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Liquid Particle Interaction with a Normal Shock Wave

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LIQUID PARTICLE INTERACTION WITH A NORMAL SHOCK WAVE

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DEDICATION

To my wife, Anne.

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APPROVALS

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NOMENCLATURE

a	Minor diameter, microns
b	Major diameter, microns
C_D	Drag coefficient
d	Droplet diameter, microns
g	acceleration due to gravity, ft/sec^2
M	Mach number
m	Droplet mass, lb_m
n	an integer, 1,2,3,. . .,n
p	Pressure, lb_f/in^2
R_D	Droplet radius, microns
Re	Reynolds'number
s	Area, $microns^2$
t	Time, seconds
t_a	Action time, seconds
U	Relative velocity between drop and surrounding medium, ft/sec
W	Molecular weight, lb_m/mole; mass injection rate, lb_m/hr
We	Weber number
X	Mass median diameter, microns
δ	Droplet deformation parallel to flow direction, microns
μ	Microns
μ_g	Gas stream absolute viscosity, lb_m/ft-sec
μ_l	Droplet absolute viscosity, lb_m/ft-sec
ν	Kinematic viscosity, ft^2/sec

π	3.14
ρ	Droplet radius, microns
ρ_g	Gas stream density, lb_m/hr
ρ_l	Droplet density, lb_m/hr
σ	Surface tension, lb_f/ft or dynes/cm
τ	Natural period of oscillation, seconds
φ	Angular distance from stagnation point; polar angle around median vertical plane of droplet

Subscripts

avg	Average condition
b	Breakup condition
c	Critical condition
D	Droplet
g	Gas
l	Liquid
max	Maximum condition
o	Initial condition; outside
s	Stagnation point; due to surface tension
x	Upstream from shock front
y	Downstream from shock front
∞	Free stream condition

CHAPTER I

INTRODUCTION

INTRODUCTION

The atomization of a liquid by an air stream has been an item of much concern in the past, and there have been numerous experimental studies of the process made in an effort to correlate liquid and air-stream physical properties into a general model which will predict the degree of and the time required for atomization for a given physical situation. The breakup of liquid drops, a major portion of the atomization process, finds application in the field of meteorology, where the formation, breakup, and shape of raindrops is an item of much concern; in the field of internal combustion engines, where the carburization of a fuel is of major importance; in the industrial field, where the atomization of paint and plant insecticides and spray drying processes are oft-mentioned objects of concern; in the chemical industries, where the emulsification of liquid-liquid systems, the formation of froths, the production of aerosols, and dispersion processes in general are most important considerations; and in the science of rheology, wherein the motion and dispersion of liquids, gases, and solids must take into consideration various breakup mechanisms. Despite the oft-mentioned importance of such liquid atomization processes, there has been little theoretical work done in an effort to mathematically correlate the important variables and parameters common to all breakup and dispersion processes in general.

With the advent of the importance of the rocket engine and the supersonic aircraft, the applications of the subject of liquid droplet breakup have been greatly multiplied. The study of rocket combustion instability, the sustaining of a detonation wave in a gas stream, the impingement of liquid particles on a supersonic aircraft, and the erosion and ablation of materials used in the construction of rockets and aircraft are but a few of the most important fields of application of droplet breakup in our modern technology. Again it should be noted that there exists an alarming dearth of available theoretical information on droplet shattering processes in general.

With the importance of the interaction of solids-gas, liquids-gas, and liquids-liquids systems in mind, a study has been undertaken to review the existing literature on the general subject and on the droplet shattering mechanisms in particular, and to attempt to discover the correlations and/or the discrepancies in the existent theories of droplet shattering.

Objectives

The objectives of this study were to throroughly research and review the existing literature on droplet breakup processes in general, attempt to discover the correlation and/or discrepancies existing between the theories described in the literature, and then attempt to modify and/or use these theories to investigate the phenomena associated with the specific case of a liquid aluminum particle passing through a normal shock wave at a Mach number of 2.5. This thesis presents a portion of the results of a larger body of research that was done under contract between the Naval Ordinance Test Station, China Lake, California and Brigham Young University, 1964-1965.

An attempt was made to develop computer programs that would adequately describe the various breakup models that were found in the literature. This was done because there appear in the literature very few actual plots of the data representing the various equations describing mathematical models of droplet breakup. In the development of these programs, the data used in calculation applied specifically to the above-mentioned case of a liquid aluminum droplet passing through a normal shock wave at a Mach number of 2.5.

By a comparison of the various outputs from the computer programs, the various breakup theories were modified and examined over a wide range of variables in an attempt to correlate the theories into a general model for the breakup of a liquid droplet of given physical properties subjected to a given combination of flow parameters.

The various breakup theories are also discussed in some detail in this thesis. This was done so that this body of research might provide a logical beginning for further original research into the droplet breakup process and mechanism.

General Discussion of Droplet Breakup

The breakup of a liquid droplet is an extremely complex process. Because of the complexity and irregularity of the shape of a liquid drop undegoing a given breakup process, it is very difficult to adequately describe the surface configuration by means of a mathematical expression. Since many of the parameters upon which the breakup process is dependent are themselves function of the shape of the droplet, the mathematical complexity of the problem of adequately describing breakup criteria is manifoldly increased.

When a droplet falls through a stagnant medium under the influence only of gravity, the shape of the droplet is significantly influenced by the surface tension forces, the hydrostatic forces within the droplet, the shape-dependent aerodynamic forces, the effects of internal and surface circulation of the fluid droplet, the natural and induced internal droplet vibrations, the centrifugal effects of the radial outflow and inflow of liquid, the viscosities of the liquid and the medium through which is is falling, the electrostatic charge on the surface of the droplet, the effect of boundary layer separation at some point and under some complex conditions, the shedding of vortices from the windward side of the droplet as it falls, and even the diameter of the droplet itself. To attempt to correlate the effects of all of these variables into a single mathematical expression would indeed prove a formidable if not impossible task with the present state of the mathematical art.

When the drop can no longer be assumed to merely be falling through a stagnant medium, but is instead subjected to horizontal flows, the effects of passing through a normal shock wave, the turbulence effects of the free stream, the interaction with other particles undergoing breakup, the indeterminate velocity lag between the particle and the free stream, wall effects, and uncertain physical parameters of the gas stream and the droplet itself, it is seen that the problem of providing an adequate mathematical description of the breakup process is presently insurmountable. To add to the complexity, for the particular situation assumed, the aluminum particle is also burning, which makes the determination of certain physical parameters much more difficult and undermines some of the assumptions underlying the basic breakup process itself.

There exists an entire spectrum of modes of droplet breakup. At the extremes of this spectrum exist the modes of breakup known respectively as bag and shear breakup. The process of bag breakup of a liquid droplet can be explained in the following manner. As a liquid droplet is subjected to a gas flow, the droplet deforms into roughly an ellipsoidal shape, with the major axis of the ellipsoid perpendicular to the direction of flow. The deformation of the droplet can also be described as a general flattening and radial outflow of the droplet in the directions perpendicular to the direction of flow. The resulting deformation at this point has been called at various times by various authors disk-shaped, saucer-shaped, and roughly toroidal-shaped. As the deformation of the droplet continues, the center portion of the droplet begins further deformation in the direction of the relative flow velocity, which process has been variously called inflation, opening of the bag, and opening like a parachute. At this point the droplet appears as a thin film of liquid, anchored to a heavier rim of liquid around the circumference of the droplet and stretched in the flow direction until the bag is several times larger than either the original droplet or the existing circumferential ring of liquid. When some critical condition occurs, the bag breaks up into a shower of fine droplets and rim disintegrates into several larger droplets.

A description of shear breakup is as follows. As in the case of bag breakup, the droplet deforms, but in the case of shear breakup the deformation has been most generally described as lenticular, with the major axis of the lens perpendicular to the direction of the relative velocity between the droplet and the air stream. As the radial outflow of liquid proceeds, a liquid film is stripped from the extreme circumferential edge of the droplet. This film is rapidly broken up into ligaments or segments, which in turn form under the action of surface tension into drops much smaller than the original drop. When the relative velocity is high enough, the stripping action from the parent drop appears as a shower of droplets being torn from the edge of the drop.

The two extreme breakup mechanisms have many times been observed in the photographic record of experimental programs of droplet breakup. The two breakup mechanisms have been called extreme because there also exists ample photographic evidence to substantiate the claims that droplet breakup occurs by a combined bag and shear mechanism. Thus it is seen that an adequate mathematical model would have to take into consideration and also be able to predict the two different modes of breakup which might occur due to some combination of the physical parameters of the gas stream and the liquid droplet.

Although the broad spectrum of droplet breakup mechanisms, running from shear breakup at one extreme of the spectrum to bag breakup at the other end, has many times been photographically observed, there yet exists in the literature no definite criteria to predict which breakup mode will occur for a given combination of liquid and gas stream physical parameters. It also seems apparent that no all-inclusive parameters have yet been discovered to determine for a given physical situation the droplet breakup time, the critical droplet diameter for a given relative velocity, or the critical relative velocity for a given droplet diameter.

With regards to droplet breakup criteria, Bond and Newton (4)* predicted that a droplet would break up when the Bond number of the given droplet-gas stream flow situation reached a critical value, experimentally found to be from eight to twelve. Gordon (15) attempted to define a mathematical model wherein a cylindrical plug was extruded from the droplet undergoing breakup, and he stated that breakup occurred when the length of the plug extrusion reached a certain critical value. Dodd (10) assumed that bag breakup of a liquid droplet occurred when a sphere of minimum diameter was inscribed inside the bag of the droplet undergoing bag breakup. He assumed the diameter of this sphere to be approximately twice the original undistorted diameter of the drop. Hinze (17), in the first classical mathematical treatment of droplet breakup, postulated the existence of a critical Weber number to determine the conditions heralding the onset of droplet breakup. This critical Weber number was different for drops subjected to either rapid step changes in relative velocity or for slow, steadily-increasing values of the

*Numbers in parentheses refer to Cited References.

relative velocity between the gas stream and the drop. The value of the critical Weber number also varied depending on whether the drop had either small or large viscosity. He did not consider cases of intermediate droplet viscosity.

Morrell (30) stated that the breakup of a jet and the breakup of a liquid droplet occurred in approximately the same manner and under the influence of approximately the same mechanism. Applying the results of some of his previous work in the field of jet breakup (31), he stated that for shear breakup, the critical condition was a given constant value of $\rho_{avg}U^2_{avg}$ over the surface of the droplet. In an experimental work, Lane (24) theorized that breakup of a liquid droplet in a flowing airstream would occur at $U^2_{crit}D$ = constant = 612. From this criteria he deduced that drops as large as five microns can withstand a sonic relative velocity without breaking up. In his experimental work he also found that there was a lower critical velocity for what he termed the transient flow case (a step change in relative velocity) than for the flow case of a constantly increasing velocity.

Hanson, et al., (16) found in their experimental work that they could deduce no critical Weber number value to correlate their experimental droplet breakup results. Their major contribution was that the critical relative velocity depended upon the one-third power of the surface tension of the liquid droplet. Engel (12), in an extremely impressive piece of experimental work, discovered many facts that are important to the field of droplet breakup. She found that it was the flow duration behind a shock wave that had the critical effect in determining breakup time and other critical breakup parameters. By examination of the photographic record of the experimental program, she was also able to conclude that shear breakup was not due to the vaporization of the liquid droplet, the mechanism which had been earlier suggested though never formally presented in the literature, but that shear breakup was "of mechanical origin." She also found plausibility for the statements that shear breakup was due to the rupture of crests of surface waves on the droplet, the spilling off into the gas stream of the moving boundary layer of the liquid drop, and the action of vortices on the downstream face of the droplet. These vortices stripped fluid from the surface of the drop as they were shed into the flow stream.

Weiss and Worsham (42), in an effort to empirically correlate the variables affecting droplet breakup, discovered that the relative velocity between the droplet and the air stream had the greatest effect in determining the critical breakup parameters. Magarvey and Taylor (27) attempted to correlate droplet breakup parameters by formulating a droplet deformation index. Their only conclusion, however, was that the droplet broke up when the hydrostatic pressure on the windward face of the drop at the stagnation point was greater than 440 dynes/cm^2. Elzinga (11) postulated the existence of a breakup mechanism which stated that droplet breakup may occur when the natural period of vibration of the liquid drop corresponds to the frequency of the shedding of vortices from the leeward face of the droplet.

However with regards to the drop vibration breakup criteria postulated by Elzinga and also by Peskin and Lawler (32), Magarvey and Taylor (27) stated that in the free-fall of liquid drops the breakup process was not triggered by the internal vibration of the droplet. Lane (24) and Hanson (16) also stated that internal droplet vibration did not trigger breakup. Rabin, et al., (34) concluded from their experimental work that neither the bag nor shear breakup mechanism could be explained on the basis of the drop vibrational period.

In a more recent work, Rabin and Lawhead (34), and Rabin, Schallenmuller, and Lawhead (35), have, as had been done prior to this work, postulated the existence of a critical breakup velocity strongly dependent upon the flow duration behind the shock wave intersecting the plane of the droplet. Flow duration is defined as the time that the flow or velocity increase behind the shock wave persists at a point after the shock wave has passed that given point. They also suggested, but never attempted to verify, that droplet breakup might correlate on the basis of the total impulse acting on the windward face of the droplet. They also concluded after attempting to correlate their experimental results on the basis of a constant Weber number, that the theory of a constant critical Weber number for determining breakup conditions was inadequate. The best Weber number criterion that they could infer was that droplet breakup occurred at a critical value of a Weber number which was a function of the droplet diameter. They also refuted the work of Lane, who postulated that the shear breakup of a drop was synonymous to transient flow conditions. They accomplished this by discovering that transient breakup was a time-controlled process but that shear breakup was time-independent.

In the lastest available work on the mechanism and process of droplet breakup, Wolfe and Anderson (43) have shed new light on the subject by insisting that droplet breakup cannot be correlated on the basis of dimensionless parameters, but that droplet breakup is a rate process and therefore the theory of absolute reaction rates from kinetic theory must be applied to the physical variables affecting the breakup of a droplet. They have stated that the classical method of equating the surface tension forces to the aerodynamic forces in an attempt to determine critical breakup parameters is not valid for systems wherein the variation of gas stream parameters is of the order of that variation which occurs in a shock wave.

There has also been some experimental and theoretical work done in an effort to delineate between the conditions which lead to bag breakup and those which lead to shear breakup. Hanson (16) found that the only criteria he could determine which would differentiate between the two modes of breakup was that shear breakup always occurred for velocities greatly in excess of the critical velocity for a given droplet diameter. Morrell (30) argued that bag breakup occurred if the time to which the droplet was subjected to a relative velocity change was in excess of the natural period of oscillation of the liquid droplet, and that shear breakup occurred if the action time was less than the natural period of the droplet. Hanson, et al., (16) however, found bag breakup to occur even if the action time was less than the natural period. Rabin, et al., (34) stated that the mode of breakup was strongly dependent upon the flow duration behind the shock wave, shear breakup occcuring for longer flow durations and bag breakup for shorter flow durations. They also discovered in their experimental work that for velocities much in excess of the critical velocity for a given drop, shear breakup always occurred. This finding supported that of Hanson. Another of their general conclusions was, that for all ranges of variables considered, shear breakup and short breakup times occurred more frequently with the larger drops tested and that the smaller drops more frequently exhibited bag breakup.

In their experimental work Wolfe and Anderson (43) indicated that for low relative velocities, bag breakup usually occurred, whereas for high relative velocities shear breakup was usually the mode of breakup observed. They also noticed that there existed a smooth transition from one type of breakup to another as the flow variables were varied and that the transition was equally smooth from bag breakup to shear breakup as it was for shear breakup to bag breakup. One point of interest of their report regarding the criteria of flow duration behind a shock wave was that the drop could not know what the flow duration was to be and thus what frontal shape it was to assume before breakup.

Thus it appears that for the general case of a given droplet subjected to a given set of physical parameters describing a flow situation there exists no tried and proven method of determining which mode of droplet breakup--bag, shear, or some combination of the two--will occur. Since, however, a mixture of modes can and does appear, perhaps the item of concern is not in describing the mode or mixture of modes that will occur for a given situation, but in describing instead the interplay of physical parameters which lead to a certain breakup time and thus a certain mode of breakup.

The time required for a droplet subjected to given physical conditions and the conditions which determine this time are also subjects of much debate in the literature. Hinze's classical work (17) stated that the breakup times were different depending upon whether the droplet was of very high or very low viscosity. Engel (12) concluded that the breakup time was inversely proportional to the strength of the shock intersecting the droplet, directly proportional to the initial diameter of the droplet, and that the change in Mach number, indicative of the change in the strength of the shock wave, is of greater effect in reducing the breakup time than is a change in the initial diameter of the droplet.

Gordon (15) found that for droplets of low viscosity, the breakup time was directly proportional to the initial diameter of the drop and inversely proportional to the relative velocity between the drop and gas stream. For drops of high viscosity, the breakup time was found to be independent of the initial diameter and inversely proportional to the relative velocity. The experimental results of Wolfe and Anderson (43) agreed most closely with those of Gordon. This result might infer that the breakup time is very much dependent upon the viscosity and the relative velocity and not so critically dependent upon the other physical variables. This result can only be inferred, however, since it is not exactly known the range of variables other than viscosity and relative velocity that were tested during the experimental efforts of the two authors.

Since this thesis is concerned primarily with liquid droplet breakup as it interacts with a normal shock wave, it is of interest and concern to note the effect of a shock front on a liquid droplet. Upon examination of the photographic record of their experimental efforts, Engel (12), Rabin and Lawhead (34), and Wolfe and Anderson (43) have all concluded that a liquid droplet is not broken up by the presence or interaction of the shock front itself but by the flow regime behind the shock front or the change in flow properties brought about by the passage of the gas through the shock front.

The physical properties which have been given the most attention regarding the basic breakup processes are the viscosity of the liquid, the surface tension of the liquid droplet configuration, and the drag coefficient of the particle moving in the gas stream.

Wolfe and Anderson (43) have found that the effect of viscosity is to retard the deformation process as the viscosity increases, and that the resultant droplet sizes after breakup were increased as the viscosity of the tested fluids increased. Higher breakup times were also measured at higher viscosities. Gordon (15) likewise found that an increase in fluid viscosity tended to retard droplet breakup processes. As previously mentioned in this thesis, he also concluded that viscous effects within the droplet tended to dominate the other physical parameters.

Lane (24) found that the viscosity affected the breakup process only if it was very high, that is, on the order of glycerol. Hanson, et al., (16) stated that the viscosity of the liquid droplet affected breakup time only if the viscosity was high and the diameter of the droplet undergoing breakup was low.

Hughes and Gilliand (21) postulated that the drag coefficient was a function of the Reynolds' number, a surface tension parameter, an acceleration parameter, a gravity parameter, the ratio of the liquid and gas densities, and the ratio of the liquid and gas velocities. Ingebo (22) showed that for the Reynolds' number range of one to one hundred the drag coefficient was less than one for clouds of solid spheres, clouds of evaporating spheres, and clouds of non-evaporating spheres. Rabin, et al., (35) verified this value in their experimental program. Wolfe and Anderson (43) expressed concern over the uncertainty of the drag coefficient in their work.

Carlson (7), in deriving an empirical expression for the drag coefficient, found that for flow regimes "such as occur in solid propellant rocket exhausts," the drag coefficient approached one as the Reynolds' number exceeded one hundred. Way and Nicholls (41) found that there was a general decrease in drag coefficient for a burning particle, but their work was primarily for a spherical, undeformed particle for a Reynolds' number range of one hundred to one thousand. The drag coefficient did decrease as the Reynolds' number increased.

Regarding the effect of surface tension on the breakup process and parameters, Adam (1) stated that an increase in pressure surrounding a liquid drop caused a decrease in surface tension, and that, logically, the surface tension increased as the radius of the liquid drop decreased. Semenchenko (36) wrote that there was a general decrease in surface tension of the liquid metals as there was a corresponding increase in temperature of the surroundings and the liquid.

