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Drag Coefficients for Magnetically Accelerated Spheres

Jann-Wuu Liang
Brigham Young University - Provo

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DRAG COEFFICIENTS FOR MAGNETICALLY ACCELERATED SPHERES

A Thesis
Presented to the
Department of Mechanical Engineering
Brigham Young University

In Partial Fulfillment
of the Requirement for the Degree
Master of Science

By
Jann-wuu Liang
August, 1966
DEDICATION

To My Parents
APPROVALS

This thesis is accepted in its present form as fulfilling the thesis requirement toward a Master of Science degree in the Mechanical Engineering Department, Brigham Young University.

APPROVED:

29 June 1966

Date
ACKNOWLEDGMENTS

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Appreciation is also expressed to the Mechanical Engineering Department and the Research Division of Brigham Young University for financing this project.
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NOMENCLATURE

A  Projection area of sphere

a  Sphere acceleration

b_n  Coefficient of displacement-time curve-fit equation

C_d  Drag coefficient

CMD  Distance between the lens of the camera and the path of the moving sphere.

c_n  Coefficient of force-displacement curve-fit equation

D_s  Sphere diameter

F_b  Buoyant force

F_d  Drag Force

F_g  Gravitational force

F_m  Magnetic force

g  Gravitational field strength

I  Current

K  Magnetic susceptibility

N  Number of core windings

N_a  Acceleration modulus

n  Constant from force data

Re  Reynolds number

R_i  Inside magnetic radius

R_o  Outside magnetic radius

R_p  Sphere radius
<table>
<thead>
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</tr>
<tr>
<td>t</td>
<td>Time</td>
</tr>
<tr>
<td>V</td>
<td>Sphere volume</td>
</tr>
<tr>
<td>(v_{s-f})</td>
<td>Velocity of sphere relative to field</td>
</tr>
<tr>
<td>(W_t)</td>
<td>Weight of sphere</td>
</tr>
<tr>
<td>(\rho_f)</td>
<td>Fluid density</td>
</tr>
<tr>
<td>(\rho_s)</td>
<td>Sphere density</td>
</tr>
<tr>
<td>(\mu_f)</td>
<td>Fluid viscosity</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>Incident angle</td>
</tr>
<tr>
<td>(\theta_2)</td>
<td>Refraction angle</td>
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CHAPTER I

INTRODUCTION
INTRODUCTION

The objective of this study was to determine the correlation of the drag coefficient of an accelerating sphere in the liquid medium over a range of Reynolds number from 0.1 to 100,000. A magnetic accelerator was used to obtain an acceleration for the sphere, the data was recorded using a photograph and a digital computer was used to reduce the data.

As a sphere is released in a fluid, under the action of a gravitational field, if the density of the sphere is higher than that of the fluid, the sphere will be accelerated downward as it is released. The existence of fluid resistance against the motion of the sphere causes the acceleration of the sphere to decrease and the velocity of the sphere to reach a terminal value. At this time the resistant force equals the magnitude of the gravitational force. This resistant force of the fluid opposing the motion of the sphere is parallel to the relative velocity and hence is by definition the drag force. The motion of the sphere when it travels at a terminated constant velocity is called a terminal or steady state. And the portion of the motion before the terminal state is called an accelerating or unsteady state. Since the gravitational force is comparatively small, the period of acceleration of the sphere is small in fluid and hence the unsteady state portion of the motion is short. By applying a magnetic or mechanical force the time of acceleration can be made much larger.
Dimensionless Parameters

In experimental fluid dynamics research there are many dimensionless parameters used to classify and generalize experimental results. The important ones chosen for this study were: Reynolds number, drag coefficient and acceleration modulus. The Reynolds number is proportional to the ratio of the inertia force and viscous force; the drag coefficient is proportional to the ratio of drag force and inertia force; and the acceleration modulus is proportional to the ratio of acceleration force and inertia force. They are defined respectively as the following:

\[ \text{Re} = \frac{\rho \cdot \frac{V}{s-f} \cdot \frac{D}{s}}{\mu_f} \quad (1) \]

\[ C_d = \frac{F_d}{\frac{1}{2} \rho_f \cdot \frac{V^2}{s-f} \cdot A} = f(\text{Re}) \quad (2) \]

\[ Na = \frac{a \cdot \frac{D}{s-f}}{\gamma_s^2} \quad (3) \]

Where

- \( \text{Re} \) = Reynolds number
- \( C_d \) = drag coefficient
- \( Na \) = acceleration modulus
\[ \rho_f = \text{the density of the fluid} \]
\[ \mu_f = \text{the dynamic viscosity of the fluid} \]
\[ D_s = \text{the diameter of the sphere} \]
\[ A = \text{the projection area of the sphere} \]
\[ V_{s-f} = \text{the sphere velocity relative to the fluid} \]
\[ a_{s-f} = \text{the sphere acceleration relative to the fluid} \]

The dimensionless quantities formulated in Equations 1, 2 and 3 have been used by many investigators [12, 13, 14, 15]*. The advantages of these quantities are: (1) independent of the system of units and (2) considerable simplification in the extent of the experimental work. Hence those quantities were also used in this investigation.

* The numbers in the brackets refers to References
Steady State Drag

The efforts of many investigators [1, 2, 17] have produced the correlation for the steady-state drag for the motion of spheres. A plot of drag coefficient versus Reynolds number is shown in Figure 1. The drag was measured under the following idealized conditions: (1) the fluid field was infinite so that there was no wall effect, (2) the properties of the fluid were homogeneous and (3) the flow pattern was fully developed with a large scale fluctuation.

In the very low Reynolds number range, i.e., Re < 1, the motion of the sphere is such that the viscous force is considerably greater than the inertia force. By omitting the inertia force of the sphere, analytical solutions were obtained by Stokes [1] and Oseen [2] respectively. The results showed an excellent agreement with the experimental data in this region.

In the high Reynolds number region, the inertia force predominates and separation of the flow occurs around the boundary of the sphere. The drag of the moving sphere can no longer be estimated by analytical methods and the experimental method is used. The steady state drag in Figure 1 in the high Reynolds number region was measured by Schiller [17], Schmiedel [17], Liebster [17], Allen [17] and Wielselsberger [17].
Figure 1. The Steady State Drag Curve for Sphere [Ref. 3]
Scope of the Thesis

There are many factors that may affect the drag of spheres moving through a fluid medium, especially in unsteady state. The most important of these factors are given as below:

- Reynolds number [3]
- Acceleration [4]
- Turbulence [5]
- Rotation [6]
- Knudsen number [7]
- Mach number [8]
- Wall effects [9]
- Heat transfer [10]
- Surface roughness [11]

Applicable investigations do not contain enough information about the effects of each of the above factors upon the unsteady state drag of the sphere; this study was to investigate systematically the effects of two: the acceleration and the Reynolds number. The results of this investigation were to be compared with those of Walker [16]. The effects of the other factors are considered to be isolated from the results of this investigation by the carefully designed experimental apparatus. Work on the effects of the other factors, the writer leaves to other investigators.
CHAPTER II

LITERATURE SURVEY
LITERATURE SURVEY

In spite of the considerable efforts devoted to the study of unsteady-state motion of spheres, the accomplishment is limited in scope, volume and consistency. The analytical solutions obtained by the method of over-simplified theoretical hydrodynamics [18] have little practical use because the viscosity of any real fluid is never equal to zero. Basset [19] and Villat [20] have produced analytical solutions for the unsteady viscous case; but like Stokes solution, they are only valid for very slow motions.

