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Thermal Transport at Superhydrophobic Surfaces in Impinging Liquid Jets, Natural Convection, and Pool Boiling

Matthew Clark Searle

A dissertation submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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ABSTRACT

Thermal Transport at Superhydrophobic Surfaces in Impinging Liquid Jets, Natural Convection, and Pool Boiling

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This dissertation focuses on the effects of superhydrophobic (SHPo) surfaces on thermal transport. The work is divided into two main categories: thermal transport without phase change and thermal transport with phase change. Thermal transport without phase change is the topic of four stand-alone chapters. Three address jet impingement at SHPo surfaces and the fourth considers natural convection at a vertical, SHPo wall. Thermal transport with phase change is the topic of a single stand-alone chapter exploring pool boiling at SHPo surfaces.

Two chapters examining jet impingement present analytical models for thermal transport; one considered an isothermal wall and the other considered an isoflux wall. The chapter considering the isothermal scenario has been archivally published. Conclusions are presented for both models. The models indicated that the Nusselt number decreased dramatically as the temperature jump length increased. Further, the influence of radial position, jet Reynolds number, Prandtl number and isoflux versus isothermal heating become negligible as temperature jump length increased. The final chapter concerning jet impingement reports an experimental exploration of jet impingement at post patterned SHPo surfaces with varying microfeature pitch and cavity fraction. The empirical results show a decrease in Nusselt number relative to smooth hydrophobic surfaces for small pitch and cavity fraction and the isoflux model agrees well with this data when the ratio of temperature jump length to slip length is 3.1. At larger pitch and cavity fractions, the empirical results have higher Nusselt numbers than the SHPo surfaces with small pitch and cavity fraction but remain smaller than the smooth hydrophobic surface. We attribute this to the influence of small wetting regions. The chapter addressing natural convection presents an analytical model for buoyant flow at a vertical SHPo surface. The Nusselt number decreased dramatically as temperature jump length increased, with greater decrease occurring near the lower edge and at higher Rayleigh number.

Thermal transport with phase change is the topic of the final stand-alone chapter concerning pool boiling, which has been archivally published. Surface heat flux as a function of surface superheat was reported for SHPo surfaces with rib and post patterning at varying microfeature pitch, cavity fraction, and microfeature height. Nucleate boiling is more suppressed on post patterned surfaces than rib patterned surfaces. At rib patterned surfaces, transition superheat decreases as cavity fraction increases. Increasing microfeature height modestly increases the transition superheat. Once stable film boiling is achieved, changes in surface microstructure negligibly influence thermal transport.

Keywords: superhydrophobic, thermal transport, jet impingement, pool boiling, natural convection
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4.10 Images of liquid jet impingement at $Re = 1.1 \times 10^4$ on a $w = 16 \mu m$ and $F_c = 0.85$ surface. Panel (a): impingement at a 4 mm target. Panel (b): impingement at a 6 mm target. Wetting is apparent in panel (a) and is illustrated schematically below the image. Wetting is less prevalent in panel (b) and the proposed partial wetting phenomenon is illustrated schematically below the image.

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5.6 Non-dimensional temperature profiles, $\hat{\theta}$, as a function of $\delta Ra^{1/4}$. Panels (a) and (b) display results at $Ra_L = 5 \times 10^6$ and $\hat{\lambda}_T = 0, 5 \times 10^{-3}$, and $10 \times 10^{-3}$. Panels (c) and (d) display results at $\hat{\lambda}_T = 5 \times 10^{-3}$ and $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$. Panels (a) and (c) display results at $\hat{y} = 0.5$. Panels (b) and (d) display results at $\hat{y} = 1$.

5.7 Local Nusselt number, $Nu_L$, as a function of $\hat{\delta} Ra^{1/4}$. Panels (a) and (b) display results at $Ra_L = 0, 5 \times 10^{-3}$, and $10 \times 10^{-3}$ and $Ra_L = 5 \times 10^6$, and $1 \times 10^9$.

5.8 The relative decrease (percentage) in the average Nusselt Number, $(\overline{Nu}_{L,0} - \overline{Nu}_L)/\overline{Nu}_{L,0}$, plotted as a function of $\hat{\lambda}_T$ at $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$.

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6.1 The qualitative boiling curve for liquid water undergoing pool boiling at atmospheric pressure on a heated classical surface is displayed at HPo and SHPo surfaces. Heat flux, $q''_w$, is plotted as a function of surface superheat, $\Delta T$. Labels A–C indicate the onset of nucleate boiling, the critical heat flux, and the Leidenfrost point for the HPo curve. The critical heat flux and Leidenfrost point at the SHPo surface are indicated by B’ and C’. The span of the nucleate, transition, and film boiling regimes are also indicated.

6.2 Panel (a) shows the solid-liquid contact angle, $\theta$, on a hydrophilic (HPi), hydrophobic (HPo), and superhydrophobic (SHPo) surface. Panel (b) displays a schematic of water resting on a microstructured surface in the wetted (Wenzel) and non-wetted (Cassie) states.

6.3 Electron micrographs are displayed of rib and post patterned surfaces acquired using SEM. Dimensions which define the geometries are shown for both rib and post geometries where $w$ is the feature pitch, $h$ is the feature height, $d$ is the cylindrical post diameter, and $w_c$ is the cavity width on rib patterned surfaces.

6.4 Schematic illustration of pool boiling experimental setup indicating heaters, instrumentation, and insulation.

6.5 $q''_w$ as a function of $\Delta T$ for a HPo surface (panel (a)) and SHPo surface with $F_c = 0.8$, $w = 40 \ \mu m$, and $h = 15 \ \mu m$ (panel (b)). These results are compared to results from prior investigators.
6.6 Two image sequences are shown as obtained with a high speed camera acquiring images at 1000 fps (images are shown at 20 ms increments). The same rib patterned SHPo surface ($F_c = 0.8, w = 40 \mu m, \text{ and } h = 15 \mu m$) is considered in both sequences with the ribs oriented horizontally in the images. The first series (panel (a)) was obtained with the water undergoing nucleate boiling ($\Delta T = 2 \text{ deg K}$ for which $q''_w = 5 \times 10^3 \text{ W/m}^2$). The second series (panel (b)) was obtained with the liquid undergoing film boiling ($\Delta T = 22 \text{ deg K}$ for which $q''_w = 2 \times 10^3 \text{ W/m}^2$). This video is included as Supplemental Video 1.

6.7 $q''_w$ is plotted as a function of $\Delta T$ for rib patterned SHPo surfaces ($h = 15 \mu m, w = 40 \mu m$) with $F_c$ ranging through 0.5, 0.8, and 0.93.

6.8 $q''_w$ is plotted as a function of $\Delta T$ at $w = 16 \mu m$ for post patterned surfaces ($h = 15 \mu m$). In panel (a), $F_c$ varies through 0.56, 0.85, and 0.98. In panel (b), $F_c = 0.85$ and $w$ varies through 8, 16, and 24 $\mu m$. The insets plot the data at surface superheats between 0 to 25 deg K on a magnified scale.

6.9 High speed images of pool boiling at post patterned surfaces ($h = 15 \mu m, w = 16 \mu m$). $F_c$ varies through 0.56, 0.85, and 0.98.

6.10 High speed images for $\Delta T$ at $F_c = 0.85$ for post patterned surfaces ($h = 15 \mu m$). $w$ varies through 8, 16, and 24 $\mu m$. Arrows indicate the locations where the film remains in contact with the microfeatures (solid-liquid contact) and locations where a vapor film is present. This video is included as Supplemental Video 3.

6.11 $q''_w$ as a function of $\Delta T$ for rib patterned and post patterned surfaces. ($F_c = 0.85, \text{ pitch } w = 24 \mu m, \text{ and } h = 15 \mu m$)

6.12 High speed images for varying $\Delta T$ and for rib patterned and post patterned surfaces ($F_c = 0.85, \text{ pitch } w = 24 \mu m, \text{ and } h = 15 \mu m$). The ribs are oriented horizontally for the rib patterned surface. This video is included as Supplemental Video 4.

6.13 $q''_w$ as a function of $\Delta T$ for rib patterned SHPo surfaces ($F_c = 0.8, w = 40 \mu m$) for $h = 4.2 \mu m$ and $h = 15 \mu m$.

B.1 The design of the screen-printed resistance heater screen is shown.
CHAPTER 1. INTRODUCTION

Interactions between liquids and surfaces arise in many engineering applications. The use of micro and nanoscale fabrication to tailor surface properties to optimize or alter macroscale flows has opened a new field of surface engineering with broad applications in thermal management, energy, and bioengineering. This dissertation contributes to this growing field by reporting results concerning thermal transport at superhydrophobic (SHPo) surfaces.

The introduction defines SHPo surfaces and then reviews classical flows in which heat transfer at SHPo surfaces is considered. Then, a literature review surveys the current understanding of thermal transport at SHPo surfaces and explains how this dissertation fills current voids.

1.1 Background

1.1.1 Superhydrophobic (SHPo) Surfaces

SHPo surfaces are composed of hydrophobic chemistry and microscale roughness, nanoscale roughness, or a combination of both. One method of characterizing the aggregate solid/liquid interaction at these surfaces is measuring the sessile droplet contact angle. A larger contact angle indicates less adhesion between the water and the sample. In Figure 1.1, regimes of surface behavior are introduced. Hydrophilic (HPi) surfaces have contact angles less than 90°. Hydrophobic (HPo) surfaces have contact angles between 90° and 120°. SHPo surfaces have contact angles between 150° and 180°, where the lower bound of 150° is the de facto standard in the literature [1,2]. As stated earlier, the upper bound on contact angles for smooth HPo surfaces is 120°. Here, the contact angle exists due to HPo chemistry alone. The introduction of surface roughness allows the contact angle to exceed 120° and, in general, surfaces may be fabricated where sessile droplets have contact angles between 120° and 150°. However, such surfaces are not considered superhydrophobic. Superhydrophilic (SHPi) surfaces (not shown here) have contact angles near 0°. These
surfaces have similar micro/nanostructure as SHPo surfaces but have hydrophilic (HPi) chemistry instead of HPo chemistry. While SHPo surfaces may be realized using randomly-patterned surfaces, specifically-designed microfeatures patterned with photolithography are utilized in this dissertation to determine the influence of microstructure geometry. Scanning electron microscope (SEM) images of rib and post patterned SHPo surfaces considered in this work are displayed in Fig. 1.2.

![Diagram of contact angles](image)

**Figure 1.1:** Liquid droplets placed on hydrophilic (HPi), hydrophobic (HPo), and superhydrophobic (SHPo) surfaces. Apparent sessile contact angles, $\theta$, are displayed.

![SEM images](image)

**Figure 1.2:** Scanning electron micrographs of a rib patterned (left) and post patterned surface (right).

When the liquid gauge pressure is greater than the Laplace pressure (the static pressure difference between the liquid and the gas, which arises due to the liquid-gas interface curvature),
the liquid wets the cavities between microfeatures. This is called the Wenzel state and is illustrated in the left panel of Figure 1.3. The Laplace pressure, $\Delta P$, for a given microstructure may be determined using the Young-Laplace equation

$$\Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

where $R_1$ and $R_2$ are the principal radii of curvature and $\sigma$ is the surface tension [3].

When the liquid gauge pressure is less than the Laplace pressure, the water does not wet the microfeatures between cavities. This state of gas-filled cavities is known as the Cassie state [4] and is desirable due to its drag-reducing and self-cleaning properties [5, 6]. It is illustrated in the right panel of Figure 1.3. Note that the wall boundary condition is a composite of liquid-air and liquid-solid interfaces. The Cassie state is the regime of interest in this dissertation and surfaces may be assumed to be non-wetted otherwise unless otherwise noted. The composite nature of the Cassie boundary condition leads to a radical departure in the hydrodynamics and thermal transport at SHPo surfaces from that observed at smooth, non-patterned surfaces.

To introduce the effect of a SHPo surface on hydrodynamics and thermal transport, Couette flow is illustrated in the two parallel plate channels shown in Fig. 1.4. In each panel, the bottom plate is heated and the top plate moves right at a fixed velocity, $U$. For the bottom panel, the top plate is smooth and HPo, while the bottom plate is SHPo. The velocity profile, $u(y)$, and temperature profile, $T(y)$, are shown, where $y$ is the wall-normal coordinate. For the top panel, the top and bottom plates are smooth and HPo.
Figure 1.4: Couette flow at no-slip and SHPo surfaces is illustrated. Shown in the top panel is a no-slip wall with velocity and temperature profiles. The lower wall (red) is heated and the upper wall (gray) is adiabatic. In the bottom panel, the heated smooth surface is replaced by a heated SHPo surface. The velocity profile displays a slip velocity, $u_s$, and the temperature profile displays a temperature jump, $\Delta T_w$.

First, we consider the hydrodynamics. Considering the Cassie state, shown in the right panel of Fig. 1.3, it is apparent that the no-slip boundary condition prevails at the liquid-solid interface at each microfeature, while the velocity at the liquid-air interface above each cavity need not be zero and, in fact, will match the air velocity in the cavity. Further, the shear stress at this interface is negligible [7]. The aggregate effect of the no-slip and shear-free boundary condition may be modeled as an apparent slip velocity at the wall, $u_s$, and is illustrated in the bottom panel of Fig. 1.4. This is in contrast to the top panel, where the no-slip condition prevails. The relationship, $u_s = \lambda \tau_w / \mu$, relates $u_s$ to the liquid shear stress, $\tau_w$, where $\mu$ is the liquid dynamic viscosity and $\lambda$ is the hydrodynamic slip length [8]. $\lambda$ may be approximated with a Stokes flow analysis for a given surface microstructure [8, 9]. The slip length, $\lambda$, is also illustrated with dashed lines in the lower panel of Fig. 1.4. It is the distance into the wall that the velocity profile must be extrapolated to reach the no-slip condition. In general, the slip length increases as the microfeature pitch and
the cavity fraction increase, where the pitch is the center-to-center distance between microfeatures and the cavity fraction is the projected cavity area per unit area. As the slip length increases, the shear stress at the wall decreases.

These hydrodynamic effects give rise to the desirable drag-reducing and self-cleaning properties of SHPo surfaces. Drag reduction has been demonstrated in laminar and turbulent flows [5, 10–15]. Self-cleaning properties arise as water droplets roll freely across the surfaces removing contaminants [6, 16, 17].

The composite surface also changes the wall thermal boundary condition. Again we direct the reader’s attention to the Cassie state illustrated in Fig. 1.3. Heat conducts at the liquid-solid interface on the microfeatures while negligible transport occurs through the liquid/air interface spanning the cavities [18]. The aggregate effect of the alternating heating and adiabatic interface conditions may be modeled as an apparent temperature jump, \( \Delta T_w \), at the SHPo wall as illustrated in the bottom panel of Fig. 1.4. This is in contrast to the no temperature jump condition illustrated in the top panel. \( \Delta T_w \) may be related to the surface heat flux by \( \Delta T_w = \lambda_T q''_w / k \) [19], where \( k \) is the liquid thermal conductivity and \( \lambda_T \) is the temperature jump length. \( \lambda_T \) is the distance into the wall that the temperature profile would need to be linearly extrapolated to recover the no temperature jump boundary condition. As the temperature jump length increases, the heat flux through the surface decreases.

The temperature jump length may be approximated through a diffusion-dominated analysis at the microstructure and then, using this length, the temperature jump boundary condition may be applied to macroscale flows.

At this point, we turn our attention from SHPo surfaces to classical flows at smooth surfaces. Sections 1.1.2–1.1.5 explore fluid and thermal transport for the well-known processes of jet impingement, natural convection, and pool boiling. SHPo effects for these scenarios are then introduced in Section 1.2.

### 1.1.2 Jet Impingement: Hydrodynamics

Jet impingement occurs when a fluid stream is directed at a wall. Two scenarios are considered in the literature: a planar jet and an axisymmetric jet. A planar jet occurs when liquid flows through a long, narrow slot and is characterized by two-dimensional flow in Cartesian coor-
An axisymmetric jet has a circular cross section and is characterized by two-dimensional flow in the axial and radial directions of a cylindrical coordinate system. Axisymmetric flow is the phenomenon of interest in this dissertation and is characterized by radial flow away from the impingement point and formation of either a hydraulic jump or a so-called “breakup” depending on the surface type and downstream conditions. Breakup occurs on hydrophobic or SHPo surfaces under certain conditions, but never on hydrophilic surfaces. A radial cross section of an impinging jet with a hydraulic jump is shown in Fig. 1.5 and an image of jet impingement with breakup is displayed in Fig. 1.6. Impinging jets are also categorized by confinement (if the jet has a free surface or not). This dissertation considers a non-confined (free) jet surrounded by a gaseous atmosphere.

![Diagram of impinging jet](image)

Figure 1.5: A radial cross section of an impinging jet is illustrated schematically. z-axis scaled to display thin film.

The schematic illustration in Fig. 1.5 displays a radial cross section of an impinging jet. Hydrodynamic behavior is discussed in this section and thermal behavior in Section 1.1.3. As illustrated, the circular liquid stream approaches the wall with a uniform velocity, $V$, and radius, $a$. It impacts the wall, forming a stagnation region (region I), which is characterized by uniform hydrodynamic boundary layer thickness. The end of the stagnation region occurs at a radius of
nominally 0.8 jet diameters [21]. After this point, the flow enters region II and the liquid spreads into a thin film of thickness, $h$, with a developing hydrodynamic boundary layer of thickness, $\delta$, which is indicated by a dashed line. $\delta$ grows until it equals $h$ at which the free surface velocity begins to decrease. This marks the beginning of region III. Region IV begins when the thermal boundary layer, $\delta_T$, which is indicated by a dotted line, equals $h$. It has the same hydrodynamic behavior as region III and its thermal behavior will be discussed in Section 1.1.3. The momentum in the thin film decreases due to wall shear. Eventually, enough momentum is dissipated that other forces dominate. For the case of a hydrophilic surface, or a HPo or SHPo surface with an imposed downstream depth, downstream resistance dominates and the height of the liquid film rises dramatically in a hydraulic jump [22, 23]. The abrupt change in height, illustrated in Fig. 1.5 (at the end of region IV), occurs because the presence of the downstream resistance is not communicated upstream since the upstream flow velocity is greater than that of surface waves.

In the scenario of a HPo or SHPo surface with no imposed downstream depth, surface tension forces dominate and a so-called “breakup” occurs where the flowing liquid forms filaments or breaks into droplets depending on the jet Weber number [24]. An image of this phenomena is displayed in Fig. 1.6.

Modeling axisymmetric liquid jet impingement including viscous effects requires solving the axisymmetric Navier-Stokes equations. Regions III and IV may be solved with a similarity solution while region II must be solved with an assumed profile in an integral analysis as shown by Watson [25]. Region I was neglected in Watson’s analysis, but may be approximated with classical stagnation flow [21] and has also received more careful attention in a model which utilizes an
inviscid flow analysis (including surface tension) as a boundary condition for a boundary layer analysis in the stagnation region [26].

Results from the classical hydrodynamics solution for regions II–IV are shown in Fig. 1.7. Non-dimensional film height, $h/aRe^{1/9}$, boundary layer thickness, $\delta/aRe^{1/9}$, and free surface velocity, $U/U_0$, are plotted as a function of $r/aRe^{1/9}$, where $a$ is the jet radius, $Re$ is the jet Reynolds number ($Re = Q/(\nu a)$, where $Q$ is the jet flow rate and $\nu$ is the liquid kinematic viscosity), $h$ is the film height, $\delta$ is the boundary layer thickness, $U$ is the free surface velocity, and $U_0$ is the initial jet velocity. Several observations are important to make concerning this plot. First, $\delta$ increases until it merges with $h$. This occurs because viscous effects propagate throughout the film. Second, $h$ decreases and then increases since the height of the slower moving film must increase to satisfy continuity. Finally, $U$ is unaffected by the presence of the wall until the boundary layer has propagated throughout the film. At this point, $U$ begins to decrease and does so rapidly as the radial coordinate increases.

![Figure 1.7: Scaled film height, $h/aRe^{1/9}$, boundary layer thickness, $\delta/aRe^{1/9}$, and free surface velocity, $U/U_0$, are plotted as a function of scaled radius, $r/aRe^{1/9}$. After Watson [25].](image)

This analytical analysis by Watson [25] found that the flow depended on the jet Reynolds number with the film thickness decreasing and the hydraulic jump occurring at larger radii as the jet Reynolds number increased. Experiments agreed moderately well with Watson’s analysis [27].
and numerical analyses agreed well with the experiments [28, 29]. Of note was the extension of the model to include surface tension effects in the calculation of the hydraulic jump diameter [30]. Further, Liu et al. modeled jet impingement flow in the stagnation region, which was neglected by Watson, and accounted for surface tension [26].

1.1.3 Jet Impingement: Thermal Transport

Liquid jet impingement has received considerable attention for applications in quenching and high-density electronics cooling. Thermal transport in impinging jets at classical (smooth) walls has been thoroughly studied and reviewed [21]. Theoretical studies modeled isothermal and isoflux boundary conditions [31–33]. These studies indicate that thermal transport, indicated by the Nusselt number, increases with the jet Reynolds number and with the Prandtl number. A numerical study [28] reported good agreement with experimental results [34] and the analytical model for an isoflux surface proposed by Liu et al. [32, 33].

Several thermal scenarios may be considered. First, the direction of transport is established. The heat may transfer from the wall ($T_w > T_j$) to the liquid or from the liquid to the wall ($T_j > T_w$), where $T_w$ and $T_j$ are the temperatures of the wall and the incoming jet, respectively. The first scenario is the one of interest in this dissertation. Second, the wall may be heated in an isoflux (constant heat flux) or isothermal manner. Both heating scenarios are considered: the isothermal in Ch. 2 and the isoflux in Ch. 3.

Turning our attention to Fig. 1.5 again, we consider the local thermal transport in jet impingement. The stagnation region (region I) extends to a radius of 0.79 jet diameters and is characterized by a thermal boundary layer of constant thickness. Upon reaching this radius, the thermal boundary layer, which is indicated by a dotted line, begins to develop as thermal energy diffuses throughout the spreading film. The thickness of the thermal boundary layer is labeled $\delta_T$. For liquid water, the Prandtl number, $Pr$, is nominally 7. Consequently, the kinematic viscosity is much larger than the thermal diffusivity. Thus, the hydrodynamic boundary layer develops more quickly and reaches the thickness of the thin film at a smaller radii than the thermal boundary layer. The conclusion of region III occurs when $\delta_T$ equals $h$. For some isoflux flows, as discussed in Chapter 3, the thermal boundary layer thickness does not grow quickly enough to reach the film thickness prior to the hydraulic jump. In other scenarios, as illustrated in Fig. 1.5, the thermal
boundary layer thickness does increase until it equals the film thickness and the flow enters region IV. This region is characterized by an increase in temperature throughout the film.

1.1.4 Natural Convection

Natural convection is flow induced by a body force acting on initially quiescent fluid. The force is buoyancy in many scenarios and results from differences in fluid density. When a fluid is heated non-uniformly, a non-uniform density profile is established. All fluid elements are acted upon by the gravitational force, which is proportional to the fluid density. At a heated wall, the liquid near the wall is at higher temperatures than liquid further from the wall. The fluid density decreases as it is heated and near-wall elements have lower density, thus experiencing an upward force due to buoyancy.

Laminar natural convection at a vertical, smooth, isothermal wall is a canonical solution [35] and is the scenario of interest in this dissertation. This flow is depicted schematically in Fig. 1.8. Here, a vertical plate is heated isothermally at temperature, $T_w$, and is submerged in ambient fluid with temperature, $T_\infty$, where $T_w > T_\infty$. The gravitational acceleration, $g$, is directed downward, parallel to the plate. Velocity and temperature distributions form at the wall. The flow exhibits the classical boundary layer behavior where the effect of heat conduction penetrates into the liquid and an upward flow develops near the wall due to the buoyant force. Hydrodynamic and thermal boundary layer thicknesses, $\delta$ and $\delta_T$, respectively, increase with radial location and typical velocity and temperature profiles are displayed.

With appropriate similarity variables, the two-dimensional boundary layer equations (mass, momentum, and energy) corresponding to these flows may be simplified into a system of non-linear ordinary differential equations. The first complete numerical solution of these equations was carried out by Ostrach [36]. Non-dimensional velocity, $F'(\eta)$, and temperature, $H(\eta)$, profiles are plotted as a function of the similarity variable, $\eta$, in Fig. 1.9. In panel (a), $F'(\eta)$ is plotted as a function of $\eta$. The velocity is zero at the wall, reaches a maximum between $\eta = 0.5$ and 1.5 (depending of $Pr$), and approaches zero as $\eta$ increases towards 6. The velocity decreases at all $\eta$ as $Pr$ decreases. This is expected since the viscosity is increasing relative to the thermal diffusivity causing a greater shear stress, which inhibits the flow. Panel (b) displays the wall temperature profile, $H(\eta)$, as a function of $\eta$. The temperature decreases from the wall temperature ($H(\eta) = 1$)
Figure 1.8: Schematic illustration of natural convection at a heated, vertical wall. The isothermal wall temperature, $T_w$, and ambient fluid temperature, $T_\infty$, are displayed. Further, $T_w > T_\infty$. Gravitational acceleration, $g$ is directed downward, parallel to the plate. Temperature, $T$, and velocity, $v$, profiles are displayed. Growth of the hydrodynamic, $\delta$, and thermal $\delta T$, boundary layers is shown.

to approach the quiescent liquid temperature ($H(\eta) = 0$) as $\eta$ increases. The thermal boundary layer thickness (the point at which $H(\eta) = 0.01$) decreases as $Pr$ increases because the thermal diffusivity is decreasing relative to viscosity.

The analysis shows a dependence on the local Grashof number (a ratio of buoyant force to viscous force) and the Prandtl number. The Nusselt number increases with increasing Grashof and Prandtl numbers. Further, the Nusselt number increases with the Rayleigh number, which is a product of the Grashof and Prandtl numbers.

$q''_w$ increases moderately with $\Delta T$ up to the onset of nucleate boiling (point A). At this point, $q''_w$ increases rapidly as $\Delta T$ increases. This occurs because of enhanced transport due to phase change heat transfer. This regime, named nucleate boiling, is characterized by the formation of vapor bubbles at the wall, which release when the buoyant force is sufficiently large to overcome adhesion. The left inset in Fig. 1.10 schematically illustrates nucleate boiling. As $\Delta T$ increases, the
bubble density increases and it becomes more difficult for liquid to flow to the surface. A maximum heat flux is achieved at point B. Beyond this point, $q''_w$ decreases as $\Delta T$ increases. This regime is called transition boiling and is characterized by temporal oscillations between nucleate and film boiling behavior. If the system is heat flux controlled rather than temperature controlled, a sudden, order-of-magnitude increase in surface superheat occurs when the applied heat flux exceeds the critical heat flux.
1.1.5 Pool Boiling

The classical pool boiling curve is displayed in Fig. 1.10. The curve was first discovered by Nukiyama [37] who identified this relationship between the wall heat flux, \( q''_w \), and surface superheat, \( \Delta T = T_s - T_{sat} \), where \( T_s \) is the surface temperature and \( T_{sat} \) is the liquid saturation temperature. It is observed for a wide variety of working fluids and classical surface types.

![Diagram of classical boiling curve](image)

**Figure 1.10**: The classical boiling curve is displayed, where wall heat flux, \( q''_w \), is plotted as a function of the surface superheat, \( \Delta T = T_s - T_{sat} \). Nucleate and film boiling on a hydrophilic surface are schematically illustrated.

Continuing with the temperature controlled case, the heat flux continues to decrease as \( \Delta T \) increases until point C at which a local minimum is reached. This point is called the Leidenfrost point and marks the onset of stable film boiling in which a vapor film covers the surface as illustrated schematically in the right inset in Fig. 1.10. The heat transfer is much lower due to the high thermal resistance of the vapor film. The relationship of increasing heat flux with increasing surface superheat resumes as the surface superheat increases beyond the Leidenfrost temperature.
1.2 Literature Review and Motivation

The present work builds upon prior studies and fills voids in the scientific understanding of thermal transport at SHPo surfaces. First, the influence of SHPo surfaces on the hydrodynamics and thermal transport of jet impingement is considered. Subsequently, developments in modeling natural convection at SHPo surfaces are introduced. Finally, prior empirical explorations of pool boiling at SHPo surfaces are surveyed.

1.2.1 Influence of SHPo Surfaces on Jet Impingement Flow

SHPo surfaces dramatically influence the hydrodynamics of circular liquid jet impingement [22–24, 38]. As the slip length increases on an isotropic SHPo surface, the spreading film thins and the hydraulic jump or breakup occurs at a larger radius. The influence of slip length on the spreading film is clearly observed in Fig. 1.11, which is a figure adapted from prior work [20]. Normalized velocity, $\hat{U}$, is shown as a function of normalized radial location, $\hat{r}$, in the top panel. In the bottom panel, normalized hydrodynamic boundary layer thickness, $\hat{\delta}$, and normalized film thickness, $\hat{h}$, are plotted as a function of $\hat{r}$. “^” indicates normalization of lengths by jet radius and velocity by initial jet velocity.

Results are shown at normalized slip length, $\hat{\lambda}$, varying from 0 to 0.2 and for the inviscid scenario, $\hat{\lambda} = \infty$. The influence of $\hat{r}$ on $\hat{U}$, $\hat{\delta}$, and $\hat{h}$ is the same observed for $U$, $\delta$, and $h$ in Fig. 1.7 and observations concerning these trends are not repeated here. In the figure, we observe that $\hat{U}$ increases and $\hat{\delta}$ and $\hat{h}$ decrease as $\hat{\lambda}$ increases. This is similar to the influence of increasing the jet Reynolds number. The flow becomes energized and the momentum increases. Further, $h$ decreases to satisfy continuity.

As shown previously in Fig. 1.6, when no downstream depth is imposed on HPo and SHPo surfaces, the spreading film breaks into droplets instead of forming a hydraulic jump because surface tension forces dominate. Anisotropic surfaces, such as those patterned with ribs, change the shape of the hydraulic jump from circular to oval, with the major axis oriented in the direction of the greatest slip length [23, 24].
Figure 1.11: The normalized free surface velocity, $\hat{U}$, is plotted as a function of the normalized radial location, $\hat{r}$, in the top panel. “$\hat{}$” indicates normalization of lengths by jet radius and velocity by initial jet velocity. Normalized film thickness, $\hat{h}$, and normalized hydrodynamic boundary layer thickness, $\hat{\delta}$, are plotted as a function of $\hat{r}$ in the bottom panel. Results are shown at normalized slip length, $\hat{\lambda}$, spanning from 0 to 0.2 and for the inviscid scenario, $\hat{\lambda} = \infty$. After Prince [20].

1.2.2 Influence of SHPo Surfaces on Jet Impingement Thermal Transport

No previous studies addressing the influence of SHPo surfaces on jet impingement thermal transport have been performed prior to this dissertation. However, extensive analysis of thermal transport in SHPo microchannels has been accomplished [18, 19, 39–43]. The analytical and numerical results demonstrated a dramatic decrease in heat transfer due to the introduction of the temperature jump length.

Having considered this prior work, we emphasize that Chapters 2 through 4 of this dissertation fill the void concerning jet impingement thermal transport at SHPo surfaces. Models for jet impingement at isothermal and isoflux surfaces are presented in Chapters 2 and 3, respectively, and an experimental study considering the validity of the models and implications of microcavity wetting (Cassie to Wenzel transition) is presented in Chapter 4.
1.2.3 Influence of SHPo Surfaces on Natural Convection

Prior analytical studies have considered fully-developed, natural convection in parallel plate, annular, and polygonal microchannels [44–46]. In each study, one channel wall was superhydrophobic and exhibited slip length and temperature jump length and the other channel wall had no-slip and no temperature jump. Isothermal or isoflux heating conditions were applied to the heated side, while isothermal conditions were applied at the unheated side. The scenario most relevant to this dissertation is the parallel plate channel with isothermal heating of the superhydrophobic surface [44]. It was found that the flow rate and bulk fluid temperature decreased as the temperature jump length increased. The Nusselt number, which is normalized by the non-dimensional bulk fluid temperature, was not influenced by the temperature jump length but decreased as slip length increased.

Another important scenario, the developing natural convection boundary layer (external flow) at a vertical SHPo surface had not been previously considered and a model (presented in Ch. 5) was developed as part of this dissertation.

1.2.4 Influence of SHPo Surfaces on Pool Boiling

Prior experimental work considering pool boiling at SHPo surfaces has focused on the influence of nanostructured surfaces with random features. Researchers have demonstrated that film boiling on these SHPo surfaces occurs at surface superheats an order of magnitude smaller than classical (smooth) surfaces [47,48]. This phenomenon is illustrated in Fig. 1.12 where results are shown for a HPo surface (open markers) and a SHPo surface (solid triangles) [48–50]. We first consider the hydrophobic data. $q''_w$ increases as $\Delta T$ increases for $\Delta T < 12$ deg K. The results from the different researchers show good agreement [48–50]. The data obtained by Fan et al. reaches critical heat flux at $\Delta T = 12$ deg K and then decreases to the Leidenfrost point at $\Delta T = 20$ deg K. The SHPo surface maintains film boiling for all $\Delta T$. $q''_w$ increases as $\Delta T_w$ increases and the local maximum marking critical heat flux is absent. The SHPo data then reunites with the HPo data in the film boiling regime. Prior to this dissertation, the influence of surface microstructure configuration (microfeature pitch, cavity fraction, feature height, and post versus rib patterning style) had not been considered and Chapter 6 explores the influence of these parameters.
1.3 Dissertation Organization

Chapters 2–6 of this dissertation have been published or will be published as stand-alone works in archival journals. Thus, each has its own introduction and literature review. Chapter 2 introduces an integral model for heat transfer due to jet impingement at an isothermal wall with isotropic slip. It is the first model published in the literature to consider the influence of slip and temperature jump on thermal transport in liquid jet impingement. Chapter 3 presents a model for jet impingement on an isoflux wall and is submitted for publication. Chapter 4 empirically explores jet impingement thermal transport at heated SHPo surfaces and provides empirical validation for the isoflux model introduced in Ch. 3. Chapter 5 presents a model for developing natural convection at a vertical, heated SHPo surface and is the first study to report on the influence of slip length and temperature jump length in this scenario. Chapter 6 explores the influence of surface microstructure on pool boiling at SHPo surfaces and is published. Chapter 7 discusses conclusions for this work and presents recommendations for future work.

Figure 1.12: Heat flux, $q''_w$, is displayed as a function of surface superheat, $\Delta T$ (deg K). Cases on hydrophobic surfaces (HPo) are displayed with open markers [48–50], while data for a SHPo surface is displayed with solid triangles [48].

CHAPTER 2. THERMAL TRANSPORT DUE TO LIQUID JET IMPINGEMENT ON SUPERHYDROPHOBIC SURFACES WITH ISOTROPIC SLIP

This chapter is published in the International Journal of Heat and Mass Transfer [51]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

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2.2 Abstract

This paper presents an analytical investigation of thermal transport due to a steady, laminar, axisymmetric liquid jet impinging normally on a superhydrophobic (SHPo) surface maintained at constant surface temperature. At the liquid-surface boundary of the spreading thin film, an isotropic hydrodynamic slip and temperature jump are imposed to approximate the SHPo surface boundary condition. Applying an integral analysis within the thin film results in a system of differential equations which are solved numerically to obtain local hydrodynamic and thermal boundary layer thicknesses, thin film height, and local and radially averaged heat flux. The classical smooth hydrophobic scenario with no-slip and no-temperature jump showed excellent agreement with previous differential analysis of the same problem. The influence of varying temperature jump length on the local Nusselt number was obtained over a range of Reynolds and Prandtl numbers. Increasing temperature jump length results in a dramatic decrease in the local thermal transport near the impingement point. The greatest decrease occurs at small temperature jump lengths. Further, local and average Nusselt numbers are less influenced by the Reynolds and Prandtl numbers as temperature jump length increases. Overall, variations in the temperature jump length exert much more influence than variations in the hydrodynamic slip length.
2.3 Introduction

Numerous studies have demonstrated drag reduction and self-cleaning behavior on superhydrophobic (SHPo) surfaces [6, 10–13, 52, 53]. More recently, thermal transport physics have been explored as well. For a liquid flowing over a SHPo surface, thermal transport is inhibited due to insulating air-filled cavities at the wall. This has been observed in forced internal flow [19, 40, 41, 54], natural convection in microchannels [44, 45], and Marangoni convection in droplets [55]. Boiling on SHPo surfaces is inhibited primarily due to an early transition from nucleate to film boiling resulting from the higher liquid-solid contact angle and altered surface geometry [47, 48]. Conversely, condensation heat transfer is enhanced due to increased vapor-surface contact area [56, 57]. The focus of this paper is to explore the thermal transport for a liquid flow scenario which has not received attention: a laminar, liquid, axisymmetric jet impinging normally on a SHPo surface with isotropic slip.

The hydrodynamics of a jet impinging on a no-slip surface were modeled analytically by Watson [25]. These results specifically capture the development of the hydrodynamic boundary layer within the spreading thin film, the radial variation of thin film height, and estimate the hydrodynamic jump location. The model compares well with experimental data and has been improved to include surface tension in subsequent studies [27, 30, 58].

Thermal transport on a surface due to the spreading thin film of an impinging jet has also received significant prior attention. This interest has been motivated by industrial applications including cooling of electronic systems and quenching of metals and molded plastics [33, 59]. Analytical models for scenarios of constant wall temperature [31], constant wall heat flux [32], and varying wall temperature or heat flux [60, 61] have been accomplished. Additionally, numerical [28, 62–65] and experimental studies [34, 59, 66] have been performed. In all studies, it was observed that the greatest thermal transport occurs near the impingement point and decreases asymptotically with increasing radial position. Additionally, the local and average Nusselt numbers increase with increasing Prandtl number and jet Reynolds number.

SHPo surfaces differ significantly from smooth hydrophobic or hydrophilic surfaces. They are created by combining micro/nano-scale roughness with hydrophobic surface chemistry. Due to the surface hydrophobicity, water does not wet the cavities between roughness features if the Laplace pressure is not exceeded. Random or structured grid microfeatures are commonly em-
ployed. Important geometric parameters of SHPo surfaces with repeating features are the pitch, \( w \), (the distance between microscale features) and cavity fraction, \( F_c \) (ratio of the cavity area projected onto the interface to the total interface area).

If the air-filled cavities are on the micron scale, the hydrodynamic and thermal boundary conditions at a surface can be significantly altered [8, 40]. Liquid near the wall encounters a no-slip boundary condition at the liquid-solid microfeature interface and a nearly shear-free boundary condition at the liquid-air interface between microfeatures. For macroscopic flows, it is beneficial to define an aggregate slip accounting for the alternating slip and no-slip boundary conditions to obtain a uniform boundary condition. This condition allows definition of a local slip velocity, \( u_s \), which is proportional to the wall shear stress, \( \tau_w \), and is given by, \( u_s = \lambda \tau_w / \mu \), where \( \mu \) is liquid dynamic viscosity, and \( \lambda \) is the hydrodynamic slip length [8]. \( \lambda \) is a property of the wall microstructure and can be defined in the Stokes flow regime by the surface parameters, cavity fraction and pitch [8].

The jet impingement problem on SHPo surfaces with isotropic [22] and anisotropic slip lengths [23] has previously been modeled analytically and validated experimentally [38]. Increasing \( \lambda \) at fixed Reynolds number was observed to have a similar effect on the hydrodynamics as increasing the Reynolds number. The hydrodynamic boundary layer developed more slowly, the film thickness decreased, and the hydrodynamic jump radius increased.

The composite liquid-solid and liquid-gas interface also alters the thermal boundary conditions. The liquid-solid interface yields a conventional convection boundary condition, while at the liquid-gas interface the heat flux is greatly reduced. Assuming metallic microfeatures (high thermal conductivity), the thermal conductivity of the gas filling each cavity is several orders of magnitude less than that of the microfeatures, rendering the liquid-gas interface nominally adiabatic. Again for macroscopic scale flows, an aggregate boundary condition can be defined in the form of a wall temperature jump, \( \Delta T_w \). This temperature jump is proportional to the wall heat flux, \( q''_w \), as given by the relationship, \( \Delta T_w = \lambda_T q''_w / k \) [19], where \( k \) is the liquid thermal conductivity and \( \lambda_T \) the temperature jump length. The temperature jump length is the thermal analog to the hydrodynamic slip length [19, 40, 67] and is a function of microscale feature geometry.

The relationship between \( \lambda_T \) and \( \lambda \) is an area of ongoing research. For Stokes flow between parallel plates with ribs aligned with the flow, \( \lambda_T \) is very nearly equal to \( \lambda \) [19]. A study of
thermal transport in a channel with ribs oriented perpendicular to the flow observed a more complex
relationship between $\lambda$ and $\lambda_T$ [41]. In this study, the ratio of $\lambda_T$ to $\lambda$ was shown to depend on
Peclet number, relative module width (ratio of pitch to channel hydraulic diameter), and cavity
fraction. At low Peclet number and small relative module width, the ratio of $\lambda_T$ to $\lambda$ is nominally
2. However, increasing the Peclet number to the order of 1,000 and setting the relative module
width to 1 causes the ratio to decrease.

The effect of $\lambda_T$ on heat transfer in an impinging jet has not been previously explained, but
is of interest as it represents a common cooling scenario for self-cleaning surfaces. Section 2 of
this paper presents an integral analysis to quantify the effect of isotropic slip and temperature jump
($\lambda$ and $\lambda_T$) on the local and average thermal transport. Results from the model are presented and
discussed in Section 3 and conclusions are given in Section 4.

2.4 Analysis

2.4.1 Model Description

Shown in Fig. 2.1 is a schematic illustration of a vertical liquid jet impinging on a hori-
zontal SHPo surface. A non-submerged, steady jet is assumed with jet velocity, $V$; radius, $a$; and
flow rate, $Q$. Temperatures for the incoming jet and wall are $T_j$ and $T_w$, respectively, both of which
are assumed to be uniform and $T_w > T_j$. The wall temperature jump is $\Delta T = T_w - T(r, z = 0)$. A
spatially uniform hydrodynamic slip length, $\lambda$, and temperature jump length, $\lambda_T$, characterize the
superhydrophobic wall boundary condition.

For this analysis, $\lambda$ and $\lambda_T$ are approximated to be equal except in an exploratory case
where they are varied independently. This assumption may be reasonable for typical SHPo surfaces
but is an area for further research. As noted in the introduction, a recent study found $\lambda$ and $\lambda_T$ were
approximately equal for a parallel plate channel with streamwise oriented ribs and cavities [19].
The text will discuss the influence of varying $\lambda_T$ but, since $\lambda = \lambda_T$, the variation of $\lambda$ is implied.

In Fig. 2.1, the development of the hydrodynamic ($\delta$) and thermal ($\delta_T$) boundary layers are
illustrated. A coordinate system is fixed at the intersection of the jet centerline and the horizontal
surface, where the $z$-axis is in the direction of the jet centerline, oriented normal to the surface and
opposite the direction of flow. $r$ is the radial coordinate.
Three regions exist moving radially outward from the impingement point prior to the location of a hydraulic jump.\textsuperscript{1} Region I spans $r$ between the impingement point and the location, $r_0$, where the hydrodynamic boundary layer, $\delta$, merges with the film height, $h$. Region II extends from $r_0$ up to the location where the thermal boundary layer, $\delta_T$, merges with the film height ($r_1$). We note that $\delta_T < \delta$, since the jet is liquid and $Pr > 1$, where the Prandtl number takes the classical form, $Pr = \nu / \alpha$. $\alpha$ is the liquid thermal diffusivity and $\nu$ is the liquid kinematic viscosity. Finally, region III spans $r$ between $r_1$ and the location where a hydraulic jump forms.

Natural convection, radial conduction, viscous dissipation, and Marangoni convection are neglected in the analysis and liquid properties are approximated as constant. Forced convection dominates natural convection due to the relatively high jet Reynolds numbers considered. Likewise, radial advection dominates radial conduction. Since the liquid has a moderate viscosity and

\textsuperscript{1}This paper was published without including the stagnation region. The stagnation region was included in Ch. 3 to increase accuracy near the impingement point. The present isothermal results are valuable since the local results become independent of the initial conditions (the impingement region hydrodynamic and thermal boundary layer thickness) within several jet radii of the impingement point. For completeness, Fig. 1.5 includes the stagnation region, which is absent in the figure shown here.
relatively slow velocity, viscous dissipation is negligible. Due to the high jet Reynolds number, the influence of Marangoni convection is small since it is driven by variations in surface tension due to temperature variation across the meniscus between microfeatures. Heat transfer through the top liquid surface and shear stress between the water and air at this surface are also neglected and, following prior work, the initial thickness of the boundary layer was considered to be negligibly small [32].

2.4.2 Application of Conservation Laws

Conservation laws of mass and energy are applied to an annular control volume of thickness, \( \Delta r \), and height, \( H \). Control volume heights are selected such that known boundary conditions may be applied to the top control surface. Specifically, control volumes of height \( \delta_T \) (regions I and II) and \( h \) (region III) were used as shown in Fig. 2.1.

Applying conservation of mass and energy to a general control volume as shown in Fig. 2.1 yields

\[
\dot{m} = -2\pi \rho \frac{\partial}{\partial r} \left( r \int_0^H u(r,z) \, d z \right) \Delta r \tag{2.1}
\]

and

\[
-\dot{\mc}T(r,H) + q_w''(2\pi r) \Delta r - 2\pi \rho c \frac{\partial}{\partial r} \left( r \int_0^H u(r,z) T(r,z) \, d z \right) \Delta r = 0 \tag{2.2}
\]

where \( \dot{m} \) is the mass flow rate through the top surface of the control volume, \( \rho \) is the liquid density, and \( c \) is the liquid specific heat. As mentioned, the hydrodynamic problem was solved previously and is included here only as needed [22].

2.4.3 Region I

In region I, the hydrodynamic boundary layer develops until it merges with the film height. Throughout this region, the thermal boundary layer is developing as well, but with a thickness that is less than that of the hydrodynamic boundary layer. The control volume height is set to \( H = \delta_T \). At this height, \( T(r,\delta_T) = T_j \). Using these boundary conditions and substituting Eq. 2.1 into 2.2 yields
\[ q''_w = \frac{\rho c}{r} \frac{\partial}{\partial r} \left( r \int_0^{\delta_r} u(T(r,z) - T_j) \, dz \right) \] (2.3)

The velocity profile was approximated for region I, with a slip velocity boundary condition at \( z = 0 \) and a free shear condition at \( z = \delta \) and is expressed as

\[ u = \frac{V}{2} \delta + \frac{\lambda}{2} + \frac{z}{3\delta^2} \] (2.4)

The temperature profile is also approximated with a third order polynomial

\[ T = a + b z + c z^2 + d z^3 \] (2.5)

Coefficients for this region I profile are found using the following boundary conditions

\[ T(r,z = 0) = T_w - \Delta T_w \] (2.6)
\[ T(r,z = \delta_T) = T_j \] (2.7)
\[ \left. \frac{\partial T}{\partial z} \right|_{z=\delta_r} = 0 \] (2.8)
\[ \left. \frac{\partial^2 T}{\partial z^2} \right|_{z=0} = 0 \] (2.9)

Applying these boundary conditions to Eq. 2.5 yields

\[ T = T_w - \Delta T_w - \frac{3(T_w - \Delta T_w - T_j)}{2\delta_T} z + \frac{T_s - \Delta T_w - T_j}{2\delta_T^3} z^3 \] (2.10)

Using Fourier’s law, the heat flux may be obtained from Eq. 2.5 as

\[ q''_w = -k \frac{dT}{dz} \bigg|_{z=0} = \frac{3k(T_w - \Delta T_w - T_j)}{2\delta_T} \] (2.11)

The temperature jump length boundary condition \[ q''_w = (\Delta T_w k) / (\lambda_T) \] is substituted into the assumed profile (Eq. 2.11) and solving for the temperature jump gives
\[ \Delta T_w = \frac{3\lambda_T(T_w - T_j)}{2\delta_T + 3\lambda_T} \]  \hspace{1cm} (2.12)

Substituting Eq. 2.12 into Eq. 2.10 gives the following expression for the local liquid temperature

\[ T = T_w - \frac{3\lambda_T(T_w - T_j)}{2\delta_T + 3\lambda_T} - \frac{3(T_w - T_j)}{(2\delta_T + 3\lambda_T)^2} \zeta + \frac{T_w - T_j}{\delta_T^2(2\delta_T + 3\lambda_T)} \zeta^3 \]  \hspace{1cm} (2.13)

The wall heat flux is determined by substituting the new temperature profile (Eq. 2.13) into Fourier’s law and evaluating the derivative

\[ q''_w = \frac{3k(T_w - T_j)}{2\delta_T + 3\lambda_T} \]  \hspace{1cm} (2.14)

The velocity profile (Eq. 2.4), temperature profile (Eq. 2.13), and heat flux equation (Eq. 2.14) are all substituted into the energy equation (Eq. 2.3), and non-dimensionalized. Velocities are normalized by the jet velocity, \( V \), and all lengths by the jet radius, \( a \), with all non-dimensionalized variables denoted with a hat. The following nonlinear ordinary differential equation (ODE) describing the relationship between \( \hat{\delta}_T \), \( \hat{\delta} \), and \( \hat{r} \) is obtained

\[ \frac{\pi\hat{r}}{RePr} = \frac{(2\hat{\delta}_T + 3\hat{\lambda})}{40} \frac{d}{d\hat{r}} \left[ \hat{r} \frac{\hat{\delta}_T^2(7\hat{\delta}_T^2(4\hat{\delta}_T + 15\hat{\lambda}) - 2\hat{\delta}_T^3)}{140\hat{\delta}_T^2(2\hat{\delta}_T + 3\hat{\lambda})(2\hat{\delta}_T + 3\lambda_T)} \right] \]  \hspace{1cm} (2.15)

where the Reynolds number is defined as \( Re = Q/(av) = \pi aV/\nu \).

In a similar manner, the following ODE was derived for the hydrodynamic variables in region I by substituting the assumed cubic velocity profile (Eq. 2.4) into the resulting momentum integral for a similar control volume to yield [22]

\[ \frac{\pi\hat{r}}{Re} = \left( \frac{2}{3} \hat{\delta} + \hat{\lambda} \right) \frac{d}{d\hat{r}} \left[ \hat{r} \hat{\delta}_T^2 \left( \frac{13}{210} \hat{\delta} + \frac{1}{4} \hat{\lambda} \right) \left( \frac{2}{3} \hat{\delta} + \hat{\lambda} \right)^{-2} \right] \]  \hspace{1cm} (2.16)

Applying conservation of mass (Eq. 2.1), the local film height, \( \hat{h} \), as a function of \( \hat{\lambda} \) and \( \hat{\delta} \), can be expressed

\[ \hat{h} = \frac{1}{2\hat{r}} + \frac{\hat{\delta}_T^2}{4(\frac{2}{3} \hat{\delta} + \hat{\lambda})} \]  \hspace{1cm} (2.17)
A numerical solution for $\hat{h}$, $\hat{\delta}$ and $\hat{\delta}_T$ as a function of $\hat{r}$ was obtained by solving the system of equations formed by Eqs. 2.15, 2.16, and 2.17 with boundary conditions $\hat{\delta}(\hat{r} = 0) = 0$, and $\hat{\delta}_T(\hat{r} = 0) = 0$. The extent of region I was determined by numerically solving for the value of $\hat{r}$ where $\hat{\delta}$ reaches $\hat{h}$. Normalized values of the thermal boundary layer thickness and local film height at that point are labeled $\hat{\delta}_{T0}$ and $\hat{h}_0$.

Nusselt number, $Nu = \frac{q''_w a \pi}{(k(T_w - T_j))}$, and the non-dimensional temperature jump, $\Delta \hat{T} = \Delta T/(T_w - T_j)$, as functions of $\hat{r}$ may be obtained from the wall heat flux (Eq. 2.14) and the wall temperature jump (Eq. 2.12), respectively. These are given as Equations 2.18 and 2.19.

$$Nu = \frac{3\pi}{2\hat{\delta}_T + 3\hat{\lambda}_T} \tag{2.18}$$

$$\Delta \hat{T} = \frac{3\hat{\lambda}_T}{2\hat{\delta}_T + 3\hat{\lambda}_T} \tag{2.19}$$

2.4.4 Region II

In region II, the hydrodynamic boundary layer has merged with the film height and the thermal boundary layer continues to grow until it also merges. The energy equation (Eq. 2.3), assumed temperature profile (Eq. 2.13), and the heat flux equation (Eq. 2.14) are the same as region I. However, boundary conditions on the velocity profile are altered in regions II and III as the free surface velocity is now variable. The resultant cubic velocity profile with a free shear boundary condition at the film thickness yields

$$\hat{u} = \frac{\hat{U}_{fs}}{\frac{2}{3} \hat{h} + \hat{\lambda}} \left[ \hat{\lambda} + \hat{\varepsilon} - \frac{\hat{\varepsilon}^3}{3 \hat{h}^2} \right] \tag{2.20}$$

where $\hat{U}_{fs}$ is the normalized velocity at the free surface. $\hat{U}_{fs} = U_{fs}/V$ may be expressed as

$$\hat{U}_{fs} = \frac{2\hat{h} + \hat{\lambda}}{2\hat{h}\hat{r}(\frac{5}{12} \hat{h} + \hat{\lambda})} \tag{2.21}$$

which results from conservation of mass (Eq. 2.1).
Substituting Eq. 2.21 into the velocity profile (Eq. 2.20) and then substituting this result and the temperature profile (Eq. 2.13) into the energy equation (Eq. 2.3) gives the following

\[
\frac{\pi \hat{r}}{RePr} = \left(2 \hat{\delta}_T + 3 \hat{\lambda}_T \right) \frac{d}{d\hat{r}} \left[ \frac{\hat{\delta}_T^2 (7\hat{h}^2 (4\hat{\delta}_T + 15\hat{\lambda}) - 2\hat{\delta}_T^3)}{70\hat{h}^3 (5\hat{h} + 12\hat{\lambda}) (2\hat{\delta}_T + 3\hat{\lambda}_T)} \right] \quad (2.22)
\]

The film thickness height can be determined throughout regions II and III by applying the integral momentum equation that is detailed in [22] and expanded as

\[
-\frac{2\pi}{Re} = \hat{h} \left( \frac{5}{12} \hat{h} + \hat{\lambda} \right) \frac{d}{d\hat{r}} \left[ \hat{r}^{-1} \hat{h}^{-1} \left( \frac{68}{315} \hat{h}^2 + \frac{5}{6} \hat{h} \hat{\lambda} + \hat{\lambda}^2 \right) \left( \frac{5}{12} \hat{h} + \hat{\lambda} \right)^{-2} \right] \quad (2.23)
\]

The system of equations formed by the hydrodynamic ODE (Eq. 2.23) and the energy ODE (Eq. 2.22) was solved numerically to obtain \( \hat{h} \) and \( \hat{\delta}_T \) as functions of \( \hat{r} \) throughout region II. The initial conditions were \( \hat{h}(\hat{r} = \hat{r}_0) = \hat{h}_0 \) and \( \hat{\delta}_T(\hat{r} = \hat{r}_0) = \hat{\delta}_T \)0. The end of region II was determined by numerically solving for the radial coordinate where \( \hat{\delta}_T \) equaled \( \hat{h} \). This radial coordinate was labeled \( \hat{r}_1 \) and the corresponding film height was labeled \( \hat{h}_1 \). The Nusselt number and \( \Delta \hat{T} \) as functions of \( \hat{r} \) are defined in the same manner as in region 1 (Eq. 2.18 and Eq. 2.19).

### 2.4.5 Region III

In region III, the hydrodynamic and thermal boundary layers have both merged with the film height. Here the control volume height is \( H = h \) such that \( \dot{m} = 0 \). Applying this simplification to the energy equation (Eq. 2.2) and rearranging yields

\[
q''_w = \frac{\rho c}{r} \frac{\partial}{\partial r} \left( r \int_0^h uT \, dz \right) \quad (2.24)
\]

In this region, the assumed velocity profile is the same as in region II (Eq. 2.20), but the boundary conditions for the temperature profile are now

\[
T(r, z = h) = T_{fs}(r) \quad (2.25)
\]

\[
\frac{\partial T}{\partial z} \bigg|_{z=h} = 0 \quad (2.26)
\]
The assumed temperature profile is the same as region I and region II (Eq. 2.13) with the exception that \( T_j \) is replaced by \( T_{fs}(r) \) and \( \delta_T \) by \( T_w - 3\lambda_T(T_w - T_{fs})/(2h + 3\lambda_T) \) where \( T_{fs} \) is the temperature at the free surface. Further, the heat flux equation (Eq. 2.14) and temperature jump equation (Eq. 2.12) are altered with the same substitutions

\[
q'' = \frac{3k(T_w - T_{fs})}{2h + 3\lambda_T} \quad (2.28)
\]

\[
\Delta T_w = \frac{3\lambda_T(T_w - T_{fs})}{2h + 3\lambda_T} \quad (2.29)
\]

Using the assumed velocity (Eq. 2.20) and temperature (Eq. 2.27) profiles, as well as the expression for the free surface velocity (Eq. 2.21) in the governing equation (Eq. 2.24) results in the following form of the energy equation

\[
\frac{\pi\hat{r}}{RePr} = \frac{2\hat{h} + 3\hat{\lambda}}{\theta_{fs}} \frac{d}{d\hat{r}} \left[ \frac{\hat{h}\theta_{fs}(26\hat{h} + 105\hat{\lambda})}{70(5\hat{h} + 12\hat{\lambda})(2\hat{h} + 3\lambda_T)} - \frac{\theta_{fs}}{6} \right] \quad (2.30)
\]

The non-dimensional temperature is defined as \( \theta = (T - T_w)/(T_j - T_w) \) and the non-dimensional free surface temperature is \( \theta_{fs} = (T_{fs} - T_w)/(T_j - T_w) \). As defined, the non-dimensional free surface temperature is unity when the free surface temperature is equal to the jet temperature and decreases to zero as the free surface temperature approaches the wall temperature.

The system of equations formed by the hydrodynamic ODE (Eq. 2.23) and energy ODE (Eq. 2.30) was solved numerically to obtain \( \hat{h} \) and \( \theta_{fs} \) as functions of \( \hat{r} \) throughout all of region III. In this region, the initial conditions were \( \hat{h}(\hat{r} = \hat{r}_1) = \hat{h}_1 \) and \( \theta_{fs}(\hat{r} = \hat{r}_1) = 1 \).

Non-dimensionalizing Eq. 2.28 and Eq. 2.29 results in expressions for \( Nu \) and \( \Delta \hat{T} \) as functions of \( \hat{r} \) in this final region.

\[
Nu = \frac{3\pi\theta_{fs}}{2\hat{h} + 3\hat{\lambda}} \quad (2.31)
\]

\[
\Delta \hat{T} = \frac{\hat{\lambda}_T\theta_{fs}}{2\hat{h} + 3\hat{\lambda}_T} \quad (2.32)
\]
The systems of equations for regions I, II, and III were solved using LSODA (a version of the Livermore Solver for Ordinary Differential equations) as implemented by the Mathematica® solver software. The routine switches automatically between a multi-order (1-12) Adams method for non-stiff problems and a backward-difference method for stiff problems. The numerical error in determining the local Nusselt number is $\pm 1 \times 10^{-4}$.

2.5 Results

2.5.1 Comparison with Previous Results

First, we compare our solution approach to previous work of Chaudhury for jet impingement on a classical smooth no-slip surface [31]. Chaudhury showed that the Reynolds number dependence could be accounted for using the scaling shown in Fig. 2.2, where the product $\hat{r}^2 \hat{NuRe}^{-1}$ is shown as a function of $\hat{r}Re^{1/3}$. The results of Fig. 2.2 correspond to the no-slip and matching temperature boundary conditions for Prandtl numbers of 2, 5, and 8. The present results differ from those of Chaudhury by no more than 6% [31]. Small discrepancies between the models

![Figure 2.2: $\hat{r}^2 \hat{NuRe}^{-1}$ as a function of $\hat{r}Re^{1/3}$ for the classical no-slip and matching wall temperature scenario for $Pr$ ranging from 2 to 8. Present solution results (dashed lines) are compared to corresponding solutions presented by Chaudhury (solid lines) [31].](image-url)
arise because the present work approximates the velocity and temperature profiles with third-order polynomials, whereas, Chaudhury used fourth-order polynomial profiles in regions I and II and a similarity solution within region III.

A similarity solution is not possible when a uniform slip velocity exists [68], which is the influence we are investigating. Additionally, Chaudhury selected fourth-order polynomials so that the thermal boundary layer thickness would match the hydrodynamic boundary layer thickness when \( Pr = 1 \). The present work builds upon that of a previous hydrodynamic model [22], which utilized third-order polynomials and obtained excellent agreement with the prior no-slip solution [25]. The approach is similar to that taken by Chaudhury in that the same order polynomials as the hydrodynamic model are selected so that the hydrodynamic and thermal boundary layers are equal when \( Pr = 1 \).

2.5.2 Local Behavior: Influence of Slip Velocity and Temperature Jump, Reynolds Number, and Prandtl Number.

This section of the paper explores the influence of slip velocity and temperature jump on the local thermal boundary layer growth and temperature profiles. The thermal boundary layer thickness and film height, \( \hat{\delta}_T \) and \( \hat{h} \), are shown as functions of \( \hat{r} \) for varying \( \hat{\lambda}, \hat{\lambda}_T, Re, \) and \( Pr \) in Fig. 2.3. For all scenarios considered, \( \hat{\delta}_T \) increases (starting at 0) with increasing \( \hat{r} \) until it eventually merges with \( \hat{h} \). The \( \hat{\delta}_T \) curves are qualitatively similar across the entire range of temperature jump lengths: the thermal boundary layer grows similar to the classical behavior with the thickness initially increasing with the radial coordinate. The growth rate in \( \hat{\delta}_T \) decreases with increasing radial coordinate up until the existence of an inflection point in the \( \hat{\delta}_T \) vs. \( \hat{r} \) plot, which marks the transition from region I to region II. This inflection occurs because the hydrodynamic boundary layer has merged with the film thickness in region II. The magnitude of the inflection decreases with increasing temperature jump length, increasing \( Re \), and decreasing \( Pr \). At \( \hat{r} \) values beyond the inflection point, the growth rate of \( \hat{\delta}_T \) increases because \( \hat{h} \) is decreasing less rapidly as the jet spreads into a thin film [22].

Panel (a) of Fig. 2.3 shows the thin film height and thermal boundary layer thickness as a function of \( \hat{r} \) and for varying values of \( \hat{\lambda}_T \) ranging from \( \hat{\lambda}_T = 0 \) (the no-slip condition) to \( \hat{\lambda}_T = 0.2 \). The Reynolds number and Prandtl number for these cases are constant at \( Re = 9,000 \) and \( Pr = 5 \).
Figure 2.3: Film thickness and thermal boundary layer thickness as a function of radial position. Panel (a): $Pr = 5$, $Re = 9,000$, and $\hat{\lambda}$ and $\hat{\lambda}_T$ varying from 0 to 0.2. Panel (b): $Pr = 5$, $\hat{\lambda} = \hat{\lambda}_T = 0.1$, and $Re$ varying from 3,000 to 15,000. Panel (c): $Re = 9,000$, $\hat{\lambda} = \hat{\lambda}_T = 0.1$, and $Pr$ varying from 2 to 11.
The local values of both $\hat{h}$ and $\hat{\delta_T}$ decrease as $\hat{\lambda}$ increases. $\hat{h}$ decreases since the velocity near the wall is higher and the hydrodynamic boundary layer develops more slowly as a result of lower wall shear. The thermal boundary layer thickness develops more slowly due to increased thermal resistance at the wall caused by the temperature jump, which results in a decreased heating rate.

Panel (b) of Fig. 2.3 highlights the influence of varying $Re$ while the temperature jump length is kept constant at $\hat{\lambda}_T = 0.1$. The Reynolds number is varied from 3,000 to 15,000 and, for all cases, $Pr = 5$. Increasing $Re$ results in an increase in the momentum of the thin film, thus decreasing the thickness of $\hat{\delta}$, $\hat{\delta_T}$, and $\hat{h}$. Similar to previous work regarding slip length [22], increasing $\hat{\lambda}$ has a similar effect as increasing $Re$ which is obvious when comparing the results of panel (a) and panel (b).

The influence of variations in $Pr$ is illustrated in panel (c) for $\hat{\lambda}_T = 0.1$ and $Re = 9,000$. Of course, the hydrodynamic solution is not affected by varying $Pr$ and increasing $Pr$ expectedly causes the thermal boundary layer to grow more slowly with $\hat{r}$ as the thermal energy diffuses more slowly.

Non-dimensional temperature profiles ($\theta(\hat{z})$) at two different normalized radial locations, $\hat{r} = 10$ and $\hat{r} = 30$, are shown in Fig. 2.4 for various combinations of $Re$, $Pr$, and $\hat{\lambda}_T$. The left column (panels (a), (c), and (e)) show profiles evaluated at $\hat{r} = 10$ and the right column (panels (b), (d), and (f)) show profiles evaluated at $\hat{r} = 30$. The vertical extent of the profiles vary because all profiles terminate at the height of the thin film, $\hat{h}$, which varies with $\hat{r}$, $Re$, and $\hat{\lambda}$, as shown in Fig. 2.3. At $\hat{h}$, the temperature gradient is zero as required by the adiabatic boundary condition at the free surface. Also, due to the non-dimensionalization ($\theta = (T - T_w)/(T_j - T_w)$), $\theta = 1$ when $T = T_j$ and $\theta = 0$ when $T = T_w$. For all scenarios, as $\hat{z}$ increases (moving from the surface), $\theta$ increases, concomitant with a greater difference between the local temperature and the wall temperature.

In the left column of panels (a, c, e) at $\hat{r} = 10$, many of the profiles are not fully developed and plateau at $\theta = 1$ where $T = T_j$. This occurs because this portion of the thin film has not yet been influenced by the conditions at the wall. For the temperature profiles evaluated at $\hat{r} = 30$, the profile is located within region III where the thermal boundary layer is fully developed. Here a plateau does not exist and the profile stretches across the entire thin film.

For all profiles shown, the temperature profiles that do not pass through the origin exhibit an apparent temperature jump at the wall due to the superhydrophobic temperature jump condition.
Figure 2.4: Non-dimensional temperature profiles, as functions of wall-normal coordinate, $\hat{z}$, at two $\hat{r}$ locations. Panels (a), (c), and (e): $\hat{r} = 10$. Panels (b), (d), and (f): $\hat{r} = 30$. Panels (a) and (b): $Pr = 5$, $Re = 9,000$, and $\hat{\lambda}_T$ ranging from 0 to 0.2. Panels (c) and (d): $Pr = 5$, $\hat{\lambda} = \hat{\lambda}_T = 0.1$, and $Re$ ranging from 3,000 to 15,000. Panels (e) and (f): $Re = 9,000$, $\hat{\lambda}_T = 0.1$, and $Pr$ ranging from 2 to 11.
This temperature jump may be as large as 90% of the temperature difference between the wall and the jet (see panel (a)). In panels (a) and (b) $Re$ and $Pr$ are held constant at 9,000 and 5 respectively and $\hat{\lambda}_T$ is varied through 0, 0.05, 0.1, and 0.2.

The results of panels (a) and (b) highlight the difference in thickness of the thermal boundary layer for the no-slip and no temperature jump case ($\hat{\lambda} = \hat{\lambda}_T$) relative to the slip and temperature jump cases $\hat{\lambda} = \hat{\lambda}_T > 0$. As $\hat{\lambda}_T$ increases, there is increased resistance to thermal transport, which leads to a more slowly developing thermal boundary layer. The results of panels (c) and (d) correspond to a fixed value of $Pr = 5$ and $\hat{\lambda} = \hat{\lambda}_T = 0.1$ while $Re$ is varied through the values: 3,000, 9,000, and 15,000. As $Re$ increases, the temperature jump at the wall increases at both $\hat{r} = 10$ and $\hat{r} = 30$, and this is more pronounced at lower $\hat{r}$. Increasing $Re$ has a similar effect as increasing $\hat{\lambda}_T$ on the temperature jump at the wall, although larger $Re$ yields larger wall-normal gradients in $\theta$ while increasing $\hat{\lambda}_T$ leads to smaller gradients. The results for panels (e) and (f) correspond to $Re = 9,000$ and $\hat{\lambda} = \hat{\lambda}_T = 0.1$ while $Pr$ is varied through the values of 2, 5, and 11. As $Pr$ increases, the thermal energy diffuses more slowly from the wall into the liquid, as is typical, and $\theta$ approaches zero more slowly with increasing $\hat{r}$. Further, the results reveal that the wall non-dimensional temperature jump also increases with increasing Prandtl number.

A local $Nu$ may be obtained from the local values of $\hat{\delta}_T$ and $\theta_{fs}$ using Eq. 2.18 and 2.31. Shown in Figs. 2.5 and 2.6 is the local Nusselt number as a function of $\hat{r}$ and the results demonstrate the influence exerted by variations in slip length and temperature jump length. For all scenarios (all values of $\hat{\lambda}_T$), $Nu$ decreases with increasing $\hat{r}$ as expected. The transition from region I to II for all scenarios is indicated by a plus sign + and the region II to III transition is indicated by an X.

Results are shown for $Re = 3,000$ (panel(a)) and 15,000 (panel (b)) at constant $Pr = 5$ in Fig. 2.5 for $\hat{\lambda}_T = 0$, 0.05, 0.1, and 0.2. Results are shown for the same values of $\hat{\lambda}_T$ in Fig. 2.6, but for $Pr = 2$, 5, and 11 and at a constant $Re = 9,000$. $Nu$ increases (at fixed $\hat{r}$) with increasing $Re$ or $Pr$ for the no-slip and matching temperature cases as expected. The influence of $Re$ or $Pr$ decreases in cases with a non-zero temperature jump length. The negligible change in $Nu$ with $Re$ or $Pr$ at the stagnation point ($\hat{r} = 0$), for cases with a non-zero temperature jump length, illustrates this effect. In general, these surfaces have smaller and more uniform $Nu$ and these effects increase as the temperature jump length increases.
Figure 2.5: $Nu$ as function of $\hat{r}$ for $\hat{\lambda}_T$ ranging from 0 to 0.2. $Pr = 5$. Panel (a): $Re = 3,000$. Panel (b): $Re = 15,000$. A $Re = 9,000$ and $Pr = 5$ case is not shown here but is included in Fig. 2.6. Plus signs + indicate the region I to region II transition and Xs indicate the region II to region III transition.

We would like to further emphasize the departure from the smooth surface behavior ($\hat{\lambda}_T = 0$) to what prevails for the superhydrophobic scenarios ($\hat{\lambda}_T \neq 0$). We first consider the transport near the jet centerline and stagnation region ($\hat{r} < 15$), where the influence of temperature jump exerts the greatest influence and large variations in $Nu$ exist for the range of $\hat{\lambda}_T$ values considered. For all scenarios, increases in $\hat{\lambda}_T$ yield notably smaller values of $Nu$, compared to the smooth surface case. For example, for the $Re = 15,000$, $Pr = 5$ scenario and at $\hat{r} = 4$, $Nu$ decreases by nominally 60%, 77%, and 90% for the $\hat{\lambda}_T = 0.05$, 0.1, and 0.2 scenarios, respectively. The dependence on $\hat{\lambda}_T$
is greatest at the jet centerline and decreases moving radially outward. The relative (compared to the smooth surface case) decrease in magnitude of $Nu$ with increasing $\tilde{\lambda}_T$ is a strong function of $Re$ and a moderate function of $Pr$. In general, the relative decrease in $Nu$ (with increasing $\tilde{\lambda}_T$) is larger as either $Re$ or $Pr$ are increased, as illustrated by the data in both figures.

Two other points deserve discussion relative to the data of Figs. 2.5 and 2.6. First, as $\tilde{\lambda}_T$ increases to 0.2, the overall variation with $Nu$ varies from approximately 18 at the stagnation point to approximately 14 at $\hat{r} = 30$, in stark contrast to the variation observed in the $\tilde{\lambda}_T = 0$ scenario. The implication of this behavior is that the local heat flux becomes much more uniform for the larger temperature jump length scenarios. Indeed, at the larger $Re$, $Pr$, and $\tilde{\lambda}_T$ explored, $Nu$ is approaching a uniform value for all $\hat{r}$, an interesting and unexpected result. The second point is related to the first. At sufficiently large values of $\hat{r}$ (the exact point is dependent on $Re$ and $Pr$ but is generally between 15 and 25 radii from the stagnation point), the magnitude of $Nu$ for the $\tilde{\lambda} \neq 0$ scenarios exceeds the value for the $\tilde{\lambda} = 0$ surface. The implication here is that the heat transfer is actually increased on the superhydrophobic surfaces at large radial locations. This behavior prevails because of the dramatic decrease in heat transfer in the stagnation region, which results in a smaller temperature increase in the liquid temperature in the liquid film. Consequently, at larger radii, the driving temperature difference between the wall and the mean liquid temperature is greater relative to the smooth surface case and results in modestly higher local heat transfer rates. Further, it is important to point out that even small values of $\tilde{\lambda}_T$ (see $\tilde{\lambda}_T = 0.05$ data) result in significant departures in the local $Nu$ magnitude (relative to the smooth surface) and may be important when considering data derived from classical jet impingement experiments on reported smooth surfaces.

For small temperature jump length, the effect of increasing either $Re$ or $Pr$, causes the local Nusselt number to increase (seen both in Figs. 2.5 and 2.6). However, as the slip length and the jump length increase, changing $Re$ or $Pr$ has less effect. Consider the $\tilde{\lambda}_T = 0.2$ case in Figs. 2.5 and 2.6 for which there is little apparent change. Here heat transfer is determined by the thermal resistance at the wall instead of the flow conditions.

Shown in Fig. 2.7 is the average Nusselt number, $\overline{Nu}$, for smooth surfaces ($\tilde{\lambda}_T = 0$) as a function of $\hat{r}$ and the results demonstrate the influence exerted by variations in $Re$ and $Pr$. Figure 2.7 provides a point of comparison for superhydrophobic scenarios ($\tilde{\lambda}_T \neq 0$) which are discussed
Figure 2.6: $Nu$ as function of $\hat{\dot{r}}$ for $\hat{\lambda}_T$ ranging from 0 to 0.2. $Re = 9,000$. $Pr$ is varied through 2, 5, and 11 in panels (a), (b), and (c), respectively. As in Fig. 2.5, plus signs + indicate the region I to region II transition and Xs indicate the region II to region III transition.
Figure 2.7: $\overline{Nu}$ for no-slip and matching temperature surface as a function of the radius, $\hat{r}$, of the disk over which $Nu$ is averaged. Panel (a): $Pr = 5$. $Re$ is varied from 3,000 to 15,000. Panel (b): $Re = 9,000$. $Pr$ is varied from 2 to 11.

Further below. Results are shown for $Re = 3,000$, 9,000, and 15,000 at a constant $Pr = 5$ (panel (a)) and for $Pr = 2$, 5, and 11 at a constant $Re = 9,000$ (panel (b)). In general, $\overline{Nu}$ decreases with increasing $\hat{r}$, $Re$, and $Pr$ in agreement with well-known behavior.

A fractional reduction in the average thermal transport, $1 - \overline{Nu}/\overline{Nu}_0$, is used to quantify the reduction in $\overline{Nu}$ for the SHPo cases, where $\overline{Nu}_0$ is $\overline{Nu}$ at $\hat{\lambda}_T = 0$. A fractional reduction of 1
indicates that the thermal transport has decreased by 100%. Shown in Fig. 2.8 is $1 - \frac{Nu}{Nu_0}$ as a function of $\hat{r}$ and the results demonstrate the influence exerted by variations in the temperature jump length. Results are shown for $Re = 3,000$ and 15,000 (panel (a)) at constant $Pr = 5$ with $\hat{\lambda}_T = 0.1$ and 0.2 and for $Pr = 2, 5$, and 11 (panel(b)) at constant $Re = 9,000$ with $\hat{\lambda}_T = 0.1$ and 0.2. In general, the fractional reduction decreases with increasing $\hat{r}$. This is expected in agreement with the same reasoning provided previously for the influence of $\hat{r}$ on $Nu$. Increasing $\hat{\lambda}_T$ yields increased reduction in $\bar{Nu}$, as expected from the results for $Nu$.

Consider the disk radius $\hat{r} = 10$ position in panels (a) and (b). For the $\hat{\lambda}_T = 0.1$ case, the fractional reduction increases from nominally 0.45 to 0.65 when $Re$ increases from 3,000 to 15,000. Further, the fractional reduction increases through nominally 0.55, 0.65, and 0.7 as $Pr$ increases through 2, 5, and 11. We emphasize this unexpected result that increasing $Re$ or $Pr$ on a SHPo surface increases the fractional reduction in $\bar{Nu}$ at a fixed $\hat{\lambda}_T$ value. This behavior demonstrates coupled interactions between $\hat{\lambda}_T$ and $Re$ and $Pr$ on the magnitude of $\bar{Nu}$.

The actual Nusselt numbers may be obtained by using the value of the fractional reduction displayed in this figure in conjunction with $\bar{Nu}$ for the smooth surface plotted in Fig. 2.7 or, for select radii, from Figs. 2.9 and 2.10.

To further quantify the influence of $\hat{\lambda}_T$ on $\bar{Nu}$, the average Nusselt number is shown as a function of $\hat{\lambda}_T$ at specific $\hat{r}$ values (10, 20, 30, 40) in Fig. 2.9. For all scenarios (all values of $\hat{r}$ explored), $\bar{Nu}$ decreases with increasing $\hat{\lambda}_T$ as expected. Results are shown at $Pr = 5$ and $Re = 3,000$ (panel (a)) and for the same values of $Pr$ but at $Re = 15,000$ (panel (b)). Increasing $\hat{\lambda}_T$ reduces thermal transport dramatically when $Nu$ is averaged over the region near the stagnation point. A 66% reduction is observed in panel (b) at $\hat{r} = 10$ when $\hat{\lambda}_T$ increases from 0 to 0.1. Less reduction is observed for larger $\hat{r}$ values. Consider a disk of radius $\hat{r} = 40$ where a 25% decrease is observed as $\hat{\lambda}_T$ increases from 0 to 0.2. Similar behavior was observed across the entire range of $Re$ and $Pr$ considered.

Shown in Fig. 2.10 is the average Nusselt number at a fixed $\hat{r}$ value of 15 as a function of $\hat{\lambda}_T$. These results demonstrate the influence exerted by variations in $Re$ (panel a) and $Pr$ (panel b) for SHPo ($\hat{\lambda}_T \neq 0$) surfaces. For all scenarios (all values of $Re$ and $Pr$ explored), $\bar{Nu}$ decreases as expected with increasing $\hat{\lambda}_T$. Results are shown for $Re = 3,000, 7,000, 11,000, and 15,000$ at constant $Pr = 5$ (panel (a)) and for $Pr = 2, 5, 8, and 11$ at constant $Re = 7,000$ (panel (b)). For all
Figure 2.8: Fractional reduction in Nusselt number, $1 - \frac{Nu}{Nu_0}$, as a function of $\hat{r}$ over which $Nu$ is averaged. $Nu_0$ is the average Nusselt number for the no-slip and matching temperature condition. $\lambda_T = 0.1$ is indicated by a solid line, while $\lambda_T = 0.2$ is indicated by a dashed line. Panel (a): $Pr = 5$ and $Re$ equals 3,000 and 15,000. Panel (b): $Re = 9,000$. $Pr$ equals 2, 5, and 11.

We emphasize the surprising interaction of varying $Re$ and $Pr$ with varying $\lambda_T$. Consider the case of $\lambda_T = 0.01$ in panels (a) and (b). In panel (a) between $Re = 3,000$ and $Re = 15,000$, $\bar{Nu}$ increases by 164%. In panel (b) between $Pr = 2$ and $Pr = 11$, $\bar{Nu}$ increases by 75%. In contrast, compare the increase at $\lambda_T = 0.2$. There the increase is nominally 20% for both $Re$ (increasing...
Figure 2.9: $\overline{Nu}$ as a function of $\hat{\lambda}_T$ for range of $\hat{r}$, from 10 to 40. $Pr = 5$. Panel (a): $Re = 3,000$. Panel (b): $Re = 15,000$.

from 3,000 to 15,000) and $Pr$ (increasing from 2 to 11). At small $\hat{\lambda}_T$, varying $Re$ or $Pr$ alters $\overline{Nu}$ dramatically while at large $\hat{\lambda}_T$ variations in $Re$ and $Pr$ exercise little influence on $\overline{Nu}$ and the $\overline{Nu}$ vs. $\hat{\lambda}_T$ curves collapse together. This occurs because thermal resistance due to the surface condition becomes more significant than thermal resistance due to the convection boundary condition as $\hat{\lambda}_T$ increases.
2.5.3 Relative Importance of Slip Length and Temperature Jump Length on the Solution

This section of the paper explores the relative importance of slip velocity compared to temperature jump on the overall transport. Shown in Fig. 2.11 is the average Nusselt number as a function of $\hat{\lambda}$ and $\hat{\lambda}_T$ at $\hat{r} = 15$.

Results are shown at constant $Pr = 5$ and $Re = 7,000$ for three cases. First, $\hat{\lambda}$ is varied from 0 to 0.2 while $\hat{\lambda}_T$ is held constant at $\hat{\lambda}_T = 0$. These results are indicated by the dash-dotted line.

Figure 2.10: $\overline{Nu}$ as a function of $\hat{\lambda}_T$. $\hat{r} = 15$. Panel (a): $Pr = 5$. $Re$ varies from 3,000 to 15,000. Panel (b): $Re = 7,000$. $Pr$ varies from 2 through 11.
Second, \( \hat{\lambda}_T \) is varied from 0 to 0.2, at a constant value of \( \hat{\lambda} = 0 \) and these results are indicated by the dashed line. Third, \( \hat{\lambda} = \hat{\lambda}_T \) and these results are indicated by the solid line.

For case 1, increasing \( \hat{\lambda} \) (\( \hat{\lambda}_T = 0 \)) yields an increase in \( \overline{Nu} \) because the wall slip velocity increases with \( \hat{\lambda} \). This enhanced near-wall advection results in a larger \( \overline{Nu} \). As \( \hat{\lambda} \) increases from 0 to 0.2, \( \overline{Nu} \) increases by nominally 100\%. For case 2, increasing \( \hat{\lambda}_T \) (\( \hat{\lambda} = 0 \)) yields an increase in the wall thermal resistance and yields a 67\% decrease in \( \overline{Nu} \) as \( \hat{\lambda}_T \) increases from 0 to 0.2. For case 3, \( \overline{Nu} \) deviates only modestly from the solution where \( \hat{\lambda}_T \) was varied at a constant \( \hat{\lambda} = 0 \) (dashed curve).

