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It Is Better to Be Upside Than Sharpe!

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ABSTRACT

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Based on the assumption that returns in *Commercial Real Estate* (CRE) are normally distributed, the *Sharpe Ratio* (SR) has been the standard risk-adjusted performance measure for the past several years. Research has questioned whether this assumption can be reasonably made. The *Upside Potential Ratio* (UPR) as a risk-adjusted performance measure is an alternative to measure performance on a risk-adjusted basis but its values differ from the Sharpe Ratio's only in the assumption of skewed returns. We will provide reasonable evidence that CRE returns should not be fitted with a normal distribution and present the *Gaussian Mixture Model* (GMM) as our choice of distribution to fit skewness. We will then use a GMM distribution to measure performance of CRE domestic markets via UPR. Additional insights will be presented by introducing an alternative risk-adjusted performance measure that we will call *D-ratio*. We will show how the UPR and the D-ratio can provide a tool-box that can be added to any existing investment strategy when identifying markets' past performance and timing of entrance. The intent of this thesis is not to provide a comprehensive framework for CRE investment decisions but to introduce statistical and mathematical tools that can serve any portfolio manager in augmenting any investment strategy already in place.

Keywords: Sharpe Ratio, Real Estate, Upside Potential Ratio, Gaussian Mixture Models, D-ratio

Acknowledgments

Thanks to my dad who always believed in me against evidence. Thanks to books that have been silent friends who never cease to speak. Thanks to my grandpa who still teaches me even from the other banks of the river.

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Chapter 1. Sharpe meets Gaussian Mixture Models

We introduce the *Sharpe Ratio* (SR) and discuss its benefits and limitations. We then introduce the *Upside Potential Ratio* (UPR) as a refinement and improvement to the SR, discussing its benefits and limitations. Evidence will be presented of skewness of returns in *Commercial Real Estate* (CRE) by referring to NCREIF (*National Council of Real Estate Investment Fiduciaries*) data[1] . *Gaussian Mixture Models* (GMM) will then be presented as our choice of skewed distribution to fit CRE returns.

1.1 Sharpe Ratio

Modern Portfolio Theory (MPT) started in the 1950s with Markovitz's publication *Portfolio Selection*[2]. At the core of MPT is the paradigm that first and second moments of returns are sufficient statistics to measure performance of assets. Portfolio managers since then have been analyzing funds through the mean and standard deviation of returns, although few measures have been introduced that include higher moments of the distribution. The extensive use of mean and variance in financial literature is described in the Journal of Portfolio Management forty years after Markowitz's publication:

> We build on Markowitz's mean-variance paradigm, which assumes that the mean and standard deviation of the distribution of one-period return are sufficient statistics for evaluating the prospects of an investment portfolio.[3]

William Sharpe introduced a measure that combines mean and standard deviation into a

[1] NCREIF collects both property and fund level information from its members on a quarterly basis, and in one case on a monthly basis. This data is used to produce various indices and performance reports. The data is also available to members in masked form for research and other purposes. NCREIFs historical property and fund database go back to the Fourth Quarter 1977 and consists of over 35,000 properties and over 150 open-end and closed-end funds. For further information on NCREIF data and methodology please visit *https://www.ncreif.org/*

[2] Harry Markowitz, *The Journal of Finance*, Vol.7, No.1.(Mar, 1952), pp.77-91

[3] The Sharpe Ratio, *Journal of Portfolio Management*, 1994, p. 49

single performance measure and it has been referred to as the *Sharpe Ratio* (SR). The intuition behind the Sharpe Ratio (SR) is that volatility of returns is perceived as risk for an investor while a high returns average attracts investors. This interpretation was already discussed by Markowitz when he said:

> We next consider the rule that the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing. [4]

The SR wants to capture the performance of an asset with respect to a risk free rate benchmark return by comparing units of excess returns to units of risk taken. With the SR we want to answer the question: If I invest in this given asset rather than investing at the risk free rate of return, how many units of profits can I expect to receive for any additional unit of risk taken? Identifying a given asset's returns with A and with r_f the risk-free rate of returns, the SR was introduced in 1966 [5] as excess returns over standard deviation:

$$SR = \frac{\mathbf{E}[A] - r_f}{\sigma_A} \tag{1.1}$$

The advantage in using this measure is evident: simplicity. It involves the simplest sample parameters in statistics which computation is quite simple under assumption of normal returns. Using *Maximum Likelihood Estimators* (MLE's) on a set of sample data, the inference population statistics are :

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}, \qquad \sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N-1}$$

where N is the sample size and x_i are the sample values. The SR will return a value of 1 when each unit of reward is offset by a unit of risk. A value greater than 1 reflects a strong asset that rewards the investor with more units of profit than risk. The opposite argument

[4] Harry Markowitz, *The Journal of Finance*, Vol.7, No.1.(Mar, 1952), pp.77

[5] *The Journal of Business*, Vol. 39, No. 1, Part 2: Supplement on Security Prices (Jan., 1966), pp. 119-138

exists for positive values smaller than 1. Needless to say that an asset with a negative SR value is quickly disregarded.

One limitation to using the SR is how it interprets risk. Volatility is not inherently negative for an investor. An investor with a conservative investment profile is adverse to sensitive upside and downside swings in returns and prefers to invest in stable, low volatility assets (note that low volatility results in a high SR score hence the preference for high SR values) but the same conservative investor will prefer to invest in an asset with exclusively positive swings in returns (note that high upside volatility either is not captured by the SR or it negatively affects it). Hence, a refinement of the SR was presented by Sortino as the *Upside Potential Ratio* (UPR), which compares the upper central first moment ($\mathbf{E}[A - r_f]_+$) with the second lower central moment (σ_-), using a benchmark rate as a center of tendency. (Sample average is known as $first\ moment$ and variance is known as *second moment*. *Central* instead refers to computing mean and variance of values above a threshold which in our case is the risk free rate of returns).

$$UPR = \frac{\mathbf{E}[A - r_f]_+}{\sigma_-} \tag{1.2}$$

The UPR appears to be more insightful to an investor that wants to measure profitability of an investment against negative volatility, regardless of the investor's risk profile. But this measure gives us the same results as the SR unless we relax the underlying assumption of normality of returns. One of the limits of applying the SR is that returns are not always normally distributed and measuring volatility for a skewed distribution can be a challenging task. Note that using the same MLE formulas as above to measure the mean and variance of a sample distribution are not valid if the data violates the assumption of normality. Rather, these formulas provide a population's mean estimate and its variance but not the distribution parameters.

Research has already shown the inadequacy of assuming normality

The assumption of normality of returns is invalid for most securities including real estate [6] [7]

Finding volatility for skewed returns is a challenging task that could be avoided if we could safely assume normality of return since positive or negative volatility would coincide in symmetric distributions. The question remains: are CRE returns skewed?

1.2 CRE Returns Analysis

We now challenge the assumption of normal returns for CRE returns and show the limitations of the SR in capturing adequately the performance of a given market. To simplify our sample size, we choose the following markets : Atlanta, Austin, Baltimore, Boston, Chicago, Miami, New York, Phoenix, Pittsburgh, San Antonio, San Francisco and Washington, DC. Of these twelve, industry professionals intuitively rank some of them as gateway or primary markets (Boston, New York, San Francisco, Washington, DC), others as secondary markets (Austin, Chicago, Miami, Phoenix) and the remaining as tertiary markets (Atlanta, Baltimore, Pittsburg, San Antonio). An example of what these returns look like is provided below (we used simple kernel density estimations to plot the returns)

[6] 1997, *Journal of Real Estate Portfolio Management*, volume 3, issue 1, pp. 37-47

[7] Further evidence that returns in CRE are skewed and fat-tailed can be found in the same article under the paragraph *Normality*

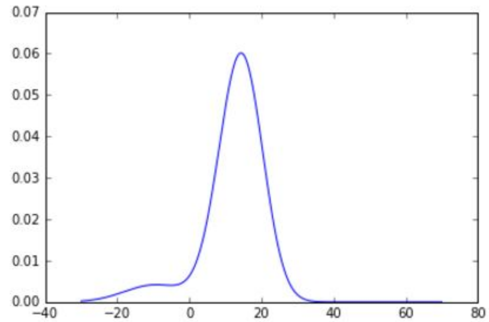


Figure 1.1: San Antonio

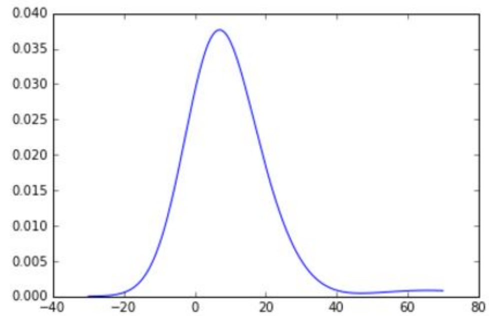


Figure 1.2: Phoenix

With the above graphs we notice that markets can present both positive and negative skewness along with bimodal behaviors. Below we provide a table that shows ranking according to SR values along with their skewness. Intuitively, we want primary markets to perform better than secondary markets and similarly we expect secondary markets to perform better than tertiary.

