2017-07-01

Teacher Response to Instances of Student Thinking During Whole Class Discussion

Rachel Marie Bernard
Brigham Young University

Teacher Response to Instances of Student Thinking

During Whole Class Discussion

Rachel Marie Gunn Bernard

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

Keith Rigby Leatham, Chair
Blake Ellis Peterson
Kate Ruth Johnson

Department of Mathematics Education

Brigham Young University

ABSTRACT

Teacher Response to Instances of Student Thinking
During Whole Class Discussion

Rachel Marie Gunn Bernard
Department of Mathematics Education, BYU
Master of Arts

While the use of student thinking to help build mathematical understandings in a classroom has been emphasized in best teaching practices, teachers still struggle with this practice and research still lacks a full understanding of how such learning can and should occur. To help understand this complex practice, I analyzed every instance of student thinking and every teacher response to that thinking during a high school geometry teacher's whole class discussion and used these codes as evidence of alignment or misalignment with principles of effective use of student mathematical thinking. I explored the teacher's practice both in small and large grains by considering each of her responses to student thinking, and then considered the larger practice through multiple teacher responses unified under a single topic or theme in the class discussion. From these codes, I moved to an even larger grain to consider how the teacher's practice in general aligned with the principles. These combined coding schemes proved effective in providing a lens to both view and make sense of the complex practice of teachers responding to student thinking.

I found that when responding to student thinking the teacher tended to not allow student thinking to be at the forefront of classroom discussion because of misinterpretation of the student thinking or only using the student thinking in a local sense to help advance the discussion as framed by the teacher's thinking. The results showed that allowing student thinking to be at the forefront of classroom discussion is one way to position students as legitimate mathematical thinkers, though this position can be weakened if the teacher makes a move to correct inaccurate or incorrect student thinking. Furthermore, when teachers respond to student thinking students are only able to be involved in sense making if the teacher turns the ideas back to the students in such a way that positions them to make sense of the mathematics. Finally, in order to allow students to collaborate a teacher must turn the mathematics to the students with time and space for them to meaningfully discuss the mathematics. I conclude that the teacher's practice that I analyzed is somewhat aligned with honoring student mathematical thinking and allowing student thinking to be at the forefront of class discussion. On the other hand, the teacher's practice was strongly misaligned with collaboration and sense making. In this teacher's class, then, students were rarely engaged in sense making or collaborating in their mathematical work.

Keywords: mathematics instruction, teaching methods, whole-class discussion (teaching technique), teacher response, classroom mathematics discourse, teachable moments

ACKNOWLEDGEMENTS

I would like to thank those who have supported me during my work on this massive project. First I would like to thank my advisor Keith, who put up with my ramblings both spoken and written and whose advice was invaluable for my completion of this thesis. I would like to thank my family and friends who supported me, and my husband Anand Bernard for his inspiration, his patience, and for helping me know when it was finally time to stop.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE: RATIONALE

While mathematical principles are universal, the conventions of mathematics itself are a human collection “of socially constructed mathematical objects and domains on the practices of mathematicians” (Dieterle, 2010, p. 314), and as such must be learned in a social environment in order to establish communication norms and the conventions as accepted by math communities at large. Many of these social experiences in which students construct mathematics take place during whole-class discussions. However, the effectiveness of student-teacher interactions in bringing about powerful learning through the use of student thinking varies greatly even among teachers with similar goals in mind (Stein, Smith, Henningsen, & Silver, 2000).

While the reasons for the variation in student thinking are diverse and varied, I consider that much of the difficulty in productive use of student thinking comes from two specific issues in math research and teaching. Firstly, we use “use” in very different, and often inconsistent, ways. Secondly, the practice of using student thinking is difficult and complex to enact. The practice of using student thinking is therefore difficult for researchers to make sense of and difficult for teachers to do well in their classrooms. Despite these difficulties and the complexity of using student thinking this problem is vital to tackle since using student thinking poorly can be detrimental to student learning.

Traditionally, the teacher’s role related to student thinking has been to assess student thinking and provide corrections where errors are identified, such as in the Initiate-Respond-Evaluate (Mehan, 1979) discourse pattern. Recent pushes for reform within classroom have emphasized alternative uses of student thinking, though there is considerable ambiguity and disagreement about the meaning of “use” (Leatham, Van Zoest, Stockero, & Peterson, 2014), and what purpose student thinking should serve within the classroom.

Because of this ambiguity of the meaning of *use,* research surrounding the use of student thinking varies with respect to both what *use* means and *how* it is explored. For instance, how teachers use student thinking could be considered with a variety of grain sizes, from immediate response following an instance of student thinking to a teacher's overall teaching practice over a school year. *Use* could be considered through exploration of the teacher's goals, or could be observed through what *actually* happens in classrooms—which may or may not align with a given teacher's goals. How teachers use student thinking has often been considered as an exploration of a teacher's general practice, with researchers identifying common goals, themes, or practices that shape an entire lesson (e.g., Stein et al., 2000). Although this expanse of research means that these aspects of *use* have been explored, the inclusive body of literature can lead to ambiguity on what the best practices for use of student thinking really are, as well as how teachers can improve on such practices. However, most research results in similar suggestions for improved teaching practices in a broad stroke, suggesting that teachers press for students to make sense of mathematics conceptually and to justify their ideas (NCTM, 2000; Doerr, 2006). However, such general statements of practice give little insight into *how* or *when* such practices should occur in the moment within a whole-class discussion.

In part due to the complexity of this student-teacher interaction, teachers often fail to use available student thinking in a way best conducive to building mathematical concepts around student ideas. Learning to orchestrate a discussion around student thinking is difficult (Peterson & Leatham, 2009; Sherin, 2002) and that when teachers respond inappropriately to student thinking there can be a resulting loss of cognitive demand and a concomitant loss of learning (Stein et al., 2000), as well as a missed opportunity for communicating an accurate representation of the nature of mathematics (Lampert, 1990).

These problems of ineffective use of student thinking on teachers' part may stem from the lack of clarity in the expectations for teachers in this student-teacher interaction. Although whole-class discussion has been encouraged as a central part of teaching in many classes (NCTM, 2000), little is actually known about what these whole-class discussions look like. Furthermore, how teachers use student thinking may vary depending on the nature of that thinking. There is little research describing the specifics of teachers' actual responses to a large variety of types of student thinking in the moment. A better understanding of the complex practice of orchestrating a classroom discussion around student thinking can help researchers recognize what elements of teacher practice around student thinking are more or less productive for effective classroom conversation. As we create a more clear and coherent view of best teaching practices around student thinking, we can help prepare future teachers and help current teachers improve their practice in order to be better aligned with best teaching practices.

In order to help fulfill this need, my research examines how teachers actually respond to instances of student thinking in the moment. By focusing on such a specific part of a teacher's "use" of student thinking, I will be able to consider many different aspects of the teacher's use of student thinking in close detail. However, because I am interested in practices specifically around whole-class discussion and those consistent with the teaching practices supported by NTCM (2000, 2007), I will also explore how these in-the-moment teacher's responses to student thinking plays out in the teacher's larger practice. In order to differentiate between more or less productive use of student thinking, as I consider both the small-scale and larger-scale use of student thinking in a teacher's classroom, I consider the extent to which a teacher's responses to student mathematical thinking align with principles of effective mathematics instruction.

CHAPTER TWO: THEORETICAL FRAMEWORK

To achieve my aim of considering the extent to which a teacher's responses to student mathematical thinking align with principles of effective mathematics instruction, I considered multiple aspects of the student thinking that arose during whole-class discussion and the teacher's responses to the student thinking. These included what the teacher was responding to, how they responded, and the effectiveness of their response. Considering these aspects allowed me to create a complex and highly detailed picture of a teacher's use of student thinking, and also allowed me to consider the quality of the teacher's response through how it allowed the student thinking to be used productively in the whole-class discussion.

My research thus required three different lenses that allowed me to get at different aspects of my purpose: one that allowed me to make sense of how the teacher responded to each instance of student thinking; a second that allowed me to view the extent to which a teacher's response aligned with specific principles for building new mathematical understandings around student ideas; and a third that allowed me to view and differentiate between types of student thinking to make sense of how a teacher might adjust her response by what was on the table. To get at the different elements involved in this exploration, my theoretical framework describes these lenses I employed to capture types of student thinking and the teacher response to that student thinking and then differentiate between these types.

Teacher Response to Student Thinking

There are many different aspects of a teacher's response to student thinking that can influence the effectiveness of the teacher's use of the student thinking at hand. For instance, a teacher's response may result in the repeating of the instance of student thinking, either by having herself repeat the thinking or having student do so. A teacher's response may result in an elaboration of the student ideas, or the evaluation of the ideas within an instance of student thinking as correct or incorrect. Another teacher response may result in the dismissal of the instance of student thinking entirely. The possible variety in how the teacher responds to student thinking is vast, and to make sense of teacher response I required a lens that allowed me to make sense of the multitude of different possible responses a teacher could respond to student thinking.

The use of student thinking in the classroom is complex, and there have been multiple frameworks developed to make sense of how teachers respond to student thinking. Before I introduce the framework I have chosen to make sense of teacher responses to student thinking, I first present an overview of frameworks that have been used in recent research with similar purpose to mine in order to justify that choice.

For over 50 years various frameworks have been created, adapted, and refined to better make sense of the interactions between student and teacher. As far back as 1966, Bellack, Kliebard, Hyman, and Smith (1966) created the Analyzing Teacher Moves Guide (ATM), which specifically focused on the moves that both students and teachers use during a whole-class discussion. In their work, Bellack et al. created a framework for categorizing both the teacher's and student's sides of the conversation, with coding schemes to differentiate between speaker, the move that teacher makes, the ideas within the move and how they relate to the lesson at hand, how those ideas are being discussed (such as by evaluating or interpreting given ideas), and the duration of those moves. The intended use of this coding was to identify patterns and trends within a class conversation rather than to promote any particular approach to classroom discourse, and was intended primarily for researchers to make sense of the discourse between teacher and student throughout a class period.

While Bellack et al. (1966) was more descriptive than prescriptive, Scherrer and Stein

(2013) adapted the ATM and reclassified the teacher moves in order to present their moves to teachers in order to help them have a more precise way of viewing and discussing the moves within their own classroom discourse. Although Scherrer and Stein kept a similar grouping as Bellack et al., which categorized moves at least partially by location within classroom interactions, they classified teacher moves more by the expected outcome of each move rather than how it related to what came before or after. For instance, some moves were grouped as those that served to introduce or reinitiate the class to a question or topic. Others were grouped by whether they served to elaborate or deepen students' understanding by furthering the discussion, or served to elicit information from the class. However, Scherrer and Stein did not utilize other codes from Bellack et al., such as who was speaking, whose ideas were central to the teacher's response, or the duration of the move. Instead, they redefined move so that it contains within it *who*, specifically, may perform that move, and contains more description about what ideas from the student thinking are used within this move as well.

This consideration of *what* the teacher response does and *who* is being invited to do it have been combined in other frameworks. Take Lineback (2015) for example, who created a framework to make sense of a specific type of teacher response called the *redirection*. In her framework, every teacher move Lineback identified focused specifically and exclusively on teacher responses to student thinking that requested a response or further action from the class or students. While Lineback's framework is a good lens for getting at the specific type of teacher responses she was interested in studying, it is limited for a more generalized consideration of *all* teacher responses to student thinking that I am interested in with my own research.

I now contrast these frameworks with the Teacher Response (TR) Coding Scheme (Peterson et al., 2017), which I utilized in order to view the entirety of a whole-class discussion and all teacher responses contained therein. The TR Coding Scheme is a lens that was created to differentiate between all teacher responses throughout the entirety of a whole-class discussion, rather than focusing on a specific type of teacher response, such as in Lineback (2015).

First, the TR Coding Scheme (Peterson et al., 2017) provides a lens for viewing what is likely to be accomplished from the teacher's response (such as the repetition of the student thinking) as identified through different moves. This identification of moves is similar to the previous frameworks discussed, but stripped of any other details of besides what is likely to be accomplished. For instance, a teacher's response may result in the student thinking being repeated, justified, elaborated on, dismissed, and so forth. This identification of what is likely to be accomplished through the teacher's response is key to being able to view what parts of the teacher's practice around student thinking is more or less effective. While the other frameworks mentioned in this section considered multiple moves by type, the TR Coding Scheme's moves were intended to be thorough and exhaustive for all possible moves in a teacher response.

Secondly, the TR Coding Scheme helps me to separately view who is being invited to act around the student thinking, whether it be the teacher, the same student who provided the student thinking, or another student or students. This aspect of the TR Coding Scheme is significant and unique, since it allows the combination of any move and any actor. Bellack et al. (1066) and Scherrer and Stein (2013) considered *who* is speaking for a given move, but through the TR Coding Scheme (Peterson et al., 2017) I not only was able to consider who was speaking, but I also considered who was being *invited* to participate for a given move. To make sense of this difference, consider this example from Scherrer and Stein (2013), where they give an example of how a teacher could respond in multiple ways to a student response:

> Teacher: How might we be able to classify this shape?
> Student: It is a quadrilateral. (p. 122)

Two of the provided possible teacher responses are “Does a quadrilateral have four sides?” and “That’s right. It has four sides.” (p. 122). The first of these responses is defined in their framework as a literal or bounded question for information, with the second being defined as *providing* information. These two moves are almost identical—providing or requesting factual information around the student thinking—with the only difference being the actor who is providing or being asked to provide that information: the students or the teacher. The TR Coding scheme’s ability to use the same move with multiple possible actors thus provides a way for me to consider all times where different actors are invited to consider student thinking, or the changes in the teacher practice that occur when the same move is used with different actors.

The identification of this actor is important because the productiveness of a teacher’s move depends greatly on who is doing the repeating, elaborating, and so forth. For instance, if a teacher’s response serves to justify the ideas from the instance of student thinking, the response to an instance of student thinking where a teacher assessed the correctness of an instance of student thinking would be a very different response than if the teacher were to invite the whole class to make that assessment. This consideration was especially relevant for later analysis so that I could view not only *what* is being done in whole-class discussions, but *who* is responsible for the actions that took place therein.

