Dynamic Radiation Heat Transfer Control Through Geometric Manipulation

Rydge Blue Mulford
Brigham Young University

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Dynamic Radiation Heat Transfer Control Through Geometric Manipulation

Rydge Blue Mulford

A dissertation submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Brian D. Iverson, Chair
Matthew R. Jones
Brent W. Webb
Dale R. Tree
Larry L. Howell

Department of Mechanical Engineering
Brigham Young University

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ABSTRACT

Dynamic Radiation Heat Transfer Control Through Geometric Manipulation

Rydge Blue Mulford
Department of Mechanical Engineering, BYU
Doctor of Philosophy

The surface area and radiative properties of an object influence the rate of radiative emission from the object’s surface and the rate of radiative absorption into the surface. Control of these variables would allow for the radiative heat transfer behavior of the surface to be manipulated in real time. Origami tessellations, being a repeated pattern of linked, dynamic surfaces, provide a framework by which dynamic control of apparent radiative properties and surface area is possible.

The panels within a tessellation form cavities whose aspect ratio varies as the device actuates. The cavity effect suggests that the apparent radiative properties of the cavity openings will vary as a function of aspect ratio. The apparent absorptivity of an accordion tessellation formed from folded shim stock is shown experimentally to increase by 10x as the tessellation actuates from fully extended to within 10% of a completely-folded state. Analytical models and Monte Carlo ray tracing are used to quantify the apparent radiative properties of an infinite V-groove for a variety of conditions, including specular or diffuse reflection and diffuse or collimated incident irradiation. For a diffuse V-groove, apparent radiative properties increase with increasing V-groove aspect ratio but do not approach unity. Highly reflective surfaces exhibit the largest relative increase in apparent radiative properties with actuation. Closed-form correlations achieve an average relative error of 2.0% or less. For a specular V-groove, apparent radiative properties approach unity as the V-groove collapses towards an infinite aspect ratio. The apparent absorptivity for a V-groove exposed to collimated irradiation shows significant variations over small actuation distances, increasing by 5x over a small actuation range. For certain conditions the apparent absorptivity of a V-groove subject to collimated irradiation decreases as the aspect ratio increases.

For an isothermal accordion tessellation the net radiative heat exchange continuously decreases as the surface is collapsed for most conditions, indicating that the reduction in apparent surface area generally dominates the increase in apparent radiative properties. Net radiative heat transfer values decrease by 7x for collimated irradiation and specular reflection over small actuation distances. Specular V-grooves subject to collimated irradiation occasionally show an increase in net radiative heat transfer as the device collapses. A non-isothermal dynamic radiative fin achieves a 3x reduction in heat transfer as the fin collapses; this value can be increased with the use of highly conductive materials and by increasing the length of the fin. The fin efficiency of a collapsible fin increases as the fin collapses. An experimental prototype of a collapsible fin is developed and tested in a vacuum environment, achieving a 1.32x reduction in heat transfer for a limited actuation range, where a numerical model suggests this prototype may achieve a 2.23x reduction in heat transfer over the full actuation range.

Keywords: heat transfer control, radiation heat transfer, origami heat transfer
ACKNOWLEDGMENTS

To begin, I offer my sincerest gratitude to my graduate committee for offering freely of their time and resources in order to help me succeed. Thank you for reviewing this lengthy dissertation and for your helpful and constructive feedback.

I also offer a heart-felt thanks to my fellow graduate students. Nothing is more valuable then a neighbor who is willing to offer a constant listening ear. Likewise, thank you to the undergraduates whose technical skills were imperative to the success of this project.

A special thank you to the many people at NASA and AFRL I have had the privilege of working with. Vivek, your boundless enthusiasm and energy not only have guaranteed the success of this project but also have built me up as an individual.

Dr. Dale Tree, thank you for instructing me in four classes. From what I understand that was not an easy task. Thank you even more for being my friend. From what I understand that is an even harder task. I also wanted to express my gratitude to Dr. Matthew Jones. Thank you for your untiring devotion to the quality of your work and the sincerity of your teaching. I would not be where I am today as a scientist and a teacher without your input.

A sincere thank you to my advisor, Dr. Brian Iverson. You never fail to find the silver lining in even the darkest storm cloud, and your optimism has always paid off with success. As I prepare to foster graduate students of my own, I find myself referring often to how you approach mentoring, writing, and research problems. Thank you for trusting me with your time during an important part of your career and for believing in me every step of the way.

Finally, I couldn’t have gone anywhere without the support of my friends and family. Thank you, Mom and Dad, for everything you have given to get me where I am today. To my wife, Lenore, this dissertation belongs to both of us. Your support during graduate school has made all of this possible. Thank you.

This work was funded by the NASA Utah Space Grant Consortium and a NASA Space Technology Research Fellowship (grant number: NNX15AP49H).
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5.1 Geometric features of the accordion tessellation, including the cavity angle \( \phi \), tessellation length \( L_P \), tessellation width \( W_P \), projected area of the cavity openings normal to collimated irradiation \( A_{a,proj} \), apparent surface area of the tessellation \( A_a \), apparent temperature \( T_a \), and the collimated irradiation angle of incidence \( \gamma \).

5.2 (a) Schematic of folded stainless-steel samples with sixteen ‘folded panels’ and two ‘mounting panels’ constrained between two copper bus bars. Bus bars were positioned using a plastic supporting base with a hole pattern associated with desired angle positions and (b) schematic of the experimental setup in a vacuum. Steady-state temperatures were measured with an infrared camera through a sapphire window, while the voltage and current associated with Joule heating were measured to determine net radiative heat transfer after accounting for losses.

5.3 (a) Normalized net radiative heat exchange \( \Pi \) as a function of \( \phi \) for a diffusely irradiated accordion tessellation with specular or diffuse reflection for two different intrinsic emissivities. The “flat” case indicates a flat, black surface, equivalent in size to the apparent area at a given cavity angle, (b) normalized net radiative heat exchange \( \Pi \) as a function of \( \phi \) for a diffusely reflecting accordion tessellation with collimated irradiation incident on the accordion tessellation at several angles \( \gamma \), and (c) normalized net radiative heat exchange \( \Pi \) as a function of \( \phi \) for a specularly reflecting accordion tessellation with collimated irradiation incident on the accordion tessellation at several angles \( \gamma \).
5.4 (a) Temperature profile for a sample positioned at $\phi = \pi/2$ with a total heating power of 3 W. The thermal image from which this profile is derived is displayed behind the temperature profile, where the temperature profile was measured along a horizontal line across the vertical center of the tessellation and (b) comparison of experimental and analytical model temperature values. Experimental temperature measurements are derived from a thermal image of the heated surface (e.g., Figure 5.4a). Predicted temperature values are determined with the analytical approach of Equation 5.9. Data from two of the seven tested fold angles are provided with the uncertainties of each measurement.

6.1 Geometry of a re-deployable, segmented fin connected to a base (at $T_b$) consisting of several rigid straight panels that are connected in series via thermally conductive hinges. The apparent surface area and apparent radiative properties of the fin vary as the fin is actuated.

6.2 (a) Results of the Segmented Fin Algorithm (SFA) compared with results from an analytical, straight, radiative fin solution (Equation 20) for $\varepsilon = 0.90$. The numerical algorithm agrees well with the analytical approach with a largest relative error of 3%, obtained at large values of $N_c$, $\varepsilon$, and $\chi$. (b) The apparent emissivity of a V-groove ($\varepsilon_a$) as calculated using the SFA compared with a published correlation (Equations 6.21 and 6.22). The largest error between the SFA and correlation is 2% which is similar to the range of errors given for the correlation.

6.3 Non-dimensional temperature ($\theta$) profiles as a function of non-dimensional panel position ($\chi$) for a four panel, hinged radiator at two values of the conduction-radiation interaction parameter ($N_c$). Results are displayed for increasing values of the actuation angle ($\phi$), showing the influence of fin deployment position on the temperature profile. (a) Behavior observed for an intrinsic emissivity of 0.9. When conduction dominates ($N_c < 1$), the profile approximates that of a straight radiative fin. When radiation dominates ($N_c > 1$) the influence of radiative exchange between panels causes significant variation in the panel temperature profiles. (b) Behavior observed for an intrinsic emissivity of 0.1. As a result of the nearly reflective surface, conduction dominates inter-panel heat exchange and the profiles continuously decrease as $\chi$ increases.

6.4 Non-dimensional fin heat transfer ($\Pi_1$) and fin efficiency ($\Pi_2$) as a function of actuation angle ($\phi$) for four different values of the conduction-radiation interaction parameter ($N_c$). (a) Behavior observed for an intrinsic emissivity of 0.9. The greatest variation in heat transfer is seen for fins where conduction dominates and heat transfer varies significantly over the full actuation angle range. Fin efficiency decreases as the fin extends towards an open configuration. (b) Behavior observed for an intrinsic emissivity of 0.1. Unlike the case of a nearly black fin (Figure 6.2a), the heat transfer and efficiency of reflective panels vary over a relatively small actuation angle range ($\phi \leq 60^\circ$).
6.5 (a) Turn-down ratio ($\Psi$) of a dynamic fin with four panels as a function of the conduction-radiation interaction parameter ($N_c$) for two different values of intrinsic emissivity. Black fins achieve larger possible turndown ratios than reflective fins, although this is a function of $N_c$. The inset of the figure displays the variation of turn-down ratio as a function of emissivity for $N_c = 10^{-3}$. (b) Turn-down ratio ($\Psi$) of a nearly-black, dynamic fin as a function of the number of panels ($N_P$) considering four different values of the conduction-radiation interaction parameter ($N_c$).

6.6 (a) Non-dimensional temperature ($\theta$) profiles as a function of non-dimensional panel position ($\chi$) for an intrinsic emissivity of 0.9 and a conduction-radiation interaction parameter ($N_c$) of 0.01. Results are displayed for three different values of the hinge-conductance radiation parameter ($N_\kappa$) and for increasing values of the actuation angle ($\phi$), showing the influence of thermal hinge conductance and fin deployment position on the temperature profile. When $N_c$ and $N_\kappa$ are on the same order the influence of the hinge on the temperature profile is almost negligible. A large value for $N_\kappa$ (relative to $N_c$) results in a significant temperature drop across the hinges and nearly isothermal temperatures along the panels. (b) Turn-down ratio ($\Psi$) of a nearly black dynamic fin as a function of $N_c$ and $N_\kappa$. Overall, $N_c$ has a stronger influence on the turn-down ratio than $N_\kappa$.

7.1 A portion of the External Active Thermal Control System on the International Space Station [4]. One of the single-deployment radiators utilized in the thermal control system, a series of connected panels arranged in series, is displayed in the image. Unlike the analysis in this work, heat is pumped to the radiators in this image through a pumped fluid loop.

7.2 (a) The panel subsystem, consisting of four solid aluminum panels coated in a spectrally-selective paint typical of spacecraft radiators. The panels are connected with woven copper straps. The left-most panel is connected to a solid aluminum block with embedded heaters. The inset image at the bottom right is a cross-section of the thermal hinge connecting the panels. (b) The actuation subsystem, consisting of two scissor extension mechanisms constructed from aluminum struts, are connected via steel rods with fiberglass sleeves. The entire system actuates up or down with a stepper motor. (c) The combination of the panel and actuation subsystems, forming the complete radiator.

7.3 To determine the hinge conductance, three thermocouples are placed on panel 2 and panel 3 at a distance 2 cm apart. A linear regression derived from these measured temperatures gives an estimate of the temperature in the panels immediately adjacent to the hinge, $T_{2,h}$ and $T_{3,h}$. Likewise, the derivative of the linear regressions is used with Fourier’s law to find the heat flux at the center of each panel. The ratio of the average heat flux value to the difference between $T_{2,h}$ and $T_{3,h}$ gives the hinge conductance as a function of hinge temperature.
7.4 (a) Temperature of the protected component \( (T_b) \) as a function of radiator angle for a radiator with variable geometry. Radiative cooling power of the actuated radiator \( (q_{rad}) \) as a function of the radiator angle is also reported using the right axis. Uncertainty of each temperature measurement is not depicted but is \( \pm 0.44 \, \text{K} \) for each data point. (b) Temperature of the protected component \( (T_b) \) as a function of radiator cooling power \( (q_{rad}) \) for the actuated \( (\phi = \text{Variable}) \) and stationary \( (\phi = 147^\circ) \) tests. In the actuated test, the radiator position varied as a function of radiator cooling power. In the stationary test, the radiator was fully extended \( (\phi = 147^\circ) \) and the component temperature was allowed to decrease as the radiator cooling power decreased. (c) Temperature of the radiator as a function of position along the radiator for three radiator angle positions, where \( x = 0 \) corresponds to the center of the protected component. Data points at 10, 22, 35, and 48 cm correspond to the center of panels 1 – 4, respectively.

7.5 (a) Radiator cooling power as a function of radiator angle as measured experimentally and calculated via the numerical model (Chapter 6). The shaded area represents the region of uncertainty associated with the numerical results. Experimental results consistently fall within the error bounds of the numerical model, with a largest relative error (relative to experimental results) of 2.6\%. (b) The mid-point temperatures of panels 1, 2, and 4 as a function of radiator angle as determined experimentally and as calculated by the numerical model. The largest relative error between numerical and experimental results is 1.2\%.
CHAPTER 1. INTRODUCTION

1.1 Motivation

Control of radiation heat transfer is limited by the static nature of the properties that govern thermal radiation. As found in Equation 1.1, emissivity ($\varepsilon$), emitting surface area ($A$), and temperature ($T$) are the three variables available to an engineer for tailoring the rate of thermal emission from a surface. The irradiation incident on a surface depends on environmental factors and therefore cannot always be controlled. Therefore, the rate at which radiative energy is absorbed into a surface may be manipulated through control of surface absorptivity or surface geometry. For applications with static thermal environments, or environments that do not change with respect to time, it is possible to identify single values for each variable whereby the system operates ideally. However, when the thermal environment or system heat transfer requirements change in real time, a single value for each variable does not achieve ideal heat transfer behavior for all points in time.

$$q_{\text{rad}} = \varepsilon A \sigma T^4$$

(1.1)

As an example, consider a spacecraft in Low Earth Orbit (LEO). On this spacecraft, specialized surfaces (radiators) emit heat originating from electronics or from solar/Earth albedo irradiation into deep space via thermal radiation. The quantity of waste heat that must be rejected varies in real time as a result of onboard power usage fluctuations and variations in environmental irradiation [5]. However, the emissivity and surface area of the radiator are fixed values, and radiators are designed to function within a narrow band of temperature values [5]. As such, spacecraft radiator surfaces are coated with a high emissivity coating and sized large enough to reject the maximum heat load the spacecraft is expected to experience. When the spacecraft is generating significant waste heat (i.e. when subjected to full solar irradiation and maximum onboard power usage) the radiators operate as expected. However, when spacecraft waste heat drops below the
maximum value, the radiators emit an excessive amount of heat due to the large size of the radiator panels. Such behavior may cause the temperature of components aboard the spacecraft to fall below established survival limits.

To mitigate this issue, onboard heaters placed in strategic locations are activated when component temperatures drop below an established threshold to warm the spacecraft components so that they function within operating limits. This methodology is often referred to as cold-biasing and it has been utilized since the early days of space exploration. However, cold-biasing requires heaters, thermostats, additional battery capacity and additional solar panel capacity to be placed aboard the spacecraft, accounting for approximately 10% of the spacecraft’s weight budget. Likewise, the heaters consume upwards of 10% of the spacecraft’s total power budget at full power [5]. This weight and power, currently budgeted for cold-biasing, could instead be used for scientific instrumentation or telecommunications.

From the absorption perspective, variability in irradiation conditions is exhibited in the application of building thermal control. Solar irradiation on a building’s outer surface varies significantly on both annual and daily time scales. Further, use of this solar resource varies; irradiation is highest in summer months when very little thermal energy may be desired for heating but lowest in winter months when it may be desirable to absorb as much radiative energy as possible. In a similar manner to spacecraft, building thermal control systems account for variability in solar loads using actively-controlled devices, such as air conditioners, heat pumps, or combustion/electric heaters, which may require limited or non-renewable energy resources to operate.

1.2 Radiative Heat Transfer Control

For both application scenarios, control of radiation heat transfer could be used to limit the consumption of resources currently required to account for variation in radiative heat loads. For the spacecraft thermal control application, controlling the radiator emissivity, emitting surface area, or the path by which heat is delivered to the radiators in real time would provide dynamic control of radiator heat loss and reduce survival heating power while maintaining spacecraft component temperatures within designated ranges. For a building, modifying the absorptivity or absorbing surface area of the external building to meet current system heat input requirements would allow the building to maximize the utility of solar heat loads. To this end, several different approaches to
dynamic radiative heat transfer control have been explored. These technologies provide dynamic control either through variation of intrinsic radiative surface properties, manipulation of the heat rate to the radiator, or control of emitting or absorbing surface area. A brief review of these technologies, focusing on technologies intended for use in spacecraft applications, is provided in Sections 1.2.1 - 1.2.3, where technologies are compared using the turn-down ratio ($\Psi$), which is defined as the ratio of the largest possible radiative heat load absorbed or emitted divided by the smallest possible heat load absorbed or emitted.

1.2.1 Variable Emissivity Devices

The emissivity of a spacecraft’s radiator surface might be directly controlled through the use of variable emissivity coatings. These coatings are generally categorized according to their activation mechanism. Thermochromic devices demonstrate a variation in surface emissivity due to a change in material phase activated by temperature fluctuations, whereas electrochromic surfaces utilize a voltage differential to switch between reflecting and emitting states.

Vanadium Dioxide (VO$_2$) and its variants (including V$_2$O$_5$) are the best known of the thermochromic devices. First described in 1966 [6], vanadium compounds without doping experience a phase transition around 68 $^\circ$C and the spectral radiative properties of the device differ significantly in the two states, specifically in the far infrared wavelengths. Several studies [7, 8] have explored the use of VO$_2$ films for spacecraft radiators and have demonstrated turn-down ratios ($\Psi$) of 3 and 10 respectively. When combined with optical resonating cavities, these coatings can achieve a turn-down ratio of 7 [9]. Thermochromic devices remain in development, with no radiator coatings utilized in a spacecraft to date. The implementation of these devices is complicated by the fact that the performance of thermochromic materials decreases with exposure to atomic oxygen which has a significant presence in spacecraft orbits [10].

Electrochromic devices utilize a voltage differential to activate a variation in the radiative surface properties of a film. The general principle of functionality is the same for all electrochromics but the specific methodology varies between devices. Generally, a substance with an emissivity and absorptivity near unity is sandwiched between identical layers of conductive materials. The emitting substance is either a porous membrane, a gel, or a liquid which allows for particles of a reflective substance to travel through the substance. A voltage differential is created
between the two identical sandwiching layers and the reflective substance is electrochemically deposited onto one of the conductive layers (or inside of the emitting substance). If the polarity of the voltage potential is switched, the reflective substance will deposit onto the opposite conductive layer. In this manner the interstitial emitting substance is either revealed to the cold surroundings or is protected from the cold surroundings via the deposited reflective layer. Several methods of achieving this functionality have been proposed, including the use of conductive polymers [11,12], liquid electrolytes [13], amorphous WO$_3$ [14,15], polyimide films [16], and tungsten oxide [17]. Turn down ratios for electrochromic devices have been shown to vary from 1.78 to 10.4. However, this control mechanism suffers from a number of undesirable performance characteristics that limits their application, including rapid decay in vacuum conditions or dissociation [13].

1.2.2 Heat Rate Control

Radiative heat transfer might also be controlled indirectly by varying the heat rate being delivered to the radiating surface. For a spacecraft, the rate of heat generation cannot be controlled directly (hence the need for cold biasing) but the pathway from the source to the radiator can be manipulated in order to limit emission from the radiator surface. As an example, many spacecraft utilize heat pipes or flexible copper straps to transport heat from the site of heat generation to the radiator [5]. By controlling the rate of heat transfer entering the radiator, the emission from the radiator may be tailored to real-time requirements.

A large number of proposed, prototyped or currently operational technologies exist that use this control methodology. The earliest examples of heat rate control include the stagnation radiator which was first used during the Apollo program [18]. A number of heat pipes within a radiator panel are designed to freeze as the radiator temperature drops, decreasing the effective emitting area of the radiator panel. Updates to this technology are being proposed even today [19]. Variable-conductance heat pipes, sometimes called loop heat pipes, use a fluid reservoir to change the amount of working fluid within a heat pipe architecture that has been formed into a complete loop [20–22]. Likewise, pumped fluid loops have been used in the spacecraft industry [23] despite long-term reliability concerns, where the flow rate of the pump may be controlled in real time to vary the heat rate being delivered to the spacecraft radiators. Thermal switches are also used in this application, including switches with liquid droplets that vary in contact resistance [24] or
compliant mechanisms that create or eliminate a conduction bridge [25]. Heat rate control devices exhibit turn-down ratios generally in the range of 5.

Although effective, controlling the rate of heat transfer to a radiator surface is not always a possibility. As an example, pumped fluid loops are weight intensive and found mostly in large, manned spacecraft such as the International Space Station. Loop heat pipes and stagnation radiators are likewise weight intensive and design redundancy (a requirement for spacecraft subsystems) is difficult to implement.

1.2.3 Variable Geometry Devices

As can be seen from Equation 1.1, radiative heat transfer rates may also be controlled through manipulation of surface area. Devices that operate based on variations to surface area generally control the size, shape, location, and/or orientation of component geometries in real-time to increase or decrease radiative surface area, or to conceal/reveal a surface with nearly-black radiative surface properties with a moving geometry constructed from a highly reflective surface. Using these techniques, variable geometry devices exhibit control of emitting surface area and provide control of radiative surface properties, often by concealing or revealing a black surface. Actuation of device geometries is accomplished either through passive methods (e.g. bi-metallic strips, shape memory alloys, or the expansion of a working fluid) or active methods (e.g. motorized equipment).

For spacecraft thermal control applications, the louver is the most recognizable and most utilized variable-geometry technology. Using the same operational principle as venetian blinds, a series of reflective panels has been arranged in parallel and placed over a radiator surface. Each panel rotates individually about its center axis using a temperature-activated, bi-metallic coil. At cold temperatures, the panels are situated such that they exist in the same plane, forming a flat, reflective surface that conceals the radiator surface from the cold surroundings of deep space. As the temperature of the radiator increases, the bi-metallic springs rotate the panels, exposing the nearly-black radiator surface to deep space. Louvers have been used extensively for spacecraft thermal control for decades [5] and are the only heritage spacecraft hardware that uses geometry manipulation to directly control radiative heat transfer. Louver’s generally produce a turn-down ratio around 3.7 [5].
A variety of experimental technologies that utilize variations in geometry have been proposed and tested over the past several decades. Initial attempts focused on the use of flexible, deployable surfaces filled with a gaseous working fluid [26–28]. The working fluid expanded as the temperature of the system increased, causing the deployable surface to open and revealing additional radiative area covered in a black film. More recent efforts have focused on the use of rigid or semi-rigid panels. Nagano has developed a rigid aluminum panel that is attached to a surface via a hinge with inherent shape-memory alloy components [29]. One side of the aluminum panel has a reflective finish while the other side has a black finish. At cold temperatures, the panel is held such that only the reflective surface is exposed to the surroundings. As the temperature of the hinge increases, the shape memory alloy causes the hinge to open, revealing a portion of the black surface and increasing emission from the aluminum panel. Experimentation has demonstrated a turn-down ratio of 2.8 for this technology. Likewise, Bertagne has developed both compliant and rigid surfaces with shape memory components which cause the panels to expand and contract with temperature [30]. As before, the internal surface features a black coating whereas the external surface utilizes a reflective finish. The technology has the potential to achieve turn-down ratios on the order of 12. Finally, Athanasopoulos has demonstrated the use of small motifs, or repeating patterns, with intrinsic shape memory alloy components that act in parallel to create large variations in surface emissive power through small changes in motif geometry [31]. As before, this technology utilizes the modification of geometry to conceal or reveal a highly emitting surface and does not appreciably modify the emitting surface area of the device.

Variable geometry devices to date have relied almost exclusively on the revealing and concealing of a high emissivity surface in order to obtain a variation in radiative heat transfer. Absent from the literature is a device where large variations in surface area may be controlled in order to effect large changes in radiative heat transfer. Such devices might be lacking due to the inherent difficulties in developing large surface area mechanisms on spacecraft, where both the weight and volume of any component must be minimized. However, recent research in the use of active origami tessellations has demonstrated the potential for surfaces with large variations in emitting surface area to be deployed and retracted in space [32].
1.3 Origami and Heat Transfer

Origami tessellations consist of a patterned unit of geometry that may be tiled to form a surface. A tessellation contains rigid sections (panels) that are connected with creased fold lines (hinges). At the application of a force, either compression or tension, the geometry of a tessellation morphs by folding or unfolding along fold lines until the extent of the geometry is reached. Origami, therefore, provides a dynamic or static architecture by which large surfaces might be stored and deployed. Applications of origami in the field of spacecraft include deployable solar arrays [33], antennas [34], and sun-shields [35].

The intersection of origami and heat transfer provides the opportunity by which a novel, dynamic spacecraft radiator might be developed. As demonstrated in other applications, origami principles provide the framework by which control of large deployable geometries is possible. Likewise, an origami tessellation can exist in one of an infinite number of geometry states between the extremes of the tessellation geometry. A deployable spacecraft radiator that utilizes a dynamic origami architecture would feature large variations in emitting surface area, providing the potential for large variation in radiative heat transfer. The emitting surface area of the device could be positioned dynamically corresponding to current heat loss requirements, resulting in a tailored waste heat load of the spacecraft. Further, an origami-inspired, dynamic radiator would also feature radiative property control capabilities through the means of the cavity effect, a phenomenon concerning the apparent emission and apparent absorption of a cavity.

1.4 The Cavity Effect

1.4.1 Apparent Radiative Properties

Incident radiative energy streaming into a cavity opening experiences multiple reflections while inside the cavity geometry. At each reflection event, a fraction of the radiative energy, equivalent to the absortivity of the cavity walls, is absorbed. After a number of reflections the remaining energy exits the cavity. As the energy was reflected (and absorbed) over multiple instances, the amount of energy absorbed by the cavity will exceed the amount of energy that would have been absorbed by a flat surface of the same material, where only one reflection event would have occurred. This augmentation of energy absorption is termed the cavity effect. For absorption of
energy, the strength of the cavity effect is often quantified using apparent absorptivity ($\alpha_a$), or the ratio of energy absorbed by the cavity to the total energy incident on the cavity opening. The apparent absorptivity describes the radiative energy absorption potential of a pseudo-surface that is stretched across the cavity opening.

The cavity effect is also apparent for radiative emission from a cavity opening. For any isothermal cavity, radiative emission from a certain point on the internal cavity walls either exits the cavity opening directly or is absorbed or reflected by a different point on the cavity walls. Internal reflections have the effect of concentrating emission from the cavity walls onto the cavity opening, augmenting the emission from the cavity opening above that for a flat surface with the same surface area as the cavity opening and the same emissivity as the internal cavity walls. The ratio of energy emitted from a cavity opening to the emission from a flat, black surface with the same surface area as the cavity opening is called the apparent emissivity ($\varepsilon_a$), and may be used to quantify the strength of the cavity effect.

Knowledge of apparent radiative properties are helpful for modelling the emission and absorption of a cavity geometry in a larger system. With respect to this work, collapsing origami tessellations form finite cavities as panels actuate into a folded position. These cavities augment the emission from the panel openings which, in turn, influence the radiative behavior of a deployable radiator. To this end, it is necessary to understand how the cavity effect applies to cavities that are found in collapsible origami tessellations. The following subsections (1.4.2 - 1.4.4) provide (1) an overview of the techniques used to determine apparent radiative properties, (2) general results that apply to all cavity geometries, and (3) specific results for the infinite V-groove geometry which has special relevance to the cavities found within origami tessellations.

1.4.2 Analysis Methods

Analytical, numerical, and experimental approaches have been utilized in published works that quantify the cavity effect. Initial works in this topic necessarily attempted to find closed-form solutions of analytical models describing the emission from a cavity. In a review by Williams [36], four separate analytical models are reported and compared. Undoubtedly the most successful early approach is the ‘Integral Equation’ method, which was developed by Buckley [37] and is described in full detail with solution methods in Section 5.6 of Modest [38]. The basis of this approach is the
net radiation method described in Section 5.3 of [38]. Written analytically as Equation 1.2, this method involves determining the radiosity $J(x)$ at a point $x$ in the cavity by adding the emission from that point ($\varepsilon_\sigma T^4(x)$) to the reflected irradiation arriving at point $x$ from all other points on the cavity surface ($y$), where the irradiation is integrated over the full cavity ($S$). The subscripts of the differential view factor used in the integral ($dF_{x-y}$) have been switched due to the application of reciprocity. In application, Equation 1.2 is not integrated using a differential view factor but instead is integrated with respect to a differential position, where the view factor is expressed as a function of the position variable.

$$J(x) = \varepsilon_\sigma T^4 + (1 - \varepsilon) \int_S J(y) dF_{x-y} \quad (1.2)$$

Equation 1.2 is a Fredholm integral equation of the second kind [38]. A closed-form solution is rarely possible; a number of researchers have attempted solutions that utilize an approximation of the kernel, being the term $J(y)$ as found in Equation 1.2 [37, 39], estimate the integral using numerical quadrature [40] or limit the number of reflections [41]. Numerical solutions gained popularity as computational tools became available, with researchers turning to the method of successive approximation [1, 42, 43], approximating the integral as a series [44], or subdividing the cavity into small but finite segments over which the radiosity is assumed uniform (zonal approximation method) [45]. An excellent review of solutions to Equation 1.2 is provided by Sparrow [46]. It should be noted that this approach requires the cavity to be isothermal, as the temperature found in the emission term is not expressed as a function of position.

Once a solution to Equation 1.2 has been determined, the radiosity may be used to calculate the total radiative heat loss from the cavity opening. Dividing the radiosity of the cavity opening by emission from a flat black surface, equivalent in size and temperature to the cavity surfaces, to give the cavity apparent emissivity.

Even as numerical solutions to Equation 1.2 became prevalent, another numerical approach was found using Monte Carlo ray tracing. Ono [47] and later Prokhorov [48] developed algorithms that traced packets of rays geometrically as they were emitted and reflected inside a defined cavity geometry. Random numbers were compared with the cavity material’s absorptivity or reflectivity to determine if rays were absorbed or reflected upon impact with cavity walls, and random numbers
were likewise used to generate a new direction for reflected ray packets upon impact and reflection. Ray tracing results were used to approximate the view factors associated with Equation 1.2 and the equation can then be solved with this new information [49]. A thorough review of Monte Carlo ray tracing papers (and other analytical approaches) is provided by Prokhorov [50].

A review of experimental methods used to determine apparent radiative properties has not been completed, although a very brief summary of experimental studies is given by Bedford [51]. Generally, experimental approaches include measuring directional radiosity from a diffusely reflecting cavity opening when the cavity is (1) either diffusely illuminated via an integrating sphere and a blackbody emitter or (2) heated to a temperature well above ambient conditions. Approaches usually differ in the methods used to measure the radiation, including the use of spectrophotometers [52], photomultiplier tubes [53], or Golay cells [53]. Experimental results agree fairly well with analytical or numerical methods, with reported average errors of 0.7% for a diffuse model [54]. For specular surfaces, experimental results and analytical/numerical methods show greater disagreement, with one researcher reporting experimental values as much as three times greater than analytical values [54].

1.4.3 General Behavior

The cavity effect exhibits standard behaviors that apply to all cavity shapes. To illustrate these general behaviors the V-groove will be used as an example. Generally, apparent radiative properties are proportional to the aspect ratio of a cavity. A V-groove exists as a flat surface when the cavity angle ($\phi$ as depicted in Figure 1.1) is a value of 180° and the apparent radiative properties adopt the value of the intrinsic radiative properties. As $\phi$ collapses from fully open (180°) to fully closed (0°), the aspect ratio of the panel length to the V-groove opening width steadily increases and the apparent radiative properties likewise increase. For a cavity with specularly reflecting surfaces, the apparent radiative properties will approach a value of unity as the aspect ratio approaches an infinite value ($\phi = 0^\circ$) irregardless of the intrinsic radiative properties of the cavity surfaces. Diffusely reflecting cavities do not achieve a value of unity at an infinite aspect ratio but the apparent radiative properties still increase above the intrinsic properties of the cavity surface. Further, cavities with highly reflective surfaces experience a greater relative increase in apparent radiative properties than highly absorbing cavity surfaces [1].
The apparent radiative properties of a cavity are a function of several variables, including the geometry of the cavity (shape and dimensions), intrinsic radiative surface properties of the cavity material, the nature of reflection from the cavity surfaces (specular or diffuse) and, for apparent absorptivity, the incident direction of irradiation streaming into the cavity opening. Most engineering models developed to predict apparent radiative properties require cavity geometry and intrinsic radiative properties as inputs. Generally, separate models are developed for cavities with specular or diffuse reflection, although some models account for a combination of the two reflection modes [55]. Further, Ohwada’s proof stipulates that the apparent emissivity of a cavity is equivalent to the apparent absorptivity when diffusely irradiated, regardless of reflection type (specular or diffuse) [3]. For the case of apparent absorptivity, separate models are developed for cavities experiencing diffuse or collimated irradiation. Collimated irradiation may be further subdivided into two categories, full-illumination and partial-illumination (see Figure 1.1). For the case of full-illumination, the irradiation incidence angle is such that all surfaces of the cavity experience irradiation. Partial-illumination occurs when irradiation only falls on a portion of the internal cavity geometry due to shading.
1.4.4 Infinite V-groove Geometry Literature Review and Results

A number of researchers have studied the cavity effect, specifically for the V-groove geometry. Nearly all of these studies were completed in the decade between 1960 and 1970 and utilize spacecraft heat transfer as a common motivation. Sparrow and Lin published the most complete work [1], producing four separate models that describe apparent absorptivity for each combination of reflection (specular or diffuse) and irradiation modes (diffuse or collimated). For diffuse reflection, both models rely on numerical solution of the integral equation given in Equation 1.2, whereas specular reflection models utilize trigonometry and basic ray tracing techniques, leading to closed-form solutions. For the case of specular reflection with collimated irradiation, Sparrow and Lin only report the model that corresponds to full illumination. These models were informed partially by an earlier study by Sparrow [56] and preceded by an approximation of the integral equation by Daws [40] via a relaxation method which was shown to be inaccurate near the V-groove apex. A number of additional studies on the V-groove quickly followed, including Kelly [57] who used a simple substitution to correct Daws’ model at the V-groove apex. Psarouthakis developed a model that assumed uniform radiosity across a V-groove panel and found good agreement with experimental results [58]. Treuenfels, in the same year, applied the Gouffe cavity approximation [59] (again assuming uniform cavity radiosity) to a V-groove cavity. Within the same decade, Perlmutter [60], Zipin [61, 62] and then Black [53, 63] investigated the directional properties of emission and absorption of V-groove cavities. Perlmutter was the first to suggest and study the use of V-grooves for concentration of emitted energy into certain directions. Zipin studied the directionality specific to specular V-grooves, using a geometric approach to determine the direction of travel for a ray exiting a cavity given an initial angle of incidence with respect to the cavity opening. Black continued the work of Perlmutter, truncating the V-groove and using multiple sets in parallel in order to maximize emission normal to the cavity opening and limit absorption from incident irradiation at large angles of incidence. Black used an approach similar to Zipin but was able to compare results with experimental data, showing excellent agreement. Masuda [64], a decade later, added a fin to the truncated base of the V-groove, further increasing the directional characteristics of the V-groove. Finally, Prokhorov [65] implemented Monte Carlo ray tracing to determine the apparent radiative properties for V-grooves arranged in a radial pattern, concluding that non-isothermal conditions have a more significant impact on results than previously predicted.
1.5 Summary and Organization

In review, control of radiative heat transfer is possible through manipulation of emitting surface area, the rate at which heat is delivered to the surface, or radiative surface properties or any combination of these individual methods. Previous work has focused on the use of thin films to control radiative surface properties, thermal switches to control the delivery of heat to radiative surfaces, or the use of deployable systems that utilize variations in geometry to conceal or reveal a black surface. As such, devices that utilize both variations in radiative surface properties and significant variations in emitting surface area have not been explored in depth. The use of origami tessellations provide the possibility of radiative heat transfer control through manipulation of both emitting surface area and apparent radiative surface properties in real time, where variations in the aspect ratios of inherent V-groove structures achieves variations in apparent radiative properties via the cavity effect. A review of cavity effect literature reveals that the phenomenon is based on the interreflections of radiative energy within a cavity geometry, and it is possible to increase apparent radiative properties from intrinsic values to unity even for highly reflective surfaces. Models predicting the apparent radiative properties specific to the V-groove have been developed by several researchers.

This introduction has motivated the work that is contained within this dissertation, which documents efforts to study the basic relationship between surface area, apparent radiative properties and heat transfer for the accordion fold origami tessellation. First, in Chapter 2, the increase of apparent radiative properties for an actuated metal shim stock is validated experimentally. This work then seeks to refine existing models and develop new models that describe the cavity effect for a V-groove surface. This is done for a diffusely reflecting cavity in Chapter 3 and for a specularly-reflecting cavity in Chapter 4, building on previous work by Sparrow [1]. Chapters 2 and 3 focus on determining total, hemispherical apparent radiative properties and do not consider spectral effects. With the apparent radiative properties quantified and enabled for numerical calculation, the influence of both a changing surface area and apparent radiative properties is explored for an isothermal accordion tessellation in Chapter 5. An isothermal surface is first explored in order to demonstrate the potential of an actuating origami surface. With the heat transfer for an isothermal surface quantified, the collective results are then used in a prediction to determine the heat transfer for an actively-controlled deployable radiative fin in Chapter 6. In Chapter 7, results
are provided for the capstone experiment of this work, where an origami spacecraft radiator prototype is used for thermal control in a simulated spacecraft scenario. Finally, Chapter 8 summarizes important conclusions from each chapter and discusses future work.
CHAPTER 2. DYNAMIC CONTROL OF RADIATIVE SURFACE PROPERTIES WITH ORIGAMI-INSPIRED DESIGN

This chapter is published in the Journal of Heat Transfer [66]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

2.1 Contributing Authors and Affiliations

Rydge B. Mulford, Matthew R. Jones, Brian D. Iverson
Department of Mechanical Engineering, Brigham Young University, Provo, Utah, 84602

2.2 Abstract

Thermal management systems for space equipment commonly use static solutions that do not adapt to environmental changes. Dynamic control of radiative surface properties is one way to respond to environmental changes and to increase the capabilities of spacecraft thermal management systems. This paper documents an investigation of the extent to which origami-inspired surfaces may be used to control the apparent absorptivity of a reflective material. Models relating the apparent absorptivity of a radiation shield to time-dependent surface temperatures are presented. Results show that the apparent absorptivity increases with increasing fold density and indicate that origami-inspired designs may be used to control the apparent radiative properties of surfaces in thermal management systems.

2.3 Nomenclature

\[ A \text{ area (m}^2\text{)} \]
\[ C \text{ specific heat (J kg}^{-1}\text{K}^{-1}\text{)} \]
\[ CV \text{ control volume} \]
\[ D \text{ width of cavity opening (m)} \]
$f(\phi, t)$ simplifying term used in integrating factor

$G$ irradiation (W m$^{-2}$)

$h$ convection heat transfer coefficient (W m$^{-2}$ K$^{-1}$)

$h_r$ linearized radiation heat transfer coefficient (W m$^{-2}$ K$^{-1}$)

$k$ thermal conductivity (W m$^{-1}$ K$^{-1}$)

$L$ length of cavity (m)

$q$ heat rate (W)

$S$ conduction shape factor (m)

$t$ time (s)

$T$ temperature (°C or K)

$U$ overall heat transfer coefficient (W m$^{-2}$ K$^{-1}$)

$w$ thickness of the thin-foil (m)

$\alpha$ intrinsic absorptivity

$\alpha_a$ apparent absorptivity

$\varepsilon$ intrinsic emissivity

$\eta$ combined parameter (s$^{-1}$)

$\mu$ integrating factor

$\rho$ density

$\rho_r$ reflectivity

$\sigma$ Stefan Boltzmann constant (W m$^{-2}$ K$^{-4}$)

$\tau$ combined heat transfer coefficient time constant (s)

$\phi$ V-groove cavity angle

$\theta$ temperature difference (°C or K)

**Subscripts**

[ ]$_{cond}$ conduction

[ ]$_{conv}$ convection

[ ]$_{rad}$ radiation

[ ]$_{SS}$ steady-state
2.4 Introduction

Dynamic control of radiative surface properties is highly desirable when designing systems that operate in environments where radiation is the dominant mode of heat transfer and radiative heat loads vary significantly. Such is the case for spacecraft in geosynchronous orbit or the exterior surfaces of terrestrial structures. However, static, intrinsic radiative surface properties do not adapt to changing thermal environments, which results in less-than-ideal operation for a significant fraction of a component’s lifetime. Dynamic control of radiative surface properties would provide the ability to adapt surface behavior to the changing radiative environment.

Variation in the radiative environment occurs in several terrestrial and extraterrestrial applications. As an illustration of adaptive radiation control, consider a satellite in geosynchronous orbit [67]. Satellite surfaces often exhibit a radiative surface property spectral distribution ideal for minimizing the net rate of heat transfer to a satellite when solar irradiation is present [5]. However, these static surfaces transfer heat to deep space when the satellite is shaded by the earth from the sun, cooling the satellite to unacceptable temperatures. Therefore, heaters must be used to prevent the satellite temperature from decreasing to unacceptable levels [5, 68, 69]. The ability to vary the radiative surface properties of these radiators would allow optimized performance for varying conditions and potentially eliminate the need for heavy, power-consuming thermal management solutions for spacecraft [69]. Additional applications that would benefit from dynamic surface behavior include architectural exterior surfaces [70], cloaking of IR signatures [71], and solar energy applications [72, 73].

Various technologies have been investigated in order to vary radiative surface properties [68, 74, 75]. Specifically, the use of surface coatings and thin films [14], electrowetting [24], and electrostatic actuation [25] has been explored. Louvers have also been used for thermal control but macro versions are generally bulky and not suitable for small satellites [5]. Electrochromic surfaces are a promising technology that can vary their emissivity through a wide range by application of a small voltage [13]. These devices, however, require time to adjust to changing environments and exhibit wide fluctuation in their spectral emissivity [76]. Thermochromic materials exhibit a change in emissivity with surface temperature. For these materials, no electrical or mechanical actuation is required, however, the radiative surface properties are entirely dependent on the surface temperature making it difficult for use in thermal management [77]. Geometrical modifications
have also been considered, including the incorporation of microcolumn arrays to increase absorptive properties [78] and radiator plates with specialized fractal geometries to increase emissive properties [79].

One dynamic solution yet to be considered in the literature involves the use of origami-inspired, dynamically variable, V-groove cavities. Multiple reflections within a cavity lead to increased apparent absorption and emission relative to a smooth surface of the same material. Apparent absorptivity is the ratio of irradiation absorbed by a surface to the irradiation incident on the surface [1] and can differ from a surface’s intrinsic absorptivity (absorptivity of a flat surface). This increase in apparent absorption and emission for high aspect ratio cavities has been termed the cavity effect [80]. Several cavity geometries have been investigated to quantify the extent of the cavity effect on radiative properties relative to a flat surface. Cylindrical, conical, spherical, rectangular, and V-groove cavities are among the surface topographies that have been studied theoretically [38, 80].

As an example of the cavity effect, consider the V-groove cavity shown in the inset of Figure 2.1a. As the angle ($\phi$) of the opening decreases, the aspect ratio ($L/D$) increases and the apparent absorptivity and emissivity values increase [1, 38, 80]. The apparent radiative surface properties can approach those of a black surface when the surface is comprised of high aspect ratio cavities, independent of the intrinsic surface properties. Highly reflective surfaces transition from flat-surface behavior to blacklike behavior at small cavity angles, while the transition occurs over a wider range of cavity angles for surfaces with lower intrinsic reflectivity.

Many origami-based structures are capable of utilizing the cavity effect. The Miura-ori [81] and even basic origami folds, such as the accordion fold [82, 83], may be used to create a surface topography comprised of V-groove cavities (see Fig. 2.1b). Origami has been shown to be an effective approach to controlling the motion of a compliant mechanism used for actuation and positioning [84, 85] with possible space applications including deployable solar arrays [33]. During deployment, origami structures transition from a folded to an expanded surface. A change in surface topography of this nature enables dynamic control of the apparent radiative surface properties through the cavity effect.

