed, and has reached a steady state; (2) friction is negligible; and (3) both heat and gravity are not factors.

Since waked turbines result in both decreased energy production and reduced turbine lifespan, wakes become an important consideration in wind farm design [12]. Turbines placed outside of each other's wake avoid these negative effects, and as a result increase their wind farm's AEP. Strategic turbine placement, then, is an important factor affecting a wind farm's AEP and COE.

## 1.5 Engineering Wake Models

To better understand the impact turbine wakes have on the aerodynamics of a wind farm, engineers have created computational models, termed *engineering wake models*, to mimic the actual fluid effects experienced in the flow field. Creating an engineering wake model to accurately and simply simulate both wake and aerodynamic turbulence is a multivariate problem and, in a general sense, is a decision between increased model realism and the computational resources required to account for it. The two ends of the complexity spectrum that have been taken to create such models are: (1) simplified theoretical wake models that are computationally inexpensive and have been formulated to correlate to the empirical measurements of real turbine waked regions [11, 13, 14]; and (2) computationally expensive but formulaically accurate models, such

as employing the Reynolds-Averaged Navier-Stokes (RANS) equations, Large-Eddy Simulations (LES), or Direct Numerical Simulation (DNS), all of which far better increase the precision and granularity of their respective results [15].

With regard to the first approach, many theoretical wake models have been created, varying in both their complexity and their perspectives of the simulation problem. While some account for 3-Dimensional aerodynamic phenomena, the simplest and most widely used wake model was created by N.O. Jensen [16] and is 2-Dimensional. Jensen created two variants, a “top hat” model and a cosine model, depicted in fig. 1.10, and demonstrated that his models matched reasonably with empirical data [16].

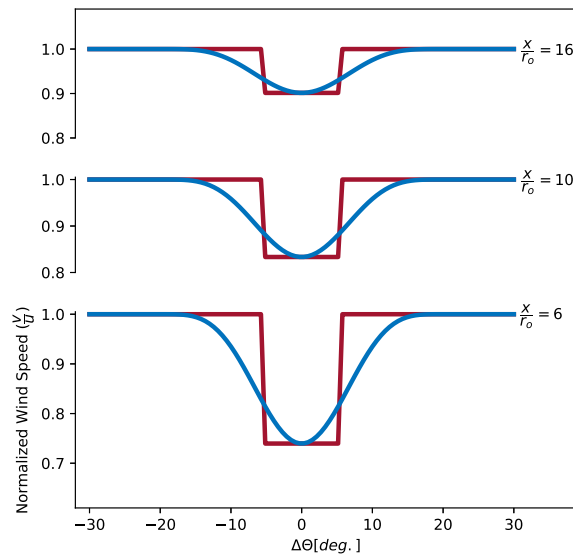


Figure 1.10: Jensen’s models against empirically measured wake [16]

The Flow Redirection and Induction in Steady-state (FLORIS), another simple theoretical wake model, can be implemented either 2- or 3-Dimensionally and is a direct improvement on the Jensen “top hat” model. It discretizes three different regions in the wake in order to mimic observed wind speed patterns in waked regions [17], and permits yaw control to affect the wake’s direction, as depicted in fig. 1.11. Improvements to the FLORIS model made to enable gradient-based optimization incorporate a cosine function [18]. In another approach, Larsen et. al created a 3-D Dynamic Wake Meandering (DWM) model [13, 19, 20]. Though not widely used, it was formulated to model the turbulent wake in three dimensions as it twists, or “meanders” (as illustrated by figs. 1.12 and 1.13), and demonstrates the 3-dimensionality of turbine wake in space. A

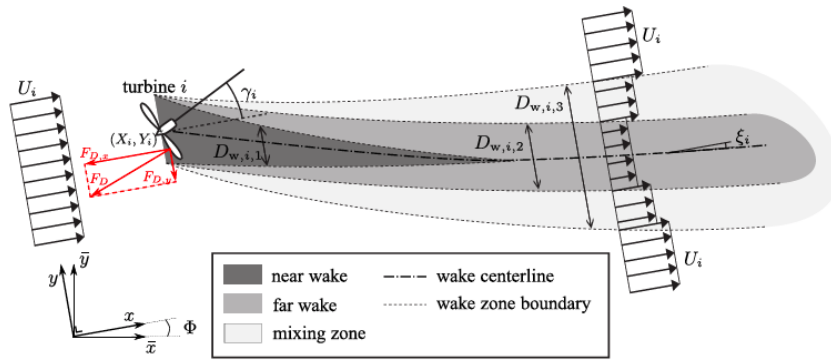


Figure 1.11: FLORIS Yawed Wake Model [17]

Gaussian-curved 3-D wake model was also created [21–24], and was determined to be the correct balance between simplicity and realism for use in the case studies. This model’s continuity enables use of both gradient-based and gradient-free optimization algorithms. The implementation of this wake model in the case studies is described in depth in chapter 2.

On the opposite end of the engineering wake model spectrum, optimizations using the RANS equations take on the order of many hours to a day to converge to a solution. More complex still, LES models can take several days to converge. Finally, a DNS analysis attempted by Soren et. al in 2011 at what can be assumed to be higher Reynolds numbers [15], and was determined to be infeasible in terms of computational time.

Despite the lengthy time requirements for such granular methods, a common practice is to first achieve a quicker optimized solution using one of the simpler models described, then implement a single LES on the optimized result to validate the proposed wind farm layout.

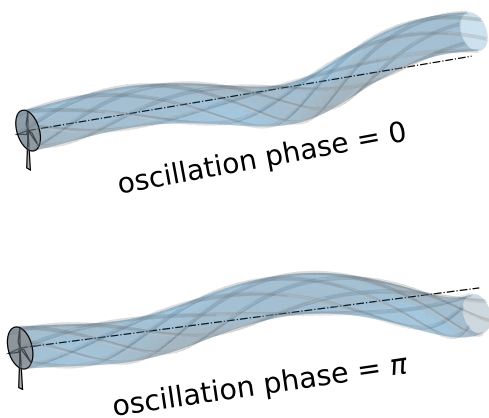


Figure 1.12: DWM interpreted in 3-dimensions.

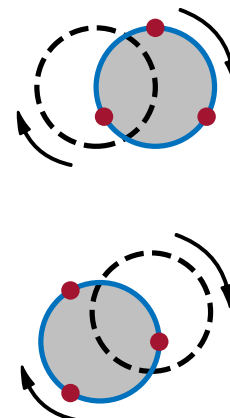


Figure 1.13: DWM in the Trefftz plane.



## 1.6 Optimization

The wind farm layout optimization problem aims to find the best turbine placement within a given farm’s geography. The term “best” here is dependent on the engineer’s ultimate objective (e.g., increased noise reduction, lowered ground footprint, etc.), the most common of which tends to either be maximizing windfarm AEP or minimizing windfarm COE. The variables involved in wind farm layout optimization are numerous and expand significantly with the inclusion of additional turbines. As a result, computer simulations are necessary to calculate all of the second and third-order effects of variable manipulation. Finding a function’s maximum or minimum evaluation, through perturbation of input variables, is called an *optimization*.

Optimization algorithms can be broadly categorized as gradient-free or gradient-based. Gradient-free algorithms can be used if derivatives cannot be obtained but are usually slower than gradient-based algorithms and do not always scale well. Some gradient-free algorithms are able to escape local optimums and are generally easier to implement. Gradient-based algorithms, however, require the governing functions to be continuous, differentiable, and that derivatives can be calculated. They scale well to large problems and are effective for finding local optimums, but have difficulties with functions that are “noisy” or when discontinuities are present [25]. Gradient-based algorithms can, however, greatly reduce computational time if derivatives can be provided inexpensively [26].

## 1.7 Computational Derivatives

Though “gradient-free” optimization is a useful and well-explored branch, if obtainable, derivatives are powerful tools used to help discover the optima of a function. The three main methods used for determining derivatives are [25]: (1) *symbolic differentiation* using the calculus rules of differentiation, (2) *numeric differentiation* using approximations like finite difference, and (3) *algorithmic differentiation* sometimes also called “automatic” differentiation, which strictly analyzes code. Each method has its strengths and weaknesses but, depending on the problem formulation, all three options may or may not be available to the engineer.

### 1.7.1 Symbolic Differentiation

Used for simple algebraic expressions, symbolic differentiation applies the basic calculus rules of differentiation to symbolically determine the exact gradient for a given function. For example, to discover the minimum of the simple algebraic formula such as,

$$f(x) = 3(x + 1)^2 - 2 \quad (1.12)$$

a corresponding differential formula can be straightforwardly calculated. Solving this differential formula for the  $x$ -value delivering  $\frac{df}{dx} = 0$  is coincident with the minimum of the original objective function. The formulation of this specific example is given in fig. 1.14.

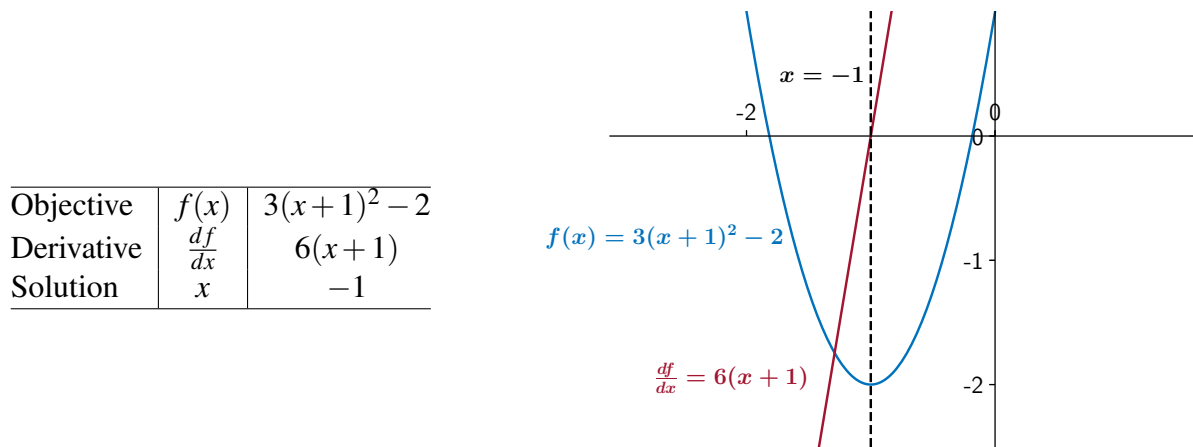


Figure 1.14: Depiction of a minimization optimization of a simple function using the derivative.

A widely used, but more complicated optimization benchmark, is the *Rosenbrock Function*, named after British engineer Howard Rosenbrock. It is a function of two variables, so symbolic gradient calculations require calculating partial derivatives. The Rosenbrock Function's corresponding differential equations are not as readily apparent as the previous example, but are provided both formulaically and graphically in fig. 1.15.

As the number of variables increases, the ability to calculate and solve symbolic derivatives becomes more complicated, made apparent by a comparison of figs. 1.14 and 1.15. In scenarios where exact derivatives are not readily available, the techniques of numerical differentiation or algorithmic differentiation can be implemented instead.

Objective	$f(x,y)$	$(a-x)^2 + b(y-x^2)^2$
Partial x	$df/dy$	$2b(y-x^2)$
Partial y	$df/dx$	$x(4bx^2 - 4by + 2) - 2a$
Solution	$(x,y)$	$(a, a^2)$

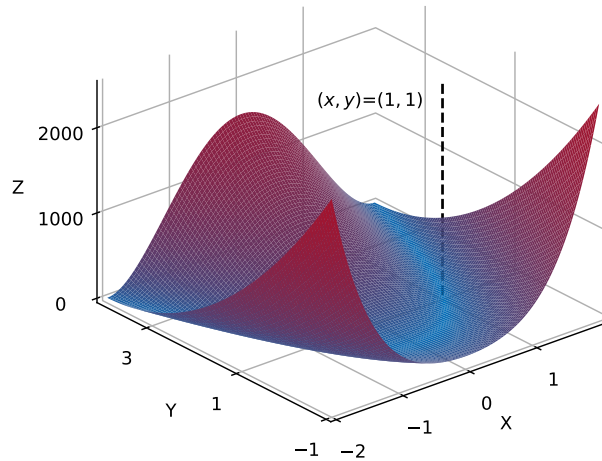


Figure 1.15: Depiction of the Rosenbrock problem, plotted with values  $a = 1$ ,  $b = 100$ , with global minimum at  $(x, y) = (1, 1)$ .

## 1.7.2 Numerical Differentiation

Numerical differentiation is a method of calculating values in close proximity to a given point of interest and approximating the gradients of the objective in the immediate area based on an interpolation. It is simple to implement but can become inaccurate due to round-off and truncation errors [27] or use of a perturbation size of inappropriate length.

A popular method for numerical differentiation is using the *central difference* method, defined as:

$$\frac{\partial f}{\partial x} \approx \frac{f(x + \varepsilon) - f(x - \varepsilon)}{2\varepsilon} \quad (1.13)$$

and is also called the “two-sided difference.” For a target function  $f(x)$ , the central-difference method samples values  $\varepsilon$  away from  $x$  and approximates the local gradient. Unfortunately, this can get quite computationally expensive since it is necessary to evaluate  $f$  at the points  $x$  and  $x \pm \varepsilon$  for every function call. A multivariate objective function further increases the number of computations required for each perturbation very quickly. Using the example formula previously mentioned in eq. (1.12), this concept can be demonstrated graphically, as seen in fig. 1.16.

Figure 1.16 illustrates that, as the values of  $\varepsilon$  become smaller and smaller, the approximation becomes more and more precise. However, as more trailing decimals are introduced, the issues

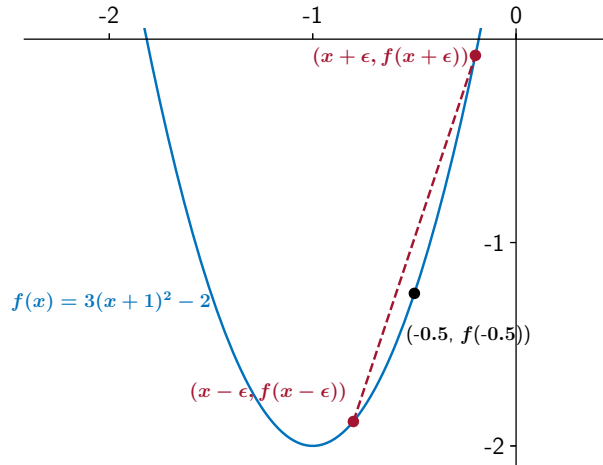


Figure 1.16: Central difference used on the example eq. (1.12), with an  $\epsilon = 0.3$  and  $x = -0.5$ . Note that the approximated slope between perturbed values  $x \pm \epsilon$  very closely estimates the tangent line at our point of interest.

of machine precision and truncation error limit the calculation accuracy. As such, the precision of numerical differentiation is limited by the computing system operating it.

### 1.7.3 Algorithmic Differentiation

While symbolic differentiation calculates exact gradients and numerical differentiation approximates the gradient based on perturbed input values, algorithmic differentiation works much like numerical differentiation, manipulating the actual code instead of the original target function. Algorithmic differentiation “is founded on the observation that any [coded] function, no matter how complicated, is evaluated by performing a sequence of simple elementary operations involving just one or two arguments at a time” [25]. By perturbing values and inputting them into each line of code, the computer is able to exploit the *chain rule* and build a surrogate derivative function for the code based on how the program was syntactically written [27].

This method can be easily illustrated with an example, in this case, again using the Rosenbrock Function described in fig. 1.15. It is a function of 4 variables and could be coded to read:

$$f(x_1, x_2, x_3, x_4) = (x_1 - x_2)^2 + x_3 * (x_4 - x_2^2)^2 \quad (1.14)$$

While algorithmic differentiation would recognize the *independent variables*  $x_1, x_2, x_3, x_4$ , it would then create *intermediate variables*  $x_5, x_6, \dots$  as needed. We could thus rewrite  $f$  in arithmetic terms as follows:

$$x_5 = x_1 - x_2$$

$$x_6 = x_5^2$$

$$x_7 = x_2^2$$

$$x_8 = x_4 - x_7$$

$$x_9 = x_8^2$$

$$x_{10} = x_3 * x_9$$

$$x_{11} = x_6 + x_{10}$$

Though the line of code expressed in eq. (1.14) has been broken down for explanation, algorithmic differentiation software does not require the engineer to do this manually. Identification of intermediate quantities and construction of the final derivative,  $x_{11}$ , is carried out by the algorithmic differentiation software itself.

While this analytical method of calculating derivatives saves more execution time than an option like numerical differentiation, proper implementation takes time to set up correctly. There are also some pitfalls to this method due to it being, by nature, a code analysis. For example, methods like recursion can confuse an algorithmic differentiation algorithm and convoluted code can give an improper surrogate derivative [28].

A simple example provided in Nocedal's text on optimization [25] is for the function  $f = x - 1$ . Though convoluted, it would not be incorrect to code this function as:

```
if (x1 = 1.0) then f = 0.0 else f = x1 - 1.0
```

However, the algorithmic differentiation would evaluate  $\frac{df}{dx_1}(1) = 0$ , instead of seeing it as a continuation of all other input values of  $x_1$ .

Algorithmic differentiation is an improvement over numerical differentiation in that it avoids some of the rounding errors inherent in numerical differentiation, as well as cutting down

on the number of function calls since it doesn't need to check and interpolate values at  $f(x)$  and  $f(x \pm \epsilon)$  for every evaluated point. Algorithmic differentiation approaches the exactness of symbolic differentiation by deriving differential expressions from the written code. However, by virtue of a code analysis, algorithmic differentiation is limited in certain situations unique to code transcription and has its own unique shortcomings as a method.

## 1.8 Wind Farm Layout Optimization

Wind farm layout design refers to the process of determining the placement of individual wind turbines, their requisite electrical connections, and the engineering infrastructure necessary to support them within a selected site [12]. More particularly, the phrase *wind farm layout optimization* describes the application of methods or algorithms on turbine placement in order to maximize or minimize some objective (e.g., power output, noise production, environmental impact, cost of energy, annual energy production, etc.).

In terms of energy production, many factors play a role in determining the amount of energy a wind farm produces including turbine specifications, geographic constraints, wind resource availability, and wake interactions due to placement of turbines. Turbine power losses due to wake interactions can be as high as 30% - 40% for farms with turbines spaced within 3 rotor diameters of each other [29]. As such, wind farm layout optimization is an essential step in wind farm development in order to maximize power production.

Current methods for determining optimal wind farm layouts include perturbing individual turbine locations independently [18, 30, 31], parameterizing turbines into pre-determined, discrete grid points [32, 33], and interdependent positioning using relative locations [34]. Perturbing individual turbines' locations allows the largest exploration of the design space, but requires comparatively heavy amounts of computation due to the number of design variables that can be required. Parameterizing turbine locations onto grid locations results in the trade-off of abbreviating the design space but can also overlook global optima not available to the discretization. Coordinating turbine locations into interdependent variables is a newer, largely unexplored method, but has demonstrated promising results in terms of low computational costs, quick convergence, and relatively good results [34].

Once the variable parameterization approach is chosen, engineering wake model and optimization algorithm selection will determine the efficiency of optimal layout discovery. It is up to the designer to decide between the numerous trade-offs associated with optimization algorithms, wake model approximations, and the resulting computational efficiency.

## **1.9 IEA Task37**

The International Energy Agency is the international hub for discourse on all things related to energy. Created in 1974, it provides “analysis, data, policy recommendations, and real-world solutions to ... provide secure and sustainable energy for all” [35]. Under the IEA is the Technology Collaboration Programme (TCP), which, as of 2018, supports 16 task groups focusing on aspects of wind energy research, development, and deployment [36].

Part of the IEA TCP’s 2019-2024 strategic plan is Wind Task 37 (IEA37), an initiative which seeks to analyze wind power plants as holistic systems via a systems engineering approach [1]. IEA37 aims to be a “forum of wind reference systems (wind turbines and plants), development, and benchmarking activities in multi-disciplinary design analysis and optimization (MDAO)” [37]. The four case studies discussed herein fall under its “benchmarking activities” mission. Further, though the case studies were created under IEA37, they are intended for anyone interested in wind energy and optimization research as a reference point or problem set.

IEA37 released an explanatory graphic in 2018 (shown in fig. 1.17), which demonstrates the various elements of a wind farm it aims to integrate. Of the elements depicted in fig. 1.17, case studies 1 and 2 involved only plant layout and energy production. Case studies 3 and 4 included the same elements as 1 and 2, in addition to complex wind inflow and more complicated plant layout requirements. Future case studies under IEA37 will continue to add more aspects and complexity, meeting the holistic aims of the Task.

## **1.10 Need**

Optimizing turbine placement within a wind farm is a complex problem characterized by many local optima. The large number of inter-dependent variables involved in wind farm layout optimization creates a design space that is difficult to solve reliably. Sub-optimal turbine placement

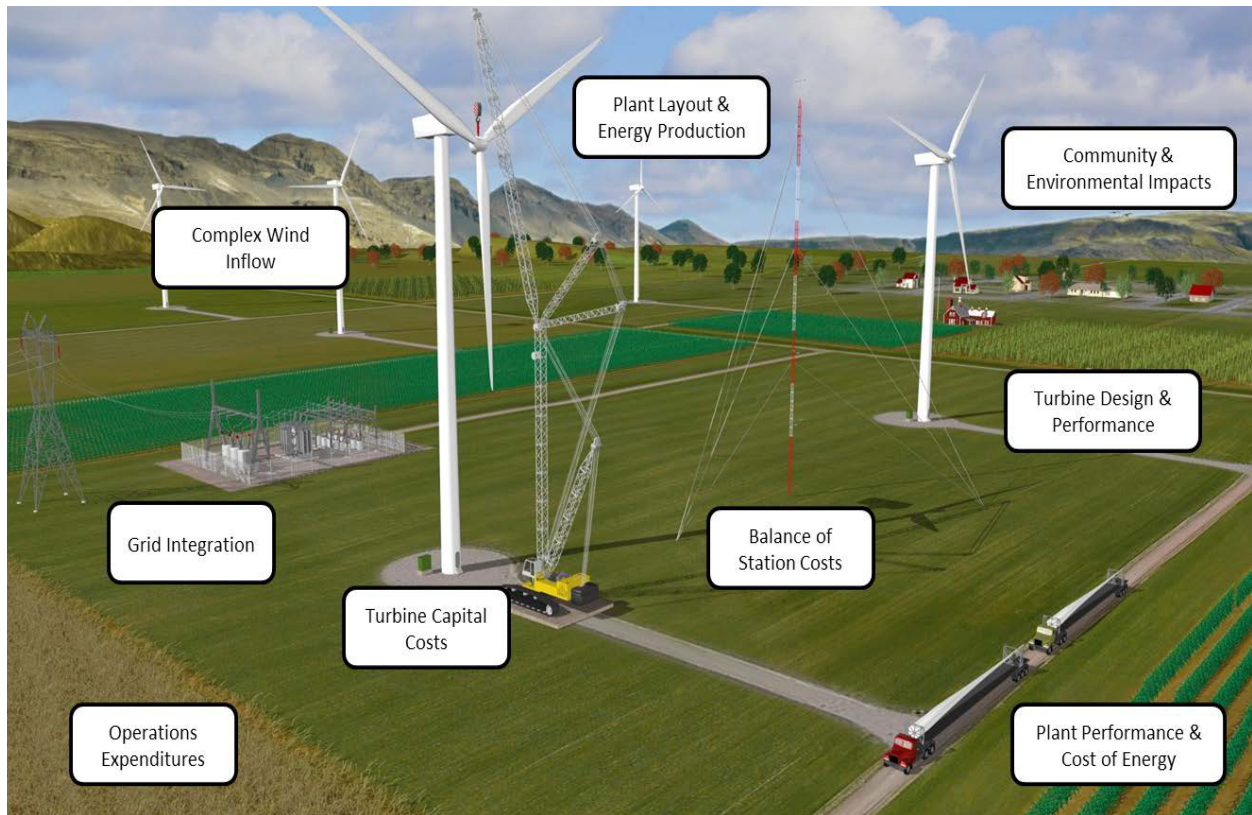


Figure 1.17: Visual depiction of all of the aspects IEA37 aims to integrate [36].

results in lost energy and potentially millions of forfeit dollars over the course of a wind farm’s typical 20-year lifespan [12]. This error could result from either an inaccurate wake model or inefficient optimization algorithms. Mistakes in either of these two areas can be avoided with a clearer understanding of best model and optimization practices.

The two main approaches used to improve the methods of discovering better turbine placement locations in wind farm layout optimizations have been: (1) improving the quality of individual models; and (2) improving the formulation of the optimization problem, in addition to the algorithms used to perform the optimization itself.

Though many studies have been propagated using a given engineering wake model or optimization algorithm as the control variable measured across different wind farm scenarios (the independent variable), the author struggled to find a study where the wind farm scenario was fixed (control variable) allowing him to measure the performance of many different wake models or optimization algorithms (independent variable) under that scenario. To conduct such a study, the most important factors to measure needed to first be determined, as well as which aspects of a wind farm layout optimization problem set would be most compelling for comparative performance.



A wind farm scenario permitting many different solutions, all of which deliver similarly high AEP would not be very instructive. For example, a large farm boundary with so few turbines that each could be placed so as to not interact with any other turbine's wake would not reveal much in terms of viable solutions to such a problem. Conversely, a wind farm layout optimization problem in which the "best" layout is so obvious that any optimization algorithm, regardless of idiosyncrasies, could discover it would also be uninformative.

Constructing wind farm scenarios that met these criteria was an optimization problem in and of itself. A description of the process of rounds of testing, implementation of user feedback, and the synthesis of work done by other researchers to arrive at the four released case studies is ultimately the purpose of this document.

## CHAPTER 2. CASE STUDY 1 & CASE STUDY 2

### 2.1 Problem Formulation

The first goal of this case study set was to isolate variability of the optimization algorithm. To achieve this, we pre-coded a representative wake model as the control variable and permitted participants to use any optimization strategy to alter turbine locations in an effort to yield the best AEP for the farm. This was called case study 1 (cs1).

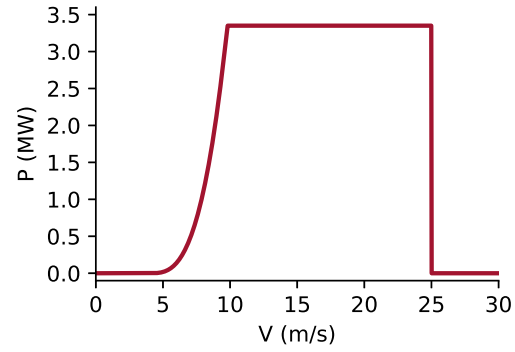
The second goal, isolating engineering wake model variability, proved more complicated. A wake model's compatibility with gradient-based or gradient-free optimization methods dictates which optimization algorithm can be applied. As such, designing a case study that restricted participants to a single optimization algorithm would unnecessarily limit the scope of the engineering wake models studied. For this reason, we designed the second case study (cs2) to permit participant selection of wake model in addition to optimization algorithm.

Though testing for different variables, certain wind farm attributes were common to both case studies. An explanatory list of these variables is described below.

#### Wind Turbine Specifications

For uniformity and simplicity, we decided to prescribe only one type of wind turbine, the IEA 3.35-MW reference turbine, for use in all farms. The IEA 3.35-MW is designed as a baseline for onshore wind turbine specifications, and its specifications are open source [38]. The properties of the 3.35-MW turbine used in case studies 1 and 2 that are necessary for calculation of the simplified version of Bastankhah's Gaussian wake model are shown in fig. 2.1. The turbine's data was given to participants as `iea37-335mw.yaml` and is included in the appendix.

$$P(V) = \begin{cases} 0 & V < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V < V_{rated} \\ P_{rated} & V_{rated} \leq V < V_{cut-out} \\ 0 & V \geq V_{cut-out} \end{cases}$$



Rotor Diameter	130	m
Turbine Rating	3.35	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	9.8	m/s
Cut-Out Wind Speed	25	m/s

Figure 2.1: Attributes for IEA's 3.35-MW onshore reference turbine

## Farm Geography

In order to focus on optimization method and EWM variability, as well as avoid introducing too many unnecessary variables, we made all wind farms in case studies 1 and 2 on flat and level terrain. To reduce boundary impacts on farm design, we chose a radially symmetric farm boundary. Had we not taken this precautionary measure, the results might have included optimal turbine layouts with turbines clustered at the farm corners or stuck in protrusions.

To reduce necessary geometric computations, turbine  $(x,y)$  hub locations were restricted to on or within the wind farm's boundary, instead of restricting turbine radii. To mimic blade-on-blade collision concerns, turbines could be no less than two rotor diameters apart from any other turbine.

Variability in the number of turbines contained in each wind farm caused variability in the number of design variables, thus affecting optimization algorithm performance. To study how increased farm size (i.e., design space complexity) impacted the performance of algorithms, and to demonstrate trends of scalability, three wind farms of increasing size and turbine quantity were specified for case study 1. In order to allow participants to implement even grid turbine arrange-

ments, the total number of turbines on each farm was a perfect square, roughly doubling in size from scenario to scenario. In our testing, 1, 4, and 9 turbines were too few to have enough local optima to sufficiently test algorithm performance. As such, we selected 16, 36, and 64 turbines for each scenario in case study 1, respectively.

Case study 2 permitted participants to implement their own wake models. As such, comparisons between user-submitted layouts would not be fair, as they were reached using different approximations of reality. To create a universal standard between user submissions, it was intended to run an LES on all submissions for an AEP analysis that was universally comparable. Since LES is so computationally expensive, though a 9-turbine farm is relatively small, that size was deemed necessary to permit the implementation of as costly a wake calculation method as LES. Though an LES was never ultimately implemented, case study 2's farm size of 9-turbines is a result of intending to use it.

Based on the selected turbine numbers, each farm's diameter needed to be restrictive enough to avoid simply placing all turbines on the boundary, while also permitting meaningful turbine movement by the optimizers. The magnitudes of the boundary radii were determined by placing the requisite number of turbines for each scenario in evenly spaced concentric rings, while maintaining a minimum turbine spacing of five rotor diameters both laterally and between adjacent rings.

The following algorithm was created and used to determine both farm boundary radii and turbine placement of the example layouts for all farms in the studies:

1. Place one turbine in the center of the farm.
2. Place turbines one by one along the next concentric circle, until no more will fit laterally, maintaining a minimum of 5 diameter spacing in all directions.
3. Move to the next concentric circle, until the number of turbines is exhausted.
4. If the outermost ring has less than 5 turbines, move said turbines into the penultimate ring, expanding it as little as possible to accommodate the additional turbines while still maintaining the lateral spacing requirement.
5. Expand radius of outermost ring to the nearest 100 *m*.

6. Evenly expand the radii of the inner turbine rings for uniformity.

Using this method, the boundary radii for the scenarios in case study 1 were determined to be 1300 m, 2000 m, and 3000 m, respectively.

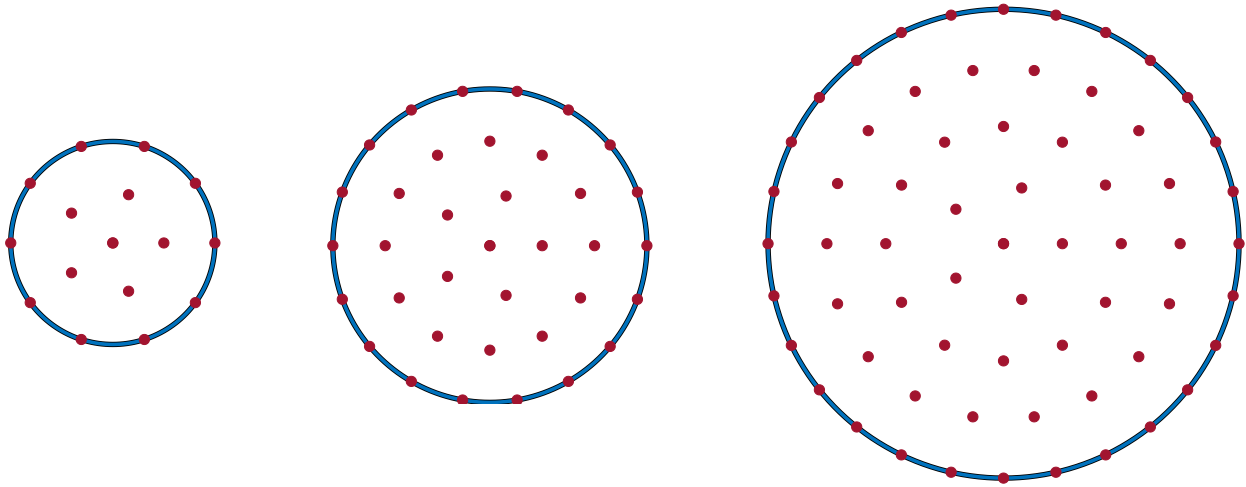


Figure 2.2: Example layouts for cs1's 16, 36, and 64 turbine farms

### Example Layouts

The example layouts resulting from the aforementioned algorithm are displayed graphically in fig. 2.2. Though primarily for case study 1, they were provided to participants of both case studies 1 and 2 in .yaml format for 4 reasons:

1. To provide an example of the .yaml schema formatting being developed by IEA Task 37 and used to collect results.
2. To help participants visualize the different farm sizes and geography.
3. To verify AEP calculation if participants implemented the wake model in a different language.
4. To provide participants with an optimization starting point if desired.

The three example layout files for case study 1 were `iea37-ex16.yaml`, `iea37-ex36.yaml`, and `iea37-ex64.yaml`, all included in the appendix. It was explicitly stated in the announcement

document that these layouts were intended to be used as examples only. Participants were not required to use them as starting points for their optimizations, though they had the option to do so.

## **Wind Attributes**

In creating these case studies, a wind direction frequency distribution (displayed graphically in a wind rose) that met the following two criteria was necessary:

1. Creates many local optima
2. Prevents dissimilar turbine layouts from reaching the same AEP values

The absence of local optima in a design space enables even ineffective optimizers to find a superior result. Inversely, in a design space where many local optima are present, inferior designs are very likely. Since the presence of several local optima is a feature observed in many wind farm optimization problems, we sought to create similar design spaces with the case study scenarios, as they provided the ability to test the exploratory capabilities of the submitted optimization algorithms.

We intentionally made all case study 1 and case study 2 freestream velocities uniformly equivalent to the IEA 3.35-MW wind turbine's rated wind speed, 9.8 m/s. As can be seen by the 3.35-MW turbine's power curve in fig. 2.1, any wake effects would move air velocities below the turbine's rated wind speed, therefore increasing power production variability between turbines in the farm. With greater variability in power production, more local optima can be experienced by participant optimizers. Conversely, if the incoming wind speed was faster than the rated wind speed (i.e., 13 m/s), optimizers would not be incentivized to alter turbine location based on wake effects. In that scenario, more turbine layouts, though different in location, would produce identical power outputs since waked turbines experiencing velocities between their rated wind speed and cut out speed still produce their full rated power.

Similarly, the selection of the wind rose was a major factor in the frequency and magnitude of local optima resulting from turbine placement. In our tests, uni-directional wind roses often pushed turbines into a single row perpendicular to the major wind direction, suggesting a relative lack of local optima. Similarly, omnidirectional roses tended toward a pattern of simply spreading the turbines out uniformly, which also suggested a lack of local optima. Based on these results,

we intentionally found a wind rose with neither pattern, as a different pattern would have relatively more local optima, enabling us to demonstrate the difference in capabilities of the various optimization strategies used by participants.

If many unique layouts produced high and similar AEP values, they would not create much room for comparison between their respective abilities and the participant optimization methods that created them. When many layouts give similarly superior results, algorithms are not incentivized to find one “best” result. To find a wind rose that could meet both of these criteria, we experimented with four different patterns of wind direction frequency:

1. Bi-modal uniaxis, simulating a canyon’s geography;
2. Quad-directional, experienced at some onshore airports;
3. Tri-directional, experienced by some coastline geographies; and
4. Bi-modal off-axis, experienced in both on- and off-shore locations.

To increase authenticity, we searched wind history data for real-world locations that embodied these patterns. We compared data from several airports during our search for viable candidates, using that years’ available wind rose history up to what was then present (01 January to 27 June 2018). Said history was subsequently binned for 16 directions. For the quad-, tri-, and bi-directional off-axis wind roses, we examined recorded data from Denver International, LaGuardia, and Santa Barbara airports, respectively. These wind roses are depicted in fig. 2.3(a)-(c) as polar coordinates, where a greater magnitude in the radial direction from the origin indicates a higher wind frequency from that specific direction. The database employed to find the wind rose candidates was the cli-MATE database from the University of Illinois.

To analyze the effects these wind roses would have on optimization performance, simulations were run using the simplified Gaussian wake model (described in section 2.1.1) and the optimization package, SNOPT. We ran each wind rose through 1000 random-start optimizations using these parameters. These three optimizations generally produced similar results, depicted as histograms in fig. 2.3(d)-(f), with AEP distributions following a bell-curve pattern. There are slight differences between the optimizations, however, in that:

- A bi-modal, uniaxial wind rose (not depicted) gave turbine arrangements that tended toward a straight line, perpendicular to the axis of the wind. Since it lacked local optima, our first disqualifying criteria, it was not analyzed further.
- A quad-directional wind rose (fig. 2.3(a)) gave turbine arrangements that tended toward a grid pattern. Similarly, since this wind rose did not have many local optima to trap optimizers, it was also disqualified.
- A tri-directional wind rose (fig. 2.3(b)) gave many grid-like arrangements and delivered a flatter bell-curve than that of a competitor, and lacked meaningful outliers on the high end of its AEP values. Since a better option with apparently more local optima was available (i.e., the bi-modal off-axis option), this wind rose was not selected.

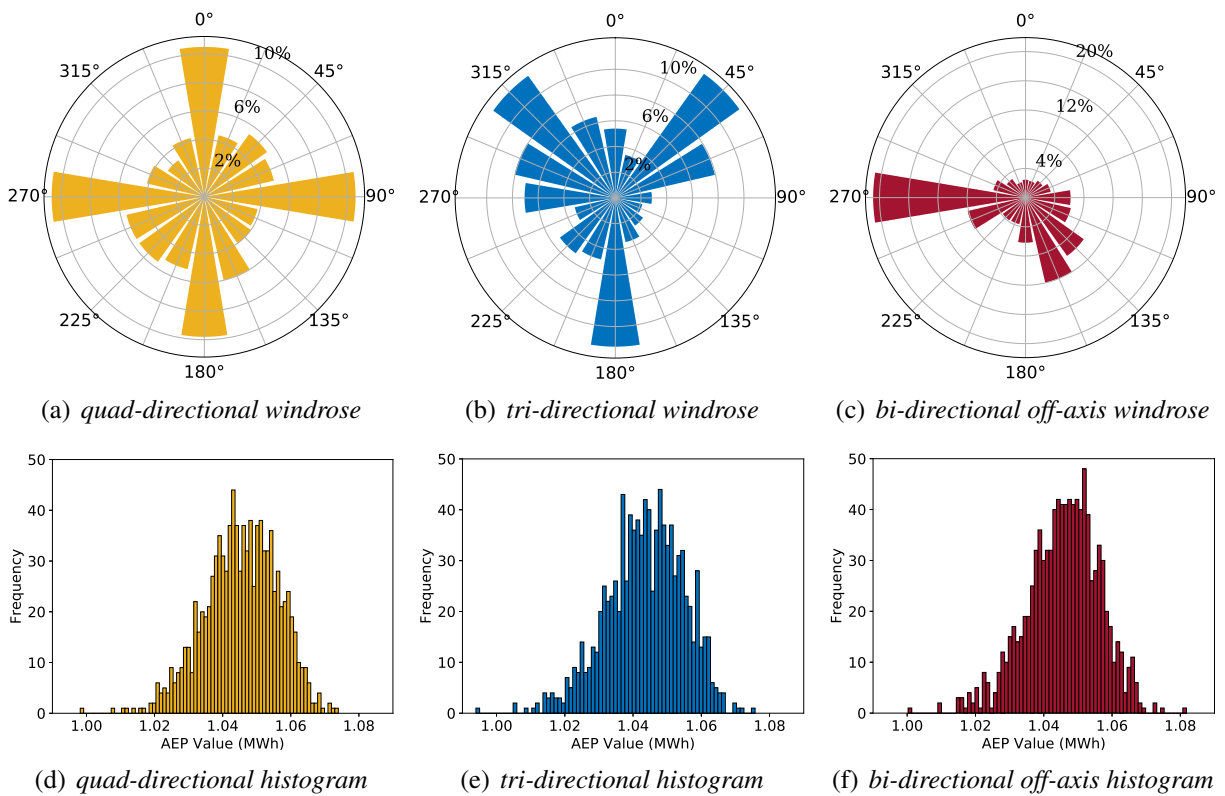


Figure 2.3: Quad-directional, Tri-directional, and Bi-modal off-axis wind roses and histograms



- A bi-modal off-axis wind rose (fig. 2.3(c)) gave few results with high AEP values removed from the rest, a fact we interpreted to be indicative of the presence of many local optima. Since this wind rose best met the selection criteria, it was used for case studies 1 and 2.

The wind rose we selected for final publication with case studies 1 and 2 is shown in fig. 2.3(c). The wind rose data was given to participants as `iea37-windrose.yaml`, and is included in the appendix.

## Data File Types

One request made by members of IEA37 work package (WP) ten (10) was to implement the IEA37 WP 10's ontology open source `.yaml` schema for all necessary data. This prescribed a format including:

- Farm turbine attributes;
- Farm turbine locations; and
- Farm wind frequency and wind speeds.

At the time we created the case studies, IEA37 WP 10 had not yet completed development of the schema. Though some analysis and alterations were necessary, we implemented the most recent iteration and modified it to meet the needs of the case studies. Necessary participation data in this modified schema format, for both the wind rose and wind turbine, was supplied to all participants. We also included example reporting files of final turbine locations, as well as a Python parser for the `.yaml` schema. The Python parser was given to participants as `iea37-aepcalc.py`, and is included in the appendix.

### 2.1.1 Case Study 1: Optimization Only

The purpose of case study 1 was to determine the best optimization practices for wind farm layout optimization using a single engineering wake model. The wake model selected needed to be a generalized wake model that both gradient-based and gradient-free optimization algorithms could use and computationally inexpensive in comparison to LES and DNS methods.

Table 2.1: Relevant variables for eq. (2.1).

Variable	Value	Definition
$\Delta V/V_\infty$	eq. (2.1)	Normalized wake velocity deficit
$C_T$	8/9	Thrust coefficient
$x_i - x_g$	-	Dist. from hub ( $x_g$ ) to point of interest ( $x_i$ ), in wind direction
$y_i - y_g$	-	Dist. from hub ( $y_g$ ) to point of interest ( $y_i$ ), $\perp$ to wind direction
$\sigma_y$	eq. (2.2)	Standard deviation of the wake deficit $\perp$ to wind direction
$k_y$	0.0324555	Coefficient based on a turbulence intensity of 0.075 [18, 24]
$D$	130 m	Turbine diameter [38]

## Wake Model

One potential model that had been used in previous papers released by our research lab was a simplified version of Bastankhah’s Gaussian wake model [23, 39]. This model is widely used in the field, and permits performance of both gradient-based and gradient-free algorithms. The governing equations for the velocity deficit in a waked region under this model are described in eqs. (2.1) and (2.2):

$$\frac{\Delta V}{V_\infty} = \begin{cases} \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y_i - y_g}{\sigma_y}\right)^2\right), & \text{if } (x_i - x_g) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (2.1)$$

$$\sigma_y = k_y \cdot (x_i - x_g) + \frac{D}{\sqrt{8}} \quad (2.2)$$

The variables used in eqs. (2.1) and (2.2) are described in table 2.1.

The two piecewise cases in the wake velocity equation are needed because wakes are assumed to only affect points downstream. Hub coordinates are used for all location calculations. For turbines placed in multiple wakes, the total velocity deficit was calculated using the square root of the sum of the squares, i.e.:

$$\left(\frac{\Delta V}{V_\infty}\right)_{total} = \sqrt{\left(\frac{\Delta V}{V_\infty}\right)_1^2 + \left(\frac{\Delta V}{V_\infty}\right)_2^2 + \left(\frac{\Delta V}{V_\infty}\right)_3^2 + \dots} \quad (2.3)$$

In case study 1, turbulence intensity was treated as a constant of 0.075, with a corresponding  $k_y$  value of 0.0324555 [24, 39]. Varying turbulence intensity had numerous effects, drawing

attention away from the main purpose of case study 1, which was to observe the differences between optimization strategies. While testing the wake model shown in eq. (2.1), a readily apparent effect of increasing the turbulence intensity was a widening of the wake cone. In this case, second and third order effects are unknown. As such, for case study 1, we used a very low intensity in an attempt to minimize the unknown optimization impacts.

## **AEP Algorithm**

To calculate overall AEP using files in the released `yaml` schema, we coded an AEP algorithm in Python to enable comparable calculations used while parsing participant submissions. For those who wished to use a programming language other than Python, we described the algorithm used to create the Python file in the case study announcement document. The steps of the AEP algorithm closely follow those previously described in section 1.2.1. Though the algorithm released to all participants was in Python, it was also recoded into Matlab and Julia for internal use in the FLOWLab at BYU. All three versions are included in the appendix.

### **2.1.2 Case Study 2: Combined Case**

The intent of case study 2 was to assess the effects different optimization methods and physics model approximations would have on turbine location recommendations. Case study 2 differs from case study 1 in that no wake model was provided and only one wind farm size was to be optimized. Participants were free to choose their preferred engineering wake model and optimization method combination.

## **Wake Model**

Unlike case study 1, the participant-reported AEP values in case study 2 were not comparable since each participant could use a different wake model to calculate AEP. To help make fair comparisons and conclusions, we coordinated a cross-comparison of results between participants. For the cross-comparison, each participant provided their optimal turbine layout in the standardized `.yaml` format, which was subsequently distributed to the rest of the participants. Participants used their own wake model to calculate the anticipated AEP of the other participant's proposed

farm layouts had they used the same model. From this portion of the case study, we hoped to learn if any participants' results were consistently superior to the other wake models, which we hypothesized would be indicative of a superior wake model and optimization algorithm combination.

## **Farm Attributes**

The wind farm size for case study 2 was limited to nine turbines. This was originally done to limit computational time requirements when assessing results in a standardized LES, though a full LES analysis of the participant submissions was never actually implemented. The previously described method under section 2.1 was used to determine the boundary radius, which for the 9-turbine wind farm was 900 *m*. For the sake of simplicity, the wind rose and wind speed for case study 2 were the same as those in case study 1.

Participant results that were submitted were then parsed, analyzed, and presented in a report of findings at the 2019 American Institute of Aeronautics and Astronautics (AIAA) Science and Technology (SciTech) in San Diego, California [40]. Said report is summarized in section 2.2.

## **2.2 Results**

One intention for the four case studies is for them to propagate throughout the wind farm layout optimization community and be used as a primer for newcomers to the field. In this regard, results should be ongoing and continuous as there is no “right” way to achieve the global maximum in each case; various methods, each with their own strengths and weaknesses, could potentially reach it. Discovery of new optimization and computing mechanisms will hopefully breathe continual life into participant performance of the case studies.

When case studies 1 and 2 were originally announced, there was a specific deadline by which results had to be submitted. There were 10 submissions for case study 1, with one participant submitting two sets of results, each using a different optimization method. For the purpose of anonymity, each submission was assigned a number. Each submission is referred to below by its submission number (i.e., *sub1*, ..., *sub10*, etc.). For case study 2, there were 5 participant submissions. Though not a requirement, all five participants also submitted results for case study 1. For continuity of method analysis between the studies, we assigned these participants the same

submission numbers for both case studies. (i.e., *sub1* for both case study 1 and case study 2 are from the same individual, *sub2* from case study 1 and case study 2 are from the same individual, and so on through *sub5*). For full disclosure, the author supplied the data in *sub5* for both case studies 1 and 2.

### 2.2.1 Case Study 1: Optimization Only

Participants ran the optimization algorithm of their choice using the supplied AEP function or a functional equivalent in another language. The Python supplied module, which uses the simplified Bastankhah wake model, was used for all AEP calculations. The AEP results and rankings are given in tables 2.2 to 2.4.

Submissions are ranked from highest to lowest resultant AEP values. Also included in the tables are the submission number (sub#), whether a gradient-based (G) or gradient-free (GF) optimization method was used, and the relative percentage increase of AEP (Increase) from the example layout's AEP.

### Case Study 1 Results General Trends

As a general trend, gradient-based methods performed better in discovering a relative optimum, especially for smaller farm sizes. Some gradient-based algorithms (*sub10*, *sub3*) improved in comparative AEP ranking as the number of design variables increased, while others (*sub5*, *sub8*) degraded. Simultaneously, one gradient-free algorithm (*sub3*) increased in effectiveness as design variables increased, while others (*sub6*, *sub7*, *sub9*) competed for lowest comparative performance, regardless of farm size.

Despite these multivariate results, one clear front-runner did emerge. Regardless of wind farm size, *sub4*'s algorithm consistently discovered turbine placements that delivered an AEP superior to all other participants. A summary of *sub4*'s method is included in the **Analysis of Best Results**.

Also worth noting is the fact that, as the number of design variables increased, the relative disparity between proposed optimal AEPs likewise diverged. For the 16-turbine case, the highest

Table 2.2: 16 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Increase
1	SNOPT+WEC	4	G	418924.4064	14.17 %
2	fmincon	5	G	414141.2938	12.86 %
3	SNOPT	8	G	412251.1945	12.35 %
4	SNOPT	1	G	411182.2200	12.06 %
5	Preconditioned Sequential Quadratic Programming	2	G	409689.4417	11.65 %
6	Multistart Interior-Point	10	G	408360.7813	11.29 %
7	Full Pseudo-Gradient Approach	3	GF	402318.7567	9.64 %
8	Basic Genetic Algorithm	7	GF	392587.8580	6.99 %
9	Simple Particle Swarm Optimization	6	GF	388758.3573	5.95 %
10	Simple Pseudo-Gradient Approach	9	GF	388342.7004	5.83 %
11	(Example Layout)	-	-	366941.5712	-

Table 2.3: 36 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Increase
1	SNOPT+WEC	4	G	863676.2993	17.05 %
2	Multistart Interior-Point	10	G	851631.9310	15.42 %
3	Preconditioned Sequential Quadratic Programming	2	G	849369.7863	15.11 %
4	SNOPT	8	G	846357.8142	14.70 %
5	SNOPT	1	G	844281.1609	14.42 %
6	Full Pseudo-Gradient Approach	3	GF	828745.5992	12.31 %
7	fmincon	5	G	820394.2402	11.18 %
8	Simple Pseudo-Gradient Approach	9	GF	813544.2105	10.25 %
9	Basic Genetic Algorithm	7	GF	777475.7827	5.37 %
10	Simple Particle Swarm Optimization	6	GF	776000.1425	5.17 %
11	(Example Layout)	-	-	737883.0985	-

Table 2.4: 64 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Increase
1	SNOPT+WEC	4	G	1513311.1936	16.86 %
2	Preconditioned Sequential Quadratic Programming	2	G	1506388.4151	16.36 %
3	Multistart Interior-Point	10	G	1480850.9759	14.35 %
4	SNOPT	1	G	1476689.6627	14.03 %
5	Full Pseudo-Gradient Approach	3	GF	1455075.6084	12.36 %
6	SNOPT	8	G	1445967.3772	11.66 %
7	Simple Pseudo-Gradient Approach	9	GF	1422268.7144	9.82 %
8	Simple Particle Swarm Optimization	6	GF	1364943.0077	5.40 %
9	fmincon	5	G	1336164.5498	3.18 %
10	Basic Genetic Algorithm	7	GF	1332883.4328	2.93 %
11	(Example Layout)	-	-	1294974.2977	-

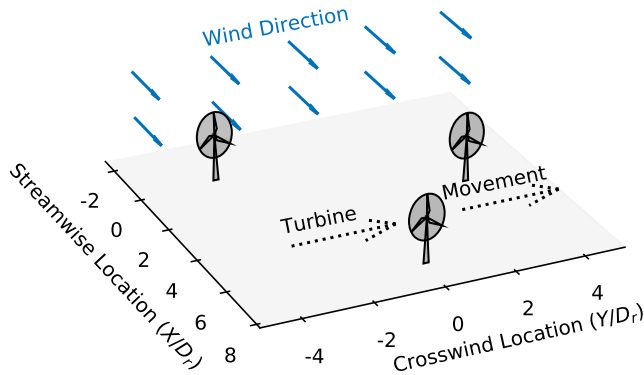


Figure 2.4: A simple 3-turbine wind farm used to demonstrate the effects of WEC’s relaxation factor,  $\xi$ , on the wind farm layout design space. [39]

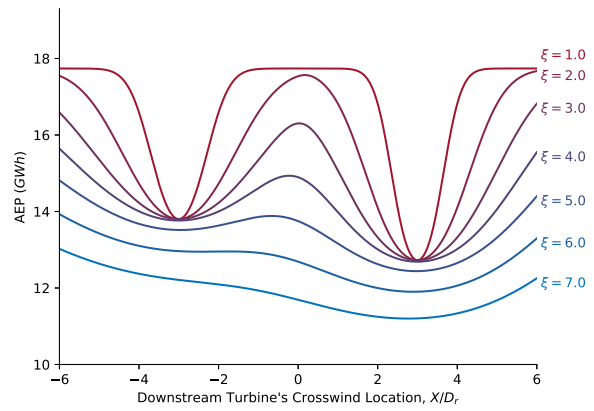


Figure 2.5: The impact of the wake relaxation factor,  $\xi$ . One turbine was moved across the wakes of two upstream turbines (see fig. 2.4).

result was 7.88% better than the lowest. For the 36 and 64 cases, the highest result was 11.45% and 13.54% better than the lowest, respectively.