Finally, the TR Coding Scheme also provides a lens to make sense of how the ideas in the teacher response relate to the ideas in the student thinking. The TR Coding Scheme first provides a lens to consider how closely the words, actions, and ideas in the teacher’s response relate to the student thinking. Next, it provides a lens to consider how the mathematical ideas within the student thinking are used considering the potential for mathematical learning in the context of the student thinking. For instance, if a student’s comment opens up an opportunity for a teacher to discuss the types of units used in surface area versus volume, the TR Coding Scheme provides a lens to capture to what extent teacher's response takes advantage of that opportunity. The TR Coding Scheme is unique in the consideration of how the teacher's response takes advantage of the learning opportunities given the ideas within the student thinking, and provides me a lens where I can first detail out what is actually happening throughout all of the teacher responses in the class and then step back to notice the effects of these teacher responses.

So it is that while the aspects I view in order to classify how teachers respond to student thinking have been studied at one time or another, in these previous studies none have studied so many of the aspects of teacher responses that I do at the same time through the TR Coding Scheme. In my thesis I not only consider how the ideas within the teacher response are related to the student thinking such as in Bellack et al. (1966), but I also consider how the mathematical ideas within the student thinking are used in the teacher's pedagogical practice. My study not only notes the teacher's moves such as Scherrer and Stein (2013), but applies moves that are unconnected to actor so that I can capture a response where the teacher justifies a mathematical idea just as easily as one where the teacher requests the class for a justification. Furthermore, my *actor* code allows me to differentiate between whether the teacher directs a move towards the entire class compared to the same student who presented the student thinking or another student within the class. Finally, while my use of the TR Coding (Peterson et al., 2017) allows me the ability to identify and isolate all instances where the students are being asked to consider student thinking as in Lineback (2015), it also provides me the liberty to make sense of *all* teacher responses to student thinking—not just those that turn the ideas back to the students for consideration. The TR Coding Scheme thus provided me a tool that allowed me to get at part of my research purpose: to make sense of how teachers respond to student thinking. The teacher's move, the actor for that move, and how the student ideas are used in a teacher's response serves to create a lens that allowed me to create a robust picture of the variation and classifications of teacher responses.

Quality of Teacher Use of Student Thinking

With the lens for viewing teacher response is in place, my focus shifts to the next necessary element of my research purpose: to make sense of the productivity or quality of a teacher's practice of responding to student thinking. A consideration of what "quality" means around teacher's responding to student thinking is necessary, as Leatham, Van Zoest, Stockero, and Peterson (2014) noted that "use" of student thinking is ill-defined and can vary broadly in meaning and purpose.

Previous research has measured the quality of teacher use of student thinking in a variety of ways. Doerr (2006) differentiated between the quality of teachers' practices around student thinking by focusing on how different teachers attended to the meaning of the mathematics. She measured the quality of teacher's practice around the lens that teachers are responsible to "respond in ways that enable students to further develop their emerging models" (p. 267) of mathematics. Responses that were marked as more effective at this facilitation were viewed as better than those who impeded or cut off the growth of emerging models.

Stein, Smith, Henningsen, and Silver (2000) measured the quality of teacher practice by noting the change in cognitive demand throughout the implementation of a task. Teacher actions, including responses to instances of student thinking, were considered more productive if the action created or sustained a situation of high cognitive demand for the students.

Rather than focusing on cognitive demand or development of mathematical ideas, Anderson (2009) focused on how a teacher's responses to a student's thinking had psychological and social effects that ended up inviting or excluding him from the mathematical doers and learners in the classroom. Teacher responses to student thinking that included the student as part of the community of mathematical learners and doers were considered of higher quality than those that did not.

While useful for their respective studies, these three frameworks for viewing the quality of responding to student thinking did not align with my purpose. For one, each of these studies focused on a single specific aspect of a teacher's practice around the use of student thinking. Doerr's (2006) was purely with the aim of considering the development of models. Anderson's (2009) focused on how students were excluded or included in the mathematical community. Stein et al. (2000)'s was both too broad and too narrow for what I wanted to view, since it did not focus specifically on teacher responses to student thinking, and at the same time focused on cognitive demand. I wanted to be able to view multiple aspects of a teacher's response at once and note different effects of the teacher's response at the same time.

In order to view the quality of a teacher's practice around student thinking at the level of detail that I desired, I used "four core principles of quality mathematics instruction" as identified by Van Zoest et al. (2016, p. 1284). According to Van Zoest et al., these four core principles are: "mathematics is at the forefront, students are positioned as legitimate mathematical thinkers, students are engaged in sense-making, and students work collaboratively" (p. 1284).

These four core principles are consistent with the idea that students should be creators and authorities of mathematics built on their own ideas, which are drawn together through public consideration of student thinking. This conceptualization of quality mathematics instruction is consistent with the most effective teaching practices as identified in *Principles to Actions* (NCTM, 2014). These principles are also consistent with measures of instructional quality found in other literature. Consider Lineback (2015), who focused on multiple aspects of responding to student as a foundation for how she considers the quality of teacher responses. Lineback stated that for the most productive conversations to occur “students [must] have the freedom to express and debate *their own ideas*,” and the teacher must “allow those ideas to direct future class activity” (p. 420). Furthermore, Lineback argued that most productive use of student thinking requires “time and support for *the community to further consider and develop that thinking*” (p. 420), which implies a level of sense making that brings the whole class together to make sense of a mathematical idea. Lineback’s framework was thus founded on similar principles as the four core principles from Van Zoest et al. (2016), even if they were not clearly separated and defined. This consistency in two independent studies in defining these key elements of effective teacher practice around the use of student thinking adds validity to both frameworks.

Through Van Zoest et al.’s (2016) four core principles, I was able to view how any teacher response was consistent or inconsistent to four distinct and specific principles of best teaching practices. Van Zoest et al. noted that these four principles could be violated by specific teacher reactions. Respective to the principles, teachers could abandon student mathematics by allowing their own ideas to take the lead in class discussions, marginalize the students’ contributions, relegate students to workers of trivial mathematics such as computations, and restrict students rather than encourage them to be involved and wrestle with each other’s thinking. On the other hand, teachers could respond to student thinking so they were honoring one, multiple, or all of these principles at the same time.

It should be noted that temporary misalignment of a teacher’s response with the four core principles of quality mathematics instruction is not “bad” and does *not* signify a teaching practice that is overall misaligned with the core principles. At times, temporary misalignment with one or more of the core principles is appropriate in order to create space for more productive avenues of conversation.

In considering whether a teacher's response might be consistent with a practice around these four core principles, I consider which teacher responses align or misalign with these core principles, and how specific moves in the instance can either assist or impede a broader alignment in the teacher's overall practice. While the effectiveness of a teacher's use of student thinking may not be evident in a single move, by considering how multiple student-teacher interactions are consistent or inconsistent with these core principles I am able to recognize how the general teaching practices are consistent or inconsistent with quality mathematics instruction.

In short, the four core principles of quality mathematics instruction provide a lens through which I can unpack a teacher's practice of responding to student thinking. Specifically, this lens allows me to consider how and to what extent a teacher's practice adjusts and changes as she responds to the different student thinking that occurs during whole-class thinking. By considering the frequency and patterns of alignment with the core principles in teacher responses throughout the class I am able to draw conclusions about the teacher's larger practice. In the next section I describe a framework for examining when the teacher's practice *should* be consistent with these four principles.

The MOST Analytic Framework

The four core principles of quality mathematics instruction (Van Zoest et al., 2016) were identified as principles that a teacher's practice should be aligned with in response to certain high-leverage instances, which Leatham et al. (2015) called *mathematically significant pedagogical opportunities to build on student thinking* (MOSTs). Van Zoest et al.'s theory was that when teachers were engaged in a practice that was aligned with all four principles as they responded both locally and generally to one of these high-leverage instances of student thinking, they were engaged in *building*, which was the most effective use of such an instance. In this study I use the MOST construct to identify times when the teacher's practice *should* be aligned with all four core principles. Being able to identify MOSTs thus affords me a method of recognizing more or less effective teacher responses to student thinking dependent on what student thinking the teacher is responding to, which is a critical aspect of productive response (Stein et al., 2000).

The MOST analytic framework requires that each instance of student thinking be analyzed to determine whether it (1) is inferable, (2) has a mathematical point that could be taught towards, (3) is appropriate to the level of learning of the students in the classroom, (4) is central to the goals of the class, course, or mathematics as a whole, (5) creates or sustains intellectual need, and (6) occurs at a time where the teacher would want to and be able to pursue that student idea. If an instance of student thinking fails on one of these requirements, then it is a non-MOST instance of student thinking. However, if an instance fulfills all of these considerations as described, then it would be considered a MOST.

Research Questions

While other studies have measured the quality of teacher responses in various ways, and Lineback (2015) especially has drawn close to the same roots that my measurements have sprouted from, none have approached the measurement of teacher responses with the level of inclusion, flexibility, and fine-grained detail that mine does. Through my combination of frameworks, I am able to draw conclusions about the entire collection of teacher responses to student thinking during whole-class discussions where previously research may have only focused on specific types of teacher response. I am also able to explore the effects of minute changes in teacher responses, and from there make conclusions about multiple aspects of those teacher responses and how they facilitate or impede the development of mathematical understandings around student ideas. With these three lenses defined, I clarify my primary research question as we move forward: To what extent does a teacher's practice align with four core principles of quality mathematics instruction as defined by Van Zoest et al. (2016)? As I answer this question, I will also address the following related questions:

- What types of teacher responses are associated with alignment or misalignment with each of the core principles?
- How are teacher responses to individual instances of student thinking related to the teacher's larger practice?
- When is the teacher's practice simultaneously aligned with all four core principles?
- Is the teacher's practice aligned when it should be (such as around a MOST)?

CHAPTER THREE: LITERATURE REVIEW

With my framework defined and my research question stated, I now discuss results of related research around how teachers respond to student thinking and the quality of those responses. Some research has considered teacher's use of student thinking by looking at the teacher's response directly after the student thinking, while others have considered the teacher's larger practice around responding to student thinking over multiple responses. In my research I am interested in both aspects of how a teacher uses student thinking, and so I arrange my literature review with consideration to these two grain sizes.

Teachers' Immediate Responses to Student Thinking

Studies that explored the quality of immediate teacher response have focused on the quality of different moves, actors, and uses of the ideas from the student thinking. By considering these different aspects, previous research has been able to draw conclusions on some of the more effective ways of responding to student thinking in the moment.

Lineback (2015) was able to differentiate between more or less effective teaching practices between multiple teachers by applying her framework, which focused on teacher responses to student thinking. She found that teacher responses that center on having the students in the class justify, elaborate on, or personally consider the student thinking are among the most effective teacher responses around that student thinking. In contrast, she noted that instances where the teacher's response serves to merely use student thinking as a steppingstone to further the teacher's own ideas, or define a term or phrase in the student's thinking, is a minimal way of using the student thinking and was therefore less effective.

Although Doerr (2006) considered the effect of teacher responses on how students were supported or hindered in their development of mathematical models, rather than effective student-to-student discourse like Lineback (2015), her results nonetheless overlapped with respect to the most effective in-the-moment teacher moves. Doerr generally concluded that pushing students to justify or elaborate on their ideas and on each other's ideas helped them critique, adapt, and polish their models. Furthermore, she found that when the teacher focused more on making sense of student thinking she was more likely to help that student thinking develop rather than replace it with her own.

Similar results were reported by Stein et al. (2000), who considered teacher responses to student thinking in combination with task implementation and cognitive demand. Although most of their findings focused on aspects of the task, they also identified teacher moves that resulted in higher or lower cognitive demand. They noted that teacher moves that focused on memorization or on procedures without connections were low-level demands, and loss of cognitive demand was often associated with a shift from sense making to a focus on correctness of an answer. On the other hand, "sustained press for justification" (p. 16), a focus on conceptual learning, and "doing mathematics" (complex problem solving towards the fulfillment of a task) were linked to high cognitive demand. One can deduce from this work that teacher responses to student thinking that focus on procedures, memorized facts, and correctness of answers are less effective than those that focus on problem solving, justification, and conceptual understandings.

Finally, although Anderson's (2009) research was not intended to differentiate between different types of teacher responses, she nonetheless identified a particular type of teacher response that had damaging effects to students. In her study, the teacher's failure to respond to a particular student, Nate, in such a way that his ideas were held up as legitimate, resulted in denying Nate the access to the same opportunities to learn as his peers and excluded him from the mathematical community of learners.

Across all of these studies we find that teacher responses that focus on justification, elaboration, and community consideration of each other's thinking are more effective teaching practices, even with variation in overall goals. Furthermore, these studies found that a focus on procedure, answers, or the teacher's ideas over the student's ideas led to a loss of learning opportunity. Upon review of this literature, I expected that my results would reflect these same findings and collection of most-effective moves. At the same time, I expected to be able to be more specific with the types of responses that were more or less effective by depending not just on the moves, but also on the actor, as well as on the extent to which the move uses the ideas from the student thinking both in the moment and in the teacher's larger practice.

However, it is absurd to think that just because justification can be an effective move that it should be used at any time during a lesson. My consideration of types of student thinking will contribute to understanding *when* these moves are most effective. Peterson and Leatham (2009) noted how student teachers struggled to identify and react appropriately to student thinking in the moment. They noted that student teachers either lacked experience to recognize teachable moments in the student comments, or were unsure what to do once they identified a teachable moment. Through my research I thus intended to not only identify elements of effective teacher responses, but also to recognize when these effective responses should be enacted.

Teachers' Practice Responding to Student Thinking

In my research, I first considered the quality teacher responses to student thinking in the moment and then considered multiple teacher responses together to draw conclusions about the teacher's general practice around student thinking. Previous research has done similarly, and I share some of their findings here.

First of all, while Anderson's (2009) research focused on the effect of a teacher responding to a single student in the moment, she also drew conclusions on the overall effect of the class experience on that student, and then extrapolate to predict how such a practice might affect that student's permanent identity. Since the teacher did not respond to Nate's thinking in a way that honored him as a legitimate mathematical thinker, Anderson noted that as the class progressed Nate was excluded more and more from mathematical conversations. His motivation and attempts to learn decreased. Anderson concluded that the teacher's in-the-moment responses to Nate's work would lead to a long-term identity formation of Nate as an unmotivated and incapable student. My results will likewise lead to a discussion of the overall effects in mathematics learning due to in-the-moment teacher responses to student thinking.

