Particle Dynamics in the Mixing of a Particle-Laden Stream with a Secondary Stream in a Duct

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PARTICLE DYNAMICS IN THE MIXING OF A PARTICLE-LADEN STREAM WITH A SECONDARY STREAM IN A DUCT

A Thesis
Presented to the
Department of Mechanical Engineering
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
William Doyle Cranney
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This thesis, by William Doyle Cranney, is accepted in its present form by the Department of Mechanical Engineering of Brigham Young University as satisfying the thesis requirement for the degree of Master of Science.

April 20, 1970
Date

Typed by Katherine Shepherd
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CHAPTER I

INTRODUCTION

The purpose of this thesis is to review the history and discuss the present work being done in particle-gas flow, with application to air-augmentation of a particle-carrying, fuel-rich gas jet. A drawing of the model considered is shown in Figure 1, with the area of interest labeled. The thesis is intended as a thorough literature review and analysis.

Fig. 1. --Model considered in thesis

The results of this work are to be used as a source for the beginning
of an entire model development study. While there are many theoretical
treatments of particle dynamics, the main sources for this thesis is experi-
mental work. These sources are used as either supporting or refuting
given analyses or treatments presented.

The thesis is the result of work supported by the Naval Weapons
Center at China Lake, California, under Contract No. N 60530-68-C-0626
to Brigham Young University. The contract deals with, specifically, the
mixing and combusting region of an air-augmented, particle-carrying gas
stream.

In this contract, researchers deal with various aspects, including:
gaseous jet mixing, particle dynamics (represented by this thesis), particle
and droplet combustion, and the entire problem as a whole, that is, mixing
and combustion of a particle-laden jet with a secondary stream in an enclosed
duct.

Related to the treatment of particle dynamics are several important
areas. These include particle drag, heat transfer analysis and prediction,
diffusion of particles into the secondary stream, similarity of the particle
velocity and concentration profiles and validity of the continuum flow assump-
tion, and finally, the present models available, including their assumptions
and treatments of the preceding topics. Each one of these topics will be
covered as individual chapters with a final chapter to conclude and summar-
ize.
CHAPTER II

PARTICLE DRAG

The study of particle drag has long been pursued; yet in the areas of turbulence, combustion, and acceleration, little knowledge or agreement has been realized. The analysis seems to be an extremely complicated one. Consistent and complete physical measurements limit any drag analysis in an effort to come up with a clean analysis.

In the literature, the drag force upon a sphere is most often described as a non-dimensionalized "drag coefficient," $C_d$. It is usually defined, and will be in this thesis, as:

$$C_d = \frac{D}{\frac{1}{2} \rho V^2 A}$$

where $C_d =$ Drag coefficient.

$D =$ Drag force on sphere due to motion with respect to surrounding fluid, not including buoyant, gravitational, electrostatic, or other non-velocity dependent forces.

$\rho =$ Density of the surrounding fluid.

$V =$ Velocity of sphere relative to undisturbed free stream.

$A =$ Projected frontal area of sphere, $\pi r^2$.

The drag forces upon a sphere have been defined and separated from
other forces, such as gravity and buoyant forces, by allowing only those forces which are a result of the relative fluid sphere motion. In this context, the forces acting upon the sphere at rest are not included as drag force. This drag is then termed by Olsen (1)* as "dynamic drag."

The drag forces are composed of two physically different forces, a pressure force and a shear force. The pressure forces act perpendicular to the surface and must be integrated around the surface of the sphere to get the total pressure force. The shear forces, the result of the viscous nature of the accompanying fluid, act parallel to the surface. As in the case of the pressure forces, the shear forces must be integrated to get the resultant shear force. The total dynamic drag is, then, the sum of the integrals of viscous shear force and pressure force.

In 1850, G. G. Stokes originated a purely theoretical solution of the drag coefficient as a function of Reynolds number. He solved a much simplified Navier-Stokes equation which in turn expressed that the drag force is composed of one-third pressure force and two-thirds shear force. His solution neglected the inertial terms in the equation, which become important at higher velocities (2). Stokes' analysis resulted in a drag coefficient equal to twenty-four divided by the Reynolds number based on sphere diameter, \( C_d = \frac{24}{Re_D} \). The generally accepted range of accuracy is for Reynolds numbers less than one, \( Re_D < 1.0 \). In addition, Stokes' flow assumes that

*Numbers in parentheses refer to list of references.*
the flow is incompressible and laminar.

In practice, the most often used and best known drag coefficient is the steady state drag, usually presented as a "standard drag" curve (2). It represents the results of experimental data collected for many years. Though its values were generated only for steady state conditions, its applications are found in many non-steady state conditions.

