Computational Investigation of the Effects of Rotor-on-Rotor Interactions on Thrust and Noise

Austin R. Schenk
Brigham Young University

Follow this and additional works at: https://scholarsarchive.byu.edu/etd

Part of the Engineering Commons

BYU ScholarsArchive Citation
Schenk, Austin R., "Computational Investigation of the Effects of Rotor-on-Rotor Interactions on Thrust and Noise" (2020). Theses and Dissertations. 8611.
https://scholarsarchive.byu.edu/etd/8611

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact scholarsarchive@byu.edu, ellen_amatangelo@byu.edu.
Computational Investigation of the Effects of Rotor-on-Rotor Interactions on Thrust and Noise

Austin R Schenk

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

Andrew Ning, Chair
  Julie Crockett
  Steven E. Gorrell

Department of Mechanical Engineering
Brigham Young University

Copyright © 2020 Austin R Schenk
All Rights Reserved
ABSTRACT

Computational Investigation of the Effects of Rotor-on-Rotor Interactions on Thrust and Noise

Austin R Schenk
Department of Mechanical Engineering, BYU
Master of Science

Recent advancements in electric propulsion systems have made electric vertical takeoff and landing aircraft a reality, and one that is seen as a partial solution to the growing issue of urban traffic congestion. Designing an aircraft with multiple smaller motors and rotors spread across the wings—referred to as distributed electric propulsion (DEP)—has shown great potential in helping improve electric aircraft performance by offering increased propulsive efficiency, augmented lift, and structural load distribution. For these reasons, DEP is one configuration that is currently being implemented into multiple prototype designs (e.g. NASA’s Maxwell X-57, Airbus Vahana, Opener BlackFly, and Joby S2). However, while a DEP configuration has many potential benefits, it complicates the aerodynamics by introducing complex rotor-on-rotor interactions which can significantly affect noise generation. In this study we use unsteady Reynolds-averaged Navier–Stokes (RANS) simulations (STAR-CCM+) with an aeroacoustic solver (PSU-WOPWOP) to quantify thrust fluctuations and noise generation for two distinct rotor-rotor configurations. The configurations investigated in this study are: 1) coplanar rotors with a varying tip separation distance and 2) one rotor downstream of the other at varying distances for a fixed tip separation distance. Both configurations are investigated using an APC 10x7E and DJI-based 0.24 m rotor. It was found that tip-to-tip separation distance has a stronger influence on noise generation than the downstream separation distance does. A one diameter change in tip separation distance resulted in a ~15 dBA change in noise while a three diameter change in downstream separation distance only resulted in a ~9 dBA change in noise for the same rotor. Changes in thrust fluctuations were found to predict trends in noise generation well for multi-rotor configurations. Additionally, it was shown that when rotors are located less than 10% of the diameter apart from each other, noise can be decreased by up to 9 dBA by moving one rotor ~0.5 diameter downstream of the other.

Keywords: rotor-on-rotor, rotor-rotor, propeller, rotor, noise, CFD, aeroacoustic, thrust
ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Andrew Ning, for his continual support, trust, and encouragement throughout this research project. A deep and heartfelt thank you goes out to all the members in the FLOW Lab for their help and friendships, which made the whole graduate school journey worthwhile. I would like to especially thank Eduardo Alvarez for his insights, patience, and direction during this entire project, without which this project would not have been possible. Additionally, for Kevin Moore, whose research and initial work this project built upon.

I also express my gratitude to the multiple undergraduate researchers, particularly Lindsey Lawless and Charlie Tainter, and Kevin Cole for their help running and refining the experimental procedure. I express my sincere appreciation to the all the donors through whose generosity the beautiful BYU facilities and Fulton Supercomputing Lab were made possible. To my family, I express my love and appreciation for their support through the many years of school. Thank you.
# TABLE OF CONTENTS

**LIST OF TABLES** ................................................................. vi

**LIST OF FIGURES** ............................................................. viii

**NOMENCLATURE** ................................................................. xii

**Chapter 1 Introduction** .................................................... 1

- 1.1 Background .................................................................... 2
- 1.1.1 Performance .............................................................. 3
- 1.1.2 Noise ......................................................................... 5
- 1.1.3 Common Methods ....................................................... 6
- 1.2 Contributions .................................................................. 6
- 1.3 Thesis Outline ............................................................... 7

**Chapter 2 Methodology** ........................................................ 9

- 2.1 Rotor Geometry Digitization ........................................... 9
- 2.2 Empirical Testing ............................................................. 12
  - 2.2.1 Wind Tunnel ............................................................. 13
  - 2.2.2 Propeller and Motor ................................................ 14
  - 2.2.3 Propeller Thrust, Torque, and Speed Measurements ........ 15
  - 2.2.4 Freestream Speed Measurement .................................. 16
  - 2.2.5 Atmospheric Measurements ...................................... 16
- 2.3 Computational Fluid Dynamic Simulations ......................... 17
  - 2.3.1 Domain ................................................................. 17
  - 2.3.2 Motion ................................................................. 18
  - 2.3.3 Boundary Conditions .............................................. 19
  - 2.3.4 Meshing ............................................................... 19
  - 2.3.5 Physics Models and Solvers ...................................... 28
  - 2.3.6 Dynamic Similarity ............................................... 29
  - 2.3.7 Resources ............................................................ 31
- 2.4 Aeroacoustics Solver ........................................................ 32
  - 2.4.1 PSU-WOPWOP Inputs/Outputs .................................. 33
  - 2.4.2 CFD ................................................................. 34

**Chapter 3 Validation and Verification of Methods** ..................... 35

- 3.1 Wind Tunnel Testing ..................................................... 35
  - 3.1.1 Uncertainty Analysis ............................................... 36
- 3.2 Computational Fluid Dynamic Simulations ......................... 37
  - 3.2.1 Verification .......................................................... 37
  - 3.2.2 Validation ............................................................ 38
- 3.3 Noise Validation ........................................................... 41
## Chapter 4  Results .......................................................... 45

4.1 Single Propeller Loading .................................................. 45
  4.1.1 Performance ......................................................... 45
  4.1.2 Noise .................................................................. 45

4.2 Rotor-Rotor Tip Separation Sweep ....................................... 49
  4.2.1 Performance ......................................................... 49
  4.2.2 Flow Field .......................................................... 50
  4.2.3 Noise .................................................................. 54

4.3 Rotor-Rotor Downstream Separation Sweep ......................... 56
  4.3.1 Performance ......................................................... 56
  4.3.2 Noise .................................................................. 59

## Chapter 5  Summary .......................................................... 65

5.1 Conclusions ................................................................ 65

5.2 Future Work ................................................................ 66

REFERENCES .................................................................... 69

Appendix A  Uncertainty Analysis Report ............................ 73

A.1 Uncertainty Analysis .................................................... 73
LIST OF TABLES

2.1 Rotor radii, hub diameters, and hub thicknesses. . . . . . . . . . . . . . . . . . . . . 10

A.1 $u_0$ and $u_i$ values for measured quantities. Note that $u_i$ values might be different for each measurement set, so only the maximum values are listed here. . . . . . . . . . . . 74
### LIST OF FIGURES

1.1 From left to right: NASA X-57 demonstrator, Joby Aviation concept plane, A3 (Airbus) full-scale prototype. ................................................................. 2

1.2 Left: Tip separation distance configuration. Right: Downstream distance configuration. .................................................. 7

2.1 All rotor CAD models used in this research. .............................................................. 10

2.2 Chord distribution and leading edge position for the APC 10x7E, Ning DJI, and DJI 9443 rotors. $x$ and $z$ are based on a coordinate system at the center of the hub where $x$ is the distance from the axis aligned with the rotor blades and $z$ is the distance normal to the plane of rotation. All distances are normalized by the radius. ........................................... 11

2.3 Twist distribution for the APC 10x7E, Ning DJI, and DJI 9443 rotors. ............................................................. 11

2.4 APC 9x4.5 3D printed propeller (top) comparison to original (bottom). ............................................................. 12

2.5 APC 9x4.5E 3D printed propeller vs actual propeller thrust and torque results as collected in the BYU wind tunnel with no corrections or post-processing. .................................................. 13

2.6 BYU 3’x4’ open circuit wind tunnel. ........................................................................ 14

2.7 80/20 structure used to mount both 1580 RC Benchmark test stands in wind tunnel. .................................................. 15

2.8 RC Benchmark 1580 test stand model. ........................................................................ 16

2.9 Example images of the flow field data CFD can provide. Left: vorticity (local rotation of a fluid particle) flow-field for a single rotor. Right: turbulent kinetic energy (kinetic energy of eddies within the flow) flow-field for two rotors. .................................................. 18

2.10 Left: Simulation domain. Right: Rotating volume within domain. .................................................. 19

2.11 Boundary conditions used in all simulations. Left: velocity inlet, freestream, and pressure outlet boundaries. Right: colored surface is the surface to which the internal interface boundary is applied. .................................................. 20

2.12 Final mesh surface target size and the prism layer settings used to resolve the rotor boundary layers. Left: Cross-section prism layer cells along the rotor surface, Right: Rotor surface as viewed from rotor hub out to blade tip. .................................................. 23

2.13 Final meshing scheme used for the fluid domain. Top: Outer domain boundary meshed with a cross-section along the rotor axis of rotation, Bottom Left: Cross-section in the plane of rotation, Bottom Right: Cross-section along the axis of rotor rotation. .................................................. 24

2.14 Cross-section along the axis of rotation of an adaptive test mesh at the initial state, after two mesh refinement, and after five refinement (left to right). The top row is a scalar representation of the field where the red cells will be refined in the next mesh. The bottom row shows the mesh at each stage (NOTE: This is not the meshing scheme used in the final results). .................................................. 27

2.15 Cross-section in the plane of rotation of an adaptive test mesh at the initial state, after two mesh refinement, and after five refinement (left to right). The top row is a scalar representation of the field where the red cells will be refined in the next mesh. The bottom row shows the mesh at each stage (NOTE: This is not the meshing scheme used in the final results). .................................................. 27