While Bellack et al.'s (1966) research on class discourse was more descriptive that prescriptive, from their analysis of multiple classrooms they were able to note general patterns in teacher-student interactions that consistently appeared over multiple classrooms. They found that teachers tended to take responsibility for structuring, soliciting, and reacting to student thinking, with students being relegated to the confining and repetitive role of responding to teachers' soliciting, with little attention to evaluation or deeper consideration of ideas. Bellack et al. found that teacher responses to student thinking in these interactions consisted of the teachers reacting to student thinking or asking a following question for students to respond to in a mostly rote manner. With such a strict and unvarying part for the teacher to play in responding to this student thinking, there was rarely, if ever, teacher responses with requests for students to consider each other's thinking, be involved in sense making, or collaborate around the mathematics.

Through their observations of multiple teachers, Scherrer and Stein (2013) identified four types of classroom discourse. First, they described a type of classroom conversation where the teacher began with an open, high-level question and then narrowed the conversation to a specific point or topic. Second, they described a type of class discussion where the whole discussion began and continued with open-ended questions, with the teacher never narrowing to a point. Third, they described a classroom where the teacher began with a bounded or limited question and then "expand[ed] the intellectual space by asking more open-ended questions" (p. 118). Finally, they described a type of discussion where the teacher again began with a bounded or limited question but never allowed the conversation to open up, keeping it restrained with bounded or limited questions. While not focusing specifically on individual teacher responses in the moment, these descriptions detail certain ways that teachers can respond to student thinking in the larger picture. For instance, a teacher's practice that matched this last model as she used student thinking might generally use student thinking as steps towards their goals, without opening the ideas up for further development or direction from the students. On the other hand, the second model could represent at teacher's practice around student thinking as one that might toss student thinking to the class in a general way, but never narrow in on the problematic aspect of the mathematics.

I expected that through my own analysis of a teacher's practice around student thinking I would be able to identify patterns of how the teacher either opened up or limited the involvement of the students with the mathematics. My results likewise will be able to describe sections of whole-class discussion by what happens at the beginning, as well as how that conversation progresses and ends. One difference is that while Scherrer and Stein (2013) focus more on how a discussion begins and progresses, my results will have a strong focus on how sections of discussion end, since it is by following these discussions through to their conclusion that I can conclude whether the entire discussion was more or less productive.

Finally, Lineback (2015) found that a teacher's effectiveness in their redirections was reflected through the frequency and duration of higher-level responsiveness codes throughout a lesson. Similarly, in my own research I consider the frequency and duration of alignment with the different principles of effective instruction as adapted from Van Zoest et al. (2016) over the course of the classroom discussion to draw conclusions about the teacher's overall effectiveness in responding to student thinking. In this way, both Lineback's and my research draw multiple immediate teacher responses together to create a larger picture of the teacher's practice and be able to measure the effectiveness of the teacher's practice around student thinking from this.

In this larger scale, I thus expected my own results to add to the understanding of classroom discussion and orchestration of classroom discussion around student thinking. Particularly, I expect to add to an understanding of how sections of classroom discussion begin, continue, and end, along with what elements of those sections are more or less effective as defined in my framework.

CHAPTER FOUR: METHODS

In order to answer my research question, I selected a video of a mathematics lesson from a collection of videos of over a dozen different teachers that had previously been recorded for the exploration of teacher's use of student thinking in the classroom. I chose the specific teacher I did, who will be referred to by the pseudonym Mrs. Hunt, because she appeared as one of the best from the collection of teachers when it came attending to and using student thinking to build a conceptual understanding of the mathematics. Although Mrs. Hunt was only at the end of her first year of teaching and taught at a low socio-economic school, she was able to draw out and address varied student thinking in a variety of ways. However, while she was an exemplar from the collection of teachers, her use of student thinking was sometimes more effective and sometimes less so. This situation was ideal, since I wanted to capture teacher responses that varied from more to less effective but also to gain a better understanding of better practices around using student thinking. The video of Mrs. Hunt's classroom recorded a full hour and a half session of a high school geometry class in a low socio-economic area. A summary of the lesson follows.

Lesson Summary

For the reader to better understand the remainder of this thesis, I present a summary of the analyzed lesson. I first discuss the learning goals for the class, including what the students were supposed to understand, and how they were to learn it. I then describe the tasks and individual problems the students worked on as they played out in the lesson.

The purpose of this geometry lesson was to have students discover and create the formulas for finding the surface area and the volume of a sphere. They were to create the formula for surface area of a sphere after discovering how a sphere's surface area could cover four great circles of that sphere. The formula of a pyramid was to be created by considering how a sphere's volume could be derived by considering the sphere's volume as a sum of infinitely many tiny pyramids with vertices at the center of the sphere and the bases on the surface of the sphere.

In order to get at these learning goals, Mrs. Hunt had the students begin with a warm up activity involving finding the volume of a pyramid and the area of a circle. Mrs. Hunt then went over the objectives for the day, as detailed in the previous paragraph, before presenting the students the definition of a great circle of a sphere. She then gave the students oranges and had them trace out the great circles of their oranges on a piece of paper. They then predicted how many great circles their orange peel would fill, and then peeled their orange to test their predictions. When they returned to whole-class discussion, the students shared their findings and the teacher guided them through constructing the formula for the surface area of any sphere from their results. They then practiced finding the surface area of a sphere of radius 7.2 and a complex 3D shape that consisted of a cube with side length of 3 with a hemisphere on one of its faces. For these practice problems Mrs. Hunt had the students work at their desks before choosing students to come up to the board and share their solutions. During this sharing she and the class discussed problematic concepts that arose, such as what dimension of units were used for surface area. When the class could not collectively find the correct solution for the complex 3D shape, Mrs. Hunt went to the board and walked the class through correcting a student's incorrect solution.

Mrs. Hunt then shifted the conversation to the volume of a sphere. A student shared the formula, and then as a class the teacher walked the students through the derivation of the formula as a sum of many small pyramids with their apexes at the center of the sphere and their bases on the surface. This conversation was carried out with Mrs. Hunt at the board, with the students following the teacher step-by-step through the derivation. Mrs. Hunt then gave the students problems to practice using the formula for volume of a sphere. These three problems consisted of students finding the volume of a sphere of radius 6, the volume of a hemisphere with radius 3, and the volume of a cylinder with a height of 9 and a radius of 4 with a hemisphere taken out of the top. Again, individual students were selected to share their solutions at the board, and again they were discussed along with the teacher filling in to help when they struggled or had questions. The class ended with the students being given a final problem where they were to find the surface area and volume of cylinder of height 8 with a hemisphere on top with a radius of 5.

Data Analysis

My analysis of the video of Mrs. Hunt's classroom consisted of me applying the frameworks I introduced in my theoretical framework involving the MOST Analytic Framework (Leatham et al., 2015), the TR Coding framework (Peterson et al., 2017), and the four core principles of quality mathematics instruction (Van Zoest et al., 2016). In this section, I detail out the actual application of these three frameworks as I applied them in three phases of my data analysis. I prepared for the application of these frameworks by coding specific aspects of her class in Studiocode (SportsTec, 1997-2017). I first coded what types of activities the class was engaged in, such as individual or group work versus whole-class discussion. A research group then marked every instance of student thinking and teacher response to student thinking and transcribed the dialogue of each.

As I selected my video from a collection that may have already been coded in some of the frameworks that I use, I went through what had already been done with that coding and made adjustments or additions as needed. Validity and reliability of my results were also checked through conversations and discussions with my advisor surrounding specific instances, especially more difficult or complicated ones.

In applying the three frameworks, I began with the identification of MOSTs and non-MOSTs. Next, I identified every instance of teacher response, as well as the duration of take-up (the duration of time where the class discussion is influenced by the instance of student thinking) of the instance immediately following the instance of student thinking. I then coded each of these teacher responses using the TR Codebook (Peterson et al. 2017). After this, I went through each teacher response and used evidence from the context, the MOST codes, and TR codes to determine the extent to which the teacher's response was aligned with each of the four core principles of quality mathematics instruction (Van Zoest et al., 2016). Once the principle analysis was done for every individual teacher response, I repeated this coding for how the teacher's practice aligned with each core principle over clusters of responses as they were unified by a theme or topic of the discussion. Finally, I used the combination of the alignment at the cluster stage to draw conclusions about how Mrs. Hunt's overall practice aligned with the four core principles throughout her whole class discussion.

Student Mathematical Thinking

The first phase of my data analysis involved analyzing student thinking instance-by-instance and separating them by type according to the MOST analytic framework (Leatham et al., 2015). To begin this analysis, I went through the entire recording of Mrs. Hunt's class and identified when the class was involved in whole-class discussion. I then proceeded to identify and transcribe all instances of student thinking, which include any significant student action (verbal or otherwise) that was made public in the whole-class discussion. I then analyzed each instance according to the MOST Analytic Framework (see *Figure 1*). If an instance of student thinking fails to meet any of the six criteria, then analysis ends and it is thus a non-MOST.

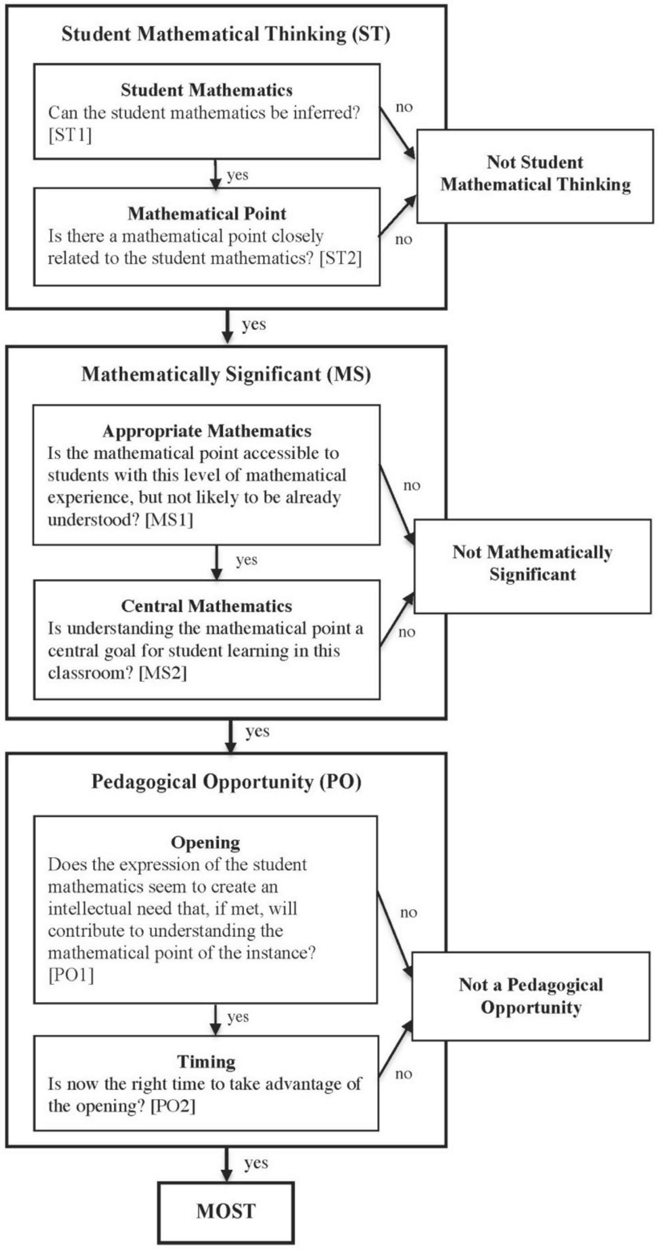

Figure 1. MOST Analytic Framework flowchart. Reprinted from Leatham et al., 2015, p. 103.

There are two criteria an instance must satisfy in order to be considered Student Mathematical Thinking. First, one must infer the "student mathematics." As I coded Mrs. Hunt's class, if there was more than one possible interpretation of an instance of student thinking, or no definite interpretation at all, then the meaning of the student thinking was considered unable to be inferred and the instance was therefore not a MOST. Secondly, to differentiate between

MOSTs and non-MOSTs I considered whether there was a main mathematical idea that could be learned through the consideration of the instance of student thinking, or whether the instance of student thinking had a "mathematical point." If the instance of student thinking did not have a mathematical point, then it was a non-MOST instance of student thinking.

There are two more criteria an instance must fulfill in order to be considered Mathematically Significant. First, the mathematical point of the instance must be appropriate in difficulty given the context. I determined whether the mathematical point of the student thinking was either too easy or too difficult considering the current level of understanding of the class, or whether the mathematics was at an appropriate level for students to be able to struggle with and gain new understanding. If the mathematical point of the instance was not appropriate, then it wound not be considered Mathematically Significant and would thus be considered a non-MOST. Second, the mathematical point of the instance of student thinking must be central to the learning goals of the class. In considering whether the mathematical point from the instance of student thinking was central to the learning goals, I determined whether the mathematical point of the instance aligned with the mathematical goals for the day's lesson, the unit, or key mathematical concepts. The farther away the student's mathematics was from the lesson's goal, the more important the mathematics needed to be to be considered as central to the class goals (Leatham et al., 2015). If the mathematical point in an instance of student thinking was not close enough to the learning goals of the mathematics class, then it was not coded as a MOST.

Finally, there are two criteria for an instance of student thinking to be considered a pedagogical opportunity. The first of these is whether an instance of student thinking was compelling so as to press students to have a desire to make sense of some aspect of the student thinking at hand. If this intellectual need was not present around the instance of student thinking, then the instance was not coded a MOST. The last consideration was whether the instance occurred at a time where it would be appropriate for the teacher to pursue it. For instance, if an instance of student thinking interrupted another student, or had a mathematical point that would fit better at a different point in the lesson or unit, then the timing of the student thinking was not ideal for the teacher to pursue. If the timing was not appropriate for the teacher to pursue the ideas of the student thinking, then the instance was not coded as a MOST.

If an instance satisfied all of these criteria as described, then it was coded as a MOST. Take the following instance as an example. After students derived the formula for surface area of a sphere, one student asked, "What would the units be? Feet squared? Or cubed?" (I 49[1]) In coding this instance I first would identify the student thinking to be, "Would the units for the surface area be feet squared or cubed?" Secondly, I would identify the mathematical point that the teacher can teach towards given this instance of student thinking: "3-dimensional shapes have volume, which is a 3-dimensional measure with cubic units; they also have a 2-dimensional measure of surface area measured in square units." Next, I would note that the mathematical point is at an appropriate level for the students, as is evidenced by the fact that the student is asking and it is something students often struggle with when dealing with surface area of three-dimensional objects. I would also check that this mathematical point is central to the goal of the class, and in this case it is clearly relevant to the mathematics at hand and mathematics in general. The last two checks for opening and timing come next. For this instance there is definitely intellectual need, with one student feeling the need to ask revealing the need and uncertainty, and the student thinking comes at a time when it would be appropriate to be

[1] Throughout this thesis individual instances of student thinking will be identified in the format "I #," with the number describing the sequence in which the instances occurred. Similarly, clusters will be labeled as cluster I #-#, with the numbers representing the instances of student thinking that fall within the cluster.

discussed. Because it fulfills all elements of the MOST Analytic Framework, I could thus identify this instance as a MOST.

Teacher Responses

Once the analysis of student thinking was completed for each instance, I moved on to the second phase of my data analysis—the analysis of the teacher response. Through applying the TR coding framework (Peterson et al., 2015), I analyzed the teacher responses for (a) what the teacher was allowing or requesting to be done around the instance of student thinking, (b) who was being asked to consider the instance of student thinking in the teacher response (c) how the student who contributed the instance of student thinking was likely to recognize their words and ideas in the teacher's response, and (d) how the teacher response used the *mathematical point* from the instance of student thinking (Peterson et al., 2017). To capture these aspects of the teacher response, I focused on 5 codes from the TR coding: *move*, *actor, student actions*, *SM Ideas*, *Move MU-MP match, and Practice MU-MP match* (Peterson et al., 2015).

The *move* corresponds to the type of teacher response that follows directly after the instance of student thinking, where we can infer what "instructional intent" (Peterson et al., 2017) the move performs. The descriptions for each move are found in Table 1. There may be times when the teacher performs two moves contiguously, such as repeating the student comment before asking the student to elaborate on his or her idea. In these cases I coded each of these moves separately, with their own separate actors, moves, and other codes that follow.

Table 1

Descriptions of Teacher Moves (Peterson et al., 2017)

Move	Description
Dismiss	Teacher rejects the student thinking, not making it an object of consideration.
Adjourn	Teacher does not make the student thinking the object of consideration, with the implication that the student thinking may be returned to later in the lesson.
Validate	Teacher affirms student participation.
Monitor	Teacher elicits students' self-assessment of their understanding.
Evaluate	Teacher provides or asks for an assessment of the student thinking.
Correct	Teacher asks for or describes a correction to the student thinking.
Repeat	Teacher repeats or rephrases the instance of student thinking.
Clarify	Teacher provides or asks student to clarify a specific part of their thinking.
Literal	The teacher asks for or provides factual information that can be answered briefly and without extension into elaboration.
Adapt	Teacher uses student thinking in a way that is different from the student's original idea.
Elaborate	Teacher expands on or asks for an elaboration of the student thinking.
Connect	Teacher asks for or provides a connection between different student ideas, representations, concepts, solutions, etc.
Collect	Teacher moves to gather more solutions, strategies, or ideas.
Justify	Teacher provides or asks for a justification for the student thinking.
Allow	Teacher allows student-to-student discourse.

The identification of the *actor* takes note of whether the *teacher,* the *same student(s), other student(s),* or the *whole class* is set up to publicly consider the student thinking through the teacher response to the student thinking. For instance, if a teacher repeats a student comment and then clarifies what she thinks the student means for the rest of the class, the actor would be the *teacher*. On the other hand, if the teacher responds by asking the class, "What do you think Ben means by that? Can someone put that in their own words?" then the actor would be the *whole class*, as the whole class is being asked to publicly consider the student thinking.

Next, *student actions* notes how the teacher response reflects the student thinking, and if so whether it does so *explicitly* or *implicitly*. This coding is considered as if from the student's point of view, and considers whether the student would recognize his or her actions or words in the teacher response. *SM Ideas* is determined by considering whether the students would recognize if their ideas were being made the object of consideration, and may be coded *core* (the main idea of the student comment is made the object of consideration), *peripheral* (the object of consideration is peripheral to the main student idea), *other* (an idea that is unrelated to the student thinking is made the object of consideration), or *not applicable* (the teacher makes it clear that no student idea from the instance will be considered at this time). For instances where the teacher move is too general or vague to infer its relation to the student idea, the *SM Ideas* is coded as *cannot infer*.

For an example of this code, consider the situation where, in responding to an instance of student thinking, the teacher asks the class what the student meant, and in so doing repeats exactly what the student said. In this case, *student actions* would be *explicit* and *SM Ideas* would be coded *core*. On the other hand, if the teacher uses some significant word or phrase the student introduced but then took the conversation beyond the student ideas from the instance of student thinking, then the while the *student actions* would remain *explicit, SM Ideas* would be considered *peripheral* or *not*, depending on how far from the student ideas the teacher's response went.

For instances of student thinking where the SM cannot be inferred, *SM Ideas* are coded *core* only if the teacher's move does not involve any interpretation of the student thinking. For example, if after an instance of student thinking with no inferable SM the teacher's response were to point to the student, nod, then applaud, this is an implicit evaluation of the student ideas even if the ideas are not rephrased or reworded. Otherwise, instances with no inferable SM are coded *other* (if the teacher pursues an idea that is clearly unrelated to the ideas in the instance of student thinking), *cannot infer* (the default code unless there is evidence for other codes), or *not applicable* (the teacher does not pursue a mathematical idea from the instance).

Finally, the *Move MU-MP Match* compares the mathematical point of the student thinking to the *mathematical understanding* (MU) of the teacher response, which is the mathematical understanding that the teacher seems to be going toward in their move. The *Move MU-MP Match* can be coded as *core* (the MU and MP are equivalent or closely related), *peripheral* (the MU and MP are related, but not closely), *other* (the MU and MP are not related), or *not applicable* (there is no MU, such as when the teacher dismisses the student thinking or her move is non-mathematical in nature). If the teacher's MU is not inferable but the teacher is going for something mathematical, then we use the code *cannot infer.* In the absence of an MP for the instance of student thinking, I use the SM instead and include a note with the coding (ex. core to SM). While using the SM does not allow me to analyze how the teacher's use of the student thinking relates to the potential of the mathematics with in the instance, it allows me to view how the teacher uses the student thinking considering the general topic of the student thinking.

The *Practice MU-MP match* coding is similar to the *Move MU-MP match*, except that the analysis goes beyond the teacher's move in order to capture how the instance of student thinking is used in the teacher's larger practice during the take up, or the classroom discourse during the time that the student thinking is affecting the classroom discussion. Take-up was identified as starting directly after an instance of student thinking ends, and stopping when it is clear that the teacher has left the student thinking behind. It is possible that take-up of an instance can overlap multiple instances as the teacher discusses or draws together multiple instances of student thinking within the discussion of a single topic. However, it is possible that the take up is no longer than the move itself, such as when the teacher does not do anything with the student thinking beyond the initial move.

Using this framework, I went through each teacher response for all of the instances of student thinking and analyzed it according to these codes, thus providing differentiation and categorization of teacher responses to all instances of student thinking in the whole class discussion.

Principle Alignment

The third phase of analysis involved analyzing how well the teacher's practice aligns with the four core principles, as were introduced in the theoretical framework. To get at the alignment of the teacher's practice with these principles, I used TR coding as well as contextual details to build evidence for how well each teacher response aligns with each core principle. This phase consists of multiple cycles of analysis, as I determined alignment with the four core principles of quality mathematics instruction (Van Zoest et al., 2016) at multiple grain sizes. In this section, I first describe the details of these multiple cycles. I then describe the general methods for determining alignment at the smaller-grain levels. To finish, I describe how I determined alignment at the larger grain-sizes and the types of conclusions I can make about the teacher's practice given her alignment codes at each level.

Van Zoest articulated these four principles as principles of practice. Thus, the principle alignment considers the teacher's practice during these interactions as the teacher responds to instances of student thinking. I can therefore argue whether particular teacher responses or collections of responses are aligned with each principle. Through the course of my analysis, the data were considered in different grain sizes: immediate (individual moves in the teacher's response to an instance of student thinking), small (the teacher's response to a single instance of student thinking), medium (the teacher's responses during a cluster of student-teacher exchanges, usually unified by a single topic or task), and large (the teacher's responses throughout entirety of the whole-class discussion). These different grain sizes were then used to draw a conclusion on the teacher's general practice around these four principles.

These different levels are necessary for viewing different aspects of the teacher's practice in order to understand her overall practice around responding to student thinking. The move level helps us understand specific aspects of the teacher's ways of responding to student thinking, especially those as described in the TR codes. While the instance level is very similar—and in the cases of single-move responses, identical—to the move level of analysis, the separation of the move and instance levels allows me to make sense of the effect of moves individually and then as they might be compounded in multi-move responses to student thinking. Drawing on these smaller levels of analysis, the cluster level further compounds the effects of multiple teacher responses into the effect of teacher responses to student thinking during specific parts of the lesson. The final, largest level of the teacher's practice once again compiles the effect of all clusters in the whole-class discussion to draw a conclusion about the teacher's overall practice around responding to student thinking.

While the TR codes and alignment with the principles of productive use have many correlations, none of the TR codes were considered "absolute" rules for coding, but rather as evidence to argue how well the student-teacher interaction aligned with these four practices. Therefore, while the TR coding can be used as evidence to argue one way or another, other significant details about the exchange should have been and were considered as they arose and as they affected the alignment toward each principle.

In describing how a teacher-student interaction is aligned with a principle I use the following codes: *aligned*, *misaligned*, and *neutral*. *Aligned* means that there is ample evidence that the teacher's practice is consistent with a given principle. *Misaligned* signifies that there is evidence that the teacher's practice is not aligned with the principle. A *neutral* code implies that:

1) There is not enough in-the-moment evidence to determine how well the teacher practice is aligned to the principle. In this case, the alignment cannot be inferred, as it is dependent on the larger context (such as the subsequent instance of student thinking and teacher response) to understand how well it is aligned with these principles.
2) The in-the-moment evidence is ambiguous such that an argument can be constructed for contradicting levels of alignment from the same evidence.
3) There is some evidence that the teacher's practice is aligned with the principle, but there is also evidence that the response is not as aligned with the principle than it should or could be ("hit and miss").

Before the application of these codes, the data were analyzed in clusters of smaller conversations that were comprised of multiple instances of student thinking and corresponding teacher responses, and were grouped by similar topic, themes, or tasks throughout the whole-class discussion. Within these clusters, every teacher response to each instance of student thinking was coded for alignment considering how well the teacher response was aligned with the each principle individually. A given teacher response could be coded as aligned, misaligned, or neutral to one, multiple, or all principles. If a teacher response contained multiple moves, then those moves were broken down further and analyzed individually to recognize their alignment with each principle, and then afterwards considered together for the overall alignment for that whole teacher response.

Since my research question explores how a teacher's practice aligns with the all of the principles of productive use, my research considered not only how often the teacher was aligned with each principle individually, but also how the alignment with each principle coincided with the other principles. It is only when the teacher's practice is aligned with all four principles that she would be considered to be engaging in a teaching practice that is aligned with building.

As I moved outward to analyzing chunks of the teacher's practice over multiple responses, I used evidence from the instance level to draw conclusions about the teacher's practice in each cluster of teacher responses. Using this clustering of student-teacher interactions, I eventually was able to "zoom out" to the teacher practice for the duration of the entire whole-class discussion and defend how the teacher's overall practice is consistent with these practices.

Principle alignment at the move level. At this level, I coded each individual teacher move included in a teacher response for alignment with the principles, regardless of whether there was a single move in the entire teacher response or multiple moves. For each principle, I include general questions I asked around each teacher move or response, evidence gathered from the TR codes, and contextual evidence.

Mathematics principle: The main mathematical idea of the instance of student thinking is at the forefront of the whole-class discussion. In deliberating how a teacher's response aligned with this principle, I considered the following questions as guides for my coding: *Is the student mathematics at the forefront of the teacher response? Can the student who created the instance tell that their idea is still on the table? Are student ideas driving the conversation (aligned) or is the teacher replacing student ideas with their own or just using student ideas to keep the momentum in the conversation that they are steering (misaligned)?*

SM Ideas from the TR coding captures how the ideas the teacher is allowing to be the object of consideration immediately following the instance of student thinking relates to the ideas in that instance of student thinking. Thus, the *SM Ideas* codes provided strong evidence for how the teacher's response aligned with this principle.

Firstly, if the coding for *SM Ideas* was *core*, then this was evidence that the mathematics principle may have been *aligned*, since the teacher's response was making the main idea of the student thinking the object of consideration. There were only a few specific exceptions where the *SM Ideas* were *core* but there was more evidence that the mathematics principle was not aligned. One of these exceptions was if the teacher's response consisted of a repeat of the student ideas that were originally just a repeat of the teacher's ideas, making it so even while the teacher response may have been *core* to the *SM Ideas* as it was seen from the student's perspective, the actual ideas that were on the table are the teacher's rather than the student's. For instance, during the analysis of my data, at one point the teacher (Mrs. Hunt) put the learning objectives on the board for the students to consider. She then called on a student to put the objectives in her own words, and the student read the objectives from the board with a few word adjustments. The teacher's response to this instance of student thinking is to repeat what the student said. In this case, the ideas the teacher repeats are still the teacher's ideas, even if they were said by the student. This was evidence that the teacher response may have been *misaligned.*

If *SM Ideas* are *peripheral*, then I had to look elsewhere for more evidence for the alignment of the teacher's response with this principle since the *peripheral* code provides mixed evidence in whether the teacher's or student's ideas were at the forefront of the class discussion. The *MU-MP match*—especially the *Move MU-MP* match—provided further evidence for the alignment of the teacher response with this principle. If *SM Ideas* were not coded *core*, I considered how the mathematics contained within the instance was being used by the teacher, which was captured in the MU-MP matches. If the *Move MU-MP* match was *not applicable* (often denoting a non-mathematically focused response on the teacher's part) or *cannot infer* (where the mathematical understanding the teacher is pursuing is not clear), then the *MU-MP match* coding may not have provide any further evidence about this principle. But if the *Move MU-MP* match as *other*, this is evidence that the teacher's response may be *misaligned* to this principle, since the teacher was either pursuing an MU that was unrelated to the MP in the instance of student thinking. Similarly, if the *Move MU-MP* match is *core,* this signified that the teacher was pursuing the MU that matched the MP of the instance of student thinking, which was a way of putting the ideas from the instance of student thinking at the forefront of the class discussion. Therefore, if the *Move MU-MP match* was *core*, then this was evidence that the teacher's response was aligned to this principle. This code could therefore act as evidence that the teacher's response was misaligned towards the mathematics principle.