Many results of the significant studies of sphere drag coefficients, including Stokes' and the steady state drag curve, are shown in Figure 2. As noted earlier, some of these studies show a wide variation in coefficients.

One of the early experimental investigators was R. G. Lunnon (3). His work involved the dropping of spheres of various density and diameters down a mine shaft. The displacements and corresponding times were recorded and presented in an article published in 1926 (3). Since then, several investigators have analyzed his data in an effort to predict the motion of particles subjected to drag.

As technology was progressing, industry became interested in the flow of solid-liquid mixtures in pipes. The main concern was that of predicting the increased pressure loss due to the presence of the solid particles. A complicated analysis was not desired nor required in order to come up with the desired predictions. Soo (4) gives a brief discussion of this era in gas-solid flow.

With the advance of rocket technology and increased thrust, a great impetus was given to the study of sphere drag. The advent of certain metals
Figure 2. Drag coefficient versus Reynolds number.
to solid rocket fuels produced a particle-laden exhaust stream. These particles lagged the changing velocity and temperature of the gas in the accelerating and cooling nozzle flow. This then caused losses in expected thrust. Because of the particle losses, nozzle sections had to be designed in a manner that would best optimize the gas-solid flow. A knowledge of the drag upon particles by the gas became important.

The next paragraphs discuss, in chronological order, the significant work in particle drag since 1954. Most of the following will also be presented in Figure 2.

Drag coefficients for burning kerosene drops were reported in 1954 by Bolt and Wolf (5). In their experiment, however, the Reynolds numbers were very low, less than 1.0. There is a significant amount of scatter present in their data, as shown in Figure 2. The experiment, however, showed a slight decrease in drag coefficient from steady state for burning drops as compared to non-burning drops.

Among the first investigators to realize the significance of losses generated in rocket nozzles due to solid-gas lag were Gilbert, Davis, and Altman (6). In their work, they analyzed the flow properties of linearly accelerated gas-particle flows. The flow was analyzed to determine the propulsion losses resulting from the velocity lag between particles and the accompanying gas. They assumed that the particle lag, due to the accelerating gas stream, followed according to Stokes' drag law. Their model was also one-dimensional and there were no interactions between the walls and
the flow. It was an early first step in realizing the importance of two-phase flow losses in nozzles.

Ingebo, in 1955, published experimental work which was concerned with the drag of a cloud of particles accelerating in an air stream (7). In his work he used both solid and liquid spheres. The accelerations reached by the particles approached upwards of 60,000 feet per second squared. The major conclusion of the experiment was that for a Reynolds number between six and four hundred, and a sphere diameter between 20 and 120 microns, the empirical drag coefficient is the following:

\[ C_d = \frac{27}{R_e D^{0.84}} \]

His curve appears with those of other important experimenters in Figure 2.

As stated previously, the drag of a sphere is due to two forces, a shear force and a pressure force. The shear exists due to the viscous nature of the fluid with which the sphere is flowing. In the case of the pressure drag, the flow surrounding the sphere is unable to completely recover the pressure on the back side of the sphere as was present on the front side. This difference in pressure causes a net force in the opposing direction. A main factor in the degree of pressure recovery is the ability of the flow to remain attached around the periphery of the sphere at increasing velocities. If the flow is able to attach to the rear surface, the drag is drastically reduced. On any aerodynamic body the desired response is that of the flow remaining attached to the rear surface of the object, which reduces the drag
to a minimum. Aerodynamic bodies are tapered in order to reduce this separation.

As in the case of the transition from a laminar to a turbulent boundary layer on a flat plate, the same event occurs on the surface of a sphere. The effect of a turbulent boundary layer on a sphere at a Reynolds number of approximately $10^5$ is to cause the flow around the sphere to remain attached further around the periphery. This allows a greater pressure recovery, which reduces the drag force. This is the explanation for the sudden drop in drag coefficient versus Reynolds number for steady state drag at a value of Reynolds number approximately equal to $10^5$, as shown in Figure 2.

Roughness on the surface of a sphere can cause this same transition to occur, but earlier than otherwise. For the proper range of Reynolds numbers, roughness can have the effect of reducing the drag on a sphere. The prime example is the golf ball. The dimples cause the boundary layer to become turbulent earlier than would a smooth sphere, and thus, for the particular range of Reynolds numbers encountered by the ball, the drag is reduced, whereby the ball goes much farther in flight.