Table 1.1: Sharpe ranking and Skewness (Office property type)

Ranking	Market	Skewness	Sharpe Ratio
1	San Antonio	-1.38	1.10
2	Pittsburgh	-0.97	1.03
3	Baltimore	-0.73	0.98
4	Washington, DC	-0.35	0.90
5	Austin	-0.75	0.66
6	New York	0.02	0.65
7	Boston	-0.73	0.61
8	Phoenix	1.92	0.57
9	Miami	0.94	0.56
10	San Francisco	-0.19	0.54
11	Chicago	0.62	0.51
12	Atlanta	0.76	0.43

The top three performing markets for the Office type properties were San Antonio, Pittsburgh and Baltimore, which are all tertiary markets. Interestingly, the top performing markets with the SR are also the markets with the highest negative skewness. This inadequacy of the SR in measuring performance in presence of negative excess returns had already been observed

It turns out that the SR will frequently provide a biased result when excess negative returns are present [8]

The Sharpe Ratio is a very useful measure of investment performance. Because it is based on mean-variance theory, and thus is basically valid only for quadratic preferences or normal distribution, skewed investment returns can lead to misleading conclusions. [9]

There is clearly a need for a mathematical and statistical tool to better capture skewness and moments of non-normal distributions in CRE and literature seems to have not yet presented reliable and satisfactory results.

1.3 Gaussian Mixture Model

We now introduce Gaussian Mixture Models, as a statistical tool in fitting skewed distribution. A Gaussian Mixture Model (GMM) is a skewed distribution obtained by a weighted average of normal densities. Although we know that the sum of normal random variables produces a normal variable, hence it is still symmetric, this is not the case when we have a weighted sum of normal densities:

$$\phi_{GMM}(x) = \sum_{i=1}^{n} w_i \phi_i(x). \tag{1.3}$$

Referring to a multivariate gaussian distribution density as f we have :

$$f(x) = \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{\det(\Sigma)}} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})^T\right)$$
$$GMM(x) = \sum_{i=1}^{n} w_i \frac{1}{(2\pi)^{\frac{n}{2}}\sqrt{\det(\Sigma_i)}} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_i)\Sigma_i^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_i)^T\right)$$

[8] 2014, *Journal of Financial Service Professionals*, volume 68, issue 3, 12-14
[9] William T. Ziemba, *Journal of Portfolio Management*, vol.32, no.1 (Fall 2005):108-122

where w_i are positive weights that add up to 1 and Σ_i is the covariance matrix. Finding the means and standard deviations of each component can be a daunting task if we are trying to fit data. It's beyond the scope of this research to explain how this is done but it suffices to say that GMM parameters are found via machine learning algorithms along with the EM algorithm[10].

Chapter 2. Upside Potential Ratio derived

We now provide the UPR explicit formula by choosing to fit returns with a GMM density. Before we do so, we provide a few intermediary results. Given $\mu, \sigma^2, \phi(x)$ and Φ to represent respectively the mean, variance, density and cumulative distribution function of a normal distribution and r to represent the benchmark return against which we are measuring performance, we compute partial first moment, partial second upper and partial second lower moments.

Here the upper first moment for a normal distribution is computed:

[10] Casella, Berger, *Statistical Inference*, Second Edition, p.326

$$
\begin{aligned}
\int_r^\infty x\phi(x)\,\mathrm{d}x &= \frac{1}{\sqrt{2\pi\sigma^2}}\int_r^\infty xe^{-\frac{(x-\mu)^2}{2\sigma^2}}\,\mathrm{d}x \\
&= \frac{1}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^\infty (\mu+t\sigma)e^{-\frac{t^2}{2}}\,\mathrm{d}t \\
&= \frac{\mu}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^\infty e^{-\frac{t^2}{2}}\,\mathrm{d}t + \frac{\sigma}{\sqrt{2\pi}}\int_{\frac{r-\mu}{\sigma}}^\infty te^{-\frac{t^2}{2}}\,\mathrm{d}t \\
&= \mu\left(1-\Phi(r)\right) + \frac{\sigma}{\sqrt{2\pi}}\int_{-\frac{(r-\mu)^2}{\sigma^2}}^{-\infty} e^v(-\,\mathrm{d}v) \\
&= \mu\left(1-\Phi(r)\right) + \frac{\sigma^2}{\sigma\sqrt{2\pi}}\left(e^{-\frac{(\mu-r)^2}{2\sigma^2}}\right) \\
&= \mu\bar{\Phi}(r) + \sigma^2\phi(r)
\end{aligned}
$$

where $\bar{\Phi} = 1-\Phi(r)$. Here is an auxiliary integral that we'll use to derive second upper and lower moments:

$$
\begin{aligned}
\frac{1}{\sqrt{2\pi}}\int_a^\infty te^{-\frac{t^2}{2}}\,\mathrm{d}t &= \frac{1}{\sqrt{2\pi}}\int_{-\frac{a^2}{2}}^{-\infty} e^v - \mathrm{d}v \\
&= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\frac{a^2}{2}} e^v\,\mathrm{d}v \\
&= \frac{1}{\sqrt{2\pi}}e^{-\frac{a^2}{2}}
\end{aligned}
$$

The second lower moment for a normal distribution is now derived :

$$\begin{aligned}
\int_{-\infty}^{r} x^2\phi(x)\,\mathrm{d}x &= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{r} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,\mathrm{d}x \\
&= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} (\mu+t\sigma)^2 e^{-\frac{t^2}{2}}\sigma\,\mathrm{d}t \\
&= \frac{\mu^2}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} e^{-\frac{t^2}{2}}\,\mathrm{d}t + \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t^2 e^{-\frac{t^2}{2}}\,\mathrm{d}t + \frac{2\mu\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t e^{-\frac{t^2}{2}}\,\mathrm{d}t \\
&= \mu^2\Phi(r) + \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t^2 e^{-\frac{t^2}{2}}\,\mathrm{d}t + \frac{2\mu\sigma}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty} t e^{-\frac{t^2}{2}}\,\mathrm{d}t - \int_{\frac{r-\mu}{\sigma}}^{\infty} t e^{-\frac{t^2}{2}}\,\mathrm{d}t\right) \\
&= \mu^2\Phi(r) + \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t^2 e^{-\frac{t^2}{2}}\,\mathrm{d}t + 2\mu\sigma\left(0 - \frac{e^{-\frac{(r-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}}\right) \\
&= \mu^2\Phi(r) + \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t^2 e^{-\frac{t^2}{2}}\,\mathrm{d}t - 2\mu\sigma^2\phi(r) \\
&= \mu^2\Phi(r) + \sigma^2\Phi(r) + \mu\sigma^2\phi(r) - r\sigma^2\phi(r) - 2\mu\sigma^2\phi(r) \\
&= (\mu^2+\sigma^2)\Phi(r) - (\mu+r)\sigma^2\phi(r)
\end{aligned}$$

where we have used this auxiliary result:

$$\begin{aligned}
\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t^2 e^{-\frac{t^2}{2}}\,\mathrm{d}t &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} t\,\mathrm{d}\left(-e^{-\frac{t^2}{2}}\right) \\
&= \frac{1}{\sqrt{2\pi}}t\left(-e^{-\frac{t^2}{2}}\right)\Big|_{-\infty}^{\frac{r-\mu}{\sigma}} - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}}\left(-e^{-\frac{t^2}{2}}\right)\mathrm{d}t \\
&= \frac{r-\mu}{\sigma\sqrt{2\pi}}\left(-e^{-\frac{(r-\mu)^2}{2\sigma^2}} - 0\right) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{r-\mu}{\sigma}} e^{-\frac{t^2}{2}}\,\mathrm{d}t \\
&= -\frac{r-\mu}{\sigma\sqrt{2\pi}}e^{-\frac{(r-\mu)^2}{2\sigma^2}} + \Phi(r) \\
&= (\mu-r)\phi(r) + \Phi(r)
\end{aligned}$$