As was true in the case of steady-state motion, the drag study of the unsteady state essentially depends on experimental investigations. Figure 2 shows a summary of the results of some earlier investigators.

Lunnan [9, 21] conducted his experiment in both air and water with spheres of steel, rubber and wax in the range of Reynolds from $10^3$ to $10^4$. The drag coefficient of his experiment for unsteady-state motion showed a little higher than that of steady-state motion.

In order to make an extreme accelerating motion, Bagliarello [13] obtained results by drawing fairly large spheres through water by a fine wire with a constant weight in air. The results he obtained for the drag coefficients varied from 0.4 to 30.0 times the steady-state drag at the same Reynolds numbers. The maximum acceleration obtained by this method was 20,000 ft/sec/sec. But, because the sphere was towed.
Figure. Range of Drag Data
by a wire, the flow pattern might have been changed because of the existence of the wire. So, his data is somewhat questionable.

Recently, Hitschfeld [28] studied the motion of water droplets falling through the air. His results were not in the form of drag coefficients and Reynolds numbers. It provided no comparison with the steady-state drag curve.

Fledderman and Hansen [22] injected small droplets to be accelerated in an air stream. Their results were in the ranges of Reynolds number from 30 to 110 and the drag coefficients were of 1,000% smaller than the steady-state motion at the same Reynolds. Since, for liquid-gas flow, there are many factors such as turbulence, evaporation of the liquid droplets, heat transfer, shape, etc., that may affect drag, his results have been questioned by many investigators.

Using the same equipment as that of Fledderman and Hansen, Ingebo again studied the injection of small droplets and solid spheres into a gas stream. The maximum acceleration was 60,000 ft/sec². Results obtained by Ingebo were quite different from that of Fledderman and Hansen. The correlation of drag coefficients and Reynolds numbers is a single line under the standard drag curve with the following empirical formula:

\[ C_d = \frac{27}{0.84 \cdot Re} \]  

(4)
Boyack [27] studied the free fall of steel spheres in water. To check his data reduction methods, some of Lunnon's data was obtained and reduced by Boyack. Excellent agreement was found between these results with the range extended. Boyack's results were in the Reynolds number range from $7 \times 10^3$ to $1.5 \times 10^4$ and the drag coefficient was higher than the steady-state drag.

More recently Walker [16] obtained some enlightening data in studying magnetically accelerated spheres in glycerine solutions. His findings are significant to this study. The range of Reynolds number does not cover the region less than 5.00 or the region greater than 5,000. The careful consideration given the test apparatus design and data reduction techniques produced results with high reproducibility. The results show no apparent effect of acceleration on the drag of spheres moving in an unsteady-state motion. Figure 3 shows his experimental results. An error of neglecting the ball weight was discovered in his data reduction. Figure 3 is the corrected data.
Figure 3  Drag Coefficient vs. Reynold Number (Ref 16)
CHAPTER III

TEST APPARATUS
TEST APPARATUS

The apparatus for this investigation was essentially designed and built by Scott [27] and Nielsen [22] under the direction of Dr. John M. Simonsen. Some minor modifications were made as necessary. The design philosophies of the apparatus were: (1) to obtain an extreme unsteady-state motion for the sphere which was accelerated by the force other than gravitational force over a wide range of Reynolds number, (2) to record accurately the data for the motion of the sphere so as to be able to find drag coefficients and Reynolds numbers for the sphere under unsteady-state, and (3) to prevent any factors that might cause inaccuracy in the results.

For the above purposes, a magnetic accelerator was built to obtain an extreme unsteady-state motion; many spheres of different sizes and many kinds of liquid of different viscosity were provided to obtain a wide range of Reynolds number of the motion. The other instruments of the apparatus were provided to obtain photographic data for the motion of the sphere and to reduce error factors.

Figure 4 is a picture of the test apparatus in its operating position. The description of the major pieces of the apparatus is presented in the following.
Figure 4. Instrumentation
Magnetic Accelerator

The basic structure of the magnetic accelerator is shown in Figure 5. It is an electromagnet with the strength of the field mainly controlled by 24 core wound units. The test region is above the center pole and under the arc of the magnet. The maximum magnetic force applied on the steel sphere was about 30 times the weight of the sphere. The theoretical equation for the magnetic force applied on the sphere in the test region was given by Scott [5] as:

\[ F_m = \frac{V K (NI)^2}{S^3 \left( \ln \frac{R_o}{R_i} \right)^2} \]  

(5)

Where

\[ N = \text{the number of core windings} \]
\[ I = \text{The current} \]
\[ V = \text{the sphere volume} \]
\[ K = \text{the magnetic susceptibility of the sphere} \]
\[ S = \text{the radial position of the sphere in the field} \]
\[ R_o = \text{the outside magnet radius} \]
\[ R_i = \text{the inside magnet radius} \]

Experimental measurements by Nielson [22] showed that this equation was approximately correct.
Figure 5. The basic construction of magnetic accelerator.
Test Chamber

The test chamber was a rectangular vessel with the dimensions of length, width and height equal to 2, 3.5 and 7 inches respectively. It was constructed in the main of non-magnetic metals with front side made of select plate glass which enabled the camera to photograph the motion of the sphere with low distortion. An engine-divided scale was fastened to the side wall of the vessel to project the scale onto the film plane. The scale was adjusted parallel to the motion of the sphere and both were at the same distance to the camera. A picture of the test chamber is shown in Figure 6.

Sphere Drop Mechanism

The main purpose of this mechanism was to release the sphere with no initial force or rotation when it was released. A vacuum pump, a solenoid valve and a copper tube were the main parts of this mechanism.

The sphere was held in the initial drop position by a vacuum applied through a copper tube. The sphere was released by opening the solenoid valve to the atmosphere breaking the vacuum. The valve was activated by the sequence control. Figure 7 shows a schematic picture of the particle drop mechanism.
Figure 6. Test Chamber
Figure 7. Schematic Particle Dropping Mechanism
Force Field Calibration Equipment

The magnetic force applied on the steel sphere was not constant in the test chamber. It varied with the position of the sphere somewhat as predicted by equation (5). The magnetic field was calibrated for each sphere which was to be tested in the experiment to obtain the exact relationship. An empirical equation for the force in the magnetic field was found and used in the data reduction.

Figure 8 shows the force-field calibration equipment. A modified analytic balance which was made of non-magnetic materials was used to measure the magnetic and gravitational forces acting on the spheres. One of the two plates of the balance was replaced by a length-adjustable copper rod and the sphere was held at the lower end of it. The exact position of the sphere was read by a cathetometer.

The analytic balance was made by Denver Fire Clay Company and the cathetometer was made by Wm. Greatner & Corporation.

Lighting System and Sequence Control

The stroboscopic lighting system and sequence control was designed by Dr. John M. Simonsen to provide the light source and precise control of all processes and sequences. A schematic of the system is shown in Figure 9. The frequency of the flashes of light was controlled by a Hewlett-Packard function generator. The period of the flashes was measured using an Erie 400 model crystal controlled counter.
Figure 8. Force Calibration Equipment
Figure 9.  Schematic Square Control and Light System
The sequence control was activated by pressing the start button. Timer units then furnished the signal to drop the sphere, open the camera shutter, turn on the 3000-volt power supply, and start the stroboscopic lighting system, all in the proper sequence, and then to turn off all circuits.

