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Aspects of Engaging Problem Contexts From Students' Perspectives

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ABSTRACT

Aspects of Engaging Problem Contexts from Students' Perspectives

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Too many students have negative feelings towards mathematics which is causing them to disengage in their classrooms. This has led to student under-achievement. This study attempts to better understand how teachers can help students to reengage with mathematics by using more engaging contexts to develop mathematical content.

The study began with the characteristics realistic, worthwhile, enjoyable or motivating as a framework for posing engaging problem contexts, which were synthesized from the current research literature. As students discussed what made contexts engaging, my understanding of what engaging problem contexts looked like expanded. The characteristics realistic and worthwhile were combined. Students felt contexts were more realistic and worthwhile when the contexts were authentic, purposeful and related to their everyday lives or a potential career situation. Furthermore, students felt context was enjoyable when it was interactive or included a good story. Finally, students discussed their frustration with repetition within problem contexts. Even if certain types of problems were engaging at first, if they saw them over and over again, they became unengaged. Students wanted to see a variety of new ideas and different kinds of contexts. This study better informs teachers and curriculum writers on what to include/exclude to make contexts more engaging for students.

Keywords: engaging problem context, realistic, worthwhile, enjoyable, motivating, variety

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CHAPTER ONE: INTRODUCTION

Rationale

It is concerning for me as a mathematics teacher to read reports that many students have negative feelings towards mathematics. For example, according to a 2005 Associated Press Poll (AP-AOL news, 2005), 40% of adults hated mathematics in school. Furthermore, in a survey of middle school students, over 50% said they would rather eat broccoli than do math (Boaler, 2008). Published papers echo the sentiment with titles like, "I Would Rather Die: Reasons Given By 16-Year-Olds For Not Continuing Their Study of Mathematics" (Brown, Brown & Bibby, 2008). It is clear that math is not a positive experience for many students.

Students give two reasons for negative feelings towards mathematics. First, they do not see mathematics as interesting or enjoyable (Brown et al., 2008; Nardi & Steward, 2003). Nardi and Steward found that students mentioned "fun" as the key ingredient missing from math classes. Second, they cannot see a relationship between classroom mathematics and real life (Boaler, 1998; Boaler 2008, Boaler & Selling, 2017). Boaler and Selling (2017) found that students who saw a connection between mathematics in the classroom and real-world mathematics were far more likely to have had positive experiences and fared better in their careers.

Two instances stick out to me when I recall being a first year mathematics teacher. The first was an invitation to a statistician and a pilot to come speak to my classes about how they used mathematics in their careers. Math became more real to my students, not something solely meant for the classroom. It became something that someone they had met really used, something that bettered the world we lived in. The second was observing my colleague, a physicist, while teaching Algebra concepts. The topic he introduced that day was logarithms. He began with posing a real-world problem that was impossible to answer without learning the math he would teach that day. He created a need for students to understand the topic. I found out this was how he strived to introduce every lesson. It was a powerful tool in his classroom, and one that I quickly adopted for mine. Students rarely asked, “Where does this apply?” because they already knew the answer.

I am aligning myself with researchers that say context is important in introducing mathematical concepts (Freudenthal, 2012, Van den Heuvel-Panhuizen, 2003). The connections these problems summon for students to the real world, the motivation, the enjoyment they experience, how worthwhile the student finds the problem, matters.

In this thesis, I will use the word “context” to mean the situation in which a mathematical problem is embedded. To clarify, consider the following problem: “Given a fixed amount of fencing, what dimensions of a rectangular field would maximize the area inside?” The context is about fencing and area. Within this context, there may arise a “problematic situation… that is unsolvable by [the student’s] current knowledge” (Harel, 2013, p. 122). In the fencing example, it might be the need for stating, with certainty, which dimensions do maximize the area. This problematic mathematical situation leads to the mathematics to be learned. In this study, I am focused only on the *problem context*, and not on the problematic mathematical situation encountered within that context.

Students are telling researchers that math should be enjoyable (Nardi & Steward, 2003) and interesting (Brown et al., 2008). Many researchers feel it is important that students see mathematics in the classroom that is applicable to mathematics used in real life situations (Freudenthal, 2012; Reddish, 2005; McDermott, 1987). Mathematics contexts should enable students to better understand the world they live in, and have the capacity to make improvements. These descriptions contribute to making contexts more engaging for students.

Yet, research on context usage and problem posing has not focused much on a student's point of view in terms of what makes problem contexts engaging. When are students engaged? What specifically within mathematics problem context is engaging to them? There is some research that informs us about students' views of contexts. For example, Schukajlow, Leiss, Pekrun, Blum, Muller & Messner (2012) had students compare pure math problems to problems with a real world context. They introduced a problem and then asked students about their interest in working on the problem as well as asking about the extent to which they agreed with the statement "I would enjoy solving the problem shown" (p. 226). Students did not solve the problems, rather Schukajlow et al. were interested in whether students would enjoy or be interested in solving the problem based on the kind of problem that was posed. While Schukajlow et al. (2012) did judge contexts on their incentive to work on a specific problem, they did not look at what about a particular problem made students interested in solving it. I discuss this study further in my literature review. My research will focus solely on students' opinions of contexts in an effort to help better understand what makes problem contexts engaging for students.

CHAPTER TWO: BACKGROUND

Literature Review

Problem Posing as a Way to Introduce Topics Through Problem Contexts

Many teachers consider problem posing when introducing mathematical topics (Crespo & Sinclair, 2008). In this section I first discuss the way in which my study defines problem posing and then previous research on problem posing and how my study differed.

In the body of research on problem posing there are two ideas. The first idea is how teachers help children to pose problems (Brown & Walter, 2005). The second idea is how do teachers pose effective problems (Crespo & Sinclair, 2008). I am working with the second idea: how teachers pose effective problems.

Currently the research on problem posing as well as research that uses problem posing within teacher experiments focuses on problematic mathematical situations (Lavy & Shriki, 2010; Kontorovich, Koichu, Lcikin & Berman, 2012). For example, Crespo and Sinclair (2008) discussed how prospective teachers chose "mathematically 'good' problems" (p. 397) for their classes. Their study surrounded 22 prospective teachers enrolled in a teacher preparation mathematics methods course. One question they posed to prospective teachers to think about was "What makes questions mathematically interesting…and worth solving?" (Crespo & Sinclair, 2008, p. 399). They did not focus on the context of the problem, rather the problematic mathematical situation that arose from the question. In response to the question the researchers discussed with prospective teachers the idea of tasty and nutritious problems. Nutrition refers to the mathematics we wish to teach through a particular problem, and tasty refers to posing problems that encourage students to bump into problematic mathematical situations that are interesting, worthwhile and fun.

Teachers in this study were given a task and asked to judge whether questions posed about the task were tasty. Crespo and Sinclair (2008) were interested in what rationale teachers used in order to judge problems on how tasty they were. Teachers mainly labeled problems tasty if a problem had surprising results, results that were not obvious, or had difficult results.

Again, Crespo and Sinclair's (2008) research focused on the problematic mathematical situations. This is an essential research topic and should continue to be developed. However, research on problem posing has not focused much on context usage. My research will focus on students' points of view in terms of how engaging the context of a problem is. Are students engaged? What makes one problem context more engaging to students than another? What specifically within the context is engaging them? It is my hope that my research will shed light on this topic.

Using Contexts in Mathematics Classrooms

In striving to compile ideas for what engaging mathematics might look like, I began with my reasons that students were disengaging from mathematics as detailed in my introduction. To recap, the literature on using contexts tends to revolve around the issues of applicability to the real world (Boaler, 2015; Boaler & Selling, 2017) or personal enjoyment (Brown, Brown, & Bibby, 2008; Nardi & Steward, 2003). I started in these two areas, as they seemed to be those that would help students to engage more with mathematics. As I read through research related to these areas, I tried to organize them in terms of the themes I saw across the literature.

In this section I describe the literature and the themes I saw, and then in the next chapter I compile these themes into a theoretical framework I used for this study. I begin by describing research related to "realistic" contexts, and explain a sub-theme within realistic contexts that I call "worthwhile." I then describe research related to "enjoyable" contexts. I conclude this section by describing a final theme I saw in the literature pertaining to "motivating" contexts. In the next chapter

I then combine the four characteristics of *realistic, worthwhile, enjoyable,* and *motivating* into a framework on engaging problem contexts and state my definitions for them, based on what I found in the literature.

Realistic contexts. In 2001 the National Research Council identified 5 strands of mathematical proficiency. The fifth strand is "The tendency to see sense in mathematics, to perceive it as … useful" (National Research Council, 2001, p.131). Many researchers are arguing for mathematics that students can apply outside of the classroom, that students can see a use for, and that they can see a need for someone to solve the problem (Redish, 2005; Van den Heuvel-Panhuizen & Drijvers, 2014; Gravemeijer, 2004). In particular, researchers in the Realistic Mathematics Education community, an instruction theory developed in the Netherlands, and proponents of STEM education were instrumental in helping me to develop ideas about what realistic contexts consist of (e.g., Van den Heuvel-Panhuizen & Drijvers, 2014; Reddish, 2005). These researchers are frustrated with the lack of real world connections in the classroom.

Realistic Mathematics Educators believe real world contexts should be the prominent part of classroom mathematics. I agree with the work of researchers within the Realistic Mathematics Education (RME) effort that state "Rich, 'realistic' situations [should be] given a prominent position in the learning process" (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 521). A common thread in the research is that *students* should see realistic application problems often.

Realistic Mathematics Educators further clarify that not only should students see meaningful real world contexts, students should also be able to imagine the situation to really understand the problem and what it looks like so as to be able to recognize it in life (Van den Heuvel-Panhuizen & Drijvers, 2014). As Van den Huevel-Panhuizen (2014) stated, "The problems are experientially real in the students' minds" (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 521).

Further, Corey (2014) states that part of the reason students do not see mathematics as useful is that they are unaware of how to apply it in a situation. Real world contexts could help students to be more aware of how math gets applied in real situations. “Most people never succeed in putting their theoretical knowledge to practical use” (Freudenthal, 1968, p. 4). Again, teaching real world contexts could help students to apply the mathematics they are learning in the classroom to other situations. Reddish (2005), a physics educator, states that lack of mathematical understanding among high performing students is a common concern among physics instructors. He argues that more applicable mathematics needs to be seen in the classroom.

Overall, these researchers believe students should be learning mathematics through the lens of real world problems, because doing so can make problems more engaging, helping overcome the negative feelings some students have for the subject. As such, I take *realistic* as a possible characteristic of an engaging problem and use it in my framework that I compile at the end of this chapter.

Worthwhile contexts. Through my own experience and as I read through research pertaining to realistic contexts, I noticed a sub-theme emerging; students need to see that a context is valuable. For example, according to NCTM standards, curriculum must be focused on important mathematics and students should be able to recognize that the math they are learning is worthwhile (NCTM, 2014). Students should engage in challenging tasks that support meaningful learning (NCTM, 2014; Brown, Brown, Reardon, & Merrill, 2011; Ejiwale, 2013; Cotabish, Robinson, Dailey & Hughes, 2013). A meaningful real world problem context has the power to improve quality of life, further innovation and/or increase economic stability (Brown et al., 2011). A meaningful problem context allows for us to better understand, explore, and engage with the world, and then have the capacity to change that world for the better (Obama, 2015).

One possibility for contexts that make a positive difference for others is found in science, technology, engineering, and mathematics (STEM) education. In a study on STEM educational awareness, Brown et al. (2011) argued that STEM teaching and learning focuses on innovative ways to help solve human wants and needs. Focusing mathematics contexts on innovative ways to help solve human wants and needs will help students to see how mathematics can make a positive difference.

In my personal experience over the years, I also noticed that students in my classrooms at times talked about mathematics that made a positive difference in the lives of people. I felt this was an important element to helping students to engage with a real world context, and that it connected with this part of the research literature's call for important, worthwhile mathematics.

Thus, contexts that provide ways to help solve human wants and needs may be more engaging to students. I decided to call this particular type of problem context a *worthwhile* context, and consider it as a second possible characteristic to add to the framework of engaging problem contexts.

Enjoyable contexts. Nardi and Steward (2003), Brown et al. (2008), Schukajlow et al. (2012) and other researchers helped me to understand the importance of enjoyment in contexts. Many studies (Nardi & Steward, 2003; Brown, Brown & Bibby, 2008; Boaler & Selling, 2017) have documented that students do not think mathematics is enjoyable or fun. Some of these researchers asserted that in order to have students continue on in their study of mathematics once it became optional, students needed to see mathematics as enjoyable. In this section I present some of the literature where researchers argue for the importance of enjoyable mathematics and share three studies researchers have done that concerned enjoyment.

Numerous studies of student affect have shown that students who enjoy mathematics experience higher academic achievement and are more likely to pursue higher mathematics courses

(Schukajlow & Krug, 2014; Pekrun, Goetz, Titz, & Perry, 2002; Martin, Anderson, Bobis & Way, 2012; Brown, Brown & Bibby, 2008). Students that experience enjoyment in the mathematics classroom are also more likely to persist in the face of failure, and increase their confidence levels (Gottfried, 1985; Pokay & Blumenfeld, 1990).

Brown, Brown & Bibby (2008) questioned 986 16-year old students, in England and Wales, about their decision to not enroll in a mathematics class as they continued their schooling. The authors gave students six categories of reasons for discontinuing mathematics classes to choose from: "boring, not needed for future degree/ career, not useful in life, prefer other courses, do not enjoy/ like it or too difficult" (p. 6). One third of all students named "do not enjoy/ like it" (p. 6) as a major factor in their decision to not continue taking math classes. The only category to exceed this amount was "too difficult" (p. 6). While the authors found that a major reason students do not continue their study of mathematics is lack of enjoyment, they did not provide a definition of what they meant by enjoyment.

Nardi and Steward (2003) interviewed 70 students who were in typical 9th grade classes. These students were identified as average students who may have the potential for higher achievement in their mathematics class. They used previous research to assert that this problem of students possibly not reaching their potential occurred because of students' negative feelings towards mathematics. Therefore, the interviews focused on students attitudes towards mathematics. Through observation and interviews, Nardi and Steward asserted that some of the reasons students may not be performing their best in class is because they find math tedious, isolating, procedural based, elitist and not personalized. Furthermore, student input caused Nardi and Steward to name fun as a major factor for students to be more engaged with mathematics and continue their study of it. Nardi and Steward referred to enjoyment as fun that was not frivolous in manner, rather this fun that students described was central to learning in the classroom. Unfortunately, this study did not define fun and the description of enjoyment was vague.

Schukajlow et al. (2012) argued that mathematics problems should be enjoyable for students. Therefore, they collected three different types of problems and asked 224 9th grade students whether they would enjoy solving them (students did not actually have to solve the problems). These problems included modeling problems, dressed up word problems and intra-mathematical problems. Modeling problems and dressed up word problems had a connection to the real world, while intra-mathematical problems did not. The authors used a five-point Likert scale for students to indicate whether they would enjoy working on these problems. Based on their findings they reported, "teachers cannot assume that it is sufficient to simply select reality-related problems for triggering students' positive emotions [enjoyment]…" (p. 227). While Schukajlow et al. talked about enjoyment being associated with positive outcomes in the classroom, they gave a vague description of enjoyment.

These studies provide evidence that often students do not enjoy mathematics and this is connected to why they may not continue taking math classes or have negative feelings towards mathematics. Although all three studies are not specifically about contexts, some of the reasons students view mathematics as they do could be a lack of enjoyment within contexts. It is reasonable to think that lack of enjoyment could be affiliated with the kinds of contexts students are seeing. I hypothesized that if students enjoy a specific problem context then they may be more likely to engage with that context. As such, I considered *enjoyable* to be an important third characteristic to add to my framework for engaging contexts, as described at the end of this chapter.

Motivating Contexts. Both within the research regarding both realistic and enjoyable contexts, and in additional research, I realized that there was another theme that needed to be fleshed out into a possible fourth characteristic of engaging problem contexts. In this section I present some of the literature where researchers argue for the importance of mathematics being "motivating" to students. Note that a problem context could be realistic, or maybe even consist of a game meant to be "fun," without being very motivating, which is why I believe this needs to be addressed as a separate theme. Much of the research I review in this subsection used the language of "interesting," but for the purposes of my study I am considering "interesting" to be interchangeable with the word motivating.

Students are more likely to self-regulate their learning process when they regard problems as interesting (Pintrich, 1999). Teachers deal with fewer classroom management problems in classrooms where students view math problems as interesting (Boaler, 2008). Similar to enjoyment, field studies have shown that students that experience interest in the classroom experience higher academic achievement (Schukajlow & Krug, 2014). Schukajlow and Krug describe interest as a relationship between a person and an object. The authors argue that a person that is mentally engaged with an object is interested in that object. Interested learners are more concerned with sense-making in mathematics and less likely to use shallow strategies like memorizing (Schiefele & Schreyer, 1994).

Recall that Brown, Brown & Bibby (2008) questioned 986 16-year old students, in England and Wales, about their decision to not enroll in mathematics classes as they continued their schooling. The authors gave students six categories of reasons for discontinuing mathematics classes to choose from. The third most common category was boring. Consider one student's statement about her decision to not continue her study of mathematics "Throughout school life, I have not been interested in maths. I may be reasonably good at it, but feel the lack of interest would not survive the subject" (p. 9). Students perceptions of mathematics as uninteresting kept them from pursuing further mathematics classes.

Recall from the section on enjoyment that Schukajlow et al. (2012) argued that mathematics problems should be enjoyable for students. They likewise argued that mathematics problems should be interesting for students. Therefore, the 224 9th grade students they surveyed were also asked about their interest in working on the solutions to problems (students did not have to solve the problems). Based on their findings they reported, "teachers cannot assume that it is sufficient to simply select reality-related problems for triggering students'…interest" (p. 227). Schukajlow et al. described interest as a relationship between a person and mathematics. However, they did not give students this definition nor collect data on how students viewed the word interest in their study.

Krapp (1999) described interest as a relationship between a person and some topic or content of their life-space. He stated that this is sometimes referred to as a person-object relationship. He also characterized interest as having affective and cognitive aspects. Interest implies a readiness to engage with a high level of effort. Hidi and Harackiewicz (2000) described motivation as a willingness to work on something for a longer period of time, pay closer attention and learn more.

Overall, it appears then that being interested, or *motivated*, is an essential piece of becoming engaged with mathematics. A sense of motivation could help students to become engaged. As such, I included this as a fourth possible characteristic in my framework for engaging contexts.

Gap in the Current Literature and Statement of the Research Questions

Researchers such as Boaler (1998), Brown et al. (2008) and Nardi and Steward (2003) are concerned about students disengaging with mathematics because they are not viewing it as realistic, worthwhile, enjoyable or motivating. While researchers seemingly argue for the importance of mathematics that is seen as realistic, worthwhile, enjoyable or motivating, studies have yet to inform research on *students' perspectives* for viewing contexts in these ways. That is, we know that a problem could be realistic to help it be engaging, but what do *students* see in a problem that helps it have the characteristic of being realistic?

Boaler (1998) found that students who saw strong connections between the math they learned in the classroom and real world experiences liked math, saw it as interesting, useful and fun. Students who were unable to connect real world experiences with classroom mathematics found math tedious, impractical and boring. "The students…all spoke very strongly about their complete inability to make use of any school-learned methods in real situations, because they could not see any connection between what they had done in the classroom and the demands of their lives outside the classroom" (Boaler, 1988, p. 58). But in order to capitalize on this knowledge, we need to ask: when do *students* see that connection?

Nardi and Steward (2003) found that many students are doing just enough to get by in their math classes because they viewed mathematics as "lacking in relevance with the world outside school and their own needs, interests and experiences" (Nardi & Steward, 2003, p. 346). Nardi and Steward lament that these students are present but often underachieving. Re-engagement of this type of learner is critical. But, again, in order to use these results, it is important to know: when do *students* see a problem as enjoyable?

Students' engagement in mathematics is important. One underlying message of the National Council of Teachers of Mathematics (2014) is the need for students to engage in mathematics. Disengagement is considered a factor in the decline in enrollment in mathematics classes (Brown et al., 2008; Bobis, 2000; Forgazs, 2006). The ideas described above gave an initial hypothesis of what engaging contexts looked like. Exposing students to contexts that are realistic, worthwhile, enjoyable, motivating or some combination of these characteristics could be a step in helping to re-engage them. Therefore, when teachers work to introduce mathematics topics by posing engaging problems to their students it may be helpful to use one or more of these characteristics. One gap in the research is understanding when *students* perceive problem contexts as realistic, worthwhile, enjoyable or motivating. In order to address this gap in the research literature, my study is centered on the following two research questions:

(1) What aspects of problems do students see as making them more realistic, worthwhile, enjoyable or motivating?

(2) When comparing problems meant to introduce a mathematics topic, what reasons do students give on their own for finding one problem context more engaging than another?

By answering these research questions I hope to shed light on the larger issue of creating mathematics problem contexts that are more engaging for students.

Theoretical Framework

In my literature review, I discussed how researchers have indicated the importance of contexts being realistic, worthwhile, enjoyable, or motivating. I pulled these ideas together to create a framework for what might make mathematics problem contexts engaging. In the following subsections I develop my framework by providing definitions for each characteristic: realistic, worthwhile, enjoyable and motivating. I then end by explaining how I see these four characteristics as being part of making problem contexts engaging.

Realistic

This first characteristic was fairly well defined in the literature and the definition came easily from it. Based on the research provided in the literature review, I define a realistic application problem as:

A realistic context is a problem context that the student could see someone (myself or others) having the need and taking the time to solve outside of the classroom.

Recall RME's statement that a problem should be "experientially real in the students' mind" (van den Heuvel-Panhuizen & Drijvers, 2014, p. 521). That is why this definition is written specifically with the student's perception in mind. Additionally, within this definition, in order for a problem to be perceived in a students' mind as realistic, the student must really believe that someone outside of a classroom situation would actually have the need to solve the problem and take the time to find the solution.

Worthwhile

Based on the research provided in the literature, as well as on my own experience, I define my second characteristic, worthwhile, as follows:

> *A worthwhile problem is a problem context whose solution is seen by the student as having the capacity to make a positive difference for someone (themselves or others).*

Recall that this second definition should be considered as a subset of the *realistic* characteristic. Thus, I assert that one way one could make a real world context even more engaging is by making it seen by students as worthwhile.

Enjoyable

I now explain the third characteristic of engaging problem contexts: enjoyable. Because the notion of enjoyment was not well defined in the literature, I describe a little bit more here in order to craft my own definition for what an enjoyable context is. First, emotions influence our actions (Frenzel, Pekrun & Goetz, 2007; Larson & Richards, 1991; Pekrun, Goetz, Daniels, Stupnisky & Perry 2010). Therefore, positive emotions can be a force for good in the mathematics classroom. In a study on affect by Nardi and Steward (2003), students named the emotion 'enjoyment' important to engaging in mathematics. Enjoyment is a positive emotion referring to one's experiencing pleasure, fun, excitement, or entertainment. Building on the relationship that "enjoyment" has with emotional states of being, I define the third characteristic, enjoyable, as follows:

An enjoyable context is a problem context that is likable from the student's perspective, in that it evokes positive feelings in the student towards the problem.

Thus, while other characteristics may be more intellectual in nature, an enjoyable problem is inherently connected to feelings or emotions.

Motivating

I now describe the final characteristic in my framework, which is neither a subset nor superset of any of the previous characteristics, but that can exist in conjunction with one or more of the others. Based on the literature, my fourth characteristic, *motivating*, is defined for my purposes as follows:

A motivating context is one that invokes an intellectual interest or desire to think about it, and to learn the mathematics needed to solve the problem.

To clarify, this is not the same thing as "intellectual need" for a specific mathematical topic as Harel (2013) would describe. Intellectual need deals more with epistemological issues as to why a certain mathematical entity exists in the first place and why it should be known. For my purposes, motivation pertains to the context itself, and not the origins of the mathematical ideas, and simply deals with whether a student has interest in continuing to examine the problem context or not.

Engaging Problem Contexts

Having detailed my four characteristics, I now explain how I see "engaging problem contexts" more generally. First, I clarify that I do *not* mean by this framework that a problem context must have all four characteristics to be "engaging." Rather, I see engaging contexts along a spectrum, where the more characteristics it has, the higher the chances are that it will be seen as engaging by students. As an example, a reasonably engaging context might be realistic and also motivating. Or it might be exceptionally enjoyable without being realistic. It is possible that a problem context could have all four characteristics, but it is not necessary for it to have all four. In this way, my research question that deals with students' perspectives on what makes problem contexts realistic, worthwhile, enjoyable, or motivating is essentially a question that seeks to know from students' points of view what makes problem contexts more engaging overall. By learning what students perceive as making a context have one or more of these characteristics, teachers can be more confident in choosing contexts that will likely engage students.

However, I am also completely leaving myself open to the possibility that my framework based on the literature review has missed an important characteristic beyond these four. As such, part of the research is intended to see what reasons students might give that go beyond these four characteristics in terms of making problem contexts engaging. The second research question, which focuses on the reasons students give for believing some contexts to be more engaging than others, can then be interpreted as an attempt to identify whether these four characteristics seem to capture most of what makes contexts engaging, or whether there are additional characteristics to be considered.

CHAPTER THREE: METHODS

In this section, I orient the reader to the methods I used in order to better understand what makes context engaging for students. I discuss the setting and participants, data collection, the choice of problems and data analysis.

Setting and Participants

To accomplish my goals in this study, I determined that it would be useful to interview students about specific problem contexts, as well as to have them describe problem contexts from their own experience that were engaging or not. I decided it would be necessary to narrow the problem contexts to a particular topic area to not have too many ideas inside the interview. Otherwise it would be difficult to identify themes pertaining to the characteristics in my framework. However, I wanted to identify a single topic to have the students focus on that is not too specific to only one class, to help the results have greater possibility of being more applicable. I determined that the topic of "optimization" would be good for the study, since it is encountered in multiple different classes like Algebra and Calculus. I therefore created my problem contexts, described within my interview design, around this topic.

Having chosen the topic, I then needed to decide which students to enroll in the study. I decided to select students from the same level of mathematics to streamline the data. I decided to recruit students in Calculus I classes because (a) as older students, they may be better able to articulate their thinking and (b) as students with more mathematical experience, they have greater exposure to different types of problems and could draw on that exposure in the study. Furthermore, I selected Calculus I students because of the findings of the Mathematical Association of America's national study on Calculus I classes (Bressoud, Carlson, Mesa, & Rasmussen, 2013). This study looked at student's attitudes towards mathematics before and after taking Calculus I at the collegiate level.

Students within the study reported being less confident in mathematics and having less desire to continue on in mathematics after taking Calculus. Thus, even if my results did not end up being as generalizable as I hoped, it could still benefit a key class in undergraduate education. Therefore, I recruited students in Calculus I courses at a northwest university.

While I kept topic and mathematical level constant to help streamline the data, within the calculus student population I did seek to achieve some diversity of students in my sample. In particular, I sought students with different attitudes toward mathematics, because a study on engagement would do well to have students with a mixture of views of mathematics. In the classes I sought to recruit students from, I gave the students a 2-3 minute survey (see Figure 1) meant to ascertain the students' attitudes towards mathematics, and to find out how useful the students viewed mathematics. The survey also asked the students to state their current or planned major. The survey included six statements from the Fennema-Sherman Usefulness of Mathematics Scale (Fennema & Sherman, 1976) to identify how useful students consider mathematics. The survey also included five statements from the Kaput Center for Research & Development in STEM Education to discover student's attitude towards mathematics (Brookstein, Hegedus, Dalton, Moniz & Tapper, 2011). I supplemented these with a final statement of my own, to have the same number of positive and negative statements. The survey results produced a "usefulness" and "affect" score for each of the students. Appendix B provides greater detail on how students were scored in these two categories. For recruitment, I attempted to recruit students with higher, middle, and lower scores in these areas. However, most students that completed the survey had fairly high "usefulness" scores, resulting in less diversity in that area. On the other hand, I did have students with high, medium, and low "affect" scores.

	Strongly Disagree	Disagree	Agree	Strongly Agree
1. I study mathematics because I know how useful it is				
2. Studying mathematics is a waste of time				
3. Mathematics is a worthwhile and necessary subject				
4. I like math				
5. Mathematics will not be important to me in my life's work				
6. I do not have positive feelings towards mathematics				
7. I see mathematics as fun.				
8. Mathematics does not interest me				
9. Mathematics is of no relevance to my life				
10. I find joy in solving non-routine problems in mathematics				
11. I will need mathematics for my future work				
12. In the past, I have not enjoyed mathematics				

Figure 1. Pre-research Survey

Students willing to participate in the study were asked to include their name, contact information, and their planned major (see Appendix B). Students were informed that if chosen to participate in a one hour interview they would be compensated $15. There was no compensation for the survey. The students' names were not connected to the information that was provided in the hour interview. Names provided in my research are pseudonyms.

To begin selection for participation, I divided the approximately 35 surveys into two piles based on the median affect score of the responses, one pile for students above the median, one pile for students below the median. Beginning with the pile of low affect scores, I went through and looked at majors. When I found a repeated major I put that survey aside. I did specifically look for students who were not in STEM fields first. There were not many. I tried to recruit the students who were not in STEM fields first, knowing that based on the numbers, I would not exceed half of interviewees outside of STEM fields. I repeated this process with the pile of high affect scores. It worked out that I had a fairly even amount of males and females. I would have returned to the original surveys and chosen someone similar in affect score and major, but with a different gender if I had needed to.

Students were contacted via the contact information they provided. I contacted each student once. If I did not hear a reply within three days, I assumed that student was uninterested in participating in the research. I then purposefully selected another student. As needed, contacting continued until I found 12 students willing to participate. I contacted 12 students to ensure at least 10 interviews. Ten interviews is consistent with many qualitative research studies. One student had to cancel. Eleven students were interviewed.

I recruited students with the goal of getting as much diversity as possible in attitudes, beliefs, and majors as well as a fairly mixed group of males and females. The 11 students contacted were those who were purposefully chosen as detailed below. The interview participants are summarized in table 1.

Table 1

Pseudonyms of Students Chosen

	Usefulness Score	Affect Score	Major
Sarah (F)	20	11	Communications
Madisun (F)	20	15	Biochemistry
Rachel (F)	19	16	International Relations
Grant (M)	24	22	Applied & Computational Mathematics Emphasis
Caleb (M)	24	22	Data Science
Rebekah (F)	24	20	Undeclared
Savannah (F)	24	23	Mathematics
Trey (M)	23	18	Pre-management
Jeff (M)	19	18	Undeclared
Dan (M)	21	17	Physics
Doug (M)	21	12	Computer Science

Data Collection

Qualitative research (see Maxwell, 2011b) fit well with my interest in student perspectives. My qualitative study consisted of one-on-one interviews with the 11 students described in the previous section, which were videotaped and audio-recorded. The interviews were 45-75 minutes in length.

The Interview Design

In this section I present the design of my interview (for the interview protocol itself, see Appendix C). The interview consisted of two parts. In part I, students compared three pairs of problems based on which problem in each pair they found most engaging. In part II, the students were asked to discuss what made problems realistic, worthwhile, motivating, or enjoyable by giving an example problem they had experienced in a math class of each characteristic. I now describe each of the two parts of the interview and the purpose of each part.

Interview parts: Part I. In part I, I gave students three sets of problem pairs to compare against one another. They discussed which problem within the pair they found more engaging and why. Students at this point in the interview were completely unaware of my framework so that they were not influenced in terms of what characteristics they focused on. By doing this, I could see what types of ideas they brought up on their own. This is the reason I did this part of the interview first, in that students would not know the specific types of characteristics I was interested in exploring.

Part one of the interview consisted of showing students six optimization problems (see the subsection "Choosing the problems" later in this section for details on how the problems were chosen and developed). These problems were each given to the students as a short video in which an instructor presents the problem. That is, a professor gave the problems verbally, while drawing a corresponding image on the whiteboard. Each problem took about 60-90 seconds to present. Students were also given a paper copy of the problems that they could read or refer back to. The written versions are shown in Appendix A. Note that the video presentations matched the written versions verbatim. These six problems were shown to the students in pairs, and they discussed one pair at a time. Each problem pair contained one problem from common mathematics textbooks, and another problem of Dr. Jones or my creation.

To select problems, I began by finding three optimization problems found in commonly used Calculus I textbooks. I call these problems: farmer problem, river problem, and parabola problem. In order to give the reader an idea of how common these problems appear in textbooks see table 2. If the problem is found in the examples in the expository part of the optimization section, I denote that with "EX" in the table. If it is found in the homework section, I denote that with "HW".

Table 2

Common Problems in Calculus Textbooks

	Farmer Problem	River Problem	Parabola Problem
Stewart (2015)	EX	EX	EX
Thomas, Weir & Hass (2009)	HW		HW
Smith & Minton (2008)	EX and HW	Similar problem in HW	EX and HW
Briggs, Cochran & Gillett (2015)	EX and HW	Similar problem in EX	HW
Zill & Wright (2011)	EX and HW	EX and similar problem in HW	EX and HW

To then give students fodder to talk about, for each textbook problem I created a problem with a different context (see Table 3). In the three new problems I tried to purposefully improve the contexts as compared to the textbook problems. Doing so provided opportunities to see whether or not such problem contexts were viewed as more engaging by the students, and why the students reported them as such. It was possible that students would see my contexts as better than the textbook problems, or vice versa, but either result would provide valuable information from the students' perspectives about what makes one context more engaging than another context.

Table 3

Reasons Why New Problems May be More Engaging Than Textbook Problems.

Abbreviated textbook problem	Abbreviated equivalent problem with different context	Reasons the different context might be more engaging
A farmer has 2,400 feet of fencing and he wants to make a rectangular field that borders a straight river… What are the dimensions of the field that will give it the greatest area?	You're adding on to your home, and you're going to knock out a wall in the back to add an open room … You have enough material for 60 feet of new wall… What dimensions would give the greatest area for this new room?	1. More realistic because closer connection to lived experience. 2. More motivating because the context has changed from a farm to a home, which may be more relatable.
You are going to launch a boat from point A on the bank of a straight river. You want to reach point B which is downstream on the opposite bank of the river… Where should you land on the opposite bank in order to reach point B as soon as possible.	You are working with a humanitarian aid group to bring safe water supply to villages. One village is located at the edge of a mountain range and its nearest water supply is up in the mountains…What path should the pipe follow in order to minimize the cost of laying the pipe?	1. More worthwhile because it deals with helping people. 2. More motivating because the context concerns humanitarian work rather than speed on a river.
Consider the graph of y 2=2x and the point (1,4). Every point on the parabola is a certain distance from the point (1,4). Find the point on the parabola that is closest to the point (1,4).	You're working for NASA and a comet's going to make a close pass by Earth… The path of the comet can be modeled by the graph of y = sqrt(x+5) (Earth at origin). To tell the public which day would be the best day to view the comet… find the point along the graph that is closest to the origin.	1. More realistic because it deals with a real-world situation. 2. More enjoyable because it deals with a fun situation (comet-viewing)

A few lessons were learned from pilot students that I tried to incorporate into the problems used in this part of the study. In my pilot studies, the students were quite concerned that the math of a problem is not too complicated, the problem is not too lengthy, and the problem is understandable.

I tried to choose problem pairs in which each pair had essentially the same mathematics required to solve both problems to help students get away from worrying about the mathematics involved. Each pair consisted of two problems with similar mathematical solutions, and for which only the context was intended to differ. Students were told that the difficulty of the math needed to solve each of the problems within the pairs was the same. This was done in order to help students focus on the context of the problem, not how difficult or challenging the math might be. I found in pilot studies that students focused too much on how difficult the mathematics of the problem was when discussing engaging context. When I gave them problem pairs and told them that the mathematics difficulty was exactly the same, students were able to focus on the context of the problem.

Pilot interviews showed that a non-understandable context made it difficult to discuss any other characteristic of the problem. Thus, it was assumed that being understandable was important, and I gave time during the interview to make sure each problem appeared to be understood by the student. Doing videos in conjunction with printed problems also avoided potential issues in reading comprehension that might affect student understanding.

Finally, students preferred concise problems in the pilot interviews. Therefore, problems were made as concise as possible.

The discussion of these problems was very student led. Other than asking which problem students would prefer a teacher to use to introduce a math lesson, I allowed students to direct the conversation and pursue the topics they were interested in. This helped me to answer my second research question, “When comparing problems meant to introduce a mathematics topic, what reasons do students give on their own for finding one problem context more engaging than another?” This part of the interview helped me to understand the reasons that students initially gave for finding one problem context more engaging than another.

Students were informed at the beginning of part I of the interview that this study was to help teachers pose more engaging problems within mathematics lessons. Students were asked to compare each of the two problems within the pairs in this way, “Given that the mathematics needed to solve these problems is exactly the same, which context would you prefer your teacher use to introduce a new math topic to you?” Through follow-up questions students were given the opportunity to discuss their thoughts, specifically the reasons why they found one problem context more engaging than another.

Part one of the interview took approximately 20-25 minutes. I took about 7 minutes on each pair of problems. Each time students were shown a pair of problems they were reminded that if they did not understand something they should please ask questions until understanding was reached. It took about 2-3 minutes to show a video of the pair of problems being presented by a professor and for the student to ask any questions about it. This allowed about 5 minutes for questioning on each pair of problems. I concluded part one of the interview by having students order all six problems from the most engaging to the least engaging and talk about why they ordered it in this manner. This gave students another opportunity to expound on what made contexts engaging or not.

Interview parts: Part II. The second part of the interview served a very different function from the first part, and was intended to be analyzed first, despite coming later in the interview. Part two was meant to better understand when students viewed contexts as having the characteristics in my framework, so the characteristics were given explicitly. In the second part of the interview, we examined each characteristic (realistic, worthwhile, enjoyable, and motivating) one at a time.

For each characteristic, I gave the student my definition from my framework and asked them to come up with a problem they felt was a strong example of that characteristic. Once they had come up with a problem and described it to me, I asked them to be explicit about what made that problem have that specific characteristic. I also allowed students to discuss problems that were bad examples of that characteristic. Furthermore, I asked them to give advice for teachers who are striving to pose problems that students would be likely to find embody that characteristic. After discussing problems relevant to each characteristic, I asked the students to give an example of a problem that they believed to be the most engaging problem they had encountered in a math class. While I assumed that realistic, worthwhile, enjoyable, motivating or some combination of these characteristics was an important part of engaging contexts, I asked about overall engaging contexts to provide students an opportunity to bring up other characteristics or aspects of engaging contexts.

There were three purposes for this second part of the interview. First, it was done to make sure that there was an opportunity to discuss characteristics that we know are important, but may not always be brought up initially by students. In pilot studies, when I let students completely lead the conversation, sometimes it consisted of discussing the voice of the teacher, the length of the problem, and math in general. This part of the interview ensured that students would discuss realistic, worthwhile, enjoyable, motivating and overall engaging contexts. Second, this part helped flesh out when students viewed contexts as realistic, worthwhile, enjoyable, motivating or some combination of these. By asking students to give an example of a problem for each of these characteristics, I would better understand when students view contexts in these ways. Third, by asking about "overall engaging" contexts, I provided students an opportunity to bring up other characteristics or aspects of engaging contexts. This second part of the interview was intended to give more clarity and more coherence in terms of what engaging problem contexts consisted of. In this way, part two of the interview could help answer my first research question, "What aspects of problems do students see as making them more realistic, worthwhile, enjoyable, motivating and overall engaging?" Overall, part two of the interview took approximately 35-40 minutes.

The order the interview parts were reported on. In my results section, I report on part II of my interview first, then part I. While this is opposite the order that the interview was done, there is a reason for this ordering. The second part of the interview allowed me to analyze what specific characteristics made problem contexts have the engaging characteristics in my framework. It also allowed an opportunity for new characteristics to emerge. Once this more open analysis was completed, I then wanted to compare these results to the first part of the interview as a way to test whether the results were accurately capturing what the students were saying in part one. It helped strengthen the results that came out of the second part of the interview and helped make sure I was not missing anything.

In this case, why did I not simply do the second part of the interview first? By giving students the characteristics explicitly, it would certainly influence what they would think about. However, I wanted their problem-by-problem comparisons to come from their own thinking and not be directly influenced by the explicit statement of the characteristics.

Data Analysis

Coding

Coding involved eight phases. Seven of the phases came from a paper on thematic analysis (Braun & Clarke, 2006, p. 87). One of the phases, (phase 7) I created based on what I wanted to do with the aspects of engaging context. I called these phases one through eight. Each phase is outlined in table 4. In this section I provide details about phases one through eight and how they were implemented in my study.

Table 4

Framework for Phases 1-8 of Thematic Analysis

Phase	**Description of the process**
1.Familiarization with the data:	I immersed myself in the data by reading and re-reading until I was familiar with the depth and breadth of the content
2. Coding the data for part II of the interview:	Coded the reasons students gave for contexts being more or less engaging in part II of the interview.
3. Searching for aspects in part II:	Collated codes from part II into potential aspects of the characteristics of engaging context.
4. Reviewing potential aspects of engaging contexts for part II:	I began to generate a thematic 'map' of potential aspects of engaging contexts from part II of the interview.
5. Defining and naming potential aspects of engaging contexts for part II:	Ongoing analysis refined the specifics of each aspect, and the overall story the analysis told, generated clear definitions and names for each aspect. Here a better understanding evolved for what engaging contexts looked like.
6. Coding the data for part I of the interview:	Coded the reasons students gave for contexts being more or less engaging as they compared problem contexts against one another.
7. Comparing codes to aspects of engaging context.	Compared the codes found in part I of the interview to the aspects of engaging contexts developed in part II of the interview. Checked to see if the aspects of engaging contexts that evolved in part II of the interview described why students found one problem context more engaging than another in part I.
8. Producing the report:	The final opportunity for analysis. Selection of vivid, compelling examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report of the analysis.

Phase 1: Familiarizing Myself with the Data

After each interview, I promptly watched and transcribed the interview and added notes from the interview. I immersed myself in the data by reading and re-reading until I was familiar with the depth and breadth of the content (Braun & Clarke, 2006).

Phase 2: Coding Data for Part II of the Interview

In phase 2, I initially coded part two of the interview. This part included the reasons students explicitly stated that they saw contexts as realistic or not, worthwhile or not, enjoyable or not, motivating or not and overall engaging. Students were asked to provide examples of problems that fit under each of those characteristics. In this stage I first color coded reasons students gave for strong and weak contexts. Each reason given for strong or weak contexts was considered a unit of analysis. Reasons for strong contexts were reasons students gave for finding a problem realistic, worthwhile, enjoyable, motivating or overall engaging. Reasons for weak contexts were reasons students gave for not finding a problem realistic, worthwhile, enjoyable, motivating or engaging. For each reason, I looked at why that reason seemed to matter to the student. What was it that made a reason justify strong or weak contexts. What specifically in the problem elicited that response? Then, I broke the information up according to the codes. Table 5 shows the overall format this phase of analysis took on.

Table 5

Example Problems Students Gave, Why They Mattered, What Elicited the Response

	Example	Why	What
Realistic			
Not Realistic			
Worthwhile			
Not Worthwhile			
Enjoyable			
Not Enjoyable			
Motivating			
Not Motivating			
Engaging			

Phase 3: Searching for Potential Aspects of Engaging Contexts in Part II of the Interview

Once I coded the comments, I began to look for themes. I left myself open to categorizing new data (Strauss & Corbin, 1998). I began to see potential themes that students brought up repeatedly that helped to strengthen context and make it more engaging. If three or more students brought up an aspect, I considered that a theme. Codes were analyzed and sorted into potential themes. I did not abandon any codes at this stage, even if they did not fit into the initial themes I saw arising. I looked for themes that answered my second research question, "What aspects of problems do students see as making them more realistic, worthwhile, enjoyable, motivating and overall engaging?"

Phase 4: Reviewing Potential Aspects of Engaging Contexts for Part II

During this phase, I began to generate a thematic map of potential aspects of engaging contexts. It became evident that some of the aspects were not really aspects due to not enough data to support them or the data being too diverse. As I reviewed and refined, I went back and repeated steps two through four. This process was repeated several times as I sometimes found two aspects overlapped too much and needed to be combined, or one aspect was too broad and needed to be divided.

I tried to make sure that data within the different aspects cohered together meaningfully, and that there were clear and identifiable distinctions between the aspects (Braun & Clarke, 2006). I also strived to make sure that I was telling the student's story. Within this phase, I looked to make sure a coherent pattern had immerged within the aspects. I made sure my aspects all fit and built on informing teachers about strengthening problem contexts by making them more realistic, worthwhile, enjoyable, motivating and overall engaging.

Throughout the process, I discussed the aspects of engaging contexts with Dr. Jones, getting his approval on the coding process and the aspects that seemed to be arising from the codes. Several of the aspects that ended up being combined were a result of deliberation with Dr. Jones.

Phase 5: Defining and Naming Themes for Part II

Here I identified what was interesting about the themes and why. I tried to make sure that each aspect gave a narrative that informed teachers on engaging problem contexts. I tried to make sure that all the aspects built on one another to tell an overall story of how contexts could be more engaging. Each aspect was considered individually and collectively in giving that narrative.

Phase 6: Coding Part I of the Interview

After coding part II of the interview, I looked at the reasons students gave for problems being more engaging in part one of the interview. What aspects of the problems did students compare and what made one problem context more engaging or preferable than another in their mind? Initially, I coded this part as I had initially coded part II of the interview. (See Table 5), with one exception. I left space for the possibility of new aspects of engaging context that my framework did not cover.

Phase 7: Comparing Codes to Framework

I compared the codes found in part I of the interview to the aspects of engaging context developed in part II of the interview. I checked to see if the aspects developed in part II of the interview described why students found one problem context more engaging than another. I looked at my first research question, "When comparing problems meant to introduce a mathematics topic, what reasons do students give on their own for finding one problem context more engaging than another?" Did students give similar reasons on their own for finding one problem context more engaging than another as they did for finding a problem realistic, worthwhile, enjoyable, motivating and overall engaging? Did similar themes arise? Did different themes arise?

Phase 8: Producing the Report

In phase eight I wrote up my results. I chose to share the particularly informative student narratives about what made problem contexts engaging, as well as what made problem contexts less engaging. These narratives helped to answer my research questions and holes in the literature. With the information gathered, I am better able to describe to teachers ways to make problem contexts more engaging.

CHAPTER FOUR: RESULTS

Recall that students first saw the three problem pairs given the prompt "given the math is exactly the same, which context is most engaging for you?" Then students discussed what problem was most engaging and why.

After this, students were shown the definitions of realistic, worthwhile, enjoyable and motivating. They gave examples of problems they had seen for each characteristic and advice on how to make problems more realistic, worthwhile, enjoyable or motivating.

However, in this chapter, I first give the results from the second part of the interview. This part of the interview provided details about what helped problems have each of the characteristics in the framework, as well as an additional characteristic not included in my original framework. Following a description of these results, I return to the first part of the interview . Students reasons for finding one problem context more engaging than another were strikingly similar to the characteristics and aspects of engaging context they described in part II of the interview.

Part II of Interview

In this section I attempt to answer my first research question: "What aspects of problems do students see as making them more realistic, worthwhile, enjoyable, motivating and overall engaging?" Five aspects of engaging problem contexts emerged: related to my everyday life or a potential career idea, authentic, purposeful, interactive and a good story. However, as students discussed what made problem contexts realistic, worthwhile, enjoyable, motivating and overall engaging, one new characteristic emerged, variety. In this section I first describe the new characteristic, and then I describe the aspects of problems that emerged from the data.

Variety

Ten out of eleven students brought up the idea of variety. Variety was an unexpected theme because it did not fit anywhere within my framework. Thus, I consider this to be a new “fifth” characteristic that can help a problem be engaging. Ten out of eleven students brought up the importance of variety. Students did not feel engaged when they saw the same problems over and over again. In this section I discuss the importance of variety from the point of view of the students.

To explore what is meant by “variety,” and to lead to my definition of this new characteristic, consider these comments from Sarah and Rachel. When asked about a problem context that was meant to be realistic but did not feel realistic to her, Sarah brought up rocket problems. She stated, “We were always doing rocket problems.” She was concerned that it was the same type of problem over and over again. Repetition made it less engaging for her. Next, within Rachel’s discussion of realistic contexts, she stated, “It is more engaging when it is something I have never thought about but also realistic.” Note that Rachel found realistic problems more engaging when the problems were new, something she had not previously thought about.

Consider this comment from Jeff. When asked for advice on posing worthwhile problem contexts, he stated, “Different interests will dictate engagement of students. Each person will have a different perspective of what is worthwhile to them.” Note that Jeff believed that different interests would determine the engagement of the students. Therefore, if teachers want to help many students to be engaged they need to recognize that one kind of problem may not have the power to do that. It is important to show a variety of problems or a variety of contexts (new or different problems).

Similar ideas also showed up while discussing motivating and enjoyable contexts. Consider these statements from Sarah and Savannah while discussing motivating contexts. Sarah suggested teachers should be creative when they are striving to pose problems that students could find motivating. Her teacher made up a motivating problem for her about a parachute that she had never seen before. She stated, “that would be kind of motivating because that is something different.” When asked about contexts that were not motivating, Savannah said, “repetition without a new way of how this applies to real life.” Additionally, Savannah, Jeff, and Rebekah conversed about variety while discussing enjoyable contexts. After Savannah gave an example of an enjoyable problem, she was asked what made it enjoyable for her. She responded, “That was something I had never thought of.” When asked for advice on posing problems that students would find enjoyable, Jeff stated, “Variety is important because each person has unique interests. Know about different interests in your classroom.” When Rebekah gave advice on posing enjoyable problem contexts, she said, “Have a variety of questions so that they are not just all pure math problems. Spice it up with other things, put a new spin on something. It is more enjoyable when we incorporate other things like science, literature. Give it a story. Bring in different things that can make it more enjoyable than just a straight math problem. Those can get pretty boring. Bring in different concepts.”

Students wanted to see problems they had never seen before, different, new, novel, surprising. It is difficult for students to see a problem as engaging when they have seen that type of problem over and over again. Students felt problems were unengaging when the problems were repetitive. Based on this data, I define a problem as having the characteristic “variety” when it is new or different compared to problems students have typically seen in the past.

To flesh out this new characteristic more, consider Caleb’s advice. Caleb was concerned about the lack of variety within Calculus problems.

Caleb: There are so many ways that derivatives and integrals can apply and that does not really come across when teachers teach it. The applications of Calculus are huge. It is used everywhere. It motivates you more to learn the math if you can see that this really applies to so many worthwhile, realistic, enjoyable situations. I can [see it] and it does now make a difference in the world [sic].

Caleb was concerned that derivatives and integrals could be applied in many ways and students often do not see that in the classroom. Caleb felt that as teachers strive to pose problems that are new and different, they help students to see the many ways that math topics apply, and this helps students engage more.

Madisun explained similar ideas. She used the same problem context for both an enjoyable problem and a realistic problem, a problem she has seen in her Chemistry class. Her repetition of the problem emphasized the idea that she was very engaged with this problem. During this discourse, I brought up a problem she had mentioned earlier and labeled unengaging - an apple problem. I asked her to compare the unengaging problem with the problem that really engaged her. Note the difference between the two and why that mattered to Madisun.

TS: Can you give me an example of a realistic problem you have seen in a math classroom?

Madisun: The oil slick one was realistic because someone could do that. If I know how big the spill is I can clean it up. Derivatives were used to find how big the spill was.

TS: You mentioned some apple problems that one could say are realistic but it did not seem that you felt that it was really realistic, or it did not engage you. Talk to me a little about that. Why was the oil problem engaging but not the apple problem?

Madisun: I guess it was because it made me feel like they [teachers] thought I was smart enough to do it [the oil problem]. Whereas the apple problem was like, you aren't smart enough to give you anything real or difficult, so here is this apple problem. The apple problems were repetitive. Apple pie, apples, I don't like apples anymore.

TS: So you like diversity.

Madisun: Yeah

TS: What else?

Madisun: When I was a little kid apples were fine. But as you get older you want to see it in different ways. So instead of seeing it repetitively, with the same problem over and over, you want to see different applications of the same [math] problem.

Madisun did not feel that apple problems were engaging. Note that one reason Madisun did not feel apple problems were engaging is because those problems were repeated often. This resulted in her eventual dislike of apple problems. Madisun wanted to see different things, new applications. It is important to her that it is not the same problem over and over again, even if it is engaging at first.

One important point to make here is that Madisun was surprised that the concept of derivative would come up in her Chemistry class. She seemed quite surprised that one could apply the derivative to solving a Chemistry problem. She was excited about this. She seemed so intrigued by the idea that derivatives could actually help people to know how quickly oil was leaking from a pipe, and therefore how much oil they would need to clean up in a spill. This was definitely a new idea for her. Madisun used this idea of "variety" to describe something that students have not seen before. She did not want to see the same problem over and over again. Madisun emphasized that she wanted to see different applications of math.

As a final example of the importance of this new characteristic, supporting its placement in my framework alongside the other characteristics, consider Caleb's experience taking a Statistics class abroad in India. Note the importance of variety within the discussion.

TS: What advice do you have for teachers who are trying to write problems that students view as worthwhile?

Caleb: We did stats in our Calc class in India. We learned about normal distributions. And how most things in life follow that pattern. So then we started talking about what follows that pattern: height, etc. Then we got into worthwhile world application problems. I cannot remember. But it was directly relatable. One was a study in the US. They were trying to fine tune a medicine. For some reason it followed a normal distribution. How much chemical do we put in to do the most help. You are saving lives right there. We talked about all the ways you can calculate probabilities and make inferences. That sparked me to want to do statistics. That made it seem more worthwhile. I could picture all of these situations in real life. With companies, health care, all kinds of things that made it feel worthwhile.

Note that it was the many applications of probabilities and making inferences that sparked in him a desire to study statistics. His teacher's ability to expose Caleb to a range of different problems including companies, health care, and other situations in real life "made it seem more worthwhile." Notice how often Caleb used the word all, "all of these situations", "all the ways", and "all kinds of things." It was the variety of contexts that engaged Caleb in the mathematics.

Having laid out the case that variety deserves to be a new characteristic in my framework, I now return to the other characteristics in my framework: realistic, worthwhile, enjoyable, and motivating. I describe aspects of problems that helped them have one (or more) of these characteristics.

Realistic/Worthwhile

Students' problem examples under worthwhile were all realistic. Only two students gave example problems under realistic that were not worthwhile, showing that while this can happen, these two characteristics are tightly connected. As students discussed the characteristics worthwhile or realistic, they almost always discussed the same aspects. Therefore, I combined these characteristics of engaging contexts into one characteristic: realistic/worthwhile. Students discussed three aspects of realistic/worthwhile problems: relate it to everyday life/ potential careers, authenticity and explicit purpose. All three were needed in conjunction with one another for students to find the problem realistic/worthwhile.

Relate it to everyday life/ potential careers. Research states that students should see realistic content that relates to their everyday lives (Bonotto, 2005; Van Den Heuvel-Panhuizen, 2003). Students within this study echoed this sentiment. For example, consider the problems Sarah and Rebekah said were the most realistic problems they had seen in a classroom. Sarah used a "compound interest" example. She stated that because she wanted to save money in the future, she felt this type of problem was enabling. Rebekah used a problem having to do with buying a car. She felt this problem helped her to better understand how the payment process worked and what to look for. Consider these statements from Grant and Savannah. Grant stated, "Physics was engaging because we were always dealing with real world problems. It is cool because the problems have to do with what we experience in life." Savanna said realistic contexts included "experiences I have had in my life." This theme was strongly represented in the data. I have kept this sub-theme short because prior research states this information. Many students are engaged by mathematics that relates to their everyday lives.

However, in this study students talked about wanting to see more than just problems that related to their everyday lives. They wanted teachers to broaden their horizons by showing ways people might use mathematics within career fields. Therefore, I focus my thoughts on contexts that relate to different career fields. Consider the problem Madisun said was the most realistic problem she had seen in a classroom. Madisun used the "Oil Spill" problem. She talked about using related rates in Calculus to find how much oil had been spilt in the ocean. She was excited that mathematics could answer that question. This is a problem context that relates to a potential career, not everyday life experiences.

Students brought up "ideas within statistics" as being very realistic. Consider this statement from Dan. "Statistical problems and economics problems are the most realistic in math class." The ideas of statistics and economics problems being realistic was brought up by five students. These are not ideas that necessarily relate to students' everyday lives. Students are pushing past this idea, wanting to see mathematics within different career fields.

Doug took it one step further, not even wanting to see problems that related to his everyday life. Consider his statement.

> Doug: I think that a goal driven person, someone who is thinking about the future is going to be engaged in a math problem that is something they are interested in doing in the future. Something that is applicable to their field of work or something they are going to be doing in the future. Because when we are kids going through grade school to high school we hang out with our friends and we do sports. So there are not a lot of real world problems you can give to students unless they have a futuristic approach.

Doug did not think problems dealing with his everyday life, which included hanging out with his friends and playing sports, were realistic. He wanted to see problems dealing with potential career ideas.

Caleb also felt potential career ideas were important in posing problems students would likely find realistic. Caleb suggested teachers use real world data. Caleb confirmed what research says about using real world data and relating math to personal experiences. But, he pushed past this idea to look at potential career ideas.

TS: What advice do you have for teachers who are striving to pose problems that students will view as realistic?

Caleb: … Using real data sets is super realistic. These are real numbers. Economics class, looked at CIA data book. That was more realistic. When you are in class doing statistics and trying to solve some sort of poverty issue in Losoto, like it is a bit of a stretch to put yourself in the shoes and maybe there are other things that are at play, but you know it is actually happening and that is what realistic means. So experience, or current event, or cultural issue, social media – realistic because students are on that everyday. They have seen it, or you have, or you know others who have been in the situation, or general knowledge, data sets- that is realistic.

Caleb readily admitted that solving a poverty issue in Losoto was the kind of problem that could be difficult for students to relate to. However, the problem was still very realistic to him because "it is actually happening." Where is it happening? Statisticians collect data that enables professionals to work on helping to understand poverty issues in Losoto and around the world. Caleb mentioned looking at the CIA data book in Economics class. Again, this idea would be difficult to relate to students everyday lives, but it does relate to potential career ideas. Economists are looking at data books to examine economic issues. These ideas were realistic to Caleb because people are using them to solve real world problems within their career fields. Caleb wanted his horizons broadened. He wanted to better understand how, when, and where others use the mathematics he was learning.

Next, Doug had ideas about how simply stating that math is related to careers is insufficient. I asked him if he had seen a problem that was meant to be realistic but did not feel realistic to him. He began by saying "all the time", and mentioned a couple of times this had happened, and then switched gears to talk about missed opportunities. Opportunities teachers had to introduce realistic contexts, but did not. He did this by considering posters that line the walls in his math classes. Posters that described the many different ways mathematics is used and the career fields that use it. Consider his thoughts.

TS: Have you ever seen a math problem that was meant to be realistic but did not feel realistic to you?

Doug: …I have seen that in my math classrooms. The classrooms had these posters hanging up that said "where do you use this math?" to try to help motivate students. So students are able to understand why they are learning a particular math topic. They had signs hanging up that said "you use this in this field, you use that is this field" But that did not translate into math problems we did, which definitely would engage the students more.

Notice Doug's concern. He was frustrated that while posters hang describing where math topics apply in various career fields, this was not translating into what is being demonstrated in the classroom. He could have learned about where different career fields are applying math through problems his teachers posed to him, but he did not. If these kinds of problems were posed, he stated that this "would engage the students more."

Students likewise described worthwhile problems as problems that gave them power to know how to apply a mathematical understanding to a future situation. Most of the example problem contexts in this section had to do with future career ideas. Nine students were able to give specific problems they felt were worthwhile. Of the nine problems, five problems had to do with future career ideas.

Consider Rachel's thoughts when asked about an example of a worthwhile math problem and advice in making math problems worthwhile. Note the potential career ideas.

Rachel: There was one that we did, maximizing the profit. What would be the best price for the customer and the business? That makes a positive difference for economic reasons. It's what is best for the customer and the businesses. That is worthwhile for me.…It can make such a great difference for your students to see problems that could relate to their future. Also, use things that are going on right now. If there are current events that could align with mathematical problems I think that would be very interesting, engaging, and worthwhile.

Notice her very explicit statement in reference to a potential career idea, "It can make such a great difference for your students to see problems that could relate to their future." This kind of problem was very worthwhile to Rachel. Note that Rachel also mentioned current events. Current events having to do with how people are applying mathematics in career situations can be an excellent way of giving students realistic/worthwhile scenarios. While current events are happening now, they give students a better understanding of how people are currently applying mathematics which can enable students to have a better understanding of how it might be applied in the future. A good example of this is "the oil spill" problem that Madisun referred to previously. Oil spills do happen. We see this in the news. In this situation, engineers would need to figure out how much oil was spilling in order to know what man-power and equipment needed for clean-up.

In summary, the students talked about two different kinds of problems that engaged them: problems that relate to their everyday lives and problems that relate to potential career ideas. Both ideas were important to most students. I focused this section on potential career ideas because of the previous research on relating mathematics to students everyday lives (Bonotto, 2005; Van Den Heuvel-Panhuizen, 2003). Students want problems that broaden their horizons by informing them of different ways mathematics is being applied in career fields.

Authentic. Recall that research states students should see realistic scenarios in math classrooms (Redish, 2005; Van den Heuvel-Panhuizen & Drijvers, 2014; Gravemeijer, 2004). My framework includes the characteristic realistic. A realistic application problem in this research is defined as one in which a student can think: I could see someone (myself or others) having the need and taking the time to solve this problem outside of the classroom. The students in my study confirmed this idea.

However, the students in this study suggested that for a realistic problem to actually be engaging, they need to believe that someone may have a need to solve the scenario *and* believe that the math used in the problem was necessary. I call such problems "authentic," which I define as a problem that students believe could come up in the real world and that really needs the math used to solve it. Consider this next statement from Dan, who was asked if he had ever seen a math problem that was meant to be realistic but did not feel realistic to him. He brought up a physics class example. He was given a problem where the velocity and force on an object were given and he had to calculate the mass. He stated, "I cannot visualize that scenario. It would be so easy to get the mass. That is annoying because it is not realistic. Don't construct these kinds of problems." Dan was frustrated because it is usually so easy to measure the mass of an object. Thus, despite being a, literally, "realistic" problem, he did not perceive the math as necessary to solve it. It was not authentic to him to find the mass in this way. The easiest way would be to take the object and measure the mass. He discussed the importance of authenticity. He later stated, "Problems try too hard to feel realistic…It feels manufactured when they are trying to pull math out of something that does not feel like it needs math to solve it. It becomes quickly unrealistic when a student can see this is unnecessary." Dan was not engaged with this problem because the problem did not feel authentic to him.

Couching a problem inside a real-world activity does not necessarily make it authentic. While it may be a real world problem, and need the math used to solve it, it also needs to be a believable situation to students. Consider this example from Doug. Doug was asked about unrealistic problem contexts.

TS: Have you ever seen a math problem that was meant to be realistic but did not feel realistic to you?

Doug: All the time. I would say, this is stupid. No one does this. Sports problems, analyzing the arch of your shot, the quickness of movement. When I go to do these activities, I don't do math. I don't see math in my day to day activities. It has to be, "When I go to do this activity, I am going to actually have to use math!"

Sporting activities are certainly in the real world, like the arch of one's shot, or the quickness of movement. Yet, this problem still did not feel engaging to Doug, because he did not feel the situation was believable, making it inauthentic. He was concerned that in an attempt to relate mathematics to students lives, teachers often pose problems that are not credible. Doug wanted to feel that the math he was using really was necessary to solve a problem that was important. He didn't do math when he played sports and these kinds of problems felt unauthentic to him. He did not feel it was necessary to understand the math behind these ideas and it kept him from being engaged. Therefore, these problems felt unauthentic to Doug because they failed the definition of authenticity. Doug did not think that these problems would come up in the real world. "No one does this."

Note this positive example of an authentic problem given by Sarah. She expressed concern that ofttimes in the classroom students are unsure about whether the math they are learning would ever be used in a real situation. In this example that she gave, she felt confident that the math she was learning would be needed and that this was an important context.

TS: Can you give an example of a realistic problem that you have seen in a math class?

Sarah: In Algebra II we learned about compound interest. It's been a while since I've used it, but I remember I felt like 'ok this is something important because if I want to save money in the future and know what the best way to go about that is, I would actually use this'.

TS: What specifically made that realistic to you?

Sarah: I would use it. People use it. My parents have used it and that is something that you see.

Notice that Sarah felt this was authentic because she would use it, she was aware of others who had used it, and it was "something that you see." Sarah was sure that this is a problem that could come up in the real world. Also, compound interest is needed in order to understand how to save money in the future. If Sarah wanted to figure out how much she needed to save in order to reach a certain goal, and she wanted to include interest paid, she would have to understand how to compute compound interest. This problem was authentic to Sarah because she believed it would be used in the real world and the math she had been taught was necessary to solve the problem.

In summary, students brought up the idea of needing the mathematics to solve the problem of a believable situation, which I call authenticity. Therefore, when attempting to pose problems that students will likely view as realistic/worthwhile, teachers should strive to pose problems that students find authentic.

Explicit purpose. Recall that past research states that students should see worthwhile problems in their classrooms (NCTM, 2014; Brown et al., 2011; Ejiwale, 2013; Cotabish et al., 2013). Students in this study confirmed this idea. Madisun discussed a gold mine problem that was worthwhile to her because "Someone would need to know when to stop digging, when it was not profitable anymore." Knowing when a business venture is going to slow down and not be profitable anymore would make a positive difference in a business owner's life. Many students referenced the Humanitarian Aid problem from the interview. Students felt the problem was engaging because solving such a problem could make a positive difference to people who were without water.

However, students in this study contributed to this characteristic by adding the importance of making that positive difference explicit in the problem. As teachers look at problems to pose to their students, it may be easy to see a problem as making a positive difference, but do our students view it as such? How can teachers be more confident in choosing problems that students are more likely to find purposeful, and therefore more engaging? In this study, students stressed the importance of feeling that the problem they did had an important purpose and that the purpose was made explicit. Consider this statement from Doug. Doug stressed the importance of the purpose of the problem when he said, "Giving students the purpose of the problem is really important."

A purposeful, explicit reason builds on the characteristic authentic. Authenticity is necessary in order for a student to find a problem purposeful. Therefore, it is assumed that the problems are authentic, and I describe the importance from the point of view of the students of needing to make the purposeful reason explicit.

To help flesh out this problem feature, consider Jeff's confirmation of the importance of a good reason being given in this example. Again, prompted by the question, "Can you give an example of a problem that you have seen in a math class that was meant to be realistic but felt unrealistic to you?" Jeff responded,

Jeff: Marbles in a bag. I pull out this many. Why? Is he doing this just for the heck of it? It may be realistic, but it is not engaging to anybody because there is no reason to be doing it. There is no reason given for him putting them in the bag. There is no reason given that he pulled a certain amount out.

One could reason that part of the problem with this example is that it is unrealistic or inauthentic. However, marbles do constitute a real-world context, and the mathematics is likely needed to solve some probability situations. Notice that Jeff is frustrated because no reason for why one might be pulling marbles out of a bag is given. He states that this may be a realistic problem, but he does not feel this could be engaging for anyone. Note his words, "Why?", "there is no reason given", and "Is he doing this just for the heck of it?" Jeff's main irritation lies with the fact that no explicit purpose is given for doing the problem. In Jeff's opinion, this is a poor context for teaching the math topic because no reason is given to be doing it.

To show an example of a problem that does have an explicit purpose, Rachel discussed what she considered to be the most engaging problem she had seen in a math classroom.

TS: What is the most engaging math problem you have ever seen in a math classroom.

Rachel: The one that I can remember was in high school. It was a historically based problem. It had to do with the Berlin Airlift, when they flew in supplies to Berlin. It was something about the speed at which they could drop them and where they would land. To me that was so engaging. This actually happened. People had to figure out where the supplies would fall. I think it was so engaging to me because of that story and the historical emphasis. Historical emphasis you don't really see in math, so that was just different. I remember it was a really cool problem because I had not seen anything like that before. It was so different from other problems.

TS: What made it different?

Rachel: It was more of a story. I could really get interested in that. It wasn't just the numbers. It had a background story. It had the real life connection, it actually happened in history and they actually had to do the math to solve this. It was this positive outlook. They calculated it so that they would not hurt people and the drop would be most effective. I didn't even like the mathematics at the time but it was really cool and engaging.

Note that Rachel felt an explicit purpose had been given for solving this problem. The supplies needed to be dropped in an area and at a speed that would not do damage or hurt anyone, and would be accessible for the people in need. Therefore this problem gave an explicit purpose for Rachel because clear, articulated, valued evidence was given as to why it was important to find a solution to the problem. Note that Rachel did not even like the math she was doing at the time, but that this problem was "really cool and engaging" for her. An explicit purpose helped Rachel to engage with a problem despite a dislike of the mathematics. Rachel was empowered to better understand how mathematics made a positive difference for the people of Berlin. Concerning this problem, Rachel stated, "Having that Berlin problem is so interesting because you think, 'oh, math really is in all walks of life. Like even in the random piece of history people were having to do math.'"

Realistic/worthwhile conclusion. It is important to note that these ideas of potential career/ relate to everyday life, authenticity and explicit purpose were largely discussed in conjunction with making a problem more realistic/worthwhile. Thus, I consider these to be aspects of problems that can help them be realistic/worthwhile. For example, Rachel's Berlin example engaged her through a historically authentic problem. Career professionals had needed to solve this problem in the past. As previously argued, an explicit purpose had been given. Rachel was engaged because she was given a context that she found authentic and had given an explicit purpose. Caleb discussed a problem in his statistics class about a study done on fine-tuning a medicine. The problem followed a normal distribution and statisticians calculated the needed amount of a chemical for doing the most good. He stated, “You are saving lives right there.” Note that the problem is embedded in a potential career idea. This problem is authentic because it was an actual study and did require the statistics used to solve it. This problem gave an explicit purpose for Caleb, namely that lives were saved. Caleb was engaged because he was given a potential career context that he found authentic and had given an explicit purpose. Of the nineteen example problems that students gave under the characteristics realistic or worthwhile, fifteen contexts included authenticity, an explicit purpose, and were found in a potential career/related to my everyday life context.

Enjoyable

Next I describe aspects of problems that helped them to be enjoyable. Recall that in my framework, an enjoyable problem was defined as one that is likable or fun and that invokes positive feelings towards the problem. The data confirmed these findings. Students talked about seeing problems that were fun and how that helped them engage more with the math. In this study, I was able to contribute to this area by identifying two possible traits a problem might have that can help it feel enjoyable to students, which I describe in this section: hands-on activity and a good story.

Hands-on activity. Students felt problems were more enjoyable when they included hands-on activities. Note that while part of hands-on activities may simply deal with how an instructor carries out an activity, it is true that some problem contexts seemed to lend themselves better to this than others, meaning the context itself may have a role. Here I describe students’ input on how the context of a hands-on problem helped them to see it as more enjoyable. Contexts that permitted hands-on activity was cited by the students as more enjoyable. It appeared that certain contexts might have a stronger hands-on nature than others. For example, when asked about advice for creating problems that students will enjoy more, Rebekah explained the following.

> Make it more hands on. My mom home-schooled by brother. She did barbie bungie jumping with him. Needed the right number of rubber bands to keep Barbie from dying. It was hands-on because my mom and brother actually did this. It was fun and exciting because it is different. It is not sitting down with a pencil and writing it out. You are doing the math and then looking to see if it actually works. If I did not do the math correct, the experiment will not work. Doing a problem and then seeing whether it works or not.

Notice that the context of this problem is about keeping Barbie from dying while bungie jumping. Rebekah's brother would need to find out how many rubber bands would be needed to keep Barbie safe while bungie jumping. This context lends itself to being hands-on because the students have trial runs with objects similar in size and shape to Barbie and then see who is able to keep Barbie safe. Rebekah thought that as teachers used hands-on activities that math problems would be more enjoyable. This problem was enjoyable because they used Barbie and bungie jumping, a hands-on activity. Trey echoed this sentiment when asked for advice on making problem contexts more enjoyable. He explained,

> Trey: If I can make a better catapult with math, then I want to do the math. Or make cars that are the fastest. How many wheels, how heavy, how wide to make it the fastest. Students are engaged. Here I am taking fun activities and looking at the math needed to improve it.

Notice that, again, these contexts lend themselves to being hands-on. Making the most effective catapult, or cars that are the fastest, make it easy to have hands on activities. Building, creating, working with his hands and using math to make his creations better is enjoyable for him.

Grant talked about an Algebra project that his class did over the course of the semester. It had to do with creating blueprints for building a home, putting in carpet, a fence, calculating the amount of materials needed. Over the course of the semester as students learned the math necessary to do parts of the project, they applied their new-found knowledge. “He [the teacher] pointed out at the beginning that none of us could solve this problem. This project helped us to see how much we had learned that could apply to this real world problem. That was very, very cool.” By the end of the project, Grant and his classmates had blueprints for building a home. This problem context lent itself to a hands-on activity that engaged Grant. Again, note that while the instructor implementation may be a factor, the context of home blueprints, and then creating the blueprints, helped the problem be enjoyable.

A good story. Eight out of eleven students in this study brought up the idea of a good story as being enjoyable. Consider these statements from Rachel and Rebekah. When Rachel was asked how to pose problem contexts that students would find more enjoyable, she responded, “Spice it up with other things, put a new spin on something. So I think it is more enjoyable when we incorporate other things like science, literature. Give it a story.” When Rebekah was asked about engaging contexts, she responded that contexts were more engaging when a teacher draws her “into a story using different math problems. Build a scenario.” Students gave examples of two different types of stories that were enjoyable for them: teacher’s personal experience and absurd stories. Students seemed especially engaged when the experience came from the life of their teacher. In this section, I begin by sharing an example of a good story shared by Caleb. This gives the reader a general idea of a good story. Then I share students' examples of teacher's personal experiences and absurd stories.

Caleb had been asked about the most engaging problem he had ever seen. He said this particular problem inspired him to pursue a major in statistics so it had a strong impact on him.

TS: What is the most engaging problem you have ever seen in a math classroom and what made it engaging for you. It does not have to be an application problem, any problem. Take your time to think about it.

Caleb: A probability problem, the Movie Line Problem. The question is "What is the likelihood that someone in the line of 40 people shares your birthday?" It was interesting, funny, kind of realistic but kind of out there. The novelty made it engaging, it was something you had never seen before. It took you by surprise. That is something that you could never intuitively find. What is the likelihood that someone shares my birthday would be super hard to figure out intuitively. Probability actually solved that. That is what it is made to do. That definitely was super engaging. The answer surprised me, because it is pretty likely. We did end up doing it in class on the first day. We did it in class and the question was asked "How quickly before someone says your birthday (or there is a repeat)" It was the third person who spoke who had someone in the class with their birthday.

Notice that Caleb used the word "funny", "interesting" and "kind of out there" alluding to the idea that this was a good story context for him. An element of fun - namely surprise and novelty - within the storyline helped Caleb to engage in the problem. Overall, this was definitely an enjoyable context.

Teacher's personal experience. Students reported that they enjoyed hearing teacher's personal experience. For example, Caleb described a funny story his math teacher had shared from his own life.

> Caleb: If a teacher can bring in something from their own life. Just here at [names university] in Calculus my teacher was talking about a potato gun, shooting the potato gun and rates of change. Where does it peak?
>
> He built one with his son. His son was trying to figure out where the potato would land. The son wanted to shoot the potato gun over a gazebo and into a river to make a splash while someone in the family was proposing and he was trying to figure that out. But when he did it, the potato hit the gazebo, exploded and ruined the moment. That was funny. Whenever they give examples from their life it usually ends up being funny and relatable.

Notice that Caleb believes these kinds of personal experiences can end up being funny and relatable. Students brought up this idea that personal experiences of teachers are enjoyable to hear and engage with. When Rachel was asked to give an example of an enjoyable problem, she answered with a personal story from her teacher.

> Rachel: In my Calculus lab we do labs and my Calculus teacher gave us this problem about his son who was trying to make the lantern from Tangled. He needed to figure out how to maximize the surface area and try to make it float. Even though I don't like Calculus, I thought "this is a really interesting problem. I kind of enjoy this."

Rachel was interested and enjoyed a problem her teacher brought in from his own life experience. The story about his experience in using calculus outside of the classroom interested Rachel and helped her to enjoy the problem.

Absurdity. Students claimed that problems that were kind of "out there" (quote from Savannah) were enjoyable. Caleb stated that sometimes you want to pose a problem that "isn't related at all to the math classroom." In particular, it appeared that students found an occasional absurd problem funny and enjoyable. First, consider this dialogue with Dan. I asked Dan for advice for creating problems that students would find motivating.

> Dan: In 8th grade we had this problem about finding ratios and bike tires. It was an absurd problem. The diameter of the tire was massive. The problem became silly and that made it motivating for me. In fact everyone wanted to do the problem because it was so unrealistic. Being way out there became funny. Funny problems are the most enjoyable and motivating.... It is so out there that it makes it fun to do. It makes you want to keep going when it is more absurd. However, [absurdity] should not be overused. Variety is important. I need some realistic problems. But occasionally putting in ridiculous problems is funny.

Note Dan's word usage here: "like", "feel", "fun", "cool", "silly", "funny." While the discussion is about intellectual stimulation (motivation), Dan talks about emotional stimulation, or enjoyment. Dan was engaged by the silly, absurd problems. Other students brought up this idea, echoing the sentiment that sometimes it is enjoyable to see problems that are ridiculous and out there.

Note Dan's caution here though, that such problems should not be overused. While this problem was motivating and enjoyable for Dan, he did not always want to see these kinds of problems. Therefore, it may be important that teachers use this type of problem sparingly.

Second, consider this example from Caleb.

TS: Let's begin with motivating contexts. Motivating contexts peak your interest intellectually. You would like to know the answer, how to solve the problem. Can you think of a problem you have seen in a math class that motivated you?

Caleb: Sure, yeah, so I remember one. It was a related rates problem… There was a street light and you were walking down the street and there was some sort of a criminal behind you trying to sneak up on you. It was all in 2D, you were on the sidewalk and they were on the sidewalk. You don't know that the criminal is behind you but at some point the criminal's shadow is going to cast into your view, right. So you are trying to see first, at what point the shadow will be in view. Then, once it is in view, how fast will you need to run to be able to escape him? Knowing there is only going to be one spot when the shadow will cross into your view. If we can find that spot then we can figure out, ok, how fast do you need to run to get away?

Caleb had been shown other contexts of related rates problems but this was the first context that engaged him. Notice that this problem is not authentic, nor does it have a purposeful, explicit reason for solving it. This problem has an absurd context. A criminal is walking underneath the basking light of a street lamp that gives him a shadow. At some point in time that shadow will be in your view and you will need to run fast. It was a good story that helped Caleb to engage more with related rates. Absurd stories contributed to students enjoyment of contexts and being engaged.

Enjoyable conclusion. To summarize, problems that had teachers' personal stories or a fun, absurd story, as well as context that lent itself to being hands-on, helped a problem context have the characteristic of being "enjoyable." These kinds of problems could help students to be more engaged with a context.

Motivating

In discussions of motivating contexts, the majority of students essentially reverted back to realistic and enjoyable contexts. No additional characteristics or problem features emerged. For example, when Madisun was asked about motivating problems, she gave the example: “We calculated how much carbon monoxide was in your cell. You want to know it so that you will not die.” This is a realistic problem context that was motivating for Madisun. Additionally, two of the “absurd stories” from the previous section evolved out of the students’ conversations after being asked about motivating contexts. Their explanations clearly showed that these two examples did belong under the characteristic enjoyable as well. Therefore, motivation could result from an enjoyable or realistic/worthwhile problem context. My study did not show that motivating contexts existed independent of being enjoyable or realistic/worthwhile, as my framework suggested.

However, there was one noteworthy new idea that came up during the discussion of motivating contexts that I wish to describe, even though it was not shared across the students. Rachel discussed a problem context that was motivating for her. In her class, her teacher sought to explain why Calculus was created in the first place.

> Rachel: Going off of the phrase ‘intellectually interesting’, my calculus teacher when he was introducing limits and derivatives he read from the book about Isaac Newton and the other founder whose name I cannot remember. He talked about the history and what was happening in their world that drove them to want to understand this. And to me that was interesting. This is where they started, this is what they were trying to figure out. And now we are here. I thought that was more motivating for me to try to answer future questions. It was more interesting to read or just to hear a little bit about the history about what they were trying to understand and what questions they were

trying to answer. It was just a small section from a book but he read about what they were trying to understand and questions they were trying to solve. It was just very interesting to me.

TS: So what made that interesting for you?

Rachel: We don't really talk about the historical basis of math, so it was different for me that we even talked about it. And second, it is interesting to think about what caused this math to come about. A lot of times in class we just do the math. It is interesting to think about why this math is here. Why did someone think about this math. How did we get this? How do limits come about. Good basis for understanding why it is here, how it started. Very interesting.

What made the problem context motivating to Rachel was her teacher's use of *historical development* of a mathematical idea. He reviewed what was going on in the world at that time that drove Newton and Leibniz to work on these concepts. This example is a description of what mathematicians were struggling to understand when Calculus was founded and why it was important to them. This was a motivating problem for Rachel.

Overall Engaging Problem

At the end of each interview, students were asked to talk about the most engaging problem they had ever seen. Students were informed that the problem did not have to be an application problem, just a very engaging problem, if possible the most engaging one they had ever seen. Not all students were able to come up with such a problem. Just over half of them described what a very engaging problem might look like, but were unable to provide a specific example. When observing the themes that students brought up for these contexts being engaging, all of the reasons fell under these characteristics: variety, realistic/worthwhile and enjoyable. In this way, the characteristics from the framework, with the addition of "variety" from this study, seemed to capture well what would engage students. To conclude my results from part two of the interview, I wish to share one of these "overall engaging" problems because it does a nice job of capturing the characteristics of variety, realistic/worthwhile, and enjoyable. In this example I hope to (a) illustrate that an engaging problem can have multiple characteristics (though it does not have to), and (b) give educators ideas about what such problems might look like.

In this example Sarah was asked about the most engaging problem she had ever seen. She actually brought up this specific problem twice during the interview (while discussing enjoyable and while discussing overall engaging), suggesting that it really was a very engaging problem to her.

Sarah: I think the most engaging problem, and the whole class was engaged, was when we were in Algebra II and we were finding out the time of death. I don't know if it was because we were sophomores in high school and less mature. But it was fascinating, and we all guessed what time it would be so we were all engaged with that. And then it was like, they pulled us in. They made it interesting by asking, "What do you think?" And then she showed us through the math to find it and then asked, "Ok, let's see who is closest?"

TS: Why were you pulled in? How did the teacher pull you in.

Sarah: I think she told us a story about this dead guy, told us his temperature and asked how we could solve the mystery of the time of death. So it made it more, it was entertaining because it was more of a storyline. And we got to guess, our opinion mattered. Even if you were super wrong, it was fun. And then she showed the math, we did the problem, and we knew how to do it. That was interesting to see the math behind that.

First, Sarah thought this problem was enjoyable. Sarah brought this problem up as an example of an enjoyable problem that she had seen as well as the most engaging problem she had seen. Sarah was pulled in by the problem because of a good story, “she told us a story about this dead guy.”

Second, this problem was realistic/worthwhile for Sarah. This problem does relate to a potential career idea as certain types of investigators might use mathematics to help solve for the time of death. Therefore, it is an authentic problem as it does come up in the real world and requires the math used to solve it. Also, a purposeful, explicit reason can be given. For example, in order to catch criminals it is important to have an idea of when a person died. That way investigators are able to know what time people should have alibies for.

Third, earlier in the interview, within the dialect about why it was an enjoyable problem Sarah stated “I didn’t know math could do this.” Clearly she had been enabled to apply mathematics in a way that she did not previously know. This suggests that the problem had the characteristic of variety, as well. It was a new problem context for Sarah.

Overall, this problem exemplified the characteristics of variety, realistic/worthwhile, and enjoyable. The problem context included these characteristics because it was applicable to a potential career, authentic, gave an explicit purpose, had a good story and was a new or different idea.

Part I of Interview

In this section I attempt to answer my first research question: “When comparing problems meant to introduce a mathematics topic, what reasons do students give on their own for finding one problem context more engaging than another?” It turned out that the results generated from part two of the interview did an excellent job of capturing the reasons students found certain problems more engaging than others. All of the characteristics (realistic/worthwhile, enjoyable, and variety) came up, as well as the features of everyday lives/potential careers, authenticity, explicit purpose, and personal/absurd story. There was one exception: hands-on activities. This idea did not come up. Perhaps because students were not given a context that lent itself to a hands-on activity, or perhaps because we did not include a hands-on activity in my study. However, a different reason was given: interactivity. This idea is described within my discussion of enjoyable contexts.

Having said that, it is also true that the majority of the reasons students gave for finding one problem context more engaging than another fell under the characteristic realistic/worthwhile. This is likely due to the types of problems I picked, as they all were based on real-world situations. While there was some effort to create problems that might also be seen as enjoyable, definitions of enjoyable from the literature were too vague, making it difficult to know what might make problems be enjoyable. Similarly, since variety was not yet conceived of as a characteristic going into the study, I did not intentionally use this characteristic, though it turned out that variety still showed up often in the students' responses.

In the following subsections I go through each characteristic to show how it was used by the students in their reasons for why certain problems were more engaging than others. As I do so, I also draw on the aspects of engaging contexts found in the previous results section concerning what helped problems have these characteristics.

Variety

This characteristic was cited frequently by students during part one of the interview. For example, it came up in the first interview I did, with Sarah. Sarah was asked to put all six problems in order from most engaging to least engaging. She ordered them in this way: Home-Add On, Humanitarian Aid, Comet, Farmer, River, and Parabola. She described to me why she ordered them in this way. Consider her reasoning for why the parabola problem was the least engaging problem.

TS: The Parabola problem is last. Why is that?

Sarah: It was just like an everyday math problem. Seeing the math book. It wasn't jumping out at me 'oh this would be fun to do'. It wasn't a good challenge. There wasn't anything special about it. It was just something you would see in a textbook. Or your math homework or on a review sheet. Something like that.

TS: So for you an everyday math problem is not engaging. Is that fair for me to say?

Sarah: Yeah, that's all you do. Everyday from kindergarten all the way to college. It's like standardized math problems everyday. Nothing jumps out at you.

Notice the concern with the repetition for Sarah in words like, "everyday math problem", "that's all you do" and "Nothing jumps out at you." These "everyday" problems were not engaging for Sarah. Sarah repeated this idea throughout the interview process. Then other students brought it up as a major factor in the engagement of a problem. By interview three I knew it would have to be a theme in the research because of how often it had been brought up.

"Variety" within problem contexts is something new, something students have not seen over and over again. Consider Doug's response after being shown and asked about the Home/Farmer problems. He identified that the house problem as more engaging and discussed why. This dialogue takes place after his thoughts on why the house problem was more engaging.

TS: Why was the farmer problem less engaging for you?

Doug: Farmer Problem [sic]. It is similar, both have purpose, both are trying to find area for something that will benefit them in the scenario. I have solved a lot of fence problems and I did not like those problems. I have seen a lot of the fencing problems.

TS: So was it the repetition? Because you had done it over and over? Anything else about it? Is there another reason why the fence problem is not engaging for you? If you had only seen a couple, do you think you still would not have been engaged? Was the repetition not engaging or the context of a "fence problem" not engaging?

Doug: When you first start solving story problems with fencing that was more engaging, but after a while of doing a lot of the same problems it makes it so the math is not as fun. Because you are like, "oh, same scenario. Why am I doing this again?"

Notice that the farmer problem was less engaging for Doug because he had seen a lot of fence problems. Lack of a variety within the problem context made it less engaging. Notice also that the interviewer tried to question whether anything else was contributing to Doug being less engaged with the fence problem. Doug's answer reflected that the real issue was the repetition. Repetition had made the problem context less fun. Repetition caused Doug to question the purpose of the problem. Doug found it difficult to engage in a problem context that he had seen over and over again. Problems that were repeated over and over were less engaging for Doug.

Other students brought up this idea as well. Consider these statements from Savannah, Sarah and Dan. Savannah brought the idea of variety up in her discussion of the Humanitarian Aid problem. "I like something different, something out of the norm." Sarah brought up this idea of variety as well. When asked if she would like to see more problems like the ones she found engaging, she answered that as she sees problems over and over again (no matter how engaging she finds them initially) they become less engaging. Dan chose the home problem over the farmer problem because, "I like the new aspect." He had not seen a problem like that previously.

In conclusion, as students discussed what made one problem context more engaging than another, variety was a factor. Students were more engaged with different kinds of problems, problems that were out of the norm. Students were less engaged with problems that were repeated over and over again.

Realistic/Worthwhile

Recall that realistic/worthwhile mathematics is: authentic, purposeful, explicit math that relates to an everyday life or potential career situation. Again, students brought these ideas up as they discussed what made one problem context more engaging than another.

Everyday life. The idea of relating mathematics to one's everyday life situation came up a great deal, especially during the discussion of the home/farmer problems. Consider these statements from Trey, Rebekah and Rachel. Trey stated, "I do not relate to the farm as well as I do to a home. We all know how a home works, not how a farm works." Rebekah said, "I can relate that [the home problem] to my personal life more than a farmer. I am not a farmer. Lots of people add on to their houses or build a house." Rachel echoed their sentiments. "Me personally, I've never been on a farm. I do not have previous experience with a farm. I actually live in a home and can think about that." Consider the dialogue with Sarah after seeing the home and farmer problems.TS: Which one of these problems is more engaging?

Sarah: I like the house one because my family has personally done that. It is more relatable. More people are going to be doing that. So to me that would actually interest me anyway. Most of us are going to be homeowners someday. It is more relatable. I find that more engaging.

Note that Sarah found problem contexts that she related to more engaging. This is an everyday life context. She related well to the problem because her family had actually added on to their house. She could visualize others wanting to do so as well. "Most of us are going to be homeowners someday." Sarah related more to the context of a home than to the context of a farm. This made the home problem more engaging for her. Others felt similarly. Eight out of eleven students felt the home problem was more engaging than the farmer problem, two students were not sure, and one student felt the farmer problem was more engaging than the home. Students felt more engaged because they could relate to the home-add on scenario better than the farmer scenario.

Most students felt more engaged with the context of Humanitarian Aid than the River Problem (seven out of eleven, one student was unsure and three out of eleven thought the river context was more engaging). Of the three that were more engaged with the river problem, the idea of relatability was a factor for two of them. One of the students lived on a river and the other competed in river races. The context of speed and water was something that they related to. Consider this example from Rebekah.

> Rebekah: The river problem is the most engaging. That one is more relatable to me. I live on a river. A lot of people do triathlons and might want to know their speeds. The humanitarian aid problem is less relatable to me because I do not have to worry about water supply.

Notice that Rebekah related better to the context of the river problem because she lived on a river and because she did triathlons. The context of speed was more engaging to her than the context of a short water supply. Being able to relate to the problem made it more engaging for Rebekah. For Rebekah, this was an everyday problem that was engaging because of her experience growing up by a river and the topic of speed.

Potential career. The idea of potential careers came up a great deal, especially during the discussion of the humanitarian aid and river problems. In this section I discuss why the idea of a "potential career" made these problem contexts more engaging for students. Consider these statements from Doug, Grant and Rachel. Doug stated, "The Humanitarian Aid one is a scenario that a civil engineer might look at. They want to keep costs low…super interesting." Grant said, "It pulls me in, because I might use it in a job." Rachel felt similarly. Rachel did not hesitate to tell me that the

Humanitarian Aid problem was far more engaging for her than the river problem. She was excited about this problem. Consider this statement from her. “For this one it was a lot easier to decide because I personally am very interested in Humanitarian Aid. So when it has a story like this behind it I think, ‘Oh I could use this math in the future’. This could be a real world problem that I have with my job, or whatever I do in the future.” Notice that Rachel is very engaged with this potential career context. In the following example, Madisun echoes the previous students’ sentiments.

> Madisun: People actually as a job go out and see the most cost-efficient way to put in pipes…It’s something you could actually see yourself or people in the engineering department doing.

Note that Madisun found this problem context having to do with a potential career idea more engaging. She could visualize engineers or others using this scenario within their employment. This makes the Humanitarian Aid problem more engaging than the river problem for Madisun. Others felt similarly. For those who felt the Humanitarian Aid problem was more engaging, the idea of a potential career idea was instrumental (as well as a purposeful, explicit reason which I discuss in that section).

Some similar reasons were given for students finding the comet problem more engaging than the parabola problem. Consider these statements from Caleb and Dan. Caleb felt the comet problem was engaging because “There is an opportunity for you to place yourself inside the problem, either as a scientist or one of the people at home.” The potential career idea contributed to Caleb finding the Comet problem more engaging. Dan stated that the comet problem “Made me think about working at NASA, I could put myself in this position. I like the idea of getting to tell the public which day would be best to view the comet.” The comet problem provided Dan and Caleb with an opportunity to put themselves into a potential career position. This helped them to feel more engaged with the problem.

All eleven students felt the comet problem was more engaging than the parabola problem. The potential career idea contributed to their engagement.

Authentic. The idea of authenticity came up often during this part of the interview as well. The majority of students found the river problem unauthentic, and the humanitarian aid and comet problems very authentic. Doug felt that the river problem was unauthentic. “When I am going down a river I am not going to be looking at, okay if I go at this angle it gives me this speed to get to this point.” Doug felt the river problem was unauthentic because the scenario was not believable to him. He did not think people would calculate the best angle for speed in this scenario. Consider the response Grant gave to why the river problem was less engaging for him.

> Grant: If I am rowing, I don’t think I would ever take the time to calculate my speeds [sic]. I think the business prospect makes humanitarian aid win as far as engaging. I can totally see how this applies. Whereas I don’t think my rowing speed, that problem is not super realistic. For that reason I would go for Humanitarian aid. I like minimizing cost. I can see how that would be a real way to apply what you are learning.

Note Grant’s last statement, “I can see how that would be a ***real*** way to apply what you are learning.” Grant saw the Humanitarian Aid problem as a real way to apply what you are learning, an authentic way. The river problem did not feel authentic to him. He would never take the time to calculate his speed in this context.

Contrast their feelings about the river problem being unauthentic to these statements from Madisun, Grant, and Caleb about the Comet and Humanitarian Aid problems. Concerning the Comet problem Madisun stated, “It is interesting to note that you can figure out how close it is to the earth. The fact that you can do that makes me want to know how that works.” Madisun was excited that mathematics could solve this real world problem. She believed it could come up in the real world.

This was a genuine scenario for her and optimization is a great approach to solving the problem. Consider these comments from Grant and Caleb about the Humanitarian Aid problem. Grant stated, "Minimizing costs conceptually makes sense." Caleb echoed his thoughts. "So the idea of minimizing cost and being a humanitarian problem made it engaging. Of course you want to minimize cost." Note the wording here, "makes sense" and "Of course you want to minimize costs." These students believed they had been given an authentic scenario. This kind of problem could come up in the real world and optimization would help them to solve the problem. Consider this example from Trey. Trey discussed why the Humanitarian Aid problem was engaging for him.

> Trey: The Humanitarian Aid problem is a lot more interesting to think about. It is a problem I can actually think about caring about solving… I can think about the water going to the village, how do I get there cheaply with these obstacles? I can think of a scenario of having limited money to help others. I want to get water to as many as I can, I need to save money because that allows me to help more people.

Notice that Trey felt this problem was authentic. It was evident that he believed this was a scenario that could come up in the real world. Note that he often said, "I can think about…." He was able to visualize this scenario. It was genuine for him. Optimization will help students find that minimum cost of laying the pipe and is often a more efficient way of doing so than the Algebra method. It is not difficult to assume that Trey had been given a scenario in which he believed it was worth his time to work the problem and that he would be engaged. If we want students to be engaged in a problem enough to persist, it may be important to give genuine problem contexts where the math used is needed. This was an authentic problem for Trey.

Explicit purpose. The idea of needing an explicit purpose in order to engage with a problem came up a great deal. Consider Caleb's frustration with the parabola problem. "There was no scenario in which I would use this in the future. I would have to come up with that on my own. It was not intuitive." Caleb was concerned that no purposeful, explicit reason was given for solving the problem. As he put it, "I would have to come up with that on my own." Caleb did not want to come up with that purposeful reason on his own. He would like to see it within the problem context. In contrast, students felt that the comet problem did provide them with an explicit purpose to solve the problem. Consider this example from Jeff.

> Jeff: The Comet problem was more engaging. It had more. This is a reason for actually trying to do this math. The parabola problem was, we want to know this for no reason at all. It made more sense to be solving the comet problem because we want to be able to tell people when the best time to view the comet is.

Note that for Jeff, the Comet problem provided an explicit purpose for wanting to solve the problem, namely, "we want to be able to tell people when the best time to view the comet is." His wording made evident that he was motivated to do the math here as he stated, "This is a reason for actually trying to do this math." Jeff was more engaged with this context than the parabola context because he cared about the reason given to solve the Comet problem. Doug echoed this sentiment. "I can see the purpose. Astronomers are looking to see when the graph is closest to the origin. That made sense." Doug explicitly stated that he was able to see the purpose. For him, a purposeful, explicit reason for solving the problem had been given. Doug later expounded on the contrast between the Parabola problem and the Comet, Home Add on, and Humanitarian Aid problems. He began with the parabola problem and then moved to the other problems.

> Doug: You are looking at the graph of an equation with nothing to relate it to. The purpose for solving the equation and doing the math is really important. To teach people why we are doing the math. These equations are applicable in life [points to the Comet, Home

Add On and Humanitarian Aid problems]. So why don't we teach people and give them a skillset so they can go home and be a mini astronomer, architect, city planner. Give them the skills to solve real world problems.

Note that for Doug, the Comet, Home Add On and Humanitarian Aid problems provided an explicit purpose for solving them. Namely, students would be better prepared to contribute to society with skillsets relating to astronomy, architecture and city planning. Doug felt this could engage students with better understanding of how math is being applied in real world situations. This was a purposeful, explicit reason for Doug.

Many students felt the river problem did not offer them a purposeful, explicit reason. Consider these statements from Trey and Sarah. Trey stated, "I do not have a context in which I would actually care about the time. Why do I care how fast it goes. I will row or not, time does not make sense to me in that context as something I actually care about." Note that Trey did not care how fast he got from point A to point B because no purposeful, explicit reason had been given for him to care. Sarah said, "who cares if you can row across a river fast and then run down. Versus, ok people actually need water." Notice that for Sarah, the Humanitarian Aid problem was more engaging in part because she saw a purposeful, explicit reason given for solving the problem, namely, "people actually need water." Trey and Sarah felt no purposeful reason was given for solving the river problem.

Consider this example from Dan. Dan was asked to order the six problems from the most engaging to the least engaging. In discussing why he found the Humanitarian Aid problem the second most engaging problem, Dan talked about the value of the reason given.

Dan: I like the idea that math helps here, it helps people get water. If I were to do this problem I would feel that I saved a village. It is rewarding. This feels like there is

> pressure to solve the problem. If I don't solve the problem people are hurting, if I do I am helping them.

Note that Dan felt he had been given a purposeful, explicit reason for solving the problem. In his words, "If I were to do this problem I would feel that I saved a village." Saving people is a purposeful, explicit reason for doing the problem. Sarah felt similarly. She stated that the humanitarian aid problem was more engaging than the river problem because it was more purposeful. When asked to expound on why that was important to her she responded, "It is helping other people. Because this is a real life issue, people really do need water. If people do not have the math behind this, they may not end up with the most efficient system or even be able to get the water." Students throughout the interview discussed the importance of the purpose given to solve the Humanitarian Aid problem.

In conclusion, students often found one problem context more engaging than another because they found the context realistic/worthwhile. Students felt problem contexts were realistic/worthwhile when the contexts were authentic, gave a purposeful, explicit reason for solving it, and were either a related to an everyday life or a potential career idea.

Enjoyable

While not as frequent as realistic/worthwhile, ideas did come up pertaining to the characteristic of being enjoyable. The following subsections describe how interactivity and good stories came up.

Interactivity. In part one of the interview the idea of interactivity - being able to put oneself in the context of the problem - was used as a reason for finding one problem context more engaging than another. Note that each problem context was written so that one could put themselves inside the problem context, except for the parabola problem which did not have a story. Students felt that the contexts of the Comet and Humanitarian Aid problem better helped them to put themselves inside the problem. Consider these statements from Caleb and Dan. Caleb felt the comet problem was engaging because "There is an opportunity for you to place yourself inside the problem, either as a scientist or one of the people at home." Being able to place himself inside the context of the problem helped Caleb find the Comet problem more engaging. Dan stated that the comet problem "Made me think about working at NASA, I could put myself in this position. I like the idea of getting to tell the public which day would be best to view the comet." The comet problem provided Dan and Caleb with an opportunity to put themselves in the problem. Other students felt similarly about the Comet problem. It provided an opportunity to put themselves in a context.

Sarah and Rachel felt similarly about the Humanitarian Aid problem. She stated, "I can picture I am a humanitarian aid person." Rachel stated, "This problem was my favorite because I could see myself actually doing something like this in the future." Again, these students brought up this idea of being able to put themselves into the context of helping get a needed water supply to a village. This was a context they could see themselves in and wanted to see themselves in. This was a pretty common sentiment throughout the interviews. Students felt more engaged with problems that they could easily put themselves into the context.

This aspect of engaging context does overlap with the characteristic realistic/worthwhile. When students found a problem authentic, purposeful, and either saw it in their everyday life or as a potential career idea, students were more likely to be able to put themselves into the context.

A good story. In this section I stick to the general idea of a good story as I did not share any personal experiences with students or absurd problems. However, students did reason about one problem being more engaging than another because of a better story. Consider this statement from Madisun about why she felt the house problem was more engaging. "The story drew me in more."

Consider these thoughts from Madisun and Rachel. Concerning the comet problem, Madisun stated, "It is like a sci-fi movie. Oh, that is really cool. You can use math to figure out how close to the earth it is." Madisun saw the comet problem as a good story, "like a sci-fi movie." This contributed to the comet problem being rated as the most engaging problem of the six she was shown. Rachel also found the comet problem more engaging.

> Rachel: I like the comet problem more because it has the story and maybe other math people would just want to get down to the nitty gritty so they can just have the specifics. But I like having a story to work with so I can see 'Oh, I could actually use this math in the future'. Again, feeling that it is not pointless to learn this. That it can be related to real-world problems. And I think it is just more exciting. We have a story about a comet. That is just more engaging than just looking at numbers in an equation.

Note that Rachel was engaged more with the comet problem because of the good story. It made the problem more exciting and engaging for her.

Often critiques of the river and farmer problems had to do with lack of a good story. Consider Rachel's critique of these two problems. "They kind of lack an exciting part of the story." Lack of an exciting story kept Rachel from being as engaged. Some students offered suggestions for how to make the river problem more engaging. These included making the points landmarks, and giving good reasons for speed within the context of the problem. These suggestions would have added to making the story better.

In contrast, Savannah felt the River problem was already exciting. She stated that it was more engaging because she thought it was more exciting, more entertaining than the Humanitarian Aid problem. When comparing the river problem to the humanitarian aid problem Savannah stated, "Speed is associated with activity and movement and having fun. Cost is associated with getting bills paid and that is not much fun." Note that for Savannah the context of speed was more exciting than the context of cost. She went on to rank the River problem as the most engaging problem and gave a suggestion for improvement. Reflect on why it was engaging for her.

Savannah: I put the river one as the most engaging. I did this because I think that is exciting. I could see myself using that in a triathlon or something like that. Calculating the best route for the fastest speed.

TS: Speed was exciting for you, and which path would make it fastest. [she nods in agreement here.] Talk to me about why that is again.

Savannah: I think it is because in our society we have the connotation that speed is adventure. We associate it with spies and race cars and all that sort of thing. I see that as, even though we are in math class we can have fun with it, with spies and racing.

TS: So if I add spies and racing does this problem become more engaging for you? Does that make sense? Is there anything that you would say, 'If you brought this in, that would make it more engaging for me'.

Savannah: I think it would be because I like heroes and I like spies. I like stuff like that. That is fun.

Notice that for Savannah the context of speed was exciting. She began to think of racing, heroes and spies. Savannah valued fun, and the River problem felt more fun for her. She built an exciting story for herself within the context of the problem. Note that she felt that the problem could be improved by adding spies, heroes and racing. She created an exciting story and this made the context fun for her, which led her to being engaged.

Student Ratings of the Six Problems

Students finished part I of the interview by rating the problems from the most engaging to the least engaging. The results of these ratings are found in table 6 below. Some of the results were not surprising. The parabola problem was rated the least engaging by all of the students. Given that this problem was not authentic; gave no purposeful, explicit reason for solving it; was not interactive; did not lend itself to being a hands-on activity and did not include a good story; this was expected. However, some results were surprising. Notice that the river problem was rated number one by two students. I had not anticipated this. I had hypothesized that students would find the river problem less engaging. This emphasized the importance of variety in contexts. Different students have different interests and in order to engage them with contexts, teachers need to show many new and different types of contexts.