In addition to surface roughness, the action of the free stream turbulence introduces turbulence into the boundary layer, the same effect as surface roughness. Free stream turbulence causes the boundary layer on the sphere to become turbulent earlier than normal. Realizing this, Torobin and Gauvin (8) attempted to find possible correlation between free stream turbulence intensities and particle drag. In their article, published in 1960,
they state that interest in this area of particle drag was due to a major difficulty encountered in drag experiments. Investigators could not reproduce previous results with any significant degree of accuracy. In most of these cases, the effect of free stream turbulence had been neglected.

As might be suspected, the result of their work with turbulence showed that "increasing intensities cause a systematic regression of the transition region of the drag coefficient curve towards lower Reynolds numbers, together with a moderate increase of the drag coefficients for both the subcritical and supercritical Reynolds numbers" (8). Figure 2 shows the range of this transition region described by Torobin and Gauvin.

A study of the displacement and shattering of particles was written by Ranin, et al. (9) in 1960. In their experiment, a shock tube was used to determine the particle break-up characteristics and position data. The results showed that the drag coefficient was reasonably close to that of steady state for small particles, but particles with larger diameters had a significant increase in drag. It was shown pictorially that these larger particles were deforming into discs. This deformation began to occur at sphere diameters greater than one hundred microns. The resulting drag coefficients for the larger particles were shown to approach the coefficients for that of discs of the same frontal area. This fact was noted in the article, but is overlooked by many who quote his data as a comparison for drag coefficients.

The effect of combustion and acceleration of the drag coefficient was dealt with by Crowe, Nicholls, and Morrison in 1963 (10). Their
experiment consisted of burning particles (gunpowder) and non-burning particles flowing in a shock tube. For their experiment they state that the rate of burning did not produce any significant change in drag coefficients of the burning particles. The same was true of the effect of acceleration upon drag, where little change was noted. At the conclusion of their article, they compare their data and results with other prominent investigators in the field of drag coefficients. The work of Ingebo and of Torobin and Gauvin agreed reasonably with their conclusive position, while Rabin, et al., was stated to have been in some error. The authors fail to realized, as has been pointed out previously, that much of Rabin's data is for deformed spheres which have a larger drag coefficient than the standard drag.

Rudinger (11), in 1963, reported some additional experiments with clouds of particles in a shock tube. His data and corresponding analysis give a resulting drag coefficient of:

\[ C_d = 6000 \text{ Re}^{-1.7} \]

A plot of this equation appears with that of others in Figure 2.

A very recent article, written in 1968 by Selburg and Nicholls (12), discusses experimental work with burning and inert spheres. A shock tube was used to determine the drag coefficients of the particles under observation. In their experiment, the drag coefficients calculated were slightly higher than the standard drag curve, especially for the case of the burning spheres. In analyzing the reasons for the difference in drag over the
corresponding steady state curve for smooth spheres, the authors concluded by photographic examination that the particles have rough surfaces. On this point an assumption was made that quite possibly the surfaces on particles in rocket motors have a roughness, also causing an increase in expected drag of the particles. This would be advantageous, because the particles would then not lag the accelerating gas stream to the same degree as would smooth spheres.

During the search of the literature for significant articles on particle drag, two survey articles were found to be very useful. Hoglund (13) wrote of recent advances in gas-particle nozzle flow. His article was published in 1962 and covers the period until approximately that time. The article is quite generally referenced by other more recent articles on the subject. In 1965, S. L. Soo (4) wrote about the history and applications of solid-gas flow. In this publication, the articles referenced and discussed by Soo are "intended to be illustrative rather than comprehensive" (4).

This completes the literature review section of particle drag. The remainder of this chapter will be devoted to the recommendation of a drag coefficient suitable to air-augmented combustion and mixing.

Though the problem of a suitable drag coefficient has been an old one, it still remains a formidable one. The decision to accept any drag coefficient is dependent upon the range of Reynolds numbers encountered. In addition, items such as the turbulence and burning characteristics of the gas and particles play a significant role in determining the best coefficient.
The first important matter is then to find the range of Reynolds numbers. In the scientific literature, there exist several one-dimensional models of gas-particle flow systems. These may be used to predict the velocity lag and corresponding Reynolds number between the particle and the gas stream. One computer program was used which involved an analysis of rocket nozzle performances with solid particles present in the exhaust (14). This program shows that for a maximum lag between the particles and the gas, the relative Reynolds number is about equal to twenty-four. This is then a rough number to work with in evaluating a suitable drag coefficient.

Traditionally, in the field of rocket propulsion, the drag coefficient used for liquid droplets is that of Ingebo (7). The steady state drag curve and the drag formula of Ingebo follow almost exactly at very low Reynolds numbers, which is the range in our problem according to the previous discussion.