Rabin, et al., (35) concluded that the surface tension value was lower for burning droplets than it was for non-burning ones and that this might be due to the vapor-phase burning of the droplet. Hinze (17) theorized that the critical breakup velocity for a given droplet diameter was proportional to the one-half power of the surface tension of the droplet, but Hanson, et al., (16) stated that this dependence was surface tension raised to the one-third power. Rabin and Lawhead (34) could make no differentiation between the powers upon examination of their data.

It is concluded, then, that the difficulty in determining the exact effect of physical parameters upon the breakup conditions is a major obstacle in the attempts made to define an adequate mathematical model of droplet breakup. It is also difficult to definitely ascertain just when the droplet has broken up when a sequence of photographs of the breakup process is examined.

CHAPTER II

LITERATURE SEARCH AND REVIEW

LITERATURE SEARCH AND REVIEW

An extensive and comprehensive review of the existing literature was made in an effort to discover the existing work that had been done on droplet breakup theories and mechanisms. Several significant works, possibly milestones in the field of droplet breakup, were found and thoroughly researched with regards to content and applicability to the major problem of this thesis. Since it is hoped that this thesis may someday prove to be a beginning point for further basic research into the field of droplet breakup, it has been decided to include in this thesis a summary of the significant points of each of the major works. Emphasis will be placed upon the mode of attack on the droplet breakup problem, and the results of the major applicable theories will be presented in a later section of this thesis.

Triebnigg's Estimate of Critical Size

Triebnigg (40) estimated the critical size at which a droplet would break up at a given relative velocity by merely equating the average air pressure on the face of the droplet, assuming that this pressure constituted the total flow resistance of the droplet, to the surface tension pressure of the spherical droplet, or

$$C_D \tfrac{1}{2} \rho_g U^2 = \frac{2\sigma}{R_D} \quad .$$

Hence,

$$R_{D_{crit}} = \frac{4\sigma}{C_D \rho_g U^2} \quad ,$$

This formula assumed a spherical shape for the droplet falling at a terminal velocity through the air with no pressure variation around the perimeter of the droplet. This equation is possibly the first attempt to estimate the critical parameters of droplet breakup.

Free Fall Breakup of Large Drops: Magarvey and Taylor

The authors of this paper (27) describing an experimental study of the free fall breakup of drops of ths size order of magnitude of raindrops have dealt heavily with the mechanism of breakup and the resultant droplet size distribution after breakup.

In an experimental effort designed to discover a reliable breakup criterion, the authors used an index of deformation and a hydrostatic pressure determination on the lower surface of the falling drop. The index of deformation was a plot of the ratio of minor axis diameter over major axis diameter,(assuming a spheroidal drop shape) versus an equivalent droplet diameter, and the hydrostatic pressure at the lower surface of the droplet was determined by a measurement of terminal droplet velocity. This attempt to discover a reliable breakup criterion, however, resulted only in the conclusion that when the hydrostatic pressure reached 440 dynes/cm^2 and the deformation index was less than 0.3, the droplet would break up. Severe difficulty in accurately measuring the terminal velocity (and thus the hydrostatic pressure) because of droplet instabilities led to the failure to establish a reliable criterion.

The photographic evidence of the authors distinctly showed a bag breakup occurring in the free-fall conditions. The photographic evidence was interpreted on the basis of a force equilibrium situation. In this force balance the hydrostatic force in the droplet just inside the lower surface of the deformed droplet was equated to the sum of surface tension pressure and the aerodynamic force at the stagnation point of the droplet. The lower deformed surface of the droplet was assumed to be a plane surface. The variation of the respective magnitudes of these forces as the drop deformed due to a force imbalance gave only a qualitative representation of the bag breakup situation.

Symbolically represented, the above situation is:

(1) hydrostatic pressure within the drop:

$$p = h\rho_1(g + a)$$

where h = distance between the approximately parallel surfaces of the drop (top and bottom)

a = an acceleration vector

(2) pressure increment due to surface tension:

$$\Delta p_s = \sigma(1/R_1 + 1/R_2)$$

where R_1 and R_2 = principal radii of curvature at a given point

(3) stagnation pressure at lower surface of droplet:

$$\Delta p_i = \tfrac{1}{2}\rho_g U^2 \quad .$$

Hence combining (1), (2), and (3),

$$h\rho_1(g + a) = \sigma(1/R_1 + 1/R_2) + \tfrac{1}{2}\rho_g U^2 \tag{4}$$

or, for a plane surface,

$$h\rho_1(g + a) = \sigma(\cancel{1/R_1 + 1/R_2} \rightarrow 0) + \tfrac{1}{2}\rho_g U^2 \quad . \tag{5}$$

For an increasing velocity and decreasing h as the drop flattens, the equilibrium conditions of equation (5) cannot be satisfied and the drop breaks up. This argument was used by the authors in their previous attempts to correlate the breakup parameters. An examination of the equations and the logic underlying each force effect gives a qualitative depiction of the bag breakup process. As the aerodynamic pressure increases so as to overcome the effects of the hydrostatic pressure, a bulge would form on the falling droplet. As the bulge increased, the surface tension effects would again become significant and the hydrostatic pressure would be of less effect as the droplet began to "inflate." Finally, the aerodynamic forces for a liquid droplet of sufficiently low viscosity and surface tension travelling sufficiently fast would cause the droplet to inflate and ultimately break up.

The authors also discussed the possibility of droplet vibration as a means of breakup since this phenomenon had been theorized by other researchers in this field. The experimental evidence showed that the droplet vibrations were confined to a plane perpendicular to the direction of motion and that these vibrations were not a major factor in the droplet breakup mechanism.

As far as the resultant droplet size distribution is concerned, photographic evidence showed that the number of fragments increased with an increase in parent droplet diameter. Actual photographic counts showed that several hundred smaller droplets are often produced from the breakup of a larger drop, and that the bursting of the "canopy" of the inflated droplet produced the smallest droplets while the larger fragments were a result of the breakup of the "rim" of the inflated droplet. The vibration nodes, or "lobes," around the rim of the droplet, never more than four in number according to the photographic evidence, seemed to account for the number of large fragments into which the rim broke up, the number of lobes being equal to the number of large fragments of the rim existing after breakup.

The study concluded that drops as large as twenty millimeters diameter exhibited the characteristics of bag breakup and that smaller droplets exhibited the same breakup mode but only after a greater fall distance. The authors also stated that a droplet of less than ten millimeters diameter cannot be broken up in a free fall, and that droplet breakup cannot be triggered by internal vibrations of the droplet.

Raindrop Size, Shape, and Falling Speed: Spilhaus

For the situation of a water raindrop falling at a constant terminal velocity the author (37) derived expressions relating the surface tension pressure to the aerodynamic pressure on a drop (assuming an ellipsoidal drop shape), the terminal velocity of the falling droplet, and the variation of the drag coefficient of the droplet due to droplet deformation. The item of primary interest in this work is the variation of the drag coefficient with droplet shape. An expression derived by the author gave this variation as

(6) $$C_D = C_O[K - h(K - 1)]$$

where

C_O = the coefficient of resistance for a sphere (0.21 to 0.3 for the range of Renolds' numbers concerned)

$K = C_{fp}/C_O$

C_{fp} = drag coefficient for a flat plate

$h = b/a$ = ratio of major axis to minor axis of the assumed ellipsoidal shape (fineness ratio of the ellipse).

In the derivation of this equation the author of the paper did make a rather significant error. When he calculated the pressure difference due to surface tension pressure for the ellipsoidal cross sectional shape, he assumed that $\Delta p = 2\sigma/a$, whereas the correct expression should have been $\Delta p = \sigma(1/a + 1/b)$. This mistake was also noticed by MacDonald (26). Hence the derived expression for the drag coefficient would not correspond to the described physical situation although it might still be useful since the droplet shape was approximated anyway.

Calculations using equation (6), however, for extreme values of the parameters C_O, C_{fp}, and h give values of C_D for the assumed ellipsoidal drop shape that are much lower (e.g., 30-50%) than the values of C_D presently being used by authors in the calculation of droplet critical velocities, diameters, and breakup times. This variation from experimental values of C_D may have been due to the mistake made in the derivation. Rabin and Lawhead (34) in their experimental work have measured drag coefficients for inert and burning droplets that were not in agreement with the values suggested by Spilhaus.

The Shape and Aerodynamics of Large Raindrops: MacDonald

The author (26) of this paper postulated that the equilibrium shape of a large raindrop falling at terminal velocity through an infinite medium is due to surface tension of the droplet, the hydrostatic pressure gradients within the drop, the external aerodynamic pressures on the drop surfaces, the electrostatic charges on the drop surfaces, and the internal circulation of the drop. By means of an order-of-magnitude argument, he concluded that only the first three effects are significant for large falling raindrops.

The equilibrium shape of a falling raindrop is that shape for which the aerodynamic pressure plus the surface tension pressure just equals the internal hydrostatic pressure at all points on the droplet surface. Since the theoretical determination of these quantities would be very difficult, if not impossible with the present state of the mathematical art, the pressures mentioned above were deduced from observation and measurement of photographs of falling liquid drops. The author also concluded that the study of photographic evidence clearly indicated that boundary layer separation existed at a point along the droplet surface, and that hence this separation did not favor the production of strong internal vorticular circulation, thus minimizing the effect of droplet internal, centrifugal effects.

The major contribution of this paper was the argument by which all pressures on the drop except the external aerodynamic pressure, the surface tension pressure, and the hydrostatic pressure of the drop could be neglected in calculating the shape of the falling droplet. It should be mentioned, however, that all of the effects neglected in inferring the shape of a falling raindrop could play significant roles in determining the shape of droplets subjected to physical conditions other than those of merely falling through a stagnant medium.

Critical Speeds and Sizes of Liquid Globules: Hinze

In his classical work, the first mathematical effort to explain droplet breakup, Hinze (17) stated that a droplet is broken up if the translatory speed of the droplet relative to the gas stream exceeded a certain critical value, or, inversely, if at a given speed the size of the moving droplet is greater than some critical size, the droplet will break up. The theory stated that the relative magnitudes of the dynamic pressure force and the surface tension force are the criteria for determining droplet breakup. Combining the dynamic pressure force and surface tension pressure into a dimensionless variable led to the definition of the Weber number. It is thus the relevant value of the Weber number that is used as the breakup criterion in Hinze's theory. The critical value of this Weber number must be experimentally determined.

Hinze assumed that the tangential forces caused by the viscous aerodynamic effects of the airstream acting on a droplet surface can be neglected in comparison to the normal component of force caused by the velocity pressures of the ambient fluid when the Reynolds' number is greater than one thousand.

In Hinze's original paper (17) two different flow situations leading to droplet breakup were considered: (a) the droplet is suddenly being exposed to a gas stream of constant speed, and (b) the droplet is being exposed to a gas flow uniformly increasing in speed from zero to a constant value.

Hinze, in a previous paper (18), linearized the hydrodynamic equation of the motion of the drop and derived formulae for the slight deformations of a droplet caused only by the normal forces acting on the droplet surface. However, the critical deformation necessary for droplet breakup was much larger than the slight value of deformations permitted by his theory; hence the deformation theory provided only a theoretical model for the breakup process. Actually, photographic evidence (2, 43) has shown that deformation of the droplet undergoing breakup may deviate substantially from the theoretical value predicted by Hinze.

The actual pressure distribution around the surface of the droplet for a Reynolds' number range of 1000 to 200,000 was approximated by

(7) $$P_\varphi = \tfrac{1}{2}\rho_g U^2(-5/4 + 9/4\cos\varphi) \quad \text{for} \quad 0 \le \varphi \le \pi/3$$
$$P_\varphi = -11/32\rho_g U^2 \qquad\qquad \text{for} \quad \pi/3 \le \varphi \le \pi$$

assuming an irrotational potential flow. Expanding the above equation into zonal harmonics gives

(8) $$P_\varphi = \tfrac{1}{2}\,{}_g U^2 \quad -0.500 + 0.1915\,P_1(\cos\varphi) + 0.551\,P_2(\cos\varphi) + 0.415\,P_3(\cos\varphi) + 0.178\,P_4(\cos\varphi) - 0.020\,P_5(\cos\varphi) + \ldots$$

Hinze remarked that the first term within the braces would cause an expansion of the droplet; it must be concluded that this term was to be ignored due to the assumed incompressibility of the droplet. The second term would cause a bodily displacement of the droplet without deformation; this term was also neglected.

Since the formulae for the pressure distribution have their maxima at $\varphi = 0$, the stagnation point of the droplet, and since the maximum deformation was considered to be decisive for droplet breakup, the following derived expressions were restricted to the maximum deformations which would occur at the stagnation point of the droplet.

Let the pressure distribution over the droplet surface be

(9) $$P = \tfrac{1}{2}\rho_g U^2 \sum C_n P_n \cos\varphi$$

where C_n stands for the values in the preceeding formula and U is an arbitrary function of time. The viscous deformation formulae derived in Hinze's previous paper (18) are restricted to cases of only slight viscosity effect (i.e., $\mu_1^2/\sigma\rho_1 R_e \ll 1$) or of a very great viscosity effect (i.e. $\mu_1^2/\sigma\,\rho_1 R_e \gg 1$) for the internal flow of the droplet. According to these formulae the deviation of the droplet shape from the spherical at the stagnation point was:

(10) $$\frac{\delta}{R} = -\sum C_n \frac{\rho_g}{\rho_1}\frac{n}{2w_n R_D}\int_0^t U^2 \left\{\exp\left(\frac{-n(n-1)\mu_1(t-t_1)}{\rho_1 R_D}\right)\right\}\sin w_n(t-t_1)\,dt_1$$

where $$w_n^2 = \frac{(n-1)(n)(n+2)}{\rho_1 R_D^3}$$ for slight viscosity effect

and $$\frac{\delta}{R} = \frac{-C_n\, n(2n+1)\,\rho_g}{4(n-1)(2n^2+4n+3)\mu_1}\int_0^t U^2 \exp\left\{\frac{n(n+2)(2n+1)\,\sigma(t-t_1)}{-2(2n^2+4n+3)\mu_1 R_D}\right\}dt_1$$

for great viscosity effect.

For a droplet exposed to a step change in velocity and with coefficients C_n as given in (8), the evaluation of the integral (10) gave:

(12) $$\frac{\delta}{R} = -\frac{\rho_g U_c^2 R_D}{\sigma}\Big[0.069(1-\{\exp(-2\mu_1\, t/\rho_1 R^2)\}\cos w_2 t) + 0.021(1-\{\exp(-6\mu_1 t/\rho_1 R^2)\}\cos w_3 t) + 0.005(1-\{\exp(-12\mu_1 t/\rho_1 R^2)\}\cos w_4 t) + \ldots\Big]$$

for slight viscosity effect.

The absolute maximum of δ/R was reached roughly for $w_2 t = 0.8\pi$, and since $\mu_1^2/\sigma\rho_1 R \ll 1$,

(13) $$\left(\frac{\delta}{R}\right)_{max} = -0.17\,\frac{\rho_g U_c^2 R}{\sigma}$$

for slight viscosity effect.

Now if $We = \rho_g U_c^2 R/\sigma$ reached its critical value (as determined by experiment), the critical value of δ/R can be computed.

For great viscosity effects ($\mu_1^2/\sigma\rho_1 R >> 1$) the evaluation of (11) with C_n as determined by (8) gave:

(14) $$\frac{\delta}{R} = -\frac{\rho_g U_c^2 R}{\sigma}\left[0.069\left(1-\left\{\exp\left(-\frac{20}{19}\frac{\sigma}{\mu_1 R}t\right)\right\}\right) + 0.021\left(1-\left\{\exp\left(-\frac{35}{22}\frac{\sigma}{\mu_1 R}t\right)\right\}\right) + 0.005\left(1-\left\{\exp\left(-\frac{36}{17}\frac{\sigma}{\mu_1 R}t\right)\right\}\right) + \ldots\right]$$

or for the maximum value (corresponding to t approaching infinity),

(15) $$\left(\frac{\delta}{R}\right)_{max} = -0.095\,\frac{\rho_g U_c^2 R}{\sigma}$$

for great viscosity effect.

For the case of a droplet exposed to a gas flow uniformly increasing in speed from zero velocity (such a situation would occur in a falling drop), the forces initially acting on the droplet were assumed to be primarily viscous tangential forces. However, if the droplet was large enough, (i.e., larger than a few millimeters) the normal pressure forces would dominate as the velocity increased (R_D becomes larger). For this case the equation of the motion of the droplet was:

(16) $$\frac{dU}{dt} + \frac{3C_D}{8R_D}\frac{\rho_g}{\rho_1}U^2 = g$$

Hinze assumed that this equation was applicable during the entire period of falling (continual velocity increase). A solution to (16) satisfying the boundary conditions of the problem ($U = 0$ at $t = 0$), substitution of this result into (10) and (11), and assuming a slight viscosity effect gave for the maximum deformation:

(17) $$\left(\frac{\delta}{R}\right)_{max} = -0.095 We_{max}\ .$$

Here it is seen that for the same $(\delta/R)_{crit}$ the corresponding critical Weber number is much greater for case (b) than case (a).

For a great viscosity effect, a similar solution also gave:

(18) $$\left(\frac{\delta}{R}\right)_{max} = -0.095\ We_{max} \quad .$$

In an attempt to estimate the breakup time for a droplet suddenly exposed to a constant velocity air stream, some authors proposed to consider the natural vibrational period of the droplet as a rough measure of the breakup time. This could be correct, however, if the air velocity was just equal to the critical speed and if viscosity effects were ignored. In this instance, the breakup time was calculated as:

(19) $$t_b = 0.8\pi\sqrt{\frac{\rho_l R_D^3}{8\sigma}} \quad .$$

For most breakup time considerations, however, the gas velocity was very much greater than the critical speed, so that the breakup time was much less than that predicted by (19). Hinze, neglecting viscosity, derived:

(20) $$t_b \approx \frac{R}{U}\sqrt{\frac{1}{0.75}\,\frac{\rho_l}{\rho_g}\left(\frac{\delta}{R}\right)_{crit}}$$

The above expression and the deformation expression (12) were based on an external pressure distribution for actual turbulent flow at high Reynolds' numbers, a state which might not be present at t = 0 (i.e., the flow requires a certain time T to become fully developed). Since only in the case that the breakup time is very much greater than T may (20) hold true, the time T was estimated from the time needed for the generation of vortices behind the droplet after the inception of flow. This led to a relation of the form:

(21) $$\frac{t}{T} \cong \sqrt{\frac{\rho_l}{\rho_g}}$$

and hence $T \sim R/U$ which is quite large.

For a large viscous effect, small values of the breakup time were estimated by developing the exponential function in (14) into a series and disregarding all terms but those linear in time. This led to:

(22) $$t \approx \frac{10\mu}{\rho_g U^2} \left(\frac{\delta}{R}\right)_{crit}$$

which is independent of droplet size. This result holds only for small breakup times at large Weber numbers and for great viscous effects.

On the Disintegration of Drops in an Airstream: Dodd

Dodd (10) developed a theory to predict the distortion and disintegration of a water drop which was exposed to an airstream of continuously increasing relative velocity. Assuming that a spherical droplet was distorted roughly into a lenticular whape by the aerodynamic forces of a moving airstream, Dodd assumed a relative velocity between the droplet and the airstream low enough in magnitude to assure a bag-breakup mechanism. He also assumed the existence of a non-uniform pressure distribution around the surface of the sphere that is described by the experimental work of Hinze to be:

(23) $$p = \rho_g U^2 (9\cos^2\varphi - 5)/8 \qquad \text{for } 0 \le \varphi \le 1/3\pi$$

$$p = -11\rho_g U^2/32 \qquad \text{for } 1/3\pi \le \varphi \le \pi$$

where

φ = angular distance from the stagnation point of the sphere.

Dodd examined the work of Lane, whose efforts led to the following critical condition for droplet disintegration:

(24) $$d(U)^2 = \text{constant} .$$

Dodd postulated the following breakup theory. As the relative velocity was increased, the drop deformed into the shape shown (Fig. 1) for bag-breakup conditions.

