Heat Transfer in Shock Boundary Layer Interaction Regions

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HEAT TRANSFER IN SHOCK
BOUNDARY LAYER INTERACTION REGIONS

A Thesis
Presented to the
Department of Mechanical Engineering
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
Rex W. Shumway

August 1965
This thesis, by Rex W. Shumway, is accepted in its present form by the Department of Mechanical Engineering of Brigham Young University as satisfying the thesis requirement for the degree of Master of Science.

Aug. 23 1965
Date
DEDICATION

To my wife, Linda.
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# TABLE OF CONTENTS

| APPROVALS | ii |
| ACKNOWLEDGMENT | iv |
| LIST OF TABLES | vii |
| LIST OF FIGURES | viii |
| LIST OF PLATES | x |
| NOMENCLATURE | xi |

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Objectives</td>
<td>1</td>
</tr>
<tr>
<td>Discussion of the Problem</td>
<td>2</td>
</tr>
<tr>
<td>II. LITERATURE RESEARCH</td>
<td>4</td>
</tr>
<tr>
<td>Experimental Flow Patterns</td>
<td>4</td>
</tr>
<tr>
<td>Normal Shocks</td>
<td>4</td>
</tr>
<tr>
<td>Concave Corner Generated Shocks</td>
<td>8</td>
</tr>
<tr>
<td>Reflected Shocks</td>
<td>10</td>
</tr>
<tr>
<td>Lambda Shocks</td>
<td>16</td>
</tr>
<tr>
<td>Theoretical Flow Patterns</td>
<td>23</td>
</tr>
<tr>
<td>Hammitt's Boundary Layer Thickness Change</td>
<td>24</td>
</tr>
<tr>
<td>Production of Turbulent Energy</td>
<td>25</td>
</tr>
<tr>
<td>Pressure Rise Required for Separation</td>
<td>27</td>
</tr>
<tr>
<td>Experimental Heat Transfer Data</td>
<td>28</td>
</tr>
<tr>
<td>Free Flight Rocket-Fin Test: Chauvin and Buglia</td>
<td>28</td>
</tr>
<tr>
<td>Rocket Nose Cone Experiment: Rumsey and Lee</td>
<td>30</td>
</tr>
<tr>
<td>Heat Transfer and Flow Separation: Schaefer and Ferguson</td>
<td>31</td>
</tr>
<tr>
<td>Extensive Flow Separation: Becker and Korycinski</td>
<td>32</td>
</tr>
<tr>
<td>Needham's Mach 9.7 Test</td>
<td>37</td>
</tr>
<tr>
<td>The Douglas Reflection Shock Test</td>
<td>39</td>
</tr>
<tr>
<td>Boeing's Mach 16 Test</td>
<td>39</td>
</tr>
</tbody>
</table>
Chapter

III. RESULTS

Empirical Correlation

Theoretical Prediction

Errors

Consideration of Mass Addition to the Boundary Layer

IV. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Recommendations

APPENDIX A METHOD OF OBTAINING PRESSURE RATIO FROM PRESSURE COEFFICIENT DATA

APPENDIX B METHOD OF EVALUATING NEEDHAM'S HEAT TRANSFER DATA

APPENDIX C DERIVATION OF BOUNDARY LAYER THICKNESS CHANGE IN A SHOCK ZONE

LIST OF CITED REFERENCES
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Test Parameters</td>
<td>41</td>
</tr>
<tr>
<td>2. List of Results</td>
<td>50</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Illustrative normal shock boundary layer interaction model</td>
<td>6</td>
</tr>
<tr>
<td>2. Laminar and turbulent wall pressure rise in a shock region on a flat plate</td>
<td>7</td>
</tr>
<tr>
<td>3. Oblique shock velocity components</td>
<td>7</td>
</tr>
<tr>
<td>4. Inviscid flow in a compression corner</td>
<td>10</td>
</tr>
<tr>
<td>5. Shock reflected off a laminar boundary layer</td>
<td>12</td>
</tr>
<tr>
<td>6. Separated and non-separated turbulent boundary layer with reflected shock interaction</td>
<td>13</td>
</tr>
<tr>
<td>7. Reflected shock generation</td>
<td>15</td>
</tr>
<tr>
<td>8. Reflected shock boundary layer thickness increase</td>
<td>15</td>
</tr>
<tr>
<td>9. Typical lambda shock structure</td>
<td>17</td>
</tr>
<tr>
<td>10. Illustration of boundary layer nomenclature</td>
<td>24</td>
</tr>
<tr>
<td>11. Variation of boundary layer thickness with pressure ratio and Mach number for a shock reflected from a boundary layer</td>
<td>26</td>
</tr>
<tr>
<td>12. Pressure rise across a shock required to separate the boundary layer</td>
<td>28</td>
</tr>
<tr>
<td>13. Instrumented rocket fin and flap</td>
<td>29</td>
</tr>
<tr>
<td>14. Cone-cylinder-flare model</td>
<td>31</td>
</tr>
<tr>
<td>15. Tripped boundary layer heat transfer and pressure distribution on a cone-cylinder-flare</td>
<td>33</td>
</tr>
<tr>
<td>16. Effect of Reynolds' number on separation and transition point</td>
<td>35</td>
</tr>
<tr>
<td>17. Heat transfer and pressure distribution on a cone-cylinder-flare</td>
<td>36</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>18. Boeing's test set-up</td>
<td>40</td>
</tr>
<tr>
<td>19. Film coefficient ratio versus pressure ratio across shocks with IA to IA, IA to TU, and TU to TU boundary layers</td>
<td>44</td>
</tr>
<tr>
<td>20. Film coefficient ratio versus pressure ratio across shocks with LA to IA and TU to TU boundary layers</td>
<td>45</td>
</tr>
<tr>
<td>21. Film coefficient ratio versus pressure ratio across Prandtl-Meyer expansions</td>
<td>46</td>
</tr>
<tr>
<td>22. Theoretical variation of Stanton number versus Reynolds' number</td>
<td>48</td>
</tr>
<tr>
<td>23. Experimental Stanton number ratio versus Reynolds' number ratio</td>
<td>52</td>
</tr>
<tr>
<td>24. Theoretical boundary layer thickness change with no mass addition</td>
<td>54</td>
</tr>
</tbody>
</table>
### LIST OF PLATES

<table>
<thead>
<tr>
<th>Plate</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal shock in a duct with thin boundary layers at Mach 1.18</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Normal shock in a duct after boundary layers have been removed</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Shock boundary layer interaction in a duct at Mach 1.7</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>Normal shock in duct mouth</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Oblique shock. Turbulent boundary layer. Mach 1.55</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>Oblique shock. Laminar boundary layer. Mach 1.55</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>The three flow regimes for a compression corner</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>Reflected shock. Turbulent boundary layer</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>Reflected shock. Laminar boundary layer</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>Lambda shock on a wing. Schlieren photograph, and pressure distributions at two elevations</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>Pressure distributions at $X = 180 \text{ mm}$ and $X = 222.5$ while the main lambda shock was at about 200 mm. Lambda shock isobars</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>Lambda shock boundary layer velocity profiles. Fluctuations in pressure are indicated by dots connected with arrows</td>
<td>21</td>
</tr>
<tr>
<td>13</td>
<td>Lambda shock pressure distribution at various distances from the wall. Flow over a wing section</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>Pressure and heat transfer distributions and schlieren photographs of flow over a compression corner at various wedge angles</td>
<td>38</td>
</tr>
</tbody>
</table>
NOMENCLATURE