Researchers in the field compare and discuss variations between their own individual results and the steady state drag curve. An inherent conclusion is that the steady state drag curve is still considered as the valid drag coefficient. Because of the closeness of the steady state curve to that of Ingebo, and the need for a formula which can be calculated, the choice of drag formula for our model is that of Ingebo, \( C_d = 27 \, \text{Re}_D^{-0.84} \). A close examination of Figure 2 will bear out this conclusion, noting the range of Reynolds numbers up to approximately twenty-five.
CHAPTER III

THERMAL RESPONSE

In the analysis of the air-augmented model, temperature response of the particles is important. In order for the particles to burn they must first heat up, then ignite and continue to burn. The ignition and combustion processes are not covered in this thesis; only a consideration of the heating period will be analyzed.

At the beginning of the section being analyzed, the fuel-rich, gas-particle stream starts to mix with the oxidizer (the secondary stream). As the fuel-rich gas mixes with the oxidizer itcombusts, raising the temperature of the gas mixture. Because of the particle mass and associated heat capacity, they will not change temperature as rapidly as the surrounding gas and, therefore, exhibit a temperature difference, or, as commonly referred to, a thermal lag. This is analogous to the velocity lag caused by the drag, previously discussed in this thesis. Drag involved a transfer of momentum due to a velocity difference while the heat transfer involves a transfer of energy due to a thermal difference. It will be shown later how the similarity between the two compares.

Similar to the non-dimensionalized drag coefficient, $C_d$, the heat
transfer to a particle is usually characterized by a dimensionless number called the Nusselt number, $N_u$. For the particular case of the heat transfer to a sphere, the Nusselt number is defined as the following:

$$N_u = \frac{h_c D}{k_f}$$

where

- $h_c =$ The convective heat transfer coefficient, $\dot{q} / A (T_s - T_f)$
- $D =$ The diameter of the sphere.
- $k_f =$ The thermal conductivity of the surrounding fluid.
- $\dot{q} =$ The rate of heat transfer, BTU / Hr.
- $A =$ The surface area of the sphere.
- $T_s =$ The surface temperature of the sphere.
- $T_f =$ The temperature of the surrounding fluid.

Because many different convective heat transfer coefficients exist, it is noted that this coefficient is based upon the difference in temperature between the instantaneous temperature of the sphere, taken as a lumped thermal capacitance, and the bulk temperature of the surrounding fluid. The heat transfer will vary as the temperature of the sphere varies.

If the fluid is considered as infinite and at rest, an analysis similar to the drag calculation for Stokes flow gives a value for the Nusselt number equal to 2.0 (15). The Nusselt number equal to 2.0 is sometimes referred to as Nusselt number based upon Stokes flow. Just as for the drag coefficient in Stokes flow, the heat transfer coefficient is accurate only for very
One of the early correlations of heat transfer data was made by Froessling (16) and was later modified by McAdams (17) to give an empirical relationship between the Nusselt number, Reynolds number, and Prandtl number for laminar flow. The correlations were made so that for very low Reynolds numbers the Nusselt number approached 2.0, just as theory predicted. The following represents the correlation of McAdams:

\[ \text{Nu} = 2.0 + 0.6 \text{Re}^{0.5} \text{Pr}^{0.33} \]

where; \( \text{Re} \) = The Reynolds number based upon sphere diameter.

\( \text{Pr} \) - The Prandtl number.

Hsu and Sage (18), in 1957, and later Sato and Sage (19), in 1958, ran experiments to determine heat transfer to silver spheres behind a grid and also in the wake of a free jet. They stated that turbulence intensity has a marked effect upon the rate of heat transfer at Reynolds numbers greater than about four thousand. The results in their articles showed as much as a two-fold increase for turbulence intensities of 15 per cent.

A very recent article, written in 1967 by Lavender and Pei (20), adds significantly to the field of heat transfer to a sphere in a turbulent flow field. In their work an experimental apparatus was built to correlate and predict heat transfer rates from spheres in turbulent flow fields.

In an effort to include the effects of all possible variables, a great
number of physical measurements were made in their experiments. The turbulence intensity was found to be a domineering factor in considering the rate of heat transfer from spheres. They analyzed the data from which developed an equation which predicted the heat transfer for their own experiments.