Again, contextual evidence not captured in the TR coding could and should have been considered as to how it supported or countered the evidence as provided by the TR coding. While the use of the TR coding as I have described was helpful in collecting evidence to argue the alignment of the teacher's response to this principle, details found within the context may have been strong enough evidence to make a code aligned or misaligned even if the TR coding provided evidence otherwise. For example, when the teacher's response may have corrected student thinking by inserting or replacing part of the student thinking with her own, thus approaching the MP of an instance but through the teacher's ideas more than the student's, this context provided evidence that the teacher's response was misaligned with this principle even though the *MU-MP match* would have been coded core. Another case where the *MU-MP match* was core but the principle alignment was misaligned was in cases where the language of the teacher's response was confused by unclear use of a word such as "unit" so that the teacher was talking past the student even while using their ideas. In any situation where the teacher was "driving" the conversation with students' comments being used as little more than participation or rote answers to keep the teacher's ideas moving, this was evidence that the teacher's response was misaligned to the mathematics principle.

Note that when only select aspects of student thinking are at the forefront of the class discussion, even when some of the ignored or dismissed seems insignificant, then the principle alignment for the mathematics principle may be *neutral*, since student thinking is indeed at the forefront of the class discussion but the teacher is picking and choosing what parts of that student thinking are considered and what are ignored. Such a *neutral* situation is likely to occur when there is relatively complicated student thinking and the teacher clearly discards part of it, whereas in cases when there are multiple good student mathematical ideas available and the teacher hones in on just one of them, the response is likely to be *aligned*.

Legitimacy Principle: Students are positioned as legitimate mathematical thinkers. If a teacher's response was aligned with this principle, there must have been evidence that the response legitimized what the student was thinking. In deliberating how a teacher's response aligned with the legitimacy principle, I considered the following questions as guides for my coding: *Does the teacher respond in a way that honors student mathematics? Does the teacher response treat student mathematics as legitimate ways of thinking?*

This principle alignment was considered both from the possible perspective of the student who created an instance and also from the perspective of an outside observer. In other words, I considered whether the teacher's response would have made the student feel as if their ideas were being honored, but as an outside observer I also considered how the student's thinking was actually used.

As the MU-MP matches identified how well the teacher's mathematical idea she was teaching toward aligned with the MP of the student's thinking, these codes provided evidence for how the teacher treated the student thinking as legitimate by taking advantage of the learning opportunity the student thinking was opening up to them. Because of this, I gathered evidence for alignment with the legitimacy principle from both MU-MP matches, since these codes took into account how the teacher used the mathematical potential from the instance of student thinking. Therefore, a *Move MU-MP match* of *core* was strong evidence that the student's mathematics was being treated as legitimate in the teacher's response, and a *Move-MU match* of *other* was evidence to the contrary. Since many instances have the *Move MU-MP match* of *cannot infer*, I also considered the *Practice MU-MP* codes, as these codes provided evidence of how the teacher honored the student thinking in his or her broader practice during the take-up of the student thinking. Notice that the *Practice MU-MP* could have strong evidence for being *aligned* to this principle when the *Move MU-MP match* may be *cannot infer*, leaning towards a *neutral* alignment to this principle. In such cases, it could be that the teacher was not being explicit about the MU at the specific move, but the overall practice focused on the MP as it arose in the given instance. This would be good evidence that the teacher's practice was *aligned* to the legitimacy principle in this instance. Thus, both the *Move MU-MP match* and *Practice MU-MP match* should be considered for evidence of how well a given instance of student is utilized in the classroom.

Often, a response that was aligned for the mathematics principle would also be aligned for the legitimacy principle, since having the student mathematics at the forefront of the class discussion was a way of honoring it and recognizing it as legitimate thinking. However, there are some exceptions.

If a teacher was correcting an instance of student thinking, then the teacher may have addressed the student thinking, albeit briefly, thus making the response aligned to the mathematics principle. However, since the teacher's move may have somewhat dismissed the student's wrong answer and replaced it with the correct one, then the student's ideas were not being honored and thus the teacher's response was misaligned to the legitimacy principle.

There were also times when the teacher made leaps in ideas that the students may not have followed, but such that the students still may have recognized the teacher's response as honoring their thinking. For instance, if a student asked if surface area was in square or cubic feet, and the teacher responded: "Good question. When we're talking about surface area, what, what dimension are we looking at?" The second move in this response: "When we're talking about surface area, what, what dimension are we looking at?" was misaligned for the mathematics principle because the students may not have understood the connection between their question (which was about the units for the surface area) and the teacher's response (which was about the dimension). However, because of the teacher's prior affirmation of their question and then this immediate question, even if the student did not see the connection we postulate that they still have felt as if their work were being honored.

It is therefore important in the second principle to note that there are times when the student mathematics was not in the forefront of the classroom and yet the student mathematics was still treated as legitimate (mainly in times when the teacher makes leaps that we can follow but the class might not, such as described in the previous paragraph). This type of situation where the second principle is *aligned* and the first is *misaligned* was much less common than vice e versa, since usually if the student thinking as being honored then it as quite clearly at the forefront of the discussion, and the reverse was not necessarily true. On the other hand, it is also possible for the situations where the teacher might take up student mathematics, but does not treat it as legitimate—such as focusing on it to correct a student error.

Since honoring the student thinking should also be considered from the student perspective, we should also consider the teacher's intonation and recognition of the student idea as evidence for the alignment of the teacher's response with the legitimacy principle. Beyond the TR coding, evidence was therefore taken from the surrounding context that the student thinking itself was being honored beyond participation, but as legitimate and reasonable mathematics.

Sense making principle: Students are engaged in sense making of mathematics. The sense making principle captures whether the teacher allows students to interact with the mathematics in such a way that students are able to experience mathematics as a logical, sensible system. In deciding how a teacher's response aligned with this principle, I considered the following questions as guides for my coding: *Are the students invited to engage in the mathematics during the whole-class discussion? Do the ways in which they are engaged with the mathematics require them to be sense makers of that mathematics?*

In coding this principle, I considered whether students are being invited to make sense not just of mathematical calculations as dictated by the teacher, but to make sense of and reason with the mathematics itself. Two elements that provided evidence for the teacher's response's alignment to this principle ere the *actor* and the *move* from the TR coding. By considering the *actor*, I could see who was being invited to consider the student thinking of the instance. If the teacher was the actor, this was evidence that the students weren't engaged in sense making unless the move made it clear that sense making was at hand (for instance, if a teacher adjourned the student thinking for students to privately consider the ideas before discussion, then this was evidence of sense-making).

If the actor was *same student*, *whole class*, or *other student(s),* then this was evidence that students might be engaging with the mathematics. However, *what* the students were being asked to do around the mathematics (captured in the *move*) gave further evidence on whether the students were being invited to be involved in not just in the mathematics, but in actual sense-making around the mathematics.

Some moves that we theorized might require the students to make sense of the mathematics were *evaluate*, *repeat* (such as an other student or whole-class repeat which requires a rephrasing or interpretation of the student thinking), *clarify*, *justify*, and *elaborate*. So long as the teacher response placed the students to be sense makers as they interacted with the mathematics, then this was evidence that the teacher's response as *aligned* with this principle. If the *actor* was merely being asked to perform rote procedures or give answers that require no deeper sense making then this is evidence that the teacher's response was *misaligned* to the legitimacy principle.

Note that because an *allow* move is non-specific with how the teacher wants the actor to interact with the student thinking, it would always be coded *neutral* for the alignment with the sense making principle.

Collaboration principle: Students are working collaboratively. This fourth principle captures how the students interact with each other's thinking in a given exchange. Therefore, the evidence I gathered for how well the teacher's response aligned with this principle came from answering: *How well does the teacher facilitate discourse such that students are invited and* *pushed to interact with each other's mathematical thinking? Does the teacher explicitly invite students to consider each other's thinking?*

As the *actor* coding for TR coding captured whom the teacher invited to consider the student ideas that were brought forward in the instance, the actor codes gave strong evidence of how well the exchange was aligned with this fourth principle. When the *actor* code was *whole class*, this was strong evidence that the exchange was *aligned* with this principle. If the *actor* code was *other student(s)* this also served as evidence for the teacher's response to be more aligned with the collaboration principle*. Same student* coding may serve as evidence for this exchange to be *misaligned*, since a teacher responses with an actor of *same student* is not pushing students to consider *each other's* thinking. A *teacher* code for the *actor* was evidence for the exchange to be *misaligned* with this principle.

Notes regarding principle alignment coding with particular moves. While the coding of most teacher responses required the consideration of context, some specific moves and actors could be theoretically considered in order to recognize consistent, specific coding for the principle alignment analysis.

First, consider the situation where the teacher's move has been coded as *whole class-allow*. From observation and consideration, *allow* codes have been separated into two different sub-codes: *specific allow* and *non-specific allow. Specific allow* instances are allows where the teacher identifies what she wants the students to consider as they respond to the instance of student thinking. An example of this is if a student, Tommy, just shared a method at the board and the teacher invited the class to have student-to-student discourse by explicitly inviting the class to respond directly to Tommy—such as by asking "Questions for Tommy?" This type of move is still an *allow*, since the teacher is making space for students to talk to each other. I call it a *specific allow* because the students are specifically being invited to talk to each other around a specific topic, such as the student's work at the board. These types of teacher responses will always be aligned with the mathematics and legitimacy principles, since the teacher is treating the student work at the board as legitimate as she keeps it at the forefront of conversation, as well as the collaboration principle as she invites the whole class to consider the student thinking. However, the sense making principle would be considered as *neutral* since in a *specific allow* the teacher is still just creating space for students to talk to each other, not specifying what they should be doing with the student thinking.

Non-specific allows, however, are allows where the teacher makes space for or allows students to talk to each other without a specific focus. For instance, a teacher may stand back and allow students to speak to each other with no teacher interference. Or, a teacher may call on a specific student who raised their hand after an instance of student thinking and call on that student to respond in a non-specific way, such as simply calling their name. It can be argued that in the principle analysis for *non-specific allows* each principle would be coded as *neutral*. The reasoning for this is that the teacher move of *whole class-allow* has the teacher stepping back to allow the class discussion to proceed uninterrupted. Since we are coding the teacher move and *not* what comes after, we see that a *non-specific whole class-allow* has no influence on whether or not the instance of student mathematical thinking is at the forefront of the ongoing conversation, whether it is honored, whether the students decide to engage in sense making around it, or whether it's publically considered. So we see that we can theoretically conclude that for most cases of *whole class-allow* the teacher's response is hands-off, and so we have no evidence one way or another about how the conversation in the classroom will be influenced by the next student comment. As such, each code for the four principles in the principle alignment coding would be *neutral.*

Dismisses and adjourns can be similarly theorized as consistent in their principle alignment coding. Since a *dismiss* has a teacher making it clear that a student's ideas will not be considered, the student thinking is not at the forefront of the classroom discussion or being honored. Also, students are clearly not being asked to make sense of mathematics through this teacher response or to explicitly consider each other's thinking. Because this reasoning is consistent for every *dismiss* move, all dismisses can be recognized as misaligned for all 4 principles. Similarly, adjourns as a move on their own act as dismisses and are misaligned for the 4 principles. If an adjourn follows another move (ex. validate or a repeat), then the first move will be coded appropriately, but the adjourn move would be misaligned for all 4 principles.

Finally, while I cannot make such succinct conclusions for all *repeat* moves, I can still theorize on how a *repeat* move might be aligned or is misaligned with the core principles. All *repeat* moves put student thinking at the forefront of the classroom discussion and honor that student thinking, even if it is but briefly. Thus all repeats, no matter the actor, are aligned with the mathematics and legitimacy principles. A *repeat* move is, however, misaligned for the sense making and collaboration principles so long as the actor is *teacher* or *same student*. This is because there is no sense making expected from the students of the class, and no other students are being invited to explicitly consider the student thinking. However, if the actor were *other student* or *whole class*—such as if the teacher were to invite an individual student or the class to try to rephrase what was said in their own words—then there would be evidence of sense making and there would be definite evidence of the class being asked to explicitly consider the student thinking. Thus a *repeat* will always be aligned for the mathematics and legitimacy principles, and the sense making and collaboration principles can be determined by considering the actor as coded in the TR coding.

Principle alignment at the instance, cluster, and class levels. Once the alignment coding was completed at the smallest-grain level (move), I proceeded with the three larger-grain levels of data compilation in order to draw accurate conclusions about how well the teacher's practice aligned with each principle for that cluster as a whole. I began by looking at multi-move instances, where the teacher had multiple moves within her response to a single instance of student thinking. I considered the combined moves in each multi-move response to see how all of the moves together made the teacher's response for a given instance more or less aligned with each principle, taking into account context and overall affect of the teacher response. I then recorded these alignments so that each teacher response to every instance of student thinking had a single alignment code for each principle. Once this was completed for each instance, I moved outward and considered how the combined responses affected the alignment of the teacher's practice with the core principles throughout each cluster of responses as separated by topics or tasks during the class discussion.

I analyzed these clusters similarly to how I analyzed the instance-by-instance analysis, in that I sought out evidence to argue whether the teacher's cluster of responses were *aligned, misaligned,* or *neutral* considering a given principle. During this analysis, I considered the alignment of each principle at the instance level, but also considered the flow of the conversation during the cluster to consider how the principle as upheld throughout. At this cluster level of analysis, I also noted the interplay between the alignments of the different principles to observe how the teacher's alignment with one principle may affect the alignment in the others.

While condensing these codes from *instance* to *cluster*, I first looked at the majority code from the smaller-grain *instance* coding, and then considered instances that did not have the majority code, but seemed to have a significant impact on how the students might have felt for the whole cluster. As I moved through a cluster of responses, I paid especial attention to the latter teacher responses of a cluster, as often it was how a teacher ended a conversation around a topic that had a greater impact on their practice's alignment with the core principles.

To summarize the above, the process of the multi-grain principle analysis began with the principle alignment coding of multi-move responses being condensed into general principle alignment coding for each instance. Second, these alignments of the teacher responses were considered at a cluster level, to note how the teacher's practice aligned with each principle through each topic or task throughout the whole-class discussion. I included for each cluster a general summary of what is happening in the classroom over the topic and task, including any prominent happenings that stand out from the viewing and the instance-by-instance coding. Finally, this phase of analysis was completed with a general consideration of how well the teacher's responses aligned with each principle throughout the entire class by considering how the teacher's practice aligned at the smaller-grain levels, along with noted evidence of key turning points or responses.

CHAPTER FIVE: RESULTS

My research question asks to what extent a teacher's responses to student thinking align with four core principles of quality mathematics in her use of student thinking. I present my results starting at the smallest grain sizes before moving forward to the larger grain sizes.