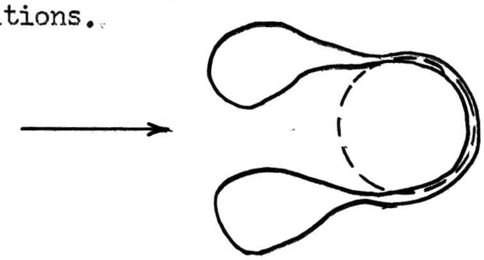


Figure 1
Drop deformation as postulated by Dodd

Drawing a dotted sphere through the forming "bag," he contended that the critical velocity for bursting is that velocity which makes the radius of this circle a minimum. He assumed that the equation:

(25) $$p_i - p_o = c_1 \rho_g U^2$$

where p_i = pressure inside the bag and

p_o = pressure just outside the bag

held for all stages of bag breakup. By equating the pressure difference between the inside and outside surfaces of the bag to the surface tension pressure, he obtained:

(26) $$p_i - p_o = 4\sigma/r \ .$$

Combining (24), (25), and (26), he obtained:

(27) $$rU^2 = 4\sigma/c_1\rho_g \ .$$

If r_m is the minimum radius of the bag (inscribed circle),

(28) $$r_m U^2{}_{crit} = 4\sigma/c_1\rho_g$$

which gives an expression for the critical breakup velocity. Assuming from photographic evidence that the minimum radius was approximately twice the radius of the original drop, he could theoretically calculate the magnitudes of the critical relative velocities.

The constant c_1 was approximated from the given assumed pressure distribution over a solid sphere. For the given distribution the pressure is positive for $0 \leq \theta \leq 43^\circ$ and is negative and practically constant for $43^\circ \leq \theta \leq 180^\circ$. If p_o is taken as this constant value and p_i is taken as the average pressure over the positive region of the sphere, we obtain $c_1 = 0.238 - (-.344) = 0.582$.

Usually the relative velocity U is not a known quantity, but rather V, the air velocity, is known as a function of position s_o; u, the droplet velocity, is known from the equations of motion for the drop but is related to V due to the aerodynamic drag on the drop. For one regime of droplet breakup, the relative velocity will increase to the critical velocity (28) and there will then be a rapid increase in the droplet size. Hence the critical velocity criteria divides the droplet into two phases.

Consider the motion wherein the bag exists as a part of the moving droplet. Let m be the mass of the entire drop. Dodd supposed that a fraction f of the mass was contained in the hollow sphere he chose to represent the bag; the rest of the liquid was contained in the rim to which the bag was anchored. Now the equations of motion for the sphere were (assuming rectilinear motion):

(29) $$\frac{ds}{dt} = u$$

(30) $$m \frac{du}{dt} = mg + \tfrac{1}{2} C_D \pi r^2 \rho_g \, (V - u)^2$$

where r = radius of the hollow sphere.

For vertical motion, the positive directions of s, u, and V were all downwards.

Let A be a small area on the surface of the bubble, and let δ_r be the thickness of the shell. The mass of this volume is $A\delta_r$ (taking ρ_{water} to be unity); its acceleration (radially) is d^2r/dt^2. Now, the relation between acceleration and the acting force is:

(31) $$A\delta_r \times \frac{d^2r}{dt^2} = A\left[p_i - p_o - \frac{4\sigma}{r}\right] = A\left[c_1 \rho_g (V - u)^2 - \frac{4\sigma}{r}\right].$$

Now the total volume of the shell is:

(32) $$4\pi r^2 \delta_r = f_m \quad .$$

Hence

(33) $$\frac{d^2r}{dt^2} = \frac{4\pi r^2}{f_m}\left[c_1 \rho_g \, (v-u)^2 - \frac{4\sigma}{r} \right] \quad .$$

Equation (33) gave the drop behavior after the passing of the critical velocity condition, The Mechanics of Drops: Hughes and Gilliand

The authors of this paper considered a force balance for the vertical motion of a drop falling through a stagnant continuous medium of infinite extent (21), whereby, from F = ma,

$$\rho_1 \frac{\pi d^3}{6} \frac{dU}{dt} = \frac{\pi}{6} d^3 g \left(\rho_1 - \rho_g\right) - C_D \frac{\pi}{4} d^2 \frac{\rho_g V^2}{2} \qquad (34)$$

where d = droplet diameter.

The definition of a drag coefficient is given by:

$$C_d = \frac{\text{Drag Force}}{(\text{Frontal Area})\ (\rho_g U^2/2g)} \qquad (35)$$

The geometrical shape of the droplet is usually an unknown quantity for a falling droplet. For a falling droplet the value of C_d in equation (34) was allowed to vary in order to adjust the drag coefficient based on a solid spherical drop, which was assumed in equation (34), to the drag coefficient of the actual but usually unknown geometrical configuration for the falling droplet.

In an attempt to determine the drag coefficient as a function of the variables affecting the geometrical configuration and physical state of the drop, the authors resorted to a dimensional analysis technique. This dimensional analysis yielded the following results;

$$C_d = f(Re,\ Su,\ Wt,\ Ac,\ \rho_g,\ \rho_1,\ \mu_g/\mu_1) \qquad (36)$$

where

$$Re = dU\rho_g/\rho_g = \text{the Reynolds' number} \qquad (37)$$

$$Su = g\ d\rho_g/\mu_g^{\,2} = \text{the surface tension group} \qquad (38)$$

$$Wt = d\left[\frac{4}{3} \frac{g\rho_g \left|\rho_1 - \rho_g\right|}{\mu_g^2}\right]^{\frac{1}{2}} = \text{the gravity group} \qquad (39)$$

$$Ac = \frac{d}{V^2} \frac{dU}{dt} = \text{the acceleration group} \qquad (40)$$

are all dimensionless parameters.

Equation (36) did not take into account the effect of the walls containing the breakup process, the possibility of a continuous phase motion, or freestream turbulence effects. The effects of these parameters on the drag coefficient are a major source of disagreeement among researchers in this field.

Hughes and Gilliand remarked that since the drag coefficient was a function of the shape of the particle, distortions of the droplet had a marked effect on the motion of the distorted fluid droplet. It had been observed by other researchers in this field that droplet distortions are of two types: (1) those of an equilibrium nature, and (2) those of an oscillating nature resulting from droplet vibration.

It is well-known that the fluid pressure on the surface of a moving sphere is not uniform over the surface of the droplet. However, within the drop, except for small amounts of internal circulation due to the distortion of the droplet and a small gravitational head, the pressure is uniform. Thus there exists a pressure difference across the droplet surface which must be balanced by the surface tension force in an equilibrium situation according to the equation:

(41) $$p_d - p_{surface} = (1/r_1 + 1/r_2)$$

where r_1 and r_2 are the perpendicularly-intersecting radii of curvature of the drop (usually parallel to or coincident with the major and minor axes of the drop). Thus the shape of the drop is described by the variation of r_1 and r_2 over the drop surface.

Since $p_{surface}$ depends itself on the shape of the drop (and is not know exactly even for a sphere) a theoretical calculation of the droplet surface aerodynamic pressure or, inversely, the droplet shape, is yet to be solved. Hence the inverse problem is usually the item of concern; that is, given a distorted shape the task will be to calculate $p_{surface}$ for given values or r_1 and r_2.

Hughes and Gilliand stated that the usual method of obtaining $p_{surface}$ was to assume that the spherical droplet is distorted into a spheroid with its minor axis in the direction of motion. The equation of the surface is the

(42) $$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1 \quad .$$

The distortion from the spherical shape can be measured in terms of the fineness ratio, h = b/a, or the eccentricity. For an oblate spheroid, $e = (1 - h^2)^{\frac{1}{2}}$. Given the volume of the spheroid as $4/3\ a^2b$, in terms of the equivalent spherical diameter,

(43) $$a = \tfrac{1}{2}dh^{-1/3}; \quad b = \tfrac{1}{2}dh^{2/3} \quad .$$

Using elliptic functions, the surface area of this ellipsoid can be expressed as:

(44) $$\frac{S_{drop}}{V_{drop}} = \frac{6}{d}\ \frac{1}{2h^{2/3}} \left\{ 1 + \frac{h^2}{\sqrt{1-h^2}} + \ln\left[\frac{1 + h + \sqrt{1-h^2}}{1 + h - \sqrt{1-h^2}}\right] \right\} \quad .$$

According to the authors, this equation showed that the correction of the drag doefficient from the spherical case is not severe as long as h is greater than 0.8. For a prolate spheroid a similar S_d/V_d relationship was derived.

Hughes and Gilliand attempted to determine whether a spheroidal shape corresponded to the actual shape defined by equation (41). A derivation using standard geometrical theorems led to

(45) $$\frac{d}{4}\left[\frac{1}{\rho_1} + \frac{1}{\rho_2}\right] = \frac{h^{5/6}}{2}\left[\frac{1 + h^2 + (1-h^2)\ (z/b)^2}{h^2 + (1-h^2)(z/b)^2)^{3/2}}\right]$$

From equation (41) it is seen that the left hand side of (12) is proportional to $p_{drop} - p_{surface}$. Since p_{drop} is essentially uniform, neglecting internal circulation and gravitational effects, the variation of $d/4\ (1/\rho_1 + 1/\rho_2)$ should correspond to the variation of $p_{surface}$. The authors plotted this data, and the resulting plot showed the reasonable appearance of the graph of p_s on the assumption of a spheroidal shape for the falling droplet.

Theoretical Studies of Mechanisms in the Atomization of Liquids: Peskin and Lawler

In the authors' discussion (32) of the mechanism of droplet breakup they mentioned a theory advanced by Elzinga (11) in an effort to explain droplet breakup in liquid-liquid systems. This theory is essentially that vortices are periodically shed from a moving droplet into the continuous medium behind the drop, and that the vortex-shedding induces an alternate acceleration-deceleration of the droplet which thus causes the droplet to oscillate at some frequency. When the frequency of the oscillation induced by the vortex shedding becomes equal to the lowest natural frequency of the liquid drop, breakup of the drop occurs. Such a matching of oscillation frequencies is theoretically possible at some drop size since the lowest natural frequency of the drop decreases as its diameter increases while the vortex shedding frequency increases as the particle diameter increases. Elzinga did plot a dimensionless vortex discharge frequency (Strouhal number) versus Reynolds' number for some of his data and found a positive correlation which thus brought credance to his theory.

Peskin and Lawler extended this theory to account for resonant conditions occurring at the frequency of vortex shedding at higher (than the lowest) natural frequencies of the droplet. In considering only primary modes of vibration for a droplet, the breakup criteria is limited to only a minimum diameter, that is, that diameter which corresponds to the frequency of vortex sheddding. However, if one considered the droplet as being capable of excitation at modes higher than its lowest natural frequency, the frequency of vortex shedding corresponding to a larger diameter drop than the one being considered could be applied to the given drop at higher natural vibration frequencies.

Given a situation where the frequency of vortex shedding is equal to the nth natural frequency of oscillation of the liquid droplet, the authors postulated that the number of droplets into which the initial drop shattered was equal to n, and upon examination of the existing data, found that this approximated the breakup conditons. The authors did not apply this theory to a liquid particle-flowing gas situation because of a lack of available experimental data given the frequency of vortex shedding for such a system.

The diameter, D_n, at which resonance will occur for any such system previously described is:

(46) $$D_n = \frac{205\,\sigma}{\rho_g U^2}\left[\frac{\mu_1 U}{\sigma} \times \frac{\rho_1}{\rho_g}\right]^{0.765} (n^3+n^2-2n)^{0.235}, \quad Re< 2000.$$

Hence the drop diameter D_n which will be excited to a mode of vibration n that will tend to break it up is given by equation (46); it is observed that D_n varies directly with surface tension and inversely with the kinetic energy per unit volume at the moment of breakup. This might lend support to the formulation (46), since surface tension would retard breakup whereas aerodynamic forces would tend to assist in the breakup.

Also, for any given system of droplets of known initial diameter, it would be possible to calculate the resultant droplet size distribution after breakup assuming the nth natural frequency at which breakup of the drops occurs resulted in n droplets per initial drop. Known variables would have to include the relative velocity between the droplet and the airstream, the physical properties σ, ρ_g, ρ_1, and μ_1 and the minimum stable droplet diameter, given by the authors to be

(47) $$D_{crit} = 9\sigma/\rho_g U^2 \quad .$$

The work of Hu and Kintner (20) was also referenced, wherein they determined the critical diameter above which a droplet must break up by

(48) $$D_{crit} = \left\{1.452 \times 10^{-2}\left[\frac{\sigma}{\rho_1 - \rho_g}\right]\right\}^{\frac{1}{2}} \quad .$$

The authors stated that the theories presented in this paper have been applied to relatively low speed processes (e.g., droplet velocities on the order of those relevant to spinning disk atomization, ultrasonic atomization, and electrostatic atomization). However, for high speed shock processes, it was postulated that since the time to which the droplet is submitted to velocity changes (the action time) is much less than the natural period of oscillation of the droplet, other droplet breakup mechanisms would prevail and become most important; that is, the process causing breakup would occur before the drop could possibly undergo even one complete oscillation at the lowest natural droplet frequency.

Atomization in High Velocity Airstreams: Weiss and Worsham

Weiss and Worsham conducted an extensive experimental study of the resultant droplet sizes obtained upon injecting liquids into airstreams of constant, moderate (200-300 fps) velocity. They found that the relative velocity between the droplet and the airstream was the flow parameter having by far the largest effect on resultant droplet size distributions. The variation of other flow parameters and physical proprties of the liquid drop did have an effect on the resultant droplet size distribution, but the net effect was negligible (upon examination of their experimental results) when compared to the effect of relative velocity variance.

An empirical correlation was made of their results. The equation is:

$$\frac{X\rho_g U^2}{\sigma} = 0.61 \left[\frac{U\mu_l}{\sigma}\right]^{2/3} \left[1 + 10^3 \frac{\rho_g}{\rho_l}\right] \left[\frac{V\rho_l \sigma_4 \mu_g}{\mu_l^4}\right]^{\frac{1}{2}} \qquad (49)$$

Solving the empirical equation of Weiss and Worsham for the relative velocity gave:

$$V = 0.61 \left[\frac{\mu_l \ \sigma \ g_e}{\rho_g \ X \ x \ 3.048}\right] \left[1 + 10^3 \ \frac{\rho_g}{\rho_l}\right] \left[\frac{W \ \rho_l \ \mu_g g_e}{3600 \mu_l^4}\right]^{3/4} \qquad (50)$$

This empirical result was examined in an effort to discover correlations between the extensive experimental work of Weiss and Worsham and the data of other researchers in this field. See Cited Reference (42).

Mechanism and Speed of Breakup of Drops: Gordon

Gordon investigated the droplet bag breakup mechanism, a process which he described as ". . .a process where the drops flatten, become bowl-shaped, inflate like a parachute, and finally burst." He postulated that drops smaller than a certain critical initial size were stable and would not break up (15).

Gordon's work is essentially a supplement to the investigations of Hinze. Hinze predicted, considering both small and large viscosity effects, the critical speed, size, and the breakup time for the bag breakup of a droplet. Gordon, in addition to the cases considered by Hinze, obtained the breakup times for the cases of intermediate droplet viscosity and surface tension effects. Gordon stated that an exact mathematical solution would require a complete knowledge of the aerodynamic pressures on the drop as a function of space and time. This pressure distribution depends upon drop shape, which shape is in turn governed in part by the external pressure distribution. Within the drop the effects of hydrostatic pressure, inertia (internal circulation effects), and the viscosity must be balanced at every point, and the shape of the surface of the drop is influenced by the surface tension forces. Gordon made no attempt to analyze these effects in detail. He did, however, attempt to analyze the effects by considering their respective orders of magnitude and to further the understanding of the physical processes involving the effect of the breakup parameters.

Gordon assumed that in the bag breakup process a cylindrical plug was extruded from the drop in the direction of flow. This extrusion was caused by the dynamic air pressure on the front stagnation point of the drop and was retarded by the surface tension, viscosity, and internal inertial circulation effects of the droplet. The air stagnation pressure, $\frac{1}{2}\rho_g U^2$, is ultimately the disturbing force which causes the breakup. Gordon also stated without explanation that actually the average pressure on the front of the droplet is less than the stagnation pressure, but that this effect is somewhat counteracted by the low pressure due to the separation behind the cylinder. This frontal pressure reduction could possibly be accounted for on the basis of surface circulation effects.

The surface tension forces tend to keep the drop spherical in the absence of other forces. The presence of external forces (e.g., pressure, frictional aerodynamic shear, etc.) tends to cause the drop to deviate from the spherical shape. For the bag breakup phenomena, the front of the drop is flattened, and the radius of curvature of the back side of the drop is increased. Because of this change from the spherical geometry, the surface tension will vary from point to point on the drop surface. Also, during the breakup process, the surface area of the droplet will be increased (due to the inflation of the bag), and this process requires an expenditure of energy. According to Gordon's mathematical model of a cylinder extrusion, two new surfaces are formed, one at each end of the cylinder. The energy required to form each new surface is equal to the area of the surface times the surface tension. Hence the resisting force is equal to the surface tension multiplied by the cylinder circumference $2\pi r$ for each area, or $2 \times 2\pi r = 4\pi r$ for the total force. Dividing this force by the cylinder area πr^2 gives the resisting surface tension pressure. Further assuming that the cylinder radius is of the order of magnitude of half the droplet diameter and substitution into the above equation gives the equation for the resisting pressure, $8\sigma/D$.

Viscous effects sometimes tend to retard breakup. Gordon assumed that the viscous retarding pressure is proportional to the speed of the breakup. The back pressure for liquid flowing through a tube is $16L\mu_1/D$, where L is the tube length. Assuming that this case is analagous to the mathematical model, the magnitude of the retarding viscous pressure is $16\mu_1 U/D$.

Combining the dynamic, surface tension, and viscous pressures, the acceleration of the cylindrical plug can be calculated, if it is assumed that the rest of the drop remains motionless.

(51) $$\frac{dv}{dt} = \frac{p}{\rho_1 D} = \frac{1}{\rho_1 D}\left[\tfrac{1}{2}\rho_a U^2 - \frac{8\sigma}{D} - \frac{16\mu_1 U}{D}\right]$$

where A = cross-sectional area of the extruded plug.

Solving (51) for the instantaneous velocity of the plug and the resulting equation for the instantaneous displacement of the cylinder as a function of time, and setting the displacement of the cylindrical plug equal to the droplet diameter D, the total breakup time of the droplet is expressed by:

(52) $$\frac{2(16\mu_1)^2}{\rho_1 D^2(\rho_g U^2 - 16\sigma/D)} = \frac{16\mu_1 t}{\rho_1 D^2} - 1 + \exp(-16\mu_1 U \rho_1 D^2).$$

This equation, even if it were analytically solvable for the breakup time, might yield a breakup time that is too low, since the experimental evidence of other researchers in this field indicated that the cylinder displacement may be five times the diameter. Gordon, however, postulated that the drop became thinner as it blew up so that the retarding forces were small, and the breakup time depended only on the first stages of breakup.

Equation (52) is non-dimensionalized by the transformations:

$$\bar{t} = (t\rho_g U^3/32\sigma)(\rho_g/\rho_1)^{\frac{1}{2}}$$

$$\bar{D} = D\rho_g U^2/16\sigma$$

$$\bar{u} = (\mu U/\sigma)(\rho_g/\rho_1)^{\frac{1}{2}}$$

and becomes:

$$\frac{2\bar{u}^2}{\bar{D}(\bar{D}-1)} = \frac{2\bar{u}\bar{t}}{\bar{D}^2} - 1 + \exp(-2\bar{u}\bar{t}/\bar{D}^2).$$

This non-dimensionalized equation can be used in estimating breakup times for a range of droplet and gas stream parameters. A plot showed that the viscosity parameter tended to dominate the other physical parameters, since for high viscosity the breakup time becomes independent of droplet diameter.

The critical diameter predicted by the theory is:

$$D = 16\sigma/\rho_g U^2 \tag{53}$$

(equation (52) has a singularity at this value as t approaches infinity).

For a droplet larger than critical size and negligible viscosity,

$$t_b = (2D/U)(\rho_l/\rho_g)^{\frac{1}{2}} \tag{54}$$

which shows that the breakup time is directly proportional to the initial droplet diameter, and inversely proportional to the relative velocity.