Table 6

Ratings of All Six Problems

	Home	Humanitarian Aid	Comet	Farmer	River	Parabola
Sarah	1	2	3	4	5	6
Madisun	3	2	1	5	4	6
Rachel	3	1	2	4	4	6
Grant	3	1	3	2	5	6
Caleb	2	4	3	5	1	6
Rebekah	1	4	3	5	2	6
Savannah	4	3	2	5	1	6
Trey	1	2	3	4	5	6
Jeff	4	1	3	2	5	6
Dan	3	2	1	4	5	6
Doug	3	1	2	5	4	6

The Lingering Issue of Understandability

As discussed earlier in the method's section, I conducted pilot studies before holding formal interviews. Within the pilot studies I found that students were often less engaged when a problem was lengthy, difficult to understand, or when they were overly concerned about the effort needed to solve the problem. We tried to address these issues by keeping problems short and to the point, making the problems straightforward, and posing the problems in pairs of equal mathematical difficulty. I successfully helped students to steer clear of the length issue. Not one student brought up length as an issue in the formal interviews. Mathematics was brought up a little, but minimally, and always in conjunction with understandability, which seemed to be the real issue. Students still discussed understandability as an issue with some of the problems. In this section, I address students' thoughts on understandability and how that led to finding certain problems more engaging than others.

Consider Caleb's reasoning for finding the River problem context more engaging than the humanitarian aid problem context.

> Caleb: I think the river problem is more engaging. I think the humanitarian problem is definitely interesting. I think it kind of gets confusing. I mean, the mountains are so ambiguous, right. You could have different heights of mountains, you know, like we do here [referring to the local terrain]. That is a little abstract. And I think the river problem is more straightforward because you can picture it in your head, that it is very flat, right. You can either go straight across, or you go downstream. I think because it is simpler. The question for the river problem is "how fast can you get there?" I think that is way more straightforward than cost of laying pipe.

Note that for Caleb, understanding of the problem context played a large part in his choosing that context as the most engaging. Caleb kept coming back to this idea of understanding. He felt that the River problem was more straightforward and simple than the Humanitarian Aid problem. He rated the river problem as the most engaging of all six of the problems and gave this reason, "The river was so easy to think about, it was the most understandable." He was not the only one to struggle with understandability. Dan changed his mind back and forth between voting for the Humanitarian Aid problem and the River problem as the most engaging of the two. He struggled to get away from the idea of understanding and this led him to be unsure of his choice. He struggled to get away from understanding with the farmer and house problems as well. In this research, both of these answers were considered ties because of the back and forth.

I do want to caution the reader in this section. Please do not mistake understandability as synonymous with engagement. While the parabola problem was often pronounced 'simple' and 'straightforward' by students, this problem did nothing to engage students, even those who rated problems highly based on understanding. Therefore, the point here is that a lack of understanding what the problem is asking will contribute to a student not engaging with a problem context.

CHAPTER FIVE: DISCUSSION

Recall that this study was centered on the research questions: (1) What aspects of problems do students see as making them more realistic, worthwhile, enjoyable, motivating and overall engaging? (2) When comparing problems meant to introduce a mathematics topic, what reasons do students give on their own for finding one problem context more engaging than another? By answering these research questions I hoped to shed light on the larger issue of creating mathematics problem contexts that are more engaging for students. This research identified a new characteristic (variety), and determined that two characteristics were closely related and should be merged into one (realistic/worthwhile). I also found six aspects of engaging problem contexts, including three that helped problems be realistic/worthwhile (everyday life/potential careers, authenticity, and explicit purpose), as well as three that helped them be enjoyable (context lends itself to a hands-on activity, interactivity, or a good story). This research helps define what is meant by realistic/worthwhile context and enjoyable context. In this section I discuss the contributions of this study, the usefulness of the data, and limitations/directions for future studies. Below I have included table 7 to help flesh out definitions and understanding of the data.

Table 7

Framework Definitions

Characteristics of engaging problem contexts + aspects that help a context have that characteristic	***Definitions***
Characteristic: Realistic/Worthwhile	A realistic context is a problem context that the student could see someone (myself or others) having the need and taking the time to solve outside of the classroom.
• Aspect #1: Authentic	A problem that students believe could come up in the real world and really needs the math students are using to solve it.
• Aspect #2: Purposeful, explicit reason	Clear, articulated reason given as to why one is working on the application problem.
• Aspect #3: Potential careers or everyday life	A relationship to either what the student already knows from their own lives, or also expanding past their current experience to possible career fields.
Characteristic: Enjoyable	An enjoyable context is a problem context that is likable from the student's perspective, in that it evokes positive feelings in the student towards the problem.
• Aspect #1: Hands-on Activity	The context lends itself to having a hands-on activity.
• Aspect #2: Good Story	An exciting or absurd story as well as any personal experiences the teacher has had with applying the math.
• Aspect #3: Interactive	There is a place for the student to put themselves into the context.
Characteristic: Variety	A problem context with variety is one that is new or different as compared to the problem contexts students have previously experienced.

Contributions of this Study

In my framework I began with realistic, worthwhile, enjoyable, motivating or some combination of these characteristics to have an engaging problem context. This framework was incomplete in two key ways. First, it did not include the characteristic variety. Second, while research states that students should see mathematics that is realistic, worthwhile, enjoyable or motivating; research did not discuss when students saw context in these ways. This made it more difficult to identify what engaging problem contexts looked like. These data help better solidify how to choose engaging problem contexts. In this section I review the new characteristic “variety” and the aspects of realistic/worthwhile and enjoyable contexts.

Variety was missing from the initial framework and was added as a characteristic of engaging contexts. Students want to see many different kinds of contexts. Students were engaged with new ideas, seeing different ways that math applied or could be used. Students were disengaged when they saw the same contexts over and over again. Engaging contexts include many different kinds of contexts and contexts that are new to students. My study suggests teachers need to provide new contexts for their students as well as pose a variety of questions from different contexts to fully engage the range of their students.

In addition to providing a new characteristics, students helped flesh out what realistic/worthwhile contexts looked like. In particular, my study suggests that a way to help problems be realistic/worthwhile is for them to be authentic, to have an explicit purpose for solving them, and to include a potential career or everyday life situation. Students want contexts that include a potential career or everyday life situation. As mentioned previously, prior research informs about the importance of everyday life situations (Bonotto, 2005; Van Den Heuvel-Panhuizen, 2003). But, prior research had not expounded on the importance of potential career situations. Further, students want to believe the math is really necessary to solve the problem and to understand better why that problem is a worthwhile one for people to examine.

My study helped us understand that enjoyable context can include those that are interactive or contain a good story. Prior research had been vague in defining enjoyment or fun. This research contributes to defining enjoyment within problem contexts. Students desired to see problem contexts that included a good story: in general, through teacher's personal experiences and through absurd stories. Students also wanted context to be interactive by being able to put themselves into the problem context, or have context that lent itself to hands-on activities. Recall that many students asserted that the reason they saw one problem context as more engaging than another was because they could see themselves within the more engaging context. This study helped to piece the prior research into a coherent whole and put educators into a better position to describe what makes contexts more engaging for students.

The Usefulness of this Study

Recall from the literature review that Schukajlow et al. (2012) judged contexts for possible enjoyment and interest. Interestingly, Schukajlow et al. found that there was no correlation between enjoyment of and interest in a problem and whether or not that problem had a connection to the real world. In my study, I did not use pure math problems, but used mainly problems that connect to the real world. Therefore, this study does not contradict the former study. But it does raise some questions. Is it possible that whether or not students will enjoy or be more interested in a real world problem versus a pure math problem has to do with the context of that problem? Would students have been more interested in the real world problems Schukajlow et al. presented to them if they had strived to pose context that included realistic/worthwhile or enjoyable aspects? They did not say that they attempted in any way to pose real world problems that students would likely find engaging.

Consider one of the problems with a connection to the real world that Schukajlow et al. (2012) posed to students.

Playground

The Traudt family is vacationing on a farm near Schwandorf in the Bavarian Forest. There is a playground there for the children, Lina and Maria. The playground's greatest attraction is the rope slide. The steel cable of the rope slide is stretched horizontally between two posts 10 meter apart. When Lina reached the middle, the steel cable stretched so much because of her weight that it dropped by 45 cm.

How long is the steel cable now?

Figure 2. Example Problem from Schukajow et al. 2012

Note that this is a context with a connection to the real world. Therefore, I will judge this context using the aspects of realistic/worthwhile contexts where they overlap with enjoyable contexts (since Schukajlow et al. were looking at enjoyment of and interest in a problem). Note that I am unable to judge whether or not this context provides a new or different scenario because I am unaware what students in this audience have previously seen. Therefore, I judge this context solely on aspects of realistic/worthwhile and enjoyable contexts.

Based off of the data in this research, students are unlikely to find this context engaging. Recall that aspects of realistic/worthwhile contexts include authenticity, an explicit purpose, and a potential career or everyday life idea. Aspects of enjoyable contexts include context that lends itself to a hands-on activity, a good story or interactivity. As I will explain below, this problem fails to give these aspects of engaging context.

First, this context is not a potential career or everyday life idea. This context does not include a potential career idea. While some might argue that a playground could relate to a student's life, it is unlikely that the given scenario of measuring how long the cable on the rope slide is would relate to the student's life. Therefore, this problem does not give a potential career or everyday life idea.

Second, authentic situations are problems that students believe come up in the real world and need the math students are using to solve it. It is unlikely that students would believe that this situation would come up in the real world. Even if it relates to their real life, why would the need to find the length as explained in this context come up in real life? Furthermore, measuring the length of the cable would be easier than using the Pythagorean theorem to find the measurement. It is doubtful students would find the math they are using necessary to do the problem. This problem is not authentic.

Third, purposeful, explicit reasons for solving a problem include clear, articulated, valued evidence given as to why one is working on the application problem. No purposeful, explicit reason is given for solving this problem. It is not clear why it is important to solve for the length in this problem. How is solving for the length helpful? What positive difference is it making? Why should a student value the solution? This problem fails to give a purposeful, explicit reason for solving the problem.

Finally, there is little attempt at making this context enjoyable. While this context could lend itself to a hands-on activity, no activity is done. There is not a good story. This context does not easily lend itself to a student being able to put him/herself in the context of the problem. This problem is unlikely to be enjoyable for students.

Perhaps Schukajlow et al (2012) may have had a different outcome if they had attempted to make the problem more realistic/worthwhile and enjoyable. I hypothesize that students within their research would have found problems that included realistic/worthwhile and enjoyable contexts more engaging than pure math problems. Given the lack of so many aspects of engaging contexts, it is unlikely that this context engaged students.

Limitations and Directions for Future Research

The results of this research are not fully generalizable due to the nature of this study. The sample size of the study was small and participants were volunteers, not randomly selected. However, it is a step in the right direction towards better understanding when students view context as engaging. Teachers and curriculum writers are better informed in choosing problem contexts that students are likely to find engaging. In this section I suggest four possible future studies to inform research on posing engaging context and some final thoughts.

First, due to the vague definitions of enjoyable in former research, I struggled to pose an 'enjoyable' problem context in part one of the research. Future research can be conducted to flesh out enjoyable contexts. This study began to help educators understand enjoyable context. It is important to share problems that encourage hands-on activities, include a good story or are interactive. Researchers should hypothesize more on what constitutes as enjoyable context, experiment with students and provide more information on when students are viewing context as enjoyable.

Second, future research could help develop the idea of a good story by simply asking students to provide examples of mathematics problem contexts that included a good story. In this research I found that a good story is enjoyable and can include absurd contexts or the teacher's own experience. However, recall that students identified the comet problem in part I of the interview and Caleb gave the movie-line problem in part II of the interview as examples of a good story. These were not a teacher's experience or an absurd story. Therefore, there are other contexts that constitute as good stories. Further research could help educators develop this aspect of engaging contexts.

Third, my interviews did not discover any new aspects of engaging contexts under the characteristic motivating. One student brought up historical development of a mathematical topic, but this idea was not brought up by other students. Further studies could be done to better understand motivating context. I believe there is something there that needs to be probed further. My study did not do an effective job of this.

Fourth, while students mentioned some problems that felt repetitive to them—the apple problem, rocket problem—future studies are needed to better understand what types of problems are being viewed over and over by students. Students want to see a variety of problems, contexts that are new or different. Teachers need to understand what students are regularly seeing in their classrooms in order to be prepared to show a variety of problem contexts.

Finally, this research is a first step in better understanding what constitutes as engaging contexts. This research does not claim to have nailed this down. Rather, this research begins to point teachers and curriculum writers in a direction that will help as they strive to get students engaged with problem contexts. Further studies need to be done in order to help expound on what engaging contexts look like.

Conclusion

In conclusion, many students have negative feelings towards mathematics, which is causing them to disengage in their classrooms. This study attempted to better understand how teachers and curriculum writers can help students to reengage with mathematics by using more engaging contexts to develop mathematical content. Students in this research were more engaged with problem contexts that were realistic/worthwhile, enjoyable and new (variety). As teachers pose problems to students with the goal of engaging them, they should pose problems that are realistic/worthwhile, problems that are enjoyable, and problems that are both realistic/worthwhile and enjoyable. Teachers should keep in mind that even if a particular problem context is initially engaging for students, students will be unlikely to find that context engaging if it is shown to them over and over again. Students like to see problem contexts that are new and different. Research should continue to flesh out ideas for engaging context in order to help students engage more in mathematics classrooms.

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APPENDIX A

Problems For Students Transcribed From Video

Farmer:

A farmer has 2,400 feet of fencing. He wants to make a rectangular field that borders a straight river. It wouldn't make sense to do a really long, not wide field or a really wide not long field. It will make sense to do something in the middle. What are the dimensions of the field that will give it the greatest area?

Home Add-On:

You're adding on to your home. You are going to knock out some wall in the back and add an open room on to increase open floor space. You have plenty of left over material from previous projects for floor and roofing, but you only have enough material for 60 feet of new wall. You wouldn't want to make a very long and narrow room, or a very wide, short room. So given that you only have 60 feet of new wall, what dimensions would give the greatest area for this new room.

Humanitarian:

You are working with a humanitarian aid group to bring safe water supply to villages. One village is located at the edge of a mountain range. It's nearest water supply is up in the mountains. The plan is to make a simple pipe that will run from the source to the village. We could lay pipe directly from the source to the village, but the cost of laying the pipe through the mountains is higher. Another option is to have the pipe run straight out of the mountains and then have the pipe run to the village, but that does make a longer pipe. What path should the pipe follow in order to minimize the cost of laying the pipe?

River:

You are going to launch a boat from point A on the bank of a straight river. You want to reach point B which is downstream on the opposite bank of the river. You could row directly across the river to point C and then run to point B. Or you could row directly to point B. Or you could row to some point D between B and C and then run the rest of the way to B. If you can run a little bit faster than you can row, where should you land on the opposite bank in order to reach point B as soon as possible.

Parabola:

Given the graph of the parabola $y^2 = 2x$ and the point (1,4). Every point on the parabola is a certain distance away from the point (1,4). Find the point on the parabola that is closest to the point (1,4).

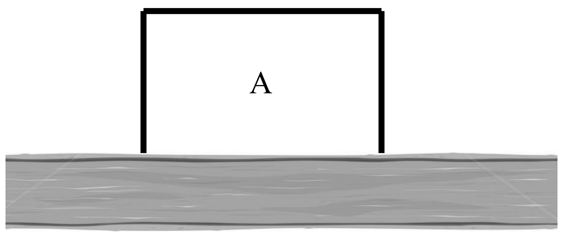
Comet Problem:

You are working at NASA. A comet is going to make a fairly close pass over earth and we want to get a good look at it. Suppose that earth is located at the origin, then the path of the comet can be modeled by the graph with equation y = $\sqrt{x + 5}$. Now in order to tell the public which day will be the best day to view the comet, we want to know at what point along the comet's path it will be closest to the earth. Find the point along the graph of this function that is closest to the origin.

Problem Solutions

Farmer Problem:

A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river (see picture). He needs no fence along the river. What are the dimensions of the field that has the largest area?



We are looking for when the width of the fence (x) times the length of the fence (y) yields a maximum.

Xy=max

$2x + y = 2400$

$y = 2400 – 2x$

substitution: $x(2400 – 2x) = max$

derivative set = to 0: $2400 – 4x = 0$; $x = 600$

then $y = 1200$ and the max are is 720,000 square feet.

House Problem:

You are adding on to your home by knocking out a wall and making an additional room. You have plenty of leftover materials for the floor and roofing from an earlier project, but you only have enough materials to make 60ft. of wall. What dimensions of the wall would create the greatest area for your new add on?

We are looking for when the width of the new wall (x) times the length of the new wall (y) yields a maximum.

Xy = max

2x + y = 60

Y = 60 – 2x

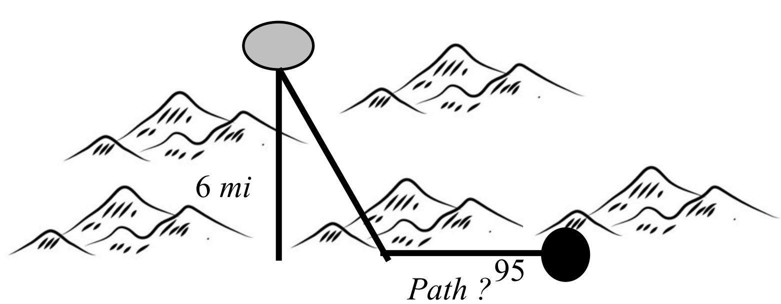
Substitution: x(60 – 2x) = max

Derivative set = to 0: 60 – 4x = 0; x = 15

Then y = 30 and the max area is 450 square feet.

