The Estimation of the RapidScat Spatial Response Function

Samuel Gary Bury
Brigham Young University

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The Estimation of the RapidScat Spatial Response Function

Samuel Gary Bury

A thesis submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of

Master of Science

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ABSTRACT

The Estimation of the RapidScat Spatial Response Function

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Master of Science

RapidScat is a pencil-beam wind scatterometer which operated from September 2014 to August 2016. Mounted aboard the International Space Station (ISS), RapidScat experiences significant altitude and attitude variations over its dataset. These variations need to be properly accounted for to ensure accurate calibration and to produce high resolution scatterometer images. Both the antenna pose and the one-way antenna pattern need to be validated.

The spatial response function (SRF) is the two-way antenna pattern for a scatterometer combined with the processing and filtering done in the radar system electronics, and is dominated by the two-way pattern. To verify the pointing of the RapidScat antenna, the RapidScat SRF is estimated using on-orbit data. A rank-reduced least squares estimate is used, which was developed previously for the Oceansat-2 (OSCAT) scatterometer [1]. This algorithm uses a small, isolated island as a delta function to sample the SRF. The island used is Rarotonga Island of the Cook Islands. The previously developed algorithm is updated to estimate the SRF in terms of beam azimuth and elevation angle rather than in kilometers on the ground. The angle-based coordinate system promotes greater understanding of how the SRF responds to biases and errors in antenna geometry.

The estimation process is simulated to verify its accuracy by calculating the SRF for several thousand measurements in the region of Rarotonga. The calculated SRFs are multiplied by a corresponding synthetically created surface and integrated to yield simulated backscatter measurements, with added white noise. The SRF estimation algorithm is then performed. The results of the simulation show that the SRF estimation process yields a close estimate of the original SRF. The antenna pointing is validated by introducing a fixed offset in azimuth angle into the simulation and observing that the SRF is correspondingly shifted in the azimuth-elevation grid. The SRF computed from real data shows that there is an azimuth rotation angle bias of about $0.263^\circ$ for the inner beam and about $0.244^\circ$ for the outer beam.

Since the SRF is dominated by the two-way antenna pattern, it can be modeled as the product of two identical one-way antenna patterns which are slightly offset from each other due to antenna rotation during the transmit/receive cycle. A method is developed based on this model to derive the one-way antenna pattern from the estimated SRF. Using a Taylor series expansion the one-way antenna pattern is computed from the SRF. The derived pattern recovers the SRF with small error, but there is significant error in the inferred one-way pattern when compared to the pre-launch estimated RapidScat one-way antenna pattern.

Keywords: scatterometer, antenna pattern, spatial response function
ACKNOWLEDGMENTS

There have been many who have helped me on my way to completing this thesis. My family and friends have been a constant support to me through thick and thin. There were times I thought I would not make it, but the support group around me kept me going. My lab-mates in the MERS lab have been great to work with and I am a better man for knowing them. They are the smartest group of people I know. I am also very grateful to the Electrical and Computer Engineering faculty here at Brigham Young University. They have expected a lot and have helped me to rise to the challenge of their classes. I particularly need to thank Dr. Cammy Peterson and Dr. Daniel Smalley for their participation on my masters committee.

My greatest and most sincere thanks go to Dr. David Long for being an incredible advisor, mentor, and friend. He hired me as a research assistant when I was at a crossroads in my education. As I have worked and learned in the MERS lab I have grown into a qualified electrical engineer capable of contributing to the world in a meaningful way. Dr. Long taught me to have patience in the research process, to work hard, and to think critically about engineering problems. Dr. Long’s love of engineering coupled with his commitment to integrity and love of people is an example I will never forget.
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CHAPTER 1. INTRODUCTION

A scatterometer is a space-borne radar instrument which measures the normalized radar cross-section ($\sigma^o$) of the Earth’s surface. RapidScat is the most recent scatterometer operating in Ku-band. As such, it is hoped that RapidScat data will contribute to the extensive existing scatterometer data set after being cross-calibrated and the antenna pointing is validated.

This thesis is about validating the pointing of the RapidScat antenna. This chapter provides background on the history of scatterometry. The measurement geometry of a conically scanning pencil beam scatterometer, particularly RapidScat, is discussed. The mathematical formulation of radar backscatter measurements is given in order to introduce the spatial response function (SRF). A history of how the SRF has been previously used in scatterometer research is given. The thesis statement regarding the use of the SRF in validating the RapidScat antenna pointing is discussed. Finally, the organization for the rest of this thesis is presented.

1.1 Scatterometer History

The first wind scatterometer was designed to observe the correlation between $\sigma^o$ and ocean winds. This was NASA’s Seasat-A Satellite Scatterometer System (SASS) launched in June 1978. SASS was operational until October 1978 [3], and provided the first scatterometer data from which global radar measurements of ocean wind were produced [4]. The Earth Remote Sensing (ERS) Scatterometer (ESCAT) was the next operational space-borne scatterometer, launched by the European Space Agency (ESA) aboard ERS-1 in July 1991 and ERS-2 in April 1995. These instruments were in use until March 2000 and September 2011, respectively [5]. In September 1996, shortly after the launch of ERS-2, the NASA Scatterometer (NSCAT) was launched to continue the Ku-band ocean vector wind mission begun by SASS [3]. NSCAT was operational from September 1996 to June 1997. SASS, ESCAT, and NSCAT all used fixed fan-beam antennas [3].
The Seawinds instrument on QuikSCAT, referred to as QuikSCAT, was the first wind scatterometer to employ a rotating pencil beam antenna design. QuikSCAT was fully operational from July 1999 to November 2009 and continues in a post-operational mission to the present day. QuikSCAT successfully produced a ten-year global scatterometer data set spanning 1999-2009 [6]. The scientific community has benefited greatly from having a long data set that is consistent in quality and coverage. Another Seawinds instrument was launched aboard the Advanced Earth Observing Satellite II (ADEOS II) in 2003. It operated for ten months until a failure in a solar panel caused the mission to end early.

The Oceansat-2 Scatterometer (OSCAT) was the third pencil-beam scatterometer to be launched. OSCAT was launched aboard the Oceansat-2 satellite in September 2009 by the India Space Research Organization (ISRO) to replicate the success of the QuikSCAT mission. OSCAT failed in April 2014 [7] [8]. A second ISRO scatterometer named ScatSat was launched in 2016.

Scatterometers were originally designed to measure ocean vector winds. However, since the first scatterometer was launched, the data provided by these sensors has played a part in various other climate studies, including: soil moisture mapping [9], rain forest vegetation density [10], hurricane tracking [11], oil spill clean-up [12], and iceberg tracking [13]. These research areas follow naturally because \( \sigma^\circ \) is a measure of surface roughness and scattering characteristics which is effective in these studies. Over time, algorithms have been developed to produce images from scatterometer data and to enhance the resolution of \( \sigma^\circ \) images [14].

RapidScat is the most recent NASA scatterometer [15]. It originally served as the engineering model for QuikSCAT and was launched as an independent sensor in September 2014. Due to the success of the QuikSCAT mission, RapidScat was designed to resemble QuikSCAT as closely as possible despite the differences in platform. A table comparing the operating parameters of RapidScat and QuikSCAT is shown in Fig. 1.1. RapidScat is mounted on the International Space Station (ISS) which follows an equatorial orbit, unlike any preceding scatterometer platform. The ISS also introduces a lower altitude and larger attitude variations into the RapidScat measurement geometry. While RapidScat has the same primary scientific objectives as QuikSCAT - to measure ocean vector winds - other interesting qualities of the Earth including diurnal vegetation cycles in the Amazon have already been observed with RapidScat \( \sigma^\circ \) data [16]. These studies are uniquely suited to RapidScat because it has an equatorial orbit.
1.2 Measurement Geometry for RapidScat

This thesis is primarily concerned with RapidScat, a conically scanning pencil-beam scatterometer flown on the ISS. The measurement geometry of this particular class of scatterometer is explained in this section.

RapidScat uses a 0.75-meter-diameter rotating dish to scan the Earth’s surface with an elliptically shaped beam footprint. The surface is scanned in a swath directly nadir to the radar as the ISS orbits. An illustration of this measurement system is shown in Fig. 1.2. RapidScat uses both a vertically polarized (V-pol) beam and a horizontally polarized (H-pol) beam. The H-pol beam is the inner beam and the V-pol beam is the outer beam.

As the antenna rotates about the nadir vector, the radar alternates between transmit mode and receive mode. So, at a certain time, a pulse is transmitted and, at a short time later, the power which is backscattered by the Earth’s surface is received. The antenna illuminates the Earth at both of these times but the illuminated area on the surface, the footprint, is shifted in azimuth between the time of transmission and reception due to the rotation of the antenna during the pulse time-of-flight (ToF). The beams also shift due to spacecraft motion, but this effect is very small, and

<table>
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<tr>
<th>Parameter</th>
<th>RapidScat</th>
<th>QuikSCAT</th>
<th>Unit</th>
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<td>m</td>
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<td>PRI</td>
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<td>46, 54</td>
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<td>Look angle, 2 beams</td>
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<td>1410, 1800</td>
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<td>-31.2, -32.2</td>
<td>dB</td>
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can be reasonably neglected on a pulse-by-pulse basis. The product of these two one-way antenna patterns yields an effective two-way antenna pattern that is illustrated in Fig. 1.3. The darker area where the two ellipses, antenna patterns, overlap is the two-way antenna pattern. This two-way antenna pattern dominates the spatial response function which is described in greater detail in the next section. The specifics of the shifted one-way patterns is addressed later in this thesis.

1.3 The Spatial Response Function

The spatial response function (SRF) is the two-way antenna pattern for a scatterometer combined with the processing and filtering done in the radar system electronics. The SRF is also referred to as the impulse response function (IRF), the measurement response function (MRF), or the measurement point-spread function. The physical meaning of the SRF as a weighting function for $\sigma^\circ$ [14] is discussed in this section.

Here, the SRF is derived from the radar equation to show that it is a weighting function. The power received by a radar is

$$P_R = P_b + P_n,$$

where $P_b$ is the backscattered power and $P_n$ in the noise power from interfering microwave sources and the radar system electronics. The monostatic radar equation for backscattered power, $P_b$, is
Figure 1.3: An illustration of the shift in the antenna pattern with the change in the antenna position between transmit-time and receive-time. The darker area corresponds to the area of the main lobe of the two-way antenna pattern.

given as

\[ P_b = \frac{P_T \lambda^2}{(4\pi)^3} \int_A \frac{G_R(\vec{v})G_T(\vec{v})\sigma^\circ(\vec{v})}{r^4(\vec{v})} d\vec{v}, \]  

(1.2)

where \( P_T \) is transmit power, \( \lambda \) is wavelength, \( r \) is slant range, \( A \) is the area illuminated by the footprint, \( G_R \) and \( G_T \) are the one-way gain patterns at receive time and transmit time respectively, \( \sigma^\circ \) is the backscatter coefficient, and \( \vec{v} \) represents the location of element \( d\vec{v} \) within \( A \) over a well-defined coordinate system. The radar detects a spatially weighted average of the surface backscatter coefficient which varies over the illuminated area. The signal-only weighted spatial average of \( \sigma^\circ \) is \( \bar{\sigma}^\circ \) as seen,

\[ \bar{\sigma}^\circ = \frac{P_b}{P_T} \frac{(4\pi)^3}{\lambda^2} \left/ \int_A \frac{G_R(\vec{v})G_T(\vec{v})}{r^4(\vec{v})} d\vec{v} \right., \]  

(1.3)

which can be rewritten as

\[ \bar{\sigma}^\circ = \frac{P_b}{X}, \]  

(1.4)
where $X$ is the X-factor commonly used in scatterometer data post-processing [2],

$$X = \frac{P_T \lambda^2}{(4\pi)^3} \int_A \frac{G_R(\vec{v})G_T(\vec{v})}{r^4(\vec{v})} d\vec{v}. \quad (1.5)$$

Ashcraft and Long [14], derived an expression for the SRF as follows by substituting Eqs. (1.2) and (1.5) into (1.4):

$$\bar{\sigma}^\circ = \int_A \frac{G_R(\vec{v})G_T(\vec{v})}{r^4(\vec{v})} \sigma^\circ(\vec{v}) d\vec{v} / \int_A \frac{G_R(\vec{v})G_T(\vec{v})}{r^4(\vec{v})} d\vec{v}, \quad (1.6)$$

A weighting function for $\sigma^\circ$, $h'$, which is related to the SRF, can be defined as

$$h'(\vec{v}) = \frac{G_R(\vec{v})G_T(\vec{v})}{r^4(\vec{v})}, \quad (1.7)$$

so that,

$$\bar{\sigma}^\circ = \int_A h'(\vec{v}) \sigma^\circ(\vec{v}) d\vec{v} / \int_A h'(\vec{v}) d\vec{v}. \quad (1.8)$$

Eq. (1.7) implies that the value of the SRF varies with changing gain and slant range over the footprint defined in $\vec{v}$. So, a different SRF needs to be defined for every measurement due to varying observation geometry. Instead, it is beneficial to remove the dependence of the SRF on slant range in order to eventually estimate a nominal SRF. Appendix B explains the justification for using a constant slant range per measurement so $r^4(\vec{v})$ can be written as simply $r^4$.

The function $h'(\vec{v})$ is normalized to integrate to 1 by dividing $h'$ by $h_0$ where

$$h_0 = \int_A \frac{G_R(\vec{v})G_T(\vec{v})}{r^4} d\vec{v}. \quad (1.9)$$

This process yields a normalized weighting function, $h$, which is the SRF,

$$h(\vec{v}) = \frac{h'(\vec{v})}{h_0}, \quad (1.10)$$

such that

$$\int_A h(\vec{v}) d\vec{v} = 1. \quad (1.11)$$
The spatial variable $A$ is the area on the Earth’s surface which is illuminated by the main lobe of the two-way antenna pattern seen in Fig. 1.3. The SRF is integrated over this area because it is dominated by the two-way antenna pattern.

The relationship between the SRF, $h$, and the observed backscatter is seen by the inner-product

$$\bar{\sigma}^o = \int_A h(\vec{v})\sigma^o(\vec{v})d\vec{v},$$

(1.12)

where $h$ is the SRF.

The backscattered power is estimated by

$$\hat{P}_b = P_R - \hat{P}_n,$$

(1.13)

where $\hat{P}_n$ is an estimate of the noise power. Using $\hat{P}_b$ and Eq. (1.4), an estimate of $\bar{\sigma}^o$ is

$$z = \frac{\hat{P}_b}{X} = \sigma^o + e,$$

(1.14)

where $e$ is the noise-like error in the estimation process. The value $z$ is recorded as the radar measurement of the surface $\sigma^o$. Substituting Eq. (1.12) into Eq. (1.14), $z$ is expressed in terms of the SRF, $\sigma^o$, and estimation error

$$z = \int_A h(\vec{v})\sigma^o(\vec{v})d\vec{v} + e.$$

(1.15)

1.4 The Spatial Response Function in Scatterometer Research

Accurate knowledge of the SRF is required to apply high-resolution image reconstruction algorithms to backscatter measurements [14] as well as to calibrate scatterometers for wind retrieval [4]. Early and Long [17], and Williams and Long [18] describe the use of the aperture function, or SRF, in sampling scatterometer data for use in image reconstruction. $\sigma^o$ data is “aperture-filtered” because the SRF is used as a low-pass filter over the $\sigma^o$ of the surface. Thus, knowledge of the shape, size, and frequency response of the SRF is critical for implementing these image processing algorithms.
In the case of QuikSCAT, the known antenna pattern permits the SRF for each measurement to be calculated and tabulated as a function of the antenna rotation angle, and orbit position [14]. Joshua Bradley developed a method of estimating the SRF for Oceansat-2 (OSCAT) using an island target and a rank-reduced least squares approach [19]. Bradley’s method is effective for estimating the OSCAT SRF. Improvements in OSCAT image reconstruction using the estimated SRF are discussed in [19].

1.5 Thesis Statement

The previously developed methods for estimating the scatterometer SRF required stability in the attitude of the radar platform. Despite the success of Bradley’s estimation algorithm in the case of OSCAT, RapidScat has proven to be more difficult due to large variations over time in attitude geometry. RapidScat’s varying geometry suggests that a different approach needs to be applied. Changing the coordinate system over which the SRF is estimated from a ground-based coordinate system to an angle-based coordinate system accounts for the variations in geometry. This yields an estimate of the SRF which is more robust to changes in attitude. This thesis presents a method of estimating the SRF for RapidScat which employs an angle-based coordinate system. The usefulness of the SRF in validating the pointing of RapidScat is demonstrated. A method of deriving the one-way antenna pattern is also developed.

The contributions that my work adds to the field of scatterometry are:

• An alternate method for estimating the SRF of a pencil-beam scatterometer that is more robust to changes in platform altitude.

• A demonstration of how the antenna pointing of a pencil-beam scatterometer can be validated using an estimate of the SRF.

• A method of deriving the one-way antenna pattern for a pencil beam scatterometer from the SRF.
1.6 Thesis Organization

This thesis is organized in the following order. Chapter 2 provides the mathematical structure for performing a rank-reduced least squares procedure to estimate the SRF. Chapter 3 describes the methods of estimating the SRF using an angle-based coordinate system. Chapter 4 shows the way in which the SRF is used to validate the antenna pointing for RapidScat through simulation. Chapter 5 derives a method of approximating the one-way antenna pattern from the estimated SRF. Chapter 6 summarizes the results of this thesis and draws comparisons from previous work done on estimating the SRF. Chapter 7 concludes with a description of the contributions this thesis adds to the scatterometer research community and suggests areas of future work on this topic.
CHAPTER 2. SPATIAL RESPONSE FUNCTION ESTIMATION MODEL

Joshua Bradley developed a procedure for estimating the OSCAT SRF where he employs the technique of sampling the SRF with carefully selected island targets [19]. Due to the fact that an island is much brighter than the dark ocean background, a small, isolated island is an approximation of a delta function. So, the radar measurements taken over an island can be viewed as samples of the SRF. These measurements are compiled into a matrix which is inverted to estimate the SRF. This same method can be used to estimate the RapidScat SRF. The mathematical details of the process are given in this chapter.

This chapter shows the way in which $\sigma^\circ$ measurements are compiled into a matrix in order to invert the radar equation and solve for the SRF. The inversion process using a singular value decomposition (SVD) is described. The mathematical development contained in this chapter closely follows a similar development in [19] but is generalized to enable expressing the SRF in elevation and azimuth angle as well as surface displacement.

2.1 Estimation Model

Following the derivation of the SRF and the expression of Eq. (1.15), the $m$-th measurement, $z_m$, taken by a scatterometer can be modeled as

$$z_m = \int_A h(\bar{\nu}) \sigma^\circ(\gamma_m, x_m, \bar{\nu}) d\bar{\nu} + e_m,$$

where the normalized radar backscatter, $\sigma^\circ$, is a function of $\gamma_m$, which encapsulates the parameters of the observation geometry including azimuth angle and incidence angle, and $x_m$ which is the orbital location of the radar. Here, the SRF, $h$, and $\sigma^\circ$ can be treated separably since only $\sigma^\circ$ is a function of $\gamma_m$ and $x_m$. The $e_m$ term is the noise-like error in the estimate of the signal only power for the $m$-th measurement. The bound of integration, $A$, is the area illuminated by the two-way
antenna pattern. Both $h$ and $\sigma^\circ$ are defined over the coordinate system $\vec{v}$. In [19], $\vec{v}$ was defined in azimuth and elevation in kilometers on the Earth’s surface [19]. An alternative approach is presented in this thesis which defines $\vec{v}$ in azimuth and elevation angle independent of the surface. This is explained in greater detail in Chapter 3 of this thesis. For the sake of notational simplicity, the derivation in this chapter uses $\vec{v}$.

Eq. (2.1) can be discretized as a sum over a sufficiently fine resolution sample grid for the SRF,

$$z_m \approx \sum_{k=1}^{K} h(\vec{v}_k) \sigma^\circ(\gamma_m, \vec{x}_m, \vec{v}_k) \Delta \nu + e_m,$$

(2.2)

where $K$ is the number of grid elements and $\Delta \nu$ is the constant area of each grid element (defined in the coordinate system of $\vec{v}$). This expression can be expressed as a vector inner product,

$$z_m \approx S_m^T h + e_m,$$

(2.3)

where

$$S_m = \begin{bmatrix} \sigma^\circ(\gamma_m, \vec{x}_m, \vec{v}_1) \\ \sigma^\circ(\gamma_m, \vec{x}_m, \vec{v}_2) \\ \vdots \\ \sigma^\circ(\gamma_m, \vec{x}_m, \vec{v}_K) \end{bmatrix} \Delta \nu,$$

(2.4)

$$h = \begin{bmatrix} h(\vec{v}_1) \\ h(\vec{v}_2) \\ \vdots \\ h(\vec{v}_K) \end{bmatrix}.$$ 

(2.5)

Collecting $M$ measurements over the $K$ element grid yields a $M \times K$ matrix $Q$ where each row represents the $\sigma^\circ$ for each grid element for the $m$-th measurement. Many measurements need to be used in order to sample each element, $d\vec{v}$, of the coordinate system, $\vec{v}$.
In
\[
\mathbf{z} = \begin{bmatrix}
\mathbf{z}_1 \\
\mathbf{z}_2 \\
\vdots \\
\mathbf{z}_M
\end{bmatrix}
\approx
\begin{bmatrix}
\mathbf{S}_1^T \\
\mathbf{S}_2^T \\
\vdots \\
\mathbf{S}_M^T
\end{bmatrix}
\mathbf{h} + \begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_M
\end{bmatrix} = \mathbf{Qh} + \mathbf{e},
\] (2.6)

\(\mathbf{h}\) is the discretized SRF. We can calculate a least-squares estimate of the SRF, \(\hat{\mathbf{h}}_{LS}\), by performing a pseudo-inverse on \(\mathbf{Q}\), i.e.,
\[
\hat{\mathbf{h}}_{LS} = \mathbf{Q}^\dagger \mathbf{z},
\] (2.7)

where \(\mathbf{Q}^\dagger\) is the Moore-Penrose pseudo-inverse \(\mathbf{Q}^\dagger = (\mathbf{Q}^H \mathbf{Q})^{-1} \mathbf{Q}^H\). In order to improve the conditioning of \(\mathbf{Q}\) to yield a closer approximation of the inverse, a rank-reduced approximation using the SVD of \(\mathbf{Q}\) is done for the pseudo-inverse so that
\[
\mathbf{z} = \mathbf{U} \Sigma \mathbf{V}^H \mathbf{h} + \mathbf{e},
\] (2.8)

\[
= \tilde{\mathbf{Q}} \mathbf{h} + \mathbf{e},
\] (2.9)

where \(\mathbf{U}\) is a unitary \(M \times M\) matrix, \(\mathbf{V}\) is a unitary \(K \times K\) matrix which has as its columns the singular vectors of \(\mathbf{Q}\), and \(\Sigma\) is a \(M \times K\) matrix with the singular values of \(\mathbf{Q}\) down the main diagonal. The SVD of \(\mathbf{Q}\) can be written as
\[
\mathbf{Q} = \begin{bmatrix}
\mathbf{U}_1 & \mathbf{U}_2
\end{bmatrix}
\begin{bmatrix}
\Sigma_1 & \mathbf{0} \\
\mathbf{0} & \Sigma_2
\end{bmatrix}
\begin{bmatrix}
\mathbf{V}_1^H \\
\mathbf{V}_2^H
\end{bmatrix}
= \mathbf{U}_1 \Sigma_1 \mathbf{V}_1^H + \mathbf{U}_2 \Sigma_2 \mathbf{V}_2^H. \tag{2.10}
\]

The matrices \(\mathbf{U}_1, \Sigma_1,\) and \(\mathbf{V}_1\) are the portions of \(\mathbf{U}, \Sigma,\) and \(\mathbf{V}\) which correspond to the largest singular values of \(\mathbf{Q}\) which represent the signal. \(\mathbf{U}_2, \Sigma_2,\) and \(\mathbf{V}_2\) correspond to all of the smaller singular values which represent noise. \(\mathbf{Q}\) is approximated by setting the small singular values to zero, which reduces the rank. We are then left with
\[
\tilde{\mathbf{Q}} \approx \mathbf{U}_1 \Sigma_1 \mathbf{V}_1^H, \tag{2.11}
\]

\[
\hat{\mathbf{h}}_{SVD} = \mathbf{Q}^\dagger \mathbf{z} = \mathbf{V}_1 \Sigma_1^{-1} \mathbf{U}_1^H \mathbf{z}. \tag{2.12}
\]
The precise number of singular values to use in the estimate is a tuning parameter for the SRF estimate that is discussed more in Appendix A.

2.2 Summary

This chapter describes the way in which samples of the SRF are used to invert the radar equation to solve for the SRF. Each measurement is modeled as a vector inner product and many measurements are combined to sample the whole grid over which the SRF is defined. Performing an SVD on the matrix of measurements and eliminating noise terms yields an estimate of the SRF.
CHAPTER 3. ESTIMATION OF THE SPATIAL RESPONSE FUNCTION

Several steps are required to estimate the RapidScat SRF using on-orbit data. These steps for RapidScat are generally the same as the steps for estimating the OSCAT SRF [19]. However, several changes have been made to the process in order to accommodate the highly variable observation geometry of RapidScat. The goal of estimating the SRF for RapidScat is to estimate a mean antenna pointing bias in the presence of the varying altitude and attitude of the ISS, which are assumed to be correctly reported. As a clarifying note, the variable measurement geometry is not necessarily the cause of this pointing bias, but it makes it difficult to estimate the SRF using the previously developed method and therefore makes it difficult to estimate the pointing bias. Thus, the altitude and attitude variations from measurement to measurement motivate the development of the angle-based method used in this thesis. The most crucial differences between the angle-based method and the previously developed ground-based method are manifest in creating the landmap with which the SRF is sampled. In [19], Bradley used a landmap in kilometers on the ground (ground-based). A new coordinate system in beam azimuth and elevation angles (angle-based) is useful for RapidScat because it accounts for geometric variations from measurement to measurement.

This chapter describes each of the steps in the process of estimating the SRF. First, the methods used to collect measurements for the estimation process are explained. The creation of the landmap to sample the SRF is discussed. The differences which were made in the case of the RapidScat SRF estimation are explained and the differences between the two sampling coordinate systems are presented. Then, the procedure for removing the ocean $\sigma^o$ bias from the land contribution to $\sigma^o$ is reviewed. Finally, the rank-reduced least squares estimation of the SRF described in depth in Chapter 2 is briefly addressed as the concluding step in the estimation process.
3.1 Selecting Measurements

In order for Eq. (2.12) to yield a good estimate of the SRF, as many measurements as possible should be combined into the $Q$ matrix. A collection of $\sigma^o$ measurements over Rarotonga Island, capital of the Cook Islands, are used in the estimation of the RapidScat SRF. Rarotonga is one of the islands identified by Joshua Bradley to approximate a delta function because it is small, isolated, elliptical, and in a region of the Pacific Ocean with relatively mild weather [19]. Rarotonga is also nearer to the equator than the other islands used by Bradley. This qualifies it as a good target for RapidScat which has an equatorial orbit. A few key statistics about Rarotonga are given in Table 3.1. The high resolution QuikSCAT image in Fig. (3.1) shows Rarotonga in the center with other islands several hundred kilometers away to the East.

Table 3.1: Rarotonga Island Information

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>-159.776°</td>
</tr>
<tr>
<td>Longitude</td>
<td>-21.229°</td>
</tr>
<tr>
<td>Max. Width</td>
<td>11 km</td>
</tr>
<tr>
<td>Area</td>
<td>67.39 km²</td>
</tr>
</tbody>
</table>

Figure 3.1: A high resolution QuikSCAT egg backscatter image of the region around Rarotonga. Rarotonga is the bright dot in the center of the image.

The RapidScat $\sigma^o$ measurements which fall into this region are found by searching all of the RapidScat Level-1B (L1B) data by date [20]. Almost a year of data is used, specifically Julian Day (JD) 276 of year 2014 to JD 220 of year 2015. This date range covers the time for which
RapidScat was in a high signal-to-noise ratio (SNR) state. Other SNR states are avoided in order to simplify the estimation process by maintaining the same level of quality in the data.

During the finding process, the measurement quality flags (sigma0_qual_flag, frame_err_status, and the frame_qual_flag) are checked to ensure that only the highest fidelity measurements are used [20]. If any of these flags are set, the measurement or the frame are thrown out. This flag check is a change required for RapidScat compared to OSCAT because there were so few erroneous measurements in the OSCAT dataset that the estimation was not significantly affected. RapidScat has various states which make the flags all the more important. For example, there are times when RapidScat is turned off in order to avoid radiating the ISS.

Another change made to simplify the measurement finding process is the elimination of the use of SIR images. Setting a simple threshold on latitude and longitude to determine if a measurement falls within the specified region is adequate, and more straightforward than converting to pixel values in a SIR image.

For all measurements that are good quality and are located within the appropriate region, key information - regarding the geometry, $\sigma^\circ$, and so forth - is written to a setup file for easy access in the next processing steps.

### 3.2 Building the Sample Landmap

The vector $S_m$ from Eq. (2.3) can be written as

$$S_m = [\text{landmap}]^T \sigma^\circ \Delta \nu,$$

where landmap is a vector indicating which grid elements within the SRF correspond to land, $\sigma^\circ$ is the scalar backscatter coefficient of the surface for the $m$-th measurement, and $\Delta \nu$ is the scalar area of each grid element in the landmap. Assuming $\sigma^\circ$ across the island target is constant for any given measurement, the contribution of each pixel to $\sigma^\circ$ is averaged over the island. So, pixels that are ocean are given a constant value and pixels which are land are given another constant value.

In order for the approximation of the island as a delta function to hold, and for each $S_m$ to correspond to one sample of the SRF, all of the power in a $\sigma^\circ$ measurement must come from the island. This means that if a pixel in the landmap is ocean, then the pixel value is set to zero. If a
pixel is land, then the pixel value needs to be some non-zero value resulting in the sum of \( S_m \) to the land component of \( \sigma^o \). The following reasoning describes how to choose the land pixel value for the \textit{landmap}.

The land and ocean components of \( \sigma^o \) are seen by rewriting Eq. (3.1) as

\[
S_m = [\text{landmap}]^T (L_F \sigma^o_L + O_F \sigma^o_O) \Delta v, \tag{3.2}
\]

where \( \sigma^o_L \) is the backscatter coefficient of land and \( \sigma^o_O \) is the backscatter coefficient of the ocean. \( L_F \) and \( O_F \) are the land fraction and ocean fraction respectively where \( L_F = 1 - O_F \). The total land component of \( \sigma^o \) is \( L_F \sigma^o_L \). \( L_F \) and \( O_F \) can be written

\[
L_F = N_L/N, \tag{3.3}
\]
\[
O_F = N_O/N, \tag{3.4}
\]

where the total number of pixels in the grid is \( N \) and the number of land and ocean pixels are \( N_L \) and \( N_O \). Section 3.3 describes how to remove \( \sigma^o_O \) from the total \( \sigma^o \) so

\[
S_m = [\text{landmap}]^T (L_F \sigma^o_L) \Delta v. \tag{3.5}
\]

By setting each land pixel in the \textit{landmap} to \( 1/N_L \), the sum of the vector \( S_m \) is

\[
\sum_{i=1}^{N} \text{landmap}[i] L_F \sigma^o_L \Delta v = \left[ N_L \left( \frac{1}{N_L} \right) + N_O(0) \right] L_F \sigma^o_L = L_F \sigma^o_L. \tag{3.6}
\]

Thus, if the grid contains any land, \( S_m \) sums to \( L_F \sigma^o_L \), and is \( 0 \) otherwise.

The methods for identifying land pixels within the \textit{landmap} are described in this section. This section also explains the choice in coordinate system over which the \textit{landmap}, and therefore the SRF, is defined.

\subsection*{3.2.1 Review of Method for OSCAT}

When Bradley developed the original version of this algorithm for OSCAT, the coordinate system \( \vec{v} \) was in (azimuth,elevation) displacement in kilometers on the ground. The origin of his
coordinate system for the SRF estimate is the position at which the boresight vector of the antenna is incident on the ground. Bradley fixed the orientation of the island in his formulation, with the vertical axis being North-South and the horizontal axis being East-West, and used the reported antenna rotation angle to rotate the SRF into the same orientation as the island in the search grid. The figure below illustrates this coordinate system.

![Coordinate System Illustration](image)

Figure 3.2: An illustration of the coordinate system used to build the landmap in the OSCAT SRF estimation, for the $i$-th and the $j$-th measurements. The SRF is rotated into the North-East coordinate system and then the landmap is constructed from gridding the rotated frames.

This method extensively used SIR images to define distances from the island in pixels, the location of the SRF relative to the island, and the search grid over which the $S_m$ vectors are compiled. The SIR images also provided a useful standard of pixel size on which to define the size of the footprint on the ground. The landmap is constructed for each measurement by selecting a certain number of pixels from the center location of the SRF. The SRF estimate is then computed in kilometers on the ground. This works for OSCAT because the geometry (attitude, altitude, slant range) is fairly stable; which means the shape and size of the OSCAT beam footprint (i.e., the SRF) on the ground is nearly constant.
3.2.2 The Method for RapidScat

RapidScat has a lot of variability in attitude and altitude so the beam footprint dimensions on the ground cannot be assumed to be constant like that of OSCAT. This is problematic because, the Bradley estimation procedure uses the size of the beam on the ground in kilometers to define distances and to decide if a measurement is an ocean measurement or a land measurement. These challenges motivate the use of a different coordinate system.

The huge variations in slant range and attitude can be accommodated by computing the SRF estimate using a fixed boresight angle and a coordinate system in azimuth and elevation angle. Instead of using the angle bearing from the spacecraft to rotate the measurement, the island is located within the search grid using the boresight angle and the angle from the spacecraft to the island. Then the island location can be specified as an azimuth and elevation angle coordinate relative to the boresight of the antenna. The landmap is made by converting the angle coordinates of each grid element into latitude and longitude and determining if the (lat,lon) is land or not. Fig. (3.3) illustrates how the island is located relative to the boresight. For two measurements, compare Fig. (3.2).

This coordinate system is implemented using the BYU X-factor computation routines [2]. First, the latitude and longitude for each measurement center (boresight) is converted into an azimuth and elevation angle from the nadir vector of the spacecraft. Then the known location in latitude and longitude of the island is used to compute the same angles for the island relative to the spacecraft. Subtracting the boresight angles from the island angles sets the boresight at the origin of an angle-based coordinate system in beam azimuth and elevation from the antenna boresight. The island, with the calculated angles, is then located within the angle grid centered on the antenna boresight at the time the transmitted pulse is incident on the surface.

Once the island location is specified in the angle-based coordinate system, each pixel in the grid is tested for land. This is done by converting the azimuth and elevation angles of each grid element to latitude and longitude and then testing for land using the world land map. If a grid element is land, it is marked with a 1, and a 0 otherwise. This grid, essentially a local landmap, is then normalized to sum to 1 and is written column by column into the landmap of Eq. (3.1). The normalization occurs because the integration of power received at the antenna is the reported $\sigma^\circ$ which is being multiplied by the local land map. Thus, the integration of the land map must
equal 1. The normalization is also useful to make every measurement including a piece of the island equal in weight for the overall estimation process and to remove any dependence on the dimensions of the footprint on the ground.

### 3.3 Removing the Ocean Bias from $\sigma^o$

Measurements over a small island target inevitably include contributions to $\sigma^o$ from both the land, $\sigma^o_L$, and from the ocean, $\sigma^o_O$. The combination of $\sigma^o_L$ and $\sigma^o_O$ can be understood by

$$\sigma^o = L_F \sigma^o_L + O_F \sigma^o_O,$$

(3.7)

where $L_F$ is the land fraction and $O_F$ is the ocean fraction, related by $L_F = 1 - O_F$. Since the approximation of the island as a delta function requires that all of the backscattered power is due to $\sigma^o_L$, $\sigma^o_O$ must be negligible. Bradley used a spatially varying quadratic fit on $\sigma^o_O$ to estimate the ocean contribution to $\sigma^o$ over land. Measurements that fall outside a certain distance threshold are
considered ocean and $\sigma^O$ is estimated (the estimate is denoted by $\hat{\sigma}^O$) and subtracted out of $\sigma^\circ$ as

$$L_F\hat{\sigma}^O_L = \sigma^\circ - O_F\hat{\sigma}^O,$$

(3.8)

where $\hat{\sigma}^O$ is the estimated $\sigma^O$. The estimation and subtraction is done for every pass because $\sigma^O$ can be assumed constant for one pass but not for multiple passes due to weather conditions. This process yields $\sigma^O_L$ to be used in Eq. (3.1) to compute $S_m$.

This method is used to estimate $\sigma^O$ for both OSCAT and RapidScat, but there are differences in how a measurement is determined to be an ocean measurement or a land measurement. For OSCAT, the measurement is considered ocean if its center is a number of pixels away from the island in a SIR image grid. The pixel angles on the ground are treated as fixed on the small area grid due to the small angle approximation. In the case of RapidScat, however, the ocean measurements are determined by an angle distance between the boresight and the island. Fig. (3.4) shows an illustration of the measurements that are used around the island to construct the $Q$ matrix. The ocean measurements are also shown whose $\sigma^O$ is estimated and subtracted off of measurements including the island. The ocean contribution to $\sigma^O$ is estimated by using measurements that are $2^\circ$-$4^\circ$ away from the island center. The (small) ocean $\sigma^O$ value is subtracted from the measurements so that the ocean $\sigma^O$ can be treated as “zero”. Only measurements found immediately around the island are used in the estimate.

### 3.4 Solving for the SRF

After removing the ocean bias from each measurement and building the corresponding landmap, each $S_m$ represents the contribution of one measurement to the SRF estimate. The $S_m$ vectors are compiled as rows to create the $Q$ matrix seen in Eq. (2.6). This is the matrix which is inverted using singular-value decomposition to solve for the SRF. The results of the entire estimation process are found in Chapter 6.
3.5 Conclusion

This process yields a good estimate of the RapidScat SRF defined in beam azimuth and elevation angle. The primary difference made to the SRF estimation process for RapidScat is the change in coordinate system. This new coordinate system is an improvement for RapidScat because the SRF geometry variations are all accounted for in changing the way that the landmap is built. The angle grid is able to include changes in geometry, like slant range for example, because as the slant range changes, the range of latitude and longitude covered by a single grid element adjusts appropriately. In other words, the value of $\Delta v$ from Eq. 3.1 is fixed in the angle-based coordinate system and it is only treated as a fixed value in the ground-based coordinate system when in reality the footprint of the two-way antenna pattern is different for every measurement. The angle-based SRF estimate is also very helpful because of the clarity of intuition regarding how the antenna pointing might influence the SRF. If there are biases in the antenna pointing, they would be seen as a constant shift in the center of the SRF.
CHAPTER 4. SIMULATION: SRF ESTIMATION ERROR AND ANTENNA POINTING VALIDATION

During the post-launch calibration phases of the RapidScat mission, a potential antenna azimuth pointing bias has been observed in the on-orbit data. The antenna pointing can be validated with the aid of the observed SRF. While a procedure for estimating the OSCAT SRF was previously developed, the estimate was defined in kilometers on the ground without any direct connection to the antenna pointing angles. Switching the coordinate system for estimating the SRF to an angle-based coordinate system, the SRF estimate can be better used to analyze pointing biases for RapidScat. A bias in either azimuth or elevation corresponds to a linear shift in the SRF within the angle-based coordinate system.

If there is a non-zero angle bias, the reported (lat,lon) location of the nominal boresight on the ground is slightly different from the actual location of the measurement. This is due to the wrong angle being used to determine the location of the boresight on the Earth’s surface. The offset in (lat,lon) corresponds to an apparent change in the location of the boresight relative to surface features. This has the effect of “offsetting” the apparent location of the SRF in the estimate because every sample of the SRF will be shifted (see Chapter 3 for an explanation of how the SRF is sampled and estimated). Essentially, when the origin of the angle-based coordinate system is offset from the true boresight location, this causes the mapping from latitude and longitude to beam angles to be in error. On the other hand, if the estimated SRF is “centered,” the implication is that the antenna geometry is unbiased.

A simulation is conducted to validate the antenna pointing using the SRF estimation procedure. In this simulation, a synthetic $\sigma^o$ surface is used. The true SRF is calculated using the BYU X-factor code [2]. Using the true SRF to observe the synthetic surface yields a set of $\sigma^o$ measurements created in a controlled experiment. This simulation helps to analyze two errors with regard to the SRF: the beam dimensions and amplitude of the estimated SRF, and the effect of a pointing error on the SRF estimate. In order to quantify the beam dimension and amplitude errors, the SRF
is estimated using real geometry and simulated $\sigma^o$ measurements. This estimate is compared to the true SRF. The antenna pointing is validated by inserting a pointing error, simulating $\sigma^o$, and then analyzing the resulting SRF estimate.

This chapter discusses the simulation used to interpret the error in the estimation process and to validate the pointing of the RapidScat antenna. The SRF estimation error is discussed. The pointing section of the simulation shows that a bias in antenna azimuth correlates to a shift in the SRF estimate. For clarification, the words “shift” and “offset” are used interchangeably and refer to the SRF; the word “bias” refers to the data or the antenna pointing geometry. The simulation analysis described in this chapter yields similar results for both the inner beam (H-pol) SRF and the outer beam (V-pol) SRF.

4.1 Simulate $\sigma^o$ Measurements

To evaluate the SRF estimation process, a synthetic surface is simulated by assigning a constant $\sigma^o$ to every land pixel and another value to every ocean pixel. The simulated $\sigma^o$ value is -8 dB for land and -25 dB for ocean. These numbers were estimated using $\sigma^o$ images created from actual data. For each measurement, every pixel within a $4^\circ \times 4^\circ$ grid, centered on the measurement boresight, is associated with a latitude and longitude and checked against the world landmap. The pixels determined to be land or ocean are assigned the land $\sigma^o$ or ocean $\sigma^o$. So, the $\sigma^o$ value for every pixel in any given measurement is

$$\sigma^o(x, y) = \begin{cases} 
\sigma^o_{\text{land}}, & (x, y) \sim \text{land}, \\
\sigma^o_{\text{ocean}}, & (x, y) \sim \text{ocean}.
\end{cases}$$

(4.1)

For every RapidScat measurement collected in the finding process of Section 3.1, the latitude and longitude are used to evaluate the SRF at each point in the $4^\circ \times 4^\circ$ grid. The true SRF is calculated in the X-factor code [2] using the known one-way antenna patterns and pointing from real data. The true SRF is seen in Fig. (4.1). This pattern is symmetric and is centered in the beam angle coordinate system, as intuition suggests should be the case.
Figure 4.1: (a) The upsampled and smoothed true RapidScat SRF, for the inner beam, calculated pixel by pixel in the X-factor routines (regular units) [2]. (b) Contour plot of the true RapidScat SRF (dB).

The simulated $\sigma^\circ$ measurement value is found as the product of the SRF and the synthetic surface. This computation is shown by Eq. (4.2),

$$
\sigma^\circ = \frac{\sum_{x,y} SRF(x,y) \sigma^\circ(x,y)}{\sum_{x,y} SRF(x,y)} + n,
$$

where $n$ is white noise added to each measurement based on whether it includes land or ocean.

$$
n = \begin{cases} 
n_{land} & \sim N[0,\text{VAR}(\sigma^\circ_{land})], \quad \text{land}, \\
n_{ocean} & \sim N[0,\text{VAR}(\sigma^\circ_{ocean})], \quad \text{ocean}. \end{cases}
$$

Any measurement located within a $2^\circ$-$4^\circ$ radius, in [azimuth, elevation], from the island target is considered an “ocean” measurement. Measurements within $2^\circ$ from the island are “land” measurements. The noise variance is estimated by examining the variance of real data for land and ocean separately.

Figs. (4.2a) and (4.2c) show histograms of actual RapidScat land and ocean measurements, for the inner beam, from which the variance of land $\sigma^\circ$ is estimated to be 16.85 dB and the variance
of ocean $\sigma^o$ is estimated to be 33.34 dB. Fig. (4.2) shows that the simulated values of $\sigma^o$ are a good representation of actual measurements.

Figure 4.2: Histograms of actual $\sigma^o$ measurements, (a) and (c), and simulated $\sigma^o$ measurements, (b) and (d). (a) Real land $\sigma^o$. (b) Simulated land $\sigma^o$. (c) Real ocean $\sigma^o$. (d) Simulated ocean $\sigma^o$.

4.2 New $\sigma^o$ and Real Geometry

For the first step in the simulation, the geometry reported in the real data is used to calculate new $\sigma^o$ values for every measurement, as described in the previous section. The new $\sigma^o$ values are
then used to estimate the SRF. This yields the SRF estimate seen in Fig. (4.3). Notice the centered and nearly symmetric pattern. The dashed lines in Fig. (4.3b) indicate that the boresight, the peak value of the SRF, and the center of the -3 dB contour coincide almost perfectly.

![Figure 4.3: (a) An upsampled and smoothed estimate of the RapidScat SRF using simulated $\sigma^\circ$ values and real geometry (regular units). (b) Contour plot of the estimated RapidScat SRF (dB).](image)

4.3 SRF Estimation Error

There are two primary ways to analyze the error for an estimate of the SRF. This section first analyzes the root mean square error (RMSE), which is an average error for the magnitude of the SRF estimate. Then, the error in beam dimensions is analyzed to determine the correctness of the shape of the estimate.

4.3.1 Root Mean Square Error

The differences between the true and estimated SRFs for the inner and outer beams are shown in Fig. (4.4a) and Fig. (4.4b) respectively. The root mean square error (RMSE) is defined
by

\[ RMSE = \sqrt{\frac{x_0^2 + x_1^2 + \ldots + x_N^2}{N}}, \]  

(4.4)

where \( x_n \) is the difference between element-\( n \) in the true SRF and element-\( n \) in the estimated SRF, and \( N \) is the total number of elements. Evaluating Eq. (4.4) for the inner beams yields an RMSE of 0.045 (-13.51 dB). For the outer beam, the RMSE is 0.019 (-17.15 dB). This result suggests that the SRF can be successfully estimated with very small error.

![Figure 4.4: The difference between the true SRF and the estimated SRF with simulated data, in regular units. (a) Inner beam. (b) Outer beam.](image)

### 4.3.2 SRF Dimension Error

Comparing the true SRF to the estimated SRF from simulated data, the accuracy of the SRF beam dimensions can be analyzed. An estimate with beam dimensions close to those of the true SRF is considered good. In this section, the azimuth and elevation widths of the -3 dB and the -6 dB contours of the true and the estimated SRF are compared. The width of the -3 dB contour is the standard half-power beam width, while the -6 dB contour, for the SRF, should be similar in dimension to the -3 dB contour for the one-way antenna pattern. The estimated beam dimensions are displayed in Table 4.1 and Table 4.2 for the inner and outer beam dimensions respectively. For
both the inner and outer beam, the true SRF is smaller in azimuth and larger in elevation. The largest difference is seen for the inner beam -6 dB elevation width where the true pattern is \( \sim 0.3^\circ \) (12%) larger than the estimated pattern. For both beams the estimated SRF is fatter in azimuth by about 0.05°-0.1° and skinnier in elevation by about 0.2°-0.3° for the inner beam and about 0.02° for the outer beam.

Table 4.1: Inner beam (H-pol) width comparison (in degrees)

<table>
<thead>
<tr>
<th></th>
<th>-3 dB</th>
<th>-6 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth</td>
<td>2.03</td>
<td>2.12</td>
</tr>
<tr>
<td>Elevation</td>
<td>1.72</td>
<td>1.54</td>
</tr>
</tbody>
</table>

Table 4.2: Outer beam (V-pol) width comparison (in degrees)

<table>
<thead>
<tr>
<th></th>
<th>-3 dB</th>
<th>-6 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth</td>
<td>1.86</td>
<td>1.97</td>
</tr>
<tr>
<td>Elevation</td>
<td>1.52</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Several factors contribute to the dimension errors. These factors include the size of the moving average filter used to smooth the estimates and the number of singular values included in the estimation process (see Appendix A).

### 4.4 Antenna Pointing Validation

To confirm the hypothesis that the estimated SRF is sensitive to a linear shift from a pointing bias, the next step in the simulation analysis is to perturb the geometry by artificially imposing an azimuth pointing bias in the data. Perturbing the azimuth angle correspondingly adjusts the reported latitude and longitude for each measurement. This shifts the origin of the SRF estimation grid. Using the perturbed geometry, the SRF is recomputed, along with creating a new landmap for each measurement, and \( \sigma^\circ \) measurements are again simulated. The SRF estimation procedure then continues as normal with the originally reported location of the measurements. This simulates the
case where there exists an antenna pointing bias that is not reflected in the reported data. The SRF estimation yields an azimuth offset version of the SRF seen in Fig. (4.5). There is a clear positive shift in azimuth angle in the estimated SRF of the same magnitude and direction as the imposed bias. This implies that an SRF estimate, in terms of azimuth and elevation angle, can in fact be used as a tool to analyze pointing biases.

Figure 4.5: (a) An upsampled and smoothed estimate of the RapidScat SRF using simulated $\sigma^\circ$ values and perturbed geometry, an azimuth bias of 0.5$^\circ$ (regular units). (b) Contour plot of the estimated RapidScat SRF (dB).

The shift in the SRF estimate is corrected by recomputing the geometry of each measurement with an azimuth-correction bias with the same magnitude and direction as the shift. This bias is introduced in the SRF estimation process when building the sample landmap (see Section 3.2). Fig. (4.6) shows the SRF estimate is better centered after correcting for an azimuth bias.

As mentioned in the introduction to this chapter, offsetting the azimuth angle offsets the location of surface features relative to the boresight location. The mapping from latitude and longitude to beam angles within the SRF is thus affected. This is illustrated in Fig. (4.7) where there is a difference in the (azimuth, elevation) location of the landmap for a particular measurement for no azimuth bias versus an azimuth bias. Notice that the azimuth bias of 0.5$^\circ$ directly corresponds to the land mass shifting 0.5$^\circ$ toward the center of the SRF. This effectively changes the location
Figure 4.6: (a) An upsampled and smoothed estimate of the RapidScat SRF using simulated $\sigma^\circ$ values and perturbed geometry, an azimuth bias of 0.5° (regular units). (b) Contour plot of the estimated RapidScat SRF (dB).

within the SRF that is being sampled by the land. Also notice, that the shape of the land mass is slightly different. This is the result of the antenna observing the land from a different angle.

Figure 4.7: (a) An example of the landmap used to compute $\sigma^\circ$ with no geometry biases. (b) The same landmap with perturbed geometry, an azimuth bias of 0.5°.
Beside a shift in the SRF, there are differences in the SRF estimates between the offset and non-offset versions (compare Fig. (4.3) and Fig. (4.5)). The actual shape of the estimate varies because of the shift in the location of the boresight relative to land shown in Fig. (4.7). The shifting landmap modifies the $\sigma^\circ$ for each measurement including land. Histograms of the differences seen in $\sigma^\circ$ caused by a pointing bias are shown in Fig. (4.8), without the added noise. The variations in $\sigma^\circ$ are summarized in Table 4.3.

<table>
<thead>
<tr>
<th></th>
<th>Inner Beam</th>
<th>Outer Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>3.54</td>
<td>3.45</td>
</tr>
<tr>
<td>Average</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.37</td>
<td>0.39</td>
</tr>
</tbody>
</table>

This result implies that the shape of the estimated SRF is dependent on the azimuth bias. So for real data, if there exists an offset in the estimated SRF, there is probably some error in the shape of the estimate. Recomputing the geometry of each $\sigma^\circ$ measurement and re-estimating the SRF is the way to minimize this error. The difference in $\sigma^\circ$ illustrated in Fig. (4.8) is much more pronounced for the regional case used in this thesis because the island and ocean $\sigma^\circ$ are so distinctly different. The differences in $\sigma^\circ$ over regions with more consistent surface features are probably not as significant.

4.5 Summary

The simulation described in this chapter shows that the angle-based SRF estimation algorithm produces a close estimate to the true SRF, and that an estimate of the SRF, when done in an angle-based coordinate system, can be used to detect antenna pointing biases. The first part of the simulation, using reported geometry parameters, demonstrates that the angle-based SRF estimation process yields an estimated pattern properly centered in beam angle coordinates. The RMSE error of the amplitude for the estimated SRF is 1-5%. This estimate is compared to the true SRF and is found to achieve a maximum dimension error of 12%. The second part of the simulation shows that when there is a bias in the antenna pointing, specifically an azimuth bias, which is not
Figure 4.8: Normalized histograms of the difference in simulated $\sigma^\circ$ when there is no pointing bias and when there is an azimuth bias of 0.5°. (a) Inner beam. (b) Outer beam.

Reflected in the reported geometry, the SRF estimate is offset in azimuth by an amount close to, if not equal to, the bias. The SRF can be correctly estimated by recomputing the geometry for the measurements to include an azimuth pointing bias of the amount the SRF is shifted.
The spatial response function (SRF) is dominated by the two-way antenna pattern so an estimate of the SRF can be used to estimate the two-way antenna pattern. The two-way pattern is the product of the one-way gain pattern when a pulse is transmitted with the one-way gain pattern when the backscatter from the pulse is received. In the radar equation,

\[ P_R = \frac{P_T \lambda^2}{(4\pi)^3} \int_A \frac{G_R(\bar{V})G_T(\bar{V})\sigma^\circ(\bar{V})}{r_R^2(\bar{V})r_T^2(\bar{V})} d\bar{V}, \]  

which is another form of Eq. (1.2), each one-way pattern is represented by \( G_T \) and \( G_R \) for the transmit and receive patterns respectively. The corresponding \( r_T \) and \( r_R \) are the slant ranges. \( A \) is the illuminated area of the two-way pattern on the ground. In the development in Sec. (1.3) the SRF is shown to be related to Eq. (5.1) by

\[ P_R = \int_A SRF(\bar{V})\sigma^\circ(\bar{V}) d\bar{V}, \]  

which, with \( \bar{V} \) being defined in terms of beam azimuth angle, \( \phi \), and beam elevation angle, \( \theta \), can be written as

\[ P_R = \int\int SRF(\phi, \theta)\sigma^\circ(\phi, \theta) d\phi, d\theta. \]  

The SRF is then,

\[ SRF(\phi, \theta) = \frac{G_R(\phi, \theta)G_T(\phi, \theta)\lambda^2 P_T}{(4\pi)^3 r_T^2(\phi, \theta)r_R^2(\phi, \theta)}. \]  

Aside from the transmit and receive gains (the one-way antenna patterns), the parameters used in Eq. (5.1) can all be assumed constant from transmit time to receive time. The slant range terms can be treated as constant over the footprint as discussed in Appendix B.

This chapter discusses the development of two methods to derive the one-way antenna pattern from the estimated SRF. The gains are treated separately and are assumed to be identical.
but shifted due to antenna rotation. The problem of deriving the antenna pattern is setup in detail in Section 5.1. An approach to this derivation using the Fourier transform is given in Section 5.2, and a derivation using a first-order Taylor series approximation is given in Section 5.3. Section 5.4 summarizes the results of the derivation.

5.1 Problem Setup

Noting that the azimuth shift is essentially one-dimensional, it is helpful to initially explore a simplified version of the pattern estimation problem. To help recognize the connection to standard signal processing techniques, the problem is formed in terms of time-varying signals. The two-way measurement response can be modeled by

\[ f(t) = g(t)g(t - \tau), \]  

\[ (5.5) \]

where \( f(t) \) is the product of the transmitted and received pulses (the SRF), \( g(t) \) is the one-way antenna pattern, and \( \tau \) is the shift in azimuth angle between the transmitting antenna position and the receiving antenna position. The problem to solve is this: find \( g(t) \) given \( f(t) \) and \( \tau \). In order to create symmetry about the origin, Eq. (5.5) can be rewritten as

\[ f(t) = g \left( t + \frac{\tau}{2} \right) g \left( t - \frac{\tau}{2} \right), \]  

\[ (5.6) \]

The first-order approximation of the shift \( \tau \) is calculated using a locally-flat Earth assumption and the parameters in Table 5.1. Figs. (5.1) and (5.2) illustrate the geometry used in calculating \( \tau \).

<table>
<thead>
<tr>
<th>Table 5.1: RapidScat Scanning Parameters [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>H-pol</strong></td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Slant Range</td>
</tr>
<tr>
<td>Antenna Rotation Rate</td>
</tr>
<tr>
<td>Pulse Time of Flight</td>
</tr>
<tr>
<td>Calculated Shift(( \tau ))</td>
</tr>
</tbody>
</table>

35
Figure 5.1: An illustration of the shift in the antenna pattern with the change in the antenna position between transmit-time and receive-time. The darker area corresponds to the main lobe of the SRF. \( \tau \) is the shift in azimuth between the two one-way antenna patterns.

Figure 5.2: An illustration of the geometry used to calculate the shift in azimuth angle.

In order for this analysis to be extended to two dimensions (2-D), the slant range terms are assumed essentially constant over the footprint. Otherwise, the variations in \( r \) would alter the timing offset of the transmit and receive antenna patterns in the SRF. The following development does not consider this variation.

In two-dimensions, the problem can be written as

\[
f(\phi, \theta) = g(\phi, \theta)g(\phi - \varphi, \theta - \vartheta),
\]

(5.7)
where \( f(\phi, \theta) \) is the product of the transmitted and received pulses (the SRF), \( g(\phi, \theta) \) is the one-way antenna pattern, \( \phi \) is the shift in azimuth angle between the transmitting position of the antenna and the receiving position of the antenna, and \( \theta \) is the shift in elevation angle. Assuming that for a single transmit/receive cycle the shift in elevation between the two antenna positions is negligible, \( \theta = 0 \). The problem is to solve for \( g(\phi, \theta) \) given \( f(\phi, \theta) \) and \( \phi \). In order to make the problem symmetric about the origin, Eq. (5.7) can be rewritten as

\[
 f(\phi, \theta) = g\left(\phi + \frac{\phi}{2}, \theta\right) g\left(\phi - \frac{\phi}{2}, \theta\right). \tag{5.8}
\]

For illustration purposes in this chapter, a multi-variate normal (MVN) function with zero mean and 0.5 variance is used to represent the SRF. This function was chosen as an appropriate function for testing the following derivation methods because it exhibits similar geometric characteristics to an SRF. Fig. (5.3) shows this test function.

![Figure 5.3: a) A 3-D perspective of the MVN function used to represent the SRF. b) A 1-D slice through the center of the MVN function.](image)

### 5.2 Fourier Transform Approach

The solution for \( g(t) \) from Eq. (5.5) is initially developed using the Fourier transform. This approach has an interesting development which is worth analyzing, but eventually yields noisy results. An alternate formulation is provided in a later section.
5.2.1 One-Dimensional

To estimate \( g(t) \) from \( f(t) \), the first step is to take the natural log of Eq. (5.6) to turn the multiplication problem into an addition problem, which can be manipulated to solve for \( g(t) \),

\[
\ln[f(t)] = \ln \left[ g \left( t + \frac{\tau}{2} \right) g \left( t - \frac{\tau}{2} \right) \right],
\]
(5.9)

\[
\ln[f(t)] = \ln \left[ g \left( t + \frac{\tau}{2} \right) \right] + \ln \left[ g \left( t - \frac{\tau}{2} \right) \right],
\]
(5.10)

\[
f' = g'_{t+\tau/2} + g'_{t-\tau/2},
\]
(5.11)

where \( f' = \ln[f(t)] \), \( g'_{t-\tau/2} = \ln[g(t - \tau/2)] \), and \( g'_{t+\tau/2} = \ln[g(t + \tau/2)] \). This linear problem can be modeled with the shift in azimuth angle as a phase shift in the Fourier Transform domain

\[
\mathcal{F} \{ f' = g'_{t+\tau/2} + g'_{t-\tau/2} \};
\]
(5.12)

\[
F'(j\omega) = e^{j\omega\tau/2} G'(j\omega) + e^{-j\omega\tau/2} G'(j\omega),
\]
(5.13)

\[
F'(j\omega) = G'(j\omega)[e^{j\omega\tau/2} + e^{-j\omega\tau/2}],
\]
(5.14)

\[
F'(j\omega) = 2\cos(\omega\tau/2) G'(j\omega),
\]
(5.15)

where \( \mathcal{F} \) denotes the Fourier transform. Solving further:

\[
G'(j\omega) = \frac{F'(j\omega)}{2\cos(\omega\tau/2)},
\]
(5.16)

\[
G'(j\omega) = \frac{1}{2} F'(j\omega) \sec(\omega\tau/2).
\]
(5.17)

From which it follows,

\[
g'(t) = \frac{1}{2} [f'(t) \ast \mathcal{F}^{-1}\{\sec(\omega\tau/2)\}] .
\]
(5.18)

Unfortunately, the inverse Fourier transform of \( \sec \) does not formally exist; though it can be approximated as a sequence of delta functions. If the Fourier transform did exist, the solution would fall out as follows:
Let \( h(t) = \frac{1}{2}[f'(t) \ast \mathcal{F}^{-1}\{\sec(\omega \tau/2)\}] \). Then,

\[
\begin{align*}
g'(t) &= h(t), \\
\ln[g(t)] &= h(t), \\
g(t) &= e^{h(t)}.
\end{align*}
\]

Thus, \( e^{h(t)} \) is the estimate of the antenna pattern.

The results of attempting the antenna pattern derivation in this way are shown in Fig. (5.4). The recovered SRF looks close in size to the true SRF, but the antenna pattern itself exhibits strange behavior with what look like step functions or impulses. This is the result of the limitations of the approximate inverse Fourier transform of the \( \sec \) function. So, the theoretical complications manifest themselves clearly in practice.

![Simulated 1-D Fourier Transform Results](image)

**Figure 5.4:** The 1-D version of the derived antenna pattern and the recovered SRF using the Fourier transform approach.

### 5.2.2 Two-Dimensional

As described in Section 5.1, the direction of the shift in the antenna pattern from the transmit location to the receive location is assumed to be in the azimuth direction only. In this development, \( \phi \) is the azimuth direction, \( \varphi \) is the shift in azimuth, and \( \theta \) is the elevation angle.
Take the natural log of Eq. (5.8) to turn the multiplication problem into an addition problem, which can be manipulated to solve for \( g(\phi, \theta) \).

\[
\ln[f(\phi, \theta)] = \ln\left[ g\left(\phi + \frac{\phi}{2}, \theta \right) g\left(\phi - \frac{\phi}{2}, \theta \right) \right], \tag{5.22}
\]

\[
\ln[f(\phi, \theta)] = \ln\left[ g\left(\phi + \frac{\phi}{2}, \theta \right) \right] + \ln\left[ g\left(\phi - \frac{\phi}{2}, \theta \right) \right]. \tag{5.23}
\]

Simplifying notation, Eq. (5.23) can be written as

\[
f' = g'_{\phi + \phi/2} + g'_{\phi - \phi/2}, \tag{5.24}
\]

where \( f' = \ln[f(\phi, \theta)] \), \( g'_{\phi - \phi/2} = \ln[g(\phi - \phi/2, \theta)] \), and \( g'_{\phi + \phi/2} = \ln[g(\phi + \phi/2, \theta)] \). Now, this linear problem can be modeled with the shift in azimuth angle as a phase shift in the Fourier Transform domain,

\[
\mathcal{F}\{f' = g'_{\phi + \phi/2} + g'_{\phi - \phi/2}\}, \tag{5.25}
\]

\[
F'(j\omega, \theta) = e^{(j\omega\phi/2)}G'(j\omega, \theta) + e^{(-(j\omega\phi/2)}G'(j\omega, \theta), \tag{5.26}
\]

\[
F'(j\omega, \theta) = G'(j\omega, \theta)[e^{(j\omega\phi/2)} + e^{(-(j\omega\phi/2)}], \tag{5.27}
\]

\[
F'(j\omega, \theta) = 2\cos(\omega\phi/2)G'(j\omega, \theta), \tag{5.28}
\]

where \( F'(j\omega, \theta) \) is the Fourier transform of \( f'\phi, \theta \) and \( G'(j\omega, \theta) \) is the Fourier transform of \( g'(\phi, \theta) \). Solving further:

\[
G'(j\omega, \theta) = \frac{F'(j\omega, \theta)}{2\cos(\omega\phi/2)}, \tag{5.29}
\]

\[
G'(j\omega, \theta) = \frac{1}{2}F'(j\omega, \theta)\sec(\omega\phi/2), \tag{5.30}
\]

From which it follows,

\[
g'(\phi, \theta) = \frac{1}{2}[f'(\phi, \theta) * \mathcal{F}^{-1}\{\sec(\omega\phi/2)\}]. \tag{5.31}
\]
Again, the inverse Fourier transform of \( \sec(x) \) does not formally exist. If it did, the solution would be:

Let \( h(\phi, \theta) = \frac{1}{2}[f'(\phi, \theta) \ast \mathcal{F}^{-1}\{\sec(\omega \varphi/2)\}] \). Then,

\[
\begin{align*}
g'(\phi, \theta) &= h(\phi, \theta), \\
\ln[g(\phi, \theta)] &= h(\phi, \theta), \\
g(\phi, \theta) &= e^{h(\phi, \theta)}.
\end{align*}
\] (5.32) (5.33) (5.34)

Thus, \( e^{h(\phi, \theta)} \) is the estimate of the antenna pattern.

5.3 Taylor Series Approximation

An alternative approach using the Taylor series approximation of the shifted signals yields a much cleaner and better result for the estimate of the antenna pattern. This approach is given in this section.

5.3.1 One-Dimensional

The one-dimensional problem given by Eq. (5.6) can be approximated with the first-order Taylor series expansion:

\[
\begin{align*}
g\left(t + \frac{\tau}{2}\right) &\approx g(t) + \dot{g}(t) \frac{\tau}{2}, \\
g\left(t - \frac{\tau}{2}\right) &\approx g(t) - \dot{g}(t) \frac{\tau}{2},
\end{align*}
\] (5.35) (5.36)

where \( \dot{g}(t) \) is the first derivative of \( g(t) \). This approximation is illustrated in Fig. (5.5) where the value of the arbitrary function \( f(t) \) evaluated at a shift, \( \Delta t \), is approximated using the first derivative of the non-shifted function and its value at \( f(t) \).
Substitute Eqs. (5.35) and (5.36) into (5.6) and simplify,

\[ f(t) \approx \left( g(t) + \dot{g}(t) \frac{\tau}{2} \right) \left( g(t) - \dot{g}(t) \frac{\tau}{2} \right), \]  
\[ f(t) \approx g^2(t) + g(t) \left( \dot{g}(t) \frac{\tau}{2} - \dot{g}(t) \frac{\tau}{2} \right) - \left( \dot{g}(t) \frac{\tau}{2} \right)^2, \]  
\[ f(t) \approx g^2(t) - \left( \dot{g}(t) \frac{\tau}{2} \right)^2. \]  
\[ 5.37 \]
\[ 5.38 \]
\[ 5.39 \]

Solving for \( g(t) \),

\[ g^2(t) \approx f(t) + \left( \dot{g}(t) \frac{\tau}{2} \right)^2. \]  
\[ 5.40 \]

It can be argued that \( \left( \dot{g}(t) \frac{\tau}{2} \right)^2 \approx \dot{f}(t) \frac{\tau}{2} \), where \( \dot{f}(t) \) is the first derivative of \( f(t) \). It is obvious from Eq. (5.6), that

\[ f \left( t + \frac{\tau}{2} \right) = g(t)g(t + \tau). \]  
\[ 5.41 \]

The first-order Taylor series approximations of \( f(t + \tau/2) \) and \( g(t + \tau) \) are

\[ f \left( t + \frac{\tau}{2} \right) \approx f(t) + \dot{f}(t) \frac{\tau}{2}, \]  
\[ g(t + \tau) \approx g(t) + \dot{g}(t) \tau. \]  
\[ 5.42 \]
\[ 5.43 \]
Solving Eq. (5.42) for \( \dot{f}(t) \) and substituting using Eq. (5.41):

\[
\dot{f}(t) \frac{\tau}{2} \approx f \left( t + \frac{\tau}{2} \right) - f(t),
\]

(5.44)

\[
\dot{f}(t) \frac{\tau}{2} \approx g(t)g(t + \tau) - f(t).
\]

(5.45)

Substituting Eqs. (5.43) and (5.39) into (5.45) and simplifying:

\[
\dot{f}(t) \frac{\tau}{2} \approx g(t)\left[ g(t) + \dot{g}(t)\tau \right] - \left[ g^2(t) - \left( \frac{\dot{g}(t)\tau}{2} \right)^2 \right],
\]

(5.46)

\[
\dot{f}(t) \frac{\tau}{2} \approx g^2(t) + \dot{g}(t)g(t)\tau - g^2(t) + \left( \frac{\dot{g}(t)\tau}{2} \right)^2,
\]

(5.47)

\[
\dot{f}(t) \frac{\tau}{2} \approx \dot{g}(t)g(t)\tau + \left( \frac{\dot{g}(t)\tau}{2} \right)^2.
\]

(5.48)

As shown below, assuming \( \dot{g}(t)g(t)\tau = 0 \) introduces a negligible amount of error into the derivation. See Fig. (5.6). Thus,

\[
\dot{f}(t) \frac{\tau}{2} \approx \left( \frac{\dot{g}(t)\tau}{2} \right)^2.
\]

(5.49)

Using this approximation in Eq. (5.40) and solving for \( g(t) \):

\[
g^2(t) \approx f(t) + \dot{f}(t) \frac{\tau}{2},
\]

(5.50)

\[
\dot{g}(t) \approx \sqrt{f(t) + \dot{f}(t) \frac{\tau}{2}}.
\]

(5.51)

Thus, the estimate of the antenna pattern is \( \hat{g}(t) \) with an error proportional to \( \dot{g}(t)g(t)\tau \). The estimate can be improved by using a higher order Taylor series expansion, though this is not done here.

As an aside, a simulation is performed to show that the error introduced into the derivation by Eq. (5.49) is very small. This simulation uses a zero-mean Gaussian function with variance of 0.5 as the true \( g(t) \). Eq. (5.40) is solved using the approximation of Eq. (5.49). Fig. (5.6) illustrates that the error between the true result and the approximate result is in fact small. However, the error increases with variance and shift. It is intuitive that the error increases with the shift \( \tau \) because, as the shift increases between antenna patterns, the antenna pattern becomes less similar to the square-
root of the SRF. The shift used in this simulation is representative of an actual shift between \( G_T \) and \( G_R \).

![Figure 5.6](image)

(a) The Gaussian test functions used to simulate \( G_T \) and \( G_R \) and their product, the SRF. The variance of the one-way patterns here 0.5 and they are zero-mean. b) A plot of the error incurred by making the approximation in Eq. (5.49).

The result of estimating the antenna pattern using this approach with the 1-D slice is shown in Fig. (5.7). The recovered SRF looks like an almost perfect recovery of the original SRF. This approach proves to work well especially compared to the Fourier transform approach. A 2-D development of this method using the first-order Taylor series approximation follows.

### 5.3.2 Two-Dimensional

As described in Section 5.1, the direction of the shift in the antenna pattern from the transmit location to the receive location is assumed to be in the azimuth direction only. In this development, \( \phi \) is the azimuth direction, \( \varphi \) is the shift in azimuth, and \( \theta \) is the elevation angle. Eq. (5.8) can be approximated with the first-order Taylor series expansion as follows:

\[
g\left(\phi + \frac{\varphi}{2}, \theta\right) \approx g(\phi, \theta) + g(\phi, \theta) \frac{\varphi}{2}, \tag{5.52}
\]

\[
g\left(\phi - \frac{\varphi}{2}, \theta\right) \approx g(\phi, \theta) - g(\phi, \theta) \frac{\varphi}{2}, \tag{5.53}
\]
Figure 5.7: The 1-D version of the derived antenna pattern and the recovered SRF using the Taylor series approximation approach. Each curve here is normalized to 1 for ease of comparison. The recovered and original SRF closely coincide.

where $\dot{g}(\phi, \theta)$ is the first derivative of $g(\phi, \theta)$. Substitute Eqs. (5.52) and (5.53) into (5.8) and simplify.

\begin{align*}
    f(\phi, \theta) &\approx \left( g(\phi, \theta) + \dot{g}(\phi, \theta) \frac{\theta}{2} \right) \left( g(\phi, \theta) - \dot{g}(\phi, \theta) \frac{\theta}{2} \right), \\
    f(\phi, \theta) &\approx g^2(\phi, \theta) + g(\phi, \theta) \left( \dot{g}(\phi, \theta) \frac{\theta}{2} - \dot{g}(\phi, \theta) \frac{\theta}{2} \right) - \left( \dot{g}(\phi, \theta) \frac{\theta}{2} \right)^2, \\
    f(\phi, \theta) &\approx g^2(\phi, \theta) - \left( \dot{g}(\phi, \theta) \frac{\theta}{2} \right)^2.
\end{align*}

Solving for $g(\phi, \theta)$,

\begin{equation}
    g^2(\phi, \theta) \approx f(\phi, \theta) + \left( \dot{g}(\phi, \theta) \frac{\theta}{2} \right)^2.
\end{equation}

The argument that $\left( \dot{g}(\phi, \theta) \frac{\theta}{2} \right)^2 \approx f(\phi, \theta) \frac{\theta}{2}$ extends from the previous section and is developed here. It is obvious from Eq. (5.8) that

\begin{equation}
    f \left( \phi + \frac{\theta}{2}, \theta \right) = g(\phi, \theta)g(\phi + \theta, \theta).
\end{equation}
The first-order Taylor series approximations of \( f(\phi + \varphi, \theta/2) \) and \( g(\phi + \varphi, \theta) \) are

\[
f\left(\phi + \frac{\varphi}{2}, \theta\right) \approx f(\phi, \theta) + f(\phi, \theta) \frac{\varphi}{2}, \quad (5.59)
g(\phi + \varphi, \theta) \approx g(\phi, \theta) + \dot{g}(\phi, \theta) \varphi. \quad (5.60)
\]

Solving Eq. (5.59) for \( \dot{f}(\phi, \theta) \) and substituting using Eq. (5.58):

\[
\dot{f}(\phi, \theta) \frac{\varphi}{2} \approx \frac{f(\phi + \varphi, \theta) - f(\phi, \theta)}{\varphi}, \quad (5.61)
\]

\[
\dot{f}(\phi, \theta) \frac{\varphi}{2} \approx g(\phi, \theta)g(\phi + \varphi, \theta) - f(\phi, \theta). \quad (5.62)
\]

Substituting Eqs. (5.60) and (5.56) into (5.62) and simplifying:

\[
\dot{f}(\phi, \theta) \frac{\varphi}{2} \approx g(\phi, \theta)[g(\phi, \theta) + \dot{g}(\phi, \theta) \varphi] - \left[ g^2(\phi, \theta) - \left( \dot{g}(\phi, \theta) \frac{\varphi}{2} \right)^2 \right], \quad (5.63)
\]

\[
\dot{f}(\phi, \theta) \frac{\varphi}{2} \approx g^2(\phi, \theta) + g(\phi, \theta)\dot{g}(\phi, \theta) \varphi - g^2(\phi, \theta) + \left( \dot{g}(\phi, \theta) \frac{\varphi}{2} \right)^2, \quad (5.64)
\]

\[
\dot{f}(\phi, \theta) \frac{\varphi}{2} \approx \dot{g}(\phi, \theta)g(\phi, \theta) \varphi + \left( \dot{g}(\phi, \theta) \frac{\varphi}{2} \right)^2. \quad (5.65)
\]

The error from approximating \( \dot{g}(\phi, \theta)g(\phi, \theta) \varphi = 0 \) is small and can be neglected. See Fig. (5.6).

Thus,

\[
\dot{f}(\phi, \theta) \frac{\varphi}{2} \approx \left( \dot{g}(\phi, \theta) \frac{\varphi}{2} \right)^2. \quad (5.66)
\]

Using this approximation in Eq. (5.57), \( g(\phi, \theta) \) is solved as:

\[
g^2(\phi, \theta) \approx f(\phi, \theta) + \dot{f}(\phi, \theta) \frac{\varphi}{2}, \quad (5.67)
\]

\[
\dot{g}(\phi, \theta) \approx \sqrt{f(\phi, \theta) + \dot{f}(\phi, \theta) \frac{\varphi}{2}}. \quad (5.68)
\]

Thus, the estimate of the antenna pattern is \( \dot{g}(\phi, \theta) \) with an error term proportional to \( \dot{g}(\phi, \theta)g(\phi, \theta) \varphi \).

The result of implementing this method with a simulated SRF can be seen in Fig. (5.8).

The derived one-way antenna pattern is a smooth function with a centered peak and is fatter than the SRF used to derive it. This is consistent with the 1-D results.
5.4 Summary

Two different methods of deriving the one-way antenna pattern from a simulated SRF have been developed. These separate approaches solved Eq. (5.5) for $g(t)$, the antenna pattern, given $f(t)$, the SRF, and $\tau$, a shift in azimuth. The method of using the Fourier transform proved to be theoretically difficult. Using a first-order Taylor series approximation has been shown to work theoretically and has yielded decent results during testing.
In the earlier chapters, this thesis describes the estimation of the RapidScat SRF and the ways in which the SRF can be used to help validate RapidScat data. The RapidScat SRF estimation method is discussed in Chapter 3. The SRF estimation simulation is then discussed in Chapter 4. Finally, a method for deriving the one-way antenna pattern is developed in Chapter 5. The results presented in these earlier chapters are demonstrated with simulated data. In this chapter, these methods are applied to real data.

6.1 RapidScat Spatial Response Function Estimation Results

The process explained in Chapter 3 yields a data-driven estimate of the SRF using on-orbit data. The RapidScat data used spans almost a year, JD 276 of 2014 to JD 220 of 2015. This date range covers most of the time for which RapidScat was in its initial high SNR state. The following results come from egg measurements for ascending and descending passes and uses both aft and fore azimuth looks. This section shows the results for the estimated RapidScat SRF in angle-based coordinates first. The results in ground-based coordinates are then shown for comparison.

6.1.1 Angle-Based Coordinates

The results of the SRF estimation in the coordinate system defined by antenna beam angles are shown in Figs. (6.1) and (6.2). The results are upsampled and smoothed in order to reduce the influence of noise and to make it easier to compare the quality of estimates. There is a clear shift in the positive azimuth direction seen in these estimates that is highlighted by the dashed lines in Figs. (6.1b) and (6.2b). This is similar to that seen in the simulation conducted in Chapter 4 when an artificial azimuth bias is imposed on the data. Further examining the shift in these estimates, the H-pol estimate is shifted by about 0.263° and the V-pol estimate is shifted by about 0.244°. By inserting a correction factor equal to these shifts into the estimation process, the SRF is recomputed.
to be properly centered. The resulting SRF estimates are seen in Figs. (6.3) and (6.4). Notice the improved centering of the estimate.

Figure 6.1: (a) An estimate of the RapidScat SRF for the horizontally polarized (H-pol) inner beam (regular units). (b) Contour plot of the H-pol SRF estimate (dB).

Figure 6.2: (a) An estimate of the RapidScat SRF for the vertically polarized (V-pol) outer beam (regular units). (b) Contour plot of the V-pol SRF estimate (dB).
Figure 6.3: (a) A corrected estimate of the RapidScat SRF for the horizontally polarized (H-pol) inner beam. (b) Contour plot of the corrected H-pol SRF estimate.

Figure 6.4: (a) A corrected estimate of the RapidScat SRF for the vertically polarized (V-pol) outer beam. (b) Contour plot of the corrected V-pol SRF estimate.

The proper centering of the SRF estimate after inserting an azimuth bias correction factor suggests that there exists a pointing bias in the RapidScat antenna geometry. The observed location
of the SRF estimate within the angle-based coordinate system, using real data, is consistent with a biased boresight.

The other metric used in Chapter 4 to determine the quality of the SRF estimate is the -3 dB and -6 dB dimensions of the main lobe. Table 6.1 and Table 6.2 outline the widths of the main lobe for the predicted SRF and the widths of the azimuth biased and unbiased estimates using actual data. The -3 dB and -6 dB contours of the estimated SRF are essentially the same size when azimuth angle biased or unbiased, but the estimated SRFs are consistently fatter in azimuth by about 0.3° and fatter in elevation by about 0.1°. For comparison, Figs. (6.5) and (6.6) show the predicted SRF for the inner and outer beams respectively.

Table 6.1: Inner beam (H-pol) width comparison (in degrees)

<table>
<thead>
<tr>
<th></th>
<th>-3 dB</th>
<th>-6 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Biased</td>
</tr>
<tr>
<td>Azimuth</td>
<td>2.03</td>
<td>2.31</td>
</tr>
<tr>
<td>Elevation</td>
<td>1.72</td>
<td>1.80</td>
</tr>
</tbody>
</table>

Table 6.2: Outer beam (V-pol) width comparison (in degrees)

<table>
<thead>
<tr>
<th></th>
<th>-3 dB</th>
<th>-6 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Biased</td>
</tr>
<tr>
<td>Azimuth</td>
<td>1.86</td>
<td>2.13</td>
</tr>
<tr>
<td>Elevation</td>
<td>1.52</td>
<td>1.68</td>
</tr>
</tbody>
</table>
Figure 6.5: (a) The predicted RapidScat SRF for the horizontally polarized (H-pol) inner beam. (b) Contour plot of the H-pol SRF. Compare Fig. (6.3)

Figure 6.6: (a) The predicted RapidScat SRF for the vertically polarized (V-pol) outer beam. (b) Contour plot of the V-pol SRF. Compare Fig. (6.4)

Figs. (6.3), (6.4), (6.5), and (6.6) show that the estimated SRF is very similar in shape and dimension to the predicted SRF for both the inner and outer beams. Variations in dimension are due
to several factors including: the order of the smoothing filter used, inaccuracies in the geometry of the data, and the number of singular values used in the estimate. The dimensionality of the SRF estimate and the behavior of the SRF to a bias correction are consistent with simulation. This consistency suggests that the estimation process and the bias correction are good methods for calculating a well-behaved SRF estimate. The difference between the estimated SRF and the predicted SRF is shown in Fig. (6.7). The RMSE, for the inner beam, is about 0.042 (-13.80 dB) and, for the outer beam, about 0.053 (-12.73 dB).

Figure 6.7: The difference between the predicted SRF and the estimated SRF with real data, in regular units. (a) Inner beam. (b) Outer beam.

6.1.2 Ground-Based Coordinates

In Chapter 3, several differences are mentioned between the previously developed ground-based SRF estimation algorithm and the angle-based method described in this thesis. Here, the estimates using the ground-based method are shown to help illustrate the way the two approaches differ. Figs. (6.8) and (6.9) show the RapidScat SRF estimate in a kilometer grid where the x-axis is the beam azimuth in kilometers and the y-axis is the beam elevation in kilometers.
Notice the elongation of the SRF in the elevation direction. This due to the way in which the footprint is projected onto the ground. This elongation is not seen in the angle-based estimates.
The differences between the ground-based and angle-based grids are illustrated in Figs. (6.10) and (6.11). Also notice that Figs. (6.8b) and (6.9b) do not indicate the azimuth bias that was readily detected using the angle-based estimates. This served as a motivation to develop the angle-based method.

![Figure 6.10](image1.png)

Figure 6.10: Side view illustration of the coordinate system difference between the angle-based and ground-based methods.

![Figure 6.11](image2.png)

Figure 6.11: A 3D view illustration of the coordinate system difference between the angle-based and ground-based methods.

The ground-based SRF estimation algorithm assumes a constant slant range and footprint size in kilometers on the ground. Since RapidScat does not have a constant footprint size because of the highly variable slant range, the ground-based method is subject to additional error from these
variations. Using an angle-based grid simplifies the estimation pointing analysis because there is no need for computing the slant range. It is important to recognize that the SRF estimate computed with the angle grid is still the nominal SRF with a nominal slant range, attitude, etc. because the SRF is still being sampled on the ground. The variable slant range of the ISS still has the effect of blurring (broadening) the SRF because the space on the ground covered by each pixel in the land map changes with each variation in slant range. These variations are accommodated by the normalization of land pixels during the creation of the sample landmap for each measurement (see Section 3.2).

6.2 One-way Antenna Pattern Derivation

In Chapter 5, two methods for deriving the one-way antenna pattern from the SRF are developed and tested using a MVN function in place of the estimated SRF. Using real data to estimate the SRF, the one-way antenna pattern can then be derived for real data. The results using the Fourier transform approach are shown for completeness despite being theoretically difficult. These are followed by the derivation results using the Taylor series approximation approach. Using a first-order Taylor series approximation has been shown to work theoretically and has yielded decent results during testing. The results of the one-way antenna pattern derivation are analyzed using a cross-sectional view of the SRF. Figs. (6.12) and (6.13) show a 3D view of the SRF and a cross-section through the center of the 3D SRF for the inner and outer beams respectively.

Figure 6.12: a) A 3-D perspective of the SRF for the inner beam (H-pol). b) A 1-D slice through the center of the inner beam SRF (H-pol).
Figure 6.13: a) A 3-D perspective of the SRF for the outer beam (V-pol). b) A 1-D slice through the center of the outer beam SRF (V-pol).

6.2.1 Fourier Transform Approach

The results of deriving the one-way antenna pattern using the Fourier transform are shown for the inner and outer beams in Figs. (6.14a) and (6.14b). The noisiness of the one-way pattern to the left of the peak is a result of the numerical inverse Fourier transform of sec which doesn’t formally exist. As the earlier experiments in Chapter 5 suggested, using the Fourier transform proves to be an inadequate technique for deriving the one-way antenna pattern.

6.2.2 Taylor Series Approach

The results of deriving the one-way antenna pattern using the Taylor series approximation approach for the inner and outer beams are shown in Figs. (6.15a) and (6.15b). Employing the 2D version of the Taylor series approach yields the results seen in Figs. (6.16a) and (6.16b).

Figs. (6.15) and (6.16) show a one-way antenna pattern which is wider than the SRF as is expected. If multiplied by a shifted version of itself the derived antenna pattern yields an almost exact replica of the estimated SRF. The recovered SRF has an RMSE of 0.012 (-19.39 dB) for the inner beam and 0.012 (-19.14 dB) for the outer beam. This approach proves to work well especially compared to the Fourier transform approach.

The derived pattern is compared to the known one-way antenna pattern. Figs. (6.17) and (6.18) show the known and derived one-way patterns, respectively, for the inner beam and
Figure 6.14: The 1-D version of the derived antenna pattern and the recovered SRF using the Fourier transform approach. (a) H-pol. (b) V-pol.

Figure 6.15: The 1-D version of the derived antenna pattern and the recovered SRF using the Taylor series approximation approach. Each curve here is normalized to 1 for ease of comparison. (a) H-pol. (b) V-pol. The recovered and original SRFs coincide.

Figs. (6.19) and (6.20) are the known and derived patterns for the outer beam. For both the inner and outer beam, the derived one-way pattern is fatter than the known pattern. The fatness is greater in the azimuth than elevation. Figs. (6.21a) and (6.13) show that the greatest difference between
Figure 6.16: An oblique view at the 3D results of the derived antenna pattern and the recovered SRF using the Taylor series approximation (left pattern only). (a) H-pol. (b) V-pol

the known and derived patterns is in azimuth. The difference is very small however near the peak. The RMSE for the derived inner beam one-way pattern is 0.143 (-8.44 dB) and 0.133 (-8.77 dB) for the outer beam. The dimensions for each beam are summarized in Table 6.3. The derived patterns are about 0.8° fatter in azimuth than the known patterns and about 0.3°-0.5° fatter in elevation. This error can be attributed to several things including: approximations made in the Taylor series derivation (see Chapter 5), the order of the Taylor series expansion, and the slant range used to compute to the shift between the two one-way antenna patterns, i.e., the nominal slant range. Jitter in reported measurement locations due to attitude noise may also contribute to the fattening of the derived one-way antenna pattern.

<table>
<thead>
<tr>
<th></th>
<th>Inner Beam</th>
<th>Outer Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Known</td>
<td>Derived</td>
</tr>
<tr>
<td>Azimuth</td>
<td>2.41</td>
<td>3.22</td>
</tr>
<tr>
<td>Elevation</td>
<td>2.01</td>
<td>2.51</td>
</tr>
</tbody>
</table>
Figure 6.17: (a) The known RapidScat one-way antenna pattern for the horizontally polarized (H-pol) inner beam (regular units). (b) Contour plot of the H-pol one-way antenna pattern (dB).

Figure 6.18: (a) The derived RapidScat one-way antenna pattern for the horizontally polarized (H-pol) inner beam (regular units). (b) Contour plot of the H-pol one-way antenna pattern (dB).
Figure 6.19: (a) The known RapidScat one-way antenna pattern for vertically polarized (V-pol) inner beam (regular units). (b) Contour plot of the V-pol one-way antenna pattern (dB).

Figure 6.20: (a) The derived RapidScat one-way antenna pattern for vertically polarized (V-pol) inner beam (regular units). (b) Contour plot of the V-pol one-way antenna pattern (dB).
Figure 6.21: (a) The difference between the known and derived one-way antenna patterns for the inner beam (regular units). (b) The difference between the known and derived one-way antenna patterns for the outer beam (regular units).