In forming the formula for correlation, the authors started with the basic equation for laminar flow of McAdams (17):

\[ N_u = 2.0 + 0.6 \, Re^{0.5} \, Pr^{0.33} \]

To the Reynolds number the authors added a turbulence factor, the turbulence intensity, to get an equation in the following form:

\[ N_u = 2.0 + A \, Re^{0.5} \, Pr^{0.33} \, Re_T^{B} \]

where: A and B are constants to be fitted to the data.

\[ Re_T = \text{The turbulent Reynolds number, } Re, \times I_T. \]

\[ I_T = \text{The turbulence intensity, } (\frac{V_i^2}{V})^{1/2}/\bar{V}. \]

\[ V_i = \text{The instantaneous velocity of the stream.} \]

\[ \bar{V} = \text{The average of the instantaneous velocity.} \]

Just as Torobin and Gauvin (8) found that the transition from laminar to turbulent flow affected the drag quite drastically, Lavender and Pei found that the heat transfer was also affected by this transition. Two sets of values for A and B had to be used, one for the heat transfer before transition
and the other set after transition. A plot of the resulting equations and the data, in their article, showed that the prediction of heat transfer had a range of error of ±10 per cent.

During these tests made by Lavender and Pei, the Prandtl number was very close to constant in value. Therefore, the authors included this value into the resulting coefficients, A and B. If the missing Prandtl number were to be reinstated into the equation, the resulting values for A and B were:

\[ A = 0.717 \quad B = 0.035 \quad R_{e_T} < 1000 \]
\[ A = 0.166 \quad B = 0.225 \quad R_{e_T} > 1000 \]

An important comparison between the drag coefficient and the Nusselt number can be made at low Reynolds numbers. If, in each case, the actual coefficients can be divided by the Stokes coefficients, the result would give a comparison of the two modes of transfer, energy and momentum. The results of just such a comparison appear in Figure 3.

In observing the plot of these two ratios, a definite similarity is seen to exist. This similarity does not continue at higher values of Reynolds number, however, because the mechanics of the two modes of transfer change appreciably.

In a discussion of the heat transfer to a sphere, there exists not only the convective heat transfer to consider, but also the internal temperature distribution with its associated heat transfer analysis.

In most articles reviewed, the surface and center temperatures
Fig. 3. -- Similarity between drag coefficient and Nusselt number
were usually treated as being equal. This treatment was usually referred to as a lumped capacitance model, whereas the entire mass of the particle was considered to be at one temperature only, at any instant in time.

The following mathematical analysis was made to discover the validity of the assumption of a lumped thermal capacitance. For aluminum particles with a diameter equal to $10 \mu m$ and a thermal diffusivity of $1.6 \text{ cm}^2/\text{sec}$, the time required for the center of the particle to reach 99 per cent of the surface temperature with a step increase in temperature at the surface was $8.15 \times 10^{-8}$ seconds.

Channapragada, et al. (21), reported that in their theoretical model of an air-augmented rocket motor, the mean residence time was $1 \times 10^{-4}$ seconds. In considering the shortness of time required for the particles to assume a center temperature the same as that temperature at the surface, the assumption of a lumped thermal capacitance is considered a valid one.
CHAPTER IV

PARTICLE DIFFUSION

Particle diffusion in a turbulent gas stream is one of the important topics related to particle dynamics. Considering the system of two coaxial jet streams, particles which emerge from the center or primary stream will spread or diffuse into the outer or secondary stream as they progress down the duct length. The amount of spread will determine various concentrations as a function of position. It will also determine the amount of oxygen available to the burning particles. This analysis, then, will affect the necessary duct length required for the combustion of the particles.

The diffusion of particles is accomplished through turbulent mixing. In regard to the phenomena of turbulence, little is known of the causes while much is being learned about the effects. Turbulence has usually, in times past, been treated as a characteristic randomness associated with the fluid velocity and direction. It greatly enhances any mixing process, such as the one being considered.

In the gas-particle literature the diffusion of particles is usually treated as a function of the diffusion of the accompanying gas. The most common portrayal of particle diffusivity is in the form of a ratio of the
diffusivity of the particles to that of the gas. If the particles follow the gas exactly in its diffusion, then the ratio of each diffusivity is one. If, however, the particles exhibit a characteristic lag, due to the drag and inertia of the particle mass, then the ratio may ultimately approach zero. A mathematical expression of the preceding is as follows:

\[ 0 < \frac{\varepsilon_p}{\varepsilon_g} < 1 \]

where: \( \varepsilon_p \) = the eddy diffusivity of the particles, and
\( \varepsilon_g \) = the eddy diffusivity of the gas.

The variables affecting the value of the ratio should include the same variables which affect the drag coefficient of the particles. Some of these include: the size of the particles, the density of both the gas and the particles, the viscosity of the gas stream, and the size and frequency of the fluctuations in the turbulent gas stream.