### Analysis of Best Results

For all three farm sizes, the most successful method was implemented by *sub4*, using a gradient-based method. Coded in Python and FORTRAN, it combined SNOPT [41] with a method called Wake Expansion Continuation (WEC) [39]. Of its 200 optimizations, *sub4* had one optimization start from the provided example layout and the remaining 199 use randomized turbine starting locations within the farm boundary.

The WEC method is specifically designed to reduce the multimodality found in wind farm layout optimization. In the paper announcing discovery of the method [39], it is shown to convert design spaces with many local minima into curves approaching convexity, allowing gradient-based optimizations to more easily find the best solutions. An example of this “relaxation” to convexity is included in figs. 2.4 and 2.5, modified and published with permission.

The effect of the WEC method on a simple design space is shown in figs. 2.4 and 2.5. WEC smooths out the local optima, making better solutions available to gradient-based optimization methods. As the authors stated, “Larger values of  $\xi$  allow the smaller local optima to disappear completely. Smaller values of  $\xi$  allow for more accurate wake widths but with an increase in the number and magnitude of local optima.” [39]. Though many factors contribute to the discovery of results, we suspect that the WEC method for reducing the multimodality of the design space is the

biggest reason why *sub4*'s optimizations found superior layouts compared to the other methods used.

## Case Study 1 Results Discussion

Though *sub4* consistently found the superior AEP relative to the other participants, *sub2*'s results demonstrated a trend that gradually closed the gap as the number of design variables increased. For the 16-turbine case, *sub4* was 2.5% better than *sub2*'s results. For the 36 and 64 cases, *sub4* was 1.68% and 0.46% better, respectively. It should be noted, however, that with the current average U.S. rate of roughly \$0.13 for a kWh (or \$133 per MWh) [42], the income difference between the AEPs of *sub4* and *sub2* in the 64-turbine case equates to a difference of a little under \$1 million per year. Since *sub2*'s Preconditioned Sequential Programming (PSQP) method steadily closed the gap, a future study should test even larger wind farm sizes. This could determine if the PSQP algorithm will eventually outperform the SNOPT+WEC method when a certain number of design variables are reached or if there is an upper limit or convergence to this trend.

Though the majority of participants used random starts for each optimization, *sub2*'s method of "warm starting" performed progressively well, improving as the number of design variables increased. Beginning with a starting set of turbine coordinates, *sub2* rotated the layout in  $\pi/6$  steps. These rotations created the initial geometry for subsequent iterations. Though not necessarily "intuitive" starts, they are more strategically designed than pure randomized locations.

Translating the provided AEP target function proved helpful in speeding up the computations and allowed for greater exploration. At least two participants translated the target Python file into FORTRAN, one used Julia, and another altered the file within Python by converting loops into vectorized statements. In testing, these reimplementations sped up the analysis time by at least an order of magnitude.

In a survey, participants were asked to self-report time and iteration count for their respective optimizations. Unfortunately, the survey was not clearly worded, resulting in different participant interpretations. Some reported the time for their best optimization run, while others provided their total time including exploratory runs, multi-starts, or other iterative approaches (the latter was intended). Also, because users were not notified in advance that this information would be requested in the survey, some of the numbers were not recorded during optimization and were



simply estimated. As an example, self-reported optimization time for the 64-turbine case is shown in fig. 2.6 labeled by submission number. Given the limitations in reporting described above, no real conclusions can be drawn from it, but the data is provided to give a general sense of the algorithmic times.

### 2.2.2 Case Study 2: Combined Physics Model/Optimization Algorithm

For case study 2, participants ran both the optimization algorithm and wake model of their choice. There were no restrictions on programming language for either the wake model or optimization algorithm but results of optimal turbine layouts were to be submitted in the .yaml format supplied in the case study 1 examples.

Because participants used different wake models, AEP values reported cannot be fairly compared between participants. Results were therefore adjudicated using cross-comparison calculations. The cross-comparison calculations display some interesting trends.

Tables 2.5 to 2.9 show how each submission's wake models ranked the proposed optimal turbine layouts for the other 4 submissions. Each submission's ranking of its own layout is in **bold**. The penultimate column in each table is the submission number of the layout being cross-compared (cc-sub#). For example, submission 4's analysis of submission 2's layout would be found in *sub4*'s table, with 2 in the cc-sub# column. The last column is the percentage difference (Difference) from the reporting submission's layout. Here, a positive value indicates a better AEP, and a negative value indicates a worse one.

### Case Study 2 General Trends

We expected participants to rank their own layout as superior to the others. Each wake model accounts for different fluids phenomena so, what one wake model considers an optimal layout, another may not. An example of this could be seen if one engineering wake model were to predict a wake deficit due to some factor, such as vorticity or turbulence. A turbine placed downstream under such a model would, under a more simplistic wake model that did not account for these phenomena (such as the Jensen's model [16]), feel the full brunt of the wake and deliver a suboptimal AEP.

Table 2.5: Cross-comparison results of *sub1*

Rank	Wake Model	Algorithm	AEP	cc-sub#	Difference
1	Bastankhah	SNOPT+WEC	262350.319	4	0.624 %
2	Simplified Bastankhah	fmincon	262282.416	5	0.598 %
3	FLORISSE 3D	SNOPT	<b>260722.295</b>	<b>1</b>	-
4	Bastankhah	Full Pseudo-Gradient Approach	260640.906	3	-0.031 %
5	Park2	PSQP	248215.024	2	-4.797 %

Table 2.6: Cross-comparison results of *sub2*

Rank	Wake Model	Algorithm	AEP	cc-sub#	Difference
1	Bastankhah	SNOPT+WEC	250464.9732	4	5.975 %
2	Simplified Bastankhah	fmincon	250249.0259	5	5.884 %
3	Bastankhah	Full Pseudo-Gradient Approach	247812.0522	3	4.853 %
4	FLORISSE 3D	SNOPT	240309.5850	1	1.678 %
5	Park2	PSQP	<b>236342.799</b>	<b>2</b>	-

Table 2.7: Cross-comparison results of *sub3*

Rank	Wake Model	Algorithm	AEP	cc-sub#	Difference
1	Simplified Bastankhah	fmincon	247109.5234	5	0.590 %
2	Bastankhah	SNOPT+WEC	246942.3767	4	0.522 %
3	Bastankhah	Full Pseudo-Gradient Approach	<b>245659.4124</b>	<b>3</b>	-
4	Park2	PSQP	242431.5431	2	-1.314 %
5	FLORISSE 3D	SNOPT	237548.6622	1	-3.302 %

Table 2.8: Cross-comparison results of *sub4*

Rank	Wake Model	Algorithm	AEP	cc-sub#	Difference
1	Bastankhah	SNOPT+WEC	<b>257790.1924</b>	<b>4</b>	-
2	Simplified Bastankhah	fmincon	257663.4068	5	-0.049 %
3	Bastankhah	Full Pseudo-Gradient Approach	255063.8201	3	-1.058 %
4	FLORISSE 3D	SNOPT	251776.7157	1	-2.333 %
5	Park2	PSQP	239612.8223	2	-7.051 %

Table 2.9: Cross-comparison results of *sub5*

Rank	Wake Model	Algorithm	AEP	cc-sub#	Difference
1	Simplified Bastankhah	fmincon	<b>251771.9067</b>	<b>5</b>	-
2	Bastankhah	SNOPT+WEC	251697.7126	4	-0.029 %
3	Bastankhah	Full Pseudo-Gradient Approach	249829.2199	3	-0.772 %
4	FLORISSE 3D	SNOPT	246503.8323	1	-2.092 %
5	Park2	PSQP	239482.6767	2	-4.881 %

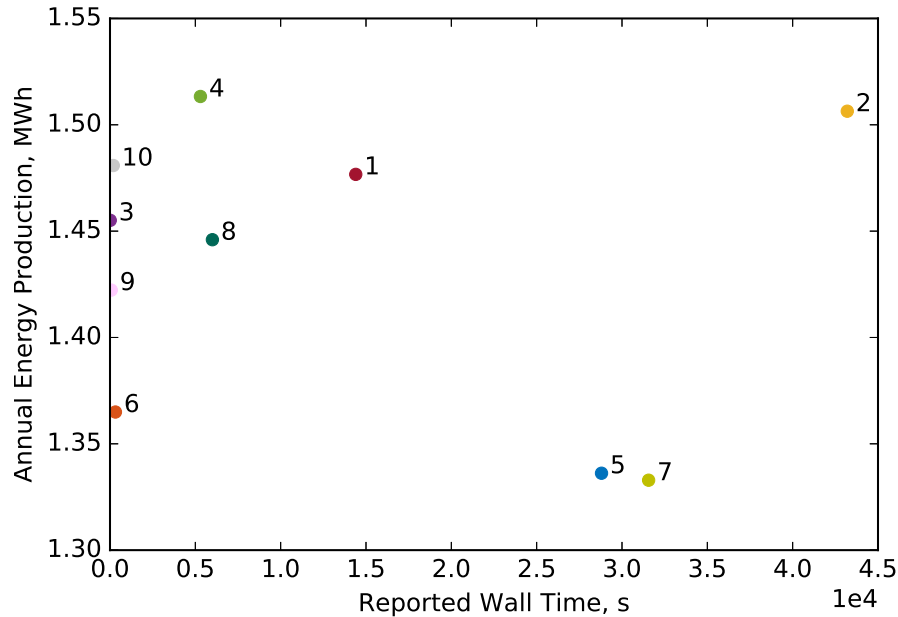


Figure 2.6: AEP vs wall time, 64-turbine scenario. Submission numbers placed next to reported values.

Unexpectedly, only *sub4* and *sub5* found their own layouts to be superior to the other participants. Furthermore, all other participants also found *sub4* and *sub5*'s layouts superior to their own, though to varying degrees. Three participants (including *sub4*) found *sub4* to have the highest AEP-producing layout. The other two participants found *sub5* to have the highest AEP-producing layout.

### Analysis of Best Results

Within expectations, *sub4* and *sub5* ranked their own layouts superior to all other participant results. Two correlations are important to note regarding *sub4* and *sub5*. First, both used variations of the same wake model. From case study 1, *sub5* used the simplified Gaussian wake model previously described [23,39]. Though *sub4* also used the Gaussian wake model [23], it combined said model with the model created by Niayifar and Port-Agel [24] and further supplemented it with the WEC method described earlier. Furthermore, *sub4* also accounted for wind shear and local turbulence intensity while *sub5* did not. The second correlation worth mentioning is that,

despite using very similar wake models, *sub4* and *sub5* used different gradient-based optimization algorithms that reached very similar conclusions.

As can be seen in the figures included in the Appendix, *sub4* and *sub5* found nearly identical optimal turbine placements. Though appearing identical, the actual coordinates do indeed differ, enough so to result in different AEP calculations, shown in the tables above.

Without baseline data such as an LES analysis, the conclusions that are able to be drawn from the cross-comparison analysis are limited. Reasons why both *sub4* and *sub5* were found by the other participants' wake models to have superior placement could include: (1) more efficient optimization methods; (2) better coupling between optimization method and wake model; and (3) wake model superiority. The reason that these optima existed within the other wake models yet were nevertheless undiscovered in their optimizations is inconclusive.

Both *sub4* and *sub5* used similar wake models but very different optimization methods. Coding in MATLAB, *sub5* did 1,000 random starts and used MATLAB's `fmincon`, which uses a finite difference method to find gradients, to optimize for a minimum. Using a combination of Python and FORTRAN, *sub4* ran one optimization with a user-selected initial turbine configuration and randomized the turbine starting locations for the other 199. SNOPT's SQP algorithm that uses algorithmic differentiation to obtain gradients was *sub4*'s implemented optimizer.

Of note, from trends seen above in case study 1, *sub5*'s optimization methods demonstrated superior performance for small design variable sizes but comparatively degraded as the wind farm size increased. The superior performance of this wake model and optimization method combination for a small farm may not be representative of performance on larger wind farms.

## CHAPTER 3. CASE STUDY 3 & CASE STUDY 4

### 3.1 Problem Formulation

Feedback from the participants in case studies 1 and 2 was positive. When polled, participants dichotomously requested that future case studies were either much simpler or much more complicated. For case studies 3 and 4 (cs3 and cs4), we decided to incrementally increase the complexity of the problems without making it unreasonably complicated for those participants who found case studies 1 and 2 to be difficult.

Of the elements proposed for inclusion in case studies 3 and 4, the two elements implemented were:

- a non-uniform boundary with concavities
- disjoint boundary regions

#### Farm Geography

At the suggestion of an early participant and in an effort to increase realism, we used a boundary geometry from a real-world wind farm instead of artificial boundaries like those used in case studies 1 and 2. The Borssele wind farm, off the coast of Borssele, Netherlands, is an offshore wind farm comprised of multiple regions built sequentially, with shipping and transit lanes between said regions. In 2016, an announcement [43] was published calling for proposals to populate Borssele Wind Farm Sites III and IV. This document's *Appendix C: Boundaries and Coordinates* [44] was the source of the geometry for sites III and IV, forming the framework for the boundaries used in case studies 3 and 4. An extracted graphic from this appendix is included in fig. 3.1.

The case study 4 boundary involved five discrete regions from Borssele sites III and IV. To ease the complexity of entry, we decided the boundary for case study 3 would be extracted from

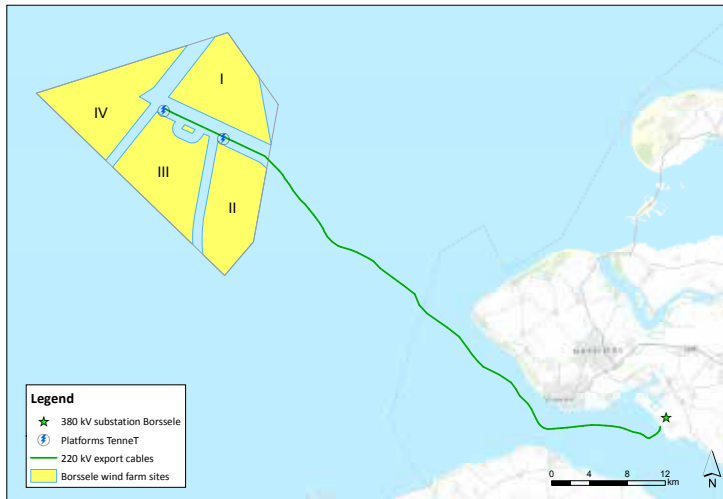


Figure 3.1: Location of Borssele wind farm offshore of Holland. Notice the marked regions III and IV, which were used in case studies 3 & 4. [43]

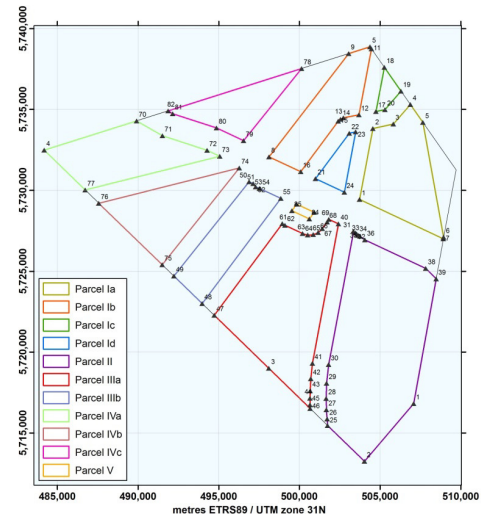


Figure 3.2: The released UTC coordinates of Borssele parcels I-IV [44]

one of the sites contained in case study 4, satisfying the “non-uniform boundary with concavities” requirement, while avoiding the “disjoint boundary regions” problem.

Some modifications to the original site coordinates were required in order to accommodate case studies 3 and 4. In the real world, the only limits on the number of turbines in any given wind farm are finances and geography. In the theoretical problems we created, however, the limiting resource was computing power. Each turbine that was added to the formulation of the problem increased the number of design variables at play in the optimization formulation. As a result, the farm dimensions needed to be scaled to match the limited number of turbines most computers can handle, rather than the other way around. Perfect squares were once again used for the number of turbines on each farm. Optimizations exceeding 100 turbines could become unwieldy, so we selected the largest perfect square that is less than 100 (i.e., 81) for the number of turbines to optimize in case study 4.

For case studies 3 and 4, our contact with IEA37 requested we use their 10-MW reference turbine, due to the fact that it was designed as an offshore turbine specifically for use in farms akin to the Borssele scenario. Given that this turbine has a diameter of 198 *m*, 81 of these turbines arranged in the Borssele farm’s listed geography would result in an average of eight turbine diameters between neighboring turbines. This sparsity would create minimal wake effects between turbines. Because of this, the boundaries needed to be tightened in order to create sufficient local minima for instructive optimization algorithm performance. We therefore scaled down the

Borssele boundaries such that the farm density had an average of five turbine diameters between neighboring turbines, instead of eight. The boundary was then offset such that the bottom left coincided with the coordinate system origin, allowing the farm to lay entirely in quadrant 1 of a Cartesian coordinate system.

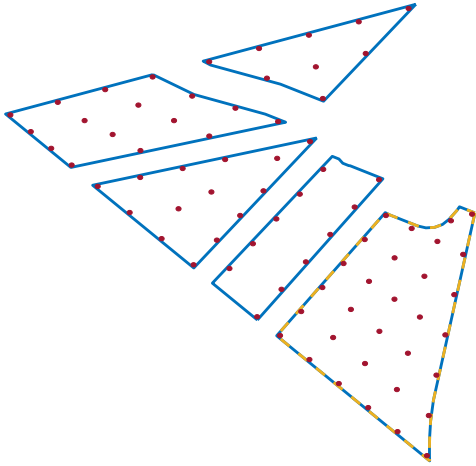


Figure 3.3: Case Study 4 initial example turbine layout coordinates with turbines about 8 diameters apart. Turbine diameters drawn to scale.

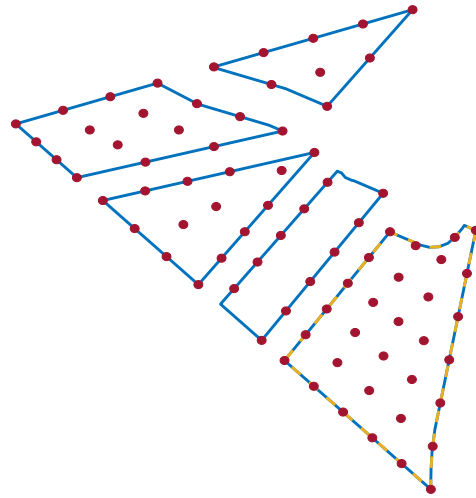
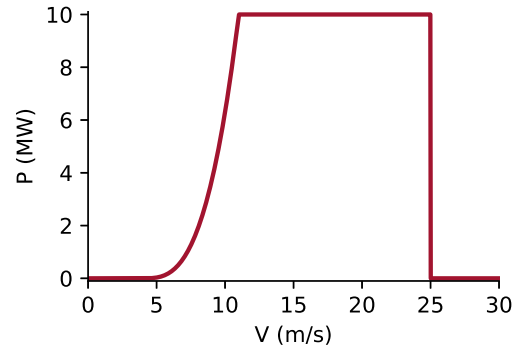


Figure 3.4: Scaled boundary coordinates such that turbines had an average 5 diameters apart. Turbine diameters drawn to scale.

Once this scale was decided, the closest number of turbines resulting in a turbine density of about five turbine diameters between adjacent turbines for the case study 3 boundary (Borssele parcel IIIa) was 25, the number of turbines ultimately predicated for that study.

Case studies 3 and 4 mimicked case studies 1 and 2 in that the first study supplied participants with a wake model in order to solely analyze their optimization algorithm, while the second study of the pair permitted participants to determine both wake model and optimization algorithm. In practice, however, all participants involved in case study 4 decided to use the wake model provided in case studies 1 and 3, so case study 4 turned into a de facto optimization only case. Given the complexity of the disjointed boundaries with concavities, this actually proved helpful to compare optimization methods for these real-world problems.

$$P(V) = \begin{cases} 0 & V < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V < V_{rated} \\ P_{rated} & V_{rated} \leq V < V_{cut-out} \\ 0 & V \geq V_{cut-out} \end{cases}$$



Rotor Diameter	198	m
Hub Height	119	m
Turbine Rating	10	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	11	m/s
Cut-Out Wind Speed	25	m/s

Figure 3.5: Attributes for IEA’s 10-MW offshore reference turbine

## Wind Turbine Specifications

As stated previously, we again decided to use only one type of wind turbine, the IEA 10-MW reference turbine, in case studies 3 and 4. The IEA 10-MW, designed as a baseline for offshore wind turbine specifications, is open source [45]. The equation for the IEA 10-MW turbine’s power curve is the same as that used for the 3.35-MW in case studies 1 and 2, but with different values for magnitude. All defining characteristics relevant for our purposes are shown here:

## Example Layouts

As was done for case studies 1 and 2, example layouts were created and propagated in order to assist participants with visualizing the farm and understanding the input and output syntax used in the `.yaml` files. The pattern for turbine placement in the case study 4 boundary was not formulaic as that used for the first two case studies, though a slightly novel approach was chosen.

Case study 3 was allotted 25 turbines for its single boundary due to spacing constraints. For case study 4, however, only the total number of farm turbines was predicated, not how many should



be in each parcel. The case was written in such a way that dynamic turbine apportionment would be a problem for each participant to solve. An exception to this was the example layout, for which we pre-determined the number of turbines in each region based on relative region area. To make this calculation, we took the usable area in each region, normalized it by the total usable area of the farm, and multiplied that percentage by the total number of wind turbines permitted in the farm. These calculations gave us a numerical value for each area’s apportioned turbines. Rounding each of these numbers to the nearest whole number placed one more turbine in the farm than should have been (i.e., 82 instead of 81), so we manually took out one turbine from region IIIa, as it visually appeared to be the densest. The number of turbines apportioned to each region following this method is in table 3.1 and the relevant code is `ApportionTurbines.m` in the appendix.

Table 3.1: Turbine apportionment per region for cs4’s example layout

Region	#
IIIa	31
IIIb	11
IVa	16
IVb	4
IVc	9

Next we needed to determine where to place the turbines within each boundary. If the boundary were a symmetric square or rectangle, the turbines could be placed in an even grid, as these patterns tend to give good AEP optimization results. Though region IIIb had a rectangular shape, the other 4 regions had irregularities that would prevent such an arrangement. As can be seen in fig. 3.2, the boundaries are defined by several coordinates, especially region IIIa’s.

Our solution was to select four “corners” for each region and spline the “sides” between these corners. We then mapped the splined sides onto the sides of a unit square and placed the turbines on the grid intersections. For region IIIa, some turbines had to be manually placed in the top right, as the grid method did not work with the concavity on the North end of the region. The code used for the automated placement is `placeTurbinesBorselle.m` in the appendix.

Finally, a discrepancy between our turbine placement limitations and the requirements in the original Borssele release documents [43] existed. Our case studies permit turbine hubs to be placed anywhere within and on the boundaries, yet the Borssele release document dictates that turbine rotors must remain entirely within the boundary. To remedy this, we took our shrunken version of the farm (shrunken to increase turbine density) and moved every boundary line inward by the radius of our reference turbine. This solution maintained the relative dimensions and spacing of the original farm, while allowing participants to move turbine hubs up onto the boundary lines.

### **Wind Attributes**

Feedback from case studies 1 and 2 suggested a desire for a more realistic wind resource. This was likely due to the fact that, in the first two case studies, wind speed was constant and binned over only 16 directions. This was not the case for case studies 3 and 4. We instead conducted a convergence study on the effects that the number of wind directional bins had on the accuracy of AEP calculations. Using the case study 4 example layout depicted in fig. 3.4, we calculated AEP with the use of between 1 and 360 bins. The calculation of 360 bins was considered a “true” AEP value to measure others against. These results are depicted in fig. 3.6.

Made apparent in fig. 3.6 is the fact that, beyond 60 bins, AEP calculations are not within significant percentages of full 360–bin calculations. For this reason, 60 bins were originally selected to be published with the case studies. However, when the use of 60 bins was presented to the IEA37 working group, they were unsure that even this many bins would be necessary. Sixty wind directions multiplied by sixty wind speeds equates to three-thousand six hundred calculations for every iteration, which could become prohibitive for some systems. For that reason, and because 20 bin AEP calculations were still within 0.025% of “truth,” we released case studies 3 and 4 with 400 wind data points that resulted from multiplying 20 wind directions by 20 wind speeds. The wind data points were originally taken from a 12-binned wind rose offering wind speed probabilities for any given direction, with each bin forming a *Weibull distribution*.

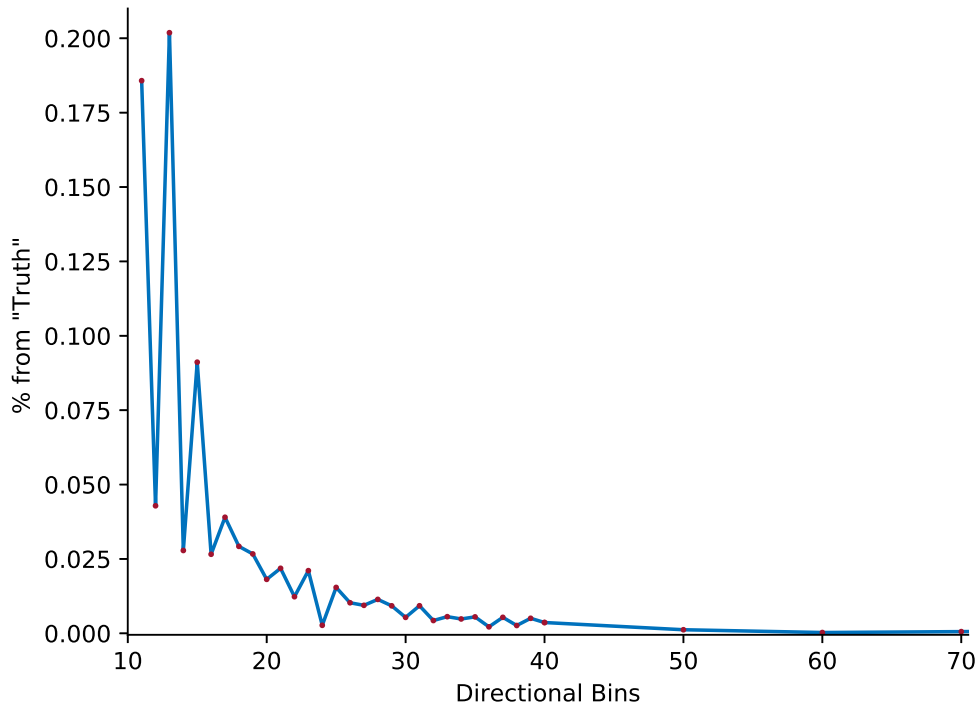


Figure 3.6: Bin convergence study for wind resource of cs4. Using extrapolated data, the left axis is percent difference from the AEP calculation using 360 bins. Note that the result from even the worst case of 13 bins is still within 0.25% of the result from a 360 bin rose.

### Weibull distribution

Weibull distributions were named after Swedish mathematician, Waloddi Weibull, and are mainly used in statistics and probability computations. They are defined by a very specific formula but, based on perturbations of two primary variables ( $\lambda$  and  $k$ ), can create a wide range of shape distributions. The formula to construct a Weibull distribution is given in eq. (3.1):

$$f(x|a, b) = \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-a} e^{-(x/\lambda)^k} \quad (3.1)$$

Here  $\lambda > 0$  is the scale parameter,  $k > 0$  is the shape parameter, and  $e$  is Euler's number 2.71828.... An example of different distribution shapes given by perturbing  $k$  for  $\lambda = 1$  is given in fig. 3.7.

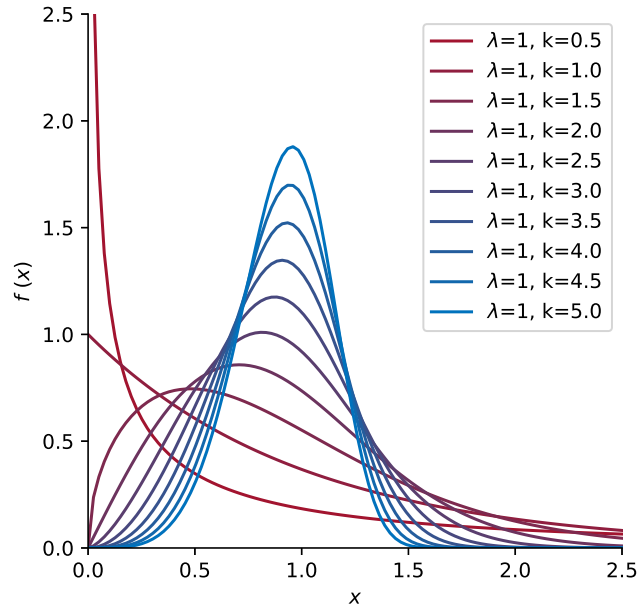


Figure 3.7: Demonstration of the different shapes produced by perturbing the shape parameter of the Weibull function.

From the 12 original binned wind directions with continuous wind speeds, computed from the  $k$  and  $\lambda$  assigned to each direction, we needed to extrapolate more bins. The algorithm used to do this followed three steps:

1. Copy the 12 binned data three times in order to enable smooth curves at either edge (simulate a radial rose).
2. Create a 3-d spline of the surface of the three-dimensional data (fig. 3.8).
3. Take as many bins as needed (ex. 60 bins in fig. 3.9).

Originally, 60 bins were computed for the wind roses used in case studies 3 and 4, but the same algorithm was later used to re-compute the 20 bins that were ultimately published. The data for the published bins are included in the appendix in `iea37-windrose-cs3.yaml`. The code which created that data file, in addition to figs. 3.8 and 3.9 is included in the appendix as `makeWindRoseCS3.m` and `extrapolateWeibull.m`.

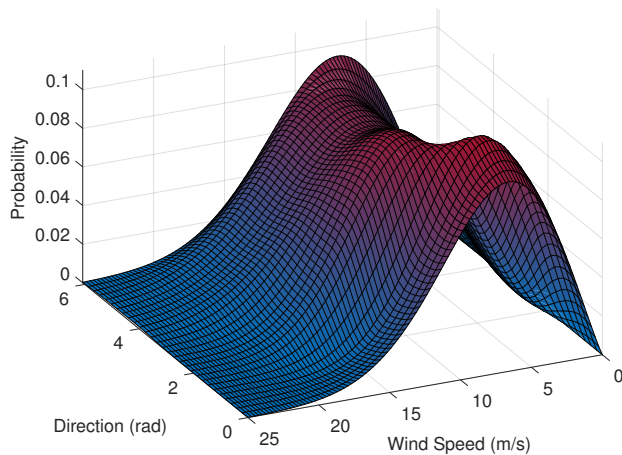


Figure 3.8: The wind speed distribution across the 360° wind rose, splined so as to permit any number of bins to be discretized.

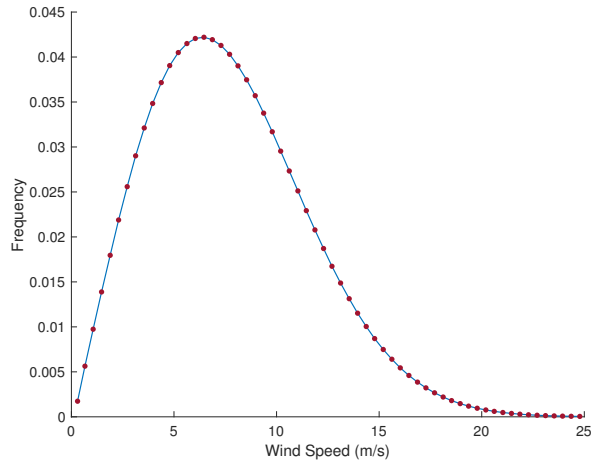


Figure 3.9: A single slice of the directional wind speed frequency depicted in fig. 3.8, taken at 36°, discretized for 60 wind speed bins.

## Data File Types

The .yaml file type was again used for all input and output files. The schema from IEA37 WP X was incomplete, so the previously used fields from case studies 1 and 2 were further abbreviated in order to increase readability of the supplied input files. We again wrote example reporting files of final turbine locations as well as a Python parser for the .yaml schema, both of which were made available to the participants. They are included in the appendix.

### 3.1.1 Case Study 3: Concave Boundary

The region specified for case study 3 was selected so as to permit participants to invent and implement methods for dealing with wind farm boundaries that include concavities. Participants using gradient-free optimization algorithms would not have difficulty with this but, at the time this study was created, the BYU FLOW Lab had no method for accurately using gradient-based optimizers on concave wind farm boundaries.

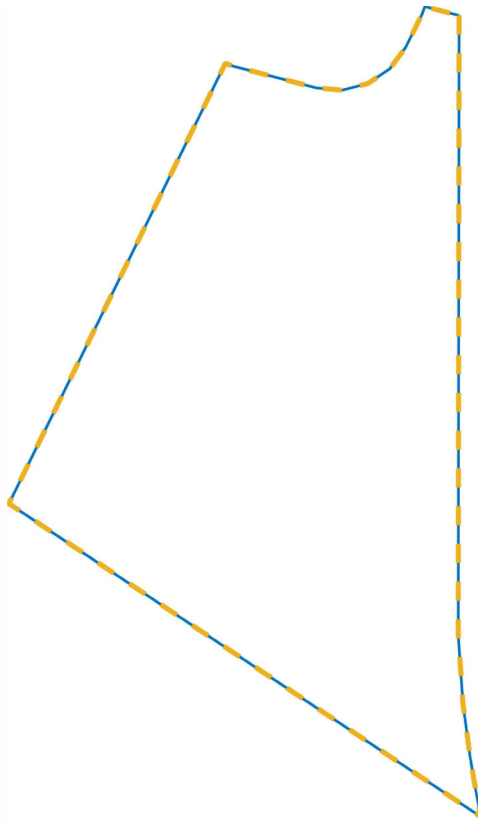


Figure 3.10: The cs3 boundary, made from a scaling of the Borssele region IIIa.

As can be seen in fig. 3.10, Borssele region IIIa was used for case study 3. It has a major concavity on the North side of the boundary and a slight concavity in the South West corner.

### **Wake Model and AEP Algorithm**

Both the wake model and AEP algorithms for case studies 3 and 4 were the same as those used in case study 1, described previously.

#### **3.1.2 Case Study 4: Discrete Boundary Regions**

Building on the algorithms for the boundaries containing concavities that were developed in case study 3, we selected the boundary for case study 4 to add the complication of discrete regions. Areas that are off-limits can occur in real world wind farms for many reasons, be it due to the necessity of shipping lanes, cabling requirements, highways, terrain obstructions, or turbine

foundational requirements. Our individual solution to this for the purpose of the example layouts was to user-select a pre-apportioned number of turbines per region. Since case study 4 is, as this is being written, ongoing, more creative solutions to this problem are currently being developed. The case study 4 boundary with discrete regions is depicted graphically in fig. 3.11.

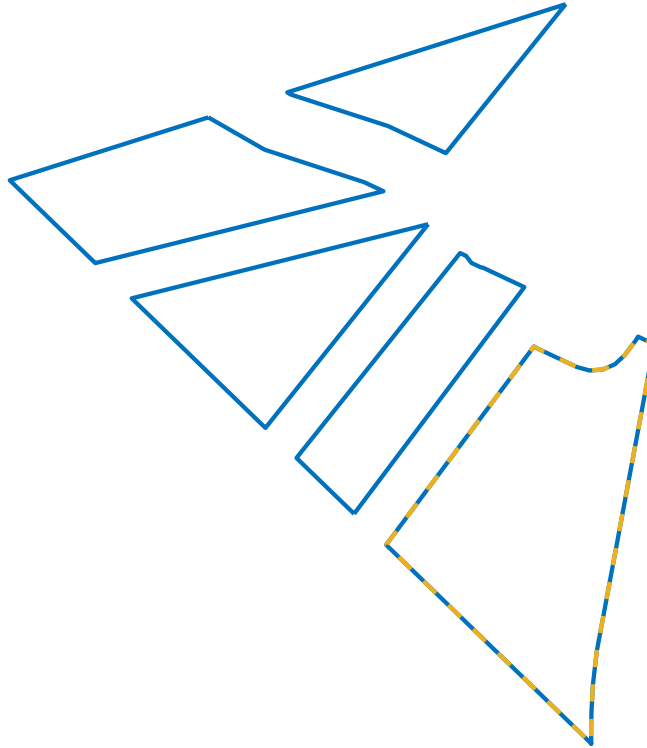


Figure 3.11: The cs4 boundary, made from a scaling of the Borssele regions III and IV. Note region IIIa used for cs3 is outlined in dashed yellow.

### **Wake Model and Submission**

Both the wake model and AEP algorithms for case studies 2 and 4 were described for case study 1. BYU's optimized layout submissions are coded in the Julia language, in the FLOWFarm.jl framework, and are currently still under development.

### **3.2 Results**

Case studies 3 and 4, like case studies 1 and 2, were intended to have specific announcement and result submission dates, despite remaining in existence for use in perpetuity. Initial

participatory interest in case study 3 and case study 4 did not, however, meet minimum requirements for statistically significant results. The project's intention thus shifted from a competitive to a collaborative focus. A joint paper written by the participants attempting case studies 3 and 4 is currently in production, with the goal of publication in an academic journal.

### **3.2.1 Interesting Methods**

Though the names of individuals must remain anonymous and the final number of participants is still growing, two participants' initial results are of particular note and will be here examined. The first developed a method for dynamic turbine apportionment in case study 4 and created a video explaining how their optimizer dynamically moves turbines into viable regions. At the beginning of the optimization the turbines are distributed randomly across the case study's entire area, including the no-go zones between discrete regions. Then, as the optimizer works, the turbines are slowly pushed from the off-limits shipping lanes into the nearest discrete region using a penalty function, iteration by iteration. While turbines tend to end up on the boundaries of their nearest discrete region, this method displayed how the turbine apportionment problem can be automated to obviate the need for engineer intervention.

The second novel method submitted involved design variable reduction. A simplistic optimization formulation would assign an  $x$ - and  $y$ - coordinate for every turbine, causing the optimizer in case study 4 to manipulate two variables per turbine. Given that there are 81 turbines in case study 4, the optimizer was perturbing 162 variables every iteration. This participant noticed that optimal turbine layouts tend to place turbines along the wind farm boundary in the optimizer's attempt to push the turbines as far away from each other as possible. Therefore, using a single boundary turbine as the primary indicator, this participant parameterized all other turbines placed along the boundary to automatically shift their location and spacing based off the location of the primary turbine, thus greatly reducing the number of design variables required. For example, in an 81-turbine wind farm scenario with 30% of the turbines placed on the boundary, a 162-design variable problem becomes a 116-design variable problem as 24 turbines are now controlled by the location of only one. This participant is still exploring other variable reduction methods, the results of which will be explained in the forthcoming journal paper.



### 3.2.2 Author's Attempt

Though participation in case studies 3 and 4 is ongoing, the author did a simplistic attempt at both case studies to serve as a comparison or guide for others to improve upon. First, the issue of convex boundaries needed to be addressed. Subsequently, a method for solving the disjoint area problem with a gradient-based optimizer was implemented.

#### Concave Boundary Problem

The simplest method capable of addressing the concavities apparent in case study 3 is the exploitation of the  $(x,y)$  coordinate system by creation of partitions of viable areas. Termed the Boundary Partition Method (BPM), its partitioning is depicted on the case study 3 boundary in fig. 3.12.

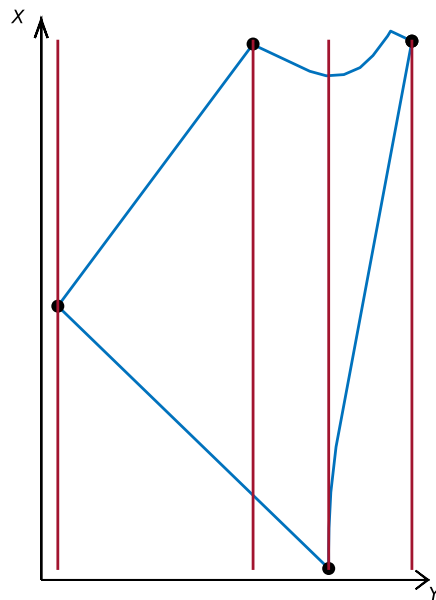


Figure 3.12: An example of the BPM on case study 3's boundary. If a turbine's  $x$ -coordinates falls between any two red vertical partition lines, the turbine's  $y$ -coordinate is checked if it also lies between the upper and lower blue lines of that partition.

Using BPM, turbines that fall between any  $x$ -partitioned region are analyzed to see if their  $y$ -coordinate falls between the splines calculated from vertices of the upper and lower boundary lines. Though computationally costly in that each turbine needs to be checked against all partitions

in the region, this method is effective for boundaries with simple concavities such as those in case study 3.

Another way to solve this problem is through the use of a method termed *ray casting*. For this method, a line is drawn, originating from the turbine of interest, in any fixed direction and the number of boundary lines crossed by this line is recorded. It has been proved that if this number is odd, the turbine of interest lies inside the closed boundary. If even, the turbine lies outside regardless of the presence of boundary concavities.

Yet another method involves the *winding number algorithm*. From the location of the turbine of interest, the angle to each boundary vertex is calculated and summed. It has been proved that if the sum is non-zero, the turbine of interest lies inside the polygon regardless of boundary concavities or irregularities. A zero sum indicates that the point of interest lies outside the closed boundary.

### **Disjoint Boundary Problem**

The simplest method to solve the disjoint boundary problem with a gradient-based optimizer is by pre-assigning turbines to each boundary region and limiting turbine movement to their assigned region. This denies the optimizer the ability to move turbines to a potentially more efficient location in a neighboring disjoint area, but also eliminates the requirement of a gradient-based optimizer to decide to push a turbine across a non-viable area.

A method that explores more of the design space includes permitting turbines to be initialized anywhere in the farm, pushing the turbines into a valid boundary area via a gradual penalty function. This method has been implemented by a current participant in case study 4. It will be interesting to compare the performance of this implementation against the other submitted gradient-free methods.

### **Simple Optimized Layout**

The author implemented two of the simplest methods to demonstrate algorithmic performance. Using the previously described partition method and pre-assignment of turbines to disjoint boundary regions, an AEP improvement of 3.37% was achieved using the Sparse Nonlinear Opti-

mizer (SNOPT) suite, shown in fig. 3.13. Using random turbine starting locations, 20 optimizations were completed. The best AEP result from these optimizations is also shown in fig. 3.13.

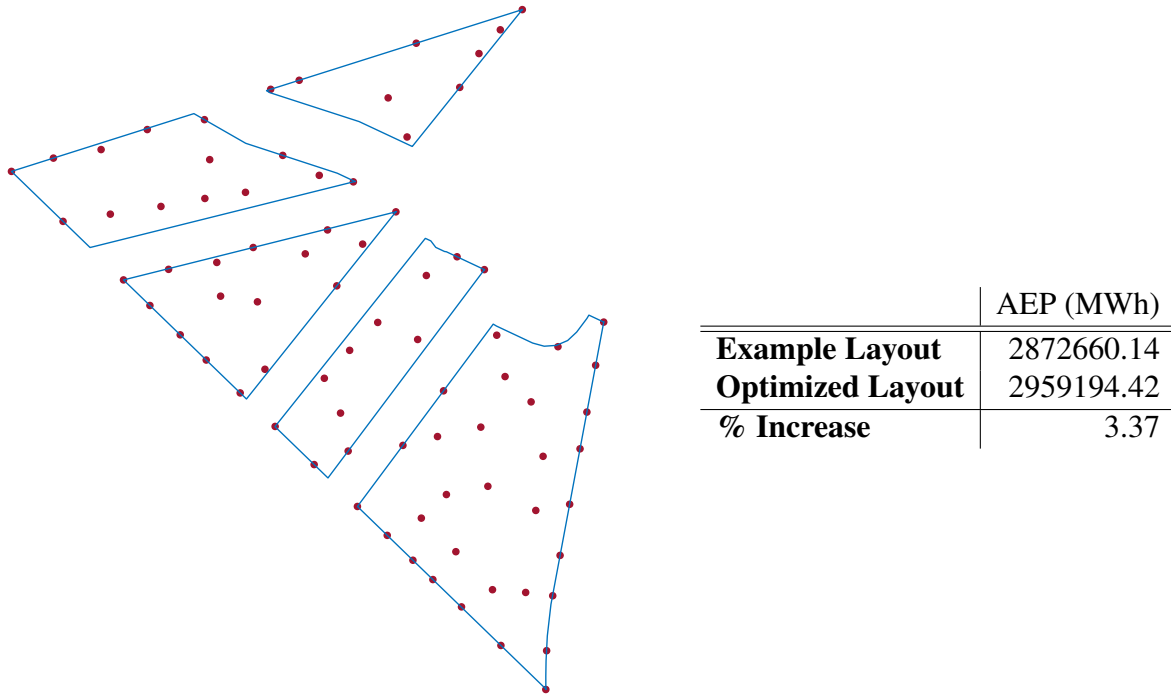


Figure 3.13: Case study 4 optimized turbine layouts using BPM and turbine region assignments from random starting points.

SNOPT’s AEP result with these two methods showed a slight improvement over the supplied example layout. Though SNOPT has demonstrated good performance in many scenarios [46], BPM is not computationally efficient when acting on complicated boundaries where complex splines need to be calculated. Furthermore, pre-apportionment of turbines does not explore the entirety of the design space and will overlook a global optimum if apportioned incorrectly. More improvement could be made through the implementation of a better boundary function such as the winding number algorithm or a penalty function which gradually pushes turbines into viable regions and can explore more of the design space.

### 3.2.3 Lessons Learned

Formulating Case Studies 1-4 was an incremental and collaborative process. Feedback from case studies 1 and 2 directly affected design choices for case studies 3 and 4. Regardless, there were some unique difficulties that, if attempted again, the author would avoid.

The undertaking of creating a series of case studies to analyze the engineering concepts involved in wind farm layout optimization turned into more of an experiment in social studies that originally supposed. Though the author had knowledge of engineering wake models and optimization methods, he was the limits of his knowledge when it came to effective survey construction and questionnaire formulation. That the responses to case study 1's exit questionnaire regarding processing time (depicted in fig. 2.6) was as varied as it was due to participant's various interpretations of question's wording was one symptom of the problem. If the case study formulation was to be attempted again, besides engineering expertise, it would be recommended that the author also research questionnaire formulation and best practices for survey creation.

Initial results from case studies 3 and 4, though not yet published, indicate that all participants are achieving relatively similar AEP results. Though steps were taken to increase wind farm turbine density, this is most likely due to a wind farm turbine density that is still too sparse to have as high a number of local optima as initially desired. Yet this outcome still has yielded interesting analysis, in that it is permitting a better comparison of the time and computational cost of each method. Though arriving at the same number, case study 4's scenario is proving to reveal how quickly some methods come to the same AEP conclusion, a factor not originally intended by the problem formulation.

The most important changes from case studies 1 and 2 to 3 and 4, were the inclusion of variable directional wind speeds and disjoint boundary regions. The first alteration serves to increase computational time for both gradient-based and gradient-free methods to something more realistic. The second was included specifically to hinder gradient-based methods from, on a whole, outperforming gradient-free methods as much as they did in case studies 1 and 2. Interesting strategies are being implemented to account for this new obstacle, and the author hopes that the creativity required to overcome this factor will improve the field of wind farm optimization.

Finally, the largest challenge in synthesis of these case studies has been incentivizing user participation. IEA37 served as a solid base of users who had self-selected as both interested and

motivated to attempt wind farm layout optimization problems. However, the author sees there being great benefit to involving users who may not have a background in wind energy, but are nevertheless skilled at mathematical optimization. Yet short of petitioning individual universities or researchers, it has proven difficult to generate interest outside the IEA37 sphere. Though no solution to this issue is readily apparent, the author believes that the case studies would benefit greatly from participation by skilled individuals not already in the unique field of wind farm layout optimization.

## CHAPTER 4. CONCLUSION AND FUTURE WORK RECOMMENDATIONS

This document catalogues the construction of four case studies created with the intent to better understand the effects of engineering wake models and optimization algorithms as applied to the wind farm layout optimization problem. Case study 1 focused on optimization methods and received 10 submissions, while case study 2 studied the combination of engineering wake model and optimization method and received five submissions. Case study 3 focused on solving the concave boundary problem. Case study 4 presents a scenario in which all aspects of the previous 3 case studies are combined, in addition to the introduction of the discrete boundary problem.

Formulation of these studies was an iterative process that required collaboration from researchers outside of BYU's lab, as well as many internal discussions on how best to move forward. I, specifically, contributed the following to the work:

- Examined multiple real-world wind roses for candidates to use in our studies.
- Conducted a convergence study on multiple wind rose patterns.
- Created and implemented the algorithm that generated the example turbine layouts and ideal diameters for each farm size.
- Parsed through IEA37's WPX's .yaml schema and modified it for our case studies.
- Coded the wake model, AEP calculator, .yaml interpreter, and all data files necessary for participation.
- Coordinated user participation and submissions for case studies 1 and 2
- Analyzed results and presented findings at AIAA's SciTech in 2019.
- Created the wind resource for case studies 3 and 4 using interpolated Weibull data.
- Selected and modified the Borssele wind farm boundary for use in case studies 3 and 4.

- Conducted a convergence study to determine the ideal number of wind resource bins.
- Determined turbine apportionment and placement for example layouts in case studies 3 and 4.
- Modified the case study 1 AEP calculator to include the expanded wind rose with variable wind speeds.
- Coordinated initial user participation and results analysis.
- Made recommendations to the IEA37 group on factors to be included in future case studies.

Beyond these contributions, some interesting analyses from the results of case studies 1 and 2 can be made.