For large viscosity and small surface tension,

$$t_b = 32\mu_l/\rho_g U^2 \tag{55}$$

which shows that the breakup time is independent of the initial droplet diameter and is inversely proportional to the relative velocity. This independence of breakup time and initial velocity is rather surprising since all other calculations show a pronounced effect of droplet diameter on breakup time.

Since (52) is not analytically solvable, a useful approximation is:

$$t = \frac{2D\rho_l^{\frac{1}{2}}}{(\;_g U^2 - 16\sigma/D)^{\frac{1}{2}}} + \frac{32\mu_l}{\rho_g U^2 - 16\sigma/D} \quad . \tag{56}$$

Gordon stated without verification that this approximation is never too small and is at most 37% too large.

Critical Conditions for Drop and Jet Shattering: Morrell

Hinze's analysis applied to a non-viscous liquid suddenly exposed to a constant velocity gas stream has given the criteria for drop breakup, that is:

$$\frac{\delta/R}{\rho_g U_c^2 R/\sigma} = -.017 \quad . \tag{57}$$

Assuming the critical displacement to be minus R, Morrell (30) assumed the critical Weber number for breakup to be about six. (See equation (57)). Morrell, in another paper (31), analyzed the case for a liquid jet and found that for a constant velocity flow the critical condition for jet breakup was

(58) $$\frac{\delta/R}{\rho_g U^2 R/\sigma} = -0.20 \quad .$$

Hence he assumed that a drop and a jet should behave in approximately the same way with regard to breakup criteria. It was also shown that for an exponential decay of dynamic pressure (i.e., $U^2 = \rho_o U_o^{\,2} e^{-at}$) the ratio of displacement to Weber number decreases as the action time, $t_o = 1/a$, decreases. The maximum values of this ratio were plotted by Morrell as a function of $\tau/2\pi t_a$, where τ is the natural period of oscillation of the jet:

(59) $$\tau = 2\pi \, (\rho_1 R^2/6\sigma)^{\frac{1}{2}}$$

where ρ_1 = liquid density.

This expression was assumed to be approximately correct for a sphere if the corresponding natural period of a liquid sphere was used:

(60) $$\tau = 2\pi \, (\rho_1 R^3/8\sigma)^{\frac{1}{2}} \, .$$

Morrell defined a function $f(\tau/2\pi t)$ as follows:

(61) $$f(\tau/2\pi t_a) = \frac{\delta/R \;\Big/ \left(\frac{\rho_o U_o^{\,2} R}{\sigma}\right) t_a}{\delta/R \;\Big/ \left(\frac{\rho_o U_o^{\,2} R}{\sigma}\right) t_a = \infty} \quad .$$

He then assumed that the critical condition for drop breakup with a finite action time should be

(62) $$f\left(\frac{\tau}{2\pi t_a}\right) \frac{\rho_o U_o^2 R}{\sigma} = k$$

assuming that δ/R is not a function of t_a. This function was plotted by Morrell. For a droplet, assuming that the critical value of δ/R is unity, K equals 6. In general K is approximately equal to $6(\delta/R)_{crit}$ for a sphere.

Morrell also discussed the breakup of a liquid drop by what he termed a stripping action or shear breakup. He quoted the work of Taylor

(39) who calculated the liquid boundary layer thickness and the stripping rate from the boundary layer. Taylor concluded, however, that the calculated breakup time and the experimental breakup time were significantly different.

Taylor's theoretical study was based on the assumption that the liquid sheet stripped from the circumference of the drop undergoing shear breakup separated from the drop surface when the frictional force on the sheet equaled or exceeded the liquid surface tension force. Using this assumption, he derived an expression

$$\frac{\rho_0 U^2}{\frac{\zeta T \alpha}{\beta}} = \text{constant} \qquad (63)$$

where ρ_0 = the average value of ρ over the droplet surface

U_0 = the average value of U over the droplet surface

ζ = the ratio of the actual tensile strength of the drop to the ideal value of the tensile strength

T = the absolute temperature of the liquid sheet

α = the thermal expansion coefficient of the liquid

β = the compressibility of the liquid.

Hence for shear breakup, a critical value of $\rho_o U_o^2$ (rather than a critical Weber number as was the case for bag breakup) should be the criterion for breakup.

As a completion of his analysis, Morrell set forth the conditions under which each type of breakup should occur. For the model he assumed, if t_a is greater than the natural period of oscillation of the drop, the liquid drop should experience bag breakup. If t_a is less than the natural period of oscillation, the droplet will experience shear breakup.

Shatter of Drops in Streams of Air: Lane

Lane (24) stated that a relationship of the form U^2d = constant would be expected to adequately express critical breakup velocities for liquid drops on the assumption that a liquid sphere placed in a steady stream of air would break up when the force due to the variation of the aerodynamic pressure over the drop surface exceeded the surface tension pressure of the droplet. This relationship naturally resulted from the expression equating the drag force on the droplet to the surface tension pressure for a sphere, or

(64) $$C_D \tfrac{1}{2} \rho_g U^2 = 4\sigma/d.$$

Lane also stated that observations from his experimental work indicated that the viscosity of the droplet affected the breakup process only when the viscosity was very high (e.g., of the order of the viscosity of glycerol).

From the experimental evidence examined by Lane, it appeared that the expression $U^2 d = 612$ was true for breakup over a wide range of droplet diameters. If this relation held true over a wide range of droplet diameters, water droplets five microns in diameter would remain intact at a sonic relative velocity. Results of further experimental work of Lane indicated that drops even larger than five microns are able to withstand such large relative velocities without breakup.

An increase in the relative velocity between the droplet and the gas stream resulted in the production of increasingly finer droplets resulting from the breakup process only up to a certain point. At relative velocities beyond this point, even well above sonic velocities, one-half of the mass of the resulting spray of fine droplets had diameters greater than 15 microns for wide ranges of initial droplet diameters.

Lane also found that the velocities required to insure droplet breakup in the transient (step change in velocity) air blasts were lower than in the steady (steadily increasing velocity) air stream. For smaller drops the divergence between the critical steady and the critical transient velocities increased. Also it was noticed that the resultant droplet mass mean diameter decreased with an increase in relative velocity.

It should be noted at this point that the results of Lane's experimental work has since been opposed by the theories and experimental results of Hanson and Domich.

Shock Tube Investigation of the Breakup of Drops by Air Blasts: Hanson, Domich, and Adams

The droplet breakup investigation of Hanson, Domich, and Adams (16) considered two situations which cause a droplet to shatter. The first case was termed by them the "steady" case, or that situation in which a droplet was subjected to a steadily increasing relative velocity. The second case, or "transient" case, was the situation that existed when a droplet was suddenly exposed to a change in relative velocity. Building upon the work of Hinze (17), Lane (24), and Merrington and Richardson (28), in considering these two cases, the authors proceeded to investigate the breakup mechanisms and the effect of physical parameters (e.g., surface tension and viscosity of the drop) upon droplet breakup for droplets in the 100 to 1000 micron size range.

The underlying philosophy of their experimental program was that it was reasonable to assume the existence of a critical velocity for a given droplet diameter. This critical velocity was defined as the relative velocity between the gas stream and the droplet just necessary to induce the droplet to break up. In an effort to discover a verifiable U_c versus d curve, droplets of fluids of differing physical properties were suspended in an acoustic field and subjected to an air blast produced in a shock tube. High speed motion pictures were taken of the droplets of different sizes as they were being deformed and broken up by and after the passage of the shock wave.

An examination of the resulting photographic evidence brought forth many interesting points. First, bag breakup was observed even with the "transient" case. This finding was in opposition to the work of Lane (24) who stated that bag breakup occurred only in the "steady" case. The findings of the authors, though, showed that bag breakup would occur in the transient case except for those velocities which are greatly in excess of the critical velocity for a given droplet diameter. The work of Rabin and Lawhead (34) supported this conclusion. Second, it was noticed that for the more viscous droplets the bag breakup mechanism was more "complicated," that is, the shape of the bag deviated considerably from the spherical shape of the bag of less viscous fluids subjected to the same mode of breakup, and the rupture of the bag resulted in the formation of fluid ligaments rather than the small spherical droplets common to the bag breakup of less viscous liquids. Third, the breakup curves of critical velocity versus diameter were plotted for drop diameters in the range of 90 to 700 microns. For this range of drop diameters, U_c was between 40 and 250 feet per second. A least squares data fit brought forth the following equation correlating critical velocity and diameter.

(65) $$U_c{}^2 D = 6.21 \times 10^6 \text{ (water)}$$
$$= 2.71 \times 10^6 \text{ (alcohol)}$$

This equation is analogous to the empirical relation of Lane (24), that is, $U_c{}^2 D$ = constant.

The authors reviewed Hinze's theory (17) which stated that breakup occurred when the dynamic pressure of the gas stream at the stagnation point of the droplet exceeded the surface tension pressure by a certain factor. Forming the ratio of the dynamic pressure of the air and the surface tension pressure, it is found that:

(66) $$\frac{\text{dynamic pressure of air}}{\text{surface tension pressure}} = \frac{\rho_g U^2/2}{2\sigma/r} = \frac{\rho_g U^2 r}{4\sigma} \quad .$$

Defining the Weber number as

(67) $$We = \rho_g U^2 r/\sigma$$

and defining, as did Hinze, for a drop deviating but slightly from the spherical shape,

(68) $$\rho_g U_c^{\,2} r/\sigma = We_{crit} ,$$

we find that

(69) $$U_c^{\,2} D = 2\sigma We_{crit}/\rho_g$$

which is justification for the empirical equation (65).

A plot of equation (65) yields the We_{crit} for various surface tensions and densities. A summary of this information can be found in Table 1.

Table 1
CRITICAL WEBER NUMBERS

Liquid	U_c, ft/sec	D,	We_{crit}
Water	84.3	600	3.60
Water	109.5	410	4.23
Water	157.3	270	6.00
Water	238.5	120	6.55
Methyl Alcohol	60.0	625	5.98
Methyl Alcohol	84.3	330	6.34
Methyl Alcohol	109.5	230	7.62
Methyl Alcohol	157.3	118	8.41

Table 1. Critical Weber Numbers as Determined by Hanson, Domich, and Adams

Considering the effects of surface tension, equation (69) would give for identical diameters

(70) $$\frac{(U_c)_{water}}{(U_c)_{alc}} = \left[\frac{\sigma_{water}}{\sigma_{alc}}\right]^{\frac{1}{2}} = 1.79 \quad .$$

Equation (70), however, did not correlate the experimental data of the authors. Had the exponent had been 1/3 instead of 1/2, the correlation would have been much better. This result seemed to indicate a more complicated surface tension effect than had been predicted in Hinze's theory.

The authors also attempted to correlate the effect of liquid viscosity on droplet breakup. Using the theory of Hinze in which he predicted that

(71) $We_{crit} = 6$ (slight viscosity, i.e. $2\mu^2/\rho_1 \sigma D << 1$)

(72) $We_{crit} = 10$ (great viscosity, i.e. $2\mu^2/\rho_1 \sigma D >> 1$)

as a basis of comparison, they found that their experimental data did not fit the above relationships. In fact, they found that the critical Weber number was not constant for liquids of approximately the same viscosity, but that it increased with decreasing diameter for each of the experimental liquids and that for very high viscosity, the divergence between Hinze's theory and their experimental data was considerable.

The authors attempted a simple correlation of data on the basis of WeRe = constant, or

(73) $$(WR)_{crit} = \frac{\rho_g}{2\sigma\nu_g} (U_c{}^3 D^2)$$

which led to, under the conditions that $(WR)_{crit}$, ρ_g, σ, and ν_g are constant

(74) $$U_c \alpha \sigma^{1/3}$$

for a given droplet diameter. This simple correlation gave some justification to their previous 1/3 power of viscosity versus critical velocity data fit. Assuming that $U_c \alpha \sigma^{1/3}$, the authors, by means of further data plots, showed that for $\nu_g < 10$ cstk viscosity had only a small effect on the range of drop sizes studied. When $\nu_g = 50$ cstk, then U_c was increased about 25% for all droplet diameters, and for $\nu_g = 100$ cstk, the effect of viscosity was even more pronounced with a decrease in droplet diameter.

Fragmentation of Waterdrops in the Zone Behind an Air Shock: Engel

In an extensive and elaborate experimental program conducted by the author (12), a wealth of photographic evidence showing the minute details of the breakup of a water droplet due to the passage of a shock wave was accumulated. For Mach numbers of 1.3, 1.5, and 1.7 and drop diameters 1 to 3 millimeters a very definite shear breakup mechanism due to the interaction of the shock wave and the droplet was shown. In general, the times required to induce the different stages of shear breakup and to totally shatter the droplet were inversely proportional to the strength of the shock wave and directly proportional to the initial droplet diameter. It appeared, however, that a change in the Mach number of the shock was more effective in increasing the rate of droplet breakup than was a change in initial droplet diameter.

The experimental data taken considered only the variation of breakup time with critical droplet size and shock strength. The effects of liquid viscosity, surface tension, and liquid density were not taken into consideration in the experimental program. One conclusion that could very definitely be made was that the passage of the shock itself did not induce breakup, but that the strength and duration of flow behind the shock were the controlling breakup parameters. The author theorized that the reaction time of the liquid droplet should decrease as the mass of water that is involved decreased and the shock strength increased.

With regards to the mechanism of shear breakup, it was concluded that the characteristic streaming mist emitting from the outer periphery of a drop undergoing shear breakup was not due to vaporization of the liquid but instead was due to "mechanical origins." The mechanical origins considered were a mist produced by sound waves, the stripping off of surface layers of water by the tangential aerodynamic forces existing around the periphery of the drop, and the breaking off of the crests of surface waves. An examination of the above postulations resulted in the acceptance of those mechanisms which took into account the action of the rapid airstream on the surface of the droplet, that is, the breaking off of wave crests, the spilling off of the moving boundary layers at the equator of the drop, and the stripping of water from the downstream face of the droplet by vortex actions.

After the examination of data from similar experiments by other researchers in the field of droplet breakup, Engel concluded that the fragmentation mechanism is dependent upon drop diameter, relative air flow velocity, and the density, surface tension, and viscosity of the liquid drop. Generalizing the results of the present study might result in spurious inferences since it appears that not only the rate of breakup but also the mechanism by which it occurs is very strongly dependent upon the variables mentioned above, and all of these variables were not included in the investigation.

The Motion and Shattering of Burning and Non-burning Propellant Droplets: Rabin and Lawhead

Rabin and Lawhead conducted a shock tube study of the effect of shock waves on the breakup of burning and non-burning liquid fuel droplets. They observed both the bag and the shear-type breakup mechanisms for both the burning and non-burning droplets. They also discovered that the type of breakup mechanism and the critical velocity required to induce breakup were correlated in some manner with the duration of the "flow plateau" following the shock front. No general correlation of these two quantities, however, was formulated. The critical velocity for burning droplets was reported by the authors to be slightly lower than the critical velocity for the non-burning droplets of the liquid fuel they were examining. This difference, they postulated, was due to the difference in the surface tension for the two cases, the surface tension for the burning droplet being lower than for the non-burning droplet.

A general conclusion obtained from an examination of the photographic experimental results showed that for flow velocities which are considerably greater than the critical velocity required to induce drop breakup, the shear-type breakup mechanism will always occur. Another conclusion of the experimental program was that the drops are broken up by the flow behind the shock wave and not by the shock front itself.

Since Rabin and Lawhead discovered that the critical velocity for breakup is reduced with an increase in duration of the flow plateau behind the shock front, it was postulated that the drop breakup could be proportional to the impulse (i.e., force multiplied by the time which the force acts) acting on the droplet. Their line of reasoning, however, was not verified in their report.

The authors pointed out that the theory of Hinze (17) predicted that the critical velocity should be directly proportional to the surface tension raised to the 1/2 power. Experimentally, other authors (e.g., Hanson (16)) have found a 1/3 power dependence. However, the data scatter of the experimental work of the authors made it impossible to confirm either the 1/2 or the 1/3 power dependence.

A further attempt to correlate experimental data in terms of Weber number also proved unfruitful. Rabin and Lawhead concluded that no simple relationship existed between the critical droplet diameter and a critical Weber number for either burning or non-burning droplets.

Perhaps the major contribution of the first report (34) of the experimental work of the authors was the data gathered on the drag coefficients of the liquid droplets. Upon examining the photographic record of the droplet breakup process, they were able to measure the droplet position as a function of time. From this data the drag coefficients were computed. For smaller droplets (less than 100 microns) the drag coefficients appeared to agree with those previously reported by Ingebo (22). However, for larger droplets, there is considerable departure from his data. This effect is possibly due to the fact that for droplets less than 100 microns diameter, the photographic record showed that the droplets deformed only slightly from the spherical shape (for velocities less than the critical velocity) while for drops greater than 100 microns, the droplet deformed into the usual disk shape even for velocities less than the critical velocity for the given drop diameter. Another significant result of the authors' work was that the drag coefficients for burning droplets are slightly lower than for non-burning droplets. This change may be due to the reduced pressure field around the burning droplet due to the vapor phase burning which in turn decreases the pressure drag of the droplet,

Displacement and Shattering of Propellant Droplets--Final Summary Report: Rabin, Schallenmuller, and Lawhead

The report by the above authors (35) summarized an extensive experimental program investigating the shattering of burning and non-burning droplets by a normal shock wave at both atmospheric and elevated pressures (i.e., in general, pressures above the critical pressures of the test liquid). In this program, liquid propellant droplets were suspended by means of a solenoidal retraction of a wire in shock tubes of cross sections 1" x 1" and 2½" x 2½". Within the shock tubes, the duration of gas flow behind the shock wave was varied by using different lengths of pressure section within the shock tube. This variance of flow duration or "flow plateau," was used to vary physical conditions to which the droplet was subjected during the experimental program. The solenoidal retraction of the wire upon which a propellant droplet was suspended resulted in the formation of two droplets within the test section, a "primary" droplet of 500-1600 micron size and a "satellite" droplet of 50-300 micron size. In its entirety, the test program investigated the effects of flow velocity, flow duration, chamber pressure, and surface tension on the shattering of burning and non-burning liquid droplets. The general and specific findings of the experimental investigation will be summarized in the following paragraphs.

The photographic evidence indicated both bag and shear methods of breakup. In general, the larger droplets exhibited shear breakup and shorter breakup times and the smaller droplets exhibited bag breakup for a given velocity and duration of flow. There were also instances in which the droplet appeared to begin the type of deformation leading to shear breakup but then only violently oscillated with no fragments being torn from the droplet.

A major finding of the experimental work was a verification of an earlier postulation, namely, that the passage of the shock front does not shatter the droplet. It is the flow that follows the shock front that causes the droplet to break up. The actual experimental procedure was confined to weak shocks because the authors theorized that the critical flow velocities were in the low velocity ranges, and the previous experimental work of the authors (34) clearly indicated the existence of a critical velocity for a given droplet diameter.

Regarding critical velocities, it was stated that there presently exists no satisfactory explanation to account for the selection of either bag or shear breakup near the critical velocity and the author of this thesis presently supports this view. It was discovered, however, that a flow velocity much greater than the critical velocity for a particular droplet diameter always causes the shear type breakup to occur. The typical critical velocities of this experimental procedure were rather low (e.g., V_{crit}= 60-100 ft/sec for the propellants RP-1, DECH (diethylchlorohexane) at one atmospheric pressure; V_{crit}= 10-15 ft/sec, DECH, 34 atmospheres pressure). Flow durations for both cases were 1.0 to 2.5 milliseconds.

There was a rather substantial decrease in the critical velocity for a droplet of given size as the flow duration was increased. This fact led to the postulation of a critical droplet diameter for a given flow duration. The time required for a droplet to deform sufficiently from its original spherical shape to a shape inducing breakup (the deformation time) was found to be inversely proportional to the droplet diameter. Therefore, droplets below the critical diameter can deform as the gas velocity decays in magnitude, but droplets above the critical size do not have time to deform and shatter. The deformation time was assumed to be inversely proportional to the gas flow velocity; it appeared that a greater flow velocity would be required to shatter a droplet above the critical size than would be required for a droplet smaller than critical size.