Origami-inspired, cavity effect surfaces have the potential to significantly affect thermal control when radiative heat transfer is the dominant mechanism. Specifically, all possible apparent
absorptivity or emissivity values are obtainable between the material’s intrinsic value and unity. Further, the desired apparent absorptivity or emissivity condition can be achieved rapidly. Finally, surface degradation due to prolonged exposure may be accommodated simply by changing the fold density, which would extend the operating lifetime of a system. Active control of surface properties by topography manipulation enables the flexibility needed to respond to dynamic changes in the operating conditions.

The relationship between surface geometry and apparent absorptivity for origami-inspired surfaces other than accordion folds does not exist in the literature. The V-groove surface has been studied analytically [1,40,53,61,64], allowing for the relationship between fold angle and apparent absorptivity to be characterized (see sample surface in Figure 2.1b). However, this same research cannot be used to characterize the Miura-ori fold [81], or other tessellated surfaces that are not comprised of 2D topographies. The objective of the work presented here is to demonstrate that the total, apparent absorptivity of a V-groove, origami-inspired surface can be controlled by varying its fold density and that experimental techniques may be used to accurately quantify the total, apparent absorptivity of a V-groove surface when irradiated by collimated, blackbody radiation. V-groove surfaces are the focus of this study because theoretical models are available for comparison. However, the methods described enable the determination of apparent absorptivity for any origami-
inspired surface. As the variation in total apparent absorptivity with cavity angle is the topic of interest, the spectral nature of the apparent absorptivity will not be pursued in this study.

First, a thermal model for determining the apparent absorptivity of a V-groove structure is presented. This model utilizes experimentally obtained temperatures and overall heat loss coefficients to calculate the apparent absorptivity. The experimental procedures used to obtain the temperatures and heat loss coefficients are described, and data are presented for a flat surface and folded surfaces with five different cavity angles. The data are used in conjunction with the thermal model to calculate the apparent absorptivity for all six data sets. Calculated apparent absorptivities for flat surfaces are compared with measurements obtained using an emissometer. Calculated apparent absorptivities for an accordion fold geometry are compared with Sparrow’s V-groove analysis [1] in order to validate the experimental method.

2.5 Thermal Models

Consider a surface that is uniformly irradiated by a blackbody source in quiescent air and isothermal surroundings. As illustrated in Figure 2.2, the irradiated portion of the surface loses heat to the surroundings by convection and radiation and to the non-irradiated portion of the surface by conduction.

2.5.1 General Approach

Applying an energy balance for the system illustrated in Fig. 2.2 gives

\[ \alpha_d GA - (q_{\text{cond}} + wq_{\text{rad}} + 2q_{\text{conv}})_{\text{loss}} = mC \frac{dT}{dt} \]  (2.1)

where \( \alpha_d \) is the apparent absorptivity, \( G \) is the irradiation, \( m = \rho Aw \) is the mass in the control volume, and \( C \) is the specific heat. Since the surface is thin and has a high thermal conductivity, the illuminated portion of the surface is approximately isothermal. Radiative heat exchange with the surroundings is represented as shown in Equation 2.2. Assuming an average convective heat transfer coefficient (Equation 2.3) and modeling the conduction loss using a shape factor (Equation 2.4), the heat loss terms may be grouped as shown in Equation 2.5.
\[ q_{\text{rad}} = \varepsilon \sigma A \left[ T(t)^4 - T_\infty^4 \right] \left[ T(t)^2 + T_\infty^2 \right] \left[ T(t) + T_\infty \right] \quad (2.2) \]

\[ q_{\text{conv}} = \tilde{h} A (T(t) - T_\infty) \quad (2.3) \]

\[ q_{\text{cond}} = S k (T(t) - T_\infty) \quad (2.4) \]

\[ q_{\text{loss}}(t) = \left[ \frac{S k}{A} + 2\tilde{h} + 2\varepsilon \sigma \left[ (T(t) + T_\infty) \left( T(t)^2 + T_\infty^2 \right) \right] \right] A \left[ T(t) - T_\infty \right] = U(t) A \theta(t) \quad (2.5) \]

Here, \( U(t) \) is an overall heat transfer coefficient associated with the losses and \( h(t) \) is the relative temperature difference between the surface and the surroundings. Note that both the radiative and convective heat transfer coefficients depend on the time-dependent temperature; therefore, \( U(t) \) also varies non-linearly with time and temperature. For applications where convection may be neglected, \( U(t) \) can easily be modified to reflect this assumption.

Figure 2.2: Schematic of the control volume and energy terms used to model the relationship between the measured time-dependent, surface temperatures, and the apparent absorptivity of the surface.

The overall heat transfer coefficient may be used to validate the assumption that the temperature of the surface within the irradiated control volume is isothermal. The Biot number (\( Bi = U_{SS} L_c / k \)) was calculated using the steady-state, overall heat transfer coefficient with \( L_c \).
equal to the surface thickness. Determination of the steady state, overall heat transfer coefficient is described in subsection 2.5.2. This resulted in $Bi << 0.1$, validating the lumped capacitance assumption for the thin metallic foils used in this work.

The irradiated surface can be considered flat or folded (Figure 2.2). As the fold density increases, the cavity angle ($\phi$) decreases and the mass of the surface within the control volume increases, for a fixed control volume size. The dependence of mass in the control volume on the cavity angle may be modeled using the ratio of volumes for a flat surface divided by that for a folded surface in the same control volume (see Figure 2.2). This ratio is provided in Equation 2.6 for the simple accordion fold. Each origami-inspired surface geometry will yield different mass compensation factors

$$\frac{V_{\text{flat}}}{V_{\text{folded}}} = \frac{A_{\text{flat}}}{A_{\text{folded}}} = \frac{1}{\sin(\phi/2)}$$ \hspace{1cm} (2.6)

The volume ratio (mass compensation factor) is unity for a flat surface and is greater than unity for folded surfaces, as expected. Inclusion of this ratio with the mass term on the right side of Equation 2.1 accounts for the changing mass in the control volume for non-flat surfaces.

To further simplify the general energy balance, the material properties and mass compensation factor were combined into a single term, $\eta(\phi) = U_{SS} \sin(\phi/2)/\rho w C$. The final, generalized expression for the scenario given in Figure 2.2 is shown in the below equation

$$\frac{d\theta}{dt} + \left[ \frac{\eta(\phi) U(t)}{U_{SS}} \right] \theta(t) = \frac{\alpha_a(\phi) \eta(\phi) G}{U_{SS}}$$ \hspace{1cm} (2.7)

### 2.5.2 Apparent Absorptivity

Ultimately, it is desirable to obtain a relationship between the apparent absorptivity of a folded surface and the cavity angle of the surface. Consider the steady-state operation of an irradiated origami surface. For this condition, Equation 2.7 simplifies to Equation 2.8, where $\theta_{SS}$ is the temperature difference between the surface and surroundings at steady-state. In order to calculate the apparent absorptivity at steady-state, the overall heat transfer coefficient as well as the resulting temperature difference would be required (assuming a known irradiation condition, $G$). While $U$ could be modeled with significant simplifying assumptions, it is preferential to measure $U$
directly for the given set of conditions. However, it is not possible to measure $U$ when the surface is irradiated, as this would require the unknown apparent absorptivity. If we consider transient cooling of the surface without irradiation, $U$ can be determined experimentally.

\[ \alpha_a(\phi) = \frac{U_{SS}(\phi) \theta_{SS}(\phi)}{G} \]  

(2.8)

Consider a transiently cooled surface at the instant irradiation from the blackbody is terminated after reaching steady conditions, resulting in the homogeneous form of Equation 2.7. For this condition, the unknown $\alpha_a$ associated with the irradiation term is absent from the energy balance and $U(t)$ can be found as a function of measured surface temperature and material properties as follows:

\[ U(t) = -\frac{\rho w C}{\sin(\phi/2)} \frac{1}{\theta(t)} \frac{d\theta}{dt} \]  

(2.9)

The temperature data for a transiently cooled surface can be obtained by performing experiments with alternate heating (irradiation) and cooling to near ambient conditions. Further, $U$ obtained for a cooling surface can also be used as the overall heat transfer coefficient for an irradiated surface if $U$ can be expressed as a function of the temperature difference, $\Delta T$. The temperature difference of the cooling curve was plotted with respect to time, giving $\Delta T(t)$. This curve was then used to transform $U(t)$ into $U(\Delta T(t))$ such that an empirically obtained $U$ can be used in the calculation of the apparent absorptivity when the surface is irradiated. This allowed the irradiated $U_{SS}$ value to be evaluated from transient cooling temperature data.

Cyclical heating also provides the opportunity to develop additional methods for calculating $\alpha_a(\phi)$ in addition to the steady-state approach described above. Since obtaining $U$ requires measuring $\theta(t)$, we can use these temperature measurements to estimate the apparent absorptivity based on the time-dependent energy balance, Equation 2.7. This equation (combined with known thermal capacitance, sample thickness, irradiation condition, and time-dependent, overall heat transfer coefficient) can be solved for the apparent absorptivity as a function of cavity angle using the following methods:
1. **Integrating Factor Method:** This approach solves the first-order differential equation using an integrating factor to obtain apparent absorptivity in terms of the time dependent surface temperature (which can be obtained experimentally).

2. **Direct Method:** This approach solves for the apparent absorptivity directly from Equation 2.7 in terms of the surface temperature and its time derivative, both of which can be obtained from time-dependent temperature measurements.

Each approach has approximations associated with them. For example, the integrating factor method requires an assumption of the functional form of \( U(t) \) and the direct method will require a derivative of the temperature data curve fit. However, agreement among the three approaches (including steady-state) would provide support for the validity of each. The three solution methods above each require known surface temperatures, which can be obtained experimentally. Thus, the apparent absorptivity solution becomes an inverse problem from known temperatures. Solving for the apparent absorptivity under steady conditions provides the avenue to verify with steady, V-groove analyses in the literature. These steady-state approaches can then be used to verify that the unsteady inverse problems converge to the correct value at long times.

**Approach 1: Integrating Factor Method.** The integrating factor method was used to solve the first-order, non-homogeneous differential Equation 2.7. This method required the use of an integrating factor, as given in the below equation

\[
\mu = \exp \left( \int \frac{\eta(\phi)}{U_{SS}} U(t) dt \right) \tag{2.10}
\]

To find a closed-form solution of the integral given in Equation 2.10, an expression for \( U(t) \) was developed. When a surface that experiences thermal cycling is suddenly irradiated, the surface temperature increases monotonically and asymptotes toward steady-state conditions. The temperature difference between steady and transient values decreases exponentially with increasing time. As \( U(t) \) is related directly to the temperature difference, the form of \( U(t) \) can be described with a decaying exponential. A model of this heating process and the associated \( U(t) \) are shown in Equation 2.11, where \( U_{SS} \) is the overall heat transfer coefficient once the surface has reached a steady, maximum temperature and \( U_o \) is an initial overall heat transfer coefficient offset due to an initial
temperature difference. The $\tau$ term is a time constant used to adjust the shape of the exponential curve to experimental data.

\[ U(t) = (U_{SS} - U_o) \left[ 1 - \exp \left( -\frac{t}{\tau} \right) \right] + U_o \quad (2.11) \]

Using Equation 2.11, the integrating factor was determined. The final form of the integrating factor is given in the below equation

\[ \mu = \exp \left( \eta \tau \exp \left[ -\frac{t}{\tau} \left( 1 - \frac{U_o}{U_{SS}} \right) \right] \right) \exp(\eta t) = f(\phi, t) \exp(\eta t) \quad (2.12) \]

where $f(\phi, t)$ is the first exponential term found in Equation 2.12. The integrating factor was then multiplied by both sides of Equation 2.7, resulting in the following, after simplification:

\[ f(\phi, t) \exp(\eta t) \theta = \frac{\alpha_o \eta G}{U_{SS}} \int_0^t f(\phi, t) \exp(\eta t) dt \quad (2.13) \]

To evaluate the above expression, a value for $s$ was determined by obtaining $U(t)$ for a heated surface and V-groove angle $\phi \cong 11^\circ$ ($L/D = 5$) using the approach described above. This fold density was selected because it was the largest fold density tested and would therefore provide the largest values for $U(t)$ and $\tau$. The empirically based curve for $U(t)$ was fitted by the form given in Equation 2.11 to find that $\tau = 0.9788$ s for $\phi \cong 11^\circ$. This $\tau$ value causes the term $f(\phi, t) = \exp[\eta(\phi)U_{SS}\tau \exp(-t/\tau)]$ to have a maximum value of 1.16 and a minimum value of 1.0. As such, $f(\phi, t)$ was assumed constant over time, allowing it to be factored out of the integral in Equation 2.13 and removed from both sides of the equation, greatly simplifying the form of Equation 2.13.

As will be seen in section 2.7, this approximation causes the final apparent absorptivity value to vary slightly with time, beginning with an initial offset and converging to the steady-state value. However, a closed-form solution of Equation 2.13 was not possible without this approximation. The assumption of a constant $f(\phi, t)$ with time for small cavity angles ($\phi \leq 11^\circ$) must be confirmed before utilizing the integrating factor method as it may exhibit greater variation than that observed for the cavity angle range of this work. Equation 2.13 was solved for $\theta(t)$, where $\theta_o$ is the initial temperature difference relative to ambient.
\[ \theta(t) = \frac{\alpha_G G}{U_{SS}} \left[ 1 + \exp(-\eta(\phi)t) \left( \frac{\theta_o U_{SS}}{\alpha_G} - 1 \right) \right] \]  

(2.14)

Finally, Equation 2.14 was rearranged to calculate the apparent absorptivity

\[ \alpha_a = \frac{U_{SS}}{G} \left[ \theta - \exp(-\eta(\phi)t)\theta_o \right] \frac{1}{1 - \exp(-\eta(\phi)t)} \]  

(2.15)

**Approach 2: Direct Method.** A direct method of solving for the apparent absorptivity in terms of experimental data, material properties, and the time derivative of temperature (which can be obtained from a curve fit of experimental data) was also developed. Rearranging Equation 2.7 directly, the resulting expression for apparent absorptivity is

\[ \alpha_a(t) = \frac{U_{SS}}{G \eta(\phi)} \frac{d\theta}{dt} + \frac{U(t)}{G} \theta(t) \]  

(2.16)

The integrating factor and direct methods (Equations 2.15 and 2.16) are expected to converge to the steady-state absorptivity value given by Equation 2.8. As such, steady-state results will be used for confirmation of the inverse model approaches and additional validation with the literature.

In addition to the geometric and material properties, the three apparent absorptivity solution approaches are a function of \( G, \theta(t), \) and \( U(t) \). For a known irradiation condition, empirically based values for \( \theta(t) \) and \( U(t) \) are necessary to calculate apparent absorptivity with any of the three developed methods. The following section, 2.6, outlines the methods for the experimentally obtained temperatures and overall heat transfer coefficient for various conditions.

**2.6 Experimental Setup**

To evaluate the derived expressions for apparent absorptivity provided by the thermal models, experimental measurements for the temperature of cyclically irradiated surfaces were performed over a range of fold densities (cavity angles). The following subsections, 2.6.1, 2.6.2, and 2.6.3, outline the experimental conditions for measurement of (1) the steady-state and transient temperature measurements of folded surfaces and (2) the radiative heat flux provided by the black-
body. An uncertainty analysis was also performed on the absorptivity models using the uncertainty and least count values associated with measurements performed in this section.

2.6.1 Transient Temperature Measurement

Change in the apparent radiative properties of a surface was demonstrated through experimentation with a folded thin-foil heated by a blackbody cavity. A sheet of aluminum shim stock (alloy 1145) of thickness 25.4 µm was folded into an accordion pattern with a pitch of 2.54 cm (see Figure 2.1b). The folded surface was constrained within a test fixture that allowed the cavity angle to be varied from 180 deg (flat) to \( \phi \approx 11^\circ \) \( (L/D \approx 5) \), without removing the surface from the fixture. Testing at various cavity angles was performed on a single surface to indicate the ability to control absorption through topographical changes governed by collapsing and expanding origami folds. The center of the surface was positioned concentric with the opening of a blackbody source (Land R1200P), 15.4 cm away from the aperture. Two K-type thermocouples (30 gauge) were placed in small indentations on the backside of one fold (see Figure 2.3) and secured by means of thermal epoxy (Duralco 132). Thermocouples were placed immediately adjacent to the two folds and midway between the two peaks at the same vertical location. A third thermocouple monitored the room temperature throughout the testing.

The surface was alternately exposed to and then shielded from the cavity irradiation by opening and closing a shutter. The blackbody emitter, set to 1000 °C, was allowed to reach steady-state operation before beginning the heating and cooling cycles. A two-color pyrometer (Omegascope OS3750) was used to confirm the temperature of the blackbody cavity. At a cavity set point of 1000 °C, the pyrometer indicated a cavity temperature of 1000 °C ± 1 °C. An insulated shutter was placed in front of the blackbody aperture and actuated with a piston linear actuator (Figure 2.3). This insulated shutter acted to shield the blackbody radiation during cooling cycles. A LABVIEW program was used to control the piston linear actuator through use of an NI 9481 SPST digital output module attached to a power supply at 15 V. When activated, this digital output module controlled a five-port (SMC VF3320), solenoid-actuated air valve, pressurizing the piston cylinder and causing the shutter to open and the surface to be irradiated. When deactivated, the cylinder was depressurized and the shutter was moved back into place by the spring-loaded air piston. Thermocouple readings (sampled at 3 Hz) were also recorded using the LABVIEW
software. The shutter was opened for 100 s during the heating phase and closed for 80 s during the cooling phase. Thermal cycling was performed for approximately 1 hr with thermocouple data being collected continuously. Response time for the actuation was less than 0.5 s to open and close the shutter or less than 0.75% of the heating or cooling cycle times.

Figure 2.3: Schematic of the test configuration and temperature measurement for cyclic heating of folded or flat thin-foils using a blackbody cavity. The heat flux gauge used to determine the radiation flux from the blackbody was positioned in the plane where the sample is located in this schematic.

2.6.2 Steady-State Temperature Measurement

To determine the steady-state temperature of each fold density, temperature data from the transient temperature measurement were utilized. The heating cycle time of 100 s used in the transient temperature procedure was of sufficient length to achieve a temperature change of $< 0.5$ °C per minute for the largest fold density, $\phi \cong 11$ deg, which required the most time to reach steady-state conditions. As such, all fold densities achieved steady-state conditions before the end of each heating cycle, allowing the last temperature data point before the cooling cycle to be designated as the steady-state temperature.
2.6.3 Flux Measurement

A Vatell HFM-7 E/H heat flux gauge was mounted in a custom housing and attached to a three-axis optical rail system for positioning. The heat flux gauge was placed 15.4 cm away from the aperture of the blackbody cavity. The gauge was moved in 5 mm increments (X and Y) in a plane parallel to the front plane of the blackbody. Data over a circular irradiated area of $A_B = 0.002 m^2$ (radius = 2.5 cm) were averaged to determine the irradiation value for $G$ used in subsequent inverse models for the apparent radiative properties.

2.6.4 Uncertainty

Uncertainty and least count values for measurements and experimental parameters are provided in Table 2.1. The last column of Table 2.1 provides the source of the uncertainty value. To quantify the cavity angle ($\phi$) measurement uncertainty, the folded sample was assembled at a selected cavity angle. All V-grooves of the surface were measured with calipers and the actual angle for each V was calculated. The average angle of all V-grooves was compared with the nominal cavity angle to find the error of the cavity angle measurement. An uncertainty analysis was performed on the three apparent absorptivity calculation methods using the uncertainty values given in Table 2.1. The root sum square of the partial derivative multiplied by the uncertainty of each parameter was used to calculate the total uncertainty for absorptivity.

Table 2.1: Uncertainty and least count values for each measurement or parameter used in absorptivity calculations. The source of each uncertainty value is listed in the column titled “Reference.” The designation instrument least count indicates that the uncertainty value was given in the documentation provided with the instrument.

<table>
<thead>
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<th>Parameter</th>
<th>Uncertainty</th>
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<th>Source</th>
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</thead>
<tbody>
<tr>
<td>$G$</td>
<td>31.5</td>
<td>W m$^{-2}$</td>
<td>Instrument least count</td>
</tr>
<tr>
<td>$C$</td>
<td>4.5</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
<td>[56]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>6.8</td>
<td>kg m$^{-3}$</td>
<td>Resolution least count</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.5</td>
<td>K</td>
<td>Instrument least count</td>
</tr>
<tr>
<td>$w$</td>
<td>1.27</td>
<td>$\mu$m</td>
<td>Resolution least count</td>
</tr>
<tr>
<td>$t$</td>
<td>10</td>
<td>$\mu$s</td>
<td>Resolution least count</td>
</tr>
<tr>
<td>$\phi$</td>
<td>4</td>
<td>deg</td>
<td>See Section 2.6.4</td>
</tr>
</tbody>
</table>
2.7 Results

2.7.1 Flat-Surface Absorptivity

Figure 2.4a illustrates the time-dependent temperatures for the thermocouples attached to the backside of a flat surface during cyclic heating and cooling. During experimentation, the heating/cooling cycles were repeated until quasi-steady behavior was observed, as shown in Figure 2.4a. After 1000 s (designated by the solid, black vertical line), note the consistency of the maximum temperature after heating and the minimum temperature after cooling for fixed periods of time. The two thermocouples attached to the backside of the folded test section differed by a maximum of 0.3 °C with an average temperature difference of 0.10 °C along the face of the folds. This small temperature difference supports the assumption of temperature uniformity over the control volume. The repeated nature of the cyclic heating and cooling provided numerous measurements to establish repeatability.

Figure 2.4b illustrates a curve fit to the average of eight separate heating and cooling cycles after reaching quasi-steady operation for the same flat-surface data presented in Figure 2.4a. All curve fits for all fold densities yielded $R^2$ values (indicating goodness-of-fit) of greater than 0.95 and were calculated using the MATLAB exponential curve fit routines. An average cooling curve over multiple cycles was used to evaluate the overall heat transfer coefficient, $U(t)$, according to Equation 2.9. The results for $U(t)$ as a function of time during cooling are given in Figure 2.5 for a flat case and a representative folded case. This data were transformed to give $U(\Delta T(t))$, allowing the empirically obtained $U$ values to be applied to the heating portion of the cycles.

Before apparent absorptivity values can be calculated, a value for irradiation ($G$) from the blackbody radiator at 1000 °C is necessary. Heat flux measurements are illustrated in Figure 2.6 along a vertical and horizontal line with the intersection of these lines corresponding to the axis of the blackbody cavity. Measurements were taken at 5 mm increments in the horizontal and vertical axes over the 25 mm radius circular area around the blackbody axis and averaged with equal weighting to obtain a flux value of 950 W m$^{-2}$ at a distance of 15.4 cm from the blackbody cavity aperture, as shown in Fig. 6. For all apparent absorptivity calculations, 950 W m$^{-2}$ was used. It should be noted that results obtained here for apparent absorptivity using the inverse model
are specific to the flux value (950 W m$^{-2}$) used in experimentation because the spectral variation of apparent absorptivity was not incorporated into the inverse model.

After transient temperature data, $U(\Delta T(t))$ and a value for $G$ were obtained, Equations 2.8, 2.15, and 2.16 were used to calculate the apparent absorptivity of a flat surface. Results for a flat sample are given in Figure 2.7. As can be seen, the direct method (Eq. (16)) and the integrating factor method (Equation 2.15) converge to the exact steady-state value, 0.028. The integrating factor solution begins at a larger value than the steady-state approximation and exponentially decays to the steady-state value. This time-dependent behavior is due to the approximations utilized in obtaining the integrating factor solution. Regardless, we observe very good agreement among the three different approaches.

### 2.7.2 Flat-Surface Absorptivity Validation

In an effort to validate the flat-surface inverse model for absorptivity, a flat sample of the aluminum shim stock material used to create the surfaces for the testing described above was analyzed with an ET-100 Emissometer at room temperature. The sample was placed on a horizontal surface and the emissometer was placed directly in contact with the sample. An integrating sphere in the emissometer collected all reflected radiation from the surface of the test sample when ir-
radiated at a near-normal angle. The reflectance measurements were recorded over six discrete wavelength bands in the infrared region. Using the blackbody fraction, the total intrinsic absorptivity for the surface was calculated using Equation 2.17 for blackbody irradiation at 1273 K.

\[
\alpha = \sum_{i=1}^{6} F_i (1 - \rho_{ri}) \tag{2.17}
\]

Here, \( F_i \) is the blackbody fraction associated with each wavelength band over which the reflectivity was measured with the emissometer.

Table 2.2 displays the spectral hemispherical reflectance results of a flat, aluminum surface tested with the ET 100 Emissometer. The total, hemispherical absorptivity, using Equation 2.17, was calculated as 0.028 ± 0.001 (an average of three measurement tests). The steady-state model approach calculated the apparent absorptivity of a flat surface (in this case, the intrinsic absorptivity) to be 0.028 ± 0.011. These two values are in near exact agreement, within the error associated with both measurements. This agreement confirms the steady-state model approach through an independent measurement. Further, it gives confidence in the inverse models; the direct and integrating factor methods both yielded a value of 0.028, respectively, for long times, giving similar values to that found by the emissometer.
Figure 2.6: Horizontal and vertical flux distributions measured over a 25 mm radius centered about the blackbody cavity axis. The average value over this region (950 W m\(^{-2}\)) is also indicated.

Figure 2.7: Results for the three inverse solution methods for a flat, Al surface with measured surface absorptivity of \(\alpha = 0.028\). Error bounds for the steady-state method are \(\pm 0.0172\).
2.7.3 Folded Surface Absorptivity

Figure 2.8 compares heating and cooling curves for several folded, thin-foil cases. Each curve is an average of 7 or more cycles after reaching quasi-steady behavior. Five different fold conditions are shown: $\phi \approx 60^\circ$, $28^\circ$, $19^\circ$, $14^\circ$, and $11^\circ$ ($L/D \approx 1, 2, 3, 4, \text{and} 5$) in addition to an unfolded (flat) baseline case. For the flat-surface baseline, the amount of absorbed radiation increased the surface temperature by 2.25 °C relative to the minimum temperature of the cooling curve. Surfaces with $\phi \approx 60^\circ$ ($L/D \approx 1$) showed an increase of 3.30 °C. The increase becomes more pronounced for the smaller cavity angles ($\phi_2 = 28^\circ$, $\phi_3 = 19^\circ$, $\phi_4 = 14^\circ$, and $\phi_5 = 11^\circ$), despite the increased amount of material in the heated control volume. The extent of the cavity effect on the surface temperature is clearly evident.

![Figure 2.8](image)

Figure 2.8: Heating and cooling curves averaged over eight cycles for surfaces that range in cavity angle from a flat surface to a surface with $\phi \approx 11^\circ$ ($L/D \approx 5$), indicating the increase in surface temperature for the same heating condition resulting from an increased apparent absorptivity with reducing cavity angle. Temperature data were collected at a rate of 3 Hz.

The three approaches used to calculate the apparent absorptivity of a flat surface were used to also find the apparent absorptivity of folded surfaces. Figure 2.4a shows the sample temperature data obtained for a folded surface with $\phi_4 = 14^\circ$ and the $U(\Delta T(t))$ curve as calculated from this temperature data is given in Figure 2.5. Using this information, the steady-state method, direct
method, and integrating factor method approaches yielded apparent absorptivity results as presented in Figure 2.9. The results show an increase in apparent absorptivity from 0.028 for the flat surface to 0.21 for the folded surface with $\phi_4 = 14^\circ$. This represents an increase by almost one order of magnitude with smaller cavity angles resulting in even greater increases for apparent absorptivity. The steady-state results and the direct method results correlate well; the integrating factor method begins with an initial offset and converges to the steady-state value. This discrepancy is due to neglecting the time dependence of $f(\phi, t)$ and approximating as unity. Results from

Table 2.2: Three separate spectral reflectivity measurements over six discrete spectral bands for a flat Al surface using an ET-100 Emissometer. Results obtained for the total intrinsic absorptivity values (using Equation 2.17) for each data set are also included in the last column.

<table>
<thead>
<tr>
<th>Test #</th>
<th>1.5-2.0</th>
<th>2.0-3.5</th>
<th>3.0-4.0</th>
<th>4.0-5.0</th>
<th>5.0-10.5</th>
<th>10.5-21.0</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.965</td>
<td>0.969</td>
<td>0.966</td>
<td>0.977</td>
<td>0.982</td>
<td>1.005</td>
<td>0.0289</td>
</tr>
<tr>
<td>2</td>
<td>0.967</td>
<td>0.972</td>
<td>0.971</td>
<td>0.973</td>
<td>0.983</td>
<td>1.010</td>
<td>0.0266</td>
</tr>
<tr>
<td>3</td>
<td>0.965</td>
<td>0.969</td>
<td>0.973</td>
<td>0.977</td>
<td>0.980</td>
<td>0.986</td>
<td>0.0286</td>
</tr>
</tbody>
</table>
all the three methods (Equations 2.8, 2.15, and 2.16) for all tested fold densities at the last time step are given in Table 2.3. As fold density increases, the apparent absorptivity of the surface likewise increases. The direct method, steady-state method, and integrating factor method produce similar results for all fold densities.

### 2.7.4 Folded Surface Absorptivity Validation

The V-groove analysis developed by Sparrow et al. was used to validate the results of the inverse model approaches for folded surfaces [1, 86]. Assuming a spectral reflector, the apparent absorptivity can be calculated as a function of cavity angle and the intrinsic surface absorptivity, assuming the irradiation is perpendicular to the surface (see Equation 2.18). Equation 2.19 defines the percentage of a V-groove cavity wall illuminated by the $n^{th}$ reflection from the opposite wall and Equation 2.20 provides the number of reflections experienced by a ray inside the V-groove, rounded to the nearest lower integer.

\[
\alpha_a = 1 - (1 - \alpha X')(1 - \alpha)^{n-1} \tag{2.18}
\]

\[
X' = \frac{\sin \left[ \left( n - \frac{1}{2} \right) \phi \right]}{\sin \left( \frac{\phi}{2} \right)} \tag{2.19}
\]

\[
n = \left( \frac{180}{\phi} \right) + \frac{1}{2} \tag{2.20}
\]

Equation 2.8 is a closed-form solution against which the inverse modeling can be benchmarked.

The apparent absorptivity as a function of cavity angle for a surface absorptivity of 0.028 using Sparrow’s analysis is shown in Figure 2.10. The step-like nature of this curve is a result of rounding the number of reflections (Equation 2.20) to a whole number (as a non-integer number of reflections are not possible) and from calculating the portion of the wall that is illuminated (Equation 2.19). As can be seen, the apparent absorptivity of the surface approaches unity as the opening angle of the V-grooves approaches zero. Note that for a small intrinsic absorptivity (highly
reflecting, opaque surface), the drastic increase in apparent absorptivity is concentrated in the low cavity angle range whereas for higher intrinsic absorptivity, this transition to a highly absorbing surface occurs at higher cavity angles.

Table 2.3: Results from Equations 2.8, 2.15, and 2.16 for all tested fold densities. The results obtained from the last time step for the integrating factor method and the direct method are reported. The apparent absorptivity of the surface increases with decreasing cavity angle, validating the cavity effect. Absorptivity values using Sparrow’s analysis [1] are also provided.

<table>
<thead>
<tr>
<th>$L/D$ ratio</th>
<th>$\phi$ (deg)</th>
<th>Equation 2.8</th>
<th>Equation 2.15</th>
<th>Equation 2.16</th>
<th>Sparrow model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (flat)</td>
<td>180</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>1</td>
<td>53</td>
<td>0.065</td>
<td>0.066</td>
<td>0.065</td>
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<td>2</td>
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<td>0.142</td>
<td>0.145</td>
<td>0.143</td>
<td>0.165</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
<td>0.185</td>
<td>0.189</td>
<td>0.186</td>
<td>0.247</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>0.218</td>
<td>0.225</td>
<td>0.219</td>
<td>0.322</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>0.297</td>
<td>0.310</td>
<td>0.305</td>
<td>0.365</td>
</tr>
</tbody>
</table>

The steady-state method results (using Equation 2.8) are compared to the analytical solution developed by Sparrow et al. (Equation 2.18) for a cavity angle of $\phi \cong 180^\circ$, $60^\circ$, $28^\circ$, $19^\circ$, $14^\circ$, and $11^\circ$, as shown in Figure 2.10. The two models exhibit the same trend of increasing apparent absorptivity with smaller cavity angle. The disagreement between Sparrow’s V-groove model and the present inverse model increases as the cavity angle decreases, with the inverse model predicting smaller apparent absorptivity values than Sparrow’s model. Table 2.3 provides the result from all three approaches for each tested cavity angle and Sparrow’s value for each tested cavity angle. Steady-state results for varying fold density are primarily within measurement error and have an average error of approximately 18%.

Discrepancy between the inverse model results and Sparrow’s results can be attributed to several conditions. First, the cavity angle of a test surface is difficult to position accurately in the small angle range. This results in increasing uncertainty (shown in Figure 2.10) as the angle approaches zero due to the significant change in apparent absorptivity observed for small angles. Second, during heating of the folded surfaces, the temperature of the metal foil asymptotically approaches the final steady-state temperature; the values used at the end of the 100-s heating cycle may not have fully reached steady conditions, leading to a higher apparent absorptivity if left for
Figure 2.10: Apparent absorptivity as a function of cavity angle for Sparrow’s analytical V-groove model (Equations 2.18 and 2.20) and the inverse steady-state model (Equation 2.8) of this work for a folded Al thin-foil with intrinsic absorptivity of $\alpha = 0.028$. Sparrow’s model for intrinsic absorptivities of $\alpha = 0.3$ and 0.6 is also presented to show the more gradual increase toward unity of these higher intrinsic absorptivity surfaces.

longer heating times. Finally, while the fold density can be fairly accurately controlled, each fold in the array exhibited some difference in cavity angle with respect to its neighbors due to the thinness of the material. This variation results in varying cavity angle as mentioned above, as well as some variance in the amount of mass appearing in the mass compensation factor. Nevertheless, the average error between the approaches is 18% with differences primarily within measurement error.
2.8 Conclusions

Thermal modeling and experimentation have shown that the apparent absorptivity of an origami-inspired, V-grooved, surface may be controlled by varying the cavity angle. Surface temperatures exhibit higher maximum temperatures for the same incident heat flux and heating time when the fold density is high, corresponding to a higher apparent absorptivity due to the cavity effect. The inverse models developed here accurately predict the apparent absorptivity as a function of the cavity angle, intrinsic surface properties, and irradiation. The methods used to measure apparent absorptivities were verified by comparison with flat-surface emissometer measurements and classical V-groove analytical models. Future work includes investigating alternative surface materials, more elaborate origami folding patterns, and the development of methods to examine diffuse, non-conductive materials. Total surface emission and the trade-off associated with a fixed amount of material are also being explored.
CHAPTER 3. TOTAL HEMISPHERICAL APPARENT RADIATIVE PROPERTIES OF THE INFINITE V-GROOVE WITH DIFFUSE REFLECTION

This chapter is published in the Journal of Thermophysics and Heat Transfer [2]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

3.1 Contributing Authors and Affiliations

Rydge B. Mulford, Nathan S. Collins, Michael S. Farnsworth, Matthew R. Jones, and Brian D. Iverson

Department of Mechanical Engineering, Brigham Young University, Provo, Utah, 84602

3.2 Nomenclature

\( A, B, C \) V-groove surfaces for view factor relationships
\( A_p \) opening area (m\(^2\))
\( A_t \) emitting area (m\(^2\))
a length of partially illuminated surface (m)
\( D, E, G \) correlation constants
\( F_{x-y} \) view factor from surface x to y
\( N_a \) number of rays absorbed by the cavity surfaces
\( N_e \) number of rays that escape from the cavity opening
\( N_t \) total number of rays present in the simulation
n sample size
\( q_a \) absorbed heat rate (W)
\( q_e \) escaped heat rate (W)
\( q_t \) total heat rate (W)
s sample standard deviation
\( \alpha \) intrinsic absorptivity of cavity material
\( \alpha_a \) apparent absorptivity of a cavity
\( \gamma \) collimation angle of incident irradiation (rad)
\( \varepsilon \) intrinsic emissivity of cavity material
\( \varepsilon_a \) apparent emissivity of a cavity
\( \Lambda_1, \Lambda_2, \Lambda_3 \) correction functions
\( \rho_a \) apparent reflectivity of a cavity
\( \sigma \) Stefan-Boltzmann constant (W m\(^{-2}\) K\(^{-4}\))
\( \phi \) V-groove cavity angle

### 3.3 Introduction

Dynamic control of radiative surface properties enables optimization of thermal management systems for spacecraft, radiative cooling systems, and other applications [66, 87, 88]. Various methods of altering the absorption or emission from a surface have been investigated [89–91]. Use of origami-inspired tessellated surfaces to control apparent radiative surface properties is a promising technology [77, 92, 93]. Realizing the full potential of tessellated surface to dynamically control apparent radiative surface properties requires convenient methods of calculating apparent properties as a function of tessellation geometry and intrinsic radiative surface properties. This chapter focuses on the use of geometry to affect total, hemispherical properties of V-grooves comprising diffuse, gray surfaces.

When radiant thermal energy enters a cavity, multiple reflections result in greater absorption than that of an equivalent flat surface. Likewise, multiple reflections concentrate radiation emitted from the cavity walls, which increases emission from the cavity above that of an equivalent flat surface. This behavior is termed the cavity effect, and it is quantified using apparent absorptivity and apparent emissivity.

The accordion tessellation is an ideal candidate for an origami-inspired variable emissivity device. A V-groove is easy to manufacture, and its geometry is changed with simple linear actuation. Models of the apparent radiative surface properties of V-grooves were developed by Sparrow
Figure 3.1: Representations of (a) V-groove cavity dimensions and nomenclature, and (b) full or partial illumination

and Lin [1,55], as well as other models stemming from these initial publications [38,53,61–63,94]. These models are presented as nested integral equations that must be solved simultaneously.

This chapter presents a series of correlations that allow rapid computation of apparent radiative properties of isothermal, diffusely reflecting, infinite V-grooves that are exposed to diffuse and collimated irradiation. The case of specular reflection with collimated or diffuse irradiation is treated separately [95].

These correlations, which are based on extensive Monte Carlo simulations, give the apparent radiative properties as a function of cavity angle ($\phi$ as shown in Figure 3.1a), intrinsic surface properties, and the collimation angle ($\gamma$ as shown in Figure 3.1a) where applicable. Presented as series solutions, these correlations are simpler to implement than the nested integral equations reported previously. Although the periodic structure of tessellated surfaces is similar to that of a diffraction grating, the length scale of a V-groove is orders of magnitude greater than wavelengths associated with thermal radiation. Therefore, the diffraction and effects associated with near field radiative transfer are negligible [90,91,96].
3.4 Methodology

3.4.1 Monte Carlo Ray Tracing

Monte Carlo ray tracing [50, 97, 98] is a straightforward numerical method that may be used to quantify the apparent emissivity and apparent absorptivity of arbitrarily shaped cavities for diffuse reflection/emission and uniform radiative surface properties [99]. Applying the principle of conservation of energy to a cavity gives an expression for the apparent emissivity of the cavity. Because the ratio of escaped energy to emitted energy \((q_e/q_t)\) is equal to the ratio of escaped rays to emitted rays \((N_e/N_t)\), the apparent emissivity of the cavity is found through an energy balance, giving Equation 3.1:

\[
\varepsilon_a = \varepsilon \left( \frac{A_t}{A_p} \right) \left( \frac{N_e}{N_t} \right)
\]  (3.1)

To obtain apparent absorptivity, a specified radiative heat rate, which is proportional to \(N_t\), is emitted diffusely or at some collimation angle \(\gamma\) from the opening \(A_p\) into the cavity. After one or more interactions with the cavity walls, the radiant energy is either absorbed or reflected through the opening. Again, application of the conservation of energy gives an expression for the apparent absorptivity:

\[
\alpha_a = 1 - \frac{N_e}{N_t}
\]  (3.2)

To obtain the ratio \(N_e/N_t\) found in Equation 3.1 or Equation 3.2, a ray tracing program was created in Java following an algorithm as described in [100] with ray–plane collision equations found in [101]. Two scenarios were tested with this Java program to obtain the apparent emissivity/absorptivity of an infinite V-groove at a given cavity angle. First, a total of \(N_t = 300,000\) rays were emitted diffusely from the cavity walls to determine the apparent emissivity for an isothermal cavity; this apparent emissivity was equivalent to the apparent absorptivity for a diffusely irradiated cavity as given by Ohwada’s proof [3]. Second, a total of 150,000 collimated rays were emitted from the cavity opening into the cavity to determine the apparent absorptivity for collimated irradiation. The second test was performed at nine discrete values for \(\gamma\), starting at zero and increasing by increments of \(\pi/18\) rad (10°). Tests for both cases were performed over the entire range of
cavity angles $\phi$, from $\pi/180$ to $179\pi/180$ rad ($1\text{ to }179^\circ$) in increments of $\pi/180$ rad ($1^\circ$). Tests were performed for 19 different intrinsic emissivity values (from 0.05 to 0.95 in increments of 0.05). Each combination of emissivity, cavity angle, and collimation angle was tested 20 times, and the average of the 20 tests was presented as the final result. The standard error of the mean ($SE = s/\sqrt{n}$ [102]) for each set of 20 tests was used as an estimate of the error of the ray tracing result. The total number of rays was selected such that the standard error of the mean for all tests remained below a value of 0.0005.

**3.4.2 Data Correlation**

**Diffuse-Emission Apparent Emissivity Model and Diffuse-Irradiation Apparent Absorptivity Model**

A radiative heat transfer model to predict the ratio $N_e/N_t$ was developed by Psarouthakis [58] and is given in Equation 3.3, where $F$ is the view factor from one V-groove surface to the other. Because the view factor is $F = 1 - \sin(\phi/2)$, the model simplifies to Equation 3.4 [38,103]:

$$\frac{q_e}{q_t} = \frac{N_e}{N_t} = (1-F)[1 + F(1-\varepsilon) + F^2(1-\varepsilon)^2 + F^3(1-\varepsilon)^3 + \ldots + F^n(1-\varepsilon)^n]$$

$$\frac{N_e}{N_t} = \sum_{n=0}^{\infty} (1-\varepsilon)^n \left[ 1 - \sin\left(\frac{\phi}{2}\right) \right]^n \sin\left(\frac{\phi}{2}\right)$$

(3.4)

This model assumes that the radiosity from each wall is uniform. This assumption eliminates the integral equation found in Sparrow and Lin’s model [1], but it introduces some error as the radiosity varies with position [56]. A correction function $\Lambda_1$ is introduced to offset the error introduced by the constant radiosity assumption. Substituting Equation 3.4 and the correction function into Equation 3.1 gives

$$\varepsilon_a = \alpha_a = \varepsilon \Lambda_1(\varepsilon, \phi) \sum_{n=0}^{\infty} (1-\varepsilon)^n \left[ 1 - \sin\left(\frac{\phi}{2}\right) \right]^n$$

(3.5)
Fully Illuminated Apparent Absorptivity Model ($\gamma \leq \phi/2$)

The nature of collimated irradiation is such that models for two separate scenarios must be developed. In the first scenario of $\gamma \leq \phi/2$, the collimated radiation is incident on both surfaces (Figure 3.1b). In this case, the derivation of the diffuse-radiosity model is nearly identical to that of Psarouthakis [58], although an additional reflection must be included because energy is entering the cavity as opposed to being emitted from the cavity walls [Equation 3.6]:

$$\frac{N_e}{N_t} = \sum_{n=0}^{\infty} (1 - \varepsilon)^{n+1} \left[ 1 - \sin \left( \frac{\phi}{2} \right) \right]^n \sin \left( \frac{\phi}{2} \right)$$  \hspace{1cm} (3.6)

Substituting Equation 3.6 into Equation 3.2 and introducing a correction factor $\Lambda_2$ to account for nonuniform radiosity, the apparent absorptivity of a fully illuminated V-groove exposed to collimated irradiation ($\gamma \leq \phi/2$) is given by

$$\alpha_a = 1 - \Lambda_2(\alpha, \phi, \gamma) \sum_{n=0}^{\infty} (1 - \alpha)^{n+1} \left[ 1 - \sin \left( \frac{\phi}{2} \right) \right]^n \sin \left( \frac{\phi}{2} \right)$$  \hspace{1cm} (3.7)

Partially Illuminated Apparent Absorptivity Model ($\gamma > \phi/2$)

When $\gamma > \phi/2$, then collimated radiation is incident only on a portion of one side of the V-groove as illustrated in Figure 3.1b. The ratio $N_e/N_t$ may again be determined using Psarouthakis’s uniform radiosity approach [58], although partial illumination must now be considered. The number of rays that exit the cavity $N_e$ is modeled by summing the percentage of rays that exit (reflectivity multiplied by view factor from the surface to the opening) after each successive reflection. The first four terms in this summation are given in Equaiton 3.8, where the terms $a$, $A$, $B$, and $C$ are illustrated in Figure 3.1b:

$$N_e = N_t \rho F_{a-C} + N_t \rho F_{a-B \rho F_{B-C}} + N_t \rho F_{a-B \rho F_{B-A \rho F_{A-C}}} + N_t \rho F_{a-B \rho F_{B-A \rho F_{A-B \rho F_{B-C}}}} + \ldots$$  \hspace{1cm} (3.8)

By including an infinite number of internal reflections, the ratio of the number of rays escaping the cavity to the total number of rays incident on the cavity opening is given by Equa-
The view factors $F_{A-B}$ and $F_{B-A}$ appearing in Equation 3.8 are identical by symmetry. Likewise, the view factors $F_{A-C}$ and $F_{B-C}$ are identical by symmetry:

$$\frac{N_e}{N_i} = \rho F_{A-C} + \sum_{n=2}^{\infty} \rho^n F_{A-B} F_{B-C} F_{A-B}^{n-2}$$ (3.9)

The view factor $F_{a-C}$ is determined using Hottel’s crossed-strings method [103] [Equation 3.10], and $F_{a-B} = 1 - F_{a-C}$ is obtained from the summation rule:

$$F_{a-C} = \frac{1}{2} \left[ 1 + \sin \left( \frac{\phi}{2} \right) + \cos \left( \frac{\phi}{2} \right) \tan \gamma - \frac{\cos (\phi / 2)}{\cos \gamma} \right]$$ (3.10)

A third correction function $\Lambda_3$ is introduced to offset errors due to the uniform radiosity assumption. Combining Equations 3.9 and 3.10 with Equation 3.2 gives an expression for the apparent absorptivity when a V-groove is exposed to collimated irradiation with $\gamma > \phi / 2$:

$$\alpha_a = 1 - \Lambda_3(\alpha, \phi, \gamma) \left[ (1 - \alpha) F_{a-C} + \sum_{n=2}^{\infty} (1 - \alpha)^n (1 - F_{a-C}) \sin \left( \frac{\phi}{2} \right) \left( 1 - \sin \left( \frac{\phi}{2} \right) \right)^{n-2} \right]$$ (3.11)

Table 3.1: Summary of errors for isothermal, diffusely reflecting V-groove correlations

<table>
<thead>
<tr>
<th>Equation numbers</th>
<th>Average relative error (standard deviation)</th>
<th>Maximum relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5, 3.12</td>
<td>0.3% (0.1%)</td>
<td>1.3%</td>
</tr>
<tr>
<td>3.7, 3.13</td>
<td>0.2% (0.2%)</td>
<td>0.6%</td>
</tr>
<tr>
<td>3.11, 3.14 - 3.17</td>
<td>2.0% (1.2%)</td>
<td>6.0%</td>
</tr>
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3.5 Correction Function Results and Discussion

3.5.1 Correlations

Correction functions were determined as functions of the intrinsic surface property ($\alpha$ or $\varepsilon$), geometry ($\phi$), and irradiation condition ($\gamma$) by correlating Equation 3.5, 3.7, or 3.11 with ray tracing results. The Levenberg-Marquardt algorithm was used to fit exponential basis functions to
approximately 10,000 discrete data points and obtain the following expressions for \( \Lambda_1 \), \( \Lambda_2 \), and \( \Lambda_3 \). Table 3.1 provides the average relative error and the maximum relative error for each correlation presented in this chapter.