Having calculated first upper moment and second lower (and upper) moments for a normal distribution, we can now compute the first upside central moment (which measures the units of excess profit above a benchmark rate) and the second downside central moment (which measures the units of standard deviation below a benchmark rate) for a GMM and indicate them respectively as $\mu_1^+(r)$ and $\mu_2^-(r)$. They are analytically defined as follows :

$$\mu_1^+(r) = \int_r^\infty (x-r)\phi(x)\,\mathrm{d}x \qquad \textit{and} \qquad \mu_2^-(r) = \int_{-\infty}^r (x-r)^2\phi(x)\,\mathrm{d}x.$$

Combining all previous results we have the explicit upper first moment:

$$\begin{aligned}
\mu_1^+(r) &= \int_r^\infty (x-r)\phi(x)\,\mathrm{d}x \\
&= \int_r^\infty (x-r)\Sigma_{i=1}^n w_i\phi_i(x)\,\mathrm{d}x \\
&= \sum_{i=1}^n w_i \int_r^\infty (x-r)\phi_i(x)\,\mathrm{d}x \\
&= \sum_{i=1}^n w_i \left[\int_r^\infty x\phi_i(x)\,\mathrm{d}x - r\int_r^\infty \phi_i(x)\,\mathrm{d}x\right] \\
&= \sum_{i=1}^n w_i \left[\mu\bar{\Phi}(r) + \sigma^2\phi_i(r) - r\bar{\Phi}(r)\right] \\
&= \sum_{i=1}^n w_i \left[(\mu - r)\bar{\Phi}(r) + \sigma^2\phi_i(r)\right]
\end{aligned}$$

And the lower second moment:

$$\begin{aligned}
\mu_2^-(r) &= \int_{-\infty}^{r} (x-r)^2 \phi(x)\,\mathrm{d}x \\
&= \int_{-\infty}^{r} (x-r)^2 \Sigma_{i=1}^{n} w_i \phi_i(x)\,\mathrm{d}x \\
&= \sum_{i=1}^{n} w_i \int_{-\infty}^{r} (x-r)^2 \phi_i(x)\,\mathrm{d}x \\
&= \sum_{i=1}^{n} w_i \left(\int_{-\infty}^{r} x^2 \phi_i(x)\,\mathrm{d}x + r^2 \int_{-\infty}^{r} \phi_i(x)\,\mathrm{d}x - 2r \int_{-\infty}^{r} x\phi_i(x)\,\mathrm{d}x \right) \\
&= \sum_{i=1}^{n} w_i \left[\int_{-\infty}^{r} x^2 \phi_i(x)\,\mathrm{d}x + r^2 \Phi_i(r) - 2r \left(\mu_i - \int_{r}^{\infty} x\phi_i(x)\,\mathrm{d}x \right) \right] \\
&= \sum_{i=1}^{n} w_i \left[\int_{-\infty}^{r} x^2 \phi_i(x)\,\mathrm{d}x + r^2 \Phi_i(r) - 2r\mu_i + 2r \int_{r}^{\infty} x\phi_i(x)\,\mathrm{d}x \right] \\
&= \sum_{i=1}^{n} w_i \left[\int_{-\infty}^{r} x^2 \phi_i(x)\,\mathrm{d}x + r^2 \Phi_i(r) - 2r\mu_i + 2r \left(\mu_i \bar{\Phi}(r) + \sigma_i^2 \phi_i(r) \right) \right] \\
&= \sum_{i=1}^{n} w_i \left[\int_{-\infty}^{r} x^2 \phi_i(x)\,\mathrm{d}x + r^2 \Phi_i(r) - 2r\mu_i + 2r\mu_i \bar{\Phi}(r) + 2r\sigma_i^2 \phi_i(r) \right] \\
&= \sum_{i=1}^{n} w_i \left[\Phi(r) \left(\mu_i^2 + \sigma_i^2 \right) - \phi_i(r)\sigma_i^2 \left(\mu_i - r \right) + r^2 \Phi_i(r) - 2r\mu_i \Phi(r) \right] \\
&= \sum_{i=1}^{n} w_i \left[\Phi_i(r) \left(\mu_i^2 + \sigma_i^2 - 2r\mu_i \right) - \sigma_i^2 \phi_i(r) \left(\mu_i - r \right) + r^2 \Phi_i(r) \right] \\
&= \sum_{i=1}^{n} w_i \left\{ \Phi_i(r) \left[\left(\mu_i - r \right)^2 + \sigma_i^2 \right] + \phi_i(r)\sigma_i^2 \left(r - \mu_i \right) \right\}
\end{aligned}$$

We will be introducing soon an application for the upper second moment but present here its explicit formula:

$$\mu_2^+(r) = \sum_{i=1}^{n} w_i \left\{ \bar{\Phi}_i(r) \left[\left(\mu_i - r \right)^2 + \sigma_i^2 \right] + \phi_i(r)\sigma_i^2 \left(\mu_i - r \right) \right\}$$

The computation is quite similar to the negative we just derived hence we skip the computation.

2.1 Explicit formula and comparison to Sharpe

After having computer partial first and second moments, we can now combine the previous intermediary results to give an explicit formula for the UPR :

$$
\begin{aligned}
UPR &= \frac{\mu_1^+(r)}{\left(\mu_2^-(r)\right)^{\frac{1}{2}}} \\
&= \frac{\sum_{i=1}^n w_i \left[(\mu - r)\bar{\Phi}(r) + \sigma^2 \phi_i(r)\right]}{\left(\sum_{i=1}^n w_i \left\{\Phi_i(r)\left[(\mu_i - r)^2 + \sigma_i^2\right] + \phi_i(r)\sigma_i^2 (r - \mu_i)\right\}\right)^{\frac{1}{2}}}
\end{aligned}
$$

Comparing now the ranking of markets using the UPR and the SR: This is the ranking

Table 2.1: UPR ranking vs SR

Ranking	Market	UPR	Sharpe Ratio
1	Austin	1.06	0.77
2	New York	0.73	0.79
3	Washington, DC	0.71	0.23
4	San Francisco	0.69	0.65
5	San Antonio	0.68	1.30
6	Miami	0.66	0.69
7	Atlanta	0.65	0.53
8	Chicago	0.61	0.63
9	Phoenix	0.60	0.71
10	Boston	0.59	0.73
11	Baltimore	0.56	1.20
12	Pittsburgh	0.55	1.27

we would intuitively expect, displaying the majority of primary markets highest, secondary markets in mid-ranking and tertiary markets (the ones that performed well with the SR!) at the bottom.

Chapter 3. D-ratio

3.1 Introduction to the D-ratio

While UPR can be interpreted as location with respect to benchmark compared to negative moment, we introduce the D-ratio as positive moment compared to negative moment. It's more of a comparison between second moments differnt from the UPR which compares first moment with second moment. The D-ratio instead compares units of positive volatility vs negative volatility. It measures the relative strength of positive skewness centered at a benchmark rate, with respect to the strength of negative skewness.

$$DR = \left(\frac{\mu_2^+(r)}{\mu_2^-(r)}\right)^{\frac{1}{2}} \tag{3.1}$$

by previous calculation the explicit formula becomes

$$DR = \left(\frac{\sum_{i=1}^n w_i \left\{\bar{\Phi}_i(r)\left[(\mu_i - r)^2 + \sigma_i^2\right] + \phi_i(r)\sigma_i^2\,(\mu_i - r)\right\}}{\sum_{i=1}^n w_i \left\{\Phi_i(r)\left[(\mu_i - r)^2 + \sigma_i^2\right] + \phi_i(r)\sigma_i^2\,(r - \mu_i)\right\}}\right)^{\frac{1}{2}}. \tag{3.2}$$

The image below represents a visual summary of the UPR and the D-ratio. The UPR compares the excess returns above a benchmark (blue arrow) to the standard deviation of returns below the benchmark (red arrow). The D-ratio compares the standard deviation above the benchmark (green arrow) to the deviation below the benchmark (red arrow). This technique of decomposing returns in their mean and standard deviation components is not new but the calculations are. The old way of calculating the components of standard deviation involved using MLE's on a truncated set of returns. It would truncated returns above the benchmark to compute negative standard deviation and viceversa. Using MLE's in such a way requires the assumption that all returns are equally probable which is far from being a reasonable assumption. GMM's are a useful probabilistic density that allows us to assign the right probability to each return and to decompose standard deviation in its components more efficiently.

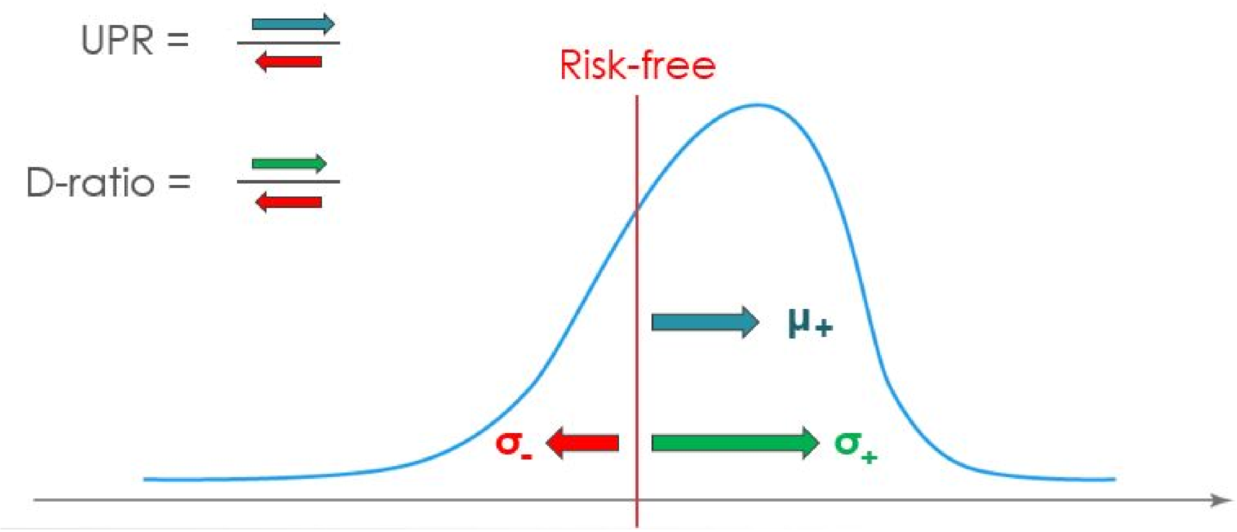


Table 3.1: Ranking by D-ratio

Ranking	Market	D-ratio
1	Austin	1.87
2	Phoenix	1.41
3	Miami	1.35
4	Atlanta	1.30
5	New York	1.24
6	Chicago	1.15
7	Washington, DC	1.14
8	San Francisco	1.13
9	San Antonio	1.02
10	Boston	0.96
11	Baltimore	0.92
12	Pittsburth	0.90

Below the ranking we obtain with the D-ratio, which is quite close to the UPR ranking hence another reasonable performance measure.

Chapter 4. Markets timing and investment strategy

4.1 Investment strategy and intuitions

Table 4.1: D-ratio, UPR and Excess Returns

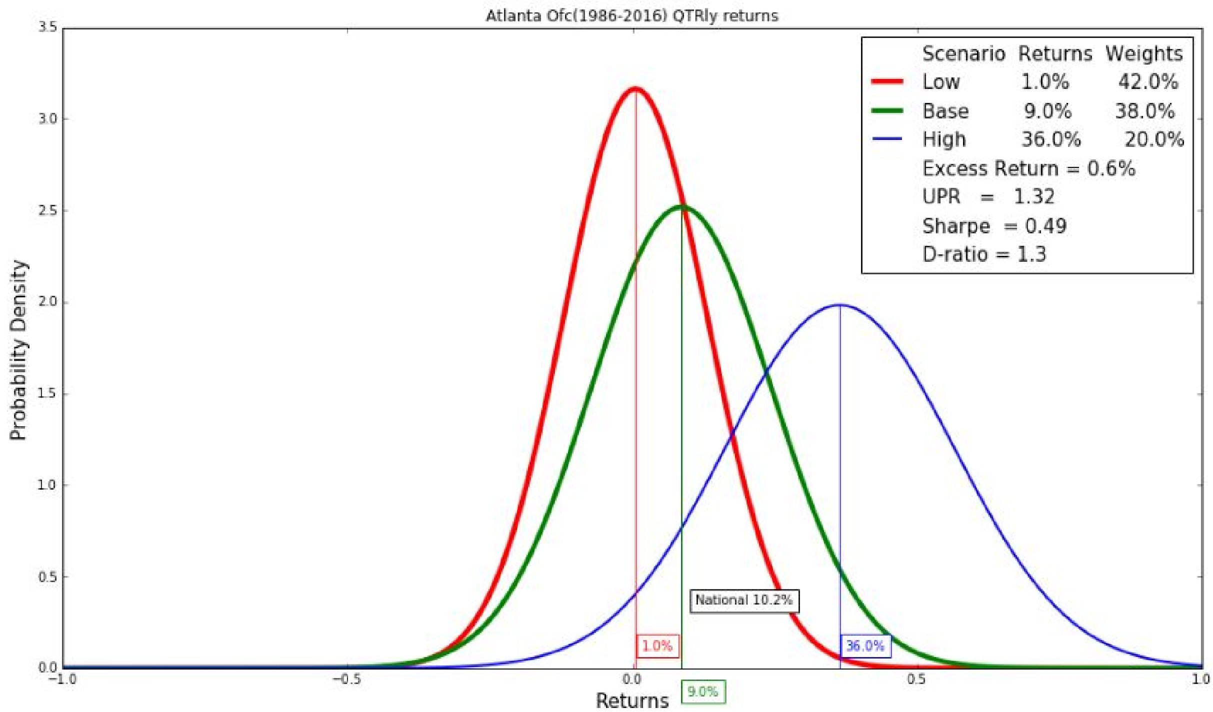
Market	D-ratio	UPR	Excess R	Score
Austin	1.87	1.1	3.54	9
New York	1.24	0.73	0.70	8
San Francisco	1.13	0.69	0.62	8
Washington, DC	1.14	0.71	0.27	7
Miami	1.35	0.66	-0.62	7
Atlanta	1.30	0.66	-0.77	7
San Antonio	1.02	0.68	0.53	6
Chicago	1.15	0.61	-1.1	5
Phoenix	1.41	0.60	-1.38	5
Boston	0.96	0.60	0.12	4
Pittsburgh	0.90	0.55	-0.96	3
Baltimore	0.92	0.56	-0.98	3

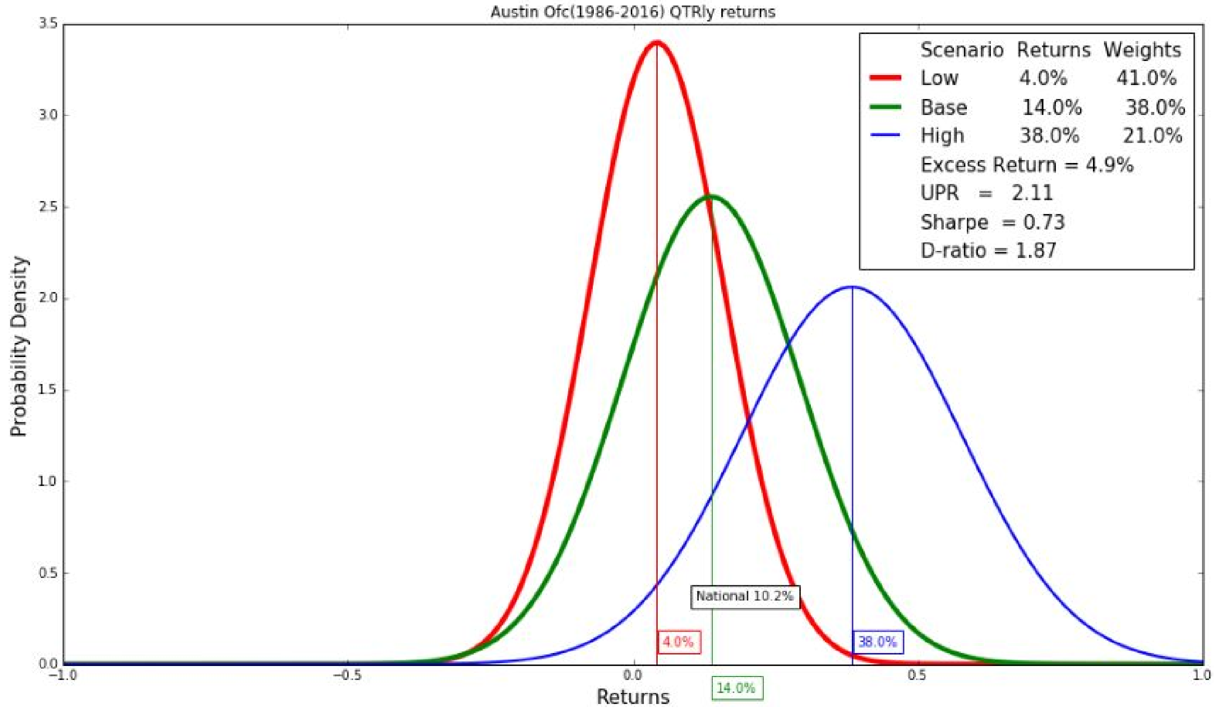
With the tools we have now at hand, we can decompose market returns in its main components of mean and standard deviation. For a *REIT* (Real Estate Investment Fund) whose purpose is to preserve capital and achieve above market returns it can be useful to identify markets that measure returns above a national average and that have a higher chance of upside scenarios than downside. Below we show an example on using the decomposition we reached with the UPR, D-ratio and Excess Returns (numerator of the UPR). This allows us to identify markets that outperform the national average (positive Excess Returns) and that have favorable reward-per-risk trade-offs (UPR and D-ratio). We have divided the markets in a top, middle and bottom tier for each performance measure (Excess Returns, UPR and D-ratio). Then we score each tier (high=3, medium=2, low=1) and added up the scores according to their individual scores. What comes out is that Austin, New York, San Francisco and Washington, DC are the top 4 markets an investor should evaluate on a risk adjusted performance. This matches what institutional investors believe are the gateway markets. Miami, Atlanta, San Antonio and Boston can be tactical investments when the timing is right. Phoenix, Chicago, Pittsburgh and Baltimore are discouraged investments. In this table, we see that the D-ratio and Excess returns do a good job at describing a market in its main performance components (excess returns vs upside and downside volatility comparison). To help us with tactical moves on secondary markets, if barriers of entrance in gateway markets are too high, then the UPR and GMM come to the rescue to help us identify the timing for each market.

4.2 Markets Analysis

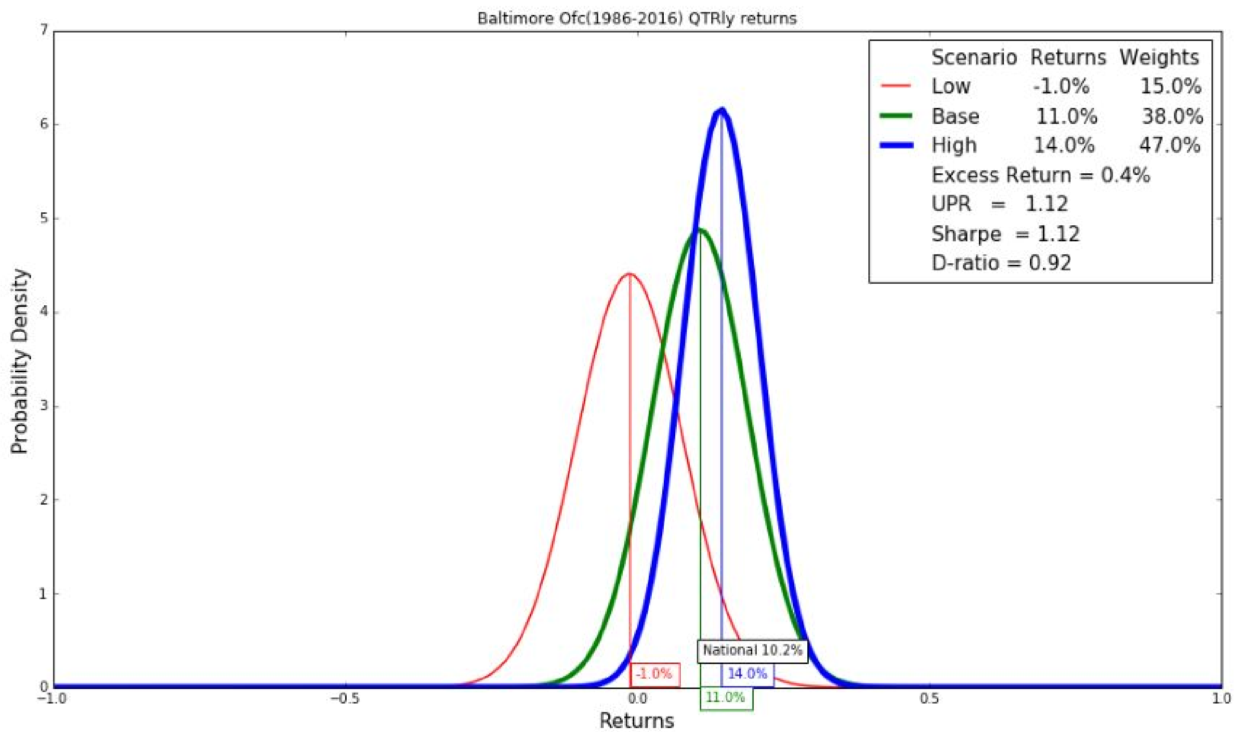
Below we present the results of fitting returns with a GMM distribution. We choose 3 components to decompose market returns into their business cycle components. The component with the higher mean will represent the peak of a cycle, the middle one represent the recovery phase of a cycle and the low mean component represents a trough. Each of these components has a weight assigned that can be interpreted as the time-weight of each cycle phase. Comparing the GMM model for Atlanta and Phoenix we notice that Atlanta returns are below national average level about 80% of the time. The information that this can provide to a fund manager is that if properties in Atlanta are reporting returns above national level, this is not likely to be a stable behavior and there is approximatively a 2 year window to sell-high in Atlanta. In Phoenix the situation is slightly different. Returns are below the national level only 50% of the time but the time-weight of far-above national average returns is 3% indicating that using Phoenix as a strategical investment is risky and delicate. New York, on the other hand, outperforms the market 74% of the time. Through GMM models and UPR ranking we can decompose risk and cycle phases for each market. It allows a fund manager to identify risks inherent to each market, it allows flexibility in how to measure risk and how to diversify a portfolio's risk. It allows static analysis, by evaluating a market in comparison to the national average by UPR, Excess Returns and the

D-ratio or to analyze markets collectively by building an optimal portfolio with respect to a more intuitive definition of risk as proposed by the lower second moment (denominator of the UPR and D-ratio). Given the current portfolio selection framework available, based on mean and standard deviation of returns, this offers tools to augment any existing investment strategy.