One of the earlier treatments of the diffusion of particles in a turbulent gas stream was that of Longwell and Weis (22). Their article, published in 1953, makes an approximation to the velocity fluctuations by assuming a sinusoidal fluctuation of gas velocity. The drag of the particle is assumed to be given by Stokes drag law. Without going into the theory of the approximation here, the result of Longwell and Weis shows that the ratio of the eddy diffusivity of the drop, \( \varepsilon_p \), to that of the gas, \( \varepsilon_g \), is equal to:
\[
\frac{\varepsilon_p}{\varepsilon_g} = \frac{b^2}{(w^2 + b^2)}
\]

where: 
\(b = 3 \times \mu \times \text{dia. of particle} / \text{mass of particle}.
\)

\(w = \) The frequency of oscillations of the particles and the gas.

\(\mu = \) The viscosity of the gas.

The article illustrates the use of this equation by assuming a kerosene drop diameter of 45 microns flowing in a fully developed air stream with a velocity of 300 feet per second. The frequency is calculated, for a six-inch diameter duct, as equal to approximately 300 radians per second. With all of the above parameters calculated, the ratio of diffusivities turns out to be equal to 0.35. Later, calculations using this technique will be shown which cover the range of practical importance to the particular air-augmented model under study.

Regarding the diffusivity of the particles, Longwell and Weis discovered that there is not any "substantial change of diffusivity with a fourteen-fold change in absolute pressure" and that the "Reynolds number is not a proper correlating function for eddy diffusivity" (22).

Soo, Ihrig, and Kouh (23) wrote, in 1960, about turbulent two-phase motion. They experimented with the diffusion of solid particles in a duct of three-square-inch cross section. In their work, the velocities ranged from twenty to one hundred feet per second. For particles of the size of 100 and 200 microns in diameter, and at the larger velocities
encountered, the ratio of particle eddy diffusivity to gas eddy diffusivity was found to be approximately equal to 0.02. The relatively low value was the result of large inertias of the particles which tend to make the particles follow the bulk flow of the stream rather than the turbulent fluctuations present. One of the results of the test was noted in that for particle loadings of up to 0.06 pounds of particles per pound of gas the particles have little or no effect upon the flow of the gas stream. The gas then behaves as though the particles were not present at all.

One of the more recent articles was written in 1966 by Goldschmidt and Eskinazi (24). Their article dealt with the diffusion of solid particles in a plane jet. At first it seemed to be a significant contribution to the problem, but proved of little value because of the range of variables in the test. The particles used had an average diameter of three microns and the velocity of the gas at the greatest value was approximately 35 feet per second. Because of the relatively low velocity and small particle diameters, the particles tended to follow the stream exactly during the diffusion process. This gave a value for the ratio of eddy diffusivity of the particles to that of the gas equal to one.

Generally, in the area of particle diffusivity, very little experimental data have been found to characterize particle eddy diffusion coefficient.

As stated previously, Longwell and Weiss theorized that the ratio of the eddy diffusivity of the particles to that of the gas could be estimated, using their assumptions listed (22). Using their resulting equation given
previously, a general degree of ability to predict this ratio can be attained for an air-augmented rocket.

If a frequency of 1000 radians per second were to be impressed upon both a one- and ten-micron diameter particle, the ratio of diffusivities would be as follows:

<table>
<thead>
<tr>
<th>Size</th>
<th>$\frac{\varepsilon_p}{\varepsilon_g}$</th>
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<tr>
<td>1 $\mu$</td>
<td>0.735</td>
</tr>
<tr>
<td>10 $\mu$</td>
<td>0.004</td>
</tr>
</tbody>
</table>

The ratio, $\varepsilon_p/\varepsilon_g$, is approximately inversely proportional to the fourth power of the diameter, by the analysis of Longwell and Weiss. The trend only can be predicted and proven technique presently exists for predicting the particle eddy diffusivity with random turbulence. No experimental data are available which encompass the range of Reynolds numbers in this thesis.

For confined jet solutions, where $p$ is a required parameter, it is recommended that the ratio of $\varepsilon_p/\varepsilon_g$ be varied between zero and unity until optimum agreement with measured system parameters is obtained. If this proves to be too large a task, the method of Longwell and Weiss can be used to estimate $\varepsilon_p/\varepsilon_g$. In using this method, the frequency could be found on the assumption that the largest eddy of concern will be equal in size to the primary flow channel diameter. Combining with the corresponding velocity of the jet will then produce a frequency required in the
calculation of diffusivity.

In conclusion, work is now only being undertaken (25) and may at some later date contribute to the prediction of the eddy diffusivity of particles in the spreading of particles entering from one duct and diffusing into another confined duct.
CHAPTER V

SIMILARITY AND CONTINUUM

Similarity in particle dynamics involves the act of non-dimensionalizing important physical variables so that items such as velocity versus position or concentration versus position can be plotted on one graph. The plot would consist of only one line or curve, representing the entire range of interest, such as position. This has been shown to be possible already in the gas phase flow, where velocity and position appear as one plot. It is desired to find out, then, if the behavior of the particles allows them to be non-dimensionalized and shown as one common graph.