a* Speed of sound where M = 1.0

$c_w$ Specific heat of wall material

$c_p$ Specific heat at constant pressure

$C_p$ Pressure coefficient

c Speed of sound

$\partial$ Partial derivative

$d$ Total derivative

$f$ Defined by $\frac{T}{T_o} = f - (k-1)\frac{M^2}{2}$

G Mass velocity $= \rho V$

$g$ Gravitational constant

$k_f$ Conductivity of fluid

$k$ Ratio of specific heats

$K \frac{\bar{V}}{V_{\infty}}$

L Total length

mm Millimeters

$M$ Mach number $= \frac{V}{c}$

$M^*$ Local velocity/speed of sound at $M = 1$

$P$ Pressure

$Pr$ Prandtl number $= \frac{\mu c_p}{k_f}$

$q''$ Heat flux

$R$ Radius

$R^*$ Recovery factor

$^\circ_R$ Degrees Rankine
Re  Reynolds number = \( \rho Vx/\mu \)

s  Entropy

S  Separation

St  Stanton number = \( h/\rho Vc_p \)

th  Wall thickness

T  Temperature

t  time

u  Velocity component tangent to the wall

u'  Velocity fluctuation tangent to the wall

v  Velocity component normal to the wall

v'  Velocity fluctuation normal to the wall

V  Local velocity

W  Local velocity

x  Distance from leading edge measured tangent to the wall

X  Distance from leading edge measured tangent to the wall

y  Distance measured normal to the wall

Z  Distance measured normal to the wall

LA to LA
  Laminar boundary layer on each side of shock zone

TU to TU
  Turbulent boundary layer on each side of shock zone

LA to TU
  Transition in the shock zone

\( \alpha \)  Wall deflection angle

\( \beta \)  Cone half angle

\( \delta \)  Boundary layer thickness

\( \delta^{**} \)  Boundary layer momentum thickness
\( \varepsilon \)  
Emissivity

\( \theta \)  
Shock angle

\( \mu \)  
Absolute viscosity

\( \rho \)  
Density

\( \sigma \)  
Stefan-Boltzmann constant

\( \bar{\ldots} \)  
Averaged

Subscripts

aw  Adiabatic wall

d  Downstream of shock

D  Diameter

f  Fluid

i  Inside

l  Length

n  Normal

o  Stagnation

t  Tangent

u  Upstream of shock

w  Wall

x  Distance from leading edge

\( \infty \)  
Free stream
CHAPTER I
INTRODUCTION

The interest in aerodynamic heating problems has been increasing rapidly in the last decade. This interest is no longer concentrated on the heating of simple shapes for which adequate heat transfer theory has been developed. Because of the increasing Mach number ranges of aircraft, the need for information on the aerodynamic heating of various components has become imperative. Only when the magnitude of the problem has been exposed can the design for reducing the severity of local heating be optimized. Definition of these problems is of interest, not only because of the flow phenomena, but because of possible design limitations of re-entry vehicles and supersonic aircraft inlets, wings, rudders, etc.

Objectives

Some of the aerodynamic heating research have been on a particular type of problem called shock heating. This is heating caused by a shock wave impinging on or being generated by a wall or mechanical component. There is a mutual interaction between the boundary layer of fluid existing next to the wall and the shock wave. The heat flux to the wall in this interaction area may be thirty or forty times higher than the heat flux in adjacent areas. The object of this thesis is to correlate the available experimental data concerning this heating problem to find if there is a way to quantitatively predict the increase in heat transfer.
in a shock boundary layer interaction region. While doing this, because of the effect of the flow pattern on the heat transfer, a study of information relating to the flow pattern in the interaction regions will also be made.

**Discussion of the Problem**

When fluid flows over a solid surface, viscous drag causes the fluid velocity to decrease to zero at the wall-fluid interface. The normal distance from the wall to where the fluid velocity reaches free stream magnitude is called the velocity boundary layer thickness.

There is also a thermal boundary layer next to the wall. For fluids such as air, with Prandtl numbers near one, the velocity and thermal boundary layers are of the same magnitude. The usual concept of a thermal boundary layer is that the fluid next to the wall tends to come to a temperature equilibrium with the wall. As this boundary layer fluid then flows downstream it has an insulating effect since in order for it to have any further energy exchange with the wall it must first have an energy exchange with other fluid particles. The thickness of the thermal boundary layer increases as the fluid flows along the wall. When the flow becomes turbulent, the magnitude of fluid particle interaction increases, thereby decreasing the insulating effect.

In other words, the convective conductance or film heat transfer coefficient \( h \) increases when a fluid undergoes transition from laminar to turbulent conditions. The heat flux to the wall, therefore, increases above what it would have been if the flow had remained laminar. The heat flux \( q'' \) to the wall is related to the film coefficient by

\[
q'' = h(T_{aw} - T_w)
\]

where \( T_w \) is the wall temperature and \( T_{aw} \) is the temperature the wall would be if it were adiabatic.
When the fluid enters a shock boundary layer interaction region there is an alteration of the flow pattern. This new flow pattern causes an increased particle interaction and energy exchange on a macroscopic scale, thereby increasing the heat transfer to the wall.

In uniform flow the common occurrence is for the film coefficient to decrease with distance except at a transition point. The velocity boundary layer thickness also commonly increases with distance. However, in a shock boundary layer region several combinations of changes may occur.

In the shock region there are many interacting variables, such as: pressure rise and temperature gradients, intermixed areas of subsonic and supersonic flow, boundary layer separation, and transition from a laminar to a turbulent boundary layer. In speaking of flow patterns and pressure jumps in a shock boundary layer interaction region, Ackeret et al., (11) says:

It has been known for some time that these jumps are merely Riemann's compression shocks which cause a discontinuous transition from supersonic to subsonic velocity. Unfortunately, there is no prospect of a possibility of calculating these phenomena . . . since one has to deal with differential equations which undergo basic changes in character at the (priori unknown) boundary between the subsonic and supersonic regions.

Men such as Ackeret have made valuable contributions to the experimental study of shock boundary layer interaction flow patterns, with others contributing directly to the heat transfer problem. Knowing the type of flow pattern and conditions of the boundary layer in the shock region is a necessary aid in the understanding of the heat transfer data; therefore, flow pattern data will be given.

* Numbers in parentheses refer to Cited References.
CHAPTER II

LITERATURE SEARCH

Experimental Flow Patterns

Some of the important characteristics of interest in a shock boundary layer interaction region are velocity profiles, pressure gradients, and separation. The theoretical problem is to obtain analytical expressions for the variation of these characteristics in a shock zone. Only after the hydrodynamic or velocity profile variation problem is resolved can the heat transfer problem be solved analytically. A study of the experimental flow patterns will disclose why a limited amount of analytical work on this problem has been completed.

Normal Shocks

A shock is a flow discontinuity which allows the flow to go from a supersonic speed to a higher pressure subsonic or supersonic speed in a very short distance. Since a shock cannot exist where the flow is subsonic and the viscous effects make the velocity approach zero as the wall is approached, the shock cannot penetrate completely through the boundary layer to the wall.

Plate 1 shows a normal shock in a duct at a low Mach number. Plate 2 illustrates a shock extending to the duct walls after the original boundary layer has been removed and the new boundary layer has zero thickness.

The flow between the wall and the Mach 1.0 line (see Figure 1)
Plate 1.

Plate 2.
Normal shock in a duct after boundary layers have been removed. Ref. 6.

Plate 3.
Shock boundary layer interaction in a duct at Mach 1.7. Ref. 6.
must undergo the same pressure rise as the free stream without the aid of a flow discontinuity to convert kinetic energy to pressure. Therefore, the distance the subsonic zone takes to make this pressure rise must necessarily be much greater than that of the free stream. This zone of pressure increase in the boundary layer extends both upstream and downstream from the shock in the subsonic region. For a laminar boundary layer the pressure rise at the wall takes about fifty boundary

layer thicknesses depending on the degree of flow separation. A turbulent boundary layer has more kinetic energy than a laminar layer and can attain the free stream pressure in a much shorter distance. Therefore, the upstream shock wave influence on a turbulent boundary layer is approximately five boundary layer thicknesses. See Figure 2.

One effect of the shock on the fluid in the boundary layer is to change its velocity in direction as well as magnitude. When fluid

Figure 1. Illustrative normal shock boundary layer interaction model.
Figure 2. Laminar and turbulent wall pressure rise in a shock region on a flat plate. (Ref. 10)

Figure 3. Oblique shock velocity components.

crosses a shock, its velocity in the direction normal to the shock is decreased to satisfy continuity \(\rho_1 V_{n1} = \rho_2 V_{n2}\). Since \(\rho_2\) is greater than the \(\rho_1\), \(V_{n2}\) is less than \(V_{n1}\). See Figure 3. To satisfy the mo-
momentum equation in the tangential direction $(\rho_1 V_{n1}) V_{tl} = (\rho_2 V_{n2}) V_{t2}$.

Therefore $V_{tl} = V_{t2}$ and the vector addition of $V_{n2} + V_{t2} = V_2$ shows that the flow is turned in a direction more nearly parallel to the shock. In the boundary layer interaction region there may be zones free of shocks (next to the wall), oblique shocks, and normal shocks. This creates zones of low pressure high velocity, and high pressure low velocity.

The other aspect of the interaction is the effect that the boundary layer has on the shock. Not only is the shock unable to extend to the wall because of the subsonic zone, but its direction is also modified. The pressure rise in the boundary layer upstream from the main shock has the same effect on the flow as an increase in wall slope. Oblique shocks are generated which change the flow direction. This is illustrated by the streamline in Figure 1. The oblique shocks combine to form the main shock. This mutual shock boundary layer interaction is particularly noted in Plates 3 and 4.

Concave Corner Generated Shocks

Shocks induced by a sudden change in wall slope may be called concave corner, compression corner, ramp or wall wedge induced shocks. Recently, Needham (20) has reported some heat transfer data taken from the region of a concave corner generated shock.

If the flow were inviscid, the flow pattern for subsonic and supersonic conditions would appear as in Figure 4. For supersonic flow, all streamlines are turned parallel to the ramp by a single shock which goes to the wall. For subsonic inviscid flow the streamlines are hyperbolas which approach the two wall directions asymptotically.

For viscous flow there exists both a subsonic and a supersonic portion which merge together. Here again the pressure build-up in the
Plate 4.
Normal shock in duct mouth.
Pressure probe at right. (Ref. 10)

Plate 5.
Oblique shock. Turbulent boundary layer. Mach 1.55. (Ref. 10)

Plate 6.
Oblique shock. Laminar boundary layer. Mach 1.55. (Ref. 10)
subsonic boundary layer starts ahead of the corner and generates compression waves which merge into the main shock. Plates 5 and 6 show this phenomena for a turbulent and a laminar boundary layer. The type of boundary layer and extent of separation greatly affect the flow pattern and consequently the heat transfer from the fluid to the wall. Plates 5 and 6 show the shock beginning in front of the corner, but Plate 7(a) 7(b) and 7(c) show that if there is separation, the shock may begin downstream from the corner at the point of separation reattachment. Plate 7 also illustrates the pressure plateau of the separated (S) region. Of significance is the observation that the peak pressure occurs just downstream of the point where the corner generated shock is instigated.

Viscous supersonic flow in a concave corner has a flow pattern similar to Figure 1.

**Reflected Shocks**

One case for which there is considerable heat transfer data is an oblique shock impinging on a boundary layer and reflecting. If the up-
Plate 7. The three flow regimes for a compression corner. (Ref. 15.)
Figure 5. Shock reflected off a laminar boundary layer. (Ref. 2)

Stream boundary layer is laminar, even though there may be separation, the flow pattern is fairly well understood. See Figure 5. Streamline 1 is bent away from the wall in Zone A and then rushes into the subsonic boundary layer which has been unable to obtain the pressure rise of the free stream. It may be caught up in the back flow if there is separation. Streamline 2 is first bent away from the wall in Zone A thus aiding the separation process. It then crosses a shock which bends it back parallel to the wall. Additional turning is required in Zone B because of the low pressure separated region and this induces a Prandtl-Meyer expansion wave. Or, in the words of Shapiro (2), "When a shock is inci-
dent on a free boundary at constant pressure it reflects in unlike sense." The expansion wave turns the flow back toward the wall. The final shock turns the flow parallel to the wall again. Streamline 3 follows a path similar to streamline 2.

The shock interaction of a turbulent boundary layer is less complex and, as has been pointed out, occurs over a shorter distance than it does for a laminar boundary layer. Figure 6 shows a separated and a non-separated turbulent boundary layer. Reflection from a turbulent boundary layer is much more like a regular reflection (one with no boundary layer effects) than the laminar boundary layer reflection.

![Diagram of shock interactions](image)

Figure 6. Separated and non-separated turbulent boundary layers with reflected shock interaction (Ref. 2)

Plates 8 and 9 are schlieren photographs of shocks reflected from turbulent and laminar boundary layers.

Barry, Shapiro and Neumann (3) have done experimental work on reflected shocks. Their test apparatus is illustrated in Figure 7.
Plate 8.
Reflected shock. Turbulent boundary layer.
(Ref. 10)

Plate 9.
Reflected shock. Laminar boundary layer.
(Ref. 10)
Figure 7. Reflected shock generation.

Figure 8. Reflected shock boundary layer thickness increase.
These authors found that the pressure rise across the shock was slightly less than the predicted normal reflection pressure rise if the boundary layer was laminar. If the boundary layer was turbulent the pressure rise matched the theory. The plotted pressure rise and boundary layer thickness change versus plate deflection angle $\theta$ was given in the preceding reference. These two graphs are replotted in Figure 8 to show pressure rise versus boundary layer thickness change. This figure illustrates the authors' findings that the boundary layer thickness increases with distance and the increase is less at higher Reynolds' numbers. Barry, Shapiro, and Neumanns' data on reflected shock boundary layer increase plotted in Figure 8 was taken from schlieren photographs.

**Lambda Shocks**

Lambda shocks are a modification of other types of shocks near the boundary layer interaction region. They occur when there is extensive mutual interaction between the shock and the boundary layer. Figure 9 illustrates a lambda shock structure near a wall.

At point A, when the boundary layer needs all its momentum to make the free stream pressure rise, the fluid is turned away from the wall by the oblique shock generated upstream. If the oblique shock angle ($\theta$) or the free stream Mach number is large enough, the pressure at B may be sufficiently low that back flow (separation) may occur. The flow accelerates toward the low pressure region (B). If its velocity exceeds Mach 1.0 it will shock again, helping to turn the flow parallel to the wall in Zone C.

Lambda shock flow phenomena are common on airplane wings and they may be present in ducts, especially when a shock occurs in a duct with a relatively thick boundary layer.
The duct normal shock in Plate 3 has a lambda shock structure near both the upper and lower wall. When the flow passes through the shock, the fluid that flows through the most normal part of the shock is at a higher pressure than the fluid next to the wall. It therefore expands into the lower pressure regions. If the boundary layer is thick, the relative volume of the low pressure region to the high pressure region may be great enough that, in equalizing the pressure, the flow may go supersonic. When this happens a series of shock structures may be established to form the familiar diamond pattern. The boundary layer thickens at each shock and appears as a supersonic diffuser acting on the core flow. Weak shocks downstream of the main shock are visible in Plate 3.

Ackeret, et al. (11) have published detailed pressure distribution
data on such shock patterns. See Plates 10, 11, 12, and 13. No experimental heat transfer data is available for this particular shock pattern but it is similar to the reflected shock for which heat transfer data is available.

In a low pressure region caused by air flowing over a wing section the flow may pass Mach 1.0 several times. Plate 10 illustrates such a shock pattern. The valleys in the pressure curve are places of accelerating flow. A scale on the schlieren photographs shows graphically just where the low pressure areas are. The Mach number is greater than one when the pressure ratio \( p/p_0 \) is less than approximately 0.52. At an elevation of 20 mm the pressure ratio went below 0.52 three times and there are three distinct shock patterns in the photograph.

Plate 11 shows the lines of constant pressure in a lambda type shock. The flow is generally high speed next to the wing. At \( X = 230 \text{ mm} \) there is an island of supersonic flow illustrating increasing speed on the left of the island and decreasing speed on the right of the island. Plate 12 shows some back flow existing very close to the wall in the region from \( Z = 0 \) to \( Z = 3 \text{ mm} \). The incoming flow in this case is laminar and therefore separates at much lower pressure gradients than a turbulent boundary layer. The thickness of the boundary layer is from \( Z = 0 \) to the \( Z \) position where the slope of the velocity profile \( \frac{\partial M^*}{\partial Z} \) is zero. The plate shows a constantly thickening boundary layer from 2 mm at \( X = 145 \text{ mm} \) to 10 mm at \( X = 265 \text{ mm} \). Plate 13 gives the pressure variations at six different elevations. It shows that the main fluctuations in pressure occur at the lower elevations around \( Z = 20 \) to 40 mm.

The Ackeret pressure information shows that in evaluating the heat transfer to a wall in a shock interaction region, it must be realized that the wall values of pressure and mass velocity may be quite different
Plate 10. Lambda shocks on a wing. Schlieren photograph and pressure distribution at two elevations. (Ref. 11)
Plate 11. (top) Pressure distributions at x=180 mm and x=222.5 mm while the main lambda shock was at about 200 mm. (bottom) Lambda shock isobars. (Ref. 11)
Plate 12. Lambda shock boundary layer velocity profiles. Fluctuations in pressure are indicated by dots connected with arrows. (Ref. 11)
Plate 13. Lambda shock pressure distribution at various distances from the wall. Flow over a wing section. (Ref. 11.)
from the free stream values. This also indicates why transition usually
occurs near a shock boundary layer region. The greatly fluctuating
velocity and pressure in a shock boundary layer region induces a turbu-
lent boundary layer if one does not already exist. Plates 1 and 9 show
the laminar boundary layer in a duct becoming turbulent upon passing
through a shock region.

**Theoretical Flow Models**

The pages prior to this have been devoted to demonstrating the
complexities of the shock boundary layer interaction regions. Because
of the complex nature of the flow, very little theoretical work has been
published. The principle analytical work has been on the pressure rise
necessary to induce separation and the boundary layer thickness varia-
tion.

A common method of solving for the heat transfer to the wall would
be to solve the boundary layer momentum equation to obtain the velocity
profile. This would then be related to the temperature profile by an
analogy method. In deriving the constant property boundary layer momen-
tum equation (see Figure 10 for nomenclature), i.e.

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}
\]

the assumption is made that \(u > v\) and \(\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}\). These assump-
tions lead to the fact that \(\frac{\partial p}{\partial y} = 0\). Plates 10, 11, 12, and 13 illustrate
that the pressure gradient normal to the surface may not be zero and that
\(\partial u/\partial x, \partial v/\partial x\) and \(\partial v/\partial y\) may all become significant compared to \(\partial u/\partial y\).
Therefore the boundary layer momentum equation is not valid for the gen-
eral case of shock boundary layer interactions.
Hammitt's Boundary Layer Thickness Change

There is some question as to the mechanism by which the boundary layer is able to overcome the adverse pressure gradient in a shock zone. Hammitt (1) says:

The only explanation which has been suggested in the literature is that momentum, added to the boundary layer through turbulent mixing, overcomes the additional adverse pressure gradient. Since the distances are short very high shear forces or mixing rates would be required to add much momentum to the layer. Before looking for a mechanism by which large quantities of momentum can be fed into the boundary layer it should be determined whether such a mechanism is required to explain the observed pressure rises. The forces caused by ordinary turbulent mixing rates are small compared with the pressure and momentum forces in the boundary layer and are negligible in a first order theory. Until it has been shown that large increases in mixing occur through the shock interaction region, it is reasonable to assume that these mixing rates have the usual values encountered in the turbulent boundary layers.

Hammitt made the assumptions in his work that no extraordinary turbulent mixing added momentum to the boundary layer, that wall friction was negligible, and that the turbulent boundary layer is a one parameter family. The parameters used were \( f \) and \( K \) where \( f \) and \( K \) were related to each other by empirical data. \( K \) was defined as the average velocity in the boundary layer divided by the free stream velocity. The quantity \( f \) was introduced to satisfy the equation

\[
\frac{T}{T_0} = f - \frac{k-1}{2} \frac{V^2}{M}.
\]
Hammitt combined the $f$ and $K$ relationships with the equation of state, continuity, and momentum equation to calculate the boundary layer thickness change across a shock interaction region. Some of Hammitt's results are shown in Figure 11.

Figure 11 shows that boundary layer changes for reflected shocks are less than one. This does not agree with the work of Barry, Shapiro and Neumann. See Figure 8. Hammitt has one data point taken from his reference 9 which supports his theory. This point was taken from total pressure measurements instead of photographs. Hammitt says:

The Schlieren pictures show a considerably thicker boundary layer behind the interaction region. This difference is probably caused by the side wall effects on the Schlieren pictures. For this reason optical measurements of boundary layer thickness behind interaction regions are unreliable.

In an article by Hammitt, Vas and Hight (4) the statement is made that "the boundary layer contains enough momentum to overcome the pressure rise without gaining momentum from the free stream." They believe that there is merely a redistribution of momentum. They state that "... the stream tubes near the wall gain momentum from those towards the center of the layer and that the outer part of the boundary layer is relatively unaffected." Plate 13 suggests this same idea because the greatest pressure fluctuations are close to the wing. However, flow next to a wing has naturally higher pressure gradients than the free stream. In a duct this is not true.

Production of Turbulent Energy

Back, Massier and Gier (5) have pointed out that the level of turbulent transport in a rocket nozzle is decreased because the gases are accelerating. The turbulent kinetic energy production term is:

$$-ar{u'v' \frac{\partial u}{\partial y}} - (\bar{u'^2} - \bar{v'^2}) \frac{\partial u}{\partial x}$$
Figure 11. Variation of boundary layer thickness with pressure ratio and Mach number for a shock reflected from a boundary layer. (Ref. 1)
The first term is positive since if \( u' \) is positive \( v' \) is negative and if \( u' \) is negative \( v' \) is positive. In a rocket nozzle \( \frac{\partial u}{\partial x} \) is positive so the second term subtracts from the first term. However, in a shock interaction zone the average value of \( \frac{\partial u}{\partial x} \) is negative. According to this analysis there will be an increase in turbulent mixing in a shock boundary layer interaction region.

**Pressure Rise Required for Separation**

Tyler and Shapiro (7) have contributed to solving the problem concerning mass addition to the boundary layer. They did this while trying to predict the pressure necessary to separate a turbulent boundary layer. In their analysis they assume that, since the pressure rise occurs in an extremely short distance, skin friction and the mass flow brought into the boundary layer from the free stream are negligible.

The criteria Tyler and Shapiro used for deciding that their analysis was accurate was the available experimental data. Some of the experimental data and the above authors' theoretical results are plotted in Figure 12 with the coordinates of pressure coefficient and Reynolds number. Since their theory gives values of pressure rise supportable without separation lower than experimental values they conclude that skin friction and mass added to the boundary layer are of some importance. They state: "... the present analysis should give too low a value for \( P/C_p \) inasmuch as the incoming momentum augments the pressure rise supportable by the boundary layer." The results were of the same order of magnitude as experimental results but slope upward with increasing Reynolds number instead of downward.

The experimental data given in Figure 12 has been empirically correlated by Donaldson and Lange (8). Their equation for both laminar and
turbulent flow is given in the figure. The subscript \( d \) stands for post-shock conditions and \( u \) stands for pre-shock conditions. The difference between the laminar and the turbulent line graphically indicates the difference in kinetic energy of the two types of boundary layers.

![Graph showing pressure rise across a shock](image)

Figure 12. Pressure rise across a shock required to separate the boundary layer.

**Experimental Heat Transfer Data**

**Free Flight Rocket Fin Test: Chauvin and Buglia**

Chauvin and Buglia (9) instrumented a rocket fin which had a trailing edge flap. The flap caused an oblique shock across which temperature measurements were made by the use of thermocouples. See Figure 13.

The method used by these and most other experimenters to obtain the convective film coefficient was to relate it to the wall temperature-time curve obtained from the data. They estimated that radiation and axial conduction were negligible. An energy balance shows that the heat
energy into the fin is equal to the change in stored energy.

\[ h (T_{aw} - T_w) = \rho_w (th) c_w (dT_w/\Delta t) \]

The adiabatic wall temperature \( T_{aw} \) was found by the use of the recovery factor \( R \).

\[ R = \frac{T_{aw} - T}{T_o - T} \]

A turbulent recovery factor, \( R = Pr^{1/3} \), was used. Velocity data was obtained by Doppler radar, and altitude and flight-path were measured by tracking radar. Atmospheric conditions and wind directions and velocities were measured by radiosondes launched near the time of the flight.

Until the rocket reached about \( M = 1.8 \) the data was not reduced because of the loss in accuracy due to low heating rates. From \( M = 1.8 \) to \( M = 2.6 \) the film coefficients on the windward side of the flap (B) were about 2.5 times those on the fin (A) for a flap deflection angle of...
On the leeward side of the flap (D) the film coefficient value was about one-third as large as the value at point C.

Their data was given in terms of the free stream Stanton number $St = h/\rho Vc$ and Mach number $M$. Shock theory was used to convert this data to film coefficient ratio versus pressure ratio across the fin flap junction. The results are given in Chapter III.

Rocket Nose Cone Experiment: Rumsey and Lee

Rumsey and Lee (12) made a free flight test with an instrumented cone-cylinder-flare model placed on the nose of a four stage rocket. Figure 14 gives an example of a cone-cylinder-flare model. Their method of obtaining the film coefficient at various axial locations differed slightly from the test Chauvin and Buglia made. Although they neglected axial conduction and solar radiation they did account for internal and external radiation. Their film coefficient equation was

$$h = \frac{\sigma T_w^4 \left( \frac{dT}{dt} + \frac{\sigma T_w^4}{\rho c_w (th) w} + \frac{\sigma (T_w^4 - T_l^4)}{r_w^2 \frac{1}{G_i} + \frac{1}{G_i} - 1} \right)}{T_{aw} - T_w}$$

Telemetered axial and normal acceleration data was taken to find if the rocket was stable at the time the heat transfer data was recorded. This was necessary because of fluctuating angles of attack incurred due to separation and firing of the various stages. The angle of attack affects the shock strength and heat transfer to the wall. Reliable data was obtained just as the rocket reached a Mach number of 4.9. The fourth stage was to fire at that moment and accelerate the rocket to a Mach
number of 9.0. The fourth stage failed to fire, so only one Rumsey-Lee data point is given in Chapter III.

Figure 14. Cone-cylinder-flare model.

**Heat Transfer and Flow Separation: Schaefer and Ferguson**

Schaefer and Ferguson (13) also used a cone-cylinder-flare model but they conducted their test in a wind tunnel where they could maintain a uniform Mach number of 4.98. The pressure distribution in the flare shock region was found to be dependent on the type of boundary layer separation. There are three types: (1) pure laminar separation where transition occurs downstream of reattachment (LA to LA), (2) transitional separation where transition occurs between separation and reattachment (LA to TU), and (3) turbulent separation where transition is upstream of separation (TU to TU).

The authors varied the flare angle (10°, 17°, 24°, 56°); the free stream Reynolds number from 1.6-4.5 x 10^5/ft; and the wall temperature from $T_w/T_{aw} = 0.18$ to 1.0. They found that the length of separation
decreased with wall cooling, increasing Reynolds number, and decreasing flare angle. The authors found that decreasing the flare angle decreased the length of separation because the shocks in their experiment were all of the weak shock variety, having decreasing strength with decreasing deflection angle. Decreasing the flare angle made the pressure gradient at the cylinder flare junction smaller so that separation was less likely.

A sample of these authors' experimental data is given in Figure 15. They found that the stagnation point heat transfer equation of Reshotko (14) followed the data on the flare if it was assumed that a new boundary layer started at the 'cylinder-flare junction. The Reshotko equation accounts for velocity gradients and temperature changes. Schaefer and Ferguson do not say what assumptions they made in using the Reshotko equation.

**Extensive Flow Separation: Becker and Korycinski**

Becker and Korycinski (16) were particularly interested in heat transfer in separated regions because of conflicting views as to whether a separated region had an insulating effect. Stalder and Nelson (17) had reported a separation heat transfer test using a model consisting of a spherical nose with a spike on the end. This reference stated that the effect of the spike was to cause separation and increase the heat transfer above the laminar level found in unseparated flow without the spike. Becker and Korycinski did not feel this evidence was conclusive because of the possibility that transition to turbulent flow may have been triggered by the spike.

An interesting statement made by Becker and Korycinski concerning the possibility of using theory to evaluate the heat transfer is:
Figure 15. Tripped boundary layer heat transfer and pressure distribution on a cone-cylinder-flare. 24° flare.
Aside from the foregoing questions pertaining to the separated zone, the heat transfer in attached flow on a flare or flap cannot be calculated theoretically at present because of inadequacies in current theories for predicting boundary-layer profile changes through the phenomena found at the body-flare juncture."

The authors used a cone-cylinder-flare model of similar dimensions to the Schaefer and Ferguson model. Their tests were made in a wind tunnel at \( M = 6.8 \). Two graphs are given as examples of their results. See Figures 16 and 17.

Figure 16 shows that the transition point moved upstream as the Reynolds number increased. The opposite happened to the separation point, however, because of the increased kinetic energy in the boundary layer. When the incoming boundary layer was turbulent no separation could be detected. On one test for which the Reynolds number was above \( 0.6 \times 10^6 \), no separation was observed at the cylinder flare junction even though the local pressure coefficient was greater than 0.7. This does not conform to the separation data previously mentioned. When the points of separation and transition were in the same area they fluctuated back and forth so much that accurate location readings were difficult. A schlieren aided in the detection of separation and transition.

When the boundary layer over the separated zone was laminar (at low Reynolds number) and transition did not occur before reattachment, the heat transfer to the wall under the separation was approximately half as much as it was on the upstream side of the separated zone. In this laminar case, the film coefficient jumped by a factor of 3.2 across the shock zone, while the pressure rose by a factor of 2.75. Figure 17 shows a case of the pressure rising across the shock zone by a factor of 24.3 while the film coefficient jumps by a factor of 40.0. These are minimum pre-shock to maximum post-shock values. The film coefficient ratio is
Figure 16. Effect of Reynolds number on separation and transition point. 10° flare. (Ref. 16)
Figure 17. Heat transfer and pressure distribution on a cone-cylinder-flare. 30° flare. (Ref. 16)
obtained by merely dividing the upstream \((u) \text{St} \times \text{Re}_D^{0.5}\) into the downstream \((d)\) value. Since the Stanton and Reynolds numbers are based on free stream wind tunnel conditions, this reduces to the film coefficient ratio \(h_d/h_u\). Appendix A shows the method used to obtain the pressure ratio from the pressure coefficient.

Like Ferguson and Schaefer, these authors found that the heat transfer to the wall in the flare could be approximated reasonably well by assuming that the boundary layer started at the cylinder-flare junction.

**Needham's Mach 9.7 Test**

Recently Needham (20) ran a shock boundary layer interaction test in the Imperial College (London) Hypersonic Gun Tunnel. He obtained very congruous data which is given in Plate 1^4. He does not give the stagnation temperature or pressure at which the test was made, nor does he indicate what generated the upstream oblique shock. Appendix B shows how Needham's data was converted to the parameters used in this thesis and the assumptions made in doing so.

Plate 1^4 shows clearly the increase in boundary layer separation as the pressure gradient increases. The effects of separation are evident in the pressure and heat flux plot and schlieren photographs. In the schlieren photographs, the separation appears as a white zone of increasing size around the six inch mark.

Also of interest is the boundary layer thickness change across the shock. The first shock is generated at the start of the pressure increase upstream from the corner. This first shock combines with the other shocks generated in the separated zone to become the main wedge shock. The union occurs at about the 7.5 inch mark, and at this point
Plate 14. Pressure- and heat-transfer distributions and schlieren photographs of flow over a compression corner at various wedge angles. $M_\infty = 9.7$, $Re_\infty = 1.48 \times 10^5$/in. (Ref. 20)
the boundary layer is extremely thin. The heat flux reaches a peak in this area. The combination of the two effects suggests that perhaps the method of predicting the heat flux by assuming that a new boundary layer begins at the corner could be used, as some authors have indicated.

The Douglas Reflection Shock Test

Sayano at Douglas Aircraft Company (18) has done extensive research on reflected shock waves interacting with turbulent boundary layers. They performed tests at various Mach numbers, total pressures, total temperatures and impinging shock angles. Pressure and heat transfer data, pre-shock velocity profiles, and boundary layer thicknesses were obtained for a shock impinging on both a cylinder and a flat plate. Measurements were made on the side of the cylinder as well as on the top. The Douglas test apparatus was the same as that shown in Figure 7.

They found at the higher Mach number that the boundary layer thickened upstream of the impingement point and that it became thin downstream. Quantitative measurements of the boundary layer thickness change were not made. At low Mach numbers no boundary layer thickness change was visible in their schlieren photographs. The Douglas heat transfer and pressure followed the same pattern as the Needham data except that the peak film coefficient values were not as high as the peak pressure values. They found that their data correlated well with the empirical equation

$$\frac{h_{\text{peak}}}{F_0} = \left(\frac{P_{\text{peak}}}{P_0}\right)^8$$

Boeing's Mach 16 Test

Boeing Aircraft Company (19) made a study of shock boundary layer
interaction on a flat plate at Mach 16.0. They made this test to obtain information which might aid in the design of hypersonic aircraft. They found that heat flux increases of thirty times the undisturbed values could occur in the shock region.

A sketch of the Boeing test arrangement is shown in Figure 18. Their experimental data shows an extremely sharp peak in the Stanton number versus distance from the shock generator graph. As the angle was increased the Stanton number peak moved closer to the shock generator. This is probably partly due to the fact that the boundary layer thickness on the generator decreased and also because $d\theta/d\alpha$ is less than 1.0 under the test conditions used.

![Figure 18. Boeing's test setup.](image)

Table 1 gives some of the variables relating to the seven tests described.
### TABLE I

**TEST PARAMETERS**

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<tr>
<th></th>
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<th>β°</th>
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* Wind Tunnel

** Free Flight
CHAPTER III

RESULTS

**Empirical Correlation**

One of the objectives of this thesis was to find a method of predicting the heat flux to the wall in a shock-boundary layer interaction region. By using existing equations the local heat flux can be calculated along the wall up to the interaction zone. The problem was to expose the parameters which would affect the change in heat flux in the interaction zone and present them in a usable form.

Besides the significant parameters, the form of the experimental data given in the literature was a determining criteria for correlating the variables. All the data, except that presented in the Needham and Douglas references was given in terms of the Stanton number. In order to obtain the heat flux variation from the Stanton number, or the Stanton number from the heat flux, it was necessary to know the local wall temperature. None of the references gave wall temperature information. Since Needham's was the only data given in terms of heat flux (see Plate 14), a constant wall temperature was assumed to convert his heat flux ratio to film coefficient ratio. Appendix B shows how the film coefficient ratio was derived from Needham's heat flux ratio.

The ratio method was chosen to display the parameters. By giving the ratio downstream to the upstream parameters in the shock zone the results can be displayed in a general and usable form. The ratio of
h_d/h_u was calculated for each set of experimental data. Since the maximum heat flux is of greatest interest, h_d is the peak downstream film coefficient. The maximum heat flux can be calculated for any specific case by the equation \( q'' = \left( \frac{h_d}{h_u} \right) (h_u) (T_{aw} - T_w) \). The \( h_u \) can be ascertained from one of the theoretical equations given in the next section.

A proven method to evaluate the film coefficient downstream of the post-shock peak value (h_d) is unknown. A suggested method for reflected shock cases is to assume that the film coefficient decreases with distance from the peak value at the same rate it decreases from this value at the leading edge of a flat plate. When the film coefficient value calculated this way reaches the pre-shock value the calculations could be resumed using the pre-shock prediction equation. This method may also be reasonable for concave corner generated shocks if a stagnation effect term were added, or a stagnation equation like Reshotko's could be used.

Figure 2 and the Douglas experiment suggest a method to empirically predict the film coefficient ratio. Figure 2 shows that the wall pressure rises slightly above the predicted value. This could be caused by a downward component of velocity connected with mass addition to/or "momentum relocation" in, the boundary layer. Pressure rise also gives a measure of the shock strength which is expected to be an important parameter. Figures 19, 20, and 21 show the experimental relationship between pressure ratio and film coefficient ratio in a shock region. The peak downstream values were used.

Figure 19 contains experimental data for pressure ratios greater than one. There is almost a linear relationship between pressure ratio and film coefficient ratio for the LA to IA and TU to TU cases. The reflected shock data of Douglas (18) falls consistently below the one-
Figure 19. Film coefficient ratio versus pressure ratio across shocks with LA to LA, LA to TU, and TU to TU boundary layers.
Figure 20. Film coefficient ratio versus pressure ratio across shocks with LA to LA and TU to TU boundary layers.
Figure 21. Film coefficient ratio versus pressure ratio across Prandtl-Meyer expansions.
to-one line after the pressure ratio becomes greater than five. More experimental data is needed to establish this. Experimental data is also needed for LA to LA and TU to TU concave corner induced shocks to verify the linear relationship at pressure ratios above eight.

Figure 20 is a magnified view of LA to LA and TU to TU data for pressure ratios between one and eight. The data varies by as much as twenty-five per cent on either side of the one-to-one line.

Figure 21 illustrates the attempt made to provide film coefficient ratio data for pressure ratios less than one. The Chauvin-Buglia points are calculated from the Prandtl-Meyer expansion data measured on the leeward side of the rocket fin sketched in Figure 13. The Jack-Diaconis (22) point, and the Zakkay-Toba-Kuo (23) points were obtained from measurements taken in the expansion zone of a cone-cylinder junction. Sternberg (24) in his report called "The Transition from a Turbulent to a Laminar Boundary Layer" has experimental proof that a new laminar sub-layer builds up on the downstream side of a convex corner. This explains the reduction in film coefficient.

Theoretical Prediction

Transition in the interaction region was obviously one of the major factors which could cause film coefficient variation. Even in undisturbed flow, transition causes a large increase in film coefficient. This is illustrated in Figure 22.

The Reynolds number based on local stream properties outside the boundary layer does not change greatly through a shock region. This is because the viscosity increases approximately the same amount as the mass velocity \( G = \rho V \), and the length dimension change is small. If the boundary layer remains laminar (LA to LA) or remains turbulent (TU
Figure 22. Theoretical variation of Stanton number versus Reynolds number. Pr = 0.72. (Ref. 26)
to TU), minor changes in Stanton number would be expected in the shock zone.

From a theoretical viewpoint, the two LA to IA values given in Figure 20 would be expected to be higher than the TU to TU values because the slope of the laminar curve in Figure 22 is greater than the slope of the turbulent curve. If transition occurred in the shock zone (LA to TU) the alteration would be from the laminar curve to the turbulent curve and large changes in Stanton number would arise. An even greater jump in film coefficient would be expected for oblique shock cases since mass velocity also increases and mass velocity is in the denominator of the Stanton number.

The three equations used in the theoretical prediction of film coefficient change across a shock are given below. They were obtained by solving for the film coefficient ratio from the Stanton number ratio using the equations of Figure 22. The distance variation in the Reynolds number is neglected.

\[
\frac{h_d}{h_u} = \frac{0.89}{0.5} \left( \frac{\mu_d}{\mu_u} \right)^{0.5}
\]

\[
\frac{h_d}{h_u} = \left( \frac{G_d}{G_u} \right)^{0.5} \left( \frac{\mu_d}{\mu_u} \right)^{0.5}
\]

\[
\frac{h_d}{h_u} = \left( \frac{G_d}{G_u} \right)^{0.8} \left( \frac{\mu_d}{\mu_u} \right)^{0.2}
\]

The viscosity was found from viscosity-temperature tables and the temperature and mass velocity were calculated for each case. Table 2 contains the results of this theoretical prediction. The values obtained
### TABLE 2
**LIST OF RESULTS**

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<tr>
<td>TU to TU</td>
<td>1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>Douglas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TU to TU</td>
<td>1.72</td>
<td>2</td>
</tr>
<tr>
<td>TU to TU</td>
<td>3.8</td>
<td>9.2</td>
</tr>
<tr>
<td>Boeing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LA to TU</td>
<td>15</td>
<td>27</td>
</tr>
<tr>
<td>IA to TU</td>
<td>7.25</td>
<td>16.6</td>
</tr>
</tbody>
</table>
using the theoretical "no pressure gradient" equations are generally about one-half the experimental values.

**Errors**

Considering the number of assumptions made to obtain the predicted values, the accuracy is encouraging. For instance, to obtain ratios across the shock at the cylinder flare junction on the cone-cylinder-flare models cone shock theory (21) was used to evaluate the stream properties after the nose shock. Then Prandtl-Meyer expansion theory was used to obtain the property changes across the cone-cylinder junction. Since the diameter of the cylinder was small, three dimensional effects would decrease the accuracy at this point. The property changes across the flare induced shock could be evaluated by using the experimental pressure ratio, cone theory or concave corner shock theory. Using the latter theory usually yielded answers for \( \frac{h_d}{h_u} \) nearer to experimental values, but three dimensional effects again introduced error. The predicted values given in Table 2 were derived by using the experimental pressure ratio to obtain the property values in the shock zone. These values were employed in one of the three preceding equations to evaluate the predicted \( \frac{h_d}{h_u} \) ratio.

The confidence level in the local properties calculated from the Boeing data is not very high. At Mach 16 the weak shock angles are small, consequently two shocks and two boundary layers were interacting in the measurement area. This fact may make the calculated properties inaccurate.

Naturally there was error in the given experimental data. The major source of this error was in calculating the derivative of the temperature-time curve. Schaefer and Ferguson (13) claim that their
results are accurate within ± 20 per cent.

Another source of error may be in experimentally specifying the state of the boundary layer in the shock zone. If the Prandtl number and Reynolds number are assumed to be constant across the shock zone, and the boundary layer undergoes transition, the Stanton number ratio is:

\[
\frac{S_{td}}{S_{tu}} = \frac{0.0288 \, \text{Re}^{-2}}{0.332 \, \text{Re}^{-0.5}} = 0.089 \, \text{Re}^{-3}
\]

For a Reynolds number of $5 \times 10^6$ the expected Stanton number change would be 9.1 for an LA to TU case. Figure 23 shows a plot of the experimental Stanton number ratios versus the experimental Reynolds number ratios given in Table 2. One of the Becker and Korycinski (16)
values given as a TU to TU case has a Stanton number ratio of 9.3 which would indicate that it may have been an IA to TU case. If the IA to TU equation is used to calculate the film coefficient ratio the predicted value is 41.6. The experimental value for this case is 40 and the predicted value in Table 2 using the TU to TU equation is 12.3. This indicates that perhaps transition did not occur upstream of the shock zone as Becker and Korycinski have indicated.

Consideration of Mass Addition to the Boundary Layer

A theoretical attempt to predict the change in boundary layer thickness across a shock boundary layer interaction region was discussed in Chapter II. Some of the results derived by Hammitt (1) were given in Figure 11. Ritter and Kuo (25) have also published work in this area for very weak shocks.

Hammitt neglected mass addition but he used some empirical relations in his derivation. If there is mass addition to the boundary layer in a shock region, the accuracy of Hammitt’s calculations would be improved by using empirical data.

Figure 24 shows the results of calculating the boundary layer thickness change without using empirical data.

The equations used to obtain the information plotted in Figure 24 were the equation of state, energy equation, continuity equations, and a Ts equation. These equations reduced to the following expression for the boundary layer thickness $\delta$:

$$\frac{d\delta}{\delta} = (1 - \frac{1}{kM}) \frac{k}{2} \frac{dM^2}{d\delta} + \left( \frac{k}{k-1} \right) \frac{d\delta}{\delta} \frac{dS}{cp} + \frac{dT}{T} .$$

If friction is neglected this equation reduces to:
Figure 24. Theoretical boundary layer thickness change with no mass addition.
\[
\frac{\delta_d}{\delta_u} = \frac{M_u}{M_d} \left( M_d^2 - M_u^2 \right) / 2
\]

With the aid of a computer this equation was solved for various upstream and downstream Mach numbers. The Mach numbers were then related to the pressure ratio by using oblique shock theory. Appendix C gives the details of the derivation of the above equations along with the computer program.

The values described by the equation above are lower than those given by Hammitt, and are probably not as accurate. They seem especially unrealistic at the higher Mach numbers and pressure ratios. This indicates that at high Mach numbers and pressure ratios, mass addition to the boundary layer becomes an important factor. Skin friction is perhaps never very important in a shock zone because of short distances and separation tendencies.
CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. Large fluid mixing rates occur in a shock-boundary layer interaction region. This macroscopic mixing causes the post-shock heat flux to be much larger than the pre-shock values.

2. The type of boundary layer in the shock zone affects the amount of heat flux increase. If transition occurs in the shock zone the increase is much greater than if transition takes place upstream or downstream from the shock.

3. According to existing data, if transition does not occur in the shock zone, the empirical relationship \( \frac{h_d}{h_u} = \frac{P_d}{P_u} \) is the most accurate method for prediction of the film coefficient increase. The limited data shows that, if the boundary layer remains laminar, the film coefficient rise is slightly greater than if it is turbulent on both sides of the shock zone.

4. If transition occurs in the shock zone, the theoretical equation

\[
\frac{h_d}{h_u} = 0.089 \left( \frac{d}{u} \right)^{0.8} \left( \frac{\mu_d}{\mu_u} \right)^{0.2}
\]

is the best way at present to predict the film coefficient increase.

5. The type of shock also affects the heat flux increase. Re-
fleeted shocks do not cause as great an increase in heat flux as concave corner generated shocks.

6. If the boundary layer separates, the heat flux is highest at the point of reattachment. Under the separated zone the heat flux may be lower than the upstream value.

**Recommendations**

1. Although it is evident that the heat flux downstream from a shock boundary layer interaction region is larger than the upstream value, exactly how far downstream this condition prevails is unknown. If the increase was only a local "spike," axial conduction in the wall could help relieve the thermal stress of the wall material. The concave corner shock data does not indicate that the heat flux along the wall comes to a peak and then falls to its pre-shock value. Further research is needed to determine how far downstream from the shock the high heat flux remains.

2. There is no known available experimental data on heat flux variations in the boundary layer zone of a normal shock wave in a duct or a lambda shock on a wing section. Carefully controlled tests need to be made for these two types of shocks. The smallest possible temperature sensing devices should be used in the tests so that closely spaced data points can be obtained.