These results indicate that \( \hat{\lambda}_T \) has a much greater influence on \( \overline{Nu} \) than a simultaneous change in \( \hat{\lambda} \) and that the increased thermal resistance caused by \( \hat{\lambda}_T \) has a larger impact on the thermal transport than the enhanced near-wall advection due to the increased wall slip velocity.

### 2.6 Conclusions

SHPo surfaces exhibit an apparent wall slip velocity and temperature jump. Although the effects of wall slip have been quantified for classical flows, interest in the influence of SHPo surfaces on thermal transport has recently emerged. A model has been developed to predict thermal
transport for an axisymmetric, laminar jet impinging on a SHPo surface with isotropic slip length and temperature jump length. An integral approach was applied to solve the mass, momentum, and energy equations within the thin film of a spreading impinged jet. The velocity and temperature profiles were approximated with third-order polynomials.

The results quantify local and average Nusselt numbers on heated superhydrophobic surfaces over a range of temperature jump lengths ranging from 0 to 0.2, jet Reynolds numbers varying from 3,000 to 15,000, and Prandtl numbers ranging from 2 to 11.

The introduction of an isotropic SHPo surface has the following effects on thermal transport:

• The local thermal boundary layer thickness decreases since it grows more slowly as a result of the increased thermal resistance at the wall.

• Thermal transport is reduced dramatically in the vicinity of the stagnation point and this decrease becomes more significant as the level of superhydrophobicity increases.

• The Nusselt number is sensitive to the introduction of even a small temperature jump length and decreases dramatically (The average Nusselt number decreases by up to 50% at $\hat{\lambda}_T = 0.05$).

• The local and average Nusselt numbers become less dependent on $Re$ and $Pr$ with increasing temperature jump length.

• Average Nusselt numbers are observed to decrease by up to 60-80% for realizable SHPo surfaces and flow regimes, relative to smooth surfaces.

Additionally,

• The overall thermal transport is a much more dependent on the temperature jump length than the hydrodynamic slip length.
CHAPTER 3. THERMAL TRANSPORT DUE TO LIQUID JET IMPINGEMENT ON SUPERHYDROPHOBIC SURFACES WITH ISOTROPIC SLIP: ISOFLUX WALL

This chapter has been submitted to the International Journal of Heat and Mass Transfer [69]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

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3.2 Abstract

Thermal transport due to a liquid water jet impinging an isoflux superhydrophobic surface with isotropic slip was modeled analytically. An integral analysis of the transport equations resulting in a system of ordinary differential equations was solved numerically. Impingement on superhydrophobic surfaces greatly reduces the heat transfer that occurs relative to a smooth surface due to gas trapped in cavities on the surface. This results in an apparent slip velocity and temperature jump at the surface. Local and average Nusselt numbers are presented as a function of radial position (0 to 45 jet radii), jet Reynolds number \(3 \times 10^3 \text{ to } 1.5 \times 10^4\), liquid Prandtl number (2 to 11), normalized slip length (0 to 0.2), and normalized temperature jump length (0 to 0.2). All results are compared to classical (no-slip, no temperature jump) behavior on a smooth surface. Although local Nusselt numbers for the isoflux scenario are greater than the corresponding isothermal case, the difference in Nusselt number between these two heating conditions becomes negligible as the temperature jump length increases to quantities realizable on superhydrophobic surfaces.
3.3 Introduction

A common scenario for cooling in single phase heat transfer applications is liquid jet impingement on a heated surface. This yields some of the highest heat transfer coefficients encountered in single phase convection applications [21]. The schematic illustration in Fig. 3.1 shows a radial cross section of a liquid jet with velocity \( V \), temperature \( T_j \), and radius \( a \) impinging on a horizontal surface. A coordinate system is fixed at the impingement point with radial coordinate, \( r \), and axial coordinate \( z \), which increases moving upward from the impinged surface. The film thickness, \( h \), the hydrodynamic boundary layer thickness, \( \delta \), and the thermal boundary layer thickness, \( \delta_T \), are shown developing outward from the stagnation point. In general, the wall may have an arbitrary wall heating condition. The present analysis considers the case of a uniform wall heat flux, \( q''_w \), which is relevant for multiple cooling applications.

Prior investigators have considered liquid jet impingement on an isoflux surface for a classical surface [21, 26, 32, 33, 70]. These studies have considered heat transfer both in the stagnation and radial flow regions. An axisymmetric integral analysis of the mass, momentum, and energy equations with assumed polynomial velocity and temperature profiles allows solution of the boundary layer thicknesses and local Nusselt number [32, 33, 70]. Excellent agreement with experimental results was demonstrated [32, 33]. These studies find that the heat transfer of the impinging liquid jet is determined by the jet Reynolds number, \( Re = Q/\nu a \), and the Prandtl number, \( Pr = \nu/\alpha \). \( Q \) is the jet flow rate, \( a \) is the jet radius, \( \nu \) is the kinematic viscosity, and \( \alpha \) is the thermal diffusivity. For \( Pr > 1 \) (the case of interest here), \( \delta > \delta_T \), and the following flow regions (indicated in Fig. 3.1 panels (a) and (b)) are encountered as the jet spreads: (I) the stagnation region where the flow turns and \( \delta \) and \( \delta_T \) are constant with respect to position, (II) a region where \( \delta \) and \( \delta_T \) grow, (III) a region where \( \delta_T \) grows and \( \delta \) equals \( h \), and, if \( 1 < Pr < 5 \), (IV) a region where \( \delta \) and \( \delta_T \) equal \( h \) (panel (a)). As shown by Liu and Lienhard, if \( Pr > 5 \), the thermal boundary layer no longer grows to the size of the film and this is denoted in panel (b) [32].

Impinging jets on superhydrophobic (SH) surfaces yield altered hydrodynamics and thermal transport [22–24, 51]. It is of interest to study thermal transport on these surfaces to determine how it is influenced when SH surfaces are utilized for their desirable properties of drag reduction [10–12, 15] and self-cleaning [6, 16, 71, 72]. SH surfaces are created by combining micro/nano-
Figure 3.1: An impinging liquid jet is illustrated schematically with a radial cross section spanning from the jet centerline (left) to beyond the hydraulic jump (right). The jet initially has a uniform velocity, $V$, and temperature, $T_j$. The liquid impinges on a horizontal surface which is heated with an isoflux boundary condition maintaining a uniform heat flux, $q''_w$. The hydrodynamic boundary layer thickness, $\delta$, thermal boundary layer thickness, $\delta_T$, and film height, $h$, are shown. The flow of the spreading jet is subdivided into four regions, numbered I–IV. All four regions are present in panel (a) ($1 \leq Pr \leq 5$). In panel (b) $Pr > 5$ and region IV is absent since $\delta_T$ is always smaller than $h$. 

$1 \leq Pr \leq 5$

(a) 

$Pr > 5$

(b)
roughness with hydrophobic chemistry. This combination leads to static solid-liquid contact angles greater than 150° [1].

Drag reduction and self-cleaning behavior are the result of the composite boundary condition present on these surfaces. Due to surface roughness and material hydrophobicity, liquid water is prevented from penetrating the cavities between microfeatures when the liquid pressure is sufficiently small. Instead, the cavities remain filled with air and/or water vapor and a meniscus spans each cavity.

The relevant boundary conditions for modeling fluid flow over these surfaces are a no-slip condition at the top of each microfeature and a free shear condition at each gas-liquid meniscus. An aggregate, single macroscale boundary condition can instead be introduced at the wall as an apparent slip velocity \( u_s \), which is proportional to the wall shear \( \tau_w \), such that

\[
  u_s = \frac{\tau_w \lambda}{\mu}
\]

Here \( \mu \) is the dynamic viscosity and \( \lambda \) is the slip length, which may be physically interpreted as the distance into the wall that the apparent velocity profile must be extrapolated to reach the no-slip condition. A slip velocity at the wall leads to a decrease in wall shear stress and an overall drag reduction.

Another effect of the menisci over the cavities is a decrease in aggregate surface energy for liquid water droplets, resulting in increased contact angle and droplet rolling behavior. This dynamic allows easy removal of contaminants from SH surfaces by rolling droplets [16].

Thermal boundary conditions on SH surfaces are similarly influenced. Metallic microfeatures have thermal conductivities three orders of magnitude higher than that of the air/water vapor mixture which fills the cavities. Thus, it is appropriate to model the gas-liquid interface as adiabatic and match the temperature at the solid-liquid interface. Again, these alternating boundary conditions may be modeled with an aggregate boundary condition, where the apparent wall temperature jump may be expressed as

\[
  \Delta T_w = q''_w \lambda_T / k,
\]

where \( q''_w \) is the wall heat flux [19,40,67]. Here, \( k \) is the thermal conductivity of the liquid and the temperature jump length, \( \lambda_T \), is the thermal analog of \( \lambda \).

The first region shown in Fig. 3.1 is the stagnation region, where the liquid impinges normally on the substrate and the flow accelerates in the radial direction. In a classical stagnation flow, a uniform flow normal to the surface impinges a substrate and this provides a good estimate of the dynamics near the impingement point in liquid jets. The SH boundary conditions of slip and temperature jump have previously been considered for classical stagnation flow from an analyti-
cal perspective. Here similarity solutions have been obtained for axisymmetric and planar flows stagnating on a surface with isotropic and anisotropic slip length and both stationary and moving plates have been considered [73–75]. Heat transfer from isothermal surfaces has been modeled under conditions of isotropic slip and temperature jump [74] and these prior solutions are utilized in the present work to approximate the heat transfer within the stagnation region of the impinging jet (Sec. 3.4.3). These prior results indicate heat transfer increases with increasing slip length and decreases with increasing temperature jump length.

Within the radial flow region, the hydrodynamics of jet impingement on a SH surface with isotropic and anisotropic slip have been modeled analytically [22–24]. These studies found that increasing the hydrodynamic slip length yields a similar effect as increasing the jet Reynolds number. Specifically, the thickness of the spreading film decreased and the location of a hydraulic jump (or film breakup with droplets) moved outward as the slip length increased.

Only a single study has explored the scenario of a jet impinging on a heated SH surface. Specifically, the constant wall temperature condition surface with isotropic slip was considered [51]. This study showed that the Nusselt number drops dramatically as the slip length and temperature jump length increased. Additionally, the dependence of thermal transport on flow conditions (Reynolds number and Prandtl number) vanished as temperature jump length increased and variations in the temperature jump length yield much stronger influence on the transport than equivalent changes in the hydrodynamic slip length.

In the present study, we consider a jet impinging at a SH surface with isotropic slip length and temperature jump length where a uniform heat flux is applied to the surface (isoflux condition). Both the stagnation and radial flow regions are considered, with the stagnation region modeled by a prior solution [74] and the radial flow region modeled by an integral analysis defined here. This scenario has not been previously considered and is valuable for multiple cooling applications. In the following sections, the analytical methodology is presented and the results are benchmarked with prior work. Local and average Nusselt numbers are presented for varying radial coordinate, Reynolds number, Prandtl number, slip length, and temperature jump length. These results are discussed and conclusions from the work are summarized.
3.4 Analysis

3.4.1 Model Description

The scenario of a cool liquid jet impinging on a SHPo surface may be modeled by performing an integral analysis of each of the four regions described earlier and applying slip and temperature jump boundary conditions at the wall. The hydrodynamic solution now depends on $\lambda$ as well as $Re$ and the thermal transport solution depends on the hydrodynamic variables, $\lambda_T$, and $Pr$.

$\lambda$ and $\lambda_T$ are determined by analysis of the diffusion-dominated flow near the wall. The present analysis intends that microscale analyses be applied to determine $\lambda$ and $\lambda_T$ for a certain microstructure. The appropriate macroscale solution of the impinging jet is presently obtained for a range of $\lambda$ and $\lambda_T$. Calculation of $\lambda$ is well-defined [8, 15]. More recent studies have performed diffusion-dominated thermal analyses to determine $\lambda_T$ for a variety of microfeature geometries [19, 40, 54, 67]. These have been accompanied by computational simulations where $\lambda_T$ has been calculated from the flow field [41, 42]. These studies indicate that the ratio of $\lambda_T$ to $\lambda$ is on the order of 1. In the subsequent analysis, we assume that the ratio is 1 or, equivalently, $\lambda = \lambda_T$. The equations developed here may easily be implemented to solve for any ratio $\lambda_T/\lambda$ as illustrated in Section 3.5.4.

All regions were solved for varying $Re$ ($3 \times 10^3$ to $1.5 \times 10^4$), $Pr$ (2 to 11), $\hat{\lambda}$ (0 to 0.2), and $\hat{\lambda}_T$ (0 to 0.2) for $\hat{r}$ varying from 0 to 45 where the hat indicates normalization with respect to $a$. All lengths in this study are normalized in this manner. The differential equation solver utilized was an explicit eighth-order Runge-Kutta method as implemented in the numerical solver software Mathematica®. The code implementing this solution is included in Appendix A. The resulting numerical error in determining the local Nusselt number is at most $\pm 1 \times 10^{-4}$.

3.4.2 Fundamental Equations

Integral forms of conservation (mass and energy) are applied to annular control volumes of radius $r$, thickness $\Delta r$, and height $H$ (see Fig. 3.1) yielding
\[ \dot{m} = -2\pi \rho \frac{\partial}{\partial r} \left( r \int_0^H u(r,z) \, dz \right) \Delta r \] (3.1)

and

\[ -\dot{m} c T(r,H) + q''_w(2\pi r) \Delta r - 2\pi \rho c \frac{\partial}{\partial r} \left( r \int_0^H u(r,z) T(r,z) \, dz \right) \Delta r = 0 \] (3.2)

where \( \dot{m} \) is the mass flow rate through the top surface of the control volume, \( u \) is the local liquid radial velocity, \( T \) is the local liquid temperature, \( \rho \) is the liquid density, and \( c \) is the liquid specific heat. The hydrodynamic solution for this scenario was obtained previously and results for the velocity field are included as needed [22].

### 3.4.3 Region I: Stagnation Region

The stagnation region is modeled using a similarity solution obtained previously for axisymmetric stagnation flow on a non-moving surface with isotropic slip velocity and temperature jump [74]. The solution satisfies the differential boundary layer equations for mass, momentum (axial and radial directions), and energy. At the wall, the slip velocity and temperature jump boundary conditions are applied. The slip length and temperature jump length parameters in the solution (\( \lambda_W \) and \( \beta \)) are related to the slip length and temperature jump length utilized in the present work (\( \lambda \) and \( \lambda_T \)) by

\[ \lambda_W = \sqrt{CRe/\pi \lambda} \] (3.3)

\[ \beta = \sqrt{CRe/\pi \lambda_T} \] (3.4)

The stagnation Nusselt number as defined in the present paper may be obtained from the prior results [74] as

\[ Nu = -\sqrt{\pi CRe} \frac{\partial \theta_s}{\partial \eta} \bigg|_{\eta=0} \] (3.5)

\( C \) is a dimensionless constant which is related to the radial velocity gradient and was obtained previously to be 0.458 for an impinging jet [26]. \( \theta_s \) and \( \eta \) are similarity variables from the prior results which indicate respectively, the temperature profile and the wall normal coordinate. Definitions for \( \theta_s, \eta, \lambda_W, \) and \( \beta \) and details concerning the solution may be obtained from the prior work [74]. To
avoid confusion with the temperature profile, \( \theta \), defined with this work, these similarity variables are not listed in the nomenclature.

Although the above similarity solution was obtained for an isothermal surface, within the stagnation region, isoflux and isothermal Nusselt numbers will be the same since the boundary layer thicknesses are uniform [21].

\( \delta_s \) and \( \delta_{Ts} \), the constant hydrodynamic and thermal boundary layer thicknesses, are used as the initial conditions at the end of the stagnation region, \( \hat{r}_s = 0.9 \), for the subsequent region. These boundary layer thicknesses are calculated from the similarity solution in the classical manner by numerically solving for the location at which the velocity and temperature profiles have reached 99\% of their respective free stream values, \( \eta = \eta_{99\%} \) and \( \eta_T = \eta_{T99\%} \), respectively, with the equations being solved expressed as

\[
\frac{\partial f}{\partial \eta} \bigg|_{\eta=\eta_{99\%}} = 0.99
\]

and

\[
\theta(\eta_{T99\%}) = 0.01
\]

\( \eta_{99\%} \) and \( \eta_T = \eta_{T99\%} \) are then substituted into

\[
\hat{\delta}_s = \eta_{99\%} \sqrt{\frac{\pi}{CRe}}
\]

(3.8)

\[
\hat{\delta}_{Ts} = \eta_{T99\%} \sqrt{\frac{\pi}{CRe}}
\]

(3.9)

yielding \( \hat{\delta}_s \) and \( \hat{\delta}_{Ts} \).

The end of the stagnation region is defined following the prior work of Liu et al. [26], in which an analytical model including surface tension for the inviscid region of the impinging jet was presented. The model found that true stagnation flow (characterized by a linear rise in the free stream velocity) extended to \( \hat{r} = 0.7 \) and that modeling the flow in this manner was a reasonable approximation to \( \hat{r} = 1.5 \). The value of \( \hat{r} = 0.9 \) was selected because it was within this range, near \( \hat{r} = 0.7 \), and minimized the discontinuity between results for the stagnation flow and radial flow models.
3.4.4 Region II: Developing Boundary Layers

Upon leaving the stagnation region, the flow is primarily in the radial direction and $\delta$ and $\delta_T$ increase as the radial coordinate increases. Since $Pr > 1$, $\delta$ grows more quickly than $\delta_T$. Region II concludes when $\delta$ merges with $h$. The height of the control volume in Region II is set to $H = \delta_T$ where the local temperature is equal to the jet temperature, $T(r, z = \delta_T) = T_j$. This control volume is displayed in panels (a) and (b) of Fig. 3.1. Substituting Eq. 3.1 into Eq. 3.2 yields the following equation for region II

$$q''_w = \frac{\rho c}{r} \frac{\partial}{\partial r} \left( r \int_0^{\delta_T} u(T(r,z) - T_j) \, dz \right)$$ (3.10)

The velocity profile was previously obtained for this region to be

$$u = \frac{V}{\frac{2}{3}\delta + \lambda} \left[ \lambda + z - \frac{z^3}{3\delta^2} \right]$$ (3.11)

The temperature profile is approximated as a third order polynomial.

$$T = a + bz + cz^2 + dz^3$$ (3.12)

To which the following boundary conditions are applied

$$-k \frac{\partial T}{\partial z} \bigg|_{z=0} = q''_w$$ (3.13)

$$T(r, z = \delta_T) = T_j$$ (3.14)

$$\frac{\partial T}{\partial z} \bigg|_{z=\delta_T} = 0$$ (3.15)

$$\frac{\partial^2 T}{\partial z^2} \bigg|_{z=0} = 0$$ (3.16)

The choice of a third-order polynomial is necessary since the hydrodynamic boundary layer is a third-order profile [22] and this allows $\delta$ to equal $\delta_T$ when $Pr = 1$. Applying these boundary conditions to Eq. 3.12 and solving for coefficients a–d yields


\[ T - T_j = \frac{q''_w}{3k} \left( 2\delta_T - 3z + \frac{z^3}{\delta_T^2} \right) \]  (3.17)

The assumed velocity (Eq. 3.11) and temperature (Eq. 3.17) profiles are substituted into Eq. 3.10. The resulting relationship is

\[
\frac{\pi \hat{r}}{RePr} = \frac{\partial}{\partial \hat{r}} \hat{r} \frac{\delta^2}{140 \delta^2 (2 \delta + 3 \lambda)} \left( 2 \delta^2 + 7 \delta^2 (4 \delta_T + 15 \lambda) \right)
\]  (3.18)

Prior hydrodynamic solutions to the integral mass and momentum equations gives the following equations [22]

\[
\hat{h} = \frac{1}{2 \hat{r}} + \frac{\delta^2}{4 (\frac{2}{5} \delta + \lambda)}
\]  (3.19)

and

\[
\frac{\pi \hat{r}}{Re} = \left( \frac{2}{3} \delta + \lambda \right) \frac{d}{d\hat{r}} \left[ \hat{r} \delta^2 \left( \frac{13}{210} \delta + \frac{1}{4} \lambda \right) \left( \frac{2}{3} \delta + \lambda \right)^{-2} \right]
\]  (3.20)

\( \delta \) and \( \delta_T \) are obtained as functions of \( \hat{r} \) by numerically solving the system of differential equations defined by Eqs. 3.18 and 3.20 subject to the initial conditions \( \delta(\hat{r} = \hat{r}_s) = \delta_s \) and \( \delta_T(\hat{r} = \hat{r}_s) = \delta_{Ts} \). \( \hat{h} \) is found by substituting the results for \( \delta \) into Eq. 3.19. The endpoint of region II is found by numerically finding \( \hat{r} \) where \( \delta \) equals \( \hat{h} \). This radius is labeled \( \hat{r}_0 \) and the values of \( \delta_T \) and \( \hat{h} \) at this point are labeled \( \delta_{T0} \) and \( \hat{h}_0 \), respectively.

We seek to obtain the local Nusselt number, \( Nu \), which is defined in the normal manner as

\[ Nu = q''_w a \pi / (k(T_w(r) - T_j)) \]  (3.21)

Recalling that there is a temperature jump at the wall, we can obtain a relationship which is valid in any region between the wall temperature, \( T_w(r) \), and the temperature given by a profile at \( T(r, z = 0) \), the difference of which is the temperature jump, \( \Delta T_w = T_w(r) - T(r, z = 0) \). The temperature jump is proportional to the heat flux, \( \Delta T_w = (q''_w \lambda_T) / k \) [19]. Combining these two results yields

\[ T_w(r) = q''_w \lambda_T / k + T(r, z = 0) \]  (3.22)
which is valid in all regions. \( Nu \) may be obtained for region II by substituting for \( T_w(r) - T_j \) in Eq. 3.21. Eq. 3.17 is evaluated at the wall yielding

\[
T(r, z = 0) - T_j = \frac{2 \dot{q}_w^\prime \delta_T}{3k}
\]

(3.23)

\( T_j \) is subtracted from both sides of Eq. 3.22 and Eq. 3.23 is substituted yielding

\[
T_w(r) - T_j = \frac{\dot{q}_w^\prime \lambda_T}{k} + \frac{2 \dot{q}_w^\prime \delta_T}{3k}
\]

(3.24)

Finally, this result is substituted into Eq. 3.21 resulting in \( Nu \) as a function of \( \hat{\delta_T} \)

\[
Nu = \frac{\pi}{\hat{\lambda_T} + \frac{2}{3} \hat{\delta_T}}
\]

(3.25)

### 3.4.5 Region III: Further Developing Thermal Boundary Layer

After region II, \( \delta_T \) continues to grow until it merges with \( h \). The energy equation (Eq. 3.10) and temperature profiles (Eq. 3.17) remain the same as those in region II but will be applied to the control volume shown in region III of Fig. 3.1 panels (a) and (b). The velocity profile must be modified since the top boundary is a shear free condition instead of matched velocity. The velocity at the top surface, \( U_{fs} \), becomes a function of \( r \) such that the velocity profile is

\[
\hat{u} = \frac{\hat{U}_{fs}}{\frac{2}{3} \hat{h} + \hat{\lambda}} \left[ \hat{\lambda} + \hat{\varepsilon} - \frac{2^3}{3 \hat{h}^2} \right]
\]

(3.26)

where the hats on \( u \) and \( U_{fs} \) indicate normalization by \( V \) and all velocities are normalized in this manner. \( \hat{U}_{fs} \) was obtained previously [22] by applying conservation of mass (Eq. 3.1) as

\[
\hat{U}_{fs} = \frac{\frac{2}{3} \hat{h} + \hat{\lambda}}{2 \hat{h} \hat{\rho}(\frac{2}{12} \hat{h} + \hat{\lambda})}
\]

(3.27)

Eqs. 3.27, Eq. 3.26, and Eq. 3.17 are substituted into the energy equation (Eq. 3.10) yielding
\[
\frac{\pi \dot{r}}{RePr} = \frac{\partial}{\partial \dot{r}} \left( \frac{\delta_r^2(2\delta_r^3 + 7h^2(4\delta_r + 15\lambda))}{70h^3(5h + 12\lambda)} \right)
\] (3.28)

The momentum equation for this region was obtained previously [22] as

\[
-\frac{2\pi}{Re} = \dot{h}(\frac{5}{12}\dot{h} + \dot{\lambda}) \frac{d}{d\dot{r}} \left[ \dot{r}^{-1}\dot{h}^{-1} \left( \frac{68}{315}\dot{h}^2 + \frac{5}{6}\dot{h}\dot{\lambda} + \dot{\lambda}^2 \right) \left( \frac{5}{12}\dot{h} + \dot{\lambda} \right)^{-2} \right]
\] (3.29)

The system of differential equations defined by Eq. 3.28 and Eq. 3.29 is solved numerically to obtain \( \dot{h} \) and \( \dot{\delta}_T \) as functions of \( \dot{r} \) subject to the initial conditions \( \dot{h}(\dot{r} = \dot{r}_0) = \dot{h}_0 \) and \( \dot{\delta}_T(\dot{r} = \dot{r}_0) = \dot{\delta}_{T0} \). \( \dot{h} \) and \( \dot{\delta}_T \) were solved through 45 jet radii (well beyond the intersection point between \( \dot{\delta}_T \) and \( \dot{h} \), if it occurs). \( Nu \) is determined by the same equation obtained for region II (Eq. 3.25). As discussed earlier in Section 3.4.1, it is possible for \( \dot{\delta}_T \) to not intersect \( \dot{h} \). A numerical root finding routine was applied to determine if this intersection did occur. If so, the radius was labeled \( \dot{r}_1 \) and the value of \( \dot{h} \) obtained at this point was labeled \( \dot{h}_1 \). These values were used as the initial conditions for the differential equations defined in region IV. Region IV was not solved if \( \dot{\delta}_T \) did not reach \( \dot{h} \).

### 3.4.6 Region IV: Both Boundary Layers Merged

In region IV, \( \delta_T \) has merged with \( h \) and the control volume height is set to \( H = h \) with \( \dot{m} = 0 \). This control volume is shown in Fig. 3.1 panel (a) region IV. Substituting these results into the energy equation Eq. 3.2 yields

\[
q''_w = \frac{\rho c}{r} \frac{\partial}{\partial r} \left( r \int_0^h uT \, dz \right)
\] (3.30)

The velocity profile remains the same as that derived in region III (Eq. 3.26) but new boundary conditions must be applied to the temperature profile so that the top boundary condition is now adiabatic (assuming negligible evaporation) instead of a fixed temperature. Consequently, the free surface temperature, \( T_{fs} \), varies with \( r \). To satisfy these new conditions, the second and third boundary conditions defined for the temperature profile in region III (Eq. 3.14 and Eq. 3.15) are replaced with the following boundary conditions

\[
T(r, z = h) = T_{fs}(r)
\] (3.31)
\[ \frac{\partial T}{\partial z} \bigg|_{z=h} = 0 \]  

(3.32)

Applying these boundary conditions, the temperature profile for region IV is obtained as

\[ T(r,z) = T_{fs} + \frac{q''_{w}}{3k} \left( 2h - 3z + \frac{z^3}{h^2} \right) \]  

(3.33)

Substituting \( \hat{U}_{fs} \) (Eq. 3.27) into the velocity profile (Eq. 3.26) and this result and the temperature profile (Eq. 3.33) into the energy equation (Eq. 3.30) yields

\[ \frac{\pi \hat{r}}{RePr} - \frac{\partial}{\partial \hat{r}} \left( \theta_{fs} + \frac{\hat{h}}{2} + \frac{71\hat{h}^2}{280(5\hat{h} + 12\hat{\lambda})} \right) \]  

(3.34)

where \( \theta_{fs} = k(T_{fs} - T_j)/(q''_w a) \) is the normalized free surface temperature. The system of differential equations defined by Eq. 3.29 and Eq. 3.34 is solved numerically for \( \hat{h} \) and \( \theta_{fs} \) as functions of \( \hat{r} \) subject to the initial conditions \( \hat{h}(\hat{r} = \hat{r}_1) = \hat{h}_1 \) and \( \theta_{fs}(\hat{r} = \hat{r}_1) = 0 \).

\( Nu \) may be determined by substituting for \( T_w(r) - T_j \) in Eq. 3.21. To find this, first Eq. 3.33 is evaluated at \( \hat{z} = 0 \) yielding

\[ T(z = 0) = T_{fs} + \frac{2q''_w h}{3k} \]  

(3.35)

Then Eq. 3.35 is substituted into Eq. 3.22 and \( T_j \) is subtracted from both sides

\[ T_w(r) - T_j = \frac{q''_{w} \lambda_T}{k} + \frac{2q''_w h}{3k} + T_{fs} - T_j \]  

(3.36)

Finally, Eq. 3.36 is substituted into Eq. 3.21 yielding

\[ Nu = \frac{\pi}{\lambda_T + \frac{2}{3} \hat{h} + \theta_{fs}} \]  

(3.37)

3.5 Results

The results section proceeds as follows: results obtained here for the no-slip scenario are compared with prior results. Then, the influence of temperature jump length on the thermal boundary layer thickness, Nusselt number, and local temperature distributions are explored. It is shown that the Nusselt number results for both the isothermal and isoflux wall scenarios merge together.
as the temperature jump length increases. Finally, results are presented for the average Nusselt number.

### 3.5.1 Comparison with Previous Results

Local Nusselt number is shown as a function of \( \hat{r} \) in Fig. 3.2 for the no-slip and no-temperature jump scenario at \( Pr = 5 \) and at \( Re = 3 \times 10^3, 9 \times 10^3, \) and \( 1.5 \times 10^4 \). Excellent agreement is shown with prior analytical solutions obtained by Ma and Zhao and \[70\] and Liu and Lienhard \[32\]. Similarly good agreement was also demonstrated at \( Pr = 2 \) and \( Pr = 11 \), although these results are not shown here. The goodness of the agreement with prior results (average difference of 2% relative to Liu and Lienhard and 6% relative to Ma and Zhao with greater difference at discontinuities) for the no-slip case provide a benchmarking of our modeling approach and justify the selection of the approximate temperature profiles.

### 3.5.2 Local behavior: Influence of Slip Velocity and Temperature Jump, Reynolds Number, and Prandtl Number.

This section of the paper explores the influence of slip and temperature jump on local features in the thermal boundary layer. Shown in Figure 3.3 are \( \hat{h} \) and \( \hat{\delta}_T \) as functions of \( \hat{r} \). Results are shown in three panels, with \( \lambda_T \), \( Re \), and \( Pr \) respectively varying in each panel, while the other parameters remain fixed. The transitions in the thermal boundary layer from region I to II, region II to III, and region III to IV are marked with an asterisk, plus sign, and diagonal cross mark, respectively. These transition locations correspond to radial locations \( r_s \), \( r_0 \), and \( r_1 \). In panel (a), \( \lambda_T \) varies from 0 to 0.2 (\( Pr = 5, Re = 9 \times 10^3 \)) and the results reveal that an increase in \( \lambda \) (accompanied by an equivalent increase in \( \lambda_T \)) results in a notable decrease in \( \hat{\delta}_T \).

This decrease is expected from hydrodynamic considerations, since an increase in \( \hat{\lambda} \) increases the momentum in the boundary layer and leads to a thinning of the layer to satisfy continuity. Note that \( \hat{\delta}_T \) never merges with the height of the thin film, \( \hat{h} \), for the \( \lambda_T = 0 \) scenario, and thus region IV does not exist for this case. However, as \( \lambda_T \) increases, \( \hat{\delta}_T \) merges with \( \hat{h} \) at \( \hat{r}_1 \) (indicated by a diagonal cross mark) and region IV does exist. Increasing \( \lambda_T \) results in merging of the boundary layer thickness with the film height at smaller \( \hat{r}_1 \).
Figure 3.2: The present solution for the local Nusselt number for the no-slip and no temperature jump scenario at $Pr = 5$ and $Re$ varying from $3 \times 10^3$ to $1.5 \times 10^4$ is plotted as a function of $\hat{r}$. Previous solutions by Ma and Zhao [70] and Liu and Leinhard [32] are included for comparison.

Shown in panel (b) of Fig. 3.3 are results with $Pr = 5$, $\hat{\lambda}_T = 0.1$, and $Re$ varying from $3 \times 10^3$ to $1.5 \times 10^4$. The $\hat{\delta}_T$ variation with $\hat{r}$ for this $\hat{\lambda}_T$ exhibits classical behavior as $Re$ is increased (decreasing $\hat{\delta}_T$ due to the decreasing hydrodynamic boundary layer thickness). $\hat{\lambda}_T = 0.1$ is sufficiently large that $\hat{\delta}_T$ merges with $\hat{h}$ for each $Re$ shown. The influence of variation in $Pr$ is demonstrated in panel (c), where $Re = 9 \times 10^4$, $\hat{\lambda}_T = 0.1$, and $Pr$ varies from 2 to 11. The $Pr = 5$ case (indicated by an open, vertical diamond) is identical to the case shown with a solid triangle in panel (a). $\hat{\delta}_T$ decreases as $Pr$ increases and $\hat{r}_1$ (indicated by a diagonal cross mark) increases as well. This occurs since the thermal diffusivity decreases relative to the kinematic viscosity as $Pr$ increases. At $Pr = 11$, $Pr$ has become sufficiently large such that $\hat{\delta}_T$ no longer merges with $\hat{h}$, even at large $\hat{r}$.

Next we explore the influence of superhydrophobicity ($\hat{\lambda}_T$) on the non-dimensional temperature profile, $\theta$. $\theta = k(T - T_j)/(q''a)$ is shown as a function of the normalized vertical coordinate, $\hat{\xi}$, in Fig. 3.4 for several scenarios. In each panel of the figure, $\theta$ is plotted as a function of the $\hat{\xi}$-coordinate from $\hat{\xi} = 0$ to $\hat{\xi} = \hat{h}$ (the height of the film). In the left three panels (a, c, e), the radial coordinate is fixed at $\hat{r} = 10$ and, in the right three panels (b, d, f), $\hat{r} = 30$. We note that the axis ranges in the left three panels are different than the axis ranges in the right three panels. The horizontal axis range of the right, middle panel (panel d) is greater than that of the panels above.
Figure 3.3: The normalized local thermal boundary layer thickness and film height as functions of $\hat{r}$. In panel (a), $Pr = 5$ and $Re = 9 \times 10^3$, with $\hat{\lambda}_T$ increasing from 0 to 0.2. In panel (b), $Pr = 5$ and $\hat{\lambda}_T = 0.1$, with $Re$ varying from $3 \times 10^3$ to $1.5 \times 10^4$. In panel (c), $Re = 9 \times 10^3$ and $\hat{\lambda}_T = 0.1$, with $Pr$ varying from 2 to 11. The region I to II transition, region II to III transition, and region III to IV transition are marked with an asterisk, plus sign, and diagonal cross mark, respectively. Some transition marks are omitted for clarity in panel (a).
Figure 3.4: Non-dimensional temperature profiles as a function of \( \hat{z} \). \( \hat{r} = 10 \) in the left three panels (a, c, and e) and \( \hat{r} = 30 \) for the right three panels (b, d, and f). In the top two panels (a,b), \( Re = 9 \times 10^3 \) and \( Pr = 5 \), with \( \hat{\lambda}_T \) varying from 0 to 0.2. In the middle row of panels (c,d), \( Pr = 5 \) and \( \hat{\lambda}_T = 0.1 \), with \( Re \) varying from \( 3 \times 10^3 \) to \( 1.5 \times 10^4 \). In the bottom two panels (e,f), \( Re = 9 \times 10^3 \) and \( \hat{\lambda}_T = 0.1 \), with \( Pr \) varying from 2 to 11.

and below (panels (b) and (f)) to allow the \( Re = 1.5 \times 10^4 \) scenario to be displayed. The top panels (a, b) provide profiles at \( Re = 9 \times 10^3 \) and \( Pr = 5 \) with \( \hat{\lambda}_T \) varying from 0 to 0.2. The second row of panels provides profiles at \( Pr = 5 \) and \( \hat{\lambda}_T = 0.1 \), and \( Re \) varying from \( 3 \times 10^3 \) to \( 1.5 \times 10^4 \). In the bottom row (e, f), \( Re = 9 \times 10^3 \) and \( \hat{\lambda}_T = 0.1 \) and \( Pr \) varies from 2 to 11.
As defined, $\theta$ approaches zero as the local temperature approaches the incoming jet temperature. For all $\theta$ profiles shown in Fig. 3.4 at $\hat{r} = 10$ (panels a, c, and e), the radial location corresponds to region III of the thin film. Recall that in region III, the hydrodynamic boundary layer has merged with the height of the thin film, while the thermal boundary layer has not. Thus, the profiles shown all level off at zero at sufficiently large $\hat{z}$, with the wall-normal location where this occurs being the thermal boundary layer thickness. $\theta$ thus remains zero through the rest of the thin film up to $\hat{z} = \hat{h}$. The variation in the curve lengths for each of the profiles shown corresponds to the differences in film thickness for each scenario. The results of panel (a) show that at fixed $Re$ and $Pr$, the magnitude of the $\theta$ profiles decrease with increasing $\hat{\lambda}_T$. It is beneficial to recall that here we have assumed that $\hat{\lambda}$ increases concomitant with $\hat{\lambda}_T$. With increasing $\hat{\lambda}$, the flow is energized and the momentum near the wall increases relative to the classical scenario. In the classical boundary layer, this would lead to a decrease in thermal resistance, corresponding to a thinner boundary layer. Increasing $Re$ leads to a similar decrease in $\theta$, resulting from similar dynamics (thinner boundary layer). Increasing $Pr$ also leads to a decrease in $\theta$ because as $Pr$ increases, the ratio of the diffusion rate of momentum normal to the wall to the thermal diffusion rate increases. Thus, the thermal boundary layer becomes thinner relative to the hydrodynamic boundary layer, decreasing thermal resistance at the wall. The above behaviors are illustrated in panels a, c, and e of Fig. 3.4.

Of course, the magnitude of $\theta$ increases with $\hat{r}$ as illustrated by comparing the profiles at $\hat{r} = 10$ (left panels) to the profiles at $\hat{r} = 30$ (right panels). The profiles at $\hat{r} = 30$ correspond to region IV of the thin film (excluding the no-slip case in panel (b) and the $Pr = 11$ case in panel (f), which remain in region III). Here, $\theta$ decreases as $\hat{z}$ increases but does not reach zero when $\hat{z} = \hat{h}$ because thermal effects have propagated through the entire film. Now $\theta_{f,s}$ increases with increasing $\hat{r}$, concomitant with an increasing temperature at the top of the film (due to increased thermal transport moving radially outward).

Attention is now turned to the local Nusselt number, which is presented as a function of $\hat{r}$ in Figs. 3.5 and 3.6. The Nusselt number definition for the isothermal and isoflux cases is identical, where $Nu = q''_w a \pi / (k(T_w(r) - T_j))$ as given earlier in Eq. 3.21. Fig. 3.5 includes three panels where $Re$ is varied from $3 \times 10^3$ to $1.5 \times 10^4$, with $Pr$ held constant at $Pr = 5$. Figure 3.6 shows two panels at $Re = 9 \times 10^3$ and with $Pr = 2$ (top) and 11 (bottom). Here we focus on the influence
of $\lambda T$ and results are shown in both Figs. 3.5 and 3.6 for $\lambda T$ varying from 0 to 0.2. In both figures, $Nu$ results are shown for the isoflux boundary condition corresponding to the current analysis and for the isothermal boundary condition scenario. The isothermal analysis utilizes the same model for the stagnation region and then utilizes the integral analysis reported previously in regions II through IV [51].

Several important observations may be made concerning the results shown in Figs. 3.5 and 3.6. First, $Nu$ decreases as $\hat{r}$ increases. Second, for all $\lambda T$, $Nu$ increases with increasing $Re$ and $Pr$. These two observations are consistent with classical jet impingement behavior and are not discussed further.

We now turn our attention to the influence of $\lambda T$ and the type of thermal boundary condition (isoflux or isothermal) on the magnitude and variation of $Nu$. For fixed values of $Re$ and $Pr$, $Nu$ decreases dramatically as $\hat{\lambda} T$ increases over the stagnation and initial radial flow region, while a modest increase is observed at larger $\hat{r}$. These results are expected based on the prior study that considered jet impingement on an isothermal superhydrophobic surface [51]. Note that $\hat{r}$ at which the transition from decreasing $Nu$ to increasing $Nu$ occurs depends on $Re$, $Pr$, and $\lambda T$. For example, the $Re$ dependence is apparent by comparing panels (a)–(c). We recall that $Pr = 5$ for these results and consider the $\lambda T = 0.1$ case. The radii marking the transition are $\hat{r} = 17$, 24, and 31 at $Re = 3 \times 10^3$, $9 \times 10^3$, and $1.5 \times 10^4$, respectively. The increase in $Nu$ at large $\hat{r}$ is caused by a decrease in the wall temperature relative to the no-temperature jump scenario. This increase in $Nu$ is small but appears in the average isoflux results discussed in Section 3.5.3.

For the isoflux case, $Nu$ decreases with increasing $\hat{\lambda} T$ due to the increased temperature difference (caused by the temperature jump) between the local wall temperature and the temperature of the impinging jet. For the isothermal case, this occurs because the local heat flux decreases (caused by the increased resistance of the air-filled cavities).

The data of Figs. 3.5 and 3.6 reveal that the decrease in $Nu$ with increasing $\hat{\lambda} T$ is greatest at small $\hat{r}$ (in the stagnation zone). To explain this result, consider the expression for the local Nusselt number given by Eq. 3.25, which has the same form regardless of the thermal boundary condition. When $\hat{\lambda} T = 0$, $Nu$ is simply inversely proportional to $\hat{\delta} T$. In the stagnation zone, $\hat{\delta} T$ has a relatively small value but then increases as $\hat{r}$ increases, leading to the classical decrease in $Nu$. When $\hat{\lambda} T > 0$, the denominator of Eq. 3.25 is increased by the magnitude of $\hat{\lambda} T$ and, in the stagnation zone, where
Figure 3.5: Local Nusselt number, $Nu$, as a function of $\hat{r}$ at $Pr = 5$ and $Re = 3 \times 10^3$ (top), $9 \times 10^3$ (middle), and $1.5 \times 10^4$ (bottom). $\lambda_T$ varies from 0 to 0.2 in each panel. The region I to II transition, region II to III transition, and region III to IV transition are marked with an asterisk, a plus sign, and a diagonal cross mark, respectively.
Figure 3.6: Local Nusselt number, Nu, as a function of $\hat{r}$ at $Re = 9 \times 10^3$ and $Pr = 2$ (top) and $Pr = 11$ (bottom). Results are shown for both the isoflux (current analysis) and isothermal boundary conditions [51] with $\hat{\lambda}_T$ varying from 0 to 0.2. $Pr = 2$ in panel (a) and $Pr = 11$ in panel (b). The region I to II transition, region II to III transition, and region III to IV transition are marked with an asterisk, a plus sign, and a diagonal cross mark, respectively.

$\hat{\delta}_T$ is a small value, this yields an incremental decrease in $Nu$. Recall that $\hat{\lambda}_T = 0.2$ is approaching the upper limit for realizable normalized temperature jump lengths and, for this value, the Nusselt number in the stagnation zone decreases by more than an order of magnitude.

The functional dependence of $Nu$ on $\hat{\lambda}_T$ and $\hat{\delta}_T$ shown in Eq. 3.25 explains other important behaviors that are demonstrated in Figs. 3.5 and 3.6. As $\hat{\lambda}_T$ increases, the resistance to thermal transport at the wall increases. Thus, depending on the magnitude of $\hat{\lambda}_T$, it can exercise much greater influence than $\hat{\delta}_T$ on the magnitude of $Nu$. Consequently, as $\hat{\lambda}_T$ increases, the $Nu$ profiles
show less dependence on $\hat{r}$. This is true for both thermal boundary conditions, where the data level off, showing less dependence on $\hat{r}$. Further, as $\hat{\lambda}_T$ increases, the influence of $Re$ and $Pr$ on the value of $Nu$ also diminish. Recall that variations in $Re$ and $Pr$ give rise to variations in the thermal boundary layer development and growth. Thus, when $\hat{\lambda}_T$ is large compared to $\hat{\delta}_T$, the influence of $Re$ and $Pr$ vanishes.

As a final point, we note that isothermal and isoflux results merge together as the temperature jump length increases. The Nusselt number for the isothermal case is always less than the isoflux case, consistent with classical behavior [35]. However, the present results demonstrate that increasing $\hat{\lambda}_T$ decreases and, ultimately eliminates, the influence of the heating condition (isoflux or isothermal) for similar reasoning as described above. When $\hat{\lambda}_T = 0$, $Nu$ for the two conditions are equal in the stagnation zone, but begin deviating immediately as $\hat{r}$ increases. When $\hat{\lambda}_T > 0$, the radial location where the two values of $Nu$ deviate moves outward. At $Re = 1.5 \times 10^4$, the two $Nu$ profiles deviate from each other at $\hat{r} \approx 1, 12, 25,$ and $36$ for the $\hat{\lambda}_T = 0, 0.05, 0.1,$ and $0.2$ scenarios, respectively. This point of departure depends on both $Re$ and $Pr$, since these parameters dictate the growth of $\hat{\delta}_T$.

Further insight into this behavior is obtained when considering the relative (percentage) difference between the isoflux and isothermal Nusselt numbers. This is expressed as $1 - Nu_T/Nu_F$, where $Nu_T$ is the local isothermal Nusselt number and $Nu_F$ is the local isoflux Nusselt number. Figure 3.7 presents the results of Fig. 3.5 using this relative difference.

Of course, the relative difference between the isoflux and isothermal Nusselt number is zero within the stagnation region, since the thermal transport here is identical [21]. Upon entering the radial flow region, the relative difference increases with increasing radius. This occurs because of the two terms in the denominator of Eq. 3.25. Moving outward with $\hat{r}$ results in an increased boundary layer thickness and, since the layers grow at different rates for the two thermal boundary conditions, the relative difference in $Nu$ increases. The temperature jump length exercises the same influence on $Nu$ for both thermal boundary conditions. Thus, as $\hat{\lambda}_T$ is increased, the relative difference in $Nu$ values increases more slowly with increasing $\hat{r}$. The results of Fig. 3.7 allow estimation of the radial extent over which the local $Nu$ values are the same for a range of parameters. Increasing $Re$ yields a decrease in the relative difference as well. In general, as $Re$ increases, the
Figure 3.7: The relative (percent) difference between the isoflux, $Nu_F$, and isothermal, $Nu_T$, local Nusselt numbers relative to the isoflux case is plotted as a function of $\hat{r}$. $Pr = 5$ and $Re$ varies from $3 \times 10^3$ to $1.5 \times 10^4$. $\hat{\lambda}_T$ varies from 0 to 0.2 in each panel.
size of the boundary layer thickness decreases as well and so in Eq. 3.25, the $\hat{\delta}_T$ term exercises less influence. Although not shown here, variations in $Pr$ yield similar behavior.

### 3.5.3 Average Results

In this section, we consider the radially averaged Nusselt number. Following standard approaches [21], for isothermal scenarios, the average Nusselt number is obtained as

$$\overline{Nu} = \frac{\int_A Nu(T_w - T_j) dA}{\int_A (T_w - T_j) dA} = \frac{\int_A Nu dA}{\int_A dA}$$  \hspace{1cm} (3.38)$$

and, for isoflux scenarios, the average Nusselt number is obtained as

$$\overline{Nu} = \frac{\int_A q'' w dA}{\int_A q''_{w} dA} = \frac{\int_A dA}{\int_A \frac{1}{Nu} dA}$$  \hspace{1cm} (3.39)$$

Figure 3.8 presents the average Nusselt number for the classical no-slip and no-temperature jump scenario so that comparisons may be made with the slip and temperature jump solutions. $\overline{Nu}$ is shown for both thermal boundary conditions as a function of $\hat{r}$ for scenarios where $\hat{\lambda}$ and $\hat{\lambda}_T$ equal zero and the Nusselt number for this scenario is written with the subscript “0” ($\overline{Nu}_0$). In panel (a), $Pr = 5$ and $Re$ varies from $3 \times 10^3$ to $1.5 \times 10^4$ and, in panel (b), $Re = 9 \times 10^3$ and $Pr$ varies from 2 to 11.

As has been well established previously, $\overline{Nu}_0$ increases with $Re$ and with $Pr$ because increasing either of these parameters decreases the thickness of the thermal boundary layer. Also, isothermal $\overline{Nu}_0$ is lower than isoflux $\overline{Nu}_0$ for small $\hat{r}$ but becomes greater at large $\hat{r}$. This is a result of averaging the local Nusselt number using the standard approach, where the local Nusselt number is weighted by the temperature difference between the wall and the jet, $T_w - T_j$, and radius (due to the circular area), as shown in Eqs. 3.38 and 3.39. For the isothermal case, $T_w - T_j$ is constant so $Nu$ is equally weighted due to temperature difference as $\hat{r}$ varies. For the isoflux case, $T_w - T_j$ increases with $\hat{r}$ and, consequently, $Nu$ at large $\hat{r}$ receives more weighting and, since these $Nu$ are smaller, $\overline{Nu}_0$ decreases. We display these classical results for completeness and to provide a basis of comparison for the slip and temperature jump scenarios below.
Figure 3.8: Average Nusselt number (for no-slip and no temperature jump scenarios), $\overline{Nu_0}$, as a function of $\hat{r}$. Isoflux (solid line) and isothermal (dashed line) results are shown for comparison. Panel (a) displays solutions at $Pr = 5$ with $Re$ varying between $3 \times 10^3$ and $1.5 \times 10^4$. Panel (b) displays solutions at $Re = 9 \times 10^3$ with $Pr$ varying between 2 and 11.

The relative decrease (percentage basis) in $\overline{Nu}$ is shown in Fig. 3.9 for $\hat{\lambda}_T$ varying from 0 to 0.2. $Pr = 5$ and $Re$ varies from $3 \times 10^3$ to $1.5 \times 10^4$ for both thermal boundary conditions. In each panel, the relative decrease is shown as a function of $\hat{r}$ at $\hat{\lambda}_T = 0.1$ and $\hat{\lambda}_T = 0.2$.

For all scenarios, the relative decrease in $\overline{Nu}$ becomes smaller with increasing radius. This occurs because the influence of the temperature jump length is greatest near the impingement point and, as the averaging radius increases, the area over which temperature jump length plays a negligible role increases. Thus, the impact of the temperature jump yielding a decreased $Nu$ near the impingement point is rapidly overcome. Additionally, as expected, the percentage decrease in
Figure 3.9: The relative decrease (percent) in $\overline{Nu}$ as a function of $\hat{r}$. In each panel, the relative decrease is shown for $\hat{\lambda}_T = 0.1$ and $\hat{\lambda}_T = 0.2$ compared to the $\hat{\lambda}_T = 0$ case. $Pr = 5$ for all results and $Re = 3 \times 10^3$, $9 \times 10^3$, and $1.5 \times 10^4$ in panels (a) through (c).
\( Nu \) is greater at higher \( \hat{\lambda}_T \) because there is more thermal resistance at the wall, leading to either a decrease in heat flux for the isothermal case or an increase in wall temperature for the isoflux scenario.

At small \( \hat{r} \), the percentage decrease for the isothermal and isoflux scenarios are nearly identical. The radial location where the behavior starts to deviate is a function of \( Re \), with increasing \( Re \) yielding a larger radial position where the results for the two boundary conditions deviate. An unexpected observation is made at large \( \hat{r} \). Here, the percentage decrease remains positive while approaching zero as \( \hat{r} \) increases for the isothermal scenario. In contrast, for the isoflux scenario, the percentage decrease in \( Nu \) becomes negative under certain conditions. This is more pronounced at smaller \( Re \) and indicates that \( Nu \) is actually increasing relative to the no-slip and no temperature jump case at large \( \hat{r} \). The \( \hat{r} \) location where this occurs is smaller for \( \hat{\lambda}_T = 0.1 \) than \( \hat{\lambda}_T = 0.2 \) and decreases as \( Re \) and \( Pr \) decrease. As noted earlier, the results with slip and temperature jump for both isoflux and isothermal scenarios become modestly larger than their no-slip and no temperature jump counterparts at large \( \hat{r} \). For the isoflux results, this difference receives greater weighting when isoflux \( Nu \) is averaged than when isothermal \( Nu \) is averaged. Consequently, the modest increase in \( Nu \) at large \( \hat{r} \) with increasing \( \hat{\lambda}_T \) affects \( Nu \) for the isoflux results and not the isothermal results. We note that the magnitude of \( Nu \) is small at large \( \hat{r} \) and, consequently, the actual increase in \( Nu \) is also very small.

Figure 3.10 presents \( Nu \) as a function of \( \hat{\lambda}_T \). Both isoflux and isothermal results are shown with panel (a) displaying results at \( Re = 3 \times 10^3 \) and, in panel (b), \( Re = 1.5 \times 10^4 \). Results are shown where \( Nu \) is averaged over 10, 20, and 30 radii.

As noted previously, \( Nu \) decreases as \( \hat{\lambda}_T \) increases. For the isoflux case, this occurs because the difference between the average wall temperature and the jet temperature increases and, for the isothermal case, the average heat flux decreases. At small \( \hat{\lambda}_T \), \( Nu \) is most sensitive to changes in averaging distance because the local \( Nu \) has not yet attained the more uniform profile achieved at larger \( \hat{\lambda}_T \). Similar to observations made concerning the average results for the no-slip scenario, \( Nu \) decreases as the averaging radius increases since the area with smaller \( Nu \) exercises greater influence. As \( Re \) increases \( Nu \) increases for all cases because the thermal boundary layer thickness becomes thinner. However, as illustrated in panel (b), at large \( \hat{\lambda}_T \), the results become independent of the averaging distance due to the uniform behavior of \( Nu \) at large \( \hat{\lambda}_T \). Lastly, the difference in
Figure 3.10: $\overline{Nu}$ as a function of $\hat{\lambda}_T$ at $Pr = 5$ for $\hat{r}$ varying from 10 to 30. In panel (a), $Re = 3 \times 10^3$ and, in panel (b), $Re = 1.5 \times 10^4$.

$\overline{Nu}$ between the isoflux and isothermal results decreases as $\hat{\lambda}_T$ increases. This trend is expected since the local Nusselt number curves merge together with increasing $\hat{\lambda}_T$.

For completeness, Fig. 3.11 displays $\overline{Nu}$ (averaged over 15 jet radii) as a function of $\hat{\lambda}_T$ for both the isoflux and isothermal conditions. Panel (a) displays results at $Pr = 5$ and $Re = 3 \times 10^3$, $9 \times 10^3$, and $1.5 \times 10^4$. Panel (b) displays results at $Re = 9 \times 10^3$ and $Pr = 2$, 5, and 11. The physical reasoning that has already been given will not be repeated. These results are an additional compelling illustration that the influence of heating condition (isoflux or isothermal) and flow properties ($Re$ and $Pr$) disappear as $\hat{\lambda}_T$ increases.
Figure 3.11: $\overline{Nu}$ ($Nu$ averaged over 15 jet radii) as a function of $\hat{\lambda}_T$. Solutions for both isoflux and isothermal results are shown. Panel (a) presents results at $Pr = 5$ and $Re$ varying from $3 \times 10^3$ through $1.5 \times 10^4$. Panel (b) presents results at $Re = 9 \times 10^3$ for $Pr$ varying from 2 through 11.

### 3.5.4 Relative Importance of Slip Length and Temperature Jump Length on the Solution

On realizable SHPo surfaces, $\hat{\lambda}$ and $\hat{\lambda}_T$ exist as some ratio of each other (typically on the order of 1), depending mainly on the surface microstructure and weakly on the flow conditions [41]. It is beneficial to consider two hypothetical bounding scenarios: one where $\hat{\lambda}_T = 0$ and $\hat{\lambda} \neq 0$ and the other where $\hat{\lambda} = 0$ and $\hat{\lambda}_T \neq 0$. Results corresponding to these two conditions provide theoretical limits for $\overline{Nu}$ at a SHPo surface. A single scenario at $Pr = 5$ and $Re = 9 \times 10^3$ is shown in Fig. 3.12, where $\overline{Nu}$ (averaged over 15 jet radii) is presented as a function of $\hat{\lambda}$ and/or $\hat{\lambda}_T$. Three cases are shown: $\hat{\lambda}_T = 0$ and $\hat{\lambda} \neq 0$, $\hat{\lambda} = 0$ and $\hat{\lambda}_T \neq 0$, and $\hat{\lambda} = \hat{\lambda}_T$. When $\hat{\lambda}_T$ and $\hat{\lambda}$ are
equal and vary together, the curve tends towards the no-slip ($\hat{\lambda} = 0, \hat{\lambda}_T \neq 0$) result where only $\hat{\lambda}_T$ varies. This indicates that the influence of increased thermal resistance resulting from the larger $\hat{\lambda}_T$ is much greater than the enhanced advection due to the slip boundary condition.

![Graph showing Nu as a function of $\hat{\lambda}$ and/or $\hat{\lambda}_T$. Three scenarios are considered $\hat{\lambda} = 0$ and $\hat{\lambda}_T \neq 0$, $\hat{\lambda}_T = 0$ and $\hat{\lambda} \neq 0$, and $\hat{\lambda}_T = \hat{\lambda}$. Pr = 5 and Re = 9 × 10^3 for all cases.]

In contrast, when $\hat{\lambda}_T = 0$, much greater $\overline{Nu}$ is achievable due to the slip boundary condition enhancing advection in the absence of temperature jump. Here, the isoflux surface has a larger $\overline{Nu}$ than the isothermal surface (nominally 7% relative to the isothermal surface). $\overline{Nu}$ for the isoflux scenario is greater than the isothermal scenario because the slip length increases the slip velocity at the surface and, in the absence of the thermal resistance imposed by a temperature jump length, the isoflux and isothermal heating conditions exhibit different convection coefficients. However, the $\hat{\lambda}_T = 0$ situation is not physically realizable since all non-wetted SHPo surfaces will exhibit a temperature jump due to the presence of air-filled cavities. Thus, this result is provided solely for illustration.

### 3.6 Conclusions

An integral analysis of a liquid jet impinging at an isoflux superhydrophobic surface has been performed. This analysis demonstrates that the introduction of isotropic slip length and tem-
temperature jump length to a surface with an isoflux heating condition has similar impact on the thermal transport as the introduction of the same conditions to a surface with an isothermal heating condition.

- The local and average Nusselt numbers are dramatically reduced with the average Nusselt number (over 15 jet radii) decreasing by up to 80%.

- Local Nusselt number profiles level out, decreasing most significantly with increasing temperature jump length at small radii.

- Flow conditions (Reynolds and Prandtl numbers) exert smaller influence on thermal transport.

We emphasize this work’s novel observation that the thermal resistance added by increasing the temperature jump length quickly overcomes the influence of the wall heating condition (isoflux or isothermal) on the Nusselt number such that this dependence on heating condition vanishes at large $\hat{\lambda}_T$. 
CHAPTER 4. EXPERIMENTAL EXPLORATION OF THERMAL TRANSPORT DUE TO LIQUID JET IMPINGEMENT ON SUPERHYDROPHOBIC SURFACES

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4.2 Abstract

This paper presents an experimental exploration of thermal transport due to jet impingement on post patterned superhydrophobic surfaces. Results are obtained for a smooth hydrophobic surface and superhydrophobic surfaces with varying microfeature pitch ($w = 8, 16, \text{ and } 24 \mu m$) and cavity fraction ($F_c = 0.56 \text{ and } 0.85$) and are compared to results derived from a previously published analytical model. Further, the Reynolds number varies from $1.1 \times 10^4 \text{ to } 1.7 \times 10^4$ and the surface heat flux varies from $2.5 \times 10^4 \text{ to } 4.9 \times 10^4 \text{ W/m}^2$. Experimental results obtained for surfaces with smaller pitch and cavity fraction display a significant decrease in Nusselt number (up to 20% reduction) and agree well with the analytical model predictions when the ratio of the temperature jump length to the slip length is $3.1 \pm 0.3$. Surfaces with larger pitch and cavity fraction yield a decrease in the Nusselt number that is less than that predicted by the model. We propose that this phenomenon is a result of dynamic, partial wetting of the microcavities.

4.3 Introduction

Impinging liquid jets achieve large heat transfer coefficients allowing them to effectively cool high-density electronics and quench materials [21]. Superhydrophobic (SHPo) surfaces dramatically alter the hydrodynamics of jet impingement [22–24]. Further, analytical models suggest
that the thermal transport at SHPo surfaces radically departs from classical behavior [51, 69]. These predictions have not yet been empirically explored.

To introduce this work, we summarize important classical observations concerning jet impingement hydrodynamics and thermal transport. Next, SHPo surfaces are introduced and the phenomena which lead to radical departure from classical behavior are discussed. Finally, we introduce how this work resolves unanswered questions concerning thermal transport at SHPo surfaces.

Important flow features are illustrated schematically in Fig. 4.1 which depicts a radial cross section of an impinging jet. A cylindrical coordinate system is introduced with axial coordinate, $z$, and radial coordinate, $r$. Four features are important to note. First, a stagnation region forms directly beneath the jet [26]. In this region (indicated region I in Fig. 4.1), the pressure is greater than atmospheric pressure since jet kinetic energy is being converted to pressure rise in the stagnating flow.

Second, viscous effects penetrate into the liquid. The distance of this penetration is the boundary layer thickness, $\delta$, which is shown as the upper horizontal line in the stagnation region.
As the flow moves away from the impingement point, the pressure decreases to atmospheric pressure, nominally within one jet diameter [26].

Third, the return to atmospheric pressure marks the beginning of the radial flow regime (regions II and III). $\delta$ begins to develop since the pressure gradient is absent. The boundary layer grows until it merges with the liquid film height, $\zeta$. At this point, region III begins. The effects of the wall have diffused through the entire film and the free surface velocity begins to decrease with radial position.

Fourth, if a downstream depth is imposed, the thin film flow transitions to downstream conditions by means of a circular hydraulic jump. Rapidly moving flow slows at this abrupt increase in height [58]. Surface tension dominates gravity for flows on hydrophobic or SHPo surfaces without an imposed downstream depth and, instead of a hydraulic jump, the spreading film breaks up into droplets for sufficiently large Weber number [24].

Obtaining the heat transfer coefficient as a function of radius is an objective of exploring thermal transport in an impinging jet. Models for thermal transport due to jet impingement at surfaces with isothermal and isoflux heating have been presented [31–33]. Experimental work has reported the local heat transfer coefficient for laminar and turbulent jets impinging isoflux surfaces [34, 59]. The heat transfer coefficient is largest in the stagnation region where it is independent of radius. In the radial flow region, the coefficient decreases rapidly as radius increases. Greater decrease with increasing radius occurs while the thermal boundary layer thickness is developing, with less decrease occurring once the thermal boundary layer has merged with the film thickness. The wall heating condition (isoflux or isothermal) influences thermal transport with the isoflux (uniform heat flux) scenario maintaining a modestly larger heat transfer coefficient than the isothermal scenario.

Having considered thermal transport at classical surfaces, we turn our attention to SHPo surfaces, which combine hydrophobic chemistry and micro/nanoscale patterning. Water does not wet the cavities between microfeatures if the liquid pressure is less than the Laplace pressure. Consequently, the working fluid encounters liquid-air and liquid-solid interfaces.

The no-slip condition is maintained at the liquid-solid interface. At the liquid-air interface, the velocity need not be zero and the shear imposed by the air on the liquid is vanishingly small [7]. The aggregate effect is an apparent slip velocity at the wall, $u_s$, which is proportional to shear stress,
and is expressed \( u_s = \lambda \tau_w / \mu \), where \( \lambda \) is the hydrodynamic slip length and \( \mu \) is the dynamic viscosity [8, 9].

Modeling of jet impingement at SHPo surfaces demonstrates that the hydraulic jump radius increases and the film thickness decreases as slip length increases. The effect of increasing \( \lambda \) is similar to that of increasing the Reynolds number [22–24].

The presence of air-filled microcavities at SHPo surfaces alters thermal transport. The thermal conductivity of the microfeatures for metallic substrates is several orders of magnitude greater than the thermal conductivity of the air. Thus, the liquid-air interfaces are nominally adiabatic while heat conducts primarily through the microfeatures and into the liquid via the liquid-solid interfaces. The aggregate effect of these alternating boundary conditions is an apparent temperature jump at the wall, \( \Delta T \). \( \Delta T \) is proportional to the wall heat flux, \( q''_w \), and may be expressed \( \Delta T = q''_w \lambda_T / k \), where \( \lambda_T \) is the wall temperature jump length and \( k \) is the liquid thermal conductivity [19, 40].

Analytical models predict that thermal transport dramatically decreases as temperature jump length increases [51, 69]. Further, the influence of \( Re, Pr, r \), and heating conditions (isothermal or isoflux) decreases as \( \lambda_T \) increases.

This paper proceeds as follows: We first describe the experimental facility, sample fabrication, and empirical method. Next, we discuss the conduction model, which was implemented to account for conjugate heat transfer in the silicon wafer on which the SHPo surface was fabricated. Finally, we present the dependence of the local temperature profile and the average Nusselt number on the surface microstructure geometry.

### 4.4 Experiments

#### 4.4.1 Experimental Facility

The experimental facility is illustrated schematically in Fig. 4.2 and principle components are the water supply, test section (Fig. 4.3), power supply, and cameras.

The pressurized water supply is considered. A pressure vessel with an air bladder was filled with deionized water and the air bladder was pressurized with compressed air. The house air supply was regulated before entering the tank. Water left the tank through tubing, which directed
Figure 4.2: An illustrative schematic displays the jet impingement setup.

it to a variable area flow meter (rotameter) with a valve. Tubing after the flow meter directed the flow through a syringe and blunt needle creating a jet directed normally towards the heated test surface. The water spread across the surface, falling over the wafer edge, and was collected by a custom 3D printed enclosure, which directed the flow into a drain. The needle was inserted into a stiff aluminum sleeve (syringe holder) to ensure that it remained oriented normal to the surface.

The needle had a diameter of 2.25 mm and a length of 127 mm. The ratio of length to diameter was 56, which is much longer than the entrance length to diameter ratio, 21, for the highest jet Reynolds numbers considered.

The sample wafer (100 mm in diameter) was secured with two weights, on opposite sides of the wafer, which slid down vertical tracks on the enclosure walls and pressed down on the wafer through silicone gasket material. The wafer was supported from beneath by a silicone foam gasket resting on a steel pipe. The steel pipe was supported by the 3D-printed enclosure.

The wafers were exchangeable. Different SHPo micropatterns were fabricated on each. A silver electric resistance heater, nominally 0.25 Ω, 51 mm in diameter, was fabricated by screen printing on the side opposite the micropatterning. Lead wires were attached to the electric resistance heater at tabs with silver epoxy (Atom Adhesives AA-DUCT 2979). Then, the side with the
heater was painted flat black (Rust-Oleum® 248903). The dried paint had an emissivity of 0.97. Details concerning the fabrication process are included in Appendix B.

The resistance heater leads were connected to a power supply with a maximum output of 20 V and 120 A. A voltmeter measured the voltage drop across the heater directly with two independent leads carrying negligible current. Current was measured by the power supply ammeter which was tested prior to use.

A thermal camera (FLIR SC6100) was mounted vertically on a track beneath the wafer so that it viewed the flat black (heater) side of the wafer. A digital single lens reflex (DSLR) camera
(Nikon D5200) was positioned to view the impinging jet from above, nominally 30 degrees from the surface normal.

### 4.4.2 SHPo Surfaces

Standard micro-machining techniques (contact photolithography and deep reactive ion etching) were utilized to fabricate micropost arrays on silicon wafers. Following etching and photore sist liftoff, a 550 nm thick oxide layer was grown on the wafer for electrical passivation. Then silver heaters were screen printed and fired on the bottom side (side without microstructure). A 100 nm chromium film was deposited on the microfeatured side so that the subsequent film, amorphous fluoropolymer (Teflon™ AF 1601) with a nominal thickness of 100 nm applied by spin coating, could adhere. Details concerning the fabrication process are included in Appendix B.

The micropost arrays had a square grid with cylindrical posts. A SEM image displays the post microstructure at these surfaces in Fig. 4.1. Geometric parameters diameter \(d\), height \(h\), and pitch \(w\), the center to center distance between surface microfeatures, are shown. The nominal post height, \(l\), was 20 \(\mu m\).

An important parameter to characterize the surfaces is cavity fraction. Cavity fraction, \(F_c\), is the ratio of projected cavity area to surface area. For circular posts, \(F_c\) can be calculated, \(F_c = (w^2 - \pi d^2 / 4) / w^2\).

Two other parameters are important: the slip length, \(\lambda\), and sessile contact angle, \(\phi\). Slip lengths were calculated using the formula

\[
\lambda = w \left( \frac{e}{\sqrt{1 - F_c}} + f \right)
\]

where \(e = 0.1555\) and \(f = 0.132\) are empirically fitted constants [76]. This formula reports slip length for channel flow past post patterned superhydrophobic surfaces. It is utilized for jet impingement flow because it is the best available correlation. The reported contact angles were calculated utilizing the Cassie equation [20].

The cavity fraction, pitch, slip length, and contact angle of surfaces considered in this study are reported in Table 4.1. Note that the first surface type listed is smooth hydrophobic and all subsequent surfaces are micropatterned surfaces. Pitch and cavity fraction have a tolerance of
± 0.3 μm and ± 0.02, respectively. φ has a tolerance of ± 3°. Utilizing propagation of error, λ has a tolerance of ± 2 μm.

The wafer was masked at the photolithography step so that a smooth hydrophobic circle, 6 mm in diameter, was formed at the wafer center. Jets were directed at this circle ("target") to prevent wetting due to the high stagnation pressure.

### Table 4.1: Surfaces considered for jet impingement experiments

<table>
<thead>
<tr>
<th>Label</th>
<th>Type</th>
<th>$F_c$ (±0.02)</th>
<th>$\mu$ (±0.3 μm)</th>
<th>$\lambda$ (±2 μm)</th>
<th>$\phi$ (±3°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>HPo</td>
<td>N/A</td>
<td>N/A</td>
<td>0</td>
<td>119</td>
</tr>
<tr>
<td>SH1</td>
<td>SHPo</td>
<td>0.56</td>
<td>16</td>
<td>5.9</td>
<td>141</td>
</tr>
<tr>
<td>SH2</td>
<td>SHPo</td>
<td>0.85</td>
<td>8</td>
<td>4.3</td>
<td>157</td>
</tr>
<tr>
<td>SH3</td>
<td>SHPo</td>
<td>0.85</td>
<td>16</td>
<td>8.5</td>
<td>157</td>
</tr>
<tr>
<td>SH4</td>
<td>SHPo</td>
<td>0.85</td>
<td>24</td>
<td>12.5</td>
<td>157</td>
</tr>
</tbody>
</table>

#### 4.4.3 Method

Acquiring experimental data points consisted of imaging a standard for spatial calibration, measuring jet temperature, and adjusting flow rate and heater power for each sample.

The variable area flow meter (rotameter) was calibrated by collecting water for one minute and measuring the accumulated mass, subtracting the container’s initial mass, and dividing by density.

A wafer with circular standards, printed on both sides was imaged by the thermal camera (FLIR® SC6100) and the top view camera (Nikon D5200) and these images were utilized to convert pixel distances to physical distances. These images were acquired each time a camera was moved. The thickness of the flat black paint layer negligibly altered the accuracy of the thermal camera temperature measurement because the temperature drop across the paint layer was small. The heat flux through the layer is limited by the large thermal resistance of the quiescent air below the heated plate. Further, the Biot number for the paint layer is $Bi_{paint} = h_{air}L_{paint}/k_{paint} = 0.07$, where $h_{air} = k_{air}/L_{air}$ is the convection coefficient to the air (which is conduction dominated), $L_{paint}$ is the paint thickness, and $k_{paint}$ is the paint thermal conductivity. Further, we note that axial temperature variation in the wafer is negligible as shown in Section 4.5.1.
A sample wafer was installed and the flow rate was set. A thirty-second IR video was acquired with the water jet impinging on the unheated wafer with a flow rate between 12 and 18 mL/s. This procedure was performed each time the tank was refilled so that the jet initial temperature could be determined. Experiments were completed within a few hours of taking the initial temperature measurement during which the change in tank temperature was negligible. For all videos, the thermal camera frame rate was 30 frames per second (fps). The top view camera frame rate was 60 fps.

Experimental data was acquired following these calibrations. The desired flow rate was achieved by adjusting the rotameter valve. Then, the power supply was turned on and the desired power output set by adjusting the output voltage. A thirty-second video was recorded with the thermal camera. Finally, a short video was acquired with the top view camera.

Uncertainties in flow and heating conditions are reported in Table 4.2. Directly measured variables in the experiment were temperature, current, voltage, nozzle diameter, heater diameter, and volumetric flow rate. The errors in the other parameters were obtained with propagation of error.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Symbol</th>
<th>Values</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Rate</td>
<td>mL/s</td>
<td>Q</td>
<td>12, 15, 18</td>
<td>±0.2 mL/s</td>
</tr>
<tr>
<td>Power</td>
<td>W</td>
<td>P</td>
<td>50, 75, 100</td>
<td>±2</td>
</tr>
<tr>
<td>Heat Flux</td>
<td>W/m²</td>
<td>q''</td>
<td>(2.5, 3.7, 4.9)×10⁴</td>
<td>±0.1 ×10⁴</td>
</tr>
<tr>
<td>Nozzle diameter</td>
<td>mm</td>
<td>d</td>
<td>2.25</td>
<td>±0.01</td>
</tr>
<tr>
<td>Initial liquid temperature</td>
<td>K</td>
<td>T_j</td>
<td>295–299</td>
<td>±1</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>N/A</td>
<td>Pr</td>
<td>6.6</td>
<td>±0.2</td>
</tr>
<tr>
<td>Reynolds number ((d = 2.25 \text{ mm}))</td>
<td>N/A</td>
<td>Re</td>
<td>(1.1, 1.4, 1.7)×10⁴</td>
<td>±0.1 ×10⁴</td>
</tr>
<tr>
<td>Weber number ((d = 2.25 \text{ mm}))</td>
<td>N/A</td>
<td>We</td>
<td>141, 220, 317</td>
<td>±20</td>
</tr>
</tbody>
</table>

### 4.4.4 Data Reduction

The video acquired for the unheated case was averaged for twenty frames and then subtracted from the IR images of heated surfaces.

The temperature profile of the jet was nominally axisymmetric but exhibited slight angular dependence due to surface imperfections and temporal fluctuations. To account for this asymmetry,
we spatially averaged twenty equally-spaced radial profiles. Lines indicating the coordinates where temperature data was extracted are drawn on an IR image in Fig. 4.4. For each data point, the profiles for twenty frames were accumulated and averaged.

Figure 4.4: A thermal image of a heated silicon wafer with uniform heat flux, \( q''_w = 3.7 \times 10^4 \), impinged by a liquid jet (\( Re = 1.4 \times 10^4 \)) is displayed. Line segments mark where radial temperature profiles were obtained.

4.5 Conduction Model

Radial conduction in the silicon wafer is non-negligible and a model was developed to account for this effect.

4.5.1 Analysis

Shown in Fig. 4.5 is a schematic illustration of the conduction problem which was solved to obtain the wafer temperature profile, \( T(r) - T_j \), when a convection coefficient profile, \( h(r) \), is specified at the top surface. The hydraulic jump or breakup radius is \( R_1 \), the heater radius is \( R_2 \),
and the wafer radius is $R_3$. In general, the breakup radius may be larger than the heater radius and the model developed here was implemented for both scenarios ($R_1 < R_2$ and $R_1 > R_2$) although only the $R_1 < R_2$ scenario is shown here.

![Figure 4.5: Conduction model utilized to model axisymmetric conjugate heat transfer in the silicon wafer.](image)

An axisymmetric model is considered since the impinging jet is circular and imposes an axisymmetric heat transfer coefficient distribution. Since the Biot number for the silicon wafer impinged by the jet is small ($Bi_{silicon} = h_{max}t/k_s = 0.07$, where $h_{max}$ is the convection coefficient at the stagnation point, $t$ is the wafer thickness, and $k_s$ is the silicon thermal conductivity), there is negligible temperature variation in the axial direction. Thus, the conduction is one-dimensional in the radial direction. A radial cross section of the computational domain is illustrated in Fig. 4.5. The domain is divided into regions: a cylindrical volume from $r = 0$ to $r = R_1$ (region A), an annular volume from $r = R_1$ to $r = R_2$ (region B), and an annular volume from $r = R_2$ to $r = R_3$ (region C).

The convection coefficient obtained by the impingement model for a given $Re$, $Pr$, $\hat{\lambda}$, and $\hat{\lambda}_T$, where “ $\hat{\quad}$ ” indicates normalization by jet radius, is appropriate for $r$ less than the breakup radius, $R_1$, which is predicted by a previously validated experimental model [24]. A uniform convection coefficient is applied at radial locations greater than the hydraulic jump or breakup radius.
radius. This coefficient is unknown and the appropriate value is determined iteratively. Loss due to free convection in the air beneath the heated wafer is negligible because upward flow is inhibited in an enclosure with a heated top. The wafer rests on silicone foam insulation on a circular pipe with a height of 4 cm. Without the convection flow, heat loss from the wafer into the air occurs primarily due to conduction, which is at most 0.2% of the power applied to the silver resistance heater.

An energy balance is applied to a differential ring control volume in each region, \( \Delta V = 2\pi rt \Delta r \). For illustration, the control volume for region A is displayed as a dashed line in Fig. 4.5. In the control volume analysis, \( q_r \) is the heat transfer rate in the radial direction, \( q_{\text{conv}} \) is the heat transfer due to convection, \( q_{\text{heater}} \) is the heat transfer supplied by the heater, and \( q_r + \Delta r \) is the radial heat transfer rate after a radial increment, \( \Delta r \). The results of this analysis are ordinary differential equations for regions A–C in Eq. 4.2–4.4, respectively.

\[
\begin{align*}
q''_h &= h(r)(T(r) - T_j) - k_s t \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (4.2) \\
q''_h &= h_2(T(r) - T_j) - k_s t \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (4.3) \\
0 &= h_2(T(r) - T_j) - k_s t \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (4.4)
\end{align*}
\]

where \( q''_h \) is the heat flux applied by the heater, \( t \) is the wafer thickness, \( k_s \) is the thermal conductivity of the wafer, \( h(r) \) is the convection coefficient as a function of radius, \( T(r) \) is the temperature of the wafer as a function of radius, \( h_2 \) is the constant convection coefficient imposed after the breakup diameter, and \( r \) is the radial position.

A finite temperature is required at \( r = 0 \). Temperature and heat rate are required to match at the cylindrical surface between region A and region B and region B and region C. The edge of the wafer is adiabatic.

The differential equations and boundary conditions are discretized utilizing finite-differencing and the resulting system of algebraic equations is solved utilizing a numerical linear algebra solver to obtain the radial temperature distribution. Code implementing this analysis is included as Appendix C.
4.5.2 Inverse Solution

Predicting the Nusselt number from the results requires an inverse solution of the conduction model. A parameterized profile for \( h(r) \) was obtained by identifying the functional form, which best matched the analytical model presented in Chapter 3. This approximate temperature profile and the conduction model were solved repeatedly in an optimization scheme to obtain the parameters at which the predicted temperature profile agreed best with the empirical data.

Assumed \( h(r) \) Profile

\( h(r) \) is modeled as the sum of two decaying exponentials.

\[
h(r) = C_1 e^{-k_1 r} + C_2 e^{-k_2 r} + C_3
\]

where \( C_1, C_2, C_3, k_1, \) and \( k_2 \) are arbitrary parameters. Fitting the function to the convection coefficient profiles predicted by the model using least squares showed excellent agreement across a wide range of parameters (\( 3 \times 10^3 \leq Re \leq 1.5 \times 10^4, 2 \leq Pr \leq 11, \) and at \( \hat{\lambda}_T = 0 \) and \( \hat{\lambda}_T = 0.05 \)).

Optimization

The conduction model was put in functional form receiving the \( h(r) \) profile parameters \( (C_1, C_2, C_3, k_1, \) and \( k_2) \) and the empirical temperature profile. It returned

\[
y = \sum \frac{(\theta_m(r) - \theta_e(r))^2}{\theta_e(r)}
\]

where \( \theta_m(r) = T_m(r) - T_j \) is the wall temperature profile predicted by the model and \( \theta_e(r) = T_e(r) - T_j \) is the wall temperature profile obtained experimentally. \( y \) is an objective function which approaches zero as the difference between \( \theta_e(r) \) and \( \theta_m(r) \) decreases. The Nelder-Mead simplex method, implemented in MATLAB® [77], was utilized to obtain the optimal values of \( C_1, C_2, C_3, k_1, \) and \( k_2 \).

For illustration, Fig. 4.6 shows results for a specific scenario \( (Re = 1.1 \times 10^4 \) and \( q'_w = 4.9 \times 10^4 \) W/m\(^2\)). Panel (a) displays the Nusselt number profile found by optimization, which
Figure 4.6: Panel (a) displays the local Nusselt number as a function of \( \hat{r} \) for a single experimental case, \( Re = 1.1 \times 10^4 \) and \( q''_w = 4.9 \times 10^4 \text{ W/m}^2 \). Panel (b) displays the difference between the wafer temperature and the jet temperature, \( T - T_j \), for the same case as a function of \( \hat{r} \).

minimizes the difference between the predicted temperature profile and empirical profile (panel (b)).

A sensitivity study was performed to determine the influence of \( C_1, C_2, C_3, k_1, \) and \( k_2 \) on the temperature profile obtained when solving the conduction analysis with a given \( h(r) \) profile. Optimal parameters for \( h(r) \) for the no-slip surface at \( Re = 1.41 \times 10^4 \) and at \( 1.69 \times 10^4 \) were found by solving the inverse conduction problem.

Then the \( h(r) \) parameters were each increased by 10% while the other parameters were held constant at their optimal values and the conduction problem was solved for each scenario.
The temperature profile predicted by the model depended most strongly on one of the exponential terms with a 10% change in the leading coefficient altering the temperature profile by up to 9% and a 10% change in the exponent yielding a 7% change in the temperature profile.

4.6 Results and Discussion

Empirical results are now presented and utilized in conjunction with a previously published isoflux impingement model [69] to obtain the temperature jump length to slip length ratio. This is accomplished using average Nusselt number results, which are presented at the conclusion of this section. First, the local wall temperature profiles for smooth hydrophobic surfaces are considered as a benchmark. Next, the wall temperature profiles for superhydrophobic surfaces are reported. Finally, the influence of superhydrophobic surfaces on the average Nusselt number is presented and the temperature jump length to slip length ratio is estimated.

4.6.1 Local Temperature Profiles

No-slip Condition (Benchmark)

In Fig. 4.7, the non-dimensional wall temperature, \( \theta = k(T - T_j)/(q''_w a \pi) \), is plotted as a function of normalized radial position, \( \hat{r} = r/a \), at \( Re = 1.1 \times 10^4 \), \( Re = 1.4 \times 10^4 \), and \( 1.7 \times 10^4 \). We emphasize that the wall temperature varies only in the radial direction and is uniform in the \( z \)-direction as shown in Sec. 4.5.1. Experimental results are indicated by solid lines and model results by dashed lines. The wall temperature profile was obtained by solving the conduction model defined in Sec. 4.5.1 subject to the jet impingement convection coefficient condition predicted by the isoflux impingement model (Chapter 3) under the same experimental conditions. The nominal error in the empirical value of \( \theta \), \( \pm 1 \times 10^{-3} \), is indicated by error bars on markers.

Of course \( \theta \) increases with increasing \( \hat{r} \) since the local convection coefficient is decreasing and \( \theta \) decreases with \( Re \) at all \( \hat{r} \) due to enhanced advection at the surface. We observe excellent agreement between the experiments and the model, where the average error of the model is 7% relative to the experiments.
Figure 4.7: \( \theta = k(T - T_j)/(q''_w a \pi) \) is plotted as a function of normalized radial position, \( \hat{r} = r/a \). Experimental results are plotted as solid lines and the model results as dashed lines. The jet diameter is 2.25 mm. Three jet Reynolds numbers, Re, are considered with values of \(1.1 \times 10^4\), \(1.4 \times 10^4\), and \(1.7 \times 10^4\). Error bars on the markers indicate a nominal error of \(\pm 1 \times 10^{-3}\).

**Influence of Superhydrophobic Surfaces**

Having benchmarked the results, we consider the influence of superhydrophobic surface microstructure on jet impingement thermal transport. In Fig. 4.8 panel (a), the model prediction for the non-dimensional wall temperature, \( \theta \), is presented as a function of \( \hat{r} \) at \( \hat{\lambda}_T = 0, 0.012, \) and \(0.016\). Model results including the effect of the target are indicated with dashed lines, while dotted lines indicate model results without the target.

In panel (b), experimental results are plotted on identical \( \theta \) and \( \hat{r} \) axes for the HPo surface and two SHPo surfaces, SH1 (\(w = 8 \mu m\) and \(F_c = 0.85\)) and SH2 (\(w = 16 \mu m\) and \(F_c = 0.56\)) surface. Error bars indicate a nominal error of \(\pm 1 \times 10^{-3}\).

The temperature jump lengths selected for panel (a), excluding the no-slip case, are approximate values and were calculated with the microstructure parameters for the surfaces considered in panel (b), SH1 (\(w = 8 \mu m\) and \(F_c = 0.85\)) and SH2 (\(w = 16 \mu m\) and \(F_c = 0.56\)), by utilizing a slip length correlation for channel flow, \( \hat{\lambda} = w/a(e/\sqrt{1 - F_c} + f) \) where \( e = 0.1555 \) and \( f = 0.132 \) [76], and approximating that \( \hat{\lambda}_T = 3.1 \hat{\lambda} \). \( \hat{\lambda}_T/\hat{\lambda} = 3.1 \) was determined using average results and this approach is discussed further in Sec. 4.6.2. Further, we note that the channel flow correlation for slip
Figure 4.8: The non-dimensional wall temperature, $\theta$, is plotted as a function of normalized radial position, $\hat{r}$. The jet diameter is 2.25 mm and $Re = 1.4 \times 10^4$. Model results are shown in panel (a) at temperature jump lengths normalized by jet radius, $\hat{\lambda}_T$, of 0, 0.012, and 0.016. Results indicated with dashed lines include the influence of the smooth hydrophobic “target” while results with dotted lines do not model the target. Experimental results are shown in panel (b) for HPo and SHPo surfaces SH1 ($w = 8 \, \mu m$ and $F_c = 0.85$) and SH2 ($w = 16 \, \mu m$ and $F_c = 0.56$). Error bars on the markers indicate a nominal error of $\pm 1 \times 10^{-3}$. 

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length is the best available and allows dependence on slip length and cavity fraction to collapse to a single axis.

At this point, we emphasize that the model results and experimental results are displayed on different panels, as is done in the present figure (Fig. 4.8) and in a later figure (Fig. 4.11), to indicate that the temperature jump length, $\hat{\lambda}_T$, is not known but is instead found empirically by matching the model with the experiments, utilizing the approach described later (Sec. 4.6.2).

Two observations are important to make concerning Fig. 4.8 panel (a). First, $\theta$ increases as $\hat{\lambda}_T$ increases due to the increasing thermal resistance at the surface, which results from the greater pitch and cavity fraction. Second, the influence of the 6 mm diameter hydrophobic target, is limited to $\hat{r} < 5$. We observe a decrease in $\theta$ due to the hydrophobic target for the $\hat{\lambda}_T > 0$ cases over $0 \leq \hat{r} \leq 2$, which becomes less than 5% for $\hat{r} > 5$. The decrease in $\theta$ results from the increased heat transfer coefficient at the smooth hydrophobic target region. Despite the substantial decrease in $\theta$, up to 50% near the impingement point, the average Nusselt number (introduced in Section 4.6.2) over 15 jet radii increases by less than 1%. The error estimate in the average Nusselt number is for the worst case scenario: the largest Reynolds number considered $1.69 \times 10^4$ and the largest normalized slip length, 0.0052. This combination leads to the greatest difference in the Nusselt number between the hydrophobic region and the superhydrophobic region.

Having considered the model results, we now turn our attention to the empirical results (panel (b)). The slip lengths predicted by the empirical correlation, $w/a(e/\sqrt{1-F_c+f})$, are 0.0038 and 0.0052 for SH1 and SH2, respectively. When multiplied by 3.1, these yield the temperature jump length at which the model results were evaluated in panel (a). $\hat{\lambda}_T/\hat{\lambda} = 3.1$ is a result of comparing the analytical results to the empirical results in Section 4.6.2. The empirical results indicate that $\theta$ increases as $w/a(e/\sqrt{1-F_c+f})$ increases, where the correlation is utilized to collapse the dependence of $\theta$ on $w$ and $F_c$. It is not set equal to $\hat{\lambda}$ to emphasize that $\hat{\lambda}$ for jet impingement is being approximated by a correlation for channel flow. One final comment is important to make concerning panel (b). The decrease in $\theta$ at small $\hat{r}$ predicted by the model when the target is present (panel (a)) is not clearly observed in the empirical results. We attribute the masking of this effect to experimental error.

Comparing panels (a) and (b) shows that the $\theta$ vs $\hat{r}$ trends match within $\pm 3 \times 10^{-3}$ and indicates good agreement between the model and experiments under the stated approximations.
4.6.2 Average Nusselt Numbers

Having considered the influence of the temperature jump length on the local temperature profiles, we now consider its influence on the average Nusselt number, \( \overline{Nu} \).

In Fig. 4.9, \( \overline{Nu}/\overline{Nu}_0 \) is plotted as a function of \( w/a(e/\sqrt{1-F_c+f}) \) where \( e = 0.1555 \) and \( f = 0.132 \) [76]. \( \overline{Nu}_0 \) is the Nusselt number for the no-slip and no temperature jump case. Results for the HPo case and SHPo cases (SH1, SH2, SH3, and SH4) are displayed. We recall that \( w = 16 \) \( \mu \)m and \( w = 24 \) \( \mu \)m for SH3 and SH4, respectively and that \( F_c = 0.85 \) for both surfaces.

\[
\overline{Nu}/\overline{Nu}_0 = 1
\]

for the HPo surface since all \( \overline{Nu} \) are normalized by \( \overline{Nu}_0 \). \( \overline{Nu}/\overline{Nu}_0 \) decreases as \( w/a(e/\sqrt{1-F_c+f}) \) increases at each \( Re \) for the SH1 and SH2 surfaces. For the SH3 and SH4 surfaces, \( \overline{Nu}/\overline{Nu}_0 \) begins to increase with \( w/a(e/\sqrt{1-F_c+f}) \). However, the model (displayed later in Fig. 4.11 panel (a)) indicates that \( \overline{Nu}/\overline{Nu}_0 \) should continue to decrease as \( \hat{\lambda}_T \) increases.

For all \( Re \) at SH3 and SH4, \( \overline{Nu}/\overline{Nu}_0 < 1 \). The higher Nusselt number indicates that the surface temperature is higher, although it is lower than the model prediction. We identify that the increase in \( \overline{Nu} \) may occur due to wetting of the microcavities and note that SH3 and SH4 are more...
susceptible to this phenomenon due to higher pitch and cavity fraction. If complete wetting of the cavities were occurring, then we would expect $\frac{\overline{Nu}}{Nu_0}$ to be at least 1, or slightly greater, since the posts would act as fins and increase the overall convection coefficient. Observing videos of the impinging jets indicate that some spatially, heterogeneous wetting occurs on SH3 and SH4 but the area wetted is small. We propose that partial wetting, the time-dependent displacement and deformation of the liquid-air interface contributes to the departure of $\frac{\overline{Nu}}{Nu_0}$ from the model and further justify this in the discussion which follows.

Images of impinging jets are shown in Fig. 4.10 to display wetted and non-wetted states. In each panel, jet impingement is shown at $Re = 1.4 \times 10^4$ on surfaces with $w = 16 \mu m$ and $F_c = 0.85$. Panel (a) displays impingement at a surface with a 4 mm diameter target region, while panel (b) displays impingement at a 6 mm diameter target. The 4 mm target, shown in panel (a), is an exception and is utilized here to illustrate wetting. 6 mm diameter targets, similar to that shown in panel (b), were utilized for all scenarios in this paper to reduce wetting. The darker regions are areas where the cavities have wetted (liquid-filled microcavities) and the lighter regions are areas where the cavities remain non-wetted (air-filled microcavities). The wetting is illustrated schematically below the image, where regions of wetting are shown at right and non-wetted regions are shown at left corresponding to behavior shown in the image. The relative difference in intensity of light reflected from the wetted and non-wetted regions depends on the angle of observation. Wetting can be detected by visual observation only when both states exist and this frequently occurs due to small surface imperfections.

In panel (b), the viewing angle (from the surface normal) has decreased and the wetted regions appear bright and the non-wetted regions appear dark. Further, we emphasize that the surface has a 6 mm target. We observe that wetting (bright region) is less prevalent and this behavior is consistent across the range of experimental conditions considered for SH3 and SH4. We propose that partial wetting is causing the observed decrease to be less than that predicted by the model. Partial wetting is the time-dependent displacement and deformation of the liquid-air interface and is illustrated in Fig. 4.10 panel (b), where liquid has penetrated to different depths in the microcavities and, in general, these interface shapes and positions vary temporally.
Wetting Non-wetting

Figure 4.10: Images of liquid jet impingement at $Re = 1.1 \times 10^4$ on a $w = 16 \mu m$ and $F_c = 0.85$ surface. Panel (a): impingement at a 4 mm target. Panel (b): impingement at a 6 mm target. Wetting is apparent in panel (a) and is illustrated schematically below the image. Wetting is less prevalent in panel (b) and the proposed partial wetting phenomenon is illustrated schematically below the image.

Numerical modeling indicated that significant displacement and deformation in the liquid-gas menisci spanning the microcavities occur due to turbulent pressure fluctuations [78] and we identify that this is likely to occur at the higher $Re$ considered in this work.

We now utilize the empirical average Nusselt number in conjunction with the analytical impingement models to predict the temperature jump length. In Fig. 4.11 panel (a), $Nu/Nu_0$ predicted by the model is plotted as a function of $\hat{\lambda}_T$ at $Re = 1.1 \times 10^4$, $1.4 \times 10^4$, and $1.7 \times 10^4$.

$Nu/Nu_0$ is plotted as a function of $w/a(e/\sqrt{1-F_c}+f)$ in panel (b) at the same $Re$ for the HPo surface and SH1 and SH2. We note that this data was previously shown in Fig. 4.9. SH3 and SH4 are excluded from this plot because they experience wetting and cannot be compared to the model, which assumes the non-wetted state.

As noted earlier, the temperature jump lengths selected for panel (a), excluding the no temperature jump scenario, were calculated at the microstructure parameters for the surfaces considered in panel (b), SH1 ($w = 8 \mu m$ and $F_c = 0.85$) and SH2 ($w = 16 \mu m$ and $F_c = 0.56$), by evaluating $\hat{\lambda} = w/a(e/\sqrt{1-F_c}+f)$ where $e = 0.1555$ and $f = 0.132$ and with $\hat{\lambda}_T = 3.1\hat{\lambda}$. 

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Figure 4.11: Average Nusselt number relative to no-slip scenario, $\bar{Nu}/\bar{Nu}_0$. In panel (a), model results are plotted as a function of $\hat{\lambda}_T$ at $\hat{\lambda}_T = 3.1\hat{\lambda}$. Results at three Reynolds numbers are displayed: $Re = 1.1 \times 10^4$, $Re = 1.4 \times 10^4$, and $Re = 1.7 \times 10^4$. In panel (b), experimental results for SH1 ($w = 8 \, \mu m$ and $F_c = 0.85$) and SH2 ($w = 16 \, \mu m$ and $F_c = 0.56$) are plotted as a function of $w/a(e/(1-F_c)^{1/2} + f)$ where $e = 0.1555$ and $f = 0.132$. 
As a first order approximation, \( \hat{\lambda}_T = \gamma \hat{\lambda} \), where \( \gamma \) is an arbitrary constant. \( \hat{\lambda}_T / \hat{\lambda} = \gamma = 3.1 \pm 0.3 \) was determined by solving the model at \( \hat{\lambda}_T / \hat{\lambda} \) varying from 1 to 4 (at increments of 0.1) and selecting the value of \( \hat{\lambda}_T / \hat{\lambda} \) for which \( \overline{Nu}/\overline{Nu}_0 \) predicted by the model best fit the data. The tolerance of \( \pm 0.3 \) is added to account for variation in \( \overline{Nu}/\overline{Nu}_0 \) due to \( Re \) dependence. Further, we note that \( \hat{\lambda}_T / \hat{\lambda} \) for microchannel flow [41] has the same order of magnitude as \( \hat{\lambda}_T / \hat{\lambda} \) obtained here.

Two observations are important to make concerning panel (a). First, \( \overline{Nu}/\overline{Nu}_0 \) decreases as \( \hat{\lambda}_T \) increases because the thermal resistance is increasing. Second, \( \overline{Nu}/\overline{Nu}_0 \) decreases as \( Re \) increases. This occurs because the temperature jump length causes a larger relative increase in thermal resistance when the advection for the no-slip case is larger (lower resistance).

We now compare the model results in panel (a) with the experimental results shown in panel (b). The trend in \( \overline{Nu}/\overline{Nu}_0 \) is similar. \( \overline{Nu}/\overline{Nu}_0 \) decreases from 1 to 0.77 as \( \hat{\lambda}_T \) increases from 0 to \( 15 \times 10^{-3} \) (panel (a)) or by the same amount as \( w/a(e/\sqrt{1-F_c}+f) \) increases from 0 to 0.0052 (panel (b)). This shows that \( \hat{\lambda}_T / \hat{\lambda} = 3.1 \pm 0.3 \) is a good estimate for the jet impingement scenario. Further, it validates the model’s prediction that \( \overline{Nu}/\overline{Nu}_0 \) decreases with \( \hat{\lambda}_T \). Although the model predicts that \( \overline{Nu}/\overline{Nu}_0 \) decreases with increasing \( Re \) at large \( \hat{\lambda}_T \), the experiments do not validate this because the observed variation in the model is less than the experimental error.

### 4.7 Conclusions

Jet impingement thermal transport at SHPo surfaces was studied by performing experiments at \( Re = 1.1 \times 10^4 \), \( 1.4 \times 10^4 \), and \( 1.7 \times 10^4 \) on smooth hydrophobic surfaces and post patterned SHPo surfaces with pitch varying from \( 8 \mu m \) to \( 24 \mu m \) and cavity fraction at 0.56 and 0.85. Results were reported as local temperature profiles and average Nusselt numbers.

The following conclusions are made concerning the experimental results and the model.

- The average Nusselt number decreases relative to the smooth surface at surfaces with small cavity fraction and pitch.

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1 The reader may ask why \( \hat{\lambda}_T / \hat{\lambda} = 1 \) was utilized for the models in Chapters 2 and 3 when the present experiment demonstrates that \( \hat{\lambda}_T / \hat{\lambda} = 3.1 \). \( \hat{\lambda}_T / \hat{\lambda} \) was unknown when the Chapters 2 and 3 analyses were completed. However, it was expected to be on the order of 1 [19]. Further, subsections in these chapters, Sections 2.5.3 and 3.5.4, demonstrate that the temperature jump length dominates the influence of the slip length on the Nusselt number. Consequently, the model predictions for the local Nusselt number are largely independent of the slip length.
• The model predicts a similar decrease with increasing temperature jump length for $\frac{\lambda_T}{\lambda_c} = 3.1 \pm 0.3$. Further, the experiments validate the trend of decreasing average Nusselt number with increasing temperature jump length predicted by the model.

• For micropatterned surfaces with larger pitch and cavity fraction, the average Nusselt number is less than the smooth hydrophobic surface but is greater than that predicted by the model. Since wetting is not visually prevalent at these surfaces, we hypothesize that this results due to partial wetting of the microcavities.
CHAPTER 5. THERMAL TRANSPORT DUE TO BUOYANT FLOW PAST A VERTICAL, HEATED SUPERHYDROPHOBIC SURFACE WITH UNIFORM STREAM-WISE SLIP

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5.2 Abstract

An analytical investigation of thermal transport due to laminar, buoyant flow past a vertical superhydrophobic (SHPo) surface was performed. The surface temperature is uniform and is greater than the temperature of the surrounding liquid. Uniform stream-wise hydrodynamic slip and temperature jump were imposed at the wall to model the SHPo surface. Applying an integral analysis within the boundary layer results in a system of differential equations, which was solved numerically to obtain boundary layer thickness and local and average values of the Nusselt number. The classical smooth hydrophobic scenario with no-slip and no temperature jump showed excellent agreement with previous analysis of the same problem. Solutions were obtained for laminar Rayleigh number ranging from $10^4$ to $10^9$ at a Prandtl number of 7. There was a modest decrease in the boundary layer thickness due to the increased slip length and the flow velocity decreased due to the increased temperature jump length. The local Nusselt number decreased as the temperature jump length increased, with greater reduction at larger Rayleigh numbers and near the lower edge. The decrease in the average Nusselt number relative to surfaces with no-slip and no temperature jump may be as much as 50%.
5.3 Introduction

Natural convection is a common phenomenon and is utilized in a variety of working fluids to cool electronics, power transformers, and internal combustion engines. It may also be an undesirable source of cooling in buildings (e.g. windows) and marine environments.

Consider a vertical wall of length, $L$, immersed in water as schematically illustrated in panel (a) of Fig. 5.1. Gravitational acceleration $g$ acts downward, parallel to the plate. The water far away from the wall is at a uniform temperature, $T_\infty$, and the wall is maintained at a uniform surface temperature, $T_w$, where $T_w > T_\infty$. The classical Prandtl number, $Pr = \nu/\alpha$ (where $\nu$ is the viscosity and $\alpha$ is the thermal diffusivity), is $Pr \approx 7$ since water is the working fluid. A coordinate system with an origin at the plate’s lower edge is introduced with the streamwise coordinate, $y$, and the wall normal coordinate, $x$. The hydrodynamic boundary layer thickness, $\delta(y)$, and thermal boundary layer thickness, $\delta_T(y)$, increase with $y$. As illustrated, $\delta$ grows more quickly than $\delta_T$ because $Pr > 1$. Typical velocity, $v(x,y)$, and temperature, $T(x,y)$, profiles are shown. Note that the temperature profile decreases monotonically, decreasing from $T_w$ to $T_\infty$ at $x = \delta_T$. The velocity profile satisfies the no-slip condition at the wall, increases to a maximum velocity, and then decreases returning to zero at $x = \delta$.

This classical problem was solved by Ostrach who presented solutions over a large range of $Pr$ ranging from 0.01 to 1000 [36]. The non-dimensional parameters governing this flow are the Grashof number and $Pr$. The Grashof number with respect to $L$ is $Gr_L = g\beta(\delta_T - \delta_f) L^3 / \nu^2$ (where $\beta$ is the volumetric thermal expansion coefficient). The product of $Gr_L$ and $Pr$ is the Rayleigh number, $Ra_L = g\beta(\delta_T - \delta_f) L^3 / \nu \alpha$. Results are presented as a function of $Ra_L$ in this analysis since the similarity solution, as well as the integral analysis in this work, are valid so long as the flow is laminar which condition holds for $Ra_L < 10^9$ [79]. Three observations are important to make concerning classical natural convection boundary layer flow. First, the thermal transport increases as $Ra_L$ increases because the buoyant force that drives the natural convection becomes larger. Second, the thermal transport increases as $Pr$ increases because the thermal boundary layer becomes thinner. Finally, the thermal transport is greatest at the lower edge and decreases as the thermal boundary layer thickness increases moving up the plate (increasing $y$).

Superhydrophobic (SHPo) surfaces have received much attention for their drag-reducing and self-cleaning behavior [6, 10, 11, 13, 23]. These surfaces are realized through micro/nano-
Figure 5.1: Schematic illustrations of natural convection boundary layers on heated vertical walls: HPo wall (panel (a)) and SHPo wall (panel (b)). The hydrodynamic and thermal boundary layer thicknesses, typical profiles, and a control volume for analysis are shown.

Structuring with hydrophobic surface chemistry. Hydrophobicity is characterized by solid-liquid contact angle. Smooth surfaces with a solid-liquid contact angle greater than 90° are hydrophobic and the maximum realizable contact angle for these surfaces is 120°. SHPo surfaces reach higher contact angles by combining hydrophobic chemistry with surface micro/nanostructure achieving contact angles ranging from 150° to nearly 180° [1].

SHPo surfaces modify the hydrodynamic and thermal wall boundary conditions. The water does not wet the cavities between microfeatures if the fluid pressure is less than the Laplace pressure and the cavities remain air-filled. The flow encounters the no-slip boundary condition at the liquid-solid interface at the top of the microfeatures. The velocity need not be zero at the liquid-air interface and a slip boundary condition arises which is nearly shear-free. The aggregate effect of these alternating boundary conditions is an apparent slip velocity, $v_s$, which is observed at the wall. $v_s$ is proportional to the wall shear stress, $\tau_w$, and is expressed as $v_s = \lambda \tau_w / \mu$, where $\lambda$ is the hydrodynamic slip length and $\mu$ is the liquid viscosity [8].
A velocity profile with slip velocity, \( v_s \), is illustrated schematically for the natural convection scenario in panel (b) of Fig. 5.1, where the wall has slip length, \( \lambda \). Note that the slope of the velocity profile has decreased at the wall since the shear stress resulting from the slip boundary condition decreases.

More recently, SHPo surfaces have received attention for thermal transport enhancing behavior in condensation heat transfer \cite{53,56,57} and thermal transport inhibiting behavior in forced convection \cite{18,19,39–41,51,54,67}, natural convection \cite{44–46}, and pool boiling \cite{47,48,69,80}.

Thermal transport at SHPo surfaces departs radically from classical flows due to the presence of air-filled cavities at the surface. If the microfeature material is metal, its conductivity is several orders of magnitude greater than the air in the cavities. Thus, heat conducts primarily through the microfeatures and enters the liquid at the liquid-solid interface, while the liquid-air interface is nominally adiabatic. The aggregate effect of these alternating boundary conditions is an apparent temperature jump at the wall, \( \Delta T_w \), which is proportional to the wall heat flux, \( q_w'' \), and this relationship is expressed as \( \Delta T_w = \lambda_T q_w'' / k \), where \( \lambda_T \) is the wall temperature jump length and \( k \) is the liquid thermal conductivity \cite{19,40,67}.

A temperature profile with temperature jump, \( \Delta T_w \), is illustrated schematically for the natural convection scenario in panel (b) of Fig. 5.1, where the wall now has temperature jump length, \( \lambda_T \).

Analytical models have been presented for fully-developed natural convection in three micro-channel geometries: vertical parallel plates, annuli, and polygonal ducts \cite{44–46}. These studies considered the influence of the slip and temperature jump boundary conditions. In the parallel plate study, one plate was SHPo and exhibited a slip length and temperature jump length. The other plate maintained the no-slip and no temperature jump conditions. The primary consideration was how the slip and temperature jump boundary conditions influenced the mass flow rate. The model demonstrated that there is a critical temperature jump length above which the flow rate increases by heating the side without a temperature jump rather than heating the side with a temperature jump. The papers reporting the annulus and polygonal duct geometries considered a heated inner wall (core) with the outer wall at ambient temperature and demonstrated that there is an optimal core diameter for maximum flow rate, which is a function of the slip and temperature jump lengths.
While the influence of SHPo surfaces on natural convection in internal flow has received attention, the scenario of a developing, natural convection boundary layer on a vertical, heated SHPo surface has not yet been considered and we present an original model here.

5.4 Analysis

The present work expands upon an integral analysis published by Bejan [81]. We start by approximating the velocity and temperature profiles with the expressions shown below.

\[
v = V \frac{x}{\delta} \left( 1 - \frac{x}{\delta} \right)^4 \quad (5.1)
\]

\[
T - T_\infty = (T(x = 0) - T_\infty) \left( 1 - \frac{x}{\delta T} \right)^2 \quad (5.2)
\]

where \( V \) is a scaling velocity which allows for variation in \( v \) with \( y \).

The velocity profile satisfies the boundary conditions: \( v(x = 0) = 0 \), \( v(x = \delta) = 0 \), and

\[
\left. \frac{\partial v}{\partial x} \right|_{x=\delta} = 0 \quad (5.3)
\]

The temperature profile satisfies the boundary conditions: \( T(x = 0) = T_w \), \( T(x = \delta_T) = T_\infty \), and

\[
\left. \frac{\partial T}{\partial x} \right|_{x=\delta_T} = 0 \quad (5.4)
\]

The \( x = 0 \) boundary conditions for both velocity and temperature profiles are modified to account for slip and temperature jump and these boundary conditions are expressed

\[
v(x = 0) = \lambda \left. \frac{\partial v}{\partial x} \right|_{x=0} \quad (5.5)
\]

\[
T_w - T(x = 0) = -\lambda_T \left. \frac{\partial T}{\partial x} \right|_{x=0} \quad (5.6)
\]

Applying these conditions, appropriate velocity and temperature profiles are obtained

\[
v = V \left( \frac{x}{\delta} + \frac{2\lambda + \delta}{2\lambda + \delta} \right) \left( 1 - \frac{x}{\delta} \right)^4 \quad (5.7)
\]
\[ T - T_\infty = (T_w - T_\infty) \left( 1 - \frac{2\lambda T}{\delta_T + 2\lambda T} \right) \left( 1 - \frac{x}{\delta_T} \right)^2 \] (5.8)

Conservation of momentum and energy are applied to a rectangular control volume of thickness, \( \Delta y \), height, \( X \), and unit depth which is shown in panel (b) of Fig. 5.1. Applying the Boussinesq approximation, \( (\rho_\infty - \rho(x,y)) \approx g\beta(T(x,y) - T_\infty) \), where \( \rho_\infty \) is the density of the quiescent liquid and \( \rho(x,y) \) is the density at a point, yields

\[ \frac{d}{dy} \int_0^X v^2 \, dx = g\beta \int_0^X (T - T_\infty) \, dx - v \frac{dv}{dx} \bigg|_{x=0} \] (5.9)

\[ \frac{d}{dy} \int_0^X v(T - T_\infty) \, dx = -\alpha \frac{dT}{dx} \bigg|_{x=0} \] (5.10)

The assumed velocity and temperature profiles are substituted into the conservation equations and the resulting equations are evaluated.

A non-dimensionalization is applied to the resulting equations. All lengths are normalized by the plate length, \( L \), and all velocities by the classical free convection scaling velocity, \( U \), which is defined \( U = \sqrt{g\beta(T_w - T_\infty)L} \). These normalizations are indicated by “\( \hat{\cdot} \)”. Additionally, \( Gr_L \) arises naturally from this non-dimensionalization. We define the temperature difference between any point and the quiescent fluid to be \( \theta = T - T_\infty \). When this difference is expressed for the wall temperature, a subscript “\( w \)” is added, \( \theta_w = T_w - T_j \). This allows the normalized temperature to be written \( \hat{\theta} = \theta / \theta_w = (T - T_j) / (T_w - T_\infty) \).

This analysis results in two ordinary differential equations (one resulting from the momentum balance and one resulting from the energy balance) with three unknowns: \( \delta \), \( \delta_T \), and \( \hat{V} \). To obtain closure, we recall that the ratio of the hydrodynamic boundary layer to the thermal boundary layer, \( q = \delta / \delta_T = \hat{\delta} / \hat{\delta_T} \) is solely a function of \( Pr \). We substitute \( \hat{\delta_T} = q\hat{\delta} \) into the equations yielding

\[ \frac{d}{d\hat{y}} \left( \hat{V}^2 \hat{\delta} (\hat{\delta}^2 + 15\hat{\delta}\hat{\lambda} + 81\hat{\lambda}^2) / 495(\hat{\delta} + 2\hat{\lambda})^2 \right) = -\frac{\hat{V}}{\sqrt{Gr_L}} \left( \hat{\delta} - 2\hat{\lambda} / \hat{\delta}(2\hat{\lambda} + \hat{\delta}) \right) + \frac{\hat{\delta}^2}{3q(2q\lambda_T + \hat{\delta})} \] (5.11)
\[
\frac{d}{d\hat{y}} \left( \frac{\hat{V} \hat{\delta}^2 (A\hat{\delta} + B\hat{\lambda})}{840q^6(\hat{\delta} + 2\hat{\lambda})(\hat{\delta} + 2q\hat{\lambda}_T)} \right) = \frac{1}{Pr\sqrt{GrL}} \frac{2q}{2q\hat{\lambda}_T + \hat{\delta}}
\]

(5.12)

where

\[
A = 5 + 2q(-16 + 7q(6 + q(-8 + 5q)))
\]

(5.13)

and

\[
B = 2(5 + 14q(-2 + q(4 + q(-2 + 5q(-1 + 2q))))))
\]

(5.14)

These equations may be solved numerically for \(\hat{\delta}\) and \(\hat{V}\) as functions of \(\hat{y}\) with initial conditions \(\hat{\delta}(\hat{y} = 1 \times 10^{-4}) = 1 \times 10^{-4}\) and \(\hat{V}(\hat{y} = 1 \times 10^{-4}) = 1 \times 10^{-4}\) and parameters \(Pr, q, GrL, \hat{\lambda},\) and \(\hat{\lambda}_T\). The initial conditions are set to be small but non-zero to avoid a numerical singularity at the lower edge. We obtain \(q = 3.73\) from the similarity solution for the no-slip and no temperature jump condition at \(Pr = 7\) [36].

The local wall heat flux is obtained by applying Fourier’s law to the temperature profile (Eq. 5.8).

\[
q''_w = \frac{2kq\theta_w}{\delta + 2\lambda_Tq}
\]

(5.15)

We obtain the local heat transfer coefficient, \(h\), by dividing Eq. 5.15 by \(\theta_w\) yielding

\[
h = \frac{2kq}{\delta + 2\lambda_Tq}
\]

(5.16)

The local Nusselt number is defined \(Nu_L = hL/k\). Substituting Eq. 5.16 into this definition yields

\[
Nu_L = \frac{2q}{\delta + 2q\lambda_T}
\]

(5.17)

The length-averaged Nusselt number, \(\overline{Nu}_L\), is calculated

\[
\overline{Nu}_L = \int_0^1 Nu_L d\hat{y}
\]

(5.18)

The \(y\)-coordinate Nusselt number is defined \(Nu_y = h\theta/y/k\) and is obtained for comparing the no-slip and no temperature jump results with the similarity solution. Substituting from Eq. 5.16
and applying the non-dimensionalization yields

\[
Nu_y = \frac{2q_y^*}{\delta + 2q\lambda_T^*}
\]  

(5.19)

The differential equation solver utilized was an explicit eighth-order Runge-Kutta method as implemented in the numerical solver software Mathematica®. Code implementing this analysis is included as Appendix D. The resulting numerical error in determining the local Nusselt number is at most \( \pm 7 \times 10^{-3} \).

5.5 Results and Discussion

We now introduce the results of this analysis. All results were obtained for \( Pr = 7 \). Water is the appropriate working fluid for superhydrophobic surfaces and this Prandtl number is equal to that of liquid water at atmospheric pressure and room temperature. First, we show agreement between the classical similarity solution and the present integral analysis for the no-slip and no temperature jump scenario. Next, we present local boundary layer thickness, scaling velocity, and velocity and temperature profiles. The local analysis concludes with results for the local Nusselt number. We discuss the decrease in Nusselt number with temperature jump length and identify the interesting result that this decrease is greater as \( Ra_L \) increases. Finally, we present the relative decrease in the average Nusselt number relative to the average Nusselt number for the no-slip scenario.

5.5.1 Benchmark: Nusselt Number for No-slip and No Temperature Jump Results

\( Nu_y \) is plotted as a function of \( Ra_y = g\beta \theta_w y^3/\nu \alpha \) in Fig. 5.2. The similarity solution (indicated by a solid line) shows that \( Nu_y \) increases with \( Ra_y \) following a 1/4 power law relationship [36]. Solutions obtained utilizing the present integral analysis at \( \lambda = 0, \lambda_T = 0, \) and \( Ra_L = 1 \times 10^4, 5 \times 10^6, \) and \( 1 \times 10^9 \) are plotted as line segments whose start and end are indicated by markers. Since the present integral analysis has at most a 1.1% error from the similarity solution, these results form a benchmark for the present analysis and justify the selection of the approximate temperature and velocity profiles.
Figure 5.2: \( N_u \) plotted as a function of \( Ra_y \). The solid line is obtained from the similarity solution [36]. Markers indicate the start and end of line segments at \( Ra_L = 1 \times 10^4 \), \( Ra_L = 5 \times 10^6 \), and \( Ra_L = 1 \times 10^9 \).

### 5.5.2 Local Behavior: Influence of Temperature Jump Length and Rayleigh Number

To begin, we show that the solution was obtained over a realizable range of temperature jump lengths. Slip lengths as large as 50 \( \mu \)m have been observed [82]. Assuming the smallest length scale of interest, \( L \), to be nominally 1 cm, then a maximum normalized slip length is \( 5 \times 10^{-3} \). Prior numerical analysis for microchannels suggest that \( \lambda_T/\lambda = 2 \) across a wide range of conditions [41].\(^1\) We assume this relationship for the present analysis. Thus, \( \hat{\lambda}_T \) varies from 0 to \( 10 \times 10^{-3} \). We note that references to variation in \( \hat{\lambda}_T \) will imply variation in \( \hat{\lambda} \).

We first consider the hydrodynamic boundary layer thickness, \( \hat{\delta} \), shown as a function of the normalized streamwise coordinate, \( \hat{y} \), in Fig. 5.3. Results are displayed at \( Ra_L = 1 \times 10^4 \), \( 5 \times 10^6 \), and \( 1 \times 10^9 \) and \( \hat{\lambda}_T = 0, 5 \times 10^{-3}, \) and \( 10 \times 10^{-3} \).

\(^1\)The reader may ask why \( \lambda_T/\lambda = 1 \) was utilized for the analytical models in Ch. 2 and 3, when Ch. 4 demonstrated that \( \lambda_T/\lambda = 3.1 \) for jet impingement flow and \( \lambda_T/\lambda = 2 \) was utilized for the present analysis. We emphasize that the temperature jump length is unknown for these flows so a suitable approximation is to utilize results from channel flow [41]. Further, we note that the jet impingement flow temperature jump length is likely to be different than the temperature jump length for a developing natural convection boundary layer. Finally, as shown in Section 2.5.3 and Section 3.5.4, the influence of the temperature jump length is much greater than the influence of the slip length on the Nusselt number. Thus, the influence of variation in the temperature jump length to slip length ratio on the Nusselt number prediction is small.
Figure 5.3: Boundary layer thickness, $\hat{\delta}$, presented as a function of $\hat{y}$ at $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$, and $\hat{\lambda}_T = 0, 5 \times 10^{-3}$, and $10 \times 10^{-3}$.

Of course, $\hat{\delta}$ increases with $\hat{y}$ following the classical boundary layer trend, where the increase is sudden at small $\hat{y}$ and decreases as $\hat{y}$ increases. Further, we note that $\hat{\delta}$ decreases with $Ra_L$ as expected.

$\hat{\delta}$ decreases modestly as $\hat{\lambda}_T$ increases with the greatest relative decrease (21%) occurring at the leading edge for the $Ra_L = 1 \times 10^4$ scenario. Recall that $\hat{\lambda}_T = \hat{\lambda}/2$, thus an increase in $\hat{\lambda}_T$ yields an increase in $\hat{\lambda}$. Increasing $\hat{\lambda}$ has a similar impact on the flow as increasing $Ra_L$ and this is similar to prior observations that $\hat{\delta}$ decreases as $Re$ or $\hat{\lambda}$ increase in jet impingement flow [22]. Further, the modest decrease in the magnitude of $\hat{\delta}$ is expected, since $\hat{\lambda}$ is small.

Next, we consider the scaling velocity, $\hat{V}$, which is plotted as a function of $\hat{y}$ in Fig. 5.4 at $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$ and $\hat{\lambda}_T = 0, 5 \times 10^{-3}$, and $10 \times 10^{-3}$.

$\hat{V}$ increases with $\hat{y}$ as expected, with the rate of growth decreasing as the flow moves upward. For the no-slip scenario, $\hat{V}$ has no dependence on $Ra_L$.

The dependence of $\hat{V}$ on $\hat{\lambda}_T$ and $Ra_L$ is coupled. At constant $Ra_L$, $\hat{V}$ decreases as $\hat{\lambda}_T$ increases. This occurs because the buoyant force, which drives the upward acceleration, decreases due to the smaller temperature difference between the fluid near the wall and the ambient pool. The decrease in $\hat{V}$ causes the velocity at all points in the profile, $v(x)$, to decrease.
Figure 5.4: Boundary layer scaling velocity, $\hat{V}$, as a function of $\hat{y}$ at $Ra_L = 1 \times 10^4$, $5 \times 10^6$ and $1 \times 10^9$ and $\hat{\lambda}_T = 0$, $5 \times 10^{-3}$, and $10 \times 10^{-3}$.

Further, $\hat{V}$ decreases more at higher $Ra_L$ as $\hat{\lambda}_T$ increases. This appears counterintuitive until we recall that the scaling velocity, $V$, is the product of $\hat{V}$ and $U$, where $U$ is the free convection scaling velocity, $U = \sqrt{g\beta \theta_u L}$. Although $\hat{V}$ decreases with $Ra_L$ when $\hat{\lambda}_T > 0$, $U$ always increases with a greater magnitude than $\hat{V}$ as $Ra_L$ increases. Consequently, $V$ increases. The physical reasoning behind the decrease in $\hat{V}$ with $\hat{\lambda}_T$ is that the resistance imposed by $\hat{\lambda}_T$ on thermal transport increases as the driving force, $Ra_L$, increases.

Having considered trends in $\hat{V}$ and $\hat{\delta}$, we study the impact of $\hat{\lambda}_T$ on the local velocity and temperature profiles, $\hat{v}(\hat{x}Ra_L^{1/4})$ and $\hat{\theta}(\hat{x}Ra_L^{1/4})$, respectively. We recall that $\hat{x}$ is the normalized wall normal coordinate. The scaling by $Ra_L^{1/4}$ is to allow comparison between profiles at different $Ra_L$ on the same plot.

In Fig. 5.5, $\hat{v}$ is plotted as a function of $\hat{x}Ra_L^{1/4}$. In panels (a) and (b), $Ra_L = 5 \times 10^6$ and $\hat{\lambda}_T = 0$, $5 \times 10^{-3}$, and $10 \times 10^{-3}$. $\hat{v}(\hat{y} = 0) = 0$ for the no-slip results (solid line) but the $\hat{\lambda}_T = 5 \times 10^{-3}$ and $10 \times 10^{-3}$ results exhibit a finite slip velocity at the wall, which increases as $\hat{\lambda}_T$ increases. Also, note that the gradient in $\hat{v}$ with respect to $\hat{x}Ra_L^{1/4}$ decreases indicating that the wall shear stress is decreasing. Further, the maximum velocity decreases modestly and the profile shifts toward the wall.
In panels (c) and (d), \( \hat{\lambda}_T = 5 \times 10^{-3} \) and \( Ra_L = 1 \times 10^4, 5 \times 10^6, \) and \( 1 \times 10^9 \). Here we consider the influence of \( Ra_L \) at an intermediate temperature jump length \( \hat{\lambda}_T = 5 \times 10^{-3} \) and again observe a coupled dependence of \( \hat{v} \) on \( \hat{\lambda}_T \) and \( Ra_L \). The effect of increasing \( Ra_L \) when \( \hat{\lambda}_T > 0 \) is similar to the effect of increasing \( \hat{\lambda}_T \). The maximum velocity decreases modestly, the slip velocity increases, and the profile shifts towards the wall. We note that the gradient at the wall is with respect to \( \hat{x}Ra_L^{1/4} \) and that \( Ra_L^{1/4} \) is varying between cases. Thus, inferring the trend in wall shear stress with \( Ra_L \) from the profile slope at the wall is not appropriate. However, when \( \hat{v} \) is plotted against \( \hat{x} \) (not shown here), the velocity gradient with respect to \( \hat{x} \) at the wall is observed to increase with \( Ra_L \) as expected.

![Figure 5.5: Non-dimensional velocity profiles, \( \hat{v} \), as a function of \( \hat{x}Ra_L^{1/4} \). Panels (a) and (b) display results, \( Ra_L = 5 \times 10^6 \) and \( \hat{\lambda}_T = 0, 5 \times 10^{-3}, \) and \( 10 \times 10^{-3} \). Panels (c) and (d) display results at \( \hat{\lambda}_T = 5 \times 10^{-3} \) and \( Ra_L = 1 \times 10^4, 5 \times 10^6, \) and \( 1 \times 10^9 \). Panels (a) and (c) display results at \( \hat{y} = 0.5 \). Panels (b) and (d) display results at \( \hat{y} = 1 \).](image)

In panels (a) and (c), \( \hat{y} = 0.5 \) and, in panels (b) and (d), \( \hat{y} = 1 \). Of course, \( \hat{v} \) and \( \hat{\delta} \) increase as \( \hat{y} \) increases.
In Fig. 5.6, \( \hat{\theta} = \frac{(T - T_\infty)}{(T_w - T_\infty)} \) is plotted as a function of \( \hat{\theta} Ra_L^{1/4} \). In panels (a) and (b), \( Ra_L = 5 \times 10^6 \) and \( \hat{\lambda}_T = 0, 5 \times 10^{-3}, \) and \( 10 \times 10^{-3} \). \( \hat{\theta} (\hat{x} = 0) = 1 \) for the no temperature jump case (solid line) but the \( \hat{\lambda}_T = 5 \times 10^{-3} \) and \( 10 \times 10^{-3} \) results exhibit finite temperature jumps at the wall (indicated by \( 0 \leq \hat{\theta}(\hat{x} = 0) < 1 \)) which increase as \( \hat{\lambda}_T \) increases. We also emphasize that \( \hat{\theta} \) decreases as \( \hat{\lambda}_T \) increases and the gradient of \( \hat{\theta} \) with respect to \( \hat{x} Ra_L^{1/4} \) decreases, indicating reduced heat flux at the wall.

![Figure 5.6: Non-dimensional temperature profiles, \( \hat{\theta} \), as a function of \( \hat{x} Ra_L^{1/4} \). Panels (a) and (b) display results at \( Ra_L = 5 \times 10^6 \) and \( \hat{\lambda}_T = 0, 5 \times 10^{-3}, \) and \( 10 \times 10^{-3} \). Panels (c) and (d) display results at \( \hat{\lambda}_T = 5 \times 10^{-3} \) and \( Ra_L = 1 \times 10^4, 5 \times 10^6, \) and \( 1 \times 10^9 \). Panels (a) and (c) display results at \( \hat{y} = 0.5 \). Panels (b) and (d) display results at \( \hat{y} = 1 \).](image)

In panels (c) and (d), \( \hat{\lambda}_T = 5 \times 10^{-3} \) and \( Ra_L = 1 \times 10^4, 5 \times 10^6, \) and \( 1 \times 10^9 \). Here we consider the influence of \( Ra_L \) at an intermediate temperature jump length (\( \hat{\lambda}_T = 5 \times 10^{-3} \)) and observe a coupled dependence of \( \hat{\theta} \) on \( \hat{\lambda}_T \) and \( Ra_L \). We note that \( \hat{\theta} \) decreases as \( Ra_L \) increases and exhibits a larger temperature jump. Once again, it is not appropriate to infer the influence of
$Ra_L$ on the heat flux from the profile slope because $\hat{\theta}$ is plotted as a function of $\hat{x}Ra_L^{1/4}$ and $Ra_L^{1/4}$ varies. However, when $\hat{\theta}$ is plotted versus $\hat{x}$, the slope at the wall increases as $Ra_L$ becomes larger.

In panels (a) and (c), $\hat{y} = 0.5$ and, in panels (b) and (d), $\hat{y} = 1$. Of course, $\hat{\delta_T}$ increases as $\hat{y}$ increases.

Having considered the hydrodynamic and thermal transport phenomenon in the local velocity and temperature profiles, we conclude our local analysis by considering the influence of $\hat{\lambda_T}$ and $Ra_L$ on local Nusselt number, $Nu_L$.

In Fig. 5.7, $Nu_L$ is plotted as a function of $\hat{y}$ at $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$ and $\hat{\lambda_T} = 0$, $5 \times 10^{-3}$, and $10 \times 10^{-3}$. As expected from classical results, $Nu_L$ decreases with $\hat{y}$ because the thermal boundary layer thickness is increasing. $Nu_L$ increases with $Ra_L$ for all values of $\hat{\lambda_T}$ because the advection increases due to the increase in buoyancy relative to viscosity and this causes the thermal boundary layer to thin.

![Figure 5.7: Local Nusselt number, $Nu_L$, as a function $\hat{y}$ at $\hat{\lambda_T} = 0$, $5 \times 10^{-3}$, and $10 \times 10^{-3}$ and $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$.](image)

Two coupled dependencies of $Nu_L$ on the parameters are important to point out. First, as $\hat{\lambda_T}$ increases, $Nu_L$ decreases and this decrease becomes greater at larger $Ra_L$. As $Ra_L$ increases, the driving temperature difference increases. The surface resistance becomes greater as $\hat{\lambda_T}$ becomes larger. The combination of these two effects causes greater reduction in $Nu_L$ concomitant with
decreased $q'''_w$ as $\hat{\lambda}_T$ increases at higher $Ra_L$. Second, the decrease in $Nu_L$ with $\hat{\lambda}_T$ is greater at smaller $\hat{y}$. This is explained by observing that the locations along the no-temperature jump plate with the greatest convection coefficient (largest $Nu_L$) will be impacted the most by the added thermal resistance arising from increasing $\hat{\lambda}_T$.

5.5.3 Average Results

$Nu_L$ is averaged over the plate length ($\hat{y} = 1$) and denoted $\overline{Nu}_L$. In Fig. 5.8, the relative decrease (percentage), $(\overline{Nu}_{L,0} - \overline{Nu}_L)/\overline{Nu}_{L,0}$ (where $\overline{Nu}_{L,0}$ is the average Nusselt number for the no temperature jump result), is displayed as a function of $\hat{\lambda}_T$ at $Ra_L = 1 \times 10^4, 5 \times 10^6$, and $1 \times 10^9$. The relative decrease becomes larger as $\hat{\lambda}_T$ increases. We also observe that the relative decrease increases as $Ra_L$ increases. This is a surprising result which indicates that the resistance imposed by the temperature jump length boundary condition depends not only on the wall boundary condition ($\hat{\lambda}_T$) but also on the driving temperature difference, $T_w - T_j$. Further, we see that the resistance to heat transfer increases in a non-linear manner with increasing $Ra_L$. At the largest $Ra_L$, the slope decreases as $\hat{\lambda}_T$ increases and this trend continues if the solution is solved at higher non-realizable $\hat{\lambda}_T$, since the relative decrease is at most 100%, which corresponds to complete insulation. As a

![Graph showing relative decrease in average Nusselt Number](image)

Figure 5.8: The relative decrease (percentage) in the average Nusselt Number, $(\overline{Nu}_{L,0} - \overline{Nu}_L)/\overline{Nu}_{L,0}$, plotted as a function of $\hat{\lambda}_T$ at $Ra_L = 1 \times 10^4, 5 \times 10^6$, and $1 \times 10^9$. 

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final note, predicted relative reduction in $\overline{Nu}_L$ can become large as $\hat{\lambda}_T$ and $Ra_L$ increase. At the highest $\hat{\lambda}_T$ and $Ra_L$ considered ($\hat{\lambda}_T = 10 \times 10^{-3}$ and $Ra_L = 1 \times 10^9$), relative decrease exceeded 50%.

In Fig. 5.9, the relative decrease is plotted as a function of $\hat{y}$ at $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$ and at $\hat{\lambda}_T = 5 \times 10^{-3}$ and $10 \times 10^{-3}$. Fig. 5.9 is included for completeness. One trend is important to observe here, the relative decrease becomes smaller as the averaging distance increases because the temperature jump length has the largest impact near the plate’s lower edge.

![Figure 5.9](image)

Figure 5.9: The relative decrease in the average Nusselt number, $(\overline{Nu}_{L,0} - \overline{Nu}_L)/\overline{Nu}_{L,0}$, is plotted as a function of $\hat{y}$ at $Ra_L = 1 \times 10^4$, $5 \times 10^6$, and $1 \times 10^9$ and $\hat{\lambda}_T = 5 \times 10^{-3}$ and $10 \times 10^{-3}$.

### 5.6 Conclusions

An integral analysis of buoyant flow induced by a vertical, heated superhydrophobic plate has been completed. The model was exercised for a range of realizable temperature jump lengths ($0 \leq \hat{\lambda}_T \leq 10 \times 10^{-3}$) and for a range of laminar Rayleigh numbers ($1 \times 10^4 \leq Ra_L \leq 1 \times 10^9$).

It is observed that as the temperature jump length increases:

- There is a modest decrease in the boundary layer thickness (up to 21%) due to the increased slip length.
• The scaling velocity decreases and the influence of $Ra_L$ on this parameter becomes coupled with the influence of the temperature jump length. For fixed $Ra_L$, the velocity profile decreases at all points.

• The local Nusselt number decreases with greater decrease at larger $Ra_L$ and near the lower edge.

• Average Nusselt number decreases and the decrease is much more significant as $Ra_L$ increases. A relative decrease in $\overline{Nu_L}$ exceeding 50% is achieved for large $Ra_L$ and $\hat{\lambda}_T$ ($Ra_L = 1 \times 10^9$ and $\hat{\lambda}_T = 10 \times 10^{-3}$).
CHAPTER 6. INFLUENCE OF MICROSTRUCTURE GEOMETRY ON POOL BOILING AT SUPERHYDROPHOBIC SURFACES

This chapter is published in the International Journal of Heat and Mass Transfer [69]. The format of this paper has been modified to meet the stylistic requirements of this dissertation.

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6.2 Abstract

Periodically patterned superhydrophobic surfaces with rectangular rib and circular post arrays were utilized as heat transfer surfaces in a boiling apparatus with the water pool undergoing saturated pool boiling. The surface microstructures were geometrically defined by cavity fraction (the ratio of projected cavity area to surface area), pitch (the center to center distance between microfeatures), and feature height. Surface heat flux and surface superheat, the difference between the heated surface and the pool saturation temperature, were measured for each surface. Ten different micropatterned surfaces with post or rib geometries were considered with cavity fraction varying from 0.5 to 0.98, pitch varying from 8 to 40 \( \mu \)m, and microfeature height at 4 \( \mu \)m or 15 \( \mu \)m. The surface heat flux was measured across a range of surface superheats spanning 2 to 38 deg K. It is demonstrated for the first time that the transition from nucleate boiling to film boiling on rib patterned surfaces depends strongly on the cavity fraction. Increasing the microstructure height from 4 \( \mu \)m to 15 \( \mu \)m modestly increases the transition temperature. Nucleate boiling is more suppressed on post patterned surfaces than on rib patterned surfaces. Further, the rib structured surfaces exhibit a sudden transition from nucleate to film boiling while the post structured surfaces exhibit a gradual transition, with the vapor film growing slowly across the surface. Once
stable film boiling is reached, the surface microstructure negligibly influences the heat flux for all surfaces.

6.3 Introduction

Pool boiling heat transfer is influenced by many parameters [79, 83]. Perhaps the least understood are surface chemistry and micro/nanostructure. The classical approach for considering pool boiling heat transfer is to empirically obtain the relationship between the heat flux at the wall \(q''_w\) and the surface superheat \(\Delta T\), the difference between the wall temperature and the liquid pool temperature [37, 84]. Multiple boiling curves may be obtained for a range of parameters including different substrates, liquids, gravitational acceleration, and ambient pressure. In this paper, the influence of superhydrophobic surface microstructure on pool boiling is characterized.

Superhydrophobic (SHPo) surfaces are realized through the combination of hydrophobic chemistry and micro/nanoscale surface roughness. The development of artificial SHPo surfaces is fairly recent [85] and the influence of these surfaces on pool boiling is an active area of inquiry. This work considers a specific scenario which has not yet been studied: periodically micropatterned SHPo surfaces of both rib and post geometries. To begin, a brief overview of classical boiling behavior is considered.

Heat flux as a function of the surface superheat has been observed for a wide variety of surface and working fluid combinations [83,86]. This relationship, the “so-called” boiling curve, is illustrated qualitatively for liquid water undergoing saturated pool boiling at atmospheric pressure in Fig. 6.1 for HPo and SHPo surfaces. The general variation in \(q''_w\) as a function of the surface superheat, \(\Delta T\), is shown. In general, some portion of the pool may be at a temperature below the liquid saturation temperature. These scenarios are called “sub-cooled pool boiling” and are in contrast to “saturated pool boiling” where the pool is uniformly at the saturation temperature. The scope of this work is limited to saturated pool boiling.

Distinct regimes of physical phenomena are observed in the liquid as \(\Delta T\) rises. These regimes are labeled in Fig. 6.1, as well as three important points which are marked A through C. Nucleate boiling begins at \(\Delta T \approx 7\) deg K [79], and a dramatic increase in \(q''_w\) occurs due to phase change and bubble-induced liquid motion. As \(\Delta T\) increases, the density of nucleation sites becomes sufficiently large that bubble interaction occurs and the bubble release frequency increases
Figure 6.1: The qualitative boiling curve for liquid water undergoing pool boiling at atmospheric pressure on a heated classical surface is displayed at HPo and SHPo surfaces. Heat flux, $q''_w$, is plotted as a function of surface superheat, $\Delta T$. Labels A–C indicate the onset of nucleate boiling, the critical heat flux, and the Leidenfrost point for the HPo curve. The critical heat flux and Leidenfrost point at the SHPo surface are indicated by $B'$ and $C'$. The span of the nucleate, transition, and film boiling regimes are also indicated.

until the consecutive bubbles merge into vapor columns. The rate of increase in $q''_w$ with $\Delta T$ decreases as the fraction of the surface area covered by the vapor increases. A local maximum is reached at the critical heat flux (point B, nominally $\Delta T \approx 30$ deg K [79]). Two different phenomena may be observed as $\Delta T$ increases further, depending on how heat is supplied to the surface. If heat is supplied at constant $q''_w$, a sudden transition to the film boiling regime is experienced and $\Delta T$ increases by 1–2 orders of magnitude [37]. Subsequently, $q''_w$ increases with increasing $\Delta T$ and film boiling prevails.

If the surface is maintained at constant temperature, a transition boiling regime will be entered and $q''_w$ will decrease with increasing $\Delta T$ until a minimum heat flux is reached [84]. This minimum heat flux and its corresponding surface superheat is known as the Leidenfrost point (point C, $\Delta T \approx 120$ deg K [79]) and is accompanied by the formation of a stable vapor film. $q''_w$ once again begins to rise as $\Delta T$ increases beyond this point, with film boiling existing everywhere on the surface.
The influence of a SHPo surface (dashed line) is shown, where the effect is to move the critical heat flux ($B'$) and Leidenfrost ($C'$) points to lower $\Delta T$, dramatically decreasing the heat flux by inhibiting nucleate boiling.

Contemporary contributions explaining the influence of micro/nanostructure and surface chemistry on pool boiling are now introduced. Surface-water interaction is typically categorized into four surface types, ranked according to decreasing intermolecular interaction with liquid water: superhydrophilic, hydrophilic, hydrophobic, and superhydrophobic. The static contact angle is used to characterize surfaces in this work. Superhydrophilic surfaces combine hydrophilic chemistry and micro/nanostructure and have contact angles which approach $0^\circ$. Hydrophilic surfaces have hydrophilic chemistry and have contact angles spanning $0^\circ$ to $90^\circ$, hydrophobic surfaces have hydrophobic chemistry and have contact angles spanning $90^\circ$ to $120^\circ$, and superhydrophobic surfaces have hydrophobic chemistry and micro/nanostructure with contact angles which span $150^\circ$ to nearly $180^\circ$ [1]. Panel (a) of Figure 6.2 illustrates schematically water droplets resting on these surfaces, excluding the superhydrophilic case in which a contact angle near $0^\circ$ would be observed and the surface would exhibit hydrophilic microfeatures.

Due to roughness features on the nanometer to micrometer scale and material which is chemically hydrophobic, water interacts little with a superhydrophobic surface and the liquid cohesion dominates the three phase interaction. As long as the ambient pressure is sufficiently small, the cavities between features do not fill with water and the surface is said to be in the Cassie state (schematically illustrated in panel (b) of Figure 6.2). This state is prerequisite for many unique SHPo surface properties and is the state considered in this study. The apparent contact angles reported in this work are nominally those which would be obtained with Cassie’s equation [4].

Deviations from classical hydrodynamic and thermal transport behavior are observed on superhydrophobic surfaces. Hydrodynamics are altered by the microscopic alternating shear and nearly shear free boundary conditions at the wall. From a macroscopic perspective, it appears as though a slip velocity prevails at the wall. The overall effect is a reduction in friction drag. This has been demonstrated through experiment and analysis in a variety of flows [10–12]. Further, a recent work considering the trajectories of superheated spheres reported trajectory stabilization and drag reduction on superhydrophobic spheres relative to smooth hydrophilic spheres [87]. Reduced
adhesion allows water to roll over the surfaces with small resistance washing away contaminants and yielding “self-cleaning” surfaces [6].

The presence of the air-filled cavities also dramatically alters thermal transport. The thermal conductivity of the air and water vapor contained in the cavities is three orders of magnitude less than for the metal substrate. Thus, the water encounters alternating matching temperature and nearly adiabatic boundary conditions. From a macroscopic scale, it appears that a finite jump in temperature occurs at the wall, reducing the wall heat flux [19, 41, 88].

Boiling transport is also altered by the smaller adhesion force acting on the liquid water. This change explains why nucleate boiling is suppressed on SHPo surfaces. Less thermal energy is required to overcome the liquid attraction to the surface and form a vapor film. This leads to a dramatic decrease in the surface superheat at which transition to film boiling occurs [47]. A stable film can be maintained on the SHPo surface through the range of $\Delta T$ where nucleate and transition boiling would normally be observed on smooth surfaces. Correspondingly, $q''_w$ decreases...
by up to an order of magnitude within this range. Previous researchers have considered horizontal surfaces [80], quenched spheres [47,48], and immersed vertical cylinders [47].

Several conclusions may be obtained from these prior studies. First, superhydrophobic surfaces alter nucleation site density, bubble growth, and bubble detachment. Bubble nucleation begins at lower surface superheats [80]. Second, film boiling occurs at much lower surface superheats [80] since the frequency of bubble interaction increases due to the larger bubble size and, also, the surface interacts less with water. Third, little nucleate boiling is observed [80] and it is completely suppressed on high contact angle surfaces [47,48]. Consequently, for high contact angle cases, the heat flux curve increases monotonically without a local maximum at the critical heat flux or a local minimum at the Leidenfrost point [47,48].

Previous investigators have characterized randomly structured superhydrophobic surfaces by contact angle. However, doing so obscures the influence of two independent surface parameters: the chemical hydrophobicity of the surface and the geometry of the surface microstructure. This work highlights the influence of the latter, by studying superhydrophobic surfaces with known periodic surface structure defined by cavity fraction, pitch, and height and obtaining the heat flux versus surface superheat relationship for each surface.

6.4 Experimental Method

6.4.1 Superhydrophobic Surfaces

Rib and post decorated surfaces were fabricated by utilizing deep reactive ion etching to form high aspect ratio cavities in photoresist-masked silicon wafers. These microstructured surfaces were spin-coated with amorphous fluoropolymer(Teflon™ AF 1601) after depositing a thin chromium film for adhesion by electron beam evaporation. Details for fabricating these surfaces are included in Appendix B.

Electron micrographs of rib and post patterned surfaces acquired using a scanning electron microscope (SEM) are shown in Fig. 6.3. Dimensions used to define the rib and post array geometries are indicated on the images where \( w \) is the feature pitch, \( h \) is the feature height, \( d \) is the diameter for cylindrical posts, and \( w_c \) is the cavity width on rib surfaces.
Figure 6.3: Electron micrographs are displayed of rib and post patterned surfaces acquired using SEM. Dimensions which define the geometries are shown for both rib and post geometries where \( w \) is the feature pitch, \( h \) is the feature height, \( d \) is the cylindrical post diameter, and \( w_c \) is the cavity width on rib patterned surfaces.

The cavity fraction, \( F_c \), is the ratio of projected cavity area to surface area and is expressed
\[
F_c = \frac{w_c}{w} \text{ for rib patterned surfaces and } F_c = 1 - \left( \pi d^2 \right)/(4w^2) \text{ for post patterned surfaces.}
\]

The superhydrophobic surfaces considered in this study are reported in Table 6.1. Each surface is characterized by surface type and three geometric parameters. Two pitches were considered on rib patterned surfaces: 24 \( \mu \text{m} \) and 40 \( \mu \text{m} \). Cavity fraction varied through 0.5, 0.8, 0.85, and 0.93. Depths of 4 \( \mu \text{m} \) and 15 \( \mu \text{m} \) were considered for the \( F_c = 0.8 \) and \( w = 40 \mu \text{m} \) surface. The variation in the reported depth for rib patterned surfaces (16, 15, and 17 \( \mu \text{m} \) on the \( F_c = 0.5, 0.8, \) and 0.93 surfaces, respectively) has negligible influence relative to the change in cavity fraction. We also note that heights have a tolerance of \( \pm 1.5 \mu \text{m} \).

Table 6.1: The SHPo surfaces considered in this study are reported. Each surface is characterized by micropattern (rib or post) and three geometric parameters: cavity fraction \( (F_c) \), pitch \( (w) \), and height \( (h) \). Sessile contact angle, \( \theta \), is a function of \( F_c \) and \( w \). It is reported here for reference.

<table>
<thead>
<tr>
<th>Micropattern</th>
<th>( F_c (+0.08 -0.03) )</th>
<th>( w (\pm 0.3 \mu \text{m}) )</th>
<th>( h (\pm 1.5 \mu \text{m}) )</th>
<th>( \theta (\pm 3^\circ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rib</td>
<td>0.5</td>
<td>40</td>
<td>16</td>
<td>139</td>
</tr>
<tr>
<td>Rib</td>
<td>0.8</td>
<td>40</td>
<td>4</td>
<td>150</td>
</tr>
<tr>
<td>Rib</td>
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<td>40</td>
<td>15</td>
<td>152</td>
</tr>
<tr>
<td>Rib</td>
<td>0.85</td>
<td>24</td>
<td>15</td>
<td>152</td>
</tr>
<tr>
<td>Rib</td>
<td>0.93</td>
<td>40</td>
<td>17</td>
<td>155</td>
</tr>
<tr>
<td>Post</td>
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<td>15</td>
<td>151</td>
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<td>Post</td>
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<td>15</td>
<td>151</td>
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<tr>
<td>Post</td>
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</tr>
<tr>
<td>Post</td>
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<td>24</td>
<td>15</td>
<td>157</td>
</tr>
<tr>
<td>Post</td>
<td>0.98</td>
<td>16</td>
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</tr>
</tbody>
</table>
On post patterned surfaces, cavity fraction ranged through 0.56, 0.85, and 0.98 and pitch varied through 8, 16, and 24 \( \mu \text{m} \). All post cavities were etched to a nominal depth of \( h = 15 \ \mu \text{m} \).

Sessile contact angles are reported for all surfaces considered. These contact angles are provided for reference and are a function of the surface microstructure and the chemical hydrophobicity, which is characterized by the sessile contact angle, or Young’s contact angle, of a droplet on a smooth surface with identical chemistry.

The relationship between cavity fraction and apparent contact angle (that which is observed macroscopically) is well defined as established by the Cassie equation and the Wenzel equation. The Wenzel equation reports apparent contact angles when the cavities are wetted and the Cassie equation reports apparent contact angles when the microcavities between features are air or vapor filled.

The Wenzel equation is

\[
\cos(\theta) = r\cos(\theta_Y)
\]

where \( r \) is the ratio of wetted area to projected area and \( \theta_Y \) is the Young’s contact angle [20]. For hydrophobic surfaces (\( \theta_Y > 90^\circ \)), the apparent contact angle increases as \( r \) increases.

The Cassie equation is

\[
\cos(\theta) = r_w(1 - F_c)\cos(\theta_Y) - F_c
\]

where \( r_w \) is the roughness ratio for the wetted area which is 1 for nonwetted microposts and microribs [20].

The sessile contact angle increases with \( F_c \). In the case of \( F_c \) approaching 1, the contact angle approaches \( 180^\circ \). When \( F_c \) approaches 0, the Cassie equation reduces to the Wenzel equation.

Considering the experimental data in Table 6.1, we observe that the contact angle increases with cavity fraction, although in some cases the observed increase is within experimental uncertainty. The increase in contact angle with pitch on the post patterned surfaces is coincidental and is within measurement uncertainty.

Two smooth hydrophobic (HPo) surfaces were considered as well and these surfaces had apparent contact angles of nominally \( 119^\circ \).
We emphasize that the contact angles reported are for sessile droplets and that a range of dynamic contact angles will exist in boiling phenomenon.

6.4.2 Experimental Setup

A schematic illustration of the experimental setup is shown in Fig. 6.4. A heater assembly was fabricated by inserting electric cartridge heaters into an aluminum block. A commercial heat flux sensor (Captec, sensitivity 17.2 \(\mu\)Vm\(^2\)/W), sized to match the area of the test surface in contact with the boiling liquid (40 mm square) was secured to the top of the heater assembly by placing it on the heater block and beneath an aluminum plate (3.2 mm thick), which was clamped to the heater block using PTFE screws.

Figure 6.4: Schematic illustration of pool boiling experimental setup indicating heaters, instrumentation, and insulation.

Thermal contact resistance was reduced by applying thermal compound to both sides of the sensor. The test surface was placed on thermal compound on top of an aluminum plate. This plate
had a thin gauge type-K thermocouple installed with 1 mm of aluminum separating it from the test surface.

A square polycarbonate reservoir was clamped to a heater assembly by steel screws torqued to 0.34 N-m (3 in-lb), obtaining a water tight seal with a silicone gasket between the plastic wall and test surface. A 6.4 mm thick PTFE lid with a 3.2 mm diameter steam vent (7 mm off center) was placed over the polycarbonate walls and attached with additional screws. The pool saturation temperature was measured by a K-type thermocouple inserted vertically downward into the boiling water through a hole centered on the PTFE lid. The thermocouple was fixed to the lid and the junction was located 11 ± 2 mm above the test surface. Deionized water was the working fluid for all experiments. Melamine foam sheet insulation (0.0375 W/m²K, 25.4 mm thick) was stacked in layers surrounding the setup and cut so that at least a 50.8 mm thick layer of insulation covered all surfaces of the experimental apparatus. On/off temperature control was utilized to maintain the surface temperature within ±1 deg K of the set point. Temperatures were measured by cold-junction compensated type-K thermocouples and heat flux was measured with the previously mentioned heat flux sensor.

The process of obtaining an experimental data point is now described. Deionized water (without degassing) was boiled for five minutes in a Pyrex beaker with a loose fitting lid on a hot plate. This preheated the water to the saturation temperature decreasing the time necessary for the setup to achieve steady state operation. Meanwhile, the cartridge heaters were turned on and the desired surface temperature was set on the on-off temperature controller. The setup was then allowed to reach a temperature just below the saturation point. The preheated deionized water was poured into the reservoir. Heat was then supplied through the cartridge heaters until the desired surface temperature was approached and steady state was achieved. The steam vent in the lid assured that the test chamber remained at atmospheric pressure.

A PC-based data acquisition system recorded the heat flux and the surface superheat. This recording was continued until at least five on-off cycles with repeatable oscillations in the heat flux signal had been accomplished. The data was then averaged over the last three cycles and the average heat flux was plotted as a function of the average surface superheat.

Heat flux and surface superheat were recorded as the reservoir depth decreased in a preliminary study. It was found that reservoir height had a negligible effect as long as the pool was
deeper than 5 mm. The pool was kept above this height for each test. Previous investigators also
identified that reservoir height had negligible influence as long as it was sufficiently large. The
critical height reported was 2.4 mm [89]. Since the test chamber pressure remained constant and
the influence of pool height was negligible over the pool depths considered, we conclude that the
experimental conditions were repeatable.

6.4.3 Error Analysis

The uncertainty in the heat flux sensor measurement was ±3% of the measured value as
reported by the manufacturer. This error was calculated at each temperature set point. Heat losses
from the setup through all surfaces were estimated to be approximately 75 W/m² using standard
natural convection correlations [79] and all measurements were adjusted by this amount to account
for the loss. This loss is nominally constant because the driving temperature difference between
the saturated pool and the ambient air is constant for all tests.

The error in each temperature measurement was ±1.1 deg K. The error in the surface
superheat measurement, which is the difference between the surface temperature and the saturation
temperature, is obtained by root sum of squares to be ±1.6 deg K.

Sessile contact angles were measured on each surface before and after the tests. The ac-
curacy of these measurements was ±3°. Data was discarded for post patterned surfaces if any
of the contact angle measurements changed by more than ±5.5°. A slightly larger variation was
observed for rib patterned surfaces. Data was discarded for these surfaces when the sessile contact
angle changed by more than ±9°.

Microstructured surfaces were fabricated with fixed $F_c$, $w$, and $h$. Measurements of $w$ are
within ±0.3 µm of the stated value. Measurements of $h$ are within ±1.5 µm, and measurements
of $F_c$ are within −0.03 to +0.08.

For each combination of $w$, $F_c$, and $h$ considered, experimental data sets were acquired for
each surface type on two identical surfaces. These results were averaged and presented with error
bars which equal the sum of the error resulting from the difference between the two data sets and
the measurement uncertainty. The experimental error for all temperature measurements (±1.6 deg
K) is constant and thus temperature (horizontal) error bars only include the influence of variation

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between the replicate tests. All error bars (horizontal and vertical) which were smaller than the marker size were omitted.

6.5 Results and Discussion

This section of the paper presents the results from the experiments and is structured as follows. First, we compare our results to existing data from other investigators that have considered pool boiling at both HPo and SHPo surfaces. Although, we note that our work is the first to consider pool boiling on horizontal SHPo surfaces. Second, we highlight the influence of cavity fraction on the overall boiling curve. In general, the transition from nucleate to film boiling occurs at lower surface superheat with increasing cavity fraction for rib patterned SHPo surfaces, while for post patterned surfaces, the influence of cavity fraction is much smaller. Third, we compare the behavior for rib and post patterned surfaces and note that the transition to complete film boiling always occurs at lower surface superheat for the post patterned surfaces. Lastly, we consider the depth of the microstructures, where the data reveal that the transition temperature increases with cavity depth.

6.5.1 Pool Boiling Behavior on Superhydrophobic Surfaces Relative to Hydrophobic Surfaces

Shown in the top panel (a) of Fig. 6.5 is our $q''_w$ data as a function of $\Delta T$ for the smooth hydrophobic (HPo) surface. Also shown in the figure are results from Jo et al. [49] and Bourdon et al. [50], who considered horizontal surfaces as well. Data from Fan et al. [48] are also shown, although their results correspond to a heated sphere immersed in a pool. Table 6.2 provides contact angles, geometry, and coating information for these prior works. The contact angles are all nominally the same as that measured for our surface (119°) and over the range $\Delta T < 12$ deg K (within the nucleate boiling regime) our $q''_w$ vs. $\Delta T$ data show good agreement with the results from these prior studies, with $q''_w$ increasing linearly with increasing $\Delta T$. The data of Fan et al. show a peak in $q''_w$ at about 12 deg K, followed by a decrease as transitional boiling conditions begin to exist. Data at $\Delta T > 12$ deg K were not collected for the present HPo surface due to limitations in the heat flux sensor and the largest heat flux measured was nominally $2 \times 10^4$ W/m$^2$. 

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Figure 6.5: $q''_w$ as a function of $\Delta T$ for a HPo surface (panel (a)) and SHPo surface with $F_c = 0.8$, $w = 40 \mu m$, and $h = 15 \mu m$ (panel (b)). These results are compared to results from prior investigators: Fan et al. [48], Jo et al. [49], Bourdon et al. [50], and Vakarelski et al. [47].
Table 6.2: Parameters of pool boiling studies in the literature. For each study, the geometry, surface type, sessile contact angle $\theta$, and nominal microfeature height $h$ and spacing $w$ are reported.

The advancing contact angle is reported if the sessile contact angle is not available and these values are indicated by “*”.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Geometry</th>
<th>Type</th>
<th>Coating Material</th>
<th>$\theta$ (°)</th>
<th>$h$ (µm)</th>
<th>$w$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan et al. [48]</td>
<td>sphere (10 mm dia.)</td>
<td>HPo</td>
<td>Trichlorosilane</td>
<td>119</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Jo et al. [49]</td>
<td>horizontal plate (rectangle, 10 mm by 15 mm)</td>
<td>HPo</td>
<td>Teflon</td>
<td>123</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Bourdon et al. [50]</td>
<td>horizontal plate (circle, 20 mm dia.)</td>
<td>HPo</td>
<td>Trichlorosilane</td>
<td>114*</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Vakarelski et al. [47]</td>
<td>vertical cylinder (24 mm dia. immersed 40 mm)</td>
<td>HPo</td>
<td>Trichlorosilane</td>
<td>115*</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Fan et al. [48]</td>
<td>sphere (10 mm dia.)</td>
<td>SHPo</td>
<td>Glaco Mirror Coat</td>
<td>165</td>
<td>0.9</td>
<td>2</td>
</tr>
<tr>
<td>Vakarelski et al. [47]</td>
<td>vertical cylinder (24 mm dia. immersed 40 mm)</td>
<td>SHPo</td>
<td>Glaco Mirror Coat</td>
<td>170*</td>
<td>0.3</td>
<td>2</td>
</tr>
</tbody>
</table>

Shown in the bottom panel (b) of Fig. 6.5 are data for SHPo surfaces. Data are shown for a rib-cavity patterned surface with $F_c = 0.8$, a pitch of 40 µm, and a cavity depth of 15 µm. For comparison, data from Fan et al. [48] (heated sphere) and Vakarelski et al. [47] (heated cylinder) are also shown. The contact angles for our rib-cavity horizontal surface was nominally 152°, while for the surfaces of Fan et al. and Vakarelski et al., the contact angles were nominally 165° and 170°.

Shown in Table 6.2 is contact angle information, coating material, and nominal feature height and spacing of the randomized surface features for these prior studies. We note that in addition to the microscale parameters specified here the coating utilized by Fan et al. and by Vakarelski et al. also exhibited a second tier of roughness at the nanoscale.

At small values of $\Delta T$ (< 5 deg K), the magnitude of $q''_w$ is similar for all surfaces, although for all surfaces the heat flux is lower (but within experimental accuracy) than exists for the HPo surfaces shown in panel (a). At $\Delta T \approx 5$ deg K, $q''_w$ for the $F_c = 0.8$ surface is a maximum (critical heat flux) at a value of nominally $7 \times 10^3$ W/m$^2$, which is about a factor of 3–4 lower than for the HPo surfaces. For $\Delta T > 5$ deg K, the data for the different surfaces deviates markedly. $q''_w$ is much lower for the $F_c = 0.8$ surface and it is increasing at a much smaller rate with increasing $\Delta T$, than for the two other surfaces. Indeed, complete film boiling conditions exist for the $F_c = 0.8$ surface, with its attendant much lower heat flux, while the conditions on the other two surfaces appear to be more transitional in nature.
It should be noted that the $F_c = 0.8$ surface differs from the other two SHPo surfaces in several important ways. First, the surface features for the other two surfaces are not uniformly structured, but are randomly sized and distributed. This is in contrast to the periodic structure of long coherent alternating ribs and cavities. Second, the characteristic spacing and height of the features for the other two surfaces are much smaller ($2 \mu m$ spacing for both Vakarelski et al. and Fan et al. and height of $0.3 \mu m$ for Vakarelski et al. and $0.9 \mu m$ for Fan et al.) than for the $F_c = 0.8$ surface (pitch of $40 \mu m$ and a cavity depth of $15 \mu m$). Third, the present heat transfer results and those of Vakarelski et al. consider a steady state scenario while the results of Fan et al. consider transient quenching. Finally, the results of Fan et al. and Vakarelski et al. considered curved surfaces, a sphere and a cylinder, respectively, while the present results consider a planar surface. The impact of these differences gives rise to the large difference in observed behavior in the $q_w''$ vs. $\Delta T$ data. The comparison is included to illustrate that the present results achieve the same order of magnitude and have similar trends.

We now consider high-speed imaging of the phenomena discussed above to further explore the boiling dynamics. Figure 6.6 displays images of bubble nucleation, growth, and detachment on the rib-cavity SHPo surface ($F_c = 0.8$, $w = 40 \mu m$, and $h = 15 \mu m$) in nucleate (panel (a)) and film (panel (b)) boiling conditions. Both sets of four images span 60 ms with images acquired at 20 ms increments.

Before discussing the images, we specify four states of interest that may be observed. 1) The Wenzel state with cavities wetted and nucleate boiling. 2) The Cassie state with gas or vapor in the cavities, nucleate boiling, and liquid contact at the tops of microfeatures. 3) A state of nucleate boiling with partial wetting between the Wenzel and Cassie states. 4) Film boiling with no liquid-solid contact. Frequently, the state varies with location. Film boiling over some regions (no liquid-solid contact) and nucleate boiling over other regions (liquid-solid contact). The regions undergoing nucleate boiling may be in the Cassie state, the Wenzel state, or the intermediate state of partial wetting.

For the scenario shown in panel (a), $\Delta T = 2$ deg K and $q_w'' = 5 \times 10^3$ W/m$^2$. The images show multiple vapor bubbles with large solid-liquid contact angles that have formed on the surface. These bubbles grow into bell shapes, merge with neighboring bubbles, and as the bubbles rise, the midsection becomes thinner until the vapor neck pinches off and the bubble is released. The
Figure 6.6: Two image sequences are shown as obtained with a high speed camera acquiring images at 1000 fps (images are shown at 20 ms increments). The same rib patterned SHPo surface \( (F_c = 0.8, w = 40 \mu m, \text{ and } h = 15 \mu m) \) is considered in both sequences with the ribs oriented horizontally in the images. The first series (panel (a)) was obtained with the water undergoing nucleate boiling \( (\Delta T = 2 \text{ deg K for which } q''_w = 5 \times 10^3 \text{ W/m}^2) \). The second series (panel (b)) was obtained with the liquid undergoing film boiling \( (\Delta T = 22 \text{ deg K for which } q''_w = 2 \times 10^3 \text{ W/m}^2) \). This video is included as Supplemental Video 1.
bubble then rises, leaving a small quantity of vapor trapped at the surface. During this process, water is touching the surface at several locations in the images, and new bubbles are nucleating and growing. The liquid-solid interface may in general be in either the Wenzel or Cassie state [90] and this cannot be completely determined from the images alone.

Next, we consider the film boiling regime in (panel (b)), where $\Delta T = 22$ deg K and $q''_{w} = 2 \times 10^3$ W/m$^2$. Bubbles formed in this regime depart in a similar manner as those originating on the surface in the nucleate regime. However, the growth is notably different. Here the bubbles grow, not as a result of smaller bubbles merging with larger bubbles, but rather from the growth of perturbations in the film. These perturbations and resulting bubbles are much more widely spaced. We emphasize here that film boiling is occurring at a surface superheat of 22 deg K, which is much lower than the typical Leidenfrost point that prevails on classical surfaces (nominally greater than 100 deg K).

The reader may be interested in how the vapor film is differentiated from the regions of solid-liquid contact in the images. We can identify that bubbles are forming on the surface in the nucleate regime (panel (a)) as opposed to forming on the vapor film (panel (b)). This becomes apparent as we observe small bubbles on the surface, which are completely absent in panel (b) where film boiling prevails. The film is more reflective to the illuminating light than the regions where there is solid-liquid contact. Thus, the regions covered by film tend to appear relatively brighter and the regions with solid-liquid contact appear darker (arrows indicating these two scenarios are shown on the images). Further, surface capillary waves are often visible on film covered surfaces and are absent where the water is in contact with the microfeatures. We also note that wetting of the cavities is unlikely since the capillary length is 2 mm, two orders of magnitude larger than the largest cavity width but we do not rule out the possibility of wetting completely.

The video from which these images are taken is included as Supplemental Video 1. One phenomenon that is apparent in the video but not the extracted images of Fig. 6.6, is the growth of low profile bubbles along the ribs which merge with the larger bubbles with circular contact regions.

By way of summary for this section, the present results illustrate the following important points. First, our results for HPo surfaces show good agreement with prior pool boiling results for HPo surfaces exhibiting similar contact angles. Second, film boiling is achieved at much lower $\Delta T$
on the rib-cavity structured SHPo surface ($\approx 6$ deg K) than on HPo or hydrophilic surfaces. Third, the magnitude of $q''_w$ for the rib-cavity SHPo surface in the film boiling regime is much lower than the heat flux that prevails at similar $\Delta T$ for HPo or randomly structured SHPo surfaces with much smaller feature sizes.

6.5.2 Influence of Superhydrophobic Microfeature Characteristics on Boiling

Influence of Varying Cavity Fraction for Rib Surfaces

In this section, we explore the influence that cavity fraction, microfeature type (ribs vs. posts), and spacing between microfeatures (pitch) exert on the boiling dynamics. Shown in Fig. 6.7 are $q''_w$ vs. $\Delta T$ data for rib patterned SHPo surfaces with $w = 40$ $\mu$m and $h = 15$ $\mu$m and for cavity fractions of $F_c = 0.5$, 0.8, and 0.93. The results of Fig. 6.7 show that the transition from nucleate to film boiling occurs at lower $\Delta T$ as $F_c$ increases. Transition occurs at a $\Delta T$ value between 8 deg K and 17 deg K for the $F_c = 0.5$ surface and between 6 deg K and 13 deg K for the $F_c = 0.8$ surface. As noted previously, the heating system was designed for temperature control and, in the transition region, the dynamics of decreasing heat flux with increasing surface temperature result in unstable operating conditions. If there is a transition from nucleate to film boiling for the $F_c = 0.93$ surface, it occurs at $\Delta T < 4$ deg K, with film boiling prevailing over nearly the entire range of surface temperatures considered.

Influence of Varying Cavity Fraction for Post Surfaces

Boiling dynamics on post patterned SHPo surfaces follow slightly a different trend than observed for rib structured surfaces. Shown in Fig. 6.8 are $q''_w$ vs. $\Delta T$ data for post patterned surfaces. Note that these plots maintain the same scale on the vertical axis as Fig. 6.7 to allow quick comparison between conditions considered. Further, insets display data at surface superheats of 0 to 25 deg K on a magnified scale. The top panel shows results at three cavity fractions ($F_c = 0.56$, 0.85, and 0.98) and fixed pitch of $w = 16$ $\mu$m and fixed post height of 15 $\mu$m. Over the entire range of $\Delta T$ considered, $q''_w$ is nearly identical for the $F_c = 0.56$ and 0.85 surfaces and film boiling conditions appear to prevail over the entire range of surface superheats. In contrast to the
rib structured surfaces (at the same cavity fractions), nucleate boiling is never observed for the $F_c = 0.56$ and 0.85 surfaces. Interestingly, $q''_w$ for the $F_c = 0.98$ surface is modestly higher than for the other two cavity fractions for $\Delta T < 20\text{ deg K}$, although it is always much lower than exists for nucleate boiling on a smooth hydrophilic or HPo surface. The authors hypothesize that Cassie to Wenzel transition may cause the increased heat transfer for this scenario since surfaces with post patterns have a greater propensity for wetting than rib patterned surfaces of equal cavity fraction due to an increased cavity width along the post-to-post diagonal. In addition, for the same pitch, the likelihood of wetting increases with increasing cavity fraction because of increased feature-to-feature spacing. If the surface is wetted, then the surface area in contact with the liquid would increase allowing for higher average heat flux. Further, a recent paper described a phenomenon whereby critical heat flux would occur at a much higher surface superheat if boiling initiated in the Wenzel state instead of the Cassie state [91]. In the Wenzel state, vapor bubbles nucleate at the base of the cavities. As the surface superheat increases, the vapor pressure in the cavity increases, driving the liquid out. The three-phase contact line at each cavity wall moves upward until it reaches the top edge. The contact line pins at this edge as the contact angle adjusts to the abrupt change. This requires much higher surface superheat before the vapor bubbles spread.
Figure 6.8: $q''_w$ is plotted as a function of $\Delta T$ at $w = 16 \mu m$ for post patterned surfaces ($h = 15 \mu m$). In panel (a), $F_c$ varies through 0.56, 0.85, and 0.98. In panel (b), $F_c = 0.85$ and $w$ varies through 8, 16, and 24 $\mu m$. The insets plot the data at surface superheats between 0 to 25 deg K on a magnified scale.
over microfeatures, initiating film boiling. This phenomenon or a similar dynamic contact line effect may explain the transition at higher surface superheat observed for the $F_c = 0.98$ surface. At $\Delta T > 20$ deg K, the $q''_w$ vs. $\Delta T$ data for all three cavity fractions are nearly identical. The error bars on the data show the repeatability of the data for tests on replicate surfaces.

Images of the boiling dynamics are shown in Fig. 6.9 for the same surfaces considered in the top panel of Fig. 6.8 at $\Delta T = 2$, 6, and 10 deg K. Supplemental Video 2 provides the high speed video from which these static images are derived and the observations noted below are more evident in the video than in the static images. The images (and supplemental videos) show that at $\Delta T = 10$ deg K film boiling conditions prevail for all three surfaces. At $\Delta T = 6$ deg K, film boiling exists on the $F_c = 0.56$ surface; while for the $F_c = 0.85$ and 0.98 surface, the boiling appears to be somewhat transitional with greater number of bubbles being produced. Lastly, at $\Delta T = 2$ deg K, the boiling on the $F_c = 0.98$ surface is more vigorous (higher production rate of vapor bubbles) than on the other two surfaces and appears to be nucleate boiling, with the behavior being transitional conditions for the $F_c = 0.56$ and 0.85 surfaces.

**Influence of Varying Pitch for Post Surfaces**

The bottom panel of Fig. 6.8 shows $q''_w$ vs. $\Delta T$ data at a fixed cavity fraction of $F_c = 0.85$ and pitches of $w = 8$, 16, and 24 $\mu$m (fixed post height of 15 $\mu$m). The $q''_w$ data for the three $w$ values are essentially identical (within experimental uncertainty) and suggest film boiling behavior prevails over the majority of the $\Delta T$ range explored. Static images (derived from Supplemental Video 3) of boiling on these same three surfaces are shown in Fig. 6.10 at $\Delta T = 2$, 6, and 9 deg K. Transitional boiling (where the vapor film is noncontiguous and covers irregularly shaped regions) exists at $\Delta T = 2$ deg K for the $w = 8$ and 24 $\mu$m surfaces, while film boiling conditions exist for the 16 $\mu$m surface. Arrows are shown pointing to regions where solid-liquid contact exists and where a vapor film exists. At higher surface superheats, film conditions generally exist for all surfaces, although some very small regions of solid-liquid behavior exist for the $w = 24$ $\mu$m surface at $\Delta T = 6$ deg K.
ΔT = 2 deg K  ΔT = 6 deg K  ΔT = 10 deg K

Figure 6.9: High speed images of pool boiling at post patterned surfaces (h = 15 μm, w = 16 μm). Fc varies through 0.56, 0.85, and 0.98. This video is included as Supplemental Video 2.

**Influence of SHPo Feature Type (Posts vs. Ribs)**

As noted above, the difference between post patterned SHPo surfaces and rib patterned surfaces influences the transition from nucleate to film boiling. This is illustrated by the data of Fig. 6.11 which shows $q''_w$ as a function of $\Delta T$ for a rib patterned surface and a post patterned surface. The cavity fraction, pitch, and feature height are identical for the two surfaces ($F_c = 0.85$, $w = 24 \mu m$, and $h = 15 \mu m$). For the rib patterned surface, film boiling conditions are reached at $\Delta T \approx 9$ deg K, while for the post patterned surface, film conditions are reached at $\Delta T \approx 5$ deg K. Above $\Delta T \approx 9$ deg K (film conditions for both), the magnitude of $q''_w$ is the same for both surfaces. At $\Delta T < 3$ deg K, the $q''_w$ data suggest nucleate conditions exist for the rib patterned surface, while transitional conditions prevail for the post patterned surface. The suppression of nucleate boiling on the post patterned surface is significant. At $\Delta T = 2$ deg K, $q''_w$ for the post patterned surface is nominally 75% less than for the rib patterned surface ($3 \times 10^3$ W/m$^2$ for the ribs compared to $8 \times 10^2$ W/m$^2$ for the posts).
\[ \Delta T = 6 \text{ deg K} \]
\[ w = 24 \mu m \]
\[ \Delta T = 2 \text{ deg K} \]
\[ \Delta T = 9 \text{ deg K} \]
\[ w = 16 \mu m \]
\[ w = 8 \mu m \]

Figure 6.10: High speed images for \( \Delta T \) at \( F_c = 0.85 \) for post patterned surfaces (\( h = 15 \mu m \)). \( w \) varies through 8, 16, and 24 \( \mu m \). Arrows indicate the locations where the film remains in contact with the microfeatures (solid-liquid contact) and locations where a vapor film is present. This video is included as Supplemental Video 3.

Images derived from Supplemental Video 4 are shown in Fig. 6.12 for the same two surfaces considered in Fig. 6.11. Images are shown at \( \Delta T = 2, 4, \) and 10 deg K, with the top row of images corresponding to the rib patterned surface. The images at \( \Delta T = 2 \text{ deg K} \) show nucleate conditions existing on the rib patterned surface and transitional behavior existing for the the post patterned surface. At \( \Delta T = 4 \text{ deg K} \), the images show that film conditions exist over nearly the entire post patterned surface, while for the rib patterned surface, the nucleate type conditions still exist. At \( \Delta T = 10 \text{ deg K} \), the images show that film conditions prevail over both surfaces. Supplemental Video 4 illustrates the above points more clearly.

The observation above that film conditions prevail on post patterned surfaces at much lower surface superheat than for rib surfaces is intriguing. For post structured surfaces, the vapor region that exists below the liquid phase is interconnected in a two-dimensional sense throughout the entire lattice of posts. In contrast, the cavity regions for rib structured surfaces are entirely separated
Figure 6.11: $q''_w$ as a function of $\Delta T$ for rib patterned and post patterned surfaces. ($F_c = 0.85$, pitch $w = 24 \mu m$, and $h = 15 \mu m$)

Figure 6.12: High speed images for varying $\Delta T$ and for rib patterned and post patterned surfaces ($F_c = 0.85$, pitch $w = 24 \mu m$, and $h = 15 \mu m$). The ribs are oriented horizontally for the rib patterned surface. This video is included as Supplemental Video 4.
from adjoining cavities and the vapor in these regions is thus isolated from the other vapor regions. We hypothesize that it is this feature that contributes to transition from nucleate to film boiling occurring at higher surface superheat for the rib patterned surfaces.

Previous studies employing randomly patterned SHPo surfaces (nominal feature spacing and nominal feature height of 2 µm and 0.3–0.9 µm, respectively) with much smaller feature sizes than considered here [47, 48] found that film conditions were formed at nearly any surface superheat above 0 deg K. This is consistent with the behavior we observed for rib patterned surfaces with \( F_c = 0.93 \) and \( w = 40 \) µm and for post patterned surfaces with \( F_c = 0.56 \) and \( w = 16 \) and \( F_c = 0.85 \) and \( w = 8, 16, \) and 24 µm. We note that the cavity fraction and pitch values we considered are much larger than those reported by prior investigators for randomly structured surfaces. Further, our data show that the onset of film boiling depends not only on whether or not a surface is superhydrophobic or not, but also on the cavity fraction and what type of surface features exist. We expect that “hole” patterned SHPo surfaces would exhibit behavior more similar to rib patterned surfaces than post patterned surfaces. We recommend further investigation of the influence of the type of the microstructure features on film boiling and the related processes of evaporation and bubble nucleation.

**Influence of Cavity Depth for Rib Surfaces**

This last section considers the influence of cavity depth on the transition from nucleate to film boiling for rib structured surfaces. Shown in Fig. 6.13 is \( q''_w \) as a function of \( \Delta T \) with \( F_c = 0.8 \) and \( w = 40 \) µm. For one surface, the cavity depth was 15 µm and for the other surface it was 4 µm. Here the transition from nucleate to film boiling occurs at a lower surface superheat as the size of the surface features decreases. The first data point in the film boiling regime is at \( \Delta T = 7 \) deg K for the \( h = 4 \) µm surface and at \( \Delta T = 14 \) deg K for the \( h = 15 \) µm surface. This behavior is consistent with the observation from prior work with much smaller feature heights (0.3–0.9 µm) that show the film boiling regime exists at very small values of the surface superheat, \( \Delta T \to 0 \) deg K. High speed imaging was analyzed for these surfaces and the videos confirm the above observations. At \( \Delta T > 14 \) deg K, where film boiling prevails for both surfaces, the value of \( q''_w \) vs. \( \Delta T \) is nearly identical.
Figure 6.13: $q''_w$ as a function of $\Delta T$ for rib patterned SHPo surfaces ($F_c = 0.8$, $w = 40 \mu m$) for $h = 4.2 \mu m$ and $h = 15 \mu m$.

6.5.3 Summary

The microstructure of superhydrophobic surfaces influences the boiling curve. As observed on the classical pool boiling curve, a local maximum occurs at the critical heat flux and a local minimum occurs at the Leidenfrost point. While our experimental study is not amenable to determining the exact location of these points (the system was on-off controlled and was unstable in transition boiling), we approximate these points, first, for the surface superheat of the critical heat flux, as the temperature where the highest heat flux is achieved in the nucleate boiling region. Second, for the Leidenfrost point, the surface superheat at which the minimum heat flux is obtained in the film boiling regime. The surface superheats of the critical heat flux and of the Leidenfrost point as a function of the surface microstructure parameters are presented in Table 6.3. In addition to the measurement error ($\pm 1.6$ deg K, see Section 6.4.3) and the error in repeatability (expressed on Table 6.3 where significant), a sampling error due to recording data points every 1 deg K is observed. These errors are one-sided with an error of 0 to +0.5 deg K occurring for $\Delta T_C$ and -0.5 to 0 deg K for $\Delta T_L$. 

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Table 6.3: Critical heat flux and Leidenfrost point temperatures for all surface microstructures considered. The measurement error in $\Delta T$ is $\pm 1.6$ deg K. Repeatability error is reported where significant.

(a) Rib patterned surfaces: varying $F_c$

<table>
<thead>
<tr>
<th>$F_c$</th>
<th>w</th>
<th>h</th>
<th>$\Delta T_C$</th>
<th>$\Delta T_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>40</td>
<td>16</td>
<td>8±2</td>
<td>18</td>
</tr>
<tr>
<td>0.8</td>
<td>40</td>
<td>15</td>
<td>5±1</td>
<td>14</td>
</tr>
<tr>
<td>0.93</td>
<td>40</td>
<td>17</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

(b) Post patterned surfaces: varying $F_c$

<table>
<thead>
<tr>
<th>$F_c$</th>
<th>w</th>
<th>h</th>
<th>$\Delta T_C$</th>
<th>$\Delta T_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.56</td>
<td>16</td>
<td>15</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>0.85</td>
<td>16</td>
<td>15</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>0.98</td>
<td>16</td>
<td>15</td>
<td>9</td>
<td>22</td>
</tr>
</tbody>
</table>

(c) Post patterned surfaces: varying $w$

<table>
<thead>
<tr>
<th>$F_c$</th>
<th>w</th>
<th>h</th>
<th>$\Delta T_C$</th>
<th>$\Delta T_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>8</td>
<td>15</td>
<td>6±3</td>
<td>13</td>
</tr>
<tr>
<td>0.85</td>
<td>16</td>
<td>15</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>0.85</td>
<td>24</td>
<td>15</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

(d) Rib patterned surfaces: varying $h$

<table>
<thead>
<tr>
<th>$F_c$</th>
<th>w</th>
<th>h</th>
<th>$\Delta T_C$</th>
<th>$\Delta T_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>40</td>
<td>4</td>
<td>3±1</td>
<td>7±1</td>
</tr>
<tr>
<td>0.8</td>
<td>40</td>
<td>15</td>
<td>5±1</td>
<td>14</td>
</tr>
</tbody>
</table>

(e) Rib patterned vs. post patterned

<table>
<thead>
<tr>
<th>$F_c$</th>
<th>w</th>
<th>h</th>
<th>$\Delta T_C$</th>
<th>$\Delta T_L$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>24</td>
<td>15</td>
<td>3</td>
<td>9±2</td>
<td>Rib</td>
</tr>
<tr>
<td>0.85</td>
<td>24</td>
<td>15</td>
<td>4</td>
<td>6</td>
<td>Post</td>
</tr>
</tbody>
</table>

The influence of surface microstructure on the surface superheat of the critical heat flux, $\Delta T_C$, and on the Leidenfrost point, $\Delta T_L$, can be considered. On rib patterned surfaces, $\Delta T_C$ and $\Delta T_L$ decrease with increasing cavity fraction. These trends are apparent in Table 6.3. On post patterned surfaces, the surface superheat of the critical heat flux and the Leidenfrost point increase with increasing cavity fraction as is visible in Table 6.3b. The authors hypothesize that this increase in the Leidenfrost point with increasing cavity fraction is due to wetting of the cavities which becomes more likely on post patterned surfaces with high cavity fraction. In Table 6.3c, we see that pitch does not have a monotonic influence on the critical heat flux, surface superheat, or the Leidenfrost temperature. However, in Table 4d, the critical heat flux surface superheat and the Leidenfrost temperature increase as the depth of the cavities increase. Finally, comparing the critical heat flux temperature and the Leidenfrost temperature in Table 6.3e, the critical heat flux temperature on the post patterned surface is equal, within measurement uncertainty, to the surface superheat at which the rib patterned surface achieves critical heat flux. The Leidenfrost temperature is higher for the
rib patterned surfaces. Further, we note, observing Fig. 6.11, that the critical heat flux on the rib patterned surface is at least three times greater than that on the post patterned surface indicating that critical heat flux behavior is nearly absent for the post patterned surfaces.

6.6 Conclusions

Heat flux as a function of surface superheat was obtained empirically for ten micropatterned SHPo surface types with rib or post geometries. The results were compared to those obtained by previous investigators for boiling on smooth HPo and randomly roughened SHPo surfaces. The following observations are made concerning the influence of SHPo surface microstructure on pool boiling:

- Transition from nucleate to film boiling on rib patterned surfaces occurs at lower surface superheats as the cavity fraction increases.

- Changes in cavity fraction and pitch do not influence boiling on post patterned surfaces.

- The surface superheat at which transition from nucleate to film boiling occurs increases as the microfeature height increases from 4 \( \mu \text{m} \) to 15 \( \mu \text{m} \).

- Post patterned surfaces suppress nucleate boiling more than rib patterned surfaces. Rib patterned surfaces exhibit a sudden transition while post structured surfaces exhibit non-contiguous vapor films which grow slowly across the surface until film boiling is achieved.

- While surface microstructure strongly influences the surface superheat of the transition point, it negligibly influences heat flux once stable film boiling is achieved.
CHAPTER 7. CONCLUSIONS

This dissertation has explored thermal transport at superhydrophobic surfaces in jet impingement, natural convection, and pool boiling. Chapters 2–3 analytically explored jet impingement heat transfer at superhydrophobic surfaces. Chapter 4 empirically considered thermal transport at superhydrophobic surfaces due to jet impingement and compared the results to the isoflux model presented in Chapter 3. Chapter 5 analytically explored thermal transport in a developing natural convection boundary layer at a vertical superhydrophobic wall. Chapter 6 empirically considered the influence of surface microstructure on pool boiling at superhydrophobic surfaces. Key conclusions from the jet impingement work (Chapters 2–4) will first be summarized followed by conclusions from the natural convection model and pool boiling work. Finally, considerations for the work as a whole and recommendations for future work will be presented.

7.1 Jet Impingement at Superhydrophobic Surfaces

Thermal transport in liquid jets impinging at superhydrophobic surfaces had not been previously considered. Chapter 2 analytically explored thermal transport due to a circular impinging jet at a superhydrophobic surface maintained at constant temperature. Laminar boundary layer flow was considered with varying jet Reynolds number ($Re = 3 \times 10^3$ to $1.5 \times 10^4$). Further, temperature jump length was varied ($\hat{\lambda}_T = 0$ to 0.2). Chapter 3 analytically explored the same hydrodynamic scenario and instead imposed an isoflux wall boundary condition. Three important conclusions may be drawn from these models. First, the thermal transport decreased dramatically as temperature jump length increased. Second, the influence of temperature jump length was much greater than the influence of slip length. Finally, the influence of jet Reynolds number, Prandtl number, and isoflux versus isothermal heating became negligible as temperature jump length increased.

Chapter 4 empirically explored jet impingement at post patterned (isotropic) superhydrophobic surfaces with varying microfeature pitch ($w = 8, 16,$ and $24 \, \mu m$) and cavity fraction
($F_c = 0.56$ and $0.85$). Further, jet Reynolds number varied from $1.1 \times 10^4$ to $1.7 \times 10^4$. Three conclusions are important to make concerning these results. First, the average Nusselt number of a superhydrophobic surface with small microfeature pitch and cavity fraction is smaller than the average Nusselt number of a smooth hydrophobic surface. Second, the empirical results agree well with the analytical model presented in Chapter 3 and suggest that the ratio of temperature jump length to slip length is $3.1 \pm 0.3$. Finally, at larger pitch and cavity fraction, the empirical results deviate from the analytical model and show an increase in local Nusselt number relative to surfaces with smaller pitch and cavity fraction. It is important to note that the Nusselt number for these surfaces is always less than the smooth hydrophobic scenario.

### 7.2 Natural Convection at Superhydrophobic Surfaces

Prior studies had considered fully developed natural convection in microchannels but developing natural convection flow had not been considered. Natural convection at a vertical, isothermal wall was analytically explored for a range of laminar scenarios with varying Rayleigh number ($Ra_L = 1.1 \times 10^4$ to $1.7 \times 10^9$). The Nusselt number decreased as temperature jump length increased with the greatest decrease near the lower edge of the plate. Further, the Nusselt number experienced a greater decrease at higher Rayleigh number as temperature jump length increased illustrating the surprising result that resistance to heat transfer depended not only on the surface temperature jump length but also on the driving temperature difference.

### 7.3 Influence of Surface Microstructure on Pool Boiling

Prior studies had shown that superhydrophobic surfaces dramatically alter pool boiling behavior, moving the critical heat flux and Leidenfrost point to lower surface superheats. Further, at large contact angles, nucleate boiling was eliminated altogether [47,48]. However, these studies neglected to explore the influence of surface microstructure and this has been accomplished in the present work.

Pool boiling was explored for surface superheat spanning 2 to 38 K at smooth hydrophobic surfaces and superhydrophobic surfaces. Surfaces spanning a range of microstructure parameters, cavity fraction ($F_c = 0.5$ to 0.98), pitch ($w = 8$ to 40 $\mu$m), and microfeature height ($h = 4$ and
15 \mu m), were considered and heat flux as a function of surface superheat was obtained for each surface.

The following observations were made concerning the influence of these parameters on the transition from nucleate to film boiling. At rib patterned surfaces, transition superheat decreases as cavity fraction increases. Increasing microstructure height modestly increases the transition superheat. Nucleate boiling is more suppressed on post patterned surfaces than rib patterned surfaces. Further, rib patterned surfaces exhibit a sudden transition while post patterned surfaces experience a gradual transition. Once stable film boiling is achieved, the surface microstructure negligibly influences thermal transport.

7.4 General Conclusions and Future Work

So long as microfeatures remain non-wetted (Cassie state), superhydrophobic surfaces inhibit heat transfer in jet impingement, natural convection, and pool boiling. The mechanism of heat transfer reduction in flows without phase change (e.g. jet impingement and natural convection) is the increased surface thermal resistance resulting from trapped air in the microcavities. In contrast, the dominant mechanism of heat transfer reduction in pool boiling is film boiling at surface superheats much lower than smooth hydrophobic surfaces, which occurs due to the lower liquid adhesion at the surface.

Modeling and exploring thermal transport reduction in these flows is important because superhydrophobic surfaces have demonstrated drag reduction and self-cleaning and may be utilized for these properties in scenarios where heat transfer is required. These models and empirical results show how thermal transport will be inhibited.

Further research to improve our understanding of thermal transport at superhydrophobic surfaces will be valuable. The author proposes three areas for potential research. The first concerns jet impingement. Post patterned superhydrophobic surfaces were considered and these exhibited isotropic slip lengths. Prior analytical consideration of hydrodynamics at rib patterned superhydrophobic surfaces demonstrate elliptical hydrodynamic jumps and breakup transitions as well as angular dependence on film thickness, boundary layer growth, and free surface velocity [23, 24]. Rib patterned surfaces are a common microgeometry and the trapped air in the microcavities is more stable. The thermal transport for these scenarios has not been considered and a model with
supporting experiments is a valuable contribution. Second, while the analytical model for natural convection has been proposed it has not been experimentally validated. A model for the isoflux scenario and experimental validation performed in a similar manner as the present jet impingement experiments would be valuable. Finally, the pool boiling results demonstrated that surface microstructure can dramatically influence the surface superheat at which the critical heat flux occurs. To the author’s knowledge, no models have been proposed to predict critical heat flux as a function of pitch and cavity fraction for rib or post patterning. This would be another valuable contribution.
REFERENCES


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APPENDIX A.  JET IMPINGEMENT MODEL CODE

The numerical solutions of the isothermal and isoflux jet impingement models (Ch. 2 and Ch. 3, respectively) were implemented in Mathematica® as Wolfram Language packages IsothermalJetSolution and IsofluxJetSolution, respectively, and are included in Sections A.1 and A.2. The implementation of the Wang stagnation model [74], WangStagnationModel is included in Section A.3 since it is required by these codes. We note that the code utilized to obtain the isothermal results in Ch. 2 did not include the stagnation model and is not presented here. However, it was modified to include the stagnation region to obtain the results for Ch. 3 and is presented as Sec. A.1.

A.1 Isothermal Model

(* Numerical solution to jet impingement on an isothermal surface with uniform slip. All lengths normalized by "a", the jet diameter. All velocities normalized by "Vj", the impinging jet velocity. Each length and velocity is assumed to have a hat, i.e. was non-dimensionalized even though shown without the hat. R is the radius normalized by a to integrate to.*)

BeginPackage["IsothermalJetSolution"]

model::usage = "Numerical solution to jet impingement on an isothermal surface with uniform slip."
r::usage = "Radial coordinate"

Begin[ "Private"]

model[Re_, Pr_, \[Lambda]_, \[Lambda]T_, rmin_, rs_, r_, r1f_, r2f_, r3f_, R_] := Block[{momEq1, energyEq1, hEq1, s1, momEq2, energyEq2, s2, momEq3, energyEq3, s3, r, r0, r1, rDelta, rDeltaNew, rDeltaT, rDeltaTNew, out, h0, rDeltaT0, h, h1, rThetafs, rThetafsNew, rThetaw, Nu, NuD, Nur, NuBar, NDaccGoal, NDprecGoal, NDwPrec, NDMethod, NDmaxStepFrac, NDInterpolation, NDDiff, RegionIV, tol, \[Epsilon], \[Lambda]star, \[Lambda]Tstar, rDeltaStarSolnStag, \[CapitalLambda], B, A, Am, AsNew, dummy1, dummy2, \[Xi], solnStag, NuDStag, ReD, stagToRadialTrans, largeNumber, largeNumberT, max\[Eta], Const, prec, rDeltaTstar, NuStag, rDeltaS, rDeltaTs},

NDaccGoal = 10;
NDprecGoal = 10;
NDwPrec = MachinePrecision;
NDDiff = 8;
tol = 1*^-10;

(*Stagnation Region*)
largeNumber = 5;
largeNumberT = 5;
max\[Eta] = 5;
Const = 458/1000; (*Matching Liu1993 for inviscid solution*)
prec = MachinePrecision;
\[Lambda]star = \[Lambda]/2;
\[Lambda]Tstar = \[Lambda]T/2;
(* Wang Model *)

$$\Delta s = 2 \Delta \text{starSolnStag};$$
$$\Delta Ts = 2 \Delta \text{Tstar};$$
$$\Xi \text{solnStag} = \Delta \text{Tstar} / \Delta \text{starSolnStag};$$
$$\text{NuDStag} = \text{NuStag} \times 2 / \pi;$$

(* Region I *)

momEq1 = \[\pi\] r / Re = (2 / 3 \[\Delta\] [\[\Delta\]] r + \[\Lambda\]) / (140 \[\Delta\] r)^3;

energyEq1 = \[\pi\] / (Re \[Pr\]) = (-2 r \[\Delta\] [\[\Delta\]] r (3 \[\Lambda\] T + 2 \[\Delta\] T [\[\Delta\]][r]) / (140 \[\Delta\] r)^3);
AccuracyGoal -> NDaccGoal, PrecisionGoal -> NDprecGoal,
WorkingPrecision -> NDwPrec, MaxStepFraction -> 1/1000,
InterpolationOrder -> All;

out = FindRoot[ Evaluate[(\[Delta]/. s1[[1]])[r] == 1/(2*r) +\[Delta][r]^-2/(4*(2/3\[Delta][r] +\[Lambda]))]/. s1[[1]],{r,2}];
r0 = r /. out;
h0 = Evaluate[(\[Delta][r0]/. s1)[[1]];
[Delta]T0= Evaluate[(\[Delta]T[r0]/. s1)[[1]];

(* Region II *)
momEq2 = (2 \[Pi])/Re == (2 (680 h[r]^4+7560 r \[Lambda]^3 Derivative[1][h][r]+1890 \[Lambda]^2 h[r] (4 \[Lambda]+5 r
Derivative[1][h][r])+54 \[Lambda] h[r]^-2 (175 \[Lambda]+67 r
Derivative[1][h][r]+54 h[r]^-3 (4257 \[Lambda]+680 r
Derivative[1][h][r]) )/(105 r^2 h[r] (12 \[Lambda]+5 h[r])
^-2);
Derivative[1][h][r]-8 \[Lambda] Derivative[1][\[Delta] T][r])
Derivative[1][\[Delta] T][r]-6 \[Delta] T[r]]
\[ (r)^2 (5 (5 \Lambda +2 \Lambda T) \text{Derivative}[1][h][r]-16 \Lambda \text{Derivative}[1][\Delta T][r]) +9 \Lambda \Delta T [r] (-25 \Lambda T \text{Derivative}[1][h][r]+4 (5 \Lambda +6 \Lambda T) \text{Derivative}[1][\Delta T][r])))/(35 r h[r]^4 (12 \Lambda +5 h[r]^2 (3 \Lambda T+2 \Delta T[r]))); \]

\[ s2 = \text{NDSolve}[[\text{momEq2, energyEq2, } h[r0]==h0, \Delta T[r0]==\Delta T0], \{h, \Delta T\}, \{r,r0,r2f\}, \text{Method}\rightarrow \{"ExplicitRungeKutta", \text{DifferenceOrder}\rightarrow \text{NDDiff}\}, \text{AccuracyGoal}\rightarrow \text{NDaccGoal}, \text{PrecisionGoal}\rightarrow \text{NDprecGoal}, \text{WorkingPrecision}\rightarrow \text{NDwPrec}, \text{MaxStepFraction}\rightarrow 1/1000, \text{InterpolationOrder}\rightarrow \text{All}]; \]

RegionIV = True;
Quiet[Check[out = \text{FindRoot}[\text{Evaluate}[h[r] == \Delta T[r]/.s2[[1]]], \{r, r0+0.1\}], RegionIV = False]];
(*Uses Newton's method by default*)

If[RegionIV, (*To evaluate if there is a region IV*)
   r1 = r/.out;
h1 = \text{Evaluate}[h[r1]/.s2][[1]];]

(*Region III*)
\text{momEq3 = momEq2;}
\text{energyEq3 = \Pi}/(Re*Pr) = -(18 (-630 \Lambda^2 \Lambda T-312 \Lambda \Lambda T h[r]+(71 \Lambda-65 \Lambda T) h[r]^2) \Theta fs[r] \text{Derivative}[1][h][r]+(3 \Lambda T+2 h[r]) (12 \Lambda +5 h[r]) (1260 \Lambda \Lambda T+525 (\Lambda+\Lambda T) h[r]+272 h[r]^2) \text{Derivative}[1][\Theta fs][r])/(210 r (3 \Lambda T+2 h[r]) (12 \Lambda +5 h[r]^2 \Theta fs[r]));
s3 = NDSolve[{momEq3, energyEq3, h[r1]==h1, \[Theta]fs[r1] == 1}, {h, \[Theta]fs}, {r, r1, r3f}, Method -> {"ExplicitRungeKutta", DifferenceOrder -> NDDiff}, MaxStepFraction -> 1/1000, AccuracyGoal -> NDaccGoal, PrecisionGoal -> NDprecGoal, WorkingPrecision -> NDwPrec, InterpolationOrder -> All];

(*Piecewise equations to return*)

h = Piecewise[{{Null, r<rs}, {1/(2*r)+\[Delta][r]^2/(4*(2/3\[Delta][r]+\Lambda)/s1, r<r0}, {h[r]/.s2, r0<r<r1}, {h[r]/.s3, r>r1}}];
\[Theta]fsNew = Piecewise[{{0, r<r1}, {\[Theta]fs[r]/.s3, r1<r}}];

(*Define \[Delta]New after intersection is found*)
\[Delta]New = Piecewise[{{\[Delta]starSolnStag*2, r<rs}, {\[Delta][r]/.s1[[1]], rs<r<r0}, {Null, r>r0}}];

(*Redefine \[Delta]TNew*)
\[Delta]TNew = Piecewise[{{\[Xi]solnStag*\[Delta]starSolnStag*2, r<rs}, {\[Delta]T[r]/.s1[[1]], rs<r<r0}, {\[Delta]T[r]/.s2[[1]], r0<r<r1}}];

(*uses r normalized by a and Nu using \[Pi] a*)
Nu = Piecewise[{{NuDStag*\[Pi]/2, r<rs}, {\[Pi]/\[Lambda]T + 2/3\[Delta]T[r]/.s1[[1]], rs<r<r0}, {\[Pi]/\[Lambda]T + 2/3\[Delta]T[r]/.s2[[1]], r0<r<r1}, {3*\[Pi]*\[Theta]fs[r]/(3*\[Lambda]T + 2*h[r]/.s3)[[1]], r1<r}}];

(*Add smoothing curve fit from stagnation region to radial flow region.*)
\[ \text{NuSubs} = \text{NuDStag} \cdot \pi /2; \]

\[ \text{NuSubm} = \text{Nu} /. r \to \text{rm}; \]

\[ \text{NuPrimeSubm} = D[\text{Nu}, r] /. r \to \text{rm}; \]

\[ \text{num} = \text{NuSubs} - \text{NuSubm} - 1/(2(-\text{rs} + \text{rm})) \cdot \text{NuPrimeSubm} ((-\text{rs}^2 + 2 \cdot \text{rs} \cdot \text{rm} - \text{rm}^2)); \]

\[ \text{den} = -3((-\text{rs}^2 + \text{rm}^2)/(2(-\text{rs} + \text{rm}))((-\text{rs}^2 + 2\text{rs}\cdot \text{rm} - \text{rm}^2)) + (-2\text{rs}^3 + 3\text{rs}^2 \cdot \text{rm} - \text{rm}^3); \]

\[ d = \frac{\text{num}}{\text{den}}; \]

\[ c = 1/(-2\text{rs} + 2\text{rm}) \cdot (\text{NuPrimeSubm} + 3d \cdot (\text{rs}^2 - \text{rm}^2)); \]

\[ b = -2c \cdot \text{rs} - 3d \cdot \text{rs}^2; \]

\[ a = \text{NuSubs} + 2c \cdot \text{rs}^2 + 3d \cdot \text{rs}^3 - c \cdot \text{rs}^2 - d \cdot \text{rs}^3; \]

\[ \text{transitionCurve} = a + b \cdot r + c \cdot r^2 + d \cdot r^3; \]

\[ \text{Nu} = \text{Piecewise} \{ \{ \text{Nu}, r < \text{rs} \}, \{ \text{transitionCurve}, \text{rs} < r < \text{rm} \}, \{ \text{Nu}, r > \text{rm} \} \}; \]

\[ (*\text{Nu}_d \text{ as function of } r \text{ normalized by } a*) \]

\[ \text{Nud}=2/\pi \cdot \text{Nu}; \]

\[ (*\text{Nu}_r \text{ as function of } r \text{ normalized by } a*) \]

\[ \text{Nur}=r/\pi \cdot \text{Nu}; \]

\[ , \]

\[ (*\text{To evaluate if there is no regionIV}*) \]

\[ r1 = \text{Null}; \]

\[ h = \text{Piecewise} \{ \{ \text{Null}, r < \text{rs} \}, \{ 1/(2 \cdot r) + \delta[r]^2/(4 \cdot (2/3) \cdot \delta[r] + \lambda) /. \text{s1}, r < r < \text{r0} \}, \{ h[r] /. \text{s2}, r > \text{r0} \} \}; \]

\[ \theta_{\text{fsNew}} = \text{Null}; \]

\[ (*\text{Redefine } \delta[*]) \]
\[ \Delta \text{New} = \text{Piecewise}[\{{\Delta \text{starSolnStag}*2}, r< rs\},\{({\Delta \text{r}[r]/. s1)[[1]],rs<r<r0},\{\text{Null}, r>r0}\}]; \]

(*Redefine \[\Delta \text{TNew}\]*)

\[ \Delta \text{TNew} = \text{Piecewise}[\{{\Xi \text{solnStag}*\Delta \text{starSolnStag}*2}, r< rs\},\{({\Delta \text{T}[r]/. s1)[[1]],r<r0},\{({\Delta \text{T}[r]/. s2)[[1]],r0<r}\}]; \]

(*uses r normalized by a and Nu using \[\Pi\] a*)

\[ \text{Nu}=\text{Piecewise}[\{{\text{NuDStag}*\Pi}/2, r<rs\},\{({\Pi}/({\Lambda \text{T} + 2/3}\Delta \text{T}[r]/. s1)[[1]],rs<r<r0},\{({\Pi}/({\Lambda \text{T} + 2/3}\Delta \text{T}[r]/. s2)[[1]],r0<r}\}]; \]

\[ \text{NumSubs} = \text{NuDStag*}\Pi/2; \]
\[ \text{NuSubm} = \text{Nu}.r-> \text{rm}; \]
\[ \text{NuPrimeSubm} = \text{D}[\text{Nu},r]/.r->\text{rm}; \]
\[ \text{num} = \text{NuSubs} - \text{NuSubm} - 1/(2(-rs + \text{rm}))*\text{NuPrimeSubm*(-rs}^2 + 2*rs*\text{rm} - \text{rm}^2); \]
\[ \text{den} = -3*(-rs}^2 + \text{rm}^2)/(2*(-rs + \text{rm}))*(-rs}^2 + 2rs*\text{rm} - \text{rm}^2) + -2*rs}^3 + 3rs}^2 *\text{rm} - \text{rm}^3; \]
\[ \text{d} = \text{num/den}; \]
\[ \text{c} = 1/(2*rs + 2*rm)*(\text{NuPrimeSubm} + 3*d*(rs}^2 - \text{rm}^2)); \]
\[ \text{b} = -2*c*rs -3*d*rs}^2; \]
\[ \text{a} = \text{NuSubs} + 2*c*rs}^2 + 3*d*rs}^3 - c*rs}^2 - d*rs}^3; \]

\[ \text{transitionCurve} = \text{a+ b*r + c*r}^2 + d*r}^3; \]
\[ \text{Nu} = \text{Piecewise}[\{{\text{Nu}, r<rs},\{\text{transitionCurve},rs<r<\text{rm}\},\{\text{Nu},r>\text{rm}\}]}]; \]

(*\text{Nu}_d as function of r normalized by a*)
\begin{verbatim}
Nud = 2/\[Pi\]*Nu;

(*Nu_r as function of r normalized by a*)
Nur = r/\[Pi\]*Nu;

NuBar = NIntegrate[Nu*2*r,\{r,rmin,R\}, Method ->"GlobalAdaptive",
   Method->"GaussKronrodRule"},AccuracyGoal->10,
   PrecisionGoal->10,WorkingPrecision->MachinePrecision]/R^2;

{rs,\[Delta]starSolnStag,\[Xi]solnStag,NuDStag, r0,r1,h,\[Delta]
   New,\[Delta]TNew, \[Theta]fsNew,Nu,Nu0,Nur,NuBar,RegionIV}

End[]
EndPackage[]

A.2 Isoflux Model

(*Numerical solution to jet impingement on an isoflux surface
 with uniform slip. All lengths normalized by "a", the jet
 radius. All velocities normalized by "Vj", the impinging jet
 velocity. Each length and velocity is assumed to have a hat
, i.e. was non-dimensionalized even though shown without the
 hat. Note that the criterion that the thermal boundary
 layer merges with the film thickness needs to be determined.
 For the no-slip case this corresponds to Pr < 4.859. R is
 the radius normalized by a to integrate to.*)
\end{verbatim}
SetDirectory[NotebookDirectory[]];
Needs["WangStagnationModel"]

BeginPackage["IsofluxJetSolution", {"WangStagnationModel"]

model::usage = "Numerical solution to jet impingement on an isoflux surface with uniform slip."
r::usage = "Radial coordinate"

Begin["Private"]

model[Re_, Pr_, \[Lambda]_, \[Lambda]T_, rmin_, rs_, rm_, r1f_, r2f_, r3f_, R_] := Block[{momEq1, energyEq1, hEq1, s1, momEq2, energyEq2, s2, momEq3, energyEq3, s3, r, r0, r1, \[Delta], \[Delta]New, \[Delta]T, \[Delta]TNew, out, h0, \[Delta]T0, h, h1, \[Theta]fs, \[Theta]fsNew, \[Theta]w, Nu, NuD, Nur, NuBar, NDaccGoal, NDprecGoal, NDwPrec, NDMethod, NDmaxStepFrac, NDInterpolation, NDDiff, RegionIV, tol, \[Epsilon], \[Lambda]star, \[Lambda]Tstar, \[Delta]starSolnStag, \[CapitalLambda], B, A, AsNew, dummy1, dummy2, \[Xi] solnStag, NuDStag, ReD, stagToRadialTrans, largeNumber, largeNumberT, max[\[Eta], Const, prec, \[Delta]Tstar, NuStag, \[Delta]s, \[Delta]Ts, Nu0, Nu1],