The continuum assumption has been accepted by many of the articles reviewed in this thesis. The property of continuum is that the particles act as though they were one body, evenly dispersed and continuous. The previous discussion on particle drag and particle heat transfer is dependent upon the continuum assumption being true.

With this brief introduction, similarity and continuum will be treated in the pages following. First, similarity will be discussed.

In 1966, Goldschmidt and Eskinazi (24) ran tests with the object of predicting the spread of two-phase flow in turbulent jets. In their tests the
liquid phase was composed of Safflower oil because of its low rate of evaporation under the particular conditions of the test. Of concern to the present analysis was the fact that they were able to develop a similarity profile for the velocity of the particles as well as for the gas. Several plots are made of velocity, position, and concentration profiles. As stated earlier in the section on diffusion, the velocity of the gas stream and the corresponding small size of the particles caused the particles to follow the gas stream exactly, during every turbulent fluctuation. Since there already exist plots of gas phase, the plots of the particle velocity and concentration profiles come as no great surprise.

The only conclusion possible, concerning the article by Goldschmidt and Eskinazi (24), is that although similarity profiles are valuable in air-augmented combustion, their article is of little value because of the fact that the profiles of the particles are exactly the same as that of the gas. The low velocity and small size of the particles involved are not comparable to air-augmentation.

In searching the experimental literature, very little was found on the subject dealing with particle similarity. A theoretical model, however, was found which discussed the very topic in question. Channapragada, et al. (26), wrote in 1967 about air-augmented rocket motors. In their theoretical treatment of particle similarity they stated:

It is observed from Figures 7-9 that the gas velocity and temperature distribution profiles are dissimilar from the near to the far field. It is interesting to note that the particle velocity lag and loading
distribution vary appreciably at different axial locations, indicating that the dissimilar nature of the flow field is very important on particle properties but not on the gas properties (26).

In the absence of any other sources, namely those involving experimental data, the conclusion reached is simple, and follows that of Channapragada, et al. (26). Due to the uneven particle loading and the velocity lag variation across the section of the diameter of the duct, there exist no similarity profiles of velocity, temperature, or concentration distribution, unless the particles are so small or velocities low enough to permit the particles to flow exactly as the gas.

Continuum is assumed by many authorities at present because one of the following two conditions may exist: (a) the flow, in actuality, may be a continuum, or (b) for the case of simplification, the flow is close to a continuum and is therefore assumed as being so.

Carlson and Hoglund (27) wrote a theoretical treatment of the particle heat transfer and drag in rocket nozzles in 1964. In their article, a computer program was used to evaluate the continuum flow assumption. The conditions present were typical of a rocket nozzle. The nozzle evaluated was described as one having "condensed-phase particles (ranging from 1 to 10 μ) in the nozzle of a small (1000 lb thrust) rocket motor operating at 400 psia" (27). They showed four regions of flow present in their analysis: first, there exists the continuum flow; second, the slip flow regime; third, the transition flow; and fourth, the free molecule flow. In their analysis the model never reaches the free molecule regime. For their given
diameter particles, the model reached the slip flow regime at the throat and moved into the transition regime in the diverging section of the nozzle. Considering the conditions for an air-augmented rocket leads to the conclusion that our assuming that continuum does indeed exist is true.

In order for the present discussion to be more exact, an analysis was made to find out in precise terms the physical parameters constituting the continuum flow regime. The following is a description of the analysis.

The basic requirement for a continuum is that the minimum length of interest, 1, must be greater than the average distance between particles, b. Assuming that the particles are evenly spaced spheres, the distance b is given by the following equation:

$$b = \frac{d_p}{2} \left( \frac{4 \rho_p}{3 m_r \rho_g} \right)^{\frac{3}{2}}$$

where: $m_r =$ Mass fraction of particle to gas.

$\rho_p =$ Density of particle material.

$\rho_g =$ Density of gas.

$d_p =$ Diameter of particle.

An example of the preceding equation and its usefulness is the following. For a gas at $3000^\circ$R and 50 psia containing aluminum particles, the plot of mass ratio versus mean distance between particles is plotted in Figure 4. The mean distance between particles is the quantity referred to as b, and should be much less than the minimum length of interest, 1, in order for
Fig. 4.--Mass ratio versus mean distance between particles
the continuum assumption to hold true.