"Steady" and "transient" flow conditions were defined based on the natural period of vibration of a liquid drop. "Steady" flow conditions existed if the flow plateau following the shock persisted longer than one-half the natural period of oscillation of the liquid drop oscillating in its lowest mode. "Transient" flow conditions existed if the flow plateau was less than one-half the natural period of the drop. These conclusions were reached by considering the droplet as an idealized spring-mass system with a step-up and decaying forcing function and then solving the resulting differential equation of motion of the system.

By solving the equation of motion, the deformation of the droplet was found to be:

(75) $$x(t) = \frac{2F_o}{k} \left[\psi(t) - \psi(t-\delta) \right]$$

where F_o = magnitude of the applied force

δ = duration of gas flow

ω_o = angular frequency of oscillation

$$\psi(t) = \begin{cases} \sin^2 \tfrac{1}{2}\omega_o t, & t \geq 0 \\ 0 & t < 0. \end{cases}$$

From equation (75) the effect of the flow duration can be seen. For a given mass and deformation (x = constant), as δ is increased, the value of $\left[\psi(t) - \psi(t-\delta)\right]$ increased until it reaches a maximum of unity at $\delta = \pi/\omega_o$; therefore, F_o must decrease for the given x. Thus, for a greater gas flow duration, a smaller force is required to deform the droplet. If the droplet breakup is considered to result from the deformation of the droplet beyond some critical value, a longer gas flow duration behind a shock wave would require a smaller force to shatter a given droplet size.

In an effort to find the criteria for determining the occurrance of either bag or shear breakup, the authors computed the natural period of vibration of the drop and compared this value with the flow duration and the observed type of breakup. The attempts to correlate the experimental data on the basis of the steady and transient flow designations defined on the basis of the natural period were unsuccessful, and to date, no satisfactory correlation is available.

At elevated pressures, only shear breakup was found to occur, but at atmospheric pressure, both types of breakup occurred. Also, after calculation of the critical pressures of the liquids tested and comparison of the characteristics of the shear breakup at pressures both above and below the critical pressures of the liquids, no significant influence of critical pressure was observed. The only observed effect of the higher pressure was a lower critical flow velocity.

Several correlations were attempted in an effort to determine the effect of test section pressure on the critical breakup velocity. These attempted correlations were: (a) shattering occurs at a constant Reynolds' number, or $V_1/V_2 = P_2/P_1$; (b) shattering occurs at a constant Weber number, or $V_1/V_2 = (P_2/P_1)^{\frac{1}{2}}$; (c) shattering occurs at $Re^n We^m$ = constant, or $V_1/V_2 = (P_1/P_2)^{x+1/x+2}$, $x = n/m$. The first two attempts did not satisfactorily fit the data while the third attempt led to an unexplained widely-varying n and m for different liquids. Hence it appeared that none of these attempts provided a satisfactory data correlation of test section pressure and critical velocity.

From the studies made at both atmospheric and elevated pressures, it did not appear that there was any significant difference in the breakup characteristics between burning and non-burning droplets. There appeared to be a slightly lower critical velocity for burning droplets than for non-burning droplets due to the lower surface tension of the burning droplets.

Concerning surface tension, no differentiation could be inferred from the test data between the critical velocity's being proportional to either the surface tension raised to the one-half power or raised to the one-third power. This correlation had been previously postulated by other researchers in this field.

These authors also attempted to correlate the test results in terms of the droplet Weber number, $We = \rho U^2 r/\sigma$. The general plot of results is illustrated in Figure 2.

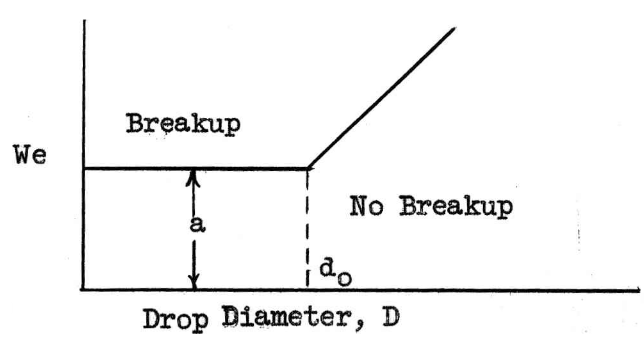


Figure 2
Rabin, et al., Constant We Correlation Attempt

For D less than d_o, We = constant = a. For D greater than d_o, We increased with increasing droplet diameter at some constant slope. For various test conditions, however, Weber number did not give an adequate general correlation.

In a further attempt at correlation, the authors plotted We versus the ratio of the flow duration divided by natural period. The general shape of this plot is illustrated in Figure 3.

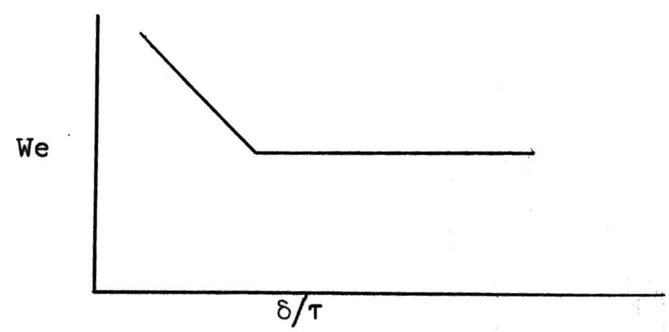


Figure 3
Rabin, et al., We vrs. δ/τ Correlation Attempt

The break in the curve occurred in the general vicinity of $\delta/\tau=0.5$, or symbolically, a break between the steady and the transient flows.

These authors recognized the inadequacy of a constant We theory and thus attempted a new data correlation. The basic concept of their hypothesis of data correlation is that transient breakup and shear breakup are not synonymous. Examination of their data showed that transient breakup must be a time-controlled phenomena whereas shear breakup is rather time-independent. Rabin, et al., postulated that shear breakup occurs when the tangential component of the aerodynamic forces on the droplet is greater than the surface tension forces. Since

(76) $F_f = \frac{1}{2}C_D\rho_g U^2 A$, and

(77) $F_\sigma = k_1 \left(\frac{\sigma}{r}\right)$, then

(78) $C_D = \frac{\rho_g U^2 r}{\sigma} = k_2$, but

(79) $C_D \sim Re^{-\frac{1}{2}}$ (Appendix B, Rabin, et al.)

and therefore

(80) $WeRe^{-\frac{1}{2}} = k_3$.

The constant k_3 was experimentally determined, and the equation (80) gave excellent agreement for all flow cases of a non-burning droplet if $k_3 = \frac{1}{2}$. For a burning droplet the correlation did not give such a good result, but this may have been due to the difficulty in determining the surface tension for the burning droplets.

A secondary result of the experimental process was that the droplet drag coefficient for high pressures and for either burning or non-burning droplets was approximately equal to one.

Kinetics, Mechanism, and Resultant Sizes of the Aerodynamic Breakup of Liquid Drops: Wolfe and Anderson

The authors of this report (43), after giving a short review of other classical theories of breakup mechanisms, have postulated that droplet breakup (which is a flow process) is a rate process. Eyring, in his book The Theory of Rate Processes (14), has stated that any "rearrangement of matter" can be considered to be a rate process, and hence the theory of absolute reaction rates can be applied, theoretically, to the breakup of liquid droplets. Wolfe and Anderson stated that the oft-used classical equation which equates the maximum force tending to break up the droplet to the surface tension force is valid only for small rates of stress loading and hence not for shock processes. They also theorized that in any situation in which the stress tending to break up the liquid undergoes a change in time less than that required to break up the liquid, the above-mentioned classical equation will not be true.

The unique approach of Wolfe and Anderson applied kinetic theory to the breakup process, whereas all work previous to theirs had considered the breakup process only from the hydrodynamic and mechanical approach. However, the authors stated that this does not mean that the hydrodynamics and mechanics of the problem should be ignored but only that they should be incorporated into the proper kinetic expression of the system.

The authors considered that the aerodynamic pressure drag and the aerodynamic friction drag logically were the two variables that were responsible for the two extreme types of liquid droplet breakup, that is bag breakup and shear breakup respectively. A qualitative theoretical derivation using rate process theory to relate droplet deformation to the above-mentioned aerodynamic forces resulted in an equation relating the droplet breakup times to the flow parameters of the gas stream and the physical properties of the liquid droplet, or

(81) $$t = \frac{d}{(A^2 + Bp)^{\frac{1}{2}} - A}$$

where $A = 16\mu/d\rho_1$

$B = 2/\rho_1$

$p = \frac{1}{2}\rho_g U^2 C_D - k\sigma/d$

k = constant reflecting drop curvature during breakup (determines effective surface tension pressure).

For flow and/or liquid conditions in which viscous and surface tension forces are negligible, equation (81) becomes

(82) $$t = \frac{d}{U}\left(\frac{\rho_1}{\rho_g}\right)^{\frac{1}{2}} ,$$

For extremely viscous liquids and negligible surface tension,

(83) $$t = 32\mu/\rho_g U^2 .$$

It was interesting to note at this point the similarity that existed between equations (82) and (83) and the expressions of Hinze and Gordon for similar breakup conditions.

Equation (81) can be regarded as a generalized equation for the breakup time of a liquid drop of given physical properties subjected to an aerodynamic flow of known conditions. The authors stated that it was possible to use (81) to predict the breakup time of a liquid without regard to the mechanism of breakup if we could choose a suitable value for k. After examination of available experimental data, (81) can be used if $C_D = 1$ and $k = 2$. For the use of (81), the experimental breakup time is defined as the time from the inception of the aerodynamic flow around the droplet to the instant in which the droplet begins to break up. Thus the theoretical total time required to break up the droplet will be slightly larger than the experimental values since the theoretical breakup time assumes that a complete disintegration of the droplet (complete rearrangement of matter) has occurred.

If equation (81) is to provide an adequate model of the breakup process, then it should, in the opinion of the authors, provide an explanation of both bag and shear breakup. Since the criteria that bag breakup results from pressure drag and shear breakup from friction drag have been invoked, two individual forms of equation (81) may be written, one expression containing the pressure drag stress in the pressure expression and one containing the friction drag stress in the pressure expression. It was postulated that for a liquid drop of given properties and an air stream of given properties breakup would occur by the mechanism that required the least breakup time. If the two rates were comparable, the drop should exhibit both bag and shear breakup characteristics.

Equation (81) can be made to fit both breakup cases if one assumes that the frictional drag is twice the pressure drag, an opinion which comes from many workers in this field. If the total drag stress acting on a drop during breakup is $\frac{1}{2}\rho_g U^2 C_D$, then for bag and shear breakup the pressure expression becomes

(84) $$P_b = (1/3)(\tfrac{1}{2}\rho_g U^2)C_D - K_b\sigma/d$$

(85) $$P_s = (2/3)(\tfrac{1}{2}\rho_g U^2)C_D - K_s\sigma/d$$

where K_b and K_s are constants that reflect the effect of surface tension tending to hold the drop together during, respectively, bag and shear breakup. By a best fit of experimental data, $K_b = 4$ and $K_s = 2$; these values may be used to predict breakup times for drops undergoing either bag or shear breakup. Experimental evidence has shown that for low velocities, bag breakup prevails and that for high velocities, shear breakup prevails.

It is also very desireable to be able to predict the mean drop size produced by the breakup of the original drop, although the droplet sizes produced by the primary breakup of the original drop may vary due to the secondary breakup of drops produced by the primary breakup, vaporization of primary and secondary droplets, coalescence of primary and secondary droplets, and settling or removal of the droplets by the gas stream. This report considered only the mean droplet size distribution resulting from the primary breakup, and the magnitudes of the other mentioned effects were estimated from existing knowledge. The experimental results of this study showed that the drop sizes produced by the two different breakup modes were essentially the same, a result that is intuitively somewhat surprising. However, since one postulated mechanism (shearing of a liquid film from the drop) and one equation (equation (81)) theoretically govern both types of breakup, this suggested result is not surprising from a theoretical standpoint.

The results of this study did not provide a theory that would provide a resultant droplet size distribution as a function of the liquid droplet and the gas stream parameters. However, by assuming that the mean drop size results from the breakup into optimum unstable wave lengths of the liquid boundary layer being stripped from the surface of the droplet, it has been found that the mean diameter is

(86) $$D = (6W\delta\lambda/\pi)^{\frac{1}{2}}$$

where δ = boundary layer thickness

$\lambda = 4\pi\sigma/\rho_g U^2$ (ref: Squires (38))

W = width of strips of liquid streaming from the droplet

$\delta = 4\mu_l/U(\rho_g \rho_l)^{\frac{1}{2}}$

or,

(87) $$D = \left(\frac{136\mu_l \sigma^{\frac{3}{2}} d^{\frac{1}{2}}}{\rho_g^2 \rho_l^2 U^4}\right)^{1/3} .$$

Equation (87) was derived for the case in which the aerodynamic forces are much larger than either the viscous or surface tension forces. It is theorized that this case is valid for shock processes.

It is interesting to note that equation (87) predicts both the same $1/U^{4/3}$ dependence of D and the same initial diameter to the 1/6 power dependence of D as does the empirical work of Weiss and Worsham.

Analysis of Normal Shock Waves in a Particle Laden Gas: Kriebel

In this paper the author mentioned a theoretical shock wave thickness of "several inches" for a particle laden gas flowing in a duct; however, this statement resulted merely from a matter of definition of shock wave thickness (i.e., the definition of shock wave thickness is the distance corresponding to the time required for a particle and a gas stream in which it is flowing to reach velocity equilibrium after passing through a shock wave). Hence this definition is effectively a measure of the velocity lag of the particle after it passes through a shock front.

Results of theoretical calculations show that after particles of the size range 0.5 to 5.0 microns pass through a shock wave there is a considerable velocity lag of the particles compared to the velocity of the gas. This result indicated the existence of a relative velocity between the particle and the gas stream sufficiently large to initiate particle breakup.

It was also significant to observe that as the size of the particles passing through the shock wave increased, the relative velocity between the gas stream and the particles increased at even a proportionately higher rate. For a particle of only a three micron radius the ratio of the particle velocity to the gas velocity was approximately three directly downstream from the shock, and this ratio did not significantly decrease until a distance of 1.2 inches downstream from the shock. For higher Mach numbers and particle sizes, the ratio increased at a rapid rate, thus insuring the existence of a significant particle relative velocity for a significant downstream distance; these two conditions are necessary though not sufficient to induce liquid particle breakup.

CHAPTER III

RESULTS

RESULTS

As was previously mentioned in this thesis, the various equations resulting from the theories presented in the literature to account for the droplet critical diameter, the droplet critical velocity, and the droplet breakup time were programmed for the IBM 7040 digital computer. In these programs the particle and gas stream physical properties were varied over an extreme range of values in an attempt to discover the effects of differing values of these properties on the critical breakup parameters and also to attempt to discover that correlations and/or discrepancies that might exist between the various theories as the physical property values were varied. The results of these calculations have been plotted where practical on the following pages. In other cases where a data plot would not provide a convenient representation of results, respresentative values from the outputs have been tabulated. These calculations are identified by the name(s) of the author(s), and specific reference is made to the equations presented under the name(s) of the author(s) as presented in Chapter II of this thesis.

Plots, Tables, and Explanatory Notes of Computer Outputs

Triebnigg's estimate of the critical velocity for a liquid droplet falling through a stagnant medium is presented in Figure 4. This plot is also indicative of Lane's work. Here plots of critical velocity versus droplet diameter are given for varying values of the drag coefficient. This was done because the drag coefficient for a burning croplet which is deformed from the spherical shape due to its motion is yet a matter of

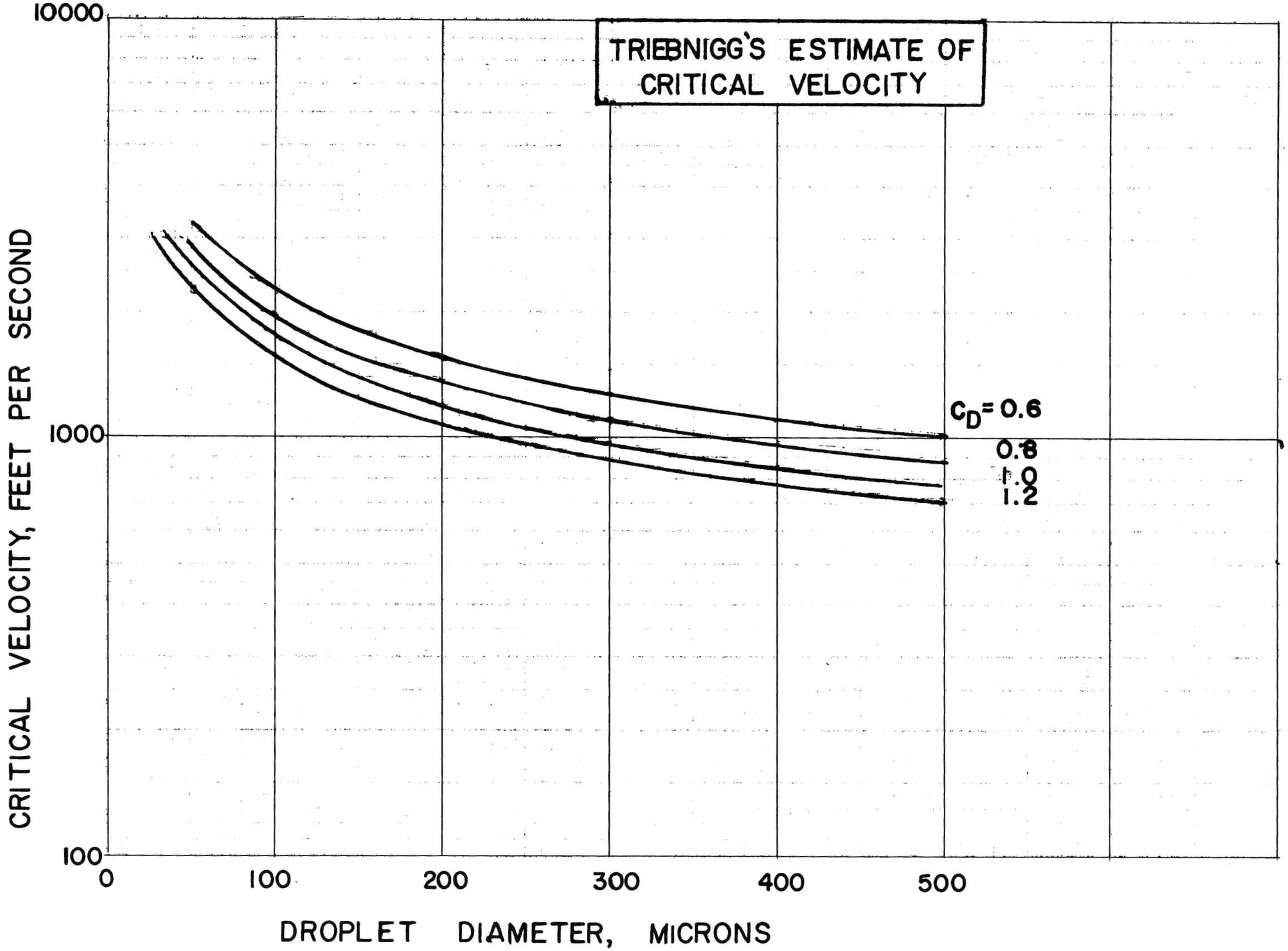

TRIEBNIGG'S ESTIMATE OF CRITICAL VELOCITY
CRITICAL VELOCITY, FEET PER SECOND
10000
1000
100
0
100
200
300
400
500
DROPLET DIAMETER, MICRONS
C_D= 0.6
0.8
1.0
1.2

Figure 4

some argument. It is seen from this plot, which is intended to serve purely as a means of comparing other more complex breakup theories to this simple case, that the critical relative velocity required to induce breakup is a rather large value even for large values of the drag coefficient. It is also seen that the drag coefficient, indicative of the magnitude of the aerodynamic forces on the droplet, exerts a rather pronounced effect on the critical velocity for a given size droplet.

The critical breakup velocities as predicted by Hinze are next summarized in Figures 5 and 6. The values of δ/R correspond respectively to different experimental values of the critical deformation of the droplet. A value of δ/R equal to one would mean that the droplet would break up if the "bag" was pushed out of it a length equal to the radius of the original undeformed droplet. It is seen upon comparison of the two plots that the viscosity of the droplet plays a very important role in the determination of the critical breakup parameters. The critical velocity for a given diameter is much greater (on the order of 700 to 800 feet per second greater) for a highly viscous droplet than for a slightly viscous one. In either case, for a droplet smaller than 50 microns radius, the critical velocity as predicted by Hinze is rather high (on the order of 3000 to 400 feet per second for a twenty-five micron radius particle) in comparison to other theories.