2.16 Final meshing scheme used for the single rotor cases. Left: Cross-section in the plane of rotation, Right: Cross-section along the axis of rotor rotation. .................................................. 29
2.17 Final meshing scheme used for the separated rotor configurations Top: Cross-section in the plane of rotation, Bottom: Cross-section along the axis of rotor rotation, Left: \( s = 0.05D \), Right: \( s = 1.00D \) ................................................................. 29
2.18 Final meshing scheme used for the staggered rotor configurations Top: Cross-section in the plane of rotation, Bottom: Cross-section along the axis of rotor rotation (left to right: \( X = 0.00D \), \( X = 1.00D \), Right: \( X = 3.00D \) ................................................................. 30
2.19 Microphone location definition. There are 360 microphones in total, but only a few are shown for illustration purposes. .................................................................................. 33
2.20 Discretization of Ning DJI (Left) and APC 10x7E (Right) rotors within STAR-CCM+ used to get rotor loading history. ................................................................. 34
3.1 Comparison of our wind tunnel APC 10x7E coefficient of thrust and power results with data published by McCrink et al. across varying advance ratios. ..................... 35
3.2 Comparison of our wind tunnel APC 10x7E efficiency results with data published by McCrink et al. across varying advance ratios. ................................................................. 36
3.3 Grid convergence results for thrust (left) and torque (right) on a single APC 10x7E propeller. .................................................................................................................................................. 38
3.4 Comparison of APC 10x7E coefficient of thrust (left) and coefficient of torque (right) CFD results to empirical data from Ohio state. ................................................................. 39
3.5 Comparison of APC 10x7E advance ratio CFD results to empirical data from Ohio state. 39
3.6 Agreement between RANS simulation and experimental CT data is within 1% for the DJI 9443 rotor. Dashed lines represent the uncertainty reported in the experimental measurements. ................................................................. 40
3.7 Comparison of our STAR-CCM+ with PSU-WOPWOP frequency domain noise prediction to published data for a single DJI 9443 rotor. ................................................................. 42
3.8 Comparison of our STAR-CCM+ with PSU-WOPWOP OASPL noise prediction results to published data for a single DJI 9443 rotor. ................................................................. 43
4.1 Convergence of the Ning DJI and APC 10x7E rotor to a thrust value of 3.123 N and 3.155 N respectively. ................................................................. 46
4.2 Left: Loading distribution with the arrows representing the location of an equivalent load. Right: Thrust fluctuations (standard deviation) divided by segment length. ........ 46
4.3 Comparison of the frequency domain for microphones located at 30°(top) and 90°(bottom) for the single Ning DJI and APC 10x7E rotors. ................................................................. 47
4.4 Comparison of the OASPL (left) and A-weighted OASPL directivity (right) for the noise generated by a single Ning DJI propeller and a single APC 10x7E rotor. ............... 48
4.5 Normalized thrust for separation cases compared to Ning et al. ................................................................. 50
4.6 Thrust fluctuations for separation cases compared to Zhou et al.. The left plot shows the actual values, while the right plot is normalized by the single rotor cases. ............... 50
4.7 Comparison of DJI propeller time-averaged velocity cross-plane at 0.1 D downstream from Iowa State Universities PIV results (left) to the CFD results (right). .......... 51
4.8 Delayed separation region seen at 0.4 D downstream. ................................................................. 51
4.9 Phase-locked vorticity distribution of DJI propeller PIV results (left) to the CFD results (right). Phase-locked at 0° and averaged over multiple revolutions. ............... 52
4.10 Phase-locked vorticity distribution of DJI propeller PIV results (left) to the CFD results (right). Phase-locked at 120° and averaged over multiple revolutions. ........................................ 52
4.11 Frequency spectrum of noise results for the tip separation distances \( (s) \) with the RANS+PSU-WOPWOP results on the top and the experimental data on the bottom for a microphone at 30°. ............................................................................ 53
4.12 Frequency spectrum of noise results for the tip separation distances \( (s) \) with the RANS+PSU-WOPWOP results on the top and the experimental data on the bottom for a microphone at 90°. ............................................................................ 54
4.13 Frequency spectrum of noise results for increasing tip separation distances \( (s) \) with the RANS+PSU-WOPWOP vs experimental data for the 90° microphone for 0.05 D (top) and 1.00 D (bottom) cases. ............................................................... 55
4.14 Frequency spectrum of RANS noise results for the tip separation distance \( (s) \) for the Ning DJI rotor at the 30° microphone (top) and the 90° (bottom). .................................................. 56
4.15 Frequency spectrum of RANS noise results for the tip separation distance \( (s) \) for the APC 10x7E (right) at the 30° microphone (top) and the 90° (bottom). .................................................. 57
4.16 A-weighted OASPL of RANS noise prediction results for increasing tip separation distances with the Ning DJI rotor on the left and the APC 10x7E on the right. .................................................. 57
4.17 Normalized average produced thrust for the Ning DJI and APC 10x7E rotors at various downstream separation distances. ............................................................................ 58
4.18 Instantaneous velocity profile for the Ning DJI rotor-rotor configuration with a 3 D downstream separation distance. ............................................................................ 59
4.19 The standard deviations of thrust comparison for the stationary/right and downstream/left rotor across all downstream separation distances. ............................................................................ 60
4.20 Wake velocity profile. Arrow indicates that the approximate location where the wake stops contracting and begins expanding. ............................................................ 60
4.21 Frequency spectrum of RANS noise results for the Ning DJI rotor 30° mic on top and the 90° mic on bottom. ............................................................................ 61
4.22 Frequency spectrum of RANS noise results for the APC 10x7E 30° mic on top and the 90° mic on bottom. ............................................................................ 62
4.23 A-weighted OASPL of RANS noise prediction results for increasing downstream separation distances with the Ning DJI rotor on the left and the APC 10x7E on the right. ............................................................ 62
4.24 A-weighted OASPL noise results for the 0, 2, and 6 R downstream separation cases with the Ning DJI rotor on the left and the APC 10x7E on the right. ............................................................ 63
4.25 Comparison of the A-weighted OASPL results at the 60° mic (left) with the averaged thrust fluctuations of the left and right rotors (right). ............................................................................ 63
NOMENCLATURE

\( c \) Speed of sound
\( C_T \) Coefficient of thrust
\( C_Q \) Coefficient of torque
\( D \) Propeller diameter
\( h_r \) Relative humidity
\( J \) Advance ratio: ratio of forward motion to angular velocity
\( L \) Characteristic length
\( n \) Rotational speed
\( N \) Number of desired boundary layer cells
\( P_v \) Vapor pressure
\( r \) Radial distance
\( R \) Specific gas constant
\( \text{Re} \) Reynolds number: ratio of inertial forces to viscous forces
\( s \) Tip separation distance
\( T \) Temperature
\( T_S \) Sutherland's temperature
\( V \) Velocity
\( Y^+ \) Y plus: ratio of inertial forces to viscous forces near a surface
\( x \) Downstream separation distance

\textit{Greek}
\( \eta \) Propeller efficiency
\( \mu \) Dynamic viscosity
\( \rho \) Density
\( \omega \) Angular velocity

\textit{Subscripts, superscripts, and other indicators}
\( [\cdot]_a \) a property of air
\( [\cdot]_c \) a property given in units of Celsius
\( [\cdot]_{\text{eff}} \) the effective value of a property
\( [\cdot]_{\infty} \) a freestream quantity
\( [\cdot]_{0,XX} \) percent of diameter at which a property is evaluated
\( [\cdot]_{sl} \) a property at sea level
\( [\cdot]_t \) a total value
\( [\cdot]_w \) a property of water vapor
CHAPTER 1. INTRODUCTION

The electric vehicle industry is projected to reach a $800 billion dollar market value within the next seven years [1] due, in part, to the challenging urban problems that electric vehicles help address, like those of vehicle pollution and traffic congestion. Also, thanks to recent developments in electric propulsion technologies, it is an industry that is taking to the skies with electric aircraft quickly becoming a reality. With millions of hours wasted on the road worldwide every day [2], development of electric air taxis is receiving significant attention as they are seen as a partial solution to this problem. However, in order to operate an aircraft in an urban setting, they must be designed with two key constraints in mind: 1) limited space for takeoff and landing and 2) urban noise regulations.

Vertical takeoff and landing (VTOL) aircraft—like a helicopter or quadcopter—and short takeoff and landing (STOL) aircraft—like an overpowered airplane—have been shown to be able to takeoff and land within the space constraints of a city [3], but there is still the challenge of noise. The two main contributors to noise generation on a traditional propeller-driven aircraft are those of their combustion engines and their propellers. While the first of these is significantly reduced with the use of electric motors rather than traditional combustion engines, the challenge of propeller noise still exists and is especially important as the International Civil Aviation Organization (ICAO) continues to enforce more strict noise regulations [4]. Also, with range still being a significant design constraint as battery technologies continue to advance, aircraft efficiency is also of key importance in any electric propulsion system.

Designing an aircraft with multiple smaller motors and rotors spread across the wings, referred to as distributed electric propulsion (DEP), is a popular design choice in many VTOL prototypes (Figure 1.1). It is being implemented in multiple designs for the potential it has to help overcome many of the challenges facing electric aircraft by offering increased propulsive efficiency [5], augmented lift [6], and structural load distribution [7]. However, while a DEP
configuration may answer many challenges faced by electric aircraft, it actually complicates the aerodynamic and aeroacoustic design by introducing complex aerodynamic interactions between closely-spaced rotor, often referred to as rotor-on-rotor interactions.

![Image](image.jpg)

Figure 1.1: From left to right: NASA X-57 demonstrator, Joby Aviation concept plane, A3 (Airbus) full-scale prototype.

1.1 Background

For an understanding of terminology, a few concepts must be reviewed in relation to how they are used in the associated literature. The term rotor is usually used to define a rotating lifting surface that has little to no freestream velocity component in the direction normal to the plane of rotation, for example as seen on a helicopter or a quadcopter. The term propeller is generally used to describe a rotating lifting surface where there is generally a significant incoming freestream velocity in the direction of the axis of rotation, as is common in traditional rotary-wing aircraft flight. However, since a more broad definition of a rotor is any mechanical part in rotation, whenever both a propeller and rotor must be referred simultaneously they will simply be called rotors. Three modes of flight are also important to understand, especially within the realm of VTOL aircraft which can function in flight modes similar to both airplanes or helicopters. The modes of flight will be defined by the freestream velocity in relation to the plane and axis of rotation. They are:

1. Hovering or static flight. This is where the freestream velocity is zero or near zero (e.g. an airborne but stationary helicopter).

2. Forward or edgewise flight. This is where the largest component of freestream velocity is in the plane of rotation with less flow in the axial direction (e.g. a quadcopter that is moving forward at a moderate to high velocity).
3. Axial flight. This is where the large majority of freestream velocity is in the direction of the axis of rotation (e.g. a plane in flight).

Flight modes 1 and 2 are common of rotors while mode 3 is typical of propellers.

1.1.1 Performance

Rotor-on-rotor aerodynamic interactions have been noted as an important area of research by several authors [8–10]. Yoon et al. [8] investigated these rotor-rotor interactions for the 25 ft diameter XV-15 rotor and found that for a coplanar counter-rotating configuration there was a 4% decrease in the coefficient of thrust as the rotors were brought from a tip separation distance of 2 D down to 0.10 D. Little explanation is given for the potential causes of these negative interactions, other than to state that, when the aircraft body is included in their simulations, there is less negative interaction between the rotors as the actual aircraft body appeared to be limiting the downstream interactions of the individual rotors. This would lead to the conclusion that the interactions between rotor wakes is a partial cause of the decrease in thrust. They also note “extremely unsteady” pressure fluctuations on the aircraft wings and body due to the rotors.

While this study on the XV-15 rotors has become a common verification case for rotor-rotor interaction studies, multiple authors [11–14] have noted the significance of Reynolds number effects on small diameter UAV-type rotors that are not predominant in large rotor, or high Reynolds number cases, such as with the XV-15 rotor. This observation has led to research focusing specifically on smaller UAV rotors with Reynolds numbers of less than 200,000, especially as the industries for both recreational-use quadcopters and VTOL urban air taxis have recently seen significant growth.

Zhou et al. [9] found comparable losses in thrust to Yoon et al. [8] when they conducted a similar study on small DJI-based (Ning-DJI) UAV rotors (240 mm diameter). They reported a loss of 2% in the coefficient of thrust for a tip separation of 0.05 D against the single rotor case with no rotor-rotor interaction. From this they concluded that the tip separation distance between rotors had an insignificant effect on the coefficient of thrust and was consequently negligible. Kaya et al. [15] performed a similar study to determine tip separation, or gap, effects between coplanar rotors and found similar results, but made the interesting observation that the effects of tip separation distance
became more observable as the rotational velocity increased. This correlation would presumably be in part due to the stronger tip vortices shed at higher Reynolds numbers. Multiple other papers have investigated these rotor-rotor interactions [14, 16, 17] using various methods. Veismann et al. [14] provides a nice succinct comparison of all these coplanar counter-rotating configuration tip separation results. Overall the data agrees well and strongly suggest that small coplanar counter-rotating UAV rotors will incur a 1-4% loss in thrust if positioned within 0.05 D of each other, rather than 1 D or more apart.

Duffy et al. [10] explored a multi-rotor system that was designed to function as a mobile crane-like lift system for objects under 2,000 lbs. In their research, they look at designs with rotors in a 2x2 configuration and a 4x4 configuration, while expressing that the end goal would be to expand this even further for larger load capabilities. With the large number of closely-spaced rotors used in their design, they suggest that rotor-rotor interactions could be important. They recommend it as an important area of future research, claiming that it would enable rotor-rotor spacing and configuration optimization for improved performance in future designs.

Chiew et al. [18] noted that rotor-rotor interactions had almost exclusively been studied in hovering flight even though forward or edgewise flight makes up a large percentage of actual quadcopter operating conditions. With less existing validation data for this mode of flight, he validated his model against the XV-15 rotor hovering study conducted by Yoon et al. [8] and the single propeller axial performance data published by Brandt et al. [11], since edgewise flight is some combination of these two. With agreement between their model and published data deemed to be good, they investigated multiple quadcopter configurations under varying edgewise flow conditions and rotor separation distances. Their results showed rotor efficiency to be masked by the much more significant drag forces produced by the additional arm structure that was needed to increase rotor separation. They suggest this may be due to their inefficient square cross-sectioned members or even just specific to their aircraft design.

Shukla et al. [12] and Theys et al. [19] also both looked at double rotor configurations, where two rotors were positioned in close proximity to each other at varying axial or planar separations distances. They investigated double rotor cases where the planes of rotation had some offset and the rotors themselves were overlapping (axial separation < diameter). Shukla et al. describes the physical interactions that occur in the flow and breaks them into vortex-vortex interactions,
vortex-vortex sheet interactions, and vortex-hub interactions for segmented regions of axial separation, while Theys et al. quantifies the thrust output for varying configuration, concluding that a 10-15% of diameter overlap is most efficient.