Results from case study 1 show that *sub4*'s use of SNOPT+WEC delivered superior results for the tested wind farms with 16, 36, and 64 turbines. Although information on this method continues to be produced, a paper written by Jared Thomas and Andrew Ning [39] includes the first discussion of this method. Regarding *sub2*'s PSQP method, though it showed a trend of increased performance that may eventually surpass SNOPT+WEC for wind farms of sizes larger than 64, further testing is required to validate this pattern.

Case study 2 demonstrated that, for wind farms of small area with few turbines, placement on the wind farm boundary delivers superior AEP. Three of the five participants reported in their cross-comparison that others found superior optima to theirs, suggesting that their optimization methods became trapped in a local optimum. It is unclear if the discrepancy was caused by a difference in the optimization approach or in the wake model's suitability for optimization. Further investigation is needed in order to compare the approaches in more detail.

Participants of earlier case studies were critical of wind farm scenarios in which non-novel layouts (such as all turbines placed on the boundary border) were optimal, due to their simplicity. The small farm radius with few turbines provided to participants in case study 2 seems to have fallen into this category. What is noteworthy, however, is the fact that three of the five participants in said study employed methods that became trapped in local optima and did not discover the superior layouts others found by using different physics approximations and optimization methods.

Even with the simple scenarios in case studies 1 and 2, the participants found very different results. Many factors could have led to these shortcomings (e.g., inferior optimization methods, lack of sufficient iterations, lack of sufficient wall time, etc.), necessitating further testing in order to discover which factors contributed significantly to the outcome. Placing turbines on the borders of concentric rings was the example method supplied for the three wind farm sizes in case study 1. These examples were created based on our presumption that maximum spacing between turbines would be relatively more efficient. As unanticipated validation, all participants of case study 2 applied layouts following this concentric pattern to the smallest farm size, despite beginning in random starting locations. This confirms our hypothesis that, for small farm sizes, border placement and concentric rings will tend to deliver optimal farm AEP.

The results obtained from case studies 1 and 2 were instructive and provided a large enough sample size to synthesize meaningful conclusions. Because of their complexities, case studies 3 and 4 may have suffered and, as such, it is regrettable that participation in these two studies was not sufficient to draw similarly meaningful conclusions.

Case studies 3 and 4 presented interesting problems to tackle. Current participation in these studies exhibits novel approaches to the problems of both concave boundaries and discrete regions. Further studies should be done on discrete boundary problem solutions and methods through which gradient-based optimizers can perform well with discontinuous boundary regions.

There are many aspects of wind farm layout optimization that have yet to be explored. Turbulence intensity effects, non-homogenous wind farm turbines, wind farms with geographic variability in the z-direction, and turbines permitted to yaw are just a few of the many directions future case studies can go, all of which have the potential to yield innovative and beneficial methods for wind farm layout optimization.



## REFERENCES

- [1] McWilliam, M. K., Zahle, F., and Dykes, K., 2017. Iea task 37 on system engineering in wind energy the aerodynamic only optimization case study, May. 2, 23
- [2] NASA, 2020. “troposphere” <https://spaceplace.nasa.gov/troposphere/en/>, Nov. 2
- [3] FEDERAL AVIATION ADMINISTRATION, 2016. *Pilot’s Handbook of Aeronautical Knowledge*. 800 Independence Avenue, SW, Washington, DC 20591. 3, 4
- [4] Shepherd, D. G., 1990. Historical development of the windmill NASA Contractor Report 4337, NASA, Lewis Research Center. 9
- [5] NASA, 2010. Museum in a box series: Wind power Public PDF. 9, 10
- [6] Wind, M. V. O. <https://mhivestasoffshore.com/mhi-vestas-launches-the-first-10-mw-wind-turbine-in-history/>. 10
- [7] Veers, P. S., Ashwill, T. D., Sutherland, H. J., Laird, D. L., Lobitz, D. W., Griffin, D. A., Mandell, J. F., Musial, W. D., Jackson, K., Zuteck, M., Miravete, A., Tsai, S. W., and Richmond, J. L., 2003. “Trends in the design, manufacture, and evaluation of wind turbine blades.” *Wind Energy*, **6**, pp. 245–259. 10
- [8] Barrett, R., and Ning, A., 2016. “Comparison of airfoil precomputational analysis methods for optimization of wind turbine blades.” *IEEE Transactions on Sustainable Energy*, **7**(3), July, pp. 1081–1088. 10
- [9] Gaertner, E., Rinker, J., Sethuraman, L., Zahle, F., Anderson, B., Barter, G., Abbas, N., Meng, F., Bortolotti, P., Skrzypinski, W., Scott, G., Feil, R., Bredmose, H., Dykes, K., Shields, M., Allen, C., and Viselli, A., 2020. Definition of the IEA Wind 15-megawatt offshore reference wind turbine Tech. rep., NREL, March. 12
- [10] Burton, T., Sharpe, D., Jenkins, N., and Bossanyi, E., 2001. *Wind Energy Handbook*. Wiley. 13, 14
- [11] Sande, B., 2009. “Aerodynamics of wind turbine wakes.” *Energy Resource Center of the Netherlands*, January, p. 46. 13, 14
- [12] Herbert-Acero, J. F., Probst, O., Réthoré, P.-E., Larsen, G. C., and Castillo-Villar, K. K., 2014. “A review of methodological approaches for the design and optimization of wind farms.” *Energies*, p. 23. 14, 22, 24
- [13] Larsen, G. C., 2009. A simple stationary semi-analytical wake model Tech. rep., Technical University of Denmark. 14, 15

- [14] Vermeer, L., Sorensen, J., and Crespo, A., 2003. “Wind turbine wake aerodynamics.” *Progress in Aerospace Sciences*, **39**, pp. 467–510. 14
- [15] Ott, S., Berg, J., and Nielsen, M., 2011. Linearised cfd models for wakes Tech. rep., Technical University of Denmark. 15, 16
- [16] Jensen, N., 1983. A note on wind generator interactions Tech. rep., RISO National Laboratory. 15, 41
- [17] Gebraad, P. M. O., Teeuwisse, F. W., van Wingerden, J. W., Fleming, P., Ruben, S. D., Marden, J. R., and Pao, L. Y., 2014. “Wind plant power optimization through yaw control using a parametric model for wake effects — a cfd simulation study.” *Wind Energy*, October, p. 20. 15, 16
- [18] Thomas, J., and Gebraad, P. “Improving the floris wind plant model for compatibility with gradient-based optimization.” p. 14. 15, 22, 34
- [19] Larsen, G. C., Aagaard Madsen, H., Larsen, T. J., and Troldborg, N., 2008. Wake modelling and simulation Tech. rep., Technical University of Denmark. 15
- [20] Larsen, G. C., Madsen, A., Helge; Bingöl, F., Mann, J., Ott, S., Sørensen, J., Okulov, V., Troldborg, N., Nielsen, M., Thomsen, K., Larsen, T. J., and Mikkelsen, R., 2007. Dynamic wake meandering modeling Tech. rep., Technical University of Denmark. 15
- [21] Parada, L., Herrera, C., Flores, P., and Parada, V., 2017. “Wind farm layout optimization using a gaussian-based wake model.” *Renewable Energy*, February, p. 10. 16
- [22] Bastankhah, M., and Porté-Agel, F., 2014. “A new analytical model for wind-turbine wakes.” *Renewable Energy*, January. 16
- [23] Bastankhah, M., and Porté-Agel, F., 2016. “Experimental and theoretical study of wind turbine wakes in yawed conditions.” *J. Fluid Mech.*, **806**, pp. 506–541. 16, 34, 43
- [24] Niayifar, A., and Porté-Agel, F., 2016. “Analytical modeling of wind farms: A new approach for power prediction.” *Energies*, September. 16, 34, 43
- [25] Nocedal, J., and Wright, S. J., 2006. *Numerical Optimization.*, 2 ed. Springer. 17, 20, 21
- [26] Kramer, O., Ciaurri, D. E., and Koziel, S., 2011. *Computational Optimization, Methods and Algorithms*. Springer. 17
- [27] Baydin, A. G., Pearlmutter, B. A., Radul, A. A., and Siskind, J. M., 2018. “Automatic differentiation in machine learning: a survey.” *Journal of Machine Learning Research*, **18**(1-43), April. 19, 20
- [28] Griewank, A., 2003. *A mathematical view of automatic differentiation*. Cambridge University Press. 21
- [29] Stanley, A. P. J., Ning, A., and Dykes, K., 2019. “Optimization of turbine design in wind farms with multiple hub heights, using exact analytic gradients and structural constraints.” *Wind Energy*, **22**(5), May, pp. 605–619. 22

- [30] Guirguis, D., Romero, D. A., and Amon, C. H., 2016. “Toward efficient optimization of wind farm layouts: Utilizing exact gradient information.” *Applied Energy*, **179**, pp. 110 – 123. 22
- [31] Feng, J., and Shen, W. Z., 2015. “Solving the wind farm layout optimization problem using random search algorithm.” *Renewable Energy*, **78**, pp. 182 – 192. 22
- [32] Parada, L., Herrera, C., Flores, P., and Parada, V., 2017. “Wind farm layout optimization using a gaussian-based wake model.” *Renewable Energy*, **107**, pp. 531 – 541. 22
- [33] Mosetti, G., Poloni, C., and Diviacco, B., 1994. “Optimization of wind turbine positioning in large windfarms by means of a genetic algorithm.” *Journal of Wind Engineering and Industrial Aerodynamics*, **51**(1), pp. 105 – 116. 22
- [34] Stanley, A. P. J., and Ning, A., 2019. “Massive simplification of the wind farm layout optimization problem.” *Wind Energy Science*, **4**(4), Dec., pp. 663–676. 22
- [35] IEA, 2020. Iea mission, <https://www.iea.org/about/mission>, August. 23
- [36] IEA, 2019. Iea technology collaboration programme 2018 annual report Year-end-review, IEA, December. 23, 24
- [37] IEA, 2018. About task 37, <https://community.ieawind.org/task37/ourlibrary>. 23
- [38] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., 2018. Iea wind task 37 on system engineering in wind energy, wp2 - reference wind turbines Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May. 26, 34
- [39] Thomas, J. J., and Ning, A., 2018. “A method for reducing multi-modality in the wind farm layout optimization problem.” In *Journal of Physics: Conference Series*, The Science of Making Torque from Wind. 34, 39, 43, 63
- [40] Baker, N. F., Stanley, A. P. J., Thomas, J. J., Ning, A., and Dykes, K., 2019. “Best practices for wake model and optimization algorithm selection in wind farm layout optimization.” In *AIAA Scitech 2019 Forum*. 36
- [41] Gill, P., Murray, W., and Saunders, M., 2005. “Snopt: an sqp algorithm for large-scale constrained optimization.” *SIAM Review*, **47**, pp. 99–131. 39
- [42] ChooseEnergy.com, 2018. Worth of kwh per state, <https://www.chooseenergy.com/electricity-rates-by-state/>, November. 40
- [43] Agency, N. E., 2016. Borssele wind farm zone wind farm sites iii and iv project and site description Press Release, August. 45, 46, 50
- [44] Agency, N. E., 2016. Borssele wind farm zone wind farm sites iii and iv appendix c: Boundaries and coordinates, August. 45, 46
- [45] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., 2018. Iea wind task 37 on system engineering in wind energy, wp2 - reference wind turbines Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May. 48

- [46] Wendorff, A. D., Botero, E., and Alonso, J. J., 2016. “Comparing different off-the-shelf optimizers performance in conceptual aircraft design.” In *AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Vol. 17, AIAA. 59

**APPENDIX A. CASE STUDY 1-2 STARTUP FILES**

# Wind Farm Layout Optimization Case Studies

IEA Task 37 on System Engineering in Wind Energy

Nicholas F. Baker, Andrew P. J. Stanley, Jared Thomas, and Andrew Ning  
Brigham Young University, Provo, Utah, USA

Katherine Dykes  
National Renewable Energy Laboratory, Golden, Colorado, USA

August 23, 2018

## 1 Introduction

Two major factors that affect wind farm layout optimization are 1) the optimization approach and 2) the wake model. This document defines two case studies designed to study these factors. One may elect to participate in either or both cases.

1. Optimization-Only Case Study: user chooses optimization approach, wake model is fixed and supplied.
2. Combined Case Study: user is free to choose both optimization approach and wake model.

Participants will (1) optimize turbine locations to maximize annual energy production, (2) submit solutions, and (3) provide details on their methodology. After all submissions are received, for the Combined Case Study participants will be expected to perform a cross comparison of other participant solutions. Data will be consolidated, processed, and made available to all participants.

## 2 Problem Definition

### Objective

The objective of each scenario is to maximize annual energy production, which we define simply as the expected value of aerodynamic power. The wind resource for each case has a wind rose binned into 16 discrete directions, with a constant wind speed. In other words:

$$AEP = \left( \sum_{i=1}^{16} f_i P_i \right) 8760 \frac{\text{hrs}}{\text{yr}}$$

where  $P_i$  is the power produced for wind direction  $i$ , and  $f_i$  is the corresponding wind direction probability.

### Design Variables

The design variables are the  $(x, y)$  locations of each turbine. All locations in this document refer to the hub location. Every turbine in the farm is identical, and explicitly defined below in **Parameters**.

### Constraints

Each wind farm scenario has a fixed circular boundary centered at  $(0, 0)$ . All turbine  $(x, y)$  locations must remain on or within this boundary. No turbine can be less than two rotor diameters from any other turbine.

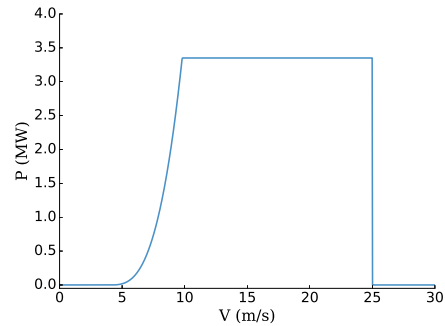
## Parameters

The wind turbine is the IEA37 3.35 MW onshore reference turbine [1] with the following characteristics:

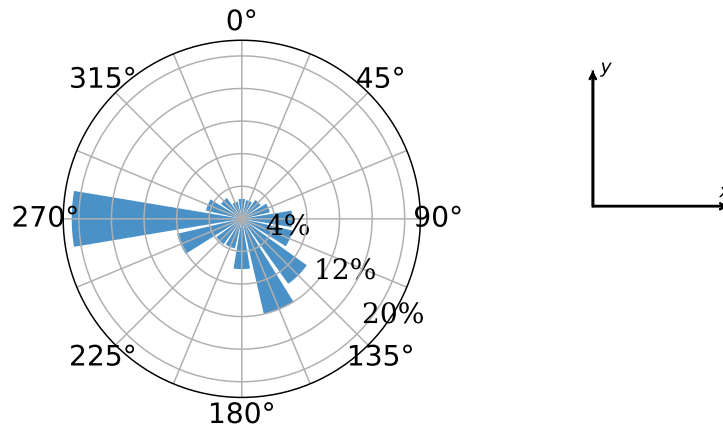
Rotor Diameter	130	m
Turbine Rating	3.35	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	9.8	m/s
Cut-Out Wind Speed	25	m/s

All turbine data is also contained in the enclosed `iea37-335mw.yaml`. The power curve is defined as:

$$P(V) = \begin{cases} 0 & V < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V < V_{rated} \\ P_{rated} & V_{rated} \leq V < V_{cut-out} \\ 0 & V \geq V_{cut-out} \end{cases}$$



The farm wind speed for all scenarios is constant at 9.8 m/s. The +y axis is coincident with 0°, and the CW wind rose is defined by 16 discrete bins tabulated in `iea37-windrose.yaml`, depicted pictorially below:



## 2.1 Case Study 1: Optimization Only

This problem defines three different wind farm sizes, and corresponding number of turbines, intended to test scalability of your optimization approach. The three scenarios are:

1. 16 turbines, boundary radius of 1,300 m.
2. 36 turbines, boundary radius of 2,000 m.
3. 64 turbines, boundary radius of 3,000 m.

For this Case Study the user is only free to choose the optimization approach. The wake model is fixed and is a simplified version of Bastankhah's Gaussian wake model [2, 3, 4]. A Python implementation is supplied for convenience (`iea37-aepcalc.py`). Alterations to this implementation are permitted, as long as the

governing physics equations are not altered. Participants may use other programming languages, but must use the same physics equations. To aid with this, the relevant equations are defined in a separate document (`iea37-wakemodel.pdf`), and example wind farm layouts with corresponding AEP values are provided in the `iea37-ex##.yaml` files to verify implementations. The example designs are only for verification, and do not need to be used as starting points in your optimization.

## 2.2 Case Study 2: Combined

This problem defines one scenario where the user is free to choose both the optimization algorithm and the wake model. The single wind farm scenario is nine turbines with a boundary radius of 900 m.

If needed by your wake model choice, the turbulence intensity is 0.075, and the wind shear is a power-law with a shear exponent of 0.15 using the hub height as the reference height.

# 3 Reporting and Evaluation

Participants will submit:

1. Optimal turbine placement solution for each scenario, using the `.yaml` format from the enclosed example layouts.
2. A survey describing your methodology and simulation environment [here](#).

Note that for both Case Studies, your `.yaml` submissions must report both total farm AEP, and farm AEP for each binned wind direction, as in the enclosed `iea37-ex##.yaml` examples.

## 3.1 Case Study 1: Optimization Only

Results will be compared by running the enclosed `iea37-aepcalc.py`, which will read the submitted `.yaml` file from each participant. Submissions must adhere to the `.yaml` format in order to receive a ranking. While other implementations may be used in the optimization, all evaluations will be done with the provided `iea37-aepcalc.py` code, so it is essential that you check that your implementation is consistent.

The command-line syntax we will use to evaluate all submitted files is:

```
$python iea37-aepcalc.py iea37-yourname-opt##.yaml
```

Where:

- `iea37-yourname-opt##.yaml` will be your submitted `.yaml` of optimal turbine locations.
  - “`yourname`” is your personal or organizational name, all lowercase with no spaces or punctuation.
  - “`##`” is the scenario size, i.e. “`opt16`” would be for the 16-turbine scenario.

The following two files must be referenced internally by your submission, as is done by the example layouts:

- `iea37-windrose.yaml` describes the binned wind rose used in both case studies.
- `iea37-335mw.yaml` lists the turbine data for the used IEA37 3.35 MW onshore reference turbine.

## 3.2 Case Study 2: Combined

Because the wake models differ in this Case Study, determining a “best” solution is generally not possible. Comparisons will be made using two approaches:

1. Every participant will evaluate every other participant’s solutions using their own wake model(s). It is essential that the `.yaml` format is adhered to so that cross-comparisons are painless.
2. Each solution will be compared using a higher-fidelity simulation, in this case large-eddy simulations (LES) using SOWFA. This simulation introduces its own modeling assumptions and is an imperfect way to compare, but does provide another piece of information on relative performance between approaches.



## 4 Enclosures

Files included with this document, needed for full participation in the Case Studies are:

- `iea37-aepcalc.py` - Python coding of AEP wake model for the Optimization Only Case Study
- `iea37-wakemodel.pdf` - description of AEP algorithm for the Optimization Only Case Study
- `iea37-windrose.yaml` - binned wind frequency for both Case Studies, in `.yaml` format
- `iea37-335mw.yaml` - data for reference turbine used in both Case Studies, in `.yaml` format
- `iea37-ex16.yaml` - 16 turbine scenario example layout
- `iea37-ex36.yaml` - 36 turbine scenario example layout
- `iea37-ex64.yaml` - 64 turbine scenario example layout

## References

- [1] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., “IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines,” Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.
- [2] Thomas, J. J. and Ning, A., “A method for reducing multi-modality in the wind farm layout optimization problem,” *Journal of Physics: Conference Series*, The Science of Making Torque from Wind, Milano, Italy, June 2018.
- [3] Bastankhah, M. and Porté-Agel, F., “A new analytical model for wind-turbine wakes,” *Renewable Energy*, January 2014.
- [4] Bastankhah, M. and Porté-Agel, F., “Experimental and theoretical study of wind turbine wakes in yawed conditions,” *J. Fluid Mech.*, Vol. 806, 2016, pp. 506–541.

# Wake Model Description for Optimization Only Case Study

IEA Task 37 on System Engineering in Wind Energy

This is an explanatory enclosure to accompany `iea37-wflocs-announcement.pdf`. For the Optimization Only Case Study, we will use the enclosed Python file `iea37-aepcalc.py` to evaluate your reported optimal turbine locations in `.yaml` format. If you desire to implement the AEP calculations in a language other than Python, the algorithm's description and wake model equations are provided below. Please insure your implementation computes the same AEP value given in each of the example layouts (`iea37-ex##.yaml`) also enclosed.

## Wake Model Equations

The wake model for the Optimization Only Case Study is a simplified version of Bastankhah's Gaussian wake model [1]. The governing equations for the velocity deficit in a waked region are:

$$\frac{\Delta V}{V_\infty} = \begin{cases} \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y_i - y_g}{\sigma_y}\right)^2\right), & \text{if } (x_i - x_g) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$\sigma_y = k_y \cdot (x_i - x_g) + \frac{D}{\sqrt{8}} \quad (2)$$

Variable	Value	Definition
$\Delta V/V_\infty$	Eq. (1)	Normalized wake velocity deficit
$C_T$	8/9	Thrust coefficient
$x_i - x_g$	-	Dist. from hub generating wake ( $x_g$ ) to hub of interest ( $x_i$ ), along freestream
$y_i - y_g$	-	Dist. from hub generating wake ( $y_g$ ) to hub of interest ( $y_i$ ), $\perp$ to freestream
$\sigma_y$	Eq. (2)	Standard deviation of the wake deficit
$k_y$	0.0324555	Variable based on a turbulence intensity of 0.075 [1, 2]
$D$	130 m	Turbine diameter [3]

The two cases in the wake velocity equation are needed because wakes are assumed to only affect points downstream. Hub coordinates are used for all location calculations. For turbines placed in multiple wakes, the total velocity deficit is calculated using the square root of the sum of the squares:

$$\left(\frac{\Delta V}{V_\infty}\right)_{total} = \sqrt{\left(\frac{\Delta V}{V_\infty}\right)_1^2 + \left(\frac{\Delta V}{V_\infty}\right)_2^2 + \left(\frac{\Delta V}{V_\infty}\right)_3^2 + \dots} \quad (3)$$

## AEP Algorithm

1. Read the following input from `.yaml` files:
  - Turbine ( $x,y$ ) locations.
  - Turbine attributes (`cut-in\cut-out\rated wind speed\rated power`).
  - Number of wind directional bins,  $\theta_i$  ( $i = 16$  for these Case Studies).
  - Wind frequency at each binned direction,  $f(\theta)$ .
  - Wind speed at each binned direction,  $V_\infty(\theta)$  (invariant for these Case Studies).
2. Calculate the power produced in the farm for one wind direction:

- (a) For each binned direction  $\theta$ , rotate the turbine locations  $(x, y)$  into the into the wind frame of reference  $(x_w, y_w)$ :

$$\begin{aligned}\Psi &= -\left(\frac{\pi}{2} + \theta\right) \\ x_w &= x \cos(\Psi) + y \sin(\Psi) \\ y_w &= -x \sin(\Psi) + y \cos(\Psi)\end{aligned}$$

- (b) Iterating through each turbine in the field to compute its power:

- Compute the wake deficit between each turbine pair Eq. (1) (there is no wake effect of a turbine on itself).
- Use Eq. (3) to calculate the total wake loss.
- Compute effective wind speed ( $V_e$ ) at each turbine:

$$V_e = V_\infty \left[ 1 - \left( \frac{\Delta V}{V_\infty} \right)_{total} \right]$$

- Use  $V_e$  and the IEA37 3.35MW power curve to calculate each turbine's power:

$$P_{turb}(V_e) = \begin{cases} 0 & V_e < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V_e - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V_e < V_{rated} \\ P_{rated} & V_{rated} \leq V_e < V_{cut-out} \\ 0 & V_e \geq V_{cut-out} \end{cases} \quad (4)$$

- (c) Sum powers from all  $n$  turbines

$$P_{farm} = \sum_{j=1}^n P_{turb,j} \quad (5)$$

3. Compute AEP using farm power for all  $m$  directions where  $P$  is the wind farm power for direction  $i$  and  $f$  is the corresponding frequency for direction  $i$ . The factor of 8760 is just to multiply by hours in a year:

$$AEP = \left( \sum_{i=1}^m f_i P_i \right) 8760 \frac{\text{hrs}}{\text{yr}} \quad (6)$$

## References

- [1] Thomas, J. J. and Ning, A., "A method for reducing multi-modality in the wind farm layout optimization problem," *Journal of Physics: Conference Series*, The Science of Making Torque from Wind, Milano, Italy, June 2018.
- [2] Niayifar, A. and Porté-Agel, F., "Analytical Modeling of Wind Farms: A New Approach for Power Prediction," *Energies*, September 2016.
- [3] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., "IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines," Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.

```

1  """IEA Task 37 Combined Case Study AEP Calculation Code
2
3  Written by Nicholas F. Baker, PJ Stanley, and Jared Thomas (BYU FLOW lab)
4  Created 10 June 2018
5  Updated 11 Jul 2018 to include read-in of .yaml turb locs and wind freq dist.
6  Completed 26 Jul 2018 for commenting and release
7  Modified 22 Aug 2018 implementing multiple suggestions from Erik Quaeghebeur:
8      - PEP 8 adherence for blank lines, length(<80 char), var names,
9      docstring.
10     - Altered multiple comments for clarity.
11     - Used print_function for compatibility with Python 3.
12     - Used structured datatype (coordinate) and recarray to couple x,y
13     coords.
14     - Removed unused variable 'sWindRose' (getTurbLocYAML).
15     - Removed unnecessary "if ... < 0" case (WindFrame).
16     - Simplified calculations for sin/cos_wind_dir (WindFrame).
17     - Eliminated unnecessary calculation of 0 values (GaussianWake, DirPower).
18     - Turbine diameter now drawn from <.yaml> (GaussianWake)
19     - Used yaml.safe_load.
20     - Modified .yaml reading syntax for brevity.
21     - Removed some (now) unused array initializations.
22 """
23
24 from __future__ import print_function    # For Python 3 compatibility
25 import numpy as np
26 import sys
27 import yaml                               # For reading .yaml files
28 from math import radians as DegToRad     # For converting degrees to radians
29
30 # Structured datatype for holding coordinate pair
31 coordinate = np.dtype([('x', 'f8'), ('y', 'f8')])
32
33 def WindFrame(turb_coords, wind_dir_deg):
34     """Convert map coordinates to downwind/crosswind coordinates."""
35
36     # Convert from meteorological polar system (CW, 0 deg.=N)
37     # to standard polar system (CCW, 0 deg.=W)
38     # Shift so North comes "along" x-axis, from left to right.
39     wind_dir_deg = 270. - wind_dir_deg
40     # Convert inflow wind direction from degrees to radians
41     wind_dir_rad = DegToRad(wind_dir_deg)
42
43     # Constants to use below
44     cos_dir = np.cos(-wind_dir_rad)
45     sin_dir = np.sin(-wind_dir_rad)
46     # Convert to downwind(x) & crosswind(y) coordinates
47     frame_coords = np.recarray(turb_coords.shape, coordinate)
48     frame_coords.x = (turb_coords.x * cos_dir) - (turb_coords.y * sin_dir)
49     frame_coords.y = (turb_coords.x * sin_dir) + (turb_coords.y * cos_dir)
50
51     return frame_coords
52
53 def GaussianWake(frame_coords, turb_diam):
54     """Return each turbine's total loss due to wake from upstream turbines"""
55     # Equations and values explained in <iea37-wakemodel.pdf>
56     num_turb = len(frame_coords)
57
58     # Constant thrust coefficient

```

```

59     CT = 4.0*1./3.*(1.0-1./3.)
60     # Constant, relating to a turbulence intensity of 0.075
61     k = 0.0324555
62     # Array holding the wake deficit seen at each turbine
63     loss = np.zeros(num_turb)
64
65     for i in range(num_turb):          # Looking at each turb (Primary)
66         loss_array = np.zeros(num_turb) # Calculate the loss from all others
67         for j in range(num_turb):      # Looking at all other turbs
(Target)
68             x = frame_coords.x[i] - frame_coords.x[j] # Calculate the x-
dist
69             y = frame_coords.y[i] - frame_coords.y[j] # And the y-offset
70             if x > 0.:                          # If Primary is downwind of the
Target
71                 sigma = k*x + turb_diam/np.sqrt(8.) # Calculate the wake
loss
72                 # Simplified Bastankhah Gaussian wake model
73                 exponent = -0.5 * (y/sigma)**2
74                 radical = 1. - CT/(8.*sigma**2 / turb_diam**2)
75                 loss_array[j] = (1.-np.sqrt(radical)) * np.exp(exponent)
76                 # Note that if the Target is upstream, loss is defaulted to zero
77                 # Total wake losses from all upstream turbs, using sqrt of sum of
sqrs
78                 loss[i] = np.sqrt(np.sum(loss_array**2))
79
80     return loss
81
82
83 def DirPower(turb_coords, wind_dir_deg, wind_speed,
84             turb_diam, turb_ci, turb_co, rated_ws, rated_pwr):
85     """Return the power produced by each turbine."""
86     num_turb = len(turb_coords)
87
88     # Shift coordinate frame of reference to downwind/crosswind
89     frame_coords = WindFrame(turb_coords, wind_dir_deg)
90     # Use the Simplified Bastankhah Gaussian wake model for wake deficits
91     loss = GaussianWake(frame_coords, turb_diam)
92     # Effective windspeed is freestream multiplied by wake deficits
93     wind_speed_eff = wind_speed*(1.-loss)
94     # By default, the turbine's power output is zero
95     turb_pwr = np.zeros(num_turb)
96
97     # Check to see if turbine produces power for experienced wind speed
98     for n in range(num_turb):
99         # If we're between the cut-in and rated wind speeds
100         if ((turb_ci <= wind_speed_eff[n])
101             and (wind_speed_eff[n] < rated_ws)):
102             # Calculate the curve's power
103             turb_pwr[n] = rated_pwr * ((wind_speed_eff[n]-turb_ci)
104                                       / (rated_ws-turb_ci))**3
105         # If we're between the rated and cut-out wind speeds
106         elif ((rated_ws <= wind_speed_eff[n])
107              and (wind_speed_eff[n] < turb_co)):
108             # Produce the rated power
109             turb_pwr[n] = rated_pwr
110
111     # Sum the power from all turbines for this direction
112     pwrDir = np.sum(turb_pwr)
113

```

```

114     return pwrDir
115
116
117 def calcAEP(turb_coords, wind_freq, wind_speed, wind_dir,
118            turb_diam, turb_ci, turb_co, rated_ws, rated_pwr):
119     """Calculate the wind farm AEP."""
120     num_bins = len(wind_freq) # Number of bins used for our windrose
121
122     # Power produced by the wind farm from each wind direction
123     pwr_produced = np.zeros(num_bins)
124     # For each wind bin
125     for i in range(num_bins):
126         # Find the farm's power for the current direction
127         pwr_produced[i] = DirPower(turb_coords, wind_dir[i], wind_speed,
128                                   turb_diam, turb_ci, turb_co,
129                                   rated_ws, rated_pwr)
130
131     # Convert power to AEP
132     hrs_per_year = 365.*24.
133     AEP = hrs_per_year * (wind_freq * pwr_produced)
134     AEP /= 1.E6 # Convert to MWh
135
136     return AEP
137
138
139 def getTurbLocYAML(file_name):
140     """ Retrieve turbine locations and auxiliary file names from <.yaml>
141     file.
142
143     Auxiliary (reference) files supply wind rose and turbine attributes.
144     """
145     # Read in the .yaml file
146     with open(file_name, 'r') as f:
147         defs = yaml.safe_load(f)['definitions']
148
149     # Rip the x- and y-coordinates (Convert from <list> to <ndarray>)
150     turb_xc = np.asarray(defs['position']['items']['xc'])
151     turb_yc = np.asarray(defs['position']['items']['yc'])
152     turb_coords = np.recarray(turb_xc.shape, coordinate)
153     turb_coords.x, turb_coords.y = turb_xc, turb_yc
154
155     # Rip the expected AEP, used for comparison
156     # AEP = defs['plant_energy']['properties']
157     #         ['annual_energy_production']['default']
158
159     # Read the auxiliary filenames for the windrose and the turbine
160     attributes
161     ref_list_turbs = defs['wind_plant']['properties']['layout']['items']
162     ref_list_wr = (defs['plant_energy']['properties']
163                  ['wind_resource_selection']['properties']['items'])
164
165     # Iterate through all listed references until we find the one we want
166     # The one we want is the first reference not internal to the document
167     # Note: internal references use '#' as the first character
168     fname_turb = next(ref['$ref']
169                      for ref in ref_list_turbs if ref['$ref'][0] != '#')
170     fname_wr = next(ref['$ref']
171                    for ref in ref_list_wr if ref['$ref'][0] != '#')
172
173     # Return turbine (x,y) locations, and the filenames for the others .yaml's

```

```

172     return turb_coords, fname_turb, fname_wr
173
174
175 def getWindRoseYAML(file_name):
176     """Retrieve wind rose data (bins, freqs, speeds) from <.yaml> file."""
177     # Read in the .yaml file
178     with open(file_name, 'r') as f:
179         props = yaml.safe_load(f)['definitions']['wind_inflow']['properties']
180
181     # Rip wind directional bins, their frequency, and the farm windspeed
182     # (Convert from <list> to <ndarray>)
183     wind_dir = np.asarray(props['direction']['bins'])
184     wind_freq = np.asarray(props['probability']['default'])
185     # (Convert from <list> to <float>)
186     wind_speed = float(props['speed']['default'])
187
188     return wind_dir, wind_freq, wind_speed
189
190
191 def getTurbAtrbtYAML(file_name):
192     """Retreive turbine attributes from the <.yaml> file"""
193     # Read in the .yaml file
194     with open(file_name, 'r') as f:
195         defs = yaml.safe_load(f)['definitions']
196         op_props = defs['operating_mode']['properties']
197         turb_props = defs['wind_turbine_lookup']['properties']
198         rotor_props = defs['rotor']['properties']
199
200     # Rip the turbine attributes
201     # (Convert from <list> to <float>)
202     turb_ci = float(op_props['cut_in_wind_speed']['default'])
203     turb_co = float(op_props['cut_out_wind_speed']['default'])
204     rated_ws = float(op_props['rated_wind_speed']['default'])
205     rated_pwr = float(turb_props['power']['maximum'])
206     turb_diam = float(rotor_props['radius']['default']) * 2.
207
208     return turb_ci, turb_co, rated_ws, rated_pwr, turb_diam
209
210
211 if __name__ == "__main__":
212     """Used for demonstration.
213
214     An example command line syntax to run this file is:
215
216         python iea37-aepcalc.py iea37-ex16.yaml
217
218     For Python .yaml capability, in the terminal type "pip install pyyaml".
219     """
220     # Read necessary values from .yaml files
221     # Get turbine locations and auxiliary <.yaml> filenames
222     turb_coords, fname_turb, fname_wr = getTurbLocYAML(sys.argv[1])
223     # Get the array wind sampling bins, frequency at each bin, and wind speed
224     wind_dir, wind_freq, wind_speed = getWindRoseYAML(fname_wr)
225     # Pull the needed turbine attributes from file
226     turb_ci, turb_co, rated_ws, rated_pwr, turb_diam = getTurbAtrbtYAML(
227         fname_turb)
228
229     # Calculate the AEP from ripped values
230     AEP = calcAEP(turb_coords, wind_freq, wind_speed, wind_dir,
231                 turb_diam, turb_ci, turb_co, rated_ws, rated_pwr)

```

```
232 # Print AEP for each binned direction, with 5 digits behind the decimal.
233 print(np.array2string(AEP, precision=5, floatmode='fixed',
234                       separator=', ', max_line_width=62))
235 # Print AEP summed for all directions
236 print(np.around(np.sum(AEP), decimals=5))
237
```



```

1 title: IEA Wind Task 37 Wind Plant Ontology version 0.1
2 description: Turbine definition for 3.35MW Onshore Reference Turbine
3   Template located at
4   <https://github.com/IEAWindTask37/ontology/blob/develop/plant_energy_turbine_
5   lookup.yaml>
6
7 definitions:
8
9   wind_turbine_lookup:
10    type: object
11    description: look-up table model fidelity for a wind turbine in a energy
12    production workflow
13
14   properties:
15     # inputs
16     wind_turbine_id:
17      type: string
18      description: identifier of wind turbine for look-up
19      default: 335MW
20
21     operating_mode_id:
22      type: identifier of operating model for look-up
23      description: normal
24
25     wind_speed:
26      type: number
27      description: The current wind speed at hub height
28      minimum: 0.0
29      maximum: 200.0
30      units: m/s
31
32     # outputs
33     c_p:
34      type: number
35      description: The current power coefficient of the wind turbine
36      minimum: 0.0
37      maximum: 0.481
38
39     power:
40      type: number
41      description: The wind turbine electrical power
42      units: W
43      minimum: 0.0
44      maximum: 3350000.0
45
46   # turbine definition
47   wind_turbine:
48    description: Description of IEA37 3.35MW Onshore Reference Turbine
49    type: collection
50
51    id:
52     type: string
53     description: 335MW
54
55    turbine_type:
56     type: string
57     description: Onshore Reference
58
59    rated_power:
60     type: number

```

```

58     description: The rotor diameter
59     units: W
60     minimum: 0.0
61     expr_max: power_curve.max(1)
62
63     hub:
64         $ref: "#/definitions/hub"
65
66     rotor:
67         $ref: "#/definitions/rotor"
68
69     operating_modes:
70         type: array
71         description: operating modes for the turbine
72         items:
73             $ref: "#/definitions/operating_mode"
74
75
76 # rotor definition
77 rotor:
78     type: object
79     description: the turbine rotor which defines swept area
80
81     properties:
82         diameter:
83             type: number
84             description: The rotor diameter
85             units: m
86             expr: radius * 2.0
87
88         radius:
89             type: number
90             description: The rotor radius
91             units: m
92             default: 65.0
93
94         area:
95             type: number
96             description: The rotor area
97             units: m*m
98             expr: 2.0 * pi * radius**2.0
99
100 # hub definition
101 hub:
102     type: object
103     description: hub of the turbine which defines the hub height
104
105     properties:
106         height:
107             type: number
108             description: The hub height
109             units: m
110             default: 110.0
111
112 # operating mode can vary and the power/thrust curves will change as a result
113 operating_mode:
114     type: object
115     description: the operating mode of the turbine
116
117     properties:

```

```

118     id:
119         type: string
120         description: unique identifier of the operating mode
121
122     name:
123         description: name of the operating mode
124         type: str
125         default: normal operation
126
127     cut_in_wind_speed:
128         type: number
129         description: The starting wind speed of the wind turbine
130         default: 4.0
131         minimum: 0.0
132         maximum: 100.0
133         expr_max: cut_out_wind_speed
134         units: m/s
135
136     cut_out_wind_speed:
137         description: The stopping wind speed of the wind turbine
138         type: number
139         units: m/s
140         default: 25.0
141         expr_min: rated_wind_speed
142         minimum: 0.0
143         maximum: 100.0
144
145     rated_wind_speed:
146         type: number
147         description: The wind speed where the turbine reaches its rated power
148         units: m/s
149         default: 9.8
150         minimum: 0.0
151         expr_min: cut_in_wind_speed
152         maximum: 100.0
153         expr_max: cut_out_wind_speed
154
155     power_curve:
156         description: The wind turbine power curve
157         type: array #ndarray([[hub:wind_speed], [power]])
158         items:
159             type: array
160             items:
161                 - $ref: "#/definitions/hub/properties/wind_speed"
162                 - $ref: "#/definitions/wind_turbine/properties/power"
163         additionalItems: false

```

```

1 title: IEA Wind Task 37 Wind Plant Ontology version 0.1
2 description: Wind resource conditions for a IEA37 WFL0 Case Studies.
3   Template located at
4   <https://github.com/IEAWindTask37/ontology/blob/master/plant_energy_inflow_loo
5 kup.yaml>
6 definitions:
7   wind_inflow:
8     type: object
9     description: inflow for current wind conditions
10
11   properties:
12     direction:
13       id: "wind_direction"
14       type: number
15       description: The wind direction in degree, with North as the 0. 16
16     bins.
17       units: deg
18       bins: [0., 22.5, 45., 67.5,
19             90., 112.5, 135., 157.5,
20             180., 202.5, 225., 247.5,
21             270., 292.5, 315., 337.5]
22       minimum: 0.0
23       maximum: 360.0
24
25     speed:
26       type: number
27       description: A wind speed, constant for these case studies.
28       default: 9.8
29       units: m/s
30
31     ti:
32       type: number
33       description: Turbulence intensity
34       default: 0.075
35
36     probability:
37       type: number
38       description: Wind directional frequency distribution for 16 bins of
39       wind rose
40       default: [.025, .024, .029, .036,
41               .063, .065, .100, .122,
42               .063, .038, .039, .083,
43               .213, .046, .032, .022]

```

```

1 input_format_version: 0
2 title: IEA Wind Task 37 Combined Case Study 16 Turbine Farm
3 description: input file for the 16 turbine wind plant model for IEA Task 37
  Combined Case Study
4   Template located at
  <https://github.com/IEAWindTask37/ontology/blob/develop/plant\_energy\_energy\_co
  nvolution.yaml>
5
6 definitions:
7   wind_plant:
8     type: object
9     description: specific plant design including turbine selection and
  placement
10    properties:
11      layout:
12        type: array
13        items:
14          - $ref: "#/definitions/position"
15          - $ref: "iea37-335mw.yaml" # Reference for the .yaml containing
  turbine attributes.
16
17    position:
18      type: array
19      items:
20        xc: [0., 650., 200.861, -525.861, -525.861, 200.861, 1300., 1051.7221,
  401.7221, -401.7221, -1051.7221,
21          -1300., -1051.7221, -401.7221, 401.7221, 1051.7221]
22        yc: [0., 0., 618.1867, 382.0604, -382.0604, -618.1867, 0., 764.1208,
  1236.3735, 1236.3735, 764.1208, 0.,
23          -764.1208, -1236.3735, -1236.3735, -764.1208]
24        additionalItems: false
25        description: an array of x-coordinates [x0, x1, ...] and y-coordinates
  [y0, y1, ...] of wind turbine positions in cartesian coordinates
26        units: m
27
28    plant_energy:
29      type: object
30      description: energy production from simplified Bastankhah Gaussian wake
  model
31
32    properties:
33      wake_model_selection:
34        type: algorithm
35        description: wake model used to calculate AEP
36        items:
37          - $ref: "iea37-aepcalc.py"
38
39      wind_resource_selection:
40        type: object
41        description: specific wind resource used to calculate AEP
42        properties:
43          type: array
44          items:
45            - $ref: "iea37-windrose.yaml" # Reference for the .yaml
  containing the wind rose
46
47      annual_energy_production:
48        type: number
49        description: binned and total (default) annual energy production for a
  wind plant given a layout and binned wind rose

```

```
50     binned: [ 9444.60012,  8497.90004, 11383.32869, 14173.40367,  
51             20979.36776, 25590.86774, 39252.85757, 43197.65856,  
52             23800.39229, 13539.36766, 15022.89800, 32644.44314,  
53             71157.32322, 18092.10102, 12326.48041,  7838.58128]  
54     default: 366941.57116  
55     units: MWh  
56
```

```

1 input_format_version: 0
2 title: IEA Wind Task 37 Combined Case Study 36 Turbine Farm
3 description: input file for the 36 turbine wind plant model for IEA Task 37
  Combined Case Study
4   Template located at
  <https://github.com/IEAWindTask37/ontology/blob/develop/plant_energy_energy_co
  nvolution.yaml>
5
6 definitions:
7   wind_plant:
8     type: object
9     description: specific plant design including turbine selection and
  placement
10    properties:
11      layout:
12        type: array
13        items:
14          - $ref: "#/definitions/position"
15          - $ref: "iea37-335mw.yaml" # Reference for the .yaml containing
  turbine attributes.
16
17    position:
18      type: array
19      items:
20        xc: [0., 666.6667, 206.0113, -539.3447, -539.3447, 206.0113, 1333.3333,
  1154.7005, 666.6667, 0.,
21          -666.6667, -1154.7005, -1333.3333, -1154.7005, -666.6667, 0.,
  666.6667, 1154.7005, 2000., 1879.3852,
22          1532.0889, 1000., 347.2964, -347.2964, -1000., -1532.0889,
  -1879.3852, -2000., -1879.3852, -1532.0889,
23          -1000., -347.2964, 347.2964, 1000., 1532.0889, 1879.3852]
24        yc: [0., 0., 634.0377, 391.8568, -391.8568, -634.0377, 0., 666.6667,
  1154.7005, 1333.3333,
25          1154.7005, 666.6667, 0., -666.6667, -1154.7005, -1333.3333,
  -1154.7005, -666.6667, 0., 684.0403,
26          1285.5752, 1732.0508, 1969.6155, 1969.6155, 1732.0508, 1285.5752,
  684.0403, 0., -684.0403, -1285.5752,
27          -1732.0508, -1969.6155, -1969.6155, -1732.0508, -1285.5752,
  -684.0403]
28        additionalItems: false
29        description: an array of x-coordinates [x0, x1, ...] and y-coordinates
  [y0, y1, ...] of wind turbine positions in cartesian coordinates
30        units: m
31
32    plant_energy:
33      type: object
34      description: energy production from simplified Bastankhah Gaussian wake
  model
35
36    properties:
37      wake_model_selection:
38        type: algorithm
39        description: wake model used to calculate AEP
40        items:
41          - $ref: "iea37-aepcalc.py"
42
43      wind_resource_selection:
44        type: object
45        description: specific wind resource used to calculate AEP
46        properties:

```

```
47     type: array
48     items:
49       - $ref: "iea37-windrose.yaml"      # Reference for the .yaml
      containing the wind rose
50
51     annual_energy_production:
52       type: number
53       description: binned and total (default) annual energy production for a
wind plant given a layout and binned wind rose
54       binned: [ 20031.56539, 18948.56110, 22909.44283, 27563.57816,
55                 39052.27825, 49767.57168, 78998.07872, 96321.85228,
56                 50479.54479, 29779.76444, 30833.38985, 63049.88078,
57                 132664.17490, 34943.30742, 25299.19167, 17240.91625]
58       default: 737883.09851
59       units: MWh
60
```



```

1 input_format_version: 0
2 title: IEA Wind Task 37 Combined Case Study 64 Turbine Farm
3 description: input file for the 64 turbine wind plant model for IEA Task 37
  Combined Case Study
4   Template located at
  <https://github.com/IEAWindTask37/ontology/blob/develop/plant\_energy\_energy\_co
  nvolution.yaml>
5
6 definitions:
7   wind_plant:
8     type: object
9     description: specific plant design including turbine selection and
  placement
10    properties:
11      layout:
12        type: array
13        items:
14          - $ref: "#/definitions/position"
15          - $ref: "iea37-335mw.yaml" # Reference for the .yaml containing
  turbine attributes.
16
17    position:
18      type: array
19      items:
20        xc: [0., 750., 231.7627, -606.7627, -606.7627, 231.7627, 1500.,
  1299.0381, 750., 0.,
21          -750., -1299.0381, -1500., -1299.0381, -750., 0., 750., 1299.0381,
  2250, 2114.3084,
22          1723.6, 1125., 390.7084, -390.7084, -1125., -1723.6, -2114.3084,
  -2250., -2114.3084, -1723.6,
23          -1125, -390.7084, 390.7084, 1125., 1723.6, 2114.3084, 3000.,
  2924.7837, 2702.9066, 2345.4944,
24          1870.4694, 1301.6512, 667.5628, 0., -667.5628, -1301.6512,
  -1870.4694, -2345.4944, -2702.9066, -2924.7837,
25          -3000., -2924.7837, -2702.9066, -2345.4944, -1870.4694, -1301.6512,
  -667.5628, 0., 667.5628, 1301.6512,
26          1870.4694, 2345.4944, 2702.9066, 2924.7837]
27        yc: [0., 0., 713.2924, 440.8389, -440.8389, -713.2924, 0., 750.,
  1299.0381, 1500,
28          1299.0381, 750., 0., -750., -1299.0381, -1500., -1299.0381, -750.,
  0., 769.5453,
29          1446.2721, 1948.5572, 2215.8174, 2215.8174, 1948.5572, 1446.2721,
  769.5453, 0., -769.5453, -1446.2721,
30          -1948.5572, -2215.8174, -2215.8174, -1948.5572, -1446.2721,
  -769.5453, 0., 667.5628, 1301.6512, 1870.4694,
31          2345.4944, 2702.9066, 2924.7837, 3000., 2924.7837, 2702.9066,
  2345.4944, 1870.4694, 1301.6512, 667.5628,
32          0., -667.5628, -1301.6512, -1870.4694, -2345.4944, -2702.9066,
  -2924.7837, -3000., -2924.7837, -2702.9066,
33          -2345.4944, -1870.4694, -1301.6512, -667.5628]
34      additionalItems: false
35      description: an array of x-coordinates [x0, x1, ...] and y-coordinates
  [y0, y1, ...] of wind turbine positions in cartesian coordinates
36      units: m
37
38    plant_energy:
39      type: object
40      description: energy production from simplified Bastankhah Gaussian wake
  model
41

```

```

42 properties:
43   wake_model_selection:
44     type: algorithm
45     description: wake model used to calculate AEP
46     items:
47       - $ref: "iea37-aepcalc.py"
48
49   wind_resource_selection:
50     type: object
51     description: specific wind resource used to calculate AEP
52     properties:
53       type: array
54       items:
55         - $ref: "iea37-windrose.yaml"      # Reference for the .yaml
56           containing the wind rose
57
58   annual_energy_production:
59     type: number
60     description: binned and total (default) annual energy production for a
61     wind plant given a layout and binned wind rose
62     binned: [ 34909.41061, 31961.97110, 38624.65424, 48717.97038,
63             73194.82922, 87963.00207, 133188.46289, 162473.35310,
64             87971.71474, 50459.68229, 51894.57832, 112009.16388,
65             247734.46985, 62077.36793, 42580.16683, 29213.50027]
66     default: 1294974.2977
67     units: MWh

```

**APPENDIX B. CASE STUDY 3-4 STARTUP FILES**

# Wind Farm Layout Optimization Case Studies 3 & 4

IEA Task 37 on System Engineering in Wind Energy

Nicholas F. Baker, Andrew P. J. Stanley, and Andrew Ning

Brigham Young University, Provo, Utah, USA

Katherine Dykes

Technical University of Denmark, Kongens Lyngby, Copenhagen, Denmark

June 4, 2019

## 1 Introduction

Two major factors that affect wind farm layout optimization are 1) the optimization method and 2) the wake model. We have thus far conducted [two case studies](#) to analyze differences in these variables, this document defines a third and fourth case study to further research these factors when given a more realistic wind farm boundary and wind resource. Case study 3 (cs3) presents a scenario with a concave boundary. Case study 4 (cs4) presents a scenario with boundaries that are discontinuous and contain concavities. For cs3 a wake model is provided, participants need only optimize turbine locations. For cs4 users are free to choose both optimization approach and wake model.

Participants will 1) optimize turbine locations to maximize annual energy production, 2) submit details regarding their optimization convergence history and methodology. After all submissions are received, participants of cs4 will be expected to perform a cross comparison of other participant solutions. Data will be consolidated, processed, and made available to all participants.

## 2 Problem Definition

### Objective

The objective of each scenario is to maximize annual energy production (AEP), which we define simply as the expected value of aerodynamic power multiplied by the hours in a year. In other words:

$$AEP = 8760 \frac{\text{hrs}}{\text{yr}} \sum_{i=1}^n \sum_{j=1}^m f_i w_{i,j} P_{i,j}$$

where  $P_{i,j}$  is the power produced for wind direction  $i$  at wind speed  $j$ ,  $n$  is the number of wind directional bins,  $f_i$  is the corresponding wind direction probability,  $m$  is the number of wind speed bins for each direction, and  $w_{i,j}$  is the probability each speed bin will occur at each direction. Participants are free to use any optimization method.