The minimum length of interest is then graphically represented in Figure 4. This length is the Prandtl mixing length given by Schlicting (2). The mixing length of Prandtl is a parameter used to describe the diffusion process of either a gas or a system of particles in a turbulent stream.

Mixing lengths may be modeled in the continuing development of a mathematical model. This analysis, then, makes it possible to calculate the continuum criteria in a solid-gas mixture.
CHAPTER VI

PRESENT MODELS

This chapter deals with the present models available, including their important simplifying assumptions. The only areas discussed will be those concerned in the analysis of the particle flow. Because of the complications involved in modeling an air-augmented combusting jet, all of the articles reported used simplifying assumptions.

Emmons (28) in 1965 wrote of a theoretical analysis of the mixing of air with a reacting gas-particle jet. The effect of the particles, however, was not considered. Emmons assumed that the particles were in both thermal and dynamic equilibrium with the gas stream. The paper does not predict the diffusion of particles, but assumes that they diffuse at the same rate as the gas. The article, though not disclosing any experimental work dealing with the gas-particle interaction, does state that this information exists in the classified literature.

A paper describing air-augmented solid propellant mixing was written in 1967 by Channpragada, et al. (26). In their analysis, the particles were assumed to have no velocity or thermal lags at the start of mixing. Actually, the entrance section should probably have the largest lags
of any location. This assumption, however, greatly simplifies the model. The temperature response of the particles is calculated using a general Nusselt number. Theoretically, according to the analysis, the temperature response should follow an exponential curve. The analysis assumes that the Nusselt number, density, and thermal conductivity remain constant over the range of conditions present. The particle cloud is treated as a continuum, which agrees with many articles read and our own previous conclusion.

A recent article, written in 1967 by Midgal and Agosta (29), does an excellent job of developing equations for the model. The article states concerning the relations between the gas and particles (drag, Nusselt number, etc.) that, "It is not the intent here to recommend the best form of these particle constitutive equations." This article, then, provides no additional information for this thesis.
CONCLUSIONS AND RECOMMENDATIONS

The preceding chapters have discussed, in detail, various important aspects of particle dynamics. These were specifically related to air-augmentation of a particle-carrying gas stream.

It has been shown that, considering the range of relative Reynolds numbers between the gas and the particles, the closest equation following the standard drag curve was that of Ingebo, \( C_d = \frac{27}{R_e^{0.84}} \). For any relation requiring the force between the particles and the gas, it is recommended that Ingebo's drag formula be used.

It is recommended that the heat transfer to a sphere in a turbulent gas stream use the relations of Lavender and Pei (20). This heat transfer relation, in terms of the Nusselt number, is

\[
N_u = 2.0 + A R_e^{0.5} P_r^{0.33} R_e T^B
\]

where the definition and explanation of terms appear in Chapter III, Thermal Response.

Predicting the particle diffusivity is a formidable problem. The best estimate, barring experimental data, is one using Longwell and Weiss'
(22) formula which appears in Chapter IV, Particle Diffusion. This technique for determining the diffusion of the particles should be used only if the diffusion proves to be very significant and all other efforts fail, for it will only give a range of probable diffusion rates.

The present models, which cover air-augmentation of a particle-carrying gas stream, fail to treat in any significant detail any of the aspects of particle dynamics. This was pointed out in Chapter VI, Present Models.

In conclusion, there is sufficient information on transport properties to recommend preliminary values for computation. However, future work is desirable to expand the knowledge available.
PARTICLE DYNAMICS IN THE MIXING OF A PARTICLE-LADEN STREAM WITH A SECONDARY STREAM IN A DUCT

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M. S. Degree, June 1970

ABSTRACT

The purpose of this thesis was to discuss all phases of particle behavior involved in the mixing of the two streams in an air-augmented solid propellant rocket.

Specific recommendations for particle drag coefficient, Nusselt number, eddy diffusivity coefficient, along with the reasons for the author's choices, were given. Also discussed were the similarity profiles of velocity and temperature distributions. The assumption of a particle-gas continuum was analyzed for applicability to the air-augmented combustion problem.

The thesis represented an in-depth literature search and analysis of any experimental work which would help predict the mixing process.

COMMITTEE APPROVAL:
LIST OF REFERENCES

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BIBLIOGRAPHY
LIST OF REFERENCES


23. Channapragada, R. S.; Anderson, R.; Duvvuri, T.; and Gopalakrishnan, A. "Mixing, Ignition, and Combustion Analysis of Air Augmented


BIBLIOGRAPHY


Grouse, S. W., Jr. "An Introduction to Two-Phase Gas-Liquid Flow," DRS 8734-3m, Massachusetts Institute of Technology, June, 1964. AD 603 659.


Rudinger, George. Private Communication, June 1968.