Dodd's minimum inscribed sphere breakup criteria for bag breakup is illustrated in Figure 7. It should be noted that the critical velocity as predicted by Dodd is considerably lower than that predicted by any of the previous plots. This may be due to the fact that Dodd's theory assumes no direct viscosity effect in determining breakup parameters.

The empirical data correlation of Weiss and Worsham (Figure 8) was solved for U in an effort to determine a critical velocity that would give

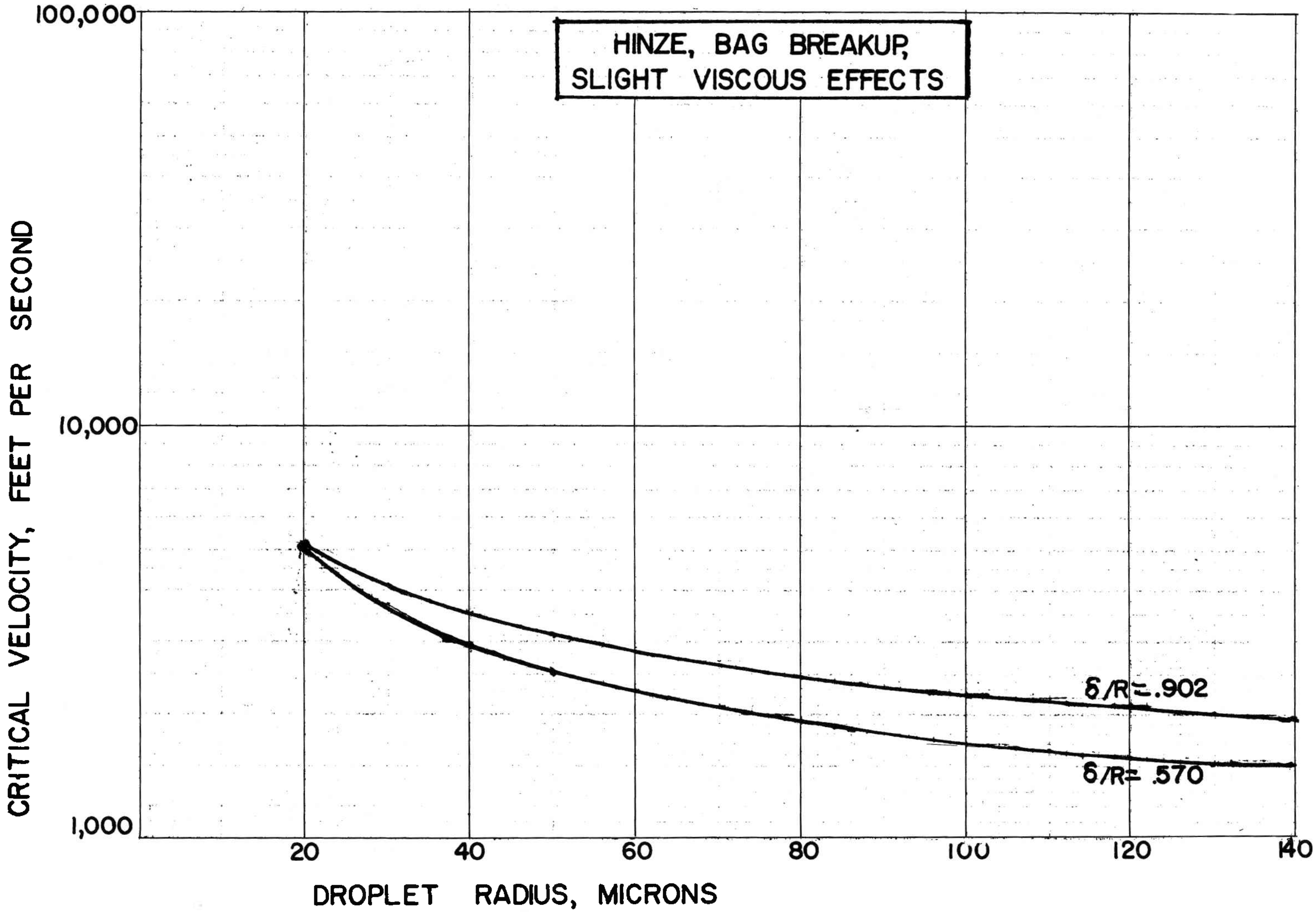

HINZE, BAG BREAKUP,
SLIGHT VISCOUS EFFECTS
CRITICAL VELOCITY, FEET PER SECOND
100,000
10,000
1,000
δ/R=.902
δ/R=.570
20
40
60
80
100
120
140
DROPLET RADIUS, MICRONS

Figure 5

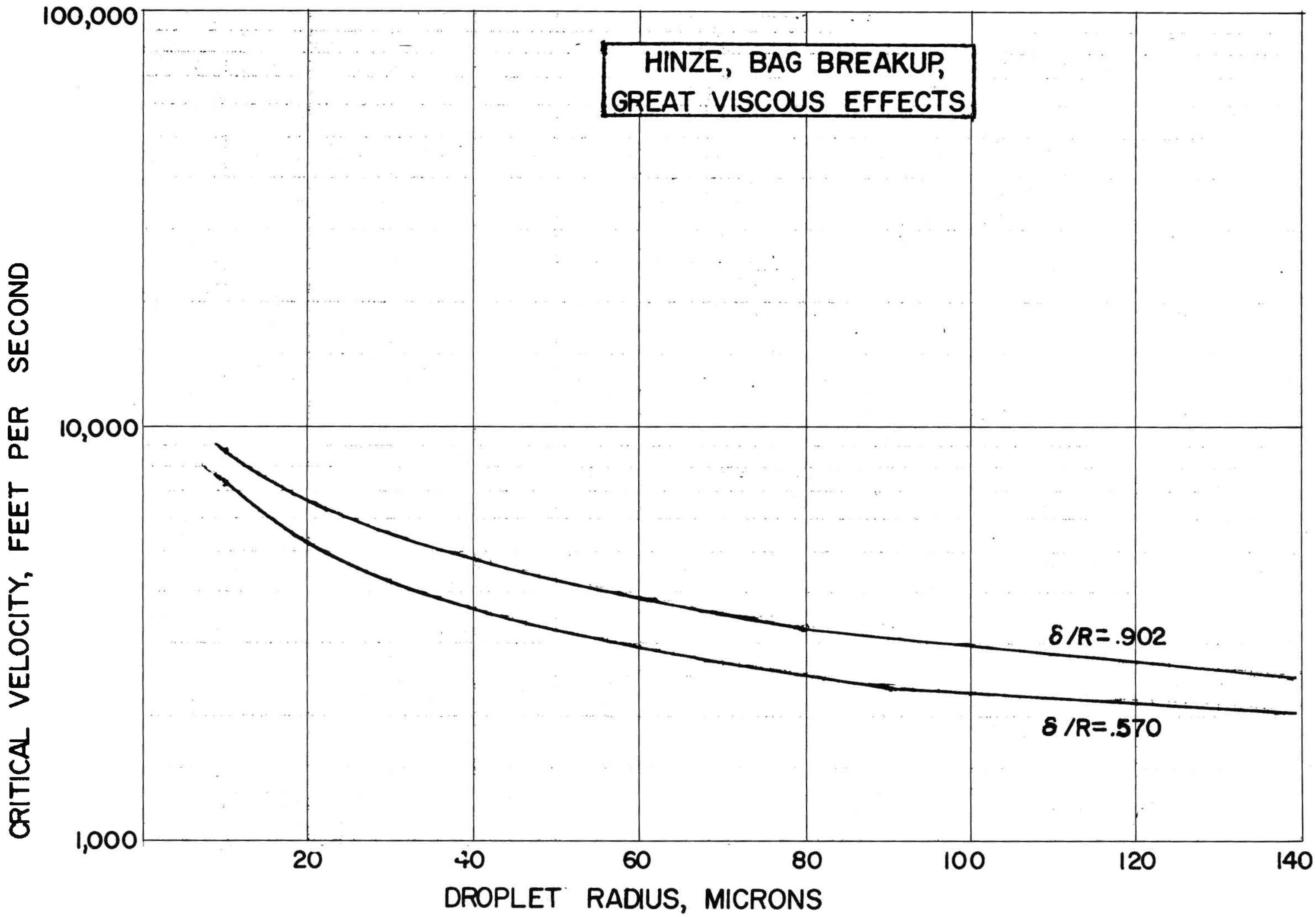

HINZE, BAG BREAKUP,
GREAT VISCOUS EFFECTS
CRITICAL VELOCITY, FEET PER SECOND
100,000
10,000
1,000
20
40
60
80
100
120
140
DROPLET RADIUS, MICRONS
δ /R=.902
δ /R=.570

Figure 6

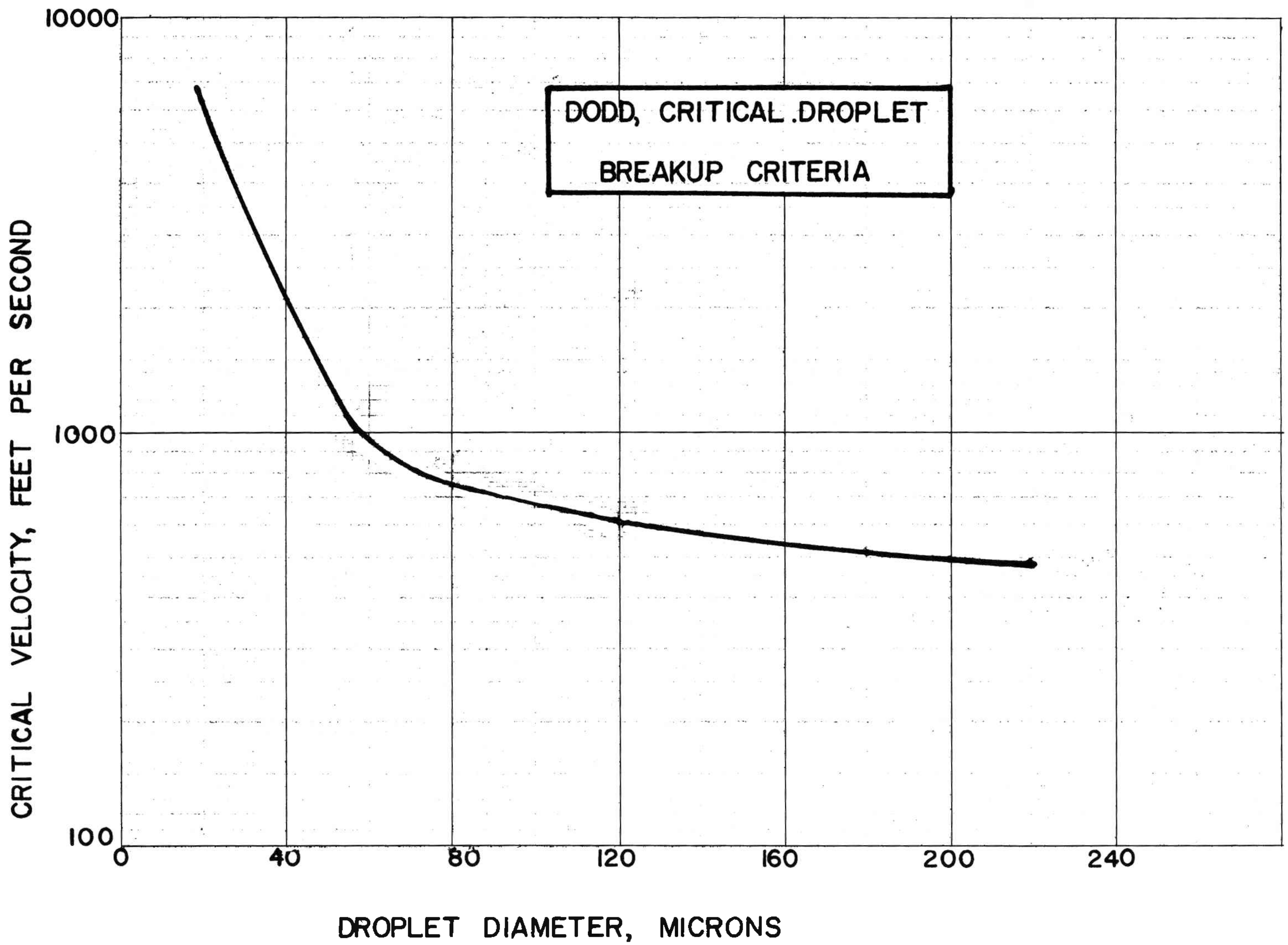
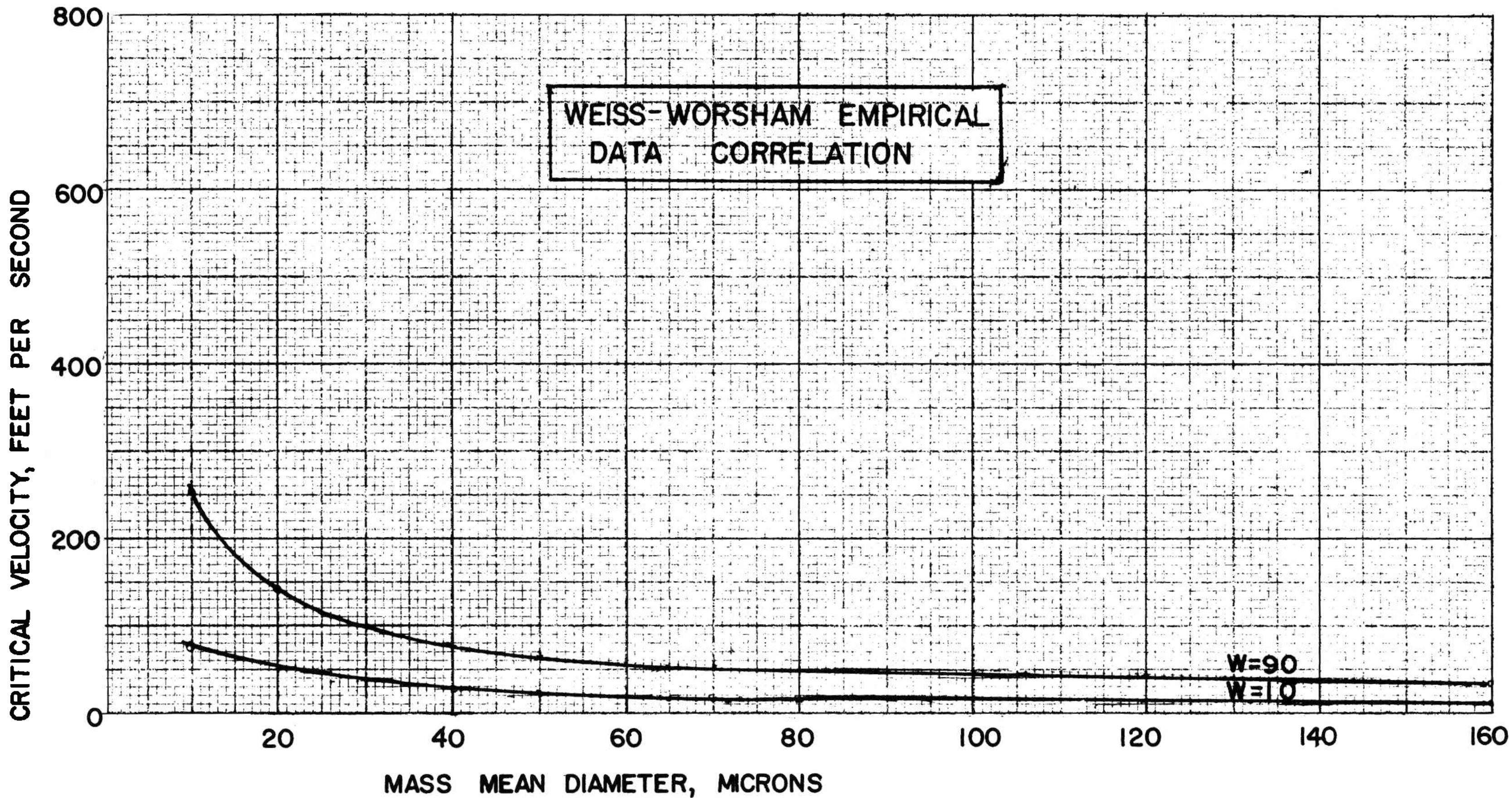

DODD, CRITICAL DROPLET
BREAKUP CRITERIA
CRITICAL VELOCITY, FEET PER SECOND
10000
1000
100
0
40
80
120
160
200
240
DROPLET DIAMETER, MICRONS

Figure 7



WEISS-WORSHAM EMPIRICAL
DATA CORRELATION
800
600
400
200
0
CRITICAL VELOCITY, FEET PER SECOND
20
40
60
80
100
120
140
160
MASS MEAN DIAMETER, MICRONS
W=90
W=10

Figure 8

a certain resultant mean droplet size after breakup. The mass injection rate, W, must be above 90 to 100 lb_m/hr before a significant critical velocity even occurs. It is noticed that this plot indicated a very low critical velocity necessary to produce a rather large (40 to 60 micron diameter) resultant mean size distribution. The reason for this is not apparent upon examination of the available work of Weiss and Worsham.

The suggested data correlation of Rabin, Schallenmuller, and Lawhead is plotted for various values of surface tension in Figure 9. This was done because of the authors' expressed concern that the surface tension of a burning aluminum droplet might be a rather difficult physical quantity to determine due to the assumed vapor phase burning of the droplet. The plot shows that there is a pronounced effect of different surface tension on the critical breakup parameters. However, since the critical velocity for a given droplet diameter decreases with a decrease in surface tension, and there is a recognized decrease in surface tension for a burning droplet as opposed to a non-burning one, this result is a significant point in the consideration of the breakup parameters of a burning aluminum droplet. It should also be noted that even for a surface tension similar to that used in the previous plots (σ= 0.048), the correlation suggested by Rabin, et al., which fits their data indicates a much lower critical velocity for a given initial droplet diameter than any of the previous theories applied to this specific case.

