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Application of the Maximum Deformation Theory of Failure to the Torsional Failure Due to Fracture of 4140 Steel

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APPLICATION OF THE MAXIMUM DEFORMATION THEORY OF FAILURE TO THE TORSIONAL FAILURE DUE TO FRACTURE OF 4140 STEEL

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DEDICATED

To my parents

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Chapter 1

INTRODUCTION

Forming is the fastest way of changing the shape of a piece of metal. It is the most economical process to use for high-volume mass production when the shape of a part is such that it can be made by one of the forming operations.

In a forming process the size or shape of a part is changed by the application of forces that produce stresses in the part which equal the yield strength. The applied forces may be tensile, compressive, shearing, bending, torsion, or any combination of these.

A forming operation is called "hot working" if the temperature of the material is above its recrystallization temperature and it is called "cold working" if the temperature of the material is below its recrystallization temperature.

Since forming is becoming an increasingly popular process and more materials are being developed with which the manufacturers have no experience, it is frequently necessary for the design engineer to be able to predict failure during forming. This can be done by estimating the maximum deformation possible for a given material.

Plastic flow will occur in a material when it is subjected to stresses equal to its yield strength. For uni-axial tensile and compressive forces on constant cross section members the stress distribution is assumed to be uniform across any transverse section. During bending and torsion the stresses vary from surface to center. When the forces are applied in more than one manner, the effective stresses and effective strains must be used.

It is frequently necessary to know the maximum amount of deformation that a material can be subjected to during forming without interposing an annealing cycle. There are several failure theories that are used to predict failure on the basis of initiation of yielding, such as the maximum-stress theory, the maximum-strain theory, the maximum-shear theory, and the maximum-strain energy theory. However, there are few theories for failure by fracture.

Datsko (1)* promotes a failure theory which predicts fracture during forming, a theory which he found to be reliable in both the laboratory and industry. This concept is the Maximum Deformation Theory, which states, "the maximum deformation that a material in a given condition can be subjected to in forming is that deformation that results in a natural tensile strain being induced in some direction in the part that is equal to the natural tensile strain at fracture of a tensile specimen of that particular condition of the material."

In forming operations the strains encountered are large as compared

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*Number in parentheses refers to the List of References.

to elastic strains and in order to calculate a finite or large strain, it is necessary to sum up infinitesimal increments of strain over which the change occurred, from the beginning to the end of the deformations.

For tension,

$$\varepsilon = \frac{\Delta \ell_1}{\ell_0} + \frac{\Delta \ell_2}{\ell_0 + \Delta \ell_1} + \frac{\Delta \ell_3}{\ell_0 + \Delta \ell_1 + \Delta \ell_2}$$

or

$$\varepsilon = \sum_{\ell_0}^{\ell_f} \frac{\Delta_\ell}{\ell}$$

and by integration,

$$\varepsilon = \int_{\ell_0}^{\ell_f} \frac{d\ell}{\ell} = \ell n \frac{\ell_f}{\ell_0}$$

where

ε = natural strain or logarthmic strain or finite strain

ℓ_f = final length

ℓ_0 = initial length

The term "natural strain" was first introduced by Ludvik as mentioned by Thomsen (2).

In plastic deformation, the volume remains constant; so

$$V_0 = V_f$$

therefore,

$$V_0 = A_0 \ell_0 = A_f \ell_f \qquad \text{or} \qquad \frac{\ell_f}{\ell_0} = \frac{A_0}{A_f}$$

where

V_0 = initial volume

V_f = final volume

A_0 = initial area

A_f = final area

ℓ_0 = initial length

ℓ_f = final length

Thus, natural strain at fracture, which is denoted by ε_f, is

$$\varepsilon_f = \ell n \frac{\ell_f}{\ell_0} = \ell n \frac{A_0}{A_f} \quad . \qquad (1)$$

Also,

$$A_r = \left(\frac{A_0 - A_f}{A_0}\right) \times 100 \qquad (2)$$

where A_r = percent reduction in area, which is defined as the maximum change in area times one hundred, divided by the original area. It can be used only when tensile deformation occurs.

From Eq. (2) above,

$$\frac{A_0}{A_f} = \frac{100}{100 - A_r}$$

and relating Eqs. (1) and (2), we get

$$\varepsilon_f = \ell n \frac{100}{100 - A_r} \tag{3}$$

An example of an experimental study that was made by Datsko (1) to verify his theory is presented next. The first relationship derived and verified experimentally was the minimum radius to which a sheet or plate can be bent. When applied to this operation, the theory states that "failure will occur as a transverse crack in the outer fiber of a material being bent when the natural tensile strain in the outer fiber is equal to the natural strain at the instant of fracture of a tensile specimen of the same material."

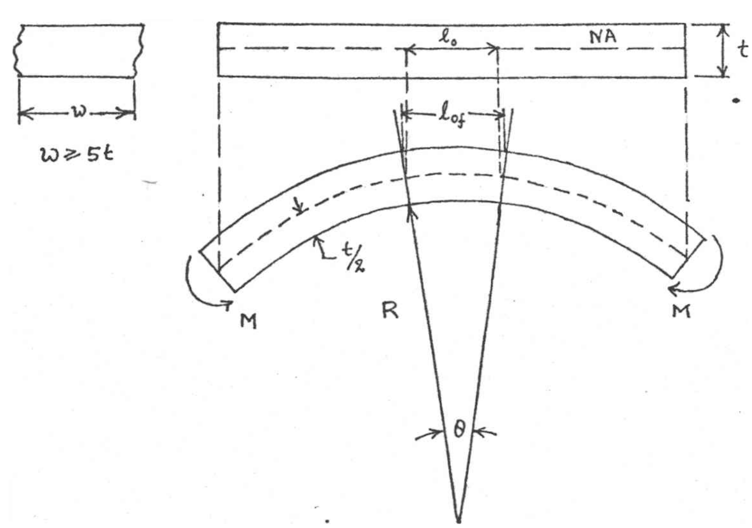

Figure 1

The Bending of a Plate

The tensile strain in the outer fiber of a bent sheet, as sketched in Figure 1, is

$$\varepsilon_{0_f} = \ell n \left(\frac{\ell_{0_f}}{\ell_0} \right) = \ell n \left[\frac{\theta (R + t)}{\theta (R + t/2)} \right] \qquad (4)$$

The natural strain at fracture of a tensile specimen is

$$\varepsilon_f = \ell n \left(\frac{100}{100 - A_r} \right)$$

By equating ε_{0_f} to ε_f, the result is

$$\left(\frac{100}{100 - A_r} \right) = \left(\frac{R + t}{R + t/2} \right)$$

By solving for R/t as a function of A_r,

$$R/t = \left(\frac{50}{A_r} - 1 \right) \qquad (5)$$

The above analysis was made assuming that neutral axis remains at mid thickness of the bent plate. This relationship was verified by laboratory tests as well as by industrial shop tests for a broad range of materials that includes brittle cast iron, high carbon steel, and precipitation hardened aluminum. For very ductile materials, with $A_r > 50$, the relationship predicts negative radii which are impossible. A modified relationship was presented by Datsko (1) which takes into account the shift in the neutral axis and does not give negative radii for $A_r > 50$.

Datsko (1) also investigated maximum deformation theory experimentally on other forming operations which include upsetting in open dies, stretching, and rolling, and his results correlated well with the analytical results.

In the present research the author applies the maximum deformation theory to a circular bar twisted under pure torsional load to fracture. The relationships derived and verified experimentally by the author are the maximum angle of twist to which a circular bar can be twisted as it relates to the strain at fracture of a tensile specimen.

Makky (3) in his mathematical analysis of the torsional failure of ductile materials at the onset of yielding, and Nadai (4) in his analysis of the torsional failure of brittle materials, have investigated that the most probable direction of fracture at the surface, the onset of yielding is in that direction which intersects the axis of the bar in a helix which makes an angle of 45° with the generators of the surface of the bar.

The author, in this research, investigated plastic torsion, that is, after yielding has taken place. The most probable surface of fracture is that surface which is perpendicular to a surface which intersects the surface of the bar in a helix which makes an angle of $45^{\circ} \leq \theta \leq 90^{\circ}$ with the generators of the surface of the bar. This angle θ is the maximum deformation angle.