1.1.2 Noise

While the aforementioned research suggests that the effects of rotor-rotor interactions on total thrust are either insignificant or minimally significant, the same is not true for their effects on noise. Zhou et al. in their above mentioned study, reported an increase of \( \sim 3 \, \text{dB} \) and a 250% increase in thrust fluctuations for the same test that resulted in only a 2% decrease in coefficient of thrust [9]. However, despite the much more significant influence that rotor-rotor interactions have on noise over performance, little research has been done to expand on the results reported by Zhou et al.. Building off the decades of single rotor noise research [20–25], is consequently a great place to begin. There are decades of studies on predicting and understanding the mechanisms through which rotors generate noise and how those noises can be approximated numerically.

Rotor noise can be broken into tonal, or deterministic components, and broadband, or non-deterministic components. The main tonal noise mechanisms for rotors are thickness noise, loading noise, blade vortex interaction noise, high-speed impulsive noise, while the main broadband noise sources are turbulence ingestion noise, blade wake interaction noise, and blade self noise [23]. One numerical approximation that is commonly used when dealing with rotor aeroacoustic is the Farassat 1A Formulation of the Ffowcs Williams-Hawkings equation. This numerical model captures noise due to three sources: 1) blade thickness, 2) rotor loading, and 3) quadrupole effects [26]. Quadrupole effects, which are the noise sources away from the rotor such as noise caused by wake interactions or vortices, however, are generally small and are consequently ignored in the majority of noise models [26–28]. In rotor noise research by Carolus et al. [24], they conclude that both thrust fluctuations along with the turbulent kinetic energy can be used to evaluate the level of broadband noise. Additionally, Zhou et al. [9] specifically mentions the rotor thrust fluctuations and downstream turbulent flow interactions as the cause of their measured increased in rotor-rotor noise as two rotors were brought closer together.

However, while there has been significant research on the effects of rotor-rotor interaction on performance and the noise generation of single rotors, very little research [9, 23] has been done
specifically on the effects that rotor-on-rotor interactions have on noise generation. Additionally, the minimal research that currently exist on the noise effects of these interactions, mainly only use experimental methods. Other tools such as computational methods and aeroacoustics solvers have not readily been applied to study rotor-rotor noise interactions.

1.1.3 Common Methods

The main numerical methods used to study these rotor-rotor interactions are blade element moment models [13, 29] and CFD models [8, 18, 30, 31], while the main empirical methods include static testing (often with particle imaging velocimetry) [9, 12], wind tunnel testing [11], and anechoic chamber testing [9, 23]. The methods used in this paper (wind tunnel testing, CFD, and aeroacoustics solver) will be described in the methodology section, while the other methods not used in this paper can be read about in more description in the aforementioned papers.

1.2 Contributions

This thesis aims to contribute a better understanding of the aerodynamic and aeroacoustic effects of rotor-rotor interactions by using methods not previously applied to these interactions. This will be done using wind tunnel testing and computational fluid dynamics (STAR-CCM+) in conjunction with an aeroacoustics code (PSU-WOPWOP). This research explores these rotor-rotor effects on small UAV-type rotors (Ning DJI [9], APC 10x7E) under low Reynolds number (less than 100,000 based the chord length at 70% radius) flow conditions as is common for these types of rotors. Rotor-rotor interactions are investigated by running two parameter correlation sweeps. The first parameter is the tip separation distance on a side-by-side (coplanar) configuration and the second parameter is the downstream distance, where one rotor is moved further downstream while keeping the same axial distance as shown in Figure 1.2. These parameters are important for the straight and swept wing designs seen in Figure 1.1. Both configurations are counter-rotating, where the right rotor (as viewed from downstream of the rotor as would be typical in an aircraft) is spinning clockwise and the left rotor is spinning counter-clockwise. Rotor thrust, loading, and predicted noise will be presented for both of these configurations and, where possible, compared to experimental measurements.
1.3 Thesis Outline

This first chapter introduced distributed electric propulsion and the current state of research into its complex rotor-rotor aerodynamic and aeroacoustic challenges. The second chapter presents all the methods used in wind tunnel testing, CFD modeling, and noise prediction. The third chapter provides verification and validation of the methods. The forth chapter contains aerodynamic and aeroacoustic results for both researched rotor-rotor configurations. The final chapter summarizes all conclusion made based on the results and provides suggestions of future work in this area of research.
CHAPTER 2. METHODOLOGY

The methodology for rotor geometry creation, empirical testing, computational fluid dynamic simulations, and noise analysis is presented here.

2.1 Rotor Geometry Digitization

The rotors used in this thesis were all small 9-10 inch UAV-type rotors that were chosen for their use in existing literature [9, 11, 13] as well as the size limits of the BYU wind tunnel. Rotor geometry was necessary for the CFD and noise analysis we intended to perform, however, due to the proprietary nature of rotor geometries, exact rotor CAD models are not available from the manufactures. When rotor data was not already available from another paper [9, 11], actual rotors were taken, methodically cut into pieces, and manually digitized. This was done by cutting the propeller into multiple chord-wise sections along the span of the blade. Photographs were then taken normal to each airfoil section as well as normal to the plane of rotation of the propeller and perpendicular to the span of the propeller in order to relate the airfoil section back to its location within the propeller.

From the image taken normal to the airfoil section the edge was selected within MATLAB or Julia and digitized. Once digitized, the airfoil profile was compared to and then fit to standard airfoil data. With the blade discretized into multiple known airfoil profiles, the twist, span, and chord lengths were then similarly extracted from the plane-normal and span-perpendicular images.

From the collected geometry information, all the extracted points were related back to a datum at the center axis of the rotor. Then, with all the points defined in the same coordinated system, a close geometric approximation of the rotor was created within a CAD package. Siemens NX was used in this case for availability of pre-written scripts (i.e. Journalling) that automated most of the data importation process. The rotors used in this research were the APC 9x4.5E, APC 10x7E, Ning DJI [9], and DJI 9443 rotors and their models can be seen in Figure 2.1 and
Figure 2.1: All rotor CAD models used in this research.

Table 2.1: Rotor radii, hub diameters, and hub thicknesses.

<table>
<thead>
<tr>
<th>Rotor</th>
<th>Radius (m)</th>
<th>Hub Diameter</th>
<th>Hub Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>APC 9x4.5E</td>
<td>0.1143</td>
<td>0.189 R</td>
<td>0.085 R</td>
</tr>
<tr>
<td>APC 10x7E</td>
<td>0.127</td>
<td>0.149 R</td>
<td>0.075 R</td>
</tr>
<tr>
<td>Ning DJI</td>
<td>0.12</td>
<td>0.100 R</td>
<td>0.040 R</td>
</tr>
<tr>
<td>DJI 9443</td>
<td>0.12</td>
<td>0.104 R</td>
<td>0.054 R</td>
</tr>
</tbody>
</table>

accessed at https://github.com/byuflowlab/schenk2020-rotor-geometries. The chord and leading edge position of the APC 10x7E, Ning DJI, and DJI 9443 rotors can be seen in Figure 2.2, their twist distributions can be seen in Figure 2.3, and their radii and hub dimensions can be found in Table 2.1. Any rotor used for experimental testing simply had the addition of a small hole through the hub to allow for it to be tightened to the motor shaft.

To verify that our rotor geometry creation process provided a reasonable approximation of the actual geometry, the digitized APC 9x4.5E propeller was printed to scale in PLA using a Prusa i3 MK3S 3D printer. Once printed, the surface was repeatedly covered in primer and then sanded until the part had a similar surface smoothness to that of the original propeller. Similar surface roughness was important as a rougher surface would increase the skin friction and by extension the
Figure 2.2: Chord distribution and leading edge position for the APC 10x7E, Ning DJI, and DJI 9443 rotors. $x$ and $z$ are based on a coordinate system at the center of the hub where $x$ is the distance form the axis aligned with the rotor blades and $z$ is the distance normal to the plane of rotation. All distances are normalized by the radius.

Figure 2.3: Twist distribution for the APC 10x7E, Ning DJI, and DJI 9443 rotors.
measured torque. The printed propeller matched the actual geometry closely and can be seen in Figure 2.4. Additionally, the propeller was tested in the wind tunnel and compared to the original manufacturer propeller to help quantify any error in the CFD models that might be introduced by the approximation of geometry. The actual APC 9x4.5E propeller and the printed version were tested on the same stand on the same day and under the same wind tunnel conditions with no corrections or post-processing to minimize variability in all other factors. The measurements were taken at multiple advance ratios, \( J = V_\infty / nD \), where the advance ratio is the ratio of freestream velocity to rotational velocity. The coefficients of thrust and torque \( C_T = T / \rho n^2 D^4 \) and \( C_Q = Q / \rho n^2 D^5 \) can be seen in Figure 2.5, which show that the printed geometry actually has both a lower coefficient of thrust and torque than the actual propeller, except for advance ratios greater than 0.3 where the printed propeller thrust slightly outperforms that of the original. The differences in geometry introduced from our digitization process for this specific propeller digitization introduced an error of over 12% in \( C_T \) and over 18% in \( C_Q \). While this process was not repeated for every rotor that was used in this study, it does provide a baseline for geometry-introduced error.

![Figure 2.4: APC 9x4.5 3D printed propeller (top) comparison to original (bottom).](image)

### 2.2 Empirical Testing

Empirical testing was used in this research on the basis of that once we had a validated procedure, we would be able to more quickly run multiple rotor-rotor configurations as compared to using a more time-intensive computational method. This would then allow us to investigate more rotor-rotor configurations and better explore the design space during the limited time frame.
Figure 2.5: APC 9x4.5E 3D printed propeller vs actual propeller thrust and torque results as collected in the BYU wind tunnel with no corrections or post-processing.

of this research. Additionally, it would allow us to create our own validation data, which would be helpful in validating later computer simulations. Our testing equipment and procedures are detailed below.

2.2.1 Wind Tunnel

All empirical testing was performed in the BYU 3’x4’ open circuit wind tunnel. The length of the testing section is 15’10” (Figure 2.6). The wind tunnel is an Eiffel type wind tunnel where the fan is located downstream and pulls the air through the test section. It utilizes a 100 HP Twin Cities Vane-Axial Fan with a diameter of 72 inches. The maximum operating speed is 110 mph (49 m/s), however our testing was ran well below this speed, generally maxing out at 56 mph (25 m/s). The fan is controlled by a heavy-duty 125 HP 480 Vac @ 157A ABB Variable Frequency Drive (VFD) which allowed the fan RPM to be incremented in 1 RPM steps. An increase of 1 RPM in the fan correlated to an approximate 0.115 mph (0.0512 m/s) increase in flow speeds. This level of control was sufficient for the testing presented here.

Good flow quality in the wind tunnel is important to ensure a less turbulent inflow to the rotors as would be common in actual flight. Additionally, good flow quality adds to the repeatability of any given test within the wind tunnel. The BYU wind tunnel uses a 3.5” thick 3/8th inch cell 3000 series aluminum honeycomb flow straightener and four turbulence-reducing screens at
the front of the wind tunnel to achieve laminar inflow. The turbulence-reducing screens are made of 0.009 inch (0.2286 mm) 316-stainless steel wire spaced at 20 wires per inch (20 mesh). This results in a max turbulence intensity of 2% across all operational speeds. The wind tunnel does not include any boundary layer correction technology. Blockage corrections are recommended for any blockage percent above 5% of the test cross-sectional area.

2.2.2 Propeller and Motor

The propellers used in all empirical testing were the APC 9x4.5E and APC 10x7E thin electric propellers. These propellers were chosen due their use in other similar studies within the literature [11, 13], as well as their previous use within the FLOW Lab [32]. The two motors that were used in this study were both Turnigy Aerodrive SK3 2836-1040kv Brushless Outrunner Motors. This motor has 14 poles, a max loading of 28 A, and a max power of 335 W. This allowed us to run our propellers up to a maximum of 8000 RPMs, which was sufficient to achieve our desired Reynolds numbers \( J = \frac{V_\infty}{nD} \) and advance ratios \( J = \frac{V_\infty}{nD} \). The motors were controlled by standard 30 A electronic speed controllers (ESC) that were then connected to the stands PCBs and controlled by the accompanying software.
2.2.3 Propeller Thrust, Torque, and Speed Measurements

Thrust, torque, and propeller speed were measured using two RC Benchmark 1580 Dynamometers mounted on a custom 80/20 adjustable stand (Figure 2.7). The 1580 stand measures thrust with one vertically-mounted load cell and torque with two horizontally-oriented load cells which are both slightly offset from the axis of rotation (Figure 2.8). All three load cells use strain gauges connected in an H-bridge configuration. The accompanying RC Benchmarks program was used to calibrate, control and record data from the test stands with a data acquisition rate for thrust and torque of 8 Hz and an angular speed data acquisition rate of 50 Hz. Calibration was performed as directed by the program and with the provided adapters by placing a 100 g weight at three specific locations on the test stand for specified stand orientations. Stands were calibrated at the beginning of each testing period. The angular speed was measured based on the feedback from the ESC and the number of motor poles. This results in an angular speed tolerance of $1 \text{eRPM}$, where $\frac{\text{RPM}}{\text{Poles}} = \text{RPM}$, and is recorded at 50 Hz. These dynamometers can measure speeds up to 13,500 RPM while using motors with 14 poles. Settings for recording the data are also set within this same program. For our test we only controlled propeller rotational speed and when to start and stop the data sample.