3. Research needs to be conducted at various pre-shock boundary layer thicknesses, Reynolds numbers, Mach numbers, and shock strengths. A series of tests need to be made for strong solution shocks, since all the available data in the references is for the weak shock type. Also of interest would be boundary layer thickness data in the shock zone obtained with pressure probes.
APPENDIX A

METHOD OF OBTAINING PRESSURE RATIO FROM PRESSURE COEFFICIENT DATA
APPENDIX A

Method of Obtaining Pressure Ratio
From Pressure Coefficient Data

\[ C_p = \frac{p - p_\infty}{(\rho_\infty v^2)/(2g)} = \frac{p/p_\infty - 1}{(\rho_\infty v^2)/(2g p_\infty)} \]

\[ \rho_\infty/p_\infty = 1/(RT_\infty) \]

\[ C_p = \frac{p/p_\infty - 1}{(v^2)/(2gRT_\infty k/k)} = \frac{p/p_\infty - 1}{M_\infty^2 k/2} \]

Therefore,

\[ p/p_\infty = C_p M_\infty^2 k/2 + 1 \]
APPENDIX B

METHOD OF EVALUATING NEEDHAM'S HEAT TRANSFER DATA
APPENDIX B

Method of Evaluating Needham's Heat Transfer Data

Example: Given $M_1 = 9.7; \theta = 7.6^\circ; q_3/q_2 = 4.0; p_3/p_2 = 3.9.$

$\theta$ was measured to be about $10^\circ$ therefore by using oblique shock
theory $M_2 = 8.0, p_3/p_2 = C_{p3} M_2^2 k/2 + 1 = 3.8$ if $k = 1.4$. This agrees
with the experimental pressure rise, therefore $M_2$ is assumed to be

1. $h_3 \frac{q''}{h_2} = q''_2 \left( \frac{T_{aw2} - T_w}{T_{aw3} - T_w} \right)$ Assume $T_w =$ constant.

$T_{aw} = R(T_0 - T) + T$. Putting this in 1 and dividing numerator and
denominator by $T_2$ yields:

2. $h_3 \frac{q''}{h_2} = q''_2 \left( \frac{R(T_0/T_2 - 1) + 1 - T_w/T_2}{R(T_0/T_2 - T_3/T_2) + T_3/T_2 - T_w/T_2} \right)$

Assume $R = 0.9$. 
\[
\left( \frac{T_2}{T_0} \right)_{M=8} = .0728
\]

\[
\frac{h_3}{h_2} = 4 \left( \frac{0.9(1/0.0728 - 1) + 1 - T_w/T_2}{0.9(1/0.0728 - 1.58) + 1.58 - T_w/T_2} \right) = \\
\left( \frac{12.4 - T_wT_2}{12.48 - T_w/T_2} \right) \approx 4
\]
APPENDIX C

DERIVATION OF BOUNDARY LAYER THICKNESS CHANGE IN A SHOCK ZONE
APPENDIX C

Derivation of Boundary Layer Thickness Change in a Shock Zone

Energy Equation (adiabatic wall)

1. \( \frac{dh}{dt} = VdV \)

Tsds Equation

2. \( \frac{dh}{dt} = Tsds - \frac{dp}{p} \)

Equation of State

3. \( p = \rho RT \)

Continuity

4. \( \rho V \delta = \text{Constant} \)

Equations 3 and 4 can be changed to the following forms:

5. \( \frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \)

6. \( \frac{d\rho}{\rho} = -\frac{dV}{V} - \frac{d\delta}{\delta} \)

Equations 1, 2, and 3 can be combined to give:

7. \( -VdV = Tsds + RT \frac{d\rho}{\rho} + RdT \)

Putting \( \frac{d\rho}{\rho} \) from equations 6 into equation 7 and solving for \( \frac{d\delta}{\delta} \) yields:

8. \( \frac{d\delta}{\delta} = VdV/(RT) + \frac{ds}{R} - \frac{dV}{V} + \frac{dT}{T} \)
Introducing Mach number and specific heat relationships into equation 8 yields:

9. \( \frac{d\delta}{d\delta} = (1-1/(kM^2))dM^2/2 + (k/(k-1))ds/c_p + dT/T \).

Upon neglecting friction and using the influence coefficients in Shapiro (27) Chapter 8, equation 9 becomes:

10. \( \frac{d\delta}{d\delta} = \frac{dM^2}{2} - \frac{dM^2}{(2M^2)} \).

Integrating this equation from \( M_u \) to \( M_d \) gives:

11. \( \frac{\delta_d}{\delta_u} = \frac{M_u}{M_d} e^{(M_d^2 - M_u^2)/2} \)

Note: In this type of analysis the boundary layer Mach numbers are average values.
BOUNDARY LAYER THICKNESS PROGRAM

D = Delta2/Delta1

REAL MACH1, MACH2

DIMENSION MACH1(5), MACH2(25), D(25)

READ (5, 100) (MACH1(I), I = 1, 5)
READ (5, 100) (MACH2(J), J = 1, 25)

FORMAT (5F10.4)

WRITE (6, 203)((I, MACH1(I), I = 1, 5)
FORMAT (///, 5(2X, 5HMACH1, I3, F7.3, 5X))
WRITE (6, 204)((J, MACH2(J), J = 1, 25)
FORMAT (///, 5(2X, 5HMACH2, I3, F7.3, 5X))

1 = 1
XA = MACH1(I)**2
DO 5 J = 1, 5
XB = MACH2(J)**2
XC = (XB - XA) / 2.0
XD = MACH1(I) / MACH2(J)
D(J) = XD * EXP(XC)

5 CONTINUE

I = 2
XA = MACH1(I)**2
DO 6 J = 6, 10
XB = MACH2(J)**2
XC = (XB - XA) / 2.0
XD = MACH1(I) / MACH2(J)
D(J) = XD * EXP(XC)

6 CONTINUE

I = 3
XA = MACH1(I)**2
DO 7 J = 11, 15
XB = MACH2(J)**2
XC = (XB - XA) / 2.0
XD = MACH1(I) / MACH2(J)
D(J) = XD * EXP(XC)

7 CONTINUE

I = 4
XA = MACH1(I)**2
DO 8 J = 16, 20
XB = MACH2(J)**2
XC = (XB) / 2.0
XD = MACH1(I) / MACH2(J)
D(J) = XD * EXP(XC)

8 CONTINUE

I = 5
XA = MACH1(I)**2
DO 9 J = 21, 25
XB = MACH2(J)**2
XC = (XB - XA) / 2.0
XD = MACH1(I) / MACH2(J)
D(J) = XD * EXP(XC)

9 CONTINUE

WRITE (6, 200)((I, D(I), I = 1, 25)
FORMAT (///, 5(2X, 1HD, I2, 2X, F6.2, 5X))
END
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LIST OF CITED REFERENCES


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68


16. BECKER, J. V., and KORYCINSKI, P. F. Heat transfer and pressure distribution at a Mach number of 6.8 on bodies with conical flares and extensive flow separation. NASA TN D-1260.


HEAT TRANSFER IN SHOCK
BOUNDARY LAYER INTERACTION REGIONS

An Abstract of a Thesis
Presented to the
Department of Mechanical Engineering
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
Rex. W. Shumway
August 1965
ABSTRACT

A literature survey was conducted to obtain the available data on a type of aerodynamic heating called shock heating. Flow pattern information relating to different types of shocks was also obtained. This thesis correlates the publications relating to the above field and presents them in graphical and tabular form.

The correlation revealed that the heat flux in a shock boundary layer interaction zone was consistently above the pre-shock and post-shock values. A shock heating prediction method was postulated depending on the point of boundary layer transition relative to the interaction zone.

An effort was also made to contribute information to the problem of mass addition to the boundary layer.

The types of shocks for which very little experimental data is available were pointed out. Further research was recommended in specific areas relating to shock heating.