### Variables

The final reported designs will be in terms of the  $(x, y)$  locations of each turbine, although participants are free to parameterize the turbine positions using any design variables they choose (e.g., pre-selected grid locations, grid spacing parameters, etc.). Note that every turbine is identical and defined below in [Parameters](#).

### Constraints

In case studies 1 and 2, all farm boundaries were circular. To make cs3 and cs4 more realistic, all boundaries are non-uniform. The cs3 and cs4 boundaries are based on the Borselle III and IV wind farms, our version is depicted graphically in [Figure 1](#). The coordinates for the boundary vertices are given in [iea37-boundary-cs3.yaml](#) and [iea37-boundary-cs4.yaml](#). All turbine hub coordinates must remain on or within these boundaries. The turbines are further constrained such that no hub can be less than two rotor diameters from any other hub, and for these farms scenarios all hub heights ( $z$  values) will be the same.

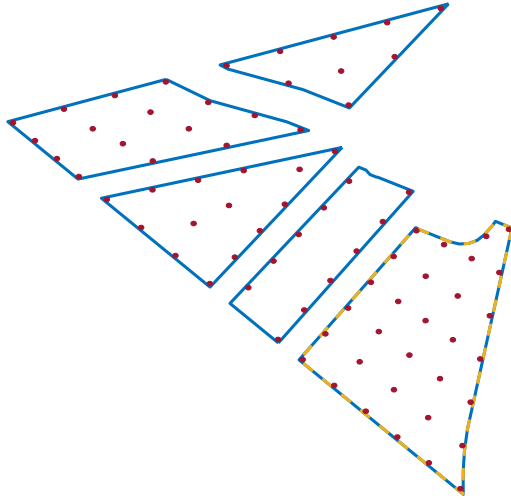


Figure 1: The wind farm boundary for cs3 (outlined in yellow) and cs4 (outlined in blue). A provided baseline turbine layout for cs4 is overlaid in red, with rotor radii to scale.

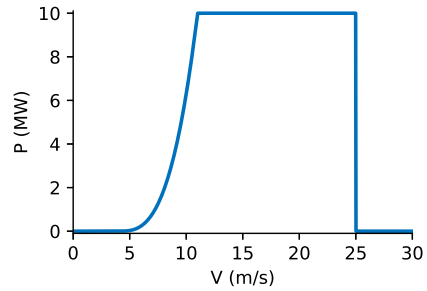
## Parameters

The wind turbine is the IEA37 10 MW offshore reference turbine [1] with the following characteristics:

Rotor Diameter	198	m
Hub Height	119	m
Turbine Rating	10	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	11	m/s
Cut-Out Wind Speed	25	m/s

All turbine data are also contained in the enclosed `iea37-10mw.yaml`. The power curve is defined as:

$$P(V) = \begin{cases} 0 & V < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V < V_{rated} \\ P_{rated} & V_{rated} \leq V < V_{cut-out} \\ 0 & V \geq V_{cut-out} \end{cases}$$



For cs3 and cs4, the supplied wind resource is binned over both wind direction and wind speed. We supply a high number of frequency probabilities associated for each wind direction (20 values), and frequency probabilities associated with each wind speed's occurrence at each direction (20 x 20 = 400 values). This is more data than needed for AEP convergence. Participants are free to use any level of discretization or sampling strategy they prefer during the optimization (though final results will be compared using all 20 x 20 bins). This data is included in `iea37-windrose-cs3.yaml`, which will be used for both cs3 and cs4. Figure 2 gives a graphical representation of the binned wind direction frequencies, and the wind speed sampling for a single wind direction.

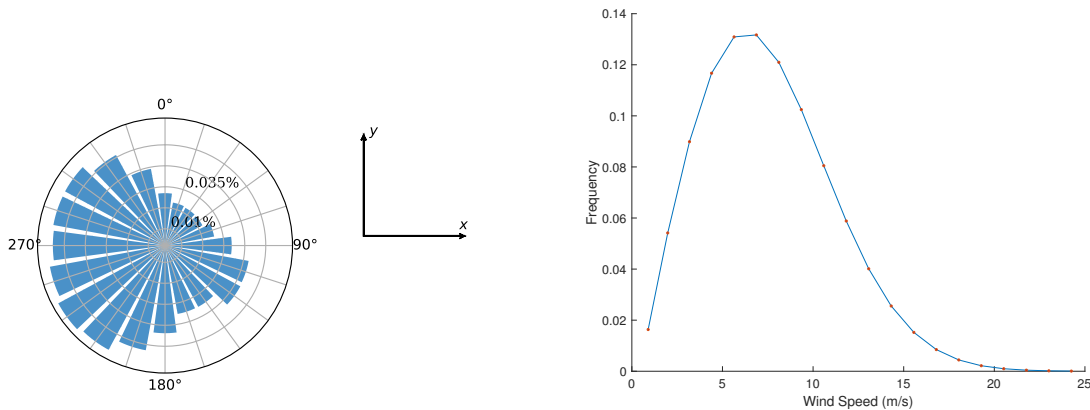


Figure 2: (left) Wind frequency distribution over the 20 bins for the windrose used in cs3 and cs4. (right) Wind speed frequency distribution at one of the wind directions (36°).

## 2.1 Case Study 3

This wind farm scenario consists of twenty-five (25) turbines with a boundary containing concavities. Participants will optimize the wind farm layout for maximum AEP with the provided Gaussian wake model, wind rose, turbine, and boundary. The wake model is supplied (coded in python) as `iea37-aepcalc.py`. Alterations to this implementation are permitted, as long as the governing physics equations are not altered. Participants may use other programming languages, but must use the same physics equations. To aid with this, the relevant equations are defined in a separate document (`iea37-wakemodel.pdf`), and baseline wind farm layouts with corresponding AEP values are provided in the `iea37-ex-opt#.yaml` files to verify implementations.

## 2.2 Case Study 4

This scenario consists of eighty-one (81) turbines with a boundary containing concavities and discontinuities. The user is free to choose both the optimization algorithm and wake model. This scenario contains discrete boundary regions. The number of turbines in a given boundary region can be changed as long as the total number of turbines is fixed. In other words, the participant or their optimization algorithm is free to move all eighty-one turbines between the five boundary regions.

# 3 Reporting and Evaluation

## 3.1 Baseline Layouts

Baseline turbine layouts for both case studies are supplied. These are provided in part to give examples of our precise reporting format. They are called `iea37-ex-opt3.yaml`, `iea37-ex-opt4.yaml`, and `iea37-ex-log3.yaml`. Like the example layouts provided for case studies 1 and 2, these baseline layouts are also meant to provide a reasonable minimum output against which results can be measured. Participants are not required to use the baseline layouts as starting points for their optimizations, though they are permitted to do so.

## 3.2 Reporting

Submissions must adhere to the `.yaml` format in order to enable easy and fast analysis of participant results. You will submit two (2) files per case study: one with your optimized turbine layout, the second a log of

your optimization convergence data. Your submitted files should be named:

```
iea37-yourname-opt#.yaml  
iea37-yourname-log#.yaml
```

Where “yourname” is your personal or organizational name, all lowercase with no spaces or punctuation, `-opt#.yaml` describes the  $(x,y)$  coordinates for your optimal turbine layout, `-log#.yaml` contains information regarding your hardware and optimization algorithm’s performance, and “#” is the case study number of the submission (i.e. “`-opt3.yaml`” contains optimized results for cs3).

Of note, we require participants to log target function evaluations for **every** function call during the optimization process. As shown in the example log given in `iea37-ex-log3.yaml`, participants must report both: (1) number of total target function calls, and (2) AEP calculation at each chronological function call. To repeat, every call to the AEP calculation should be logged whether it is formally part of the optimization or not (e.g., each call in a finite difference estimate, each call in creating an initial population, etc.)

### 3.3 Evaluation

Evaluations for cs3 will be made using the wake model target function supplied in `iea37-aepcalc.py` and convergence data reported by each participant.

Because the participant wake models in cs4 are intended to differ, determining a “best” solution for cs4 is generally not possible. Evaluations will be made using a cross-comparison approach. Every participant will evaluate every other participant’s solutions using their own wake model(s). It is essential that the `.yaml` format is adhered to so that cross-comparisons are painless.

## 4 Participation

If interested (or potentially interested) in participating in either or both case studies send an email to [the primary author](#) so we can keep you informed of updates/deadlines, etc.

## 5 Enclosures

Files included with this document, needed for full participation in the case studies are:

- `iea37-10mw.yaml` - data for the reference turbine used in both cs3 and cs4
- `iea37-aepcalc.py` - target AEP calculator using a simplified Gaussian wake model
- `iea37-boundary-cs3.yaml` - the vertices for cs3’s wind farm’s boundary
- `iea37-boundary-cs4.yaml` - the vertices for cs4’s wind farm’s boundary
- `iea37-ex-log3.yaml` - example of optimization results, denoting timing and iteration analysis
- `iea37-ex-opt3.yaml` - baseline layout for cs3, template for your submitted optimal turbine layout
- `iea37-ex-opt4.yaml` - baseline layout for cs4
- `iea37-wakemodel.pdf` - description of the AEP algorithm used in cs3
- `iea37-windrose-cs3.yaml` - 20 directional bins with 20 speed probabilities, for both cs3 and cs4

## References

- [1] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., “IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines,” Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.

# Wake Model Description for Case Study 3

IEA Task 37 on System Engineering in Wind Energy

This is an explanatory enclosure to accompany `iea37-cs3-announcement.pdf`. For case study 3 (cs3), we will use the enclosed Python file `iea37-aepcalc.py` to evaluate your reported optimal turbine locations in `.yaml` format. If you desire to implement the AEP calculations in a language other than Python, the algorithm's description and wake model equations are provided below. Please insure your implementation computes the same AEP value given in the baseline layout (`iea37-ex-opt3.yaml`) also enclosed.

## Wake Model Equations

The wake model for cs3 is a simplified version of Bastankhah's Gaussian wake model [1]. The governing equations for the velocity deficit in a waked region are:

$$\frac{\Delta V}{V_\infty} = \begin{cases} \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y_i - y_g}{\sigma_y}\right)^2\right), & \text{if } (x_i - x_g) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$\sigma_y = k_y \cdot (x_i - x_g) + \frac{D}{\sqrt{8}} \quad (2)$$

Variable	Value	Definition
$\Delta V/V_\infty$	Eq. (1)	Normalized wake velocity deficit
$C_T$	8/9	Thrust coefficient
$x_i - x_g$	-	Dist. from hub generating wake ( $x_g$ ) to hub of interest ( $x_i$ ), along freestream
$y_i - y_g$	-	Dist. from hub generating wake ( $y_g$ ) to hub of interest ( $y_i$ ), $\perp$ to freestream
$\sigma_y$	Eq. (2)	Standard deviation of the wake deficit
$k_y$	0.0324555	Variable based on a turbulence intensity of 0.075 [1, 2]
$D$	198 m	Turbine diameter [3]

The two cases in the wake velocity equation are needed because wakes are assumed to only affect points downstream. Hub coordinates are used for all location calculations. For turbines placed in multiple wakes, the total velocity deficit is calculated using the square root of the sum of the squares:

$$\left(\frac{\Delta V}{V_\infty}\right)_{total} = \sqrt{\left(\frac{\Delta V}{V_\infty}\right)_1^2 + \left(\frac{\Delta V}{V_\infty}\right)_2^2 + \left(\frac{\Delta V}{V_\infty}\right)_3^2 + \dots} \quad (3)$$

## AEP Algorithm

1. Read the following input from `.yaml` files:
  - Turbine ( $x, y$ ) locations.
  - Turbine attributes (cut-in\cut-out\rated wind speed\rated power).
  - Number of wind directional bins,  $n$  ( $n = 60$  for cs3).
  - Wind frequency at each binned direction,  $f(\theta)$ .
  - Number of speed bins to compute,  $m$  ( $m = 60$  for cs3).
  - Wind speed at each binned direction,  $w(\theta)$ .
2. Calculate the power produced in the farm for one wind direction:



- (a) For each binned direction  $\theta$ , rotate the turbine locations  $(x, y)$  into the wind frame of reference  $(x_w, y_w)$ :

$$\begin{aligned}\Psi &= -\left(\frac{\pi}{2} + \theta\right) \\ x_w &= x \cos(\Psi) + y \sin(\Psi) \\ y_w &= -x \sin(\Psi) + y \cos(\Psi)\end{aligned}$$

- (b) For each wind speed bin as  $V_\infty$ , iterate through each turbine in the field to compute its power:
- Compute the wake deficit between each turbine pair Eq. (1) (there is no wake effect of a turbine on itself).
  - Use Eq. (3) to calculate the total wake loss.
  - Compute effective wind speed ( $V_e$ ) at each turbine:

$$V_e = V_\infty \left[ 1 - \left( \frac{\Delta V}{V_\infty} \right)_{total} \right]$$

- Use  $V_e$  and the IEA37 10MW power curve to calculate each turbine's power:

$$P_{turb}(V_e) = \begin{cases} 0 & V_e < V_{cut-in} \\ P_{rated} \cdot \left( \frac{V_e - V_{cut-in}}{V_{rated} - V_{cut-in}} \right)^3 & V_{cut-in} \leq V_e < V_{rated} \\ P_{rated} & V_{rated} \leq V_e < V_{cut-out} \\ 0 & V_e \geq V_{cut-out} \end{cases} \quad (4)$$

- (c) Sum powers from all  $t$  turbines for that wind speed

$$P_{speed} = \sum_{k=1}^t P_{turb,k} \quad (5)$$

- (d) Sum powers for all  $m$  wind speed bins for that wind direction, multiplied by the probability that each speed will occur

$$P_{dir} = \sum_{j=1}^m w_j P_{speed,j} \quad (6)$$

3. Compute AEP using farm power for all  $n$  directions where  $P$  is the wind farm power for direction  $i$  and  $f$  is the corresponding frequency for direction  $i$ . The factor of 8760 is to multiply by hours in a year:

$$AEP = \left( \sum_{i=1}^n f_i P_{dir,i} \right) 8760 \frac{\text{hrs}}{\text{yr}} \quad (7)$$

## References

- [1] Thomas, J. J. and Ning, A., "A method for reducing multi-modality in the wind farm layout optimization problem," *Journal of Physics: Conference Series*, The Science of Making Torque from Wind, Milano, Italy, June 2018.
- [2] Niyayifar, A. and Porté-Agel, F., "Analytical Modeling of Wind Farms: A New Approach for Power Prediction," *Energies*, September 2016.
- [3] Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., "IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines," Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.

```

1  """IEA Task 37 Case Study 3 AEP Calculation Code
2
3  Written by Nicholas F. Baker, PJ Stanley, Jared Thomas (BYU FLOW lab) and
4  Erik Quaeghebeur (TU Delft)
5  Released 22 Aug 2018 with case studies 1 & 2
6  Modified 15 Apr 2019 for case studies 3 and 4
7  """
8  from __future__ import print_function    # For Python 3 compatibility
9  import numpy as np
10 import sys
11 import yaml                               # For reading .yaml files
12 from math import radians as DegToRad     # For converting degrees to radians
13 from math import log as ln              # For natural logarithm
14
15 # Structured datatype for holding coordinate pair
16 coordinate = np.dtype([('x', 'f8'), ('y', 'f8')])
17
18
19 def WindFrame(turb_coords, wind_dir_deg):
20     """Convert map coordinates to downwind/crosswind coordinates."""
21
22     # Convert from meteorological polar system (CW, 0 deg.=N)
23     # to standard polar system (CCW, 0 deg.=W)
24     # Shift so North comes "along" x-axis, from left to right.
25     wind_dir_deg = 270. - wind_dir_deg
26     # Convert inflow wind direction from degrees to radians
27     wind_dir_rad = DegToRad(wind_dir_deg)
28
29     # Constants to use below
30     cos_dir = np.cos(-wind_dir_rad)
31     sin_dir = np.sin(-wind_dir_rad)
32     # Convert to downwind(x) & crosswind(y) coordinates
33     frame_coords = np.recarray(len(turb_coords), coordinate)
34     frame_coords.x = (turb_coords[:, 0] * cos_dir) - \
35         (turb_coords[:, 1] * sin_dir)
36     frame_coords.y = (turb_coords[:, 0] * sin_dir) + \
37         (turb_coords[:, 1] * cos_dir)
38
39     return frame_coords
40
41
42 def GaussianWake(frame_coords, turb_diam):
43     """Return each turbine's total loss due to wake from upstream turbines"""
44     # Equations and values explained in <iea37-wakemodel.pdf>
45     num_turb = len(frame_coords)
46
47     # Constant thrust coefficient
48     CT = 4.0*1./3.*(1.0-1./3.)
49     # Constant, relating to a turbulence intensity of 0.075
50     k = 0.0324555
51     # Array holding the wake deficit seen at each turbine
52     loss = np.zeros(num_turb)
53
54     for i in range(num_turb):          # Looking at each turb (Primary)
55         loss_array = np.zeros(num_turb) # Calculate the loss from all others
56         for j in range(num_turb):      # Looking at all other turbs
57             (Target) x = frame_coords.x[i] - frame_coords.x[j] # Calculate the x-
58             dist

```

```

58         y = frame_coords.y[i] - frame_coords.y[j] # And the y-offset
59         if x > 0.: # If Primary is downwind of the
Target
60             sigma = k*x + turb_diam/np.sqrt(8.) # Calculate the wake
loss
61             # Simplified Bastankhah Gaussian wake model
62             exponent = -0.5 * (y/sigma)**2
63             radical = 1. - CT/(8.*sigma**2 / turb_diam**2)
64             loss_array[j] = (1.-np.sqrt(radical)) * np.exp(exponent)
65             # Note that if the Target is upstream, loss is defaulted to zero
66             # Total wake losses from all upstream turbs, using sqrt of sum of
sqrs
67             loss[i] = np.sqrt(np.sum(loss_array**2))
68
69         return loss
70
71
72 def DirPower(frame_coords, dir_loss, wind_speed,
73             turb_ci, turb_co, rated_ws, rated_pwr):
74     """Return the power produced by each turbine."""
75     num_turb = frame_coords.shape[0]
76
77     # Effective windspeed is freestream multiplied by wake deficits
78     wind_speed_eff = wind_speed*(1.-dir_loss)
79     # By default, the turbine's power output is zero
80     turb_pwr = np.zeros(num_turb)
81
82     # Check to see if turbine produces power for experienced wind speed
83     for n in range(num_turb):
84         # If we're between the cut-in and rated wind speeds
85         if ((turb_ci <= wind_speed_eff[n])
86             and (wind_speed_eff[n] < rated_ws)):
87             # Calculate the curve's power
88             turb_pwr[n] = rated_pwr * ((wind_speed_eff[n]-turb_ci)
89                                     / (rated_ws-turb_ci))**3
90         # If we're between the rated and cut-out wind speeds
91         elif ((rated_ws <= wind_speed_eff[n])
92              and (wind_speed_eff[n] < turb_co)):
93             # Produce the rated power
94             turb_pwr[n] = rated_pwr
95
96     # Sum the power from all turbines for this direction
97     pwrDir = np.sum(turb_pwr)
98
99     return pwrDir
100
101
102 def calcAEPcs3(turb_coords, wind_freq, wind_speeds, wind_speed_probs,
wind_dir,
103               turb_diam, turb_ci, turb_co, rated_ws, rated_pwr):
104     """Calculate the wind farm AEP."""
105     num_dir_bins = wind_freq.shape[0] # Number of bins used for our
windrose
106     num_speed_bins = wind_speeds.shape[0] # Number of wind speed bins
107
108     # Power produced by the wind farm from each wind direction
109     pwr_prod_dir = np.zeros(num_dir_bins)
110     # Power produced by the wind farm at a given windspeed
111     pwr_prod_ws = np.zeros((num_dir_bins, num_speed_bins))
112

```

```

113     # For each directional bin
114     for i in range(num_dir_bins):
115         # For each wind speed bin
116         # Shift coordinate frame of reference to downwind/crosswind
117         frame_coords = WindFrame(turb_coords, wind_dir[i])
118         # Use the Simplified Bastankhah Gaussian wake model for wake deficits
119         dir_loss = GaussianWake(frame_coords, turb_diam)
120
121         for j in range(num_speed_bins):
122             # Find the farm's power for the current direction and speed,
123             # multiplied by the probability that the speed will occur
124             pwr_prod_ws[i][j] = DirPower(frame_coords, dir_loss,
wind_speeds[j],
125                                         turb_ci, turb_co, rated_ws,
126                                         rated_pwr) * wind_speed_probs[i][j]
127             pwr_prod_dir[i] = sum(pwr_prod_ws[i]) * wind_freq[i]
128
129         # Convert power to AEP
130         hrs_per_year = 365.*24.
131         AEP = hrs_per_year * pwr_prod_dir
132         AEP /= 1.E6 # Convert to MWh
133
134     return AEP
135
136
137 def getTurbLocYAML(file_name):
138     """ Retrieve turbine locations and auxiliary file names from <.yaml>
file.
139
140     Auxiliary (reference) files supply wind rose and turbine attributes.
141     """
142     # Read in the .yaml file
143     with open(file_name, 'r') as f:
144         defs = yaml.safe_load(f)['definitions']
145
146     # Rip the (x,y) coordinates (Convert from <list> to <ndarray>)
147     turb_coords = np.asarray(defs['position']['items'])
148
149     # Rip the expected AEP, used for comparison
150     # AEP = defs['plant_energy']['properties']
151     #         ['annual_energy_production']['default']
152
153     # Read the auxiliary filenames for the windrose and the turbine
attributes
154     ref_list_turbs = defs['wind_plant']['properties']['turbine']['items']
155     ref_list_wr = (defs['plant_energy']['properties']
156                  ['wind_resource']['properties']['items'])
157
158     # Iterate through all listed references until we find the one we want
159     # The one we want is the first reference not internal to the document
160     # Note: internal references use '#' as the first character
161     fname_turb = next(ref['$ref']
162                      for ref in ref_list_turbs if ref['$ref'][0] != '#')
163     fname_wr = next(ref['$ref']
164                    for ref in ref_list_wr if ref['$ref'][0] != '#')
165
166     # Return turbine (x,y) locations, and the filenames for the others .yaml's
167     return turb_coords, fname_turb, fname_wr
168
169

```

```

170 def getWindRoseYAML(file_name):
171     """Retrieve wind rose data (bins, freqs, speeds) from <.yaml> file."""
172     # Read in the .yaml file
173     with open(file_name, 'r') as f:
174         props = yaml.safe_load(f)['definitions']['wind_inflow']['properties']
175
176     # Rip wind directional bins, their frequency, and the windspeed
parameters for each bin
177     # (Convert from <list> to <ndarray>)
178     wind_dir = np.asarray(props['direction']['bins'])
179     wind_dir_freq = np.asarray(props['direction']['frequency'])
180     # (Convert from <list> to <float>)
181     wind_speeds = np.asarray(props['speed']['bins'])
182     wind_speed_probs = np.asarray(props['speed']['frequency'])
183     # Get default number of windspeed bins per direction
184     num_speed_bins = wind_speeds.shape[0]
185     min_speed = props['speed']['minimum']
186     max_speed = props['speed']['maximum']
187
188     return wind_dir, wind_dir_freq, wind_speeds, wind_speed_probs,
num_speed_bins, min_speed, max_speed
189
190
191 def getTurbAtrbtYAML(file_name):
192     '''Retreive turbine attributes from the <.yaml> file'''
193     # Read in the .yaml file
194     with open(file_name, 'r') as f:
195         defs = yaml.safe_load(f)['definitions']
196         ops = defs['operating_mode']
197         turb = defs['wind_turbine']
198         rotor = defs['rotor']
199
200     # Rip the turbine attributes
201     # (Convert from <list> to <float>)
202     turb_ci = float(ops['cut_in_wind_speed']['default'])
203     turb_co = float(ops['cut_out_wind_speed']['default'])
204     rated_ws = float(ops['rated_wind_speed']['default'])
205     rated_pwr = float(turb['rated_power']['maximum'])
206     turb_diam = float(rotor['diameter']['default'])
207
208     return turb_ci, turb_co, rated_ws, rated_pwr, turb_diam
209
210
211 if __name__ == "__main__":
212     """Used for demonstration.
213
214     An example command line syntax to run this file is:
215
216         python iea37-aepcalc.py iea37-ex-opt3.yaml
217
218     For Python .yaml capability, in the terminal type "pip install pyyaml".
219     """
220     #-- Read necessary values from .yaml files --#
221     # Get turbine locations and auxiliary <.yaml> filenames
222     turb_coords, fname_turb, fname_wr = getTurbLocYAML(sys.argv[1])
223     # Get the array wind sampling bins, frequency at each bin, and wind speed
224     wind_dir, wind_dir_freq, wind_speeds, wind_speed_probs, num_speed_bins,
min_speed, max_speed = getWindRoseYAML(
225         fname_wr)
226     # Pull the needed turbine attributes from file

```

```
227     turb_ci, turb_co, rated_ws, rated_pwr, turb_diam = getTurbAtrbtYAML(  
228         fname_turb)  
229  
230     #-- Calculate the AEP from ripped values --#  
231     AEP = calcAEPcs3(turb_coords, wind_dir_freq, wind_speeds,  
wind_speed_probs, wind_dir,  
232         turb_diam, turb_ci, turb_co, rated_ws, rated_pwr)  
233     # Print AEP summed for all directions  
234     print(np.around(AEP, decimals=5))  
235     print(np.around(np.sum(AEP), decimals=5))  
236
```

```

1 title: IEA Wind Task 37 Wind Plant Ontology version for case studies 3 and 4
2 description: Turbine definition for 10MW Offshore Reference Turbine
3
4 definitions:
5 # turbine definition
6 wind_turbine:
7   description: Description of IEA37 10MW Onshore Reference Turbine
8   id: 10MW
9
10  rated_power:
11    units: W
12    minimum: 0.0
13    maximum: 10000000.0
14
15 # rotor definition
16 rotor:
17   diameter:
18     units: m
19     default: 198.0
20
21   radius:
22     units: m
23     default: 99.0
24
25 # hub definition
26 hub:
27   height:
28     units: m
29     default: 119.0
30
31 # operating mode can vary and the power/thrust curves will change as a result
32 operating_mode:
33   id: normal
34
35   cut_in_wind_speed:
36     units: m/s
37     default: 4.0
38
39   cut_out_wind_speed:
40     units: m/s
41     default: 25.0
42
43   rated_wind_speed:
44     units: m/s
45     default: 11.0

```

```
1 title: IEA Wind Task 37 case study 3 wind farm site
2 description:
3   - Adapted from Borssele IIIa
4   - Created by Sebastian Sanchez and Erik Quaeghebeur
5   - Modified by N.F. Baker for IEA37 case study 3
6
7 location:
8   reference_system: ETRS89
9   utm_zone: 31U
10  utm: [484178.50, 5716513.50]
11
12 boundaries:
13   IIIa:
14     - [10363.8, 6490.3]
15     - [ 9449.7, 1602.2]
16     - [ 9387.0, 1056.6]
17     - [ 9365.1, 625.5]
18     - [ 9360.8, 360.2]
19     - [ 9361.5, 126.9]
20     - [ 9361.3, 137.1]
21     - [ 7997.6, 1457.9]
22     - [ 6098.3, 3297.5]
23     - [ 8450.3, 6455.3]
24     - [ 8505.4, 6422.3]
25     - [ 9133.0, 6127.4]
26     - [ 9332.8, 6072.6]
27     - [ 9544.2, 6087.1]
28     - [ 9739.0, 6171.2]
29     - [ 9894.9, 6316.9]
30     - [10071.8, 6552.5]
31     - [10106.9, 6611.1]
```



```

1 title: IEA Wind Task 37 case study 4 wind farm site
2 description:
3   - Adapted from Borssele III and IV
4   - Created by Sebastian Sanchez and Erik Quaeghebeur
5   - Modified by N.F. Baker for IEA37 case study 4
6
7 location:
8   reference_system: ETRS89
9   utm_zone: 31U
10  utm: [484178.50, 5716513.50]
11
12 boundaries:
13   IIIa:
14     - [10363.8, 6490.3]
15     - [ 9449.7, 1602.2]
16     - [ 9387.0, 1056.6]
17     - [ 9365.1, 625.5]
18     - [ 9360.8, 360.2]
19     - [ 9361.5, 126.9]
20     - [ 9361.3, 137.1]
21     - [ 7997.6, 1457.9]
22     - [ 6098.3, 3297.5]
23     - [ 8450.3, 6455.3]
24     - [ 8505.4, 6422.3]
25     - [ 9133.0, 6127.4]
26     - [ 9332.8, 6072.6]
27     - [ 9544.2, 6087.1]
28     - [ 9739.0, 6171.2]
29     - [ 9894.9, 6316.9]
30     - [10071.8, 6552.5]
31     - [10106.9, 6611.1]
32   IIIb:
33     - [ 5588.4, 3791.3]
34     - [ 4670.7, 4680.2]
35     - [ 7274.9, 7940.8]
36     - [ 7369.9, 7896.2]
37     - [ 7455.1, 7784.3]
38     - [ 7606.5, 7713.0]
39     - [ 7638.9, 7708.4]
40     - [ 8297.1, 7398.9]
41   IVa:
42     - [ 3267.1, 10100.6]
43     - [ 4164.1, 9586.6]
44     - [ 5749.8, 9068.6]
45     - [ 6054.7, 8925.3]
46     - [ 1468.5, 7781.7]
47     - [ 107.4, 9100.0]
48   IVb:
49     - [ 6764.9, 8399.7]
50     - [ 4176.8, 5158.6]
51     - [ 2047.8, 7220.7]
52   IVc:
53     - [ 8953.7, 11901.5]
54     - [ 7048.3, 9531.5]
55     - [ 6127.7, 9962.7]
56     - [ 4578.1, 10464.9]
57     - [ 4524.1, 10498.7]

```

```

1 title: IEA Wind Task 37 case study 3, optimization log
2 description: Example target-function convergence log for the 25 turbine wind
  plant model for IEA Task 37 case study 3
3
4 hardware_summary:
5   processor:
6     manufacturer: Apple
7     model: Intel Core i7
8     speed:
9       default: 3.1
10      units: Ghz
11     num_cores: 1
12     default: 1 # This is the number of processors used. If more than one type
  used, describe fastest one.
13   RAM:
14     type: DDR3
15     size:
16       default: 16
17       units: GB
18     speed:
19       default: 1867
20       units: MHz
21
22 optimization_summary:
23   gradient_based: true # three options are: 'true', 'false', 'hybrid'
24   algorithm_name: fmincon
25   program_language: MATLAB
26   total_optimizations: 2 # This matches the number of "_log_" entries, and
  accounts for restarts or different starting layouts.
27   total_wall_time: # From start of first optimization to convergence of
  final optimization
28     default: 300
29     units: s
30
31   optimization_log_1:
32     function_calls: 26 # Note yours could be orders of magnitude more. 26 is
  used only as a starting example
33     annual_energy_production:
34       - [945163.30426]
35       - [922470.13486]
36       - [929399.58022]
37       - [950292.75484]
38       - [931307.68837]
39       - [951968.34221]
40       - [936124.6179]
41       - [944744.39259]
42       - [941138.84678]
43       - [941897.0321]
44       - [945942.94077]
45       - [941481.96524]
46       - [949603.22752]
47       - [942175.6923]
48       - [946792.22672]
49       - [943535.91008]
50       - [945067.08286]
51       - [944945.99559]
52       - [944475.40174]
53       - [946817.8821]
54       - [944232.64669]
55       - [947260.05278]

```

```
56     - [944530.87478]
57     - [945995.52061]
58     - [945203.78499]
59     - [945769.78351]
60     units: MWh
61
62 optimization_log_2:
63     function_calls: 30
64     annual_energy_production:
65         - [1017926.05459]
66         - [1020231.4478]
67         - [1017380.03366]
68         - [1020941.4921]
69         - [1017601.23056]
70         - [1019415.60343]
71         - [1018285.36295]
72         - [1018627.43565]
73         - [1019221.34578]
74         - [1018193.70791]
75         - [1020285.29836]
76         - [1018271.23152]
77         - [1019940.74798]
78         - [1018520.44561]
79         - [1019246.9473]
80         - [1018863.04768]
81         - [1018564.41906]
82         - [1019486.52658]
83         - [1018770.31388]
84         - [1020109.52476]
85         - [1018555.253]
86         - [1019594.15567]
87         - [1019009.5311]
88         - [1019241.90886]
89         - [1019263.64552]
90         - [1018618.08031]
91         - [1019937.12733]
92         - [1019045.03444]
93         - [1020030.57945]
94         - [1018773.11703]
95         - [1019438.63882]
96     units: MWh
97
```

```

1 title: IEA Wind Task 37 case study 3, baseline layout
2 description: baseline layout for the 25 turbine wind plant model for IEA Task
3 37 case study 3
4
5 definitions:
6   wind_plant:
7     type: object
8     description: specific plant design including turbine selection and
9     placement
10    properties:
11     turbine:
12       type: array
13       items:
14         - $ref: "iea37-10mw.yaml"    # Reference for the .yaml containing
15         turbine attributes.
16
17    position:
18     description: an array of x and y-coordinates [x0, y0] \n [x1, y1] \n ...
19     of wind turbine positions in cartesian coordinates
20     units: m
21     items:
22       - [10363.7833, 6490.2719]
23       - [ 9894.9437, 6316.9180]
24       - [ 8450.2895, 6455.3421]
25       - [ 9008.9311, 6043.4997]
26       - [ 9567.5726, 5631.6572]
27       - [10126.2142, 5219.8148]
28       - [ 7862.2807, 5665.8933]
29       - [ 8537.7355, 5093.7148]
30       - [ 9213.1903, 4521.5362]
31       - [ 9888.6451, 3949.3577]
32       - [ 7274.2718, 4876.4446]
33       - [ 8066.5399, 4143.9299]
34       - [ 8858.8079, 3411.4153]
35       - [ 9651.0760, 2678.9006]
36       - [ 6686.2630, 4086.9958]
37       - [ 7371.5049, 3416.8405]
38       - [ 8056.7467, 2746.6851]
39       - [ 8741.9886, 2076.5297]
40       - [ 9427.2305, 1406.3743]
41       - [ 6098.2541, 3297.5471]
42       - [ 6750.8566, 2665.4498]
43       - [ 7403.4592, 2033.3525]
44       - [ 8056.0622, 1401.2557]
45       - [ 8708.6700, 769.1637]
46       - [ 9361.2778, 137.0718]
47
48 plant_energy:
49   description: energy production data
50   properties:
51     wake_model:
52       description: wake model used to calculate AEP
53       items:
54         - $ref: "iea37-aepcalc.py"
55
56   wind_resource:
57     description: specific wind resource used to calculate AEP
58     properties:
59       items:

```

```
56         - $ref: "iea37-windrose-cs3.yaml"      # Reference for the .yaml
          containing the wind rose
57
58         annual_energy_production:
59           description: binned and total (default) annual energy production for a
          wind plant given a layout and binned wind rose
60           units: MWh
61           binned: [ 20238.63584, 15709.41125, 13286.56833, 13881.04112,
62 19232.89054,
          32035.08418, 52531.37389, 47035.14700, 46848.21422,
63 45107.13416,
          53877.69698, 68105.50430, 69587.76656, 73542.89319,
64 69615.74101,
          66752.31531, 73027.78883, 60187.14103, 59847.98304,
65 38123.29869]
          default: 938573.62950
```

```

1 title: IEA Wind Task 37 case study 4, baseline layout
2 description: baseline layout for the 25 turbine wind plant model for IEA Task
3 37 case study 4
4
5 definitions:
6   wind_plant:
7     type: object
8     description: specific plant design including turbine selection and
9     placement
10    properties:
11     turbine:
12       type: array
13       items:
14         - $ref: "iea37-10mw.yaml" # Reference for the .yaml containing
15         turbine attributes.
16
17    position:
18     description: an array of x and y-coordinates [x0, y0] \n [x1, y1] \n ...
19     of wind turbine positions in cartesian coordinates
20     units: m
21     items:
22       - [10363.7833, 6490.2719]
23       - [ 9894.9437, 6316.9180]
24       - [ 8450.2895, 6455.3421]
25       - [ 9022.1294, 6114.0806]
26       - [ 9593.9692, 5772.8191]
27       - [10165.8090, 5431.5577]
28       - [ 7979.8825, 5823.7831]
29       - [ 8642.5333, 5340.1365]
30       - [ 9305.1840, 4856.4899]
31       - [ 9967.8348, 4372.8434]
32       - [ 7509.4754, 5192.2241]
33       - [ 8074.5717, 4722.7003]
34       - [ 8639.6679, 4253.1766]
35       - [ 9204.7642, 3783.6529]
36       - [ 9769.8606, 3314.1291]
37       - [ 7039.0683, 4560.6651]
38       - [ 7672.2728, 3984.3525]
39       - [ 8305.4773, 3408.0400]
40       - [ 8938.6818, 2831.7274]
41       - [ 9571.8863, 2255.4149]
42       - [ 6568.6612, 3929.1061]
43       - [ 7277.1539, 3244.9222]
44       - [ 7985.6467, 2560.7382]
45       - [ 8694.1395, 1876.5542]
46       - [ 9402.6323, 1192.3703]
47       - [ 6098.2541, 3297.5471]
48       - [ 6750.8566, 2665.4498]
49       - [ 7403.4592, 2033.3525]
50       - [ 8056.0622, 1401.2557]
51       - [ 8708.6700, 769.1637]
52       - [ 9361.2778, 137.0718]
53       - [ 8297.1401, 7398.8969]
54       - [ 7057.9009, 7669.1317]
55       - [ 7755.3966, 6677.3823]
56       - [ 6537.0543, 7017.0094]
57       - [ 7213.6530, 5955.8679]
58       - [ 6016.2077, 6364.8870]
59       - [ 6671.9094, 5234.3534]
60       - [ 5495.3611, 5712.7647]

```

```

57     - [ 6130.1658, 4512.8390]
58     - [ 4974.5145, 5060.6424]
59     - [ 5588.4223, 3791.3245]
60     - [ 3267.0701, 10100.6305]
61     - [ 4145.9677, 9596.9976]
62     - [ 5107.1349, 9278.5659]
63     - [ 6054.6902, 8925.3415]
64     - [ 2213.8627, 9767.0876]
65     - [ 2951.0233, 9359.4397]
66     - [ 3743.0302, 8951.7917]
67     - [ 4525.9626, 8544.1437]
68     - [ 1160.6553, 9433.5446]
69     - [ 1756.0788, 8951.7917]
70     - [ 2378.9255, 8586.4789]
71     - [ 2997.2350, 8162.9460]
72     - [ 107.4479, 9100.0017]
73     - [ 561.1344, 8660.5839]
74     - [ 1014.8209, 8221.1661]
75     - [ 1468.5074, 7781.7483]
76     - [ 6764.9216, 8399.6839]
77     - [ 6034.3987, 7957.6709]
78     - [ 4878.0741, 7928.0749]
79     - [ 5729.6775, 7103.2410]
80     - [ 3934.6503, 7692.2704]
81     - [ 4573.3529, 7073.6450]
82     - [ 5212.0555, 6455.0196]
83     - [ 2991.2265, 7456.4659]
84     - [ 3842.8300, 6631.6320]
85     - [ 4694.4335, 5806.7981]
86     - [ 2047.8027, 7220.6614]
87     - [ 2757.4723, 6533.2998]
88     - [ 3467.1419, 5845.9382]
89     - [ 4176.8115, 5158.5767]
90     - [ 8953.7052, 11901.4945]
91     - [ 7846.3157, 11550.7937]
92     - [ 6738.9262, 11200.0929]
93     - [ 5631.5367, 10849.3921]
94     - [ 4524.1472, 10498.6913]
95     - [ 8001.0152, 10716.4728]
96     - [ 6893.6257, 10365.7720]
97     - [ 5806.1713, 10066.8889]
98     - [ 7048.3252, 9531.4511]
99
100    plant_energy:
101      description: energy production data
102      properties:
103        wake_model:
104          description: wake model used to calculate AEP
105          items:
106            - $ref: "iea37-aepcalc.py"
107
108        wind_resource:
109          description: specific wind resource used to calculate AEP
110          properties:
111            items:
112              - $ref: "iea37-windrose-cs3.yaml"          # Reference for the .yaml
113                containing the wind rose
114          annual_energy_production:

```

```
115     description: binned and total (default) annual energy production for
a wind plant given a layout and binned wind rose
116     units: MWh
117     binned: [ 64746.85731, 48565.54757, 37944.19712, 44280.63073,
118 55063.19289,
100113.04060, 150211.13657, 148769.25473, 136997.00284,
119 134156.63805,
173899.67085, 212214.18770, 207993.76767, 235385.31651,
120 209062.06090,
209913.66060, 213333.97207, 191039.80140, 175738.03308,
121 111754.53649]
default: 2861182.50569
```



```

1 title: IEA Wind Task 37 Wind Plant Ontology version 0.1
2 description: Wind resource conditions for a IEA37 WFL0 case studies 3 and 4.
3           Template located in branch of
<https://github.com/IEAWindTask37/ontology/tree/master/plant>
4
5 definitions:
6   wind_inflow:
7     description: inflow for current wind conditions
8     properties:
9       direction:
10        description: The wind direction frequencies. Binned in degrees, with
North as the 0.
11        units: deg
12        bins: [ 0.0, 18.0, 36.0, 54.0, 72.0,
13              90.0, 108.0, 126.0, 144.0, 162.0,
14              180.0, 198.0, 216.0, 234.0, 252.0,
15              270.0, 288.0, 306.0, 324.0, 342.0]
16        frequency: [0.0312, 0.0260, 0.0255, 0.0253, 0.0297,
17                  0.0397, 0.0506, 0.0510, 0.0415, 0.0414,
18                  0.0522, 0.0634, 0.0706, 0.0723, 0.0697,
19                  0.0668, 0.0676, 0.0677, 0.0613, 0.0464]
20        minimum: 0.0
21        maximum: 360.0
22
23     speed:
24     description: wind speed frequency samples for each binned direction
and wind speed
25     units: m/s
26     bins: [ 0.90, 1.98, 3.18, 4.40, 5.64,
27            6.87, 8.11, 9.35, 10.59, 11.83,
28            13.07, 14.31, 15.56, 16.80, 18.04,
29            19.28, 20.52, 21.77, 23.01, 24.25]
30     frequency:
31     - [0.0156401750, 0.0497090909, 0.0811024638, 0.1050883329,
0.1190301631, 0.1222668202, 0.1159445367, 0.1025108097, 0.0850102571,
0.0663764744, 0.0489239316, 0.0340999223, 0.0225045684, 0.0140748572,
0.0083476946, 0.0046969535, 0.0025085092, 0.0012722583, 0.0006121243,
0.0002800569]
32     - [0.0174786954, 0.0548443199, 0.0883795728, 0.1128729487,
0.1256551886, 0.1264602666, 0.1171083595, 0.1007717749, 0.0810568625,
0.0611779343, 0.0434372134, 0.0290638192, 0.0183497799, 0.0109410613,
0.0061655981, 0.0032843186, 0.0016551606, 0.0007890765, 0.0003560345,
0.0001520147]
33     - [0.0163365064, 0.0541606790, 0.0898797863, 0.1167076179,
0.1309060581, 0.1316880823, 0.1209647499, 0.1024541761, 0.0804744947,
0.0588288237, 0.0401142435, 0.0255527921, 0.0152204718, 0.0084822630,
0.0044241371, 0.0021610670, 0.0009880306, 0.0004230131, 0.0001690052,
0.0000640020]
34     - [0.0131561184, 0.0483094348, 0.0851957668, 0.1153680383,
0.1333342000, 0.1368182314, 0.1269601426, 0.1075589680, 0.0836297527,
0.0598535387, 0.0394933554, 0.0240452164, 0.0135111216, 0.0070060631,
0.0033520302, 0.0014790133, 0.0006010054, 0.0002250020, 0.0000780007,
0.0000250002]
35     - [0.0096451543, 0.0385656170, 0.0720491528, 0.1024286389,
0.1239279828, 0.1330241284, 0.1291130658, 0.1144128306, 0.0930354886,
0.0696031136, 0.0479667675, 0.0304604874, 0.0178212851, 0.0096011536,
0.0047590761, 0.0021690347, 0.0009070145, 0.0003480056, 0.0001230020,
0.0000390006]
36     - [0.0059162662, 0.0266481992, 0.0539554280, 0.0821266957,
0.1060917741, 0.1216774755, 0.1264536904, 0.1202924132, 0.1052787375,

```

0.0849718237, 0.0632988484, 0.0435159582, 0.0275892415, 0.0161127251,  
0.0086563895, 0.0042721922, 0.0019330870, 0.0008010360, 0.0003030136,  
0.0001050047]

37 - [0.0033912442, 0.0177062749, 0.0394378395, 0.0647356610,  
0.0894924435, 0.1095018841, 0.1212547303, 0.1228158427, 0.1143572337,  
0.0980800618, 0.0774955797, 0.0563590579, 0.0376647119, 0.0230836620,  
0.0129429319, 0.0066224768, 0.0030842221, 0.0013030938, 0.0004990359,  
0.0001720124]

38 - [0.0023011680, 0.0133949778, 0.0319683337, 0.0553040372,  
0.0799918394, 0.1020204475, 0.1174695753, 0.1234710134, 0.1190496906,  
0.1054576984, 0.0857852623, 0.0639746702, 0.0436311851, 0.0271319806,  
0.0153331193, 0.0078465728, 0.0036222644, 0.0015031097, 0.0005580407,  
0.0001850135]

39 - [0.0022121748, 0.0129870260, 0.0311634619, 0.0541482777,  
0.0786432128, 0.1007199569, 0.1164822021, 0.1230117179, 0.1192094175,  
0.1061763879, 0.0868758632, 0.0651921502, 0.0447545356, 0.0280232138,  
0.0159512601, 0.0082236497, 0.0038263023, 0.0016001264, 0.0005990473,  
0.0002000158]

40 - [0.0025653617, 0.0142190049, 0.0329346438, 0.0558158700,  
0.0795232128, 0.1003331470, 0.1147461792, 0.1202989622, 0.1162293883,  
0.1037036222, 0.0854610500, 0.0649841628, 0.0455144175, 0.0292971309,  
0.0172874375, 0.0093243147, 0.0045836463, 0.0020472887, 0.0008281168,  
0.0003030427]

41 - [0.0031360035, 0.0159851152, 0.0352022647, 0.0575904289,  
0.0799035691, 0.0988156210, 0.1114316581, 0.1158900848, 0.1118007762,  
0.1003191021, 0.0838078185, 0.0651768566, 0.0471470871, 0.0316821383,  
0.0197453185, 0.0113936460, 0.0060739437, 0.0029859555, 0.0013514325,  
0.0005611796]

42 - [0.0041012892, 0.0187200135, 0.0385729355, 0.0602413135,  
0.0807017228, 0.0971789375, 0.1075052192, 0.1105016223, 0.1061861613,  
0.0957167648, 0.0810740214, 0.0645757898, 0.0483707934, 0.0340633188,  
0.0225360739, 0.0139972258, 0.0081525383, 0.0044485678, 0.0022718220,  
0.0010838693]

43 - [0.0054756790, 0.0223183746, 0.0429681044, 0.0639713947,  
0.0826069320, 0.0966541969, 0.1046128113, 0.1059048592, 0.1009159518,  
0.0908560068, 0.0774577706, 0.0626132420, 0.0480261214, 0.0349634170,  
0.0241602941, 0.0158441129, 0.0098576243, 0.0058162187, 0.0032531563,  
0.0017237321]

44 - [0.0067183022, 0.0254914731, 0.0470020641, 0.0678182802,  
0.0854101120, 0.0978798020, 0.1041251678, 0.1039418048, 0.0979800004,  
0.0875553596, 0.0743522174, 0.0600949881, 0.0462746238, 0.0339672552,  
0.0237750746, 0.0158704234, 0.0101030039, 0.0061331436, 0.0035490271,  
0.0019578766]

45 - [0.0073336894, 0.0270762337, 0.0490933339, 0.0699511121,  
0.0871766148, 0.0989912795, 0.1044502138, 0.1035063232, 0.0969411732,  
0.0861455495, 0.0728148481, 0.0586384528, 0.0450362075, 0.0330111213,  
0.0231012718, 0.0154379222, 0.0098527350, 0.0060050281, 0.0034950005,  
0.0019418896]

46 - [0.0070345567, 0.0264421993, 0.0484514859, 0.0695431345,  
0.0871395441, 0.0993513421, 0.1051276529, 0.1043542724, 0.0977875508,  
0.0868400094, 0.0732627741, 0.0588089740, 0.0449612558, 0.0327574721,  
0.0227516116, 0.0150668944, 0.0095129807, 0.0057252195, 0.0032848635,  
0.0017962062]

47 - [0.0066654512, 0.0255148345, 0.0472692085, 0.0683955035,  
0.0862672071, 0.0989039169, 0.1051536540, 0.1048000465, 0.0985292733,  
0.0877217059, 0.0741353646, 0.0595633298, 0.0455362312, 0.0331449430,  
0.0229744701, 0.0151660553, 0.0095343801, 0.0057078060, 0.0032525879,  
0.0017640306]