When the maximum deformation theory is applied to torsional failure due to fracture, the following statement can be made:

<u>Fracture failure in a circular bar subjected to an increasing static torsional load will occur in the outermost surface of the bar in a fiber, when the natural tensile strain induced in this fiber is equal to the natural tensile</u>

strain at the instant of fracture of a tensile specimen of the same material.

Detailed mathematical derivations of the maximum tensile strain (ε_{ℓ_i}), maximum deformation angle (θ), and correlation of the maximum tensile strain (ε_{ℓ_i}) with the natural tensile strain at fracture of a tensile specimen (ε_f) and percent area reduction (A_r), by the application of maximum deformation theory, are presented next.

The circular bar sketched in Figure 2 is twisted by the application of an increasing static torsional load to fracture, and the maximum angle of twist (ϕ) is recorded.

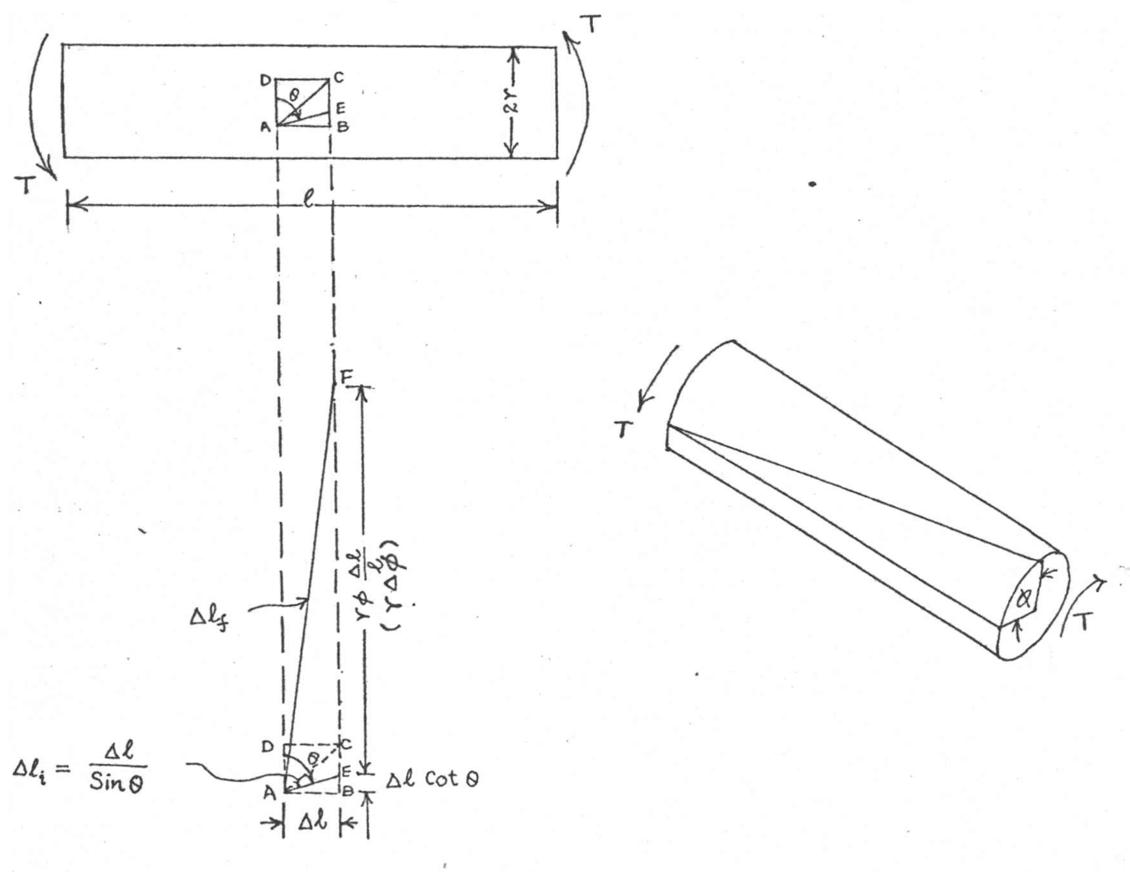

Figure 2

A Circular Bar Subjected to Torsional Load

Consider an elemental square ABCD on the surface of the bar.

For small angles of twist (ϕ) the strain of diagonal AC will be greater than the rate of strain of the base AB.

But, for large angles of twist (ϕ), the strain of base AB will be greater than the strain of diagonal AC.

For a general case, let us consider a line AE which is at an angle θ to the side AD.

Let

Δ_ℓ = length of the side of the elemental square ABCD (i.e., AB).

Δ_{ℓ_i} = initial length of the line AE.

ℓ = total length of the bar.

r = radius of the bar.

ϕ = maximum angle of twist.

Δ_{ℓ_f} = final length of line AE.

Then, the tensile strain induced in the fiber Δ_{ℓ_i} is

$$\Delta\varepsilon_{\ell_i} = \ell n \frac{\Delta\ell_f}{\Delta\ell_i}$$

$$\Delta\ell_i = \frac{\Delta\ell}{\sin\theta}$$

and

$$\Delta\ell_f = \sqrt{\Delta\ell^2 + [\Delta\ell \quad \cot\theta + r\phi\frac{\Delta\ell}{\ell}]^2}$$

Therefore, the natural tensile strain induced in the fiber ℓ_i, which is applied to the total length of the bar, is

$$\varepsilon_{\ell_i} = \ell n \{ \sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2} \cdot \sin\theta \} \qquad (6)$$

and

$$\varepsilon_{\ell_i} = \frac{1}{2} \ell n \{ 1 + (\cot\theta + \frac{r\phi}{\ell})^2 \} + \ell n \sin\theta$$

To get the maximum ε_{ℓ_i}, the derivative is taken with respect to θ and set equal to zero.

Therefore,

$$\frac{d\varepsilon_{\ell_i}}{d\theta} = \frac{-\operatorname{cosec}^2\theta \, (\cot\theta + \frac{r\phi}{\ell})}{1 + (\cot\theta + \frac{r\phi}{\ell})^2} + \cot\theta$$

Equating $d\varepsilon_{\ell_i}/d\theta$ to zero, we get

$$\cot\theta = \frac{\operatorname{cosec}^2\theta \, (\cot\theta + \frac{r\phi}{\ell})}{1 + (\cot\theta + \frac{r\phi}{\ell})^2}$$

but $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$. Therefore,

$$\cot\theta + \cot^3\theta + 2(\frac{r\phi}{\ell})\cot^2\theta + (\frac{r\phi}{\ell})^2 \cot\theta =$$

$$1 + \cot^2\theta \; [\cot\theta + \frac{r\phi}{\ell}]$$

$$= \cot\theta + \cot^3\theta + \frac{r\phi}{\ell} + \frac{r\phi}{\ell}\cot^2\theta$$

Therefore,

$$\frac{r\phi}{\ell} \cot^2\theta + \left(\frac{r\phi}{\ell}\right)^2 \cot\theta - \frac{r\phi}{\ell} = 0$$

or

$$\frac{r\phi}{\ell} \left\{\cot^2\theta + \frac{r\phi}{\ell} \cot\theta - 1 \right\} = 0$$

Either $\frac{r\phi}{\ell} = 0$ or $\cot^2\theta + \frac{r\phi}{\ell} \cot\theta - 1 = 0$. Therefore,

$$\cot^2\theta + \frac{r\phi}{\ell} \cot\theta - 1 = 0$$

and

$$\cot\theta = \left[\frac{-\frac{r\phi}{\ell} \pm \sqrt{\left(\frac{r\phi}{\ell}\right)^2 + 4}}{2}\right].$$

Taking only the positive value,

$$\cot\theta = \left[\frac{\sqrt{\left(\frac{r\phi}{\ell}\right)^2 + 4} - \left(\frac{r\phi}{\ell}\right)}{2}\right]$$

or

$$\theta = \cot^{-1}\left[\frac{\sqrt{\left(\frac{r\phi}{\ell}\right)^2 + 4} - \left(\frac{r\phi}{\ell}\right)}{2}\right] \qquad (7)$$

When $(r\phi/\ell) = 0$ (as in the case of a perfectly brittle material),

$$\theta = 45^\circ$$

and when $(r\phi/\ell) = \infty$ (as in the case of a pure ductile material which twists infinitely),

$$\theta = 90^{o}$$

The natural strain at fracture of a tensile specimen is

$$\varepsilon_f = \ell n \frac{100}{100 - A_r}$$

Applying the maximum deformation theory, we equate the maximum tensile strain induced due to torsion to the natural strain at fracture of a tensile specimen.