6.3 Summary

This chapter presents the various results attained from performing the SRF estimation techniques described in Chapter 3, the pointing validation techniques discussed in Chapter 4, and the one-way antenna pattern derivation developed in Chapter 5 on real data. The estimated SRF using real data is shown to be offset in azimuth by 0.263° for the inner beam and 0.244° for the outer beam. By applying the bias correction presented in simulation in Chapter 4, the SRF is properly centered and the dimensions of the -3 dB and -6 dB contours are minimally affected. The estimated SRF also has beam contour dimensions similar to the predicted SRF and the RMSE of the estimate compared to the predicted is 0.042 for the inner beam and 0.053 for the outer beam. Using the Taylor series approach to deriving the one-way antenna pattern yields a pattern which has a wider main lobe than the SRF but which yields the appropriate SRF when multiplied with a shifted version of itself. The SRF recovered from the derived one-way antenna pattern has an RMSE of 0.012 for both the inner and outer beams. Though this error is very small, the RMSE of the one-way patterns compared to the true RapidScat antenna patterns are 0.143 and 0.133 for the inner and outer beams respectively. The error in the one-way pattern derivation is probably great enough to disqualify the derived pattern as calibration truth.
CHAPTER 7. CONCLUSION

This thesis explores the estimation of the RapidScat SRF using an island target in an angle-based coordinate system. The angle-based method is an improvement on the previously developed ground-based method developed by Joshua Bradley [19]. This thesis verifies the SRF estimation process through simulation. An artificial azimuth bias is simulated to observe the behavior of the resulting SRF estimate. That bias is corrected by adding an azimuth offset to the boresight azimuth for every measurement. When the angle-based SRF estimation is performed on actual data, the estimated RapidScat SRF shows that there exists an azimuth bias in both the inner and outer beams. This bias is estimated and corrected to yield a better SRF estimate. This bias can also be used to correct the RapidScat antenna pointing and measurement locations in ground processing.

The azimuth bias found by estimating the SRF is assumed to be pervasive in the RapidScat data set. This means that the $(\text{lat}, \text{lon}), \sigma^\circ$ pair for all of the RapidScat measurements is slightly off. This could be corrected by recomputing the true locations of every RapidScat measurement after correcting the azimuth angle of the measurement.

This thesis also derives the one-way antenna pattern from the estimated SRF using a Taylor series expansion. The resulting one-way pattern successfully recovers the estimated SRF, but is significantly different compared to the true one-way pattern. The method of deriving the one-way pattern from the SRF requires knowledge of the slant range. This is problematic for RapidScat because of the highly variable measurement geometry associated with the ISS.

7.1 Contributions

This thesis makes the following contributions to the field of scatterometry:

- An alternate method for estimating the SRF of a pencil-beam scatterometer that is more robust to changes in platform altitude and attitude.
The angle-based method for estimating the SRF developed in this thesis does not assume stable platform geometry. This allows the SRF to be estimated despite large variations in altitude and attitude when platforms like the ISS are used. The differences in the angle-based method and the previously developed ground-based method are discussed in Chapter 3 and results are shown in Chapter 6.

- A demonstration of how the antenna pointing of a pencil-beam scatterometer can be validated using an estimate of the SRF.

In this thesis, the SRF is estimated for the purpose of estimating pointing biases. Pointing biases are estimated by the amount the estimated SRF is offset within an azimuth/elevation coordinate grid. Pointing biases are corrected by adding a bias-correction term (equal to the bias) to the biased angle for each measurement. This procedure is verified in simulation in Chapter 4 and results using actual data are shown in Chapter 6.

- A method of deriving the one-way antenna pattern for a pencil beam scatterometer from the SRF.

The SRF, which is dominated by the two-way antenna pattern, can be modeled as the product of two one-way antenna patterns which are shifted relative to each other. The shift is due to the rotation of the antenna during ToF. The one-way antenna pattern can be derived using a Taylor series expansion with the shift in the patterns estimated from the nominal slant range. This process is shown using simulated data in Chapter 5 and the results using actual data are shown in Chapter 6.

### 7.2 Future Work

There are several areas in which the work of this thesis can be extended. This section describes those areas and suggests some work that can be done.

- Estimate the azimuth bias for various date ranges to analyze any potential time-varying bias.

If there are varying biases for different date ranges, there may be some other factor influencing the pointing which was not considered previously. A time-varying bias would indicate that a different correction factor would need to be used for the several date ranges. The size
of date range used to estimate a bias would have to be large enough to still collect a lot of
data for the SRF estimation process. Thus, one could shrink the date range used in this thesis
and estimate the SRF and compare the estimated biases. The date range could be tightened
until the estimated SRF doesn’t resemble a smooth function.

• Explore the implications of an azimuth bias on the geophysical model function and wind/rain
estimation models.

The geophysical model function (GMF) is used to estimate surface qualities based on scat-
terometer measurements. Wind speed and direction and rainfall are very common parameters
that are estimated by the remote sensing community. The implications of an antenna point-
ing bias include the mislocation of radar measurements and error in measurement azimuth
angles, both of which play a role in the GMF. So, experiments can be conducted where the
pointing bias is used to update the geometry and location of a set of measurements. These
updated measurements can be used to estimate wind speeds, for example, using the current
GMF and then the wind speeds can be compared to original estimates. This comparison
can be performed for multiple parameters to analyze the overall affect of pointing biases on
gеophysical estimates.

• Explore the effect that the varying slant range has across the footprint in the one-way antenna
pattern derivation.

Appendix B justifies the approximation that slant range is constant over the two-way antenna
pattern, but this is still an approximation. The one-way antenna pattern derivation can be
extended by including the varying slant range in the elevation direction. This would make
the shift in the two one-way patterns a function of elevation angle and potentially improve
the accuracy of the estimated gain patterns.

• Measure the sensitivity of the SRF estimate RMSE to the method by which the island $\sigma^o$ is
unbiased from the ocean $\sigma^o$.

In this thesis, the contribution of ocean $\sigma^o$ on measurements including land is estimated
using a quadratic fit. There are other ways of estimating this contribution including perhaps
a cubic or fourth-order polynomial fit. The quadratic fit performed well for the purposes
of this thesis, but the RMSE of the estimated SRF may be improved by attempting other kinds of estimation. There may also be methods employed by near-coastal wind retrieval algorithms that would improve the SRF estimation process.

- Extend the angle-based SRF estimation algorithm to estimate the slice SRFs and analyze the influence of a pointing bias on individual slices.

This thesis only considered full-footprint "egg" $\sigma^\circ$ measurements. However, the footprint (two-way antenna pattern) can be broken up into slices using range and doppler processing techniques. This allows the resolution of radar data to be made more fine and enhances the resolution of reconstructed images. Each slice has its own SRF. An antenna pointing bias may have some implications as far the creation of slices, though a bias in azimuth should not be as influential as a bias in elevation. Further research can be conducted to evaluate the slice SRF.
REFERENCES


APPENDIX A. SELECTING THE NUMBER OF SINGULAR VALUES IN RANK-REDUCED ESTIMATION

In Chapter 2, the mathematical framework for estimating the SRF is developed using singular value decomposition (SVD). After decomposing the $Q$ matrix [see Eq. (2.10)], the rank of $Q$ is reduced to produce, $\tilde{Q}$, [see Eq. (2.11)]. The rank is reduced by choosing the number of singular values to include, i.e., the rank of $\Sigma_1$, and setting all other singular values to zero. This appendix explores the reasoning used to decide how many singular values to retain in the SRF estimation process. While a single set of simulated data for the SRF is used in this analysis, the discussion in this appendix applies to all sets of data used in this thesis for estimating the SRF.

In general, the singular values are ordered down the diagonal of $\Sigma$ in decreasing order:

$$
\Sigma = \begin{bmatrix}
\sigma_1 & 0 \\
0 & \sigma_2 \\
& \ddots \\
0 & & \cdots & \sigma_M
\end{bmatrix} \quad \text{with} \quad \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_M,
$$

(A.1)

where $M$ is the rank of $\Sigma$, and $\sigma_M$ is the singular value corresponding to the $M$-th singular vector.

The relationship between the magnitude of each singular value of the SRF and its placement in the diagonal of $\Sigma$ is seen in Fig. (A.1). The elbow in Fig. (A.1) indicates the point at which the singular vectors start to represent more noise than signal. This trend is general for many SVD problems but Fig. (A.1) is specific for the simulated SRF data used in this appendix. The principle components of $Q$, which are associated with the signal power, are the first several singular vectors which correspond to the largest singular values. The noise is represented by the small singular values making up $\Sigma_2$ in Eq. (2.10) which are the values after the elbow in Fig. (A.1).
Figure A.1: A plot of the magnitude of each singular value of $Q$. The magnitude decreases with each succeeding singular value as in Eq. (A.1).

With the number of singular values included in the SRF estimate, there is a trade-off in noise and higher order detail. For example, the first four singular vectors represent the four most important signal components in the SRF and are reshaped into matrices and plotted in Fig. (A.2). As a general trend, more lobes appear in higher order singular vectors. These smaller lobes can represent noise or high frequency information for the SRF. So, while including more singular values yields greater detail in the SRF, they also include more noise.

For the purposes of this thesis, the center of the SRF, which is not greatly influenced by including more singular vectors, is more important than higher order details. Thus, an appropriately conservative choice for the number of singular values to use is near the elbow; this is the primary heuristic for determining how many singular values to include in the SRF estimate. The elbow is located at roughly 20-30 singular values for the experiments conducted in this thesis. SRF estimates using 15-30 singular values are subjectively chosen as candidate best estimates because they typically bracket the elbow. Fig. (A.3) shows the estimated SRF using 15, 20, 25, and 30 singular values. Visually, these are all very similar so a secondary heuristic must be used to decide the best number of singular values to retain.
The secondary heuristic is the number of singular values where the corresponding SRF estimate has the fewest negative pixels. This is appropriate in the case of estimating the SRF because it does not make physical sense to have a negative value in a SRF. The final choice for the number of singular values to include in the SRF estimate for the data used in this appendix is 26 because the SRF using 26 singular values has fewer negative pixels than the other numbers of singular values between 15-30. By retaining the first 26 singular values the succeeding $\sim 2000$ singular values are set to zero to reduce the noise in the estimate.
Figure A.3: Estimates of the SRF using simulated data for the inner beam for various numbers of singular values (in regular units). (a) 15 singular values. (b) 20 singular values. (c) 25 singular values. (d) 30 singular values.
APPENDIX B. SLANT RANGE DEPENDENCE OF THE SRF

For any given measurement, the slant range from the radar to the surface varies over the illuminated two-way antenna pattern. The expression given by Eq. (1.7) shows that the SRF, which is defined over \( \vec{v} \), is a function of the gain \( G \) and the slant range \( r \). Both \( G \) and \( r \) are functions of \( \vec{v} \) which includes the observation geometry for \( r \). Since the geometry is different for every measurement, the SRF is unique for every measurement. This makes the estimation process described in Chapter 2 cumbersome and impractical. To enable the use of a nominal SRF rather than a unique SRF for every measurement, the dependence of \( r \) on \( \vec{v} \) can be removed.

This appendix presents a reasonable approximation for the slant range. The approximation is verified with parameters from RapidScat.

B.1 Approximation of Slant Range

Consider defining the slant range at each point within the footprint as

\[
\begin{align*}
\mathbf{r} &= \mathbf{r}_0 + \Delta \mathbf{r}, \\
&= \mathbf{r}_0 \left(1 + \frac{\Delta \mathbf{r}}{\mathbf{r}_0}\right),
\end{align*}
\]

where \( \mathbf{r}_0 \) is the nominal slant range to the center of the footprint on the ground and \( \Delta \mathbf{r} \) is the difference between \( \mathbf{r}_0 \) and the slant range to any other point within the footprint. Since \( \mathbf{r}_0 \) is much greater than \( \Delta \mathbf{r} \) the ratio \( \Delta \mathbf{r}/\mathbf{r}_0 \) is very small and \( \mathbf{r} \approx \mathbf{r}_0 \). An illustration of the way slant range varies over the footprint is shown in Fig. (B.1).

B.2 The Case of RapidScat

In order to verify this slant range approximation for RapidScat, the parameters contained in Fig. (1.1) can be used to calculate the approximate variations in slant range. Only the variations
in slant range in the elevation dimension of the beam are significant because the slant range across the azimuth dimension is almost constant. Using the law of cosines, the maximum slant range and the minimum slant range in the elevation dimension can be computed. The computed values are found in the table below. The maximum error in $\sigma^0$ resulting from the approximation $r = r_0$ is 6.12%.

Table B.1: RapidScat Slant Range Variations

<table>
<thead>
<tr>
<th>Nominal Slant Range ($r_0$)</th>
<th>600 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Slant Range</td>
<td>609.54 km</td>
</tr>
<tr>
<td>Min. Slant Range</td>
<td>590.57 km</td>
</tr>
<tr>
<td>Max. $\Delta r$</td>
<td>9.54 km</td>
</tr>
<tr>
<td>Ratio $\frac{\Delta r}{r_0}$</td>
<td>0.0159</td>
</tr>
<tr>
<td>Nominal $r^4$</td>
<td>$1.2960 \times 10^{11}$ km$^4$</td>
</tr>
<tr>
<td>Max. $r^4$</td>
<td>$1.3804 \times 10^{11}$ km$^4$</td>
</tr>
<tr>
<td>Max. Percent Error in assuming $r = r_0$</td>
<td>6.12%</td>
</tr>
</tbody>
</table>

The values in the table above show that it is a reasonable approximation to ignore the variations in slant range over the coordinate system $\vec{V}$ in the case of RapidScat.