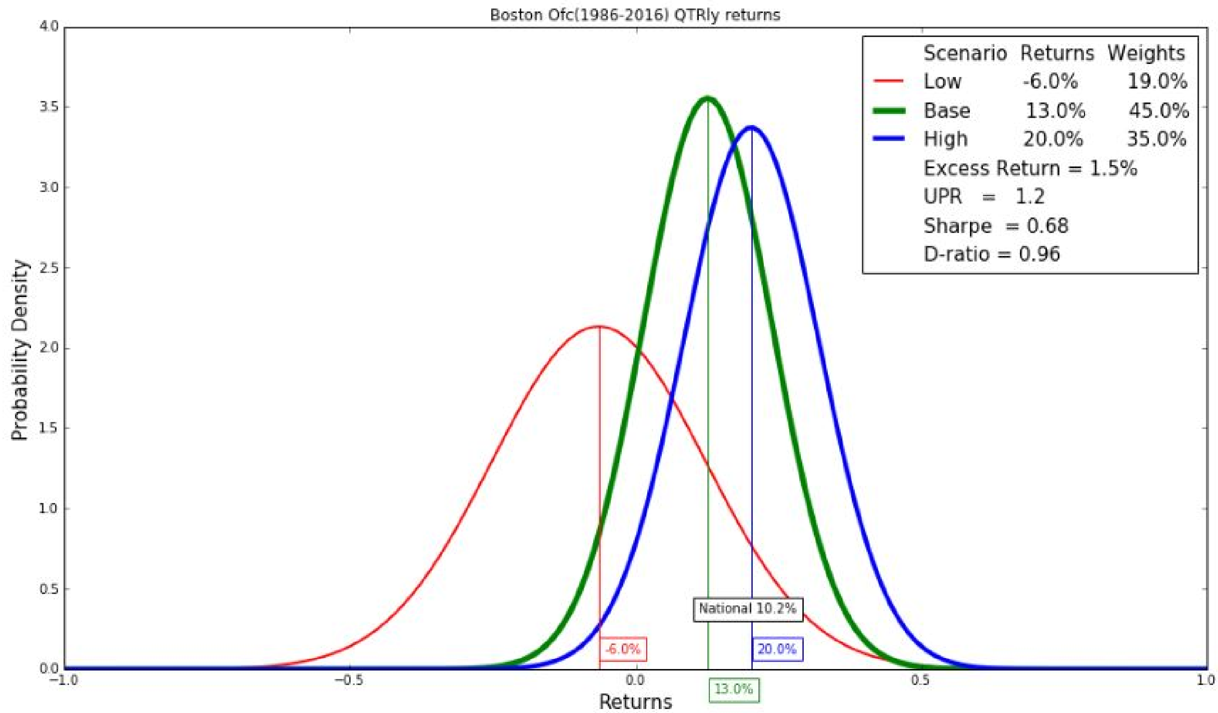


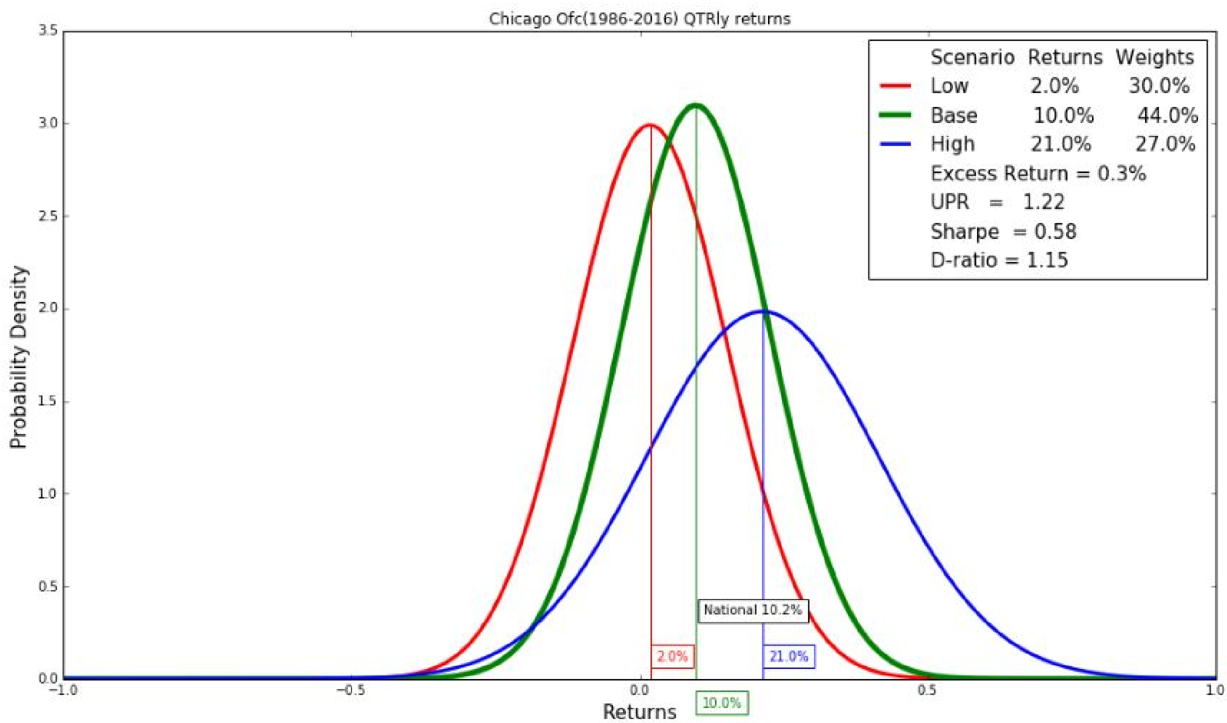


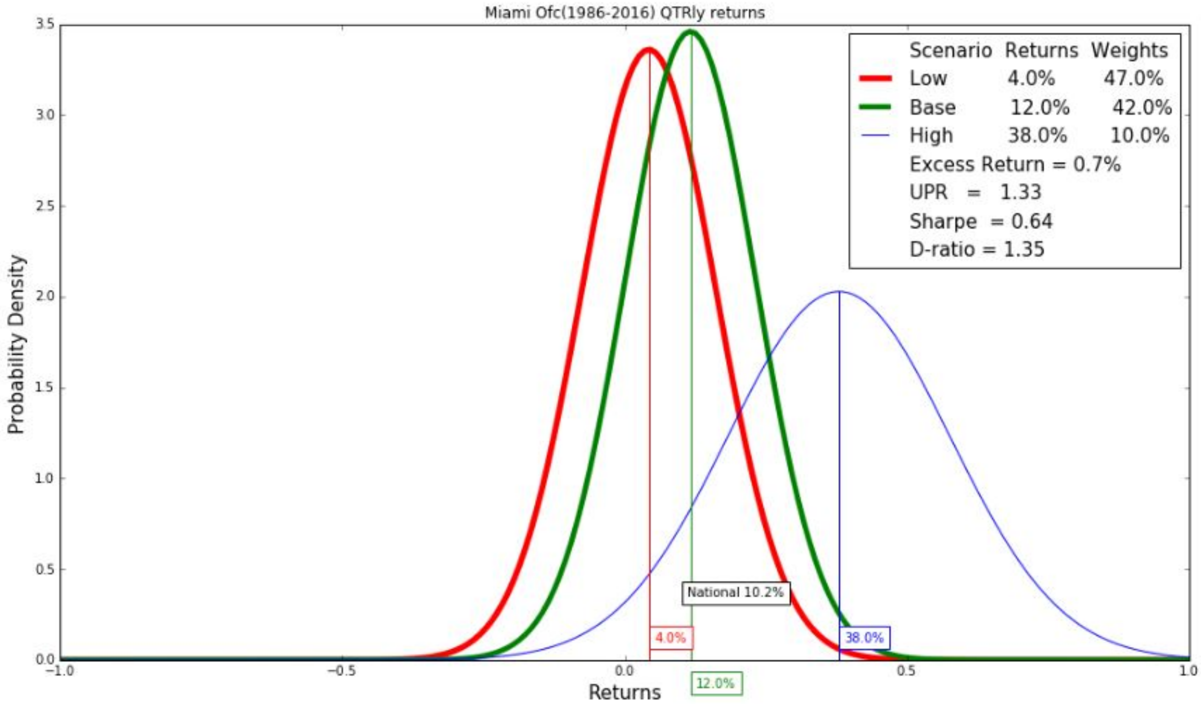
Austin Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low 4.0% 41.0%
Base 14.0% 38.0%
High 38.0% 21.0%
Excess Return = 4.9%
UPR = 2.11
Sharpe = 0.73
D-ratio = 1.87
National 10.2%
4.0%
14.0%
38.0%
Probability Density
Returns