48 - [0.0068309869, 0.0261352097, 0.0483737224, 0.0698831692,  
0.0879405691, 0.1005169558, 0.1064566396, 0.1056003878, 0.0987293423,

```
0.0873296760, 0.0732571019, 0.0583643286, 0.0442026242, 0.0318395494,  
0.0218178978, 0.0142237952, 0.0088208962, 0.0052026062, 0.0029182665,  
0.0015562753]  
49 - [0.0085671154, 0.0309959797, 0.0553338752, 0.0776406096,  
0.0951382271, 0.1059877710, 0.1094344382, 0.1058265995, 0.0964406128,  
0.0831404615, 0.0679673172, 0.0527741517, 0.0389584518, 0.0273611122,  
0.0182884589, 0.0116363811, 0.0070484996, 0.0040643244, 0.0022303731,  
0.0011652398]  
50 - [0.0119334560, 0.0399546154, 0.0677356450, 0.0909862235,  
0.1069768745, 0.1143638709, 0.1132292570, 0.1048797399, 0.0914354666,  
0.0753167464, 0.0587657923, 0.0435065370, 0.0305975533, 0.0204580678,  
0.0130120395, 0.0078762611, 0.0045374548, 0.0024883462, 0.0012997031,  
0.0006463497]  
51     minimum: 0.0  
52     maximum: 25.0  
53     units: m/s  
54  
55     turbulence_intenstiy:  
56     description: turbulence intensity  
57     default: 0.075  
58
```

```

1 title: IEA Wind Task 37 Wind Plant Ontology version 0.1
2 description: Modified wind resource conditions for a IEA37 WFL0 case study 4
  (360 bins vs previous 20).
3     Template located in branch of
  <https://github.com/IEAWindTask37/ontology/tree/master/plant>
4
5 definitions:
6   wind_inflow:
7     description: inflow for current wind conditions
8     properties:
9       direction:
10        description: The wind direction frequencies. Binned in degrees, with
  North as the 0.
11        units: deg
12        bins: [ 0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0,
13          9.0,
14            10.0, 11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0,
15          19.0,
16            20.0, 21.0, 22.0, 23.0, 24.0, 25.0, 26.0, 27.0, 28.0,
17          29.0,
18            30.0, 31.0, 32.0, 33.0, 34.0, 35.0, 36.0, 37.0, 38.0,
19          39.0,
20            40.0, 41.0, 42.0, 43.0, 44.0, 45.0, 46.0, 47.0, 48.0,
21          49.0,
22            50.0, 51.0, 52.0, 53.0, 54.0, 55.0, 56.0, 57.0, 58.0,
23          59.0,
24            60.0, 61.0, 62.0, 63.0, 64.0, 65.0, 66.0, 67.0, 68.0,
25          69.0,
26            70.0, 71.0, 72.0, 73.0, 74.0, 75.0, 76.0, 77.0, 78.0,
27          79.0,
28            80.0, 81.0, 82.0, 83.0, 84.0, 85.0, 86.0, 87.0, 88.0,
29          89.0,
30            90.0, 91.0, 92.0, 93.0, 94.0, 95.0, 96.0, 97.0, 98.0,
31          99.0,
32            100.0, 101.0, 102.0, 103.0, 104.0, 105.0, 106.0, 107.0, 108.0,
33          109.0,
34            110.0, 111.0, 112.0, 113.0, 114.0, 115.0, 116.0, 117.0, 118.0,
35          119.0,
36            120.0, 121.0, 122.0, 123.0, 124.0, 125.0, 126.0, 127.0, 128.0,
37          129.0,
38            130.0, 131.0, 132.0, 133.0, 134.0, 135.0, 136.0, 137.0, 138.0,
39          139.0,
40            140.0, 141.0, 142.0, 143.0, 144.0, 145.0, 146.0, 147.0, 148.0,
41          149.0,
42            150.0, 151.0, 152.0, 153.0, 154.0, 155.0, 156.0, 157.0, 158.0,
43          159.0,
44            160.0, 161.0, 162.0, 163.0, 164.0, 165.0, 166.0, 167.0, 168.0,
45          169.0,
46            170.0, 171.0, 172.0, 173.0, 174.0, 175.0, 176.0, 177.0, 178.0,
47          179.0,
48            180.0, 181.0, 182.0, 183.0, 184.0, 185.0, 186.0, 187.0, 188.0,
49          189.0,
50            190.0, 191.0, 192.0, 193.0, 194.0, 195.0, 196.0, 197.0, 198.0,
51          199.0,
52            200.0, 201.0, 202.0, 203.0, 204.0, 205.0, 206.0, 207.0, 208.0,
53          209.0,
54            210.0, 211.0, 212.0, 213.0, 214.0, 215.0, 216.0, 217.0, 218.0,
55          219.0,
56            220.0, 221.0, 222.0, 223.0, 224.0, 225.0, 226.0, 227.0, 228.0,
57          229.0,
58            230.0, 231.0, 232.0, 233.0, 234.0, 235.0, 236.0, 237.0, 238.0,
59          239.0,
60            240.0, 241.0, 242.0, 243.0, 244.0, 245.0, 246.0, 247.0, 248.0,
61          249.0,
62            250.0, 251.0, 252.0, 253.0, 254.0, 255.0, 256.0, 257.0, 258.0,
63          259.0,
64            260.0, 261.0, 262.0, 263.0, 264.0, 265.0, 266.0, 267.0, 268.0,
65          269.0,
66            270.0, 271.0, 272.0, 273.0, 274.0, 275.0, 276.0, 277.0, 278.0,
67          279.0,
68            280.0, 281.0, 282.0, 283.0, 284.0, 285.0, 286.0, 287.0, 288.0,
69          289.0,
70            290.0, 291.0, 292.0, 293.0, 294.0, 295.0, 296.0, 297.0, 298.0,
71          299.0,
72            300.0, 301.0, 302.0, 303.0, 304.0, 305.0, 306.0, 307.0, 308.0,
73          309.0,
74            310.0, 311.0, 312.0, 313.0, 314.0, 315.0, 316.0, 317.0, 318.0,
75          319.0,
76            320.0, 321.0, 322.0, 323.0, 324.0, 325.0, 326.0, 327.0, 328.0,
77          329.0,
78            330.0, 331.0, 332.0, 333.0, 334.0, 335.0, 336.0, 337.0, 338.0,
79          339.0,
80            340.0, 341.0, 342.0, 343.0, 344.0, 345.0, 346.0, 347.0, 348.0,
81          349.0,
82            350.0, 351.0, 352.0, 353.0, 354.0, 355.0, 356.0, 357.0, 358.0,
83          359.0,
84            360.0, 361.0, 362.0, 363.0, 364.0, 365.0, 366.0, 367.0, 368.0,
85          369.0,
86            370.0, 371.0, 372.0, 373.0, 374.0, 375.0, 376.0, 377.0, 378.0,
87          379.0,
88            380.0, 381.0, 382.0, 383.0, 384.0, 385.0, 386.0, 387.0, 388.0,
89          389.0,
90            390.0, 391.0, 392.0, 393.0, 394.0, 395.0, 396.0, 397.0, 398.0,
91          399.0,
92            400.0, 401.0, 402.0, 403.0, 404.0, 405.0, 406.0, 407.0, 408.0,
93          409.0,
94            410.0, 411.0, 412.0, 413.0, 414.0, 415.0, 416.0, 417.0, 418.0,
95          419.0,
96            420.0, 421.0, 422.0, 423.0, 424.0, 425.0, 426.0, 427.0, 428.0,
97          429.0,
98            430.0, 431.0, 432.0, 433.0, 434.0, 435.0, 436.0, 437.0, 438.0,
99          439.0,
100           440.0, 441.0, 442.0, 443.0, 444.0, 445.0, 446.0, 447.0, 448.0,
101         449.0,
102         450.0, 451.0, 452.0, 453.0, 454.0, 455.0, 456.0, 457.0, 458.0,
103         459.0,
104         460.0, 461.0, 462.0, 463.0, 464.0, 465.0, 466.0, 467.0, 468.0,
105         469.0,
106         470.0, 471.0, 472.0, 473.0, 474.0, 475.0, 476.0, 477.0, 478.0,
107         479.0,
108         480.0, 481.0, 482.0, 483.0, 484.0, 485.0, 486.0, 487.0, 488.0,
109         489.0,
110         490.0, 491.0, 492.0, 493.0, 494.0, 495.0, 496.0, 497.0, 498.0,
111         499.0,
112         500.0, 501.0, 502.0, 503.0, 504.0, 505.0, 506.0, 507.0, 508.0,
113         509.0,
114         510.0, 511.0, 512.0, 513.0, 514.0, 515.0, 516.0, 517.0, 518.0,
115         519.0,
116         520.0, 521.0, 522.0, 523.0, 524.0, 525.0, 526.0, 527.0, 528.0,
117         529.0,
118         530.0, 531.0, 532.0, 533.0, 534.0, 535.0, 536.0, 537.0, 538.0,
119         539.0,
120         540.0, 541.0, 542.0, 543.0, 544.0, 545.0, 546.0, 547.0, 548.0,
121         549.0,
122         550.0, 551.0, 552.0, 553.0, 554.0, 555.0, 556.0, 557.0, 558.0,
123         559.0,
124         560.0, 561.0, 562.0, 563.0, 564.0, 565.0, 566.0, 567.0, 568.0,
125         569.0,
126         570.0, 571.0, 572.0, 573.0, 574.0, 575.0, 576.0, 577.0, 578.0,
127         579.0,
128         580.0, 581.0, 582.0, 583.0, 584.0, 585.0, 586.0, 587.0, 588.0,
129         589.0,
130         590.0, 591.0, 592.0, 593.0, 594.0, 595.0, 596.0, 597.0, 598.0,
131         599.0,
132         600.0, 601.0, 602.0, 603.0, 604.0, 605.0, 606.0, 607.0, 608.0,
133         609.0,
134         610.0, 611.0, 612.0, 613.0, 614.0, 615.0, 616.0, 617.0, 618.0,
135         619.0,
136         620.0, 621.0, 622.0, 623.0, 624.0, 625.0, 626.0, 627.0, 628.0,
137         629.0,
138         630.0, 631.0, 632.0, 633.0, 634.0, 635.0, 636.0, 637.0, 638.0,
139         639.0,
140         640.0, 641.0, 642.0, 643.0, 644.0, 645.0, 646.0, 647.0, 648.0,
141         649.0,
142         650.0, 651.0, 652.0, 653.0, 654.0, 655.0, 656.0, 657.0, 658.0,
143         659.0,
144         660.0, 661.0, 662.0, 663.0, 664.0, 665.0, 666.0, 667.0, 668.0,
145         669.0,
146         670.0, 671.0, 672.0, 673.0, 674.0, 675.0, 676.0, 677.0, 678.0,
147         679.0,
148         680.0, 681.0, 682.0, 683.0, 684.0, 685.0, 686.0, 687.0, 688.0,
149         689.0,
150         690.0, 691.0, 692.0, 693.0, 694.0, 695.0, 696.0, 697.0, 698.0,
151         699.0,
152         700.0, 701.0, 702.0, 703.0, 704.0, 705.0, 706.0, 707.0, 708.0,
153         709.0,
154         710.0, 711.0, 712.0, 713.0, 714.0, 715.0, 716.0, 717.0, 718.0,
155         719.0,
156         720.0, 721.0, 722.0, 723.0, 724.0, 725.0, 726.0, 727.0, 728.0,
157         729.0,
158         730.0, 731.0, 732.0, 733.0, 734.0, 735.0, 736.0, 737.0, 738.0,
159         739.0,
160         740.0, 741.0, 742.0, 743.0, 744.0, 745.0, 746.0, 747.0, 748.0,
161         749.0,
162         750.0, 751.0, 752.0, 753.0, 754.0, 755.0, 756.0, 757.0, 758.0,
163         759.0,
164         760.0, 761.0, 762.0, 763.0, 764.0, 765.0, 766.0, 767.0, 768.0,
165         769.0,
166         770.0, 771.0, 772.0, 773.0, 774.0, 775.0, 776.0, 777.0, 778.0,
167         779.0,
168         780.0, 781.0, 782.0, 783.0, 784.0, 785.0, 786.0, 787.0, 788.0,
169         789.0,
170         790.0, 791.0, 792.0, 793.0, 794.0, 795.0, 796.0, 797.0, 798.0,
171         799.0,
172         800.0, 801.0, 802.0, 803.0, 804.0, 805.0, 806.0, 807.0, 808.0,
173         809.0,
174         810.0, 811.0, 812.0, 813.0, 814.0, 815.0, 816.0, 817.0, 818.0,
175         819.0,
176         820.0, 821.0, 822.0, 823.0, 824.0, 825.0, 826.0, 827.0, 828.0,
177         829.0,
178         830.0, 831.0, 832.0, 833.0, 834.0, 835.0, 836.0, 837.0, 838.0,
179         839.0,
180         840.0, 841.0, 842.0, 843.0, 844.0, 845.0, 846.0, 847.0, 848.0,
181         849.0,
182         850.0, 851.0, 852.0, 853.0, 854.0, 855.0, 856.0, 857.0, 858.0,
183         859.0,
184         860.0, 861.0, 862.0, 863.0, 864.0, 865.0, 866.0, 867.0, 868.0,
185         869.0,
186         870.0, 871.0, 872.0, 873.0, 874.0, 875.0, 876.0, 877.0, 878.0,
187         879.0,
188         880.0, 881.0, 882.0, 883.0, 884.0, 885.0, 886.0, 887.0, 888.0,
189         889.0,
190         890.0, 891.0, 892.0, 893.0, 894.0, 895.0, 896.0, 897.0, 898.0,
191         899.0,
192         900.0, 901.0, 902.0, 903.0, 904.0, 905.0, 906.0, 907.0, 908.0,
193         909.0,
194         910.0, 911.0, 912.0, 913.0, 914.0, 915.0, 916.0, 917.0, 918.0,
195         919.0,
196         920.0, 921.0, 922.0, 923.0, 924.0, 925.0, 926.0, 927.0, 928.0,
197         929.0,
198         930.0, 931.0, 932.0, 933.0, 934.0, 935.0, 936.0, 937.0, 938.0,
199         939.0,
200         940.0, 941.0, 942.0, 943.0, 944.0, 945.0, 946.0, 947.0, 948.0,
201         949.0,
202         950.0, 951.0, 952.0, 953.0, 954.0, 955.0, 956.0, 957.0, 958.0,
203         959.0,
204         960.0, 961.0, 962.0, 963.0, 964.0, 965.0, 966.0, 967.0, 968.0,
205         969.0,
206         970.0, 971.0, 972.0, 973.0, 974.0, 975.0, 976.0, 977.0, 978.0,
207         979.0,
208         980.0, 981.0, 982.0, 983.0, 984.0, 985.0, 986.0, 987.0, 988.0,
209         989.0,
210         990.0, 991.0, 992.0, 993.0, 994.0, 995.0, 996.0, 997.0, 998.0,
211         999.0,
212         1000.0, 1001.0, 1002.0, 1003.0, 1004.0, 1005.0, 1006.0, 1007.0, 1008.0,
213         1009.0,
214         1010.0, 1011.0, 1012.0, 1013.0, 1014.0, 1015.0, 1016.0, 1017.0, 1018.0,
215         1019.0,
216         1020.0, 1021.0, 1022.0, 1023.0, 1024.0, 1025.0, 1026.0, 1027.0, 1028.0,
217         1029.0,
218         1030.0, 1031.0, 1032.0, 1033.0, 1034.0, 1035.0, 1036.0, 1037.0, 1038.0,
219         1039.0,
220         1040.0, 1041.0, 1042.0, 1043.0, 1044.0, 1045.0, 1046.0, 1047.0, 1048.0,
221         1049.0,
222         1050.0, 1051.0, 1052.0, 1053.0, 1054.0, 1055.0, 1056.0, 1057.0, 1058.0,
223         1059.0,
224         1060.0, 1061.0, 1062.0, 1063.0, 1064.0, 1065.0, 1066.0, 1067.0, 1068.0,
225         1069.0,
226         1070.0, 1071.0, 1072.0, 1073.0, 1074.0, 1075.0, 1076.0, 1077.0, 1078.0,
227         1079.0,
228         1080.0, 1081.0, 1082.0, 1083.0, 1084.0, 1085.0, 1086.0, 1087.0, 1088.0,
229         1089.0,
230         1090.0, 1091.0, 1092.0, 1093.0, 1094.0, 1095.0, 1096.0, 1097.0, 1098.0,
231         1099.0,
232         1100.0, 1101.0, 1102.0, 1103.0, 1104.0, 1105.0, 1106.0, 1107.0, 1108.0,
233         1109.0,
234         1110.0, 1111.0, 1112.0, 1113.0, 1114.0, 1115.0, 1116.0, 1117.0, 1118.0,
235         1119.0,
236         1120.0, 1121.0, 1122.0, 1123.0, 1124.0, 1125.0, 1126.0, 1127.0, 1128.0,
237         1129.0,
238         1130.0, 1131.0, 1132.0, 1133.0, 1134.0, 1135.0, 1136.0, 1137.0, 1138.0,
239         1139.0,
240         1140.0, 1141.0, 1142.0, 1143.0, 1144.0, 1145.0, 1146.0, 1147.0, 1148.0,
241         1149.0,
242         1150.0, 1151.0, 1152.0, 1153.0, 1154.0, 1155.0, 1156.0, 1157.0, 1158.0,
243         1159.0,
244         1160.0, 1161.0, 1162.0, 1163.0, 1164.0, 1165.0, 1166.0, 1167.0, 1168.0,
245         1169.0,
246         1170.0, 1171.0, 1172.0, 1173.0, 1174.0, 1175.0, 1176.0, 1177.0, 1178.0,
247         1179.0,
248         1180.0, 1181.0, 1182.0, 1183.0, 1184.0, 1185.0, 1186.0, 1187.0, 1188.0,
249         1189.0,
250         1190.0, 1191.0, 1192.0, 1193.0, 1194.0, 1195.0, 1196.0, 1197.0, 1198.0,
251         1199.0,
252         1200.0, 1201.0, 1202.0, 1203.0, 1204.0, 1205.0, 1206.0, 1207.0, 1208.0,
253         1209.0,
254         1210.0, 1211.0, 1212.0, 1213.0, 1214.0, 1215.0, 1216.0, 1217.0, 1218.0,
255         1219.0,
256         1220.0, 1221.0, 1222.0, 1223.0, 1224.0, 1225.0, 1226.0, 1227.0, 1228.0,
257         1229.0,
258         1230.0, 1231.0, 1232.0, 1233.0, 1234.0, 1235.0, 1236.0, 1237.0, 1238.0,
259         1239.0,
260         1240.0, 1241.0, 1242.0, 1243.0, 1244.0, 1245.0, 1246.0, 1247.0, 1248.0,
261         1249.0,
262         1250.0, 1251.0, 1252.0, 1253.0, 1254.0, 1255.0, 1256.0, 1257.0, 1258.0,
263         1259.0,
264         1260.0, 1261.0, 1262.0, 1263.0, 1264.0, 1265.0, 1266.0, 1267.0, 1268.0,
265         1269.0,
266         1270.0, 1271.0, 1272.0, 1273.0, 1274.0, 1275.0, 1276.0, 1277.0, 1278.0,
267         1279.0,
268         1280.0, 1281.0, 1282.0, 1283.0, 1284.0, 1285.0, 1286.0, 1287.0, 1288.0,
269         1289.0,
270         1290.0, 1291.0, 1292.0, 1293.0, 1294.0, 1295.0, 1296.0, 1297.0, 1298.0,
271         1299.0,
272         1300.0, 1301.0, 1302.0, 1303.0, 1304.0, 1305.0, 1306.0, 1307.0, 1308.0,
273         1309.0,
274         1310.0, 1311.0, 1312.0, 1313.0, 1314.0, 1315.0, 1316.0, 1317.0, 1318.0,
275         1319.0,
276         1320.0, 1321.0, 1322.0, 1323.0, 1324.0, 1325.0, 1326.0, 1327.0, 1328.0,
277         1329.0,
278         1330.0, 1331.0, 1332.0, 1333.0, 1334.0, 1335.0, 1336.0, 1337.0, 1338.0,
279         1339.0,
280         1340.0, 1341.0, 1342.0, 1343.0, 1344.0, 1345.0, 1346.0, 1347.0, 1348.0,
281         1349.0,
282         1350.0, 1351.0, 1352.0, 1353.0, 1354.0, 1355.0, 1356.0, 1357.0, 1358.0,
283         1359.0,
284         1360.0, 1361.0, 1362.0, 1363.0, 1364.0, 1365.0, 1366.0, 1367.0, 1368.0,
285         1369.0,
286         1370.0, 1371.0, 1372.0, 1373.0, 1374.0, 1375.0, 1376.0, 1377.0, 1378.0,
287         1379.0,
288         1380.0, 1381.0, 1382.0, 1383.0, 1384.0, 1385.0, 1386.0, 1387.0, 1388.0,
289         1389.0,
290         1390.0, 1391.0, 1392.0, 1393.0, 1394.0, 1395.0, 1396.0, 1397.0, 1398.0,
291         1399.0,
292         1400.0, 1401.0, 1402.0, 1403.0, 1404.0, 1405.0, 1406.0, 1407.0, 1408.0,
293         1409.0,
294         1410.0, 1411.0, 1412.0, 1413.0, 1414.0, 1415.0, 1416.0, 1417.0, 1418.0,
295         1419.0,
296         1420.0, 1421.0, 1422.0, 1423.0, 1424.0, 1425.0, 1426.0, 1427.0, 1428.0,
297         1429.0,
298         1430.0, 1431.0, 1432.0, 1433.0, 1434.0, 1435.0, 1436.0, 1437.0, 1438.0,
299         1439.0,
300         1440.0, 1441.0, 1442.0, 1443.0, 1444.0, 1445.0, 1446.0, 1447.0, 1448.0,
301         1449.0,
302         1450.0, 1451.0, 1452.0, 1453.0, 1454.0, 1455.0, 1456.0, 1457.0, 1458.0,
303         1459.0,
304         1460.0, 1461.0, 1462.0, 1463.0, 1464.0, 1465.0, 1466.0, 1467.0, 1468.0,
305         1469.0,
306         1470.0, 1471.0, 1472.0, 1473.0, 1474.0, 1475.0, 1476.0, 1477.0, 1478.0,
307         1479.0,
308         1480.0, 1481.0, 1482.0, 1483.0, 1484.0, 148
```

35 230.0, 231.0, 232.0, 233.0, 234.0, 235.0, 236.0, 237.0, 238.0,  
 239.0,  
 36 240.0, 241.0, 242.0, 243.0, 244.0, 245.0, 246.0, 247.0, 248.0,  
 249.0,  
 37 250.0, 251.0, 252.0, 253.0, 254.0, 255.0, 256.0, 257.0, 258.0,  
 259.0,  
 38 260.0, 261.0, 262.0, 263.0, 264.0, 265.0, 266.0, 267.0, 268.0,  
 269.0,  
 39 270.0, 271.0, 272.0, 273.0, 274.0, 275.0, 276.0, 277.0, 278.0,  
 279.0,  
 40 280.0, 281.0, 282.0, 283.0, 284.0, 285.0, 286.0, 287.0, 288.0,  
 289.0,  
 41 290.0, 291.0, 292.0, 293.0, 294.0, 295.0, 296.0, 297.0, 298.0,  
 299.0,  
 42 300.0, 301.0, 302.0, 303.0, 304.0, 305.0, 306.0, 307.0, 308.0,  
 309.0,  
 43 310.0, 311.0, 312.0, 313.0, 314.0, 315.0, 316.0, 317.0, 318.0,  
 319.0,  
 44 320.0, 321.0, 322.0, 323.0, 324.0, 325.0, 326.0, 327.0, 328.0,  
 329.0,  
 45 330.0, 331.0, 332.0, 333.0, 334.0, 335.0, 336.0, 337.0, 338.0,  
 339.0,  
 46 340.0, 341.0, 342.0, 343.0, 344.0, 345.0, 346.0, 347.0, 348.0,  
 349.0,  
 47 350.0, 351.0, 352.0, 353.0, 354.0, 355.0, 356.0, 357.0, 358.0,  
 359.0]

48 frequency: [0.0017335066840017336, 0.0017174556961869030,  
 0.0017014047083720720, 0.0016853537205572410, 0.0016693027327424103,  
 49 0.0016532517449275794, 0.0016372007571127485,  
 0.0016211497692979176, 0.0016050987814830867, 0.0015890477936682560,  
 50 0.0015729968058534251, 0.0015569458180385940,  
 0.0015408948302237633, 0.0015248438424089324, 0.0015087928545941016,  
 51 0.0014927418667792707, 0.0014766908789644398,  
 0.0014606398911496090, 0.0014445889033347782, 0.0014430455391218136,  
 52 0.0014415021749088490, 0.0014399588106958844,  
 0.0014384154464829200, 0.0014368720822699555, 0.0014353287180569910,  
 53 0.0014337853538440265, 0.0014322419896310620,  
 0.0014306986254180975, 0.0014291552612051330, 0.0014276118969921685,  
 54 0.0014260685327792040, 0.0014245251685662396,  
 0.0014229818043532750, 0.0014214384401403106, 0.0014198950759273460,  
 55 0.0014183517117143816, 0.0014168083475014170,  
 0.0014161910018162312, 0.0014155736561310454, 0.0014149563104458595,  
 56 0.0014143389647606737, 0.0014137216190754879,  
 0.0014131042733903022, 0.0014124869277051164, 0.0014118695820199306,  
 57 0.0014112522363347447, 0.0014106348906495590,  
 0.0014100175449643730, 0.0014094001992791874, 0.0014087828535940016,  
 58 0.0014081655079088158, 0.0014075481622236300,  
 0.0014069308165384440, 0.0014063134708532583, 0.0014056961251680726,  
 59 0.0014192777302421602, 0.0014328593353162480,  
 0.0014464409403903355, 0.0014600225454644233, 0.0014736041505385108,  
 60 0.0014871857556125986, 0.0015007673606866860,  
 0.0015143489657607736, 0.0015279305708348614, 0.0015415121759089492,  
 61 0.0015550937809830370, 0.0015686753860571245,  
 0.0015822569911312122, 0.0015958385962052998, 0.0016094202012793875,  
 62 0.0016230018063534750, 0.0016365834114275626,  
 0.0016501650165016504, 0.0016810323007609405, 0.0017118995850202308,  
 63 0.0017427668692795210, 0.0017736341535388108,  
 0.0018045014377981012, 0.0018353687220573910, 0.0018662360063166814,  
 64 0.0018971032905759713, 0.0019279705748352616,  
 0.0019588378590945517, 0.0019897051433538420, 0.0020205724276131320,

65 0.0020514397118724220, 0.0020823069961317120,  
0.0021131742803910027, 0.0021440415646502924, 0.0021749088489095825,  
66 0.0022057761331688726, 0.0022394214730114990,  
0.0022730668128541254, 0.0023067121526967513, 0.0023403574925393777,  
67 0.0023740028323820037, 0.0024076481722246305,  
0.0024412935120672564, 0.0024749388519098830, 0.0025085841917525087,  
68 0.0025422295315951350, 0.0025758748714377615,  
0.0026095202112803874, 0.0026431655511230143, 0.0026768108909656400,  
69 0.0027104562308082660, 0.0027441015706508925,  
0.0027777469104935190, 0.0028113922503361453, 0.0028126269417065165,  
70 0.0028138616330768880, 0.0028150963244472600,  
0.0028163310158176315, 0.0028175657071880030, 0.0028188003985583750,  
71 0.0028200350899287460, 0.0028212697812991180,  
0.0028225044726694895, 0.0028237391640398610, 0.0028249738554102330,  
72 0.0028262085467806045, 0.0028274432381509757,  
0.0028286779295213474, 0.0028299126208917190, 0.0028311473122620908,  
73 0.0028323820036324624, 0.0028336166950028340,  
0.0028042927749565080, 0.0027749688549101826, 0.0027456449348638570,  
74 0.0027163210148175310, 0.0026869970947712060,  
0.0026576731747248800, 0.0026283492546785546, 0.0025990253346322290,  
75 0.0025697014145859036, 0.0025403774945395776,  
0.0025110535744932525, 0.0024817296544469266, 0.0024524057344006010,  
76 0.0024230818143542755, 0.0023937578943079496,  
0.0023644339742616240, 0.0023351100542152980, 0.0023057861341689730,  
77 0.0023054774613263800, 0.0023051687884837867,  
0.0023048601156411943, 0.0023045514427986013, 0.0023042427699560084,  
78 0.0023039340971134155, 0.0023036254242708226,  
0.0023033167514282297, 0.0023030080785856367, 0.0023026994057430440,  
79 0.0023023907329004510, 0.0023020820600578580,  
0.0023017733872152650, 0.0023014647143726720, 0.0023011560415300792,  
80 0.0023008473686874868, 0.0023005386958448934,  
0.0023002300230023005, 0.0023335666900023340, 0.0023669033570023674,  
81 0.0024002400240024010, 0.0024335766910024340,  
0.0024669133580024673, 0.0025002500250025004, 0.0025335866920025343,  
82 0.0025669233590025677, 0.0026002600260026008,  
0.0026335966930026340, 0.0026669333600026672, 0.0027002700270027010,  
83 0.0027336066940027340, 0.0027669433610027676,  
0.0028002800280028006, 0.0028336166950028340, 0.0028669533620028680,  
84 0.0029002900290029010, 0.0029348613873733060,  
0.0029694327457437110, 0.0030040041041141156, 0.0030385754624845207,  
85 0.0030731468208549260, 0.0031077181792253305,  
0.0031422895375957352, 0.0031768608959661404, 0.0032114322543365455,  
86 0.0032460036127069500, 0.0032805749710773550,  
0.0033151463294477600, 0.0033497176878181650, 0.0033842890461885700,  
87 0.003418860405589746, 0.0034534317629293797,  
0.0034880031212997850, 0.0035225744796701895, 0.0035447989243368784,  
88 0.0035670233690035670, 0.0035892478136702560,  
0.0036114722583369457, 0.0036336967030036340, 0.0036559211476703230,  
89 0.0036781455923370118, 0.0037003700370037006,  
0.0037225944816703903, 0.0037448189263370787, 0.0037670433710037675,  
90 0.0037892678156704563, 0.0038114922603371450,  
0.0038337167050038340, 0.0038559411496705230, 0.0038781655943372120,  
91 0.0039003900390039010, 0.0039226144836705890,  
0.0039278619219946685, 0.0039331093603187485, 0.0039383567986428280,  
92 0.0039436042369669070, 0.0039488516752909860,  
0.0039540991136150660, 0.0039593465519391450, 0.0039645939902632245,  
93 0.0039698414285873040, 0.0039750888669113830,  
0.0039803363052354630, 0.0039855837435595410, 0.0039908311818836210,  
94 0.0039960786202077000, 0.0040013260585317805,  
0.0040065734968558590, 0.0040118209351799390, 0.0040170683735040180,

```

95      0.0040090428795966020, 0.0040010173856891870,
0.0039929918917817710, 0.0039849663978743560, 0.0039769409039669400,
96      0.0039689154100595250, 0.0039608899161521095,
0.0039528644222446944, 0.0039448389283372794, 0.0039368134344298636,
97      0.0039287879405224480, 0.0039207624466150330,
0.0039127369527076170, 0.0039047114588002013, 0.0038966859648927860,
98      0.0038886604709853710, 0.0038806349770779554,
0.0038726094831705400, 0.0038636579707353458, 0.0038547064583001516,
99      0.0038457549458649574, 0.0038368034334297630,
0.0038278519209945690, 0.0038189004085593748, 0.0038099488961241806,
100     0.0038009973836889864, 0.0037920458712537930,
0.0037830943588185990, 0.0037741428463834046, 0.0037651913339482104,
101     0.0037562398215130162, 0.0037472883090778220,
0.0037383367966426280, 0.0037293852842074336, 0.0037204337717722394,
102     0.0037114822593370452, 0.0037139516420777886,
0.0037164210248185320, 0.0037188904075592750, 0.0037213597903000180,
103     0.0037238291730407610, 0.0037262985557815044,
0.0037287679385222480, 0.0037312373212629907, 0.0037337067040037340,
104     0.0037361760867444774, 0.0037386454694852203,
0.0037411148522259637, 0.0037435842349667070, 0.0037460536177074500,
105     0.0037485230004481933, 0.0037509923831889366,
0.0037534617659296795, 0.0037559311486704230, 0.0037562398215130154,
106     0.0037565484943556087, 0.0037568571671982016,
0.0037571658400407950, 0.0037574745128833875, 0.0037577831857259800,
107     0.0037580918585685733, 0.0037584005314111662,
0.0037587092042537587, 0.0037590178770963520, 0.0037593265499389445,
108     0.0037596352227815380, 0.0037599438956241310,
0.0037602525684667233, 0.0037605612413093167, 0.0037608699141519090,
109     0.0037611785869945025, 0.0037614872598370954,
0.0037417321979111495, 0.0037219771359852040, 0.0037022220740592585,
110     0.0036824670121333120, 0.0036627119502073667,
0.0036429568882814212, 0.0036232018263554753, 0.0036034467644295300,
111     0.0035836917025035843, 0.0035639366405776390,
0.0035441815786516925, 0.0035244265167257470, 0.0035046714547998016,
112     0.0034849163928738556, 0.0034651613309479100,
0.0034454062690219647, 0.0034256512070960188, 0.0034058961451700733,
113     0.0033599038916237306, 0.0033139116380773883,
0.0032679193845310465, 0.0032219271309847040, 0.0031759348774383616,
114     0.0031299426238920194, 0.0030839503703456770,
0.0030379581167993350, 0.0029919658632529920, 0.0029459736097066500,
115     0.0028999813561603080, 0.0028539891026139654,
0.0028079968490676227, 0.0027620045955212810, 0.0027160123419749387,
116     0.0026700200884285964, 0.0026240278348822537,
0.0025780355813359115, 0.0025311173092617905, 0.0024841990371876700,
117     0.0024372807651135485, 0.0023903624930394275,
0.0023434442209653065, 0.0022965259488911860, 0.0022496076768170645,
118     0.0022026894047429434, 0.0021557711326688230,
0.0021088528605947020, 0.0020619345885205804, 0.0020150163164464600,
119     0.0019680980443723390, 0.0019211797722982176,
0.0018742615002240966, 0.0018273432281499756, 0.0017804249560758548]
120     minimum: 0.0
121     maximum: 360.0
122
123     speed:
124     desctiption: wind speed frequency samples for each binned direction
and wind speed
125     units: m/s
126     bins: [ 0.90,  1.98,  3.18,  4.40,
127            5.64,  6.87,  8.11,  9.35,
128            10.59, 11.83, 13.07, 14.31,

```

129 15.56, 16.80, 18.04, 19.28,  
130 20.52, 21.77, 23.01, 24.25]  
131 frequency:  
132 - [0.0156401750, 0.0497090909, 0.0811024638, 0.1050883329,  
0.1190301631, 0.1222668202, 0.1159445367, 0.1025108097, 0.0850102571,  
0.0663764744, 0.0489239316, 0.0340999223, 0.0225045684, 0.0140748572,  
0.0083476946, 0.0046969535, 0.0025085092, 0.0012722583, 0.0006121243,  
0.0002800569]  
133 - [0.0157423150, 0.0499943814, 0.0815067476, 0.1055208116,  
0.1193982201, 0.1224997894, 0.1160091935, 0.1024141967, 0.0847906241,  
0.0660876666, 0.0486191139, 0.0338201388, 0.0222737468, 0.0139007574,  
0.0082264670, 0.0046184738, 0.0024611009, 0.0012454149, 0.0005978971,  
0.0002729434]  
134 - [0.0158444550, 0.0502796719, 0.0819110315, 0.1059532902,  
0.1197662770, 0.1227327587, 0.1160738503, 0.1023175836, 0.0845709910,  
0.0657988588, 0.0483142962, 0.0335403553, 0.0220429252, 0.0137266577,  
0.0081052394, 0.0045399941, 0.0024136927, 0.0012185714, 0.0005836699,  
0.0002658300]  
135 - [0.0159465951, 0.0505649624, 0.0823153153, 0.1063857689,  
0.1201343340, 0.1229657279, 0.1161385072, 0.1022209706, 0.0843513580,  
0.0655100510, 0.0480094786, 0.0332605718, 0.0218121036, 0.0135525579,  
0.0079840118, 0.0044615144, 0.0023662844, 0.0011917280, 0.0005694427,  
0.0002587165]  
136 - [0.0160487351, 0.0508502529, 0.0827195991, 0.1068182475,  
0.1205023910, 0.1231986972, 0.1162031640, 0.1021243575, 0.0841317250,  
0.0652212433, 0.0477046609, 0.0329807883, 0.0215812821, 0.0133784581,  
0.0078627843, 0.0043830346, 0.0023188762, 0.0011648846, 0.0005552155,  
0.0002516031]  
137 - [0.0161508751, 0.0511355434, 0.0831238830, 0.1072507262,  
0.1208704480, 0.1234316664, 0.1162678208, 0.1020277445, 0.0839120919,  
0.0649324355, 0.0473998432, 0.0327010048, 0.0213504605, 0.0132043583,  
0.0077415567, 0.0043045549, 0.0022714679, 0.0011380411, 0.0005409882,  
0.0002444896]  
138 - [0.0162530151, 0.0514208339, 0.0835281668, 0.1076832048,  
0.1212385049, 0.1236646357, 0.1163324776, 0.1019311314, 0.0836924589,  
0.0646436277, 0.0470950255, 0.0324212213, 0.0211196389, 0.0130302586,  
0.0076203291, 0.0042260752, 0.0022240597, 0.0011111977, 0.0005267610,  
0.0002373762]  
139 - [0.0163551552, 0.0517061244, 0.0839324506, 0.1081156835,  
0.1216065619, 0.1238976049, 0.1163971345, 0.1018345184, 0.0834728259,  
0.0643548199, 0.0467902079, 0.0321414378, 0.0208888173, 0.0128561588,  
0.0074991015, 0.0041475955, 0.0021766514, 0.0010843543, 0.0005125338,  
0.0002302627]  
140 - [0.0164572952, 0.0519914149, 0.0843367345, 0.1085481621,  
0.1219746189, 0.1241305742, 0.1164617913, 0.1017379053, 0.0832531928,  
0.0640660121, 0.0464853902, 0.0318616543, 0.0206579957, 0.0126820590,  
0.0073778739, 0.0040691158, 0.0021292432, 0.0010575108, 0.0004983066,  
0.0002231493]  
141 - [0.0165594352, 0.0522767054, 0.0847410183, 0.1089806408,  
0.1223426758, 0.1243635434, 0.1165264481, 0.1016412923, 0.0830335598,  
0.0637772044, 0.0461805725, 0.0315818708, 0.0204271742, 0.0125079592,  
0.0072566464, 0.0039906360, 0.0020818349, 0.0010306674, 0.0004840794,  
0.0002160358]  
142 - [0.0166615752, 0.0525619959, 0.0851453021, 0.1094131195,  
0.1227107328, 0.1245965126, 0.1165911049, 0.1015446793, 0.0828139268,  
0.0634883966, 0.0458757548, 0.0313020872, 0.0201963526, 0.0123338595,  
0.0071354188, 0.0039121563, 0.0020344266, 0.0010038240, 0.0004698522,  
0.0002089223]  
143 - [0.0167637152, 0.0528472864, 0.0855495860, 0.1098455981,  
0.1230787898, 0.1248294819, 0.1166557617, 0.1014480662, 0.0825942937,



0.0631995888, 0.0455709371, 0.0310223037, 0.0199655310, 0.0121597597,  
 0.0070141912, 0.0038336766, 0.0019870184, 0.0009769805, 0.0004556250,  
 0.0002018089]  
 144 - [0.0168658553, 0.0531325769, 0.0859538698, 0.1102780768,  
 0.1234468468, 0.1250624511, 0.1167204186, 0.1013514532, 0.0823746607,  
 0.0629107810, 0.0452661195, 0.0307425202, 0.0197347094, 0.0119856599,  
 0.0068929636, 0.0037551969, 0.0019396101, 0.0009501371, 0.0004413978,  
 0.0001946954]  
 145 - [0.0169679953, 0.0534178674, 0.0863581536, 0.1107105554,  
 0.1238149037, 0.1252954204, 0.1167850754, 0.1012548401, 0.0821550277,  
 0.0626219732, 0.0449613018, 0.0304627367, 0.0195038878, 0.0118115602,  
 0.0067717360, 0.0036767172, 0.0018922019, 0.0009232937, 0.0004271706,  
 0.0001875820]  
 146 - [0.0170701353, 0.0537031579, 0.0867624375, 0.1111430341,  
 0.1241829607, 0.1255283896, 0.1168497322, 0.1011582271, 0.0819353946,  
 0.0623331654, 0.0446564841, 0.0301829532, 0.0192730662, 0.0116374604,  
 0.0066505084, 0.0035982375, 0.0018447936, 0.0008964502, 0.0004129433,  
 0.0001804685]  
 147 - [0.0171722753, 0.0539884484, 0.0871667213, 0.1115755127,  
 0.1245510177, 0.1257613589, 0.1169143890, 0.1010616140, 0.0817157616,  
 0.0620443576, 0.0443516664, 0.0299031697, 0.0190422447, 0.0114633606,  
 0.0065292808, 0.0035197578, 0.0017973854, 0.0008696068, 0.0003987161,  
 0.0001733551]  
 148 - [0.0172744154, 0.0542737389, 0.0875710051, 0.1120079914,  
 0.1249190747, 0.1259943281, 0.1169790459, 0.1009650010, 0.0814961286,  
 0.0617555499, 0.0440468488, 0.0296233862, 0.0188114231, 0.0112892608,  
 0.0064080533, 0.0034412780, 0.0017499771, 0.0008427634, 0.0003844889,  
 0.0001662416]  
 149 - [0.0173765554, 0.0545590294, 0.0879752890, 0.1124404700,  
 0.1252871316, 0.1262272974, 0.1170437027, 0.1008683879, 0.0812764955,  
 0.0614667421, 0.0437420311, 0.0293436027, 0.0185806015, 0.0111151611,  
 0.0062868257, 0.0033627983, 0.0017025689, 0.0008159199, 0.0003702617,  
 0.0001591282]  
 150 - [0.0174786954, 0.0548443199, 0.0883795728, 0.1128729487,  
 0.1256551886, 0.1264602666, 0.1171083595, 0.1007717749, 0.0810568625,  
 0.0611779343, 0.0434372134, 0.0290638192, 0.0183497799, 0.0109410613,  
 0.0061655981, 0.0032843186, 0.0016551606, 0.0007890765, 0.0003560345,  
 0.0001520147]  
 151 - [0.0174152405, 0.0548063398, 0.0884629180, 0.1130859859,  
 0.1259469036, 0.1267507008, 0.1173226034, 0.1008652416, 0.0810245087,  
 0.0610474282, 0.0432526040, 0.0288687621, 0.0181759295, 0.0108044614,  
 0.0060688503, 0.0032219157, 0.0016180978, 0.0007687396, 0.0003456440,  
 0.0001471251]  
 152 - [0.0173517855, 0.0547683598, 0.0885462632, 0.1132990231,  
 0.1262386185, 0.1270411350, 0.1175368473, 0.1009587084, 0.0809921550,  
 0.0609169220, 0.0430679945, 0.0286737051, 0.0180020790, 0.0106678615,  
 0.0059721024, 0.0031595129, 0.0015810350, 0.0007484028, 0.0003352535,  
 0.0001422355]  
 153 - [0.0172883306, 0.0547303798, 0.0886296084, 0.1135120602,  
 0.1265303335, 0.1273315692, 0.1177510912, 0.1010521751, 0.0809598012,  
 0.0607864159, 0.0428833851, 0.0284786480, 0.0178282286, 0.0105312616,  
 0.0058753546, 0.0030971100, 0.0015439723, 0.0007280659, 0.0003248630,  
 0.0001373459]  
 154 - [0.0172248756, 0.0546923997, 0.0887129536, 0.1137250974,  
 0.1268220485, 0.1276220034, 0.1179653351, 0.1011456418, 0.0809274474,  
 0.0606559097, 0.0426987756, 0.0282835910, 0.0176543781, 0.0103946617,  
 0.0057786068, 0.0030347071, 0.0015069095, 0.0007077291, 0.0003144724,  
 0.0001324563]  
 155 - [0.0171614207, 0.0546544196, 0.0887962988, 0.1139381346,  
 0.1271137635, 0.1279124376, 0.1181795791, 0.1012391086, 0.0808950937,

0.0605254036, 0.0425141662, 0.0280885339, 0.0174805276, 0.0102580618,  
 0.0056818589, 0.0029723043, 0.0014698467, 0.0006873922, 0.0003040819,  
 0.0001275667]  
 156 - [0.0170979657, 0.0546164396, 0.0888796440, 0.1141511718,  
 0.1274054784, 0.1282028718, 0.1183938230, 0.1013325753, 0.0808627399,  
 0.0603948974, 0.0423295568, 0.0278934768, 0.0173066772, 0.0101214619,  
 0.0055851111, 0.0029099014, 0.0014327839, 0.0006670554, 0.0002936914,  
 0.0001226771]  
 157 - [0.0170345108, 0.0545784596, 0.0889629892, 0.1143642089,  
 0.1276971934, 0.1284933060, 0.1186080669, 0.1014260420, 0.0808303861,  
 0.0602643913, 0.0421449473, 0.0276984198, 0.0171328268, 0.0099848620,  
 0.0054883633, 0.0028474985, 0.0013957212, 0.0006467185, 0.0002833009,  
 0.0001177875]  
 158 - [0.0169710558, 0.0545404795, 0.0890463344, 0.1145772461,  
 0.1279889084, 0.1287837402, 0.1188223108, 0.1015195088, 0.0807980324,  
 0.0601338851, 0.0419603379, 0.0275033627, 0.0169589763, 0.0098482621,  
 0.0053916154, 0.0027850957, 0.0013586584, 0.0006263817, 0.0002729104,  
 0.0001128979]  
 159 - [0.0169076009, 0.0545024994, 0.0891296796, 0.1147902833,  
 0.1282806234, 0.1290741745, 0.1190365547, 0.1016129755, 0.0807656786,  
 0.0600033790, 0.0417757284, 0.0273083056, 0.0167851258, 0.0097116622,  
 0.0052948676, 0.0027226928, 0.0013215956, 0.0006060448, 0.0002625198,  
 0.0001080084]  
 160 - [0.0168441460, 0.0544645194, 0.0892130247, 0.1150033205,  
 0.1285723383, 0.1293646087, 0.1192507986, 0.1017064422, 0.0807333248,  
 0.0598728729, 0.0415911190, 0.0271132486, 0.0166112754, 0.0095750622,  
 0.0051981198, 0.0026602899, 0.0012845328, 0.0005857079, 0.0002521293,  
 0.0001031188]  
 161 - [0.0167806910, 0.0544265394, 0.0892963699, 0.1152163577,  
 0.1288640533, 0.1296550429, 0.1194650425, 0.1017999090, 0.0807009711,  
 0.0597423667, 0.0414065096, 0.0269181915, 0.0164374250, 0.0094384623,  
 0.0051013719, 0.0025978871, 0.0012474700, 0.0005653711, 0.0002417388,  
 0.0000982292]  
 162 - [0.0167172361, 0.0543885593, 0.0893797151, 0.1154293948,  
 0.1291557683, 0.1299454771, 0.1196792864, 0.1018933757, 0.0806686173,  
 0.0596118606, 0.0412219001, 0.0267231345, 0.0162635745, 0.0093018624,  
 0.0050046241, 0.0025354842, 0.0012104073, 0.0005450342, 0.0002313483,  
 0.0000933396]  
 163 - [0.0166537811, 0.0543505792, 0.0894630603, 0.1156424320,  
 0.1294474832, 0.1302359113, 0.1198935303, 0.1019868424, 0.0806362635,  
 0.0594813544, 0.0410372907, 0.0265280774, 0.0160897240, 0.0091652625,  
 0.0049078763, 0.0024730813, 0.0011733445, 0.0005246974, 0.0002209578,  
 0.0000884500]  
 164 - [0.0165903262, 0.0543125992, 0.0895464055, 0.1158554692,  
 0.1297391982, 0.1305263455, 0.1201077743, 0.1020803092, 0.0806039098,  
 0.0593508483, 0.0408526813, 0.0263330203, 0.0159158736, 0.0090286626,  
 0.0048111284, 0.0024106785, 0.0011362817, 0.0005043605, 0.0002105673,  
 0.0000835604]  
 165 - [0.0165268712, 0.0542746192, 0.0896297507, 0.1160685064,  
 0.1300309132, 0.1308167797, 0.1203220182, 0.1021737759, 0.0805715560,  
 0.0592203421, 0.0406680718, 0.0261379633, 0.0157420232, 0.0088920627,  
 0.0047143806, 0.0023482756, 0.0010992189, 0.0004840237, 0.0002001768,  
 0.0000786708]  
 166 - [0.0164634163, 0.0542366391, 0.0897130959, 0.1162815435,  
 0.1303226282, 0.1311072139, 0.1205362621, 0.1022672426, 0.0805392022,  
 0.0590898360, 0.0404834624, 0.0259429062, 0.0155681727, 0.0087554628,  
 0.0046176328, 0.0022858727, 0.0010621562, 0.0004636868, 0.0001897862,  
 0.0000737812]  
 167 - [0.0163999613, 0.0541986590, 0.0897964411, 0.1164945807,  
 0.1306143431, 0.1313976481, 0.1207505060, 0.1023607094, 0.0805068485,

0.0589593298, 0.0402988529, 0.0257478492, 0.0153943222, 0.0086188629,  
 0.0045208849, 0.0022234699, 0.0010250934, 0.0004433500, 0.0001793957,  
 0.0000688916]  
 168 - [0.0163365064, 0.0541606790, 0.0898797863, 0.1167076179,  
 0.1309060581, 0.1316880823, 0.1209647499, 0.1024541761, 0.0804744947,  
 0.0588288237, 0.0401142435, 0.0255527921, 0.0152204718, 0.0084822630,  
 0.0044241371, 0.0021610670, 0.0009880306, 0.0004230131, 0.0001690052,  
 0.0000640020]  
 169 - [0.0161598182, 0.0538356099, 0.0896195630, 0.1166331968,  
 0.1310409549, 0.1319730906, 0.1212978273, 0.1027377756, 0.0806497868,  
 0.0588857523, 0.0400797497, 0.0254690379, 0.0151255079, 0.0084002519,  
 0.0043645756, 0.0021231751, 0.0009665292, 0.0004120125, 0.0001639494,  
 0.0000618352]  
 170 - [0.0159831300, 0.0535105408, 0.0893593397, 0.1165587757,  
 0.1311758516, 0.1322580989, 0.1216309046, 0.1030213752, 0.0808250789,  
 0.0589426809, 0.0400452559, 0.0253852837, 0.0150305440, 0.0083182408,  
 0.0043050141, 0.0020852833, 0.0009450278, 0.0004010119, 0.0001588936,  
 0.0000596685]  
 171 - [0.0158064417, 0.0531854716, 0.0890991164, 0.1164843546,  
 0.1313107484, 0.1325431072, 0.1219639820, 0.1033049748, 0.0810003710,  
 0.0589996095, 0.0400107622, 0.0253015295, 0.0149355801, 0.0082362297,  
 0.0042454526, 0.0020473914, 0.0009235264, 0.0003900112, 0.0001538378,  
 0.0000575017]  
 172 - [0.0156297535, 0.0528604025, 0.0888388931, 0.1164099335,  
 0.1314456452, 0.1328281154, 0.1222970594, 0.1035885743, 0.0811756631,  
 0.0590565381, 0.0399762684, 0.0252177753, 0.0148406162, 0.0081542186,  
 0.0041858911, 0.0020094995, 0.0009020250, 0.0003790106, 0.0001487820,  
 0.0000553349]  
 173 - [0.0154530653, 0.0525353334, 0.0885786698, 0.1163355125,  
 0.1315805420, 0.1331131237, 0.1226301368, 0.1038721738, 0.0813509553,  
 0.0591134668, 0.0399417746, 0.0251340211, 0.0147456523, 0.0080722075,  
 0.0041263296, 0.0019716076, 0.0008805236, 0.0003680100, 0.0001437262,  
 0.0000531682]  
 174 - [0.0152763771, 0.0522102643, 0.0883184465, 0.1162610914,  
 0.1317154387, 0.1333981320, 0.1229632141, 0.1041557734, 0.0815262474,  
 0.0591703954, 0.0399072808, 0.0250502669, 0.0146506884, 0.0079901964,  
 0.0040667681, 0.0019337158, 0.0008590222, 0.0003570094, 0.0001386704,  
 0.0000510014]  
 175 - [0.0150996888, 0.0518851951, 0.0880582232, 0.1161866703,  
 0.1318503355, 0.1336831403, 0.1232962915, 0.1044393730, 0.0817015395,  
 0.0592273240, 0.0398727870, 0.0249665127, 0.0145557245, 0.0079081853,  
 0.0040072066, 0.0018958239, 0.0008375208, 0.0003460088, 0.0001336146,  
 0.0000488346]  
 176 - [0.0149230006, 0.0515601260, 0.0877979999, 0.1161122492,  
 0.1319852323, 0.1339681486, 0.1236293689, 0.1047229725, 0.0818768316,  
 0.0592842526, 0.0398382932, 0.0248827585, 0.0144607606, 0.0078261742,  
 0.0039476451, 0.0018579320, 0.0008160194, 0.0003350082, 0.0001285588,  
 0.0000466679]  
 177 - [0.0147463124, 0.0512350569, 0.0875377766, 0.1160378281,  
 0.1321201290, 0.1342531568, 0.1239624462, 0.1050065720, 0.0820521237,  
 0.0593411812, 0.0398037994, 0.0247990042, 0.0143657967, 0.0077441631,  
 0.0038880836, 0.0018200402, 0.0007945180, 0.0003240076, 0.0001235030,  
 0.0000445011]  
 178 - [0.0145696242, 0.0509099878, 0.0872775532, 0.1159634070,  
 0.1322550258, 0.1345381651, 0.1242955236, 0.1052901716, 0.0822274158,  
 0.0593981098, 0.0397693057, 0.0247152500, 0.0142708328, 0.0076621519,  
 0.0038285222, 0.0017821483, 0.0007730166, 0.0003130069, 0.0001184471,  
 0.0000423343]  
 179 - [0.0143929360, 0.0505849187, 0.0870173299, 0.1158889859,  
 0.1323899226, 0.1348231734, 0.1246286010, 0.1055737712, 0.0824027079,