Figure 2.7: 80/20 structure used to mount both 1580 RC Benchmark test stands in wind tunnel.
2.2.4 Freestream Speed Measurement

Air speed was measured using the dynamic pressure drop across the settling chamber as recommended for accuracy by Barlow et al. [33] and by the manufacturers of the wind tunnel. The front and back of the settling chamber have pressure orifices around their circumference which are then plumbed together and used as the pressure inputs to the pressure transducer. The pressure transducer used in these experiments was the OMEGA PX409-001DDU5V High Accuracy Transducer. For a typical test case there was a standard deviation of less than 4.1 kPa in pressure which correlated to a velocity of 1 m/s. Where velocity is calculated using Bernoulli’s equation, resulting in:

\[ V = \sqrt{\frac{2P}{\rho}}. \]

The pressure transducer was then connected to a Texas Instrumentation data acquisition system and interfaced with a custom LabView program for recording all data.

2.2.5 Atmospheric Measurements

To determine the dynamic viscosity and density on each day of testing, we made three measurements: temperature \((T)\), pressure \((P)\), and relative humidity \((h_r)\). To measure tempera-
ture and pressure we used a Science Associates Incorporated mercury barometer and to measure relative humidity we used an Omega RH-201 hygrometer. From these measurements both vapor pressure \( P_v \); Equation 2.1 and dynamic viscosity \( \mu \); Sutherland’s Equation; Equation 2.2 can be calculated after which density can be solved for using the ideal gas law (Equation 2.3).

\[
P_v = 610.78\exp^{17.2694T_c / (T_c + 238.3)}
\]

\[
\mu = \mu_{sl} \left( \frac{T^{3/2}}{T_{sl}} \right) \frac{T_{sl} + T_S}{T + T_S}
\]

\[
\rho = \frac{1}{T} \left( \frac{P}{R_a} - h_r P_v \left( \frac{1}{R_a} - \frac{1}{R_w} \right) \right)
\]

The density and dynamic viscosity were then used to nondimensionalize recorded values using nondimensional numbers such as Reynolds number \( Re = \rho V D / \mu \).

2.3 Computational Fluid Dynamic Simulations

While significant effort is required to create a CFD simulation that is both validated and verified, once those steps are complete that model can be modified and extended to explore a myriad of different configurations while providing rich information over the entire fluid domain. Additionally, within a CFD model all supporting structures and extraneous noise sources can be eliminated to isolate only the exact interaction in which you are interested. For the rich flow field detail (Figure 2.9), adaptability once created, and ability to isolate exact interactions, CFD was used as the primary research method in this study.

2.3.1 Domain

Since all our test cases were operating well within the subsonic regime and the Mach number being equal to less than 0.3, we used a bullet shape domain as recommended in STAR-CCM+ documentation for incompressible external aerodynamics. As recommended by Abras et al. [30], the spherical end of the domain was created with a radius of 10\( L \) where \( L \) is the length scale of the simulation. The downstream domain was cylindrical and extended 20\( L \) downstream as suggested.
Figure 2.9: Example images of the flow field data CFD can provide. Left: vorticity (local rotation of a fluid particle) flow-field for a single rotor. Right: turbulent kinetic energy (kinetic energy of eddies within the flow) flow-field for two rotors.

by multiple authors in similar modeling situations [8, 18, 30] (Figure 2.10). For simplicity, all simulations used the same characteristic length $L$ of four rotor diameters as this was the maximum expected characteristic length across all configurations. The rotor was located within this domain at the center of the spherical end.

2.3.2 Motion

For simplicity, rigid body motion was used in this simulation and all aero-structural interactions were not modeled. To define the motion of the rotor, a small cylindrical volume which extended 3 mm beyond the tips of the rotor and was 2.5 cm thick was used to completely contain the rotor (Figure 2.10). We will refer to this volume as the rotating “puck”. The entire domain was then split into two regions with the first region being the full domain minus the puck and the second region being the puck minus the rotor geometry. A rotating coordinate system is then defined at the center of the rotor and then applied to the puck to define its motion. An internal interface is created on the boundary between the puck and the rest of the domain to join the rotating and stationary domains of the simulation. For simulations with two rotors, the same process is followed but there are three resulting regions (two rotating pucks). With the described method of defining motion, the transfer of cell information occurs only across the defined internal interface boundary. While this method was used for its previous use within the FLOW Lab, an overset mesh could potentially
lead to improved results (with increased computation time) as the transfer of cell information in an overset mesh occurs more gradually over a region rather than just at a boundary.

Figure 2.10: Left: Simulation domain. Right: Rotating volume within domain.

2.3.3 Boundary Conditions

The spherical end of the domain set to be a velocity inlet boundary, the cylindrical surface was defined as a the freestream, and the backside of the domain was defined as a pressure outlet boundary (Figure 2.11). These boundary conditions were all set to velocities of zero or to the ambient pressure in order to represent hovering flight with no freestream flow for our final test cases. The boundary for defining the motion was an internal interface boundary as described in the previous section and can be seen for the double rotor configuration in Figure 2.11 as the colored surface. This type of boundary joins the two fluid volumes as though they were one continuous fluid regime, like you have with an actual rotor spinning in open space.

2.3.4 Meshing

Due to the computational expense of running an unsteady RANS simulation, an efficient mesh was of key importance in this research. How quickly simulations could be solved would
Figure 2.11: Boundary conditions used in all simulations. Left: velocity inlet, freestream, and pressure outlet boundaries. Right: colored surface is the surface to which the internal interface boundary is applied.

directly impact how many simulations could be run and more importantly how many configuration scenarios could be investigated. As such, a significant part of this research was spent on the CFD mesh and improving the refinement scheme. The meshing choices and selected models were largely chosen based on the Recommended Mesh Settings section from the STAR-CCM+ documentation on External Aerodynamics.

A polyhedral mesh with a surface remesher and a prism layer mesher were chosen for the rotating volume around the rotors for the ability of the polyhedral mesher to better handle the complicated curved surfaces of the rotors along with the larger changes in flow direction as the rotor rotated through the region. The rest of the fluid domain was meshed with the Trimmed Cell mesher and a surface remesher for the increased efficiency of a structured mesh as well as the use of similar meshes in existing papers [9,30].

**Polyhedral Mesher**

With the polyhedral mesh selected, the settings were changed to better match the given characteristic length of the simulation and provide a gradual transition in mesh size from the small cells at the rotor’s surface to the large cells at the outer domain boundary. Following the recommendations found in the documentation, similar papers, a grid convergence study (Section 3.2.1), and iterating based on post-processing visualization, the following mesh default control values were selected:
Two additional custom controls were applied to the mesh to achieve the desired mesh. The surface of the rotating region was set to have a Target Surface Size of 0.3 BS and the whole rotating volume was defined to have a volumetric refinement of 0.3 BS as well. All other settings within the default controls were left unmodified.

**Surface Remesher and Prism Layer Mesher**

The surface remesher was used to provide a more defined and accurate mesh along all surfaces within the domain. All settings were left as their defaults for the surface mesher. Since the prism layer mesher is the mesher that defines the important rotor surface boundary layers, it was especially important that the correct refinement was used. As such, the following boundary layer calculations were used to determine the boundary layer thickness, necessary number of prism layer cells, and initial prism layer thickness:

**Y+:** Y+ is essentially a Reynolds number—nondimensional ratio of fluid inertial forces to viscous forces—that is applicable near a wall. A Y+ value of one is required in order to resolve the entire viscous sublayer. However, resolving an entire surface boundary to a Y+ of 1, significantly increases computationally time. Consequently there are wall treatment functions that use equations to approximate the boundary layer. Our simulations use the All Y+ Wall Treatment model which only requires Y+ values between 30-50, and then approximates the rest of boundary layer. Using this wall treatment function, instead of resolving the entire boundary layer with elements, results in computational saving of ~2x by reducing the number of required elements in half.

While this wall model does allow for a range of Y+ values it was still important to ensure our Y+ values were around the right range. This was done by first calculating Reynolds number,
\[ \rho = \frac{1}{T} \left( \frac{P}{R_0} - h_r P_v \left( \frac{1}{R_0} - \frac{1}{R_w} \right) \right) \]  \hspace{2cm} (2.4)

\[ \text{Re}_{0.75} = \frac{\rho V_t c_{0.75}}{\mu}, \]  \hspace{2cm} (2.5)

where \( V_t \),

\[ V_t = \sqrt{(\omega r_{0.75})^2 + V_r^2}, \]  \hspace{2cm} (2.6)

is the velocity seen by the rotor blade at 75% of the radius. From these equations, we determined that the rotor cases in this research would be operating in a low Reynolds number (chord-based) regime of less than 200,000.

To calculate the total boundary layer thickness \( (T) \) we used the following equation from Schlichting’s boundary-layer theory:

\[ T = \frac{0.37 c_{0.75}}{\text{Re}_{0.75}^{0.2}}. \]  \hspace{2cm} (2.7)

Then, setting \( Y^+ \) to our desired value, as dictated by our wall model, we can derive that the initial prism layer must be a thickness of \( t \), where \( t \) is

\[ t = c_{0.75} Y^+ \sqrt{\frac{2}{\text{Re}_{0.75} c_f}}. \]  \hspace{2cm} (2.8)

To find \( c_f \), we assumed a turbulent boundary layer as defined in STAR-CCM+ and approximated it once again using Schlichting’s empirical data fit for boundary-layers to arrive at

\[ c_f = \frac{0.0592}{\text{Re}_{0.75}^{0.2}}. \]  \hspace{2cm} (2.9)

Then, since the mesher input within STAR-CCM+ requires the number of desired prism layers \( (N) \), we used the default growth rate of \( s = 1.2 \) and calculate \( N \) as

\[ N = \log_s \left( \frac{T}{t} (s - 1) + 1 \right). \]  \hspace{2cm} (2.10)

22
Additionally, the surface target cell size must be taken into account here, in order to avoid creating poor quality cells with too large of aspect ratios (length to width). This means that your target surface size needs to be around the same size as your initial prism layer cell thickness (Figure 2.12). One area of refinement for any future iterations would be to smooth out the growth rate from the cells near the rotor surface out to the fluid domain as it is not ideal to increase cell size so rapidly from one cell to the next. This rapid volume change is potentially due to the volumetric refinement that could be overriding the previously set volumetric growth rates. For our simulations, this process resulted in \( N = 5 \) prism layer cells and a total boundary layer thickness \((T)\) of 1 mm.

The resulting mesh on the rotor surface can be seen in Figure 2.12.

![Figure 2.12: Final mesh surface target size and the prism layer settings used to resolve the rotor boundary layers. Left: Cross-section prism layer cells along the rotor surface, Right: Rotor surface as viewed from rotor hub out to blade tip.](image)

**Trimmed Cell Mesher**

The fluid domain beyond the rotors is either stagnant or mostly only moving downstream, which makes a trimmed mesher the most efficient choice for the rest of the fluid domain. All default controls were set to be the same as the polyhedral mesher with the exception of the volume growth rate for the trimmed mesher being specified as very slow. The only other difference between the trimmed cell mesher and the polyhedral mesher was the addition of a surface control on all the far field domain boundary surfaces, which set the cell size on those surfaces to be approximately that.
of the characteristic length, $L$, or 1 m. The resulting structured mesh from the trimmed cell mesher can be seen in Figure 2.13.

Figure 2.13: Final meshing scheme used for the fluid domain. Top: Outer domain boundary meshed with a cross-section along the rotor axis of rotation, Bottom Left: Cross-section in the plane of rotation, Bottom Right: Cross-section along the axis of rotor rotation.
Domain Refinement Scheme

For general flow field refinement, adjoint, adaptive, and volume refinement schemes were all investigated. Adjoint meshing refinement was briefly tested in this research for its ability to directly refine cells which most directly contribute to performance value errors, but was found to be especially complicated when applied to a transient rotating mesh and was consequently eliminated as a possibility due to project time constraints.