Therefore,

$$\varepsilon_f = \varepsilon_{\ell_i}$$

and

$$\ell n \frac{100}{100 - A_r} = \ell n \sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2} \sin\theta$$

and

$$\frac{100}{100 - A_r} = \sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2} \sin\theta$$

Therefore,

$$A_r = 100 \{1 - \frac{\text{cosec}\,\theta}{\sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2}}\} \qquad (8)$$

In the experiments carried out as part of this thesis, standard tensile and torsion specimens of 0.5 inch diameter were used to determine the true fracture strain, percent area reduction, and maximum angle of twist for 4140 steel in hot-rolled, annealed, and oil-quenched conditions.

From the data of percent area reductions and true fracture strains the $(r\phi/\ell)$ ratios were calculated analytically and compared with the $(r\phi/\ell)$ ratios obtained experimentally. The results agreed within 99 percent.

Chapter 2

EXPERIMENTAL EQUIPMENT

The equipment used to obtain experimental data consisted of hot-rolled, annealed, and oil-quenched 4140 steel specimens, a universal testing machine, electric furnaces, a torsion testing machine, a hardness tester, and a micrometer callipers.

TEST SPECIMENS

Standard tensile and torsion test specimens were used for this research. The standard test specimens had dimensions as shown in Figures 3 and 4.

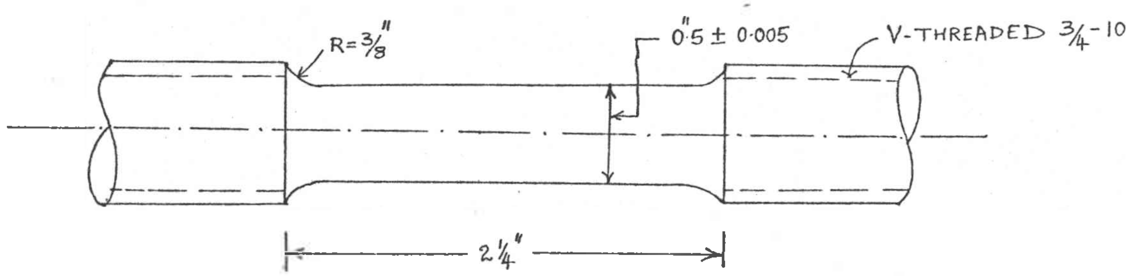

Figure 3

Standard Tensile Specimen

The test specimens were coded for identification as presented below:

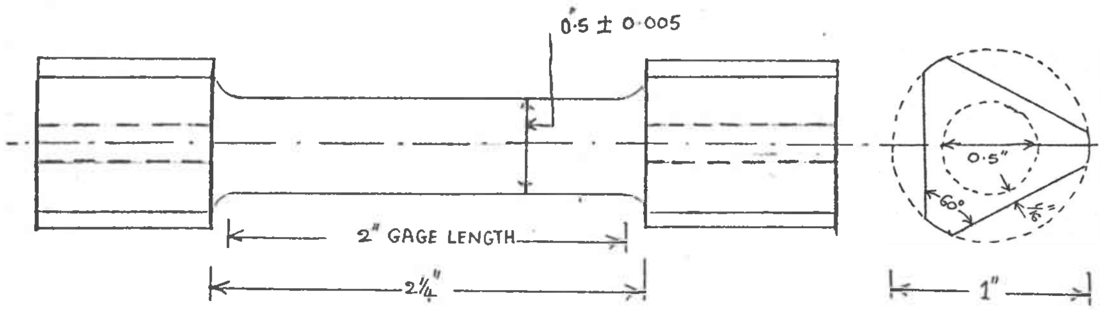

Figure 4

Standard Torsion Specimen

1. Hot-rolled 4140 steel specimens:

   a. Tension specimens : 1-1, 1-2, 1-3

   b. Torsion specimens : 2-1, 2-2, 2-3

2. Annealed 4140 steel specimens:

   a. Tension specimens : 3-1, 3-2, 3-3

   b. Torsion specimens : 4-1, 4-2, 4-3

3. Oil-quenched 4140 steel specimens:

   a. Tension specimens : 5-1, 5-2, 5-3

   b. Torsion specimens : 6-1, 6-2, 6-3

EQUIPMENT

Universal Testing Machine

A universal testing machine of the Riehle Engineering Company, serial number R-57450-1, capacity 120,000 lbs., was used to axially load the tensile specimens.

Electric Furnaces

The electric furnaces used to heat-treat specimens were products of Lucifer Electric Company, serial number 1441, temperature range 0-2300°F, and Applied Mechanics Inc., model number AMI Center C, temperature range 0-2000°F.

Torsion Testing Machine

A torsion testing machine of the Tinius-Olsen Company, model number T, MAC 08, was used to determine the maximum angles of twist for the torsion test specimens.

Rockwell Hardness Tester

A Wilson Rockwell hardness tester model 3 JR, serial number 3759, was used to measure hardnesses of the tensile test specimens. A diamond brale penetrator with 150 kg load was used ("C" scale).

Micrometer Calipers

To measure the cylindrical diameter of the tensile specimens, a 0-1 inch micrometer calipers having 0.0001 least count was used.

Chapter 3

EXPERIMENTATION

The experimentation involved processing and testing of the standard tensile and torsion specimens. The test specimens were machined from a 1-inch diameter 10-ft long hot-rolled 4140 steel bar. About twenty-four 4-1/2 inch long pieces, were cut from the 10-ft long bar with a power hack-saw. From these pieces, twelve 0.5 inch diameter tensile specimens and twelve 0.5 inch diameter torsion specimens were turned on a lathe. The ends of the tensile specimens were threaded 3/4 - 10. The ends of the torsion specimens were made triangular, the angle between two adjacent sides being 60^{o}. A shaper was used to machine the triangular ends.

The specimens which were prepared as explained above were identification coded, heat treated, and tested in three different groups--namely, hot-rolled, annealed, and oil-quenched. Heat treating and testing of the standard specimens were done by the following methods:

HEAT TREATING OF STANDARD TENSILE AND TORSION SPECIMENS

The specimens were classified as hot-rolled, annealed, and oil-quenched groups.

The first group of specimens was not heat treated as the bar from which the specimens were made was a hot-rolled 4140 steel bar.

The second group of specimens was annealed as a single batch. For annealing, all specimens were heated to 1400^{o}F and cooled overnight by turning the power off, leaving the furnace shut. The Lucifer electric furnace was used for the purpose of annealing. Annealing produced uniformity of properties and removed the effects of residual stresses due to machining.

The third group of specimens was oil-quenched as a single batch. For oil-quenching, all specimens were heated to 1450^{o}F and quenched in a medium of oil at a cooling rate of approximately 30^{o}F per second. An atmospherically controlled furnace of the Applied Mechanics Inc. company was used for this purpose. The furnace atmosphere was controlled by a regulated flow of argon gas. This prevented decarburization of the surfaces of the specimens.

TESTING OF STANDARD TENSILE SPECIMENS

The hardnesses of the tensile specimens in the three groups were determined with a Rockwell hardness tester. A diamond brale penetrator, with a 150 kg load, was used ("C" scale). From the hardness data the ultimate tensile loads for the various specimens were approximated (see Table 1) and the load scale of the universal testing machine was selected accordingly.