**Diffuse-Emission Apparent Emissivity and Diffuse-Irradiation Apparent Absorptivity Model**

The correction function needed to calculate the apparent emissivity or apparent absorptivity for diffuse irradiation using Equation 3.5 is given by Equation 3.12:

\[
\Lambda_1(\varepsilon, \phi) = 1 - (0.0169 - 0.1900\ln(\varepsilon))\exp(-1.4892\varepsilon^{-0.4040}\phi)
\]

(3.12)

**Collimated Irradiation Apparent Absorptivity**

The correction function needed to calculate the apparent absorptivity of a fully illuminated V-groove exposed to collimated irradiation \( (\gamma \leq \phi/2) \) using Equation 3.7 is given by Equation 3.13:

\[
\Lambda_2(\alpha, \phi) = 1 - (0.0169 - 0.1900\ln(\alpha))\exp(-1.4415\alpha^{-0.4240}\phi)
\]

(3.13)

The correction function needed to calculate the apparent absorptivity of a partially illuminated V-groove exposed to collimated irradiation \( (\gamma > \phi/2) \) using Equation 3.11 is given by Equations 3.14 - 3.17:

\[
\Lambda_3(\alpha, \phi, \gamma) = D - E \exp(G\phi)
\]

(3.14)

\[
D = 0.0345\gamma^{-1.1447}\alpha^2 - 0.0414\gamma^{-0.8573}\alpha + 1 - 1.7702\exp(-18.0990\gamma)
\]

(3.15)

\[
E = -3.2301\exp(-1.1420\gamma)\exp(-2.6635\gamma^{-0.0370}\alpha)
\]

(3.16)

\[
G = -2.2780\gamma^{-0.5690}\alpha^{0.1330}\gamma^2 - 0.2372\gamma - 0.5434
\]

(3.17)
3.5.2 Apparent Radiative Property Behavior

With the correlations fully defined, the general behavior of the apparent emissivity and apparent absorptivity may be investigated and the effect of each parameter described.

Apparent Emissivity

Figure 3.2a depicts the results of Equations 3.5 and 3.12 for the apparent emissivity of an isothermal, infinite V-groove as a function of cavity angle for intrinsic emissivity values of 0.05, 0.2, 0.4, and 0.6. When the surface is flat ($\phi = \pi$), the apparent emissivity is equal to the intrinsic emissivity value, as expected. As the surface collapses and the cavity depth increases, the apparent emissivity increases monotonically. However, unlike the case of specular reflection [95], the apparent emissivity does not approach a value of unity as the V-groove collapses, a phenomenon that has been reported previously [104].

The ability to control the apparent emissivity and affect net radiation heat exchange between the surface and its surroundings is greater when the intrinsic emissivity is low. As an example, the apparent emissivity of a surface with an intrinsic emissivity of 0.05 may be increased by more than 800% to a value of 0.41 by decreasing the cavity angle. However, a V-groove with an intrinsic emissivity of 0.6 only increases by 50% to 0.9 with decreasing cavity angle.

Apparent Absorptivity

The apparent absorptivity for a diffusely irradiated cavity is equivalent to the apparent emissivity of an isothermal cavity, as shown by Ohwada [3], and is given by Equations 3.5 and 3.12, with behavior as shown in Figure 3.2a. The apparent absorptivity for a cavity with collimated irradiation entering normal to the cavity opening is shown in Figure 3.3a for three intrinsic absorptivities. Because the V-groove is fully illuminated in the case of normal, collimated irradiation, Figure 3.3a indicates the behavior of only the full-illumination correlation (Equations 3.7 and 3.13). Figure 3.3b displays apparent absorptivity as a function of cavity angle for collimation angles of $2\pi/9$ rad (20°) and $7\pi/9$ rad (70°) with intrinsic absorptivities of 0.1 and 0.6. For these conditions, the behavior of both the full ($\phi/2 \geq \gamma$, Equations 3.7 and 3.13) and partial-illumination correlations ($\phi/2 < \gamma$, Equations 3.11 and 3.14 - 3.17) are shown in Figure 3.3b.
Figure 3.2: Representations of (a) apparent emissivity/absorptivity for diffuse irradiation and (b) correlation relative error.

The collimation angle has a significant effect on the apparent absorptivity of the V-groove. When the collimated irradiation is normal to the surface \( (\gamma = 0) \), the apparent absorptivity approaches unity as the V-groove collapses, regardless of the intrinsic absorptivity, as shown in Figure 3.3a. Note that this behavior differs from that observed for apparent emissivity. Furthermore, the apparent absorptivity is not equal to the apparent emissivity when a V-groove with diffuse and gray intrinsic surface properties are exposed to collimated irradiation. As shown in Figure 3.3b, the apparent absorptivity decreases as the collimation angle increases, regardless of intrinsic absorptivity. This result is consistent with the fact that the projected area of the V-groove opening decreases as the collimation angle increases, which results in less radiation entering the cavity and less absorption. Furthermore, the apparent absorptivity approaches a value less than unity because the cavity angle collapses when the collimated irradiation is offnormal. This behavior highlights the fact that absorption by a diffusely reflecting V-groove depends strongly on the direction of collimated irradiation [63, 95, 105].

3.5.3 Comparison and Error Analysis

The accuracy of each correlation is assessed by comparison with the ray tracing (RT) data from which it is derived. The agreement between the correlation and RT data is excellent for the
apparent emissivity correlation with an average relative error of 0.3% and a standard deviation of 0.1%, having a maximum relative error of 1.5%. Figure 3.2b illustrates the relative error between the RT data and the apparent emissivity correlation as a function of intrinsic emissivity, illustrating that the correlation is least accurate at the extreme values of intrinsic emissivity. The relative error as a function of cavity angle for an intrinsic emissivity of 0.95 is also shown in Figure 3.2b. As the correlation is least accurate for large intrinsic emissivities, this curve represents the upper bound of the relative error as a function of cavity angle. Note that the accuracy of Equations 3.5 and 3.12 decreases as the cavity angle decreases. Results for the apparent emissivity are equivalent to the apparent absorptivity of a V-groove with diffuse reflection experiencing diffuse irradiation.

Figures 3.3a and 3.3b provide a comparison of RT data and correlation results for the case of full illumination. The correlation (Equations 3.7 and 3.13) and RT data show excellent agreement, with an average relative error of 0.2% and a standard deviation of 0.23%, having a maximum relative error of 0.6%. The RT data and correlation results for partial illumination (Equations 3.11 and 3.14 - 3.17) are shown in Figure 3.3b for the case of $\phi/2 < \gamma$. As compared to the full-illumination correlation, the partial-illumination correlation is less accurate, with an average relative error of 2% and a standard deviation of 1.2%, having a maximum relative error of 6.0%. Figure 3.3c displays the relative error of the partial-illumination correlation with respect to ray tracing results, averaged over the cavity angle, as a function of the collimation angle and intrinsic absorptivity. The partial-illumination correlation (Equations 3.11 and 3.14 - 3.17) is least accurate for low intrinsic absorptivities and large collimation angles, with the largest errors occurring for very small cavity angles ($\phi < \pi/6$).

To determine overall accuracy, the correlations and RT data were compared against analytical results digitally extracted from plots available in [1]. The correlation results agreed well with previously published analytical results, with only slight differences apparent for small cavity angles and small intrinsic absorptivity. Where a discrepancy existed between the analytical prediction and correlation, the RT results favored the analytical prediction. Overall, the average relative error between the analytical models and all correlation results was 0.5%.
Figure 3.3: Apparent absorptivity for a) full and b) partial illumination; and c) relative error for partial illumination averaged over cavity angle.

### 3.6 Conclusions

The correlations provided in this work (Table 3.1) provide a simple, accurate, and rapid method of apparent radiative property calculation for the infinite V-groove exposed to diffuse or collimated irradiation. The average value of the relative error for all correlations proposed in this work as compared to the ray tracing data is less than 2.0%, with the maximum never exceeding 6.0%. When compared with analytical models, the average error of the correlations is 0.5%. The least accurate predictions occur for small cavity angles, intrinsic absorptivities, and collimation angles.
CHAPTER 4. TOTAL HEMISPHERICAL APPARENT RADIATIVE PROPERTIES OF THE INFINITE V-GROOVE WITH SPECULAR REFLECTION

This chapter is published in the International Journal of Heat and Mass Transfer [95]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

4.1 Contributing Authors and Affiliations
Rydge B. Mulford, Nathan S. Collins, Michael S. Farnsworth, Matthew R. Jones, and Brian D. Iverson
Department of Mechanical Engineering, Brigham Young University, Provo, Utah, 84602

4.2 Abstract
Multiple reflections in a cavity geometry augment the emission and absorption of the cavity opening relative to a flat surface in a phenomenon known as the cavity effect. The extent of the cavity effect is quantified using apparent absorptivity and apparent emissivity. Analysis of complicated thermal systems is simplified through application of apparent radiative properties to cavity geometries. The apparent radiative properties of a specularly-reflecting, gray, isothermal V-groove have been derived analytically, but these results have not been validated experimentally or numerically. Additionally, the model for apparent absorptivity of an infinite V-groove subjected to partial illumination in the presence of collimated irradiation is not available. In this work, the following existing models for a specularly-reflecting V-groove are collected into a single source: (1) the apparent absorptivity of a diffusely irradiated V-groove, (2) the apparent emissivity of an isothermal V-groove and (3) the apparent absorptivity of a V-groove subject to collimated irradiation with full-illumination. Further, a new analytical model is developed to predict the apparent absorptivity of an infinite V-groove subject to collimated irradiation with partial-illumination. A custom, Monte Carlo ray tracing solver is used to predict the apparent radiative properties for all
cases as a means of numerical verification by comparing the ray tracing data with the results from the new model in this work and the previously existing models. For diffuse irradiation, the analytical model and ray tracing data show excellent agreement with an average discrepancy of $4.4 \times 10^{-4}$, verifying the diffuse-irradiation analytical model. Similar agreement is found for collimated irradiation, where the full and partial illumination models indicate average discrepancies of $4.9 \times 10^{-4}$ and $4.6 \times 10^{-4}$ when compared with ray tracing data.

### 4.3 Nomenclature

- $m$: maximum number of reflections experienced by rays that impact the right surface
- $n$: maximum number of reflections experienced by rays that impact the left surface
- $N$: number of tests used in determining the standard error of the mean
- $N_T$: total number of rays emitted in a ray tracing test
- $N_a$: total number of rays absorbed by cavity surfaces
- $q_a$: amount of energy absorbed by the cavity surface (W)
- $q_t$: total amount of energy incident on the cavity opening (W)
- $\vec{r}_1$: incoming ray vector
- $\vec{r}_2$: outgoing ray vector
- $s$: sample size
- $SE$: standard error of the mean for $N$ completed ray tracing tests
- $\vec{v}$: surface normal used in calculating reflected ray
- $x_i$: X location of the $i^{th}$ impact of the ray used in the simple ray tracing routine (m)
- $X_n$: fraction of rays incident on the left panel that will reflect $n$ times
- $X'$: normalized length from the bottom of the V-groove along the left panel to the initial intersection of a ray
- $X''$: normalized length of the shadowed portion for the left panel
- $y_i$: Y location of the $i^{th}$ impact of the ray used in the simple ray tracing routine (m)
- $Y_m$: fraction of rays incident on the right panel that will reflect $m$ times
- $Y'$: normalized length from the bottom of the V-groove along the right panel to the initial intersection of a ray that grazes the top of the cavity when exiting
4.4 Introduction

V-groove geometries are found in a variety of mechanical designs such as deployable mechanisms [81, 106], solar absorbers [107–109], solar cells [110], thermal radiators [94] and in the natural world. The analysis of radiative heat transfer within systems that contain V-groove geometries (or any cavity geometry) is complicated by the occurrence of multiple reflections within the V-groove. The results of this study demonstrate apparent radiative properties may be used to accurately model the effects of multiple reflections in specularly reflecting V-grooves. The apparent radiative properties of diffusely reflecting V-grooves are addressed by Mulford et al. [2].

Cavity geometries emit and absorb more thermal radiation than an equivalent flat surface (equal in size to the cavity opening) with the same thermal radiation properties and temperature as the cavity walls [51]. The augmentation of emission and absorption, termed the cavity effect, is quantified with apparent emissivity and apparent absorptivity, respectively. Apparent emissivity is defined as the emission from a cavity opening divided by the emission from an equivalent black surface stretched across the cavity opening having the same temperature as the cavity walls [50]. Likewise, the apparent absorptivity is defined as the ratio of the incident radiation absorbed by the cavity to the total incident radiation on the cavity opening [1, 100].
The apparent radiative properties of a cavity are a function of the intrinsic radiative properties of the cavity walls, the type of reflection (specular or diffuse), the overall cavity shape and the cavity geometry [54]. The apparent absorptivity of a cavity is also dependent on the nature of the irradiation entering the cavity (diffuse or collimated) and the angle of incidence if the irradiation is collimated [111]. Conversely, the apparent emissivity of a cavity is strongly affected by the temperature of the cavity walls [112]. Likewise, the apparent emissivity of an isothermal cavity is equivalent to the apparent absorptivity of the same cavity subjected to diffuse irradiation, regardless of reflection type [3].

Apparent radiative behavior for a wide variety of cavity shapes subjected to various temperature profiles, irradiation conditions, and intrinsic surface properties are available in the literature. Several excellent reviews detail the different approaches used by various researchers to determine these apparent radiative properties and provide an overview of their results [50, 51, 54]. The majority of the studies performed on cavity geometries have focused on spherical [100, 113], cylindrical [37, 114, 115] and conical geometries [45] with a few papers examining V-groove or rectangular-groove geometries [1, 62]. The closed, circular shapes often emphasized in the literature (i.e. cylinder or cone) have been investigated primarily because they are the most efficient shapes for approximating blackbody emission.

A new application of the cavity effect using origami surfaces comprised of angular cavities has revitalized the study of V-groove geometries [66]. Variations to cavity geometries cause the apparent radiative properties of the cavity to likewise change. By actively controlling the geometry of a cavity, the apparent radiative properties of the cavity may be controlled, allowing for adjustments to the apparent radiative properties in real time. Actuation of origami tessellations provide the mechanism by which cavity geometry is modified, creating a variable emissivity/absorptivity surface [66]. This application has created renewed interest in the apparent radiative properties of the V-groove shape, as this shape best approximates the cavities inherent to origami tessellations. Several apparent radiative property studies have examined the V-groove geometry, including works by Sparrow [1, 116, 117], Zipin [61, 105], Hollands [94] and Daws [40]. However, these models do not encompass the full range of possible cavity angles, intrinsic surface properties and collimation angles.
This work compiles the works of previous authors related to the V-groove geometry and presents a new model that addresses partial illumination in the case of collimated irradiation to provide a suite of equations that may be used to calculate the apparent emissivity or apparent absorptivity of a specularly-reflecting V-groove. The apparent absorptivity of a cavity for the full range of possible intrinsic surface properties, V-groove cavity angles and illumination conditions (diffuse or collimated) can be predicted with these models. The apparent emissivity expression will be limited to the case of an isothermal cavity. All models reported in this paper, whether derived in this work or developed by other authors, have been verified using Monte Carlo ray tracing. The case of a diffusely-reflecting V-groove cavity is considered separately [2].

4.5 Methodology

4.5.1 Apparent Absorptivity for Diffuse Irradiation (Apparent Emissivity for Isothermal Cavity)

The apparent emissivity for an isothermal, specularly-reflecting, infinite V-groove subject to diffuse irradiation has been determined analytically by Modest [38] based on the work of Sparrow [56, 118] and Hollands [94] resulting in Equation 4.1. The maximum number of reflections \( n \) experienced by a ray is equivalent to the integer portion of \( \pi/\phi \), where the angle \( \phi \) (as shown in Figure 4.1a) is given in radians. Ohwada [3] has shown that the apparent emissivity of an isothermal cavity and apparent absorptivity of a diffusely-irradiated cavity are equivalent regardless of reflection mode. As such, Equation 4.1 can be used to predict both apparent emissivity and apparent absorptivity for these conditions.

\[
\varepsilon_a = \alpha_a = \frac{\varepsilon}{\sin(\phi/2)} \left[ 1 - \varepsilon \sum_{k=1}^{n} \rho^{k-1} \left( 1 - \sin \left( k \frac{\phi}{2} \right) \right) \right] \quad (4.1)
\]

An additional model that describes the apparent emissivity of an isothermal V-groove is available from Daws [40]. However, usage of the model relies on tabulated solutions that are only available for a limited number of cavity angles and intrinsic emissivities. Likewise, Sparrow has developed a method to calculate the radiative heat loss from a V-groove cavity [118], but an equation that gives the apparent emissivity is not derived. Finally, Sparrow has also derived a model for the apparent absorptivity of a diffusely-irradiated V-groove [1], which is equivalent to
the isothermal apparent emissivity. However, Sparrow’s model is limited to cavity angles that are given by the equation \( \phi = \pi/n \) where \( n \) is a positive integer. As such, Modest’s equation, which has only been published as a textbook example question, has been selected for its simple implementation and general applicability. Equation 4.1 has been verified with Monte Carlo ray tracing as will be shown in this work.

### 4.5.2 Apparent Absorptivity for Collimated Irradiation

The apparent absorptivity for a specularly reflecting infinite V-groove subject to collimated irradiation (at a given angle \( \gamma \), Figure 4.1b) must be separated into two scenarios, each with its own model. In the full illumination case \( (\gamma \leq \phi/2) \), incident rays completely illuminate the cavity, falling on both the left and right panels of the V-groove as shown in Figure 4.1b. In the partial illumination case \( (\gamma > \phi/2) \), rays initially impact only a portion of one V-groove panel and do not fall on the other panel as shown in Figure 4.1c. Sparrow and Lin [1] developed a model that applies to the case of a fully illuminated V-groove which is summarized in Section 4.5.2 to provide context for the partial illumination model developed in this work (Section 4.5.3).

#### Fully Illuminated V-groove

As shown in Figure 4.2a, a sample ray with a given angle of incidence \( \gamma \) (defined in the clockwise direction from the surface normal) intersects the left V-groove panel at location \( X' \) and exits the cavity after reflecting twice, leaving the cavity as close to the left V-groove panel as possible without intersection. Any ray that initially intersects the left V-groove panel at a location below \( X' \) will reflect one additional time before leaving the cavity; all rays that intersect above the point \( X' \) will experience the same number of reflections as the sample ray incident exactly at \( X' \). Figure 4.2a also depicts the ray-plane angle \( \beta \) (angle between an intersecting ray and the cavity surface) for both internal reflections and for the exiting ray. For the \( i^{th} \) reflection, the ray-plane angle \( (\beta_i) \) for a ray initially striking the left-hand panel is given by Equation 4.2 [1]. The length \( X' \), which has been normalized by the length of the V-groove panel, is determined through the law of sines as given in Equation 4.3 [1]. For the case of full illumination, the length \( X' \) is equivalent to \( X_n \) or the fraction of rays incident on the left V-groove panel that will reflect \( n \) times.
Figure 4.1: (a) An infinite V-groove with diffuse irradiation. Cavity dimensions are governed by sides of length $L$ and an included angle of $\phi$. (b) An infinite V-groove with collimated irradiation and full illumination where the irradiation falls initially on both sides of the V-groove; the required condition for $\gamma$ and $\phi$ is shown in the figure. (c) An infinite V-groove with collimated irradiation and partial illumination where the irradiation falls only on one side of the V-groove; the required condition for $\gamma$ and $\phi$ is shown in the figure. (d) A 3D depiction of an accordion tessellation

$$\beta_i = \gamma + \left( i - \frac{1}{2} \right) \phi \quad \text{(4.2)}$$

$$X' = \frac{\sin[(n - 1/2)\phi + \gamma]}{\sin(\phi/2 + \gamma)} = X_n \quad \text{(4.3)}$$

The maximum number of reflections experienced by rays initially incident on the left panel ($n$ reflections for rays that intersect below $X'$) is determined by assuming that a ray has left the cavity once the ray-plane angle of the next intersection ($\beta_{n+1}$) exceeds $\pi$. When $\beta_{n+1} > \pi$, the ray is moving vertically upwards between the two V-groove planes such that an additional reflection is not possible. Setting $\beta_i = \pi$ in Equation 4.2 and designating the $i^{th}$ reflection as $n$, the maximum number of reflections can be obtained as given in Equation 4.4, where all angles are input as
Figure 4.2: (a) Definition of ray-plane angle (β) and X'. A ray at an angle of γ enters a V-groove cavity and intersects the left cavity wall at X'. The ray is reflected from the cavity walls twice before exiting the cavity through the left-most point of the cavity opening. Any ray that enters the cavity opening further to the right of this ray, striking below the point X', will reflect one additional time before reflecting out of the cavity opening. All rays that enter to the left of the initial ray, striking above the point X', will reflect the same number of times as the indicated ray. The ray-plane angle β is depicted for both of the internal reflections and at the exit. (b) Definition of ray-vertical angle (χ) and X''. A ray enters the right-most point of the cavity opening with an initial ray-vertical angle (χ₁) equivalent to the ray’s collimation angle (γ). The ray strikes the left wall at the point X'' which represents the fraction of the left wall that is irradiated since no rays will intersect below this point initially. After an additional reflection, the ray exits the cavity opening. The ray-plane angle between the theoretical extension of the cavity wall and the exiting ray (β₃) does not exceed π, violating the condition used in the full illumination case used to determine when a ray has left the cavity.

The result of Equation 4.4 must be rounded down to the nearest integer value, and if the initial evaluation is exactly an integer then the number of reflections should be decreased by one.

\[ n = \left\lfloor \frac{\pi - \gamma}{\phi} + \frac{1}{2} \right\rfloor \]  

(4.4)

Rays that initially strike the right surface also have a reflection split point Y', which is again equivalent to the fraction of rays that reflect m times, or Yₘ. The reflection split point and total number of reflections for the right side can be determined using Equations 4.3 and 4.4, respectively. However, in both equations −γ should be used in the place of γ.
A percentage of the incident energy is absorbed ($\alpha$) and reflected ($1 - \alpha$) at each reflection event. A summation of the total energy absorbed over all reflections (i.e., the total fraction of incident energy that is absorbed after $n$ reflections) is given in Equation 4.5, assuming that all of the incident energy reflected $n$ times.

$$\alpha + \alpha(1 - \alpha) + \alpha(1 - \alpha)^2 + \ldots + \alpha(1 - \alpha)^{n-1} = 1 - (1 - \alpha)^n$$

(4.5)

In reality, only a fraction of the incident energy on one of the V-groove surfaces reflects $n$ times, that fraction being equivalent to $X_n$ or $Y_m$. To account for the unequal number of reflections, the right hand side of Equation 4.5 is factored to give $1 - (1 - \alpha)(1 - \alpha)^{n-1}$. The absorptivity of the factored term is then corrected by scaling the absorptivity by $X_n$ or $Y_m$ for the left and right sides, respectively, giving $1 - (1 - X_n\alpha)(1 - \alpha)^{n-1}$ and $1 - (1 - Y_m\alpha)(1 - \alpha)^{(m-1)}$. The corrected fraction of incident energy absorbed for each side of the V-groove is then multiplied by the incoming radiative flux and the projected area of the associated side. The sum total of both sides is divided by the product of the projected area of the V-groove cavity and the incoming radiative flux. The radiative flux terms cancel in the numerator and denominator to give the following apparent absorptivity for a fully illuminated V-groove.

$$\alpha_a = \frac{\left[1 - (1 - X_n\alpha)(1 - \alpha)^{n-1}\right] \sin(\phi/2 + \gamma) + \left[1 - (1 - Y_m\alpha)(1 - \alpha)^{(m-1)}\sin(\phi/2 - \gamma)\right]}{2\cos\gamma\sin(\phi/2)}$$

(4.6)

Partially Illuminated V-groove

Sparrow and Lin [1] stated that an apparent absorptivity model for a partially illuminated V-groove with specular reflection would be similar to the model developed for full illumination (Section 4.5.2) but did not report this model or its development. This section outlines the development of a new model to determine the apparent absorptivity of a partially illuminated V-groove following the basic approach of Sparrow and Lin. The approach is also verified in this work with Monte Carlo ray tracing.

As with the case of full illumination, the total fraction of incident energy absorbed is determined through a summation, as shown in Equation 4.5. The right hand side of Equation 4.5 must
again be scaled by $X_n$ to account for the uneven number of reflections, giving Equation 4.7. The term for the right side of the V-groove, or $1 - (1 - Y_m \alpha)(1 - \alpha)^m$, is not present in the partial illumination model because no rays are initially incident on this side. The result given in Equation 4.7 is the apparent absorptivity of a partially illuminated V-groove.

$$\alpha_a = 1 - (1 - \alpha X_n)(1 - \alpha)\!^{n-1} \quad (4.7)$$

For partial illumination, $X_n$ is not equivalent to $X'$ due to shading of the left side of the V-groove (Figure 4.1c). To find $X_n$ for a partially illuminated surface, it is first necessary to calculate the normalized length of the lowest ray’s impact point ($X''$, Figure 4.2b) using the law of sines, giving Equation 4.8. The split point $X'$ is calculated as before with Equation 4.3. However, $X_n$ is now calculated using the following formulas for one of two scenarios: (1) if $X' < X''$, the split point $X'$ falls below the lowest ray impact point $X''$ indicating that all rays striking the left surface reflect a total of $n - 1$ times and $X_n = 0$; (2) if $X' \geq X''$, then $X'$ must be scaled by $X''$, as shown in Equation 4.9, providing a value for $X_n$ that accounts for shading.

$$X'' = \frac{\sin(\gamma - \phi/2)}{\sin(\pi - \phi/2 - \gamma)} \quad (4.8)$$

$$\begin{align*}
\text{if} \quad X' \geq X'' : \quad X_n &= \frac{X' - X''}{1 - X'} \\
\text{if} \quad X' < X'' : \quad X_n &= 0 \quad (4.9)
\end{align*}$$

The maximum number of reflections that an incident ray will experience before leaving the cavity ($n$) must also be determined for a partially illuminated cavity. For the case of full illumination, a ray will continue to reflect inside of a cavity until the ray-plane angle (Equation 4.2) exceeds $\pi$, allowing for $n$ to be found with Equation 4.4. However, a ray reflecting inside of a partially illuminated V-groove could exit the cavity before the ray plane angle exceeds $\pi$. As an example, Figure 4.2b depicts a ray entering a partially illuminated cavity that is reflected twice before exiting. Upon exiting, it is clear that the ray-plane angle ($\beta_3$ in Figure 4.2b) of the exiting
ray does not exceed $\pi$ but exits regardless due to the finite length of the V-groove panels, violating the condition used in the derivation of Equation 4.4.

Since Equation 4.4 is not valid for partial illumination, the total number of reflections was determined through 2D ray tracing by tracking the path of a single ray that enters the cavity through the right-most point of the cavity opening (depicted in Figure 4.2b) and counting the number of reflections the ray experiences before exiting the cavity. The mathematical development and final equations for the 2D ray tracing method are described in Appendix A. The value $n$ was obtained using the ray tracing routine detailed in the Appendix A for all possible combinations of cavity angle ($\phi$) from 0 to $\pi$ and ray incidence angle ($\gamma$) from 0 to $\pi/2$ in increments of $\pi/180$ (i.e. $1^\circ$).

Figure 4.3 illustrates the results for $n$ from the 2D ray tracing routine for all combinations of $\phi$ and $\gamma$ as a filled contour plot, where the dotted line illustrates the boundary between full and partial illumination. Distinct linear patterns, designated as solid black lines, are visible in both the full and partial illumination regions. In the full illumination region, the equation for each solid black line is given by $\gamma = -(n - 1/2) \phi + \pi$, where $n$ is a positive integer and angles are in radians. When rearranged, this linear equation is equivalent to Sparrow’s equation used to compute the number of reflections (Equation 4.4) for full illumination. This agreement verifies Sparrow’s counting method for the full illumination case.

As observed in Figure 4.3, the $n$-counting method used for full illumination is not applicable to the partial illumination case. However, a linear relationship between $\phi$, $\gamma$ and $n$ is also evident in the partial illumination region. Interpolation of the data following the same general form as Sparrow gives Equation 4.10 which must also be rounded down to the nearest integer and angles are expressed in radians.

$$n = \left\lfloor \frac{\pi - 2\gamma}{\phi} + 1 \right\rfloor$$

(4.10)

For full illumination, Equation 4.4 was derived with an intuitive physical argument. A similar physical argument for the derivation of Equation 4.10 is less clear. However, if we rearrange Equation 4.10, we obtain a form that is very similar to the fully illuminated case derived by Sparrow, indicating that half of the reflections have occurred once the ray-plane angle exceeds $\pi/2$ (Equation 4.11).
Figure 4.3: Maximum number of reflections \( (n) \) encountered by a ray entering a V-groove cavity from the right-most location of the cavity opening as a function of ray inclination angle \( (\gamma) \) and V-groove cavity angle \( (\phi) \), determined using the 2D ray tracing routine described in the Appendix A. The dashed line depicts the relationship \( \gamma = \phi/2 \) and separates the fully and partially illuminated regions as indicated on the image. Distinct linear patterns for the number of reflections are visible and have been marked with solid black lines. Data was calculated over the full range of collimation angles \( (0 < \gamma < \pi/2) \) and cavity angles \( (0 < \phi < \pi) \) in increments of \( \pi/180 \). However, the data is displayed over a limited \( \phi \) range to better illustrate the results.

\[
\frac{n}{2} = \frac{(\pi/2 - \gamma)}{\phi} + \frac{1}{2}
\]  

(4.11)

This interpretation is further verified by considering that a ray reflects downward as long as the ray plane angle is less than \( \pi/2 \) and then reflects towards the direction of the opening once the ray plane angle has exceeded \( \pi/2 \) (as indicated in Figures 4.2a and 4.2b), with the number of downward and upward reflections differing at most by a value of one.
4.5.3 Monte Carlo Ray Tracing

In the previous sections, analytical expressions have been reported or derived for the apparent absorptivity of an infinite V-groove and for the apparent emissivity of an isothermal V-groove. Monte Carlo ray tracing was used to calculate the apparent absorptivity of the infinite V-groove for equivalent heating conditions, providing a numerical verification of the analytical models. Ray tracing, a statistical approach, can be computationally expensive when compared with deterministic methods that incorporate specular reflection [119, 120]. However, the simplicity of the application and the availability of computing power motivated the use of Monte Carlo ray tracing as a verification.

Apparent Absorptivity

Monte Carlo ray tracing is a straightforward numerical method used to solve radiation heat transfer problems with difficult geometries [50, 97, 98]. Emitted and absorbed rays for a geometry of interest are counted and related to apparent radiative properties. In this work, a direct relationship between the number of rays emitted and number of rays absorbed gives the apparent absorptivity of the tested cavity geometry.

A V-groove geometry was positioned directly below a transparent, emitting surface that provides a number of emitted rays downward into the V-groove of cavity angle $\phi$. The rays may be emitted diffusely or at a given collimation angle $\gamma$ (see dashed lines in Figures 4.1a - 4.1c). All rays emitted from the transparent surface enter the V-groove. The surfaces of the V-groove absorb a percentage of the incident rays equivalent to the intrinsic absorptivity of the material and reflect the remainder in a specular manner. Each ray is tracked until it is absorbed upon intersection with a cavity surface or escapes out of the cavity after one or more reflections. The total number of emitted rays $N_T$ and the number of absorbed rays $N_a$ are counted.

The apparent absorptivity of a surface is defined as the ratio of total absorbed energy ($q_a$) to the total energy entering the cavity opening ($q_t$) [1], as shown in Equation 4.12. To determine apparent absorptivity from ray tracing results, each ray is assumed to represent a unit of quantized energy [121, 122]. Each unit may be either completely reflected or completely absorbed at a single
reflection event. With this analogy, the ratio of absorbed rays \( N_a \) to the total number of rays \( N_t \) can be used to obtain the apparent absorptivity as shown in Equation 4.120 (see also [51]).

\[
\alpha_a = \frac{q_a}{q_t} = \frac{N_a}{N_t}
\]

(4.12)

In the case of diffuse irradiation, the apparent absorptivity of the infinite V-groove is equivalent to the apparent emissivity if the cavity is isothermal [3]. As such, ray tracing verification of Modest’s equation (Equation 4.1) was performed by determining the apparent absorptivity of the cavity and equating that value to the apparent emissivity as determined through Modest’s model.

**Ray Tracing Applications**

A custom ray tracing program was developed by the authors following the mathematical basis provided by Steinfeld [2,100] to determine the values \( N_a \) and \( N_t \) for use in Equation 4.12. For diffuse emission, the polar and azimuthal angles of the emitted ray were determined through random number generation with the polar angle weighted towards a cosine distribution. Collimated rays were emitted at the specified collimation angle. The intersection of a ray and a participating surface were determined with a line-plane intersection algorithm [101]. When a ray-surface interaction occurred, a new random number in the range \([0,1)\) was compared against the absorptivity of the surface to determine if the ray was absorbed or reflected. If reflected, Equation 4.13 [100] was used to determine a new ray direction, where \( \vec{r}_2 \) is the reflected ray direction vector, \( \vec{r}_1 \) is the incident ray direction vector and \( \vec{v} \) is the unit normal vector of the impacted surface. This 3D ray tracing program was developed separately from the 2D ray tracing method described in the Appendix A used for determining \( n \) for the case of partial illumination.

\[
\vec{r}_2 = \vec{r}_1 - 2(\vec{v} \cdot \vec{r}_1)\vec{v}
\]

(4.13)

**Testing Procedure**

The custom ray tracing program was constructed to operate in three-dimensional space for determining the apparent radiative properties of origami tessellations such as the accordion fold (Figure 4.1d), the Barreto’s Mars [106] and the Miura Ori [81]. To create the illusion of an infinite
V-groove, perfectly reflecting specular surfaces were added to the open ends of the 3D V-groove. A transparent surface was placed across the top opening of the V-groove of specified cavity angle and intrinsic emissivity. The transparent surface emitted a specified number of rays diffusely or in a collimated fashion into the cavity. The total number of rays emitted $N_t$ and number of rays absorbed $N_a$ was counted and the apparent absorptivity calculated using Equation 4.12. Data was collected for all possible combinations of $\alpha$, $\phi$, and $\gamma$ (where applicable). The surface absorptivity $\alpha$ was varied from 0.01 to 1.00 in increments of 0.01; $\phi$ from $\pi/180$ to $\pi$ in increments of $\pi/180$ ($1^\circ$); and $\gamma$ from 0 to $8\pi/9$ in increments of $\pi/18$ ($10^\circ$).

Each combination of $\phi$, $\gamma$, and $\alpha$ was evaluated a total of 20 times for $N_t$ number of rays. The standard error of the mean [102] for the 20 tests at a given value of $N_t$ was calculated using Equation 4.14, where $SE$ is the standard error of the mean, $s$ is the unbiased standard deviation of the results and $N$ is the number of samples.

$$SE = \frac{s}{\sqrt{N}} \quad (4.14)$$

The number of total rays emitted was increased to $N_t = 300,000$ rays, at which point the standard error of the mean fell at or below $5.0 \times 10^{-3}$ for all tested cases. A value of $5.0 \times 10^{-3}$ was determined to be an appropriate error threshold and all reported data was determined at this ray count.

### 4.6 Results and Discussion

#### 4.6.1 Model Summary

Table 4.1 provides the equation numbers, required inputs and references for the models developed by others and summarized in this work (i.e. for diffuse irradiation and for collimated irradiation with full illumination). The model for collimated irradiation with partial illumination was developed in this work and is also summarized in Table 4.1. Verification of these three models through Monte Carlo ray tracing is presented in Section 4.6.2 and the results of these models are presented in Section 4.6.3.
Table 4.1: Summary of the models for apparent radiative behavior summarized or developed in this work. The conditions required for the use of each model are listed as well as the desired apparent property. Equation numbers for calculation of the desired output are listed.

<table>
<thead>
<tr>
<th>Desired output</th>
<th>Conditions</th>
<th>Required inputs</th>
<th>Equation numbers</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_a$</td>
<td>Emission: Isothermal, Diffuse</td>
<td>$\varepsilon, \phi$</td>
<td>4.1</td>
<td>[38]</td>
</tr>
<tr>
<td></td>
<td>Reflection: Specular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Irradiation: Diffuse</td>
<td>$\alpha, \phi$</td>
<td>4.1</td>
<td>[38]</td>
</tr>
<tr>
<td></td>
<td>Reflection: Specular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Irradiation: Collimated, Fully illuminated ($\gamma \leq \phi/2$)</td>
<td>$\alpha, \phi, \gamma$</td>
<td>4.3, 4.4, 4.6</td>
<td>[1]</td>
</tr>
<tr>
<td></td>
<td>Reflection: Specular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Irradiation: Collimated, Partially illuminated ($\gamma &gt; \phi/2$)</td>
<td>$\alpha, \phi, \gamma$</td>
<td>4.3, 4.7 - 4.10</td>
<td>This work</td>
</tr>
<tr>
<td></td>
<td>Reflection: Specular</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.4: The apparent absorptivity of a specularly-reflecting infinite V-groove subject to diffuse irradiation as a function of cavity angle ($\phi$) and intrinsic surface absorptivity ($\alpha$). Results in this plot also apply to the apparent emissivity of an isothermal infinite V-groove as a function of cavity angle and intrinsic surface emissivity [3]. Agreement between ray tracing results and analytical results has an average discrepancy of $4.4 \times 10^{-4}$.

4.6.2 Ray Tracing Verification

Apparent Absorptivity for Diffuse Irradiation (Apparent Emissivity for Isothermal Cavity)

Figure 4.4 depicts the results of the analytical model (Equation 4.1 from Modest [38]) and ray tracing tests for a diffusely-irradiated absorbing infinite V-groove or isothermal, emitting V-groove. Results are compared for four different intrinsic surface properties. Although ray tracing was performed at 179 angles between $\phi = 0$ and $\pi$, only a selection of the ray tracing data points are shown in the figure for clarity. The average discrepancy (the absolute value of the difference between the analytical result and ray tracing result) between the analytical model and all available ray tracing data is $4.4 \times 10^{-4}$ with a standard deviation of $9.8 \times 10^{-5}$. These values are within the average ray tracing error of $5.0 \times 10^{-3}$. As such, Equation 4.1 is an accurate expression of apparent properties over the full range of intrinsic properties and cavity angle.
Collimated Irradiation

In Figures 4.5a and 4.5b, selected ray tracing results are provided to verify the results of the analytical models. The average discrepancy between all ray tracing data points for the fully illuminated region and analytical results is \(4.9 \times 10^{-4}\) with a standard deviation of \(7.8 \times 10^{-5}\). The average discrepancy is very small compared with the apparent absorptivity values, verifying Sparrow’s model as accurate over the applicable range of cavity angles and collimation angles. In the case of partial illumination, the average discrepancy between all ray tracing data points and analytical models is \(4.6 \times 10^{-4}\) with a standard deviation of \(8.1 \times 10^{-5}\). Again, these discrepancies are very small compared to the absolute apparent absorptivity values. This confirms the accuracy of the partial illumination analytical model introduced in this work.

4.6.3 Apparent Radiative Properties

Apparent Absorptivity for Diffuse Irradiation (Apparent Emissivity for Isothermal Cavity)

Figure 4.4 illustrates the analytically derived apparent absorptivity for diffuse irradiation and the equivalent apparent emissivity for isothermal panels of an infinite V-groove as a function of cavity angle and intrinsic radiative property (Equation 4.1). The influence of the cavity effect is clearly evident, where the apparent radiative property is equivalent to the intrinsic radiative property only for the flat case \((\phi = \pi)\) and increases above the intrinsic value for all other cavity angles. The ability to achieve black behavior via the cavity effect is independent of material type, with all cavities approaching unity as the cavity angle collapses regardless of intrinsic radiative property. Lower intrinsic radiative properties experience rapid apparent property variation in the small angle range, whereas V-grooves with intrinsic properties near unity experience the most rapid apparent property variation in the mid-angle range.

Apparent Absorptivity for Collimated Irradiation

Figure 4.5a and 4.5b illustrate the analytically-derived apparent absorptivity results for collimated irradiation at collimation angles of \(\pi/18\) (10°) and \(2\pi/9\) (40°), respectively. The vertical dashed line in each plot indicates the separation point between full and partial illumination (where
full illumination occurs to the right and partial illumination to the left). The analytical results were obtained with the use of the full illumination model as reported from Sparrow and Lin [1] and the partial illumination model developed in this work.

Unlike the case for diffuse irradiation, the apparent absorptivity for a specular V-groove exposed to collimated irradiation does not begin to increase immediately as the cavity angle decreases. For a collimated irradiation angle of $\gamma = \pi/18$, the apparent absorptivity remains equivalent to the intrinsic absorptivity from approximately $\phi = \pi$ to $2\pi/3$ regardless of intrinsic absorptivity. This behavior is due to the specular nature of the surface, causing all rays to be reflected away from the cavity when the surface is mostly flat ($\phi = \pi$ to $2\pi/3$). At around $\phi = 2\pi/3$ the rays reflected after initial contact with one V-groove surface begin to make contact with the opposite surface and the apparent absorptivity experiences a discontinuity in slope, suddenly increasing towards unity as the cavity angle continues to collapse towards zero. In general, the presence of collimated irradiation results in discontinuous behavior due to the similar reflection patterns experienced by all of the incident collimated rays.