Adaptive meshing is an equation-based refinement scheme which is appealing due to the ease with which it can refine geometrically complex regions based on user-defined mathematical equations. Additionally, adaptive meshing within STAR-CCM+ can be implemented to automatically and iteratively refine the fluid domain as the flow field continues to be solved. This is especially of interest as it would allow the initial transient solution to be solved more quickly with a coarse mesh and then automatically updated to a finer mesh once the transient flow has been resolved. Due to these potential time and computation benefits, significant time was spent exploring adaptive mesh refinement.

Refinement equations based on the Courant number, turbulent kinetic energy, density gradient, and velocity gradient were all considered based on their relation back to first principles which were believed to most greatly affect rotor thrust and torque values. The goal was to create a refinement criterion that created fine cells where the most change was occurring within the flow while keeping cell refinement minimal in areas where the flow was essentially stagnant or contributed less to the thrust and torque values. For this reason, a refinement based on the velocity gradient was determined to be the best option and was pursued.

A gradient, or vector derivative of a field, always points in the direction of the greatest change and its magnitude is the greatest rate of increase at a given point. With a desire to have fine cells where there were rapid changes in velocity, while maintaining coarser cells where the flow was more stagnant, we needed to define a refinement equation that could accurately relate velocity change to a distance change in three dimensions. The velocity gradient was the best option to represent the change in velocity at any given point in the domain. We also needed to normalize any velocity change by the distance over which it occurred, since a large velocity change across a small cell would be less desirable than that same velocity change across a larger cell. For simplicity, we approximate all cells as squares, where the distance over which the velocity change occurred,
or the length of the cell, was found from the cubic root of the cell volume. Combining both the velocity change and cell size together, the cell length and velocity gradient magnitude were multiplied together to create the refinement equation (Note: Dividing the values would actually better represent our refinement goal, but this was not realized until all results had been run and it was already decided not to use adaptive meshing). This refinement equation was then compared against a threshold value to create our overall refinement criteria:

\[
\text{mag}(\nabla V)l < X_{\text{threshold}}.
\] (2.11)

If the refinement criteria returned true for a given cell then it was left alone; however, if false, then the cell would be marked and divided in half during the next meshing iteration. This method was implemented in STAR-CCM+ by extracting a table of the refinement criteria values for every cell which was then used as an input into the Table Based Mesh Refinement function.

Figures 2.14 and 2.15 show the meshing progression, where the cells in red are the cells which exceed the function threshold and will be halved in the next meshing iteration. Pictured in Figures 2.14 and 2.15 are the initial mesh, the mesh after two refinement steps, and the mesh after five refinement steps, along with a scalar representation of the refinement criteria at each of those steps. The initial mesh contained \(\sim 5.0E5\) elements, the second step had \(\sim 1.9E6\) elements, and at refinement step five there were \(\sim 1.37E7\) elements. The refinement criteria resulted in a mesh that was refined around the rotor tips the most and then had refined regions extended downstream from those tips. This is to be expected, as this is the region of most extreme velocity change within the flow field. However, while the results of how the mesh looked was what we expected, the grid convergence was very noisy and did not show good convergence within a reasonable mesh cell count. Additionally, due to the rotors rotating within these simulations, it was found that the mesh refinement was dependent on the phase angle at which the rotor was located when the meshing step took place. To overcome this challenge, the refinement criteria would need to be time-averaged over multiple rotor rotations. With the poor initial results, and this added complexity and computational cost of time-averaging the flow over multiple revolutions it was decided to forego additional exploration into adaptive meshing for the current study.
Figure 2.14: Cross-section along the axis of rotation of an adaptive test mesh at the initial state, after two mesh refinement, and after five refinement (left to right). The top row is a scalar representation of the field where the red cells will be refined in the next mesh. The bottom row shows the mesh at each stage (NOTE: This is not the meshing scheme used in the final results).

Figure 2.15: Cross-section in the plane of rotation of an adaptive test mesh at the initial state, after two mesh refinement, and after five refinement (left to right). The top row is a scalar representation of the field where the red cells will be refined in the next mesh. The bottom row shows the mesh at each stage (NOTE: This is not the meshing scheme used in the final results).
The volumetric refinement, on the other hand, was extremely simple, and upon further review of the literature, was actually a very common refinement scheme among similar rotor simulations [18, 30, 34]. Our single rotor volume refinement region was a cylinder that begins 0.1 D upstream of the rotor and ends 0.6 D downstream, with a radius of 1.25 D (Figure 2.16). Similar refinement was used for both multi-rotor tip separation configuration and the staggered distance configurations. For the separation cases the area in between the two rotors was also refined (Figure 2.17) and for the staggered cases the upstream rotors wake was refined all the way downstream to where the downstream rotors refinement region ended (Figure 2.18). These adjustments were made to the multi-rotor cases to better capture rotor-rotor interactions. With the volumetric refinement regions defined, a maximum cell size of $0.3 \text{ BS}$ was then set in each of those regions and the mesh was created. One major strength of this refinement method over the other aforementioned methods was that the mesh only had to be created once. This was found to be the simpler, more repeatable, and much faster. Additionally, and more importantly, this method also resulted in a much better grid convergence as is shown in Section 3.2.1. The finalized meshes for the entire domain, the single rotor cases, the separation cases, and the staggered cases can be seen in Figures 2.13, 2.16, 2.17, and 2.18 respectively.

### 2.3.5 Physics Models and Solvers

The physics models and solvers were largely dictated by the nature of our problem and the goal to capture unsteady rotor flows. First, an implicit unsteady time solver is required in order to use a turbulence model within STAR-CCM+. An ideal gas model was selected since the rotors generally operate at a Mach number of less than 0.3, where compressibility and other air property changes are negligible. The Reynolds-Averaged Navier-Stokes SST (Menter) $k – \omega$ turbulence model was selected for its long-time use within the aerospace industry, as well as for it being better suited for solving complex body forces which were expected in these simulations. While the double equation $k – \omega$ turbulence model is more computationally expensive than other models such as the single equation Spalart-Allmaras turbulence model, this additional computational expense was seen as justified due to the small deviations in body forces that we were hoping to capture within
Figure 2.16: Final meshing scheme used for the single rotor cases. Left: Cross-section in the plane of rotation, Right: Cross-section along the axis of rotor rotation.

Figure 2.17: Final meshing scheme used for the separated rotor configurations Top: Cross-section in the plane of rotation, Bottom: Cross-section along the axis of rotor rotation, Left: $s = 0.05D$, Right: $s = 1.00D$

our simulations. One other model of note was the All Y+ Wall Treatment model. This model allows for Y+ values between 30-50 and was used for the significant savings in mesh elements that would be required to resolve the boundary layer all the way down to a Y+ value of one which was already discussed in Section 2.3.4.

2.3.6 Dynamic Similarity

Dynamic similarity is the process through which the same flow patterns can be achieved in different atmospheric conditions or for different length scales. It is a core concept in fluids research and is achieved through the matching of non-dimensional number such as Reynolds number (Re) and advance ratio (J). All the non-dimensional numbers (Eqs. 2.12-2.15) and all other key rotor flow definitions (Eqs. 2.16-2.20) as they were used in this study are defined below. One exception
Figure 2.18: Final meshing scheme used for the staggered rotor configurations Top: Cross-section in the plane of rotation, Bottom: Cross-section along the axis of rotor rotation (left to right: $X = 0.00D$, $X = 1.00D$, Right: $X = 3.00D$)

to this dynamic similarity is in the study of noise generation between two rotors of different diameters. For these cases, the total thrust values were matched as suggested by Zawodny et al. [23] for better noise comparisons.

$$J = \frac{V_\infty}{nD}$$  \hspace{1cm} (2.12)

$$M = \frac{V_\infty}{c}$$  \hspace{1cm} (2.13)

$$Re_{0.7} = \frac{\rho V_{0.7}D}{\mu} = \frac{VD}{v}$$  \hspace{1cm} (2.14)
\[ Y_+ = \frac{\rho U_\tau \Delta y_1}{\mu} \]  \hspace{1cm} (2.15)

\[ C_T = \frac{T}{\rho n^2 D^4} \]  \hspace{1cm} (2.16)

\[ C_Q = \frac{Q}{\rho n^2 D^5} \]  \hspace{1cm} (2.17)

\[ \eta = \frac{VT}{nQ} \]  \hspace{1cm} (2.18)

\[ V_{0.7} = \sqrt{V_\infty^2 + (\omega \tau_{0.7})^2} \]  \hspace{1cm} (2.19)

\[ V_\infty = JnD \]  \hspace{1cm} (2.20)

### 2.3.7 Resources

All simulations were meshed locally on a desktop workstation and then submitted to the BYU Fulton Supercomputer. The workstation has 57 Intel® Xeon(R) CPU E5-2699 v3 processors at 2.30GHz with 128 GB of memory. Meshes were generally created using 36 cores and would typically require around 40 minutes to mesh. The BYU Fulton supercomputer has a total of 23,920 CPU cores with 92 TB of memory across 1,012 compute nodes. This was a key resource that allowed for this research to take place. Typical runs consisted of requesting 192 cores across 12 nodes with 3 GB of memory per core or a total of 576 GB of memory. Double rotor meshes generally had approximately 14 million cells and would need to run for around 25 revolutions before the thrust and torque parameters were mostly converged. To ensure full convergence most simulations were ran to 50 revolutions, which required a total of \( \sim 48 \) hours of run time for the computational resources specified above.
2.4 Aeroacoustics Solver

The focus of this research was not to explore the intricacies of noise prediction, but rather to provide approximate quantifications of changes in produced noise as a result of varying rotors and rotor configurations. Consequently, developing our own noise prediction code was well beyond the scope or goal of this research and we instead used the existing PSU-WOPWOP aeroacoustics solver. It is based on the well-known theoretical Farassat’s Formulation 1A that is used in many other noise prediction codes such as NASA’s WOPWOP [35] and University of Michigan’s HELINOIR [27] aeroacoustic codes.

The Ffowcs Williams-Hawkings (FW-H) analogy, reduces the Navier-Stokes equations to

$$\Box^2 p'(x,t) = \frac{\partial}{\partial t} (p_0 v_n \delta(f)) - \frac{\partial}{\partial x_i} \left( \Delta P_{ij} \hat{n}_j \delta(f) \right),$$

where $p'$ is the acoustic pressure, and $\Box^2$ is the wave-equation operator. The first term in the right-hand side is a monopole source representing the volume displaced by the thickness of a solid body, the second term is a dipole source representing the force applied on the fluid by that body, and higher-order terms (quadrupole sources) have been neglected. This formulation has been shown to accurately predict helicopter noise radiated from complicated aerodynamic phenomena like blade-vortex interaction and high-speed impulsive effects [22]; however, it has not yet been applied to the prediction of rotor-on-rotor aeroacoustic interactions. In a recent study, Zolbayar et al. [36] coupled FW-H with blade-element momentum theory to investigate the noise of a light airplane with distributed propulsion. He was able to draw conclusions about the accumulation of noise directivity, but he recognized the need of a higher-fidelity aerodynamic solver in order to capture important sources of noise associated with rotor-on-rotor and wing-on-rotor interactions. Thus, in this study we explore whether the increased noise due to rotor-on-rotor interactions observed experimentally in Ref. 9 can be predicted by coupling RANS or VPM with the FW-H equation.

While only the integration between PSU-WOPWOP and our CFD simulations will be specified in more detail here, all the details of the noise code as it relates to our research can be found in B. Goldman’s Masters thesis [25].
2.4.1 PSU-WOPWOP Inputs/Outputs

The inputs which we are giving to PSU-WOPWOP are the blade geometry, blade loading history, and microphone locations. The thickness of the body is given to PSU-WOPWOP as a three-dimensional surface taken directly from the CAD model and the unsteady loading is given in a compact-patch approach along \(\sim 30\) blade segments as outputted from Star-CCM+. From this we get back out the tonal noise prediction at the microphone locations. All results are then post-processed using Julia and then visualized using Paraview. The microphone locations were created based on the papers which were used to validate the results. This meant that for all APC and DJI results the microphones were located at a distance of 1.44 m (or \(\sim 6\) D) away from the plane of symmetry in the side-by-side case and the DJI 9443 results were located 1.905 m away from the rotor hub. The microphones were defined along these radii from \(0^\circ\) to \(360^\circ\), with \(0^\circ\) and \(180^\circ\) being in the plane of rotation of the most forward/upstream rotor and \(90^\circ\) being directly in front of the rotors as seen in Figure 2.19.