The specimens were pulled by the universal testing machine at the rate of 0.025 inch per minute to fracture. The yield load, ultimate load, and the fracture load were recorded with the corresponding diameters. Three

Table 1

Hardness Data for Tensile Specimens
(4140 steel)

Group	Rockwell C	Brinell hardness number	Ultimate tensile stress (lb./sq. in.) 500 x BHN	Ultimate tensile load (lb.)
Hot rolled	15	200	100,000	20,000
Annealed	10	180	90,000	18,000
Oil quenched	21	230	115,000	23,000

sets of data pertaining to hot-rolled, annealed, and oil-quenched specimens were obtained (see Appendix B). The true stress-strain curves were plotted for the three groups of specimens (see Appendix B). The true fracture strains (ε_f) and hence the percent area reductions (A_r) for the various specimens were determined (see Appendix A).

TESTING OF STANDARD TORSION SPECIMENS

The standard torsion specimens of the hot-rolled, annealed, and oil-quenched 4140 steel were tested on the Tinius-Olsen torsion testing machine. The specimens were twisted at the rate of 0.05 radian per second to fracture. The tension force which was induced due to the shortening of the length while twisting was considerably eliminated by adjusting the end chucks each time a few degrees of twist was given to the specimen.

The angles corresponding to the fracture failure of the three groups of specimens were recorded (see Appendix B). The ($r\phi/\ell$) ratios for the three groups of specimens were determined (see Appendix A). The maximum angles of twist (ϕ) were the averages of the values obtained in the three groups.

Chapter 4

RESULTS AND DISCUSSION OF RESULTS

RESULTS

The following results were arrived at from the experimental data and the analytical relationships derived in Chapter 1.

1. True strains at fracture of the tensile specimens and their corresponding percent area reductions (see Table 2).

2. Average maximum angles of twist for torsion specimens and their corresponding $(r\phi/\ell)$ ratios calculated analytically, from true fracture strains and percent area reductions of tensile specimens, and obtained experimentally (see Table 3).

3. Maximum deformation angles for the analytical and experimental values of $(r\phi/\ell)$ ratios.

DISCUSSION OF RESULTS

Figure 5 shows the analytical curves of maximum deformation angle (θ), maximum tensile strain (ε_{ℓ_i}), and percent area reduction (A_r) drawn to common base of $(r\phi/\ell)$ ratio.

The $(r\phi/\ell)$ ratios obtained experimentally are superimposed on the analytical curves of $(r\phi/\ell)$ ratio versus percent area reduction and $(r\phi/\ell)$

Table 2

Data of 4140 Steel Tensile Specimens*

Characteristic	Hot-rolled	Annealed	Oil-quenched
Initial diameter (d_0) (in.)	0.505	0.505	0.505
Fracture diameter (d_f) (in.)	0.364	0.330	0.440
True fracture strain (ε_f) (in./in.)	0.655	0.850	0.276
Percent reduction in area (A_r)	48.00	57.25	24.00

*See Appendices (A and B) for calculations and tension test data.

Table 3

Data of $(r\phi/\ell)$ Ratio, Maximum Deformation Angle (θ), and Maximum Angle of Twist (ϕ) for 4140 Steel Torsion Specimens*

Sr. No.	Condition	ϕ in degrees (average value obtained in each group)	$(r\phi/\ell)$		% Error	θ	
			Experimental	Calculated		Experimental	Calculated
1	Hot-rolled	590.66	1.305	1.305	0.0	$61^{o}30'$	$61^{o}30'$
2	Annealed	793.30	1.730	1.700	0.2	$65^{o}45'$	$65^{o}30'$
3	Oil-quenched	249.33	0.545	0.520	0.5	$57^{o}30'$	$57^{o}00'$

*See Appendices (A and B) for calculations and torsion test data.

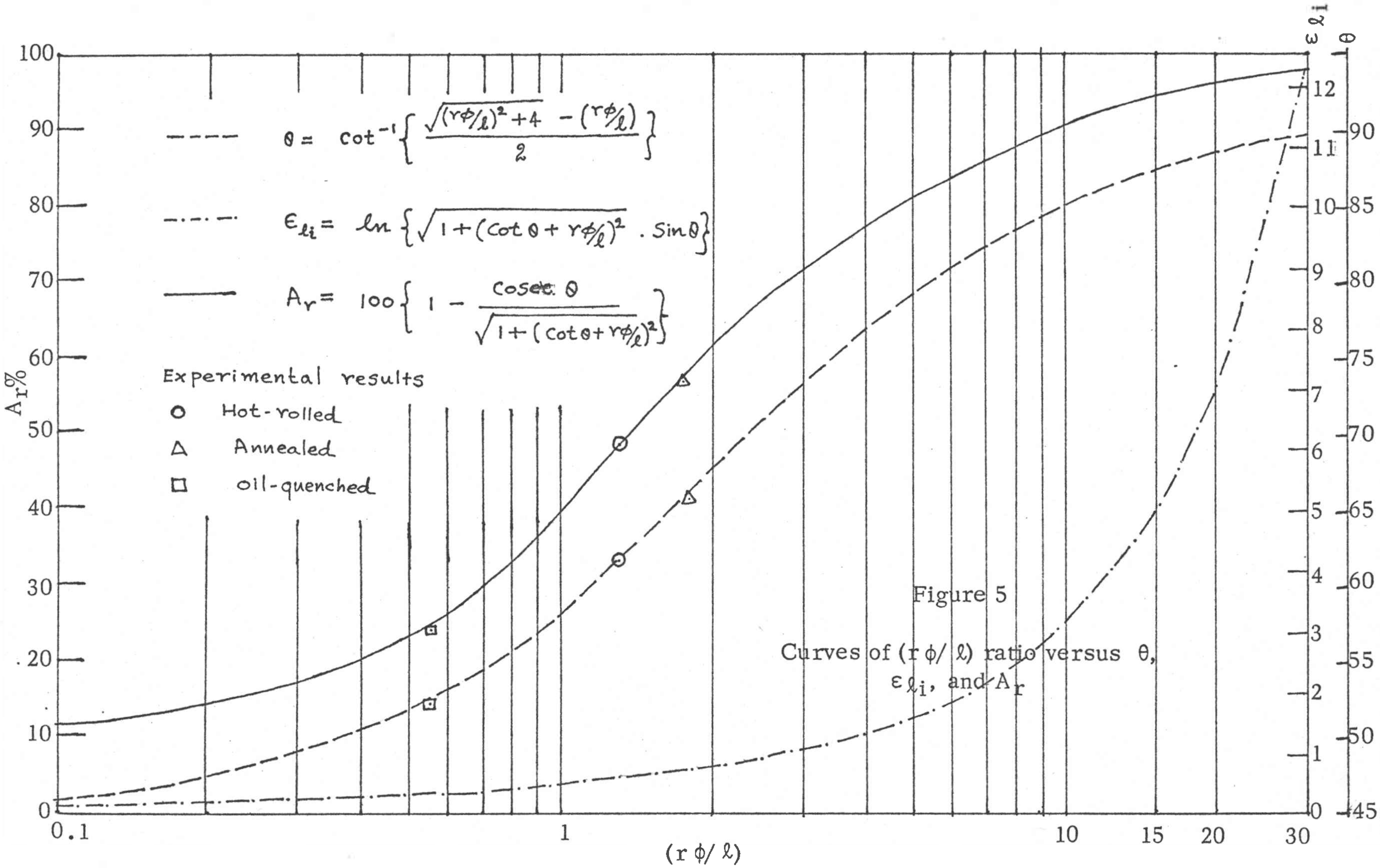

Figure 5

Curves of $(r\phi/\ell)$ ratio versus θ, $\varepsilon_{\ell i}$, and A_r

ratio versus maximum deformation angle.

It can be observed that the experimental values of $(r\,\phi/\ell)$ ratio differed from the analytical values of $(r\,\phi/\ell)$ ratio by 0 percent, 0.2 percent, and 0.5 percent for the hot-rolled, annealed, and oil-quenched 4140 steel specimens respectively.

Figure 6 shows a photograph of torsional failure of annealed 4140 steel specimen.