An interesting behavior is encountered for larger collimation angles (Figure 4.5b), where the apparent absorptivity decreases slightly with decreasing cavity angle over the range of approximately $\phi = 5\pi/6$ to $\pi/2$, as shown in Figure 4.5b. In this cavity angle range, for $\gamma = 2\pi/9$, the rays initially striking the left surface reflect one time before exiting the cavity whereas the rays that strike the right surface reflect twice. As the cavity angle decreases, the percentage of rays falling on the right side of the surface decreases while the percentage of rays falling on the left surface increases, causing the total number of reflections experienced by all rays in the cavity to decrease. This slight decrease in apparent absorptivity as the cavity angle decreases disqualifies the intuitive assumption that a deeper cavity is always a more effective absorber for the case of specular reflection and collimated irradiation.

### 4.7 Conclusions

Existing models to calculate the apparent absorptivity of a diffusely-irradiated V-groove [38] (equivalent to the apparent emissivity of an isothermal V-groove), and the apparent absorptivity of a fully illuminated cavity subject to collimated irradiation [1] were verified against ray tracing results with an average discrepancy of less than $4.9 \times 10^{-4}$. A new analytical model was developed...
Figure 4.5: (a) Apparent absorptivity of a specularly-reflecting infinite V-groove subject to collimated irradiation with an inclination angle of $\gamma = \pi/18$ ($10^\circ$) as a function of cavity angle ($\phi$) and intrinsic surface absorptivity ($\alpha$). Solid lines were calculated by Equation 4.6 for full illumination, $\phi \geq \pi/9$ ($20^\circ$), and Equation 4.7 for partial illumination, $\phi < \pi/9$ ($20^\circ$). Data indicated by the squares were obtained by Monte Carlo ray tracing; only a portion of the numerical data points are depicted here for clarity. Agreement between the analytical and numerical methods is excellent with an average discrepancy of $4.9 \times 10^{-4}$ to the right of the dotted line and $4.6 \times 10^{-4}$ to the left of the dotted line. (b) Apparent absorptivity of a specularly-reflecting infinite V-groove subject to collimated irradiation with an inclination angle of $\gamma = 2\pi/9$ ($40^\circ$) as a function of cavity angle and intrinsic surface absorptivity. Partial illumination occurs at cavity angles less than $4\pi/9$ ($80^\circ$) as indicated by the vertical dashed line. Agreement between the analytical model and numerical data is the same as in Figure 4.4. A slight increase in the apparent absorptivity is visible between the cavity angles of $5\pi/6$ ($150^\circ$) and $\pi/2$ ($90^\circ$), indicating that apparent radiative properties do not always increase as the cavity angles decreases.

Developed to calculate the apparent absorptivity for a partially illuminated, infinite V-groove subject to collimated irradiation. This model has also been verified with ray tracing results with an average discrepancy of $4.6 \times 10^{-4}$. Results show that significant control of apparent radiative properties is possible by controlling the V-groove cavity angle, confirming the use of origami tessellations as possible variable emissivity surfaces.
CHAPTER 5. CONTROL OF NET RADIATIVE HEAT TRANSFER WITH A VARIABLE-EMISSIVITY ACCORDION TESSELLATION

This chapter is published in the Journal of Heat Transfer [123]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

5.1 Contributing Authors and Affiliations

Rydge B. Mulford, Matthew R. Jones, and Brian D. Iverson
Department of Mechanical Engineering, Brigham Young University, Provo, Utah, 84602

Vivek H. Dwivedi
Associate Head, Thermal Management Group, NASA Goddard, Greenbelt, MD

5.2 Abstract

Origami tessellations have been proposed as a mechanism for control of radiative heat transfer through the use of the cavity effect. This work explores the impact of a changing projected surface area and varying apparent radiative properties on the net radiative heat transfer of an accordion fold comprised of V-grooves. The net radiative heat transfer of an accordion tessellation is obtained by a thermal energy balance at the cavity openings with radiative properties of the cavities given as a function of various cavity parameters. Results of the analytical model are experimentally confirmed. An accordion tessellation, constructed of stainless-steel shim stock, is positioned to achieve a specified fold angle and placed in a vacuum environment while heated by Joule heating. A thermal camera records the apparent temperature of the cavity openings for various fold angles. Results are compared to apparent temperatures predicted with the analytical model. Analytically and experimentally obtained temperatures agree within 5% and all measurements fall within experimental uncertainty. For diffusely irradiated surfaces, the decrease in projected sur-
face area dominates, causing a continuous decrease in net radiative heat transfer for a collapsing accordion fold. Highly reflective specular surfaces exposed to diffuse irradiation experience large turn-down ratios (7.5x reduction in heat transfer) in the small angle ranges. Specular surfaces exposed to collimated irradiation achieve a turn down ratio of 3.35 between V-groove angles of 120° and 150°. The approach outlined here may be extended to modeling the net radiative heat transfer for other origami tessellations.

5.3 Nomenclature

$A_a$ apparent area of the accordion tessellation (m$^2$)
$A_{a,proj}$ apparent projected area normal to collimated irradiation (m$^2$)
$A_{cond}$ 2x sample cross-sectional area (m$^2$)
$dT/dx$ conductive loss temperature gradient (K m$^{-1}$)
$G$ collimated irradiation heat flux (W m$^{-2}$)
$G_{\lambda,b}$ spectral irradiation from a black surface (W m$^{-2}$)
$I$ current (A)
$k_{ss}$ thermal conductivity of stainless-steel (W m$^{-1}$ K$^{-1}$)
$L_P$ length of a panel (m)
$n$ largest number of reflections experienced by collimated rays in a specular cavity
$N_{panels}$ number of panels
$P_{loss}$ power dissipated in the circuit outside of the control volume (W)
$P_s$ power dissipated in the stainless-steel sample (W)
$P_t$ total power dissipated in the circuit (W)
$q_{abs}$ irradiation absorbed by the tessellated surface (W)
$q_e$ heat emitted by tessellated surface (W)
$q_{loss}$ heat loss via conduction (W)
$q_{rad}$ net radiative heat loss from the tessellated surface (W m$^{-1}$)
$R_s$ resistance of the stainless-steel surface (Ω)
$R_t$ resistance of the total circuit including the stainless-steel surface (Ω)
$T_a$ apparent temperature of the tessellation opening (K)
\( T_{\text{surr}} \) temperature of the surroundings (K)
\( x \) horizontal location on tessellation (cm)
\( W_p \) width of a tessellation panel (m)
\( X' \) fraction of unabsorbed, collimated rays that reflect n times in a specular cavity
\( \alpha_a \) apparent absorptivity
\( \gamma \) collimated irradiation angle (rad)
\( \epsilon \) intrinsic emissivity of the sample material
\( \epsilon_a \) apparent emissivity
\( \eta \) sample heating efficiency
\( \tau_\lambda \) spectral, hemispherical transmissivity of the sapphire window
\( \tau_{3-5\mu m} \) transmissivity of the sapphire window over 3-5 \(\mu m\) band
\( \phi \) cavity angle (rad)
\( \chi \) sample length ratio
\( \Delta T_a \) temperature difference between analytical and experimental results (deg K)
\( \Pi \) normalized net radiative heat transfer

### 5.4 Introduction

The static behavior of radiative surface properties can prevent optimal operation of thermal management systems that are dominated by radiative heat transfer. As an example, consider a spacecraft in low earth orbit [5]. Variations in spacecraft power dissipation and large external heating variations can result in significant fluctuations in heat loss. However, because the spacecraft radiator’s surface properties and emitting area are inherently static, radiators are designed to reject the maximum heat load experienced by the spacecraft. Power consuming heaters are commonly utilized to maintain desired operating temperatures [5, 124], increasing the size, weight, and power of the spacecraft equipment. An alternative solution is to provide a radiator with dynamically variable radiative surface properties, giving the radiator the ability to adjust radiative heat loss in response to a change in operating conditions. Such a radiator would be possible with variable emissivity devices.
Several experimental variable emissivity devices exist [69, 125], including electrochromic [14–17, 25, 76, 126] and thermochromic [7, 8, 77] coatings. Utilizing changes to internal chemistry through applied voltages or temperature changes, these devices exhibit a change in emissivity as large as one order of magnitude [8]. However, these surfaces have not yet shown satisfactory performance in a vacuum setting [68, 124, 125]. As such, another approach to variable emissivity has been proposed [66, 87, 88], utilizing origami tessellations and the cavity effect.

The cavity effect describes the increased absorbing and emitting capabilities of cavities as compared to flat surfaces. Radiation entering a cavity of any given geometry may be reflected multiple times before potentially exiting the cavity. These inter-reflections result in a greater fraction of absorbed irradiation as compared to a flat surface with an equivalent size as the cavity opening and an absorptivity are equal to that of the cavity material. Likewise, the emissivity of the cavity opening is greater than the intrinsic emissivity of the cavity material. The extent of the cavity effect is quantified with apparent absorptivity and apparent emissivity [127]. Apparent emissivity for an isothermal cavity is defined as the ratio of the emitted heat rate from a real cavity opening as compared to the emitted heat rate from a similar cavity (same temperature and geometry) but with surfaces that have an intrinsic emissivity of unity. Apparent absorptivity is defined as the ratio of irradiation absorbed by the cavity compared to the total irradiation incident on the cavity opening [1].

Apparent radiative properties for a given cavity are a function of several parameters including cavity geometry, intrinsic radiative surface properties, reflection type (specular/diffuse), irradiation type, and boundary conditions [50, 104]. As such, the apparent radiative surface properties of a cavity may be controlled by changing one or more of these parameters. Physical actuation of origami tessellations may be used to achieve variation in apparent radiative properties through a changing cavity geometry [2, 66, 87, 88, 95]. For example, as an accordion tessellation (Figure 5.1) is collapsed, the aspect ratio of the V-groove cavities increases, causing the apparent emissivity and apparent absorptivity of the cavities to increase. Any origami tessellation with variable cavity geometries, such as the Miura-Ori or Barreto’s Mars tessellations [88, 106], is capable of variable emissivity through this same mechanism.

However, the apparent radiative properties of a tessellation are not the only changing variables affecting the net heat transfer rate. As a tessellation is actuated to modify the apparent
radiative properties, the apparent emitting area of the cavity openings likewise changes. For most tessellations, the apparent surface area decreases toward zero as apparent radiative properties increase, with some exceptions such as Barreto’s Mars tessellation [106]. Published works that explore origami tessellations and the cavity effect have only considered the effect of a tessellation change on apparent radiative properties and have not addressed the effect of a changing apparent surface area on net radiative heat transfer [2, 66, 95].

The purpose of this work is to quantify the net radiative heat exchange of an accordion origami tessellation as a function of actuation position. The accordion tessellation was selected to utilize the existing models that describe apparent emissivity and apparent absorptivity for an infinite V-groove [1, 2, 40, 95]. However, the analytical approach has been generalized to allow for application to any origami tessellation if a model of the apparent radiative properties can be obtained or developed. This work will consider specularly or diffusely reflecting surfaces as well as collimated or diffuse irradiation from the surroundings; such conditions are representative of the thermal environment encountered in space applications.

A general energy balance is developed to predict the net radiative heat transfer for any origami tessellations. This energy balance is then applied to the accordion tessellation. Next, an experimental method used to verify the results from the analytical model is described, and this is followed by a discussion of the uncertainties for both the analytical and experimental approaches. Results from the analytical model are presented for diffuse or collimated irradiation and for a diffuse or specular reflector, illustrating the net radiative heat transfer behavior of an accordion tessellation as a function of geometry. The analytical results are compared to the experimental results for validation in predicting net radiative heat exchange of the accordion tessellation. Finally, implications of the results for net radiative heat exchange with regard to use of the accordion tessellation as a dynamic radiator are discussed.

5.5 Analytical Methods

5.5.1 General Energy Balance

Consider an isothermal origami tessellation, consisting of an array of cavities, suspended in space that is in radiative equilibrium with the surroundings. Emission from the origami tessellation
is assumed to be diffuse but the tessellation surfaces may reflect either specularly or diffusely. The tessellation may receive irradiation from diffusely emitting surroundings and/or from collimated irradiation of a known heat flux \( G \) at some angle of incidence \( \gamma \) with respect to the apparent surface normal (Figure 5.1). Net radiative heat exchange \( q_{rad} \) between the tessellation and the surroundings is a balance of emitted \( q_e \) and absorbed \( q_{abs} \) heat transfer rates, as given in the following equation:

\[
q_{rad} = q_e - q_{abs} = \epsilon_a A_a \sigma T_a^4 - \left[ \alpha_a A_a \sigma T_{surr}^4 + \alpha_{a,proj} A_{a,proj} G_{collimated} \right]
\]  

(5.1)

The apparent absorptivity \( \alpha_a \) in Equation 5.1 accounts for the reflections and rereflections of radiative power streaming into the cavity. Likewise, emission from the opening is modeled by assigning an apparent temperature to the cavity opening with a given apparent emissivity \( \epsilon_a \) to account for the reflections and reabsorptions of emitted energy. The apparent area \( A_a \) is the
planar surface area of all tessellation cavity openings and applies to diffuse emission and diffuse irradiation, whereas \( A_{a,\text{proj}} \) is the apparent projected area of the array normal to the collimated irradiation (Figure 5.1). These apparent properties and the apparent area are functions of tessellation geometry.

### 5.5.2 Accordion Tessellation

The net heat rate for an accordion tessellation using Equation 5.1 may be obtained by defining the opening area of a single V-groove as a function of \( \phi \) and multiplying by the number of cavity openings on the top and bottom of the surface, giving the apparent area in Equation 5.2 as pictured in Figure 5.1. For the apparent projected area, the apparent area of only the top surface is multiplied by the cosine of the collimation angle of incidence, resulting in Equation 5.3. In Equations 5.2 - 5.3, \( W_p \) is the width of one panel and the panels are assumed to be of sufficient length \( (L_p) \) that end effects are neglected (e.g., \( L_p/W_p \gg 1 \)). Emission from the bottom side of the panels furthest to the left and right is neglected in this analysis.

\[
A_a = 2(N_{\text{panels}} - 1)W_pL_p \sin \left( \frac{\phi}{2} \right) \tag{5.2}
\]

\[
A_{a,\text{proj}} = N_{\text{panels}}W_pL_p \sin \left( \frac{\phi}{2} \right) \cos \gamma \tag{5.3}
\]

These area terms are then substituted into Equation 5.1. After simplification, the net radiative heat exchange for an accordion tessellation experiencing irradiation from a diffuse source and/or collimated irradiation may be described with the following equation:

\[
q_{\text{rad}} = 2W_pL_p \sin \left( \frac{\phi}{2} \right) \left[ (N_{\text{panels}} - 1)\sigma(\varepsilon_aT_a^4 - \alpha_aT_{\text{surr}}^4) - \alpha_aG \left( \frac{N_{\text{panels}}}{2} \right) \cos \gamma \right] \tag{5.4}
\]

### 5.5.3 Apparent Radiative Properties

Results for Equation 5.4 require the apparent radiative properties \((\varepsilon_a, \alpha_a)\) as a function of intrinsic emissivity \((\varepsilon)\), V-groove angle \((\phi)\), and irradiation incidence angle \((\gamma)\), where applica-
ble. Recalling that apparent absorptivity is dependent on the type of irradiation (diffuse irradiation versus collimated irradiation) and reflection (specular versus diffuse), four separate models for apparent absorptivity are necessary. Apparent emissivity is independent of the behavior of irradiation entering the cavity, and therefore, only two apparent emissivity models are required, one each for specular and diffuse reflection. Ohwada [3] has shown that the apparent emissivity of an isothermal, diffusely emitting cavity is equivalent to the apparent absorptivity of a diffusely irradiated cavity regardless of the reflection type. As such, the apparent absorptivity models for diffuse irradiation likewise describe the apparent emissivity behavior for both specular and diffuse reflection.

Recent works [2,95] have built upon early models by Sparrow and Lin [1] and Modest [38] to obtain closed form models of apparent absorptivity and apparent emissivity of V-grooves for all possible combinations of surface conditions and irradiation types. A summary of these models is given in Tables 5.1 and 5.2. These models assume infinite V-grooves with isothermal cavities in order for the apparent absorptivity and apparent emissivity to be equated through Ohwada’s proof [3]. Although real tessellations cannot extend indefinitely, V-grooves with panel length to width ratios of ten or greater ($L_p/W_p > 10$) exhibit apparent radiative properties to within 5% of infinite V-groove values [88].

With the appropriate areas defined, the net radiative heat transfer for an accordion fold can be obtained with Equation 5.4 using models for the apparent radiative properties from Tables 5.1 and 5.2. These properties may be dynamically altered through actuation to affect the geometry and positioning of the panels to achieve variable heat transfer control.

5.6 Experimental Methods

Experiments were performed to validate the models and assumptions utilized in the analytical predictions above. This experimentation was restricted to one of four possible irradiation/reflection modes, namely specular reflection with diffuse irradiation. Results from the experiment are compared to results from the analytical model to demonstrate the accuracy of the approach and models utilized.
Figure 5.2: (a) Schematic of folded stainless-steel samples with sixteen ‘folded panels’ and two ‘mounting panels’ constrained between two copper bus bars. Bus bars were positioned using a plastic supporting base with a hole pattern associated with desired angle positions and (b) schematic of the experimental setup in a vacuum. Steady-state temperatures were measured with an infrared camera through a sapphire window, while the voltage and current associated with Joule heating were measured to determine net radiative heat transfer after accounting for losses.

5.6.1 Experimental Setup

An accordion tessellation was folded from a single piece of polished 18-8 stainless-steel shim stock (0.0254 mm thick) measuring 25.4 cm by 7.62 cm. The folded test piece had 16 panels ($N_{\text{panels}}$) with each panel measuring $W_P = 1.27$ cm in width and $L_P = 7.62$ cm in length, giving a length to width ratio of 6. Mounting panels (2.54 cm by 7.62 cm) remained unfolded on both ends with half of each mounting panel constrained between two vertical copper bus bars (Figure 5.2a). The bus bars suspended the folded test piece vertically at one of the seven discrete fold angles ($\phi = \pi/9$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $11\pi/18$, $\pi$ or 20°, 30°, 45°, 60°, 90°, 110°, 180°) using an electrically and thermally insulating base with preset holes. When the tessellation was positioned for testing, calipers were used to measure the opening distance of each individual V-groove at three different vertical points. These measurements were used to find the average V-groove angle $\phi$ and standard deviation.

The sample and fixture were placed at the center of a cylindrical vacuum chamber measuring 70 cm in diameter and 76.2 cm in length, as shown in Figure 5.2b. Wire leads were bolted to each copper bus bar and connected to a power supply through a vacuum feedthrough. A 7.5 cm
diameter sapphire window on the vacuum chamber wall provided optical access to the chamber interior and a thermal camera (FLIR 6103, Wilsonville, OR) was used to measure the apparent temperature of the cavity openings. The vacuum chamber was pumped down to a pressure of $4 \times 10^{-5}$ Torr which ensured that convective heat transfer losses were negligible [128]. Upon achieving the desired vacuum conditions, the power supply was activated and electrical current, measured via a shunt resistor of known resistance, was passed through the shim stock to heat the sample via Joule heating. A data acquisition system monitored the applied voltage across the full circuit throughout testing. The resulting Joule heating in the folded sample caused the temperature to increase above ambient conditions, eventually reaching steady-state, defined as a temperature change of less than 0.5 deg K over a 1 h period.

5.6.2 Apparent Temperature Measurement

Several parameters were required in order to determine the apparent temperature of the tessellation cavity openings ($T_a$) through infrared thermography, including the transmissivity of the sapphire viewing window, temperature of the surroundings, and apparent emissivity of the tessellation. A T-type thermocouple attached to the wall of the vacuum chamber with thermal epoxy (Duralco 132, Cotronics Corp., Brooklyn, NY) was used to record the temperature of the enclosure/surroundings. The spectral transmissivity of the sapphire window, available from the manufacturer (Lesker, part number: VPZL-450DUSW, Jefferson Hills, PA), was multiplied by the blackbody spectral irradiation $G_{\lambda,b}$ and integrated over 3–5 $\mu$m (wavelength range of the thermal camera) to find the band transmissivity of the sapphire window for irradiation from heated accordion samples (Equation 5.5, assuming gray emission).

$$
\tau_{3-5\mu m} = \frac{\int_{3\mu m}^{5\mu m} \tau_{\lambda} G_{\lambda,b} d\lambda}{\int_{3\mu m}^{5\mu m} G_{\lambda,b} d\lambda} \quad (5.5)
$$
Table 5.1: Apparent emissivity and apparent absorptivity models for *specularly* reflecting V-groove surfaces

<table>
<thead>
<tr>
<th>Property</th>
<th>Conditions</th>
<th>Reference</th>
<th>Specular reflection model</th>
</tr>
</thead>
</table>
| $\epsilon_a$ | Diffuse emission for an isothermal gray surface with *specular* reflection | [2, 38] | \[\epsilon_a = \frac{\epsilon}{\sin(\phi/2)} \left[ 1 - \epsilon \sum_{k=1}^{n} \rho^{k-1} \left( 1 - \sin \left( \frac{k \phi}{2} \right) \right) \right] \]
\[n = \frac{\phi}{2} \text{ where } n \text{ is the integer portion}^a \text{ of } \phi/\pi\] |
| $\alpha_a$ | Diffuse irradiation on a gray surface with specular reflection | [3, 38] | Same as above |
| $\alpha_a$ | Collimated irradiation with full-illumination ($\phi/2 \geq \gamma$) on a gray surface with specular reflection | [1, 2] | \[\alpha_a = \frac{1 - (1 - \alpha'X)(1 - \alpha)^n - 1}{\frac{\phi}{2} \sin(\phi/2)} \sin \left( \frac{\phi}{2} + \gamma \right) + \frac{1 - (1 - \alpha'Y)(1 - \alpha)^m - 1}{\frac{\phi}{2} \sin(\phi/2)} \sin \left( \frac{\phi}{2} - \gamma \right) \]
\[n = \frac{\pi - \gamma}{\phi} + \frac{1}{2} \quad m = \frac{\pi + \gamma}{\phi} + \frac{1}{2} \quad \text{where } n \text{ and } m \text{ are rounded down}^a\] |
\[X' = \frac{\sin \left( (n - 1/2) \phi + \gamma \right)}{\sin(\phi/2 + \gamma)} \quad Y' = \frac{\sin \left( (m - 1/2) \phi - \gamma \right)}{\sin(\phi/2 - \gamma)} \] |
| $\alpha_a$ | Collimated irradiation with partial-illumination ($\phi/2 < \gamma$) on a gray surface with specular reflection | [2] | \[\alpha_a = 1 - (1 - \alpha'X')(1 - \alpha)^n - 1 \]
\[n < \frac{\pi - 2\gamma}{\phi} + 1 \text{ where } n \text{ is rounded down} \]
\[X' = \frac{\sin \left( (n - 1/2) \phi + \gamma \right)}{\sin(\phi/2 + \gamma)} \quad X'' = \frac{\sin \left( (m - 1/2) \phi - \gamma \right)}{\sin(\phi/2 - \gamma)} \]
\[\text{If } X' < X'', \text{ then: } X' = 0; \text{ If } X' > X'', \text{ then: } X' = \frac{X' - X''}{1 - X''} \] |

\(^a\) The number of reflections ($n$ and/or $m$) must be rounded down to the nearest whole number; if the calculation of $n$ and/or $m$ yields a whole number, that value should be deprecated by unity.
Table 5.2: Apparent emissivity and apparent absorptivity models for diffusely reflecting V-groove surfaces. Infinite summations should be carried out to at least twenty terms for accuracy [2].

<table>
<thead>
<tr>
<th>Property</th>
<th>Conditions</th>
<th>Reference</th>
<th>Specular reflection model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_a$</td>
<td>Diffuse emission for an isothermal gray surface with diffuse reflection</td>
<td>[95]</td>
<td>$\varepsilon_a = \alpha_a = \varepsilon \Lambda_1(\varepsilon, \phi) \sum_{n=0}^{\infty} (1 - \varepsilon)^n \left[1 - \sin \left(\frac{\phi}{2}\right)\right]^n$</td>
</tr>
<tr>
<td></td>
<td>Approximate with $&gt; 20$ terms</td>
<td></td>
<td>$\Lambda_1(\varepsilon, \phi) = 1 - (0.0169 - 0.1900 \ln(\varepsilon)) \exp(-1.4892 \varepsilon^{-0.4040} \phi)$</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Diffuse irradiation on a gray surface with diffuse reflection</td>
<td>[3]</td>
<td>Same as above</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Collimated irradiation with full-illumination ($\phi/2 \geq \gamma$) on a gray surface with diffuse reflection</td>
<td>[95]</td>
<td>$\alpha_a = 1 - \Lambda_2(\alpha, \phi, \gamma) \sum_{n=0}^{\infty} (1 - \alpha)^{n+1} \left[1 - \sin \left(\frac{\phi}{2}\right)\right]^n \sin \left(\frac{\phi}{2}\right)$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_2(\alpha, \phi) = 1 - (0.0169 - 0.1900 \ln(\alpha)) \exp(-1.4415 \alpha^{-0.4240} \phi)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Collimated irradiation with partial-illumination ($\phi/2 &lt; \gamma$) on a gray surface with diffuse reflection</td>
<td>[95]</td>
<td>$\alpha_a = 1 - \Lambda_3(\varepsilon, \phi, \gamma) \left[(1 - \alpha)F_{a-c} + \sum_{n=2}^{\infty} (1 - \alpha)^n (1 - F_{a-c}) \sin \left(\frac{\phi}{2}\right) \left(1 - \sin \left(\frac{\phi}{2}\right)\right)^{n-2}\right]$</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_3(\alpha, \phi, \gamma) = D - E \exp(G \phi)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D = 0.0345 \gamma^{-1.1447} \alpha^2 - 0.0414 \gamma^{-0.8573} \alpha + 1 - 1.7702 \exp(-18.0990 \gamma)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E = -3.2301 \exp(-1.1420 \gamma) \exp(-2.6635 \gamma^{-0.0370} \alpha)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$G = -2.2780 \gamma^{-0.5690} \alpha^{0.1330} \gamma^{-0.2372} \gamma^{-0.5434}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using the spectral transmissivity from the manufacturer, the calculated band transmissivity (3–5 µm) of the window ranged from 0.81 to 0.85 for irradiation values based on source temperatures of 323 K and 423 K, respectively. A transmissivity of 0.83 was used for measurements as this corresponded to the average steady-state temperature of the sample for all tested conditions (373 K). The impact of the transmissivity variation on the infrared measurement of the sample was accounted for by incorporating possible transmissivity variation with temperature into the infrared measurement uncertainty.

To verify the transmissivity value obtained using Equation 5.5, a laboratory blackbody emitter (Landcal R1200P, Dronfield, UK) with an emissivity of 0.97 was placed directly behind the sapphire window, with the blackbody cavity opening oriented parallel to the sapphire window and heated to 573 K. The thermal camera was placed on the opposing side of the sapphire window and focused to the center of the blackbody cavity. The camera emissivity parameter was set to 0.97 and the external optics transmissivity setting in the camera software was varied until the temperature measurement of the camera was identical to the set temperature of the blackbody. With the blackbody emitter set to 573 K, the external optics transmissivity setting in the camera software which gave a measured temperature of 573 K with the thermal camera was also 0.85, equivalent to that obtained from Equation 5.5.

To determine the apparent emissivity of the tessellation, the V-groove angle and the intrinsic absorptivity of the stainless-steel surface were used in the specularly reflecting, diffusely emitting V-groove apparent emissivity model [38] as reported in Table 5.1. The intrinsic, total hemispherical emissivity of the stainless-steel was measured three times with an SOC-100 emissometer and then averaged to give a value of 0.117. The obtained apparent emissivity value was used as the emissivity parameter in the thermal camera software, correcting the temperature of the cavity to account for the cavity effect.

Upon reaching steady-state conditions, temperature data were collected using the thermal camera across the vertical center of all 8 V-groove openings comprising the accordion fold. The temperature profile over all V-grooves was averaged to eliminate local irregularities (due to the specular nature of the surfaces), giving the average emitting temperature of the accordion tessellation for a given power input. Temperature profiles were collected at seven discrete total power levels (3.0, 3.5, 4.0, 4.5, 5.0, 5.5, and 6.0 W) for each angle mentioned previously.
The apparent emissivity model used in this study assumes isothermal panels comprising the cavity. Experimentally, the isothermal condition was not imposed exactly. However, the alternating nature of the accordion fold (i.e., the vertex of a V-groove on one side is the peak protrusion on the opposite side) likely results in a relatively uniform heat loss from any position on a panel, assuming a linear heat loss profile along the length of a panel [56]. This fact, combined with the use of a thermally conductive material, suggests that the panel surfaces are near isothermal when heated.

5.6.3 Experimental Losses

The results of the idealized analytical model (Equation 5.4) must account for experimental losses to enable a comparison between experimental and analytical results. To this end, an energy balance of the experimental setup, using the control volume indicated in Figure 5.2b, is defined in the first equality of Equation 5.6. The net radiative heat transfer can be obtained with this energy balance by measuring the total power dissipation in the circuit \( P_t \) and quantifying the electrical power dissipated outside of the sample control volume \( P_{loss} \) as well as the heat transferred from the sample control volume by conduction into the copper bus bars \( q_{loss} \). Equation 5.6 is rearranged to give the cavity surface temperature as a function of circuit power, experimental loss, and cavity parameters as shown in Equation 5.7.

\[
q_{rad} = P_t - P_{loss} - q_{loss} = 2(N_p - 1)W_P L_P \varepsilon_a \sigma \sin \left( \frac{\phi}{2} \right) (T_a^4 - T_{surr}^4) \tag{5.6}
\]

\[
T_a = \left[ \frac{P_t - P_{loss} - q_{loss}}{2(N_p - 1)W_P L_P \varepsilon_a \sigma \sin \left( \frac{\phi}{2} \right)} + T_{surr}^4 \right]^{1/4} \tag{5.7}
\]

Electrical power losses outside of the sample control volume \( P_{loss} \) and nonradiative heat losses from the sample control volume \( q_{loss} \) must be calculated to obtain the apparent surface temperature using Equation 5.7. Each of these losses was determined experimentally, as outlined below.
Power Losses

Electrical heating that occurs outside of the sample control volume must be quantified and subtracted from the total power dissipation to determine the electrical heating present in the stainless-steel sample. Power dissipation in the supporting circuitry is quantified with a sample heating efficiency (\(\eta\)), or the ratio of power dissipated within the control volume to the total power. In selecting a method to determine the heating efficiency, it was important to consider the resistive losses in the wires as well as the resistance losses in the electrical contacts including between the copper bus bars and the stainless-steel sample. To determine the power dissipation within the sample alone, an unfolded stainless-steel sample of length 30.5 cm was secured within the fixture with the power supply set to a voltage of 1 V. The resulting current (measured at the power supply) and total circuit resistance were recorded. The sample was then removed from the fixture and 2.5 cm of its length was cut from one end. The shortened sample was then secured again into the fixture, and the resulting current and resistance at 1 V were again recorded. This process was repeated until the total sample length between bus bars was reduced to approximately 1.5 cm. The total resistance of the circuit was then plotted as a function of sample length and a linear fit to this data (\(R^2 = 0.994\)) provided the relationship between total circuit resistance and sample length. The y-intercept of the linear fit gives the resistance of the circuit for a theoretical sample length of zero, or the resistance of the circuit outside of the control volume pictured in Figure 5.2b, including the contact resistance. The heating efficiency is defined as the ratio of power dissipated in the stainless-steel sample (\(P_s\)) divided by the total power dissipated in the full circuit (\(P_t\)), given by Equation 5.8. Since the current is the same throughout the full circuit, the heating efficiency can be expressed as a ratio of resistances, as shown in Equation 5.8, where \(R_s\) is the resistance of the stainless-steel sample and \(R_t\) is the resistance of the full circuit. For a sample length of 25 cm, used in all radiative experiments, the heating efficiency was found to be 61%. The heating efficiency multiplied by the total circuit power (\(P_t\)) gives the rate of heat generation within just the sample material, \(P_s\).

\[
\eta = \frac{P_s}{P_t} = \frac{I^2 R_s}{I^2 R_t} = \frac{R_s}{R_t} \tag{5.8}
\]
The heating efficiency ($\eta$) gives the ratio of power dissipated in the sample (including the mounting panels shown in Figure 5.2a) as compared to the total circuit power. Since the mounting panels are not included in the control volume of Equation 5.6 (as shown in Figure 5.2b), the heat generated within the mounting panels must also be subtracted from the circuit power to give the heat generation within just the control volume. The sample length ratio ($\chi$) is the ratio of sample volume that resides within the control volume (i.e., folded panels) to the total sample volume (i.e., folded and mounting panels). For a sample length of 25 cm, $\chi = 91\%$. The heat generated within just the sample control volume (or $P_t - P_{\text{loss}}$) may be determined by multiplying the heating efficiency ($\eta$) and sample length ratio ($\chi$) by the total circuit power.

**Thermal Losses**

The thermal camera was used to quantify heat lost from the accordion sample by conduction through the mounting panels. The sample fold pattern and fixturing device were designed such that the sample contained a flat portion (1.25 cm in length) immediately adjacent to the copper bus bars on either side before the tessellation folds began (Figure 5.2a). The flat mounting panels are exterior to the sample control volume for the analytical model (Figure 5.2b). To find the conduction loss through these panels, the thermal camera recorded the temperature profile along a mounting panel at the control volume edge for use in Fourier’s law. This conduction loss was then subtracted from the total heating power (Equation 5.6).

The power and heat loss terms just described are applied to Equation 5.7, giving the final form for the prediction of the apparent surface temperature, Equation 5.9. This predicted temperature was compared against the value obtained from the thermal camera. For this equation, $k_{SS}$ is the thermal conductivity of 18-8 stainless-steel at 350 K (15 W m$^{-1}$ K$^{-1}$) and $A_{\text{cond}}$ is the heat conduction area of the sample (twice the cross-sectional area of the shim).

$$T_a = \left[ \frac{\eta \chi P_t - k_{SS}A_{\text{cond}}}{2\epsilon \sigma W_p L_p N_p \sin \left( \frac{\phi}{2} \right)} \frac{dT}{dx} + T_{\text{surr}}^4 \right]^{1/4} (5.9)$$
5.7 Uncertainty

Subsections 5.7.1 and 5.7.2 outline the uncertainty estimations for the experimental temperature measurement made by the thermal camera and the analytical model (Equation 5.9).

5.7.1 Thermal Camera Uncertainty

The thermal camera uncertainty is a function of the uncertainties of the emissivity measurement, the transmissivity of the sapphire window, the temperature of the surroundings, and the inherent uncertainty within the camera. The total uncertainty of the camera measurements was calculated through a modification of the method of sequential perturbation, as follows [129]. A sample is heated in one of two extreme configurations: (1) an unfolded sample heated with the lowest tested power level (3 W) and (2) a folded sample positioned at the smallest fold angle (20 deg) heated at the largest power level (6 W). For both testing scenarios, each parameter (emissivity, transmissivity, and surrounding temperature) is adjusted individually in the camera software upward and then downward by the associated uncertainty of that parameter. The resulting change in the temperature readout is recorded, and the increase and decrease in temperature for a given parameter is averaged to find the camera temperature uncertainty for that parameter. For this work, the uncertainty of the surroundings is taken to be the uncertainty of a T-type thermocouple (±1 °C); emissivity uncertainty is taken to be the uncertainty of the intrinsic emissivity reflectometer measurement (±0.006); and the sapphire window uncertainty is equivalent to the difference between the highest and lowest transmissivity values calculated with Equation 5.5 at the extreme sample temperatures encountered in the experiment (±2%). The inherent uncertainty of the camera was also accounted for, with manufacturer specifications indicating that the camera uncertainty is 2% of the temperature value. For these uncertainties, the greatest overall apparent temperature uncertainty is observed with the sample folded to π/9 (20 deg) and heated with the highest power level with a value of 3.25 °C. This uncertainty value is used as the uncertainty of the camera apparent temperature measurement for all power levels and cavity angles.
5.7.2 Analytical Model Uncertainty

The overall uncertainty of temperatures obtained using Equation 5.9 is also calculated with the method of sequential perturbation. Essentially, each parameter is individually increased or decreased by its given uncertainty value to determine the sensitivity of the results of Equation 5.9 with respect to each individual parameter. The root sum square of all sensitivity indices gives the total uncertainty of Equation 5.9. The uncertainty of each parameter is listed in Table 5.3 along with the source from which this uncertainty is derived/obtained. Parameters not listed in Table 3 but present in Equation 5.9 do not have an associated uncertainty.

Several of the parameters listed in Table 5.3, including $P_t$, $dT/dx$, $\eta$, $\epsilon_a$, and $\chi$, are calculated from combinations of more basic measurements. The uncertainties of these values are calculated with the root-mean-squared (RMS) method, as designated in Table 5.3 [129]. Uncertainty values given in Table 5.3 marked with an asterisk vary with each power/angle combination, with the reported value being the maximum uncertainty.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uncertainty (±)</th>
<th>Units</th>
<th>Sources/notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{surr}$</td>
<td>1</td>
<td>K</td>
<td>Manufacturer provided, T-type thermocouple</td>
</tr>
<tr>
<td>$W_P$</td>
<td>$1.3 \times 10^{-5}$</td>
<td>m</td>
<td>Caliper resolution uncertainty</td>
</tr>
<tr>
<td>$L_P$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>m</td>
<td>Ruler resolution uncertainty</td>
</tr>
<tr>
<td>$k_{SS}$</td>
<td>0.25</td>
<td>W m$^{-1}$ K$^{-1}$</td>
<td>Variance between 50 and 150°</td>
</tr>
<tr>
<td>$P_t$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>W</td>
<td>RMS propagation of voltage and current measurement uncertainties</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.005</td>
<td>-</td>
<td>Standard deviation of three emissivity measurements</td>
</tr>
<tr>
<td>$\epsilon_a$</td>
<td>0.01</td>
<td>-</td>
<td>RMS propagation of $\phi$ and $\epsilon$ uncertainty</td>
</tr>
<tr>
<td>$dT/dx$</td>
<td>55.67$^a$</td>
<td>K m$^{-1}$</td>
<td>RMS propagation of camera temperature measurements and length measurements</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$3.80 \times 10^{-7}$</td>
<td>-</td>
<td>RMS propagation of line of best fit slope and intercept uncertainties</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.015</td>
<td>-</td>
<td>RMS propagation of caliper measurement uncertainty</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$0.055^a$</td>
<td>radians</td>
<td>Standard deviation of 24 angle measurements</td>
</tr>
</tbody>
</table>

$^a$ Uncertainties that vary with circuit power and cavity angle
5.8 Results

5.8.1 Net Radiative Heat Transfer

Results for the net radiative heat transfer from an accordion fold are normalized by the net radiative heat exchange at a cavity angle of \( \pi \) (flat condition), as given in Equation 5.10, in order to allow for comparison across intrinsic emissivities.

\[
\Pi = \frac{q_{\text{rad}, \phi}}{q_{\text{rad}, \phi = 180 \text{deg}}} \tag{5.10}
\]

Figures 5.3a - 5.3c provide the normalized results of Equation 5.4 evaluated for four different scenarios across the complete angle range: (1) diffuse reflector and diffuse irradiation, (2) specular reflector and diffuse irradiation, (3) diffuse reflector and collimated irradiation, and (4) specular reflector and collimated irradiation. For all results provided here, a collimated irradiation flux \( G \) of 1360 W/m\(^2\) was used as an estimation of solar irradiation [130].

Figure 5.3a provides the normalized net radiative heat exchange as a function of \( \phi \) for diffuse irradiation incident on a cavity with either diffuse or specular reflection for two different intrinsic emissivity values. To visualize the impact of the cavity effect, the net radiative heat transfer for a flat, black surface equivalent in size to \( A_a(\phi) \) is provided. As the cavity angle decreases toward zero, the net radiative heat exchange for both specular and diffuse reflection decreases to zero. This indicates that the decrease in projected surface area always dominates relative to the increase in apparent radiative properties with decreasing cavity angle. For cases where the cavity effect is especially pronounced (i.e., highly reflective surfaces), the increase in apparent radiative properties and the decrease in surface area are nearly balanced, resulting in a relatively constant net radiative heat exchange for a large angle range. This behavior significantly concentrates the change in net radiative heat exchange to small cavity angles. As an example, a specularly reflecting tessellation with an intrinsic emissivity of 0.2 experiences a turndown ratio (ratio of largest to smallest heat transfer) of 7.3 as the V-groove angle collapses from \( \pi/6 \) (30°) to \( \pi/60 \) (3°). For diffuse reflectors, the heat transfer reduction capability is diminished; a diffusely reflecting surface with an intrinsic emissivity of 0.2 has a turn-down ratio of 5.9 over the same cavity angle range. As the intrinsic emissivity of the surface increases toward black behavior, the net radiative heat trans-
Figure 5.3: (a) Normalized net radiative heat exchange $\Pi$ as a function of $\phi$ for a diffusely irradiated accordion tessellation with specular or diffuse reflection for two different intrinsic emissivities. The “flat” case indicates a flat, black surface, equivalent in size to the apparent area at a given cavity angle, (b) normalized net radiative heat exchange $\Pi$ as a function of $\phi$ for a diffusely reflecting accordion tessellation with collimated irradiation incident on the accordion tessellation at several angles $\gamma$, and (c) normalized net radiative heat exchange $\Pi$ as a function of $\phi$ for a specularly reflecting accordion tessellation with collimated irradiation incident on the accordion tessellation at several angles $\gamma$. 

91
fer curve approaches that of the shrinking flat surface, decreasing the turn-down ratio potential for this angle range even further. For an intrinsic emissivity of 0.5, the tessellation is characterized by a turn-down ratio of 4.8 (specular) and 3.2 (diffuse) over the range of $\pi/6$ (30°) to $\pi/60$ (3°).

The normalized net radiative heat transfer for collimated irradiation and diffuse reflection is given in Figure 5.3b for $\varepsilon = 0.2$ and four different collimation angles $\gamma$. The characteristic flat surface data are again plotted for comparison. When the cavity is fully illuminated ($\phi/2 \geq \gamma$), the normalized net radiative heat exchange for a given cavity angle is nearly equivalent across all possible collimation angles, giving behavior similar to the curve indicated by $\gamma = 0$ in Figure 5.3b. However, when the V-groove is partially illuminated ($\phi/2 < \gamma$), the normalized net radiative heat exchange increases above the fully illuminated case. This change in net radiative heat transfer behavior is caused by a rapid decrease in absorbing surface area with the onset of partial illumination, reducing the absorbing capability of the cavity. This effect delays the reduction in net radiative heat transfer as the cavity angle is collapsed, meaning that additional actuation is necessary to achieve the turn-down ratios experienced by the surfaces in Figure 5.3a. As the irradiation ($G$) increases, the possible turn down ratios likewise increase; the opposite behavior is observed as $G$ is reduced. The normalized heat transfer may fall into negative values if the irradiation becomes sufficiently large ($G > 1800$), indicating that the net flow of energy is into the tessellation. Note that in Figure 5.3b, the horizontal axis is plotted over a reduced range ($\phi < 2\pi/3$) to magnify the behavior of the normalized net radiative heat transfer in the small angle range. The noncontinuous jump of $\Pi$ observed when $\phi/2 = \gamma$ indicates discrepancies between the models for partial and full illumination at this transition (Table 5.2).

Results for the condition of collimated irradiation with specular reflection are depicted in Figure 5.3c for a surface intrinsic emissivity of 0.2 evaluated at collimation angles of 0, $2\pi/9$ ($40°$), and $4\pi/9$ ($80°$). The combination of specular surfaces with collimated irradiation results in groups of parallel rays that follow similar reflection patterns [2]. Of the rays that are not absorbed by the cavity walls, a fraction ($X'$ in Table 5.1) will experience $n$ total reflections before exiting the cavity, while the remaining unabsorbed rays ($1 - X'$) will experience one less reflection ($n - 1$) before exiting the cavity [2]. The number of reflections $n$ must be a whole number, and therefore, varies discretely as a function of V-groove angle. Likewise, the fraction $X'$ is a function of $n$ and also varies discretely as a function of V-groove angle. Further, $X'$ cannot exceed unity.
as it is a fraction by definition. Therefore, \( X' \) must be rounded down whenever the computed value is greater than one, resulting in an additional source of noncontinuous behavior. When fully illuminated \((\phi/2 \geq \gamma)\), the computed value of \( X' \) (via the equation given in Table 5.1) sometimes exceeds unity and the rounding operation, combined with the discrete nature of \( n \), causes the net radiative heat transfer to vary in a noncontinuous fashion. This discontinuous behavior can appear dramatic (e.g., \( \gamma = 0 \)), or exhibit smaller, “noisy” variations (e.g., \( \gamma = 2\pi/9 \) near \( \pi = \pi/2 \)). When partially illuminated \((\phi/2 < \gamma)\), the computed value of \( X' \) does not exceed unity, eliminating the noncontinuous influence of the rounding operation. For this case, the discrete variations in \( n \) and the resulting change in \( X' \) are coordinated such that the net radiative heat transfer varies in a continuous fashion, as shown by the \( 4\pi/9 \) case in Figure 5.3c.