Principle Analysis at the Move Level

In this section, I first present the frequencies of alignment with the four core principles at the move level, then describe and illustrate how certain teacher moves are associated with alignment or misalignment with each principle. Next, I relate the frequencies of alignment at the instance level to those at the move level. Finally, I present the results for simultaneous alignment across all four principles at the instance level, and describe the results from the MOST coding as they relate to simultaneous alignment.

Move Alignment for All Principles

Figure 2 displays the frequency of teacher moves that are aligned, misaligned, or neutral around each of the four core principles, as well as the percentages of moves for each of these categories. There is little difference between the neutral codes across the four principles, with a range of 8-12% of all moves being neutrally aligned with any principle. These move-level results signify that for about half of the teacher moves in the whole-class discussion, the mathematical ideas from the instance of student thinking were in the forefront of the classroom. For about the same proportion the teacher treats student mathematical thinking as legitimate. In contrast, it is rare to find a move that is aligned with either the sense making principle or the collaboration principle, as each was coded misaligned for over 80% of all moves and aligned for fewer than 10% of all moves throughout the class discussion.

	Mathematics Principle		Legitimacy Principle		Sense Making Principle		Collaboration Principle	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Aligned	156	58	139	51	16	6	22	8
Misaligned	92	34	99	37	231	85	228	84
Neutral	23	8	33	12	24	9	21	8
Total	**271**		**271**		**271**		**271**	

Figure 2. Principle alignment results at the move level.

Even at this small-grain level there is evidence that I can use to address my research question (To what extent is the teacher's practice aligned to the core principles?). This data shows that at this level the teacher's practice is strongly misaligned with the sense making and collaboration principles, and that her practice is aligned with the mathematics and legitimacy principles for about half of her responses. However, this evidence is insufficient to accurately represent how the teacher's entire practice aligns with the core principles because it alone does not take into account other critical details such as the context of each move (such as whether the move is the entire response to an instance of student thinking or is only a part of a response, or what the move is in response to).

Even as these results begin to answer my central research question of how the teacher's practice is aligned with the core principles, questions arise from the data that are related to my secondary questions. Specifically, I want to know what aspects of a teacher move affect the alignment with each principle, both positively and negatively. Specifically, what is it in the teacher's moves in response to an instance of student thinking that makes it so the teacher is not honoring the student thinking or allowing it to be at the forefront of the class discussion during that move? What about the teacher's moves makes move alignment with the sense making and collaboration principles so rare? These questions and more will be answered as I report specifics around each principle in the sections that follow.

As described in the methods chapter, one of the productive uses of the TR coding in determining principle alignment was that of ruling out moves that were definitely misaligned and then going through and considering evidence for potential alignment. I structure my discussion of each core principle in the same way: by first presenting codes that relate to moves that have definite misalignment and then relating evidence from the remaining codes that are associated with possible alignment. I present the collaboration principle first, followed by the sense making principle, the mathematics principle, and then finish with the legitimacy principle. I present the legitimacy principle (student thinking is honored) last, because it is the principle that is most dependent on context, alignment with the other principles, and all of the TR codes together as evidence for alignment. Alignment with the legitimacy principle is also closely linked with the mathematics principle (student mathematical thinking is at the forefront of class discussion), since allowing an instance of student thinking to be at the forefront of class discussion is one way of honoring it and treating it as legitimate. As such, I report the results for the mathematics principle just before the results for the legitimacy principle. Moreover, since the sense making and the collaboration principles both use the *actor* and *move* codes as evidence, but the sense making principle considers the two as more interconnected than the collaboration principle, I report the collaboration principle and the *actor* results before reporting on the sense making principle.

Move Alignment with the Collaboration Principle

The collaboration principle takes into account whether students have the chance to consider and respond to each other's mathematical thinking during the whole-class discussion. The TR code for *actor* was used to determine who was being invited to consider the instance of student thinking. The code for *move* was used to consider whether a given actor was allowed or invited to collaborate around previous student thinking.

The collaboration principle alignment as affected by actor. Of the four actors, two (teacher and same student) are associated with misalignment with the collaboration principle while the other two (other student and whole class) have potential for alignment depending on the move the actor is invited to do. As shown in *Figure 3*, *teacher* and *same student* make up about 80% of all moves. Less than 20% of all moves have the actor of *whole class* or *other student* with less than 1% of all moves coming from the actor *other student*. I present the results for how these actors affected the alignment beginning with those associated with misalignment before moving on to those that have potential for alignment.

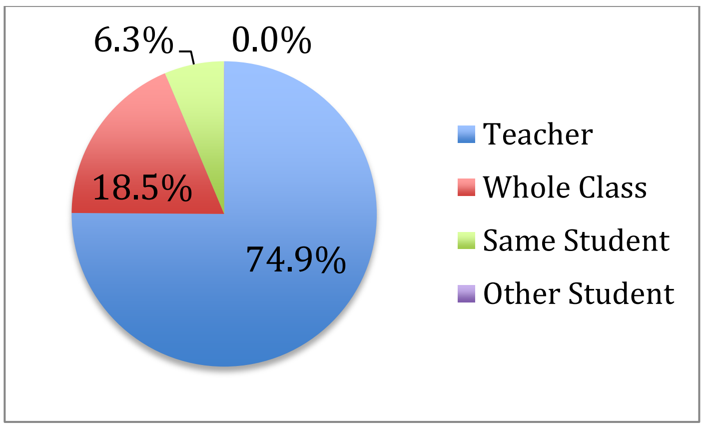

Figure 3. Distribution of actor codes for all moves.

For all moves with the actor of *teacher*, students are not explicitly asked to be involved with the student thinking on the table. Furthermore, for this proportion of moves students are not being asked to explicitly consider or do *anything*, except, perhaps, to listen to whatever the teacher is saying. For example, consider the teacher response to I 7, which consists of three moves that are all coded with the actor of *teacher:*

> Mrs. Hunt: Units squared. It actually does tell us units. Have you seen this symbol before? [circles the " symbol for inches]
> I 7: Inches.

Mrs. Hunt (move a): Inches.
Mrs. Hunt (move b): Good.
Mrs. Hunt (move c): So 25 pi *inches* squared.

For all three moves in the teacher response to I 7, the teacher is the one considering the student thinking publicly as she *repeats*, *evaluates*, and then *elaborates* on the student idea. In this case and the many others like it, the teacher is the one who interacts with the student idea, including incorporating the student statement into the broader point she wanted to make.

The evidence that 3 out of 4 of all moves have the actor of *teacher* and thus do not involve any students except as an audience during these moves is telling about Mrs. Hunt's overall alignment with the collaboration principle. Furthermore, with this recognition that any move with *teacher* actor is misaligned with the collaboration principle, we can see this kind of move makes up the large majority of the 84% of all moves that are misaligned with the collaboration principle (*Figure 2*). This means that for most of the moves that students are not invited to collaborate with each other, it is because the teacher alone is acting in response to the student thinking those moves.

The other actor code used as evidence for misalignment with this principle is *same student*, as no one but the student that created the instance is being asked to consider the student thinking of the instance and thus no collaboration is taking place for that move. In Mrs. Hunt's class, the actor was *same student* for 6% of all moves (*Figure 3*). This percentage accounts for another chunk of the 84% of all moves that are misaligned with the collaboration principle, leaving only about 3% of all moves that are misaligned with the collaboration principle that have an actor other than *teacher* or *same student.*

An example of a move with the actor of *same student,* where the same student is able to consider their own thinking but other students are not invited to collaborate, is in the response to

I 63. This instance occurred just after a student shared her solution to the surface area of a sphere with a given radius on the board in front of the class:

> Mrs. Hunt: Um, so I don't see, just in your last step right there, you said that 81*4=324π?
> I 63 (student at the board): 324.
> Mrs. Hunt: So where did the pi come from?

Here the teacher makes this students' thinking an object of consideration for this same student, as the teacher is clearly speaking directly to this student and to no one else. Thus, although this move gives this one student the opportunity to interact with his or her own mathematics, the move is *misaligned* with the collaboration principle because no one else is invited explicitly to consider the student's thinking and thus have the opportunity to collaborate around that thinking.

The two actor codes with potential for alignment with the collaboration principle are *whole class,* which is the actor for over 18% of all instances, and *other student,* which makes up less than 1% of all instance (see *Figure 3*). It is notable that this approximately 20% of moves with the actor of *other student* or *whole class* is a much larger percentage than that of aligned moves in Table 1, which showed only an 8% alignment with the collaboration principle. These percentages bring in the question of what is happening when students are invited to participate, and what are they being asked to do if they are not invited to collaborate around each other's thinking. I will use the TR code for the *move* to answer these questions in the next section.

To conclude, the collaboration principle is always misaligned with any actor that does not explicitly invite students other than the student who created the instance of student thinking. The *actor* codes of *teacher* and *same student* are thus firmly associated with misalignment with the collaboration principle, which is significant considering the prevalence of those moves. The relative lack of involvement of other students in the move thus seems to be a major reason for misalignment with the collaboration principle in Mrs. Hunt's classroom.

The collaboration principle alignment as affected by move. The only potentially aligned moves come from the 18.5% of all moves that are coded with *other student(s)* or *whole class* (*Figure 3*). Since about 8% of all moves are aligned (*Figure 2*), about half of all moves with actor of *other student(s)* or *whole class* are aligned with the collaboration principle. This already is a notable find, since it tells us that no matter what Mrs. Hunt invited the whole class or other students to do after an instance of student thinking, as long as they were doing *something* then the alignment was significantly higher than the alignment for all instances regardless of actor.

We turn to the results for the TR code of *move* to further explore how the *move* the actor is allowed or invited to do can explain the alignment or misalignment we find within the *actor* moves of *whole class* or *other student(s).* There are only 6 *moves* that Mrs. Hunt used with the actors of either *whole class* or *other student* (see *Figure 4*). *Allows* made up of almost half of all of these moves, followed by *literals,* which made up about a fourth of these moves. *Monitors* and *evaluates* both accounted for about 10%, with repeats and justifies both taking up about 2%.

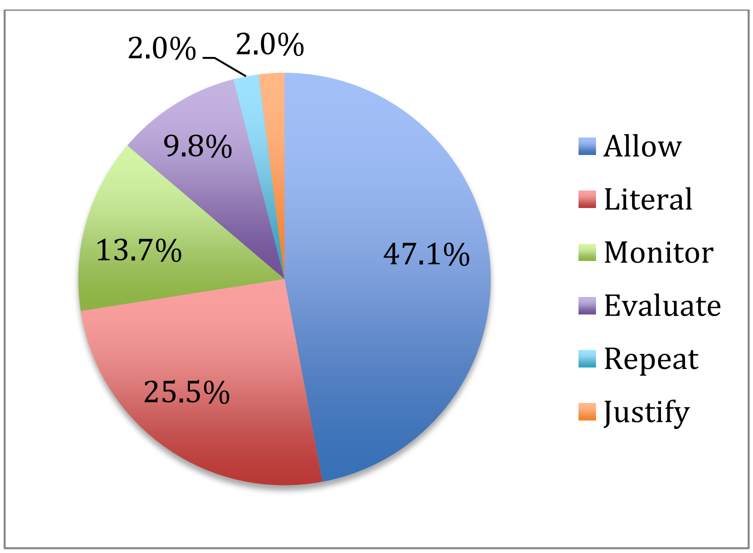

Figure 4. All *move* codes with the *actor* codes of *whole class* and *other student.*

On first glance, much of the lack of alignment with these actors comes from the *neutral* codes in alignment that come from non-specific allows. As discussed previously, a *non-specific allow* is an *allow* where the teacher does not clearly specify that students are to speak about the student mathematics of the instance. *Non-specific allows* are coded as *neutral* for all principles and thus would not be coded as *aligned* for the collaboration principle. Seventy-eight percent of all *allows* (about 37% of all *whole class* or *other student* moves and about 7% of all moves) consist of these *non-specific allows*. These *neutral* codes would thus account for the main body of moves that have the actor of *whole class* or *other student(s)* and are not aligned with the collaboration principle.

However, while the teacher's move itself may be coded *neutral*, a closer look at how the students respond in these *non-specific allow* situations (by applying the TR *move* codes to these student responses) revealed that the *non-specific allow* are far more aligned with the collaboration principle than not (see *Figure 5*). For example, for a third of all *non-specific allows,* a student either evaluated the previous student thinking (28% of *non-specific allows*) or requested an evaluation of that previous student thinking from the class (6%). Since dismisses are 11% of this data set, then for about 89% of all *non-specific allows* the students do explicitly interact with each other's ideas even without the teacher directing them to do so.

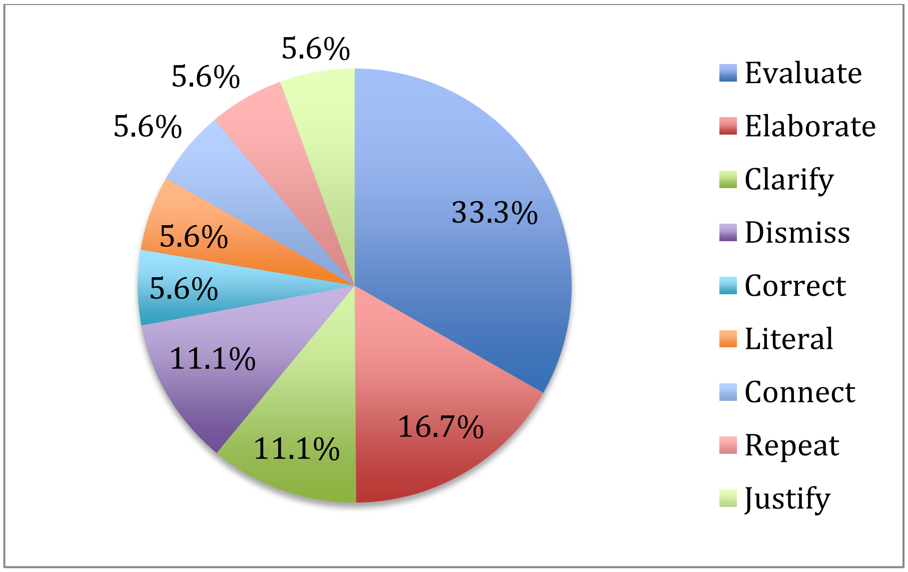

Figure 5: Types of responses of students given non-specific allows.