0.0594550384, 0.0397348119, 0.0246314958, 0.0141758689, 0.0075801408,  
 0.0037689607, 0.0017442564, 0.0007515152, 0.0003020063, 0.0001133913,  
 0.0000401676]  
 180 - [0.0142162477, 0.0502598495, 0.0867571066, 0.1158145648,  
 0.1325248194, 0.1351081817, 0.1249616784, 0.1058573707, 0.0825780000,  
 0.0595119670, 0.0397003181, 0.0245477416, 0.0140809050, 0.0074981297,  
 0.0037093992, 0.0017063645, 0.0007300138, 0.0002910057, 0.0001083355,  
 0.0000380008]  
 181 - [0.0140395595, 0.0499347804, 0.0864968833, 0.1157401437,  
 0.1326597161, 0.1353931900, 0.1252947557, 0.1061409703, 0.0827532921,  
 0.0595688956, 0.0396658243, 0.0244639874, 0.0139859411, 0.0074161186,  
 0.0036498377, 0.0016684727, 0.0007085124, 0.0002800051, 0.0001032797,  
 0.0000358340]  
 182 - [0.0138628713, 0.0496097113, 0.0862366600, 0.1156657227,  
 0.1327946129, 0.1356781983, 0.1256278331, 0.1064245698, 0.0829285843,  
 0.0596258243, 0.0396313305, 0.0243802332, 0.0138909772, 0.0073341075,  
 0.0035902762, 0.0016305808, 0.0006870110, 0.0002690045, 0.0000982239,  
 0.0000336673]  
 183 - [0.0136861831, 0.0492846422, 0.0859764367, 0.1155913016,  
 0.1329295097, 0.1359632066, 0.1259609105, 0.1067081694, 0.0831038764,  
 0.0596827529, 0.0395968368, 0.0242964790, 0.0137960133, 0.0072520964,  
 0.0035307147, 0.0015926889, 0.0006655096, 0.0002580038, 0.0000931681,  
 0.0000315005]  
 184 - [0.0135094948, 0.0489595730, 0.0857162134, 0.1155168805,  
 0.1330644065, 0.1362482148, 0.1262939879, 0.1069917689, 0.0832791685,  
 0.0597396815, 0.0395623430, 0.0242127248, 0.0137010494, 0.0071700853,  
 0.0034711532, 0.0015547970, 0.0006440082, 0.0002470032, 0.0000881123,  
 0.0000293337]  
 185 - [0.0133328066, 0.0486345039, 0.0854559901, 0.1154424594,  
 0.1331993032, 0.1365332231, 0.1266270652, 0.1072753684, 0.0834544606,  
 0.0597966101, 0.0395278492, 0.0241289706, 0.0136060855, 0.0070880742,  
 0.0034115917, 0.0015169052, 0.0006225068, 0.0002360026, 0.0000830565,  
 0.0000271670]  
 186 - [0.0131561184, 0.0483094348, 0.0851957668, 0.1153680383,  
 0.1333342000, 0.1368182314, 0.1269601426, 0.1075589680, 0.0836297527,  
 0.0598535387, 0.0394933554, 0.0240452164, 0.0135111216, 0.0070060631,  
 0.0033520302, 0.0014790133, 0.0006010054, 0.0002250020, 0.0000780007,  
 0.0000250002]  
 187 - [0.0129610648, 0.0477681116, 0.0844653994, 0.1146491828,  
 0.1328116324, 0.1366074479, 0.1270797494, 0.1079397381, 0.0841522936,  
 0.0603951818, 0.0399641005, 0.0244016203, 0.0137505751, 0.0071502348,  
 0.0034301994, 0.0015173478, 0.0006180059, 0.0002318355, 0.0000805008,  
 0.0000257780]  
 188 - [0.0127660113, 0.0472267884, 0.0837350319, 0.1139303273,  
 0.1322890648, 0.1363966644, 0.1271993563, 0.1083205083, 0.0846748345,  
 0.0609368248, 0.0404348456, 0.0247580243, 0.0139900287, 0.0072944065,  
 0.0035083686, 0.0015556823, 0.0006350064, 0.0002386691, 0.0000830008,  
 0.0000265558]  
 189 - [0.0125709577, 0.0466854652, 0.0830046645, 0.1132114717,  
 0.1317664971, 0.1361858809, 0.1273189631, 0.1087012784, 0.0851973754,  
 0.0614784678, 0.0409055908, 0.0251144282, 0.0142294822, 0.0074385782,  
 0.0035865378, 0.0015940169, 0.0006520069, 0.0002455026, 0.0000855009,  
 0.0000273336]  
 190 - [0.0123759042, 0.0461441420, 0.0822742970, 0.1124926162,  
 0.1312439295, 0.1359750974, 0.1274385700, 0.1090820486, 0.0857199162,  
 0.0620201109, 0.0413763359, 0.0254708322, 0.0144689357, 0.0075827499,  
 0.0036647071, 0.0016323514, 0.0006690074, 0.0002523361, 0.0000880010,  
 0.0000281114]  
 191 - [0.0121808506, 0.0456028187, 0.0815439296, 0.1117737607,  
 0.1307213619, 0.1357643139, 0.1275581768, 0.1094628187, 0.0862424571,

0.0625617540, 0.0418470810, 0.0258272361, 0.0147083892, 0.0077269216,  
 0.0037428763, 0.0016706859, 0.0006860079, 0.0002591697, 0.0000905011,  
 0.0000288892]  
 192 - [0.0119857970, 0.0450614955, 0.0808135621, 0.1110549052,  
 0.1301987943, 0.1355535304, 0.1276777837, 0.1098435889, 0.0867649980,  
 0.0631033970, 0.0423178261, 0.0261836401, 0.0149478428, 0.0078710933,  
 0.0038210455, 0.0017090204, 0.0007030084, 0.0002660032, 0.0000930011,  
 0.0000296670]  
 193 - [0.0117907435, 0.0445201723, 0.0800831947, 0.1103360496,  
 0.1296762266, 0.1353427469, 0.1277973905, 0.1102243590, 0.0872875389,  
 0.0636450400, 0.0427885712, 0.0265400440, 0.0151872963, 0.0080152650,  
 0.0038992147, 0.0017473550, 0.0007200089, 0.0002728367, 0.0000955012,  
 0.0000304448]  
 194 - [0.0115956899, 0.0439788491, 0.0793528272, 0.1096171941,  
 0.1291536590, 0.1351319634, 0.1279169974, 0.1106051292, 0.0878100798,  
 0.0641866831, 0.0432593163, 0.0268964480, 0.0154267498, 0.0081594367,  
 0.0039773839, 0.0017856895, 0.0007370094, 0.0002796703, 0.0000980013,  
 0.0000312226]  
 195 - [0.0114006364, 0.0434375259, 0.0786224598, 0.1088983386,  
 0.1286310914, 0.1349211799, 0.1280366042, 0.1109858993, 0.0883326206,  
 0.0647283262, 0.0437300614, 0.0272528519, 0.0156662034, 0.0083036084,  
 0.0040555532, 0.0018240240, 0.0007540100, 0.0002865038, 0.0001005014,  
 0.0000320004]  
 196 - [0.0112055828, 0.0428962027, 0.0778920924, 0.1081794831,  
 0.1281085238, 0.1347103964, 0.1281562110, 0.1113666694, 0.0888551615,  
 0.0652699692, 0.0442008066, 0.0276092558, 0.0159056569, 0.0084477800,  
 0.0041337224, 0.0018623585, 0.0007710105, 0.0002933373, 0.0001030014,  
 0.0000327782]  
 197 - [0.0110105292, 0.0423548795, 0.0771617249, 0.1074606276,  
 0.1275859562, 0.1344996129, 0.1282758179, 0.1117474396, 0.0893777024,  
 0.0658116122, 0.0446715517, 0.0279656598, 0.0161451104, 0.0085919517,  
 0.0042118916, 0.0019006930, 0.0007880110, 0.0003001709, 0.0001055015,  
 0.0000335560]  
 198 - [0.0108154757, 0.0418135563, 0.0764313575, 0.1067417720,  
 0.1270633885, 0.1342888294, 0.1283954247, 0.1121282097, 0.0899002433,  
 0.0663532553, 0.0451422968, 0.0283220637, 0.0163845639, 0.0087361234,  
 0.0042900608, 0.0019390276, 0.0008050115, 0.0003070044, 0.0001080016,  
 0.0000343338]  
 199 - [0.0106204221, 0.0412722331, 0.0757009900, 0.1060229165,  
 0.1265408209, 0.1340780459, 0.1285150316, 0.1125089799, 0.0904227842,  
 0.0668948984, 0.0456130419, 0.0286784677, 0.0166240175, 0.0088802951,  
 0.0043682300, 0.0019773621, 0.0008220120, 0.0003138379, 0.0001105016,  
 0.0000351116]  
 200 - [0.0104253685, 0.0407309098, 0.0749706226, 0.1053040610,  
 0.1260182533, 0.1338672624, 0.1286346384, 0.1128897500, 0.0909453251,  
 0.0674365414, 0.0460837870, 0.0290348716, 0.0168634710, 0.0090244668,  
 0.0044463992, 0.0020156966, 0.0008390125, 0.0003206715, 0.0001130017,  
 0.0000358894]  
 201 - [0.0102303150, 0.0401895866, 0.0742402551, 0.1045852055,  
 0.1254956857, 0.1336564789, 0.1287542453, 0.1132705202, 0.0914678659,  
 0.0679781845, 0.0465545322, 0.0293912756, 0.0171029245, 0.0091686385,  
 0.0045245684, 0.0020540311, 0.0008560130, 0.0003275050, 0.0001155018,  
 0.0000366672]  
 202 - [0.0100352614, 0.0396482634, 0.0735098877, 0.1038663499,  
 0.1249731180, 0.1334456954, 0.1288738521, 0.1136512903, 0.0919904068,  
 0.0685198275, 0.0470252773, 0.0297476795, 0.0173423780, 0.0093128102,  
 0.0046027377, 0.0020923657, 0.0008730135, 0.0003343385, 0.0001180019,  
 0.0000374450]  
 203 - [0.0098402079, 0.0391069402, 0.0727795202, 0.1031474944,  
 0.1244505504, 0.1332349119, 0.1289934590, 0.1140320605, 0.0925129477,

0.0690614706, 0.0474960224, 0.0301040835, 0.0175818316, 0.0094569819,  
 0.0046809069, 0.0021307002, 0.0008900140, 0.0003411721, 0.0001205019,  
 0.0000382228]  
 204 - [0.0096451543, 0.0385656170, 0.0720491528, 0.1024286389,  
 0.1239279828, 0.1330241284, 0.1291130658, 0.1144128306, 0.0930354886,  
 0.0696031136, 0.0479667675, 0.0304604874, 0.0178212851, 0.0096011536,  
 0.0047590761, 0.0021690347, 0.0009070145, 0.0003480056, 0.0001230020,  
 0.0000390006]  
 205 - [0.0094379938, 0.0379035382, 0.0710439459, 0.1013007532,  
 0.1229370823, 0.1323937588, 0.1289653227, 0.1147394741, 0.0937156691,  
 0.0704569308, 0.0488185498, 0.0311857913, 0.0183639493, 0.0099629076,  
 0.0049755935, 0.0022858768, 0.0009640185, 0.0003731740, 0.0001330026,  
 0.0000426675]  
 206 - [0.0092308334, 0.0372414595, 0.0700387389, 0.1001728674,  
 0.1219461818, 0.1317633892, 0.1288175796, 0.1150661176, 0.0943958496,  
 0.0713107481, 0.0496703320, 0.0319110953, 0.0189066136, 0.0103246615,  
 0.0051921109, 0.0024027189, 0.0010210226, 0.0003983423, 0.0001430033,  
 0.0000463344]  
 207 - [0.0090236730, 0.0365793807, 0.0690335320, 0.0990449817,  
 0.1209552814, 0.1311330196, 0.1286698366, 0.1153927610, 0.0950760301,  
 0.0721645653, 0.0505221143, 0.0326363992, 0.0194492778, 0.0106864155,  
 0.0054086283, 0.0025195610, 0.0010780266, 0.0004235107, 0.0001530039,  
 0.0000500013]  
 208 - [0.0088165125, 0.0359173019, 0.0680283251, 0.0979170960,  
 0.1199643809, 0.1305026500, 0.1285220935, 0.1157194045, 0.0957562106,  
 0.0730183825, 0.0513738966, 0.0333617031, 0.0199919421, 0.0110481695,  
 0.0056251457, 0.0026364030, 0.0011350306, 0.0004486790, 0.0001630046,  
 0.0000536682]  
 209 - [0.0086093520, 0.0352552232, 0.0670231181, 0.0967892102,  
 0.1189734804, 0.1298722804, 0.1283743504, 0.1160460480, 0.0964363911,  
 0.0738721997, 0.0522256789, 0.0340870071, 0.0205346063, 0.0114099235,  
 0.0058416632, 0.0027532451, 0.0011920346, 0.0004738474, 0.0001730052,  
 0.0000573351]  
 210 - [0.0084021916, 0.0345931444, 0.0660179112, 0.0956613245,  
 0.1179825799, 0.1292419108, 0.1282266073, 0.1163726915, 0.0971165716,  
 0.0747260170, 0.0530774611, 0.0348123110, 0.0210772706, 0.0117716774,  
 0.0060581806, 0.0028700872, 0.0012490387, 0.0004990157, 0.0001830059,  
 0.0000610020]  
 211 - [0.0081950312, 0.0339310656, 0.0650127043, 0.0945334388,  
 0.1169916794, 0.1286115412, 0.1280788643, 0.1166993349, 0.0977967521,  
 0.0755798342, 0.0539292434, 0.0355376149, 0.0216199348, 0.0121334314,  
 0.0062746980, 0.0029869293, 0.0013060427, 0.0005241841, 0.0001930065,  
 0.0000646689]  
 212 - [0.0079878707, 0.0332689869, 0.0640074973, 0.0934055530,  
 0.1160007789, 0.1279811716, 0.1279311212, 0.1170259784, 0.0984769326,  
 0.0764336514, 0.0547810257, 0.0362629189, 0.0221625991, 0.0124951854,  
 0.0064912154, 0.0031037714, 0.0013630467, 0.0005493524, 0.0002030072,  
 0.0000683358]  
 213 - [0.0077807102, 0.0326069081, 0.0630022904, 0.0922776673,  
 0.1150098784, 0.1273508020, 0.1277833781, 0.1173526219, 0.0991571130,  
 0.0772874686, 0.0556328080, 0.0369882228, 0.0227052633, 0.0128569393,  
 0.0067077328, 0.0032206134, 0.0014200508, 0.0005745208, 0.0002130078,  
 0.0000720026]  
 214 - [0.0075735498, 0.0319448293, 0.0619970835, 0.0911497816,  
 0.1140189780, 0.1267204323, 0.1276356350, 0.1176792654, 0.0998372935,  
 0.0781412859, 0.0564845902, 0.0377135267, 0.0232479275, 0.0132186933,  
 0.0069242502, 0.0033374555, 0.0014770548, 0.0005996892, 0.0002230084,  
 0.0000756695]  
 215 - [0.0073663894, 0.0312827506, 0.0609918765, 0.0900218958,  
 0.1130280775, 0.1260900627, 0.1274878919, 0.1180059089, 0.1005174740,

0.0789951031, 0.0573363725, 0.0384388307, 0.0237905918, 0.0135804473,  
 0.0071407676, 0.0034542976, 0.0015340588, 0.0006248575, 0.0002330091,  
 0.0000793364]  
 216 - [0.0071592289, 0.0306206718, 0.0599866696, 0.0888940101,  
 0.1120371770, 0.1254596931, 0.1273401489, 0.1183325523, 0.1011976545,  
 0.0798489203, 0.0581881548, 0.0391641346, 0.0243332560, 0.0139422013,  
 0.0073572850, 0.0035711397, 0.0015910628, 0.0006500259, 0.0002430097,  
 0.0000830033]  
 217 - [0.0069520684, 0.0299585930, 0.0589814627, 0.0877661244,  
 0.1110462765, 0.1248293235, 0.1271924058, 0.1186591958, 0.1018778350,  
 0.0807027376, 0.0590399370, 0.0398894385, 0.0248759203, 0.0143039552,  
 0.0075738024, 0.0036879818, 0.0016480669, 0.0006751942, 0.0002530104,  
 0.0000866702]  
 218 - [0.0067449080, 0.0292965143, 0.0579762557, 0.0866382386,  
 0.1100553760, 0.1241989539, 0.1270446627, 0.1189858393, 0.1025580155,  
 0.0815565548, 0.0598917193, 0.0406147425, 0.0254185845, 0.0146657092,  
 0.0077903199, 0.0038048239, 0.0017050709, 0.0007003626, 0.0002630110,  
 0.0000903371]  
 219 - [0.0065377476, 0.0286344355, 0.0569710488, 0.0855103529,  
 0.1090644756, 0.1235685843, 0.1268969196, 0.1193124828, 0.1032381960,  
 0.0824103720, 0.0607435016, 0.0413400464, 0.0259612488, 0.0150274632,  
 0.0080068373, 0.0039216660, 0.0017620749, 0.0007255309, 0.0002730117,  
 0.0000940040]  
 220 - [0.0063305871, 0.0279723567, 0.0559658419, 0.0843824672,  
 0.1080735751, 0.1229382147, 0.1267491766, 0.1196391262, 0.1039183765,  
 0.0832641892, 0.0615952839, 0.0420653503, 0.0265039130, 0.0153892172,  
 0.0082233547, 0.0040385080, 0.0018190789, 0.0007506993, 0.0002830123,  
 0.0000976709]  
 221 - [0.0061234266, 0.0273102780, 0.0549606349, 0.0832545814,  
 0.1070826746, 0.1223078451, 0.1266014335, 0.1199657697, 0.1045985570,  
 0.0841180065, 0.0624470661, 0.0427906543, 0.0270465773, 0.0157509711,  
 0.0084398721, 0.0041553501, 0.0018760830, 0.0007758676, 0.0002930130,  
 0.0001013378]  
 222 - [0.0059162662, 0.0266481992, 0.0539554280, 0.0821266957,  
 0.1060917741, 0.1216774755, 0.1264536904, 0.1202924132, 0.1052787375,  
 0.0849718237, 0.0632988484, 0.0435159582, 0.0275892415, 0.0161127251,  
 0.0086563895, 0.0042721922, 0.0019330870, 0.0008010360, 0.0003030136,  
 0.0001050047]  
 223 - [0.0057759872, 0.0261514256, 0.0531488953, 0.0811605271,  
 0.1051695891, 0.1210010538, 0.1261648593, 0.1204326037, 0.1057830984,  
 0.0857000592, 0.0640875557, 0.0442294637, 0.0281489899, 0.0164999994,  
 0.0088945307, 0.0044027636, 0.0019970390, 0.0008289281, 0.0003139037,  
 0.0001087273]  
 224 - [0.0056357082, 0.0256546521, 0.0523423626, 0.0801943585,  
 0.1042474040, 0.1203246320, 0.1258760282, 0.1205727943, 0.1062874593,  
 0.0864282946, 0.0648762630, 0.0449429693, 0.0287087382, 0.0168872736,  
 0.0091326720, 0.0045333349, 0.0020609909, 0.0008568202, 0.0003247939,  
 0.0001124500]  
 225 - [0.0054954292, 0.0251578785, 0.0515358299, 0.0792281899,  
 0.1033252190, 0.1196482103, 0.1255871971, 0.1207129848, 0.1067918202,  
 0.0871565300, 0.0656649703, 0.0456564748, 0.0292684866, 0.0172745479,  
 0.0093708132, 0.0046639063, 0.0021249428, 0.0008847123, 0.0003356840,  
 0.0001161726]  
 226 - [0.0053551502, 0.0246611049, 0.0507292972, 0.0782620213,  
 0.1024030340, 0.1189717885, 0.1252983659, 0.1208531753, 0.1072961811,  
 0.0878847655, 0.0664536776, 0.0463699804, 0.0298282349, 0.0176618222,  
 0.0096089545, 0.0047944777, 0.0021888948, 0.0009126044, 0.0003465741,  
 0.0001198953]  
 227 - [0.0052148712, 0.0241643313, 0.0499227645, 0.0772958527,  
 0.1014808489, 0.1182953668, 0.1250095348, 0.1209933658, 0.1078005420,

0.0886130010, 0.0672423849, 0.0470834859, 0.0303879833, 0.0180490965,  
 0.0098470957, 0.0049250490, 0.0022528468, 0.0009404965, 0.0003574642,  
 0.0001236180]  
 228 - [0.0050745922, 0.0236675578, 0.0491162318, 0.0763296841,  
 0.1005586639, 0.1176189450, 0.1247207037, 0.1211335564, 0.1083049029,  
 0.0893412364, 0.0680310922, 0.0477969914, 0.0309477316, 0.0184363707,  
 0.0100852370, 0.0050556204, 0.0023167987, 0.0009683886, 0.0003683544,  
 0.0001273406]  
 229 - [0.0049343132, 0.0231707842, 0.0483096991, 0.0753635155,  
 0.0996364789, 0.1169425233, 0.1244318726, 0.1212737469, 0.1088092638,  
 0.0900694718, 0.0688197995, 0.0485104970, 0.0315074800, 0.0188236450,  
 0.0103233782, 0.0051861918, 0.0023807506, 0.0009962807, 0.0003792445,  
 0.0001310632]  
 230 - [0.0047940342, 0.0226740106, 0.0475031664, 0.0743973469,  
 0.0987142938, 0.1162661015, 0.1241430415, 0.1214139374, 0.1093136247,  
 0.0907977073, 0.0696085068, 0.0492240025, 0.0320672283, 0.0192109193,  
 0.0105615195, 0.0053167631, 0.0024447026, 0.0010241728, 0.0003901346,  
 0.0001347859]  
 231 - [0.0046537552, 0.0221772370, 0.0466966338, 0.0734311784,  
 0.0977921088, 0.1155896798, 0.1238542104, 0.1215541280, 0.1098179856,  
 0.0915259428, 0.0703972140, 0.0499375080, 0.0326269767, 0.0195981936,  
 0.0107996607, 0.0054473345, 0.0025086546, 0.0010520649, 0.0004010247,  
 0.0001385086]  
 232 - [0.0045134762, 0.0216804635, 0.0458901011, 0.0724650098,  
 0.0968699238, 0.1149132581, 0.1235653792, 0.1216943185, 0.1103223465,  
 0.0922541782, 0.0711859213, 0.0506510136, 0.0331867251, 0.0199854678,  
 0.0110378019, 0.0055779059, 0.0025726065, 0.0010799570, 0.0004119149,  
 0.0001422312]  
 233 - [0.0043731972, 0.0211836899, 0.0450835684, 0.0714988412,  
 0.0959477387, 0.1142368363, 0.1232765481, 0.1218345090, 0.1108267074,  
 0.0929824137, 0.0719746286, 0.0513645191, 0.0337464734, 0.0203727421,  
 0.0112759432, 0.0057084772, 0.0026365584, 0.0011078491, 0.0004228050,  
 0.0001459538]  
 234 - [0.0042329182, 0.0206869163, 0.0442770357, 0.0705326726,  
 0.0950255537, 0.1135604146, 0.1229877170, 0.1219746995, 0.1113310683,  
 0.0937106491, 0.0727633359, 0.0520780247, 0.0343062218, 0.0207600164,  
 0.0115140844, 0.0058390486, 0.0027005104, 0.0011357412, 0.0004336951,  
 0.0001496765]  
 235 - [0.0040926392, 0.0201901428, 0.0434705030, 0.0695665040,  
 0.0941033687, 0.1128839928, 0.1226988859, 0.1221148901, 0.1118354292,  
 0.0944388846, 0.0735520432, 0.0527915302, 0.0348659701, 0.0211472906,  
 0.0117522257, 0.0059696200, 0.0027644624, 0.0011636333, 0.0004445853,  
 0.0001533992]  
 236 - [0.0039523602, 0.0196933692, 0.0426639703, 0.0686003354,  
 0.0931811836, 0.1122075711, 0.1224100548, 0.1222550806, 0.1123397901,  
 0.0951671200, 0.0743407505, 0.0535050357, 0.0354257185, 0.0215345649,  
 0.0119903669, 0.0061001913, 0.0028284143, 0.0011915254, 0.0004554754,  
 0.0001571218]  
 237 - [0.0038120812, 0.0191965956, 0.0418574376, 0.0676341668,  
 0.0922589986, 0.1115311493, 0.1221212236, 0.1223952711, 0.1128441510,  
 0.0958953555, 0.0751294578, 0.0542185413, 0.0359854668, 0.0219218392,  
 0.0122285082, 0.0062307627, 0.0028923663, 0.0012194175, 0.0004663655,  
 0.0001608444]  
 238 - [0.0036718022, 0.0186998220, 0.0410509049, 0.0666679982,  
 0.0913368136, 0.1108547276, 0.1218323925, 0.1225354616, 0.1133485119,  
 0.0966235909, 0.0759181651, 0.0549320468, 0.0365452152, 0.0223091135,  
 0.0124666494, 0.0063613341, 0.0029563182, 0.0012473096, 0.0004772556,  
 0.0001645671]  
 239 - [0.0035315232, 0.0182030485, 0.0402443722, 0.0657018296,  
 0.0904146285, 0.1101783058, 0.1215435614, 0.1226756522, 0.1138528728,



0.0973518264, 0.0767068724, 0.0556455524, 0.0371049635, 0.0226963877,  
 0.0127047907, 0.0064919054, 0.0030202702, 0.0012752017, 0.0004881458,  
 0.0001682898]  
 240 - [0.0033912442, 0.0177062749, 0.0394378395, 0.0647356610,  
 0.0894924435, 0.1095018841, 0.1212547303, 0.1228158427, 0.1143572337,  
 0.0980800618, 0.0774955797, 0.0563590579, 0.0376647119, 0.0230836620,  
 0.0129429319, 0.0066224768, 0.0030842221, 0.0013030938, 0.0004990359,  
 0.0001720124]  
 241 - [0.0033306844, 0.0174667584, 0.0390228670, 0.0642116819,  
 0.0889646322, 0.1090862487, 0.1210444439, 0.1228522411, 0.1146179258,  
 0.0984899305, 0.0779561176, 0.0567821475, 0.0379961826, 0.0233085686,  
 0.0130757201, 0.0066904821, 0.0031141133, 0.0013142058, 0.0005023139,  
 0.0001727347]  
 242 - [0.0032701246, 0.0172272419, 0.0386078944, 0.0636877028,  
 0.0884368208, 0.1086706134, 0.1208341575, 0.1228886394, 0.1148786178,  
 0.0988997992, 0.0784166555, 0.0572052370, 0.0383276534, 0.0235334752,  
 0.0132085083, 0.0067584875, 0.0031440046, 0.0013253178, 0.0005055920,  
 0.0001734570]  
 243 - [0.0032095648, 0.0169877254, 0.0381929219, 0.0631637237,  
 0.0879090095, 0.1082549780, 0.1206238711, 0.1229250378, 0.1151393098,  
 0.0993096679, 0.0788771935, 0.0576283266, 0.0386591241, 0.0237583818,  
 0.0133412965, 0.0068264928, 0.0031738958, 0.0013364298, 0.0005088700,  
 0.0001741792]  
 244 - [0.0031490050, 0.0167482089, 0.0377779493, 0.0626397446,  
 0.0873811981, 0.1078393426, 0.1204135847, 0.1229614362, 0.1154000019,  
 0.0997195366, 0.0793377314, 0.0580514162, 0.0389905948, 0.0239832884,  
 0.0134740847, 0.0068944981, 0.0032037871, 0.0013475418, 0.0005121481,  
 0.0001749015]  
 245 - [0.0030884453, 0.0165086924, 0.0373629768, 0.0621157655,  
 0.0868533868, 0.1074237073, 0.1202032984, 0.1229978346, 0.1156606940,  
 0.1001294053, 0.0797982693, 0.0584745058, 0.0393220656, 0.0242081949,  
 0.0136068728, 0.0069625035, 0.0032336783, 0.0013586538, 0.0005154261,  
 0.0001756238]  
 246 - [0.0030278855, 0.0162691759, 0.0369480042, 0.0615917864,  
 0.0863255755, 0.1070080719, 0.1199930120, 0.1230342329, 0.1159213860,  
 0.1005392740, 0.0802588072, 0.0588975953, 0.0396535363, 0.0244331015,  
 0.0137396610, 0.0070305088, 0.0032635695, 0.0013697658, 0.0005187042,  
 0.0001763461]  
 247 - [0.0029673257, 0.0160296594, 0.0365330317, 0.0610678073,  
 0.0857977641, 0.1065924365, 0.1197827256, 0.1230706313, 0.1161820780,  
 0.1009491427, 0.0807193452, 0.0593206849, 0.0399850070, 0.0246580081,  
 0.0138724492, 0.0070985141, 0.0032934608, 0.0013808778, 0.0005219822,  
 0.0001770684]  
 248 - [0.0029067659, 0.0157901429, 0.0361180591, 0.0605438282,  
 0.0852699528, 0.1061768012, 0.1195724392, 0.1231070297, 0.1164427701,  
 0.1013590114, 0.0811798831, 0.0597437745, 0.0403164778, 0.0248829147,  
 0.0140052374, 0.0071665195, 0.0033233520, 0.0013919898, 0.0005252603,  
 0.0001777907]  
 249 - [0.0028462061, 0.0155506264, 0.0357030866, 0.0600198491,  
 0.0847421414, 0.1057611658, 0.1193621528, 0.1231434280, 0.1167034622,  
 0.1017688801, 0.0816404210, 0.0601668640, 0.0406479485, 0.0251078213,  
 0.0141380256, 0.0072345248, 0.0033532433, 0.0014031018, 0.0005285383,  
 0.0001785130]  
 250 - [0.0027856463, 0.0153111098, 0.0352881141, 0.0594958700,  
 0.0842143301, 0.1053455304, 0.1191518664, 0.1231798264, 0.1169641542,  
 0.1021787488, 0.0821009589, 0.0605899536, 0.0409794192, 0.0253327279,  
 0.0142708138, 0.0073025301, 0.0033831345, 0.0014142137, 0.0005318163,  
 0.0001792352]  
 251 - [0.0027250865, 0.0150715933, 0.0348731415, 0.0589718909,  
 0.0836865188, 0.1049298951, 0.1189415800, 0.1232162248, 0.1172248462,

0.1025886175, 0.0825614968, 0.0610130432, 0.0413108900, 0.0255576345,  
 0.0144036020, 0.0073705355, 0.0034130257, 0.0014253257, 0.0005350944,  
 0.0001799575]  
 252 - [0.0026645267, 0.0148320768, 0.0344581690, 0.0584479118,  
 0.0831587074, 0.1045142597, 0.1187312936, 0.1232526232, 0.1174855383,  
 0.1029984862, 0.0830220348, 0.0614361328, 0.0416423607, 0.0257825411,  
 0.0145363902, 0.0074385408, 0.0034429170, 0.0014364377, 0.0005383724,  
 0.0001806798]  
 253 - [0.0026039669, 0.0145925603, 0.0340431964, 0.0579239327,  
 0.0826308961, 0.1040986243, 0.1185210072, 0.1232890215, 0.1177462304,  
 0.1034083549, 0.0834825727, 0.0618592223, 0.0419738314, 0.0260074477,  
 0.0146691784, 0.0075065461, 0.0034728082, 0.0014475497, 0.0005416505,  
 0.0001814021]  
 254 - [0.0025434072, 0.0143530438, 0.0336282239, 0.0573999536,  
 0.0821030848, 0.1036829890, 0.1183107209, 0.1233254199, 0.1180069224,  
 0.1038182236, 0.0839431106, 0.0622823119, 0.0423053022, 0.0262323542,  
 0.0148019665, 0.0075745515, 0.0035026994, 0.0014586617, 0.0005449285,  
 0.0001821244]  
 255 - [0.0024828474, 0.0141135273, 0.0332132513, 0.0568759745,  
 0.0815752734, 0.1032673536, 0.1181004345, 0.1233618183, 0.1182676144,  
 0.1042280923, 0.0844036485, 0.0627054015, 0.0426367729, 0.0264572608,  
 0.0149347547, 0.0076425568, 0.0035325907, 0.0014697737, 0.0005482066,  
 0.0001828466]  
 256 - [0.0024222876, 0.0138740108, 0.0327982788, 0.0563519954,  
 0.0810474621, 0.1028517182, 0.1178901481, 0.1233982167, 0.1185283065,  
 0.1046379610, 0.0848641865, 0.0631284911, 0.0429682436, 0.0266821674,  
 0.0150675429, 0.0077105621, 0.0035624819, 0.0014808857, 0.0005514846,  
 0.0001835689]  
 257 - [0.0023617278, 0.0136344943, 0.0323833062, 0.0558280163,  
 0.0805196507, 0.1024360829, 0.1176798617, 0.1234346150, 0.1187889986,  
 0.1050478297, 0.0853247244, 0.0635515806, 0.0432997144, 0.0269070740,  
 0.0152003311, 0.0077785675, 0.0035923732, 0.0014919977, 0.0005547627,  
 0.0001842912]  
 258 - [0.0023011680, 0.0133949778, 0.0319683337, 0.0553040372,  
 0.0799918394, 0.1020204475, 0.1174695753, 0.1234710134, 0.1190496906,  
 0.1054576984, 0.0857852623, 0.0639746702, 0.0436311851, 0.0271319806,  
 0.0153331193, 0.0078465728, 0.0036222644, 0.0015031097, 0.0005580407,  
 0.0001850135]  
 259 - [0.0022962239, 0.0133723138, 0.0319236186, 0.0552398283,  
 0.0799169157, 0.1019481980, 0.1174147212, 0.1234454970, 0.1190585643,  
 0.1054976256, 0.0858458512, 0.0640423080, 0.0436935935, 0.0271814936,  
 0.0153674605, 0.0078675215, 0.0036335998, 0.0015084995, 0.0005603188,  
 0.0001858470]  
 260 - [0.0022912799, 0.0133496498, 0.0318789035, 0.0551756195,  
 0.0798419920, 0.1018759485, 0.1173598672, 0.1234199806, 0.1190674380,  
 0.1055375528, 0.0859064402, 0.0641099458, 0.0437560018, 0.0272310065,  
 0.0154018016, 0.0078884702, 0.0036449353, 0.0015138893, 0.0005625970,  
 0.0001866804]  
 261 - [0.0022863358, 0.0133269858, 0.0318341884, 0.0551114106,  
 0.0797670683, 0.1018036991, 0.1173050131, 0.1233944642, 0.1190763118,  
 0.1055774800, 0.0859670291, 0.0641775835, 0.0438184102, 0.0272805195,  
 0.0154361428, 0.0079094190, 0.0036562707, 0.0015192792, 0.0005648751,  
 0.0001875139]  
 262 - [0.0022813917, 0.0133043218, 0.0317894733, 0.0550472018,  
 0.0796921446, 0.1017314496, 0.1172501590, 0.1233689477, 0.1190851855,  
 0.1056174072, 0.0860276181, 0.0642452213, 0.0438808185, 0.0273300324,  
 0.0154704839, 0.0079303677, 0.0036676062, 0.0015246690, 0.0005671533,  
 0.0001883473]  
 263 - [0.0022764477, 0.0132816579, 0.0317447582, 0.0549829929,  
 0.0796172209, 0.1016592001, 0.1171953050, 0.1233434313, 0.1190940592,

0.1056573344, 0.0860882070, 0.0643128591, 0.0439432269, 0.0273795454,  
0.0155048251, 0.0079513164, 0.0036789416, 0.0015300588, 0.0005694314,  
0.0001891808]

264 - [0.0022715036, 0.0132589939, 0.0317000431, 0.0549187840,  
0.0795422972, 0.1015869506, 0.1171404509, 0.1233179149, 0.1191029329,  
0.1056972616, 0.0861487959, 0.0643804969, 0.0440056353, 0.0274290583,  
0.0155391662, 0.0079722651, 0.0036902770, 0.0015354486, 0.0005717096,  
0.0001900143]

265 - [0.0022665595, 0.0132363299, 0.0316553280, 0.0548545752,  
0.0794673735, 0.1015147012, 0.1170855968, 0.1232923985, 0.1191118066,  
0.1057371888, 0.0862093849, 0.0644481346, 0.0440680436, 0.0274785713,  
0.0155735074, 0.0079932138, 0.0037016125, 0.0015408384, 0.0005739877,  
0.0001908477]

266 - [0.0022616155, 0.0132136659, 0.0316106129, 0.0547903663,  
0.0793924498, 0.1014424517, 0.1170307428, 0.1232668821, 0.1191206803,  
0.1057771160, 0.0862699738, 0.0645157724, 0.0441304520, 0.0275280842,  
0.0156078485, 0.0080141625, 0.0037129479, 0.0015462282, 0.0005762659,  
0.0001916812]

267 - [0.0022566714, 0.0131910019, 0.0315658978, 0.0547261574,  
0.0793175261, 0.1013702022, 0.1169758887, 0.1232413656, 0.1191295540,  
0.1058170432, 0.0863305628, 0.0645834102, 0.0441928604, 0.0275775972,  
0.0156421897, 0.0080351113, 0.0037242834, 0.0015516180, 0.0005785440,  
0.0001925146]

268 - [0.0022517273, 0.0131683379, 0.0315211827, 0.0546619486,  
0.0792426024, 0.1012979527, 0.1169210346, 0.1232158492, 0.1191384278,  
0.1058569703, 0.0863911517, 0.0646510480, 0.0442552687, 0.0276271102,  
0.0156765309, 0.0080560600, 0.0037356188, 0.0015570079, 0.0005808221,  
0.0001933481]

269 - [0.0022467833, 0.0131456739, 0.0314764676, 0.0545977397,  
0.0791676787, 0.1012257032, 0.1168661806, 0.1231903328, 0.1191473015,  
0.1058968975, 0.0864517406, 0.0647186858, 0.0443176771, 0.0276766231,  
0.0157108720, 0.0080770087, 0.0037469542, 0.0015623977, 0.0005831003,  
0.0001941816]

270 - [0.0022418392, 0.0131230099, 0.0314317525, 0.0545335309,  
0.0790927550, 0.1011534538, 0.1168113265, 0.1231648164, 0.1191561752,  
0.1059368247, 0.0865123296, 0.0647863235, 0.0443800854, 0.0277261361,  
0.0157452132, 0.0080979574, 0.0037582897, 0.0015677875, 0.0005853784,  
0.0001950150]

271 - [0.0022368951, 0.0131003459, 0.0313870374, 0.0544693220,  
0.0790178313, 0.1010812043, 0.1167564724, 0.1231393000, 0.1191650489,  
0.1059767519, 0.0865729185, 0.0648539613, 0.0444424938, 0.0277756490,  
0.0157795543, 0.0081189061, 0.0037696251, 0.0015731773, 0.0005876566,  
0.0001958485]

272 - [0.0022319511, 0.0130776820, 0.0313423223, 0.0544051131,  
0.0789429076, 0.1010089548, 0.1167016184, 0.1231137836, 0.1191739226,  
0.1060166791, 0.0866335074, 0.0649215991, 0.0445049022, 0.0278251620,  
0.0158138955, 0.0081398548, 0.0037809605, 0.0015785671, 0.0005899347,  
0.0001966820]

273 - [0.0022270070, 0.0130550180, 0.0312976072, 0.0543409043,  
0.0788679839, 0.1009367053, 0.1166467643, 0.1230882672, 0.1191827964,  
0.1060566063, 0.0866940964, 0.0649892369, 0.0445673105, 0.0278746749,  
0.0158482366, 0.0081608036, 0.0037922960, 0.0015839570, 0.0005922129,  
0.0001975154]

274 - [0.0022220629, 0.0130323540, 0.0312528921, 0.0542766954,  
0.0787930602, 0.1008644559, 0.1165919102, 0.1230627507, 0.1191916701,  
0.1060965335, 0.0867546853, 0.0650568746, 0.0446297189, 0.0279241879,  
0.0158825778, 0.0081817523, 0.0038036314, 0.0015893468, 0.0005944910,  
0.0001983489]

275 - [0.0022171189, 0.0130096900, 0.0312081770, 0.0542124866,  
0.0787181365, 0.1007922064, 0.1165370562, 0.1230372343, 0.1192005438,

0.1061364607, 0.0868152743, 0.0651245124, 0.0446921272, 0.0279737008,  
 0.0159169189, 0.0082027010, 0.0038149669, 0.0015947366, 0.0005967692,  
 0.0001991823]  
 276 - [0.0022121748, 0.0129870260, 0.0311634619, 0.0541482777,  
 0.0786432128, 0.1007199569, 0.1164822021, 0.1230117179, 0.1192094175,  
 0.1061763879, 0.0868758632, 0.0651921502, 0.0447545356, 0.0280232138,  
 0.0159512601, 0.0082236497, 0.0038263023, 0.0016001264, 0.0005990473,  
 0.0002000158]  
 277 - [0.0022317963, 0.0130554693, 0.0312618609, 0.0542409217,  
 0.0786921017, 0.1006984675, 0.1163857564, 0.1228610092, 0.1190438603,  
 0.1060390120, 0.0867972625, 0.0651805953, 0.0447967513, 0.0280939870,  
 0.0160254922, 0.0082847978, 0.0038683770, 0.0016249688, 0.0006117734,  
 0.0002057395]  
 278 - [0.0022514178, 0.0131239125, 0.0313602599, 0.0543335657,  
 0.0787409906, 0.1006769780, 0.1162893107, 0.1227103006, 0.1188783031,  
 0.1059016362, 0.0867186617, 0.0651690405, 0.0448389669, 0.0281647601,  
 0.0160997243, 0.0083459458, 0.0039104516, 0.0016498111, 0.0006244995,  
 0.0002114632]  
 279 - [0.0022710393, 0.0131923558, 0.0314586589, 0.0544262098,  
 0.0787898795, 0.1006554886, 0.1161928650, 0.1225595920, 0.1187127460,  
 0.1057642603, 0.0866400610, 0.0651574856, 0.0448811826, 0.0282355333,  
 0.0161739563, 0.0084070939, 0.0039525263, 0.0016746534, 0.0006372256,  
 0.0002171870]  
 280 - [0.0022906608, 0.0132607991, 0.0315570579, 0.0545188538,  
 0.0788387684, 0.1006339991, 0.1160964192, 0.1224088833, 0.1185471888,  
 0.1056268844, 0.0865614603, 0.0651459308, 0.0449233982, 0.0283063065,  
 0.0162481884, 0.0084682419, 0.0039946010, 0.0016994958, 0.0006499516,  
 0.0002229107]  
 281 - [0.0023102823, 0.0133292424, 0.0316554569, 0.0546114978,  
 0.0788876572, 0.1006125097, 0.1159999735, 0.1222581746, 0.1183816316,  
 0.1054895085, 0.0864828595, 0.0651343759, 0.0449656139, 0.0283770797,  
 0.0163224205, 0.0085293900, 0.0040366756, 0.0017243382, 0.0006626777,  
 0.0002286344]  
 282 - [0.0023299038, 0.0133976856, 0.0317538559, 0.0547041418,  
 0.0789365461, 0.1005910203, 0.1159035278, 0.1221074660, 0.1182160744,  
 0.1053521327, 0.0864042588, 0.0651228211, 0.0450078296, 0.0284478528,  
 0.0163966526, 0.0085905380, 0.0040787503, 0.0017491805, 0.0006754038,  
 0.0002343581]  
 283 - [0.0023495253, 0.0134661289, 0.0318522549, 0.0547967858,  
 0.0789854350, 0.1005695308, 0.1158070821, 0.1219567574, 0.1180505173,  
 0.1052147568, 0.0863256581, 0.0651112662, 0.0450500452, 0.0285186260,  
 0.0164708846, 0.0086516861, 0.0041208250, 0.0017740228, 0.0006881299,  
 0.0002400818]  
 284 - [0.0023691468, 0.0135345722, 0.0319506539, 0.0548894298,  
 0.0790343239, 0.1005480414, 0.1157106364, 0.1218060487, 0.1178849601,  
 0.1050773809, 0.0862470573, 0.0650997114, 0.0450922609, 0.0285893992,  
 0.0165451167, 0.0087128341, 0.0041628996, 0.0017988652, 0.0007008560,  
 0.0002458055]  
 285 - [0.0023887682, 0.0136030154, 0.0320490528, 0.0549820738,  
 0.0790832128, 0.1005265520, 0.1156141906, 0.1216553400, 0.1177194029,  
 0.1049400051, 0.0861684566, 0.0650881565, 0.0451344765, 0.0286601724,  
 0.0166193488, 0.0087739822, 0.0042049743, 0.0018237076, 0.0007135820,  
 0.0002515292]  
 286 - [0.0024083897, 0.0136714587, 0.0321474518, 0.0550747179,  
 0.0791321017, 0.1005050625, 0.1155177449, 0.1215046314, 0.1175538457,  
 0.1048026292, 0.0860898559, 0.0650766016, 0.0451766922, 0.0287309455,  
 0.0166935809, 0.0088351303, 0.0042470490, 0.0018485499, 0.0007263081,  
 0.0002572530]  
 287 - [0.0024280112, 0.0137399020, 0.0322458508, 0.0551673619,  
 0.0791809906, 0.1004835731, 0.1154212992, 0.1213539228, 0.1173882885,

0.1046652533, 0.0860112551, 0.0650650468, 0.0452189079, 0.0288017187,  
 0.0167678130, 0.0088962783, 0.0042891236, 0.0018733922, 0.0007390342,  
 0.0002629767]  
 288 - [0.0024476327, 0.0138083453, 0.0323442498, 0.0552600059,  
 0.0792298795, 0.1004620836, 0.1153248535, 0.1212032141, 0.1172227314,  
 0.1045278774, 0.0859326544, 0.0650534919, 0.0452611235, 0.0288724919,  
 0.0168420450, 0.0089574264, 0.0043311983, 0.0018982346, 0.0007517603,  
 0.0002687004]  
 289 - [0.0024672542, 0.0138767885, 0.0324426488, 0.0553526499,  
 0.0792787684, 0.1004405942, 0.1152284078, 0.1210525054, 0.1170571742,  
 0.1043905016, 0.0858540537, 0.0650419371, 0.0453033392, 0.0289432650,  
 0.0169162771, 0.0090185744, 0.0043732730, 0.0019230770, 0.0007644864,  
 0.0002744241]  
 290 - [0.0024868757, 0.0139452318, 0.0325410478, 0.0554452939,  
 0.0793276572, 0.1004191048, 0.1151319621, 0.1209017968, 0.1168916170,  
 0.1042531257, 0.0857754529, 0.0650303822, 0.0453455549, 0.0290140382,  
 0.0169905092, 0.0090797225, 0.0044153476, 0.0019479193, 0.0007772125,  
 0.0002801478]  
 291 - [0.0025064972, 0.0140136751, 0.0326394468, 0.0555379380,  
 0.0793765461, 0.1003976153, 0.1150355164, 0.1207510882, 0.1167260598,  
 0.1041157498, 0.0856968522, 0.0650188274, 0.0453877705, 0.0290848114,  
 0.0170647413, 0.0091408705, 0.0044574223, 0.0019727616, 0.0007899386,  
 0.0002858716]  
 292 - [0.0025261187, 0.0140821184, 0.0327378458, 0.0556305820,  
 0.0794254350, 0.1003761259, 0.1149390706, 0.1206003795, 0.1165605027,  
 0.1039783739, 0.0856182515, 0.0650072725, 0.0454299862, 0.0291555846,  
 0.0171389733, 0.0092020186, 0.0044994970, 0.0019976040, 0.0008026646,  
 0.0002915953]  
 293 - [0.0025457402, 0.0141505616, 0.0328362448, 0.0557232260,  
 0.0794743239, 0.1003546364, 0.1148426249, 0.1204496708, 0.1163949455,  
 0.1038409981, 0.0855396507, 0.0649957177, 0.0454722018, 0.0292263577,  
 0.0172132054, 0.0092631666, 0.0045415716, 0.0020224464, 0.0008153907,  
 0.0002973190]  
 294 - [0.0025653617, 0.0142190049, 0.0329346438, 0.0558158700,  
 0.0795232128, 0.1003331470, 0.1147461792, 0.1202989622, 0.1162293883,  
 0.1037036222, 0.0854610500, 0.0649841628, 0.0455144175, 0.0292971309,  
 0.0172874375, 0.0093243147, 0.0045836463, 0.0020472887, 0.0008281168,  
 0.0003030427]  
 295 - [0.0025970640, 0.0143171221, 0.0330606227, 0.0559144566,  
 0.0795443437, 0.1002488400, 0.1145620391, 0.1200540246, 0.1159833543,  
 0.1035155933, 0.0853692038, 0.0649948680, 0.0456051214, 0.0294296313,  
 0.0174239864, 0.0094392776, 0.0046664406, 0.0020994369, 0.0008571899,  
 0.0003173836]  
 296 - [0.0026287663, 0.0144152394, 0.0331866017, 0.0560130432,  
 0.0795654746, 0.1001645330, 0.1143778991, 0.1198090869, 0.1157373203,  
 0.1033275644, 0.0852773576, 0.0650055732, 0.0456958252, 0.0295621317,  
 0.0175605354, 0.0095542404, 0.0047492349, 0.0021515850, 0.0008862630,  
 0.0003317246]  
 297 - [0.0026604687, 0.0145133566, 0.0333125806, 0.0561116298,  
 0.0795866055, 0.1000802260, 0.1141937590, 0.1195641493, 0.1154912863,  
 0.1031395355, 0.0851855114, 0.0650162784, 0.0457865291, 0.0296946321,  
 0.0176970843, 0.0096692032, 0.0048320292, 0.0022037332, 0.0009153361,  
 0.0003460655]  
 298 - [0.0026921710, 0.0146114739, 0.0334385596, 0.0562102164,  
 0.0796077364, 0.0999959190, 0.1140096190, 0.1193192117, 0.1152452523,  
 0.1029515066, 0.0850936652, 0.0650269836, 0.0458772330, 0.0298271325,  
 0.0178336333, 0.0097841661, 0.0049148235, 0.0022558813, 0.0009444092,  
 0.0003604065]  
 299 - [0.0027238733, 0.0147095911, 0.0335645385, 0.0563088030,  
 0.0796288673, 0.0999116120, 0.1138254789, 0.1190742740, 0.1149992183,