Unlike all other possible combinations of surface/irradiation conditions, specular reflection with collimated irradiation causes the net radiative heat transfer to drop below the characteristic flat case for small collimation angles. Further, specularly reflecting cavities with collimated irradiation experience drastic changes in net radiative heat transfer with respect to cavity angle, with larger variations occurring for small collimation angles and highly reflective surfaces. As an example, for irradiation normal to the surface \((\gamma = 0)\) and an intrinsic surface emissivity of \( 0.2 \), the net radiative heat transfer experiences a turn-down ratio of 3.35 between the cavity angles of \( 2\pi/3 \) (120°) and \( \pi/2 \) (90°). Although this turn-down ratio is small compared to the turn-down ratios seen in Figure 5.3a, this reduction in heat transfer occurs at much larger angles (\( 2\pi/3 \) compared to \( \pi/6 \)), resulting in a significant variation in heat transfer. Again, as the collimated irradiation flux \((G)\) increases, the turn-down ratio likewise increases, and the normalized heat transfer may become negative in the small V-groove angle range. The presence of specular reflections also introduces normalized net radiative heat transfer values greater than unity in locations where the increase in apparent emissivity dominates the decrease in apparent surface area (e.g., \( \phi > 2\pi/3 \) and \( \gamma = 0 \)).

### 5.8.2 Surface Temperature

For a thin, stainless-steel sample heated resistively in a vacuum environment, a typical infrared image and associated temperature profile is given in Figure 5.4a. Irregularities in temperature are observed near fold locations resulting in peaks or troughs in the temperature profile. These irregularities are due to warping and bending of the material near each bend and the spec-
Figure 5.4: (a) Temperature profile for a sample positioned at $\phi = \pi/2$ with a total heating power of 3 W. The thermal image from which this profile is derived is displayed behind the temperature profile, where the temperature profile was measured along a horizontal line across the vertical center of the tessellation and (b) comparison of experimental and analytical model temperature values. Experimental temperature measurements are derived from a thermal image of the heated surface (e.g., Figure 5.4a). Predicted temperature values are determined with the analytical approach of Equation 5.9. Data from two of the seven tested fold angles are provided with the uncertainties of each measurement.

ular behavior of the sample. Likewise, the average temperatures of individual cavities do not appear equivalent due to variation in cavity angles across the sample and discrepancies in cavity orientation with respect to the camera lens. Further, the V-groove cavities on the left-most or right-most edges show a smaller average temperature as compared to cavities in the middle region which is likely due to the presence of conductive heat losses. To mitigate these various effects, the temperature profile was averaged over all V-grooves comprising the accordion fold to obtain the steady-state apparent temperature measurement for a given cavity angle and circuit power.

The average steady-state temperature was measured with the thermal camera and calculated using Equation 5.9 for seven different accordion fold angles at six power levels each. The results for both methods, with uncertainties, are given in Figure 5.4b for the smallest and largest cavity angles ($\pi/9$ and $\pi$, respectively), where the $x$-axis of Figure 5.4b is $q_{rad}$ as calculated using Equation 5.6. Table 5.4 gives the difference between the calculated apparent temperature using
Equation 5.9, the measured apparent temperature from the thermal camera ($\Delta T_a$), and the percent difference between these two values, averaged across all tested total power levels for the seven cavity angles. The average uncertainties of the camera measurement and Equation 5.9 are also provided in Table 5.4. The temperature results of both methods for all test cases fall within the bounds of uncertainty, as depicted in Figure 5.4b.

The uncertainty of the camera measurement is constant for all total power levels and cavity angles at a value of 3.25 °C. The uncertainty of Equation 5.9, however, varies with total circuit power and cavity angle (increasing uncertainty for larger power levels and for decreasing cavity angle). The largest uncertainty is ±15.2 deg K at a total power of 6W and a cavity angle of $\pi/9$. The average prediction uncertainty across all cavity angles and power levels is ±6.1 deg K with a standard deviation of 3.74 deg K.

Table 5.4: Temperature error and uncertainties associated with each tested cavity angle averaged over all tested power levels. Average temperature difference (Average $\Delta T_a$) between the analytical model result (Equation 5.9) and the thermal camera temperature measurement and the associated average % difference (relative to the camera measurement) are reported.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>Average % difference</th>
<th>Average $\Delta T_a$ (deg K)</th>
<th>Average uncertainty (Equation 5.9) (deg K)</th>
<th>Average uncertainty (camera) (deg K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 deg</td>
<td>0.4</td>
<td>0.46</td>
<td>13.44</td>
<td>3.25</td>
</tr>
<tr>
<td>30 deg</td>
<td>0.8</td>
<td>0.89</td>
<td>9.26</td>
<td>3.25</td>
</tr>
<tr>
<td>44 deg</td>
<td>3.3</td>
<td>3.76</td>
<td>4.96</td>
<td>3.25</td>
</tr>
<tr>
<td>56 deg</td>
<td>4.2</td>
<td>4.72</td>
<td>5.30</td>
<td>3.25</td>
</tr>
<tr>
<td>89 deg</td>
<td>1.2</td>
<td>1.23</td>
<td>4.00</td>
<td>3.25</td>
</tr>
<tr>
<td>109 deg</td>
<td>3.4</td>
<td>3.23</td>
<td>2.97</td>
<td>3.25</td>
</tr>
<tr>
<td>180 deg</td>
<td>2.7</td>
<td>0.03</td>
<td>2.70</td>
<td>3.25</td>
</tr>
</tbody>
</table>

5.8.3 Discussion

As illustrated in Table 5.4 and Figure 5.4b, the net radiative heat transfer model developed in this work agrees with experimental results with a worst-case error of 4.2% averaged over all power levels. Likewise, Equation 5.9 and thermal camera temperature measurements for all tested cavity angles and total powers agree within the uncertainties of both methods. This validates the
results of the thermal model and suggests that the generic model is correct in predicting the net radiative heat exchange of an origami tessellation. This modeling approach may be used to predict the net radiative heat exchange of an isothermal, accordion tessellation when applied to a variety of thermal control scenarios and applications.

One purpose for this technology may be to provide variation in total net radiative heat transfer for thermal management. To this end, an efficient variable radiator should (1) provide a large change in net radiative heat transfer, (2) achieve this change in net radiative heat transfer over a small actuation range, and (3) minimize the material size and weight required to meet a given power requirement.

With regard to the first requirement, we see in Figures 5.3a - 5.3c that the net radiative heat exchange of an accordion tessellation may be varied from the full normalized value ($\Pi = 1$) to a value approaching zero, a significant variation in heat transfer, no matter how the material reflects or how the environment is heating the surface (diffuse or collimated irradiation). This versatile behavior is useful in applications where collimated and diffuse irradiation are both sometimes present, such as spacecraft thermal control.

Second, diffusely reflecting, high-emissivity ($\varepsilon > 0.5$) surfaces exposed to either diffuse or collimated irradiation exhibit a gradual change in net radiative heat transfer with changing cavity angle (Figures 5.3a - 5.3b). This slow change in heat transfer with position may not be ideal for applications that require rapid variation in heat transfer. However, the net radiative heat transfer of diffusely reflecting, low-emissivity ($\varepsilon < 0.2$) surfaces increases rapidly in the small angle ranges, achieving a turn-down ratio of 7.4 between the angles of $\pi/60$ to $\pi/6$. As such, reflecting surfaces may be more suited to applications requiring rapid variations in heat transfer. With regard to collimated irradiation and specular reflection (Figure 5.3c), highly reflective surfaces ($\varepsilon < 0.2$) exposed to normal or near-normal collimated irradiation ($\gamma = 0$) show significant heat transfer variation, achieving a turn-down ratio of 3.35 ($\varepsilon = 0.2$) for a range of relatively large V-groove angles $2\pi/3 - \pi/2$ ($120^\circ - 90^\circ$). As the collimation angle increases, this same turn-down capability is possible but for a range of angles much closer to a fully collapsed condition ($\pi/6 - \pi/60$).

Finally, an efficient variable radiator should require minimal volume and mass to emit a given heat load. As an example, diffusely reflecting, highly reflective surfaces ($\varepsilon < 0.2$) show significant variation in heat transfer for very small cavity angles ($\phi < \pi/6$, Figure 5.3a). However,
the emissivity of these surfaces is quite low and the apparent area is very small compared to the total area, indicating that significantly large radiators would be required to reject a given heat load. The low emissivity values found in most of the highly variable surfaces and the existence of rapid heat transfer decrease in very small cavity angles is an impediment to utilizing these devices in an application such as aerospace where weight must be minimized. However, a specular surface exposed to normal collimated irradiation exhibits significant variation for large cavity angles ($\phi > \pi/2$) even as the emissivity is increased to larger values (e.g., $\varepsilon > 0.5$). As such, specular surfaces utilized in the presence of collimated irradiation show significant abilities to vary net radiative heat transfer while minimizing the weight necessary to reject a given heat load. Finally, a perfectly black surface shows significant variation in net radiative heat transfer while minimizing the amount of area required to emit a given heat load although the surface must be actuated over the full range of cavity angles to achieve this variation.

These results also indicate the utility of an actively controlled origami surface in maintaining or controlling the apparent emitting temperature of a surface. As shown in Figure 5.4b and the net radiative heat transfer curves in Figures 5.3a - 5.3c, the emission from an adapting surface may be tailored such that the infrared signature of the surface and the apparent temperature of the surface may be controlled to a desired value. This includes maintaining a surface at a constant apparent temperature or varying the apparent temperature so as to hide the true emitting temperature of the surface.

The accordion tessellation may also be utilized in a constant-area scenario. In this case, only a portion of the fold is exposed to the surroundings, while the remainder may be shielded. As the fold collapses or expands, the viewing window between the tessellation and the surroundings remains the same. This behavior causes the area terms of Equation 5.1 to remain constant, removing the effect of the collapsing area from the net radiative heat transfer. This scenario would cause the net radiative heat transfer to increase as the cavity angle is decreased. A disadvantage with this application is the requirement to store tessellation material without exposing it to the surroundings, potentially increasing the weight of the device substantially.

Several other origami tessellations are of possible interest as variable radiators [88]. One such tessellation is Barretto’s Mars, a fold which collapses to one side, resulting in directional behavior that is concentrated to non-normal angles. This tessellation collapses to a finite area,
eliminating the decrease of net radiative heat transfer to zero. Another possible tessellation is a modified, constant-projected area accordion tessellation. This tessellation maintains a given projected area throughout its actuation but can only collapse to a finite cavity angle, limiting its increase in net radiative heat transfer.

5.9 Conclusions

An expression to predict the net radiative heat transfer of an isothermal accordion origami tessellation (Equation 5.4), after accounting for experimental losses and averaging over all tested power levels (Equation 5.7), exhibits a 4.2% relative error or less when compared with experimental results. This equation may be used for all possible cavity angles, collimation angles, and intrinsic radiative surface properties as well as for diffuse or specular reflection and collimated and/or diffuse irradiation. A survey of the analytical results indicates that a specularly reflecting tessellation exposed to diffuse irradiation experiences large reductions in net radiative heat transfer for small cavity angle variations, achieving turn-down ratios greater than seven over a cavity angle range of $\pi/6$ for highly reflective surfaces. Diffuse reflectors with low intrinsic emissivities behave in a similar fashion but exhibit smaller turn-down ratios over a similar cavity angle range. When exposed to collimated irradiation, the turn-down ratio of diffuse reflectors is further reduced over the same cavity angle range. However, when exposed to collimated irradiation, specular surfaces experience erratic behavior, with widely varying net radiative heat transfer over relatively small changes in cavity angle for surfaces with low to moderately high emissivity. For this scenario, turn-down ratios of 3.35 and greater are possible for large cavity angles ($\phi > \pi/2$). When actuated dynamically, an accordion tessellation would provide active control of radiative heat losses, allowing for real-time thermal control in scenarios where radiative heat transfer is dominant.
6.1 Contributing Authors and Affiliations

Rydge B. Mulford, Matthew R. Jones, and Brian D. Iverson
Department of Mechanical Engineering, Brigham Young University, Provo, Utah, 84602

6.2 Abstract

Efficient heat transfer is critical in the design and optimization of thermal control systems. Static radiative heat exchangers are often simple and reliable systems but typically cannot be adapted to environmental changes. Adaptable radiative heat exchangers can be adjusted in response to variations in the thermal environment or operating conditions and have the potential for increased efficiency and reduced cost. Dynamic control of a radiative heat exchanger is possible through geometric manipulation of a segmented, self-irradiating fin, consisting of rigid panels that are linked by thermal hinges in an accordion arrangement. In this paper, a numerical model is described to predict the temperature profile and efficiency of a radiative heat exchanger, accounting for conduction and self-irradiation. Governing equations are cast in terms of the conduction-radiation interaction parameter, surface emissivity, actuation angle, and the thermal conductance of the hinges linking the panels. Results indicate that a turn-down ratio (largest possible heat rate divided by smallest possible heat rate) of greater than three is possible for realistic panel geometries and materials. Self-irradiation decreases the turn-down ratio, and there is evidence that an optimal number of rigid panels exists for any combination of panel geometry and device temperature. The maximum efficiency occurs when the plates are in the collapsed position, but the heat rate is at a minimum in this configuration. Finally, the properties and geometry of the plates are shown to have a more significant effect on the turn-down ratio than the properties of the thermal hinges.
6.3 Nomenclature

\begin{itemize}
  \item \textbf{A} coefficient matrix
  \item \textbf{b} source term column vector
  \item \textbf{Bi} biot number
  \item \textbf{F} view factor
  \item \textbf{J} radiosity (W m\(^{-2}\))
  \item \textbf{J*} non-dimensional radiosity
  \item \textbf{k} thermal conductivity of panel material (W m\(^{-1}\) K\(^{-1}\))
  \item \textbf{\kappa} hinge thermal conductance (W m\(^{-2}\) K\(^{-1}\))
  \item \textbf{L} panel length (m)
  \item \textbf{N} number of finite elements in one panel
  \item \textbf{N\(_c\)} conduction-radiation interaction parameter
  \item \textbf{N\(_k\)} hinge-conductance radiation parameter
  \item \textbf{N\(_p\)} number of panels in the fin
  \item \textbf{q'} heat transfer rate per unit depth (W m\(^{-1}\))
  \item \textbf{t} panel thickness (m)
  \item \textbf{T} temperature of an element (K)
  \item \textbf{T\(_b\)} temperature of the base (K)
  \item \textbf{x} position along the panel length (m)
  \item \textbf{\Delta x} length of an element (m)
  \item \textbf{\alpha} intrinsic absorptivity
  \item \textbf{\varepsilon} intrinsic emissivity
  \item \textbf{\varepsilon\(_a\)} apparent emissivity
  \item \textbf{\theta} temperature profile column vector
  \item \textbf{\theta} non-dimensional temperature
  \item \textbf{\Lambda} apparent emissivity correction function
  \item \textbf{\Pi\(_1\)} Heat transfer rate normalized by heat transfer from a straight, black, isothermal fin
  \item \textbf{\Pi\(_2\)} Heat transfer rate normalized by heat transfer from a folded, isothermal fin
  \item \textbf{\sigma} Stefan-Boltzmann constant (W m\(^{-2}\) K\(^{-4}\))
  \item \textbf{\phi} actuation angle (deg)
\end{itemize}
\( \chi \) non-dimensional panel position

\( \Psi \) turn-down ratio

**Subscripts and Superscripts**

\( [\text{cond}] \) heat transfer due to conduction

\( [\text{fin}] \) heat transfer from the entire fin

\( [i] \) primary element index

\( [\text{in}] \) heat transfer into an element

\( [\text{iso}] \) isothermal

\( [j] \) radiosity element index

\( [\text{left}] \) radiosity originating from the panel to the left of the element

\( [n] \) panel index

\( [\text{out}] \) heat transfer leaving an element

\( [\text{rad}] \) heat transfer due to radiation

\( [\text{right}] \) radiosity originating from the panel to the right of the element

\( [k] \) iteration number

### 6.4 Introduction

Economical and efficient heat transfer is critical in the design and optimization of thermal control systems. For radiative cooling applications, total cooling power is determined by operating temperature, material properties, and geometry of a dedicated cooling surface. Constraints on component operating temperature typically govern temperature limitations and operating conditions. Likewise, material properties are often constrained by the need for specialized materials and/or surface coatings for specific applications such as spacecraft thermal control [5] or daytime radiative refrigeration [131, 132]. Systems that leverage the low-temperature (3 K) of deep space to efficiently reject heat via radiation have recently been the focus of many researchers. These promising radiative cooling technologies require spectrally selective radiative heat exchangers that are engineered to be strongly emitting in the 8-13 μm spectral region and weakly absorbing at wavelengths outside this ‘atmospheric window.’
However, it is the geometry of dedicated cooling surfaces that is often most flexible in addressing increasing power requirements. Although the available emitting surface area of a radiative cooling device may be increased, the size, weight and power requirements of cooling panels must be considered before large surfaces may be utilized, especially with regards to applications such as spacecraft thermal control [23, 133]. In particular, radiative heat exchangers used in space are often simple and reliable systems but typically do not adapt to environmental changes. Adjustable systems, that can be adapted for variations in the thermal environment or operating conditions, have the potential for increased efficiency and reduced cost.

Recent technology developments [66, 88, 123] propose the use of origami-inspired structures, consisting of small, rigid sections connected in a tessellated pattern, to collapse large radiating surfaces into a stowable volume. Intrinsic to this technology is the capability for control of radiative heat transfer via manipulation of surface geometry in real-time. The operating mechanism of these surfaces is such that both the apparent radiative properties of the device [2,95] as well as the apparent emitting surface area of the device [123] vary as a function of deployment position. The net effect is that the total radiative cooling power tends to decrease as the tessellated surface collapses into the stored state while deploying the surface into a fully-actuated state increases the total cooling power [123]. By controlling the position of the origami structure, the radiative cooling power of the radiative heat exchanger may be adjusted to real-time system requirements.

Published studies concerning the radiative behavior of origami tessellations have been limited to isothermal surfaces, neglecting heat conduction. However, the series of panels comprising the tessellation naturally forms a segmented radiative fin that is self irradiating (Figure 6.1). A number of researchers have evaluated arrays of mutually irradiating stationary fins consisting of cylindrical, rectangular or triangular fins connected to a common planar or cylindrical base [134–140] as well as single conical or fractal-like fins that are self irradiating [79,141]. Many of these studies report fin efficiency as a function of fin design parameters and optimize the dimensions of the fin or fin array. However, the thermal performance of a self-irradiating fin with dynamic geometry has yet to be considered.

A radiative fin array is likely preferable to a single large radiative fin for achieving the greatest heat transfer per unit mass for a given base temperature. However, a radiative fin array is not always possible. As an example, Cubesat spacecraft architectures usually do not include
external surfaces designed for radiative heat exchange as all available external panel surfaces are typically covered in solar panels. A deployable radiative fin would provide radiative surface area that is easily stowed on board and then deployed upon launch. As an added benefit, the folded nature of the radiative fin then provides the opportunity for dynamic control of the radiative fin geometry to optimize heat transfer behavior.

In this study, the turn-down ratio (ratio of largest to smallest radiative heat loss) and efficiency of a segmented, dynamic radiative heat exchanger (consisting of multiple, mutually irradiating straight segments arranged in series, forming a self-0irradiating actuating fin) is described as a function of surface radiative properties and static /dynamic dimensions. This work considers the accordion tessellation, a repeating structure of V-grooves, as this geometry bears the strongest resemblance to static spacecraft radiators deployed at orbit insertion on the International Space Station [142] and other commercial and scientific satellites [22, 143–145] as well as to dynamically-actuated radiators currently in development [133, 146, 147]. The accordion fold has also been explored for solar shading applications of buildings [148]. However, the numerical approach detailed in this paper, which is designated as the Segmented Fin Algorithm (SFA), may be applied to other two-dimensional tessellations of interest. The 2D approach in this work is also applicable to three-dimensional tessellations considered viable for spacecraft missions, such as the flasher [33, 149] or square patterns, if the depth to panel length ratio is sufficiently large [88]. Surfaces are assumed to exhibit diffuse reflection in the present study, and fin geometries are restricted to isothermal conditions across the panel thickness. In order to focus the results of the paper on the fundamental radiative behavior of a dynamic radiative fin geometry, the influence of radiative sources aside from the fin geometry (such as spacecraft surfaces, building rooftops, solar inputs, etc.) are not considered in this work and may be incorporated by those using the approach outlined here. Regardless, excluding external radiative sources can be an appropriate assumption for select applications, including long, cantilevered radiators extending away from a co-planar spacecraft (e.g. ISS, CubeSats).

First, the governing equation of the numerical model is presented, including non dimensionalization of the design variables and comparison metrics of interest. The numerical solution approach used to determine an accurate temperature profile of the dynamic radiative fin is described and limiting conditions are defined. Simulation results are compared with available analytical solu-
tions for a static, rectangular radiative fin and for an isothermal V-groove to verify the performance of the model and code. Results of the numerical model are described and the influence of the actuation position, number of tessellation panels, surface emissivity, conduction-radiation interaction parameter, and hinge-conductance radiation parameter on the temperature profile, heat transfer, fin efficiency, and turn-down ratio of the device is described. Finally, results are further discussed in light of application considerations.

![Figure 6.1: Geometry of a re-deployable, segmented fin connected to a base (at $T_b$) consisting of several rigid straight panels that are connected in series via thermally conductive hinges. The apparent surface area and apparent radiative properties of the fin vary as the fin is actuated.](image)

### 6.5 Methodology

#### 6.5.1 Segmented Fin Algorithm

**Model Geometry**

A rigid, straight panel of known thermal conductivity ($k$), surface emissivity ($\varepsilon$), thickness ($t$), and length ($L$) having a unit width in-and-out of the page is connected to an isothermal base ($T_b$) via a flexible thermal hinge with a given thermal conductance ($\kappa$, reported per unit area). At the tip of the base-mounted panel, an additional thermal hinge, also of thermal conductance $\kappa$, connects the tip of the first panel to the base of another straight, rigid panel (Figure 6.1). This repeating behavior continues for $N_p$ panels, where the geometric angle between two adjacent panels is identical across the entire series and specified as the actuation angle ($\phi$). The resulting pattern of
angled panels forms an accordion tessellation, resulting in a self-irradiating, segmented, radiative fin.

**Governing equation**

Each panel is discretized into $N$ number of finite, isothermal elements which include the entire thickness of the radiator panel and with uniform finite length ($\Delta x$). The geometric center of each element contains a node at a certain temperature, designated as $T_{n,i}$, where $n$ is the panel index (as numbered from the base) and $i$ is the index of the element (as numbered from the base-side of the panel) as shown in Figure 6.1.

Consider an energy balance on a single element (Equation 6.1, inset Figure 6.1). Heat enters this element via one-dimensional conduction from the left element ($n, i-1$) and leaves via conduction to the right element ($n, i+1$); both conduction terms may be described with Fourier’s Law at the element boundary (Equation 6.2). To allow for numerical calculation, the temperature gradient in Fourier’s Law is replaced with a finite difference approximation between adjacent temperature nodes (right-hand-side of Equation 6.2), where the distance between nodes is equivalent to the length of an element, $\Delta x$. Energy leaving the element via conduction is determined in a similar manner.

$$q'_{\text{cond.in}} - q'_{\text{cond.out}} + q'_{\text{rad.in.left}} + q'_{\text{rad.in.right}} - q'_{\text{rad.out}} = 0 \quad (6.1)$$

$$q'_{\text{cond.in}} = -kt \frac{dT}{dx} \approx kt \frac{T_{n,i-1} - T_{n,i}}{\Delta x} \quad (6.2)$$

Radiant energy is incident on the element from the left ($q'_{\text{rad.in.left}}$) and right ($q'_{\text{rad.in.right}}$) panels as well as emitted from both sides of the element ($q'_{\text{rad.out}}$). The rate at which radiant energy from an element on the left panel ($n-1,j$) is absorbed by element $n,i$ is determined by multiplying the absorptivity of the panel surface, radiosity of the left element, view factor between the two elements, and the area of the left element, or $\alpha J_{n-1,j} F_{j-i} \Delta x_j$, assuming diffuse and gray surfaces. The heat originating from the entire left panel that is absorbed by element $n,i$ is determined by summing over all elements on panel $n-1$ as shown in Equation 6.3, where $N$ represents the total number of elements on an individual panel, where all panels have the same number of elements.
The absorbed heat from the right panel \( q'_{\text{rad,in, right}} \) is given by a similar expression, where the index \( j \) now indicates elements on the panel \( n+1 \) and \( J_{n-1,j} \) is replaced with \( J_{n+1,j} \). The heat emitted from the element is given by Equation 6.4, which accounts for emission from both exposed surfaces. For this work, the geometry of the base was not specified and the radiative exchange between the base and the fin was not considered.

\[
q'_{\text{rad, left,in}} = \alpha \sum_{j=1}^{N} J_{n-1,j} F_{j-i} \Delta x_j
\]  

\[
q'_{\text{rad, out}} = 2\epsilon \sigma \Delta x_i T^4_{n,i}
\]  

After substituting Equation 6.3 and 6.4 into the energy balance, the governing equation becomes,

\[
kt \frac{T_{n,i-1} - T_{n,i}}{\Delta x_i} - k t \frac{T_{n,i} - T_{n,i+1}}{\Delta x_i} + \alpha \sum_{j=1}^{N} J_{n-1,j} F_{j-i} \Delta x_j + \alpha \sum_{j=1}^{N} J_{n+1,j} F_{j-i} \Delta x_j - 2\epsilon \sigma \Delta x_i T^4_{n,i} = 0
\]  

Emission and reflection from the base geometry to which the dynamic radiative fin is attached as well as irradiation from the surroundings have been shown to impact the heat transfer behavior of a parallel radiative fin array \([134, 150]\). However, these influences were not considered in this work for two reasons. First, the focus of this paper is the fundamental heat transfer behavior of a dynamic radiative fin, focusing on the impact of self-irradiation on the fin heat transfer and temperature profiles. As such, external inputs were not considered in order to focus on the basic behavior. Second, it is difficult to identify and generalize the exact geometry of the base and the strength and directionality of radiation from the surroundings. Instead the influence of these factors on the heat transfer and temperature profiles of self-irradiating dynamic fins is left as future work. Finally, there are application scenarios where external inputs (specifically from the base) are negligible, including heavily insulated spacecraft or spacecraft with geometry such that the fin panels do not view the spacecraft significantly.

Likewise, the use of a 2D model results in error when used to predict the performance of 3D surfaces. As real radiative fins are three dimensional, it is necessary to ensure that panels of
sufficient depth are used such that the infinite depth assumption does not introduce significant error. It has been shown that 3D V-grooves with a depth to panel length \( L \) ratio of 10 or more exhibit radiative heat transfer values within 95\% of the values predicted by a 2D model [88]. Results provided in this work are best applicable to self-irradiating, segmented fins that meet this criteria. The approach established here may be extended for other 3D surfaces.

The governing equation is non-dimensionalized using the terms given in Equations 6.6 - 6.9. The local temperature \( T \) is scaled by the temperature of the base \( T_b \); the element length \( \Delta x \) is scaled by the length of a single panel \( L \); the thermal conductivity, panel length, panel thickness and base temperature are combined to form the radiation-conduction interaction parameter \( (N_c) \), being a ratio of the resistance to conduction through the fin material to the resistance to radiation heat transfer form the fin surfaces, where this a non-dimensional term is often utilized in radiative fin solutions [134]; the radiosity \( J \) is scaled by the emissive power of the base \( \sigma T_b^4 \).

\[
\theta = \frac{T}{T_b} \quad (6.6)
\]

\[
\chi = \frac{x}{L} \quad (6.7)
\]

\[
N_c = \frac{R_{\text{cond}}}{R_{\text{rad}}} = \frac{\sigma T_b^3 L^2}{k t} \quad (6.8)
\]

\[
J^* = \frac{J}{\sigma T_b^4} \quad (6.9)
\]

The intrinsic emissivity and intrinsic absorptivity are equivalent in this work as the surfaces are assumed to be diffuse and gray. Likewise, the intrinsic radiative properties of the surface were not included in non-dimensional terms in order to explore their specific effect on the heat transfer and turn-down ratio of a self-irradiated, segmented fin.

The non-dimensional form of the governing equation for an internal element is given as Equation [95] after recalling that all elements have an equivalent length \( \Delta \chi \). The view factor \( F \) used in this analysis is for exchange between finite areas on one side of a V-groove to another as
reported in the Hottel crossed strings section in Modest [38] or condition A-10 in the configuration catalog [151].

\[
\frac{1}{\varepsilon \Delta \chi^2 N_c} \left[ -2\theta_{n,i} + \theta_{n,i-1} + \theta_{n,i+1} \right] + \sum_{j=1}^{N} J_{n-1,j}^{\ast} F_{j-i} + \sum_{j=1}^{N} J_{n+1,j}^{\ast} F_{j-i} - 2\theta_{n,i}^4 = 0
\]  

(6.10)

Elements found on the ends of the radiator panels, having an index \( i = 1 \) or \( i = N \), will be subject to different boundary conditions than an interior node and therefore require a modified form of Equation 6.10. The first element (\( i = 1 \)) on any panel is connected to the last element of the previous panel, or the base in the case of the first panel, using a thermal hinge with thermal conductance \( \kappa \). Conduction terms across these thermal hinges (e.g. \( q_{\text{cond,in}} \) in Equation 6.1) can be described using Equation 6.11. For the last element on a panel (\( i = N \)), Equation 6.11 is modified by replacing subscript \( n - 1 \) with \( n \) and subscript \( n \) with \( n + 1 \); these modified forms are then substituted into the term \( q_{\text{cond,out}}' \) found in Equation 6.1.

\[
q_{\text{cond,in}}' = \kappa T (T_{n-1,N} - T_{n,1})
\]  

(6.11)

The governing equation for these boundary elements is non-dimensionalized in the same manner as Equation 6.10. However, the thermal conductance \( \kappa \) is non-dimensionalized to obtain a hinge-conductance radiation parameter \( N_\kappa \) (Equation 6.12), similar to the conduction-radiation interaction parameter but with one length scale now incorporated into the conductance. This parameter \( N_\kappa \) is a ratio of the resistance to conduction through the hinge and the resistance to radiative heat transfer form the panel surfaces. The last element of the last panel is assumed to be insulated at the tip.

\[
N_\kappa = \frac{R_{\text{cond,hinge}}}{R_{\text{rad}}} = \frac{\sigma T_b^3 L}{\kappa T}
\]  

(6.12)

Substituting Equation 6.11 into Equation 6.1 and proceeding as was done for an internal node, the non-dimensional form of the governing equation for the first element on a panel (\( i = 1 \)) is given by Equation 6.13. The governing equation for an element on the end of a panel (\( i = N \)) is given in Equation 6.14.
\[
\frac{1}{\epsilon \Delta x^2 N_{\kappa}} [\theta_{n-1,N} - \theta_{n,1}] + \frac{1}{\epsilon \Delta x^2 N_{c}} [\theta_{n,2} - \theta_{n,1}] + \sum_{j=1}^{N} J_{n-1,j}^* F_{j-i} + \sum_{j=1}^{N} J_{n+1,j}^* F_{j-i} - 2\theta_{n,i}^4 = 0
\] (6.13)

\[
\frac{1}{\epsilon \Delta x^2 N_{c}} [\theta_{n,N-1} - \theta_{n,N}] + \frac{1}{\epsilon \Delta x^2 N_{\kappa}} [\theta_{n+1,1} - \theta_{n,N}] + \sum_{j=1}^{N} J_{n-1,j}^* F_{j-i} + \sum_{j=1}^{N} J_{n+1,j}^* F_{j-i} - 2\theta_{n,i}^4 = 0
\] (6.14)

Mathematical closure for Equation 6.10, 6.13, or 6.14 is obtained with the definition of the radiosity for each exposed side of an element. The radiative heat rate leaving element \(i\) per unit depth is defined in Equation 6.15. This equation includes the emission from element \(i\) summed with the reflected portion of energy incident from opposing elements, being the product of element \(j\)'s radiosity, view factor to element \(i\), and length \((\Delta x_j)\) summed over all elements on the opposing panel. The length of every element is identical, causing element length terms to cancel and giving the definition of the radiosity for the element surface facing the right panel \((n+1)\) in equation 6.16 and non-dimensionalized in Equation 6.17. For the side of the element facing the left panel \((n-1)\), the radiosity is still given by Equation 6.17 although the subscript of \(J^*\) must be changed to \(n-1\). When one side of the element only experiences radiative heat exchange with the surroundings (e.g. the top surface of panel 1 or panel 4 in Figure 6.1 or when the panels are flat, \(\phi = \pi\)), the dimensionless radiosity reduces to only the first term of Equation 6.17.

\[
J_{n,i} \Delta x_i = \epsilon \Delta x_i \sigma T_{n,i}^4 \left[1 - \alpha \right] + \sum_{j=1}^{N} J_{n+1,j}^* F_{j-i} \Delta x_j
\] (6.15)

\[
J_{n,i} = \epsilon \sigma T_{n,i}^4 \left[1 - \alpha \right] \sum_{j=1}^{N} J_{n+1,j}^* F_{j-i}
\] (6.16)

\[
J_{n,i}^* = \epsilon \theta_{n,i}^4 \left[1 - \alpha \right] \sum_{j=1}^{N} J_{n+1,j}^* F_{j-i}
\] (6.17)
Solution method

A balance equation is written for every element on each panel. This series of equations is cast into matrix form as shown in Equation 6.18, where \( A \) is a square matrix with \( n \times N \) rows and \( n \times N \) columns containing the coefficients for the linear non-dimensional temperature variables, \( \theta \) is a column vector with \( n \times N \) entries containing the unknown temperature profile, and \( b \) is an \( n \times N \) column vector containing the source terms, being the radiosity summation terms as well as the nonlinear thermal emission term \( (2\theta^4) \).

\[
A\theta = b
\]  

(6.18)

\( A \) and \( b \) are built sequentially, beginning with the first element of the first panel and iterating along \( i \) and \( n \). The row number of \( A \) and \( b \) corresponding to element \( i \) of panel \( n \) is given by the expression \( (n-1)N + i \). Coefficients in the first row of \( A \) and \( b \), corresponding to the first element of the first panel, are generated using Equation 6.13 where the term \( \theta_{n-1,N} \), is the temperature of the base, \( \theta_b \). For the remaining elements, if the row number divided by \( N \) is a whole number, indicating an element immediately to the left of a hinge, then Equation 6.13 is used to determine coefficients for \( A \) and \( b \). Likewise, if the row number divided by \( (N-1) \) is a whole number, indicating an element to the right of a hinge, then Equation 6.14 is used to generate coefficients for \( A \) and \( b \). All other rows utilize Equation 6.10. The last row of \( A \) and \( b \), being row \( n \times N \), corresponds to the last element of the last panel and is determined using Equation 6.14 but with the second variable grouping \( (\theta_{n+1,1} - \theta_{n,N}) \) set to zero, giving an insulated tip.

The presence of a non-linear temperature term as well as temperature-dependent radiosity terms within the source term vector necessitates an iterative process in order to converge to the correct temperature profile. To proceed, an initial guess of the temperature profile is made and the radiosity profiles of the top and bottom surface of each panel are assumed, allowing the source vector to be calculated. For all tests in this work, the initial temperature profile was guessed to be isothermal and equal to the base temperature and with initial radiosity values defined using only the first term of Equation 6.17. The matrix system found in Equation 6.18 is then solved using the calculated source vector, giving a new temperature profile. The temperature for every element in the new temperature profile is compared with its equivalent in the guessed temperature profile. If
the difference between the two profiles exceeds a specified convergence criterion, then the source vector is recalculated and a new temperature profile is obtained. This procedure is repeated until the convergence criterion of $\theta_{n,i}^{k-1} - \theta_{n,i}^k = 1 \times 10^{-5}$ is met, where the superscript $k$ indicates the iteration number. To recalculate the source vector using a new temperature profile the radiosity profiles must first be updated. This is accomplished via a process similar to Gauss-Seidel iteration. Beginning at the first element on the first panel and moving towards the end of the fin, the radiosity profile at each node is updated individually using Equation 6.17 for an element surface facing the panel to the right or a modified form of Equation 6.17 for an element surface facing the panel to the left, utilizing the updated temperature profile and the most recently available radiosity information.

The iterative procedure just described requires frequent recalculation of updated temperature profiles through solution of the matrix equation given in Equation 6.18. By using a linear approximation to estimate the conduction terms, as shown in Equation 6.2, coefficients for a given row in $A$ only pertain to the current element (the coefficients found on the matrix diagonal) as well as the two elements immediately adjacent. As such, $A$ is tri-diagonal and the system is solved with simplified Gaussian elimination using the Tri-Diagonal Matrix Algorithm (TDMA) as described in [152].

6.5.2 Verification

Grid independence study

The accuracy of the converged temperature profile is dependent on the length of the element, given nondimensionally as $\Delta \chi$. To ensure an accurate solution for all tested cases, a grid refinement study was performed for the extremes in the variables; these include combinations of the smallest ($1 \times 10^{-5}$) and largest (10) $N_c$ values with the smallest (0.1) and largest (0.9) intrinsic emissivity values. For each combination, an initial element size was selected and the iterative temperature profile solution process was executed. Upon solution, the element size was decreased by a factor of two and a new temperature profile was determined, starting with the most recent temperature profile as an initial guess. The temperature profiles were compared and the process was repeated until the difference between the iterated temperature profiles for all elements fell within the bounds of an overall convergence criteria. For this study, the overall convergence criteria was
\( \theta_{n,i}^{k-1} - \theta_{n,i}^k = 1 \times 10^{-3} \) resulting in 2000 elements per panel \((N)\) to satisfy the grid independence criterion for all four tested cases. All data points in this study were conducted with \(N = 2000\).

**Limiting condition**

In developing the self-irradiating fin model, the temperature at any position on any panel was assumed to be isothermal through the thickness of the panel. When applied to a given geometry, this assumption must be validated using the Biot number to ensure accuracy. Using the radiation heat transfer coefficient obtained by linearizing the radiation rate equation [153] with surroundings at a temperature of 0 K and assuming the characteristic conduction length to be the thickness of the material, the Biot number of the element is given as Equation 6.19.

\[
Bi = \frac{\varepsilon \sigma T_b^3 t}{k} = \frac{N_c \varepsilon l^2}{L^2}
\]

(6.19)

This number compares radiative emission from the surface with conduction through the thickness. To be conservative, the Biot number analysis given as Equation 6.19 ignores radiative energy absorbed by the element and the highest temperature, \(T_b\), should be used. Results for the Biot number for a particular application as calculated in this manner should be less than 0.1 to ensure appropriate application of the isothermal condition through the thickness of the panel. Results presented in this paper are provided in terms of non-dimensional parameters, without selection of specific values for \(t\), \(\varepsilon\) and \(L\); the condition of a small Biot number should be verified before applying these results to a particular application.

**Model validation**

To test the accuracy of the SFA, numerical results using the current approach are compared with known analytical solutions for certain idealized cases. First, the actuation angle is defined as 180° and the hinge-conductance radiation parameter is set to \(\Delta \chi N_c\), resulting in a flat, straight fin of homogeneous material. The solution provided by the SFA for this scenario corresponds with the analytical solution for a straight rectangular radiating fin with negligible base interactions (available from [154], repeated in Equation 6.20).
\[ x = \left( \frac{5kt}{4\varepsilon \sigma} \right)^{1/2} \int_{T_{b}}^{T_{b}} \frac{dT}{[T^5 - T_{LsNp}^5 - (5/2\varepsilon \sigma)(T - T_{LsNp})]^{1/2}} \]  \hspace{1cm} (6.20)

To evaluate this expression, the temperature at the end of the fin is first evaluated by setting \( x = LN_{p} \) (the total length of the fin) and determining the correct \( T \) that satisfies the integral with root-finding methods so that the temperature at the tip of the fin can be obtained. A temperature value is then chosen between the tip temperature and base temperature. Equation 6.20 is evaluated by integrating from the selected temperature to the base temperature, giving the \( x \) location for the selected temperature. A new temperature value is selected and its associated \( x \) value is determined with Equation 6.20. This process is repeated until a sufficient number of points have been evaluated and the temperature profile of the fin is clear. Comparison of the temperature profiles using the current method and the analytical solution are provided in the results.

To test the accuracy of the radiative interactions, the emissivity of the surface is set to 0.3, the conduction-radiation interaction parameter is set to zero (e.g. a fin with infinite thermal conductivity) and the hinge-conductance radiation parameter is set to \( \Delta \chi N_{c} \). The heat loss from one V-groove of the resulting isothermal fin with an emissivity of 0.3 is calculated for all actuation angles and this value is divided by the heat loss from a black V-groove of the same geometry to determine the apparent emissivity [95] as calculated by the SFA. The resulting apparent emissivity value is compared with a correlation from ray tracing data for the apparent emissivity of an isothermal V-groove [2] as given in Equations 6.21 and 6.22, where the variable \( \Lambda \) is the apparent emissivity correction function.

\[ \varepsilon_{a} = \varepsilon \Lambda \sum_{m=0}^{\infty} (1 - \varepsilon)^{m} \left[ 1 - \sin \left( \frac{\phi}{2} \right) \right]^{m} \]  \hspace{1cm} (6.21)

\[ \Lambda = 1 - [0.0169 - 0.1900\ln(\varepsilon)] \exp(-1.4892\varepsilon^{-0.4040}\phi) \]  \hspace{1cm} (6.22)

### 6.5.3 Result Metrics

The SFA outputs the non-dimensional temperature distribution of the segmented fin and the non-dimensional radiosity distributions for the top and bottom fin surfaces. In order to study the effect of the parameters \( \varepsilon, N_{c}, N_{p}, \) and \( N_{\kappa} \) on the heat transfer performance of the fin, several
metrics are proposed. The first of these metrics is the fin heat transfer rate non-dimensionalized using the maximum possible radiative transfer from the fin ($\Pi_1$). To determine this value, the heat transfer per unit width across the thermal hinge (Equation 6.11) is applied to the first hinge and scaled using the total emission per unit width leaving a straight, black, isothermal radiative fin having the same length as a fully extended segmented fin, as shown in Equation 6.23.

$$\Pi_1 = \frac{q'_{\text{fin}}}{2 LNp \sigma T_b^4} = \frac{1 - \theta_{1,1}}{2 \kappa N_p}$$

Here, $\theta_{1,1}$ is the non-dimensional temperature of the first node of the first panel and $\Pi_1$ is the non-dimensional fin heat transfer rate. The non-dimensional fin heat rate is defined as the ratio of the heat rate through the fin to the heat rate through a black, fully-extended, isothermal fin of the same dimensions. The non-dimensional fin heat rate reports a relative heat transfer value and is used to determine how the heat transfer of the dynamic segmented fin varies with actuation and is a measure of how efficiently the mass of the fin is being utilized. This term is referred to as the ‘non-dimensional heat transfer’ throughout the remainder of this work.

A second non-dimensional heat transfer value is used in this work in order to examine the heat transfer behavior of a dynamic segmented radiative fin. This heat transfer value is calculated by dividing the heat rate of the segmented fin calculated using Equation 6.11 by the heat rate of the same segmented fin with an isothermal temperature profile for the same intrinsic emissivity and actuation angle. The heat loss of the isothermal fin is the summation of the emission from the V-groove cavity openings, utilizing the apparent emissivity (Equations 6.21 and 6.23), and the emission from the panel surfaces that are exposed only to the isothermal surroundings, giving Equation 6.24. Equations 6.11 is not used to find the heat loss from an isothermal fin as this equation would return a value of zero in the isothermal case. This second non-dimensional heat transfer value quantifies how closely the fin temperature profile approaches isothermal conditions and therefore indicates how effectively heat is being transferred along the length of the fin. This metric resembles the classic definition of fin efficiency and will therefore be referred to as ‘fin efficiency’ throughout the remainder of this work.
\[ \Pi_2 = \frac{q'_{\text{fin}}}{q'_{\text{fin, iso}}} = \frac{\theta_b - \theta_{i,1}}{2N_k \left[ \varepsilon + \varepsilon_a(N_p - 1) \sin \left( \frac{\phi}{2} \right) \right]} \] (6.24)

Finally, the turn-down ratio (\(\Psi\)) is defined as the ratio of the largest possible non-dimensional fin heat rate to the smallest possible non-dimensional fin heat rate (using Equation 6.23) over the full range of actuation angles while \(\varepsilon\), \(N_c\) and \(N_k\) are held constant. For this work, the turn-down ratio was determined by dividing the \(\Pi_1\) evaluated at an actuation angle of 180° by \(\Pi_1\) evaluated at an actuation angle of 5° (the smallest angle considered). A larger turn-down ratio indicates an increased range of radiative heat transfer control.