These data are significant in that they show that even though the teacher's move for a *non-specific allow* is *neutral* to the collaboration principle, the norms in the classroom encourage students to interact with each other's ideas when the teacher creates the space for it to happen. Taking these instances into account, the students' use of these *non-specific allow* moves provides an additional 6% of all moves that could be considered to be leading to alignment with the collaboration principle—bringing the sum total to about 14% of all moves that are aligned, compared to the 18.5% of all moves that are *whole class/other student(s).* Due to the the lack of direction for what students *should* talk about in the teacher's move itself, these teacher moves are coded *neutral* for the collaboration principle. However, this general practice of alignment to a principle on the students' parts despite neutrality on the teacher's part was taken into account in the consideration of alignment at the larger grain sizes, specifically the cluster and whole-class alignment.

After this consideration of *non-specific allows,* we turn to the other moves—*literal*, *monitor*, *evaluate*, *specific allow, repeat*, and *justify*—and consider which of these are correlated with the highest lack of alignment. To begin, 53% of literals with the specified actors (2.5% of all moves regardless of actors) are misaligned because the mathematics the teacher requests moves forward from the instance of student thinking rather than focusing on the student thinking itself. For instance, many literals are the teacher requesting of the class the next step in a solution, thus using the student comments more—as Lineback (2015) described—as "stepping stones" to get through a sequence of the teacher's ideas rather than allowing the students to collaborate around each other's ideas.

The most lack of alignment after *literal* moves are due to *monitor* moves, with 43% of all *whole class* or *other student(s) monitors* (1% of all instances) being either neutral or misaligned. The main cause for lack of alignment is because the teacher's move does not explicitly have the students considering each other's thinking, or her request is too vague about what the students are supposed to be monitoring their understanding of. For example, when Mrs. Hunt requested from the students the formula for the surface area of a sphere, received multiple answers in a chorus, her response was, "Cool?" as she turned to the class. In these types of vague monitors, the teacher looked to the students as if to check understanding, but her request was so vague that the students can't be sure what they are monitoring their understanding of. Because of this vagueness, it does not matter how the students respond, since any student response to the teacher's request would be uninferable.

Twenty percent of *whole class* or *other student(s) evaluate* moves (less than 0.5% of all moves) are misaligned because the teacher's question was more about having the students evaluate the *teacher's* idea instead of any student thinking. For instance, take Mrs. Hunt's response to I 159:

> Mrs. Hunt: If I add the area of this base to the area of this base to the area of this base, all the way around until I get the whole sphere what do I actually have?

I 159 [chorus]: The surface area.

Mrs. Hunt: So can I replace all of these [points to $b_1+b_2+b_3+ . . . +b_n$] with surface area?

In this case, even though the teacher requests and then uses the student thinking, the idea she is having the student evaluate in the response to I 159 comes directly from the teacher, and the requested evaluation is little more than a formality. Such a move does not have students considering each other's ideas—only the teacher's.

The rest of the moves—*specific allows*, *repeats*, and *justifies*—are always aligned with the collaboration principle throughout the whole-class discussion in Mrs. Hunt's classroom. I explore why below.

While *non-specific allows* have the teacher creating space for students to talk without specifying what they are to talk about, *specific allows* have the teacher creating a space for students to explicitly respond to the student thinking, thus making them all *aligned* for the collaboration principle. For example, in I 169, a student presents her solution at the board. After she is finished, the teacher's response is, "Questions for Karlee?" This question explicitly makes I 169 an object of consideration for the class, thus aligning the move with the collaboration principle. These *specific allows* make up 22% of all *allows* (about 10% of all *whole class* or *other student* moves or about 2% of all moves).

Repeat and *justify* moves are both associated with alignment with these actors because in all cases, the teacher explicitly requested the whole class to collaborate around a student's mathematical idea as they were invited either to repeat the student ideas in their own words or explain why a peer's claims might be true.

In conclusion, when Mrs. Hunt had moves that had the *actor* of *whole class* or *same student*, about half of those moves were aligned with the collaboration principle. Alignment was largely determined by how explicitly the teacher move communicated the expectations that the student thinking from the instance was to be discussed in the subsequent student responses. Lack of alignment was associated with a mixture of vagueness and moving on with the teacher's ideas instead of focusing on the student thinking. Connecting the effects of the *actor* and *the move* on alignment helps us understand that a teacher's response is only aligned with the collaboration principle when she invites the class to explicitly consider the student thinking in specific ways. These ways include having students self-monitor their understanding of the mathematics of the instance, evaluate correctness of the mathematics of the instance, talk to each other about the instance, explain a specific part of a student solution, or repeat or reword the ideas from the instance of student thinking.

Move Alignment with the Sense Making Principle

The TR Codes for *actor* and *move* were used as evidence to determine alignment with the sense making principle. As when I discussed the collaboration principle, here I present results as they were helpful in recognizing misalignment with the sense making principle, then discuss the remaining moves that have potential for alignment.

Sense making principle alignment as affected by actor. Students are not presented with an opportunity to engage in sense making unless they are the *actors* for a given move. So it is that *any* moves with the *actor* of *teacher*—and such moves make up 75% of all moves (see *Figure 3*)—are automatically misaligned with the sense making principle. For example, even a move that is very strongly associated with sense making, such as *justify*, does not engage students in sense making if only the teacher is the one doing the justifying, such as in the second move of the teacher's response to I 165. In this part of the class, the class is pulling together the formula for finding the volume of a sphere:

Mrs. Hunt: Volume equals?
I 165: $4\pi r^3$ divided by 3.

Mrs. Hunt (Move 1): 4 pi r *cubed*, right?
Mrs. Hunt (Move 2): Because we have r times r times r.
Mrs. Hunt (Move 3): R cubed, over 3.

In this multi-move response to I 165, the teacher's justify move between her two repeats of the student idea serves to make sense of the student's mathematical idea of I 165. While the students may be listening to the teacher's sense making, they are not being actively engaged in the sense making themselves.

Since there is potential for alignment with the *actor* codes of *other student(s), whole class*, and *same student* we turn to the results for the TR code of *move* to further explore how the *move* the actor does or is invited to do might affect the alignment with the sense making principle.

Sense making principle alignment as affected by move. I first report on all *move* codes that are associated with the actors *other student(s)* or *whole class*. I present these codes together since they are both instances where at least one other student is considering the student thinking at hand. Afterwards, I report on the results for when the *same student* is invited to make sense of the mathematics at hand.

Whole class *and* other student(s) *actor moves.* As was the case with the collaboration principle, the only moves with possible alignment with the sense making principle and with the *actor* of *other student(s)* or *whole class* are *allow*, *literal*, *monitor*, *evaluate*, *repeat*, and *justify* (see *Figure 4*). As discussed in the Data Analysis section, all *allow* moves (47% of all *whole class* or *other student* moves or about 8.7% of all moves) are automatically coded *neutral* for the sense making principle because the teacher provides no clear request for sense making. For example, when Mrs. Hunt asks the class "Questions for Suzie?" in response to I 58, this *whole* *class allow* gives no direction about whether those questions may be sense making questions or not.

Similar to how *neutral* teacher moves could be associated with student responses that were more or less aligned with the collaborative principle, from *Figure 6* we note that students themselves can choose to respond in such a way so as to involve themselves in sense making when given the opening to interact with each other's thinking in a non-specific manner. All of the student responses given a *teacher-allow* have at least some potential for the students to be sense making except for the approximately 20% of all student moves in response to an *allow* that are dismisses. However, even with this in mind, it is also important to note that the most frequent move by the students is to either give or provide an evaluation of the student work. While evaluation is certainly one way to make sense of the student thinking at hand, many of these student evaluations stop short after a determination of correctness without concern of how the students knew the solution was right or wrong. Such types of requests for evaluations are often considered to be neutral to the sense making principles because of the conflicting evidence for alignment or misalignment. Because these types of sense making situations cross multiple instances and are heavily affected by larger contexts, they were taken into account in the larger grain size analysis at the cluster level, but are still considered *neutral* in the move level of analysis.

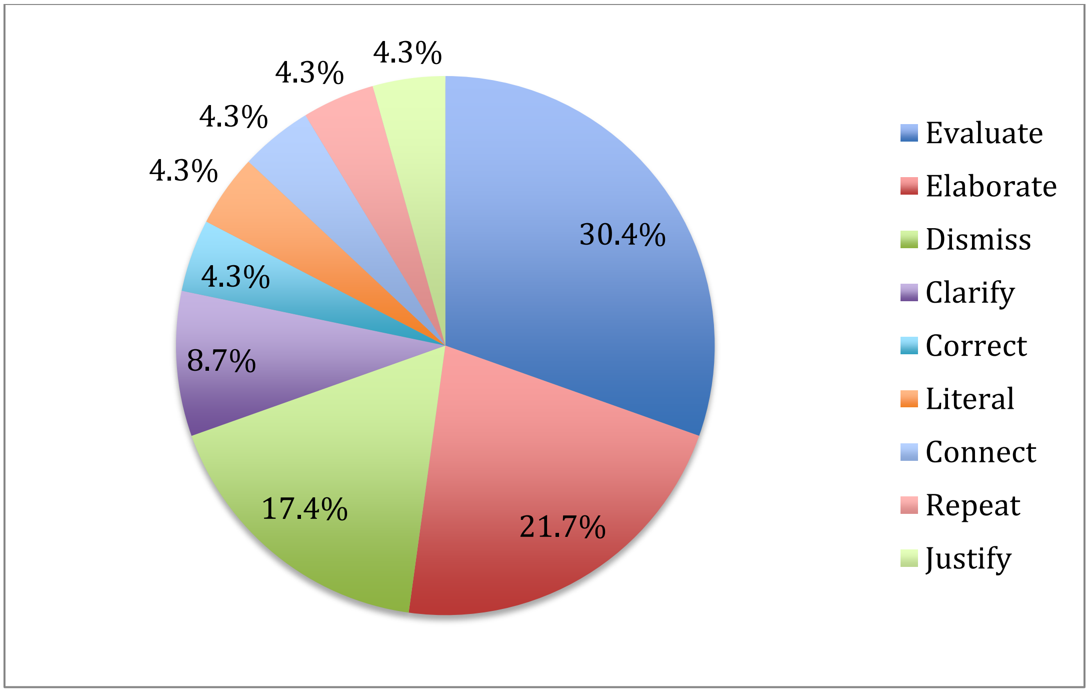

Figure 6: Types of responses of students given an *allow.*

The majority of the teacher's *monitor* moves (14% of all *whole class* or *other student* moves and about 2.6% of all moves) did not put the students in a sense-making situation. For example, a typical *monitor* in reply to a student thinking consists of the teacher asking for thumbs up or thumbs down, or has the teacher asking "We good?" Such *monitor* moves are vague, and while they may be a type of check in understanding, these moves do not invite students to be involved in sense making.

With monitors and allows accounted for, we are only left with four move codes that are at least possibly aligned with the sense making principle for the actors of *whole class* or *other student*: *literal*, *evaluate*, *repeat,* and *justify*. I now discuss what aspects of these moves may lead to lack of alignment versus alignment.

Eighty-five percent of all *whole class* or *other student literal* codes are not aligned with the sense making principle, accounting for about 4% of all moves. *Literal* moves where the teacher requests the recall of previously known and memorized facts are misaligned with the sense making principle. While these facts are mathematical, the move itself does not involve the students in sense making of the mathematics. Instead, they only involve the students peripherally, and are often used by the teacher just to keep the flow of the classroom going. On the other hand, for the 15% of moves with a *literal* code that are aligned with the sense making principle, the teacher is asking short, factual information about the meaning of a specific part of a student solution—which requires the students of the class to engage in sense making of the mathematics involved in the solution.

The 40% of *evaluate* moves that are aligned with the sense making principle take place when students are asked to evaluate the reasonableness of mathematical claims, such as when the teacher asked the whole class to evaluate the student claim that a smaller orange might change how many great circles the peeling would fill. On the other hand, if an *evaluate* is limited to students signaling agreement or disagreement or a determination of correctness of the numerical answer to a problem without consideration of the broader solution, the students are only involved in a superficial way are not actually making sense of the mathematics beyond simple agreement.

Finally, *whole class* or *other student* moves with a *repeat* or *justify* were always aligned to the sense making principle in Mrs. Hunt's practice. The *whole class* or *other student(s) repeat* moves were aligned because in all cases of this type of move the teacher was requesting the class to put together a series of ideas from a single student into a complete mathematical solution for solving a problem, which requires a level of sense making. Similarly, the *whole class* or *other student justify* moves required students to make sense of the reasoning behind a student claim. Thus, these moves were aligned with the sense making principle. All together, then, these *evaluate* and *literal* moves were occasionally aligned with the sense-making principle, with *repeat* and *justify* codes with the actor of *whole class* or *same student(s)* always aligned with the sense making principle.

Same student actor moves. There are seven moves that were used with the *same student*, as seen in *Figure 7*. All of the *same student-clarify*, *-justify*, and *-evaluate* moves were aligned with the sense making principle, as were most of the *-elaborate* moves. Contextually, these moves have the teacher pushing the students to make sense of mathematics as they grapple with and explain their own mathematical ideas. The *elaborate* moves that were not aligned were instances where the teacher returned to the same student to ask a non-mathematical question about their comment, such as to ask why a student did not speak louder about his mathematical ideas earlier in the conversation.