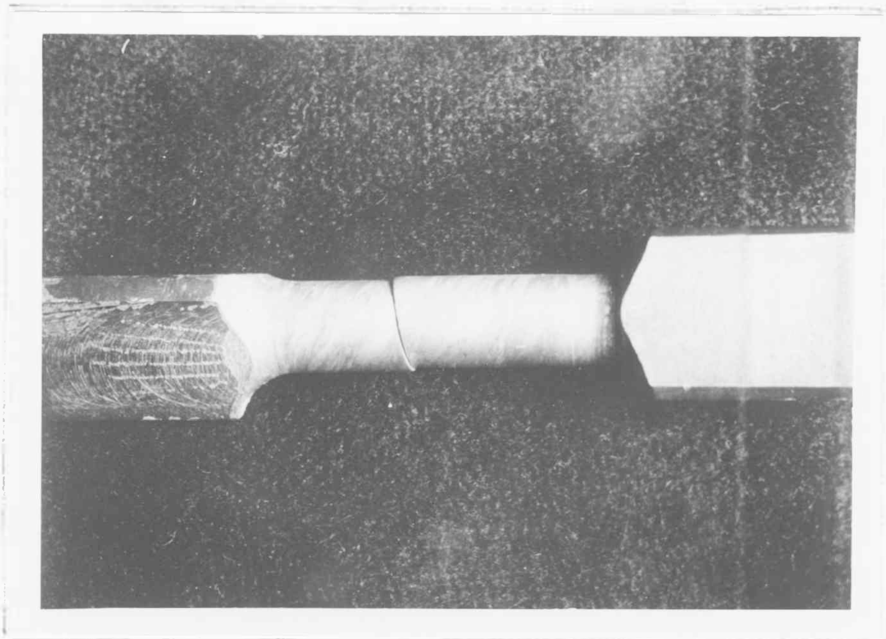

Figure 6

Photograph of torsional failure of annealed 4140 steel specimen

Chapter 5

CONCLUSION

It is possible to conclude the following from the relationships derived in Chapter 1 and the experimental results obtained.

1. The maximum deformation theory can be applied to predict torsional failure by fracture of 4140 steel, in hot-rolled, annealed, and oil-quenched conditions having percent area reductions of 48.00, 57.25, and 24.00 respectively.

Table 4 gives some insight to those interested in doing further work in the area of torsional deformation studies. The $(r\phi/\ell)$ ratios for several materials with percent area reductions ranging from 12 to 90 percent have been given in this table. These values need experimental verification.

2. The experimental data obtained and the analytical results calculated correlate within 99 percent accuracy for 4140 steel.

3. The present study shows the percent reduction of area (A_r) is an important tensile property of materials from which it is possible to predict the total torsional fracture failure of materials.

Table 4

Data of ($r\phi/\ell$) Ratio Versus Percent Reduction of Area (A_r) for Some Materials

Sr. No.	Material	A_r (%)	($r\phi/\ell$)
1	2024 Al., T-4	12.19	0.260
2	4140 steel, oil quenched*	24.00	0.520 0.545 (exp)
3	Molybdenum, ext. annealed	31.61	0.750
4	52100 steel, 1500 F ann. Cobalt base alloy, soln. treated	32.96	0.800
5	4340 steel, ann.	36.23	0.900
6	1144 steel, ann.	38.73	0.975
7	1045 steel, HR 52100 steel, spher. ann.	44.01	1.150
8	17-4 PH stainless, 1100 F age	47.79	1.300
9	4140 steel, HR*	48.00	1.305 1.305 (exp)
10	4140 steel, annealed*	57.25	1.700 1.730 (exp)
11	1024 steel, HR Vanadium, ann.	59.34	1.800
12	202 stainless, 1900 F, 1 hr. ann.	63.21	2.000
13	70-30 leaded brass, 1250 F, 1 hr. ann.	66.71	2.200
14	1002 steel, ann.	69.88	2.700
15	18-8 stainless, 1800 F 1 hr. ann.	74.84	3.500

Table 4 (continued)

Sr. No.	Material	A_r (%)	$(r\phi/\ell)$
16	70-30 brass, 1000 F, 1 hr. ann.	77.68	4.000
17	304 stainless, annealed	81.17	5.000
18	1100 Al., 900 F, 1 hr. ann.	89.97	9.500

*Test specimens were made from these materials.

LIST OF REFERENCES

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1. Datsko, Joseph. Material Properties and Manufacturing Processes. New York: John Wiley and Sons, Inc., 1966.

2. Thomsen, Eric G. Plastic Deformation in Metal Processing. New York: The Macmillan Company, 1965.

3. Makky, Sadia M. "Plastic Flow and Fracture in Round Bars under Pure Torsion," Journal of Mathematics and Mechanics, X, 1961.

4. Nadai, Arpad. Theory of Flow and Fracture of Solids. 2nd ed. New York: McGraw Hill Book Co., 1950.

5. The American Society of Metals. Metals Handbook. Novelty, Ohio: Metals Park, 1948.

6. American Society for Testing and Materials Standards. "Physical and Mechanical Testing of Metals--Non-destructive Tests," Part 13. Philadelphia: ASTM, 1967.

7. Peckner, Donald. The Strengthening of Metals. New York: Reinhold Publishing Corp., 1964.

8. Avitzur, Betzalel. Metal Forming Processes and Analysis. New York: McGraw Hill Book Company, 1968.

9. Vidosic, J. P. Metal Machining and Forming Technology. New York: The Ronald Press Company, 1964.

APPENDICES

APPENDIX A

CALCULATIONS

DETERMINATION OF TRUE FRACTURE STRAIN (ε_f) AND PERCENT AREA REDUCTION (A_r) FOR 4140 TENSILE SPECIMENS

Hot-rolled specimen:

d_0 = initial diameter = 0.505 inch

d_f = fracture diameter = 0.364 inch

Therefore, true fracture strain

$$\varepsilon_f = 2 \ln \frac{d_0}{d_f} = 2 \ln \frac{0.505}{0.364} = 0.655 \text{ in/in}$$

Percent area reduction

$$A_r = [1 - \frac{A_f}{A_0}] \times 100 = [1 - (\frac{0.364}{0.505})^2] \times 100$$

$$= 48\%$$

Annealed specimen:

d_0 = initial diameter = 0.505 inch

d_f = fracture diameter = 0.330 inch

Therefore, true fracture strain

$$\varepsilon_f = 2 \ln \frac{d_0}{d_f} = 2 \ln \frac{0.505}{0.330}$$

$$= 0.85 \text{ in/in}$$

Percent area reduction

$$A_r = [1 - \frac{A_f}{A_0}] \times 100$$

$$= [1 - (\frac{0.330}{0.505})^2] \times 100 = 57.25\%$$

Oil-quenched specimen:

d_0 = initial diameter = 0.505 inch

d_f = fracture diameter = 0.440 inch

Therefore, true fracture strain

$$\varepsilon_f = 2 \ln \frac{d_0}{d_f} = 2 \ln \frac{0.505}{0.440}$$

$$= 0.276 \text{ in/in}$$

Percent area reduction

$$A_r = [1 - \frac{A_f}{A_0}] \times 100$$

$$= [1 - (\frac{0.440}{0.505})^2] \times 100 = 24\%$$

DETERMINATION OF $(r\phi/\ell)$ RATIO FOR 4140 STEEL TORSION SPECIMENS

Hot rolled:

A_r = 48%

ε_f = 0.655 in/in

$$A_r = 100 \{ 1 - \frac{\text{cosec } \theta}{\sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2}} \}$$

$$48 = 100 \left\{ 1 - \frac{\operatorname{cosec} \theta}{\sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2}} \right\} \qquad (1)$$

and also

$$\varepsilon_f = \varepsilon_{\ell_i}$$

Therefore,

$$0.655 = \ell n \left\{ \sqrt{1 + (\cot\theta + \frac{r\phi}{\ell})^2} \cdot \sin\theta \right\} \qquad (2)$$

Solving Equations (1) and (2), we get

$$\frac{r\phi}{\ell} = 1.305, \qquad \theta = 61^\circ 30'$$

Annealed:

In a similar way for $A_r = 57.25\%$ and

$\varepsilon_f = 0.85$ in/in,

we get

$$\frac{r\phi}{\ell} = 1.7 \quad \text{and} \quad \theta = 65^\circ 30'$$

Oil quenched:

For $A_r = 24\%$ and

$\varepsilon_f = 0.276$ in/in,

we can calculate that

$$\frac{r\phi}{\ell} = 0.52 \qquad \theta = 57^\circ$$

DETERMINATION OF EXPERIMENTAL VALUES OF θ

Hot rolled:

From experiment,

$$\frac{r\,\phi}{\ell} = 1.305$$

Therefore,

$$\theta = \cot^{-1}\left\{\frac{\sqrt{\left(\frac{r\,\phi}{\ell}\right)^2 + 4} - \left(\frac{r\,\phi}{\ell}\right)}{2}\right\}$$

$$= \cot^{-1}\left\{\frac{\sqrt{(1.305)^2 + 4} - (1.305)}{2}\right\}$$

$$= \cot^{-1}(0.5430) = 61^{\circ}30'$$

Annealed:

From experiment,

$$\frac{r\,\phi}{\ell} = 1.73$$

Therefore,

$$\theta = \cot^{-1}\left\{\frac{\sqrt{(1.73)^2 + 4} - (1.73)}{2}\right\}$$

$$= \cot^{-1}(0.4503)$$

$$= 65^{\circ}45'$$

Oil quenched:

From experiment,

$$\frac{r\,\phi}{\ell} = 0.545$$

Therefore,

$$= \cot^{-1}\left\{\frac{\sqrt{(0.545)^2 + 4} - (0.545)}{2}\right\}$$

$$= \cot^{-1}(0.6371)$$

$$= 57^{o}30'$$

APPENDIX B

TENSION AND TORSION TEST DATA

Table 5

Stress Strain Data for Hot-rolled 4140 Steel
Tensile Specimen

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	10050	.495	52200	.0042
2	15050	.490	79900	.0600
3	16650	.480	92000	.0980
4	16950	.475	95750	.1170
5	17450	.470	100500	.1390
6	17250	.460	104000	.1890
7	17200	.450	108000	.2280
8	17100	.440	112500	.2760
9	17000	.430	117000	.3240
10	16900	.420	117300	.3660

Table 6

Stress Strain Data for Annealed 4140 Steel
Tensile Specimen

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	9070	.495	47100	.0042
2	13400	.490	71000	.0060
3	15500	.480	85500	.0980
4	15800	.475	89250	.1170
5	16380	.470	94250	.1390
6	16420	.460	99000	.1890
7	16480	.450	103500	.2280
8	16830	.440	110800	.2760
9	16630	.430	114800	.3240
10	16470	.425	116000	.3380
11	15740	.420	110000	.3660

Table 7

Stress-Strain Data for Oil Quenched 4140 Steel
Tensile Specimen

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	22360	.495	116000	.0042
2	22330	.490	118500	.0600
3	22300	.480	123000	.0980
4	22220	.475	126000	.1170
5	22200	.470	128000	.1390
6	22160	.460	133000	.1890

Figure 7

True Stress-Strain Curves for 4140 Tensile Test Specimens

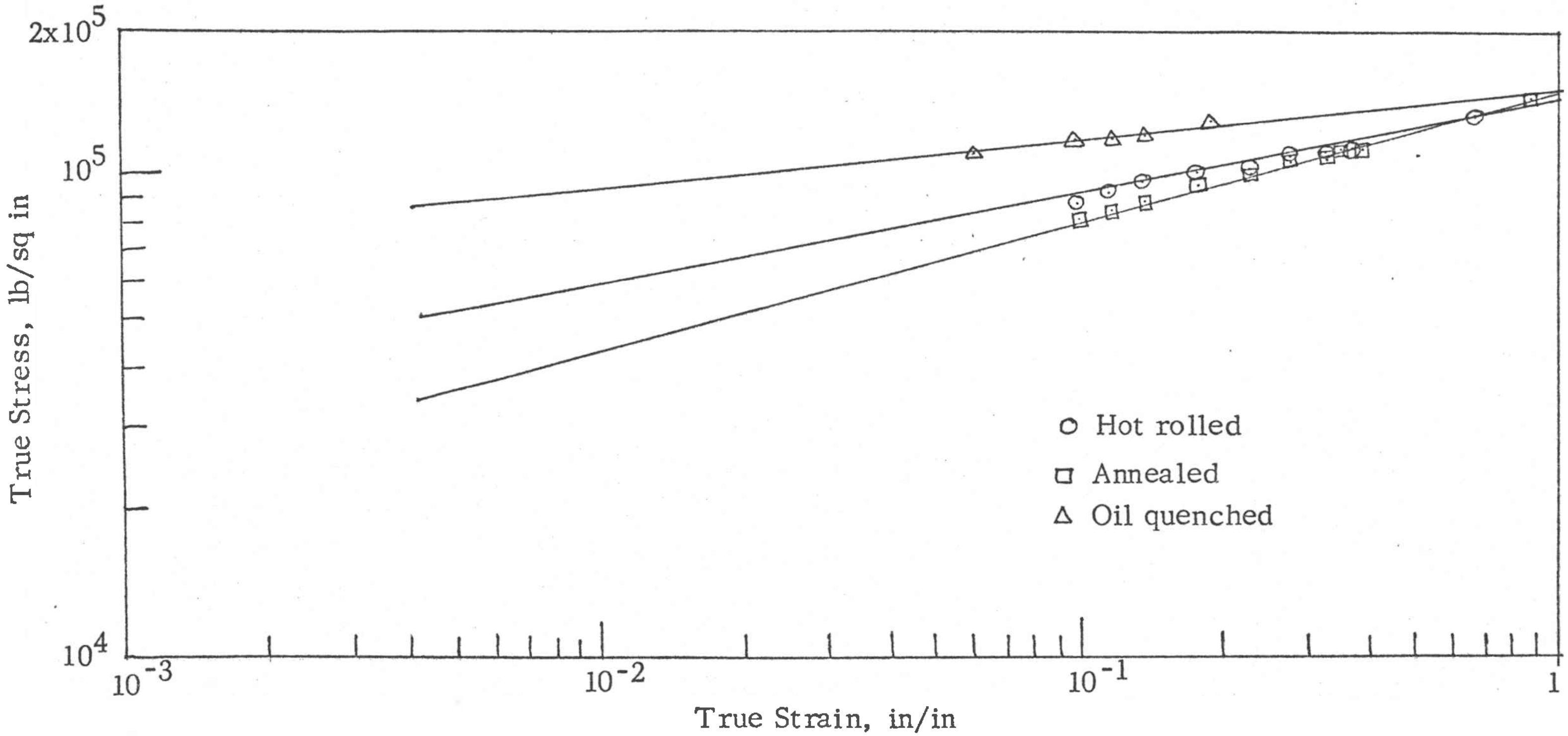


Table 8

Stress-Strain Data for Specimen 1-1

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	10000	.495	52000	.004
2	15000	.490	79250	.059
3	16650	.480	92000	.098
4	16900	.475	95500	.116
5	17500	.470	100000	.136
6	17200	.460	103750	.192
7	17100	.450	107500	.209
8	17100	.440	112500	.280
9	17000	.430	117500	.332
10	16950	.420	122500	.362

Initial Diameter of the Specimen: .505 in
Maximum Load: 17500 lbs
Fracture Diameter: .368 in
Fracture Load 15000 lbs

Table 9

Stress-Strain Data for Specimen 1-2

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	10200	.495	52250	.004
2	15150	.490	80000	.059
3	16600	.480	91750	.098
4	16950	.475	96000	.116
5	17400	.470	99750	.136
6	17300	.460	104000	.192
7	17200	.450	108000	.209
8	17100	.440	112500	.280
9	17000	.430	117200	.332
10	16900	.420	122000	.362

Initial Diameter of the Specimen: .505 in
Maximum Load: 17400 lbs
Fracture Diameter: .354 in
Fracture Load: 14800 lbs.