### 6.6 Results and Discussion

#### 6.6.1 Verification Comparisons

Figure 6.2a plots the non-dimensional temperature profile against the non-dimensional, length-wise panel coordinate \(\chi\) for both the SFA and the analytical results from Equation 6.20. Results are provided for four different values of \(N_c\) and an intrinsic emissivity value of 0.90. Given the definition of \(\chi\) as shown in Equation 6.7, whole number values of \(\chi\) represent the location of thermal hinges between adjacent panels. As seen in Figure 6.2a, SFA results are in excellent agreement with the analytical results, with slight disagreement apparent when \(\chi\), \(N_c\) and \(\varepsilon\) are large. For 0.005 \(\leq N_c \leq 10\) and 0.1 \(\leq \varepsilon \leq 0.9\), the largest discrepancy between the SFA temperature profile and the analytical temperature profile is 3%. For these cases, the temperature profile becomes less accurate as \(\chi\) increases. Since these discrepancies are negligibly small, comparison of the analytical and SFA results verifies the conduction and emission portions of the code.

To verify the behavior of re-reflected energy within the V-grooves, the SFA results for an isothermal fin are compared in Figure 6.2b with an apparent emissivity correlation given in Equations 6.21 and 6.22 over the full range of actuation angles for an intrinsic emissivity of 0.3. The average relative discrepancy between the SFA and correlation is 2% for 0.1 \(\leq \varepsilon \leq 0.9\) (step size of 0.1) over the full range of cavity angles. Perfect agreement between the SFA and correlation is not expected due to uncertainty inherent to the correlation, with correlation uncertainty approaching 1.6% for actuation angles near 0°. Since the largest SFA/correlation discrepancy is comparable
Figure 6.2: (a) Results of the Segmented Fin Algorithm (SFA) compared with results from an analytical, straight, radiative fin solution (Equation 20) for $\varepsilon = 0.90$. The numerical algorithm agrees well with the analytical approach with a largest relative error of 3%, obtained at large values of $N_c$, $\varepsilon$, and $\chi$. (b) The apparent emissivity of a Vgroove ($\varepsilon_a$) as calculated using the SFA compared with a published correlation (Equations 6.21 and 6.22). The largest error between the SFA and correlation is 2% which is similar to the range of errors given for the correlation.

to the uncertainty in the correlation and also occurs near angles of 0°, the ability of the SFA to correctly predict radiosity is considered verified.

### 6.6.2 Large Thermal Hinge Conductance

Results are first presented for the case where conduction resistance across the thermal hinge is negligible ($N_{k} = \Delta \chi N_c$), representing a single continuous material that has been folded into an accordion pattern with an isothermal boundary condition on one end. When a value of $N_{k} = \Delta \chi N_c$ is used, governing equations for the first or last element on a panel, given by Equations 6.13 and 6.14 respectively, are transformed into the governing equation for an element in the middle of the panel as given by Equation 6.10. This scenario explores the operation of a deployable fin without the added complexity of a hinge resistance. Figure 6.3a displays the temperature profile for a four-panel, hinged radiator with an intrinsic emissivity of 0.9 and conduction-radiation interaction
Figure 6.3: Non-dimensional temperature ($\theta$) profiles as a function of non-dimensional panel position ($\chi$) for a four panel, hinged radiator at two values of the conduction-radiation interaction parameter ($N_c$). Results are displayed for increasing values of the actuation angle ($\phi$), showing the influence of fin deployment position on the temperature profile. (a) Behavior observed for an intrinsic emissivity of 0.9. When conduction dominates ($N_c < 1$), the profile approximates that of a straight radiative fin. When radiation dominates ($N_c > 1$), the influence of radiative exchange between panels causes significant variation in the panel temperature profiles. (b) Behavior observed for an intrinsic emissivity of 0.1. As a result of the nearly reflective surface, conduction dominates inter-panel heat exchange and the profiles continuously decrease as $\chi$ increases.

Parameters of 0.05 or 10. Results are displayed for four different actuation angles, $\phi = 5^\circ, 30^\circ, 90^\circ, 150^\circ$. Figure 6.3b gives temperature profiles for these same $N_c$ values, actuation angles, and number of panels but for an intrinsic emissivity of 0.1.

For small $N_c$ values, indicated by the family of curves labeled $N_c = 0.05$ in Figures 6.3a and 6.3b, conduction dominates inter-panel heat exchange over radiation. As such, the temperature drop from the base to the tip of the fin is small when compared with temperature profiles for fins with large $N_c$ values. Likewise, the influence of self-irradiation on the fin temperature profile is evident in the changing tip temperature of the fin with changes in actuation angle. However, temperature profiles for all angles still approximate an exponentially decaying form except for slight perturbations in the middle two panels ($1 < \chi < 3$) for small actuation angles. Regarding emissivity, the fin with a high emissivity experiences a 5x larger reduction in tip temperature than...
for a fin with low emissivity when comparing tip temperatures in the fully extended and fully
collapsed positions.

At large $N_c$ values (e.g. $N_c = 10$ curve families in Figures 6.3a and 6.3b), radiative losses
dominate inter-panel heat exchange over conduction. As such, self-irradiation causes significant
variation in the temperature profile as compared to the exponentially-decaying straight-fin tem-
perature profile given approximately by $\phi = 150^\circ$. For large emissivities, the temperature of the
second panel ($1 < \chi < 2$) can be seen to increase as $\chi$ increases, especially for small actuation
angles, indicating that heat is conducted towards the base of the fin. Although counter-intuitive,
this behavior may be explained with an examination of the fin’s geometry. For very small actua-
tion angles ($\phi < 5^\circ$), the hottest elements at the beginning of the first panel ($0 < \chi < 0.25$) are
within close radiative proximity to the elements at the end of the second panel ($1.75 < \chi < 2$),
resulting in significant radiative transfer from the beginning of the first panel to the end of the
second panel. This elevated radiative transfer combined with the weak influence of conduction
generates a local hot spot near the end of the second panel, causing heat to conduct away from this
location in both the $+\chi$ and $-\chi$ directions. However, for reflective surfaces ($\varepsilon = 0.1$), the effect of
radiative heat transfer is decreased, resulting in monotonically decreasing temperature profiles for
all cases (see Figure 36.3b). The tip temperature reduces in value as actuation angle increases for
both emissivities.

Figure 6.4a illustrates the non-dimensional heat transfer ($\Pi_1$) of the dynamic fin as a func-
tion of actuation angle $\phi$ for an emissivity of 0.9 and Figure 6.4b displays this same information
for an emissivity of 0.1. Universally, the heat transfer increases as the actuation angle increases,
although this change reduces in value as $N_c$ increases. For nearly black surfaces, the heat transfer
curves follow an approximate sine distribution, indicating that the heat loss is dominated by the sur-
face area of the V-groove openings, which increase along a sine distribution with angle. However,
as the intrinsic emissivity decreases, the heat transfer variation becomes increasingly concentrated
towards the small angle range (see Figure 6.4b), suggesting that inter-reflections (which are signif-
icant in the small angles) dominate the total heat transfer. Finally, smaller $N_c$ values result in more
significant heat transfer control, where a nearly black surface with an $N_c$ value of 10 shows only a
3% variation in heat transfer over the full actuation angle range.
Figure 6.4: Non-dimensional fin heat transfer ($\Pi_1$) and fin efficiency ($\Pi_2$) as a function of actuation angle ($\phi$) for four different values of the conduction-radiation interaction parameter ($N_c$). (a) Behavior observed for an intrinsic emissivity of 0.9. The greatest variation in heat transfer is seen for fins where conduction dominates and heat transfer varies significantly over the full actuation angle range. Fin efficiency decreases as the fin extends towards an open configuration. (b) Behavior observed for an intrinsic emissivity of 0.1. Unlike the case of a nearly black fin (Figure 6.2a), the heat transfer and efficiency of reflective panels vary over a relatively small actuation angle range ($\phi \leq 60^\circ$).

Figures 6.4a and 6.4b also display the fin efficiencies ($\Pi_2$) for high and low emissivity fins, respectively. Universally, the efficiency of a dynamic fin decreases as the fin actuates from a collapsed to an extended state, indicating that the fin temperature profile is deviating further from isothermal conditions. This behavior is the result of conduction and radiation acting simultaneously when the fin is collapsed to move energy along the fin in the positive $\chi$ direction, causing the fin tip temperature to increase and improving fin efficiency. The influence of self-irradiation decreases significantly as the actuation angle increases, causing the fin efficiency to drop with actuation. Regarding emissivity, a reflective fin is always more efficient for all actuation angles when compared with a nearly black fin with an equivalent $N_c$ value. Finally, it is interesting to note that the largest variation in fin efficiency with actuation does not occur at the lowest or highest $N_c$ but is found to exist, for all emissivities, at an optimal value of $N_c$ found between the extremes.

The turn-down ratios ($\Psi$) for fins with four panels and intrinsic emissivities of 0.9 or 0.1 as a function of $N_c$ are shown in Figure 6.5a. Large values of $N_c$ result in turn-down ratios near unity.
regardless of emissivity, indicating a device with very little capability for radiative heat transfer control. As $N_c$ decreases the turn-down ratio increases and then asymptotes to a limiting value. For a nearly black fin ($\varepsilon = 0.9$) with four panels, the asymptotic value for $\Psi$ is 3.48. For a reflective surface ($\varepsilon = 0.1$) the asymptotic value is 2.43. The inset of Figure 6.5a illustrates the behavior of turn-down ratio as a function of intrinsic emissivity for an $N_c$ value of 0.001, where turn-down ratio increases logarithmically as a function of emissivity. Figure 6.5b plots the turn-down ratio for a dynamic fin with nearly black ($\varepsilon = 0.9$) panels as a function of the number of panels ($N_P$) for four different $N_c$ values. As expected, increasing the total deployed surface area by increasing the number of panels will generally increase turn-down ratio. However, this increase in turn-down ratio will eventually reach a maximum and then decrease as the number of panels increases.

![Figure 6.5a](image1.png)  ![Figure 6.5b](image2.png)

**Figure 6.5:** (a) Turn-down ratio ($\Psi$) of a dynamic fin with four panels as a function of the conduction-radiation interaction parameter ($N_c$) for two different values of intrinsic emissivity. Black fins achieve larger possible turndown ratios than reflective fins, although this is a function of $N_c$. The inset of the figure displays the variation of turn-down ratio as a function of emissivity for $N_c = 10^{-3}$ (b) Turn-down ratio ($\Psi$) of a nearly-black, dynamic fin as a function of the number of panels ($N_P$) considering four different values of the conduction-radiation interaction parameter ($N_c$).
6.6.3 Finite Thermal Hinge Conductance

Results are now presented for the case when $N_κ$ is non-zero, indicating the presence of a flexible thermal hinge. Figure 6.6a displays the temperature profile of a nearly black ($ε = 0.9$) dynamic fin having four panels with an $N_c$ value of 0.01 as a function of non-dimensional panel position for actuation angles of $5^\circ$, $90^\circ$, and $150^\circ$ and for three different values of $N_κ$, being 0.01, 0.5, and 10. The hinge thermal conductance value ($κ$) captures the influence of hinge thermal conductivity, geometry, and thermal interface losses in a single value. Although convenient, this method does not allow for the determination of the temperature profile within the hinge itself resulting in apparent temperature discontinuities (as shown in Figure 6.6a) representing the total temperature change across the thermal hinges.

Figure 6.6a displays temperature profiles for the case when conduction dominates radiation ($N_c = 0.01$). When $N_κ$ and $N_c$ are on the same order of magnitude, the temperature drop across thermal hinges are relatively small compared with the temperature change due to conduction along the fin panels. However, as $N_κ$ grows larger compared to $N_c$ the temperature differential across the hinges grows in magnitude while the temperature profile along the panel length flattens until nearly isothermal. For the $N_κ = 10$ curve in Figure 6.6a, the temperature drop along the length of the first panel is only 0.01% of the total temperature drop along the entirety of the fin. As seen from Equation 6.11, the temperature drop across a hinge is proportional to the rate of heat conduction through that hinge. At small angles ($φ = 5^\circ$), the temperature drop across each hinge is relatively constant, indicating that very little heat is lost by radiation into the surroundings from panels 2 and 3. However, as the radiator actuates towards an open position ($φ = 150^\circ$) the temperature drop across each hinge decreases along the length of the fin, indicating that panels 2 and 3 have a greater contribution to the total heat loss of the fin. Regarding emissivity, results are similar to the case of a large hinge conductance ($N_κ = ΔχN_c$), where temperature profiles for reflective surfaces show smaller temperature differences between fin base and fin tip. Likewise, the temperature drop across the thermal hinges are smaller for reflective surfaces. It should be noted that the non-dimensional temperature of the first element on the first panel is no longer equivalent to unity due to the presence of a thermal hinge between the first panel and the isothermal base.

When radiation dominates conduction ($N_c = 10$), the thermal hinges have little impact on the fin’s temperature profile. If $N_κ$ and $N_c$ are on the same order, then the temperature profile is
indistinguishable from the $N_\kappa = \Delta \chi N_c$ scenario. As $N_\kappa$ increases in value relative to $N_c$, the temperature profile experiences an overall decrease in value and the temperature difference between the first element and last element decreases in magnitude. The temperature drop across the thermal hinges likewise grow in magnitude as $N_\kappa$ increases in value relative to $N_c$.

Concerning heat transfer and efficiency, increasing values of $N_\kappa$ exacerbates the total heat transfer resistance of the fin, decreasing the heat transfer and fin efficiency when compared with values where $N_\kappa = 0$. The largest decrease in heat transfer and efficiency occurs between $N_\kappa$ values of 0.1 and unity, with the fully extended heat transfer decreasing by a relative difference of 64%. As with the case for infinite hinge thermal conductance, the change in fin efficiency with actuation is greatest for an intermediate value between the extremes of $N_\kappa = 0.01$ and $N_\kappa = 10$.

Figure 6.6b displays the influence of both $N_c$ and $N_\kappa$ on the turn-down ratio of the device for a nearly black fin ($\varepsilon = 0.9$). Increasing either $N_c$ or $N_\kappa$ results in a decreased turn-down capability; however, $N_c$ has a greater influence on the turn-down ratio value as compared to $N_\kappa$.

### 6.6.4 Application and Design Considerations

Fundamental design considerations regarding the use of dynamic fins are similar to those suggested for straight radiative fins. Specifically, fins with larger $N_c$ values (e.g. short, thick fins constructed from a thermally-conductive material) are more efficient and achieve greater heat transfer values than fins with lower $N_c$ values. Likewise, a low emissivity fin experiences much lower heat transfer rates but exhibits higher fin efficiencies as compared to a black fin. Several points of discussion concerning the influence of self-irradiation, actuation, and thermal hinge losses on the radiative heat transfer behavior of a dynamic fin are now provided.

First, and most noteworthy, dynamic fins are capable of turn-down ratios on the order of three or greater (Figures 6.5a and 6.5b). A turn-down ratio of 3 is on the order of turn-down ratios available by switching coatings or other devices currently under development [8, 9, 12, 13, 15]. However, the current approach using a radiative, segmented fin can utilize common materials and realistic fin dimensions. For example, a turn-down ratio of three is achievable with a dynamic fin comprised of four black panels ($L = 0.1$ m and $t = 0.005$ m) constructed from pure aluminum, coated with a nearly black coating ($\varepsilon = 0.9$) and connected to a base held at 293 K. The use of shorter or thicker panels or the use of materials with higher conductivities (such as copper
Figure 6.6: (a) Non-dimensional temperature ($\theta$) profiles as a function of non-dimensional panel position ($\chi$) for an intrinsic emissivity of 0.9 and a conduction-radiation interaction parameter ($N_c$) of 0.01. Results are displayed for three different values of the hinge-conductance radiation parameter ($N_\kappa$) and for increasing values of the actuation angle ($\phi$), showing the influence of thermal hinge conductance and fin deployment position on the temperature profile. When $N_c$ and $N_\kappa$ are on the same order the influence of the hinge on the temperature profile is almost negligible. A large value for $N_\kappa$ (relative to $N_c$) results in a significant temperature drop across the hinges and nearly isothermal temperatures along the panels. (b) Turn-down ratio ($\Psi$) of a nearly black dynamic fin as a function of $N_c$ and $N_\kappa$. Overall, $N_c$ has a stronger influence on the turn-down ratio than $N_\kappa$.

or aluminum with integrated heat pipes) would further increase the turn-down ratio. Therefore, dynamic fins could prove useful in practical applications.

From a design perspective, an effective dynamic fin should utilize high thermal conductivity materials, thermal hinges with very low thermal resistance, and nearly-black surface coatings. These conditions would minimize the mass of the system while allowing for maximum total heat transfer and the largest possible turn-down ratio. The number of panels as well as the thickness and length of the panels may be tailored to the geometry of the application, although care must be taken in the correct selection of $L$ and $N_P$. The turn-down ratio is maximized by increasing the deployed surface area and decreasing the $N_c$ value until it is sufficiently small. Increases in deployed surface area are possible both by increasing the length of the fin ($L$) and/or by increasing the number of panels ($N_P$). However, increasing the length of the panels also increases $N_c$ substan-
tially, decreasing the turn-down ratio. This appears to suggest that adding panels is advantageous when compared with increasing the panel length, but the presence of the thermal hinge must be considered. Since \( N_c \) has a more significant impact than \( N_\kappa \) on the fin’s turn-down ratio potential (as shown in Figure 6.6b), the length of the panels should first be increased until the value of \( N_c \) is less than \( N_\kappa \). Once this condition is satisfied, the number of panels may be increased. However, as shown in Figure 6.5b, the number of panels cannot be increased indefinitely (for a given \( N_c \)), as there is an optimum number of panels to achieve the maximum turn-down ratio for any given \( N_c \) value.

Second, radiative exchange between panels universally decreases the turn-down ratio potential of a dynamic fin. For the case of a black fin where conduction dominates (\( N_c < 1 \)), radiation exchange between panels for small angles provides an additional path by which energy is transferred along the panels. This inter-panel radiative exchange increases the total heat transfer at the lowest angles which in turn decreases the turn-down ratio of the device. Likewise, for a black fin where radiation dominates (\( N_c > 1 \)), inter-panel radiative exchange is even more significant and penalizes the fin by reversing the direction of heat transfer via conduction entirely (as shown in Figure 6.3b), causing the heat transfer to remain almost unchanged for the entire actuation range (as shown in Figure 6.4a). This suggests that the ideal variable geometry fin is a straight fin capable of varying its length in real time (all other properties remaining the same) which would obtain larger turn-down ratios than a folded fin of an equivalent fully deployed length.

Third, reflective fins have interesting behavior but do not make useful dynamic fins from a practical standpoint. For reflective fins where conduction dominates, radiative exchange between panels results in significant variation in heat transfer in the small actuation angles (\( 0^\circ < \phi < 60^\circ \)). This results in a four-panel fin that is capable of a turn-down ratio of 2.5 where the fin has reached 90\% of its full heat transfer potential at an actuation position of only 60\%. This small actuation range indicates a fin that is capable of rapid heat transfer control through minimal geometric manipulation. However, the extremely low emissivity of the fin surfaces significantly reduces the total heat transfer capability of each panel, requiring the surface area (and mass) of the reflective fin to increase significantly in order to match the heat transfer from a black fin. Likewise, reflective fins cannot match the turn-down ratios achieved by a black fin.
Finally, panel geometry and material ($N_c$) have a larger negative impact on turn-down ratio than the heat transfer performance of the thermal hinge ($N_\kappa$). If $N_c = 0$, the turn-down ratio is approximately 2.25 when $N_\kappa = 0.1$ (Figure 6.6b). However, for $N_c = 0.1$ and $N_\kappa = \Delta \chi N_c$, the turn-down ratio is approximately 1.75. This behavior is true for all values of $N_\kappa$ and $N_c$. This suggests, in application, that minimization of $N_c$ should be prioritized over minimization of $N_\kappa$.

With regards to future work, specular reflection should be incorporated into the model, allowing for a study of how this reflection mode impacts heat transfer behavior. Also, the model might be developed to work in three dimensions, allowing real tessellations to be explored to identify tessellations that optimize turn-down ratio and provide significant heat transfer control over the full range of actuation. Likewise, this work considered panels with uniform thickness and length. However, variations in the length, thickness, and emissivity of individual panels should be considered in optimizing the performance of self-irradiating, dynamic fins with regards to the heat transfer per unit mass of the system. The influence of external radiative inputs, including solar gains and emission/reflection from the radiative fin base structure, should be considered to more accurately depict the behavior of this device in specific applications. Finally, the use of coatings capable of emissivity variation (such as thermochromic or electrochromic coatings) on the surface of the dynamic fin should be modeled to explore the use of multiple radiative heat transfer variation mechanisms operating simultaneously.

### 6.7 Conclusion

A numerical algorithm was developed and used to explore the impact of heat conduction, self-irradiation, hinge thermal conductance, and panel emissivity on the overall performance of an actuating, accordion fin with adjustable geometry for dynamic control of radiating fins. Results show that dynamic fins are capable of turn-down ratios on the order of three or greater for realistic selections of panel geometry, panel material, and number of panels. Results have also shown that there exists an optimum number of panels such that turn-down ratio is maximized for a given panel geometry and material. Also, inter-panel radiative exchange always acts to reduce the turn-down ratio of a folding fin by providing an additional pathway by which energy moves along the length of the fin. Regarding emissivity, reflective fins may be used to rapidly adjust the heat rate, but this rapid response comes at the cost of relatively low total heat rates. Finally, panel
geometry and material selection have a more significant impact on radiator turn-down ratio than hinge construction and performance, suggesting that optimization of the panel performance should be preferred over optimization of the hinge performance.
CHAPTER 7. EXPERIMENTAL DEMONSTRATION OF HEAT LOSS AND TURN-DOWN RATIO FOR A MULTI-PANEL, ACTIVELY DEPLOYED RADIATOR

7.1 Contributing Authors and Affiliations

Rydge B. Mulford, Samuel Salt, Lance Hyatt, Matthew R. Jones, and Brian D. Iverson
Department of Mechanical Engineering, Brigham Young University, Provo, Utah, 84602

Vivek H. Dwivedi
Associate Head, Thermal Management Group, NASA Goddard, Greenbelt, MD

7.2 Abstract

A recently proposed dynamic spacecraft radiator uses origami tessellation frameworks to generate an expandable/collapsible surface capable of large variations in emitting surface area. In this work, this proposed concept is realized as an experimental prototype and its performance is analyzed. Aluminum panels are connected via a flexible hinge constructed from thin, interwoven copper wires and suspended from an actuating framework. The radiator panels are connected to a heated aluminum block. The radiator is placed in a vacuum environment with cooled surroundings (173 K) and the total radiative cooling power is determined as a function of radiator actuation position for a constant aluminum block temperature. As the radiator actuates from extended to collapsed, the heat transfer decreases and the fin efficiency increases. For a limited actuation range, the radiator exhibits a turn-down ratio (largest cooling power / smallest cooling power) of 1.32. A numerical model validated in this work predicts a turn-down ratio of 2.27 for a full actuation radiator in surroundings at 4 K. Increasing panel and hinge material thermal conductivities and utilizing eight panels yields a turn-down ratio of 6.01. The largest possible turn-down ratios for a two, four and eight panel radiator, respectively, are 2.00, 3.98, and 7.92.
7.3 Nomenclature

\(A\) surface area (m\(^2\))

\(L\) length (m)

\(P\) heater power (W)

\(T\) temperature (K)

\(k\) thermal conductivity (W m\(^{-1}\) K\(^{-1}\))

\(q_{\text{rad}}\) radiative cooling power of the radiator (W)

\(r\) radius (m)

\(t\) thickness (m)

\(w\) width (m)

\(x\) horizontal position on fully-extended radiator, measured from center of aluminum block (cm)

\(\varepsilon\) emissivity

\(\kappa\) hinge conductance (W K\(^{-1}\))

\(\phi\) radiator angle

\(\sigma\) Stefan-Boltzmann constant

Subscripts

\([\ ]_{1-4}\) panel number

\([\ ]_{b}\) aluminum block

\([\ ]_{h}\) flexible hinge

\([\ ]_{m}\) multi-layer insulation (MLI)

\([\ ]_{p}\) radiator panel

\([\ ]_{s}\) standoffs

\([\ ]_{\text{surr}}\) surroundings

7.4 Introduction

Radiative cooling of spacecraft is commonly achieved through the use of specialized radiating surfaces. Currently, spacecraft radiators are sized sufficiently large so as to emit the maximum cooling load expected to occur in the spacecraft’s lifetime [5]. However, spacecraft waste heat
loads can be highly variable, increasing or decreasing significantly with orbit position, on-board electronic waste heat generation, and distance from the sun for interplanetary missions. Due to the static nature of radiator geometry and radiative surface properties, a decrease in the radiator’s cooling load results in a decrease in the overall spacecraft temperature, causing the temperature of critical components to fall below established limits. To mitigate this effect, survival resistance heaters are placed throughout the spacecraft and activated when component temperatures approach an established threshold [5]. Although effective, use of survival heaters requires the installation of additional battery and solar panel capacity, decreasing payload weight and power capacity for scientific instruments or communication arrays. The disadvantages of survival heaters are especially evident for small, high-powered spacecraft or interplanetary missions, where large variations in cooling loads and lower thermal mass dramatically increases the spacecraft power and weight used to maintain temperature set points.

Spacecraft radiators capable of dynamic variation in emitted power offer the potential to reduce the amount of survival heating necessary to maintain spacecraft temperatures within established limits [69]. Several technologies exist or have been proposed in the literature that allow for dynamic control of spacecraft radiator heat transfer. The turn-down ratio, or the fraction of largest possible emitted energy to smallest possible emitted energy, provides a metric by which the effectiveness of these technologies might be compared. Thermochromic surfaces [6–9,77] or electrochromic surfaces [11,14,16,76] exhibit control of radiative surface properties through variations in surface chemistry via a change in temperature or voltage, respectively. Recently proposed thermochromic films have exhibited turn-down ratios as high as 7 [9]. When circulating fluids are used to transfer heat to a radiator, the heat rejection from the radiator may be controlled by varying the speed, quantity or pathway of the fluid. Examples of this approach include variable conductance heat pipes [21,22], the International Space Station (ISS) thermal control system [142], or stagnation radiators [19], with turn-down ratios on the order of 10 or less. However, these methods are not available for small spacecraft which often do not use circulating fluid loops. Finally, variations to radiator geometry to achieve a change in radiator heat loss have been proposed or demonstrated, achieving turn-down ratios of 5 or less [30,31,133,155]. These technologies increase or decrease the emitting surface area of a radiator by deploying a radiator panel through expansion of an inter-
nal gas [26, 155] or through the use of actuation mechanisms driven by a variation in temperature such as shape-memory alloys [30, 133].

Nearly all completed studies on dynamic radiator technologies described in this review use passive solutions to achieve variation in radiator cooling power. The use of actively manipulated technologies, specifically motorized surfaces, for spacecraft thermal control has not yet been explored. Likewise, the study of re-deployable radiators has so far been restricted to the actuation of single-panel radiators acting in parallel. Published works that utilize variable geometry to control radiator cooling power use geometry as a means of concealing or revealing a black surface, causing the actuating geometry to function as a gate or shutter as opposed to a means by which large variations in emitting surface area might be realized.

A recently proposed re-deployable radiator design [66, 88, 123] utilizes origami topographies to achieve dynamic control of radiative heat transfer, consisting of several flat, rigid panels connected in a tessellated pattern with the ability to fold or unfold. As the device unfolds, the emitting surface area of the radiator panels increases causing the heat transfer to also increase [123]. Conversely, as the device folds, the emitting surface area decreases with an associated change in the heat transfer. The folding nature of the panels is such that many panels may be included in the device, significantly increasing the potential variation in radiator surface area. Likewise, individual panels of a collapsing origami tessellation form cavities with their neighbors, and the aspect ratio of these cavities varies with actuation. The apparent radiative properties of the cavities formed between neighboring panels are directly related to the aspect ratio of the cavity [2, 95] in a phenomenon known as the cavity effect. As such, origami-inspired radiators are capable of controlling both surface area and apparent radiative surface properties as a function of position. However, published research concerning this new technology has focused on the development of models that describe the radiator’s apparent radiative properties or net radiative heat transfer. An experimental demonstration of the technology using geometries, surface properties, and structures more appropriate for spacecraft applications has not yet been completed.

The purpose of this work is to develop a functional origami-inspired radiator prototype for the purpose of verifying the thermal control potential of an actively-controlled, multi-panel, deployable radiator. In this paper, the design of the radiator is described first, followed by the experimental methodology used to measure performance metrics and demonstrate the thermal control
potential of this actively-manipulated device. The basic outline of the thermal model used to predict the heat transfer from the radiator is then presented, with additional details available from 6. The experimental method used to validate the thermal model is also presented. Results from the experiment and numerical model are then used to quantify the turn-down ratio, cooling power and thermal control performance of the device. The numerical model is then used to predict the turn-down ratio of deployable radiators with improved materials and technologies. Finally, results are discussed as they apply to the viability of the approach for spacecraft thermal control applications and future development considerations.

7.5 Methodology

7.5.1 Radiator Design

Previous studies demonstrating the behavior of origami radiators have used thin metal shim stocks (thickness < 0.03 mm) folded into an accordion pattern. However, for spacecraft applications, deployable radiator structures must utilize rigid panels with structural support. ISS radiators [4], pictured in a semi-extended state in Figure 7.1, provide an excellent example of a deployable radiator with space heritage; although, the ISS radiators do not actuate dynamically and heat is transferred to the radiator panels through a pumped fluid loop. The deployable radiator discussed in this work was inspired by the ISS radiator design with accommodations for active control and thermal transport via heat conduction.

The radiator developed in this work can be divided into two subsystems; the panel subsystem and the actuation subsystem. The panel subsystem, pictured in Figure 7.2a and consisting of four panels and a heated aluminum block all connected with thermally conductive flexible hinges, is responsible for conducting heat away from the heat source and radiating this energy to space through thermal emission. The actuation subsystem, pictured in Figure 7.2b, provides structure for the panel subsystem and is the mechanism by which the position of the panel subsystem is varied dynamically. When combined (Figure 7.2c), the two subsystems create a radiative fin that can be actuated in real time over a wide range of positions, from nearly collapsed to fully-extended.
Figure 7.1: A portion of the External Active Thermal Control System on the International Space Station [4]. One of the single-deployment radiators utilized in the thermal control system, a series of connected panels arranged in series, is displayed in the image. Unlike the analysis in this work, heat is pumped to the radiators in this image through a pumped fluid loop.

**Panel Subsystem**

Four aluminum panels (labeled 1 through 4 in Figure 7.2a; alloy: 1100, hardness: Brinell 30, temper: H14), measuring 16 cm wide and 10.2 cm long, with a thickness of 3.5 mm are coated on both sides with AZ-93 paint, a spectrally selective coating with an emittance in the infrared band of 0.91 and an absorptance in the solar band of 0.15 [1]. Each panel is connected in series to a neighboring panel via a flexible thermal hinge (A in Figure 7.2a). A heated aluminum block (B in Figure 2a), measuring 15 cm long, 1.2 cm thick and 2.4 cm wide, is connected to panel 1 (as labeled in Figure 2a) using a similar thermal hinge. The thermal hinge consists of fifteen nickel-coated, copper grounding straps (part number: EM 014-FB-250, manufacturer: Electric Motion Company) measuring 8 mm in width and 1 mm in thickness. The bottom side of the copper straps are epoxied to one side of each aluminum panel via thermal epoxy (Duralco 132, Cotronics Corp.) and the top side of the straps are epoxied to an aluminum pressure plate which is riveted to the radiator panel. A schematic of the thermal hinge is depicted in the bottom right of Figure 7.2a. The presence of the thermal hinge separates each aluminum panel a distance of 2.4 cm when laid flat. The thermal epoxy, pressure plate and rivets are intended to decrease the thermal contact resistance between the panels and the copper straps through the application of pressure and through the introduction of a conductive interface material (epoxy). Since this work is intended to demonstrate the use of
Figure 7.2: (a) The panel subsystem, consisting of four solid aluminum panels coated in a spectrally-selective paint typical of spacecraft radiators. The panels are connected with woven copper straps. The left-most panel is connected to a solid aluminum block with embedded heaters. The inset image at the bottom right is a cross-section of the thermal hinge connecting the panels. (b) The actuation subsystem, consisting of two scissor extension mechanisms constructed from aluminum struts, are connected via steel rods with fiberglass sleeves. The entire system actuates up or down with a stepper motor. (c) The combination of the panel and actuation subsystems, forming the complete radiator.
an actively-controlled deployable radiator for temperature control, the thermal hinge was designed for economy and manufacturability, not performance. Likewise, the geometry of the panels has not yet been optimized to achieve maximum heat rejection per unit area. Improved designs for the thermal hinge are possible and the effect of the hinge conductance will be explored in the results.

**Actuation Subsystem**

The actuation subsystem includes: (1) two articulating strut assemblies that each form a scissoring mechanism, (2) thermally insulated rods connecting the two strut assemblies for securing the panel subsystem to the actuation subsystem, and (3) a base support structure that secures the two strut assemblies to a base plate while providing means for actuation. The strut assemblies each consist of eight identical aluminum struts (one strut labelled C in Figure 7.2b), each measuring 13.4 cm long, 1 cm wide and 1.6 mm thick, and arranged in a repeating ‘X’ pattern to form a scissoring mechanism. Adjacent struts are connected with aluminum shoulder bolts (D in Figure 7.2b) passing through holes on either end and in the middle of each strut. The two strut assemblies are aligned in a parallel fashion such that the actuating motion is vertical. Rods (E in Figure 7.2b), constructed from 3.5 mm diameter stainless steel and threaded on each end, connect the two strut assemblies together by passing through equivalent holes on each strut assembly. An insulating sleeve, constructed from a fiberglass laminate (G10), is sheathed over each connecting rod to prevent heat conduction between the hinges of the panel subsystem and the framework of the actuation subsystem (E in Figure 7.2b). The strut and rod assembly is secured to a base plate (F in Figure 2b). Of the four struts that form the bottom of both scissoring mechanisms, one pair is secured to a stand on the base such that they can rotate about an axis but are unable to translate in any direction (G in Figure 7.2b). The other pair is connected with an insulated rod (H in Figure 7.2b) that passes through a slotted aluminum bar, creating a slider mechanism. As the slider rod moves along the slot away from the secured strut location, the entire mechanism collapses downward until it reaches the fully collapsed state. If the slider rod is actuated towards the secured strut location, the assembly extends upwards until it reaches the fully extended state. Actuation of the slider rod is achieved through a vacuum-rated stepper motor (I in Figure 7.2b; part number: 4118M-01-52RO, manufacturer: Lin Engineering) controlled via LABVIEW. The stepper motor turns a threaded ACME rod which passes through a bronze nut attached to the slider rod.
The panel subsystem is connected to the actuation subsystem by attaching panel 4 (as labeled in Figure 7.2a) to the top-most rod using six vacuum-rated zip ties (two zip ties pictured in Figure 7.2c, labeled J). The panels are then woven throughout the structure such that each thermal hinge is wrapped around the outside of the insulated rod at each hinge in the actuation subsystem. The aluminum block is secured to the G-10 base with G-10 spacers (K in Figure 7.2c). The angle between neighboring panels (φ as pictured in Figure 7.2a) may be tracked in real time by measuring the starting angle of the system and then counting the number of “steps” taken by the stepper motor. Although the actuating subsystem alone could achieve a minimum angle of φ = 10° and a maximum angle of φ = 175°, the introduction of the hinged panels introduced an additional constraint. As such, the actuation range of φ that could be reliably achieved during experimentation was a minimum of φ = 35° and a maximum of φ = 170°.

7.5.2 Radiator Experiment Method

Radiator Experiment Setup

The purpose of this experiment is to demonstrate the ability of an actively controlled deployable surface to maintain a component at a specified temperature. To this end, the aluminum block with embedded heaters attached to panel 1 (as depicted in Figure 7.2a) was included to represent a spacecraft component such as a computer or battery. The radiative cooling power of the protected component was measured for discrete radiator angle (φ) values while the temperature of the aluminum block was maintained at a given value using a PID controller.

As shown in Figure 7.2a and 7.2c, panel 1 was secured via a thermal hinge to a solid aluminum block which represents a temperature-sensitive spacecraft component. Two cartridge heaters, measuring 6 mm in diameter and 7.5 cm long, were embedded within the aluminum block to simulate a thermal load. Three thermocouples, secured with thermal epoxy, were evenly distributed across the top of the aluminum block. A thermocouple was also epoxied to the side of the block facing the panel subassembly and the side of the block facing away from the panel subassembly. The average of these five thermocouples provided the temperature of the aluminum block (T_b). A single thermocouple was attached to the geometric center of the upper side of each radiator panel. The entire assembly was placed in a vacuum chamber at a pressure consistently
less than $10^{-7}$ Torr and the walls and platen of the vacuum chamber were cooled to a temperature ($T_{surr}$) of 173 K. These conditions eliminated convective heat transfer and generated a large temperature difference between the radiator and surroundings.

The entire assembly was mounted onto a 12 mm thick G-10 board to prevent conductive losses into the cooled platen. Likewise, the heated aluminum block was separated from the G-10 board using two thin G-10 standoffs, with a length ($L_s$) of 4 cm and a radius ($r_s$) of 6.4 mm, to further prevent conductive losses from the aluminum block. The aluminum block was entirely shrouded from the cooled surroundings through the use of Multi-Layer Insulation (MLI). The MLI and standoffs reduced conductive and radiative losses from the block itself, causing a large majority of the heat generated by the cartridge heaters to conduct into the radiator panels through the thermal hinge. As such, the power of the cartridge heaters approximates the cooling power of the radiator once thermal losses have been accounted for. An MLI enclosure was used to cover the stepper motor and its accompanying PID controlled heater (used to prevent the motor from falling below its operational temperature range).

**Actuating Radiator Experiment**

To begin, the radiator was fully actuated to a 147° actuation angle ($\phi$ in Figure 7.2c), the vacuum system was activated and the surroundings were cooled to a temperature of 173 K using a liquid nitrogen shroud. The radiator and aluminum block temperatures were allowed to come to steady state, where steady state is defined as variation of less than 0.1 °C over the period of one hour for all monitored thermocouples. Once steady state conditions were achieved, the cartridge heaters in the aluminum block were activated. The heaters were controlled via a PID controller, where the average temperature of the five thermocouples mounted on the aluminum block provided the process variable for the controller and the temperature set point of the aluminum block was 293.5 K throughout testing. The system was again allowed to come to steady state and the heater power necessary to maintain the aluminum block at 293.5 K was recorded using a two-minute average of heater power data. Temperatures of the aluminum block thermocouples and all four radiator panel thermocouples were also recorded using a two-minute average. The radiator system was then actuated inwards (towards a more compressed state) by reducing $\phi$ from 147° to 138°. The variation in panel geometry caused the heater power to decrease while the aluminum block’s
temperature was maintained. Steady state values were again recorded and the process was repeated. Data was collected for actuation angles of $\phi = 147^\circ, 138^\circ, 121^\circ, 107^\circ, 92^\circ, 80^\circ, 65^\circ, 53^\circ, 44^\circ$ and $37^\circ$. The time required to reach steady state varied from 2 - 6 hours depending on the actuation angle of the radiator.

**Stationary Radiator Experiment**

A separate experiment was also performed to determine how the temperature of the aluminum block would change if the radiator remained in the fully extended position while the heater power decreased to predetermined set points. This test provides a comparison case that simulates the current state-of-the-art with a fixed radiator. For this test the radiator surface was maintained at a constant value of $147^\circ$ while the heater power was set to the measured power values obtained for each angle in the actuating radiator experiment. The steady state temperature of the aluminum block was then recorded for each power level. The temperature reduction of the aluminum block with each successive decrease in heater power represents a temperature difference that would be corrected for using thermostat-controlled heaters on modern spacecraft.

**7.5.3 Radiator Thermal Model**

The experiment utilized only one uniform panel geometry and one simple thermal hinge design. Further, the radiator system was not able to actuate below an angle of approximately $35^\circ$. Therefore, the full potential of a system of this kind was not entirely explored in the experimental conditions. As such, a numerical model that approximates the heat transfer and temperature profile of the radiator was used to determine data that could not be determined experimentally for this work. This numerical model was validated via comparison with equivalent experimental data.

A paper reporting the development and initial results from this model will be published separately and is provided as Chapter 6. The reader is referred to that publication for full details regarding the model. In short, each panel is subdivided into a discrete number of elements, where each element is assumed isothermal. The model also assumes that the panels are isothermal along the width of the panels, or perpendicular to the axis of conduction. Conduction between elements is calculated with a numerical approximation of Fourier’s law, and radiative heat exchange is cal-
culated by summation of radiosity terms from elements that are visible to the element of interest. Boundary conditions are applied and a governing equation is determined for each element. Using an initial guess of temperature and radiosity distributions, the resulting series of simultaneous equations is solved iteratively using the Thomas algorithm. Necessary inputs to the model include the length, width and thickness of the panels, the emissivity of the panels, the number of panels, the thermal conductivity of the panel material \((k_p)\), the thermal conductance of the thermal hinge \((\kappa)\), and the temperature of the heated aluminum block \((T_b)\).

The primary purpose of this work is demonstration of the potential for thermal control with an actively-controlled deployable surface. As such, the design and materials used in the prototype are not optimized for thermal performance. The turn-down ratio of a radiator of this kind could be further increased through the use of highly conductive panels and hinges with a high thermal conductance. Likewise, the experimental temperature of the surroundings \((173 \, \text{K})\) does not reflect the effective temperature of the surroundings experienced by a radiator pointed towards deep space onboard an orbiting spacecraft \((4 \, \text{K})\). To this end, the numerical model was used to analyze the turn-down ratio of several theoretical radiators with a surrounding temperature of \(4 \, \text{K}\). These radiators utilize the same panel geometries as the radiator described in this paper but the panel thermal conductivity, hinge conductance, and number of radiator panels are varied to determine the impact on thermal performance. Panel thermal conductivities used in this numerical test correspond to values for aluminum \((k = 237 \, \text{W m}^{-1} \, \text{K}^{-1})\) [156], copper \((k = 401 \, \text{W m}^{-1} \, \text{K}^{-1})\) [156], and in-plane graphite \((k = 1950 \, \text{W m}^{-1} \, \text{K}^{-1})\) [153]. Improvements in the thermal hinge were demonstrated by testing the model at hinge conductance values of \(\kappa = 0.6, 6 \text{ and } 60 \, \text{W K}^{-1}\) which correspond to the range of published conductance values for flexible oscillating heat pipes [157–160]. Finally, each combination of panel thermal conductivity and hinge conductance was tested for a radiator with 2, 4 or 8 panels. The reference case of an infinite panel thermal conductivity and infinite hinge conductance for radiators with 2, 4 or 8 panels was also performed as a demonstration of the best performance case possible.

### 7.5.4 Hinge Thermal Conductance

The thermal conductance for the flexible hinge constructed with copper straps and used in the current experiment was determined experimentally for use in the numerical model. The
measured hinge conductance allowed for comparison between the numerical and experimental results for validation.

To determine the hinge thermal conductance, the panel subsystem is separated from the actuation subsystem and positioned in a fully extended configuration ($\phi = 180^\circ$). The entire panel subsystem is insulated by sandwiching the panel and hinge assembly between two 5 cm thick sheets of expanded polystyrene. Three thermocouples, separated by a distance of 2 cm, are secured on the surfaces of panels 2 and 3 with aluminum tape in a straight line along the direction of heat conduction. Two thermocouples are attached to the aluminum heated block. The assembly is secured in a vertical orientation and suspended so that part of panel 4 extends into an insulated box. The insulated box is filled with an ice-water mixture or liquid nitrogen such that 4 cm of panel 4 is submerged. Heaters mounted inside the aluminum block maintain the temperature of the aluminum block at a given set point and the system is allowed to come to steady state (again defined as a change of 0.1 °C or less over an hour period for any thermocouple). The temperature profile, as measured by thermocouples along the entire assembly, was recorded over an average of 20 s of data. Data was collected for aluminum block set point temperatures ranging from 298 K to 308 K for each constant temperature bath. The experiment was performed eight times with an ice-water bath and three times with a liquid nitrogen bath.