NDaccGoal = 10;
NDprecGoal = 10;
NDwPrec = MachinePrecision;
NDDiff = 8;
tol = 1*^-10;

(*Stagnation Region*)
largeNumber = 5;
largeNumberT = 5;
max[\[Eta\]] = 5;
Const = 458/1000; (*Matching Liu1993 for inviscid solution*)
prec = MachinePrecision;
\[\Lambda\]star = \[\Lambda\]/2;
\[\Lambda\]Tstar = \[\Lambda\]T/2;

(*Wang Model*)
{\[\Delta\]starSolnStag, \[\Delta\]Tstar, NuStag} = WangSolution[Pr , Re, \[\Lambda\]star,\[\Lambda\]Tstar, Const, largeNumber, 
largeNumberT, max[\[Eta]], prec];

\[\Delta\]s = 2*\[\Delta\]starSolnStag;
\[\Delta\]Ts = 2*\[\Delta\]Tstar;
\[\Xi\]solnStag = \[\Delta\]Tstar /\[\Delta\]starSolnStag;
NuDStag = NuStag*2/\[\Pi\];

(*Region I*)
momEq1 = \[\Pi\]*r/Re == (2/3*\[\Delta\]T[r]+\[\Lambda\])*(3 \[\Delta\]r (52 \[\Delta\]r^2+288 \[\Delta\]r \[\Lambda\] 
+315 \[\Lambda\]^2)+2 (26 \[\Delta\]r^3+117 \[\Lambda\] \[\Delta\]r+315 \[\Lambda\]^2) r \[\Delta\]T[r])/(140 (2 \[\Delta\]r 
+3 \[\Lambda\])^3);
energyEq1 =\[\Pi\]/(Re*Pr) == 1/(140 r \[\Delta\]T[r]^3 (3 \[\Lambda\] +2 \[\Delta\]T[r])^2) \[\Delta\]T[r] (-2 r \[\Delta\]T[r] (105 \[\Lambda\] 
+\[\Delta\]T[r]^3+28 \[\Delta\]T[r]^3 \[\Delta\]T[r]+6 (\[\Lambda\] +\[\Delta\]T[r]) \[\Delta\]T[r]^3) Derivative[1][\[\Delta\]T][r]+ \[\Delta\]T[r] (3 \[\Lambda\]+2 \[\Delta\]T[r]) (2 \[\Delta\]T[r]^3 (\[\Delta\]T[r]+5 r Derivative[1][\[\Delta\]T][r])+7 \[\Delta\]T[r] 
^3)};
\[ r^2 (\Delta T[r] (15 \Lambda +4 (\Delta T[r])+6 r (5 \Lambda +2 (\Delta T[r])) Derivative[1][\Delta T][r])) \]

\[ s1 = NDsolve[{momEq1, energyEq1, \Delta rs == \Delta s, \Delta Trs == \Delta Ts}, {\Delta , \Delta T}, \{r, rs, r1f\},
Method -> "ExplicitRungeKutta", DifferenceOrder -> NDDiff, AccuracyGoal -> NDaccGoal, PrecisionGoal -> NDprecGoal,
WorkingPrecision -> NDwPrec, MaxStepFraction -> 1/1000, InterpolationOrder -> All];

out = FindRoot[Evaluate[(\Delta /s1[[1]])[r] == 1/(2*r) +\Delta [r]^2/(4*(2/3[\Delta [r] +\Lambda ))/. s1[[1]], \{r, 2\}];(*From Jet Solution Rev 18 Slip vs. Jump Comparison*)

r0 = r/. out;
h0 = Evaluate[\Delta [r0]/. s1[[1]];
\Delta T0 = Evaluate[\Delta T[r0]/. s1[[1]];

(*Region II*)

momEq2 = (2 \Pi)/Re == (2 (680 h[r]^4+7560 r \Lambda)^3 Derivative[1][h][r]+1890 \Lambda h[r]^2 h[r] (4 \Lambda +5 r Derivative[1][h][r])+54 \Lambda h[r]^2 (175 \Lambda +67 r Derivative[1][h][r])+h[r]^3 (4257 \Lambda +680 r Derivative[1][h][r]))/(105 r^2 h[r] (12 \Lambda +5 h[r])^2);(*From Jet Solution Rev 18 Slip vs. Jump Comparison*)

energyEq2 = \Pi/(Re*Pr) == 1/(35 r h[r]^4 (12 \Lambda +5 h[r])^2) \Delta T[r] (-\Delta T[r] (105 \Lambda h[r]^2 (6 \Lambda +5 h[r])+28 h[r]^2 (6 \Lambda +5 h[r]) \Delta T[r]+4 (9 \Lambda +5 h[r]) \Delta T[r] Derivative[1][h][r]+h[r] (12 \Lambda +5 h[r]) (5 \Delta T[r]^3+21 h[r]^2 (5 \Lambda +2 \Delta T[r])) Derivative[1][\Delta T][r]);

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\[ s2 = \text{NDSolve}\{\text{momEq2}, \text{energyEq2}, h[r0] == h0, \Delta[T][r0] == \Delta[T][r0], h, \Delta[T], \{r, r0, r2f\}, \text{Method} \to \{"\text{ExplicitRungeKutta"}, \text{DifferenceOrder} \to \text{NDDiff}, \text{AccuracyGoal} \to \text{NDaccGoal}, \text{PrecisionGoal} \to \text{NDprecGoal}, \text{WorkingPrecision} \to \text{NDwPrec}, \text{MaxStepFraction} \to 1/1000, \text{InterpolationOrder} \to \text{All}\}; \]

RegionIV = True;
Quiet[\text{Check}\[out = \text{FindRoot}\{\text{Evaluate}\{h[r] == \Delta[T][r]/.s2[[1]],\{r, r0+0.1\}\}, \text{RegionIV} = \text{False}\}\];
(*Uses Newton’s method by default*)

If[RegionIV, (*To evaluate if there is a region IV*)
r1 = r/.out;
h1 = \text{Evaluate}\{h[r1]/.s2[[1]]\};

(*Region III*)
momEq3 = momEq2; (*From Jet Solution Rev 18 Slip vs. Jump Comparison*)
energyEq3 = \[Pi]/(Re*Pr) == ((2 (630 \[Lambda]^2+312 \[Lambda] h[r]+65 h[r]^2) \text{Derivative}[1][h][r])/(12 \[Lambda]+5 h[r])^2+35 \text{Derivative}[1][\[Theta]fs][r])/(70 r);
s3 = \text{NDSolve}\{\{\text{momEq3}, \text{energyEq3}, h[r1] == h1, \[Theta]fs[r1] == 0\},\{h, \[Theta]fs\}, \{r, r1, r3f\}, \text{Method} \to \{"\text{ExplicitRungeKutta"}, \text{DifferenceOrder} \to \text{NDDiff}, \text{MaxStepFraction} \to 1/1000, \text{AccuracyGoal} \to \text{NDaccGoal}, \text{PrecisionGoal} \to \text{NDprecGoal}, \text{WorkingPrecision} \to \text{NDwPrec}, \text{InterpolationOrder} \to \text{All}\};

(*Piecewise equations to return*)
h = Piecewise[{Null, r < rs}, {1/(2*r) + \[Delta][r]^2/(4*(2/3 \[Delta][r] + \[Lambda])), .s1, r < r0}, {h[r]/.s2, r0 < r < r1}, {h[r]/.s3, r > r1}];
\[Theta]fsNew = Piecewise[{0, r < r1}, {\[Theta]fs[r]/.s3, r1 < r}];

(* Define \[Delta]New after intersection is found *)
\[Delta]New = Piecewise[{\[Delta]starSolnStag *2, r < rs}, {(\[Delta][r]/.s1)[[1]], rs < r < r0}, {Null, r > r0}];

(* Redefine \[Delta]TNew *)
\[Delta]TNew = Piecewise[{\[Xi]solnStag *\[Delta]starSolnStag *2, r < rs}, {\[Delta]T[r]/.s1)[[1]], rs < r < r0}, {\[Delta]T[r]/.s2)[[1]], r0 < r < r1}];

(* uses r normalized by a and Nu using \[Pi] a *)
Nu = Piecewise[{NuDStag *\[Pi]/2, r <= rs}, {\[Pi]/(\[Lambda][T[r]/.s1)[[1]], rs < r < r0}, {\[Pi]/(\[Lambda][T[r]/.s2)[[1]], r0 < r < r1}, {\[Pi]/(\[Lambda][T[r] + 2/3 h[r] + \[Theta]fs[r]/.s3)[[1]], r1 < r <= r}];

(* Add smoothing curve fit from stagnation region to radial flow region.*)
NuSubs = NuDStag *\[Pi]/2;
NuSubm = Nu/.r -> rm;
NuPrimeSubm = D[Nu, r]/.r -> rm;
num = NuSubs - NuSubm - 1/(2*(-rs + rm)) *NuPrimeSubm*(-rs^2 + 2*rs*rm - rm^2);
den = -3*(-rs^2 + rm^2)/(2*(-rs + rm))*(-rs^2 + 2rs*rm - rm^2) + -2*rs^3 + 3rs^2 *rm - rm^3;
d = num/den;
c = 1/(-2*rs + 2*rm)* (NuPrimeSubm + 3*d*(rs^2 - rm^2));
\[ b = -2c*rs -3d*rs^2; \]
\[ a = NuSubs + 2c*rs^2 + 3d*rs^3 - c*rs^2 - d*rs^3; \]

\[ \text{transitionCurve} = a + b*r + c*r^2 + d*r^3; \]
\[ \text{Nu} = \text{Piecewise}\left[\{\{\text{Nu}, r<rs\}, \{\text{transitionCurve}, rs<r<rm\}, \{\text{Nu}, r>rm\}\}\right]; \]

(*Nu_d as function of r normalized by a*)
\[ \text{Nud} = 2/\pi*Nu; \]

(*Nu_r as function of r normalized by a*)
\[ \text{Nur} = r/\pi*Nu; \]

\[ \text{Nu0} = \text{Nu} /. r->r0; \]
\[ \text{Nu1} = \text{Nu} /. r->r1 \]

,(*To evaluate if there is no regionIV*)
\[ \text{r1} = \text{Null}; \]
\[ \text{h1} = \text{Null}; \]
\[ \text{h} = \text{Piecewise}\left[\{\{\text{Null}, r<rs\}, \{1/(2*r)+[\Delta][r]-2/(4*2/3[\Delta][r]+[\Lambda])/s1, rs<r<r0\}, \{h[r]/.s2, r0<r\}\}\right]; \]
\[ \text{\Theta fsNew} = \text{Null}; \]

(*Redefine \[\Delta]\*)
\[ \text{\Delta New} = \text{Piecewise}\left[\{\{[\Delta]\text{starSolnStag}\times2, r<rs\}, \{([\Delta]\text{starSolnStag}\times2)\times1, rs<r<r0\}, \{\text{Null}, r>r0\}\}\right]; \]

(*Redefine \[\Delta T\text{New}\]*)
\[ \text{\Delta TNew} = \text{Piecewise}\left[\{\{[\Xi]\text{solnStag}\times[\Delta]\text{starSolnStag}\times2, r<rs\}, \{([\Delta]\text{T[r]}\times1), r0<r\}, \{([\Delta]\text{T[r]}\times1), r0<r\}\}\right]; \]
(*uses r normalized by a and Nu using \[Pi] a*)

\[ Nu = \text{Piecewise}\left[\begin{array}{ll}
{\{\text{NuDStag} \times \frac{\pi}{2} , r \leq rs}\},
{\left(\frac{\pi}{(\lambda T + 2/3 \delta T r) / . s1)} [[1]], rs < r \leq r0\},
{\left(\frac{\pi}{(\lambda T + 2/3 \delta T r) / . s2)} [[1]], r0 < r \right)}
\end{array}\right];
\]

(*Add smoothing curve fit from stagnation region to radial flow region.*)

\[ NuSubs = \text{NuDStag} \times \frac{\pi}{2}; \]
\[ NuSubm = Nu / . r \to rm; \]
\[ NuPrimeSubm = D[Nu, r] / . r \to rm; \]
\[ \text{num} = NuSubs - NuSubm - 1/(2(-rs + rm)) \times NuPrimeSubm*(-rs^2 + 2*rs*rm - rm^2); \]
\[ \text{den} = -3*(-rs^2 + rm^2)/(2*(-rs + rm))*(-rs^2 + 2rs*rm - rm^2) + -2*rs^3 + 3rs^2 * rm - rm^3; \]
\[ d = \text{num}/\text{den}; \]
\[ c = 1/(-2*rs + 2*rm)*(NuPrimeSubm + 3*d*(rs^2 - rm^2)); \]
\[ b = -2*c*rs -3*d*rs^2; \]
\[ a = NuSubs + 2*c*rs^2 + 3*d*rs^3 - c*rs^2 - d*rs^3; \]
\[ \text{transitionCurve} = a + b*r + c*r^2 + d*r^3; \]
\[ Nu = \text{Piecewise}\left[\begin{array}{ll}
{\{Nu, r \leq rs}\},
{\text{transitionCurve}, rs < r \leq rm}\},
{\{Nu, r > rm \}}
\end{array}\right];
\]

(*Nu-d as function of r normalized by a*)
\[ Nu_d = 2/\pi * Nu; \]

(*Nu-r as function of r normalized by a*)
\[ Nu_r = r/\pi * Nu; \]
\[\text{Nu}_0 = \text{Nu}/.r\to r_0;\]
\[\text{Nu}_1 = \text{Null};\]

(* Integrated function *)
\[\text{NuBar} = \frac{R^2}{\text{NIntegrate}}[\frac{1}{\text{Nu}}2*r,\{r,\text{rmin},R\}, \text{Method \to \{"GlobalAdaptive", Method \to \"GaussKronrodRule\"\}, \text{AccuracyGoal \to 8, PrecisionGoal \to 8, WorkingPrecision \to \text{MachinePrecision}}];\]

(* See derivation, 4/19/17-7*)

\{
\[\Delta\]_{\text{starSolnStag}}, \[\Xi\]_{\text{solnStag}}, \text{NuDStag}, rs, \[\Delta\]_{\text{s}}, \[\Delta\]_{\text{Ts}}, \text{NuStag}, r_0, \[\Delta\]_{\text{T0}}, h_0, \text{Nu}_0, r_1, h_1, \text{Nu}_1, h, \[\Delta\]_{\text{New}}, \[\Delta\]_{\text{TNew}}, \text{fsNew}, \text{Nu}, \text{Nud}, \text{Nur}, \text{NuBar}, \text{RegionIV}\}

\}

End[]
EndPackage[]

A.3 Stagnation Model

(* Solution to axisymmetric stagnation flow with slip as derived by C.Y. Wang 2006.*)

BeginPackage["WangStagnationModel"]

WangSolution::usage = "Provides the hydrodynamic BL thickness, thermal BL thickness, and Nu as predicted by C.-Y. Wang in 2006. Parameters are my normalization."

Begin[ "Private"]
WangSolution[Pr_, ReN_, \[Lambda]star_, \[Lambda]Tstar_, Const_, 
largeNumber_, largeNumberT_, max[\[Eta]_, prec_] := Module[
{equations, boundaryConditions, solution, \[Lambda], \[Beta], f, 
g, \[Theta], ReD, \[Eta]99, \[Eta]T99, \[Eta], \[Delta]star, \[Delta]Tstar, Nu},
ReD = 2*ReN/\[Pi];
\[Lambda] = Sqrt[2*Const*ReD]*\[Lambda]star;
equations = {f''[\[Eta]] + 2*f[\[Eta]]*f''[\[Eta]] - (f'[\[Eta]])^2 + 1 == 0, 
g''[\[Eta]] + 2*f[\[Eta]]*g''[\[Eta]] - f'[\[Eta]]* g[\[Eta]] == 0, 
\[Theta]''[\[Eta]] + 2*Pr*f[\[Eta]]*[\[Theta]''[\[Eta]] + 2*Pr*f[\[Eta]]]*\[Theta]''[\[Eta]] == 0};
boundaryConditions = {f'[0] == \[Lambda]*f''[0], g[0] == 1 + \[Lambda]*g'[0], f[0] == 0, f'[largeNumber] == 1, g[largeNumber] == 0, \[Theta][0] == 1 + \[Beta]*\[Theta]'[0], \[Theta][largeNumberT] == 0};
solution = NDSolve[Join[equations, boundaryConditions], {f, g, \[Theta], f', g', \[Theta]', f''}, \{\[Eta], 0, max[\[Eta]]\}, 
WorkingPrecision -> prec];
\[Eta]99 = \[Eta]/.FindRoot[(f'[\[Eta]]/.solution) == 99/100, \{\[Eta], 2, 0, 5\}, WorkingPrecision -> prec];
\[Eta]T99 = \[Eta]/.FindRoot[(\[Theta][\[Eta]]/.solution) == 1/100, \{\[Eta], 1, 0, 5\}, WorkingPrecision -> prec];
\[Delta]star = \[Eta]99/Sqrt[2*Const*ReD];
Nu = \[Pi]/2*Sqrt[2*Const*ReD]*-((\[Theta]''[0]/.solution)[[1]]);
\{\[Delta]star, \[Delta]Tstar, Nu\}
End[]
EndPackage[]
APPENDIX B. SAMPLE FABRICATION

This section contains pertinent information concerning the fabrication of samples utilized in the experiments performed. Many of the processes are well-documented in Adam Cowley’s dissertation [43] and those are referenced here.

B.1 Surfaces for Pool Boiling

Surfaces for pool boiling were fabricated using the standard superhyrophobic surface fabrication process. The appropriate sections in this chapter are referenced for further information. Photolithography is utilized to mask the substrate (B.3). Deep reactive ion etching (DRIE) is utilized to micromachine the cavities in the substrate (B.4). The photoresist is removed using Nano-strip® and any remaining residue is removed by an oxygen plasma “burn” (B.5). Chromium is deposited so that Teflon may adhere (B.7 and B.8). The wafers are then diced 52 mm square (B.9) and recoated with a second layer of Teflon.

B.2 Surfaces for Jet Impingement

The microfabrication process for substrates for jet impingement is similar to the microfabrication process for substrates for pool boiling but has some modifications. Photolithography is utilized to mask the substrate. To pattern the target in addition to the micropattern, the wafer is exposed with the micropattern mask and then with the target mask. Deep reactive ion etching (DRIE) is utilized to micromachine cavities. Since a resistive heater is screen printed on the back of the wafer, the wafer must be first be passivated to prevent the wafer from being energized by the heater. This is achieved by oxidizing the wafer (B.6). Then a silver resistance heater is screen printed on the side opposite the microfeatures (B.10). Following this step, the top surface is coated with chromium and Teflon. The leads are attached with conductive silver epoxy and the heater side of the wafers is spray-painted flat black (B.11).
B.3 Lithography (Negative Resist)

See the instructions in Section A.1 of Adam Cowley’s dissertation [43].

B.4 STS DRIE

See the instructions in Section A.3 of Adam Cowley’s dissertation [43].

B.5 Photoresist Removal

See the instructions in Section A.4 of Adam Cowley’s dissertation for Nano-strip® and in Section A.5 for the oxygen burn in the Planar Etcher II (PE-2) [43].

B.6 Oxidation

A wet oxide layer was grown to a thickness of 550 nm on each wafer for electrical passivation following DRIE and photoresist removal. The current furnace will soon be replaced so the processing steps (which are standard) are not published here. Consult with the BYU Cleanroom supervisor for instructions. An oxide growth calculator is available at https://cleanroom.byu.edu/OxideTimeCalc.

B.7 Chromium Deposition

The chromium should be coated to a thickness of 100 nm. See the instructions in Section A.6 of Adam Cowley’s dissertation [43] for using the Denton® E-beam Evaporator. The thermal evaporator may also be used.

B.8 Teflon Coating

A 100 nm film was applied. See instructions in Section A.7 Adam Cowley’s dissertation [43].
B.9 Dicing

See instructions in Adam Cowley’s dissertation Appendix A.8 [43].

B.10 Screen Printed Electric Resistance Heaters

This documentation was prepared in collaboration with Dewey Potts.

A 50 mm diameter electric resistance heater was designed for screen printing on the bottom of the oxidized wafers. First, the design is given in Section B.10.1. Then, the fabrication process is reported in Section B.10.2.

B.10.1 Design

The electric resistance heater was designed to produce 100 watts of heat within a circular area of 0.002 m². The design is shown in Fig. B.1. The power supply used had a maximum output of 20 volts, which required a low resistance to produce high power values ($Power = V^2/R$), where $V$ is the applied voltage and $R$ is the heater resistance. To produce a low resistance heater, the circle was divided into seven equal areas with electric resistance heaters used in parallel (each with a resistance of 1.75 Ω) to produce an overall nominal resistance of 0.25 Ω. The heater was created by screen printing lines of electrically conductive silver ink on the back of an oxidized silicon wafer. The thickness, $t$, of the fired screen printed ink was 11 μm and the resistivity, $\rho$, was $3.3 \times 10^{-5}$ Ω-mm.

Resistance of each area electrical resistance heater was calculated assuming straight heater lines with a constant rectangular cross section. The resistance of the curved sections was calculated as if they were straight lines, where the centerline was used for the length. The equation, $R = \rho l/wt$, was used to calculate resistance where $R$ is resistance, $\rho$ is resistivity (ohm-m), $l$ is the line length, $w$ is the width, and $t$ is the thickness. To produce an even heat flux, heater lines are evenly spaced over each area. Because lines had to start and end at a bus bar, only an odd integer number of lines would work. After the number of lengths was calculated, the line width was varied to achieve the desired resistance for each area. Parameters for resistance heaters in each area can be seen in Table B.1. The area resistance heater numbering is from left to right, where (in Fig. B.1) 1 is the far left, 4 is in the center, and 7 is the far right. A .dxf file of the heater, along
Figure B.1: The design of the screen-printed resistance heater screen is shown.
Table B.1: Electrical resistance heater parameters, where $l$ is the length, $w$ is the width, $R$ is the resistance, and $\rho$ is the resistivity.

<table>
<thead>
<tr>
<th>Area Number</th>
<th>$l$ (mm)</th>
<th>$w$ (mm)</th>
<th>Spacing between lines (mm)</th>
<th>$R$ (Ω)</th>
<th>$\rho$ (Ω mm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area 1</td>
<td>248.4</td>
<td>0.43</td>
<td>1.18</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Area 2</td>
<td>315.7</td>
<td>0.54</td>
<td>0.93</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Area 3</td>
<td>319.8</td>
<td>0.55</td>
<td>0.92</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Area 4</td>
<td>255.4</td>
<td>0.44</td>
<td>1.14</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Area 5</td>
<td>247.8</td>
<td>0.42</td>
<td>0.92</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Area 6</td>
<td>315.3</td>
<td>0.54</td>
<td>0.93</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Area 7</td>
<td>319.4</td>
<td>0.55</td>
<td>1.18</td>
<td>1.75</td>
<td>$3.3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

with the calculations, can be obtained by emailing deweypotts@gmail.com. The seven heaters are connected to a common positive bus bar on the bottom and common negative bus bar on the top. The bus bar resistance was not taken into account for the resistance of the heater since it added at most $0.07$ Ω to the resistance of any single area heater.

### B.10.2 Fabrication

The heater was fabricated using Ferro® 9913 screen printable silver ink. An imaged screen was purchased from RIV, Inc. Specifications for this screen are given in Table B.2. The instructions that follow detail the fabrication process.

<table>
<thead>
<tr>
<th>Screen-printing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
</tr>
<tr>
<td>Mesh Count</td>
</tr>
<tr>
<td>Wire Diameter</td>
</tr>
<tr>
<td>Emulsion Thickness</td>
</tr>
</tbody>
</table>

#### Screen-printing

1. Place wafer on screen

   (a) Retrieve oxidized silicon wafers.

   (b) Place black 3D-printed wafer aligner on screen.

   (c) Place wafer on screen with superhydrophobic (SH) side facing up.
(d) Place two pieces of blue painter’s tape on either side of the wafer.

(e) Remove 3D-printed wafer aligner.

(f) Place a Kimtech® wipe on top of the wafer.

(g) Place a piece of cardboard on top of the Kimtech® wipe.

(h) Flip the screen (as well as the Kimtech® wipe and cardboard) 180 degrees and place it on a flat surface.
   - Be careful not to move or slide anything around.

2. Apply Paste

   (a) With the spatula, apply a liberal amount of paste (Ferro® 9913, silver screen printing ink) along the edge of the squeegee (about 1 cm by 1 cm cross sectional area).

   (b) Lightly press the squeegee onto the screen about 1 cm from the top of the heater outline (align the squeegee so it is perpendicular to the heater lines)

   (c) Wipe the paste over the length of the heater outline until about 1 cm after the end of the heater outline. You should push down hard enough to get the paste into the heater outline, but the outline of the heater should not show through.

   (d) Using the reverse side of the squeegee (side with no paste), wipe over the heater with enough pressure to remove any extra paste that is over the heater outline. You should be able to see the outline of the heater through the paste.

3. Remove Wafer

   (a) Flip the screen (as well as the Kimtech® wipe and cardboard) 180 degrees and place it on a flat surface.

   (b) Hold the wafer from moving with one hand while you remove the blue tape with the other hand.

   (c) With one finger from your left hand, press down on the larger flat edge of the wafer.

   (d) Place a fingernail from your right hand under the wafer on the opposite side of the large wafer flat.
(e) Lift the wafer up with your right fingernail until the paste is no longer holding the wafer to the screen.

(f) Remove the wafer.

4. Clean Up

(a) Wipe away extra paste with spatula and place it into the clear vial with the rest of the excess paste. DON’T put the paste in the original container.

(b) Use a combination of Acetone, IPA, Kimtech® wipes, and Q-tips to remove any remaining paste. Squirt the IPA and Acetone onto the screen, wipe with Kimtech® wipes, apply more IPA and Acetone. Repeat until completely clean.

(c) Close both paste bottles tightly and tape the edges of the lids with electrical tape.

5. Dry and Fire, Location: CB 121

(a) Turn on the ventilation system.

   • The switch is on the wall to the North of the door.

(b) Preheat Lucifer furnace (or another furnace) to 257 °F.

(c) Preheat Recco furnace to 1562 °F.

(d) Let furnace heat up to 257 °F.

(e) Place wafers on glass wafer rack.

(f) Hold wafer rack with metal fork.

(g) Place wafer rack in Lucifer furnace for 15 minutes to dry.

(h) Remove wafer rack using metal fork and gloves.

(i) Place wafer rack in Recco oven for 12 minutes.

(j) Remove wafer rack from furnace with metal fork.

(k) Turn off Lucifer and Recco Furnaces.

(l) Turn off ventilation system unless someone else is using the furnaces.
B.11 Lead Wire Attachment and Spray Painting

These instructions were prepared in collaboration with Dewey Potts.

1. Attach leads

   (a) Put hot plate in fume hood.
   (b) Place a sheet of aluminum foil on top of the hot plate and wrap edges around the side.
   (c) Place stand with alligator clips behind hot plate.
   (d) Place wafer heater-side-up on hot plate so that the bus bars are on the right and left side.
   (e) Put metal weights on the right and left side of the wafer edge.
   (f) Place the bead of the thermocouple underneath one of the metal weights so that the bead is in contact with the aluminum foil.

2. Prepare wires

   (a) Cut two 18” sections of the black-white-red wire.
   (b) Remove 3/4” of insulation from both sides.
   (c) Cut two 12” sections of small diameter wire (Ethernet size).
   (d) Remove 3/4” of insulation from both sides.
   (e) Wind exposed small-diameter wire around large diameter wire.
   (f) Bend the combined wires 90 degrees at a point 3/4” from the edge of the insulation.
   (g) Place one wire in each of the alligator clips. Orient them so that the bent part of the wire is directly above and parallel with the bus bar.

3. Apply Paste

   (a) Apply paste (Atom Adhesives AA-DUCT 2979) on the bus bar where the leads will attach.
   (b) Apply paste onto the wire. Do this by removing the green nozzle, pushing the wire into the extruder and extruding paste so that the entire wire is covered with paste.
(c) Lower the alligator clip assembly until the wires are on the wafer.

(d) Use the wood end of a Q-tip to push the paste around the wire. Do not leave any air gaps.

4. Curing

(a) Rotate the power knob slowly until the temperature reaches around 125 °C.

(b) Adjust the temperature every 10 minutes to keep it as close to 125 °C as possible. Apply heat for 1 hour. After one hour, turn off hot plate. Allow to cool.

(c) To avoid scratching, pull up the entire alligator clip assembly and then remove the leads from the clips.

5. Apply flat black paint

(a) Place wafer patterned side down on clean surface.

(b) Apply primer (Rust-Oleum® 249340, High Heat, Primer): Coat 3 times 3 minutes apart. Let dry for 2 hours.

(c) Apply top coat (Rust-Oleum® 248903, High Heat, Flat Black): Coat 3 times 3 minutes apart. Let dry for 1 hours.
APPENDIX C. CODE FOR JET IMPINGEMENT EXPERIMENTS

The function \texttt{FiniteDifferenceVariableH} developed to solve the conduction problem in the wafer impinged by the liquid jet was written in Matlab® and is included in Section C.1. The code requires that the user provide \texttt{hfun(r,funParam)} and \texttt{hfun_pb(r,funParam_pb)} which receive a vector of radial coordinates, \( r \), and a structure of parameters, \texttt{funParam} or \texttt{funParam_pb} and return the convection coefficient \( h \) (as a row vector) at those locations.

C.1 Conduction Model

\begin{verbatim}
function results = FiniteDifferenceVariableH(bc1, hfun, hfun_pb, funParam, funParam_pb, qf, R1, R2, R3, t, k, Tc, ni, Theta_inf)
% This model predicts the temperature distribution in the wafer for a given surface heat transfer coefficient distribution, h(r).

% Inputs
% bc1: Boundary Condition 1: Set to 'finite' to use Tc at fixed temperature. Set to 'adia' to specify adiabatic.
% hfun \%W/m2-K function for h as a function of r. funParam, function
% parameters associated with hfun.
% hfun_pb \%W/m2-K function for h after breakup as a function of r.
\end{verbatim}
% funParam_pb, function parameters associated with hfun_pb

% qf  %W/m2 Applied heat
% t  %m Thickness of wafer
% R1 %m Radius of jet breakup
% R2 %m Radius of heater
% R3 %m Radius of wafer
% k %W/m-K Thermal conductivity
% Tc %Center Temperature
% ni %number of intervals
% Theta_inf = T_inf - T_j

% Outputs
% theta %K Temperature in wafer minus the jet temperature
% rval %m radial location
% checkq %W/m^2 the total heat leafing the wafer divided the
    the heater area. Should equal the heater heat flux.
% qf_r %W/m^2 heat flux in the radial direction as a function
    of radial location
% q_r %W heat rate in radial direction

% Solution parameters
nn = ni + 1;%number of nodes *

reverse = 0;
if R1 > R2
    temp = R2;
    R2 = R1;
    R1 = temp;
    reverse = 1;
end
% Calculations for other solution parameters.

\[ h = \frac{R_3}{n_1}; \]
\[ n_{i1} = \frac{R_1}{h}; \]
\[ n_{i2} = \frac{R_2}{h}; \]

\[ n_{1Integer} = \text{abs}(n_{i1} - \text{round}(n_{i1})) < 1e^{-10}; \]
\[ n_{2Integer} = \text{abs}(n_{i2} - \text{round}(n_{i2})) < 1e^{-10}; \]

if \( n_{1Integer} && n_{2Integer} \)

% Convert to be integer because it is close enough

\[ n_{i1} = \text{round}(n_{i1}); \]
\[ n_{i2} = \text{round}(n_{i2}); \]
\[ n_{n1} = n_{i1} + 1; \quad \% \text{node number at } R_1, \]
\[ n_{n2} = n_{i2} + 1; \quad \% \text{node number at } R_2 \]

% Setup b vector
\[ b = \text{zeros}(n_n, 1); \]

switch bc1
  case 'finite'
    b(1,1) = Tc;
  case 'adia'
end

% Setup coefficient matrix
\[ A = \text{zeros}(n_n); \]

% Boundary condition at wafer center (r = 0)
switch bc1
  case 'finite'
    A(1,1) = 1; \% Fixed Temperature
end
case 'adia'
    A(1,1:2) = [1,-1]; % Adiabatic condition.
end

% Region I internal nodes (heater region before breakup)
gI = qf/(k*t);
rI = h:h:(ni1-1)*h;
hOFrI = hfun(rI,funParam);
cI = (1/h^2 - 1./(rI*2*h));
dI = (-2/h^2 - hOFrI/(k*t));
eI = (1/h^2 + 1./(rI*2*h));
A(2:(nn1-1), 1:(nn1-2)) = diag(cI);
A(2:(nn1-1), 2:(nn1-1)) = A(2:(nn1-1), 2:(nn1-1)) + diag(dI);
A(2:(nn1-1), 3:(nn1)) = A(2:(nn1-1), 3:(nn1)) + diag(eI);

b(2:(nn1-1),1) = -gI;

% Matched heat flux between region I and region II.
A(nn1,(nn1-1):(nn1+1)) = [-1,2,-1];

if ~reverse
    % Region IIa internal nodes (heater region after breakup)
rII = (ni1+1)*h:h:(ni2-1)*h;
hOFrII = hfun_pb(rII,funParam_pb);
gII = qf/(k*t) + hOFrII/(k*t)*Theta_inf;
cII = (1/h^2 - 1.)/(rII*2*h));
\[
dII = \left( -2/h^2 - hOFrII/(k*t) \right);
eII = \left( 1/h^2 + 1./rII*2*h \right);
\]
\[
A((nn1+1):(nn2-1), nn1:(nn2-2)) = \text{diag}(cII);
A((nn1+1):(nn2-1), (nn1+1):(nn2-1)) = A((nn1+1):(nn2-1), (nn1+1):(nn2-1)) + \text{diag}(dII);
A((nn1+1):(nn2-1), (nn1+2):nn2) = A((nn1+1):(nn2-1), (nn1+2):nn2) + \text{diag}(eII);
\]

\[
\text{else}
\]
\[
\text{% Region IIb internal nodes (fin region before breakup)}
\]
\[
gII = 0;
\]
\[
rII = (ni1+1)*h:h:(ni2-1)*h;
\]
\[
hOFrII = hfun(rII,funParam);
\]
\[
cII = \left( 1/h^2 - 1./rII*2*h \right);
\]
\[
dII = \left( -2/h^2 - hOFrII/(k*t) \right);
\]
\[
eII = \left( 1/h^2 + 1./rII*2*h \right);
\]
\[
A((nn1+1):(nn2-1), nn1:(nn2-2)) = \text{diag}(cII);
A((nn1+1):(nn2-1), (nn1+1):(nn2-1)) = A((nn1+1):(nn2-1), (nn1+1):(nn2-1)) + \text{diag}(dII);
A((nn1+1):(nn2-1), (nn1+2):nn2) = A((nn1+1):(nn2-1), (nn1+2):nn2) + \text{diag}(eII);
\]
\[
b((nn1+1):(nn2-1),1) = -gII;
\]
\[
\text{end}
\]

\[
b((nn1+1):(nn2-1),1) = -gII;
\]

\[
\text{% Matched heat flux between region II and region III}
\]
\[
A(nn2,(nn2-1):(nn2+1)) = [-1,2,-1];
\]
\[
\text{% Region III internal nodes (fin region)}
\]
\[ r_{III} = (n_i2 + 1)\cdot h : h : (n_i - 1)\cdot h; \]
\[ h_{OF_{r_{III}}} = h_{\text{fun}_p b}(r_{III}, \text{funParam}_p b); \]
\[ g_{III} = h_{OF_{r_{III}}}/(k\cdot t)\cdot \Theta_{inf}; \]
\[ c_{III} = (1/h^2 - 1/(r_{III}\cdot 2\cdot h)); \]
\[ d_{III} = (-2/h^2 - h_{OF_{r_{III}}}/(k\cdot t)); \]
\[ e_{III} = (1/h^2 + 1/(r_{III}\cdot 2\cdot h)); \]
\[ A((n n_2 + 1):(n n - 1), n n_2:(n n - 2)) = \text{diag}(c_{III}); \]
\[ A((n n_2 + 1):(n n - 1), (n n_2 + 1):(n n - 1)) = A((n n_2 + 1):(n n - 1), (n n_2 + 1):(n n - 1)) + \text{diag}(d_{III}); \]
\[ A((n n_2 + 1):(n n - 1), (n n_2 + 2):n n) = A((n n_2 + 1):(n n - 1), (n n_2 + 2):n n) + \text{diag}(e_{III}); \]
\[ % \text{Adiabatic boundary condition at the end of the wafer.} \]
\[ A(n n, (n n - 1):n n) = [1, -1]; \]
\[ b((n n_2 + 1):(n n - 1), 1) = -g_{III}; \]

else
    disp('Please enter a number of steps, n_i, so that n_i1 = R_1/R_3* n_i and n_i2 = R_2/R_3* n_i are integers."
end

theta = A\b;
rval = (0:h:h*ni)';

% Numerical Integration (Check on q"
if ~reverse
    rvalA = 0:h:h*ni1;
rvalB = h*(ni1+1):h:h*ni;
else
    rvalA = 0:h:h*ni2;
end
rvalB = h*(ni2+1):h:h*ni;
end

hOFrA = hfun(rvalA,funParam);
hOFrB = hfun_pb(rvalB,funParam_pb);
fA = 2*pi*hOFrA.*theta(1:length(rvalA)).'*rvalA;
fB = 2*pi*hOFrB.*(theta(length(rvalA)+1:end)-Theta_inf).'*rvalB;
f = [fA,fB];

if ~reverse
    checkq = 1/(pi*R2^2)*h/2*(f(1) + 2*sum(f(2:end-1)) + f(end));
else
    checkq = 1/(pi*R1^2)*h/2*(f(1) + 2*sum(f(2:end-1)) + f(end));
end

% qf_rF = -k*(theta(2) - theta(1))/h;
qu_rC = -k*(theta(3:end) - theta(1:(end-2)))/(2*h);
qu_rB = -k*(theta(end) - theta(end-1))/h;
qu_r = [qf_rF; qu_rC; qu_rB];
qu_r = t*2*pi*rval.*qf_r;

% Return results structure
results = struct('theta',theta,'rval',rval,'checkq',checkq,'qu_r', qf_r, 'q_r', qu_r);
end
APPENDIX D. CODE FOR NATURAL CONVECTION ANALYSIS

The numerical solution of the natural convection problem was implemented in Mathematica® as a Wolfram Language package, NaturalConvectionModel, and is included in Section D.1. It provides the average Nusselt number for plate length, \( Y \).

D.1 Natural Convection Model

```
BeginPackage["NaturalConvectionModel"]

naturalConvectionModel::usage = "natConv[ x] solves a vertical wall natural convection problem on with fixed streamwise slip length and temperature jump length."

Begin[ "Private"]
naturalConvectionModel[GrL_,Pr_,\[Lambda]_,\[Lambda]T_, yi_, yf_, V0_, \[Delta]0_,q_,Y_] := Block[{momEq,heatEq,s,Nu,
NDaccGoal,NDprecGoal, NDwPrec,NDDiff},
NDaccGoal = 10;
NDprecGoal = 10;
NDwPrec = MachinePrecision;
NDDiff = 8;
\[ \text{Delta} \] (y) / (495 (2 \[ Lambda \] + \text{Delta} (y))^3) = \text{Delta} (y) / (2 \[ Lambda \] + \text{Delta} (y)) /

\[ \text{Pr} (2 q \[ Lambda \] T + \text{Delta} (y)) \]

\text{heatEq} = 1 / (840 q^6 (2 \[ Lambda \] + \text{Delta} (y))^2 (2 q \[ Lambda \] T + \text{Delta} (y)) (2 (5+14 q (-2+q (4+ q (-2+5 q (-1+2 q))))) \[ Lambda \] + (5+2 q (-16+7 q (6+q (-8+5 q))))) \[ Lambda \] + 2 \[ Lambda \] T + 4 \[ Lambda \] ((5+14 q (-2+q (4+q (-2+5 q (-1+2 q))))) \[ Lambda \] + 4 q (5+q (-31+7 q (11+q (-13+5 q (1+q))))) \[ Lambda \] T (2 q \[ Lambda \] T + \text{Delta} (y))^2) (2 q \[ Lambda \] T + \text{Delta} (y)) (2 (5+14 q (-2+q (4+q (-2+5 q (-1+2 q))))) \[ Lambda \] + 4 q (5+q (-31+7 q (11+q (-13+5 q (1+q))))) \[ Lambda \] T (2 q \[ Lambda \] T + \text{Delta} (y))^2 + 4 q (5+q (-2+5 q (-1+2 q))))) \[ Lambda \] T)

\text{s} = \text{NDSolve} \{ \text{momEq, heatEq, \text{Delta}[yi]==\text{Delta}[0], V[yi]==V0 \}, \{\text{Delta},V\}, \{y,yi,yf\}, \text{Method->"ExplicitRungeKutta", DifferenceOrder->"NDDiff", AccuracyGoal->NDaccGoal, PrecisionGoal->NDprecGoal, WorkingPrecision->NDwPrec, MaxStepFraction->1/1000, InterpolationOrder->All} \};

\text{Nu} = (2 q / (\text{Delta} (y) + 2 q \[ Lambda \] T) / \text{s})[[1]];

1/Y*NIntegrate[\text{Nu}, \{y,yi,Y\}, \text{Method->"GlobalAdaptive", Method->"GaussKronrodRule"}, \text{AccuracyGoal->8, PrecisionGoal->8, WorkingPrecision->MachinePrecision}]

End[]
EndPackage[]