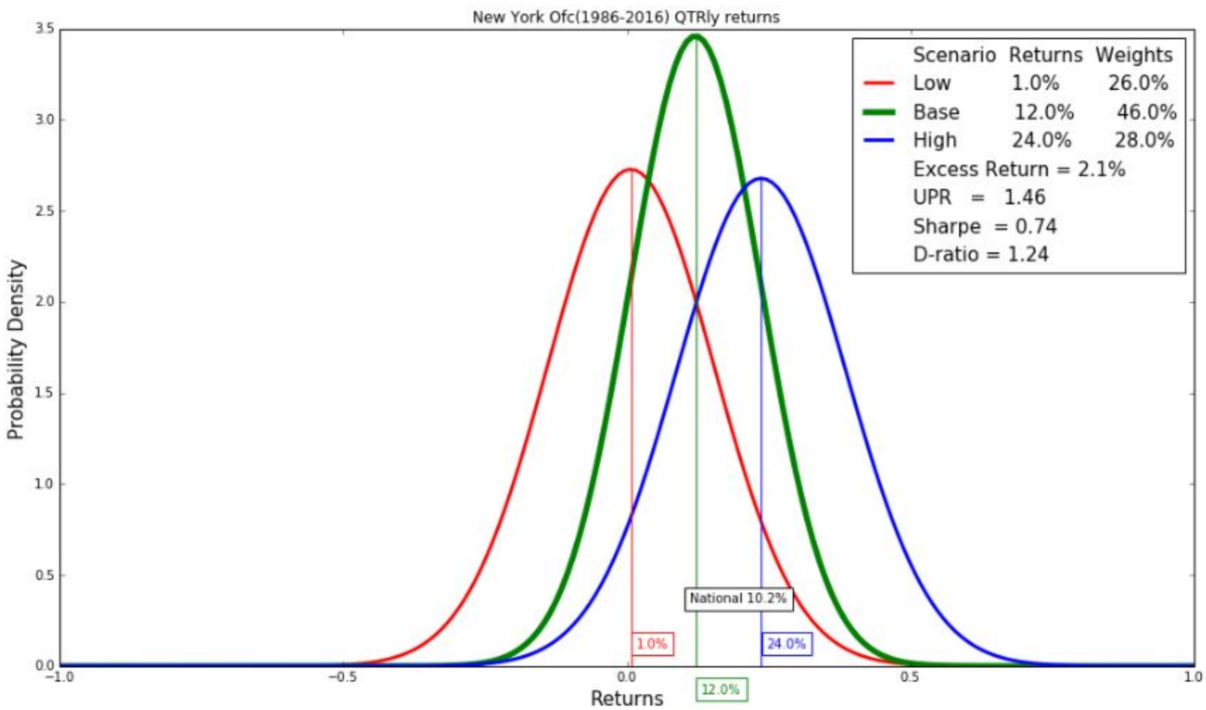


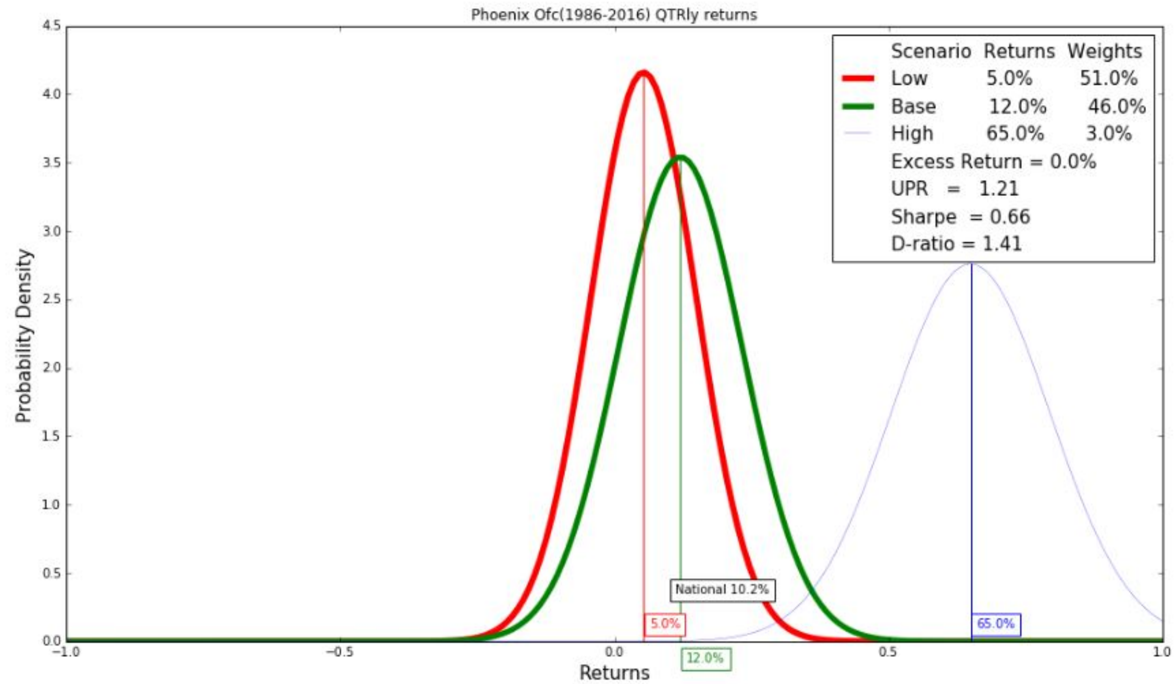
Baltimore Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low -1.0% 15.0%
Base 11.0% 38.0%
High 14.0% 47.0%
Excess Return = 0.4%
UPR = 1.12
Sharpe = 1.12
D-ratio = 0.92
National 10.2%
-1.0%
11.0%
14.0%
Probability Density
Returns


Boston Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low -6.0% 19.0%
Base 13.0% 45.0%
High 20.0% 35.0%
Excess Return = 1.5%
UPR = 1.2
Sharpe = 0.68
D-ratio = 0.96
National 10.2%
-6.0%
20.0%
13.0%
Probability Density
Returns

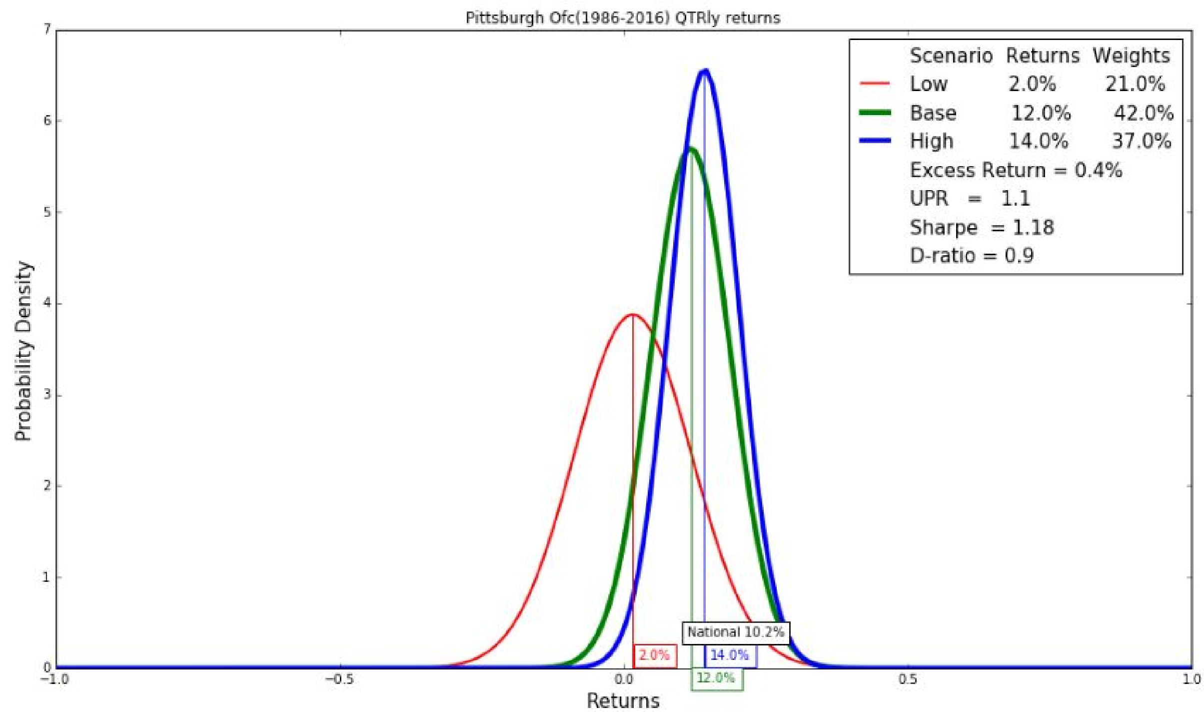

Chicago Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low 2.0% 30.0%
Base 10.0% 44.0%
High 21.0% 27.0%
Excess Return = 0.3%
UPR = 1.22
Sharpe = 0.58
D-ratio = 1.15
National 10.2%
2.0%
21.0%
10.0%
Probability Density
Returns


Miami Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low 4.0% 47.0%
Base 12.0% 42.0%
High 38.0% 10.0%
Excess Return = 0.7%
UPR = 1.33
Sharpe = 0.64
D-ratio = 1.35
National 10.2%
4.0%
38.0%
12.0%
Probability Density
Returns

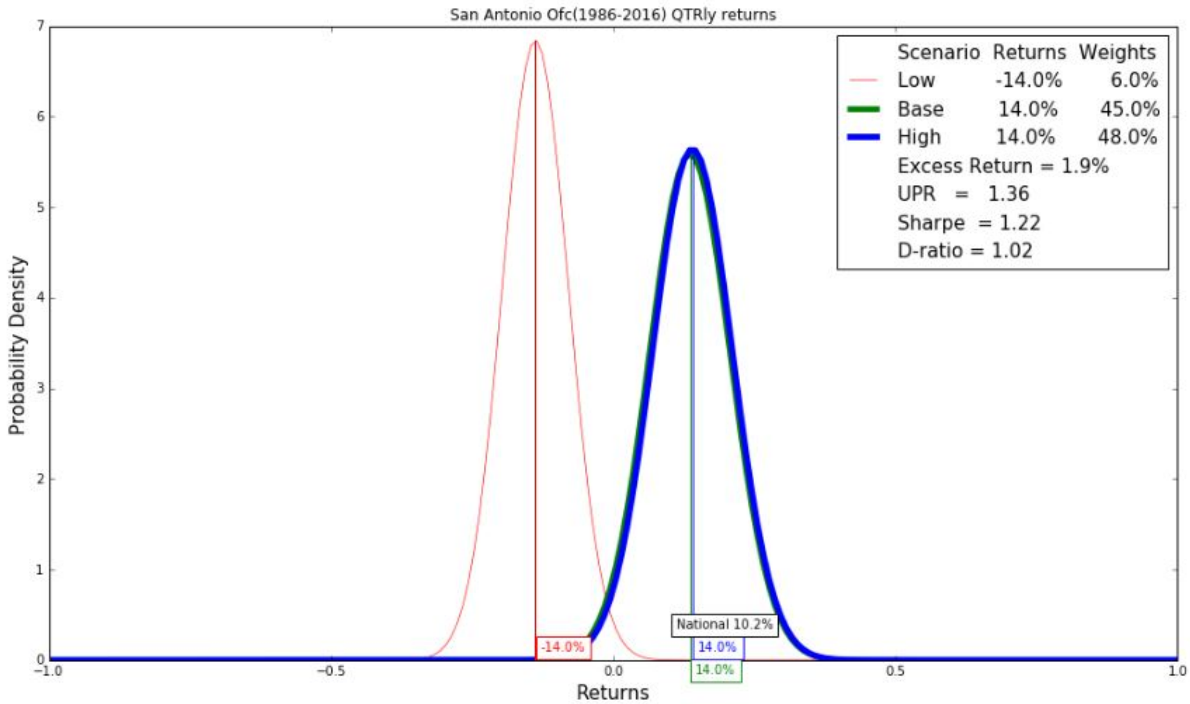

New York Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low 1.0% 26.0%
Base 12.0% 46.0%
High 24.0% 28.0%
Excess Return = 2.1%
UPR = 1.46
Sharpe = 0.74
D-ratio = 1.24
National 10.2%
1.0%
24.0%
12.0%
Probability Density
Returns