<u>Humanitarian Aid Problem:</u>

You are working with a humanitarian aid group on a semester abroad trip. A village located at the edge of a mountain range has lost its water supply and the closest water source is 10 miles along the flatlands and from there 6 miles in a straight line into the mountains. Your group decides to build a simple pipe from the water source in the mountains to the village in the flatlands. While one idea could be to dig the pipe straight from the water source to the village, it will cost your group more to lay the pipe through the mountains, because of the extra digging, and cost less to lay the pipe along the edge of the mountains, because the ground there is flatter. DO NOT USE THESE FACTS ON THE VIDEO! In fact, an estimate suggests it will cost your group \$10,000/mile in the mountains and \$4,000/mile along the edge of the mountains. USE THIS: (If the path will consist of one straight line segment through the mountains connected to another straight line segment along the edge of the mountain range, your objective is to figure out what path for the pipe minimizes the cost.) (see picture).



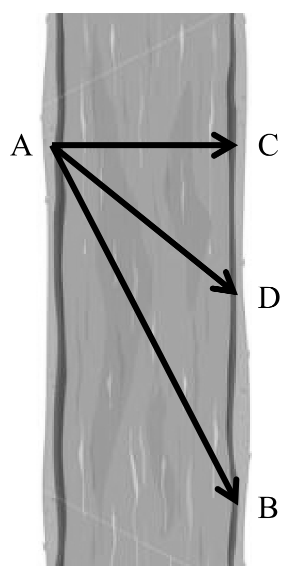
$$\$10{,}000\sqrt{(36 + x^2)} + \$4{,}000(10 - x) = cost$$

$$Derivative: \frac{10{,}000x}{\sqrt{36 + x^2}} - 4000; set\ equal\ to\ 0\ for\ a\ minimum.\ 6.25x^2 = 36 + x^2$$

$$x^2 = \frac{36}{5.25}, x = 2.61\ miles\ minimum\ cost: \$29{,}525.65 + \$65{,}430.96 = \$94{,}956.61\ dollars$$

River Problem:

You are at point A on the bank of a straight river, 3 km wide, and want to reach point B, 8 km downstream on the opposite bank, as quickly as possible (see picture). You could row your boat directly across the river to point C and then run to B, or you could row directly to B, or you could row to some point D between C and B and then run to B. If you can row 6 km/hr and run 8 km/hr, where should you land to reach B as soon as possible?



Distance = rate X time

time = distance/rate

$$\frac{\sqrt{3^2 + x^2}}{6} + \frac{8 - x}{8} = time\ to\ travel\ the\ distance$$

Derivative: $\frac{2x}{12\sqrt{(3^2+x^2)}} - \frac{1}{8}$

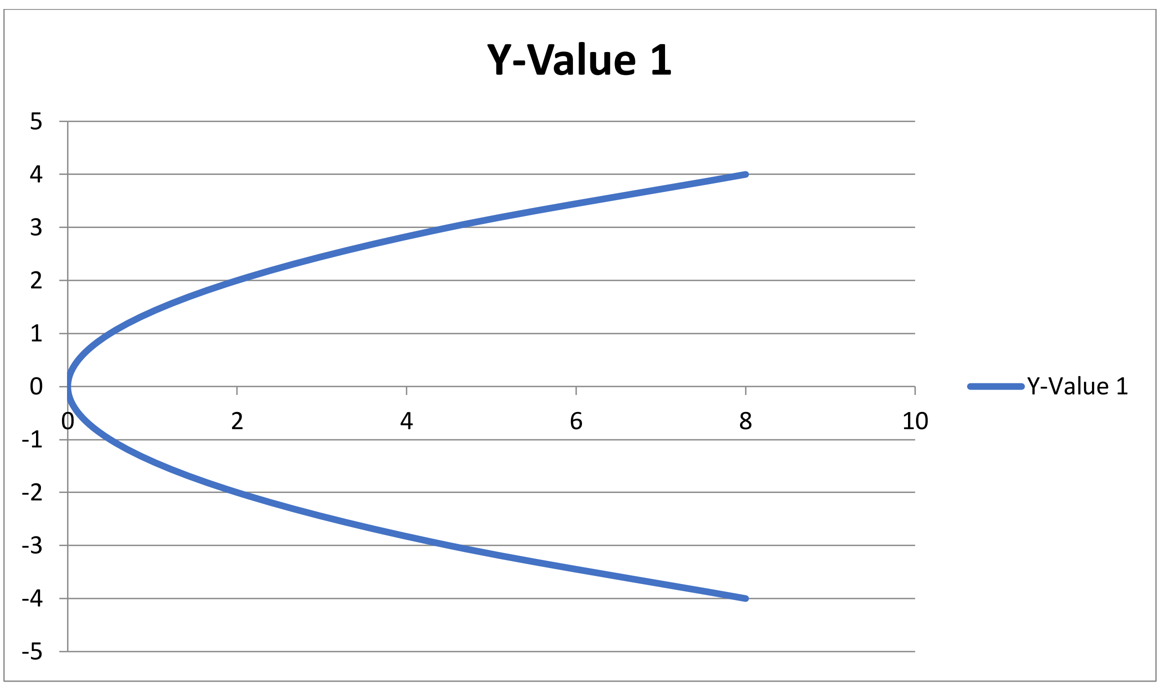
Set derivative equal to 0 to find a minimum. $\frac{4x}{3} = \sqrt{3^2 + x^2} so \frac{16x\text{^}2}{9} = 9 + x^2$

X = $9/\sqrt{7}$

Minimum time is : .756 + .575 = 1.331 hours

<u>Parabola Problem:</u>

1. Find the point on the parabola $y^2 = 2x$ that is closest to the point (1,4).



Example 3 in Stewart

Solution:

$d = \sqrt{(x-1)^2 + (y-4)^2}$

$d = \sqrt{(.5y^2-1)^2 + (y-4)^2}$

$d^2 = (.5y^2-1)^2 + (y-4)^2$ (The minimum at d^2 will be the same as the minimum at d)

If f(y) is d^2 then $f'(y) = 2(.5y^2-1)y + 2(y-4) = y^3 - 8$

So f'(y) = 0 when y = 2. The first derivative test shows that an absolute minimum occurs when y = 2. The corresponding x-value for x=.5y^2 is 2. So the point closest to (1,4) is (2,2).

Comet Problem:

You are working at NASA. A comet is going to make a fairly close pass over earth and we want to get a good look at it. Suppose that earth is located at the origin, then the path of the comet can be modeled by the graph with equation y = $\sqrt{x+5}$. Now in order to tell the public which day will be the best day to view the comet, we want to know at what point along the comet's path it will be closest to the earth. Find the point along the graph of this function that is closest to the origin.

$$\mathrm{d} = \sqrt{(\mathrm{x}-0)^2 + (\mathrm{y}-0)^2}$$

$$\mathrm{d} = \sqrt{(\mathrm{x}-0)^2 + (\mathrm{y}-0)^2}$$

If y = $\sqrt{(x+5)}$ then x = $y^2 - 5$. Using substitution we obtain…

$$\mathrm{d} = \sqrt{(\mathrm{y}^2-5)^2 + (\mathrm{y})^2}$$

$$\mathrm{d}^2 = (\mathrm{y}^2-5)^2 + \mathrm{y}^2$$

Again, the minimum at d^2 will be the same as the minimum at d. So let f(y) = d^2, then

$f'(y) = 2(y^2-5)(2y) + 2y = 4y^3 - 18y$

When this equation is set equal to 0 to look for possible minimums and maximums, the possibilities include y=0, $3/\sqrt{2}$ or -$3/\sqrt{2}$

The first derivative test shows that an absolute minimum occurs when y = $3/\sqrt{2}$

The corresponding x-value is -.5 So the point closest to the origin is (-.5, $3/\sqrt{2}$)

APPENDIX B

Pre-interview survey statements

The Mathematics As Useful Component:
Fennema-Sherman Usefulness of Mathematics Scale (Fennema & Sherman, 1976)

Overall Useful:
+ I study mathematics because I know how useful it is
- Mathematics will not be important to me in my life's work

Realistic:
+ I will need mathematics for my future work
- Mathematics is of no relevance to my life

Worthwhile:
+ Mathematics is a worthwhile and necessary subject
- Studying mathematics is a waste of time

The Positive Affect Towards Mathematics Component:
Kaput Center for Research & Development in STEM Education

Overall Positive Attitude:
+ I like math
- I do not have positive feelings towards mathematics

Enjoyable:
+ (My added statement) I see mathematics as fun.
- In the past, I have not enjoyed mathematics

Motivating:
+ I find joy in solving non-routine problems in mathematics
- Mathematics does not interest me

Survey as given to the students

Name____________________________________

Best Way to Contact________________________

Major____________________________________

We are looking for a wide range of students with different attitudes towards mathematics so please be very candid on the form.

	Strongly Disagree	Disagree	Agree	Strongly Agree
1. I study mathematics because I know how useful it is				
2. Studying mathematics is a waste of time				
3. Mathematics is a worthwhile and necessary subject				
4. I like math				
5. Mathematics will not be important to me in my life's work				
6. I do not have positive feelings towards mathematics				
7. I see mathematics as fun.				
8. Mathematics does not interest me				
9. Mathematics is of no relevance to my life				
10. I find joy in solving non-routine problems in mathematics				
11. I will need mathematics for my future work				
12. In the past, I have not enjoyed mathematics				

If you would be interested in participating in a one hour interview for mathematics research (we will not be doing mathematics) please include your name and contact information. This interview will inform research on helping to make mathematics problem context more engaging for students. Your name will not be connected to the information you provide for the research and while there is no compensation for filling out this survey, students who are chosen to participate in the one hour interview will be paid $15 for their time.

How students were scored

Below is how the tallying of scores was done. A checkmark in a given box received the score associated with that box. I wrote a (U) next to the statements about seeing mathematics as useful. I wrote an (A) next to the statements about attitudes towards mathematics. I then totaled the students' scores and attempted to recruit students with higher, middle, and lower scores.

	Strongly Disagree	Disagree	Agree	Strongly Agree
1. I study mathematics because I know how useful it is (U)	1	2	3	4
2. Studying mathematics is a waste of time (U)	4	3	2	1
3. Mathematics is a worthwhile and necessary subject (U)	1	2	3	4
4. I like math (A)	1	2	3	4
5. Mathematics will not be important to me in my life's work (U)	4	3	2	1
6. I do not have positive feelings towards mathematics (A)	4	3	2	1
7. I see mathematics as fun. (A)	1	2	3	4
8. Mathematics does not interest me (A)	4	3	2	1
9. Mathematics is of no relevance to my life (U)	4	3	2	1
10. I find joy in solving non-routine problems in mathematics (A)	1	2	3	4
11. I will need mathematics for my future work (U)	1	2	3	4
12. In the past, I have not enjoyed mathematics (A)	4	3	2	1

APPENDIX C

The Interview Protocol

Interview: Part 1

Students will be informed that this study is to help teachers pose more effective problems within mathematics lessons. Here students will be given full disclosure on how this research might be used and their rights (consent form and video consent form). Interviews will only commence if the consent form and the right to video have been signed. If students do not wish the video to be shared outside of myself and professor, I will still commence the interview and respect their wishes.

Students will be shown six application problems presented by a professor on video, two at a time. Students will be asked to compare each of the two problems in this way, "Given that the mathematics needed to solve these problems is exactly the same, and your teacher will choose one of these problems to introduce a new mathematics topic with, which context do you find more engaging?" If students ask what I mean by engaging I will follow up with this questions, "Which problem would you prefer your teacher use to introduce a new math topic to you?" I will not get into my characteristics of engaging problem context at this time, as I want to see what students come up with initially in discussing what makes mathematics problem context engaging.

As the students discuss the problems, the following are possible follow-up questions I may use. Of course, I may use other follow-up questions depending on the students' responses.

- Why do you find this context more engaging than this one?
- Why do you prefer this context over this one?
- [After a student gives a reason] Why is it that [restate reason]? (For example, why is it that you could see yourself doing this?)

- [After a student gives a reason] Why does [restate reason] matter to you? (For example, why does being able to see yourself doing this matter to you?)
- [After a student gives a reason] What part of the problem got you thinking that [restate reason]? (For example, what part of the problem got you thinking that this is something you could see yourself doing?)
- What made that [reason] important to you?

Repeat the previous 4 questions for each reason a student gives for more engaging context.

- Why do you find this context less engaging than the one?
- What is it about the problem specifically that elicited that response?
- Why is it that [restate reason] made it a weak context?

Repeat the previous 3 questions for each reason a student gives for less engaging context.

I plan to let students do most of the talking in this part of the interview. I will ask them to think out loud and remind them often that I want to hear their thoughts. Students are the experts in this interview. A student's thought is never wrong. These questions and process will be repeated with each pair of questions. Students will be given the opportunity to discuss their thoughts. Given that this discussion is student-driven, I am unable to give an account of the exact questions that may come up, but the above questions are some examples that I infer may come up.

I hypothesize that reasons relating to problem context being realistic, worthwhile, enjoyable and motivating will come up as our discussion ensues, however, I do not know. This is simply a best guess and it will be interesting to see as the interview unfolds what reasons students are giving for finding one problem context more engaging than another.

In order to ensure that the order the problems are presented in is diverse and students are not choosing preferences based on a consistent order I will present the problems according to the table below (table 8). The numbers one through 12 refer to the twelve students. The Farmer problem is referred to by “F”, House “H”, Humanitarian Aid “HA”, River “R”, Parabola “P”, and Comet “C”.

Table 8

Defining the Order Students Will See the Problems In.

	1	2	3	4	5	6	7	8	9	10	11	12
First Pair	F/H	C/P	H/F	F/H	HA/R	R/HA	P/C	H/F	HA/R	R/HA	P/C	C/P
Second Pair	HA/R	R/HA	R/HA	C/P	F/H	P/C	HA/R	P/C	C/P	F/H	H/F	F/H
Third Pair	P/C	H/F	C/P	HA/R	C/P	F/H	H/F	HA/R	H/F	P/C	R/HA	HA/R

To conclude the first part of the interview, I will ask the student to order the 6 problems from the most engaging to the least engaging. I will ask students why they ordered it that way.

Interview Part 2: Students will be reminded that this interview is about helping teachers provide/create more engaging problem context. This could be one small step in the direction of helping students to enjoy mathematics more and see it as more useful. In part two of the interview, I introduce the four characteristics and their definitions, and try to elicit information from students concerning when they see context as having each of these characteristics.

Introducing the characteristics to the student. Students will be informed that within research certain characteristics that imply strong problem context have been identified. These characteristics include (but are not limited to) realistic, worthwhile, motivating, and enjoyable. Students will be given the following definitions of these characteristics and I will go over them. They will see all of them at the same time to make sure they understand the difference between each definition. Then I will ask students to give me an example of a problem that they have seen that fits well under each of these definitions.

Realistic A problem is realistic to you if you could see someone having the need to solve the problem and taking the time outside of the classroom to do so. Does that make sense?

Worthwhile A problem is worthwhile to you if you believe it has the capacity to make a positive difference for someone (you or others). Does that make sense?

Enjoyable A problem is enjoyable to you if you like it, if you think it is fun. This is about how you *feel* about the problem in terms of emotions. Does that make sense?

Motivating A problem is motivating to you if it is intellectually interesting, you want to know the answer to the question. You would like to learn the mathematics necessary to solve it. Does that make sense?

Questions about realistic context. Research states that problems should be realistic. Students should believe someone in real life (outside of the classroom) would really need to solve the problem if it is to be considered realistic. I want to talk with you about what makes a problem realistic or unrealistic? The following are questions that I will use to probe the students' responses:

- Give an example of a realistic problem you have seen in a mathematics class.
- What specifically about that problem made it realistic to you?
- Why did that matter to you?
- Give an example of an unrealistic problem you have seen in a mathematics class.
- What specifically about that problem made it unrealistic to you?
- Why did that matter to you?
- What advice do you have for teachers who are striving to pose problems that students are likely to find realistic?

Questions about enjoyable context. Research states that students want problems to be enjoyable, they want problems they like. I want to talk with you about what makes a problem enjoyable or unenjoyable? The following are questions that I will use to probe the students' responses:

- Give an example of an enjoyable problem you have seen in a mathematics class.
- What specifically about that problem made it enjoyable to you?
- Why did that matter to you?
- Give an example of an unenjoyable problem you have seen in a mathematics class.
- What specifically about that problem made it unenjoyable to you?
- Why did that matter to you?
- What advice do you have for teachers who are striving to pose problems that students are likely to find enjoyable

Questions about motivating context. In research, Nardi and Steward (2003) found that students want problems to be motivating, they want problems that interest them, that they have a desire to know the solution to. I want to talk with you about what makes a problem motivating or unmotivating. The following are questions that I will use to probe the students' responses:

- Give an example of a motivating problem you have seen in a mathematics class.
- What specifically about that problem made it motivating to you?
- Why did that matter to you?
- Give an example of an unmotivating problem you have seen in a mathematics class.
- What specifically about that problem made it unmotivating to you?
- Why did that matter to you?

- What advice do you have for teachers who are striving to pose problems that students are likely to find motivating?

Questions about worthwhile context. In research, I found that students want problems to be worthwhile, they want problems whose solutions they think could make a positive difference for people. I want to talk with you about what makes a problem worthwhile or not? The following are questions that I will use to probe the students' responses:

- Give an example of a worthwhile problem you have seen in a mathematics class.
- What specifically about that problem made it worthwhile to you?
- Why did that matter to you?
- Give an example of a problem you have seen in a mathematics class that was not worthwhile.
- What specifically about that problem made it not worthwhile to you?
- Why did that matter to you?
- What advice do you have for teachers who are striving to pose problems that students are likely to find worthwhile?

To conclude the interview I will ask the student to describe the most engaging mathematics problem they have ever seen. I will ask them to explain what they felt was engaging about it. Furthermore, I will ask for advice on choosing engaging context.