A tabulation of critical velocities, breakup times for large droplet viscous effects, breakup times for small droplet viscous effect, and breakup times for nominal droplet viscous and surface tension effects for a range of droplet diameters as predicted by Gordon is presented in Table 2. Herein is noted a relatively high required critical velocity, but the surprising result is that the breakup times for high viscosity are much

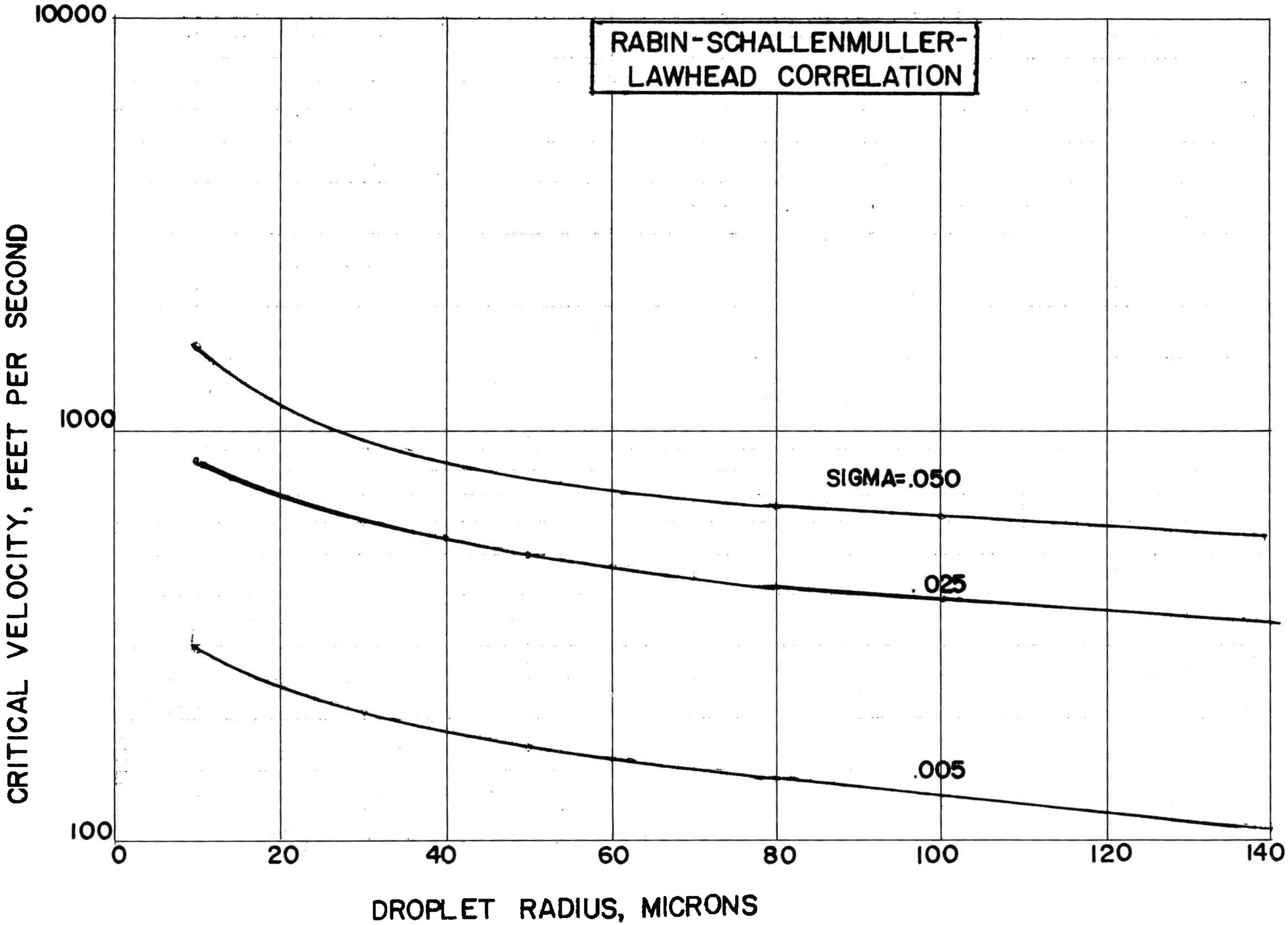

RABIN-SCHALLENMULLER-
LAWHEAD CORRELATION
10000
1000
100
CRITICAL VELOCITY, FEET PER SECOND
SIGMA=.050
.025
.005
0
20
40
60
80
100
120
140
DROPLET RADIUS, MICRONS

Figure 9

Table 2
Gordon's Theory

Diameter, microns	U_g, ft/sec	t_b, seconds high viscosity	t_b, seconds low viscosity	t_b, seconds nom. viscosity
1	10,000	< 0.000000	< 0.000000	< 0.000000
5	10,000	< 0.000000	< 0.000000	0.000004
10	7,900	< 0.000000	< 0.000000	0.000011
20	5,600	< 0.000000	< 0.000000	0.000032
25	5,000	< 0.000000	< 0.000000	0.000045
30	4,500	< 0.000000	< 0.000000	0.000060
35	4,200	< 0.000000	< 0.000000	0.000075
40	3,900	< 0.000000	< 0.000000	0.000092
50	3,500	< 0.000000	< 0.000000	0.000128
60	3,200	< 0.000000	< 0.000000	0.000168
75	2,900	< 0.000000	0.000001	0.000241
100	2,500	< 0.000000	0.000001	0.000363
150	2,000	< 0.000000	0.000002	0.000666

lower than those for either low viscosity or nominal viscosity with surface tension effects also considered. It is seen that the surface tension criteria is important in the determination of critical breakup parameters, since the case considering nominal viscosity and surface tension effects results in a much higher required breakup time for a given droplet diameter and critical velocity than do the cases for either high or low viscosity. However, the breakup time even for the case requiring the longest time interval between flow inception and droplet breakup is still on the order of less than a millisecond.

The rate process theory of droplet breakup as proposed by Wolfe and Anderson was the subject of a very lengthy computer calculation. Due to the range of parameters encountered, the representative results of this calculation are presented in Table 3. From these data it is seen that the breakup times for given initial droplet diameters and assumed critical velocities are indeed very low, that is, within the microsecond range for the small diameters (less than 40 micron diameters) considered. Using the tabulated results of theory in conjuction with the critical velocities predicted by other theories, it is possible to determine the critical breakup velocity and breakup time for droplets of nominal, low, or high viscosity. It is interesting to note that the breakup times for the high viscosity case are again much smaller than those times indicated for either the nominal or low viscosity cases. Another interesting result is that the breakup times for low and for nominal viscosity are essentially the same just beyond the critical velocity as predicted by this calculation. It was interesting to note in the computer output that the value of the critical velocity remained an imaginary number until a certain point, beyond which the critical velocity was purely real. Perhaps this is an indication that below this first real value breakup will not occur, and that the first real value can therefore be considered to be the lowest critical velocity for a given diameter.

At this point, it is of worth to indicate that the breakup times of liquid droplet as predicted by Hinze for both the high and the low viscosity cases were also in the microsecond range for small droplet diameter, and that again the cases of large droplet viscosity resulted in predicting lower breakup times than those predicted for the low viscosity consideration. The breakup time data were not convenient for plotting, but a short tabulation of representative values is presented in Table 4.

Table 4
HINZE'S THEORY

$\delta/R = .902$

Drop Diameter, microns	t_b, seconds low viscosity	t_b, seconds high viscosity
5	.00000001	< .00000000
10	.00000002	< .00000000
20	.00000007	< .00000000
25	.00000010	< .00000000
30	.00000013	< .00000000
40	.00000020	< .00000000
50	.00000028	< .00000000
60	.00000037	< .00000000
80	.00000057	< .00000000
100	.00000079	< .00000000

Table 4
Breakup times according to Hinze

Table 3: Wolfe-Anderson Rate Process Theory

Diameter, microns	U, ft/sec	t_b, seconds non. viscosity	t_b, seconds low viscosity	t_b, seconds high viscosity
1	3750*	2E-07	3E-08	3E-10
	4000	9E-08	3E-08	3E-10
	5000	3E-08	2E-08	2E-10
	7500	2E-08	1E-08	8E-11
	10000	1E-08	1E-08	5E-11
5	1650	1E-06	3E-07	2E-09
	2000	5E-07	3E-07	1E-09
	2500	3E-07	2E-07	7E-10
	4000	2E-07	1E-07	3E-10
	5000	1E-07	1E-07	2E-10
	7500	8E-08	7E-08	8E-11
	10000	6E-08	6E-08	5E-11
10	1150	3E-06	1E-06	3E-09
	1500	1E-06	7E-07	2E-09
	2000	7E-07	6E-07	1E-09
	2500	5E-07	4E-07	7E-10
	4000	3E-07	3E-07	3E-10
	5000	2E-07	2E-07	2E-10
	7500	1E-07	1E-07	8E-11
	10000	1E-07	1E-07	5E-11
20	800	8E-06	3E-06	7E-09
	1000	4E-06	2E-06	5E-09
	1500	2E-06	1E-06	2E-09
	2000	1E-06	1E-06	1E-09
	2500	9E-07	9E-07	7E-10
	4000	6E-07	6E-07	3E-10
	5000	4E-07	4E-07	2E-10
	7500	3E-07	3E-07	8E-11
	10000	2E-07	2E-07	5E-11
30	700	3E-05	5E-06	9E-09
	1000	5E-06	3E-06	5E-09
	1500	2E-06	2E-06	2E-09
	2000	2E-06	2E-06	1E-09
	2500	1E-06	1E-06	7E-10
	4000	8E-07	8E-07	3E-10
	5000	7E-07	7E-07	2E-10
	7500	4E-07	4E-07	8E-11
	10000	3E-07	3E-07	5E-11
40	550	2E-05	8E-06	2E-08
	1000	6E-06	4E-06	5E-09
	2000	2E-06	2E-06	1E-09
	4000	1E-06	1E-06	3E-10

*The first value of U for each D was the first value of U for which U was a purely real number.

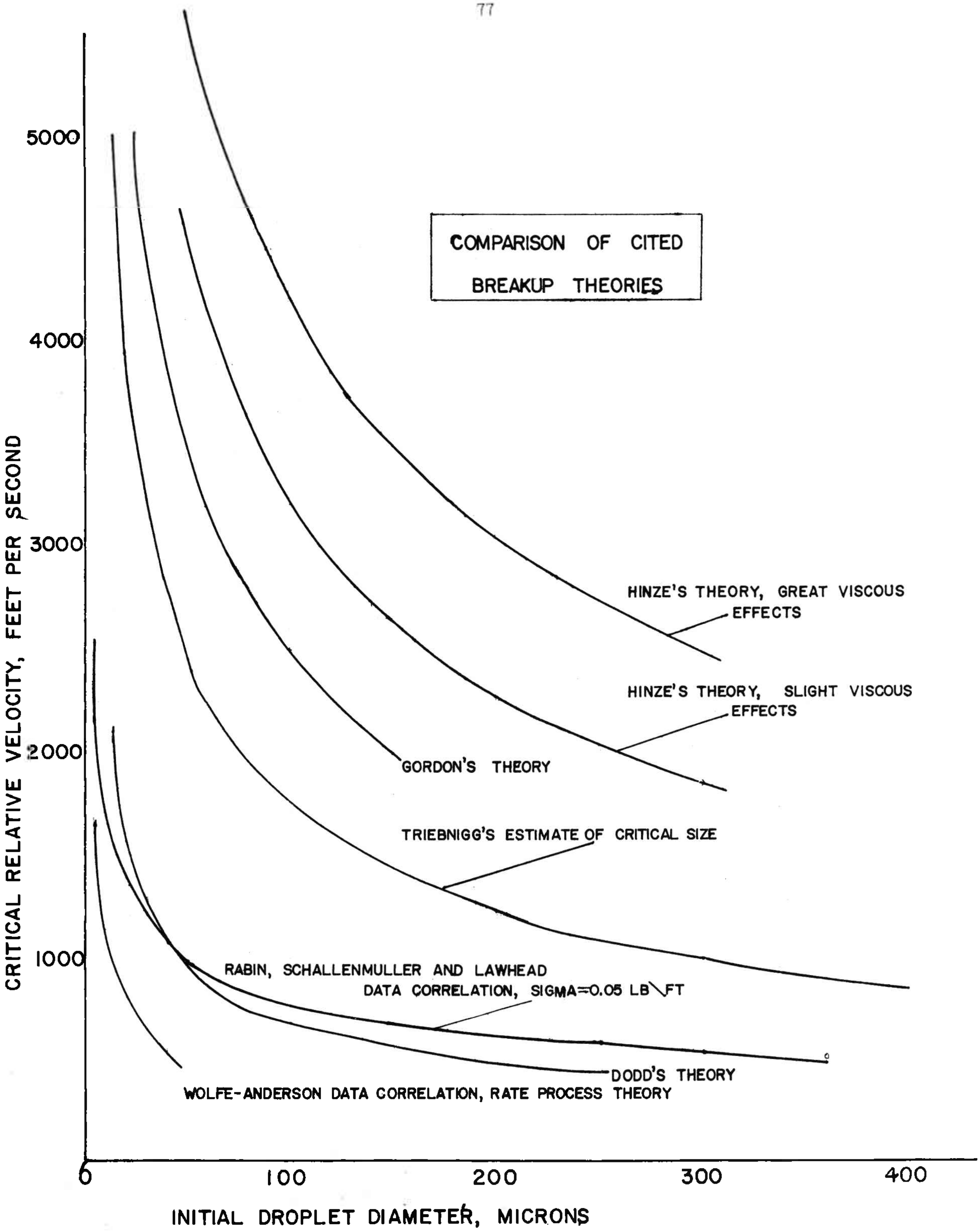

COMPARISON OF CITED
BREAKUP THEORIES
CRITICAL RELATIVE VELOCITY, FEET PER SECOND
5000
4000
3000
2000
1000
HINZE'S THEORY, GREAT VISCOUS
EFFECTS
HINZE'S THEORY, SLIGHT VISCOUS
EFFECTS
GORDON'S THEORY
TRIEBNIGG'S ESTIMATE OF CRITICAL SIZE
RABIN, SCHALLENMULLER AND LAWHEAD
DATA CORRELATION, SIGMA=0.05 LB\FT
DODD'S THEORY
WOLFE-ANDERSON DATA CORRELATION, RATE PROCESS THEORY
0
100
200
300
400
INITIAL DROPLET DIAMETER, MICRONS

Figure 10

Correlations-Discrepancies Between Various Theories

The results presented in the previous section of this chapter of those theories which lend themselves to numerical calculation show a wide range of critical breakup parameters. The best that can be expected with the present state of the art is an order of magnitude quantitative description of droplet breakup parameters and a "worst case" argument concerning a given physical situation. This will be done in the next section of this chapter.

The earlier attempts of Triebnigg would most probably not be applicable in the present case of a burning aluminum droplet passing through a shock wave. His data can best be used as a worst case argument for the present physical situation.

Hinze's results clearly show that a more viscous droplet tends to required a larger critical velocity to induce breakup at a given droplet diameter, but that at this critical velocity, the breakup time is less than that required to break up a less viscous droplet of the same size. However, if the less viscous droplet were to be subjected to the higher critical velocity required by the more viscous droplet, the breakup times would be of the same order of magnitude although it has been shown that the more viscous droplet still requires a slightly shorter time to break up than the less viscous drop. This view somewhat contradicts the view that viscosity acts purely as a damping force in determining critical breakup parameters.

It was interesting to note that both the theories of Gordon and of Wolfe and Anderson also predicted the behavior mentioned in the preceding paragraph. Some concern is expressed by the author of the present thesis that, since the cases of high viscosity breakup and low viscosity breakup were derived by means of simplifying assumptions from the same general equations in the breakup theories previously discussed, perhaps the basis of the simplifying assumptions should be subjected to a rigorous mathematical treatment to assure that the disregarded portions of the general equation do not contain important terms.

Hence, despite the differences in critical velocities as predicted by the different theories (Figure 10), differences which can qualitatively be reconciled by the different assumptions inherent in the treatment of the force balances on the droplets, the primary item of concern seems to be the effect of the physical properties, especially viscosity, on the breakup parameters. It was previously mentioned in this thesis that viscosity, in the opinion of many researchers in this field, plays a retarding role in the droplet breakup processes. However, the results of numerical calculations over a wide range of variables have shown that even though the critical velocity for a viscous droplet is greater than that for a less viscous one, the breakup time for the less viscous droplet is greater than for the more viscous one. This intuitively seems to be incongruous.

Specific Application of Results to Burning Aluminum Droplet

The case to be considered is that of a burning aluminum droplet passing through a shock wave at a Mach number of 2.5. The rest of the assumed physical properties can be found in Appendix B of this thesis. Before the direct results of the aforementioned computer calculations are applied to this physical situation, a short review of applicable material found in the literature will be made in order to substantiate some of the claims that will need to be considered in this situation.

Before the aluminum particle, assumed to be liquid and burning, intersects the plane of the shock wave, there will exist a considerable velocity lag between the flowing particle and the gas stream, or, in other words, it cannot be assumed that upstream from the shock wave there is no relative velocity between the particle and the gas stream. Hoglund (19) has stated that for particles of less than two microns diameter there will be no appreciable particle velocity lag, but for particles larger than two microns diameter, the velocity lag will be significant. Gilbert, Davis, and Altman (13) have shown that a one micron diameter particle follows the gas velocity closely, but that a ten micron diameter particle exhibits a significant velocity lag. Kriebel's work (23) implied that there existed a significant particle velocity lag even for particles of the 0.5 to 5.0 micron diameter range. He also stated that as the size of the particles increased, the relative velocity increased at even a proportionately higher rate, or, equivalently, that the velocity lag increased with an increase in the Mach number.

This fact would tend to foster the deformation of the particle to some extent even upstream from the shock front, and thus it can be theorized that this deformation of the particle before it intersects the shock front will shorten the breakup time.

Since it has been shown that the particle drag coefficient exhibits some effects in the determination of the critical relative velocity, the magnitude of this coefficient is of some interest. Carlson and Hoglund (7) have presented an empirical expression fitting their experimental data which indicates that the drag coefficient for a spherical particle in "flow regimes such as occur in solid propellant rocket exhaust" approaches one as the Reynolds' number exceeds one hundred. Ingebo's (22) data shows that for the Reynolds' number range applicable to the present physical situation the drag coefficient is approximately one. It should be noted that his data considered the drag coefficient for clouds of solid spheres, clouds of evaporating liquid droplet, and clouds of non-evaporating liquid droplets accelerating in an air stream. The physical situations that he considered could easily approximate in magnitude of effect the case presently being considered in this thesis. Way and Nicholls (41) have stated that drag coefficient of 0.5 to 1.0 has been apparent in the Reynolds' number range of 10 to 100, and that generally, a decrease in drag coefficient due to burning has been established. The work or Rabin, Schallenmuller, and Lawhead (35) showed an appreciably higher drag coefficient of 0.5, and Way and Nicholls attributed this difference to the deformation of the droplet from the spherical shape to the disk shape.

It is felt that the decrease in drag coefficient due to burning and the increase due to deformation of the disk shape after the droplet passes through the shock wave are effects that tend to at least compensate each other, if not actually cancel each other. Hence a drag coefficient of one will be assumed for the above-mentioned physical situation.

A value of the surface tension of 0.048 lb_f/ft will be used. It is felt that this would represent the highest value of the surface tension that could be used, and since it has been reported that the surface tension value decreases with a burning droplet as compared to a non-burning one, with other physical conditions held constant, this is a good assumption. Another theory that leads to the justification of the above assumption is that the surface tension decreased with an increase in temperature, and the droplet temperature may be considerably higher, and certainly no lower, than the droplet temperature assumed for this case. However, since there may exist an oxide coating, either liquid or solid, on the surface of the particle, the surface tension may show an increase due to this coating. The surface tension values for either a liquid or a solid aluminum oxide coating are presently an unknown quantity to the author. Hence there is some uncertainty regarding this particular liquid property. Nevertheless, taking into consideration the high relative velocity between the particle and the gas stream after the particle has passed through a Mach 2.5 shock wave and even considering the possible velocity lag of the particle upstream from the shock, it is theorized that this uncertainty of surface tension will be compensated by the high relative velocity existent downstream from the shock.

Hence it is postulated that for a Mach 2.5 shock, with a ratio of M_x/M_y equal to 2.5/0.512, considering even a 20 percent particle velocity lag upstream of the shock front, that there will exist a sufficiently high relative velocity downstream from the shock front for a 30 to 40 micron diameter particle to shatter, be it by bag or shear mode, in a time certainly less than one millisecond after passing through the shock.

Regarding the resultant particle mean diameter or particle size distribution after breakup, Hoglund (19) stated that there is yet no theory capable of predicting a particle size distribution after breakup. If the data of Weiss and Worsham can be applied to this specific case, a mean diameter of less than one micron seems plausible. Brown and McArty (5) have stated that for an aluminzed propellant, data has shown a one micron diameter oxide particle exists in the exhaust, but the initial solid aluminum particle was only 2 to 3 microns diameter. Wolfe and Anderson (43) have stated that the same size distribution resulted from their observations of bag and shear breakup, but they were unable to completely theorize an exact restultant mean diameter. However, photographic records of both shear and droplet breakup have shown that the resulting droplet size after breakup appears as a mist compared with the parent droplet.

The theory of Rabin, et al., quite similar to the situation theorized in this thesis, and the theory of Wolfe and Anderson, substantiated with data for a situation similar to this one, have shown the existence of a critical velocity approximately one-fifth of that assumed to exist in this given physical situation. Hence it is postulated that there will exist sufficient relative velocity in spite of an uncertainty in surface tension values to be used, to break up the aluminum droplet in a very short time duration following its passage through the shock front.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. It is concluded that there exists a wide spectrum of theories which have been developed in an attempt to provide a mathematical model for droplet breakup.
2. The equations resulting from these theories, when examined over a wide range of droplet and gas variables, show a substantial divergence in predicted critical relative velocities, critical diameters, and breakup times.
3. There also exist discrepancies which seem to be a function of the original assumptions made in fashioning a mathematical model and in the method of simplifying the resulting general equation into an analytically solvable form for the special cases of high, low, and intermediate viscosity and surface tension.
4. There seems to be in existence no reliable model to predict the occurrence of either bag or shear breakup for given physical parameters.
5. For the specific case of a burning aluminum droplet passing through a shock wave at a Mach number of 2.5, and assuming the physical parameters given in the appendix to this thesis, it is theorized that a particle 30 to 40 microns in diameter will shatter after passing through a shock wave at or above Mach 2.5 in a matter of less than a millisecond.
6. Those breakup theories correlating experimental results indicate, however, that the breakup time will be in the 0.01 to 10.0 microsecond range.