The temperature data collected from panel 2 ($T_{2,1}$, $T_{2,2}$ and $T_{2,3}$ as shown in Figure 7.3) and panel 3 ($T_{3,1}$, $T_{3,2}$ and $T_{3,3}$ shown in Figure 7.3) along with the measured distance between each thermocouple is used to generate a linear regression to estimate the temperature profile in each panel. These regressions are then used to estimate the temperature of panels 2 and 3 immediately adjacent to the hinge, $T_{2,h}$ and $T_{3,h}$, respectively (Figure 7.3). The temperatures $T_{2,h}$ and $T_{3,h}$ were not measured directly as the proximity of the hinge introduced three-dimensional temperature gradients that were not accounted for in the numerical model. The derivative of the linear regression is used with Fourier’s law to determine the heat flux at the center of each panel. The average heat flux measurement of panels 2 and 3 is used to estimate the heat flux across the hinge. The ratio of the heat flux through the hinge to the temperature drop across the hinge ($T_{2,h} - T_{3,h}$) then gives the thermal hinge conductance as a function of hinge temperature.
Figure 7.3: To determine the hinge conductance, three thermocouples are placed on panel 2 and panel 3 at a distance 2 cm apart. A linear regression derived from these measured temperatures gives an estimate of the temperature in the panels immediately adjacent to the hinge, \( T_{2,h} \) and \( T_{3,h} \). Likewise, the derivative of the linear regressions is used with Fourier’s law to find the heat flux at the center of each panel. The ratio of the average heat flux value to the difference between \( T_{2,h} \) and \( T_{3,h} \) gives the hinge conductance as a function of hinge temperature.

7.5.5 Error and Uncertainty Analysis

Heater power measurements approximate the thermal radiation heat loss from the radiator panels and may be compared with the output from the numerical model. For an accurate comparison, experimental heat losses from the aluminum block must be quantified as well as the experimental error of the power and temperature measurements. Likewise, the error of the numerical model with respect to the uncertainty in radiator dimensions and physical properties must be accounted for.

Radiator Experiment Error and Uncertainty

The heater power \((P)\) is reported as the average of 2 min of data, with data collected every 3.5 s. The uncertainty of the power measurement is given as the first standard deviation over the 2 min of heater power data. Likewise, the heat lost from the aluminum block via conduction into the base through the fiberglass standoff is quantified with with second term of Equation 7.1, using an approximation of Fourier’s Law. The temperature of the fiberglass baseboard is assumed to be
the temperature of the platen ($T_{surr}$), the radius of the standoff is $r_s = 1.27$ cm, the length of the standoff is $L_s = 2.54$ cm, and the thermal conductivity of the standoff is $k_s = 1.059$ W m$^{-1}$ K$^{-1}$. The radiative heat loss from the aluminum block to the radiative insulation and then into the cold surroundings is given as the third term in Equation 7.1, where the emissivity of the aluminum block ($T_b$) is given as 0.20 - 0.33 for heavily-oxidized aluminum [103], the emissivity of the MLI ($\varepsilon_m$) is given as 0.015 – 0.030 [161], the surface area of the block is $A_b = 0.012$ m$^2$ and the external surface area of the MLI is $A_m = 0.02$ m$^2$. These two heat loss terms are subtracted from the heat power measurement to give the radiative cooling power of the radiator ($q_{rad}$) as shown in Equation 7.1.

$$q_{rad} = P - 2k_s\pi r_s^2 \frac{T_b - T_{surr}}{L_s} - \frac{\sigma \left( T_b^4 - T_{surr}^4 \right)}{1 - \varepsilon_b A_b + \frac{1}{A_b} + \frac{2}{1 - \varepsilon_m A_m}}$$  \hspace{1cm} (7.1)

The uncertainty of an individual thermocouple measurement, such as the temperature at a given panel location or the temperature of the surroundings, is the uncertainty of a T-type thermocouple, reported by the manufacturer (Omega) as $\pm 1$° C. The uncertainty of the component temperature ($T_b$), which utilizes five temperature measurements, is given with the root-sum-square of the five temperature measurement’s uncertainties using the thermocouple uncertainty of $\pm 1$° C, giving a component temperature uncertainty of 0.44 °C.

Regarding the radiator position uncertainty, the vacuum chamber used for testing featured four viewing ports equally spaced around the perimeter of the cylindrical chamber. The height of the windows was such that a portion of the radiator was always visible along the center-line of the windows. At each radiator position, two photographic images were captured. The first image was photographed in the same plane as the scissoring mechanism. The struts of the scissor mechanism, which were a known length, were used as a reference length and the radiator angle was measured graphically by determining the angle between adjacent mechanism struts. The second image was photographed in the same plane as the fully-extended panels, similar to the image in Figure 7.2a. The stainless-steel rods, again of a known length, were used as a reference point, and the vertical distance between adjacent steel rods and the known length of the aluminum struts was used to measure the current radiator angle. Additionally, the radiator angle was determined by counting revolutions of the stepper motor, to determine changes in actuation distance and a
corresponding change in panel angle. The average of the three radiator angle measurements above (2 photographic, 1 counting revolutions) was provided as the actual radiator position while the uncertainty was reported as the average difference between the three measurements.

The total uncertainty of the radiative cooling power is determined with the root-sum-square-method. Table 7.1 provides the numerical values for the uncertainty of each variable used in Equation 7.1.

### Table 7.1: The value and uncertainty for each variable found in Equation 7.1 as reported at 273 K. Sources are provided for uncertainties found in published works.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Uncertainty (±)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$</td>
<td>1.059 (W m$^{-1}$ K$^{-1}$)</td>
<td>0.019</td>
<td>[162]</td>
</tr>
<tr>
<td>$L_s$</td>
<td>0.0254 (m)</td>
<td>0.0005</td>
<td>least-count uncertainty</td>
</tr>
<tr>
<td>$r_s$</td>
<td>0.0127 (m)</td>
<td>0.0005</td>
<td>least-count uncertainty</td>
</tr>
<tr>
<td>$\varepsilon_b$</td>
<td>0.260 (-)</td>
<td>0.060</td>
<td>[103]</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>0.023 (-)</td>
<td>0.006</td>
<td>[161]</td>
</tr>
<tr>
<td>$A_{b}$</td>
<td>0.012 (m$^2$)</td>
<td>0.000254</td>
<td>least-count uncertainty</td>
</tr>
<tr>
<td>$A_m$</td>
<td>0.020 (m$^2$)</td>
<td>0.00028</td>
<td>least-count uncertainty</td>
</tr>
</tbody>
</table>

### Table 7.2: The value and uncertainty for variables used as inputs to the numerical model. The panel thermal conductivity value is reported at a temperature of 293 K where the uncertainty of the value is the difference between the thermal conductivity at 293 K and at the lowest temperature encountered in analysis, 220 K.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Uncertainty (±)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_p$</td>
<td>0.102 (m)</td>
<td>0.00001</td>
<td>least-count uncertainty</td>
</tr>
<tr>
<td>$w_p$</td>
<td>0.159 (m)</td>
<td>0.0005</td>
<td>least-count uncertainty</td>
</tr>
<tr>
<td>$t_p$</td>
<td>0.0032 (m)</td>
<td>0.00001</td>
<td>least-count uncertainty</td>
</tr>
<tr>
<td>$k_p$</td>
<td>220 (W m$^{-1}$ K$^{-1}$)</td>
<td>2</td>
<td>[156]</td>
</tr>
<tr>
<td>$\varepsilon_p$</td>
<td>0.91 (-)</td>
<td>0.01</td>
<td>[163]</td>
</tr>
<tr>
<td>$T_b$</td>
<td>293 (K)</td>
<td>1</td>
<td>least-count uncertainty</td>
</tr>
</tbody>
</table>
Numerical Model Uncertainty

The properties used to describe the physical radiator (panel length $L_p$, panel width $w_p$, panel thickness $t_p$, panel emissivity $\varepsilon_p$, material thermal conductivity $k_p$) are inputs to the numerical model. To determine the final uncertainty of the radiator heat transfer rate as determined by the numerical model, the influence of each parameter’s uncertainty on the final result is quantified by the Method of Sequential Perturbations [129]. In this method, an individual parameter, such as the emissivity, is increased by the uncertainty of the reported value and the numerical model is executed, giving a heat transfer rate which accounts for the increased value of a single parameter. The parameter value is then decreased below the reported value by the uncertainty of the parameter and the numerical model is again executed. Half of the difference between the two model results gives the uncertainty of the result due to the single test parameter, or the sensitivity index. This procedure is performed for each parameter, and the root-sum-square of these values gives the total uncertainty of the numerical result. Table 7.2 reports the value and uncertainty for each parameter used in the numerical model. For the panel thermal conductivity, the value was determined at a temperature of 293 K using a correlation from [164] and the uncertainty is the difference between the value at 293 K and the lowest temperature encountered by a panel, being 220 K.

7.6 Results and Discussion

7.6.1 Radiator Experiment

Figure 7.4a illustrates the results of the radiator experiment where the potential for thermal control via active manipulation of a deployable surface is demonstrated. The radiative cooling power ($q_{rad}$), being the heater power minus the experimental loss terms (Equation 7.1), is given as the vertical axis on the right. The left vertical axis depicts the temperature of the protected component, or the average of the five thermocouples placed on the heated aluminum block. Both vertical axes are reported as a function of the radiator angle ($\phi$). The error of the radiative cooling power is depicted as error bars, and the component temperature uncertainty, which is not depicted, is $\pm 0.44^\circ$C. Finally, the uncertainty of each measurement with respect to the radiator angle is $\pm 3^\circ$. Figure 7.4b also depicts the temperature of the protected component for the actuated radiator, now plotted with respect to the radiator cooling power (‘$\phi – \text{Variable}$’). Likewise, the temperature of
the protected component for a stationary radiator at $\phi = 147^\circ$ (near flat) is plotted. Again, the uncertainty of each data point is $T = \pm 0.44^\circ$C for the y-axis and $\pm 3^\circ$ for the x-axis.

The temperature of the aluminum block was maintained at a constant value by actively controlling the heater power in response to variations in the radiator’s position. This approach incorporated a PID controller which significantly reduced the time required to reach steady state conditions. However, in a spacecraft application, the device would be utilized in the inverse control scenario, where the radiator’s position would be actively controlled in response to variations in the waste heat load. However, the steady-state values for aluminum block temperature and radiative cooling power as reported in Figure 7.4a would be identical for either control scenario. Figure 7.4a, therefore, illustrates the capability of an actively controlled multi-panel radiator to maintain the steady state temperature of a spacecraft component subjected to varying waste heat loads. This is evidenced by the constant temperature of the aluminum block for an actively controlled radiator scenario. In contrast, the extended, stationary radiator ($\phi = 147^\circ$) was unable to maintain the temperature of the aluminum block at a constant value as shown in Figure 7.4b. As the radiative cooling power decreased from a maximum of 14.3 W to a minimum of 10.8 W, the temperature of the aluminum block likewise decreased by a total of 23 K, a variation in temperature that may be too large for sensitive components such as batteries, optics or processors [5]. In an applied scenario, this decrease in waste heat load, amounting to 3.5 W, would need to be supplied by an equivalent increase from survival heaters attached to the protected component. This survival heater power must be applied continuously whenever the waste heat load remains below the maximum value, increasing the necessary battery and solar panel capacity of the spacecraft.

Figure 7.5a illustrates the heat transfer control potential of the device. From fully-extended ($\phi = 147^\circ$) to fully-retracted ($\phi = 37^\circ$), the experimental prototype demonstrated a turn-down ratio of 1.32 (ratio of largest to smallest cooling power). This value, however, does not reflect the true potential of the radiator. As seen in Figure 7.5a, the variation in cooling power as a function of radiator angle is greatest for small radiator angles. For example, the radiator cooling power decreased by only 0.08 W between $\phi = 147^\circ$ and $\phi = 138^\circ$. However, when the radiator angle decreased from $\phi = 45^\circ$ to $\phi = 37^\circ$, the radiative cooling power decreased by 0.75 W. Further decreases in radiator angle below the experimental limit would have resulted in significant decreases in radiative cooling power. Likewise, the vacuum chamber shroud was maintained at a temperature of 173 K.
Figure 7.4: (a) Temperature of the protected component ($T_b$) as a function of radiator angle for a radiator with variable geometry. Radiative cooling power of the actuated radiator ($q_{rad}$) as a function of the radiator angle is also reported using the right axis. Uncertainty of each temperature measurement is not depicted but is ±0.44 K for each data point. (b) Temperature of the protected component ($T_b$) as a function of radiator cooling power ($q_{rad}$) for the actuated ($\phi =$ Variable) and stationary ($\phi = 147^\circ$) tests. In the actuated test, the radiator position varied as a function of radiator cooling power. In the stationary test, the radiator was fully extended ($\phi = 147^\circ$) and the component temperature was allowed to decrease as the radiator cooling power decreased. (c) Temperature of the radiator as a function of position along the radiator for three radiator angle positions, where $x = 0$ corresponds to the center of the protected component. Data points at 10, 22, 35, and 48 cm correspond to the center of panels 1 – 4, respectively.
throughout testing. However, the radiator of a spacecraft is generally exposed to deep space at an effective temperature of 4 K, and the temperature of the surroundings impacts the turn-down ratio of the radiator. Results from the numerical model are used to predict the turn-down ratio of the radiator for a larger range of radiator angles \((5^\circ < \phi < 180^\circ)\) and for a surrounding temperature of 4 K.

Figure 7.4c depicts the temperature profile of the radiator for three different radiator angles. The temperature at \(x = 0\) cm correspond to the center of the aluminum block. Each successive \(x\)-location (10, 22, 36, and 48 cm) corresponds to the midpoint of panels 1 - 4, respectively. The uncertainty of each temperature measurement is \(\pm 1\) K. As shown in Figure 7.4c, the panel temperatures decrease as the radiator angle (and radiative cooling power) increases. This decreasing temperature trend is due to the increased exposure of the panels as the radiator extends outwards. Likewise, the difference between the temperatures of panel 1 and panel 4 increases from an initial value of 34 K at \(\phi = 37^\circ\) to a value of 42 K at \(\phi = 147^\circ\). Both of these temperature trends indicate a decrease in the radiative fin efficiency of the device as the radiator angle increases, a phenomenon explained by the decreasing influence of inter-panel radiation heat transfer as the radiator expands. At small radiator angles, the panels are exposed almost entirely to adjacent panels and energy is easily transferred along the length of the radiator via both heat conduction and thermal radiation emission and absorption. However, as the radiator expands, the panels move away from each other, removing the radiation coupling between panels and decreasing the total quantity of energy that is transferred from the radiator base to the radiator tip. As such, a competing effect is observed, where the radiator fin efficiency decreases and the radiator cooling power increases as the radiator angle increases.

### 7.6.2 Hinge Conductance Experiment

The hinge conductance test was completed eight times for the ice-water bath and three times for the liquid nitrogen bath with the aluminum block set to a random temperature between 298 and 308 K. The average hinge conductance using the ice-water bath data is 0.67 W K\(^{-1}\) with a standard deviation of 0.05 W K\(^{-1}\) at an average hinge temperature of 289 K. For liquid nitrogen the average hinge conductance is 0.48 W K\(^{-1}\) with a standard deviation of 0.09 W K\(^{-1}\) at an average hinge temperature of 230 K. These results indicate that the hinge conductance decreases as the hinge
temperature decreases, perhaps due to reduced contact pressure as the hinge components contract with a decrease in temperature. Although the hinge conductance data is not suitable to adequately predict the relationship between temperature and hinge conductance with confidence (due to a weak temperature dependence relative to the uncertainty), the full span of experimental hinge temperatures (220 - 280 K) is approximately represented by the hinge conductance measurement data. As such, the numerical model utilized a mean value for the hinge conductance of $\kappa = 0.58 \text{ W K}^{-1}$ with a 25% uncertainty determined from the spread in measured values relative to the mean. In this manner the variation in hinge conductance with hinge temperature is accounted for in the numerical model by increasing the uncertainty of the numerical result.

As a general comparison, commercially available copper straps exhibit a thermal conductance of 0.168 W K$^{-1}$ at a temperature of 77 K over a transported distance of 0.057 m [165]. Likewise, a number of researchers have developed flexible oscillating heat pipes with reported thermal conductance values (averaged over all tested power levels and charge ratios) of 0.359 [157], 1.913 [158], 5.290 [159], and 62.53 [160] over transported distances of 0.085 m, 1.070 m, 0.270 m, and 0.400 m respectively. However, these conductance values for the flexible oscillating heat pipes were obtained for temperature ranges above 293 K.

7.6.3 Numerical Model Validation and Results

Experimental/Model Validation

Figure 7.4a depicts a comparison of the radiator’s cooling power as determined by experimental measurements and the numerical model as a function of radiator angle. Numerical model input values and uncertainties are given in Table 7.2. The experimental uncertainties of the radiator cooling power are expressed as error bars for each experimental data point. For the numerical model, the uncertainty values given in Table 7.2 and the measured uncertainty of the flexible hinge were used in conjunction with the method of sequential perturbations to determine the uncertainty of the numerical model results (shaded region in Figure 7.5a). Likewise, Figure 7.5b depicts a comparison of the experimental and numerical panel temperatures for the center-points of panels 1, 2, and 4 as a function of radiator angle. Uncertainty for each data point is again expressed with error bars for experimental results and as a shaded region for numerical model results.
Figure 7.5: (a) Radiator cooling power as a function of radiator angle as measured experimentally and calculated via the numerical model (Chapter 6). The shaded area represents the region of uncertainty associated with the numerical results. Experimental results consistently fall within the error bounds of the numerical model, with a largest relative error (relative to experimental results) of 2.6%. (b) The mid-point temperatures of panels 1, 2, and 4 as a function of radiator angle as determined experimentally and as calculated by the numerical model. The largest relative error between numerical and experimental results is 1.2%.

As shown in Figure 7.5a, the numerical and experimental approaches generally agree, with a worst-case discrepancy of 0.38 W at a radiator angle of 147°, or 2.6% relative error compared to the experimental value, and all experimental data points fall within the uncertainty of the numerical model. The disagreement in radiator cooling power is greatest for large radiator angles. Likewise, the turn-down ratio as computed by the experimental data is 1.31, whereas the turn-down ratio as computed by the numerical model for the same angle range is 1.36, or 3.8% relative error. Figure 7.5b demonstrates the generally good agreement between the numerical model results and experimental results with regards to panel temperatures. All experimental data points fall within the uncertainty of the numerical model, and the largest relative error between approaches is 1.1%. These comparisons serve as a general validation of the numerical model’s ability to provide an estimate of the radiator’s cooling power and temperature profile. Likewise, the small discrepancy between the turn-down ratio values of the two approaches (3.8% relative error) suggests that the
turn-down ratio as determined by the model is sufficiently accurate for approximation of the turn-
down ratio for a specific radiator design.

Several assumptions utilized in the numerical model explain the discrepancy between the
numerical and experimental results. First and foremost, measurements indicate that the hinge
conductance may likely be a function of temperature, with the hinge conductance decreasing as
temperature decreases. Further, the temperature of the hinge connected to the aluminum block
(approximately the same temperature as the aluminum block) and the temperature of the hinge
connecting panels 3 and 4 (mean value of panel 3 and panel 4 temperatures) differed by 70 K at
$\phi = 147^\circ$ and only 47 K at $\phi = 37^\circ$. As such, the average hinge temperature and, therefore, the
average hinge thermal conductance varies as a function of radiator angle. The numerical model,
however, assumes a uniform conductance for all hinges in the radiator and uses this same value for
all tested radiator angles. To demonstrate the impact a variable hinge conductance would have on
the numerical model prediction, the average hinge conductance value ($\kappa$) was allowed to vary as
a function of radiator angle, scaling linearly from $\kappa = 0.6$ at $\phi = 47^\circ$ to $\kappa = 0.5$ at $\phi = 147^\circ$, ac-
counting for a decreasing average hinge temperature as $\phi$ increases. Compared with the numerical
heat transfer data provided in Figure 7.5a, the numerical heat rate profile resulting from a scaled
average hinge conductance value matches the experimental data more effectively than the constant
conductance scenario, where the absolute error between experimental and numerical heat transfer
for the scaled scenario was a nearly constant value of 0.22 ± 0.02 W for every tested radiator an-
gle. Therefore, it is inferred that modelling hinge conductance as a function of temperature would
increase the accuracy of the heat transfer results. Second, the two-dimensional assumption used
in the numerical model is undoubtedly responsible for some portion of the disagreement, although
the exact amount is uncertain. Previous research has shown that 2D origami heat transfer models
used to describe 3D geometries show the greatest error in the mid-range of radiator angles ($30^\circ \leq
\phi \leq 90^\circ$), not the largest angle range [88]. Third, experimental conditions may also contribute to
discrepancy between prediction and experiment. Specifically, the angle between panels 3 and 4
was not always equivalent with the angle between panels 1 and 2 or between panels 2 and 3, with a
greatest reported discrepancy of 6°. In the small angle range ($\phi < 50^\circ$), such variations in radiator
angle could introduce error [123].
**Model Results**

With the numerical model validated experimentally, it may now be used to predict the radiative cooling power for radiator angle ranges and surrounding temperatures that could not be tested experimentally (i.e. $\phi < 37^\circ$, $T_{surr} = 4$ K) in order to find the turn-down ratio potential of a fully functional device. For an equivalent radiator comprised of the same components, geometries, and surrounding temperature as was tested in this work but with the ability to actuate from $180^\circ$ to $5^\circ$, the numerical model predicts a turn-down ratio of 1.94 ($q_{\phi=180^\circ} / q_{\phi=5^\circ}$). Likewise, a radiator with the same components and geometries but now in surroundings at 4 K and featuring full actuation achieves a turn-down ratio of 2.27 ($q_{\phi=180^\circ} / q_{\phi=5^\circ}$).

The radiator’s current design has a possible turn-down ratio that falls below the values achieved by alternative variable emissivity technologies. Recently-proposed vanadium oxide variable emissivity device achieves a turn-down ratio of 7 [9]. Similarly, electrochromic coatings and other switching devices show turn-down ratios of 5 or lower [11, 12]. However, the current prototype might be improved through the use of more conductive materials for panel construction, implementation of hinges with a larger conductance value, and the inclusion of additional radiating panels. The numerical model was used to project the possible but realistic turn-down ratios of radiators with panels constructed from highly conductive materials and hinges with larger conductance values. Likewise, each combination of thermal conductivity and hinge conductance was tested for radiators with 2, 4, or 8 panels in order to demonstrate the influence of number of panels on turn-down ratio. The combination of panel thermal conductivity, hinge conductance, and number of panels for possible radiators is displayed in Table 7.3 along with the calculated turn-down ratio. The case of a radiator with infinite thermal conductivity and infinite hinge conductance is also provided as a reference bound. The temperature of the surroundings was maintained at 4 K while calculating the turn-down ratios reported in Table 7.3 to simulate conditions typical of radiators aboard an orbiting spacecraft.

As seen in Table 7.3, the implementation of panel and hinge materials with greater conductivities increases the turn-down potential of the device. For a panel constructed from a highly conductive material such as graphite ($k = 1950$ W m$^{-1}$ K$^{-1}$) and with a hinge conductance value of 60, a turn-down ratio on the order of 3.70 is expected. This turn-down ratio might be further increased by incorporating straight or oscillating [166] heat pipes inside of the panels, where the
maximum possible turn-down ratio for this geometry is 3.98. Adding additional radiator panels would significantly increase the turn-down ratio of the device, although at the cost of additional weight (as shown in Chapter 6). Likewise, decreasing the number of panels decreases the expected turn-down ratio, although turn-down ratios up to 2 are still possible for a device with two panels. The hinge conductance values used in this analysis (0.6, 6, and 60) are not representative of specific devices but represent the range of thermal conductance values available with proposed or commercially available thermal hinges [157–160].

Table 7.3: Predicted turn-down ratio, as measured from a radiator angle of 5° to 180°, for a radiator with 2, 4 or 8 panels having the same geometry as the current experiments but with four different panel thermal conductivities and four different hinge conductance values.

<table>
<thead>
<tr>
<th>κ [W K⁻¹]</th>
<th>0.6</th>
<th>6</th>
<th>60</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panels</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>237</td>
<td>1.81</td>
<td>2.27</td>
<td>2.46</td>
<td>1.95</td>
</tr>
<tr>
<td>401</td>
<td>1.82</td>
<td>2.32</td>
<td>2.55</td>
<td>1.96</td>
</tr>
<tr>
<td>1950</td>
<td>1.82</td>
<td>2.39</td>
<td>2.67</td>
<td>1.97</td>
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<tr>
<td>kₚ [W m⁻² K⁻¹]</td>
<td>1.82</td>
<td>2.41</td>
<td>2.71</td>
<td>1.97</td>
</tr>
</tbody>
</table>

### 7.6.4 Design and Application Considerations

Controlled deployable surfaces feature several application-specific advantages not illustrated by the experimental or numerical data provided by this work. First, deployable surfaces have been used in spacecraft for decades, indicating a certain degree of space heritage. As an example, the International Space Station utilizes deployable radiators, where the panels were retracted for stowage prior to launch and then deployed upon installment via a cable and winch system (Figure 7.1). Likewise, solar panels, communication arrays, and even optical components of numerous spacecraft offer deployable features to enable functionality [167]. These technologies, however, require a one-time deployment upon orbit insertion, whereas a dynamic radiator requires intermittent variation of geometry. As such, a final radiator design must be robust, accounting for wear on the hinges from continuous use while providing a measure of redundancy in the case of hinge or actuating system failure.
Second, origami-inspired dynamic radiators are functionalized entirely by variations in geometry and therefore may be constructed from simple, inexpensive materials. As such, this technology would be easily and economically scaled to match the heat transfer requirements of any spacecraft. Likewise, deployable surfaces might be combined with other radiative heat transfer control technologies that utilize separate mechanisms. As an example, an actively or passively controlled deployable radiator might be coated with a thermochromic or electrochromic surface. Such a device would utilize variations in emitting radiative area as well as variations in surface intrinsic radiative surface properties. Acting in parallel, the heat transfer control mechanisms could increase the total turn-down ratio potential while providing multiple functional states for a given radiator position.

Finally, actively-controlled surfaces have the capability of rapid mobilization, enabling real-time matching of waste heat loads. Likewise, actively-controlled surfaces may be positioned in advantageous configurations previous to variations in thermal loads. Passive technologies require a variation in temperature to achieve actuation or reconfiguration and, therefore, the change in radiative behavior can be later than is desirable.

7.7 Conclusions

In this work, the heat transfer behavior of a multi-panel actively deployed radiator was demonstrated experimentally and explored numerically. The radiative cooling power of the four-panel radiator proposed in this work decreases as the radiator collapses with a measured turn-down ratio of 1.32 for the current radiator design (restricted to actuation from $\phi = 37^\circ$ to $\phi = 147^\circ$ and surroundings at 173 K) and a turn-down ratio of 2.27 for the same radiator when actuated from $\phi = 5^\circ$ to $\phi = 180^\circ$ and utilized in cold surroundings ($T_{surr} = 4$ K). Also, the use of an actively-controlled dynamic radiator to achieve constant temperature conditions for a protected component experiencing varying waste heat loads was demonstrated. Such a device would conserve survival heater power as it is currently utilized on orbiting spacecraft. Temperature measurements verified that the fin efficiency of the device decreases as the radiator extends towards a fully-open position, the result of a decrease in the radiative transfer between panels. Finally, the use of improved panel materials, hinge materials, and number of radiator panels was explored numerically, demonstrating turn-down ratios as high as 1.99, 3.70, and 6.01 for a two, four, and eight panel radiator,
respectively, through the use of highly conductive radiator panel materials and published prototype thermal hinges. The maximum possible turn-down ratios, assuming an infinite thermal conductivity and thermal conductance, for two, four and eight panel radiators operating in surroundings at 4 K are 2.0, 3.98, and 7.92 respectively.
CHAPTER 8. CONCLUSIONS

This dissertation presents research efforts concerning the utilization of dynamic geometry to control radiative heat transfer in real time. Combining the principles of origami tessellations and heat transfer, dynamically actuated surfaces are capable of radiative heat transfer control through variation in emitting surface area and apparent radiative properties. In Chapter 2, the validity of the cavity effect in thin metal materials was validated. Experimental methods were used to demonstrate the potential for control of apparent radiative properties via control of cavity geometries. Chapters 3 and 4 further developed the relationship between geometry and apparent radiative properties for an infinite V-groove by reporting stochastic or analytical models that determine apparent radiative properties as a function of cavity geometry and surface properties. These models were determined for surfaces that reflect diffusely (Chapter 3) or specularly (Chapter 4). With apparent radiative properties defined, the impact of a varying surface area was explored in Chapter 5. An analytical model was developed and then experimentally verified that quantifies the net radiative heat transfer of an isothermal surface as a function of actuation position. This model relies on the relationships determined in Chapters 3 and 4. Studies in Chapters 2 - 5 were conditional on the use of isothermal surfaces, an assumption that often will not hold for dynamic radiative surfaces. As such, Chapter 6 explored the behavior of a non-isothermal, dynamic radiative fin attached to a constant temperature base at one end with conductive rigid panels connected by flexible hinges; turn-down ratio and non-dimensional heat transfer rates for a non-dimensional fin were provided. In Chapter 7, the model developed in Chapter 6 was verified and a prototype of a dynamic spacecraft radiator was constructed and tested.

Conclusions from each individual chapter are now summarized in section 8.1. Future work pertaining to focus areas of this research is then detailed in section 8.2. Finally, a discussion concerning the impact of completed and future research is provided.
8.1 Chapter-Specific Conclusions

8.1.1 Chapter 2: Dynamic Control of Radiative Surface Properties with Origami-Inspired Design

The apparent absorptivity of a V-groove surface constructed from thin aluminum shim stock may be controlled by varying the cavity angle. For a large fold density and equivalent heating conditions the aluminum surface achieves a higher temperature due to the increased apparent absorptivity of the material. The inverse model developed in this chapter has not been seen in the literature previously. Temporal measurements of temperature combined with the application and removal of a heating source allows the heat loss from the surface to be determined and then subtracted from the heating condition such that the apparent absorptivity may be determined. This experimental method is verified against a model available from Sparrow with good agreement. Results from the inverse model demonstrate an increase of apparent absorptivity greater than 10x with the potential for larger increases. These large increases in apparent absorptivity illustrate the true potential of dynamic geometry for control of radiative surface properties via the cavity effect.

8.1.2 Chapter 3: Total Hemispherical Apparent Radiative Properties of the Infinite V-groove With Diffuse Reflection

Monte Carlo ray tracing methods are used to determine apparent radiative properties for a diffusely-reflecting V-groove. A simple correlation relating basic ray tracing measurements (rays emitted and rays escaped) with apparent radiative properties is developed. Analytical models describing the apparent radiative properties of diffusely-reflecting surfaces generally involve the solution of integro-differential equations. As such, these models are relatively inaccessible. In this work, algebraic correlations are developed that relate cavity angle, intrinsic emissivity and collimation angle (when present) to the apparent absorptivity of a V-groove cavity. All correlations proposed in this work as compared to ray tracing data exhibit relative errors (averaged over actuation angle) of 2.0% or less, with the greatest relative error never exceeding 6.0%. Compared to analytical models, the error of the correlations are on average 0.5%. The correlations are least accurate for small cavity angles, small intrinsic absorptivities, and small collimation angles.
8.1.3  Chapter 4: Total Hemispherical Apparent Radiative Properties of the Infinite V-groove with Specular Reflection

Models describing the apparent radiative properties of a specularly-reflecting V-groove were collected and evaluated in this work. A model available from Modest as well as models developed by Sparrow were reported. These models were compared with Monte Carlo ray tracing data and were shown to be very accurate, with an average, absolute discrepancy of $4.9 \times 10^{-4}$ between the models and ray tracing data. Likewise, a new analytical model was developed in this chapter to calculate the apparent absorptivity for a partially illuminated, infinite V-groove subject to collimated irradiation. This model, based on the fully-illuminated model provided by Sparrow, has also been verified with ray tracing results with an average discrepancy of $4.6 \times 10^{-4}$. A new equation was found to describe the number of reflections an incoming collimated ray will experience within a partially-illuminated cavity and it was shown that the ray-reflection counting method in the fully-illuminated case does not apply to the partially-illuminated case as suggested in previous works. Likewise, it is shown that the apparent radiative properties for a specular V-groove subject to collimated irradiation do not always increase as the cavity angle decreases. This is the first demonstration of a decrease in apparent radiative properties despite an increase in a cavities aspect ratio. This unique behavior is the result of a reflection pattern near the base of the V-groove that causes the number of reflections that occur within the cavity to decrease for a given angle range as the surface collapses. Results show that significant turn-down ratios ($\Psi > 5$) are possible for an isothermal, specularly reflecting surface. This is especially true for a surface subject to collimated irradiation where the irradiation is entering normal to the cavity opening. Likewise, these turn-down ratios are possible over a small angle range ($\Delta \phi < 20^\circ$).

8.1.4  Chapter 5: Control of Net Radiative Heat Transfer with a Variable-Emissivity Accordion Tessellation

A general expression, giving the net radiative heat transfer as a function of tessellation actuation angle and apparent radiative properties (which are actuation angle dependent) was developed. This model was applied to an accordion tessellation with a given number of panels. The influence of a decreasing projected surface area with decreasing cavity angle was demonstrated. A survey of the analytical results indicates that the decreasing surface area almost always dominates the in-
creasing apparent radiative properties, causing the net radiative heat exchange to decrease with a decreasing cavity angle for nearly all conditions. A specularly reflecting tessellation exposed to diffuse irradiation experiences large reductions in net radiative heat transfer for small cavity angle variations, achieving turn-down ratios greater than seven over a cavity angle range of $\pi/6$ for highly reflective surfaces. Diffuse reflectors with low intrinsic emissivities behave in a similar fashion but exhibit smaller turn-down ratios over a similar cavity angle range. When exposed to collimated irradiation, the turn-down ratio of diffuse reflectors is further reduced over the same cavity angle range. However, when exposed to collimated irradiation, specular surfaces experience somewhat oscillatory behavior, with widely varying net radiative heat transfer over relatively small changes in cavity angle for surfaces with low to moderately high emissivity. Likewise, specular surfaces exposed to normal collimated irradiation show an increase in net radiative heat exchange for certain cavity angle ranges despite a decrease in cavity angle. For a specular surface under normal, collimated irradiation, turn-down ratios of 3.35 and greater are possible for large cavity angles ($\phi > \pi/2$). When compared with experimental data, the model developed in this chapter exhibits a relative error of 4.2% or less. This model demonstrates the true potential of an origami-inspired dynamic radiator, where the combined influence of both the dynamic surface area and variable apparent radiative properties has been considered.

For certain scenarios (specular reflection, collimated irradiation) the heat transfer for a given angle $\phi$ relative to the heat transfer for a flat surface ($\phi = 1180^\circ$) is shown to increase above unity. This behavior occurs only for large angles and is the result of a steady increase in apparent emissivity as the surface collapses while the apparent absorptivity remains constant. The reported heat transfer is a net value and is calculated by subtracting absorbed heat transfer from emitted heat transfer. As the apparent emissivity increases the emitted heat transfer value likewise increases while the constant apparent absorptivity causes the absorbed heat transfer to remain constant. The net effect is an increase of the net radiative heat transfer above the fully-extended value.

8.1.5 Chapter 6: Heat Transfer, Fin Efficiency, and Turn-Down Ratio of a Self-Irradiating, Dynamic, Segmented Radiative Fin

The Segmented Fin Algorithm (SFA) was developed as a numerical model to predict the non-dimensional heat transfer and turn-down ratio for a dynamic radiative fin. The influence of
material properties, self-irradiation, and hinge thermal conductance on fin turn-down ratio and heat transfer was explored numerically using the SFA. Results show that dynamic fins are capable of turn-down ratios on the order of three or greater for realistic selections of panel geometry, panel material, and number of panels. Results also show that there exists an optimum number of panels such that turn-down ratio is maximized for a given panel geometry and material. Also, inter-panel radiative exchange always acts to reduce the turn-down ratio of a folding, radiative fin by providing an additional pathway for heat transfer along the length of the fin. Regarding emissivity, reflective fins may be used to rapidly adjust the heat rate as a result of the cavity effect, but this rapid response comes at the cost of relatively low total heat rates. For black fins the heat transfer varies substantially over the full actuation range. Likewise, the cavity effect has little influence for high emissivity fins, suggesting that variations in apparent area are more important than variations due to the cavity effect. Finally, panel geometry and material selection have a more significant impact on radiator turn-down ratio than hinge construction and performance, suggesting that optimization of the panel performance should be preferred over optimization of the hinge performance.

8.1.6 Chapter 7: Experimental Demonstration of Heat Loss and Turn-Down Ratio for a Multi-Panel, Actively Deployed Radiator

Building on the work of Chapter 6, this chapter details the development of a prototype spacecraft radiator. This radiator utilizes rigid aluminum panels that are connected in series via flexible thermal hinges constructed from interwoven copper straps that are secured to panels with thermal epoxy. The radiator is connected to an aluminum block which is maintained at a given temperature with a PID controller. The cooling power of the radiator is measured as a function of radiator actuation position. The radiative cooling power of the four-panel radiator detailed in this chapter decreases as the radiator collapses with a measured turn-down ratio of 1.32 for the current radiator design (restricted to actuation angles from $\phi = 37^\circ$ to $\phi = 147^\circ$ and surroundings at 173 K). Using the SFA, the turn-down ratio of a similar radiator is determined numerically for an actuation range from $\phi = 5^\circ$ to $\phi = 180^\circ$ and surroundings at a temperature of $T_{surr} = 4$ K, giving a turn-down ratio of 2.23. The data in this chapter also validates the use of an actively-controlled dynamic radiator to achieve constant temperature conditions for a protected component experiencing varying waste heat loads. In this experiment, collapsing the radiator geometry from
\( \phi = 147^\circ \) to \( \phi = 37^\circ \) reduced the cooling power by 3 W. State-of-the-art spacecraft thermal control would have required continuous, on-board heating of 3 W, consuming battery power. Temperature measurements verified that the fin efficiency of the device decreases as the radiator extends towards a fully-extended position, a result of a decrease in the radiative transfer between panels. Finally, the SFA was used to determine the impact of improved panel materials, hinge materials, and number of radiator panels on the heat rate and turn-down ratio. Turn-down ratios as high as 1.99, 3.70, and 6.01 for a two, four, and eight panel radiator, respectively, are possible through the use of highly conductive radiator panel materials and published prototype thermal hinges. Assuming an infinite thermal conductivity and thermal conductance, the maximum possible turn-down ratios for two, four, and eight panel radiators operating in surroundings at 4 K are 2.0, 3.98, and 7.92, respectively.

8.2 Future Work

8.2.1 Apparent Radiative Properties

Published cavity effect research focused on dynamic radiative heat transfer control has concentrated specifically on V-groove cavities. However, additional tessellations exist which might prove useful in application. Current research efforts are developing apparent radiative property data and models for the Miura-Ori, finite V-groove, hinged V-groove and Barreto’s Mars tessellations. Moving forward, apparent radiative properties might be developed for the “flasher” class of tessellations. These tessellations actuate in a radial fashion about a central axis and have received significant attention with regards to deployable spacecraft structures. The apparent radiative properties of these surfaces would prove useful not just for spacecraft radiator purposes but also to quantify the heat transfer behavior of deployable spacecraft surfaces, including antenna arrays and solar-panel arrays.

Research in this dissertation pertaining to apparent radiative properties has focused on total hemispherical properties. However, cavities have been shown to exhibit directional variation in apparent radiative properties which are a function of the cavity aspect ratio. Surfaces might be developed which allow for control of both the amplitude and direction of emission and absorption of a cavity opening. As an example, V-groove surfaces exhibit directional behavior in the range \( 30^\circ < \phi < 120^\circ \). Control of cavity angle would allow for a V-groove to preferentially emit radia-
tion in certain directions when it is advantageous to do so. Likewise, the absorption of irradiation within a V-groove exhibits directional behavior in the range $30^\circ < \phi < 120^\circ$, with an increased ability to absorb irradiation entering from near-normal directions and primarily reflecting irradiation from directions near the horizon to the cavity opening. Certain origami tessellations, such as the Barreto’s Mars, collapse such that the normal to the cavity surface points in a direction other than directly vertical. Actuation of these tessellations would allow for the preferred direction of emission and absorption to be controlled in real time.

Further, current models have assumed cavity surfaces to be gray and diffuse. However, spacecraft radiators utilize spectrally selective coatings with wavelength-dependent properties. New apparent radiative property models must be developed that account for the variation of intrinsic properties with wavelength. These models could include analytical methods or utilize Monte Carlo ray tracing, where rays are now assigned a wavelength band.

Finally, in Chapter 5 the model used to predict radiative heat transfer from an isothermal surface used the apparent temperature of the cavity for calculation of cavity heat transfer. The relationship between the apparent cavity temperature and real surface temperature might be explored, varying the observing emissivity in order to quantify how the two measurements are related. This research would validate the use of this technology for radiative camouflage, demonstrating how the perceived (apparent) temperature of a surface might differ from the real temperature of the surface through the use of cavity geometries.

### 8.2.2 Dynamic Radiative Fins

Chapter 6 is the first study of radiative fins arranged in series. As such, a large number of research questions within this field remain. The Segmented-Fin Algorithm (SFA) does not currently account for heat transfer exchange with the base structure to which the fin is attached, but the base has been shown to have a significant influence on total heat transfer. A study of base-fin interactions might include the use of various base geometries and sizes to account for the variation in spacecraft design. The impact of the base on turn-down ratio and total cooling power should be quantified. Current work being performed on Chapter 7 includes completing additional tests to quantify the hinge conductance and reduce the current error of 25% on the hinge conductance value.
Further, the SFA model should be developed to operate in three dimensions. The current two dimensional model, although functional, does not allow for more complicated tessellations (such as Barreto’s Mars or “flasher” folds) to be explored. A 3D model would allow for the influence of finite V-groove panels to be explored and for a more realistic turn-down ratio to be calculated.

Fin geometries should be optimized to meet certain design criteria. For spacecraft applications, fin geometries, materials and surface properties should be optimized such that the mass of the system is minimized while the turn-down ratio and total cooling power are maximized. For terrestrial applications the economic cost of a given system might be minimized while a performance metric (such as turn-down ratio or cooling power) is maximized.

Finally, dynamic radiative fins might be combined with alternative radiative control technologies to create devices with large turn-down ratios. As an example, a dynamic, origami-inspired radiator might be coated with an electrochromic material. When the radiator’s geometry is collapsed the emissivity of the radiator surfaces could be decreased via the electrochromic surface, further decreasing the minimum cooling power. This combined approach to radiative heat transfer control would drastically increase the turn-down ratio of a radiative control device and represents the first instance of multiple radiative control methodologies acting in parallel.

8.2.3 Application

Studies of deployable surfaces for radiative heat transfer control have been mostly restricted to spacecraft applications. The research in this dissertation has likewise focused on the use of origami tessellations as applied to thermal control of spacecraft. The concepts and models developed in this work, however, are also applicable to other scenarios where radiative heat loads dominate and exhibit temporal variation. Specifically, building thermal control stands to benefit significantly from the application of dynamic geometry for radiative heat transfer control. As an example, buildings may benefit from a design strategy that would generally absorb radiative heating during the winter months but then reject this heating during summer months. Current building technologies with the intent of controlling radiative loads focus on the use of spectrally-selective surfaces to enable net cooling even under direct sunlight. However, these surfaces might cause a building’s heating load to increase. The use of dynamic geometry would enable radiative heating
loads to be utilized optimally at all times. These surfaces might be incorporated onto building rooftops, within window panes or on buildings facades.

With regard to spacecraft applications, there is still a significant amount of work to be completed before dynamic radiators are utilized throughout the industry. The prototype presented in Chapter 7 might be improved significantly through the use of highly-conductive, light-weight materials such as graphite or through the implementation of heat pipes within the panel architecture. Further, improvements to the flexible thermal hinge might be explored through the use of additional copper straps, alternative hinge attachment methods, or by utilizing flexible oscillating heat pipes as panel hinges.

8.3 Impact

The fluctuation of both the strength and direction of solar irradiation over daily and yearly time scales has significantly informed the development of human technology utilized on the Earth and in space. When properly utilized, solar thermal energy operates as a benefit for the thermal design of technology, displacing the need for energy that otherwise would be provided from limited energy sources. However, solar thermal energy often is viewed as a disadvantage to thermal design, requiring the use of additional energy in order to satisfy temperature requirements. The lack of reliable and economical technologies capable of controlling radiative heat transfer complicates the issue of properly utilizing solar thermal energy. Dynamically-controlled geometry could provide a solution to radiative heat transfer control, acting as a medium by which incident irradiation and radiative emission might be tailored to the best advantage of the application. The research in this dissertation and the suggested future work that might follow are initial steps towards accomplishing this goal.
REFERENCES


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APPENDIX A.  2D GEOMETRIC RAY TRACING METHODS AND CODE

This appendix is provided in conjunction with Chapter 4 as a description of the two-dimensional ray tracing code used to determine the total number of reflections for a V-groove cavity with a given cavity angle of $\phi$. The nomenclature used in this appendix is defined in Chapter 4.