Table 10

Stress-Strain Data for Specimen 1-3

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	10000	.495	52250	.004
2	15000	.490	79250	.059
3	16400	.480	90750	.098
4	16950	.475	96000	.116
5	17400	.470	99500	.136
6	17300	.460	104000	.192
7	17250	.450	108000	.209
8	17100	.440	112500	.280
9	17000	.430	117500	.332
10	16850	.420	122000	.362

Initial Diameter of the Specimen: .505 in
Maximum Load: 17400 lbs
Fracture Diameter: .370 in
Fracture Load: 14500 lbs

Figure 8

True Stress-Strain Curves for Hot-Rolled Specimens

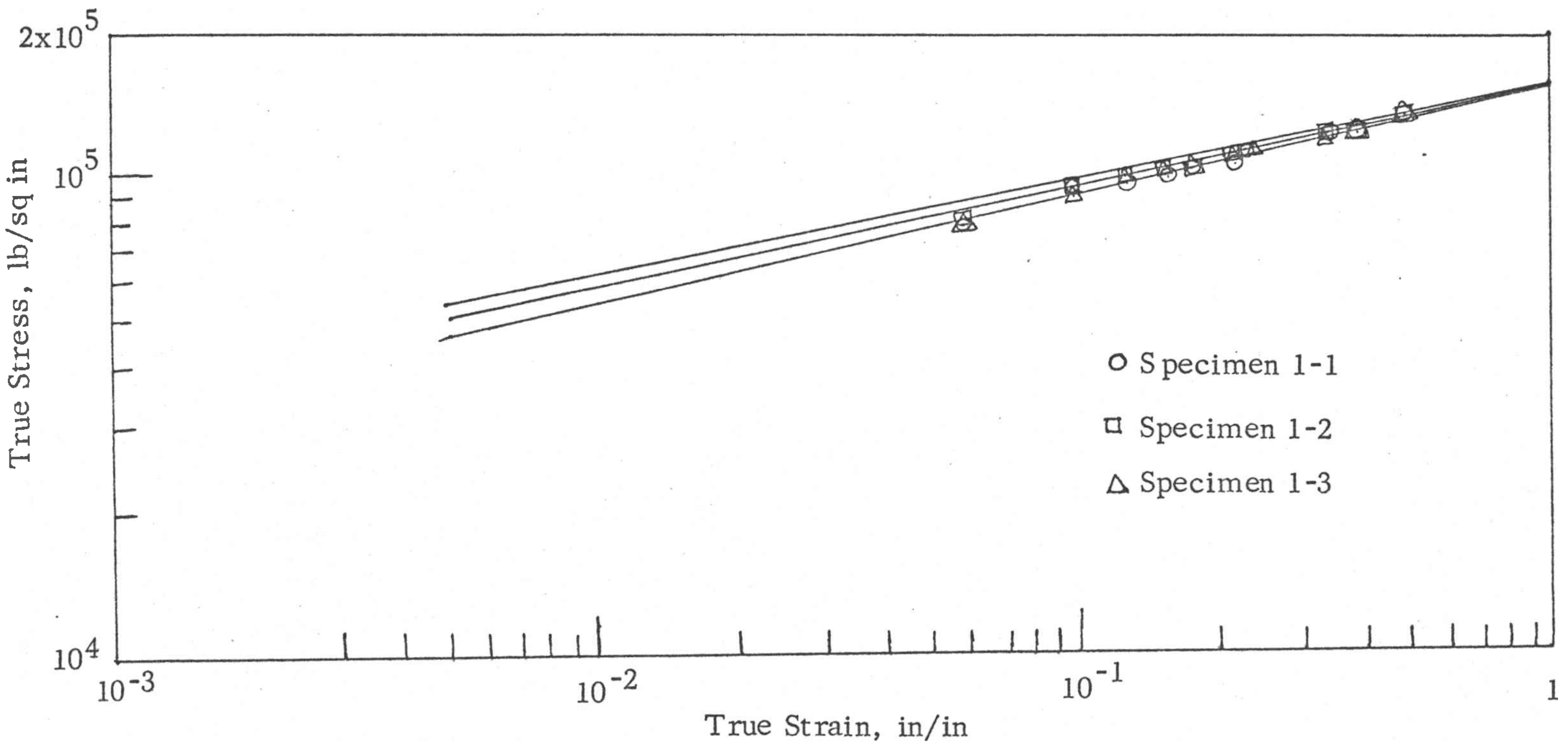


Table 11

Stress-Strain Data for Specimen 3-1

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	9050	.495	47300	.004
2	13250	.490	70000	.058
3	15450	.480	85400	.097
4	15900	.475	89800	.114
5	16200	.470	93000	.135
6	16300	.460	98000	.190
7	16300	.450	102100	.207
8	16600	.440	109000	.275
9	16500	.430	113500	.330
10	16400	.425	115500	.346
11	15750	.420	114000	.360

Initial Diameter of the Specimen: .503 in
Maximum Load: 16600 lbs
Fracture Diameter: .340 in
Fracture Load: 14250 lbs

Table 12

Stress-Strain Data for Specimen 3-2

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	10000	.495	52250	.004
2	13350	.490	70600	.059
3	15650	.480	86500	.098
4	16050	.475	90750	.116
5	16450	.470	94250	.136
6	16450	.460	99000	.192
7	16550	.450	104000	.209
8	17000	.440	105000	.280
9	16600	.430	114500	.332
10	16500	.425	116000	.348
11	15800	.420	114000	.362

Initial Diameter of the Specimen: .505 in
Maximum Load: 17000 lbs
Fracture Diameter: .333 in
Fracture Load: 14000 lbs

Table 13

Stress-Strain Data for Specimen 3-3

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	10000	.495	52250	.004
2	13300	.490	70400	.059
3	15500	.480	85750	.098
4	15850	.475	89500	.116
5	16500	.470	94500	.136
6	16500	.460	99400	.192
7	16600	.450	104500	.209
8	16900	.440	111000	.280
9	16800	.430	116000	.332
10	16500	.425	125000	.348
11	15700	.420	114000	.362

Initial Diameter of the Specimen: .505 in
Maximum Load: 16900 lbs
Fracture Diameter: .345 in
Fracture Load 13900 lbs