Recommendations

1. Since there exists such a mass of material, both applicable and not, in the field of droplet breakup, it is hoped that further work in this field will continue where this thesis has concluded (i.e., attempting to correlate and explain the available information).
2. Further research is needed to accurately and confidently determine the property values pertinent to this droplet shattering problem.
3. Research should be done to determine an accurate and reliable theory to predict the occurrence of either bag or shear breakup under given physical conditions.
4. The assumptions inherent in the general equations and the simplifying assumptions made to render the general equation analytically useable need to be critically examined from a physical-mathematical standpoint to asses their validity.
5. The rate process theory of Wolfe and Anderson shows particular promise since a force balance on the droplet is not an absolute necessity in deriving an equation for critical breakup parameters. This theory should be further investigated as to applicability to the droplet shattering problem.

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APPENDIX A

SURFACE TENSION, VISCOSITY, AND DENSITY OF LIQUID ALUMINUM

APPENDIX A

Surface Tension, Viscosity, and Density of Liquid Alumunum

1. Ref: Lyon, Richard N. (ed.) Liquid-Metals Handbook. (Office of Naval Research: Second Edition-Revised) Washington, D.C.: U.S. Government Printing Office, 1954, pp. 40-44.

Viscosity = 2.9 centipoises x 10^2 @ 700° C

Surface Tension = 520 dynes/cm°C @ 750° C

Density = 2.380 g/cm^3 @ 660° C

2. Semenchenko, V. K. Surface Phenomena in Metals and Alloys. London: Pergamon Press, Ltd., 1962, p. 398.

The surface tension of liquid aluminum:

T, °C	Surface Tension, dynes/cm
700 - 820	520
712	502
706	494
660	914
700	900
800	865

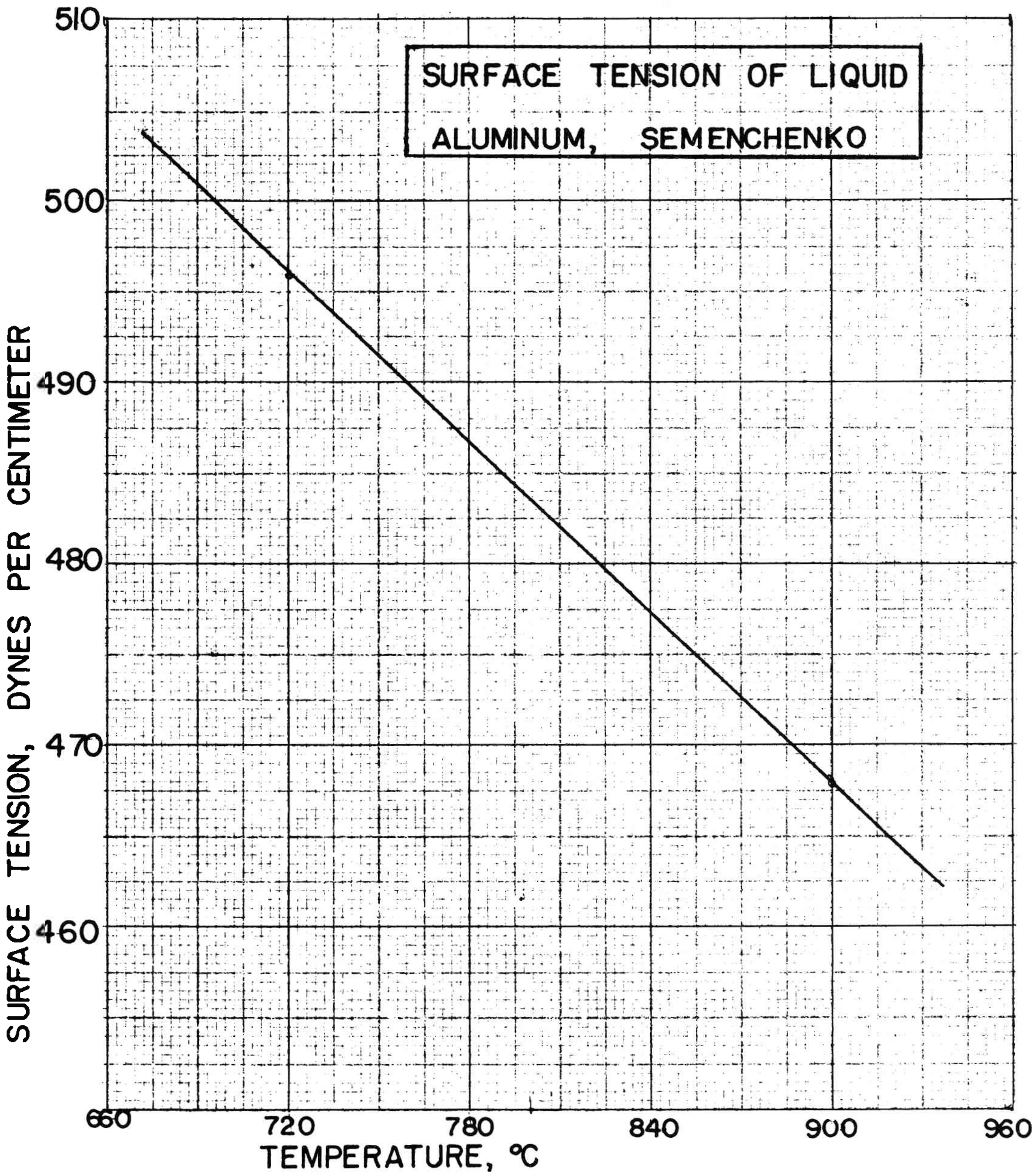
SURFACE TENSION OF LIQUID ALUMINUM, SEMENCHENKO
510
500
490
480
470
460
SURFACE TENSION, DYNES PER CENTIMETER
660
720
780
840
900
960
TEMPERATURE, °C

Figure 11

APPENDIX B

PHYSICAL PARAMETERS OF SPECIFIC CASE EXAMINED

APPENDIX B

Physical Parameters of Specific Case Mentioned

$\sigma = 700$ dynes/cm $= 0.048$ lb_f/ft

$\rho_1 = 2.38$ $g/cm^3 = 148.583$ lb_m/ft^3

$M = 2.5$

$\mu_1 = 2.9 \times 10^{-2} cp = 0.186173 \times 10^{-4}$ $lb_m/ft\text{-}sec$

$P_s = 600$ lb_f/in^2

$T_s = 6000^\circ$ R

$W = 32$ (with aluminum particles)

$W = 20$ (without aluminum particles)

<u>For $M = 2.5$:</u>

$$\rho_y = \rho_g = \left(\frac{\rho_y}{\rho_x}\right)\left(\frac{\rho_x}{\rho_{sx}}\right) \rho_{sx}$$

$$\rho_x = \frac{\rho_s}{RT_s} = \frac{600 \times 32 \times 144}{1545 \times 6000}$$

$$\frac{\rho_y}{\rho_x} = 3.3333$$

$$\frac{\rho_x}{\rho_{sx}} = 0.13169$$

and therefore:

$$\rho_g = (3.33)(0.13169)\left(\frac{600 \times 32 \times 144}{1545 \times 6000}\right) = 0.131 \ lb_m/ft^3$$

APPENDIX C

COMPUTER PROGRAMS

```
C       CHINA LAKE RESEARCH PROJECT
C       VIC FORSNES
C       TRIEBNIGGS ESTIMATE OF CRITICAL VELOCITY
C       U = VELOCITY FOR BREAKUP
C       DEFINE VARIABLES
        SIGMA =0.048
        RHOA=0.131
        CD=0.4
        R=0.000001
        DO 10 I=1,9
        WRITE (6,100) CD
        DO 20 J=1,500
        U=SQRT((128.8*3.28*SIGMA)/(R*CD*RHOA))
        WRITE (6,101) R, U
        R=R+0.000001
   20   CONTINUE
        R=0.000001
        CD=CD+ 0.1
   10   CONTINUE
  100   FORMAT (1H0, F6.1)
  101   FORMAT (2X, F12.6, 4X, F12.6)
        END
```

```
C      CHINA LAKE PROJECT
C      VIC FORSNES    THESIS CALCULATION
C      LANES ANALYSIS
       RHOA=0.131
       SIGMA=0.048
       CSUBB=0.5
       D=0.000001
       DO 10 I=1,6
       DO 20 J=1,500
       VC=SQRT((844.93*SIGMA)/(CSUBB*RHOA*D))
       WRITE(6,100) CSUBB, D, VC
   20  D=D+0.000001
       D=0.000001
   10  CSUBB=CSUBB+0.1
       CONTINUE
  100  FORMAT(2X,F6.2,4X,F10.6,4X,F13.2)
       END
```

```
C     VIC FORSNES  CHINA LAKE RESEARCH PROJECT
C     CALCULATION TO DETERMINE NATURAL PERIOD OF DROPLET
C       DEFINE VARIABLES
        RHOL = 143.5834
        SIGMA = 8993.74
        PI = 3.14
    100 FORMAT (5X,F10.7)
    101 FORMAT (10F11.7)
        DIMENSION  TAU(10), FINVAL (10)
        R = 0.0
        TA = 0.00010
        DO 10 I=1,1000
        DO 20 J=1,10
        TAU(J)=(2.0*PI)*SQRT((RHOL*R**3)/(8.0*SIGMA))
        FINVAL(J)=TAU(J)/(2.0*PI*TA)
     20 R=R+0.00010
        WRITE (6,100)  TA
        WRITE (6,101)  (TAU(J), J=1,10)
        WRITE (6,101)  (FINVAL(J), J=1,10)
        R=0.0
        TA=TA+0.00010
     10 CONTINUE
        END
```

```
C        VIC FORSNES    CHINA LAKE RESEARCH PROJECT
C        HINZE DROPLET BREAKUP    BAG DEFORMATION MECHANISM
C        CALCULATION TO DETERMINE CRITICAL VELOCITY, RADII
C                FOR SLIGHT VISCOUS EFFECTS
C        DEFINE VARIABLES
      WEMAX=6.0
      R=0.000005
      RHOA= 0.0518
      RHOL= 148.5834
      SIGMA= 0.048
      XMUL= .00001861713
      DO 10 I=1,8
      DO 20 J=1,30
      DELTAR=-0.095*WEMAX
      UC=24.9*SQRT((-DELTAR*SIGMA)/(RHOA*R))
      TB=0.3537*(R/UC)*SQRT((RHOL/RHOA)*-DELTAR)
      WRITE(6,100)DELTAR,UC,TB
      R=R+0.000005
   20 CONTINUE
      R=0.000005
      WEMAX=WEMAX=0.5
   10 CONTINUE
      WEMAX=6.0

C               FOR GREAT VISCOUS EFFECTS
      DO30 I=1,8
      DO40 J=1,30
      DELTAR=-0.095*WEMAX
      UC=33.32*SQRT((DELTAR*SIGMA)/(-RHOA*R))
      TB=((10.0*XMUL)/(RHOA*UC*UC))*DELTAR
      WRITE(6,100)DELTAR,R,UC,TB
      R=R+0.000005
   40 CONTINUE
      R=0.000005
      WEMAX=WEMAX+0.5
   30 CONTINUE
  100 FORMAT(10X,F7.3,4X,F9.6,4X,F12.4,4X,F11.8)
      CONTINUE
      END
```

```
C      CHINA LAKE RESEARCH PROJECT
C      DROPLET PRESSURE DISTRIBUTION   SPHEROIDAL CROSS SECTION
      SIGMA=0.048
      H=0.100
      Z=0.00
      DO 30 I=1,10
      DO 20 J=1,10
      IF(Z.NE,0.0) GO TO 9
      A=B/H
      R=((A*A*B)**(1.0/3.0))*0.3048
      WRITE(6,100) H,B,A
    9 H2=H*H
   90 ZB2=(Z/B)*(Z/B)
      SOLUT=((H**(5.0/6.0))/2.0)*(1.0+H2+((1.0-H2)*ZB2))/
     1((H2+((1.0-H2)*ZB2))**(3.0/2.0))
      RHO=(SOLUT/(R/2.0))*SIGMA
      WRITE(6,101) Z,SOLUT,RHO
      Z=Z+.000020
      IF(Z.LE.B) GO TO 90
      Z=0.0
   20 B=B+.000020
      B=.000001
   30 H=H+.10
      CONTINUE
  100 FORMAT(1HO,F6.2,4X,F12.6,4X,F12.6)
  101 FORMAT(2X,F12.6,4X,F12.6,4X,F12.6)
      END
```

```
C      CHINA LAKE RESEARCH PROJECT
C      VIC FORSNES
C      GORDON THEORY FOR DROPLET BREAKUP

      SIGMA=.048
      RHOA=.131
      RHOL=148.583
      XMU=.00001862
      D=.000001
      DO 10 I=1,200
C            CRITICAL DIAMETER
      VCRIT=41.113*SQRT(SIGMA/(RHOA*D))
C            FOR LOW VISCOSITY
      TBL=((.6096*D)/VCRIT)*SQRT(RHOL/RHOA)
C            FOR HIGH VISCOSITY
      TBH=(32.0*XMU)/(RHOA*VCRIT*VCRIT)
C            FOR INFLUENCE OF SURFACE TENSION AND NOMINAL VISCOSITY
      TBN=(.6096*D*SQRT(RHOL))/SQRT((RHOA*VCRIT*VCRIT)-((1690.27*SIGMA/D))
      WRITE(6,100)D,VCRIT,TBH,TBL,TBN
   10 D=D+.000001
  100 FORMAT(2X,F12.6,4X,F12.6,4X,F12.6,4X,F12.6,4X,F12.6)
      END
```

```
C     CHINA LAKE RESEARCH PROJECT
C     VIC FORSNES
C     DODD MINIMUM INSCRIBED SPHERE CRITERION
C     CRITICAL VELOCITY DETERMINATION
      RHOA=0.131
      SIGMA=0.048
      CSUBI=0.582
      RSUBM=0.000001
      DO 10 I=1,300
      VCRIT=SQRT((39.27*SIGMA)/(CSUBI*RHOA*RSUBM))
      WRITE(6,100)RSUBM,VCRIT
      RSUBM=RSUBM+0.000001
   10 CONTINUE
  100 FORMAT(2X,F10.6,4X,F14.3)
      END
```

```
C       WEISS AND WORSHAM DATA CORRELATION
C       CHINA LAKE RESEARCH PROJECT
C       VIC FORSNES
C       ATOMIZATION IN HIGH VELOCITY AIRSTREAMS
C       EMPIRICAL DATA CORRELATION
C       DIFFERENT INJECTION RATES, MASS MEAN DIAMETERS
      SIGMA=0.048
      XMUL=0.0000186
      RHOA=0.131
      RHOL=148.583
      XMUA=0.0000148
      GSUBC=32.174
      XMDIA=0.000001
      WINJ-1.0
      DO 20 J=1,10
      DO 10 I=1,1000
      TERM1=(((XMUL)**(2.0/3.0))*((SIGMA)**(1.0/3.0))*((GSUBC)**(1.0/3.0)))
     1/((RHOA)*(XMDIA)*(3.048))
      TERM2=(1.0+(1000.0*((RHOA)/(RHOA)/(RHOL))))
      TERM3=((WINJ*RHOL*SIGMA*XMUA*GSUBC)/(3600.*(XMUL**(4.0)))**(1.0/12.0))
      VELCR=((0.61)*TERM1*TERM2*TERM3)**3.0/4.0)
      WRITE(6,100)XMDIA,VELCR,WINJ
      XMDIA=XMDIA+0.000001
   10 CONTINUE
      XMDIA=0.000001
      WINJ=WINJ+10.0
   20 CONTINUE
  100 FORMAT(4X,F12.8,2X,F16.2,2X,F6.2)
      CONTINUE
      END
```

```
C      CHINA LAKE RESEARCH PROJECT
C      VIC FORSNES
C      RABIN-SCHALLENMULLER-LAWHEAD CORRELATION

      RHOA=0.131
      XMU=0.0000186171
      SIGMA=0.00
      R=0.000001
      DO 20 I=1,11
      DO 10 J=1,500
      V=((0.5*SIGMA*32.2)/((RHOA**(1.0/2.0))*(XMU**(1.0/2.0))*(R**(1.0/2.0))
     1*3.28))**(2.0/3.0)
      WRITE(6,100) SIGMA, R, V
      R=R+0.000001
   10 CONTINUE
      R=0.000001
      SIGMA=SIGMA+0.005
   20 CONTINUE
  100 FORMAT(2X,F7.3,5X,F10.6,5X,F13.6)
      END
```

```
C      CHINA LAKE RESEARCH PROJECT
C      VIC FORSNES
C      WOLFE AND ANDERSON RATE PROCESS THEORY

      GSUBC=32.17
      SIGMA=0.048
      XMUL=0.000018617
      RHOA=0.131
      RHOL=148.583
      D=0.000001
      U=0.0
      DO 10 I=1,301
      DO 20 J=1,201
      A=(16.0*XMUL)/(RHOL*D*3.28)
      B=2.0/RHOL
      P=(0.5*RHOA*U*U)-((2.0*SIGMA*GSUBC)/(3.28*D))
C           CALCULATION OF DROPLET BREAKUP TIME
      TSUBB=(D*3.28)/((SQRT((A*A)+(B*P)))-A)
C           FOR LOW VISCOSITY AND NEGLIGIBLE SURFACE TENSION
      TLOW=((D*3.28)/U)**(SQRT(RHOL/RHOA))
C           FOR HIGH VISCOSITY AND NEGLIGIBLE SURFACE TENSION
      THI=(32.0*XMUL)/(RHOA*U*U)
      WRITE(6,100)D,U,TSUBB,TLOW,THI
      U=U+50.0
   20 CONTINUE
      U=0.0
      D=D+0.000001
   10 CONTINUE
  100 FORMAT(5X,F10.6,5X,F9.1,4X,3E8.6)
      END
```

APPENDIX D

NATURAL PERIOD OF OSCILLATION OF LIQUID DROP

APPENDIX D

Natural Period of Oscillation of Liquid Drop

From Rayleigh:

$$\tau_{nat} = \left(\frac{3\pi m}{8\sigma}\right)^{\frac{1}{2}} = 2\left(\frac{\rho_1 R^3}{8\sigma}\right)^{\frac{1}{2}}$$

LIQUID PARTICLE INTERACTION

WITH A NORMAL SHOCK WAVE

An Abstract of a Thesis

Presented to the

Department of Mechanical Engineering Science

Brigham Young University

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

by

Victor George Forsnes

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APPROVALS

This abstract is accepted in its present form as fulfilling the abstract requirement toward a Master of Science degree in the Mechanical Engineering Department, Brigham Young University.

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ABSTRACT

The literature pertinent to the general field of droplet breakup was thoroughly surveyed and reviewed with regard to application to droplet breakup in shock processes. The results of this thorough literature search are presented in this thesis, both in the general bibliography and in a section of the thesis wherein the major droplet breakup theories are discussed with special attention being given to the method of attack on the problem.

The equations resulting from the major droplet breakup theories relating critical droplet breakup parameters were programmed for an IBM 7040 computer, and the values of the droplet and gas stream parameters were varied over a wide range.

An attempt was made to show the correlations and/or discrepancies between the various breakup theories. These results are presented in graphical or tabular form.

The results of the computer runs were specifically applied to the case of a burning aluminum droplet passing through a normal shock front at a Mach number of 2.5 in order to ascertain whether or not the droplet would break up.

Recommendations for further specific research into facets of the droplet breakup problem are made. Particular emphasis is given to the present lack and/or availability of physical property data for liquid aluminum.