![Microphone location definition](image.png)

Figure 2.19: Microphone location definition. There are 360 microphones in total, but only a few are shown for illustration purposes.
2.4.2 CFD

In order to extract the loading data from our CFD simulations, the rotor blades were discretized into segments of 0.5 cm to 1.27 cm in length with the finer segments being located at the tips for better loading resolution, as seen in Figure 2.20. For convenience, the DJI 9443 and Ning DJI rotors were discretized into 24 equal segments (0.01 m), after which the last three segments on each tip were split in half (0.005 m), resulting in a total of 30 segments. The APC 10x7E propeller was discretized in a similar fashion other than it was initially only split into 20 segments (0.0127 m) with the 12 post-refinement tip segments being 0.00635 m long. With the rotors discretized, three pressure-based force reports were created for each segment in the X, Y, and Z directions. Results were recorded at each time step in monitors from which the data was then exported once the run had finished. This loading data was then used as the input to PSU-WOPWOP.

Figure 2.20: Discretization of Ning DJI (Left) and APC 10x7E (Right) rotors within STAR-CCM+ used to get rotor loading history.
CHAPTER 3. VALIDATION AND VERIFICATION OF METHODS

3.1 Wind Tunnel Testing

The APC 10x7E thin electric propeller was chosen for our empirical testing validation due to there being multiple sources of published performance data for it [11,13]. Our testing was done at a Reynolds number of 1E6 and then compared to data published by McCrink et al. [13] at a similar Reynolds number of 1.5E6 (Figures 3.1 and 3.2). The discrepancy in Reynolds number was due to our original Reynolds number calculations being done based on the full diameter of the propeller rather than the 70% diameter as used by Ohio state, however, McCrink et al. [13] showed that a change of 500,000 in the Reynolds number only changed the efficiency by less than 1% for most points. This was much less than the uncertainty in our data and was consequently neglected. Overall, our results matched within ~10% for most points and were mostly contained within the uncertainty bars.

![Figure 3.1](image-url): Comparison of our wind tunnel APC 10x7E coefficient of thrust and power results with data published by McCrink et al. across varying advance ratios.

While we were able to refine our single propeller experimental procedure and get it working, we found that when we tried to extend it to the double propeller case there were multiple...
additional considerations that would need to be addressed. One issue was that the stands would need to be individual mounted and structurally isolated in order to accurately capture any rotor-rotor interaction. If the systems were not dynamically isolated, then the smaller aerodynamic interaction would likely be indiscernible in the data. In order to fully isolate the stands, extensive additional structure would be needed to modify our current setup. Zhou et al. noted the importance of isolating the propeller, especially when trying to resolve multi-rotor interactions [9]. Additionally, new test stand equipment would be needed with a higher data acquisition rate. Our current test stands only had data acquisition rates of 8 Hz which made it challenging to accurately capture the dynamic loading. For some test cases, we had to run the rotors for up to 20 minutes in order to gather numerically significant data samples. This long run time then led to all sorts the other complex challenges as the motors would begin to drift as they warmed up. All of these problems, along with the fact that the interactions we were interested in could be as small as 0.03 N, or 1% to 2%, finalized our decision to not pursue the experimental testing any further at this point in the research and instead focus fully on the computational analysis.

3.1.1 Uncertainty Analysis

The uncertainty results presented for this experimental setup were performed by fellow FLOW Lab member, Judd Mehr, and can be found in greater detail in Appendix A. The basic approach, however, was to find the uncertainty \( u \), based on the resolution of the instrument \( (u_0) \),
and the accuracy of the instrument \( (\nu_I) \) as seen in Equation 3.1.

\[
\nu = \sqrt{\nu_0^2 + \nu_I^2}
\]  

(3.1)

This resulted in uncertainty of up to 0.2 N in the thrust measurements and up to 0.03 N-m the torque measurements. The uncertainty for the coefficients of thrust and torque as well as efficiency can be seen in Figures 3.1 and 3.2.

### 3.2 Computational Fluid Dynamic Simulations

#### 3.2.1 Verification

The RANS computational fluid dynamics simulation used in this study was created in STAR-CCM+ v11.04-r8 using the polyhedral and trimmed cell meshers. In order to verify that there was sufficient cell resolution within the domain to resolve all important length scales, a grid convergence study was run on a single propeller simulation at an advance ratio of 0.6. This was performed by varying the base sizes of the mesh from 4-0.325, where the base size is a parameter to which all cells within the mesh are scaled (excluding the propeller boundary layer cells, which are fixed based on desired \( Y^+ \) values). This varying of the base size corresponded to 100 thousand and 18 million elements, respectively. Convergence of the key performance values that were to be extracted from the simulations—thrust and torque—can be seen in Figure 3.3. From this convergence study, we determined that a mesh of 7.8 millions cells would resolve the thrust values within \( \sim 1.5\% \) of the fully-converged value (determined from a Richardson extrapolation), while still not being too computationally expensive.

Figures 2.13, 2.16, 2.17, and 2.18 show cross-sectional images of the final mesh for all configurations. Due to the computational expense in solving a fully transient RANS simulation with this amount of cells, only one radius length downstream was refined as is shown to be sufficient in the following section. The cell size along the edge of the domain was 0.3 m while the average cell size on the surface of the propeller was 0.25 mm. The 7.8 million element single-rotor simulation was run on 198 cores with 400 GB of RAM for one day in order to solve 25 revolutions with a time step corresponding to 3° of rotation.
Figure 3.3: Grid convergence results for thrust (left) and torque (right) on a single APC 10x7E propeller.

3.2.2 Validation

The single propeller simulation was then validated against empirical data presented by McCrink et al. [13]. The CFD results for coefficient of thrust and torque generally followed the trend of the published data but underpredicted coefficients of thrust and torque and efficiency (Figures 3.4 and 3.5), which is similar to the underprediction seen by Chiew et al. [18] in their thrust coefficient results. Coefficient values at an advance ratio of 0.3, 0.5, and 0.6 fell up to 9% outside of the presented error bars, as can be seen in Figure 3.4. This error corresponds to a difference of 0.05 for $C_T$ and 0.001 for $C_Q$. The error is in part due to the mesh resolution used in the simulation and the offset that this introduces (Figure 3.3). However, when this offset is divided out, as is done in efficiency calculations, the results matched much closer, as shown in Figure 3.5.

To investigate other potential causes of this underprediction, additional mesh verification was run on the worst point of $J=0.5$. The first potential cause of the performance underprediction was the possibility that the boundary layer was not being sufficiently resolved. To verify if this was the case we re-meshed the simulation and increased the number prism layers used to resolve the boundary layers from 4 to 25, which gave us a $Y+$ value of around one. However, while this nearly doubled the number of cells within the mesh and significantly increased the computational time required to solve the field, it resulted in only a 1% change in the predicted thrust and torque. Consequently, we concluded that additional boundary layer resolution was not needed. Another potential cause for the under prediction was potentially an insufficient time-step resolution. Thus,
we look at the effects of time-step size by refining it. The initial simulation had a temporal resolution of 3° per time step and the refined simulation had a resolution of 0.5° per time step, which increased the computation time by a factor of six while only resulting in a difference of 1.6% in the thrust. As such, the simulations for all experiments were run at the coarser time step resolution of 3°. The last additional verification test that was run was on how far downstream from the propeller the wake was resolved. Our initial simulation refines the wake one radius downstream of the propeller where the cells then begin to slowly increase in size. Our additional wake refinement simulation refined the wake 4 radii downstream of the propeller, which increased the number of
mesh cells by 10% while resulting in a 0.04% change in the reported thrust value. Consequently additional downstream refinement was also foregone and only one radius was refined.

With none of the above simulation refinements significantly impacting how well our results matched the experimental data, we decided to try validating against another test case, published by Zawodny et al. [23]. By validating against a different, but similar propeller, we were able to see if it was likely that it was the actually APC 10x7E propeller geometry that was causing the discrepancies. Zawodny et al. reported the coefficient of thrust for the DJI 9443 rotor at an RPM of 5400 of 0.0722 and our simulation converged to a coefficient of 0.0715 which was only 1% less than the experimental measurement. Our simulation converged to this value in fewer than 10 revolutions (Figure 3.6). These results, along with the aforementioned simulation refinements that yielded no better agreement with experimental results for the APC 10x7E case, gave us much more confidence that it was not our simulation but rather the inaccuracies in our geometry digitization which was causing the underprediction of thrust. This conclusion is additionally supported by the results of our 3D printed propeller wind tunnel results that were shown in Figure 2.5.

Figure 3.6: Agreement between RANS simulation and experimental CT data is within 1% for the DJI 9443 rotor. Dashed lines represent the uncertainty reported in the experimental measurements.
3.3 Noise Validation

Due to the plethora of factors that influence and contribute to noise generation, it can be very challenging to accurately predict. The purpose of including noise results within this research is to provide approximate noise predictions and trends as they relate to rotor-rotor interactions and is not to delve into the intricacies of noise generation or prediction. The aeroacoustics solver utilized was PSU-WOPWOP, which has previously been verified and validated for a number of large rotor cases [28], however additional validation is presented here for small UAV-type rotors as is more consistent with our research cases. Additionally, PSU-WOPWOP incorporates monopole, dipole, and quadrupole noise sources, of which we are neglecting the quadrupole source. In our noise predictions we include the effects of body thickness and unsteady loading, while neglecting sources of broadband noise. Experimental noise measurements for a single DJI 9443 rotor were taken from a study by Zawodny et al. to validate against. In that study they investigated noise characterization and prediction for small UAV-type rotors [23]. The rotor was ran at 5400 rpms with a Reynolds number of ~6.75E5.

Before looking at the noise validation results we have to understand a few key acoustics terms on a very basic level. First, is the blade passing frequency (BPF), which is defined as the rotational speed ($\omega R$) multiplied by the number of blades on the rotor and is essentially how often a blade on the rotor passes by the observers location. It is generally the first peak you see in the noise data (frequency domain) and then its harmonics generally make up the other following peaks. The first three BPFs are the ones that most significantly contribute to tonal noise. Second, is the sound pressure level (SPL), which is the change in pressure away from the ambient pressure as caused by the sound wave. Third, is the overall sound pressure level (OASPL), which is an integration of the SPL across all frequencies. It is convenient for condensing a full frequency domain into one equivalent value. The one problem with the OASPL is that humans don’t perceive all the frequencies equally and that is where A-weighted values are useful as it places more emphasis on values between 1,000 to 10,000 Hz, which humans can hear, while neglecting other frequencies. Due to this, the A-weighted OASPL is a better representation of what a human would hear and turns out to be a better metric for representing rotor-rotor noise interactions. Finally, it is important to mention again that this research only aims to predict the rotor’s tonal noise values at the BPFs.
This is seen in the frequency domain as the peaks, and it is only at those points which we are attempting to predict the noise levels.

Our results using STAR-CCM+ with PSU-WOPWOP matched the experimental results within 1.5 dB for the first blade passing frequency and within about 4 dB for the second blade passing frequency (Figure 3.7). As can be seen in the frequency domain from Figure 3.7, our noise prediction is only capturing the tonal frequencies (blade passing frequencies (BPF)) and is not capturing the broadband noise. The overall sound pressure level (OASPL) at an array of microphones located 1.905 m from the rotor hub can be seen in Figure 3.8. For microphones at 45° and 315°, our OASPL prediction is within ~1 dB of the experimental data (Figure 3.8). In the plane of rotation (0° and 180°), however, our predicted noise is 19 dB less than the experimental data. This validation shows that we can capture general trends and approximate tonal noise levels, but that future work could focus on fine-tuning these predictions. Additionally, considering that research with higher-fidelity (than CFD) numerical methods at the NASA Langley Research Center underpredicted SPL’s by up to 10 dB [23] we were very pleased with how well our results matched the experimental data.

Figure 3.7: Comparison of our STAR-CCM+ with PSU-WOPWOP frequency domain noise prediction to published data for a single DJI 9443 rotor.
Figure 3.8: Comparison of our STAR-CCM+ with PSU-WOPWOP OASPL noise prediction results to published data for a single DJI 9443 rotor.
CHAPTER 4. RESULTS

Below are the performance, flow field, and noise results for the Ning DJI and APC 10x7E rotors under the following three configurations:

1. Single rotor case
2. Tip separation distance sweep
3. Downstream separation distance sweep

4.1 Single Propeller Loading

4.1.1 Performance

In order to best compare the noise production from rotors of different geometries and diameters, the total thrust output of the different rotors were matched, as suggested by Zawodny et al. [23] and as detailed in Section 2.3.6. Since the flow parameters for the Ning DJI rotor were already defined by the validation data, we matched the APC 10x7E rotor’s total thrust to it. This was done by iteratively reducing the APC 10x7E rotor’s rotational speed and interpolating until it produced close to the same thrust as the Ning DJI rotor. This resulted in the APC propeller being run at a rotation rate of 4660 rpms which resulted in 3.155 N and a Reynolds number of 7E5 and matched the Ning DJI rotor’s total thrust of 3.123 N (at 4850 rpm and a Reynolds number of 6.5E5) within 1.1% (Figure 4.1).

4.1.2 Noise

It was initially hypothesized that if a rotor carried more load towards its tips, that it would potentially lead to stronger rotor-rotor interactions, due to the stronger resulting tip vortices. As such, the rotor loading for both the Ning DJI rotor and the APC 10x7E propeller are presented in
Figure 4.1: Convergence of the Ning DJI and APC 10x7E rotor to a thrust value of 3.123 N and 3.155 N respectively.

Figure 4.2: Left: Loading distribution with the arrows representing the location of an equivalent load. Right: Thrust fluctuations (standard deviation) divided by segment length.

Figure 4.2, along with an approximation of the equivalent load location—or the location at which a single point load on that blade would need to be located in order to cause the same moment about the hub given that the point load is the magnitude of the total blade loading, for each rotor. In this figure we can see that the Ning DJI rotor carries more of its load towards the tip than the APC propeller does and that the APC loading fluctuates more than that of the DJI rotor. As discussed in Section 1.1.2, the rotor thrust fluctuations, or standard deviation of thrust, were presented by Carolus et al. [24] as a way to evaluate noise. Zhou et al. [9] also later built on that claim, and
concluded that the noise increase they observed in a rotor-rotor configuration was in part due to a corresponding increase in thrust fluctuations along the rotors. Consequently, we will also present the thrust fluctuations as an approximation for noise, after which we compare it to actual predicted noise values from the aeroacoustics solver. The single rotor cases are presented first as a baseline before moving onto the multi-rotor configurations (Figure 4.2). Both the loading distributions and the thrust fluctuations were expected to have a significant influence on noise generation for both the single and multi-rotor cases.

Figure 4.3: Comparison of the frequency domain for microphones located at 30°(top) and 90°(bottom) for the single Ning DJI and APC 10x7E rotors.

Once the CFD simulations were solved, the rotor loading time histories were extracted and then processed with the PSU-WOPWOP aeroacoustics prediction code. The noise frequency results from microphones located at 30° and 90° for both the Ning DJI and APC 10x7E rotors can be seen in Figure 4.3. The results show that the APC 10x7E rotor has a higher sound pressure level (SPL)—the sound pressure related back on a logarithmic scale to the lowest pressure change
that the human ear can perceive (~2E-5 Pa)—at both the 30° and 90° microphone locations for all blade passing frequencies. This means that the APC 10x7E propeller is louder than the Ning DJI rotor. For the first two BPFs the APC 10x7E rotor produces a SPL ~1-3 dBA higher than that of the Ning DJI rotor.

Plotting the overall sound pressure level (OASPL) at each microphone location surrounding a rotor shows the noise directivity, or the directions in which it is the loudest. However, as seen in Figure 4.4, this method shows that the Ning DJI and APC 10x7E rotors are practically identical in their noise generation despite their different geometries and diameters. This was unexpected based on the differences that were seen in the frequency domain (Figure 4.3); however, building on what was discussed in Section 3.3, it was found that this discrepancy was caused by the fact that a significant portion of the noise predicted by PSU-WOPWOP occurs at low, near zero, frequencies. This large portion of the predicted noise is not shown in Figure 4.3 due to a logarithmic scale being used. These low-frequency results, that humans can’t discern, drown out any other smaller difference that may be present between the different rotors. As such it was important to look at the A-weighted OASPL. This eliminates the lower frequencies which dominate the OASPL noise results and produces a much more insightful metric for comparing noise generation from different rotors or rotor configurations. Figure 4.4 shows the OASPL verse the A-weighted OASPL for the Ning DJI and APC 10x7E rotors. As was seen in the frequency domain, the A-weighted OASPL also shows that the APC 10x7E rotor is louder than the Ning DJI rotor, with a difference of up to
\sim 5 \text{ dBA} \ (\text{direction dependent}). \ This \ is \ a \ significant \ difference, \ considering \ that \ a \ 3 \ \text{dB} \ increase \ in 
\text{sound} \ corresponds \ to \ a \ doubling \ of \ power \ (\text{Watts}). 

The \ frequency \ and \ A-weighted \ directivity \ results \ would \ both \ suggest \ that \ the \ greater \ thrust 
fluctuations, \ or \ standard \ deviation \ in \ the \ thrust \ time \ history, \ of \ the \ APC \ propeller \ verse \ the \ DJI \ rotor, \ is \ the \ dominating \ factor \ in \ noise \ generation \ rather \ than \ the \ differences \ in \ tip \ loading. \ However 
we \ cannot \ conclude \ that \ the \ tip \ loading \ does \ not \ affect \ noise \ generation, \ only \ that \ if \ it \ does, \ its 
effects \ are \ small \ in \ comparison \ to \ the \ effects \ of \ thrust \ fluctuations.

4.2 Rotor-Rotor Tip Separation Sweep

4.2.1 Performance

For \ the \ tip-to-tip \ separation \ distance \ case, \ two \ rotors \ were \ moved \ incrementally \ from \ 1 \ D 
apart \ to \ 0.05 \ D \ apart \ (Figure \ 1.2). \ As \ seen \ in \ Figure \ 4.5, \ this \ resulted \ in \ a \ maximum \ thrust \ decrease 
of \ \sim 1\% \ when \ normalized \ by \ the \ 1 \ D \ tip \ separation \ case. \ This \ trend \ was \ observed \ for \ both \ the \ Ning 
DJI \ and \ APC 10x7E \ rotors \ and \ agrees \ well \ with \ multiple \ other \ papers \ that \ similarly \ report \ a \ 1- 
4\% \ decrease \ in \ thrust \ produced \ for \ comparable \ configurations [8, 9, 14, 15]. \ Figure \ 4.6 \ shows 
the \ thrust \ fluctuations \ for \ both \ rotors, \ in \ which \ the \ RANS \ solution \ shows \ a \ \sim 20x \ increase \ as 
the \ rotors \ are \ brought \ together. \ While \ the \ experimental \ data \ from \ Zhou \ et \ al. \ shows \ only \ a \ 3.5x 
increase \ in \ fluctuations, \ this \ could \ have \ been \ due \ to \ multiple \ different \ reasons. \ One \ potential 
cause \ for \ this \ difference \ is \ how \ the \ rotors \ interact \ with \ all \ the \ supporting \ structures \ and \ equipment 
that \ are \ necessary \ in \ the \ experimental \ testing, \ and \ which \ are \ not \ modeled \ in \ the \ CFD \ simulation. 
Considering \ that \ if \ bringing \ two \ rotors \ closer \ together \ could \ augment \ the \ thrust \ fluctuations \ by 
20x, \ then \ it \ is \ reasonable \ to \ conclude \ that \ the \ attached \ motor \ and \ supporting \ structures \ would \ also 
cause \ some \ significant \ increase \ in \ thrust \ fluctuations \ as \ well. \ Additionally, \ other \ sources \ of \ noise 
that \ were \ not \ captured \ in \ our \ CFD \ model, \ such \ as \ motor \ noise \ or \ background \ noise, \ could \ have \ a 
significant \ effect. \ While \ additional \ research \ would \ be \ needed \ to \ identify \ the \ exact \ cause \ of \ these 
discrepancies, \ the \ close \ agreement \ between \ both \ the \ Ning \ DJI \ and \ APC 10x7E \ rotors \ in \ the \ RANS 
simulations, \ as \ well \ as \ with \ the \ general \ trend \ of \ the \ experimental \ measurements, \ is \ reassuring. \ As 
mentioned \ previously, \ our \ goal \ in \ this \ research \ is \ to \ reveal \ general \ aeroacoustic \ trends \ and \ not 
exact \ numerical \ values.
4.2.2 Flow Field

The solved CFD simulations are valuable as they allow you to look at the entire flow domain rather than just preselected viewing planes as is the case with particle imagine velocimetry (PIV) [9]. This allowed us not only to see if the CFD trends were in agreement with the experimental test, but if they were not in agreement, it allowed us to see if the expected trends were occurring somewhere else within the flow field. The results from the PIV testing show a region between the two rotors that Zhou et al. referred to as a separation region (Figure 4.7). The CFD results, however, do not show this behavior at 0.1 D downstream of the rotors as the PIV results do, but
rather show it occurring farther downstream at the 0.4 D cross-plane (Figure 4.8). One potential reason for this discrepancy or delay in the CFD results is that Zhou et al. attributed its formation to the separation that was occurring over the rotor airfoils, and the CFD uses an “All Y+” boundary layer model which would likely not be as accurate for large separation regions.

Figure 4.7: Comparison of DJI propeller time-averaged velocity cross-plane at 0.1 D downstream from Iowa State Universities PIV results (left) to the CFD results (right).

Figure 4.8: Delayed separation region seen at 0.4 D downstream.
Figure 4.9: Phase-locked vorticity distribution of DJI propeller PIV results (left) to the CFD results (right). Phase-locked at $0^\circ$ and averaged over multiple revolutions.

Figure 4.10: Phase-locked vorticity distribution of DJI propeller PIV results (left) to the CFD results (right). Phase-locked at $120^\circ$ and averaged over multiple revolutions.

Perhaps one of the biggest differences and advantages of the CFD simulation is the removal of any motor, mounting structure, or supporting hardware, as was discussed above. Some interesting consequences of this are that the shear layers labelled as $a-d$ in Figure 4.9, appear to occur slightly later in the CFD flow field. As seen in Figure 4.9, the very first shear layer comes off about $0.15 \, D$ downstream, where the PIV results show it occurring at just $0.10 \, D$ downstream,
along with it being more pronounced and perpendicular to the flow. These differences are possibly
caused, in part, by the motor and stand not being included in the simulation. Apart from these small
differences, the PIV and CFD results seem to agree quite well. The leapfrogging of tip vortices
occurs near the 0.4 D mark and the tip vortices occur later in the flow for the 120° phase-locked
system (Figure 4.10) than they do for the 0° phase-locked results in both the CFD and experimental
results. Additionally the vorticity magnitudes and wake shapes are very similar between both
methods. This agreement gives additional confidence that the CFD simulations are accurately
representing the flow characteristics.

Figure 4.11: Frequency spectrum of noise results for the tip separation distances ($s$) with the
RANS+PSU-WOPWOP results on the top and the experimental data on the bottom for a microphone at 30°.
4.2.3 Noise

To investigate the effect of decreasing the tip separation between the two rotors, the CFD pressure loading history was run through PSU-WOPWOP, as it was for the single rotor cases. The frequency domain from the predicted noise results can be seen in Figures 4.11 and 4.12 alongside the experimental measurements [9]. The noise predictions match the experimental values within $\sim$6 dBA at the first BPF. However, at the rest of the BPF harmonics the predicted noise levels do not match as well. At the BPF for the microphone at 90° overpredicts the 0.05 D case by $\sim$5 dBA.
Figure 4.13: Frequency spectrum of noise results for increasing tip separation distances (s) with the RANS+PSU-WOPWOP vs experimental data for the 90° microphone for 0.05 D (top) and 1.00 D (bottom) cases while underpredicting the 1 D case as compared to the experimental measurements. Additionally, at the first BPF harmonic (also referred to as the second BPF) for the 1 D case at 30° microphone significantly underpredicted the experimental measurement by \(\sim 20\) dBA for the Ning DJI rotor and \(\sim 10\) dBA for the APC 10x7E rotor. The experimental and numerical results for a microphone at 90° are shown on the same plot for better comparison in Figure 4.13.

The frequency results for the Ning DJI and APC 10x7E rotors are compared in Figures 4.14 and 4.15 and the A-weighted OASPL results can be seen in Figure 4.16. In comparing Figures 4.14 and 4.15 both rotors show a much more significant difference in noise between the 0.05 D and 1.00 D cases for the 90° mic than they do for the 30° mic. This would suggest that the tip-to-tip separation distances are of more importance at some angles than other. This can be seen better in Figure 4.16 in the directivity plot. Based on the directionality of the rotor-rotor noise, future VTOL flight paths could potentially be optimized to ensure that the aircraft passes by buildings in such a way that the fewest possible observers experience the rotor-rotor noise at its loudest levels.
Another interesting point from Figure 4.16 is that the A-weighted OASPL tonal noise increases by up to 15 dBA directly downstream or below (270°) for the Ning DJI rotor as the separation distance is decreased from 1.00 D to the 0.05 D while it only increases by about 11 dBA for the APC 10x7E rotor. This means that the DJI rotor noise generation is more strongly influenced by tip separation distance than the APC propeller.