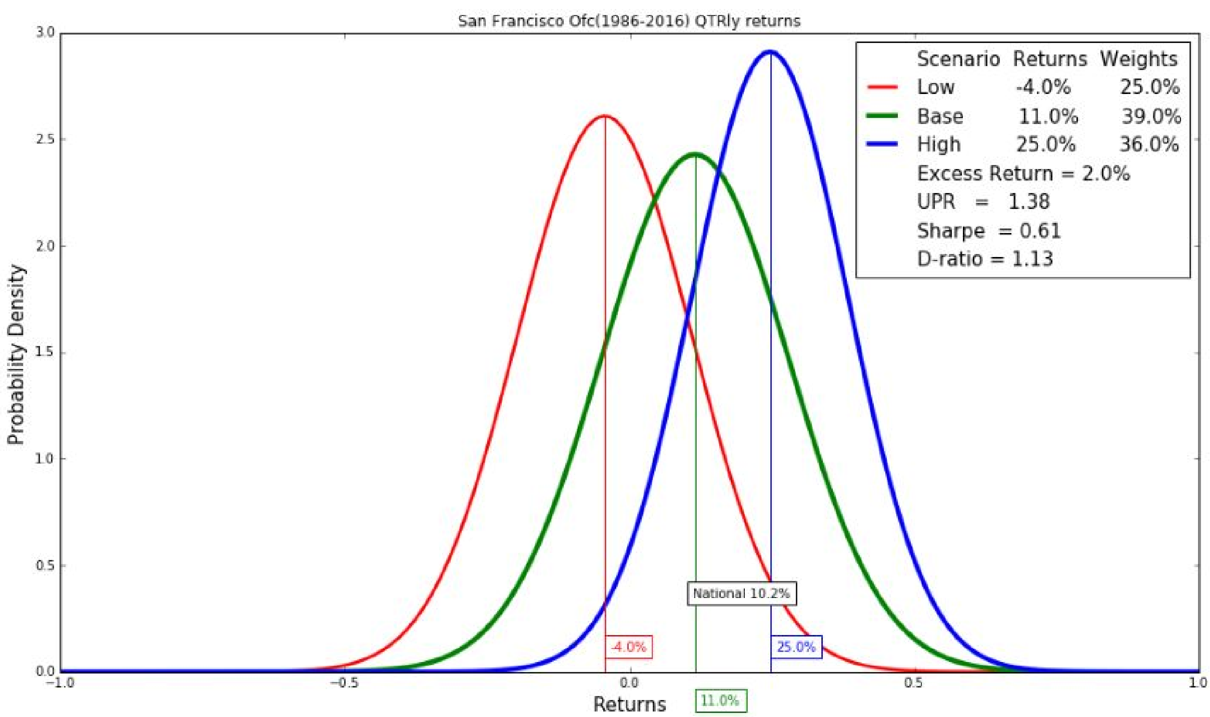


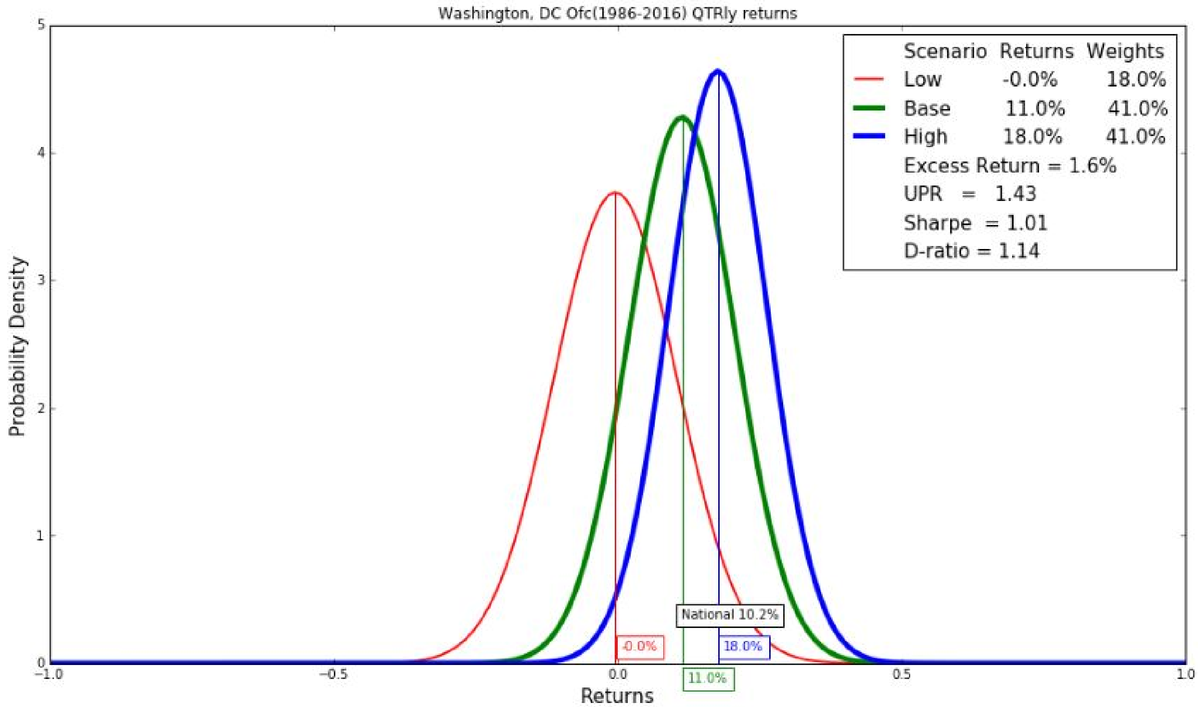
Phoenix Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low 5.0% 51.0%
Base 12.0% 46.0%
High 65.0% 3.0%
Excess Return = 0.0%
UPR = 1.21
Sharpe = 0.66
D-ratio = 1.41
National 10.2%
5.0%
12.0%
65.0%
Probability Density
Returns



Pittsburgh Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low 2.0% 21.0%
Base 12.0% 42.0%
High 14.0% 37.0%
Excess Return = 0.4%
UPR = 1.1
Sharpe = 1.18
D-ratio = 0.9
National 10.2%
2.0%
14.0%
12.0%
Probability Density
Returns


San Antonio Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low -14.0% 6.0%
Base 14.0% 45.0%
High 14.0% 48.0%
Excess Return = 1.9%
UPR = 1.36
Sharpe = 1.22
D-ratio = 1.02
Probability Density
Returns
National 10.2%
-14.0%
14.0%
14.0%


San Francisco Ofc(1986-2016) QTRly returns
Scenario Returns Weights
Low -4.0% 25.0%
Base 11.0% 39.0%
High 25.0% 36.0%
Excess Return = 2.0%
UPR = 1.38
Sharpe = 0.61
D-ratio = 1.13
Probability Density
Returns
National 10.2%
-4.0%
25.0%
11.0%


Washington, DC Ofc(1986-2016) QTRly returns
Probability Density
Returns
Scenario Returns Weights
Low -0.0% 18.0%
Base 11.0% 41.0%
High 18.0% 41.0%
Excess Return = 1.6%
UPR = 1.43
Sharpe = 1.01
D-ratio = 1.14
National 10.2%
-0.0%
11.0%
18.0%
−1.0
−0.5
0.0
0.5
1.0
0
1
2
3
4
5

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