The Spheres and Liquids

The spheres used in this experiment were precision ball bearings with the rigid diameter tolerance of \( \pm 0.0000635 \text{ cm} \) and with highly polished surfaces. The diameter range varied from 0.0778 cm to 0.4059 cm. The liquids used were glycerine solutions with different concentrations to obtain a range of viscosities.

The range of the concentration was from 0\% to 100\% and the range of viscosity was from 1 centipoise to 1500 centipoises. Spheres and liquids were so selected that the uniformly distributed experimental data would be obtained in the range \( 0.1 < \text{Re} < 100,000 \). The liquids and spheres used in this experiment are listed in Table 1 and Table 2 respectively. The tabulated viscosities of the glycerine solutions in Table 1 are standard values at a specified temperature [23]. Since the viscosity of the glycerine solution is of high sensitivity to temperature, a Hoeppler precision viscosimeter made by Fish Schurman Corporation was provided to measure the viscosity of the glycerine solution at the temperature at which the test was processed.
<table>
<thead>
<tr>
<th>Sp. Gr. 25°/25° C</th>
<th>% Glycerine</th>
<th>Viscosity (C.P.)</th>
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<tr>
<td></td>
<td></td>
<td>20°</td>
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<tr>
<td>1.262</td>
<td>100</td>
<td>1,499</td>
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<td>1.236</td>
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<tr>
<td>1.210</td>
<td>80</td>
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<td>1.182</td>
<td>70</td>
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<td>1.155</td>
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<td>1.127</td>
<td>50</td>
<td>6.05</td>
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<td>1.100</td>
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<td>3.73</td>
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<td>1.074</td>
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### TABLE 2

**SPHERES USED IN THIS EXPERIMENT**

<table>
<thead>
<tr>
<th>Sphere</th>
<th>Diameter (inch)</th>
<th>Weight (grams)</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>0.1558</td>
<td>0.2542</td>
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<tr>
<td>B</td>
<td>0.1247</td>
<td>0.1312</td>
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<tr>
<td>C</td>
<td>0.0934</td>
<td>0.0546</td>
</tr>
<tr>
<td>D</td>
<td>0.0778</td>
<td>0.0342</td>
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<tr>
<td>AA</td>
<td>0.4059</td>
<td>4.3350</td>
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CHAPTER IV

DATA ACQUISITION AND REDUCTION
DATA ACQUISITION AND REDUCTION

This chapter presents the methods and sequence by which accurate data for this investigation were obtained using the apparatus discussed in the previous chapter. The techniques and procedures used to reduce the data are also discussed.

There are many factors to be measured to obtain drag and Reynolds number of a moving sphere. They are viscosity and density of the fluid, diameter of the sphere, and intensity of the magnetic field. In this investigation, the magnetic field was kept constant. The different drag coefficients and Reynolds numbers were obtained by changing the properties of the fluid and the diameters of the spheres.

Data Acquisition

Data acquisition was accomplished in four steps: (1) demagnetization of the magnet accelerator, (2) force calibration in magnetic field, (3) time-displacement data, and (4) determination of the fluid viscosity.

(1) Demagnetization - In order to obtain a uniform and steady magnetic field in the test region, the magnetic accelerator went through a demagnetization process prior to the test to eliminate any residual magnetism left from previous test and to insure reproducible fields. The procedure of demagnetization was to reverse the field polarity repeatedly with the coil current only 80% of the previous value for each reversing of the polarity. This was continued until the coil current was less than 1 amp.
This procedure made the field reproducible as explained in one of the basic concepts of the electro-magnetic theory [28].

After demagnetization the desired current level was established and left at that level for about one hour to stabilize completely. The power level was maintained constant during the test. The other electronic instruments were also turned on and warmed up for a period of time to insure stability.

(2) Force Field Calibration - According to Equation (5) in Chapter III,

$$F_m = \frac{VK (NI)^2}{S^3 \left(\ln \frac{R_o}{R_1}\right)^2}$$

The magnetic force theoretically could be determined by measuring seven quantities. However, it was neither convenient nor accurate; hence, the magnetic force was determined by calibration. The magnetic force was assumed to be a polynomial of the inverse of distance.

$$F_m = b_0 + \frac{b_1}{s} + \frac{b_2}{s^2} + \frac{b_3}{s^3} + \ldots$$

Where

- $F_m$ is the magnetic force
- $b_n$ are constants to be determined
- $s$ is the distance from the center pole of the magnet to the position in the test region.
Force and displacement data were taken at one-half inch intervals through the test region. The force calibration equipment is shown in Figure 8, Chapter III. As explained, the magnetic force was measured by the analytical balance and the displacement was measured by the cathetometer. Both readings were of four significant figures. The force-displacement data with the FORFIT subroutine in the data reduction computing program determined the coefficients of Equation. The magnetic force acting on the sphere could be calculated at any position in the test section. Figure 10 shows a comparison of calibrated data and data calculated by Equation (6). An excellent result was observed in that the error was less than 0.1%. The FORFIT subroutine program will be discussed in detail in the data reduction section.

(3) The motion of the sphere was recorded by using multiple exposure flash and a Calumet View Camera. The multiple flashes were furnished by a stroboscopic lighting system which has been mentioned in Chapter III. As the sphere fell, the stroboscopic light flashed, the displacements of the sphere were recorded on the 4" x 5" Polaroid high-contrast, ultra-high speed film. The period between each consecutive position was measured by a crystal controlled Erie-400 interval timer to four significant figures. The flash unit allowed only a predetermined number of flashes and then turned off.
Figure 10. Typical Force Calibration

- Force (cm)
- POSITION (cm)

- Calculated data
- Measured data
A typical picture for the motion of the sphere is shown in Figure 11. The displacements of the sphere were then measured by a travelling microscope, which was made by Central Scientific Corporation, to four significant figures. Two more readings were taken on the sphere trajectory corresponding to two positions on the image of the ruler, attached to the side wall of the test chamber. These two readings provided a reduction ratio between the photograph and the actual test section.

**Time-Displacement Data** - The data was curve fit to obtain a time-displacement function. The curve-fitted functions were then used as explained later to calculate the drag coefficient and the Reynolds number of the moving sphere. The technique of curve-fitting the time displacement data is discussed in the section on data reduction; only the procedure in data acquisition is presented here.

(4) **Determination of the Fluid Viscosity** - The viscosity of a glycerine solution is very sensitive to temperature. One degree centigrade change in temperature makes it as high as 16% change in viscosity. For this reason a rolling-ball style, Hoepller Precision viscosimeter (made by Fish Schurmann Corp.) was used to determine the viscosity of the fluid at the temperature at which the test was made.

The fluid was placed in the test tube of the viscosimeter and adjusted to a desired temperature using the temperature bath. The ball was dropped and the falling time measured. The viscosity of the fluid was calculated by the following formula (25):
Figure 11. Typical Exposure of Trajectory
\[ Cy = T \times (S_b - S_f) \times B \]

Where

- \( Cy \) is absolute viscosity in centipoise
- \( T \) is time interval of the falling ball
- \( S_b \) is specific gravity of the ball
- \( S_f \) is specific gravity of the fluid at the measured temperature
- \( B \) is ball constant

Errors in data taking were minimized by following the above procedure.

Data Reduction

The previous sections of this chapter describe the techniques by which the raw data were obtained for the spheres moving in an unsteady state. This section presents the data reduction techniques to evaluate the drag coefficient and Reynolds number for the sphere. The data reduction was executed by an IBM7040 digital computer using the computer program as shown in Appendix B.