Figure 9

True Stress-Strain Curve for Annealed Specimens

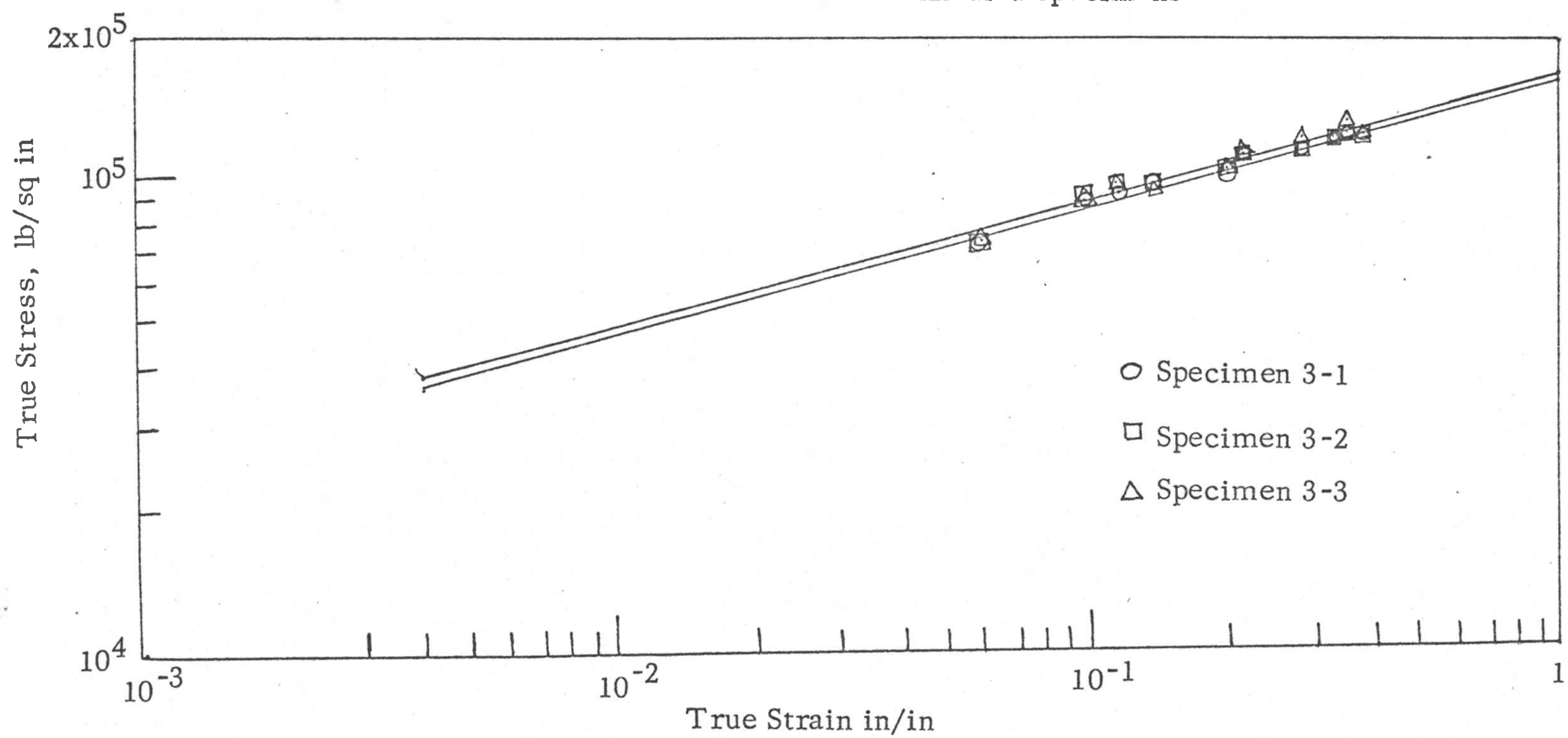


Table 14

Stress-Strain Data for Specimen 5-1

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	22500	.495	117250	.004
2	22450	.490	119000	.059
3	22400	.480	124000	.095
4	22300	.475	127000	.112
5	22250	.470	128500	.132
6	22100	.460	135000	.190

Initial Diameter of the Specimen: .503 in
Maximum Load = Yield Load: 22500 lbs
Fracture Diameter: .450 in
Fracture Load 22000 lbs

Table 15

Stress-Strain Data for Specimen 5-2

Sr. No.	Load (lb)	Diameter (in)	True Stress (lb/sq in)	True Strain (in/in)
1	22200	.495	115000	.004
2	22150	.490	119000	.059
3	22150	.480	122000	.098
4	22100	.475	124800	.116
5	22100	.470	126500	.136
6	22050	.460	132800	.192

Initial Diameter of the Specimen: .505 in
Maximum Load = Yield Load: 22200 lbs
Fracture Diameter: .440 in
Fracture Load: 22000 lbs

Table 16

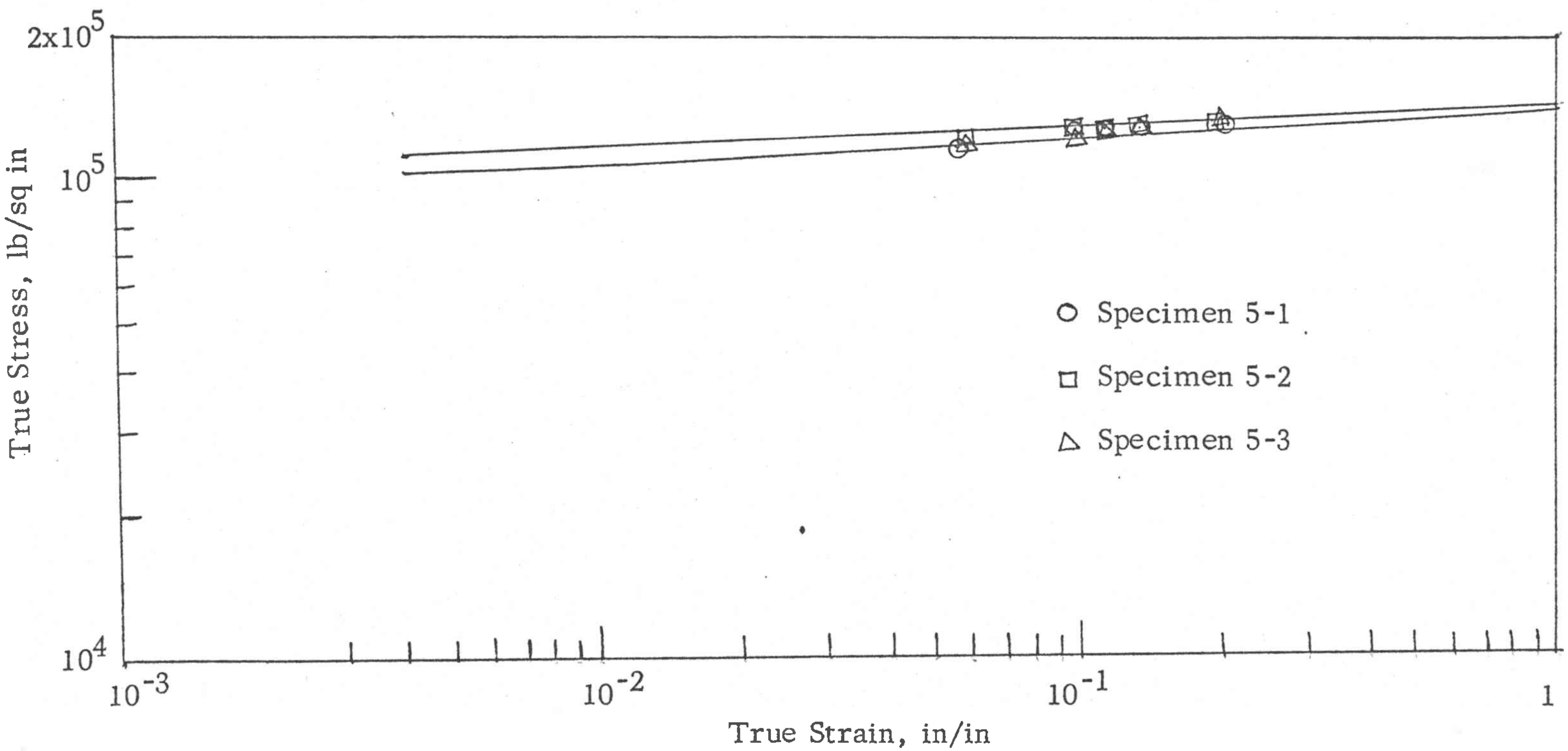
Stress-Strain Data for Specimen 5-3

Sr. No.	Load (lb)	Diameter (in)	True Stress (in/sq in)	True Strain (in/in)
1	22600	.495	118000	.004
2	22500	.490	119000	.059
3	22400	.480	124000	.098
4	22250	.475	126000	.116
5	22250	.470	127800	.136
6	22250	.460	134000	.192

Initial Diameter of the Specimen: .505 in
Maximum Load = Yield Load: 22600 lbs
Fracture Diameter: .43 in
Fracture Load: 22200 lbs

Figure 10

True Stress-Strain Curve for Oil-Quenched Specimens



APPLICATION OF THE MAXIMUM DEFORMATION THEORY OF FAILURE TO THE TORSIONAL FAILURE DUE TO FRACTURE OF 4140 STEEL

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M. S. Degree, May 1972

ABSTRACT

This study was undertaken to apply the maximum deformation theory of failure to the torsional failure due to fracture of 4140 steel in hot-rolled, annealed, and oil-quenched conditions. Analytical relationships were derived for the maximum tensile strain (ε_{ℓ_i}), the maximum deformation angle (θ), and the percent area reduction (A_r) in terms of the ($r\phi/\ell$) ratios, and were verified experimentally.

For experimentation, standard 4140 steel tensile and torsion specimens were prepared, heat treated, and tested. The experimental values of ($r\phi/\ell$) ratios obtained were compared with the analytical values of ($r\phi/\ell$) ratios. The result agreed within 99 percent accuracy for 4140 steel.

COMMITTEE APPROVAL: