Ambiguous Student Contributions and Teacher Responses to Clarifiable Ambiguity in Secondary Mathematics Classrooms

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Ambiguous Student Contributions and Teacher Responses to Clarifiable Ambiguity in Secondary Mathematics Classrooms

Alicia Marie Heninger

A thesis submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of Master of Arts

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ABSTRACT

Ambiguous Student Contributions and Teacher Responses to Clarifiable Ambiguity in Secondary Mathematics Classrooms

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Different types of ambiguous student contributions occur in mathematics classrooms. In this study I identified (1) different types of ambiguous student contributions and (2) the different ways teachers respond to one particular kind of ambiguous contribution, clarifiable ambiguity. Note that clarifiable ambiguity is ambiguity that stems from a student who uses an unclear referent in their contribution and can be clarified in the moment by the particular student. Literature has focused only on ambiguity that has the potential to further the development of mathematical concepts and has only theorized about teacher responses to this specific type of ambiguity.

This study identified an additional three types of ambiguous student contributions: Student Appropriation of Teacher Ambiguity, Irrelevant Ambiguity, and General Ambiguity. It was important to identify all the different types of ambiguous student contributions because teacher responses should likely be different to the different types of ambiguity. In addition, through analyzing the teacher responses to the clarifiably ambiguous student contributions, this study found that teachers addressed the clarifiably ambiguous student contributions about half the time. When the teachers did address the clarifiable ambiguity, the majority of the time the teacher clarified the ambiguity themselves with the most common response being the teacher honed in on the clarifiably ambiguity and asked for confirmation from the student on the accuracy of the clarification.

Keywords: classroom mathematics discourse, student mathematical contributions, ambiguous contributions, clarifiable ambiguity, teacher response
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CHAPTER ONE: RATIONALE

Mathematics education research has emphasized the importance of discourse in secondary mathematics classrooms. In fact, over time discourse has become something that is considered essential in the secondary mathematics classroom (Walshaw & Anthony, 2008). However, when discourse is not attended to properly, it can lead to problematic situations. The following is an example of one such problematic situation.

In an Algebra class students were analyzing data about multiple runners’ speed by examining a graph with different lines representing each runner’s progress. A student said, “That one has a greater speed.” The student’s use of “that” makes their statement ambiguous because other people in the classroom may not know which line is “that” line. Now, the student most likely knows which line they are referring to, so “that” could easily be replaced by the student with more specific words were they asked to do so. If the situation is not rectified, however, students could become confused as to what the class is communicating about. Thus, this type of ambiguity in discourse should be addressed in the moment to avoid possible ramifications of not addressing it.

Not clearing up this ambiguity can unnecessarily add more complexity to the already complicated classroom environment. One complexity that is added is students may believe what they inferred from what another student said is the correct inference (even if it is not) and that their own thinking is mathematically correct (again, even if it is not). This in turn may cause ramifications such as student confusion or parallel conversations, which may hinder the learning going on in the mathematics classroom (Peterson et al., 2020).

One implication of these ramifications is that students may disengage with the mathematical discourse because they may not be able to make sense of the student’s statement.
Another implication is that students’ mathematical understanding may suffer because their misinterpretation of the classmate’s statement may result in the development of misconceptions that could have been avoided if the student’s statement had been clarified. Finally, the teacher may miss opportunities to better understand student contributions. Since there may be multiple ways to interpret the student contribution, the teacher may not be sure what the student means. Thus, if clarification is not pursued, the teacher may miss an opportunity to better interpret current student understanding.

Now, returning to the example where a student said, “That one has a greater speed” in reference to a graph with different lines representing each runner’s progress, because the student could likely clarify their vague language, the teacher could ask a clarification question. For example, the teacher could ask, “You said ‘that one;’ which one is ‘that one’?” The student would likely be able to specify “that one,” making the student contribution more precise.

Some may believe that studying how teachers respond to this type of ambiguity is pointless because teachers would innately recognize the ambiguous student contribution and seek clarification. I have reason to believe, however, that teachers may not naturally seek clarification when clarification is needed. While looking at data from the Building on MOSTs: Investigating Productive Use of High-Leverage Student Mathematical Thinking project (http://buildingonmosts.org), I saw many instances where teachers did not address this type of ambiguous contribution.

For example, in a class discussion when students were trying to decide if $x$ or $x + x$ was larger a student said, “Yeah, so it’s like, it’s like one of them can be larger, but they can switch so the other one could be larger, or it could be the same.” It is unclear what “they” and “them” are referring to. The student may have meant $x$ or $x + x$, or explicit values they have
plugged in for the variables. The teacher’s response to this student’s statement was, “Okay, so, does he agree or disagree with Daniel?” The teacher did not clarify what the student was saying, but moved on to having the rest of the class evaluate the vague student contribution. The class then quickly moved on to addressing another student contribution that was unrelated to this particular student’s thinking leaving the vague student contribution unresolved. Because the unclear thinking was never addressed before the turn of the class discussion, it is unknown if the students in the class were interpreting the students statement the way the student intended it to be. Thus, by not addressing the situations, students may continue to interpret the situation in different ways, potentially leading to parallel conversations and student confusion.

A particular reason why teachers may not address ambiguity is because of the challenges of decentering (Teuscher et al., 2016). To decenter, a teacher must take on a student’s perspective, which is often a difficult task. Although the teacher may think that they understand what the particular student is trying to say, they may not recognize that students could very well be assuming different ideas from the use of an unclear referent. As the teacher takes on the student's perspective, the teacher may better interpret what the students may be understanding from the particular statements.

In summary, one situation that arises in classroom discourse is the type of ambiguity that may be rectified in the moment and that becomes problematic when it is not rectified in the moment. While it may seem obvious that teachers should clarify the ambiguous student contribution in the moment, there is reason to believe that teachers often are not responding to the ambiguous contribution during their mathematical lesson. Now, although research has conjectured how teachers may address this particular kind of ambiguity, research has not
explicitly studied how teachers are responding. Thus, this study will explore how teachers are responding to this kind of ambiguity.

I have only discussed one kind of ambiguous student contribution up until this point. This kind of ambiguity is different from the type of ambiguity literature has focused on. Other literature has described ambiguity in the mathematics classroom as desirable. Ambiguity is seen as something that has a productive role in students’ learning of mathematics (Barwell, 2005). However, the kind of ambiguity that I have been discussing does not seem to be desirable, as I have stated previously. Knowing that there is at least one kind of ambiguity in the mathematics classroom that does not fit the mold of the kind of ambiguity that has been discussed in literature begs the question of, are there other kinds of ambiguous student contributions that have not been identified in mathematics classrooms? In addition to studying teacher responses to the specific kind of ambiguous student contributions I have discussed, I will also identify other types of ambiguous student contributions that occur in a mathematics classroom.
CHAPTER TWO: BACKGROUND

Theoretical Framework

The purpose of my research is to identify different types of ambiguous student contributions and determine how teachers respond to the particular type of ambiguity described in chapter one. A word will be considered ambiguous when it reasonably may be interpreted in multiple ways. This study will identify different types of ambiguous student contributions and hone in on the particular type of ambiguous student contribution that is caused by using unclear referents that can be clarified in the moment. The particular type of ambiguity that will be honed in on is called clarifiable ambiguity (Peterson et al., 2020). The first part of the theoretical framework will define clarifiable ambiguity and discuss what elements must be present for a word to be considered clarifiably ambiguous. The second part of the theoretical framework will discuss the aspects of the teacher responses to clarifiable ambiguity.

Clarifiable Ambiguity

Before discussing clarifiable ambiguity, I want to note that using a referent is not necessarily bad. Although ambiguity is caused by unclear referents, referents are tools people use to great benefit in their everyday language. Thus, while someone may run the risk of being ambiguous if they use a referent, not all referents in all contexts cause the meaning of the overall statement to be unclear.

The following example illustrates when not using referents may cause a statement to be more unclear than if referents would have been used. A middle school mathematics class is working on a set of problems that relate to a graph named “Top 20 Women 2010 Boston Marathon Splits.” A student, when referencing the graph says, “The Top 20 Women 2010 Boston Marathon Splits graph helps us see how fast each woman is running because the Top 20
Women 2010 Boston Marathon Splits graph has the time on the x-axis and the Top 20 Women 2010 Boston Marathon Splits graph has mileage on the y-axis. So, when we look at the Top 20 Women 2010 Boston Marathon Splits graph we can see by the slope of the lines on the Top 20 Women 2010 Boston Marathon Splits graph which women had a greater speed at the different splits.” Within this student’s statement, there is a lot of repetition of the title of the graph, and in reality, people usually do not talk this way. People may reference the whole title of the graph in their first statement, after which they would typically refer to the graph by saying “it” or “the graph.” If a student continually repeated the whole title of the graph, the statement may begin to sound weird and people may get lost in what the student is saying because the main point of the statement is lost in the repetition of the title. As long as there is only one graph on the board and the whole class is aware that the class discussion is focused on that graph, using a referent instead of the whole title of the graph may not cause a problem. If there were two graphs on the board, however, then the use of a referent could cause ambiguity. Thus, while clarifiable ambiguity stems from the use of unclear referents, the use of referents is not innately bad and does not always cause ambiguity.

Clarifiable ambiguity is defined by Peterson et al. (2020) as follows:

If a word is used such that (a) it can be interpreted in multiple viable ways, (b) the existence for those interpretations causes the overall meaning of the statement in which it occurs to be ambiguous, and (c) the individual who made the statement could likely clarify their intended meaning for the word if asked, then the word is clarifiably ambiguous. (p. 5)

Thus, for a student contribution to be considered clarifiably ambiguous, all three criteria must be met. The following sections expound on how each of the three criteria are met.
First Criterion

The first criterion that must be met for a student contribution to be considered clarifiably ambiguous is that a word may be interpreted in multiple viable ways. Any statement or question can technically be interpreted in multiple ways, however all the interpretations may not be viable. If a student says, “x is a variable,” another student could interpret the statement to mean “x is ice cream.” Although “x is ice cream” is an interpretation, it is not a viable interpretation. A viable interpretation is an interpretation that a teacher or student in a mathematics classroom are likely to come up with.

Consider the following situation. A teacher writes down the expressions $4^2$, $5^2$, and $2^3$, on the board. As the class starts to discuss the expressions a student says, “That’s 8.” Although there are three expressions the student may be referring to as 8, all of them are not viable interpretations. “$4^2$ is 8” and “$2^3$ is 8” are both viable interpretations because $2^3$ is 8 and a common mathematical misconception of students is that $4^2$ is 8. $5^2$ is not a viable interpretation for “that’s” because the likelihood of the student contribution being $5^2$ is 8 is low.

Second Criterion

The second criterion that must be met for a student contribution to be considered clarifiably ambiguous is that the multiple viable interpretations of the word cause the overall meaning of the statement or question to be ambiguous. Causing ambiguity in the overall meaning of the statement or question means the different interpretations that the teacher and students could make matters because the interpretations may position the students and teacher in different places of understanding. The way in which the students interpret the statement may disrupt future mathematical dialogue because different students would have different interpretations of the mathematical idea.
The example used to highlight how criterion one can be met also illustrates how ambiguity can be problematic. The example was where three expressions, $4^2$, $5^2$, and $2^3$, are on the board and a student says “that’s 8.” If one classmate thinks “that’s” is referring to $4^2$ while another thinks “that’s” is referring to $2^3$, then these classmates will be heading in vastly different directions. The classmate who interprets “that’s” as $4^2$ believes the student is meaning $4^2$ is 8. If this claim goes unchallenged either by themselves, other students, or the teacher, one ramification that may happen could be that the student decides $4^2$ is 8 because no one said otherwise. They might think that, because the teacher is the authority in the classroom, if the teacher did not challenge the student’s claim, then the student must be correct.

What follows is an example where the multiple viable interpretations of the word do not cause the overall meaning of the statement or question to be ambiguous, or in other words, when the second criterion is not met. Suppose a student said, “the half of 32 wouldn’t be the same as the half of 48 cause they’re—they’re different numbers.” Person’s well versed within mathematics education could infer the mathematics behind the first part of what the student said, “The half of 32 wouldn’t be the same as the half of 48.” What the student meant in the second part of their statement, “they’re—they’re different numbers” is not as clear. In fact, there are multiple viable interpretations of the mathematics behind what the student said. The student could have meant that 32 and 48 are different numbers or that half of 32 and half of 48 are different numbers. Other students and the teacher in the mathematics classroom could have interpreted “they’re—they’re different numbers” in either way. However, the statement of either half of 32 and half of 48 being different numbers or 32 and 48 being different numbers is not problematic in the moment because both statements are similar, correct, and seem to be pointing students in the same direction.
Third Criterion

The third criterion that must be met for a student contribution to be considered clarifiably ambiguous is that the ambiguous word or phrase likely can be clarified by the student who made the statement or question. This means that, if given the chance, the student would likely be able to replace the ambiguous words with more specific words. Clarifiable ambiguity differs from other types of ambiguity because of this criterion. With clarifiable ambiguity students can clarify their vague language while with other types of ambiguity the students might not know how to provide clarification.

The example used to highlight how criteria one and two can be met also illustrates a situation that satisfies the third criterion, as it is likely that the student could clarify their ambiguous language if asked. The example was where three expressions, $4^2$, $5^2$, and $2^3$, are on the board and a student says “that’s 8.” The student in this situation likely knows if their “that’s” represents $4^2$ or $2^3$. So if a teacher were to say, “When you say ‘that’s’, what expression are you referring to?” the student could likely specify whether they meant $4^2$ or $2^3$.

As previously mentioned, not all uses of referents are unclear. In a similar vein, however, not all unclear referents are clarifiable. Whether ambiguity is clarifiable is dependent on whether the student can clarify the words that were ambiguous. If students do not have the understanding needed to replace their vague words, the student contribution is ambiguous, however it is not clarifiably ambiguous (Peterson et al., 2020). The following is an example of a student using an unclear referent that is not clarifiable. While contemplating the equation $3x + 5 = 26$, a student in a pre-algebra class says, “we just do the 5.” Other classmates may interpret ‘do’ in multiple viable ways such as, subtract 5 from both sides of the equation, add 5 to the other side of the equation, or subtract the 5 from the left side of the equation. Of the three different
interpretations, fellow classmates could believe “we just do the 5” results in any of the three
different equations: $3x = 2l$, $3x + 5 = 3l$, or $3x + 5 = 2l$. These three different equations that
all stemmed from the equation $3x + 5 = 26$ place students in different positions of
understanding. However, the student may not be able to clarify their intended meaning for “do.”
The student may have said “do” because they do not have a more specific, exact way of
describing that action. Thus, this example would not be clarifiably ambiguous because the
student may not possess the understanding to simply clarify the meaning of their ambiguous
statement.

**Teacher Response**

To study teacher responses to clarifiable ambiguity I used what Peterson et al. (2020)
described as a theorized teacher response to clarifiable ambiguity. They suggested teachers
respond by asking the student a question that: a) makes clear the aspects of the student
contribution they do understand, and b) focuses in on the part of speech that is clarifiably
ambiguous. The intent of this approach is to be as specific as possible. The specificity helps
guide the students to understand which part of their statement was causing the ambiguity. Also,
the specificity positions the entire class to better engage with the clarified version of the
statement. In addition, this clarification move gives students the opportunity to use mathematical
objects in their clarification. Finally, the specificity shows the students the teacher has been
listening to the student statement, is trying to understand, and wants the entire class to
understand what the student has said.

There were four aspects from the theorized teacher response I used to analyze the
different types of teacher responses:
1. **Hone:** Does the teacher’s response hone in on the clarifiably ambiguous part of the student contribution?

2. **Acknowledge:** Does the teacher’s response acknowledge what the teacher did understand from the student contribution?

3. **Clarify:** Does the teacher response clarify the ambiguity? In other words, after the teacher's response, is there now one interpretation of the student contribution?

4. **Honor:** Does the teacher's response seem to honor the student thinking?

The first two points relate to the theorized response because they are what the teacher needs to say to be considered the theorized response. The third point is important because there seems to be little point of a teacher asking for or clarifying themselves if nothing gets clarified. The final point is important because one of the purposes of clarifying ambiguous student contributions is to honor the student’s thinking. I am defining honoring student thinking as when it is likely the student felt like the teacher heard and responded to what the student said.

I used these four aspects to help determine what categories of teacher responses were most to least similar to the theorized response. The most important part of the theorized response was the teacher posing a question to the student. If the teacher did pose a question to the student then the teacher *honored* the student thinking. This is the most important aspect of the theorized response because the most likely way to get what the student meant by the vague referent would be to ask the particular student. The next most important part of the theorized response was the teacher asking for clarification on the clarifiably ambiguous portion of the student contribution. If a teacher did this, they *honed* in on the clarifiable ambiguity. *Honing* in on the vague referent seems like a likely way to *clarify* what was meant by the vague referent. However, if it is the teacher clarifying, there is a chance they will give a clarification that is not in agreement with
what the student was thinking. Thus, asking the student the question is the most important. Finally, is that the teacher makes clear what they understood from the clarifiably ambiguous contribution. If the teacher did make clear what they understood, then the teacher acknowledged. To hone in on what the teacher actually wants clarified, stating what the teacher does know would be effective. However, if honing in on the clarifiable ambiguity is left out, stating what the teacher understood may not be enough to clarify the unclear referent.

**Research Question**

The purpose of my research is to identify types of ambiguous student contributions and, in particular, how teachers respond in situations where students are clarifiably ambiguous during classroom discourse. To achieve this purpose, I will answer the following questions:

1. What types of ambiguous student contributions occur in classroom mathematical discussions?
2. How do teachers respond to clarifiable ambiguity in secondary mathematics classrooms?
Literature Review

My literature review focuses on three areas. First, the kinds of ambiguity that have been identified in literature, particularly ambiguity in a mathematics classroom. Second, literature on teacher responses to ambiguity in a mathematics classroom. Third, literature that focuses on teacher responses that serve to clarify student contributions.

Ambiguity

Previous literature has focused on ambiguity that has the potential to further the development of mathematical concepts. (Barwell, 2003; Foster, 2011; Rathouz, 2010; Rowland, 2003). Some literature has broken productive ambiguity into subsets that focus on mathematical definitions, mathematical symbols, etc. (Foster, 2011). Literature however has said little on other kinds of ambiguity that may not further student mathematical understanding. Clarifiable ambiguity has been identified by Peterson et al. (2020) as one kind of ambiguity that does not help further student mathematical understanding. However, this is only one kind of ambiguity. There may be other unidentified kinds of ambiguity that have not been focused on in the literature.

There is a gap in the literature around what different kinds of ambiguity arise in mathematics classes. This gap not only is apparent in ambiguity that does not help further student mathematical understanding, but on ambiguity that does help further student mathematical understanding. Literature has yet to breakdown when ‘productive’ ambiguity is happening in the classroom and what types of student contributions are considered to be productive ambiguity (Barwell, 2003; Rathouz, 2010; Rowland, 2003).
Teacher Response to Ambiguity

As stated in the previous section, literature has theorized about how ambiguity in a mathematics classroom can be helpful in developing new mathematical concepts. This literature suggests ways teachers may respond by being more conscious of the ambiguity, delaying student’s arrival at a formal definition, facilitating different ideas about terms, etc. (Barwell, 2003; Foster, 2011; Rathouz, 2010; Rowland, 2003). However, the literature only gives suggestions and not findings on how teachers should respond to ambiguity.

Literature that has theorized about potentially unproductive ambiguity also gives suggestions on what the ambiguity might look like and how teachers can rectify the ambiguity in the moment (Peterson et al., 2020; Rowland, 1999). Rowland and Peterson both suggested asking questions that focus in on the ambiguous words. However, while these researchers have made recommendations about teachers’ responses to unproductive ambiguity, no one has actually studied teacher responses to unproductive ambiguity.

Clarification Moves

The third area of research that is core to my study is teacher responses within mathematics classroom discourse. Note that the teacher responses discussed in the following sections are not specifically geared toward ambiguous student contributions, but to student contributions in general. I do so because research has not focused in on teacher responses to ambiguity. As researchers have studied teacher responses, they have identified clarification as a productive move teachers use (Brodie, 2011; Ellis et al., 2019; Franke et al., 2015; Franke et al., 2009; Webb et al. 2014; Zee, 1997). Clarification is usually classified as a support move (Cengiz et al., 2011; Franke et al., 2015) or as follow up questions (Broadie & Boaler, 2004; Franke et al., 2009; Webb et al., 2014; Zee, 1997). Support moves are considered to be moves that happen
after the teacher’s initial invitation move. The teacher may use different support moves to help increase student engagement with the mathematical ideas present and help establish how and when students should engage with other student contributions (Franke et al., 2015). Similarly, the purpose of follow up questions or reflective questions were to help students connect their ideas to the student idea that was currently being discussed (Zee, 1997).

Furthermore, clarification has been identified and placed into different subcategories within support moves and follow up questions. For example, a clarification move has been categorized as a scaffolding move, which is when a teacher takes up a portion of the student contribution by connecting it to other student work, written or verbal, and provides either more information or clarification of the idea (Franke et al., 2015). Clarification has also been categorized as a teacher interpreting a claim, which means the teacher shares their own interpretation of the student contribution and their interpretation is meant to help students focus in on the mathematical issues at hand (Cengiz et al., 2011). Another category clarification has been placed in is probing questions, which is where the teacher could ask students questions that allowed students to articulate, clarify, or develop ideas (Bishop et al., 2016; Brodie & Boaler, 2004). An additional categorization was probing sequences of specific questions which entails a teacher asking multiple questions about a piece of the student contribution and entails multiple student and teacher responses (Franke et al., 2009). Clarification was also included as a press move which means the teacher probes the student for further thinking on their idea by either clarifying or justifying their idea (Brodie et al., 2011). All these different categorizations included clarification statements or questions as a part of the meaning of each category. In particular, these studies mentioned how these different categories were productive moves that were observed by the researchers because they were seen to produce more student engagement.
Follow up questions help to produce more student engagement by helping students compare their strategies to other strategies (Webb et al., 2014). Franke et al. (2009) focused on teacher questions posed to follow up on the initial student contribution. They found that teacher questions can help students be more explicit and complete in their explanations. However, it was found that these teacher questions did not guarantee further elaboration. Even though these questions did not guarantee students would elaborate, it was found that the most productive way to help students further elaborate on their ideas was to ask multiple questions, such as highlighting, clarifying, or making explicit questions, that were each focused on a particular element of the student contribution.

Support moves were shown to have the potential to help increase the engagement students had in other students' ideas, connect more with the mathematical ideas present, and help establish when and how to engage with other student ideas. Although these support moves, including clarifying, are considered productive teacher moves, there was no guarantee the students would engage more with other student contributions when these moves were used (Franke et al., 2015). Researchers included the different categories that seemed to possibly have an impact on student engagement, thus the teacher move was deemed as a productive move. Although studies mentioned that support moves and follow up questions could increase student engagement and thus are productive teacher responses, there is not a focus on clarification itself. Clarification is one amongst many teacher moves that may be productive. In other words, clarification has been identified as part of teacher responses that may lead to productive classroom discourse, but has not been studied further to see how clarification specifically impacts classroom discourse. Further, while these studies identified these productive teacher moves, the studies did not show how often or how effectively these teacher moves, including
clarification, are enacted by teachers. This presents a gap in research not only in how clarification contributes specifically to productive classroom discourse, but on how often or effectively clarification is enacted in a mathematics classroom.

**Teacher Moves**

Beyond the research in support moves and follow up questions, literature has focused on theorizing about what teacher response moves may be helpful in engaging students during different parts of the mathematical lesson. However, studies have not focused on how often teachers are using any teacher response moves or the quality of the teacher response moves when they actually use the moves in their mathematical lessons.

Different studies have identified three distinct groups of teacher moves: eliciting, supporting, and extending (Tytler & Aranda, 2015, Cengiz et al., 2011, Ellis et al., 2019). These groupings are identified to help show when different groups of moves should be used during a mathematics lesson. The first group, eliciting, is a teacher response that is meant to bring the initial student idea into the classroom discourse. Studies that have focused on eliciting found that teachers use eliciting moves, especially in tasks that push for student ideas to be shared, however these moves did not further the development of student ideas (Harris et al., 2012; Mueller et al., 2014). The second group, supporting, is meant to follow up on the initial student idea. The last group, extending, is meant to continue the classroom discourse by having students consider other students' claims, justify their own claim, etc. Cengiz et al. (2011) identified the most common extending teacher move was asking students to evaluate a claim. This move along with others were meant to help students develop deeper connections among the different student contributions. However, there was no guarantee that the moves would elicit further student contributions (Harris et al., 2012). Now, while research has come to more of an agreement that
the first set of teacher moves is eliciting, and the last set of teacher moves is extending, what is entailed in the middle group of teacher moves is not as clear. It is likely that clarification falls within this middle group. Thus, the following paragraphs will focus on what research has said about the middle group of teacher moves.

The middle group of teacher moves, which I call support moves, has been categorized in different ways. The group of moves has been called supporting (Franke et al., 2015, Cengiz et al., 2011), responding and facilitating (Ellis et al., 2019), and clarifying (Tytler & Aranda, 2015). Supporting moves, which I addressed in the clarification moves section, are moves that happened after the teachers initial move and were meant to help students engage with the current student contribution (Franke et al., 2015). Now, clarification is included in support moves as a subset of a scaffolding move (Franke et al., 2015) or a subset of teacher suggesting an interpretation of the student contribution (Cengiz et al., 2011).

Ellis et al. (2019) presented responding and facilitating moves as the middle group of moves (supporting moves). Responding moves are moves that validate or correct the student contribution or encourage students to validate and correct other student contributions. Facilitating moves are moves that start to push on the student contribution by building on student ideas, presenting additional information, explanations, or other strategies. However, unlike the other groupings of moves presented by other studies, Ellis et al. considered clarifying as a possible eliciting move. This may be because their eliciting move went beyond just bringing out student ideas and included clarifying and understanding the student contribution. Other studies included clarifying and understanding the student contribution as part of the second group of moves.
Tytler and Aranda (2015) defined the middle group of moves (supporting moves) to be clarifying. They defined clarifying as attempting to make student contributions more precise. This can be done by revoicing, requesting confirmation, re-framing a question, or requesting clarification. With clarification being the name of the group, it is obvious that Tytler and Aranda considered clarification move to be one that comes after eliciting the student idea and before extending the student idea.

From these different studies it is apparent that research has not come to a concrete conclusion on what is considered to be the middle group (support moves) of teacher moves. The supporting moves presented by Franke et al. (2015) and Cengiz et al. (2011) seem to be more broad than clarifying move presented by Tytler & Aranda (2015), however the clarifying moves seem to be more detailed in what just clarifying can entail. Ellis et al. (2019) did not include clarifying as a part of the support moves, but their study of eliciting moves were more comprehensive than other studies. Although many studies have identified and categorized teacher moves, there is a gap in the research that describes the quality and frequency of the support moves. The first step is knowing what teacher moves should be sought after, and now the next step is seeing how often these moves should be used and how to be effective when using them. My research will specifically study how often and how effective teacher moves, such as clarify and other possible teacher moves, are being used in the situation of clarifiably ambiguous student contribution.
CHAPTER 3: METHODS

Data Collection

To study different types of ambiguous student contributions, and how teachers respond to clarifiable ambiguity in a mathematics classroom, I studied a data set containing student contributions that were ambiguous, and within this set of ambiguous contributions, the teacher responses to the contributions that were clarifiably ambiguous. I used 11 recorded full and partial mathematical lessons already collected by the Building on MOSTs project. Five of the recorded lessons were from high school classrooms and six were from middle school classrooms. The mathematics courses ranged from pre-algebra classes to pre-calculus classes. Examples of lesson topics include variables, proportions, percentages, graphing, surface area, and dilations. The schools where these lessons took place spanned the United States. The teachers varied in terms of their experience and skill level, but all had an expressed desire to try to use student thinking in their instruction.

While there are multiple viable methods to study ambiguous student contributions and the teacher responses to clarifiable ambiguity, I believe using a collection of recorded full and partial mathematical lessons already collected by the Building on MOSTs project was the most effective way to research these ambiguous student contributions and the particular teacher responses for two reasons. First, identifying all the ambiguous student contributions in real time would be nearly impossible. These ambiguous student contributions happen often and so much is happening in the classroom that picking up on if each contribution is ambiguous would be problematic. I would most likely miss a good amount of data which in turn would skew my results. Second, these videos provide data that show what students and teachers are saying in the moment. While I could talk with teachers about their dialogue after a mathematical lesson had
taken place, this may not produce the best data. Since time has passed, the teacher could have a
different viewpoint than they did during the lesson. Their answers to questions about their
specific teacher responses or how they interpreted a student’s comment may be different than
what they were thinking in the moment. Since I am interested in studying teacher responses in
the moment this may be problematic.

However, because I did not interview the students or teachers, I was not able to hear how
they described their interpretations of different clarifiably ambiguous student contributions. As
an observer, I made conjectures of possible interpretations the teacher and students might make
of a particular student contribution. Thus, as an observer, I was the one making the decision on
whether a student contribution had multiple viable interpretations.

To determine whether an ambiguous student contribution was clarifiably ambiguous, it
was important that I decided not only what possible viable interpretations the teacher could
make, but the students as well. The teacher and I may be equipped with similar knowledge, thus
my own interpretations may have mirrored that of the teachers. However, my knowledge and the
students’ knowledge may have been quite different. While I may have decided on one
interpretation, students may still have had multiple other interpretations of the word or phrase in
question. In the previous chapter, I used the example of “that’s 8.” Although I may have easily
known that the student could be referring to $2^3$, I may have needed to decenter (Tuescher et al.,
2016) to realize that $2^2$ is another viable interpretation of the students.

Another potential problem related to determining whether a student contribution was
clarifiably ambiguous was that I may not have been familiar with the context of the classroom or
understand the shared meaning of words the participants in the classroom hold. Because I may
not have been familiar with the context of the classroom or the shared meaning of words in the
classroom I may have interpreted student contributions to be clarifiably ambiguous when the
word or phrase may not be to the teacher and students in the classroom because of their shared
understanding. Unless I could conjecture the shared meanings the students and teacher hold,
certain student contributions may be considered clarifiably ambiguous because I was not able to
interpret the meaning from the context of the classroom. However, I had access to the recordings
of the whole classroom lesson. Thus, when I was unsure of the meaning behind student thinking
I was able to watch different parts of the lesson to help familiarize myself with the context and
some classroom norms. Within this set of data, I focused on ambiguous student contributions and
the teacher responses to clarifiably ambiguous student contributions in the mathematical lessons.

Analysis

To answer my two research questions I needed to study ambiguous student contributions.
I started out by trying to identify all ambiguous student contributions that were clarifiably
ambiguous. I used the criteria for identifying which ambiguous student contributions were
clarifiably ambiguous to sort all the contributions into their respective categories of ambiguity. I
then took the ambiguous student contributions that were clarifiably ambiguous and went on to
study their corresponding teacher responses. The following sections explain how I identified and
analyzed the different types of ambiguous student contributions and then particularly the
clarifiably ambiguous student contributions and their corresponding teacher responses.

Student Contributions

The first part of my analysis required me to identify different types of ambiguous student
contributions. Before explaining more about how I identified the different types of ambiguous
student contributions, I will explain exactly what counted as a student contribution in my study
and when the contribution was considered ambiguous.
In this study a student contribution is considered to be any observable action, verbal or nonverbal, a student does that may be mathematical (Leatham et al., 2015). For example, the student question, “How is the solution different if x is a negative number?” would be considered to be a student contribution. The student has made an observable, verbal comment that is mathematical in nature. On the other hand, the student question, “Do we put the papers in the middle of the table?” would not be a student contribution in my study. While the student contribution is observable and verbal, what the student said is not mathematical.

**Ambiguous Student Contributions**

Ambiguous student contributions fell into a broader set of student contributions which I call noninferable student contributions. Noninferable student contributions are “student contributions [that] may seem to be mathematical in nature and yet still require further information to make a reasonable inference about the student mathematics” (Leatham et al., 2015, p. 93). I was able to distinguish these noninferable student contributions from inferable contributions because the Building on MOSTs project had previously coded noninferable contributions as cannot infer (CNI contributions).

Since ambiguous means that something has multiple interpretations, student contributions that were ambiguous (including clarifiably ambiguous contributions) were considered to be noninferable. Thus, ambiguous student contributions would be among those contributions coded CNI. The CNI contributions had been previously determined to be noninferable by at least 3 other people who were well versed in mathematics education in general and who had studied student thinking in particular. In addition, I reviewed the coding on all CNI contributions to confirm or rescind that coding.
Categorizing CNI Contributions

My data set consisted of 1238 student contributions, 294 of which had been previously coded as CNI contributions. The next step of the analysis was to identify what made each student contribution noninferable. I did this by using the criteria for clarifiably ambiguous student contributions:

If a word is used such that (a) it can be interpreted in multiple viable ways, (b) the existence for those interpretations causes the overall meaning of the statement in which it occurs to be ambiguous, and (c) the individual who made the statement could likely clarify their intended meaning for the word if asked, then the word is clarifiably ambiguous. (Peterson et al., 2020, p. 5)

I used these criteria to identify which CNI contributions were clarifiably ambiguous and which were not. I wanted to make sure I could categorize every CNI contribution as clarifiable ambiguity, as another type of ambiguity, or as noninferable for a reason not related to ambiguity. The criteria for clarifiably ambiguous student contributions led to an initial categorization of other types of ambiguous student contributions. Figure 1 contains these initial reasons why a noninferable student contributions was not clarifiably ambiguous.
To be thorough I asked myself questions to determine whether each noninferable student contribution met the criteria of clarifiable ambiguity. I asked myself the following types of questions to help me see if a student contribution satisfied the first criterion:

1. What seems to be the most likely thing the student is saying?

2. If someone had some misunderstandings in this area, what would they think? Are there multiple obvious viable interpretations?

Answering questions like the first one helped me get a sense of the most probable mathematics the student is talking about. Answering questions like the second one helped me consider if there were areas related to the mathematics that students were likely to be confused about. Answering questions like the final question helped me hone in on any viable interpretations that fulfilled the first criterion of clarifiable ambiguity.

To see if the student contribution satisfied the second criterion, I asked myself the following types of questions:
1. How different are the multiple viable interpretations?

2. Are the multiple viable interpretations different enough to cause problems in the following discussion?

Answering questions like the first question helped me hone in on the differences in the interpretations. Answering questions like the second question helped me see if the differences in the interpretations presented a problem if the discussion were to continue on. If the different interpretations did present a problem, then the second criterion would be satisfied.

To see if the student contribution satisfied the third criterion, I asked myself the following types of questions:

1. Could the student likely replace the clarifiably ambiguous word(s) with a more specific word(s)?

2. What words might they use to replace the clarifiably ambiguous word(s)?

Both types of questions helped me decide whether the student could actually clarify their ambiguity. If I decided the student could clarify their thinking and specify words the student might use, then I knew the third criterion of clarifiable ambiguity was satisfied.

As the coding of the CNI contributions began I noticed that the three criteria codes did not fully account for all the ways a CNI contribution could fail to be clarifiably ambiguous. In fact, there seemed to be CNI contributions that did not rise to the level of any possible type of ambiguity. So, I added some additional codes. First, was a code for incomplete data. If the student comment was inaudible or incomplete in some way because of the technology, I did not include these contributions in the pool of CNI contributions. Thus, I created a code called incomplete data.
Next, I ran into an issue of students not completing their statement or question. Because the CNI contribution was incomplete, it is unknown whether the contribution was ambiguous or not. If the contribution would have been complete, it is possible that the contribution would be inferable. Incomplete statements did not rise to the level of an ambiguous student contribution. Thus, I created a code called incomplete statement. However, this category did not include student contributions that included a complete statement and then followed the complete statement with an incomplete statement. An example of this would be if the expressions $4^2$, $5^2$, and $2^3$ were on the board when a student said, “that’s 8, and um…” The first part of the statement is complete while the “and um” part is incomplete. In such situations the latter part of the statement was discarded and the first part was considered to be a complete contribution.

Now that I was only dealing with complete student contributions I saw one more category emerge. Sometimes what the student was saying was ambiguous because of the preceding teacher turn. Because the cause of the ambiguity did not start with the student but was caused by the teacher, I called this new category teacher ambiguity. The distinction between whether the student or the teacher was the reason why the student contribution was CNI was important because this probably changes how the teacher should respond. If the student was unclear in their statement because of the preceding teacher comment, it likely would not make sense for the teacher to ask the student to clarify what they meant. Instead, the teacher should clarify their own statement. Figure 2 shows the updated collection of reasons why CNI contributions failed to be clarifiably ambiguous.
Figure 2

Revised Codes of Failing to be Clarifiably Ambiguous

<table>
<thead>
<tr>
<th>Why Student Contributions Failed to be Clarifiably Ambiguous</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failed Criterion 1</td>
<td>There is only one viable interpretation of the student's thinking.</td>
</tr>
<tr>
<td>Failed Criterion 2</td>
<td>The different viable interpretations do not position students in different points of understanding. This code also includes when the teacher should not pursue the thinking because the thinking would not help further class discussion if pursued.</td>
</tr>
<tr>
<td>Failed Criterion 3</td>
<td>The student says something ambiguous that they cannot clear up by a clarification of a vague term.</td>
</tr>
<tr>
<td>Student Appropriation of Teacher Ambiguity</td>
<td>The ambiguity in the student thinking is due to the teacher being ambiguous in their own preceding turn.</td>
</tr>
<tr>
<td>Incomplete Statement</td>
<td>Student does not finish the statement or question they were posing.</td>
</tr>
<tr>
<td>Incomplete Data</td>
<td>Teacher or student comment was inaudible to the observer due to technology.</td>
</tr>
</tbody>
</table>

By continuing to create categories, I was able to hone in on the different types of ambiguous contributions that were coded as CNI contributions. However, there were three codes that needed to be discounted from my data before I could be just dealing with ambiguous student contributions: failed criterion 1, incomplete data, and incomplete statement. The CNI contributions that received these codes were removed because if the student contribution failed to pass criterion 1 that meant there was only one viable interpretation, thus the contribution was actually inferable and should not have been coded as a CNI. If the student contribution was coded as incomplete data, then, due to technology, what the teacher or student said was inaudible
to the observer. Because the reason why the student contribution was CNI did not stem from the
teacher or student but from a technological issue, these contributions were removed from the
data. Finally, if the student contribution was coded as an incomplete statement then I could not
determine if the contribution was ambiguous. After removing these CNI contributions from my
collection of data I was left with 201 ambiguous student contributions that were categorized
according to the type of ambiguity.

Teacher Response

After identifying why a student contribution was ambiguous, I honed in on the clarifiably
ambiguous student contributions. For these particular student contributions, I identified the
corresponding related teacher turns, which I refer to as the teacher response. I considered a
teacher turn to be “the collection of observable teacher actions” (Van Zoest et al., 2020, p. 29)
that happened after a student contribution had occurred and that continued until another
contribution happened. There were two types of teacher turns that had the possibility of being
included in the teacher response. First, the initial teacher turn after a clarifiably ambiguous
student contribution was always considered to be part of the teacher response. This is because the
initial teacher turn was the first interaction the teacher has with the clarifiably ambiguous student
contribution and thus may be a good representation of how the teacher was addressing the
contribution. The second type of teacher turn that was included in the teacher response was any
subsequent teacher turn that may be related to the clarifiably ambiguous student contribution. To
decide if the teacher turn was related to the clarifiably ambiguous student contribution, the
teacher turn had to reference the particular contribution or have some of the underlying ideas
from the particular contribution within the teacher turn. However, included in the subsequent
teacher turns, the last teacher turn to be included was when the teacher clearly moves on from the
clarifiably ambiguous student contribution. This was because this last teacher turn indicated that the teacher was no longer referencing the particular student contribution and the teacher response had ended.

One reason for considering including subsequent teacher turns in the teacher response was it gave the teacher the benefit of the doubt in case the teacher was waiting to respond or realized a little later that addressing the student contribution was necessary. Another reason for including subsequent turns as a part of the teacher response was the teacher may take more than one turn to address the clarifiably ambiguous student contribution. Thus, to try to interpret what the teacher was attempting to do with the clarifiably ambiguous statements, studying teacher responses with multiple teacher turns supported discovering how teachers were actually addressing the clarifiably ambiguous student contributions.

After the classroom discourse had clearly moved on from the clarifiably ambiguous student contribution I stopped watching the particular video and focused on all identified teacher turns that related to the clarifiably ambiguous contribution in that video. I then analyzed each of the teacher turns in that video to determine how the teacher turns made up the teacher response.

During this phase of my analysis I only identified the initial teacher turn and subsequent teacher turns that were possibly related to the clarifiably ambiguous student contribution; I did not analyze them until I had finished the particular video. There were three reasons for this choice. First, by not analyzing each identified teacher response immediately I was able to stay focused on the broader level of the class discussion, thus it helped me be able to have a better understanding of what was happening in the overall classroom setting. Second, by identifying all the teacher turns that were related to the clarifiably ambiguous student contribution I was able to identify if some of the contributions or teacher turns were related. This is important because
many times clarifiably ambiguous student contributions come in clusters. When a student says something clarifiably ambiguous, another student will use their same wording, consequently creating another clarifiably ambiguous student contribution. Thus, some teacher turns may relate to multiple clarifiably ambiguous student contributions. Seeing if teacher turns relate to multiple clarifiably ambiguous student contributions was easier if I continually identified the teacher turns that related to the particular student contribution without stopping to analyze each teacher turn. Finally, I analyzed each teacher turn after all the teacher turns had been identified in the whole video and not after I had done the whole collection of videos because I was still refining my coding process. Stopping after each video gave me time to make the needed changes. Also, just analyzing one video at a time kept the recording fresh in my mind, helping me remember classroom context, the teacher’s style of teaching, etc.

**Teacher Response Coding**

To analyze each teacher response that was addressing a clarifiably ambiguous student contribution, I used part of the Teacher Response Coding Scheme (TRC) designed by Van Zoest, et al. (2020). Van Zoest et al. have created a teacher response coding that acknowledges multiple research ideas that are already in the mathematics education field in teacher response. The TRC is meant to disentangle and make explicit three facets of teacher responses: who, what, and how. For this study, I only utilized the who and what portion of the coding scheme. I did not use the how facet because in my study I was explicitly focusing on if the teacher or student were responding or being asking to respond to the clarifiable ambiguity (who) and what the teacher was doing in response or the student was being asked to do in response to the clarifiable ambiguity (what).
In the TRC coding the *who* is “the member(s) of the classroom community publicly given the opportunity to consider the student contribution as a result of the teacher response to it” (Van Zoest et al., 2020 p. 10). The *who* is represented by the category of Actor. The actor could be the teacher, same student(s), other student(s), or whole class depending on who is given the opportunity to consider the thinking as an outcome of the teacher response. Figure 3 identifies how each different person could be the actor.

**Figure 3**

*Definitions of Codes for the Actor Category*

<table>
<thead>
<tr>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teacher</strong></td>
<td>does not publicly invite or allow anyone other than the teacher to consider the SMT.</td>
</tr>
<tr>
<td><strong>Same Student(s)</strong></td>
<td>invites or allows the contributor(s) to consider the SMT.</td>
</tr>
<tr>
<td><strong>Other Student(s)</strong></td>
<td>invites or allows a subset of students other than the contributor(s) to consider the SMT.</td>
</tr>
<tr>
<td><strong>Whole Class</strong></td>
<td>invites or allows the whole class to consider the SMT.</td>
</tr>
</tbody>
</table>

*Note.* From Van Zoest, L., Peterson, B., Rougée, A., Stockero, S., Leatham, K., & Freeburn, B. (2020). *Conceptualizing Important Facets of Teacher Responses to Student Mathematical [Unpublished manuscript].* Department of Mathematics, Western Michigan University, p. 16.

The Actor category was important to include in my analysis of the teacher response because knowing who was being asked to consider the clarifiably ambiguous student contribution brought insight into how the teacher is responding. It differentiated between different types of clarifying moves such as if the teacher is clarifying the student contribution themselves, asking another student to clarify the thinking, or asking that same student to clarify their thinking.
The *what* part of the TRC coding is “what the response gives the actor the opportunity to do with respect to the student contribution” (Van Zoest et al., 2020, p. 11). The *what* is represented by the Action category. There are fifteen different discrete codes under the category of Action (see Figure 4). These codes identified the actions the teacher could invite the actor to do. I only focused explicitly on three of these codes: repeat, clarify, and develop. Any of these codes could be seen as an attempt to clarify some aspect of the clarifiably ambiguous student contribution. The other codes were used to code the teacher turns, but if a repeat, clarify, or develop move was not a part of the coding of the teacher response, I stopped the coding process for that particular teacher response.
Figure 4

Definitions of Codes for the Action Category

<table>
<thead>
<tr>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjourn</td>
<td>indicates (either explicitly or implicitly) that the SMT(^1) will not be considered at that time, but suggests the SMT may be considered later.</td>
</tr>
<tr>
<td>Allow</td>
<td>creates an open space for interaction with the SMT.</td>
</tr>
<tr>
<td>Check-in</td>
<td>gives an opportunity for the actor to self-assess their reaction to or understanding of the SMT.</td>
</tr>
<tr>
<td>Clarify</td>
<td>gives the actor an opportunity to make the SMT more precise.</td>
</tr>
<tr>
<td>Collect</td>
<td>gives the actor an opportunity to contribute additional ideas, methods, or solutions.</td>
</tr>
<tr>
<td>Connect</td>
<td>gives the actor an opportunity to make a connection between the SMT and other ideas, representations, methods/strategies, or solutions.</td>
</tr>
<tr>
<td>Correct</td>
<td>gives the actor an opportunity to rectify the student mathematics of the SMT.</td>
</tr>
<tr>
<td>Develop</td>
<td>gives the actor an opportunity to expand the SMT beyond a simple clarification, but does not request justification.</td>
</tr>
<tr>
<td>Dismiss</td>
<td>indicates (either explicitly or implicitly) that the SMT will not be considered.</td>
</tr>
<tr>
<td>Evaluate</td>
<td>gives the actor an opportunity to determine the correctness of the mathematics of the SMT.</td>
</tr>
<tr>
<td>Justify</td>
<td>gives the actor an opportunity to contribute mathematical reasoning related to the SMT.</td>
</tr>
<tr>
<td>Literal</td>
<td>gives the actor an opportunity to provide brief factual information related to the SMT.</td>
</tr>
<tr>
<td>Repeat</td>
<td>gives the actor an opportunity to repeat (verbally or in writing) the SMT (including minor rephrasing that does not change the meaning).</td>
</tr>
<tr>
<td>Validate</td>
<td>gives the actor the opportunity to affirm the general value of the SMT and/or encourage student participation (e.g., says “thank you,” gives a thumbs up signal, asks for applause).</td>
</tr>
<tr>
<td>Move On</td>
<td>indicates (either explicitly or implicitly) that the SMT will no longer be addressed.</td>
</tr>
</tbody>
</table>

The Actions codes were important to my analysis of teacher responses because they identified what the actor is given the opportunity to do in response to the clarifiably ambiguous student contribution. These codes identified whether the actor is clarifying, moving on from, developing, etc. the student contribution. Note that the *move on* action was not part of the original TRC coding. I added this code to indicate the last teacher turn of the response. This last teacher turn was part of the response because it showed that the teacher has *moved on* from the clarifiably ambiguous student contribution.

TRC coding was applied to each teacher turn that made up the teacher response. Furthermore, if the teacher turn had more than one action or actor, the teacher turn was split according to what part of the teacher turn matched with what action and actor. Each teacher turn was coded in relation to the specific clarifiably ambiguous student contribution. This is different from how the TRC has been previously applied where it was applied to only the previous student contribution (e.g., Stockero et al., 2018; Stockero et al., 2019; Stockero et al., 2017). However, because the TRC has been designed to capture the nature of responses (and is not just specific to the previous student contribution), the coding can be applied to any previous contribution.

In the TRC coding, there was a background section and a codes section. The background section contained the prompt of the student contribution, the contribution, the teacher response, and the take up. The take up was everything from the clarifiably ambiguous student contribution through the "move on" teacher turn, with both teacher and student turns included. The code section contained the Actor and Action codes. What follows is an example of what the coding looked like for one clarifiably ambiguous student contribution.
Students had been discussing their reasons for how to decide whether x or x+x is larger.

The teacher asked, “Okay. Now, let’s generalize this. You have some other variable
expression[s] and you’re asked which one is bigger. Can you generalize this for like any situation
of the variable?” The following transcript excerpt contains the clarifiably ambiguous student
contribution and the discussion that followed:

Tracy²: “It’s unpredictable [sic].”
Teacher: “Why is it unpredictable?”
Tracy: “Because if it was negative then this one would be positive and that one would be
negative.”
Teacher: “Okay, so when you are testing two variable expressions to see how they
compare, if somebody summarized for us what we just learned. Or will you just figure it
out? Tasha?”
Tasha: “You have to know the value of x? Before you..”
Teacher “Okay, but remember earlier Danica said the value of x can be anything.”
Tracy: “You have to know if it’s positive, negative, or zero.”
Teacher: “Okay, but what if you don’t know?”

The first teacher turn is related to the clarifiably ambiguous student contribution by the teacher
directly using “unpredictable.” The next teacher turn seems to be repeating the teacher’s original
question without acknowledging the statement “It’s unpredictable” or Tracy’s response to the
teacher’s question, “Why is it unpredictable?” Thus, the teacher has moved on. The third and
fourth teacher turns are evidence that the discussion has indeed moved in another direction with
the new student contributions. So, the first and second teacher turns will be the turns that make
up the teacher response to this particular student contribution. Figures 5 and 6 show the TRC
coding for teacher turn 1 and teacher turn 2 respectively.

² All student names are pseudonyms.
Figure 5

TRC Coding for Teacher Turn 1

<table>
<thead>
<tr>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prompt:</strong></td>
</tr>
<tr>
<td>“Okay. Now, let’s generalize this. You have some other variable expression[s] and you’re asked which one is bigger. Can you generalize this for like any situation of the variable?”</td>
</tr>
<tr>
<td><strong>Student Contribution:</strong></td>
</tr>
<tr>
<td>&quot;It's unpredictable&quot;</td>
</tr>
</tbody>
</table>

| Teacher Turn: “Why is it unpredictable?” |
| **Take Up:** |
| Tracy: “It’s unpredictable [sic].” |
| Teacher: “Why is it unpredictable?” |
| Tracy: “Because if it was negative then this one would be positive and that one would be negative.” |
| Teacher: “Okay, so when you are testing two variable expressions to see how they compare, if somebody summarized for us what we just learned. Or will you just figure it out? Tasha?” |

<table>
<thead>
<tr>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actor:</strong> Same Student</td>
</tr>
<tr>
<td><strong>Action:</strong> Develop</td>
</tr>
</tbody>
</table>
Figure 6

TRC Coding for Teacher Turn 2

<table>
<thead>
<tr>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prompt:</strong></td>
</tr>
<tr>
<td>“Okay. Now, let’s generalize this. You have some other variable expression[s] and you’re asked which one is bigger. Can you generalize this for like any situation of the variable?”</td>
</tr>
</tbody>
</table>

| **Student Contribution:** |
| "It's inpredictable" |

| **Teacher Turn:** |
| “Okay, so when you are testing two variable expressions to see how they compare, if somebody summarized for us what we just learned. Or will you just figure it out? Tasha?” |

| **Take Up:** |
| Tracy: “It’s inpredictable [sic].” |
| Teacher: “Why is it unpredictable?” |
| Tracy: “Because if it was negative then this one would be positive and that one would be negative.” |
| Teacher: “Okay, so when you are testing two variable expressions to see how they compare, if somebody summarized for us what we just learned. Or will you just figure it out? Tasha?” |

| Codes |
| **Actor:** | Whole Class |
| **Action:** | Move On |

From this coding we can see the first teacher’s action was to ask Tracy to develop her idea. By the teacher asking for a development of what the student meant, this teacher turn could be seen as an attempt to clarify what the student meant by “it’s,” even if the teacher turn did not succeed in the attempt. The next teacher move did not address the student contribution. The actor was the whole class and the teacher action was a move on action.

Since the teacher did attempt to clarify the clarifiably ambiguous student contribution by trying to develop the student contribution, I classified the teacher response as addressing the clarifiable ambiguity. However, while this teacher response was an attempt to clarify, a better
way to clarify would have been for the teacher to say, “You said ‘it’s unpredictable’, what are you referring to when you say ‘it’s’?”

I applied this coding to every teacher turn that made up a teacher response to clarifiably ambiguous student contribution. When all teacher turns in the teacher response have been analyzed, I moved onto the next clarifiably ambiguous student contribution and coded that contribution’s teacher response. I did this until each teacher response in the video had been coded. Then, I moved onto the next video and started the process over again.

**Clarify Coding**

The final part of my coding was to classify the different ways teachers were clarifying the clarifiably ambiguous student contributions and the effectiveness of these clarifying actions. The effectiveness of the different clarification actions were based on the extent to which the clarifiable ambiguity was addressed. I used the theorized teacher response (Peterson et al., 2020) to analyze this effectiveness, as described in Chapter 2. This coding was only applied to teacher turns that were coded as a *Clarify, Repeat, or Develop* action since all three actions could be seen as an attempt of the teacher to clarify the clarifiably ambiguous student contribution.

I was able to partially know if the teacher turn stated the part of speech they do understand, and if the teacher focused on part of speech that was clarifiably ambiguous by the Actor code. If a teacher was enacting the ideal type of clarify action to a clarifiably ambiguous student contribution, the teacher would ask a question to the *same student*. The teacher clarifying themself or asking another student to clarify would not fit under the ideal teacher response to clarifiable ambiguity.

The following is an example of how I utilized the TRC coding to determine the effectiveness of the Teacher Response to a clarifiably ambiguous student contribution (see figure
7). When a class was discussing whether $x$ or $x + x$ is larger a student says, “$x$ plus $x$ is bigger because it has—is adding another of that $x$ value to the first one.” The teacher says in response, “Okay. Could you be more specific in telling me what you mean by that?” By the teacher asking the student to be more specific in their statement, the teacher is making a clarification move. Thus, this teacher move would be coded as clarify.

**Figure 7**

*Clarify Coding for Teacher Turn 1*

<table>
<thead>
<tr>
<th><strong>Background</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prompt:</strong></td>
</tr>
<tr>
<td><strong>Student Contribution:</strong></td>
</tr>
<tr>
<td><strong>Teacher Turn:</strong></td>
</tr>
</tbody>
</table>
| **Take Up:** | Student: “$x$ plus $x$ is bigger because it has—is adding another of that $x$ value to the first one.”  
Teacher: “Okay. *Could you be more specific in telling me what you mean by that?*” |

<table>
<thead>
<tr>
<th><strong>Codes</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actor:</strong></td>
</tr>
<tr>
<td><strong>Action:</strong></td>
</tr>
</tbody>
</table>

The teacher is addressing the same student and asks a clarifying question. The teacher is trying to clarify, but is not honing in on what particular part of the statement they want the student to clarify. A viable interpretation of use of “that” in the teacher’s statement is of the whole student contribution and not the “that” the student is likely referring to. From this coding, I identified this part of the teacher response to be asking the student a question, but did not hone
in on the clarifiably ambiguous speech or state what they understood from the student contribution.

Now, I will share this same example with a different teacher response that will show the ideal teacher response to the clarifiable ambiguity (see figure 8). Suppose again that a student says, “x plus x is bigger because it has—is adding another of that x value to the first one.” This time, the teacher says in response, “Okay. So you said x plus x is bigger, but what do you mean by ‘that’ when you said ‘adding another of that x value to the first one’?” By the teacher asking the student to be more specific in their statement, the teacher is making a clarification move. Thus, this teacher move would be coded as clarify.

**Figure 8**

*Clarify Coding for Teacher Turn 1*

<table>
<thead>
<tr>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prompt:</strong> Class discussion on whether x or x+x is larger</td>
</tr>
<tr>
<td><strong>Student Contribution:</strong> “x plus x is bigger because it has—is adding another of that x value to the first one.”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher Turn: <strong>“Okay. Could you be more specific in telling me what you mean by that?”</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Take Up:</strong> Student: “x plus x is bigger because it has—is adding another of that x value to the first one.” Teacher: “Okay. Could you be more specific in telling me what you mean by that?”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actor:</strong> Same Student</td>
</tr>
<tr>
<td><strong>Action:</strong> Clarify</td>
</tr>
</tbody>
</table>

This teacher response addresses the same student and asks a clarifying question that hones in on the word that makes the contribution ambiguous. These codes show that this teacher response is a possible ideal teacher response to the clarifiable ambiguity.
I conjectured that teachers would seldom address clarifiable ambiguity in the way Peterson et al. (2012) have suggested. I conjectured the following other possible ways teachers may seek clarification:

1. Ask the student themself to clarify.
2. Pose a question that fills in the unclear referent with a specific word and ask for confirmation of the correctness of the assumption.
3. Fill in the unclear referent with a specific word without asking the student for confirmation of the correctness of their assumption.
4. Pose a question to another student for what they think the student meant by the unclear referent.

These were a few possibilities of what teachers may do when they seek clarification. Although all the moves may be considered a clarification move, there may be certain ways of seeking clarification that were more effective than others. Now, there may be other effective or better ways than what I have conjectured to be the ideal response to clarifying student contributions in different situations.

Clarify coding was an open coding since the literature (Peterson et al., 2020) has only conjectured how to respond to clarifiable ambiguity. As I proceeded with the open coding I continued to ask myself whether the student contribution was clarified. Additional coding was determined as I bumped into different ways teachers attempt to clarify.
CHAPTER 4: RESULTS

My results come from an analysis of 201 ambiguous student contributions from 11 different classroom recordings. These 201 ambiguous student contributions were categorized according to the type of ambiguity. Twenty-eight percent of the 201 ambiguous student contributions were coded as clarifiable ambiguity. In this chapter I begin by discussing the different kinds of ambiguous student contributions. Then, I hone in on one of those kinds of ambiguity, clarifiable ambiguity, and discuss how teachers respond to it.

Categories of Ambiguous Student Contributions

Seventy-two percent of the ambiguous student contributions were not clarifiably ambiguous. The purpose of analyzing this 72% of ambiguous student contributions was to answer my first research question: What types of ambiguous student contributions occur in classroom mathematical discussions? In Figure 9, the italicized terms are the different types of ambiguous student contributions. The ambiguity either stemmed from the preceding teacher turn or from the contribution itself. If the reason the student contributions was ambiguous was because of a prior teacher statement I coded the ambiguous student contribution as Student Appropriation of Teacher Ambiguity. If the student contribution was ambiguous because of what the student said, then the student contribution was ambiguous because of Irrelevant Ambiguity, Clarifiable Ambiguity, or General Ambiguity. Figure 10 lists the types of ambiguous student contributions and defines them. In the following paragraphs I will expound upon each of these types of ambiguous student contributions.
Categories of Ambiguous Student Contributions

Types of Ambiguity

Student Appropriation of Teacher Ambiguity

Student Ambiguity

Irrelevant Ambiguity

Clarifiable Ambiguity

General Ambiguity
### Definitions for the Types of Ambiguous Student Contributions

<table>
<thead>
<tr>
<th>Type of Ambiguity</th>
<th>Frequencies (# of Student Contributions, N=201)</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Appropriation of Teacher Ambiguity</strong></td>
<td>36% (72 Student Contributions)</td>
<td>The ambiguity in the student contribution is due to the teacher being ambiguous in their own preceding turn.</td>
</tr>
<tr>
<td><strong>Irrelevant Ambiguity</strong></td>
<td>19% (38 Student Contributions)</td>
<td>The different viable interpretations of the ambiguous statement in the student contribution do not position students in different points of understanding or the teacher would not likely pursue the contribution because the thinking would not help further the class discussion if pursued. (Note that what I called “failed criterion 2” in the methods section turned into <strong>Irrelevant Ambiguity</strong>)</td>
</tr>
<tr>
<td><strong>Clarifiable Ambiguity</strong></td>
<td>28% (57 Student Contributions)</td>
<td>The student uses a vague word or phrase in the student contribution that has multiple viable interpretations, does position student differently in their understanding, and the student could likely clarify the vague word or phrase in the moment</td>
</tr>
<tr>
<td><strong>General Ambiguity</strong></td>
<td>17% (34 Student contributions)</td>
<td>The student says something ambiguous in the student contribution due to a lack of understanding that they could likely not clear up by a clarification of a vague term. (Note that what I called “failed criterion 3” turned into <strong>General Ambiguity</strong>)</td>
</tr>
</tbody>
</table>

**Student Appropriation of Teacher Ambiguity**

The first type of ambiguous student contribution is **Student Appropriation of Teacher Ambiguity**. For an ambiguous student contribution to be **Student Appropriation of Teacher Ambiguity**.
Ambiguity, the contribution is ambiguous because of the statements or questions the teacher had previously made. While at first it may appear that some of the resulting student contributions were clarifiably ambiguous, upon further inspection, it became apparent that if the teacher would have been less ambiguous with their speech, the student contribution may have been able to be interpreted.

Student Appropriation of Teacher Ambiguity is separated into three subgroups: multiple questions, teacher clarifiable ambiguity, and vague questions (see Figure 11). While multiple questions and teacher clarifiable ambiguity presented similar substantial contributions to the category, vague questions contributed the greatest amount to Student Appropriation of Teacher Ambiguity. Vague questions seemed to contain two main themes within the category: development and word definition.

Figure 11
Categories of Student Appropriation of Teacher Ambiguity

Multiple Questions

Multiple questions is when the teacher asks more than one question at a time. In such situations an observer may not know what question the student contribution is answering. An
example of *multiple questions* is the following. In a college algebra class a teacher asks, “Why did you divide 27 by 3? You set up an equation here, so you're going to solve for x, so what would you do? How do you solve for x in this case? You gotta get rid of what?” In response a student says “the 27.” In this example we can assume the student is saying “the 27” to one of the teacher questions. However, it is unclear which question the student is responding to. Before continuing with the discussion, the teacher would need to stop and clarify which question they were wanting students to answer. *Multiple questions* accounted for about 10% of all ambiguous student contributions.

**Teacher Clarifiable Ambiguity**

The next subgroup of *Student Appropriation of Teacher Ambiguity* is *teacher clarifiable ambiguity*. This subgroup is defined the same way that *Clarifiable Ambiguity* is except instead of the student making the clarifiably ambiguous statement or question, the teacher is making the clarifiably ambiguous question or statement. Thus, we can say *teacher clarifiable ambiguity* happens when a teacher makes a statement or questions that may be interpreted in multiple viable ways, positions the students at different points of understanding, and the teacher could replace the vague referent they use with a more precise term. An example of *teacher clarifiable ambiguity* is the following. During a college algebra classroom discussion the fractions ⅔ and ¾ were being discussed in relation to each other. The teacher then asks, “Is it bigger or smaller?” Students respond with “it’s smaller” and “it’s bigger.” Now, as seen by the two different responses, the ‘it’ the teacher is referring to may be interpreted by students as ⅔ or ¾. Knowing whether ⅔ or ¾ is bigger does position students at different points of understanding. Finally, the teacher could specify whether they meant ⅔ or ¾. Thus, the student contribution is ambiguous.
because of teacher clarifiable ambiguity. Similar to the subgroup multiple questions, teacher clarifiable ambiguity accounts for about 10% of all ambiguous student contributions.

**Vague Questions/Statements**

The final subgroup of Student Appropriation of Teacher Ambiguity, vague questions/statements, is when a teacher does not specify what they are wanting students to focus on in their reply. The subgroup vague questions/statements accounted for about 16% of all ambiguous student contributions. Within the subgroup vague questions/statements I found two distinct categories: development and word definition.

**Development.** A Vague Question/Statement that was categorized as development means that the teacher's question or statement is vague because the statement or question would need to be developed first in order to be able to infer what the student is meaning in the student contribution that followed. Many times it is clear that something is lacking in these statements or questions because the student's reply to the teacher turn is short and, in many cases, just a yes or no answer. The short student reply accentuates how the question or statement needs to be developed since when trying to decipher what the student means in the student contribution it is very unclear what the student is responding yes or no to. An example of a student contribution that is ambiguous due to development vague question/statement is in a pre-algebra classroom when the teacher asks the students what a ratio is and a student explains, “It's a part to part.” In response the teacher says, “Part to part comparison?” The student says, “Yeah.” Then the teacher says, “What do you guys think?” and a student says, “Yes.” Now, in deciphering the meaning behind the student’s reply “yes” we can assume the student agrees with something. However, the question arises as to what the student agrees with. This example is not teacher clarifiable ambiguity because to clarify this teacher turn the whole teacher turn may need to be developed
instead of just clarifying a vague referent. Because the teacher is not specific in asking the students what they think, an observer cannot interpret the meaning behind the student contribution. Before being able to know what the students are trying to say, the teacher's question of “what do you guys think” needs to be developed. Instead of asking the students what they think the teacher could have said, “Student A has said that ratios are a part to part comparison. What do you all think about ratios being defined as a part to part comparison?” After the question is developed an observer then may be able to decipher what the students are saying in response.

**Word Definition.** The other type of vague questions/statements is word definition. The reason the question or statement is vague is because the teacher uses a word that may need to be defined further. *Word definition* is not *teacher clarifiable ambiguity* because the word that needs to be defined further needs more than the teacher replacing the vague term with a more specific term. The word/words need to be clarified with an explanation. By defining these words the students in turn may know how to respond and the meaning behind the student contribution then could be inferred. An example of a word needing to be defined further is in a pre-algebra class the teacher says, “Okay, so if you go to one store and it costs—if they tell you it’s 10.44, and you go to another store and it’s 10 dollars and 50 cents, would that make a big difference to you?” In response the students say “no.” Now, the problem with this student contribution stems from the teacher asking the vague question, “would that make a big difference to you?” It can be assumed that the “that” being used is referring to the two different prices. Then the teacher says “big difference.” To infer the meaning behind the student contribution an observer may need to know what the teacher means by “big difference.” This phrase needs to be defined to be able to make
sense of the student contribution. Future research will hopefully explore how teachers can seek to minimize their ambiguous language during classroom discussion.

**Irrelevant Ambiguity**

The second kind of ambiguity is *Irrelevant Ambiguity*. *Irrelevant Ambiguity* is the category that encapsulates all the ambiguous student contributions that are not likely to help a teacher meet the goal of furthering that lesson’s discussion. There is not enough time during a class period to pursue every student contribution. Beyond that, pursuing every thought does not honor students’ thinking because teachers will leave other student contributions behind if the teacher grabs onto every new contribution that comes along during classroom discussion. It soon becomes a long chain of semi-related student contributions that may not lead anywhere. Thus, sometimes when these student thoughts are given, it is better to not clarify the thinking and move on or stay with the student contribution that is on the table.

Two prominent categories emerged out of *Irrelevant Ambiguity*: *similar conclusion* and *no reason to pursue* (see Figure 12).

**Figure 12**

*Subcategories of Irrelevant Ambiguity*

![Diagram](attachment:image.png)

*Similar Conclusion*

The category of *similar conclusion* entails when the different viable interpretations of an ambiguous student contribution lead the students to approximately the same conclusion. The
ambiguous student contribution is not positioning students at different places of understanding that would make future conversation confusing. Since the conclusions are so similar, it may not matter if the teacher clarifies or not. An example of this comes forward in a classroom discussion about is “x or x plus x larger?” Students had moved on to discussing what would happen when x is equal to 0. A student then said, “Oh, they’re tied.” Now, “they’re” could be referring to x plus x and x or 0 plus 0 and 0. However, because the classroom discussion at this moment is centered around x equaling 0, either interpretation of the ambiguous student contribution would likely result in the students coming to very similar interpretations of the contribution. Similar conclusions category is classified under Irrelevant Ambiguity because while the teacher could clarify this ambiguity, clearing up the ambiguity would not necessarily move the class discussion forward since the ambiguity was not a hindrance in the discussion to begin with. Similar Conclusions accounted for about 7% of all ambiguous student contributions.

No Reason to Pursue

Next, the other major category under Irrelevant Ambiguity is no reason to pursue. This category entails when the ambiguous student contribution is not a contribution that would be productive for the teacher to pursue even if it were clarified. Many times the comment is an offhand comment. Because there is no reason for the teacher to pursue the comment, the ambiguous student contribution is not deemed as something that will position the students differently. An example of this comes up in a mathematics class where they are discussing dilations. A student says, “Now you have to trace the lines from the center of dilation… to our vertices… and then you can fold it in half and stuff like that.” Right after that student contribution another student says, “I did that.” Now, the second student contribution is ambiguous because of the “that” the student uses. The student could be referring to all of what the first student said or just part of
what the first student said. Although the second student contribution is ambiguous, the teacher is unlikely to focus on the second contribution. The first student contribution is where the rich mathematics is happening, thus pursuing the first contribution and not the second contribution seems like a productive teacher move. Thus, the second student contribution would be considered *Irrelevant Ambiguity* due to *no reason to pursue* it. *No reason to pursue* accounted for about 9% of all ambiguous student contributions.

**Clarifiable Ambiguity**

A third type of ambiguous student contribution is *Clarifiable Ambiguity*. Now, all of the other types of ambiguity stemmed from trying to identify what student contributions that were coded as CNI were clarifiably ambiguous. *Clarifiable Ambiguity* is placed here between *Irrelevant Ambiguity* and *General Ambiguity* because *Clarifiable Ambiguity* is relevant. A teacher would want to pursue the student contribution and the different interpretations of the contribution do place students at different levels of understanding. However, *Clarifiable Ambiguity* does not rise to the level of *General Ambiguity* because the ambiguity can be cleared up with a more precise phrase or word unlike *General Ambiguity* which requires more development of the ambiguous student contribution. As stated in Figure 10, *Clarifiable Ambiguity* accounts for about 28% of all ambiguous student contributions.

**General Ambiguity**

A final type of ambiguous student contribution is *General Ambiguity*. *General Ambiguity* goes beyond *Clarifiable Ambiguity* because a clarification of more than one word or phrase is required to make the ambiguous student contribution clear (unlike *Clarifiable Ambiguity* which requires the replacement of a vague referent with a more precise word or phrase). *General Ambiguity* also includes ambiguous student contributions where there are no particular words or
phrases to substitute in for the vagueness. Many times, the students who said the statement or question may not know how to clarify what they said.

There are three main subgroups of General Ambiguity: general confusion, word definition, and development. Word definition and development were subgroups of Student Appropriation of Teacher Ambiguity also, and these subgroups are defined in the same way as they were in Student Appropriation of Teacher Ambiguity. However, instead of the teacher making the statement with a word that needs to be more defined or a statement that needs more development overall, the student is making the statement or question. Figure 13 shows the subgroups of General Ambiguity.
Figure 13

*Subcategories of General Ambiguity*

**General Ambiguity**

- **General Confusion**
- **Word Definition**
- **Development**

**General Confusion**

*General confusion* is a new subcategory of ambiguous student contributions that had not yet emerged from the analysis. *General confusion* is a subgroup that encapsulates statements or questions by students that provide evidence that the student does not know how to say what they are trying to say. The student remark shows that the students are just generally confused at the problem at hand. Much of the time students make broad claims or express that they do not understand what is currently being discussed. An example of this is when during a classroom discussion on percent the teacher asks a student what they would change about their answer. The student in response says, “All of it. I don’t know. All of it.” The ambiguous student contribution is one of *general confusion*. The student does not specify what is confusing them, but makes a sweeping statement about everything confusing them. *General confusion* is categorized under *General Ambiguity* because what is confusing the student is unclear and it would likely require more than a one-phrase clarification to make the statement clear. *General confusion* accounted for about 3% of all ambiguous student contributions.
**Word Definition**

The next category is *word definition*. This is when students use a word or phrase that just does not fit, like “that works.” The word “works” could not be cleared up with one or two words, but would need the student to really explain what they are trying to say with the word “works”. *Word definition* is not categorized under *Clarifiable Ambiguity* because while we need something substituted in for a vague word or phrase, that substitution would not simply be a word or phrase. To substitute in for that word would require an explanation of sorts. An example of *word definition* happens in a class discussion about a geometry task (see Figure 14). The student explained how she found rectangles A and G to be similar. Then a moment later she said, “So I figured this, I thought the same thing would apply to A and D.” Now, it is unclear what the student means by “same thing” in this context. There probably is not a simple word that the student could swap the phrase out for, but the student may need to explain what the phrase means in this situation. *Word definition* accounted for about 3% of all ambiguous student contributions.

**Figure 14**

_Dilation Task_
Development

The next category is *development*. *Development* entails the idea that the essence of the student contribution is unclear and the student needs to develop their thoughts further. If the teacher were to push the students to develop their thoughts a little more, they may be able to articulate more clearly what they are trying to convey. *Development* differs from *word definition* because *development* is focused on the overall statement while *word definition* is centered around a particular word or phrase. An example of *development* is in a lesson around geometric transformations (see Figure 15). A student says, “If you put two, if the 9EG, if you put two of them on the A, on the thing it will work.” It is unclear what the student is trying to point out from this statement. The student’s overall statement is confusing. The student makes the statement “put two of them on the A”, uses the word “thing,” and then makes the statement “it will work.” These three portions of the student statement combined makes the student contribution ambiguous. The student would have to make more than a one word clarification to make clear what they were saying, thus the example is not categorized as *Clarifiable Ambiguity* but as *General Ambiguity*. Clarifying the ambiguous student contribution would require more than defining just “thing,” but developing the whole idea of what is meant by “put two of them on the A” and “it will work.” Thus, this ambiguous student contribution would need development before the meaning behind it could be inferred. The teacher could seek development by asking the student, “I’m confused by what you mean by ‘put two of them on A’ and ‘it will work.’ Could you explain more about what ‘put two of them on A’ first and then explain what ‘it will work’ means?” *Development* accounted for about 10% of all ambiguous student contributions.
I have now presented all four types of ambiguous student contributions. In my discussion section I will discuss the types of ambiguous student contributions and implications for how teachers might respond based on the types of ambiguity.

**Teacher Responses to Clarifiable Ambiguity**

This section presents the results that answered my second research question: How do teachers respond to clarifiable ambiguity in secondary mathematics classrooms? Now, this question only focused on clarifiably ambiguous student contributions. Thus, I am switching the focus now to just clarifiably ambiguous student contributions and their corresponding teacher responses.

After analyzing the teacher response to every clarifiably ambiguous student contribution I found that 51% of the time teachers did not attempt to clarify the corresponding clarifiably ambiguous student contribution. When teachers did not attempt to clarify the clarifiably ambiguous student contribution the teacher either did not acknowledge the student contributions or did acknowledge the student contribution, but in some other way besides clarification. Of
these teacher responses that did not attempt to clarify the clarifiably ambiguous student contribution, 83% of the time the teacher immediately moved on and did not acknowledge the particular student contribution at all. An example of teachers not acknowledging the clarifiably ambiguous student contribution is during a class discussion where the class was discussing whether a general rule can be made about x or x+x being larger. A student made a comment about ‘it’ being unpredictable. The teacher then asked why it’s unpredictable. In response the student said, “Because if it was negative then this one would be positive and that one would be negative.” The teacher’s response then was, “Okay, so when you are testing two variable expressions to see how they compare, can somebody summarize for us what we just learned? Or will you just figure it out? Brooklyn?” The teacher does not mention the students' claim of something being negative or positive, but moves on to trying to get the class to summarize what they learned from the task. Now, the teacher’s intention may have been to show a connection between what the student said and their response, however many students probably could not have made the connection themselves. While both the student and the teacher were discussing the same task, it would be hard for students to see the relation between the teacher response and clarifiably ambiguous student contribution.

The other 17% of the time that the teacher response did not attempt to clarify the clarifiably ambiguous student contributions, the teacher acknowledged the student contribution by neither moving on nor clarifying, but rather by using a teacher move that was different than a type of clarify move. These teacher responses varied in how they addressed the particular clarifiably ambiguous student contribution with no pattern in what teacher moves the different teachers used to respond to the clarifiably ambiguous student contribution. An example of this is during a lesson on graphing in a pre-algebra class. When discussing what is happening on the
graph a student says, “where they kept going.” In response the teacher said, “Where they kept on going. Is it easy to tell where they were stopped on the graph?” In the first part of the teacher response the teacher repeats the student contribution. This did not qualify as a type of clarification because the teacher did not repeat the student contribution in a questioning tone or give any indication that they were trying to clarify the student contribution or wanted the student to clarify the student contribution. In the second part of the teacher response the teacher moves on. Another example in this same lesson is when multiple students were talking at once during the class discussion. All three students made comments about the graph; one student in particular made a clarifiably ambiguous claim saying, “or they went down, and back.” The student contribution is clarifiably ambiguous because their use of the word “they” could mean dots, bikers, distances, or times. In response the teacher said, “I’m hearing about 3 people. That’s good, we’re starting to talk. That’s good, good.” The teacher here is acknowledging that the student said something by making a literal comment about this student contribution and two others. After, the teacher moves on from this student comment and focuses on one of the other student contributions.

Forty-nine percent of the time the corresponding teacher response involved the teacher attempting to clarify some aspect of the corresponding student contribution. This does not necessarily mean that the teacher response clarified the particular phrase that was clarifiably ambiguous in the student contribution, but that the teacher response attempted to clarify some aspect of the particular contribution.

First, looking at whether the teacher asked a student a question, the teacher responses that attempted to clarify some aspect of the clarifiably ambiguous student contribution fell into two different categories. Either the teacher asked the student for clarification or the teacher did the
clarification themself. I will call when the teacher asked the student for clarification, Student-Contributed Clarification and when the teacher did the clarifying Teacher-Contributed Clarification. Eighty-one percent of the time the clarification was Teacher-Contributed Clarification while 19% of the time the clarification was Student-Contributed Clarification. Because asking a clarification question to the student is the most pertinent part to the theorized response, I will first discuss Student-Contributed Clarification and then follow with discussing Teacher-Contributed Clarification. In the following paragraphs I expound on the different subsets that emerged for these two types of clarification.

**Student-Contributed Clarification**

When a teacher response was a Student-Contributed Clarification the teacher asked the student for clarification. Two possible ways the teacher could ask the student for clarification would be to hone in and ask for clarification on the clarifiably ambiguous phrase in the student contribution or to not hone in and ask the student to clarify without being specific about what part of the student contribution needs to be clarified.

**Student-Contributed Generic Clarification**

All of the teacher responses that were Student-Contributed Clarifications consisted of the teacher asking a broad clarification question about the whole statement. I refer to this category as student-contributed generic clarification. An example of these clarification questions happens in a class discussion about graphing. In the midst of the class identifying what was happening at different points on the graph a student says, “And then they went up.” This student contribution is clarifiably ambiguous because “they” could mean dots, bikers, distances, or times. In response the teacher says, “So what does that mean?” Although the teacher response entails more than just this teacher turn, this initial teacher turn encapsulates what happens in the other teacher turns.
Teacher-Contributed Clarification

The other type of clarification was Teacher-Contributed Clarification. Teacher-Contributed Clarification is when the teacher clarifies the student contribution themself. Teacher-Contributed Clarification lacked the most important part of the theorized teacher response because the teacher was not asking the students for clarification but clarifying themself. However, the focal point of Teacher-Contributed Clarification was on the second most important part of the theorized teacher response, honing in on the clarifiable ambiguity in the student contribution. Two subgroups emerged when Teacher-Contributed Clarification was used. Either the teacher honed in on the clarifiably ambiguous part of the student contribution (teacher-contributed clarification of a clarifiably ambiguous statement) or instead honed in on an unambiguous part of the contribution (Teacher-contributed clarification of unambiguous statement).

Of the Teacher-Contributed Clarifications, 68% of the time teacher responses fell into the subcategory teacher-contributed clarification of a clarifiably ambiguous statement while 32% of the time teacher responses fell into the subcategory teacher-contributed clarification of an unambiguous statement. In the following sections I will first discuss the teacher responses in teacher-contributed clarification of clarifiably ambiguous statement and then discuss the teacher responses in teacher-contributed clarification of an unambiguous statement. The reason why I will discuss the subcategories in this order is because the teacher honing in on the clarifiably ambiguous part of the student contribution is more closely related to the theorized teacher response because it focused on the clarifiably ambiguity in the contribution. Thus, this type of clarification is more closely related than the teacher-contributed clarification of an unambiguous
statement which is only related to the third most important aspect of the theorized teacher response, stating what was understood from the student contribution.

**Teacher-Contributed Clarification of a Clarifiably Ambiguous Statement**

Teacher-contributed clarification of a clarifiably ambiguous statement had two subcategories: the teacher either asked for confirmation (*confirmed clarification*) of their clarification or they did not ask for confirmation (*unconfirmed clarification*). Of the teacher responses in teacher-contributed clarification of a clarifiably ambiguous statement, 53% of the teacher response fell into the subcategory *confirmed clarification* while 47% fell into the subcategory *unconfirmed clarification*. Now, a teacher response in teacher-contributed clarification of a clarifiably ambiguous statement that was also a confirmed clarification was more closely associated with the theorized teacher response because while the response containing a confirmation is not asking the student to clarify, it at least is asking for confirmation of the correctness of the clarification. When the teacher does not ask for confirmation, the main similarity the response has to the theorized response is honing in on the clarifiable ambiguity.

An example of a teacher-contributed clarification of a clarifiably ambiguous statement that was also a confirmed clarification is when students were discussing dividing and multiplying fractions during a classroom discussion. The teacher says, “K, so \(\frac{3}{4}\) divided by \(\frac{1}{2}\) gave you double and this \(1 \frac{1}{2}\) is what?” The student then says, “Bigger.” The teacher in response says, “This \(1 \frac{1}{2}\) is bigger than what we started out with, correct?” While the student could have been referencing the \(\frac{3}{4}\) or the \(\frac{1}{2}\), in the teacher response the teacher states that the bigger is referring to the number they started out with, \(\frac{3}{4}\). He then asks for confirmation which the student gave.
An example of a teacher-contributed clarification of a clarifiably ambiguous statement that was also an unconfirmed clarification is when students were discussing graphing. When discussing a graph about the distance a biker has gone a student said, “Because, you can tell by the dots that they went up!” In response the teacher says, “So what story does dots going up, what does that tell us about the day?” The “they” in the clarifiably ambiguous student contribution could refer to the bikers, the dots, the distances, or the times. The teacher decides that the student is talking about the dots and did not ask for confirmation on the correctness of the clarification.

Teacher-Contributed Clarification of an Unambiguous Statement

In teacher-contributed clarification of an unambiguous statement, the teacher makes clear what they understood about the clarifiably ambiguous student contribution. This is because they are only talking about the inferable part of the contribution, leaving out the clarifiably ambiguous part of the contribution. An example of teacher-contributed clarification of an unambiguous statement is during a classroom discussion on the task, “The price of a necklace was first increased 50% and later decreased 50%. Is the final price the same as the original price? Why or why not?” A student says:

50 is 50 percent of what number? You can’t just get 50 and then say 0 and then plus 50. You added, but 50 percent of what? ‘Cause like at the end it would end up being a decimal point 50 so it’d be point 50 of what whole number that you begin to find.

The beginning portion of the student contribution is unambiguous, however when the student starts using “it” the contribution becomes ambiguous because the “it” could be referring to the answer, 50 percent of the original price, or 50 percent. This is a complicated student contribution, but it adequately shows how the teacher only focuses on the unambiguous part of the contribution in their response. In response the teacher says, “So he’s saying 50 percent of
what? 50 is not just 50, but it’s 50 percent of what?” The teacher honed in on the first part of the student contribution which was unambiguous. The meaning behind the first portion of the student contribution can be assumed to mean “50 is 50% of what number? You can’t just get 50; take 0 and then plus 50, you’re adding, but 50% of what?” The second portion of the statement is clarifiably ambiguous. The teacher only grabs onto the first portion and clarifies what is already not ambiguous leaving the clarifiably ambiguous portion of the student contribution to still have more than one interpretation.
CHAPTER 5: DISCUSSION

I discuss the major findings of my study, what the findings mean, how this research extends past research, and future steps. I first discuss these points with respect to the types of ambiguous student contributions, and then with respect to how teachers respond to clarifiable ambiguity.

Types of Ambiguous Student Contributions

Ambiguous student contributions either stemmed from the teacher’s or the student’s verbal and physical actions. Student Appropriation of Teacher Ambiguity is the only category of the four categories to fit into the Teacher Actions while Irrelevant Ambiguity, Clarifiable Ambiguity, and General Ambiguity all fall under Student Actions.

Teacher Actions

Thirty-six percent of ambiguous student contributions stemmed from Teacher Actions while 64% of ambiguous student contributions stemmed from Student Actions. Although Student Actions were the greatest contribution to the ambiguous student contributions, the fact that Teacher Actions accounted for 36% of the ambiguous student contributions is significant. This finding suggests that teachers too should be concerned about the clarity of what they are saying.

Figure 16 shows the frequencies of all the different categories. In this figure it is very apparent how much Student Appropriation of Teacher Ambiguity contributes to the ambiguous student contributions because Student Appropriation of Teacher Ambiguity is the greatest contributor. Research has not focused on Student Appropriation of Teacher Ambiguity, leaving this very large contributor to ambiguous student contributions unstudied.
Student Appropriation of Teacher Ambiguity falls under the category of potentially unproductive ambiguity. Past research has looked at ambiguity that is productive in the classroom (Barwell, 2003). Student Appropriation of Teacher Ambiguity is not a type of ambiguity that if the student contribution is continually developed could help students come to a greater understanding. In fact, Student Appropriation of Teacher Ambiguity could be hindering a student's understanding because they might not understand what the teacher is saying or what they are asking. If teachers became more aware of the different ways they may be causing student contributions to be ambiguous they may be able to pay more particular attention to their teaching and lessen the amount of time they are being ambiguous. While I have identified three subcategories of Student Appropriation of Teacher Ambiguity
(multiple questions, teacher clarifiable ambiguity, and vague questions), my study has only gone
as far as identifying the different subcategories. Future research may want to analyze these
subcategories to further understand Student Appropriation of Teacher Ambiguity and the role
Student Appropriation of Teacher Ambiguity plays in classroom discourse.

Student Actions

Student Actions contain the other three (Irrelevant Ambiguity, Clarifiable Ambiguity, and
General Ambiguity) types of ambiguous student contributions.

Clarifiable Ambiguity

Figure 16 shows that Clarifiable Ambiguity was most frequent out of the Student Actions
categories. One fourth of the time a student contribution was ambiguous, the ambiguity was
clarifiable ambiguity. On a bigger scale, approximately one in every ten student contributions in
the 11 classroom videos were clarifyably ambiguous. There is significance in how often during
classroom discourse clarifyably ambiguous student contributions happen. Peterson et al. (2020)
discuss how clarifyable ambiguity can hinder classroom discussion. With the frequency of
clarifyably ambiguous student contributions in mathematics classrooms it seems important that
teachers become more aware of the clarifyable ambiguity happening in their own classrooms. I
will address how teachers are responding to clarifyable ambiguity in the second portion of my
discussion.

Other Student Actions Categories

As seen in Figure 16, General Ambiguity and Irrelevant Ambiguity had similar
occurrences. While these categories did not happen as frequently as Student Appropriation of
Teacher Ambiguity and Clarifiable Ambiguity, it is still important to recognize the different
categories. This is because the different types of ambiguous student contributions may call for
different types of teacher responses. For example, the way a teacher responds to a student contribution that is ambiguous because of *Student Appropriation of Teacher Ambiguity* would be different than how they would respond to a student contribution that is ambiguous because of *Clarifiable Ambiguity*. In the next few paragraphs I will theorize about how teachers may want to respond differently to different types of ambiguous student contributions.

**Possible Teacher Responses**

**Student Appropriation of Teacher Ambiguity**

If a student contribution is ambiguous because the teacher’s statement or question is ambiguous, it may not be best for the teacher to ask the student to clarify. If the teacher’s statement is ambiguous, the teacher may want to stop and clarify what they personally mean. The teacher could ask the student to clarify, however the teacher runs the risk of the new student contribution not answering what the teacher intended. This could turn the classroom discussion to an unplanned direction and cause the classroom discussion to be unproductive. Thus, it may be best if the teacher first clarifies what they are wanting students to think about.

**Irrelevant Ambiguity**

If a student contribution is ambiguous but the contribution does not position students differently or is not worth pursuing, it may be best for the teacher to do nothing with the student contribution and move on. This may be a good way to deal with *Irrelevant Ambiguity* because if all student thinking is pursued, the classroom discussion may not go anywhere. Also, by trying to honor all student thinking, a teacher may end up not honoring any student thinking. *Irrelevant Ambiguity* is the type of ambiguity that is not worth clarifying or pursuing, thus moving on to focus on more essential student thinking may be best.
**Clarifiable Ambiguity**

If a student contribution is ambiguous because the contribution positions students differently and the ambiguity can be clarified in the moment, then the theorized teacher response I discussed in the methods section may be the best teacher response. In the second half of the discussion I will discuss other possible productive teacher responses.

**General Ambiguity**

If a student contribution is ambiguous because it positions students differently and the ambiguity cannot be clarified in the moment, the teacher may want to develop the contribution. Past research has focused on this type of ambiguity, *General Ambiguity*--more particularly, the sub category of development under *General Ambiguity*. Literature suggests ways teachers may respond to ambiguity in general by being more conscious of the ambiguity, delaying student’s arrival at a formal definition, facilitating different ideas about terms, etc. (Barwell, 2003; Foster, 2011; Rathouz, 2010; Rowland, 2003). However, existing literature provides only a general sense of how to use this ambiguity productively in the classroom. I theorize that, like previous research has stated, not clarifying in the moment, but developing the student idea further may be the best course of action.

**Connections to Literature on Ambiguity**

Research has solely focused on *General Ambiguity*. The research on *General Ambiguity* and the usefulness of this ambiguity in the classroom is extremely insightful and helpful to teachers, however the issue is that there are three other types of ambiguous student contributions. This means that research has only addressed how to respond to 17% of ambiguous student contributions. Eighty-three percent of ambiguous student contributions have not been researched. Furthermore, the ways that researchers have described dealing with *General Ambiguity* does not
seem fitting for the other types of ambiguous student contributions. Thus, by completing this research, I will hopefully have given some direction to what teachers may want to do when an ambiguous student contribution arises in a classroom discussion. All of these different types of ambiguous student contributions may require different teacher moves. By being aware of the different types of ambiguous student contribution, teachers may have a better idea of how to respond and honor student thinking.

**Teacher Responses to Clarifiable Ambiguity**

In this section of my discussion I first focus on the findings on all teacher responses to clarifiably ambiguous student contributions. I discuss the positive aspects and ramifications of how the teacher addresses the particular student contribution and then narrow in on just the teacher responses that included a type of clarify move in the teacher response. In doing so, I discuss the four aspects of the theorized response (hone, acknowledge, clarify, and honor) for each category of teacher responses. I also discuss the positive aspects and ramifications of the different categories of teacher responses that emerged in my data.

My results showed that teacher responses to clarifiably ambiguous student contribution landed into three separate areas. Figure 17 shows that the teacher either did not address the particular student contribution, addressed the particular student contribution by using some type of clarify teacher move, or addressed the student contribution by using a teacher move other than a type of clarify move. In the following paragraphs I will address each of these categories.
Figure 17

*Teacher Responses to Clarifiably Ambiguous Student Contributions*

Did Not Address Clarifiably Ambiguous Student Contribution

The fact that 51% of the time the teacher did not attempt to clarify the clarifiably ambiguous student contribution is significant. Half the time that a student contribution was clarifiably ambiguous the teacher did not attempt to clarify the contribution, but let the contribution continue to have multiple meanings.

When the teacher did not address the particular student contribution, one could think that since the teacher did not focus on the contribution, the clarifiable ambiguity was not as much of a problem. However, there are a few issues with not addressing the student contribution. First, *Irrelevant Ambiguity* is not part of the category of *Clarifiable Ambiguity*. All student contributions that seemed off hand, not worth pursuing, and did not position the students differently in their understanding are not a part of the category of *Clarifiable Ambiguity*. Thus, all these clarifiably ambiguous student contributions are relevant to the classroom discussion.
Students could use a wrong intended interpretation of the student contribution as the discussion went on because the particular contribution was relevant to the classroom discussion. Also, students may feel like their thinking has not been honored because the teacher does not acknowledge their thinking. Students may start to share less in the future if they feel like the teacher does not value their thinking. Now, teachers picking up on all relevant student thinking seems like an enormous goal. But, if teachers could become more aware of clarifiably ambiguous student contributions and when they should clarify the contributions, students may feel like their ideas are honored and there may be less misconceptions in the class discussion.

An example of what happened when the clarifiable ambiguity was not addressed happened in a pre-algebra lesson about graphing. The class was discussing the graph in Figure 18. At the beginning of the conversation the teacher asked the student if they should connect the dots. In response a student said they should and gave the following rationale, “It goes up and down, up, and sideways, and up and sideways.” The student’s use of the word “it” makes this contribution clarifiably ambiguous because “it” could mean the dots, the bikers, the distances, or the times. The teacher does not address the student contribution but immediately moves on to another student.
Figure 18

*Graph of Day 2 Progress of a Bike Tour*

The discussion continues around this graph and again and again the students and teacher refer to something on the graph as “it” or “they.” The “it” or “they” seem to refer to the dots, the bikers, the distances, or the times at different times during discussion. The longer the conversation goes on the more confusing the discussion becomes because everyone is talking about a different “it” or “they.” The discussion around the graph wraps up with no one ever clarifying or asking for clarification on what “it” or “they” is. Not clarifying was problematic because this class was learning how to read graphs. With so many ways of interpreting “it” and “they,” there is a high possibility students took away different, incorrect ways of reading a graph.

**Addressed Student Contribution with a Teacher Move Other Than Clarify**

Nine percent of the time a teacher would address but not clarify the student contribution. A positive aspect to the teacher addressing the student contribution is the student may feel like their thinking has been honored. However, while addressing the student contribution, even if it is not clarifying the thinking, may be honoring the student's thinking, it may also not honor the student thinking if the teacher runs with the idea from the contribution in a completely different direction than the student intended.
Another ramification of addressing the student contribution, but not clarifying the contribution is possible misconceptions that may arise. The student contribution was clarifiably ambiguous, thus the contribution has multiple interpretations. If the student contribution is not clarified, there is still more than one interpretation of the contribution. In the proceeding class discussion students may be able to come to the intended interpretation of the student contribution and the discussion. However, if the teacher clarifies or asks for clarification of the student contribution there is a higher chance of all students coming to a similar interpretation.

**Addressed Student Contribution With a Clarify Teacher Move**

Forty-nine percent of the time the teacher addressed the student contribution by using some type of clarifying move. This means that teachers are making the decision to attempt to clarify a clarifiably ambiguous student contribution about half the time. Using a clarify teacher move may indicate that the teacher recognizes the student contribution is ambiguous and can be clarified in the moment. In each of the following subsections I discuss the extent to which the four aspects of the theorized response appeared in the different ways that teachers attempted to clarify the clarifiably ambiguous student contributions.

**Student-Contributed Honed-in Clarification**

I found no teacher responses where a *Student-Contributed Clarification* honed in on the part of the student statement that needed to be clarified. *Student-contributed honed-in clarification* seems to be the most closely related response to the theorized response because it fulfills at least the two most important requirements, asking a student a question and honing in on the clarifiably ambiguous phrase. Because I never actually found any teacher responses that were a *Student-contributed honed-in clarification*. I do not know if the teacher responses fulfilled the third requirement of the theorized response, the teacher stating what they understood from
the student contribution. I will not be going through the four aspects of the theorized response because this response never showed up in the data, thus I cannot say whether or not the Student-contributed honed in clarification actually satisfied the aspects of the theorized response.

**Student-Contributed Generic Clarification**

*Student-contributed generic clarification* did not hone in on the clarifiable ambiguity or point out what was understood from the student contribution. However, the student-contributed generic clarification was similar to the theorized response by asking the student a clarifying question, which makes this category the most similar to the theorized response.

**Hone.** This teacher turn does not hone in on the vague referent “they,” but asks a general question about what the student’s whole statement means. In fact, instead of honing in on the clarifiable ambiguity, the teacher makes the exchange between the student and teacher more vague by being clarifiably ambiguous themself by referring to something in the student contribution as “that.”

**Acknowledge.** Every teacher response in the student-contributed generic clarification category either included the portion of the claim that was not clarifiably ambiguous or referenced the whole claim (ambiguous portion and unambiguous portion) with a vague referent. This is because the teacher did not hone in on the clarifiably ambiguous part of speech but asked for a broad clarification of the whole student contribution. For example, in the same classroom discussion on identifying what is happening to the graph at different points a student says, “They’re slowing down.” This student contribution is clarifiably ambiguous because “they’re” could mean dots, bikers, distances, or times. In response the teacher says, “They’re slowing down?” While the teacher stated the words that were not ambiguous, the teacher also included the vague referent too. So, the teachers were putting the same emphasis on the clear portion of
the student contribution as they were on the unclear portion of the contribution. It is unclear how useful it is to include what the teacher understood if there is no way to distinguish it from what they did not understand.

**Clarify.** There were no cases where a student-contributed generic clarification served to clarify the clarifiably ambiguous student contribution. The teacher did not hone in on the part of the statement that was clarifiably ambiguous, so when the teacher moved on from the particular student contribution, there was still more than one viable interpretation of the contribution. Thus, the teacher responses in student-contributed generic clarification attempted to but failed to clarify the clarifiable ambiguity.

**Honor.** The student-contributed generic clarification did show evidence that the teacher is honoring the student’s thinking to some extent. The teacher is focused on what the student said as a whole and asking the student about their thinking. Honoring the student's thinking can be seen by the example given previously when the student said, “They’re slowing down.” And in response the teacher said, “They’re slowing down?” The teacher is honoring the student's thinking by acknowledging and focusing on the student contribution.

*Teacher-Contributed Clarification of a Clarifiably Ambiguous Statement*

Teacher-Contributed Clarification of a Clarifiably Ambiguous Statement honed in on the clarifiable ambiguity. This type of teacher response included the teacher making clear what they understood about half the time. This type of teacher response did result in the clarifiably ambiguous statement being clarified, and if the teacher confirmed the clarification, the teacher response honored the student thinking.

**Hone.** As stated previously, teacher-contributed clarification of a clarifiably ambiguous statement hones in on the clarifiable ambiguity. In fact, honing in is the major distinction this
category has from the other category in Teacher-Contributed Clarification. Also, because I never saw any Student-contributed Clarification that honed in, the teacher responses in the category of teacher-contributed clarification of a clarifiably ambiguous were the only teacher responses that did hone in on the clarifiable ambiguity.

**Acknowledgment.** In both of the subcategories of teacher responses, confirmed clarification and unconfirmed clarification, sometimes the teachers did make clear what they understood and sometimes the teachers did not make clear what they understood. Roughly half the time the teacher response included what the teacher understood from the student contribution (that is, the teacher used unambiguous parts of the student contribution in their response) for both confirmed clarification and unconfirmed clarification. Both the example of confirmed clarification and the example of unconfirmed clarification in the preceding paragraphs show the teachers making clear what they did understand from the student contribution. An example where the teacher did not make clear what they understood from the student contribution is the following. A class is discussing why you must take the coordinates from the same coordinate pair and not coordinates from two different coordinate pairs to satisfy an equation. A student made the following clarifiably ambiguous statement, “I feel like that’s two points not one line.” This student contribution is clarifiably ambiguous because of the student’s use of ‘that’ could mean (0,3) and (3,0), A and B, (1,5) and (0,6) are, or (3,0) and (3,3). In response the teacher said, “‘Kay so Margaret’s conjecture is taking maybe the x value. Is the x value coming from A or from B?” While the teacher response is centered on clarifying the student contribution, the teacher does not state what they understood from “I feel like that’s two points not one line.”

**Clarify and Honor.** Both subcategories, confirmed clarification and unconfirmed clarification, honed in on the clarifiably ambiguous part of the student contribution. The teacher
clarified the clarifiable ambiguity leaving only one viable interpretation by the time the teacher moves on from the student contribution. The difference between confirmed clarification and unconfirmed clarification is whether the teacher checked to see if their interpretation was what the student actually meant. In the case of unconfirmed clarification, while there is now one interpretation, there is no indication that the interpretation is what the student meant. This may be seen by the student and others in the class as not honoring the particular student’s thinking, especially by the particular student if the teacher’s clarification was wrong. In the case of confirmed clarification, there is one interpretation and the student indicated that the interpretation was what they meant. Now, because we do not have access to the student’s thoughts there is a chance the student is just agreeing because the teacher said it and they actually were not thinking the teacher’s interpretation. However, if the teacher is not going to ask the student for clarification, asking for confirmation may be a productive alternative, especially if social norms in the classroom support students in speaking up when the teacher’s interpretation is not what they originally were thinking.

Teacher-Contributed Clarification of an Unambiguous Statement

Teacher-Contributed Clarification of an Unambiguous Statement did not hone in on the clarifiable ambiguity. This type of teacher response did state the clear portion of the student contribution, the student contribution was not clarified, and the teacher response only somewhat honored the student thinking.

Hone. Teacher-contributed clarification of an unambiguous statement does not hone in on the clarifiably ambiguous part of the student contribution because the teacher responses only focus on the unambiguous part of the student contribution.
**Acknowledge.** These teacher responses do state what the teacher understood from the student contribution. This is because all the teacher does is state the unambiguous portion of the student contribution.

**Clarify.** Because the teacher response does not hone in on the part of the student contribution that is clarifiably ambiguous, by the time the teacher moves on from the particular contribution there is still more than one interpretation of the particular contribution. Thus, the particular student contribution has not been clarified.

**Honor.** Teacher responses in the Teacher-contributed clarification of an unambiguous statement category somewhat honor the student's thinking. The teacher is discussing part of the student contribution, but is ignoring the other part. If the clarifiably ambiguous part of the student contribution was the student’s main point, the student may not have felt like their thinking was honored.

**Teacher-Contributed Clarification versus Student-Contributed Clarification**

The data showed that 81% of the time the teacher was the one who attempted to clarify the clarifiably ambiguous student contribution. One benefit to the teacher clarifying the student contribution is that it is a sure way for the teacher to know the contribution has been clarified. When they continue the discussion the teacher knows that there is now one interpretation for the student contribution. However, because the teacher is the one who clarified, they cannot be sure if their clarification is the ‘true’ clarification. What I mean by ‘true’ clarification is because the student contribution is clarifiably ambiguous, the student knows the word or phrase they were meaning with their vague referent. The ‘true’ clarification is the student’s clarification since it was their thinking. When the teacher clarifies, it is unclear whether the clarification was what the
student was thinking. By the teacher clarifying it may undermine what the student was meaning and result in the teacher not honoring the student thinking.

A Teacher-Directed Clarification could honor the student thinking and get closer to the ‘true’ clarification by asking for confirmation. The most frequent of the four responses given by teachers was the teacher-contributed clarification of a clarifiably ambiguous statement with confirmed clarification (see figure 19). By the teacher asking for confirmation, the teacher is doing a better job honoring the student's thinking by asking the student if their clarification was correct. That said, asking for confirmation may be problematic in many classes, because the teacher holds the authoritative role and students might feel the need to agree with whatever clarification the teacher gives even if the clarification is not what they originally meant.

**Figure 19**

*Frequencies of Teacher Responses to Clarifiable Ambiguity*
There may be many reasons why a teacher may use a *Teacher-Contributed Clarification*. When a teacher clarifies and asks for confirmation they may believe they know what the student means, but wants to be sure the rest of the class picks up on this clarification. If the teacher clarifies and does not ask for confirmation the teacher may believe what the student meant is obvious so they clarify and move on without bringing attention to the clarification. If the teacher uses a *teacher-contributed clarification of an unambiguous statement* the teacher may believe the clarifiably ambiguous portion of the student’s statement was trivial and the teacher wants the class to focus on the unambiguous portion of the student thinking. While all of these rationales for using the different clarifications are reasonable for teacher’s to think, the student contributions they were responding to were coded as clarifiably ambiguous, and as such were deemed as having two or more viable interpretations. The student contributions were also seen as being important and not irrelevant. By the teacher clarifying they are potentially undermining what the students think, or in other words not honoring the student thinking. Now, it is understandable that teachers only have so much time in their lessons and asking students a question every time they need clarification may not be ideal.

Through the preceding paragraphs I have stated possible positive and negative aspects of *Teacher-Contributed Clarification*. Through doing this I have discussed how a *Student-Contributed Clarification* may also be beneficial. When the student is asked to clarify, the student's thinking is honored. However, only one category of teacher responses showed the teacher asking the student for clarification and in this category the teacher did not specify what the student should clarify, *student-contributed generic clarification* (see figure 19). Not specifying what the student should clarify seems problematic because the student may just repeat what they already said or go off on the unambiguous portion of their statement which still leaves
the clarifiable ambiguity unclarified. In this case, it may be beneficial for the teacher to clarify when compared to asking the student a nonspecific clarification question because if the teacher clarifies there is now one interpretation of the student contribution while the proceeding contribution may leave multiple viable interpretations.

**Connections to Literature on Clarifying Teacher Moves**

In my study I identified four different clarifying moves that teacher’s made. One clarify move was *Student-Contributed Clarification* (*student-contributed generic clarification*) while three of the clarify moves were *Teacher-Contributed Clarification* (*teacher-contributed clarification of a clarifiably ambiguous statement with confirmed clarification* or with *unconfirmed clarification* and *teacher-contributed clarification of a clarifiably unambiguous statement*). Previous research has classified their clarify teacher moves as revoicing, requesting confirmation, re-framing a question, or requesting clarification (Tytler et al., 2015). *Student-Contributed Clarification* seems similar to requesting clarification because they both are centered around asking students a question. The category of *confirmed confirmation* seems similar to requesting confirmation category due to both of them being centered on confirming. One difference between previous literature and my study is previous literature is discussing clarification related to all student contributions while I am studying clarification only for clarifiably ambiguous student contributions.

**Theorized Teacher Response**

There was no evidence of the full theorized response in any of the teacher responses. Thus, knowing if the theorized teacher response is the most beneficial way to respond is unknown. Possible reasons as to why the theorized teacher response never showed up was because many teachers’ ‘go to’ move is to clarify the student contribution themselves. When
teachers actually did ask a student a clarifying question, they may not remember what exact word or phrase they wanted clarified. Slowing down a classroom recording and analyzing each student contribution individually gives an observer a lot of time to know what word or phrase the teacher would want clarified. However, in the classroom it is much more difficult to identify which words or phrases may need to be clarified. I still believe, based on wanting to honor student thinking and getting the “true” clarification, the theorized response may be the most appropriate response to clarifiable ambiguity. Hopefully future research will find teacher responses that are similar to the theorized response.
CHAPTER 6: CONCLUSION

Different types of ambiguous student contributions occur in mathematics classrooms. In this study I identified (1) different types of ambiguous student contributions and (2) the different ways teachers respond to one particular kind of ambiguous contribution, clarifiable ambiguity. Note that clarifiable ambiguity is ambiguity that stems from a student who uses an unclear referent in their contribution and can be clarified in the moment by the particular student. Literature has focused only on ambiguity that has the potential to further the development of mathematical concepts and has only theorized about teacher responses to this specific type of ambiguity.

This study identified three additional types of ambiguous student contributions: Student Appropriation of Teacher Ambiguity, Irrelevant Ambiguity, and General Ambiguity. It was important to identify all the different types of ambiguous student contributions because teacher responses should likely be different to the different types of ambiguity. In addition, through analyzing the teacher responses to the clarifiably ambiguous student contributions, this study found that teachers addressed the clarifiably ambiguous student contributions about half the time. When the teachers did address the clarifiable ambiguity, the majority of the time the teacher clarified the ambiguity themselves with the most common response being the teacher honed in on the clarifiable ambiguity and asked for confirmation from the student on the accuracy of the clarification.

Contributions

This study has extended research on the different types of ambiguous student contributions. As stated in chapter 2, literature has thus far focused on ambiguity that helps further classroom discourse. This study has identified other ambiguous student contributions that
are different from this “productive ambiguity.” What literature has stated is productive ambiguity emerged in my data under General Ambiguity. Although General Ambiguity was present, it only accounted for a small percentage of the ambiguous student contributions. Thus, the vast majority of the ambiguous contributions in this data set did not seem to fall under the umbrella of what literature has called productive ambiguity.

A particular type of ambiguous student contribution that was valuable to identify was Student Appropriation of Teacher Ambiguity. Research has not previously focused on student contributions being ambiguous because of the preceding teacher response. Student Appropriation of Teacher Ambiguity was the most common reason for why a student contribution was ambiguous. Thus, identifying this category of ambiguous student contribution may help direct future research in knowing where to focus on in trying to minimize unproductive ambiguity in mathematics classrooms.

Finally, this study identifies actual ways teachers are responding to clarifiable ambiguity. Literature has identified productive teacher moves in response to ambiguity, however literature has not studied how teachers are actually responding to ambiguity. This study identifies how the teachers are actually responding which allows for future research to look into how these current teacher responses are beneficial or may need to change to become more helpful in clarifying ambiguity.

**Implications**

Identifying different types of ambiguous student contributions implies that research should not focus solely on productive ambiguity but also on unproductive ambiguity since most of the ambiguous student contributions are potentially unproductive. Knowing the different types of ambiguous contribution also implies that possible teacher moves should be studied and
developed to better deal with the different types of ambiguity. This is because teachers should respond differently to different types of ambiguous student contributions.

Identifying *Student Appropriation of Teacher Ambiguity* implies that there should be more of a focus on how teachers are talking. If teachers are only concerned about what their students are saying, they may miss how their own contributions may be affecting the student contributions. Research could focus on how teachers can recognize and correct their own ambiguity.

My results show that a majority of the time teachers are clarifying the clarifiably ambiguous student contributions themselves and not asking students clarifying questions as often. Research could focus on the usefulness of asking students questions versus the usefulness of clarifying themselves. Also, this research may shed light on how teachers need to work on the type or clarifying questions they ask or if they clarify themselves, when to ask for confirmation of their clarification.

**Limitations and Future Research**

This study used mathematical lessons that were recorded instead of observing and interviewing teachers and students. By using recordings, this study is limited by not gaining more access to what students and the teacher had to say about their contributions, especially with why the teacher may have clarified in the particular way they chose to clarify. However, the recordings gave sufficient evidence on what was happening in the moment in the classroom and allowed for me to identify more teacher responses and student contributions than I would have if I would have just observed.

This study is limited by the small sample of teacher responses that actually attempted to clarify ambiguous student contributions. Each of the different responses were seen multiple
times in multiple lessons, but because this study only looked at how teachers were clarifying one specific type of ambiguous student contribution the sample size became smaller. Future research may pull from a larger base of ambiguous student contributions to provide more evidence for the types of clarify moves teachers are using. This larger set may also contain the theorized teacher response. If the theorized teacher response is seen being enacted in a mathematics classroom, there may be more guidance for teachers on how to clarify clarifiably ambiguous student contributions.

**Conclusion**

This study has identified different types of ambiguous student contributions that occur in mathematics classrooms: *Student Appropriation of Teacher Ambiguity, Irrelevant Ambiguity, Clarifiable Ambiguity, and General Ambiguity*. This study has also identified how teachers respond to clarifiable ambiguity: *student-contributed generic clarification, teacher-contributed clarification of a clarifiably ambiguous statement with confirmed clarification or with unconfirmed clarification, and teacher-contributed clarification of an unambiguous statement*. Future research will hopefully study different ways to address the different types of ambiguous student contributions and how to help teachers better respond to clarifiably ambiguous student contributions.
REFERENCES