0.1027634777, 0.0850018190, 0.0650376889, 0.0459679368, 0.0299596330,  
 0.0179701822, 0.0098991289, 0.0049976178, 0.0023080295, 0.0009734823,  
 0.0003747474]  
 300 - [0.0027555756, 0.0148077083, 0.0336905174, 0.0564073896,  
 0.0796499982, 0.0998273050, 0.1136413388, 0.1188293364, 0.1147531843,  
 0.1025754488, 0.0849099728, 0.0650483941, 0.0460586407, 0.0300921334,  
 0.0181067312, 0.0100140918, 0.0050804121, 0.0023601776, 0.0010025554,  
 0.0003890883]  
 301 - [0.0027872780, 0.0149058256, 0.0338164964, 0.0565059762,  
 0.0796711291, 0.0997429980, 0.1134571988, 0.1185843988, 0.1145071503,  
 0.1023874199, 0.0848181266, 0.0650590993, 0.0461493446, 0.0302246338,  
 0.0182432801, 0.0101290546, 0.0051632064, 0.0024123258, 0.0010316285,  
 0.0004034293]  
 302 - [0.0028189803, 0.0150039428, 0.0339424753, 0.0566045628,  
 0.0796922600, 0.0996586910, 0.1132730587, 0.1183394611, 0.1142611163,  
 0.1021993910, 0.0847262804, 0.0650698045, 0.0462400484, 0.0303571342,  
 0.0183798291, 0.0102440175, 0.0052460007, 0.0024644739, 0.0010607016,  
 0.0004177702]  
 303 - [0.0028506826, 0.0151020600, 0.0340684543, 0.0567031494,  
 0.0797133910, 0.0995743840, 0.1130889186, 0.1180945235, 0.1140150822,  
 0.1020113622, 0.0846344342, 0.0650805097, 0.0463307523, 0.0304896346,  
 0.0185163780, 0.0103589804, 0.0053287950, 0.0025166221, 0.0010897746,  
 0.0004321112]  
 304 - [0.0028823849, 0.0152001773, 0.0341944332, 0.0568017361,  
 0.0797345219, 0.0994900770, 0.1129047786, 0.1178495859, 0.1137690482,  
 0.1018233333, 0.0845425881, 0.0650912149, 0.0464214562, 0.0306221350,  
 0.0186529269, 0.0104739432, 0.0054115893, 0.0025687703, 0.0011188477,  
 0.0004464521]  
 305 - [0.0029140872, 0.0152982945, 0.0343204121, 0.0569003227,  
 0.0797556528, 0.0994057700, 0.1127206385, 0.1176046482, 0.1135230142,  
 0.1016353044, 0.0844507419, 0.0651019201, 0.0465121600, 0.0307546354,  
 0.0187894759, 0.0105889060, 0.0054943836, 0.0026209184, 0.0011479208,  
 0.0004607930]  
 306 - [0.0029457896, 0.0153964118, 0.0344463911, 0.0569989093,  
 0.0797767837, 0.0993214630, 0.1125364985, 0.1173597106, 0.1132769802,  
 0.1014472755, 0.0843588957, 0.0651126253, 0.0466028639, 0.0308871358,  
 0.0189260248, 0.0107038689, 0.0055771779, 0.0026730666, 0.0011769939,  
 0.0004751340]  
 307 - [0.0029774919, 0.0154945290, 0.0345723700, 0.0570974959,  
 0.0797979146, 0.0992371560, 0.1123523584, 0.1171147730, 0.1130309462,  
 0.1012592466, 0.0842670495, 0.0651233305, 0.0466935678, 0.0310196362,  
 0.0190625738, 0.0108188318, 0.0056599722, 0.0027252147, 0.0012060670,  
 0.0004894749]  
 308 - [0.0030091942, 0.0155926462, 0.0346983489, 0.0571960825,  
 0.0798190455, 0.0991528490, 0.1121682183, 0.1168698353, 0.1127849122,  
 0.1010712177, 0.0841752033, 0.0651340358, 0.0467842716, 0.0311521367,  
 0.0191991227, 0.0109337946, 0.0057427665, 0.0027773629, 0.0012351401,  
 0.0005038158]  
 309 - [0.0030408965, 0.0156907635, 0.0348243279, 0.0572946691,  
 0.0798401764, 0.0990685420, 0.1119840783, 0.1166248977, 0.1125388782,  
 0.1008831888, 0.0840833571, 0.0651447410, 0.0468749755, 0.0312846371,  
 0.0193356717, 0.0110487574, 0.0058255608, 0.0028295110, 0.0012642132,  
 0.0005181568]  
 310 - [0.0030725989, 0.0157888807, 0.0349503068, 0.0573932557,  
 0.0798613073, 0.0989842350, 0.1117999382, 0.1163799601, 0.1122928442,  
 0.1006951599, 0.0839915109, 0.0651554462, 0.0469656794, 0.0314171375,  
 0.0194722206, 0.0111637203, 0.0059083551, 0.0028816592, 0.0012932863,  
 0.0005324977]  
 311 - [0.0031043012, 0.0158869980, 0.0350762858, 0.0574918423,  
 0.0798824382, 0.0988999280, 0.1116157982, 0.1161350224, 0.1120468102,

0.1005071310, 0.0838996647, 0.0651661514, 0.0470563832, 0.0315496379,  
 0.0196087696, 0.0112786832, 0.0059911494, 0.0029338073, 0.0013223594,  
 0.0005468387]  
 312 - [0.0031360035, 0.0159851152, 0.0352022647, 0.0575904289,  
 0.0799035691, 0.0988156210, 0.1114316581, 0.1158900848, 0.1118007762,  
 0.1003191021, 0.0838078185, 0.0651768566, 0.0471470871, 0.0316821383,  
 0.0197453185, 0.0113936460, 0.0060739437, 0.0029859555, 0.0013514325,  
 0.0005611796]  
 313 - [0.0031896305, 0.0161370540, 0.0353895242, 0.0577377003,  
 0.0799479110, 0.0987246941, 0.1112135226, 0.115907258, 0.1114888532,  
 0.1000634167, 0.0836559409, 0.0651434640, 0.0472150708, 0.0318144261,  
 0.0199003605, 0.0115382893, 0.0061894212, 0.0030672117, 0.0014025652,  
 0.0005902179]  
 314 - [0.0032432575, 0.0162889928, 0.0355767837, 0.0578849716,  
 0.0799922528, 0.0986337673, 0.1109953871, 0.1152913667, 0.1111769301,  
 0.0998077313, 0.0835040633, 0.0651100714, 0.0472830545, 0.0319467139,  
 0.0200554024, 0.0116829326, 0.0063048987, 0.0031484680, 0.0014536980,  
 0.0006192562]  
 315 - [0.0032968844, 0.0164409316, 0.0357640432, 0.0580322430,  
 0.0800365947, 0.0985428404, 0.1107772516, 0.1149920077, 0.1108650070,  
 0.0995520459, 0.0833521857, 0.0650766788, 0.0473510381, 0.0320790017,  
 0.0202104444, 0.0118275760, 0.0064203761, 0.0032297242, 0.0015048308,  
 0.0006482946]  
 316 - [0.0033505114, 0.0165928704, 0.0359513027, 0.0581795144,  
 0.0800809366, 0.0984519136, 0.1105591161, 0.1146926487, 0.1105530840,  
 0.0992963605, 0.0832003080, 0.0650432862, 0.0474190218, 0.0322112895,  
 0.0203654864, 0.0119722193, 0.0065358536, 0.0033109805, 0.0015559635,  
 0.0006773329]  
 317 - [0.0034041384, 0.0167448092, 0.0361385621, 0.0583267857,  
 0.0801252785, 0.0983609867, 0.1103409806, 0.1143932897, 0.1102411610,  
 0.0990406751, 0.0830484304, 0.0650098936, 0.0474870055, 0.0323435773,  
 0.0205205283, 0.0121168626, 0.0066513311, 0.0033922367, 0.0016070962,  
 0.0007063712]  
 318 - [0.0034577654, 0.0168967480, 0.0363258216, 0.0584740571,  
 0.0801696203, 0.0982700598, 0.1101228451, 0.1140939306, 0.1099292379,  
 0.0987849897, 0.0828965528, 0.0649765010, 0.0475549892, 0.0324758651,  
 0.0206755703, 0.0122615059, 0.0067668086, 0.0034734929, 0.0016582290,  
 0.0007354095]  
 319 - [0.0035113924, 0.0170486868, 0.0365130811, 0.0586213285,  
 0.0802139622, 0.0981791330, 0.1099047096, 0.1137945716, 0.1096173148,  
 0.0985293043, 0.0827446752, 0.0649431084, 0.0476229729, 0.0326081529,  
 0.0208306123, 0.0124061493, 0.0068822860, 0.0035547492, 0.0017093618,  
 0.0007644478]  
 320 - [0.0035650194, 0.0172006256, 0.0367003406, 0.0587685998,  
 0.0802583041, 0.0980882061, 0.1096865741, 0.1134952126, 0.1093053918,  
 0.0982736189, 0.0825927976, 0.0649097158, 0.0476909566, 0.0327404407,  
 0.0209856542, 0.0125507926, 0.0069977635, 0.0036360054, 0.0017604945,  
 0.0007934861]  
 321 - [0.0036186464, 0.0173525644, 0.0368876001, 0.0589158712,  
 0.0803026460, 0.0979972793, 0.1094684386, 0.1131958536, 0.1089934688,  
 0.0980179335, 0.0824409200, 0.0648763232, 0.0477589402, 0.0328727286,  
 0.0211406962, 0.0126954359, 0.0071132410, 0.0037172616, 0.0018116272,  
 0.0008225244]  
 322 - [0.0036722733, 0.0175045031, 0.0370748596, 0.0590631426,  
 0.0803469878, 0.0979063524, 0.1092503032, 0.1128964945, 0.1086815457,  
 0.0977622480, 0.0822890423, 0.0648429306, 0.0478269239, 0.0330050164,  
 0.0212957382, 0.0128400792, 0.0072287185, 0.0037985179, 0.0018627600,  
 0.0008515628]  
 323 - [0.0037259003, 0.0176564419, 0.0372621191, 0.0592104139,  
 0.0803913297, 0.0978154255, 0.1090321677, 0.1125971355, 0.1083696226,

0.0975065626, 0.0821371647, 0.0648095380, 0.0478949076, 0.0331373042,  
 0.0214507801, 0.0129847225, 0.0073441960, 0.0038797741, 0.0019138928,  
 0.0008806011]  
 324 - [0.0037795273, 0.0178083807, 0.0374493786, 0.0593576853,  
 0.0804356716, 0.0977244987, 0.1088140322, 0.1122977765, 0.1080576996,  
 0.0972508772, 0.0819852871, 0.0647761454, 0.0479628913, 0.0332695920,  
 0.0216058221, 0.0131293659, 0.0074596734, 0.0039610304, 0.0019650255,  
 0.0009096394]  
 325 - [0.0038331543, 0.0179603195, 0.0376366381, 0.0595049567,  
 0.0804800134, 0.0976335718, 0.1085958967, 0.1119984174, 0.1077457766,  
 0.0969951918, 0.0818334095, 0.0647427528, 0.0480308750, 0.0334018798,  
 0.0217608641, 0.0132740092, 0.0075751509, 0.0040422866, 0.0020161583,  
 0.0009386777]  
 326 - [0.0038867813, 0.0181122583, 0.0378238975, 0.0596522280,  
 0.0805243553, 0.0975426449, 0.1083777612, 0.1116990584, 0.1074338535,  
 0.0967395064, 0.0816815319, 0.0647093602, 0.0480988587, 0.0335341676,  
 0.0219159060, 0.0134186525, 0.0076906284, 0.0041235428, 0.0020672910,  
 0.0009677160]  
 327 - [0.0039404082, 0.0182641971, 0.0380111570, 0.0597994994,  
 0.0805686972, 0.0974517181, 0.1081596257, 0.1113996994, 0.1071219304,  
 0.0964838210, 0.0815296542, 0.0646759676, 0.0481668424, 0.0336664554,  
 0.0220709480, 0.0135632958, 0.0078061059, 0.0042047991, 0.0021184238,  
 0.0009967544]  
 328 - [0.0039940352, 0.0184161359, 0.0381984165, 0.0599467708,  
 0.0806130391, 0.0973607912, 0.1079414902, 0.1111003404, 0.1068100074,  
 0.0962281356, 0.0813777766, 0.0646425750, 0.0482348260, 0.0337987432,  
 0.0222259900, 0.0137079392, 0.0079215833, 0.0042860553, 0.0021695565,  
 0.0010257927]  
 329 - [0.0040476622, 0.0185680747, 0.0383856760, 0.0600940421,  
 0.0806573809, 0.0972698644, 0.1077233547, 0.1108009813, 0.1064980844,  
 0.0959724502, 0.0812258990, 0.0646091824, 0.0483028097, 0.0339310310,  
 0.0223810319, 0.0138525825, 0.0080370608, 0.0043673116, 0.0022206893,  
 0.0010548310]  
 330 - [0.0041012892, 0.0187200135, 0.0385729355, 0.0602413135,  
 0.0807017228, 0.0971789375, 0.1075052192, 0.1105016223, 0.1061861613,  
 0.0957167648, 0.0810740214, 0.0645757898, 0.0483707934, 0.0340633188,  
 0.0225360739, 0.0139972258, 0.0081525383, 0.0044485678, 0.0022718220,  
 0.0010838693]  
 331 - [0.0041776442, 0.0189199224, 0.0388171116, 0.0604485402,  
 0.0808075678, 0.0971497852, 0.1073445299, 0.1102462466, 0.1058933719,  
 0.0954467227, 0.0808731186, 0.0644667594, 0.0483516450, 0.0341133243,  
 0.0226263084, 0.0140998306, 0.0082472653, 0.0045245484, 0.0023263406,  
 0.0011194172]  
 332 - [0.0042539992, 0.0191198314, 0.0390612876, 0.0606557670,  
 0.0809134127, 0.0971206330, 0.1071838405, 0.1099908708, 0.1056005825,  
 0.0951766806, 0.0806722158, 0.0643577289, 0.0483324965, 0.0341633297,  
 0.0227165428, 0.0142024355, 0.0083419923, 0.0046005290, 0.0023808591,  
 0.0011549652]  
 333 - [0.0043303542, 0.0193197404, 0.0393054637, 0.0608629937,  
 0.0810192577, 0.0970914807, 0.1070231512, 0.1097354951, 0.1053077930,  
 0.0949066385, 0.0804713129, 0.0642486985, 0.0483133481, 0.0342133352,  
 0.0228067773, 0.0143050403, 0.0084367193, 0.0046765096, 0.0024353777,  
 0.0011905131]  
 334 - [0.0044067092, 0.0195196493, 0.0395496397, 0.0610702204,  
 0.0811251026, 0.0970623285, 0.1068624619, 0.1094801194, 0.1050150036,  
 0.0946365964, 0.0802704101, 0.0641396681, 0.0482941996, 0.0342633406,  
 0.0228970117, 0.0144076452, 0.0085314463, 0.0047524902, 0.0024898963,  
 0.0012260610]  
 335 - [0.0044830641, 0.0197195582, 0.0397938158, 0.0612774472,  
 0.0812309476, 0.0970331762, 0.1067017726, 0.1092247437, 0.1047222142,



0.0943665542, 0.0800695073, 0.0640306376, 0.0482750512, 0.0343133461,  
0.0229872462, 0.0145102500, 0.0086261733, 0.0048284708, 0.0025444149,  
0.0012616090]

336 - [0.0045594191, 0.0199194672, 0.0400379918, 0.0614846739,  
0.0813367925, 0.0970040240, 0.1065410832, 0.1089693679, 0.1044294248,  
0.0940965121, 0.0798686045, 0.0639216072, 0.0482559027, 0.0343633515,  
0.0230774806, 0.0146128548, 0.0087209003, 0.0049044514, 0.0025989334,  
0.0012971569]

337 - [0.0046357741, 0.0201193762, 0.0402821678, 0.0616919006,  
0.0814426375, 0.0969748717, 0.1063803939, 0.1087139922, 0.1041366354,  
0.0938264700, 0.0796677016, 0.0638125768, 0.0482367543, 0.0344133570,  
0.0231677151, 0.0147154597, 0.0088156273, 0.0049804320, 0.0026534520,  
0.0013327048]

338 - [0.0047121291, 0.0203192851, 0.0405263439, 0.0618991274,  
0.0815484824, 0.0969457195, 0.1062197046, 0.1084586165, 0.1038438460,  
0.0935564279, 0.0794667988, 0.0637035463, 0.0482176058, 0.0344633624,  
0.0232579495, 0.0148180645, 0.0089103543, 0.0050564126, 0.0027079706,  
0.0013682528]

339 - [0.0047884841, 0.0205191941, 0.0407705200, 0.0621063541,  
0.0816543274, 0.0969165672, 0.1060590152, 0.1082032408, 0.1035510565,  
0.0932863858, 0.0792658960, 0.0635945159, 0.0481984574, 0.0345133679,  
0.0233481840, 0.0149206694, 0.0090050813, 0.0051323933, 0.0027624892,  
0.0014038007]

340 - [0.0048648391, 0.0207191030, 0.0410146960, 0.0623135808,  
0.0817601724, 0.0968874149, 0.1058983259, 0.1079478650, 0.1032582671,  
0.0930163437, 0.0790649932, 0.0634854855, 0.0481793090, 0.0345633734,  
0.0234384185, 0.0150232742, 0.0090998083, 0.0052083739, 0.0028170077,  
0.0014393486]

341 - [0.0049411941, 0.0209190120, 0.0412588720, 0.0625208076,  
0.0818660173, 0.0968582627, 0.1057376366, 0.1076924893, 0.1029654777,  
0.0927463016, 0.0788640904, 0.0633764550, 0.0481601605, 0.0346133788,  
0.0235286529, 0.0151258790, 0.0091945353, 0.0052843545, 0.0028715263,  
0.0014748966]

342 - [0.0050175491, 0.0211189209, 0.0415030481, 0.0627280343,  
0.0819718623, 0.0968291104, 0.1055769473, 0.1074371136, 0.1026726883,  
0.0924762595, 0.0786631875, 0.0632674246, 0.0481410121, 0.0346633843,  
0.0236188874, 0.0152284839, 0.0092892623, 0.0053603351, 0.0029260449,  
0.0015104445]

343 - [0.0050939041, 0.0213188298, 0.0417472242, 0.0629352610,  
0.0820777072, 0.0967999582, 0.1054162579, 0.1071817378, 0.1023798989,  
0.0922062174, 0.0784622847, 0.0631583942, 0.0481218636, 0.0347133897,  
0.0237091218, 0.0153310887, 0.0093839893, 0.0054363157, 0.0029805634,  
0.0015459924]

344 - [0.0051702590, 0.0215187388, 0.0419914002, 0.0631424878,  
0.0821835522, 0.0967708059, 0.1052555686, 0.1069263621, 0.1020871095,  
0.0919361752, 0.0782613819, 0.0630493637, 0.0481027152, 0.0347633952,  
0.0237993563, 0.0154336935, 0.0094787163, 0.0055122963, 0.0030350820,  
0.0015815404]

345 - [0.0052466140, 0.0217186478, 0.0422355762, 0.0633497145,  
0.0822893971, 0.0967416537, 0.1050948793, 0.1066709864, 0.1017943200,  
0.0916661331, 0.0780604791, 0.0629403333, 0.0480835667, 0.0348134006,  
0.0238895907, 0.0155362984, 0.0095734433, 0.0055882769, 0.0030896006,  
0.0016170883]

346 - [0.0053229690, 0.0219185567, 0.0424797523, 0.0635569412,  
0.0823952421, 0.0967125014, 0.1049341900, 0.1064156107, 0.1015015306,  
0.0913960910, 0.0778595762, 0.0628313029, 0.0480644183, 0.0348634061,  
0.0239798252, 0.0156389032, 0.0096681703, 0.0056642575, 0.0031441192,  
0.0016526362]

347 - [0.0053993240, 0.0221184656, 0.0427239283, 0.0637641680,  
0.0825010870, 0.0966833492, 0.1047735006, 0.1061602349, 0.1012087412,

0.0911260489, 0.0776586734, 0.0627222724, 0.0480452698, 0.0349134115,  
 0.0240700596, 0.0157415081, 0.0097628973, 0.0057402381, 0.0031986377,  
 0.0016881842]  
 348 - [0.0054756790, 0.0223183746, 0.0429681044, 0.0639713947,  
 0.0826069320, 0.0966541969, 0.1046128113, 0.1059048592, 0.1009159518,  
 0.0908560068, 0.0774577706, 0.0626132420, 0.0480261214, 0.0349634170,  
 0.0241602941, 0.0158441129, 0.0098576243, 0.0058162187, 0.0032531563,  
 0.0017237321]  
 349 - [0.0055447136, 0.0224946578, 0.0431922133, 0.0641851106,  
 0.0827626642, 0.0967222861, 0.1045857200, 0.1057958006, 0.1007528434,  
 0.0906726375, 0.0772852399, 0.0624733390, 0.0479288160, 0.0349080747,  
 0.0241388930, 0.0158455746, 0.0098712565, 0.0058338256, 0.0032695936,  
 0.0017367401]  
 350 - [0.0056137482, 0.0226709411, 0.0434163221, 0.0643988264,  
 0.0829183964, 0.0967903752, 0.1045586287, 0.1056867420, 0.1005897350,  
 0.0904892682, 0.0771127091, 0.0623334360, 0.0478315106, 0.0348527324,  
 0.0241174919, 0.0158470363, 0.0098848887, 0.0058514326, 0.0032860308,  
 0.0017497482]  
 351 - [0.0056827829, 0.0228472244, 0.0436404310, 0.0646125423,  
 0.0830741287, 0.0968584644, 0.1045315374, 0.1055776835, 0.1004266266,  
 0.0903058989, 0.0769401784, 0.0621935330, 0.0477342051, 0.0347973900,  
 0.0240960909, 0.0158484980, 0.0098985209, 0.0058690395, 0.0033024681,  
 0.0017627562]  
 352 - [0.0057518175, 0.0230235076, 0.0438645399, 0.0648262581,  
 0.0832298609, 0.0969265536, 0.1045044461, 0.1054686249, 0.1002635182,  
 0.0901225296, 0.0767676477, 0.0620536300, 0.0476368997, 0.0347420477,  
 0.0240746898, 0.0158499597, 0.0099121531, 0.0058866465, 0.0033189054,  
 0.0017757642]  
 353 - [0.0058208521, 0.0231997908, 0.0440886488, 0.0650399740,  
 0.0833855931, 0.0969946428, 0.1044773548, 0.1053595663, 0.1001004097,  
 0.0899391604, 0.0765951169, 0.0619137270, 0.0475395943, 0.0346867054,  
 0.0240532887, 0.0158514214, 0.0099257853, 0.0059042534, 0.0033353426,  
 0.0017887722]  
 354 - [0.0058898867, 0.0233760741, 0.0443127576, 0.0652536899,  
 0.0835413253, 0.0970627319, 0.1044502635, 0.1052505077, 0.0999373013,  
 0.0897557911, 0.0764225862, 0.0617738240, 0.0474422889, 0.0346313631,  
 0.0240318876, 0.0158528831, 0.0099394175, 0.0059218603, 0.0033517799,  
 0.0018017803]  
 355 - [0.0059589214, 0.0235523573, 0.0445368665, 0.0654674057,  
 0.0836970576, 0.0971308211, 0.1044231722, 0.1051414492, 0.0997741929,  
 0.0895724218, 0.0762500555, 0.0616339210, 0.0473449834, 0.0345760207,  
 0.0240104865, 0.0158543448, 0.0099530497, 0.0059394673, 0.0033682172,  
 0.0018147883]  
 356 - [0.0060279560, 0.0237286406, 0.0447609754, 0.0656811216,  
 0.0838527898, 0.0971989103, 0.1043960809, 0.1050323906, 0.0996110845,  
 0.0893890525, 0.0760775247, 0.0614940180, 0.0472476780, 0.0345206784,  
 0.0239890854, 0.0158558065, 0.0099666819, 0.0059570742, 0.0033846544,  
 0.0018277963]  
 357 - [0.0060969906, 0.0239049238, 0.0449850842, 0.0658948374,  
 0.0840085220, 0.0972669994, 0.1043689896, 0.1049233320, 0.0994479761,  
 0.0892056832, 0.0759049940, 0.0613541150, 0.0471503726, 0.0344653361,  
 0.0239676844, 0.0158572682, 0.0099803141, 0.0059746812, 0.0034010917,  
 0.0018408044]  
 358 - [0.0061660252, 0.0240812071, 0.0452091931, 0.0661085533,  
 0.0841642542, 0.0973350886, 0.1043418982, 0.1048142734, 0.0992848677,  
 0.0890223139, 0.0757324633, 0.0612142121, 0.0470530672, 0.0344099938,  
 0.0239462833, 0.0158587298, 0.0099939463, 0.0059922881, 0.0034175290,  
 0.0018538124]  
 359 - [0.0062350598, 0.0242574903, 0.0454333020, 0.0663222692,  
 0.0843199864, 0.0974031778, 0.1043148069, 0.1047052148, 0.0991217593,

0.0888389446, 0.0755599325, 0.0610743091, 0.0469557618, 0.0343546515,  
 0.0239248822, 0.0158601915, 0.0100075785, 0.0060098950, 0.0034339662,  
 0.0018668204]  
 360 - [0.0063040945, 0.0244337736, 0.0456574109, 0.0665359850,  
 0.0844757187, 0.0974712670, 0.1042877156, 0.1045961563, 0.0989586509,  
 0.0886555753, 0.0753874018, 0.0609344061, 0.0468584563, 0.0342993091,  
 0.0239034811, 0.0158616532, 0.0100212107, 0.0060275020, 0.0034504035,  
 0.0018798284]  
 361 - [0.0063731291, 0.0246100568, 0.0458815197, 0.0667497009,  
 0.0846314509, 0.0975393561, 0.1042606243, 0.1044870977, 0.0987955425,  
 0.0884722060, 0.0752148711, 0.0607945031, 0.0467611509, 0.0342439668,  
 0.0238820800, 0.0158631149, 0.0100348429, 0.0060451089, 0.0034668408,  
 0.0018928365]  
 362 - [0.0064421637, 0.0247863401, 0.0461056286, 0.0669634168,  
 0.0847871831, 0.0976074453, 0.1042335330, 0.1043780391, 0.0986324340,  
 0.0882888368, 0.0750423403, 0.0606546001, 0.0466638455, 0.0341886245,  
 0.0238606789, 0.0158645766, 0.0100484751, 0.0060627158, 0.0034832780,  
 0.0019058445]  
 363 - [0.0065111983, 0.0249626233, 0.0463297375, 0.0671771326,  
 0.0849429153, 0.0976755345, 0.1042064417, 0.1042689805, 0.0984693256,  
 0.0881054675, 0.0748698096, 0.0605146971, 0.0465665401, 0.0341332822,  
 0.0238392778, 0.0158660383, 0.0100621073, 0.0060803228, 0.0034997153,  
 0.0019188525]  
 364 - [0.0065802330, 0.0251389066, 0.0465538464, 0.0673908485,  
 0.0850986476, 0.0977436237, 0.1041793504, 0.1041599220, 0.0983062172,  
 0.0879220982, 0.0746972789, 0.0603747941, 0.0464692346, 0.0340779398,  
 0.0238178768, 0.0158675000, 0.0100757395, 0.0060979297, 0.0035161526,  
 0.0019318605]  
 365 - [0.0066492676, 0.0253151898, 0.0467779552, 0.0676045643,  
 0.0852543798, 0.0978117128, 0.1041522591, 0.1040508634, 0.0981431088,  
 0.0877387289, 0.0745247481, 0.0602348911, 0.0463719292, 0.0340225975,  
 0.0237964757, 0.0158689617, 0.0100893717, 0.0061155367, 0.0035325898,  
 0.0019448686]  
 366 - [0.0067183022, 0.0254914731, 0.0470020641, 0.0678182802,  
 0.0854101120, 0.0978798020, 0.1041251678, 0.1039418048, 0.0979800004,  
 0.0875553596, 0.0743522174, 0.0600949881, 0.0462746238, 0.0339672552,  
 0.0237750746, 0.0158704234, 0.0101030039, 0.0061331436, 0.0035490271,  
 0.0019578766]  
 367 - [0.0067524904, 0.0255795154, 0.0471182458, 0.0679367709,  
 0.0855082510, 0.0979415508, 0.1041432259, 0.1039176114, 0.0979222878,  
 0.0874770368, 0.0742668080, 0.0600140695, 0.0462058229, 0.0339141366,  
 0.0237376411, 0.0158463956, 0.0100891001, 0.0061260261, 0.0035460256,  
 0.0019569884]  
 368 - [0.0067866786, 0.0256675576, 0.0472344274, 0.0680552615,  
 0.0856063901, 0.0980032995, 0.1041612840, 0.1038934180, 0.0978645752,  
 0.0873987140, 0.0741813986, 0.0599331508, 0.0461370220, 0.0338610181,  
 0.0237002076, 0.0158223677, 0.0100751962, 0.0061189085, 0.0035430241,  
 0.0019561003]  
 369 - [0.0068208667, 0.0257555999, 0.0473506091, 0.0681737522,  
 0.0857045291, 0.0980650483, 0.1041793421, 0.1038692245, 0.0978068625,  
 0.0873203912, 0.0740959892, 0.0598522322, 0.0460682211, 0.0338078996,  
 0.0236627741, 0.0157983399, 0.0100612924, 0.0061117910, 0.0035400227,  
 0.0019552121]  
 370 - [0.0068550549, 0.0258436421, 0.0474667907, 0.0682922428,  
 0.0858026682, 0.0981267970, 0.1041974002, 0.1038450311, 0.0977491499,  
 0.0872420685, 0.0740105798, 0.0597713136, 0.0459994202, 0.0337547810,  
 0.0236253406, 0.0157743120, 0.0100473886, 0.0061046735, 0.0035370212,  
 0.0019543239]  
 371 - [0.0068892431, 0.0259316844, 0.0475829724, 0.0684107335,  
 0.0859008072, 0.0981885458, 0.1042154584, 0.1038208377, 0.0976914373,

0.0871637457, 0.0739251704, 0.0596903950, 0.0459306193, 0.0337016624,  
0.0235879072, 0.0157502842, 0.0100334848, 0.0060975560, 0.0035340197,  
0.0019534358]

372 - [0.0069234313, 0.0260197266, 0.0476991540, 0.0685292242,  
0.0859989463, 0.0982502945, 0.1042335165, 0.1037966443, 0.0976337247,  
0.0870854229, 0.0738397610, 0.0596094763, 0.0458618184, 0.0336485439,  
0.0235504737, 0.0157262563, 0.0100195809, 0.0060904384, 0.0035310182,  
0.0019525476]

373 - [0.0069576194, 0.0261077689, 0.0478153357, 0.0686477148,  
0.0860970853, 0.0983120433, 0.1042515746, 0.1037724508, 0.0975760120,  
0.0870071001, 0.0737543516, 0.0595285577, 0.0457930175, 0.0335954254,  
0.0235130402, 0.0157022285, 0.0100056771, 0.0060833209, 0.0035280168,  
0.0019516594]

374 - [0.0069918076, 0.0261958111, 0.0479315173, 0.0687662055,  
0.0861952244, 0.0983737920, 0.1042696327, 0.1037482574, 0.0975182994,  
0.0869287773, 0.0736689422, 0.0594476391, 0.0457242166, 0.0335423068,  
0.0234756067, 0.0156782006, 0.0099917733, 0.0060762034, 0.0035250153,  
0.0019507713]

375 - [0.0070259958, 0.0262838534, 0.0480476990, 0.0688846962,  
0.0862933634, 0.0984355408, 0.1042876908, 0.1037240640, 0.0974605868,  
0.0868504546, 0.0735835328, 0.0593667204, 0.0456554157, 0.0334891882,  
0.0234381732, 0.0156541728, 0.0099778694, 0.0060690858, 0.0035220138,  
0.0019498831]

376 - [0.0070601840, 0.0263718957, 0.0481638807, 0.0690031868,  
0.0863915024, 0.0984972895, 0.1043057489, 0.1036998706, 0.0974028742,  
0.0867721318, 0.0734981233, 0.0592858018, 0.0455866147, 0.0334360697,  
0.0234007397, 0.0156301450, 0.0099639656, 0.0060619683, 0.0035190123,  
0.0019489949]

377 - [0.0070943722, 0.0264599379, 0.0482800623, 0.0691216775,  
0.0864896415, 0.0985590383, 0.1043238070, 0.1036756772, 0.0973451616,  
0.0866938090, 0.0734127139, 0.0592048832, 0.0455178138, 0.0333829512,  
0.0233633062, 0.0156061171, 0.0099500618, 0.0060548508, 0.0035160108,  
0.0019481068]

378 - [0.0071285603, 0.0265479802, 0.0483962440, 0.0692401681,  
0.0865877805, 0.0986207870, 0.1043418651, 0.1036514837, 0.0972874489,  
0.0866154862, 0.0733273045, 0.0591239646, 0.0454490129, 0.0333298326,  
0.0233258727, 0.0155820893, 0.0099361580, 0.0060477333, 0.0035130094,  
0.0019472186]

379 - [0.0071627485, 0.0266360224, 0.0485124256, 0.0693586588,  
0.0866859196, 0.0986825358, 0.1043599232, 0.1036272903, 0.0972297363,  
0.0865371634, 0.0732418951, 0.0590430459, 0.0453802120, 0.0332767141,  
0.0232884392, 0.0155580614, 0.0099222541, 0.0060406157, 0.0035100079,  
0.0019463304]

380 - [0.0071969367, 0.0267240647, 0.0486286073, 0.0694771495,  
0.0867840586, 0.0987442845, 0.1043779814, 0.1036030969, 0.0971720237,  
0.0864588406, 0.0731564857, 0.0589621273, 0.0453114111, 0.0332235955,  
0.0232510058, 0.0155340336, 0.0099083503, 0.0060334982, 0.0035070064,  
0.0019454423]

381 - [0.0072311249, 0.0268121069, 0.0487447889, 0.0695956401,  
0.0868821977, 0.0988060333, 0.1043960395, 0.1035789035, 0.0971143111,  
0.0863805178, 0.0730710763, 0.0588812087, 0.0452426102, 0.0331704770,  
0.0232135723, 0.0155100057, 0.0098944465, 0.0060263807, 0.0035040049,  
0.0019445541]

382 - [0.0072653130, 0.0269001492, 0.0488609706, 0.0697141308,  
0.0869803367, 0.0988677820, 0.1044140976, 0.1035547100, 0.0970565984,  
0.0863021951, 0.0729856669, 0.0588002901, 0.0451738093, 0.0331173584,  
0.0231761388, 0.0154859779, 0.0098805427, 0.0060192632, 0.0035010035,  
0.0019436659]

383 - [0.0072995012, 0.0269881914, 0.0489771522, 0.0698326214,  
0.0870784758, 0.0989295308, 0.1044321557, 0.1035305166, 0.0969988858,

0.0862238723, 0.0729002575, 0.0587193714, 0.0451050084, 0.0330642399,  
 0.0231387053, 0.0154619500, 0.0098666388, 0.0060121456, 0.0034980020,  
 0.0019427778]  
 384 - [0.0073336894, 0.0270762337, 0.0490933339, 0.0699511121,  
 0.0871766148, 0.0989912795, 0.1044502138, 0.1035063232, 0.0969411732,  
 0.0861455495, 0.0728148481, 0.0586384528, 0.0450362075, 0.0330111213,  
 0.0231012718, 0.0154379222, 0.0098527350, 0.0060050281, 0.0034950005,  
 0.0019418896]  
 385 - [0.0073170709, 0.0270410096, 0.0490576757, 0.0699284467,  
 0.0871745553, 0.0990112830, 0.1044878493, 0.1035534315, 0.0969881942,  
 0.0861841306, 0.0728397329, 0.0586479262, 0.0450320435, 0.0329970297,  
 0.0230818462, 0.0154173095, 0.0098338598, 0.0059894832, 0.0034833262,  
 0.0019337961]  
 386 - [0.0073004524, 0.0270057854, 0.0490220175, 0.0699057813,  
 0.0871724958, 0.0990312865, 0.1045254848, 0.1036005398, 0.0970352152,  
 0.0862227117, 0.0728646177, 0.0586573996, 0.0450278795, 0.0329829381,  
 0.0230624207, 0.0153966969, 0.0098149845, 0.0059739383, 0.0034716519,  
 0.0019257026]  
 387 - [0.0072838340, 0.0269705613, 0.0489863592, 0.0698831158,  
 0.0871704363, 0.0990512899, 0.1045631203, 0.1036476481, 0.0970822361,  
 0.0862612928, 0.0728895024, 0.0586668730, 0.0450237156, 0.0329688464,  
 0.0230429951, 0.0153760842, 0.0097961093, 0.0059583933, 0.0034599777,  
 0.0019176090]  
 388 - [0.0072672155, 0.0269353372, 0.0489507010, 0.0698604504,  
 0.0871683769, 0.0990712934, 0.1046007558, 0.1036947564, 0.0971292571,  
 0.0862998739, 0.0729143872, 0.0586763464, 0.0450195516, 0.0329547548,  
 0.0230235695, 0.0153554716, 0.0097772340, 0.0059428484, 0.0034483034,  
 0.0019095155]  
 389 - [0.0072505970, 0.0269001130, 0.0489150428, 0.0698377850,  
 0.0871663174, 0.0990912969, 0.1046383913, 0.1037418646, 0.0971762781,  
 0.0863384550, 0.0729392720, 0.0586858198, 0.0450153876, 0.0329406632,  
 0.0230041440, 0.0153348589, 0.0097583588, 0.0059273035, 0.0034366291,  
 0.0019014220]  
 390 - [0.0072339785, 0.0268648889, 0.0488793846, 0.0698151196,  
 0.0871642579, 0.0991113004, 0.1046760268, 0.1037889729, 0.0972232991,  
 0.0863770361, 0.0729641568, 0.0586952932, 0.0450112236, 0.0329265716,  
 0.0229847184, 0.0153142463, 0.0097394836, 0.0059117586, 0.0034249548,  
 0.0018933285]  
 391 - [0.0072173600, 0.0268296648, 0.0488437263, 0.0697924541,  
 0.0871621984, 0.0991313038, 0.1047136623, 0.1038360812, 0.0972703200,  
 0.0864156172, 0.0729890415, 0.0587047666, 0.0450070596, 0.0329124799,  
 0.0229652928, 0.0152936336, 0.0097206083, 0.0058962136, 0.0034132806,  
 0.0018852349]  
 392 - [0.0072007415, 0.0267944406, 0.0488080681, 0.0697697887,  
 0.0871601389, 0.0991513073, 0.1047512978, 0.1038831895, 0.0973173410,  
 0.0864541983, 0.0730139263, 0.0587142400, 0.0450028956, 0.0328983883,  
 0.0229458673, 0.0152730210, 0.0097017331, 0.0058806687, 0.0034016063,  
 0.0018771414]  
 393 - [0.0071841230, 0.0267592165, 0.0487724099, 0.0697471233,  
 0.0871580794, 0.0991713108, 0.1047889334, 0.1039302978, 0.0973643620,  
 0.0864927794, 0.0730388111, 0.0587237134, 0.0449987316, 0.0328842967,  
 0.0229264417, 0.0152524083, 0.0096828578, 0.0058651238, 0.0033899320,  
 0.0018690479]  
 394 - [0.0071675046, 0.0267239924, 0.0487367517, 0.0697244579,  
 0.0871560200, 0.0991913143, 0.1048265689, 0.1039774061, 0.0974113830,  
 0.0865313606, 0.0730636959, 0.0587331868, 0.0449945677, 0.0328702051,  
 0.0229070161, 0.0152317956, 0.0096639826, 0.0058495789, 0.0033782577,  
 0.0018609544]  
 395 - [0.0071508861, 0.0266887682, 0.0487010935, 0.0697017925,  
 0.0871539605, 0.0992113178, 0.1048642044, 0.1040245144, 0.0974584040,

0.0865699417, 0.0730885807, 0.0587426602, 0.0449904037, 0.0328561135,  
 0.0228875906, 0.0152111830, 0.0096451074, 0.0058340340, 0.0033665834,  
 0.0018528609]  
 396 - [0.0071342676, 0.0266535441, 0.0486654352, 0.0696791270,  
 0.0871519010, 0.0992313212, 0.1049018399, 0.1040716227, 0.0975054249,  
 0.0866085228, 0.0731134654, 0.0587521336, 0.0449862397, 0.0328420218,  
 0.0228681650, 0.0151905703, 0.0096262321, 0.0058184890, 0.0033549092,  
 0.0018447673]  
 397 - [0.0071176491, 0.0266183200, 0.0486297770, 0.0696564616,  
 0.0871498415, 0.0992513247, 0.1049394754, 0.1041187310, 0.0975524459,  
 0.0866471039, 0.0731383502, 0.0587616070, 0.0449820757, 0.0328279302,  
 0.0228487394, 0.0151699577, 0.0096073569, 0.0058029441, 0.0033432349,  
 0.0018366738]  
 398 - [0.0071010306, 0.0265830958, 0.0485941188, 0.0696337962,  
 0.0871477820, 0.0992713282, 0.1049771109, 0.1041658392, 0.0975994669,  
 0.0866856850, 0.0731632350, 0.0587710804, 0.0449779117, 0.0328138386,  
 0.0228293139, 0.0151493450, 0.0095884817, 0.0057873992, 0.0033315606,  
 0.0018285803]  
 399 - [0.0070844122, 0.0265478717, 0.0485584606, 0.0696111308,  
 0.0871457226, 0.0992913317, 0.1050147464, 0.1042129475, 0.0976464879,  
 0.0867242661, 0.0731881198, 0.0587805538, 0.0449737478, 0.0327997470,  
 0.0228098883, 0.0151287324, 0.0095696064, 0.0057718543, 0.0033198863,  
 0.0018204868]  
 400 - [0.0070677937, 0.0265126476, 0.0485228023, 0.0695884653,  
 0.0871436631, 0.0993113351, 0.1050523819, 0.1042600558, 0.0976935088,  
 0.0867628472, 0.0732130045, 0.0587900272, 0.0449695838, 0.0327856553,  
 0.0227904627, 0.0151081197, 0.0095507312, 0.0057563093, 0.0033082121,  
 0.0018123932]  
 401 - [0.0070511752, 0.0264774234, 0.0484871441, 0.0695657999,  
 0.0871416036, 0.0993313386, 0.1050900174, 0.1043071641, 0.0977405298,  
 0.0868014283, 0.0732378893, 0.0587995006, 0.0449654198, 0.0327715637,  
 0.0227710372, 0.0150875071, 0.0095318559, 0.0057407644, 0.0032965378,  
 0.0018042997]  
 402 - [0.0070345567, 0.0264421993, 0.0484514859, 0.0695431345,  
 0.0871395441, 0.0993513421, 0.1051276529, 0.1043542724, 0.0977875508,  
 0.0868400094, 0.0732627741, 0.0588089740, 0.0449612558, 0.0327574721,  
 0.0227516116, 0.0150668944, 0.0095129807, 0.0057252195, 0.0032848635,  
 0.0017962062]  
 403 - [0.0070140508, 0.0263906790, 0.0483858038, 0.0694793772,  
 0.0870910809, 0.0993264851, 0.1051290974, 0.1043790376, 0.0978287576,  
 0.0868889925, 0.0733112514, 0.0588508827, 0.0449931989, 0.0327789983,  
 0.0227639926, 0.0150724033, 0.0095141696, 0.0057242521, 0.0032830704,  
 0.0017944187]  
 404 - [0.0069935450, 0.0263391588, 0.0483201217, 0.0694156199,  
 0.0870426178, 0.0993016282, 0.1051305419, 0.1044038029, 0.0978699644,  
 0.0869379757, 0.0733597286, 0.0588927913, 0.0450251420, 0.0328005244,  
 0.0227763737, 0.0150779123, 0.0095153584, 0.0057232847, 0.0032812773,  
 0.0017926311]  
 405 - [0.0069730391, 0.0262876385, 0.0482544397, 0.0693518627,  
 0.0869941546, 0.0992767712, 0.1051319864, 0.1044285681, 0.0979111712,  
 0.0869869588, 0.0734082058, 0.0589347000, 0.0450570850, 0.0328220506,  
 0.0227887547, 0.0150834212, 0.0095165473, 0.0057223172, 0.0032794842,  
 0.0017908436]  
 406 - [0.0069525333, 0.0262361182, 0.0481887576, 0.0692881054,  
 0.0869456914, 0.0992519143, 0.1051334309, 0.1044533333, 0.0979523780,  
 0.0870359420, 0.0734566831, 0.0589766086, 0.0450890281, 0.0328435767,  
 0.0228011357, 0.0150889302, 0.0095177361, 0.0057213498, 0.0032776911,  
 0.0017890561]  
 407 - [0.0069320274, 0.0261845980, 0.0481230755, 0.0692243481,  
 0.0868972283, 0.0992270573, 0.1051348754, 0.1044780985, 0.0979935848,

0.0870849251, 0.0735051604, 0.0590185173, 0.0451209712, 0.0328651029,  
 0.0228135167, 0.0150944391, 0.0095189250, 0.0057203824, 0.0032758981,  
 0.0017872685]  
 408 - [0.0069115215, 0.0261330777, 0.0480573934, 0.0691605908,  
 0.0868487651, 0.0992022004, 0.1051363199, 0.1045028638, 0.0980347916,  
 0.0871339082, 0.0735536376, 0.0590604259, 0.0451529143, 0.0328866291,  
 0.0228258978, 0.0150999480, 0.0095201138, 0.0057194150, 0.0032741050,  
 0.0017854810]  
 409 - [0.0068910157, 0.0260815574, 0.0479917114, 0.0690968336,  
 0.0868003019, 0.0991773434, 0.1051377644, 0.1045276290, 0.0980759984,  
 0.0871828914, 0.0736021148, 0.0591023346, 0.0451848573, 0.0329081552,  
 0.0228382788, 0.0151054570, 0.0095213027, 0.0057184476, 0.0032723119,  
 0.0017836935]  
 410 - [0.0068705098, 0.0260300372, 0.0479260293, 0.0690330763,  
 0.0867518388, 0.0991524865, 0.1051392089, 0.1045523942, 0.0981172052,  
 0.0872318745, 0.0736505921, 0.0591442432, 0.0452168004, 0.0329296814,  
 0.0228506598, 0.0151109659, 0.0095224915, 0.0057174802, 0.0032705188,  
 0.0017819059]  
 411 - [0.0068500040, 0.0259785169, 0.0478603472, 0.0689693190,  
 0.0867033756, 0.0991276295, 0.1051406534, 0.1045771594, 0.0981584120,  
 0.0872808576, 0.0736990694, 0.0591861519, 0.0452487435, 0.0329512076,  
 0.0228630409, 0.0151164748, 0.0095236804, 0.0057165127, 0.0032687257,  
 0.0017801184]  
 412 - [0.0068294981, 0.0259269966, 0.0477946651, 0.0689055617,  
 0.0866549124, 0.0991027725, 0.1051420980, 0.1046019247, 0.0981996189,  
 0.0873298408, 0.0737475466, 0.0592280606, 0.0452806866, 0.0329727337,  
 0.0228754219, 0.0151219838, 0.0095248693, 0.0057155453, 0.0032669326,  
 0.0017783309]  
 413 - [0.0068089922, 0.0258754764, 0.0477289830, 0.0688418044,  
 0.0866064493, 0.0990779156, 0.1051435425, 0.1046266899, 0.0982408257,  
 0.0873788239, 0.0737960238, 0.0592699692, 0.0453126297, 0.0329942599,  
 0.0228878029, 0.0151274927, 0.0095260581, 0.0057145779, 0.0032651395,  
 0.0017765433]  
 414 - [0.0067884864, 0.0258239561, 0.0476633010, 0.0687780472,  
 0.0865579861, 0.0990530586, 0.1051449870, 0.1046514551, 0.0982820325,  
 0.0874278071, 0.0738445011, 0.0593118779, 0.0453445727, 0.0330157860,  
 0.0229001839, 0.0151330017, 0.0095272470, 0.0057136105, 0.0032633464,  
 0.0017747558]  
 415 - [0.0067679805, 0.0257724358, 0.0475976189, 0.0687142899,  
 0.0865095229, 0.0990282017, 0.1051464315, 0.1046762204, 0.0983232393,  
 0.0874767902, 0.0738929784, 0.0593537865, 0.0453765158, 0.0330373122,  
 0.0229125650, 0.0151385106, 0.0095284358, 0.0057126431, 0.0032615533,  
 0.0017729683]  
 416 - [0.0067474746, 0.0257209156, 0.0475319368, 0.0686505326,  
 0.0864610598, 0.0990033447, 0.1051478760, 0.1047009856, 0.0983644461,  
 0.0875257733, 0.0739414556, 0.0593956952, 0.0454084589, 0.0330588384,  
 0.0229249460, 0.0151440195, 0.0095296247, 0.0057116757, 0.0032597603,  
 0.0017711807]  
 417 - [0.0067269688, 0.0256693953, 0.0474662547, 0.0685867753,  
 0.0864125966, 0.0989784878, 0.1051493205, 0.1047257508, 0.0984056529,  
 0.0875747565, 0.0739899328, 0.0594376038, 0.0454404020, 0.0330803645,  
 0.0229373270, 0.0151495285, 0.0095308135, 0.0057107082, 0.0032579672,  
 0.0017693932]  
 418 - [0.0067064629, 0.0256178750, 0.0474005727, 0.0685230181,  
 0.0863641334, 0.0989536308, 0.1051507650, 0.1047505160, 0.0984468597,  
 0.0876237396, 0.0740384101, 0.0594795125, 0.0454723450, 0.0331018907,  
 0.0229497080, 0.0151550374, 0.0095320024, 0.0057097408, 0.0032561741,  
 0.0017676057]  
 419 - [0.0066859571, 0.0255663548, 0.0473348906, 0.0684592608,  
 0.0863156703, 0.0989287739, 0.1051522095, 0.1047752813, 0.0984880665,

0.0876727228, 0.0740868874, 0.0595214211, 0.0455042881, 0.0331234168,  
0.0229620891, 0.0151605464, 0.0095331912, 0.0057087734, 0.0032543810,  
0.0017658181]

420 - [0.0066654512, 0.0255148345, 0.0472692085, 0.0683955035,  
0.0862672071, 0.0989039169, 0.1051536540, 0.1048000465, 0.0985292733,  
0.0877217059, 0.0741353646, 0.0595633298, 0.0455362312, 0.0331449430,  
0.0229744701, 0.0151660553, 0.0095343801, 0.0057078060, 0.0032525879,  
0.0017640306]

421 - [0.0066746476, 0.0255492998, 0.0473305704, 0.0684781516,  
0.0863601717, 0.0989935302, 0.1052260421, 0.1048445099, 0.0985403882,  
0.0876999265, 0.0740865722, 0.0594967186, 0.0454621419, 0.0330724211,  
0.0229102161, 0.0151137075, 0.0094947421, 0.0056797393, 0.0032340145,  
0.0017524886]

422 - [0.0066838441, 0.0255837651, 0.0473919323, 0.0685607997,  
0.0864531362, 0.0990831434, 0.1052984302, 0.1048889733, 0.0985515032,  
0.0876781470, 0.0740377799, 0.0594301074, 0.0453880526, 0.0329998993,  
0.0228459621, 0.0150613597, 0.0094551041, 0.0056516727, 0.0032154411,  
0.0017409467]

423 - [0.0066930405, 0.0256182304, 0.0474532942, 0.0686434478,  
0.0865461008, 0.0991727567, 0.1053708183, 0.1049334367, 0.0985626181,  
0.0876563676, 0.0739889875, 0.0593634963, 0.0453139634, 0.0329273774,  
0.0227817081, 0.0150090120, 0.0094154661, 0.0056236060, 0.0031968677,  
0.0017294047]

424 - [0.0067022369, 0.0256526957, 0.0475146560, 0.0687260959,  
0.0866390653, 0.0992623700, 0.1054432064, 0.1049779001, 0.0985737331,  
0.0876345881, 0.0739401951, 0.0592968851, 0.0452398741, 0.0328548555,  
0.0227174540, 0.0149566642, 0.0093758281, 0.0055955394, 0.0031782943,  
0.0017178628]

425 - [0.0067114333, 0.0256871609, 0.0475760179, 0.0688087440,  
0.0867320299, 0.0993519833, 0.1055155944, 0.1050223635, 0.0985848480,  
0.0876128087, 0.0738914027, 0.0592302739, 0.0451657848, 0.0327823337,  
0.0226532000, 0.0149043164, 0.0093361901, 0.0055674727, 0.0031597208,  
0.0017063208]

426 - [0.0067206298, 0.0257216262, 0.0476373798, 0.0688913921,  
0.0868249944, 0.0994415965, 0.1055879825, 0.1050668269, 0.0985959630,  
0.0875910293, 0.0738426104, 0.0591636627, 0.0450916955, 0.0327098118,  
0.0225889460, 0.0148519686, 0.0092965521, 0.0055394061, 0.0031411474,  
0.0016947788]