4.3 Rotor-Rotor Downstream Separation Sweep

4.3.1 Performance

This downstream configuration study builds on the last sweep, by beginning with the 0.05 D tip separation case and then incrementally moving the left rotor farther and farther downstream while keeping the right rotor fixed (Figure 1.2). The change in total thrust between both rotors, as the left rotor is moved downstream, is shown in Figure 4.17. This figure shows the average
Figure 4.15: Frequency spectrum of RANS noise results for the tip separation distance ($s$) for the APC 10x7E (right) at the 30° microphone (top) and the 90° (bottom).

Figure 4.16: A-weighted OASPL of RANS noise prediction results for increasing tip separation distances with the Ning DJI rotor on the left and the APC 10x7E on the right.
total thrust (left and right rotors combined) normalized by their single rotor case. This plotting approach was taken to better visualize if the system as a whole (the left and right rotors combined) resulted in a greater thrust production than an equivalent system of two rotors that had no rotor-rotor interactions. Both rotors show the same general trend in thrust, with a minimum thrust when tip-to-tip and a maximum thrust when the right rotor was ~1 D downstream.

The APC 10x7E propellers show an overall total thrust output greater than that of the two propeller case with no interaction from around 0.5 D to 1.75 D downstream. This total thrust increase in the system of two separated rotors, is consistent with findings by Stokkermans et al. [37], although he reported a larger increase of ~5% for their specific rotor and setup. Additionally, it is possible that this slight increase in thrust was seen only for the APC 10x7E propellers since they are designed for forward flight conditions with an incoming freestream velocity (in contrast to the DJI rotor that was designed for hover conditions), and the upstream rotor induces an inflow for the downstream rotor (Figure 4.18). However, as was found in the separation case, these rotor-rotor interactions only affected the thrust output minimally, with a max change in thrust of only 1% across the entire sweep (0 D to 3 D downstream).

![Figure 4.17: Normalized average produced thrust for the Ning DJI and APC 10x7E rotors at various downstream separation distances.](image-url)
4.3.2 Noise

The thrust fluctuations for the downstream separation distance case can be seen in Figure 4.19. The rotors behave as would be expected, with the maximum fluctuations occurring when both rotors are side-by-side and then decreasing towards the single rotor case as the right rotor moves downstream. The downstream rotor’s fluctuations initially begin to decrease as it moves downstream and then begin to increase again at around 0.5-1 D downstream. This behavior is believed to be due to the shape of the upstream rotor’s wake. As seen in Figure 4.20 this same region, of 0.5-1 D downstream, is where the wake stops contracting and begins expanding. Thrust fluctuations, from rotor-rotor interactions, decrease when the adjacent wake contracts and then increase as it expands.

To see if these thrust fluctuations related back to the noise production for each simulation, we once again ran the rotor loading history through the aeroacoustic noise code. Downstream distance sweep frequency results for the DJI and APC 10x7E rotor can be seen in Figures 4.21 and 4.22. Similar to the tip separation case we see that more separation between the rotors results in a lower sound pressure level.
Figure 4.19: The standard deviations of thrust comparison for the stationary/right and downstream/left rotor across all downstream separation distances.

Figure 4.20: Wake velocity profile. Arrow indicates that the approximate location where the wake stops contracting and begins expanding.

The A-weighted OASPL directivity for the downstream distance sweep can be seen in Figure 4.23 where we see that there is about a 8 dBA increase in noise directly downstream of the rotors from the 1 R case to the 6 R case. Note that the change is from 1 to 6 R instead of from 0 to 6 R since the nose for the 1 R case is actually less than the more separated 6 R case. If we take Figure 4.23 and only plot the 0, 1, and 6 R cases it becomes easier to see that the 1 R case is actually quieter than the 6 R configuration, where the rotors actually have more distance between them (Figure 4.24). Building on this, if we compare the A-weighted OASPL results at the 270° downstream mic with the averaged thrust fluctuations from Figure 4.19, we can better
Figure 4.21: Frequency spectrum of RANS noise results for the Ning DJI rotor 30° mic on top and the 90° mic on bottom.

see how well the thrust fluctuations predict the noise generation (Figure 4.25). This side-by-side comparison strongly supports that thrust fluctuations can be used in small UAV-type multi-rotor configurations to predict aeroacoustic trends.
Figure 4.22: Frequency spectrum of RANS noise results for the APC 10x7E 30° mic on top and the 90° mic on bottom.

Figure 4.23: A-weighted OASPL of RANS noise prediction results for increasing downstream separation distances with the Ning DJI rotor on the left and the APC 10x7E on the right.
Figure 4.24: A-weighted OASPL noise results for the 0, 2, and 6 R downstream separation cases with the Ning DJI rotor on the left and the APC 10x7E on the right.

Figure 4.25: Comparison of the A-weighted OASPL results at the 60° mic (left) with the averaged thrust fluctuations of the left and right rotors (right).
CHAPTER 5. SUMMARY

In this study we explored the effects of rotor-rotor interactions on total thrust output and noise for increasing tip-to-tip and downstream separation distances. With multiple VTOL prototype designs utilizing distributed electric propulsion systems—which result in more closely spaced rotors—rotor-rotor interactions have been a growing area of research. Many studies have been conducted on rotor-rotor interactions and this research aimed to build on those studies by using CFD (STAR-CCM+) with aeroacoustic noise code (PSU-WOPWOP) to investigate additional configurations.

5.1 Conclusions

Our initial focus in this research was to quantify thrust losses for a variety of different configurations, however, when those interactions were found to be minimal, or even negligible, for most situations, our focused changed to looking at noise, which had been shown to be more significant [9]. Our results support those trends. With changes of < 2% in performance metrics, like thrust and torque, there was an associated change of over a 20x increase in thrust fluctuations and a ~15 dBA increase in overall sound pressure level (OASPL). This is especially significant, considering that a 3 dB increase represents a doubling of power (Watts) within the sound pressure wave.

- Tip separation distances have a greater effect on noise generation than downstream separation distances do, with differences of up to ~15 dBA for the tip-to-tip cases and only ~10 dBA for the downstream configuration.

- If rotors must be placed less than 0.1 D apart, then moving one rotor ~0.5 D downstream of the other can results in up to a 9 dBA decrease in OASPL, while increasing the thrust output by up to 1% (Figures 4.16 and 4.19, and 4.23).
• Thrust fluctuations can be used as a noise prediction method for rotor-rotor configurations when looking for general trends.

• The design difference between a propeller (APC 10x7E) and a rotor (Ning DJI) did not appear to have any noticeable effect on the rotor-on-rotor interactions.

• Decreasing tip separation distance for 1D to 0.05D decreased the coefficient of thrust by ~1% while increasing thrust fluctuations by ~20x and resulting in up to a 15 dBA increase in predicted OASPL.

• Decreasing the downstream separation distance from 6R to 0R resulted in a ~10 dBA change in OASPL, with the quietest separation distance being at 1R which was ~6 dBA quieter than both the 0R and the 6R case.

5.2 Future Work

We suggest three main directions for expanding upon or furthering the current study. They are: the modeling of rotors within CFD, exploration of other potential rotor-rotor configurations, and aeroacoustics prediction for rotors. Some specific ideas for future research include:

• Integration between STAR-CCM+ and PSU-WOPWOP to include the full pressure field as an input into the aeroacoustics solver and not only the blade loading and geometry data. Having a more detailed input into the aeroacoustics code that included the flow field interactions (i.e. the quadrupole noise sources) would allow for better noise predictions.

• Additional validation and refinement of the CFD simulations in order to more accurately match experimental results. Other mesh schemes such as using an overset mesh to capture motion or an adjoint refinement function to better refine the flow-field could lead to more accurate results.

• Rotor configurations other than those presented here, such as overlapping or non-parallel rotation planes or differing rotor sizes, as a better understanding of the design space would allow for more efficient multi-rotor configurations in future electric aircraft.
• Extension of presented hovering cases to forward flight with an inflow, since a vertical take-off and landing vehicle would likely operate with an inflow during the majority of its operation time and not just in hover.

• Introduction of supporting structures such as a wing, since our results suggested that rotor-structure interactions were also very important. Also in any real-world setting or aircraft there will always be supporting structures.
REFERENCES


APPENDIX A. UNCERTAINTY ANALYSIS REPORT

The following section was created by Judd Mehr and contains the details of the uncertainty analysis that was used in this report.

A.1 Uncertainty Analysis

For each of our measured values, we calculated the base uncertainty using the equation

\[ u = \sqrt{u_0^2 + u_I^2} \]  

(A.1)

where \( u_0 \) is the uncertainty based on the resolution of the instrument, and \( u_I \) is the uncertainty based on the accuracy of the instrument. A.1 contains the list of these uncertainties for each measured property. The values for dynamic pressure and ambient humidity came from the product datasheets. The \( u_0 \) values for RPM, Thrust, and Torque also came from product data sheets, and the \( u_I \) values are root mean square deviations from the mean of the data taken. The ambient pressure and temperature were taken with high accuracy analogue instruments where user inaccuracy dominates, so \( u_I \) values for those were taken to be reasonably conservative ranges for human error. In the case of temperature, the observed value was incredibly clear, and therefore the \( u_I \) value was an estimated, but likely conservative value for the thermometer accuracy. Similarly the propeller diameter could be well measured on the order of millimeters.

These uncertainties needed to be propagated to the final non-dimensional values: advance ratio, \( J \), coefficient of thrust, \( C_T \), coefficient of power, \( C_P \), and efficiency, \( \eta \). The equations for each of these are as follows.

\[ J = \frac{V_\infty}{nD} \]  

(A.2)
Table A.1: \( u_0 \) and \( u_i \) values for measured quantities. Note that \( u_i \) values might be different for each measurement set, so only the maximum values are listed here.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>( u_0 )</th>
<th>( u_i )</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Pressure</td>
<td>6.67</td>
<td>5.515</td>
<td>Pascals</td>
</tr>
<tr>
<td>RPM</td>
<td>1.0</td>
<td>2.7964</td>
<td>RPM</td>
</tr>
<tr>
<td>Thrust</td>
<td>1e-7</td>
<td>0.1936</td>
<td>Newtons</td>
</tr>
<tr>
<td>Torque</td>
<td>1e-5</td>
<td>0.02766</td>
<td>Newton meters</td>
</tr>
<tr>
<td>Ambient Pressure</td>
<td>6.67</td>
<td>300</td>
<td>Pascals</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>0.5</td>
<td>0.01</td>
<td>deg Celsius</td>
</tr>
<tr>
<td>Ambient Humidity</td>
<td>0.0031</td>
<td>0.0062</td>
<td>unitless</td>
</tr>
<tr>
<td>Propeller Diameter</td>
<td>0.0005</td>
<td>0.001</td>
<td>meters</td>
</tr>
</tbody>
</table>

where \( n \) is the propeller revolutions per second, \( D \) is the propeller diameter, and \( V_\infty \) is the free stream velocity calculated from the measured dynamic pressure by

\[
V_\infty = \sqrt{\frac{2\pi q}{\rho}} \tag{A.3}
\]

where \( q \) is the measured dynamic pressure, and \( \rho \) is the ambient air density.

\[
C_T = \frac{T}{\rho n^2 D^4} \tag{A.4}
\]

where \( T \) here is the measured thrust (not temperature).

\[
C_P = \frac{2\pi Q}{\rho n^2 D^5} \tag{A.5}
\]

where \( Q \) is the measured torque.

\[
\eta = \frac{C_T J}{C_P} \tag{A.6}
\]

In order to propagate the uncertainties from the measured values, we took the sum of the root of squares of the individual uncertainties multiplied by their partial derivatives. To demonstrate, we show the calculation for the advance ratio uncertainty.

\[
u_J = \left[ \left( u_V \frac{\partial J}{\partial V_\infty} \right)^2 + \left( u_n \frac{\partial J}{\partial n} \right)^2 + \left( u_D \frac{\partial J}{\partial D} \right)^2 \right]^{1/2} \tag{A.7}
\]
where $u_V$ follows a similar process

$$u_V = \left[ \left( u_q \frac{\partial V}{\partial q} \right)^2 + \left( u_{\rho} \frac{\partial V}{\partial \rho} \right)^2 \right]^{1/2}$$

(A.8)

and $u_{\rho}$ as well

$$u_{\rho} = \left[ \left( u_{\text{Temp}} \frac{\partial \rho}{\partial \text{Temp}} \right)^2 + \left( u_{P} \frac{\partial \rho}{\partial P} \right)^2 \right. \right.$$

$$\left. + \left( u_{P_v} \frac{\partial \rho}{\partial P_v} \right)^2 + \left( u_{h} \frac{\partial \rho}{\partial h} \right)^2 \right]^{1/2}$$

(A.9)

e tcetera, down to the base uncertainties in A.1. With these uncertainties and propagations, we were able to produce the error bars on Figures 3.1 and 3.2.