The data reduction program consisted of two basic parts: (1) the calculation of the drag coefficient and Reynolds number after knowing the equation for the motion, and (2) the generation of the equation describing the motion. Each of the theories of the parts is described below.

As was mentioned in Chapter I, Reynolds number and drag coefficient are defined as:
\[ Re = \frac{\rho_f D_s V_{s-f}}{\mu_f} \]  

(12)

and

\[ C_d = \frac{F_d}{\frac{1}{2} \rho_f V_{s-f}^2 \left( \frac{\pi}{4} D_s^2 \right)} \]  

(13)

Where

- \( Re \) is the Reynolds number
- \( C_d \) is the drag coefficient
- \( F_d \) is the drag force
- \( \rho_f \) is the density of the fluid
- \( D_s \) is the diameter of the sphere
- \( \mu_f \) is the viscosity of the fluid
- \( V_{s-f} \) is the sphere velocity relative to fluid

The drag force acting on the sphere is calculated by summing the forces on the sphere.

\[ F_d = F_m + F_g - F_b + m_s a_{s-f} \]  

(14)

Where

- \( F_m \) is the magnetic force
- \( F_g \) is the gravitational force
- \( F_b \) is the buoyancy
- \( m_s \) is the mass of the sphere
- \( a_{s-f} \) is the acceleration of the sphere
In the above equation the quantities, $D_{fi}, F^i, F^j, F^k,$ and $m$ are measured quantities and constants, while $F_m, V_{s-f}$ and $a_{s-f}$ are reduced quantities and vary either with displacement or time. By substituting these quantities into Equations 14, 13 and 12, Re and $C_d$ are established.

The data reduction program was used to construct the empirical expressions for both displacement-time and displacement-force needed to find the drag force. The displacement of the sphere was assumed to be expressible as a polynomial in time. After examining Equation 15, the magnetic force applied to the sphere was assumed to be a function of the inverse of displacement. Both of these two functions were in polynomials.

\[ S = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + \cdots \]  \hspace{5cm} (15)

\[ F_m = c + \frac{c_1}{s} + \frac{c_2}{s^2} + \frac{c_3}{s^3} + \cdots \]  \hspace{5cm} (16)

Where

- $s$ is displacement
- $t$ is time
- $b_n$ is coefficient of displacement function
- $c_n$ is coefficient of force function
Substituting the displacement-time data and force-displacement data into Equations (15) and (16) respectively, the coefficients $b_n$ and $c_n$ were solved by the computer by the least square method.

The velocity and acceleration of the spheres were obtained from the first and second derivatives of the displacement with respect to time.

\[
V = \frac{ds}{dt} \quad (17)
\]

\[
a = \frac{d^2s}{dt^2} \quad (18)
\]

Where

$S$ was the previously derived expression:

\[
S = b_0 + b_1t + b_2t^2 + b_3t^3 + \ldots \quad (19)
\]

With all the necessary data available, the drag coefficients and Reynolds numbers of the spheres were calculated and printed by the computer.
CHAPTER V

RESULTS AND DISCUSSION
RESULTS AND DISCUSSION

Representation of Experimental Data

A graphical representation of the results is presented in Figure 12 as a plot of the drag coefficient versus Reynolds number at the associated acceleration modulus a log-log coordinates. Typical results are shown in Table 3 along with data corresponding to the steady state for comparison.

Discussion of Results

This section will discuss the performance of the test apparatus and the function of the computer program as they might affect the accuracy of the final data. A comparison for these results with those of other investigators is represented as well.

The Performance of the Apparatus

In the force governing equation for the sphere

\[ F_d = F_m + F_g - F_b + ma \]  

\[ (20) \]

\( F_g \) and \( F_b \), the gravitational and buoyant forces, were constants and easy to measure with the error less than 0.01%. In a glycerine solution \( F_b \) is about one seventh of \( F_g \). \( F_m \), the magnetic force, varied from two to twenty-five times of \( F_g \) in the test section. Thus it is observed that the summation of drag force \( F_d \), and acceleration force \( ma \) varied from three to twenty-six times of \( F_g \). \( F_d \) was evaluated from the quantities of \( F_g \), \( F_b \), \( F_m \) and \( ma \). While \( F_g \) and \( F_b \) were of comparatively small and of high accuracy, the accuracy of \( F_d \) was mainly dependent on
Figure 12. Measured Drag Coefficients Plotted at the Associated Acceleration Modulus as a Function of Reynolds Number.
### TABLE 3
TYPICAL DATA OF THE FINAL RESULTS

<table>
<thead>
<tr>
<th>Reynolds Numbers</th>
<th>Drag Coefficient</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady State</td>
<td>Unsteady State</td>
<td></td>
</tr>
<tr>
<td>0.161E 00</td>
<td>0.149E 03</td>
<td>0.147E 03</td>
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<td>0.198E 00</td>
<td>0.121E 03</td>
<td>0.123E 03</td>
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</tr>
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<td>0.346E 02</td>
<td>0.261E 02</td>
<td></td>
</tr>
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<td>0.261E 02</td>
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<td>0.868E 01</td>
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<td>0.461E 01</td>
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<td></td>
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<td>0.520E 01</td>
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<td>0.419E 01</td>
<td></td>
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<td>0.190E 02</td>
<td>0.320E 01</td>
<td>0.321E 01</td>
<td></td>
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<tr>
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<td>0.209E 01</td>
<td>0.208E 01</td>
<td></td>
</tr>
<tr>
<td>0.652E 02</td>
<td>0.151E 01</td>
<td>0.149E 01</td>
<td></td>
</tr>
<tr>
<td>0.851E 02</td>
<td>0.131E 01</td>
<td>0.135E 01</td>
<td></td>
</tr>
<tr>
<td>0.102E 03</td>
<td>0.117E 01</td>
<td>0.118E 01</td>
<td></td>
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<td>0.181E 03</td>
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<td>0.852E 00</td>
<td></td>
</tr>
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<td>0.650E 00</td>
<td>0.645E 00</td>
<td></td>
</tr>
<tr>
<td>0.428E 03</td>
<td>0.574E 00</td>
<td>0.493E 00</td>
<td></td>
</tr>
<tr>
<td>0.801E 03</td>
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<td>0.493E 00</td>
<td></td>
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<td>0.104E 04</td>
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<td>0.208E 04</td>
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<td>0.400E 00</td>
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<td>0.450E 04</td>
<td>0.395E 00</td>
<td>0.399E 00</td>
<td></td>
</tr>
<tr>
<td>0.725E 04</td>
<td>0.395E 00</td>
<td>0.399E 00</td>
<td></td>
</tr>
</tbody>
</table>
that of $F_m$ and $ma$.

By thoroughly observing the precautions mentioned in Chapter IV, the magnetic field was kept constant during each test. This was proven by calibrating the field before and after each test (which usually lasted more than ten hours). The calibrations checked each other within the measuring accuracy.

Acceleration and velocity data were derived from the curve fitted displacement-time data. Any error in either displacement-data taking or time-data taking would have affected the accuracy of the evaluations of acceleration and velocity of the sphere. For this consideration, two questions arose: (1) How close did the period between two consecutive flashes approach a constant? What was the firing time of the mercury vapor lamp for each flash and did it affect the accuracy of the period? (2) The displacement data were taken by a camera. The light went through three different media, the air, the glass and the glycerine solution. Is the refraction effect appreciable?

To answer the first equation, the flash system of the experiment is discussed. The width of the pulses which controlled the firing time of the mercury vapor lamp was observed on the screen of an oscillograph to have been less than 10 micro-second. The period of the flashes was read by a crystal counter which read to one hundredth of one millisecond. The fluctuation of the periods appeared to have been $\pm 0.01$ ms. But the periods used for each different test were usually greater than 20.00 ms.
So, the maximum error could have been 0.15% which was allowable for this experiment.

As to the second question, a simple inspection was made. For, if there was any refraction effect in this problem, the divisions of the ruler on Figure 11 would not have appeared equal. By measuring each division of the ruler carefully, no appreciable difference was found. A theoretical study of this problem on the optical of view done by Boyack is shown in Appendix A.

The Computer Program

The function of the data reduction computer program was checked by the data generated by another computer program. This was programmed assuming Stokes flow (1) in the gravitational field. The drag coefficient was therefore assumed to be 24.0 derived by the Reynolds number. By these assumptions the governing force equation was simplified to a second order, linear, non-homogeneous ordinary differential equation. With the boundary conditions, the differential equation was solved analytically. The derivation and solution of the differential equation are shown as follows:

By Figure 13

\[
m \frac{d^2 s}{dt^2} = F_g - F_b - F_d
\]

(21)

Where

\[
F_g = \frac{1}{6} D_s^2 \rho_s g \quad \text{gravitational force}
\]
Figure 13. Free Body Diagram
Buoyant force

\[ F_b = \frac{1}{6} \pi D_s^3 \rho_f g \]

Drag force

\[ F_d = C_d \frac{1}{2} \frac{\rho_f}{\rho_s} V^2 \left( \frac{1}{4} \pi D_s^2 \right) \]

\[ C_d = \frac{24.0}{\text{Re}} \]

Stokes assumption

drag coefficient

\[ \text{Re} = \frac{\rho_f D_s V}{\mu_f} \]

Reynolds number

\[ V = \frac{ds}{dt} \]

Velocity

Substituting the above quantities into Equation (14), the following equation is obtained:

\[ \frac{d^2 s}{dt^2} + \frac{9 \mu_f}{8 D_s^2 \rho_s} \frac{ds}{dt} = \left( 1 - \frac{\rho_f}{\rho_s} \right) g \]  \hspace{1cm} (22)

let

\[ \frac{9}{8 D_s^2 \rho_s} \rho_f \]  = A

\[ \left( 1 - \frac{\rho_f}{\rho_s} \right) g = B \]

then

\[ \frac{d^2 s}{dt^2} + A \frac{ds}{dt} = B \]  \hspace{1cm} (23)
with the boundary conditions:

\[
\frac{ds}{dt} = 0 \quad \text{as} \quad t = 0
\]

\[
s = 0 \quad \text{as} \quad t = 0
\]

the solution of Equation (23) is:

\[
s = \frac{B}{2} e^{-At} + \frac{B}{A} t - \frac{B}{A}
\]

(24)

From equation (24), a computer program (see Appendix C) was written to produce numbers for displacement, velocity, acceleration, Reynolds number and drag coefficient of a sphere moving under Stoke's assumptions. Then the time-displacement data which were calculated by the Stoke's motion were entered into the data reduction program. A time-displacement function was curve-fitted by the least square method of the computer program, then the other quantities concerning the motion of the sphere were all calculated from the curve-fit equation. A comparison of the data for the Stokes motion calculated by both analytical methods and numerical methods is shown in Table 4. It is observed that the error in displacement and Reynolds number is less than 0.01% and the error in drag coefficient is less than 0.1%. Hence the function of the data reduction program was concluded to be of high accuracy.
TABLE 4

COMPARISON FOR THE DATA CALCULATED BY ANALYTICAL AND NUMERICAL METHOD FOR
STOKES FLOW

g = 980 cm/sec/sec, \( \Delta t = 0.3331 \) sec. \( \mathrm{Cd} \times \text{Rey} = 24.0 \)
Sphere Weight = 0.2495 gm;
Fluid: \( \rho_f = 1.126 \) \( \mu_f = 0.0605 \) cp

<table>
<thead>
<tr>
<th>No</th>
<th>Displacement (cm)</th>
<th>Reynolds Number</th>
<th>Drag Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>Numerical</td>
<td>Analytical</td>
</tr>
<tr>
<td>1</td>
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<td>0.4596E 00</td>
<td>0.2021E 03</td>
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<tr>
<td>2</td>
<td>0.1820E 01</td>
<td>0.1820E 01</td>
<td>0.3983E 03</td>
</tr>
<tr>
<td>3</td>
<td>0.4055E 01</td>
<td>0.4055E 01</td>
<td>0.5886E 03</td>
</tr>
<tr>
<td>4</td>
<td>0.7138E 01</td>
<td>0.7138E 01</td>
<td>0.7733E 03</td>
</tr>
<tr>
<td>5</td>
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<td>0.1104E 01</td>
<td>0.9525E 03</td>
</tr>
<tr>
<td>6</td>
<td>0.1575E 02</td>
<td>0.1575E 02</td>
<td>0.1126E 04</td>
</tr>
<tr>
<td>7</td>
<td>0.2123E 02</td>
<td>0.2123E 02</td>
<td>0.1393E 04</td>
</tr>
<tr>
<td>8</td>
<td>0.2746E 02</td>
<td>0.2746E 02</td>
<td>0.1459E 04</td>
</tr>
<tr>
<td>9</td>
<td>0.3442E 02</td>
<td>0.3442E 02</td>
<td>0.1618E 04</td>
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<tr>
<td>10</td>
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<tr>
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<td>0.5947E 02</td>
<td>0.2067E 04</td>
</tr>
</tbody>
</table>
Comparison with Other Investigations

These comparisons were made with other data. These are:

1. The results of this experiment in Figure 12 show an excellent agreement with Walker's data (with the correction to the data reduction program).

2. The results obtained by re-reducing the experimental data of Lunnons and Moormans [26] through the data reduction computer program shows fairly good agreement with the results of this experiment.

3. Torabin and Gauvin have pointed out that the calculations made from the Navier-Stokes equations for the moderate accelerations and low Reynolds numbers clearly indicate the drag force is independent of the previous history of flow. The results of this experiment are in agreement with their conclusions.
CHAPTER VI

CONCLUSION

AND

RECOMMENDATION
CONCLUSION AND RECOMMENDATION

Based on the results of this study, the following conclusions can be drawn concerning the effect of acceleration on the drag of a sphere moving in fluid. Recommendations will also be made for further study on this project.

Conclusions

1. Acceleration has no appreciable effect on the drag of a sphere moving in fluid. Hence, the acceleration modulus has little value in this application.

2. The drag of a moving sphere depends only on the Reynolds number.

3. The relationship between drag coefficient and Reynolds number remains the same for both accelerated spheres and non-accelerated spheres.

Recommendations

1. The test chamber should be enlarged to eliminate any wall effects.

2. A constant temperature bath should be used to provide a uniform and constant temperature within the test region.

3. The magnetic force should be adjusted to the magnitude of the gravitational force to increase the accuracy of the experiment. Less viscous liquids should be used for obtaining data at a high Reynolds number.