427 - [0.0067298262, 0.0257560915, 0.0476987417, 0.0689740402,  
0.0869179590, 0.0995312098, 0.1056603706, 0.1051112903, 0.0986070779,  
0.0875692498, 0.0737938180, 0.0590970516, 0.0450176063, 0.0326372899,  
0.0225246920, 0.0147996208, 0.0092569141, 0.0055113394, 0.0031225740,  
0.0016832369]

428 - [0.0067390226, 0.0257905568, 0.0477601036, 0.0690566883,  
0.0870109235, 0.0996208231, 0.1057327587, 0.1051557537, 0.0986181929,  
0.0875474704, 0.0737450256, 0.0590304404, 0.0449435170, 0.0325647681,  
0.0224604380, 0.0147472730, 0.0092172761, 0.0054832728, 0.0031040006,  
0.0016716949]

429 - [0.0067482190, 0.0258250221, 0.0478214655, 0.0691393364,  
0.0871038881, 0.0997104364, 0.1058051468, 0.1052002172, 0.0986293078,  
0.0875256910, 0.0736962332, 0.0589638292, 0.0448694277, 0.0324922462,  
0.0223961840, 0.0146949252, 0.0091776382, 0.0054552061, 0.0030854272,  
0.0016601530]

430 - [0.0067574155, 0.0258594874, 0.0478828273, 0.0692219844,  
0.0871968527, 0.0998000496, 0.1058775349, 0.1052446806, 0.0986404227,  
0.0875039115, 0.0736474409, 0.0588972180, 0.0447953384, 0.0324197243,  
0.0223319299, 0.0146425775, 0.0091380002, 0.0054271394, 0.0030668538,  
0.0016486110]

431 - [0.0067666119, 0.0258939527, 0.0479441892, 0.0693046325,  
0.0872898172, 0.0998896629, 0.1059499230, 0.1052891440, 0.0986515377,



0.0874821321, 0.0735986485, 0.0588306068, 0.0447212491, 0.0323472025,  
 0.0222676759, 0.0145902297, 0.0090983622, 0.0053990728, 0.0030482804,  
 0.0016370690]  
 432 - [0.0067758083, 0.0259284180, 0.0480055511, 0.0693872806,  
 0.0873827818, 0.0999792762, 0.1060223111, 0.1053336074, 0.0986626526,  
 0.0874603526, 0.0735498561, 0.0587639957, 0.0446471599, 0.0322746806,  
 0.0222034219, 0.0145378819, 0.0090587242, 0.0053710061, 0.0030297070,  
 0.0016255271]  
 433 - [0.0067850048, 0.0259628833, 0.0480669130, 0.0694699287,  
 0.0874757463, 0.1000688894, 0.1060946992, 0.1053780708, 0.0986737676,  
 0.0874385732, 0.0735010638, 0.0586973845, 0.0445730706, 0.0322021587,  
 0.0221391679, 0.0144855341, 0.0090190862, 0.0053429395, 0.0030111336,  
 0.0016139851]  
 434 - [0.0067942012, 0.0259973485, 0.0481282749, 0.0695525768,  
 0.0875687109, 0.1001585027, 0.1061670872, 0.1054225342, 0.0986848825,  
 0.0874167938, 0.0734522714, 0.0586307733, 0.0444989813, 0.0321296369,  
 0.0220749139, 0.0144331863, 0.0089794482, 0.0053148728, 0.0029925601,  
 0.0016024431]  
 435 - [0.0068033976, 0.0260318138, 0.0481896368, 0.0696352249,  
 0.0876616754, 0.1002481160, 0.1062394753, 0.1054669976, 0.0986959975,  
 0.0873950143, 0.0734034790, 0.0585641621, 0.0444248920, 0.0320571150,  
 0.0220106598, 0.0143808386, 0.0089398102, 0.0052868062, 0.0029739867,  
 0.0015909012]  
 436 - [0.0068125940, 0.0260662791, 0.0482509986, 0.0697178730,  
 0.0877546400, 0.1003377293, 0.1063118634, 0.1055114610, 0.0987071124,  
 0.0873732349, 0.0733546866, 0.0584975510, 0.0443508028, 0.0319845931,  
 0.0219464058, 0.0143284908, 0.0089001722, 0.0052587395, 0.0029554133,  
 0.0015793592]  
 437 - [0.0068217905, 0.0261007444, 0.0483123605, 0.0698005211,  
 0.0878476045, 0.1004273425, 0.1063842515, 0.1055559244, 0.0987182274,  
 0.0873514554, 0.0733058943, 0.0584309398, 0.0442767135, 0.0319120713,  
 0.0218821518, 0.0142761430, 0.0088605342, 0.0052306729, 0.0029368399,  
 0.0015678173]  
 438 - [0.0068309869, 0.0261352097, 0.0483737224, 0.0698831692,  
 0.0879405691, 0.1005169558, 0.1064566396, 0.1056003878, 0.0987293423,  
 0.0873296760, 0.0732571019, 0.0583643286, 0.0442026242, 0.0318395494,  
 0.0218178978, 0.0142237952, 0.0088208962, 0.0052026062, 0.0029182665,  
 0.0015562753]  
 439 - [0.0069274385, 0.0264052525, 0.0487603976, 0.0703141381,  
 0.0883404390, 0.1008208900, 0.1066220729, 0.1056129551, 0.0986021907,  
 0.0870969419, 0.0729632250, 0.0580537632, 0.0439112813, 0.0315907473,  
 0.0216218179, 0.0140800500, 0.0087224297, 0.0051393683, 0.0028800502,  
 0.0015345511]  
 440 - [0.0070238901, 0.0266752953, 0.0491470727, 0.0707451070,  
 0.0887403089, 0.1011248242, 0.1067875061, 0.1056255224, 0.0984750390,  
 0.0868642077, 0.0726693480, 0.0577431978, 0.0436199384, 0.0313419453,  
 0.0214257379, 0.0139363047, 0.0086239632, 0.0050761304, 0.0028418339,  
 0.0015128269]  
 441 - [0.0071203416, 0.0269453380, 0.0495337479, 0.0711760759,  
 0.0891401788, 0.1014287583, 0.1069529394, 0.1056380898, 0.0983478874,  
 0.0866314736, 0.0723754711, 0.0574326324, 0.0433285955, 0.0310931432,  
 0.0212296580, 0.0137925595, 0.0085254968, 0.0050128926, 0.0028036176,  
 0.0014911027]  
 442 - [0.0072167932, 0.0272153808, 0.0499204230, 0.0716070448,  
 0.0895400487, 0.1017326925, 0.1071183726, 0.1056506571, 0.0982207357,  
 0.0863987394, 0.0720815942, 0.0571220671, 0.0430372526, 0.0308443411,  
 0.0210335780, 0.0136488143, 0.0084270303, 0.0049496547, 0.0027654013,  
 0.0014693785]  
 443 - [0.0073132448, 0.0274854236, 0.0503070982, 0.0720380138,  
 0.0899399185, 0.1020366267, 0.1072838059, 0.1056632244, 0.0980935841,

0.0861660053, 0.0717877173, 0.0568115017, 0.0427459096, 0.0305955391,  
 0.0208374981, 0.0135050691, 0.0083285638, 0.0048864168, 0.0027271850,  
 0.0014476543]  
 444 - [0.0074096964, 0.0277554664, 0.0506937733, 0.0724689827,  
 0.0903397884, 0.1023405609, 0.1074492391, 0.1056757917, 0.0979664325,  
 0.0859332712, 0.0714938403, 0.0565009363, 0.0424545667, 0.0303467370,  
 0.0206414182, 0.0133613238, 0.0082300973, 0.0048231789, 0.0026889687,  
 0.0014259301]  
 445 - [0.0075061480, 0.0280255091, 0.0510804485, 0.0728999516,  
 0.0907396583, 0.1026444950, 0.1076146724, 0.1056883590, 0.0978392808,  
 0.0857005370, 0.0711999634, 0.0561903709, 0.0421632238, 0.0300979349,  
 0.0204453382, 0.0132175786, 0.0081316309, 0.0047599411, 0.0026507524,  
 0.0014042059]  
 446 - [0.0076025996, 0.0282955519, 0.0514671236, 0.0733309205,  
 0.0911395282, 0.1029484292, 0.1077801056, 0.1057009263, 0.0977121292,  
 0.0854678029, 0.0709060865, 0.0558798055, 0.0418718809, 0.0298491329,  
 0.0202492583, 0.0130738334, 0.0080331644, 0.0046967032, 0.0026125361,  
 0.0013824817]  
 447 - [0.0076990512, 0.0285655947, 0.0518537988, 0.0737618894,  
 0.0915393981, 0.1032523634, 0.1079455389, 0.1057134936, 0.0975849776,  
 0.0852350688, 0.0706122096, 0.0555692402, 0.0415805380, 0.0296003308,  
 0.0200531784, 0.0129300882, 0.0079346979, 0.0046334653, 0.0025743198,  
 0.0013607576]  
 448 - [0.0077955027, 0.0288356375, 0.0522404740, 0.0741928583,  
 0.0919392680, 0.1035562976, 0.1081109722, 0.1057260610, 0.0974578259,  
 0.0850023346, 0.0703183326, 0.0552586748, 0.0412891951, 0.0293515287,  
 0.0198570984, 0.0127863429, 0.0078362314, 0.0045702274, 0.0025361035,  
 0.0013390334]  
 449 - [0.0078919543, 0.0291056803, 0.0526271491, 0.0746238272,  
 0.0923391379, 0.1038602318, 0.1082764054, 0.1057386283, 0.0973306743,  
 0.0847696005, 0.0700244557, 0.0549481094, 0.0409978522, 0.0291027267,  
 0.0196610185, 0.0126425977, 0.0077377649, 0.0045069895, 0.0024978872,  
 0.0013173092]  
 450 - [0.0079884059, 0.0293757230, 0.0530138243, 0.0750547961,  
 0.0927390078, 0.1041641659, 0.1084418387, 0.1057511956, 0.0972035226,  
 0.0845368663, 0.0697305788, 0.0546375440, 0.0407065093, 0.0288539246,  
 0.0194649385, 0.0124988525, 0.0076392985, 0.0044437517, 0.0024596709,  
 0.0012955850]  
 451 - [0.0080848575, 0.0296457658, 0.0534004994, 0.0754857650,  
 0.0931388777, 0.1044681001, 0.1086072719, 0.1057637629, 0.0970763710,  
 0.0843041322, 0.0694367018, 0.0543269786, 0.0404151664, 0.0286051225,  
 0.0192688586, 0.0123551072, 0.0075408320, 0.0043805138, 0.0024214546,  
 0.0012738608]  
 452 - [0.0081813091, 0.0299158086, 0.0537871746, 0.0759167340,  
 0.0935387475, 0.1047720343, 0.1087727052, 0.1057763302, 0.0969492194,  
 0.0840713981, 0.0691428249, 0.0540164132, 0.0401238234, 0.0283563205,  
 0.0190727787, 0.0122113620, 0.0074423655, 0.0043172759, 0.0023832383,  
 0.0012521366]  
 453 - [0.0082777606, 0.0301858514, 0.0541738497, 0.0763477029,  
 0.0939386174, 0.1050759685, 0.1089381384, 0.1057888976, 0.0968220677,  
 0.0838386639, 0.0688489480, 0.0537058478, 0.0398324805, 0.0281075184,  
 0.0188766987, 0.0120676168, 0.0073438990, 0.0042540380, 0.0023450220,  
 0.0012304124]  
 454 - [0.0083742122, 0.0304558941, 0.0545605249, 0.0767786718,  
 0.0943384873, 0.1053799026, 0.1091035717, 0.1058014649, 0.0966949161,  
 0.0836059298, 0.0685550711, 0.0533952825, 0.0395411376, 0.0278587163,  
 0.0186806188, 0.0119238716, 0.0072454326, 0.0041908002, 0.0023068057,  
 0.0012086882]  
 455 - [0.0084706638, 0.0307259369, 0.0549472000, 0.0772096407,  
 0.0947383572, 0.1056838368, 0.1092690049, 0.1058140322, 0.0965677644,

0.0833731956, 0.0682611941, 0.0530847171, 0.0392497947, 0.0276099143,  
 0.0184845388, 0.0117801263, 0.0071469661, 0.0041275623, 0.0022685894,  
 0.0011869640]  
 456 - [0.0085671154, 0.0309959797, 0.0553338752, 0.0776406096,  
 0.0951382271, 0.1059877710, 0.1094344382, 0.1058265995, 0.0964406128,  
 0.0831404615, 0.0679673172, 0.0527741517, 0.0389584518, 0.0273611122,  
 0.0182884589, 0.0116363811, 0.0070484996, 0.0040643244, 0.0022303731,  
 0.0011652398]  
 457 - [0.0087541343, 0.0314936817, 0.0560228624, 0.0783820326,  
 0.0957959297, 0.1064531099, 0.1096452615, 0.1057739962, 0.0961625491,  
 0.0827058107, 0.0674561214, 0.0522592842, 0.0384939574, 0.0269776097,  
 0.0179953245, 0.0114274855, 0.0069089971, 0.0039767701, 0.0021786692,  
 0.0011364126]  
 458 - [0.0089411532, 0.0319913837, 0.0567118496, 0.0791234556,  
 0.0964536324, 0.1069184488, 0.1098560847, 0.1057213929, 0.0958844854,  
 0.0822711598, 0.0669449255, 0.0517444167, 0.0380294631, 0.0265941073,  
 0.0177021901, 0.0112185900, 0.0067694946, 0.0038892157, 0.0021269653,  
 0.0011075853]  
 459 - [0.0091281722, 0.0324890856, 0.0574008368, 0.0798648786,  
 0.0971113350, 0.1073837876, 0.1100669080, 0.1056687896, 0.0956064218,  
 0.0818365090, 0.0664337297, 0.0512295492, 0.0375649687, 0.0262106048,  
 0.0174090557, 0.0110096944, 0.0066299921, 0.0038016614, 0.0020752614,  
 0.0010787581]  
 460 - [0.0093151911, 0.0329867876, 0.0580898240, 0.0806063016,  
 0.0977690376, 0.1078491265, 0.1102777313, 0.1056161863, 0.0953283581,  
 0.0814018581, 0.0659225339, 0.0507146818, 0.0371004744, 0.0258271023,  
 0.0171159213, 0.0108007989, 0.0064904896, 0.0037141070, 0.0020235575,  
 0.0010499309]  
 461 - [0.0095022100, 0.0334844896, 0.0587788113, 0.0813477246,  
 0.0984267403, 0.1083144654, 0.1104885545, 0.1055635829, 0.0950502944,  
 0.0809672073, 0.0654113381, 0.0501998143, 0.0366359800, 0.0254435999,  
 0.0168227868, 0.0105919033, 0.0063509872, 0.0036265527, 0.0019718537,  
 0.0010211037]  
 462 - [0.0096892289, 0.0339821916, 0.0594677985, 0.0820891476,  
 0.0990844429, 0.1087798043, 0.1106993778, 0.1055109796, 0.0947722307,  
 0.0805325565, 0.0649001422, 0.0496849468, 0.0361714856, 0.0250600974,  
 0.0165296524, 0.0103830078, 0.0062114847, 0.0035389983, 0.0019201498,  
 0.0009922764]  
 463 - [0.0098762479, 0.0344798936, 0.0601567857, 0.0828305706,  
 0.0997421455, 0.1092451432, 0.1109102011, 0.1054583763, 0.0944941671,  
 0.0800979056, 0.0643889464, 0.0491700793, 0.0357069913, 0.0246765949,  
 0.0162365180, 0.0101741122, 0.0060719822, 0.0034514440, 0.0018684459,  
 0.0009634492]  
 464 - [0.0100632668, 0.0349775956, 0.0608457729, 0.0835719936,  
 0.1003998482, 0.1097104821, 0.1111210243, 0.1054057730, 0.0942161034,  
 0.0796632548, 0.0638777506, 0.0486552118, 0.0352424969, 0.0242930925,  
 0.0159433836, 0.0099652167, 0.0059324797, 0.0033638896, 0.0018167420,  
 0.0009346220]  
 465 - [0.0102502857, 0.0354752976, 0.0615347601, 0.0843134165,  
 0.1010575508, 0.1101758210, 0.1113318476, 0.1053531697, 0.0939380397,  
 0.0792286040, 0.0633665548, 0.0481403444, 0.0347780026, 0.0239095900,  
 0.0156502492, 0.0097563211, 0.0057929772, 0.0032763353, 0.0017650381,  
 0.0009057948]  
 466 - [0.0104373046, 0.0359729995, 0.0622237473, 0.0850548395,  
 0.1017152534, 0.1106411598, 0.1115426709, 0.1053005664, 0.0936599760,  
 0.0787939531, 0.0628553589, 0.0476254769, 0.0343135082, 0.0235260875,  
 0.0153571148, 0.0095474255, 0.0056534747, 0.0031887810, 0.0017133342,  
 0.0008769675]  
 467 - [0.0106243235, 0.0364707015, 0.0629127345, 0.0857962625,  
 0.1023729561, 0.1111064987, 0.1117534941, 0.1052479631, 0.0933819123,

0.0783593023, 0.0623441631, 0.0471106094, 0.0338490138, 0.0231425851,  
 0.0150639804, 0.0093385300, 0.0055139722, 0.0031012266, 0.0016616303,  
 0.0008481403]  
 468 - [0.0108113425, 0.0369684035, 0.0636017217, 0.0865376855,  
 0.1030306587, 0.1115718376, 0.1119643174, 0.1051953598, 0.0931038487,  
 0.0779246514, 0.0618329673, 0.0465957419, 0.0333845195, 0.0227590826,  
 0.0147708460, 0.0091296344, 0.0053744697, 0.0030136723, 0.0016099264,  
 0.0008193131]  
 469 - [0.0109983614, 0.0374661055, 0.0642907089, 0.0872791085,  
 0.1036883613, 0.1120371765, 0.1121751407, 0.1051427565, 0.0928257850,  
 0.0774900006, 0.0613217714, 0.0460808744, 0.0329200251, 0.0223755801,  
 0.0144777116, 0.0089207389, 0.0052349672, 0.0029261179, 0.0015582225,  
 0.0007904858]  
 470 - [0.0111853803, 0.0379638075, 0.0649796962, 0.0880205315,  
 0.1043460640, 0.1125025154, 0.1123859639, 0.1050901531, 0.0925477213,  
 0.0770553498, 0.0608105756, 0.0455660069, 0.0324555307, 0.0219920777,  
 0.0141845771, 0.0087118433, 0.0050954648, 0.0028385636, 0.0015065187,  
 0.0007616586]  
 471 - [0.0113723992, 0.0384615094, 0.0656686834, 0.0887619545,  
 0.1050037666, 0.1129678542, 0.1125967872, 0.1050375498, 0.0922696576,  
 0.0766206989, 0.0602993798, 0.0450511394, 0.0319910364, 0.0216085752,  
 0.0138914427, 0.0085029478, 0.0049559623, 0.0027510092, 0.0014548148,  
 0.0007328314]  
 472 - [0.0115594182, 0.0389592114, 0.0663576706, 0.0895033775,  
 0.1056614692, 0.1134331931, 0.1128076105, 0.1049849465, 0.0919915940,  
 0.0761860481, 0.0597881840, 0.0445362720, 0.0315265420, 0.0212250727,  
 0.0135983083, 0.0082940522, 0.0048164598, 0.0026634549, 0.0014031109,  
 0.0007040042]  
 473 - [0.0117464371, 0.0394569134, 0.0670466578, 0.0902448005,  
 0.1063191719, 0.1138985320, 0.1130184337, 0.1049323432, 0.0917135303,  
 0.0757513972, 0.0592769881, 0.0440214045, 0.0310620477, 0.0208415703,  
 0.0133051739, 0.0080851567, 0.0046769573, 0.0025759005, 0.0013514070,  
 0.0006751769]  
 474 - [0.0119334560, 0.0399546154, 0.0677356450, 0.0909862235,  
 0.1069768745, 0.1143638709, 0.1132292570, 0.1048797399, 0.0914354666,  
 0.0753167464, 0.0587657923, 0.0435065370, 0.0305975533, 0.0204580678,  
 0.0130120395, 0.0078762611, 0.0045374548, 0.0024883462, 0.0012997031,  
 0.0006463497]  
 475 - [0.0121393848, 0.0404965307, 0.0684782460, 0.0917696740,  
 0.1076465016, 0.1148029236, 0.1133801059, 0.1047481327, 0.0910785105,  
 0.0748200646, 0.0582190223, 0.0429839473, 0.0301479430, 0.0201034450,  
 0.0127529092, 0.0076996329, 0.0044247356, 0.0024207858, 0.0012615043,  
 0.0006260001]  
 476 - [0.0123453137, 0.0410384460, 0.0692208471, 0.0925531245,  
 0.1083161288, 0.1152419764, 0.1135309547, 0.1046165254, 0.0907215544,  
 0.0743233828, 0.0576722522, 0.0424613576, 0.0296983328, 0.0197488222,  
 0.0124937790, 0.0075230047, 0.0043120164, 0.0023532253, 0.0012233055,  
 0.0006056505]  
 477 - [0.0125512425, 0.0415803613, 0.0699634481, 0.0933365751,  
 0.1089857559, 0.1156810291, 0.1136818036, 0.1044849182, 0.0903645984,  
 0.0738267011, 0.0571254822, 0.0419387679, 0.0292487225, 0.0193941994,  
 0.0122346487, 0.0073463765, 0.0041992972, 0.0022856649, 0.0011851066,  
 0.0005853009]  
 478 - [0.0127571713, 0.0421222766, 0.0707060492, 0.0941200256,  
 0.1096553831, 0.1161200819, 0.1138326525, 0.1043533110, 0.0900076423,  
 0.0733300193, 0.0565787121, 0.0414161782, 0.0287991122, 0.0190395766,  
 0.0119755184, 0.0071697483, 0.0040865780, 0.0022181044, 0.0011469078,  
 0.0005649513]  
 479 - [0.0129631002, 0.0426641919, 0.0714486502, 0.0949034761,  
 0.1103250102, 0.1165591346, 0.1139835014, 0.1042217037, 0.0896506862,

0.0728333375, 0.0560319421, 0.0408935885, 0.0283495019, 0.0186849537,  
 0.0117163881, 0.0069931201, 0.0039738588, 0.0021505440, 0.0011087090,  
 0.0005446017]  
 480 - [0.0131690290, 0.0432061072, 0.0721912513, 0.0956869266,  
 0.1109946374, 0.1169981873, 0.1141343502, 0.1040900965, 0.0892937301,  
 0.0723366557, 0.0554851721, 0.0403709988, 0.0278998917, 0.0183303309,  
 0.0114572579, 0.0068164919, 0.0038611396, 0.0020829836, 0.0010705102,  
 0.0005242521]  
 481 - [0.0133749578, 0.0437480225, 0.0729338523, 0.0964703772,  
 0.1116642645, 0.1174372401, 0.1142851991, 0.1039584893, 0.0889367740,  
 0.0718399740, 0.0549384020, 0.0398484091, 0.0274502814, 0.0179757081,  
 0.0111981276, 0.0066398637, 0.0037484204, 0.0020154231, 0.0010323113,  
 0.0005039025]  
 482 - [0.0135808867, 0.0442899378, 0.0736764534, 0.0972538277,  
 0.1123338917, 0.1178762928, 0.1144360480, 0.1038268820, 0.0885798179,  
 0.0713432922, 0.0543916320, 0.0393258194, 0.0270006711, 0.0176210853,  
 0.0109389973, 0.0064632355, 0.0036357012, 0.0019478627, 0.0009941125,  
 0.0004835529]  
 483 - [0.0137868155, 0.0448318532, 0.0744190544, 0.0980372782,  
 0.1130035188, 0.1183153456, 0.1145868968, 0.1036952748, 0.0882228618,  
 0.0708466104, 0.0538448620, 0.0388032296, 0.0265510609, 0.0172664625,  
 0.0106798670, 0.0062866073, 0.0035229820, 0.0018803022, 0.0009559137,  
 0.0004632033]  
 484 - [0.0139927443, 0.0453737685, 0.0751616554, 0.0988207287,  
 0.1136731459, 0.1187543983, 0.1147377457, 0.1035636676, 0.0878659058,  
 0.0703499286, 0.0532980919, 0.0382806399, 0.0261014506, 0.0169118397,  
 0.0104207368, 0.0061099791, 0.0034102628, 0.0018127418, 0.0009177149,  
 0.0004428537]  
 485 - [0.0141986732, 0.0459156838, 0.0759042565, 0.0996041792,  
 0.1143427731, 0.1191934510, 0.1148885946, 0.1034320603, 0.0875089497,  
 0.0698532468, 0.0527513219, 0.0377580502, 0.0256518403, 0.0165572169,  
 0.0101616065, 0.0059333509, 0.0032975436, 0.0017451814, 0.0008795161,  
 0.0004225041]  
 486 - [0.0144046020, 0.0464575991, 0.0766468575, 0.1003876298,  
 0.1150124002, 0.1196325038, 0.1150394435, 0.1033004531, 0.0871519936,  
 0.0693565651, 0.0522045518, 0.0372354605, 0.0252022300, 0.0162025941,  
 0.0099024762, 0.0057567227, 0.0031848244, 0.0016776209, 0.0008413172,  
 0.0004021545]  
 487 - [0.0146105308, 0.0469995144, 0.0773894586, 0.1011710803,  
 0.1156820274, 0.1200715565, 0.1151902923, 0.1031688459, 0.0867950375,  
 0.0688598833, 0.0516577818, 0.0367128708, 0.0247526198, 0.0158479713,  
 0.0096433460, 0.0055800945, 0.0030721052, 0.0016100605, 0.0008031184,  
 0.0003818049]  
 488 - [0.0148164597, 0.0475414297, 0.0781320596, 0.1019545308,  
 0.1163516545, 0.1205106092, 0.1153411412, 0.1030372386, 0.0864380814,  
 0.0683632015, 0.0511110118, 0.0361902811, 0.0243030095, 0.0154933484,  
 0.0093842157, 0.0054034663, 0.0029593860, 0.0015425001, 0.0007649196,  
 0.0003614553]  
 489 - [0.0150223885, 0.0480833450, 0.0788746607, 0.1027379813,  
 0.1170212817, 0.1209496620, 0.1154919901, 0.1029056314, 0.0860811254,  
 0.0678665197, 0.0505642417, 0.0356676914, 0.0238533992, 0.0151387256,  
 0.0091250854, 0.0052268381, 0.0028466668, 0.0014749396, 0.0007267208,  
 0.0003411057]  
 490 - [0.0152283173, 0.0486252603, 0.0796172617, 0.1035214319,  
 0.1176909088, 0.1213887147, 0.1156428390, 0.1027740242, 0.0857241693,  
 0.0673698380, 0.0500174717, 0.0351451017, 0.0234037889, 0.0147841028,  
 0.0088659551, 0.0050502099, 0.0027339476, 0.0014073792, 0.0006885219,  
 0.0003207561]  
 491 - [0.0154342462, 0.0491671756, 0.0803598628, 0.1043048824,  
 0.1183605360, 0.1218277675, 0.1157936878, 0.1026424169, 0.0853672132,

```
0.0668731562, 0.0494707016, 0.0346225120, 0.0229541787, 0.0144294800,  
0.0086068249, 0.0048735817, 0.0026212284, 0.0013398187, 0.0006503231,  
0.0003004065]  
492     minimum: 0.0  
493     maximum: 25.0  
494  
495     turbulence_intenstiy:  
496     description: turbulence intensity  
497     default: 0.075  
498
```

**APPENDIX C. CASE STUDY 1-2 PARTICIPANT RESULTS**

Figures 1 to 3 correspond to the 16, 36, and 64 optimized wind farm layouts submitted by all participants for case study 1. Submissions are ordered from highest to lowest AEP.

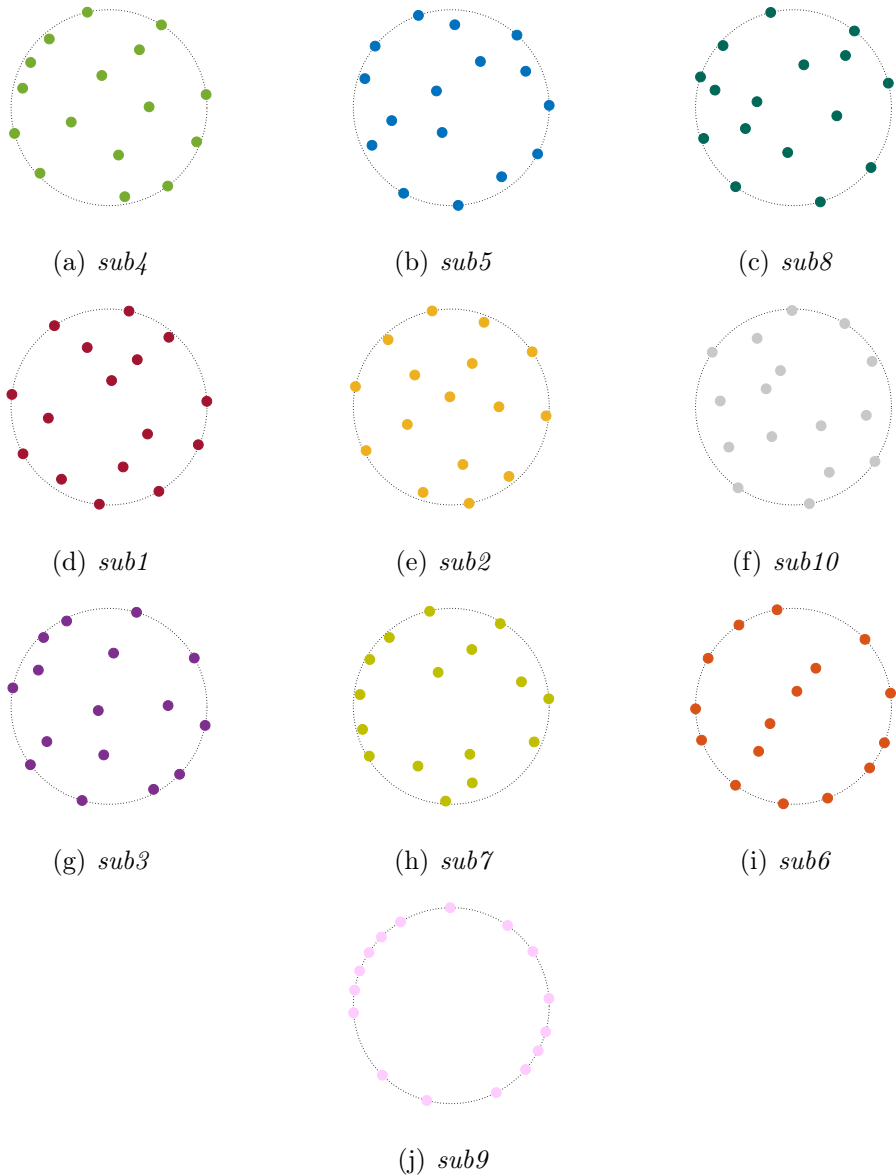


Figure 1: Optimized wind farm layout submissions for case study 1, 16 turbine scenario. Ranked from highest AEP (a) to lowest (j).



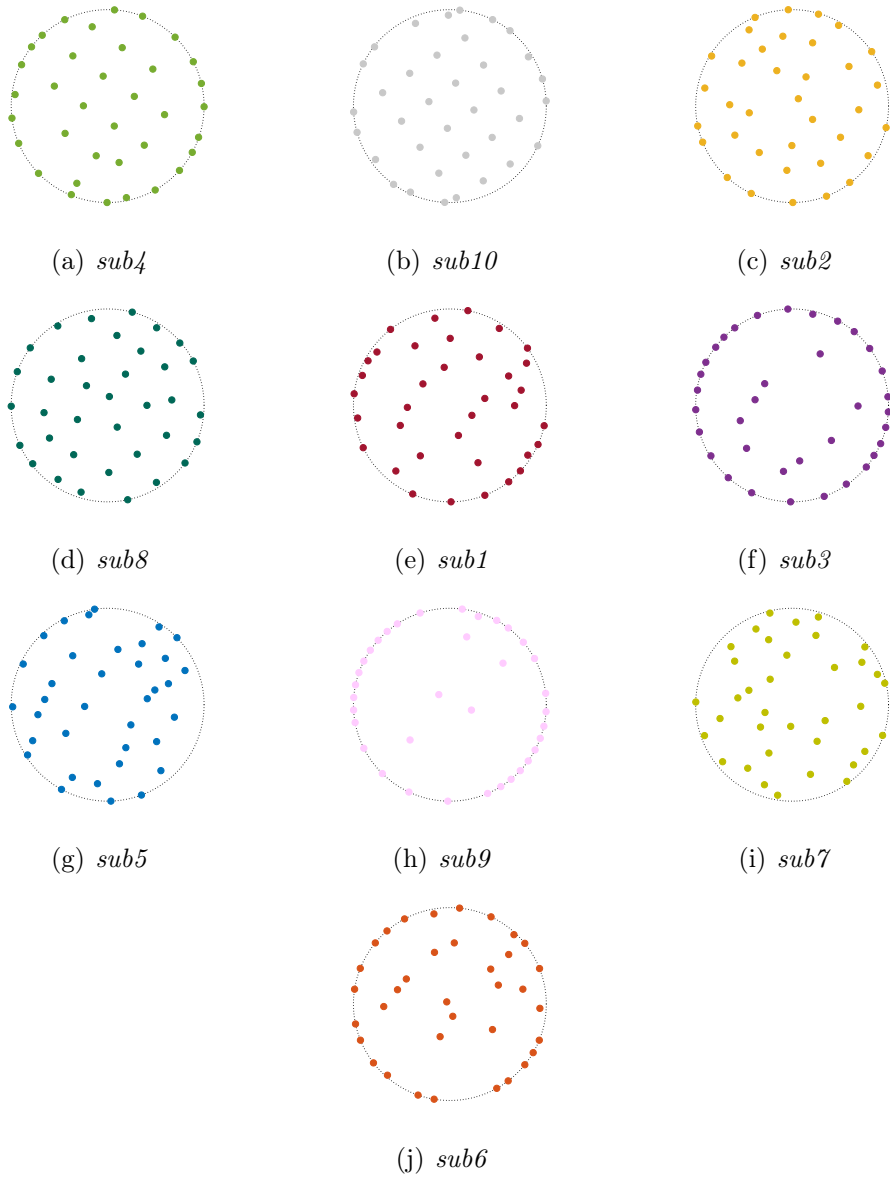


Figure 2: Optimized wind farm layout submissions for case study 1, 36 turbine scenario. Ranked from highest AEP (a) to lowest (j).

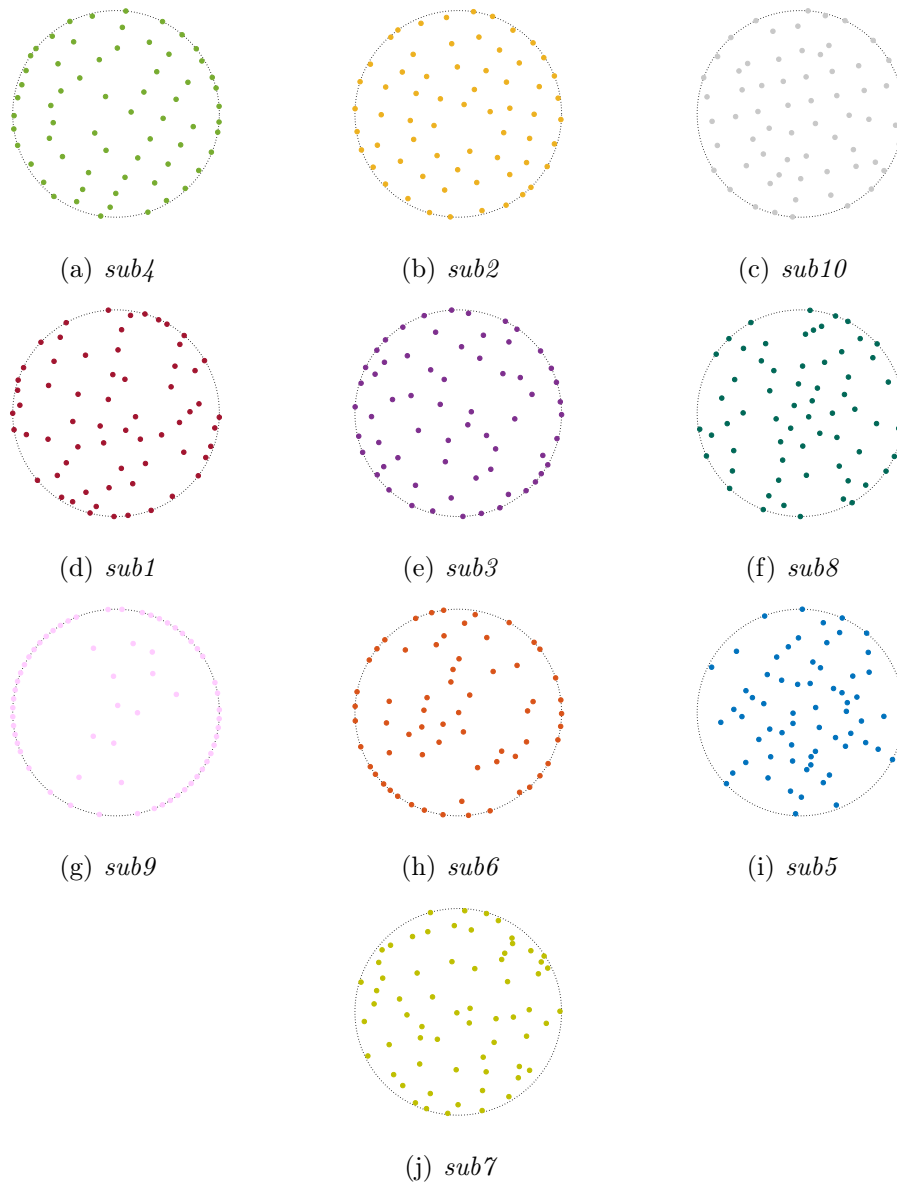


Figure 3: Optimized wind farm layout submissions for case study 1, 64 turbine scenario. Ranked from highest AEP (a) to lowest (j).

Table 1: 16 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Increase
1	SNOPT+WEC	4	G	418924.4064	14.17 %
2	fmincon	5	G	414141.2938	12.86 %
3	SNOPT	8	G	412251.1945	12.35 %
4	SNOPT	1	G	411182.2200	12.06 %
5	Preconditioned Sequential Quadratic Programming	2	G	409689.4417	11.65 %
6	Multistart Interior-Point	10	G	408360.7813	11.29 %
7	Full Pseudo-Gradient Approach	3	GF	402318.7567	9.64 %
8	Basic Genetic Algorithm	7	GF	392587.8580	6.99 %
9	Simple Particle Swarm Optimization	6	GF	388758.3573	5.95 %
10	Simple Pseudo-Gradient Approach	9	GF	388342.7004	5.83 %
11	(Example Layout)	-	-	366941.5712	-

Table 2: 36 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Increase
1	SNOPT+WEC	4	G	863676.2993	17.05 %
2	Multistart Interior-Point	10	G	851631.9310	15.42 %
3	Preconditioned Sequential Quadratic Programming	2	G	849369.7863	15.11 %
4	SNOPT	8	G	846357.8142	14.70 %
5	SNOPT	1	G	844281.1609	14.42 %
6	Full Pseudo-Gradient Approach	3	GF	828745.5992	12.31 %
7	fmincon	5	G	820394.2402	11.18 %
8	Simple Pseudo-Gradient Approach	9	GF	813544.2105	10.25 %
9	Basic Genetic Algorithm	7	GF	777475.7827	5.37 %
10	Simple Particle Swarm Optimization	6	GF	776000.1425	5.17 %
11	(Example Layout)	-	-	737883.0985	-

Table 3: 64 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Increase
1	SNOPT+WEC	4	G	1513311.1936	16.86 %
2	Preconditioned Sequential Quadratic Programming	2	G	1506388.4151	16.33 %
3	Multistart Interior-Point	10	G	1480850.9759	14.35 %
4	SNOPT	1	G	1476689.6627	14.03 %
5	Full Pseudo-Gradient Approach	3	GF	1455075.6084	12.36 %
6	SNOPT	8	G	1445967.3772	11.66 %
7	Simple Pseudo-Gradient Approach	9	GF	1422268.7144	9.83 %
8	Simple Particle Swarm Optimization	6	GF	1364943.0077	5.40 %
9	fmincon	5	G	1336164.5498	3.18 %
10	Basic Genetic Algorithm	7	GF	1332883.4328	2.93 %
11	(Example Layout)	-	-	1294974.2977	-

**APPENDIX D. RELEVANT CODE**

```

% Nick Baker
% IEA37 cs3&4
% Code to apportion turbines in each area based on their % of total area
% 18 Mar 2019
clear, close all;

addpath('./support-files/')
row = 0;
col = 0;
numBounds = 5; % Number of boundaries we're mapping
fArea = zeros(numBounds,1); % Holds the actual area for each bndry
pArea = zeros(numBounds,1); % Holds the percentage area for each bndry
numTurbs = zeros(numBounds,1); % Holds the # of turbines in each bndry
switch(numBounds)
    case 5
        numTotTurbs = 81;
    case 1
        numTotTurbs = 25;
end

% Read .csv files
[File3a, File3b, File4a, File4b, File4c] = readBorsseleBoundaries(row,col);
%plotBorsseleBoundaries(File3a,File3b,File4a,File4b,File4c);

% Find Area for each section
fArea(1) = calcPolygonArea(File3a(:,1), File3a(:,2));
fArea(2) = calcPolygonArea(File3b(:,1), File3b(:,2));
fArea(3) = calcPolygonArea(File4a(:,1), File4a(:,2));
fArea(4) = calcPolygonArea(File4b(:,1), File4b(:,2));
fArea(5) = calcPolygonArea(File4c(:,1), File4c(:,2));

% Find percent area for each section & correspond number of turbs per area
totArea = sum(fArea); % Sum the areas
for i = 1:numBounds
    pArea(i) = fArea(i)/totArea; % Find percentage area
    numTurbs(i) = numTotTurbs * pArea(i); % Find percentage of turbines
end

numTurbs = round(numTurbs,0); % Make # of turbs whole #s
numTurbs(2) = numTurbs(2) - 1; % manual adjust for correct sum
%sum(numTurbs)

function [f1,f2,f3,f4,f5] = readBorsseleBoundaries(row,col)
    f1 = csvread('borsele-boundary-iiia.csv',row,col);
    f2 = csvread('borsele-boundary-iiib.csv',row,col);
    f3 = csvread('borsele-boundary-iva.csv',row,col);
    f4 = csvread('borsele-boundary-ivb.csv',row,col);
    f5 = csvread('borsele-boundary-ivc.csv',row,col);
end

```

```

% Given coordiantes, calculates the interior area of a polygon
% Note: 1) shape must not be self intersecting.
%       2) coordinates must be clockwise or cc.
function [fArea] = calcPolygonArea(xCoords, yCoords)
    numCoords = length(xCoords);
    fSum = 0;

    % Sum from first to penultimate
    for i = 1:(numCoords-1)
        fFrstTrm = (xCoords(i) * yCoords(i+1));
        fScndTrm = (yCoords(i) * xCoords(i+1));
        fSum = fSum + (fFrstTrm - fScndTrm);
    end

    % Sum the last point (which wraps to the first)
    fFrstTrm = (xCoords(numCoords) * yCoords(1));
    fScndTrm = (yCoords(numCoords) * xCoords(1));
    fSum = fSum + (fFrstTrm - fScndTrm);

    % Div by 2 for answer
    fArea = abs(fSum / 2);
end

```

```

% Nick Baker
% IEA37 cs4
% Places turbines in initial layout for cs4
clear, close all;

addpath('../support-files/')
addpath('turb-layout/')
addpath(genpath('/Users/nbaker/Documents/MATLAB/YAMLMatlab'));
row = 0;
col = 0;          % Start on the second column (it can't read letters)
rotorRadius = 99; % m, for 10 MW NREL turbin

%-- Read .csv files --%
[bp3a,bp3b,bp4a,bp4b,bp4c] = readBorsseleBoundaries(row,col);

%-- Shrink boundaires to accomodate rotor radii --%
[os3a,os3b,os4a,os4b,os4c] = offsetBorsseleBoundaries(bp3a,bp3b,bp4a,bp4b,bp4c,
rotorRadius);

%-- Spread our Turbines --%
%[tc3a,tc3b,tc4a,tc4b,tc4c] = makeTurbineCoordsCS4(os3a,os3b,os4a,os4b,os4c);
%[tc3aCS3] = layout3aCS3(os3a);

%-- Plot farm boundaries to check --%
hold on
% plotBorsseleBoundary(os3a);
% plotBorselleTurbines(tc3aCS3, rotorRadius, 1);
% plotBorsseleBoundaries(bp3a,bp3b,bp4a,bp4b,bp4c);
plotBorsseleBoundaries(os3a,os3b,os4a,os4b,os4c);
% plotBorselleTurbines(tc3a, rotorRadius, 1);
% plotBorselleTurbines(tc3b, rotorRadius, 1);
% plotBorselleTurbines(tc4a, rotorRadius, 1);
% plotBorselleTurbines(tc4b, rotorRadius, 1);
% plotBorselleTurbines(tc4c, rotorRadius, 1);
axis equal
axis tight
hold off

%-- Present all turbines in .csv format --%
% allTurbLoc = round([tc3a;tc3b;tc4a;tc4b;tc4c],4);
% struct('position', allTurbLoc);
% WriteYaml('../support-files/cs4-baseline.yaml',allTurbLoc);
% WriteYaml('../support-files/cs3-baseline.yaml',round(tc3aCS3,4));
% WriteYaml('../support-files/cs3-outline.yaml',round(os3a,1));
% WriteYaml('../support-files/cs3b-outline.yaml',round(os3b,1));
% WriteYaml('../support-files/cs4a-outline.yaml',round(os4a,1));
% WriteYaml('../support-files/cs4b-outline.yaml',round(os4b,1));
% WriteYaml('../support-files/cs4c-outline.yaml',round(os4c,1));

function [f1,f2,f3,f4,f5] = readBorsseleBoundaries(row,col)
    f1 = csvread('../support-files/borsele-boundary-iiia.csv',row,col);
    f2 = csvread('../support-files/borsele-boundary-iiib.csv',row,col);
    f3 = csvread('../support-files/borsele-boundary-iva.csv',row,col);
    f4 = csvread('../support-files/borsele-boundary-ivb.csv',row,col);
    f5 = csvread('../support-files/borsele-boundary-ivc.csv',row,col);
end
function [tc3a,tc3b,tc4a,tc4b,tc4c] = makeTurbineCoordsCS4(os3a,os3b,os4a,os4b,os4c)

```

```
[tc3a] = layout3aCS4(os3a);  
[tc3b] = layout3b(os3b);  
[tc4b] = layout4b(os4b);  
[tc4c] = layout4c(os4c);  
[tc4a] = layout4a(os4a);  
end
```



```

% Nick Baker
% IEA37 cs3&4
% Wind Rose interpolation from discreet points
% 18 Mar 19
clear, close all
addpath('./support-files/')
%addpath(genpath('/Users/nbaker/Documents/MATLAB/YAMLMatlab'));
addpath(genpath('C:/Users/Captain Baker/Documents/MATLAB/YAMLMatlab/'));

% Make matrix of points
nNumOrigBins = 12;
nNumFinalBins = 20;
maxWindSpeed = 25;
oldBins = zeros(nNumOrigBins, 4);

% Hard code data from original file
% [dirDeg, Lambda, k, freq%]
oldBins(1,:) = [0, 8.65, 2.11, 5.1];
oldBins(2,:) = [30.0, 8.86, 2.05, 4.3];
oldBins(3,:) = [60.0, 8.15, 2.35, 4.3];
oldBins(4,:) = [90.0, 9.98, 2.55, 6.6];
oldBins(5,:) = [120.0, 11.35, 2.81, 8.9];
oldBins(6,:) = [150.0, 10.96, 2.74, 6.5];
oldBins(7,:) = [180.0, 11.28, 2.63, 8.7];
oldBins(8,:) = [210.0, 11.50, 2.40, 11.5];
oldBins(9,:) = [240.0, 11.08, 2.23, 12.0];
oldBins(10,:) = [270.0, 10.94, 2.28, 11.1];
oldBins(11,:) = [300.0, 11.27, 2.29, 11.4];
oldBins(12,:) = [330.0, 10.55, 2.28, 9.6];

%-- Extrapolate our frequencies --%
Freqs = (oldBins(:,4)/100)'; % Make a decimal from a percentage
[~,~,newDirs,newFreqs] = extrapolateFrequencies(Freqs,nNumFinalBins); % Extrapolate our
points.

%-- Extrapolate our Weibull distributions --%
[~,~, newWeibVars] = extrapolateWeibull([oldBins(:,2),oldBins(:,3)],nNumFinalBins,
maxWindSpeed);
newDirs = round(rad2deg(newDirs),0);
newWeibVars = round(newWeibVars,2);
%csvwrite('./support-files/new-WeibVars.csv',newWeibVars);
newFreqs = round(newFreqs,4)';
%csvwrite('./support-files/new-freqs.csv',newFreqs);
%sum(newFreqs)
%-- Plot and write the data --%
% To plot the Wind Frequency distribution
plotWindRoseFreq(newFreqs,newDirs);
%writeWindroseYAML('./support-files/new-iea37-cs3-windrose-test.yaml', newDirs,
newWeibVars, newFreqs);

```

```

function [f,newDirsRad,newWeibVars] = extrapolateWeibull(oldWeibVars,numNewDirs,maxMagnitude)
%--- Given a Weibull distribution of windspeeds around a windrose, splines and reslices new windspeed distributions.
%-- <oldWeibVars> (:,1) k values (:,2) Lambda values or [k, Lambda] --%
%-- <numNewDirs> number of new directions desired to slice --%

%-- Setup --%
%-- Make times 3 for smooth splines on either side --%
oldWeib3 = [oldWeibVars;oldWeibVars;oldWeibVars];
newWeibVars = zeros(numNewDirs,2);
numOldDirs3 = length(oldWeibVars)*3; % Multiply by three for our duplicates
oldDirs3 = linspace((-2)*pi,4*pi-6*pi/numOldDirs3,numOldDirs3);
newDirsRad = linspace(0,2*pi-(2*pi/numNewDirs),numNewDirs)';

x = oldDirs3;
y = linspace(0,maxMagnitude); % Y-axis is wind speeds (0 to ~25 m/s)
z = zeros(length(x),length(y)); % Z-axis is our Weibull distribution
figure(2)
hold on
for i = 1:numOldDirs3
    z(i,:) = wblpdf(y,oldWeib3(i,1),oldWeib3(i,2));
    plot(y,z)
end
hold off
%-- Debug visualization --%
figure(1)
surf(x,y,z')
%-- End debug visualization --%

%-- Spline for 3D surface --%
[fpp]=csaps({x,y},z); % Spline 3-D surface of all directions
f = @(i,j) fnval(fpp,[i;j]); % Turn the peicwise polynomial into a function

[~,newWeibVars] = getWeibSlices(f,oldWeibVars, numNewDirs,maxMagnitude);
%-- Debug visualization --%
% xNew = linspace(0,2*pi-(2*pi/numNewDirs),numNewDirs)';
% zNew = zeros(length(xNew),length(y));
% for i = 1:numNewDirs
%     zNew(i,:) = wblpdf(y,newWeibVars(i,1),newWeibVars(i,2));
% end
% figure(2)
% surf(xNew,y,zNew')
% font_size = 15;
% set(gca,'FontSize',font_size)
% xlabel('Direction (rad)', 'FontSize', font_size%, 'FontWeight', 'bold')
% ylabel('Magnitude (m/s)', 'FontSize', font_size%, 'FontWeight', 'bold')
% zlabel('Probability', 'FontSize', font_size%, 'FontWeight', 'bold')
end

```