A.1 Methods

A ray at a given point of origin ($x_1$, $y_1$) and ray-vertical angle $\chi_1$, measured clockwise from the vertical axis (Figure 4.2b in Chapter 4), intersects the left surface of the V-groove at the location $X'''$. The ray-vertical angle $\chi_1$ is equivalent to the collimation angle $\gamma$ and the point of origin is the right-most point of the cavity opening, where the coordinate system origin is located at the V-groove apex (Figure 4.2b). The ray is described mathematically as a line using Equation A.1, where $x_i$ and $y_i$ represent a known point on the ray line (in this case $x_1$ and $y_1$), and $\chi_i$ is the ray-vertical angle of the ray (in this case $\gamma$). The left and right sides of the V-groove are also defined mathematically as lines (see Equations A.2 and A.3, respectively).

In order to determine the intersection point of the ray and a cavity wall, the $x$ intersection point of the line defining the ray (Equation A.2) and the line defining the V-groove side that the ray is travelling towards (Equation A.2 or A.3) is determined by equating the $y$ terms of Equation A.1 and Equation A.2 for a left-side intersection point or Equation A.2 and Equation A.3 for a right-side intersection point. A ray collides with a V-groove side of length $L$ (Figure 4.1a in Chapter 4) if an $x$ intersection value falls within the range $[-L\sin(\phi/2), 0]$ for a left-side intersection or $[0, L\sin(\phi/2)]$ for a right-side intersection. If an intersection occurs, the ray is reflected specularly using Equation A.4 for a left-side intersection or Equation A.5 for a right side intersection to determine the ray-vertical angle of the reflected ray ($\chi_{i+1}$).
\[ y = \frac{1}{\tan \chi_i} x + y_i - \frac{1}{\tan \chi_i} x_i \quad (A.1) \]

\[ y = \frac{-x}{\tan(\phi/2)} \quad \text{left side} \quad (A.2) \]

\[ y = \frac{x}{\tan(\phi/2)} \quad \text{right side} \quad (A.3) \]

\[ \chi_{i+1} = \chi_i + \left[ \pi - 2 \left( \chi_i + \frac{\phi}{2} \right) \right] \quad (A.4) \]

\[ \chi_{i+1} = \chi_i + \left[ \pi - 2 \left( \chi_i - \frac{\phi}{2} \right) \right] \quad (A.5) \]

Once a ray has been reflected, a new line is calculated defining the direction of the reflected ray (Equation A.1), where \( x_i \) and \( y_i \) are now the latest intersection point of the ray (\( x_2 \) and \( y_2 \) as shown in Figure 4.2b) and \( \chi_i \) is the new ray-vertical angle (\( \chi_2 \) in Figure 4.2b). Once again, the intersection point is calculated and a new ray line is calculated if a reflection occurs. This pattern continues until the ray leaves the cavity as determined by the \( x \) intersection values. The number of reflections that the ray experienced is then counted and assigned to the value \( n \).

### A.2 Code

1 #%% This file calculates the number of reflections that will occur in a specular V-groove for a given cavity angle and angle of incidence for the collimated irradiation.

2

3 #%% Preamble

4

5 import math

6 import matplotlib.pyplot as plt

7

8 # This code will be used to visualize the reflection of rays in an infinite V-groove.
def Left(x, phi):  # Get y location for a given x for left side
    y = -math.cos(phi/2)/math.sin(phi/2)*x
    return y

def Right(x, phi):  # Get y location for a given x for right side
    y = math.cos(phi/2)/math.sin(phi/2)*x
    return y

def intercept(gamma, y, x):  # Give the intercept of the ray
    b = y - (1/(math.tan(gamma))) * x
    return b

def xintleft(phi, b, gamma):  # Get x location for intersection between ray and left side
    if phi/2 == gamma:
        x = 0
    else:
        x = b/(-math.cos(phi/2)/math.sin(phi/2) - 1/math.tan(gamma))
    return x

def xintrightright(phi, b, gamma):  # Get x location for intersection between ray and left side
    if phi/2 == gamma:
        x = 0
    else:
        x = b/(math.cos(phi/2)/math.sin(phi/2) - 1/math.tan(gamma))
return x

def leftreflection(phi, gamma):
    gammanew = gamma + (math.pi - 2*(gamma + phi/2))
    return gammanew

def rightreflection(phi, gamma):
    gammanew = gamma + (math.pi - 2*(gamma - phi/2))
    return gammanew

#%% Reflection Counter (trace and theory)

# This code will count the number of reflections of the ray that is reflected the most

ntheory = []
ntrace = []
nequation = []

# Find theoretical number of reflections
for phi in range(1, 2*int(math.degrees(gammainitial))):
    phi = math.radians(phi)
    n = 1
    X = []
    xeta = []
    gamma = gammainitial
    # Find the impact point and impingment angle for the first reflection. This reflection
# is guaranteed to happen
X.append(math.sin(gamma - phi/2)/math.sin(math.pi - phi/2 - gamma))
xeta.append(gamma + (n - 0.5)*phi)

# Now we will loop for the remaining reflections

while True:

    # First calculate the xeta of the next possible reflection
    xeta.append(gamma + ((n + 1) - 0.5)*phi)

    # Then find the impact point of the next possible reflection. Depends on what xeta for the previous
    # reflection (hence the -2) is
    if xeta[-2] < math.pi/2:
        print('Downwards Reflection')
        X.append(-2*X[-1]*math.sin(phi/2)*((math.sin(math.pi/2 + phi/2 - xeta[-1]))/(math.sin(xeta[-1]))) + X[-1])

    elif xeta[-2] >= math.pi/2:
        print('Upwards Reflection')
        X.append(-X[-1]/(2*math.sin(phi/2)*math.sin(xeta[-1] - math.pi/2 - phi/2)/math.sin(math.pi + phi - xeta[-1])) - 1))

    # Now check to see if this reflection actually happened or if it left the cavity
    if X[-1] >= 1: # If the expected impact is outside of the panels
break

eif xeta[-1] > math.pi: # If the xeta of the impact is larger than 180 degrees
    break

eif X[-1] < 0 : # If the impact point is in the negatives, which is not possible
    break

eelse: # If none of these conditions apply, then a reflection did indeed occur and loop again
    n = n + 1

ntheory.append(n)

### Next Approach

xint = [0.99]
xint = [math.sin(phi/2)*i for i in xint]

for xinttemp in xint:
    pointstempx = [] # Will be used to keep track of points for given ray
    pointstempy = []
    gamma = gammainitial
    pointstempx.append(xinttemp)
    pointstempy.append(math.cos(phi/2))

    while True:
# First determine intersection points for right and left sides

\[
b = \text{intercept}(\gamma, \text{pointstempy}[-1], \text{pointstempx}[-1])
\]

\[
x_{\text{right}} = \text{xintright}(\phi, b, \gamma)
\]

\[
x_{\text{left}} = \text{xintleft}(\phi, b, \gamma)
\]

# Now check if one, none or both of them are inside of the bounds that constitute the actual sides

\[
\text{if } (x_{\text{right}} \geq 0 \text{ and } x_{\text{right}} \leq \text{math.sin}(\phi/2)) \text{ and } (x_{\text{left}} > 0 \text{ or } x_{\text{left}} < -\text{math.sin}(\phi/2)): \text{ # Right impact only}
\]

\[
x_{\text{last}} = \text{pointstempx}[-1]
\]

\[
\text{if } \text{len}(\text{pointstempx}) == 1:
\]

\[
\begin{align*}
& \text{pointstempx.append}(x_{\text{right}}) \\
& \text{pointstempy.append}(\text{Right}(x_{\text{right}}, \phi)) \\
& \gamma = \text{rightreflection}(\phi, \gamma) \\
& \text{print}(\text{`}\text{Right Reflection}\text{'}\text{)}
\end{align*}
\]

\[
\text{else:}
\]

\[
\text{if } x_{\text{last}} < 0: \text{ # If the last reflection was on the left side}
\]

\[
\begin{align*}
& \text{pointstempx.append}(x_{\text{right}}) \\
& \text{pointstempy.append}(\text{Right}(x_{\text{right}}, \phi)) \\
& \gamma = \text{rightreflection}(\phi, \gamma) \\
& \text{print}(\text{`}\text{Right Reflection}\text{'}\text{)}
\end{align*}
\]

\[
\text{if } x_{\text{last}} > 0 : \text{ # If the last reflection was on the right side}
\]

\[
\text{print}(\text{`}\text{End of Ray}\text{'}\text{)}
\]
```python
    pointstempy.append(math.cos(phi/2))
    pointstempx.append((math.cos(phi/2) - b)/(1/
                     math.tan(gamma)))

    break

if (xleft <= 0 and xleft >= -math.sin(phi/2)) and (xright
    < 0 or xright > math.sin(phi/2)):  # Left impact only

    xlast = pointstempx[-1]

    if len(pointstempx) == 1:

        pointstempx.append(xleft)
        pointstempy.append(Left(xleft,phi))
        gamma = leftreflection(phi, gamma)
        print('Left Reflection')

    else:

        if xlast > 0:  # If the last reflection was on the
            right side

            pointstempx.append(xleft)
            pointstempy.append(Left(xleft,phi))
            gamma = leftreflection(phi, gamma)
            print('Left Reflection')

        if xlast < 0:  # If the last reflection was on the
            left side

            print('End of Ray')
            pointstempy.append(math.cos(phi/2))
            pointstempx.append((math.cos(phi/2) - b)/(1/
                              math.tan(gamma)))

        break
```

if (xright >= 0 and xright <= math.sin(phi/2)) and (xleft <= 0 and xleft >= -math.sin(phi/2)): # Left and right impact

# First check and see which side the last impact was on

xlast = pointstempx[-1]

if xlast < 0: # Do a Right reflection
    pointstempx.append(xright)
    pointstempy.append(Right(xright,phi))
    gamma = rightreflection(phi,gamma)
    print('Right Reflection - Double Intersection')

if xlast > 0: # Do a Left reflection
    pointstempx.append(xleft)
    pointstempy.append(Left(xleft,phi))
    gamma = leftreflection(phi,gamma)
    print('Left Reflection - Double Intersection')

if (xleft > 0 or xleft < -math.sin(phi/2)) and (xright < 0 or xright > math.sin(phi/2)): # No impact
    pointsx.append(pointstempx)
    pointsy.append(pointstempy)
    print('No Reflection')

break

n = len(pointstempx) - 2
ntrace.append(n)
if gammainitial < phi/2:
    ntemp = (math.pi - gammainitial)/phi + 1/2
    if ntemp.is_integer() == True:
        ntemp = ntemp - 1
    else:
        ntemp = math.floor(ntemp)

if gammainitial >= phi/2:
    ntemp = (math.pi - 2*gammainitial)/phi + 1
    if ntemp.is_integer() == True:
        ntemp = ntemp - 1
    else:
        ntemp = math.floor(ntemp)
nequation.append(ntemp)
print('Phi = ' + str(phi) + ' completed!')

plt.figure()
error = [ntrace[i] - nequation[i] for i in range(0,len(ntrace))]
plt.plot(range(1,len(error)+1),error,'ks')
APPENDIX B. SEGMENTED FIN ALGORITHM

This appendix is provided in conjunction with Chapter 6 as a description of the two-dimensional numerical heat-transfer code used to determine heat transfer rates and temperature profiles for a radiative fin consisting of solid segments connected by a flexible hinge with a known thermal conductance.

B.1 Code

```python
# Preamble

# This is the Segmented-Fin Algorithm, a python code used to predict the heat loss
# and temperature profile from a radiative fin consisting of solid panels
# connected by a flexible thermal hinge. The angle phi is the angle between
# panels such that the panels are mutually-irradiating. The base is not modeled
# in this algorithm

import scipy.optimize as optimize
import numpy as np
import math
import copy
import matplotlib.pyplot as plt
import tdma

plt.rcParams['mathtext.default'] = 'regular'
plt.rcParams('font', size = 12)
plt.rcParams('axes', titlesize = 24)
```
plt.rc('axes', labelsize = 24)
plt.rc('xtick', labelsize = 18)
plt.rc('ytick', labelsize = 18)
plt.rc('legend', fontsize = 14)
plt.rc('figure', titlesize = 16)

def view_factor(Dparams, Sparams, n, i, j, N):
    wcount = Sparams['CVX']
w = Sparams['dx']
phi = Dparams['phi']

    if ((N+1) % 2 == 0 and n == 0) or ((N+1) % 2 != 0 and n != 0):
        x1 = wcount*w - (j+1)*w
        x2 = wcount*w - j*w
        y1 = i*w
        y2 = (i+1)*w
    else:
        x1 = w*j
        x2 = w*(j+1)
        y1 = w*wcount - (i+1)*w
        y2 = w*wcount - i*w

    F = ((x1**2 - 2*x1*y2*math.cos(phi) + y2**2)**(1/2) + (x2**2 - 2*x2*y1*math.cos(phi) + y1**2)**(1/2) - (x2**2 - 2*x2*y2*math.cos(phi) + y2**2)**(1/2) - (x1**2 - 2*x1*y1*math.cos(phi) + y1**2)**(1/2))/(2*(x2-x1))

    return F
def view_factor_panel_to_opening(Dparams, Sparams, n, i, N):
    # CHECK THE VIEW FACTOR HERE

    wpanel = Dparams['wpanel']
    w = Sparams['dx']
    phi = Dparams['phi']

    if len(Sparams['dy']) == 1:  # If there is only one layer
        
        # Then we will compute both the top and bottom values and
        pass them at the same time

        if (N + 1) % 2 == 0:  # If we are on the right side of the top
            D1 = (wpanel**2 + ((i + 1) * w)**2 - 2 * wpanel * (i + 1) * w * math.cos(phi))**(1/2)
            D2 = wpanel - i * w
            S1 = (wpanel**2 + (i * w)**2 - 2 * wpanel * (i * w) * math.cos(phi))**(1/2)
            S2 = wpanel - w * (i + 1)
            Ftop = (D1 + D2 - (S1 + S2)) / (2 * w)
        else:
            Ftop = 1
    elif Dparams['Ntotal'] % 2 != 0 and N == Dparams['Ntotal'] - 1:  # If there are an odd number of panels and we are on the last one
        Ftop = 1
    else:
        D1 = w * (i + 1)
        D2 = (wpanel**2 + (wpanel - i * w)**2 - 2 * wpanel * (wpanel - i * w) * math.cos(phi))**(1/2)
        S1 = i * w
S2 = (wpanel**2 + (wpanel - (i + 1)*w)**2 - 2*wpanel*(wpanel - (i + 1)*w)*math.cos(phi))**(1/2)

Ftop = (D1 + D2 - (S1 + S2))/(w*2)

if N == 0:  # If we are on the very first panel
    Fbottom = 1  # Then everything leaves

elif Dparams['Ntotal'] % 2 == 0 and N == Dparams['Ntotal'] - 1:  # If there are an even number of panels and we are on the last one
    Fbottom = 1  # Then everything leaves

elif (N + 1) % 2 == 0:  # If we are on the left side of the bottom inside a Vgroove
    D1 = w*(i + 1)
    D2 = (wpanel**2 + (wpanel - i*w)**2 - 2*wpanel*(wpanel - i*w)*math.cos(phi))**(1/2)
    S1 = i*w
    S2 = (wpanel**2 + (wpanel - (i + 1)*w)**2 - 2*wpanel*(wpanel - (i + 1)*w)*math.cos(phi))**(1/2)
    Fbottom = (D1 + D2 - (S1 + S2))/(w*2)

else:  # If we are on the right side of the bottom inside a Vgroove
    D1 = (wpanel**2 + ((i + 1)*w)**2 - 2*wpanel*((i + 1)*w)*math.cos(phi))**(1/2)
    D2 = wpanel - i*w
    S1 = (wpanel**2 + (i*w)**2 - 2*wpanel*(i*w)*math.cos(phi))**(1/2)
    S2 = wpanel - w*(i + 1)
Fbottom = (D1+D2 - (S1+S2))/(2*w)

F = [Ftop,Fbottom]

# Top panels

elif n == 0:

    if (N+1) % 2 == 0:  # If we are on the right side of the top

        D1 = (wpanel**2 + ((i+1)*w)**2 - 2*wpanel*((i+1)*w)*math.cos(phi))**(1/2)
        D2 = wpanel - i*w
        S1 = (wpanel**2 + (i*w)**2 - 2*wpanel*(i*w)*math.cos(phi))**(1/2)
        S2 = wpanel - w*(i+1)

        F = (D1+D2 - (S1+S2))/(2*w)

    elif Dparams['Ntotal'] % 2 != 0 and N == Dparams['Ntotal'] - 1:  # If there are an odd number of panels and we are on the last one

        F = 1

else:  # If we are on the left side of the top

    D1 = w*(i+1)
    D2 = (wpanel**2 + (wpanel - i*w)**2 - 2*wpanel*(wpanel - i*w)*math.cos(phi))**(1/2)
    S1 = i*w
    S2 = (wpanel**2 + (wpanel - (i+1)*w)**2 - 2*wpanel*(wpanel - (i+1)*w)*math.cos(phi))**(1/2)

    F = (D1+D2 - (S1+S2))/(w*2)
# Bottom panels

```python
elif n == len(Sparams['dy']) - 1:

    if N == 0:  # If we are on the very first panel

        F = 1  # Then everything leaves

    elif Dparams['Ntotal'] % 2 == 0 and N == Dparams['Ntotal'] - 1:  # If there are an even number of panels and we are on the last one

        F = 1  # Then everything leaves

    elif (N+1) % 2 == 0:  # If we are on the left side of the bottom inside a Vgroove

        D1 = w*(i+1)
        D2 = (wpanel**2 + (wpanel - i*w)**2 - 2*wpanel*(wpanel - i*w)*math.cos(phi))**(1/2)
        S1 = i*w
        S2 = (wpanel**2 + (wpanel - (i+1)*w)**2 - 2*wpanel*(wpanel - (i+1)*w)*math.cos(phi))**(1/2)

        F = (D1+D2 - (S1+S2))/(w*2)

    else:  # If we are on the right side of the bottom inside a Vgroove

        D1 = (wpanel**2 + ((i+1)*w)**2 - 2*wpanel*((i+1)*w)*math.cos(phi))**(1/2)
        D2 = wpanel - i*w
        S1 = (wpanel**2 + (i*w)**2 - 2*wpanel*(i*w)*math.cos(phi))**(1/2)
        S2 = wpanel - w*(i+1)
```

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\[ F = \frac{(D_1+D_2 - (S_1+S_2))}{(2\times w)} \]

\[ return \ F \]

def view_factor_opening_to_panel(Dparams, Sparams, n, i, N):
    F = view_factor_panel_to_opening(Dparams, Sparams, n, i, N)
    if len(F) == 1:
        F = F * (Sparams['dx']/(2*Dparams['wpanel']*math.sin(Dparams['phi']/2)))
    else:
        F = [Fitem * (Sparams['dx']/(2*Dparams['wpanel']*math.sin(Dparams['phi']/2))) for Fitem in F]
    return F

def summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N):
    Jtopsummation = 0
    Jbottomsummation = 0
    summation = 0
    if len(Sparams['dy']) == 1: # If there is only one layer
        if N == 0: # The first panel on the bottom doesn't see any other panels
            Jbottomsummation = Jbottomsummation + 0
```python
elif N == (Dparams['Ntotal'] - 1) and Dparams['Ntotal'] % 2 == 0: # If there are an even number of panels than the last panel doesn't see any other panels

    Jbottomsummation = Jbottomsummation + 0

elif (N + 1) % 2 == 0: # If we are on an even panel BOTTOM

    for j in range(0, len(Jbottom[N+1])):

        F = view_factor(Dparams,Sparams,n,i,j,N)

        Jbottomsummation = Jbottomsummation + F*Jbottom[N+1][j]*Sparams['dx']

else: # If we are on an odd panel BOTTOM

    for j in range(0, len(Jbottom[N-1])):

        F = view_factor(Dparams,Sparams,n,i,j,N)

        Jbottomsummation = Jbottomsummation + F*Jbottom[N-1][j]*Sparams['dx']

if Dparams['Ntotal'] % 2 != 0 and N == (Dparams['Ntotal'] - 1): # If the total number of panels is NOT even and we are on the last panel (that only sees space) TOP

    Jtopsummation = Jtopsummation + 0

elif (N+1) % 2 == 0: # If we are on an even panel TOP

    for j in range(0, len(Jtop[N-1])):

        F = view_factor(Dparams,Sparams,n,i,j,N)

        Jtopsummation = Jtopsummation + F*Jtop[N-1][j]*Sparams['dx']
```

Jtopsummation = Jtopsummation + F*Jtop[N-1][j]*Sparams['dx']

else: # If we are on an odd panel TOP
    for j in range(0, len(Jtop[N+1])):
        F = view_factor(Dparams, Sparams, n, i, j, N)
        Jtopsummation = Jtopsummation + F*Jtop[N+1][j]*Sparams['dx']

summation = [Jtopsummation, Jbottomsummation]

elif n == len(Sparams['dy']) - 1: # If we are on the bottom
    if N == 0: # The first panel on the bottom doesn't see any other panels
        summation = summation + 0
    elif N == (Dparams['Ntotal'] - 1) and Dparams['Ntotal'] % 2 == 0: # If there are an even number of panels than the last panel doesn't see any other panels
        summation = summation + 0
    elif (N + 1) % 2 == 0: # If we are on an even panel
        for j in range(0, len(Jbottom[N+1])):
            F = view_factor(Dparams, Sparams, n, i, j, N)
            summation = summation + F*Jbottom[N+1][j]*Sparams['dx']
else: # If we are on an odd panel

    for j in range(0, len(Jbottom[N-1])):

        F = view_factor(Dparams, Sparams, n, i, j, N)

        summation = summation + F*Jbottom[N-1][j]*Sparams['dx']

# Top Code

elif n == 0:

    if Dparams['Ntotal'] % 2 != 0 and N == (Dparams['Ntotal'] - 1): # If the total number of panels is NOT even and we are on the last panel (that only sees space)

        summation = summation + 0

    elif (N+1) % 2 == 0: # If we are on an even panel

        for j in range(0, len(Jtop[N-1])):

            F = view_factor(Dparams, Sparams, n, i, j, N)

            summation = summation + F*Jtop[N-1][j]*Sparams['dx']

    else: # If we are on an odd panel

        for j in range(0, len(Jtop[N+1])):

            F = view_factor(Dparams, Sparams, n, i, j, N)

            summation = summation + F*Jtop[N+1][j]*Sparams['dx']

    return summation
# Control Volume Functions

def full_middle(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):

    Jsum = sum(summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N))
    F = view_factor_opening_to_panel(Dparams, Sparams, n, i, N)
    Fopenpanelbottom = F[1]
    Fopenpaneltop = F[0]

    Sstar = Dparams['e'] * Dparams['Lopen'] * BCparams['sigma'] * (BCparams['Tsurr'] ** 4) * (Fopenpanelbottom + Fopenpaneltop) + Dparams['e'] * Jsum - 2 * BCparams['sigma'] * Sparams['dx'] * Dparams['e'] * T[N][n][i] ** 4
    dSdT = -8 * Dparams['e'] * BCparams['sigma'] * Sparams['dx'] * T[N][n][i] ** 3

    ap = (2 * Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx'] - dSdT
    ae = -(Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx']
    aw = -(Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx']
    b = Sstar - dSdT * T[N][n][i]

    parameters = {'ap': ap, 'ae': ae, 'aw': aw, 'b': b}

    return (parameters)

def full_right_end(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):

    Jsum = sum(summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N))
    F = view_factor_opening_to_panel(Dparams, Sparams, n, i, N)
    Fopenpanelbottom = F[1]
    Fopenpaneltop = F[0]
Sstar = Dparams['e']*Dparams['Lopen']*BCparams['sigma']*(BCparams['Tsurr']**4)*(Fopenpanelbottom + Fopenpaneltop) + Dparams['e']*Jsum - 2*BCparams['sigma']*Sparams['dx']*Dparams['e']*T[N][n][i]**4

\[
dSdT = -8*Dparams['e']*BCparams['sigma']*Sparams['dx']*T[N][n][i]**3
\]

\[
ap = (Dparams['k'][0]*Sparams['dy'][0])/Sparams['dx'] - dSdT
\]

\[
aw = -(Dparams['k'][0]*Sparams['dy'][0])/Sparams['dx']
\]

\[
ae = 0
\]

\[
b = Sstar - dSdT*T[N][n][i]
\]

parameters = {'ap':ap, 'aw':aw, 'ae':ae, 'b':b}

return(parameters)

def full_right(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):

Jsum = sum(summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N))

F = view_factor_opening_to_panel(Dparams, Sparams, n, i, N)
Fopenpanelbottom = F[1]
Fopenpaneltop = F[0]

Sstar = Dparams['e']*Dparams['Lopen']*BCparams['sigma']*(BCparams['Tsurr']**4)*(Fopenpanelbottom + Fopenpaneltop) + Dparams['e']*Jsum - 2*BCparams['sigma']*Sparams['dx']*Dparams['e']*T[N][n][i]**4

dSdT = -8*Dparams['e']*BCparams['sigma']*Sparams['dx']*T[N][n][i]**3

\[
ap = (Dparams['k'][0]*Sparams['dy'][0])/Sparams['dx'] + Sparams['Khingelayer'][0] - dSdT
\]

\[
aw = -(Dparams['k'][0]*Sparams['dy'][0])/Sparams['dx']
\]

\[
ae = -Sparams['Khingelayer'][0]
\]

\[
b = Sstar - dSdT*T[N][n][i]
\]
parameters = {'ap':ap, 'aw':aw, 'ae':ae, 'b':b}

return(parameters)

def full_left(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):

    Jsum = sum(summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N))
    F = view_factor_opening_to_panel(Dparams, Sparams, n, i, N)
    Fopenpanelbottom = F[1]
    Fopenpaneltop = F[0]

    Sstar = Dparams['e'] * Dparams['Lopen'] * BCparams['sigma'] * (BCparams['Tsurr']**4) * (Fopenpanelbottom + Fopenpaneltop) + Dparams['e'] * Jsum - 2 * BCparams['sigma'] * Sparams['dx'] * Dparams['e'] * T[N][n][i]**4
    dSdT = -8 * Dparams['e'] * BCparams['sigma'] * Sparams['dx'] * Dparams['e'] * T[N][n][i]**3

    ap = (Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx'] + Sparams['Khingelayer'][0] - dSdT
    ae = -(Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx']
    aw = -Sparams['Khingelayer'][0]
    b = Sstar - dSdT * T[N][n][i]

    parameters = {'ap':ap, 'aw':aw, 'ae':ae, 'b':b}
    return(parameters)

def full_left_T(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):

    ap = 1
    aw = 0
    ae = 0
    b = BCparams['Tset']
parameters = {'ap':ap, 'aw':aw, 'ae':ae, 'b':b}

return(parameters)

def full_left_q(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):
    Jsum = sum(summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N))
    F = view_factor_opening_to_panel(Dparams, Sparams, n, i, N)
    Fopenpanelbottom = F[1]
    Fopenpaneltop = F[0]
    Sstar = Dparams['e'] * Dparams['Lopen'] * BCparams['sigma'] * (BCparams['Tsurf']**4) * (Fopenpanelbottom + Fopenpaneltop) + Dparams['e'] * Jsum - 2 * BCparams['sigma'] * Sparams['dx'] * Dparams['e'] * T[N][n][i]**4
    dSdT = -8 * Dparams['e'] * BCparams['sigma'] * Sparams['dx'] * T[N][n][i]**3
    ap = (Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx'] - dSdT
    ae = -(Dparams['k'][0] * Sparams['dy'][0]) / Sparams['dx']
    aw = 0
    b = Sstar - dSdT * T[N][n][i] + Sparams['qlayer'][0]
    parameters = {'ap': ap, 'aw': aw, 'ae': ae, 'b': b}
    return(parameters)

def full_left_Tc(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N):
    Jsum = sum(summation(Dparams, Sparams, BCparams, T, Jtop, Jbottom, n, i, N))
    F = view_factor_opening_to_panel(Dparams, Sparams, n, i, N)
    Fopenpanelbottom = F[1]
    Fopenpaneltop = F[0]
Sstar = Dparams['e']\times Dparams['Lopen']\times BCparams['\sigma']\times (BCparams['Tsurr']^4)\times (Fopenpanelbottom + Fopenpaneltop) + Dparams['e']\times Jsum - 2\times BCparams['\sigma']\times Sparams['dx']\times Dparams['e']\times T[N][n][i]^4

dSdT = -8\times Dparams['e']\times BCparams['\sigma']\times Sparams['dx']\times T[N][n][i]^3

ap = \frac{(Dparams['k'][0]\times Sparams['dy'][0])}{Sparams['dx']} + Sparams['Khingelayer'][0] - dSdT

ae = -\frac{(Dparams['k'][0]\times Sparams['dy'][0])}{Sparams['dx']}

aw = 0

b = Sstar - dSdT\times T[N][n][i] + Sparams['Khingelayer'][0]\times BCparams['Tc']

parameters = {'ap':ap, 'aw':aw, 'ae':ae, 'b':b}

return(parameters)

# %% Temperature Profile Function

""

In calculating the temperature profile we assume a unit depth into the page. This is because we are assuming that the temperature is uniform in and out of the page. So, the term Lpanel does not affect the temperature profile of the radiator. However, Lpanel will be used in finding the total heat transfer away from the surface.

""

def temp_profile(Dparams, BCparams, Sparams, Told, Jtopold, Jbottomold):
# Initialize the old and new T and J lists

```python
T = copy.deepcopy(Told)
Jtop = copy.deepcopy(Jtopold)
Jbottom = copy.deepcopy(Jbottomold)
```

# Step through every CV. Different solution methods will be used depending on how many rows of CV's we have.

```python
iterations = 1  # for tracking number of iterations
converged = False
```

```python
while converged == False:
    # If we have one CV vertically than we will use matrix inversion.
    if len(Sparams['dy']) == 1:
        A = np.zeros([Dparams['Ntotal']*Sparams['CVX'], Dparams['Ntotal']*Sparams['CVX']])
        b = np.zeros([Dparams['Ntotal']*Sparams['CVX'], 1])
        matindex = 0
        for N in range(Dparams['Ntotal']):
            for n in range(len(Sparams['dy'])):
                for i in range(Sparams['CVX']):
                    # We are now down to one individual control volume. We must now decide which of 12 cases the control volume fits into. First, we must determine if there is only one layer or multiple layers
```
if len(Sparams['dy']) == 1:  # if there is only one layer we will use a unique set of functions
    # and TDMA to find the answer

    if i == Sparams['CVX'] - 1 and N < Dparams['Ntotal'] - 1:  # If we are on the right-most CV but not on the last panel
        results = full_right(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)

        A[matindex][matindex] = results['ap']
        A[matindex][matindex+1] = results['ae']
        A[matindex][matindex-1] = results['aw']
        b[matindex] = results['b']

    elif i == Sparams['CVX'] - 1 and N == Dparams['Ntotal'] - 1:
        results = full_right_end(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)

        A[matindex][matindex] = results['ap']
        A[matindex][matindex-1] = results['aw']
        b[matindex] = results['b']

    elif i == 0 and BCparams['BC'] == 0 and N == 0:  # left most CV of first panel with 0 BC
        results = full_left_T(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)

        A[matindex][matindex] = results['ap']
        A[matindex][matindex+1] = results['ae']
elif i == 0 and BCparams[‘BC’] == 1 and N == 0:  # Left most CV of first panel with 1 BC
    results = full_left_q(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)
    A[matindex][matindex] = results[‘ap’]
    A[matindex][matindex+1] = results[‘ae’]
    b[matindex] = results[‘b’]

elif i == 0 and BCparams[‘BC’] == 2 and N == 0:  # Left most CV of first panel with 2 BC
    results = full_left_Tc(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)
    A[matindex][matindex] = results[‘ap’]
    A[matindex][matindex+1] = results[‘ae’]
    b[matindex] = results[‘b’]

elif i == 0 and N > 0:  # left most CV of any but the first panel
    results = full_left(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)
    A[matindex][matindex] = results[‘ap’]
    A[matindex][matindex+1] = results[‘ae’]
    A[matindex][matindex-1] = results[‘aw’]
    b[matindex] = results[‘b’]

else:  # Any middle panel
results = full_middle(Dparams, BCparams, Sparams, T, Jtop, Jbottom, n, i, N)

A[matindex][matindex] = results['ap']
A[matindex][matindex+1] = results['ae']
A[matindex][matindex-1] = results['aw']
b[matindex] = results['b']

matindex = matindex + 1

else: # If there is more than CV vertically than we will use the other set of functions and Gauss-Seidel iteration to find the answer

# This section would include energy balances for systems that are internal to panels, but that is not included in this appendix

if len(Sparams['dy']) == 1: # If we have the one row of CV's than solve the matrix

T = tdma.TDMA(A, b)
Tcopy = copy.deepcopy(T)

# Reconstruct the T individual lists

iterator = 0
Treconstruct = []

for N in range(0, Dparams['Ntotal']):

Tpanel = []

for x in range(0, Sparams['CVX']):

Tpanel.append(Tcopy[iterator])
iterator = iterator + 1

Treconstruct.append([Tpanel])

T = Treconstruct

for N in range(0,(Dparams['Ntotal'])):
    for i in range(0,Sparams['CVX']):
        F = view_factor_opening_to_panel(Dparams,Sparams,
        n,i,N)
        Jtotal = summation(Dparams,Sparams,BCparams,T,
        Jtop,Jbottom,n,i,N)

        Ftop = F[0]
        Fbottom = F[1]

        Jsumtop = Jtotal[0]
        Jsumbottom = Jtotal[1]

        Jtop[N][i] = ((1-Dparams['e'])*Dparams['Lopen']*
        Ftop*BCparams['sigma']*BCparams['Tsurr']**4 + (1-Dparams['e'])*
        Jsumtop + Dparams['e']*BCparams['sigma']*Sparams['dx']*T[N][n][i]
        **4)/Sparams['dx']

        Jbottom[N][i] = ((1-Dparams['e'])*Dparams['Lopen']*
        Fbottom*BCparams['sigma']*BCparams['Tsurr']**4 + (1-Dparams['e'])*
        Jsumbottom + Dparams['e']*BCparams['sigma']*Sparams['dx']*T[N][n][i]
        **4)/Sparams['dx']

        heatrate2CVnew = heatlossCV(Dparams,Sparams,T)
        heatrate2CVold = heatlossCV(Dparams,Sparams,Told)

        print('Last T convergence: ' + str(abs(T[-1][0][-1] - Told
        [-1][0][-1])) + '')
if abs(heatrate2CVnew - heatrate2CVold) < Sparams['convergencecriteria'] and abs(T[0][0][0] - Told[0][0][0]) < Sparams['convergencecriteria'] and abs(T[-1][0][-1] - Told[-1][0][-1]) < Sparams['convergencecriteria']:

    converged = True

if iterations % 100 == 0: # If the number of iterations is divisible by 100

    print('')
    print('Iteration number: ' + str(iterations))
    print('Heat Rate Residual CV: ' + str(abs(heatrate2CVnew - heatrate2CVold)))
    print('Temperature Tip Residual: ' + str(T[0][0][0] - Told[0][0][0]))
    print('Temperature Base Residual: ' + str(T[-1][0][-1] - Told[-1][0][-1]))
    print('Heat Rate CV: ' + str(heatrate2CVnew))
    print('Heat Rate Radiosity: ' + str(heatlossradiosity(Dparams,Jtop,Jbottom)))
    print('Component Temperature CV: ' + str(heatrate2CVnew/Dparams['Khinge'] + T[0][0][0] - 273))

if converged == False: # Reset the old profiles to be the new profiles and iterate again

    Told = copy.deepcopy(T)
    Jbottomold = copy.deepcopy(Jbottom)
    Jtopold = copy.deepcopy(Jtop)

    print('Iteration number: ' + str(iterations) + ' complete. Iterating Again \r')

    iterations = iterations + 1
return (T, Jtop, Jbottom, iterations)

def plotT(T):
    x = [i for i in range(int(len(T) * int(len(T[0][0]))))]
    Tplot = [(T[N][0][i]) / (T[0][0][0]) for N in range(0, len(T)) for i in range(0, len(T[0][0]))]
    plt.figure()
    plt.plot(x, Tplot, 'ks')
    plt.xlabel('X (Unitless)')
    plt.ylabel('T (Celsius)')
    plt.show()

def heatlossCV(Dparams, Sparams, T):
    # THIS SHOULD ACTUALLY SUM OVER ALL LAYERS IN A FOR LOOP
    heatloss = Dparams['k'][0] * sum(Sparams['dy']) * Dparams['Lpanel'] * (T[0][0][0] - T[0][0][1]) / (Sparams['dx']) + 2 * Sparams['dx'] * 5.67 * 10 ** (-8) * Dparams['Lpanel'] * T[0][0][0] ** 4
    return (heatloss)

def heatlosshinge(Dparams, Sparams, BCparams, T):
    heatloss = Sparams['Khingelayer'][0] * (BCparams['Tc'] - T[0][0][0]) * Dparams['Lpanel']
    return (heatloss)

def heatlossradiosity(Dparams, Jtop, Jbottom): # Return the heat loss from the surface
    qloss = 0
if len(Sparams['dy']) == 1: # If there is only one layer, use the following code. This is necessary because of how the view factors works

    for N in range(0,Dparams['Ntotal']):

        for i in range(Sparams['CVX']):

            Ftotal = view_factor_panel_to_opening(Dparams,Sparams,0,i,N)

            Ftop = Ftotal[0]
            Fbottom = Ftotal[1]

            qloss = qloss + Jtop[N][i]*Sparams['dx']*Dparams['Lpanel']*Ftop
            qloss = qloss + Jbottom[N][i]*Sparams['dx']*Dparams['Lpanel']*Fbottom

    return(qloss)

#%%% Calculate turn-down ratio

klist = [237, 401, 1950, 100000]
hingeconductance = 100000 #6 # 60 #100000
turndownratio = []

for kitem in klist: # m is the iterator used to move between scenarios, where each scenario corresponds to a given index of the klist, Khinge

    phi = 180

    # Radiator Design Parameters
Dparams = {'Lpanel': 0.159, 'tlist': [0.003175], 'Ntotal': 8, 'wpanel': 0.102, 'e': 0.91, 'klist': [kitem], 'phi': math.radians(phi), 'Khinge': hingeconductance} # Khinge should be reported as W/K on this line. It is converted into per unit area later in the code

# Uncertainties of the radiator design parameters

Uparams = {'Lpanel': 0.00001, 'wpanel': 0.0005, 'e': 0.01, 'tlist': 0.00001, 'klist': 4, 'Khinge': Dparams['Khinge']*0.25}

# Boundary Condition / Heat Transfer Parameters

BCparams = {'BC': 2, 'sigma': 5.67*10**-8, 'Tsurr': 4, 'q': 16.5, 'Tc': 293.5, 'Tset': 0 + 273} # Tc is the temp of the component and Tset is the temp of the first CV for the different boundary conditions

Dparamscopy = copy.deepcopy(Dparams)

# BC = 0 is constant temperature on first panel node. BC = 1 is constant heat flux at first node of panel. BC = 2 is constant protected component temperature

# Solver Parameters

Sparams = {'Tinitial': BCparams['Tset'], 'convergencriteria': 0.00001, 'CVX':1000, 'CVY': 1, 'relaxation': 1}

# First we will find the actual solution using the real parameters

# Calculated Parameters

Sparams['dx'] = Dparams['wpanel']/Sparams['CVX']
Dparams['Lopen'] = 2*math.sin(Dparams['phi']/2)*Dparams['wpanel']  
### RYDGE YOU CHANGED 'Lpanel' to 'wpanel'  
dy = []  
k = []  
Khingelayer = []  
qlayer = []  

for j in range(0,len(Dparams['tlist'])):  
    for i in range(0,Sparams['CVY']):  
        dy.append(Dparams['tlist'][j]/Sparams['CVY'])  
        k.append(Dparams['klist'][j])  
        Khingelayer.append(Dparams['Khinge']*dy[-1]/(Dparams['Lpanel']*sum(Dparams['tlist']))))  
        qlayer.append(BCparams['q']*dy[-1]/(Dparams['Lpanel']*sum(Dparams['tlist']))))  

Sparams['dy'] = dy  
Dparams['k'] = k  
Sparams['Khingelayer'] = Khingelayer  
Sparams['qlayer'] = qlayer  

Told = [[[Sparams['Tinitial'] for i in range(Sparams['CVX'])] for n in range(len(Sparams['dy']))] for N in range(Dparams['Ntotal'])]  

#Told = [[[Tinitial - 30/(Ntotal*itotal)*(i + itotal*(N)) for i in range(itotal)] for n in range(ntotal)] for N in range(Ntotal)]  

Jtopold = [[0 for i in range(Sparams['CVX'])] for N in range(Dparams['Ntotal'])]  

Jbottomold = [[0 for i in range(Sparams['CVX'])] for N in range(Dparams['Ntotal'])]  

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results = temp_profile(Dparams, BCparams, Sparams, T0, Jtopold, Jbottomold)

heatrate180 = (heatlosshinge(Dparams, Sparams, BCparams, results[0])

# Radiator Design Parameters

phi = 5

Dparams = {'Lpanel': 0.159, 'tlist': [0.003175], 'Ntotal': 4, 'wpanel': 0.102, 'e': 0.91, 'klist': [kitem], 'phi': math.radians(phi), 'Khinge': hingeconductance} # Khinge should be reported as W/K on this line. It is converted into per unit area later in the code

# Uncertainties of the radiator design parameters

Uparams = {'Lpanel': 0.00001, 'wpanel': 0.0005, 'e': 0.01, 'tlist': 0.00001, 'klist': 2, 'Khinge': Dparams['Khinge']*0.25}

# Boundary Condition / Heat Transfer Parameters

BCparams = {'BC': 2, 'sigma': 5.67*10**-8, 'Tsurr': 173, 'q': 16.5, 'Tc': 293.5, 'Tset': 0 + 273} # Tc is the temp of the component and Tset is the temp of the first CV for the different boundary conditions

Dparamscopy = copy.deepcopy(Dparams)

# BC = 0 is constant temperature on first panel node. BC = 1 is constant heat flux at first node of panel. BC = 2 is constant protected component temperature

# Solver Parameters
Sparams = {'Tinitial': BCparams['Tset'], 'convergencecriteria': 0.00001, 'CVX': 1000, 'CVY': 1, 'relaxation': 1}

# First we will find the actual solution using the real parameters

# Calculated Parameters

Sparams['dx'] = Dparams['wpanel']/Sparams['CVX']
Dparams['Lopen'] = 2*math.sin(Dparams['phi']/2)*Dparams['wpanel']

### RYDGE YOU CHANGED 'Lpanel' to 'wpanel'

dy = []
k = []
Khingelayer = []
qlayer = []

for j in range(0, len(Dparams['tlist'])):
    for i in range(0, Sparams['CVY']):
        dy.append(Dparams['tlist'][j]/Sparams['CVY'])
        k.append(Dparams['klist'][j])
        Khingelayer.append(Dparams['Khinge']*dy[-1]/(Dparams['Lpanel'])*sum(Dparams['tlist'])))
        qlayer.append(BCparams['q']*dy[-1]/(Dparams['Lpanel'])*sum(Dparams['tlist'])))

Sparams['dy'] = dy
Dparams['k'] = k
Sparams['Khingelayer'] = Khingelayer
Sparams['qlayer'] = qlayer

Told = [[Sparams['Tinitial'] for i in range(Sparams['CVX'])] for n in range(len(Sparams['dy'])) ] for N in range(Dparams['Ntotal'])] }
# Told = [[[ Tinitial - 30/(Ntotal*itotal)*(i + itotal*(N)) for i in range(itotal) ] for n in range(ntotal) ] for N in range(Ntotal) ]

Jtopold = [[ 0 for i in range(Sparams['CVX']) ] for N in range(Dparams['Ntotal']) ]

Jbottomold = [[ 0 for i in range(Sparams['CVX']) ] for N in range(Dparams['Ntotal']) ]

results = temp_profile(Dparams,BCparams,Sparams,Told,Jtopold,Jbottomold)

heatrate10 = (heatlosshinge(Dparams,Sparams,BCparams,results[0]))

turndownratio.append(heatrate180/heatrate10)

print('Thermal Conductivity: ' + str(klist))

print('Hinge Conductance: ' + str(hingeconductance))

print('Turn-Down Ratio: ' + str(turndownratio))