4. By using an interferometer, a study should be made to examine the flow patterns around an accelerating sphere.
5. Computer solutions of the Navier-Stokes equations should be sought.
LIST OF REFERENCES
LIST OF REFERENCES

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APPENDIX A

REFRACTION ERROR
APPENDIX A

Refraction Error

The following analysis was made to determine the error of the sphere's reflected image when refracted through the test liquid, glass, and air to the camera. The index of refraction of each liquid used in the investigation was close enough to glass so that they could be considered as one medium. Referring to Figure 14, the error Y which would be expected at successive points was derived:

\[
Y_{\text{app}} = Y + SY
= CMD \tan \theta_1
\]

\[
Y = (CMD - \ell) \tan \theta_1 - \ell \tan \theta_2
\]

therefore

\[
SY = \ell (\tan \theta_1 - \tan \theta_2)
\]

The error at successive points Y was calculated and found to vary linearly. This is seen in Figure 15. The above error is negligible since linearity would be present for both the sphere's position and the scale length.
Figure 14. Refraction Error Diagram
Figure 15. Linearity of refraction error
APPENDIX B

DATA REDUCTION PROGRAM
APPENDIX B

DATA REDUCTION PROGRAM

$1BTC MAIN DECK
C PARTICLE PROJECT DATA REDUCTION
C MAIN DRIVER
COMMON D(8),E(8),K1,K2,K3,K4,N1,N2,M1,C(0),K0,K3,DIAM,M3
COMMON ACM(1000),KEE(1000),CDU(1000),LL
DIMENSION RUN(10)
5 READ(5,30)K1,K2,N1,N2,LL
M3=0
LL=0
30 FORMAT(4I3,10A6)
WRITE(6,40)RUN,K1,K2,N1,N2
40 FORMAT(1H1,3X,4HDATA,2X,31HREDUCTION FOR PARTICLE PROJECT,10X,10A
16//12X,4HK1=,I3,7X,4HK2=,I3,7X,4HM1=,I3,7X,4HM2=,I3)
45 K5=1
CALL FORFIT
35 CALL DISFIT
CALL REDUC
M3=M3+1
IF(K5.LE.(K2-K3+1)) GO TO 35
264 REWIND 4
WRITE(4,403)LL
403 FORMAT(3E15.6)
CALL LEON
CALL ORDER
GO TO 5
END

$1BTC DISFIT DECK,LST
SUBROUTINE DISFIT
COMMON D(8),E(8),K1,K2,K3,K4,N1,N2,M1,C(0),K0,K3,DIAM,M3
DIMENSION SD(35),DT(35),S(35)
IF(K5.NE.1)GO TO 65
READ(5,30)X0,X4,X3,UN
30 FORMAT(4F10.0)
READ(5,31)K3,K4
31 FORMAT(21X)
DO 45 I=1,K2
READ(5,32)SD(I)
45 DI(I)=I
20 FORMAT(F10.0)
WRITE(6,32)K3,K4
40 WRITE(6,34)X0,X4,X3,DN
34 FORMAT(1H0,3X,1H1,6X,3HDT(I),12X,3HSU(I),10X,3HX3=F15.6,5X,3HX0=
13 FORMAT(1H1, 20H DATA WILL BE TAKEN 13, 25 MCARDS AT A TIME, SKIPPING 13, 18 MCARDS FOR EACH RUN)
WRITE(6, 55) (I, DT(I), SD(I), I = 1, K2)
55 FORMAT(1H+, 3X, I3, F13.6, 3X, D13.6)
DO 70 I = 1, 8
70 E(I) = 0.0
J1 = 2
R30 = R3 - R0
RATIO = 2.54 * DN / (X4 - X0)
K6 = K5 + K3 - 1
DO 75 I = K5, K6
75 S(I) = (SD(I) - X3) * RATIO + K30
CALL CURFIT(K5, K6, N2, S, DT)
N3 = N2 + 1
DO 90 I = 1, N3
90 E(I) = C(I)
CONTINUE
K5 = K5 + K4
RETURN
END
$IBFTC FORFIT DECK, LIST
SUBROUTINE FORFIT
COMMON D8(8), E(8), K1, K2, K3, K4, K5, N1, N2, WT, C(8), R0, R3, DIAM, M3
DIMENSION F(10), S(10), A(10, 10), X(10), Y(10)
IF(K1.EQ.0) GO TO 10
READ(5, 20) WT, DIAM
READ(5, 20) R0, R3
IF(K1.EQ.1) GO TO 30
READ(5, 20) (S(I), F(I), I = 1, K1)
20 FORMAT(2F10.0)
WRITE(6, 50)
50 FORMAT(1H1, 16H THIS IS NEW DATA)
DO 888 I = 1, 8
888 D(I) = 0.0
GO TO 40
10 WRITE(6, 51)
51 FORMAT(1H1, 30H THIS IS REUSE OF PREVIOUS DATA)
GO TO 40
30 WRITE(6, 52) WT
DO 99 I = 1, 8
99 D(I) = 0.0
52 FORMAT(1H1, 34H THIS IS FREE FALL GRAV. ONLY FORCE/ 710X, 4HWT = ,
1F10.4)
RETURN
40 WRITE(6, 54) R0, R3
54 FORMAT(1H0, 4X
1F13.6)
IF (KI.EQ.0) GO TO 60
WRITE(6,600) R0, WT(I), F(I), S(I), I = 1, K1

600 FORMAT (1H7, 32HDATA READ IN R0, WT, I, F(I), S(I), I = 1, K1)
WRITE(6, 601)

601 FORMAT (1H7, 47HMAGNETIC FORCES AND CORRESPONDENT DISPLACEMENTS/)
D0100 I = 1, K1
F(I) = F(I) - WT
S(I) = S(I) - R0

100 WRITE(6, 602) I, F(I), S(I)
602 FORMAT (1H7, 15E15.4)
D0200 I = 1, 10
D0200 J = 1, 10
200 A(I, J) = 0.
D0210 I = 1, K1
210 A(I, I) = F(I)
D0220 J = 2, 7
D0220 J = 1, K1
IF (J J GE. K1 + 2 - J) GOTO 220
A(I, J) = A(I, J) - A(I - 1, J - 1)
220 CONTINUE
WRITE(6, 603)

603 FORMAT (1H7, 35HDifference Table for Magnetic Force/)
WRITE(6, 604)

604 FORMAT (1H7, 8X, 5H S(I), 10X, 5H F(I), 10X, 5HDEL 1, 10X, 5HDEL 2, 10X,
15HDEL 3, 10X, 5HDEL 4, 10X, 5HDEL 5, 10X, 5HDEL 6/)
605 FORMAT (1H7, 8E13.6)
D0300 I = 1, K1
230 WRITE(6, 605) S(I), (A(J, I), J = 1, 7)
319 DO 320 I = 1, 8
320 C(I) = 0.
21 M1 = M1 + 1
22 M2 = M2 + 2
23 DO 35 I = 1, M1
24 DO 34 J = 1, M2
25 L = I + J - 2
26 A(I, J) = 0.
27 IF (J - M2) 28, 31, 31
28 DO 29 M = 1, K1
29 X(M) = I * S(M)
Y(M) = F(M)
Q3 = L
IF (X(M) NE. 0. AND. Q3 NE. 0.) GO TO 401
29 A(I, J) = A(I, J) + X(M) ** L
GO TO 34
401 A(I, J) = A(I, J) + 1.0
402 GO TO 34
403 A(I, J) = A(I, J) + Y(M)
404 GO TO 33
31 I = I - 1
DO 33 M=1,K1
Q4=11
IF(X(M)*EQ.0.0)AND.X4*EQ.0.0)GO TO 403
A(I,J)=A(I,J)+Y(M)*X(M)**11
CONTINUE
CONTINUE
CONTINUE
C
MATRIX SOLUTION
DO 699 I=1,M1
   L1=I+1
   DO 60 J=L1,M2
      SOL=A(I,J)/A(I,I)
      IF(SOL.GT.6.0,J3,58)
      DO 59 L=J,M2
         A(J,L)=A(J,L)-A(I,L)*SOL
      CONTINUE
   CONTINUE
   DO 68 I=1,M1
      L3=M2-1
      SUM=0.0
      IF(1-L3,68,68,65)
      L2=M1-I+2
      DO 67 L=L2,M1
         SUM=SUM+C(L)*A(L3,L)
      C(L3)=(A(L3,M2)-SUM)/A(L3,L3)
      ERROR+VALUE COMPUTATION
      SUM=0.0
      WRITE(6,611)
      FORMAT(1H—,3X,1HC0EFFICIENTS OF CURFIT EQUATION)
      WRITE(6,116)
      FORMAT(1H0,2X,126H (1) (2) (3) (4) (5) (6) (7) (8) (9)
           (10) )
      WRITE(6,103) C
      FORMAT(11E12.4)
      WRITE(6,6666)
      FORMAT(1H—,3X,1HLIST OF MAG-FORCES COMPARISON)
      WRITE(6,104)
      FORMAT(11M0,5X,1MI,7X,4MS(1),10X,OMF(I)MEAS,OA+OMF(I)CALC,9X,2MERR
           1R,/) 
   DO325 I=1,K1
      FEX=0.0
      DO 75 J=1,N1
         L=N1+2-J
         FEX=(FEX+C(L))*X(I)
      FEX=FEX+C(1)
      ERR=Y(I)-FEX
      SUM=SUM+ERR*ERR
      X(I)=1./X(I)
      WRITE(6,117)I,X(I),Y(I),FEX,ERR
      WRITE(6,117)I,X(I),Y(I),FEX,ERR
117  FORMAT(1H+, (4,1,4E15.5))
    WRITE(6,105)
105  FORMAT(1H0, 3X, 26HSTANDA-D DEVIATION OF DATA/)  
    DEV=1
326  DEVSUSRT(SUM)/DEV  
327  WRITE(6,118)DEV  
118  FORMAT(1H+, (E14.5))  
DO 119  I=1,8  
119  D(I)=C(I)  
80  CONTINUE  
    RETURN
END  

$IBFTC DIFF DECK
SUBROUTINE DIFF(X, Y, K, K8)
  COMMON D(8), E(8), K1, K2, K3, K4, K5, N1, N2, wT, C(6), K0, K3, DIAM, M3
  DIMENSION X(35), Y(35), A(7,35)
  M4=M3+1  
  DO 10 U=1,7  
  DO 10 J=1,35  
  A(I,J)=0.0  
  DO12 I=K, K6  
  A(1, I)=Y(I)  
  DO 65 U=2,7  
  DO 65 K7=K, K8  
  IF(K7.GE.K8-J+2) GO TO 65
  A(J, K7)=A(J-1, K7)-A(J-1, K7+1)  
  CONTINUE
  WRITE(6,81)M4  
81  FORMAT(1H1, 3X, 31HDIFFERENCE TABLE FOR DATA SET NO,13, //8X, SHA(I)  
  11X, 5HY(I), 10X, SHDEL 1, 1OX, SHDEL 2, 10X, SHDEL 3, 1OX, SHDEL 4, 1OX,  
  25SHDEL 5, 1OX, SHDEL 6/)  
  DO 75 I=K, K8  
  WRITE(6,80)X(I), A(1, I), A(2, I), A(3, I), A(4, I), A(5, I), A(6, I), A(7, I)  
80  FORMAT(1H+, 8E12.6)  
85  RETURN
90  END

$IBFTC CURFIT DECK
SUBROUTINE CURFIT(K, K8, N, X)
  COMMON D(8), E(8), K1, K2, K3, K4, K5, N1, N2, wT, C(6), K0, K3, DIAM, M3
  DIMENSION X(35), Y(35), A(7,35)
  CALL DIFF(X, Y, K, K8)  
  C COMPUTATION OF COEFFICIENTS
  IF(N.GE.(K8-K+1))GO TO 405
319  DO320 I=1,8  
320  C(I)=0.0  
21  M1=N+1  
22  M2=N+2

66
DO 35 I=1,M1
DO 34 J=1,M2
L=I+J-2
A(I,J)=0.0
IF(J-M2)23,31,31
DO29 M=K,K8
Q3=L
IF(X(M) .EQ. 0.0 .AND. Q3 .EQ. 0.0 ) GO TO 401
A(I,J)=A(I,J)+X(M)**L
GO TO 34
A(I,J)=A(I,J)+1.0
GO TO 34
A(I,J)=A(I,J)+Y(M)
GO TO 33
WRITE(6,406)
FORMAT(1H3,39H NOT ENOUGH DATAPTS FOR POWER CHosen)
CALL EXIT
11=I-1
DO33 M=K,K8
Q4=I1
IF(X(M) .EQ. 0.0 .AND. Q4 .EQ. 0.0 ) GO TO 403
A(I,J)=A(I,J)+Y(M)*X(M)**I1
CONTINUE
CONTINUE
CONTINUE
C MATRIX SOLUTION
DO 601 I=1,M1
L1=I+1
DO 60 J=L1,M2
SOL=A(I,J)/A(I,I)
IF(SOL)58,60,58
DO 59 L=J,M2
A(J,L)=A(J,L)-A(I,L)*SOL
CONTINUE
CONTINUE
DO 68 I=1,M1
L3=M2-I
SUM=0.0
IF(I-1)66,68,65
L2=M1-I+2
DO 67 L=L2,M1
SUM=SUM+C(L)*A(L3,L)
68 C(L3)=(A(L3,M2)-SUM)/A(L3,L3)
C ERROR+VALUE COMPUTATION
168 SUM=0.0
WRITE(6,611)
611 FORMAT(1H-,3X,31H COEFFICIENTS OF CURFIT EQUATION)
CALL EXIT
WRITE(6,116)
116 FORMAT(1H0,2X,126H (0) (1) (2) (3) (4) (5) (6) (7) (8) (9))
67
2 (10)/
70 WRITE(6,103) C
103 FORMAT(11E12.4)
WRITE(6,666)
6666 FORMAT(1H-3X,31HLIST OF DISPLACEMENT COMPARISON)
79 WRITE(6,104)
104 FORMAT(1H6,3X,1H1,7X,4H*Y(I),10X,4HY(I),MEAS,6X,4HY(I),CALC,9X,PREM
1R,**)
71 DO 325 I=K,1
72 FEX=FEX+U
73 DO 76 J=1,N
74 L=N+2-J
75 FEX=(FEX+C(L))*X(I)
76 FEX=FEX+C(I)
77 ERR=Y(I)-FEX
78 SUM=SUM+ERR*ERR
325 WRITE(6,117) I,X(I)*Y(I),FEX, ERR
117 FORMAT(1H6,3X,1HSTD.12.4)
80 WRITE(6,105)
105 FORMAT(1H6,3X,26HSTANDAR
106 deviation of data/)
326 DEV=1
327 DEV=(SQRT(SUM)/DEV)
328 WRITE(6,118)DEV
118 FORMAT(1H6,14.5)
RETURN
END

$ IBFC REDUC DECK LIST
SUBROUTINE REDUC
C DATA REDUCTION
COMMON D(8),E(8),K1,K2,K3,K4,K5,N1,N2,WT,C(8),RO,K3,DIA,M3
COMMON ACMM(1000),REET(1000),CDD(1000),LL
DIMENSION B(8)
EQUIVALENCE (B,E),D(WT)
90 IF(M3.EQ.0)GOTO 329
83 READ(5,119) PRESS, TEMP, WM, VIS
85 READ(5,119)TPN, PER
84 IF(PRESS) 90, 91, 90
91 READ(5,119)DEN
119 FORMAT(5F10.0)
329 WRITE(6,120)
TMIN=K5-K4
TMAX=K5+K4-1
120 FORMAT(1H6,3X,28HINPUT DATA ON FLUID AND DIAL//3X,2MPRESS,7X,4HTFP
1P,7X,6HMUL WT,7X,4HDIA,M,7X,OMT-STAR,6X,OMT-STUP,6X,OMN-PTS,2X,OMB,
2X,LL MASA,7X,4HSV, 10X, 4X, MENS, 8X, 5MPER)
92 GO TO 330
90 DEN=PRESS*WM/3464.2*(TEMP+459.7)
330 WRITE(6,121)PRESS, TEMP, WM, DIA, TMIN, TMAX, TPN, WM, VIS, DEN, PER
121 FORMAT(1H6,10E12.4)
DIA=2.54*DIAM
T=THIN
WRITE(6,123)
FORMAT(1H0,10X,4HTIME,10X,2HCD,10X,3HRLY,10X,3HACM,6X,8HPOSITION,7X
1,3HVEL,10X,3HACK,11X,2HFU,11X,2HFMM/)
R=0.
S=B(1)
V=B(2)
DO2061=1,8
S=S+B(I+1)*T**1
IF(I.EQ.1)GO TO 206
Z2=I
V=V+Z2*B(I+1)*T**(I-1)
IF(I.LE.2)GO TO 206
Z3=I-1
AB=AB+Z3*Z2*B(I+1)*T**(I-2)
CONTINUE
V=V/PER
AB=AB/(PER*PER)
FM=D(1)
DO33331=2,8
FM=FM+D(I)/S**(I-1)
CONTINUE
FD=FM+DM*AB/979.8+WT
VOL=0.523*(DIA**3)
FD=FD+DM*AB/979.8+WT
VOL=0.523*(DIA**3)
SUBROUTINE LEON
DIMENSION CD(1000), RE(1000), ACM(1000)
REWIND 4
READ(4, 401) K
401 FORMAT(15)
READ(4, 402) (ACM(I), RE(I), CD(I), I=1, K)
402 FORMAT(3E15.6)
WRITE(6, 601) (J, ACM(J), RE(J), CD(J), J=1, K)
601 FORMAT(1HI, 3X, ACM(I), 9X, SHK(I), 1DX, SMCD(I)///(1D, 3E15.6)) T=1
WRITE(6, 602)
602 FORMAT(1H1, 11HDATA OUTPUT///6X, SHACM(I), 9X, SHRE(I), 1DX, SMCD(I)///)
111 L=K-1
100 DO300 I=1, L
IF(REF•GT•ACM(I)•AND•REF•LT•ACM(I+1)) GOTO 200
IF(REF•LT•ACM(I)•AND•REF•GT•ACM(I+1)) GOTO 200
GOTO 300
200 C=CD(I)+(REF-ACM(I))/(ACM(I+1)-ACM(I))*((CD(I+1)-CD(I))
R=RE(I)+(REF-ACM(I))/(ACM(I+1)-ACM(I))*((RE(I+1)-RE(I))
WRITE(6, 603) REF, R, C
603 FORMAT(1H1, 3E15.6)
300 CONTINUE
T=T+1
IF(T•GE•21•) GOTO 999
REF=T•REF
GOTO 111
999 RETURN
END

SUBROUTINE ORDER
SUBROUTINE ORDER
DIMENSION CD(1000), RE(1000), ACM(1000)
REWIND 4
READ(4, 401) K
401 FORMAT(15)
READ(4, 402) (ACM(I), RE(I), CD(I), I=1, K)
402 FORMAT(3E15.6)
100 L=0
J=K-1
DO300 I=1, J
IF(RE(I)-RE(I+1))200, 300, 300
200 R=RE(I+1)
C=CD(I+1)
A=ACM(I+1)
RE(I+1)=RE(I)
CD(I+1)=CD(I)
ACM(I+1)=ACM(I)
REF(I)=R
300 CONTINUE
T=T+1
IF(T•GE•21•) GOTO 999
REF=T•REF
GOTO 111
999 RETURN
END
CD(I)=C
ACM(I)=A
L=L+1
300 CONTINUE
IF(L.NE.0) GOTO 100
WRITE(6,600) RUN
600 FORMAT(1H1,12HORDERED DATA,4X,10A6)
WRITE(6,601)(J,ACM(J),RE(J),CD(J),J=1,K)
601 FORMAT(1H0,3X,1HI,6X,6HACM(I),9X,2HRE(I),10X,5HCD(I)/(15,3E15.6)
WRITE(6,602)(J,ACM(J),RE(J),CD(J),J=1,K)
602 FORMAT(1H-,/(15,3E15.3))
RETURN
END
APPENDIX C

PROGRAM CHECKING PROGRAM
DIMENSION S(21), V(21), RE(21), CD(21), A(21)
T=0.
READ(S,501)VIS,DP,ROF,PM,DELTAT
501 FORMAT(5F10.4)
RP=2.84*DP/2.
PV=4./3.*1416*RP*KP*KP
RUP=PM/PV
C=4.5*VIS/(RP*KP*KP)
B=(1.-ROF/RUP)*980.
DO200 I=1,20
S(I)=B/C*T-B/C+C/C*EXP(-C*T)
V(I)=B/C-B/C*EXP(-C*T)
A(I)=B*EXP(-C*T)
RE(I)=ROF*2.*RP*V(I)/VIS
CD(I)=2.4./RE(I)
150 WRITE(6,501)I, T, S(I), DELTAT, A(I), V(I), RE(I), CD(I)
601 FORMAT(1H13,5E16.4)
WRITE(7,601) S(I)
701 FORMAT(F10.4)
T=T+DELTAT
200 CONTINUE
END
DRAG COEFFICIENTS FOR MAGNETICALLY ACCELERATED SPHERES

An Abstract of a Thesis

Presented to the

Department of Mechanical Engineering

Brigham Young University

In Partial Fulfillment

of the Requirement for the Degree

Master of Science

By

Jann-wuu Liang

August, 1966
ABSTRACT

A survey of previous work was conducted to collect and review some of the information concerning the effect of the acceleration upon the sphere drag coefficients. The correlated data available were presented graphically.

Equipment was modified to provide accurate recording of the motion of an accelerated sphere. The technique in obtaining valid results was presented.

Tests were conducted in glycerine solutions of different concentrations with the range of thickness varied 0% to 100%. The drag coefficients were found for Reynolds Number varied from 0.1 to 100,000. The results were tabulated and graphically presented.

Recommendations were made to study this investigation theoretically to have a further understanding of the phenomena of the drag of an accelerated sphere.
APPROVED:

29 June 1966
Date