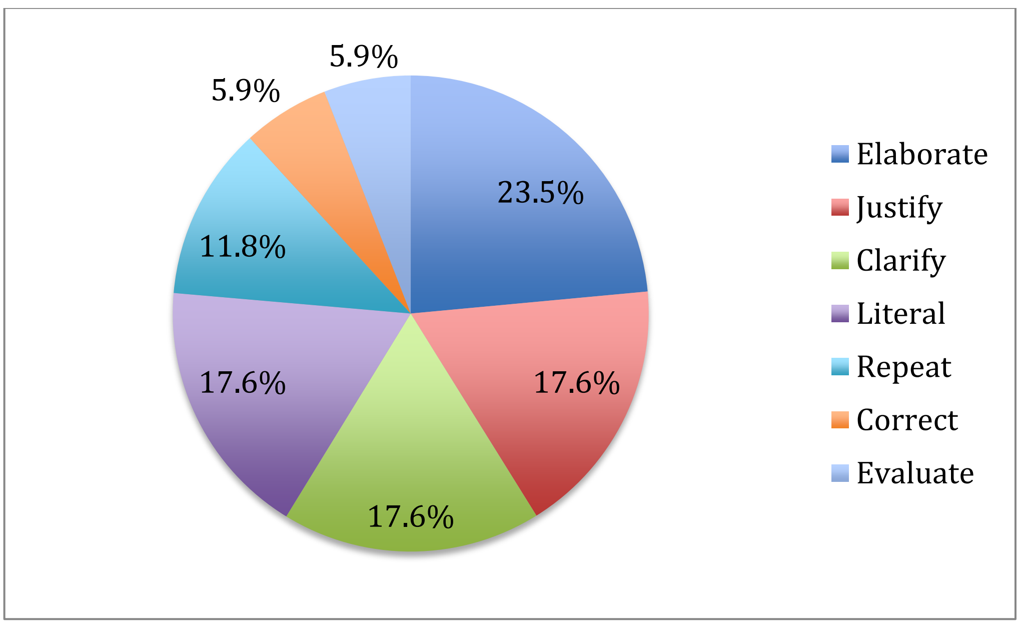

Figure 7: All move codes for *same student.*

On the other hand, the *same student-repeat*, *-correct*, and *-literal* moves were never aligned with the sense making principle. The lack of alignment with the *repeat* moves was largely due to the fact that students are not involved in sense making when they are merely repeating their own ideas from the previous instance. For *correct* moves, despite the *actor* being *same student*, these corrects often had the teacher requesting the student to correct a specific part of their thinking. In these cases, the sense making was generally taken from the student's hands and done by the teacher instead. Finally, the *same student-literals* were also often driven by teacher's ideas for factual information that was connected to the student's original idea, but did not extend beyond recitation of known information into sense making.

All of these *move* results for *whole class, other student(s),* and *same student* show that although it was rare for students to be involved in the sense making of mathematics during the whole class discussion in this class, such an occurrence could happen when the teacher was pushing a student to make deeper sense of their own mathematics. In these cases, students made sense of their own mathematics mostly as the teacher pushed them to clarify, justify, or elaborate on their ideas, or consider the correctness of their own thinking. Any other student sense making was done when the teacher's response set up the whole class as the actor, usually having them evaluate ideas on the table, use short, factual information to explain the meaning of specific parts of a student solution, repeat a student idea in their own words, or justify the thinking of one of their fellow students.

Move Alignment with the Mathematics Principle

Through the principle analysis we found that the teacher was aligned with the mathematics principle almost 60% of the time. In the TR Coding, we used the codes *Student Actions* and *Student Ideas* as evidence for how a teacher's move was aligned to the mathematics principle, since these codes capture whether and how the ideas that the teacher allows to be the object of consideration relate to the main ideas of the student thinking and how explicit the teacher is in using the words or gestures the student used. I present the data around these codes to make sense of how the results are associated with alignment with the mathematics principle.

Mathematics principle alignment as affected by *Student Actions.* Through the *Student actions* code we found that the more explicit Mrs. Hunt was in using the students words or actions in a move in response to that student's instance of student thinking, the more likely that move was to be aligned with the mathematics principle. Conversely, the less explicit she was with their words or actions, the less likely that move was to be aligned with the mathematics principle.

Student Actions codes of *explicit*, which make up about 40% of all moves (*Figure 8*), are almost always aligned with the mathematics principle—with 96% of all *explicit* moves *aligned*. In fact, the only times that an *explicit* is not aligned with the mathematics principle is in cases where the teacher is explicitly repeating a student that itself was repeating an idea that originated with the teacher—such as in I 9 where a student rephrases the learning goals of the day from the teacher's projector and the teacher repeats what the student said—or where the teacher is explicitly repeating the student idea while at the same time correcting it and replacing the student idea with her own. In both cases, although the teacher explicitly uses the student's words, the mathematical ideas at the forefront of the class discussion are the teacher's, not the student's.

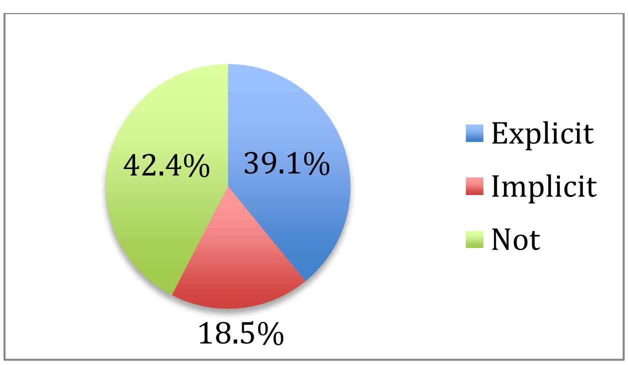

Figure 8: Student Actions codes for all moves.

Student Actions codes of *implicit* (about 20% of all moves—see Figure 8) are the next most frequently aligned with the mathematics principle, with 78% aligned. The misalignment is found with instances where the teacher is using the student idea, but extending beyond it so that the teacher's ideas more than the student's are at the forefront. For example, when the teacher is asking the class how many great circles their orange peels took up, this exchange occurred:

> I 32: Yeah, it's like 3 ½.
> Mrs. Hunt: And I'm going to say that there's a little bit over overlap in these, that it—you probably would have gotten 4 as well.

This teacher response implicitly uses the student words by saying that they would have gotten 4 great circles—not 3½--if their peelings had not overlapped. However, since she is correcting and essentially dismissing their findings, it is very much misaligned with the mathematics principle as her own ideas replace what the students reported. The 78% that are *aligned* with the mathematics principle, however, have the teacher implicitly using the student ideas yet still keeping their mathematical thinking at the forefront rather than replacing it with her own.

Student Actions codes of *not* (about 40% of all moves—see Figure 8) are most often associated with a lack of alignment with the mathematics principle. About 70% of *not* moves are misaligned with the mathematics principle, largely because all *dismiss* and *adjourn* moves are coded as *not* for *Student Actions*. Once these moves are accounted for, the non-*dismiss* or -*adjourn* moves that are coded *not* for *Student Actions* are almost equally split between *aligned*, *neutral*, and *misaligned* codes. Every neutral is due to *non-specific allows*, which, similar to the argument during the collaboration principle, may have students responding in ways that are more or less core to the student ideas in the mathematics and will be taken into account in the larger cluster analysis. Those that are misaligned with the mathematics principle are largely responses where the teacher takes over explanation for a student answer or only uses the student thinking to help progress her own ideas. The moves that have *Student Actions* codes of *not* and are still aligned are generally cases where the teacher evaluates the student thinking as correct or validates the student thinking without being specific about what is being validated or evaluated. For example, after a student makes a comment in I 102, the teacher responds with “Ding, ding, ding!” which does not use the student words or actions implicitly or explicitly, but still keeps the student thinking from the instance at the front of the classroom.

Thus we see that the more explicit the teacher is in using the student’s words and/or actions in the responses to an instance, the more likely the response is to have the student mathematical thinking at the forefront of the class discussion.

Mathematics principle alignment as affected by *SM Ideas*. Alignment with the mathematics principle is correlated almost completely with a *core* code for *SM Ideas*, which account for almost 60% of all moves (see *Figure 9*). The majority of moves with the *SM Ideas* of *core* are aligned with the mathematics principle, since if the teacher response is using the same main mathematical ideas from the instance then the student mathematical thinking is at the forefront of the classroom discussion. Less than 2% of all *core SM Ideas* are misaligned, and this misalignment is due to situations where the student is following with the teacher’s ideas and only filling in the blanks as expected—thus keeping the teacher’s ideas at the forefront rather than any student ideas.

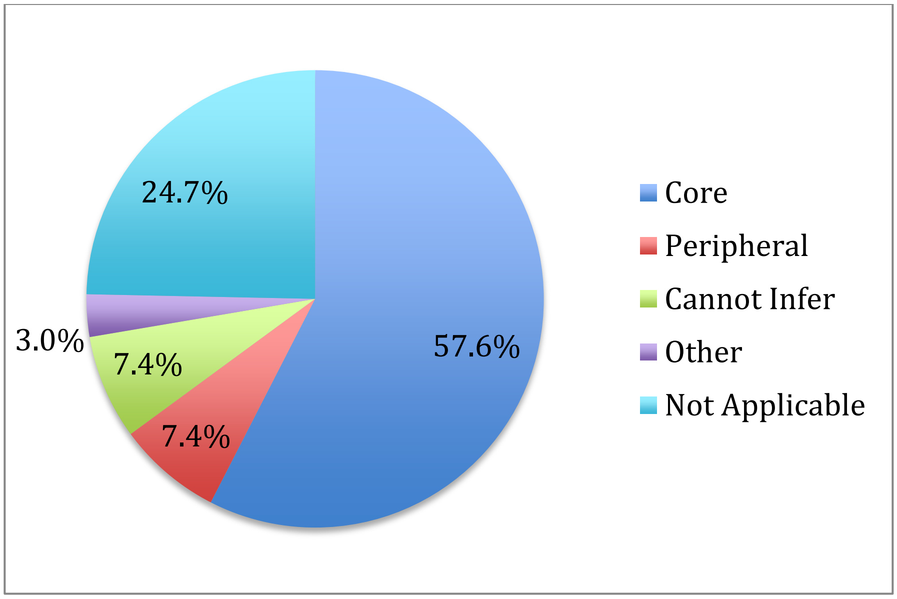

Figure 9: SM Ideas codes for all moves.

Considering these results, it makes sense that the 57% of all moves that were coded SM-Ideas *core* and are aligned with the mathematical principle is very close to the 58% total of all the teacher moves that were aligned with the mathematics principle. The difference is made up from the few *peripheral* codes where the student ideas were still considered influential enough to the classroom discussion to be considered at the forefront, or the response was such that the student ideas were at the forefront even if it was but briefly.

Peripheral codes (about 7% of all instances, see Figure 8) are most often misaligned or neutral with respect to the mathematics principle, but also have a scattering of about a third of all such codes being *aligned*. This variety is often due to the variation included with the *peripheral* code, since *peripheral* just notes that the mathematical ideas the teacher takes up are related to, but not core, to the student ideas. Any move that is *peripheral* to the student thinking yet still close enough related to the student thinking that the student would still recognize her main idea as at the forefront of the conversation would be *aligned* to the mathematics principle. If it so happens that the teacher's thinking replaces the student thinking even as it addresses it—such as when the teacher focuses on a student's idea in order to correct it—the *peripheral* code results with the student thinking being put at the forefront of the conversation in order to be replaced by the teacher's correct answer. This conflicting evidence around the mathematics principle leads to a *neutral* alignment with the mathematics principle.

The remaining *SM Ideas* codes of *cannot infer, other, and not applicable*—which make up about 35% of all moves—are associated with complete lack of alignment with the mathematics principle (see *Figure 9*), since in these moves the teacher's response is either unrelated to the mathematics of the student thinking or it is unclear how their comment relates to the mathematics from the student comment. In either case, the mathematics of the student thinking is not in the forefront of the classroom discussion for the Mrs. Hunt's response.

Move Alignment with the Legitimacy Principle

Throughout her whole-class discussion, Mrs. Hunt was aligned with the legitimacy principle for about 50% of all moves. Much of this alignment can be explained by close association of alignment to mathematics principle, since having student mathematics at the forefront of the class discussion is one way to honor student thinking or treat it as legitimate. I will present the results of the correlation between the mathematics and the legitimacy principle, and then I will present how the evidence of the TR code of *Move MU-MP Match* was associated with alignment with the legitimacy principle.

Correlation between the mathematics principle and the legitimacy principle. The mathematics principle and the legitimacy principle are closely related since, as noted previously, allowing student thinking to be at the forefront of class discussion is one way of treating it as legitimate mathematics. Alignment with the mathematics principle is almost always associated with alignment with the legitimacy principle, with 88% of all instances that are aligned with the mathematics principle also aligned with the legitimacy principle. This leaves 12% of moves that are aligned with the mathematics principle but are not aligned with the legitimacy principle. These moves can be grouped into two different types that cause misalignment to the legitimacy principle.

Firstly, 84% of the moves that are aligned with the mathematics principle while not aligned with the legitimacy principle are such that the teacher allowed student ideas to briefly take the forefront of the classroom as the teacher set herself up as the authority to dictate correctness or clarify imprecision instead of allowing students to do this work. As the teacher took over control of the mathematics in these ways, the student thinking was not treated as legitimate.

The other 16% of moves that are not aligned with the legitimacy principle despite being aligned with mathematics principle are moves that use the student thinking—thus putting the student thinking at the forefront of the class discussion at least temporarily—but react to the student thinking in such a way that the teacher takes over the work around the mathematical ideas or that it is credited to an outside source rather than originating from the students themselves. For example, as the teacher is about to launch into a whole-class exploration of the reasoning behind the formula to find the volume of a sphere, she asks if the students know how to find the volume of a sphere. A student speaks up in I 128, giving the exact formula, and the teacher's immediate reaction is: "It is. Did you learn that before this class?" She gives the student no chance to explain any further mathematical understanding, but only tells the student: "We're going to figure that out in just a minute." While Mrs. Hunt's response allows the student thinking to briefly rise to the forefront of the whole-class conversation, her response also feels somewhat like a dismiss in that she does not follow-up or use the student mathematical thinking to frame or purpose the activity they are beginning or allow the student to take any significant role in the conversation forward as the teacher walks them through derivation of the formula.

On the other hand, 97% of all instances that are aligned with the legitimacy principle are also aligned with the mathematics principle, since if the teacher is treating the student thinking as legitimate it makes sense that the mathematics from the student thinking would be at the forefront of the classroom. The few instances that are not aligned with the mathematics principle despite being aligned for the legitimacy principle are so coded for two main reasons. The first case is when the mathematics in the teacher response might be clearly connected from the teacher's perspective, but not necessarily from the student's point of view. In such cases, students might recognize that their ideas were being honored despite not having their mathematical ideas clearly at the forefront of the response. The second case are instances where the teacher validated the student thinking in such a way that the student might feel like their mathematics was being honored even if the mathematics itself was not still on the table.

From these data we see clearly that the mathematics principle is associated with alignment with the legitimacy principle and vice versa. Still, there are misaligned moves that show key aspects of the teacher's practice where she denies students the opportunities to analyze, correct, or clarify their thinking on their own. This correlation serves as evidence that while she often does honor student thinking by allowing it to rise to the front of the classroom, there are also significant—perhaps more influential to the students—parts of her practice that place the teacher as the mathematical authority rather than the students as creators of a sensible and reasonable mathematics.

Legitimacy principle alignment as affected by Move MU-MP Match. The results from the *Move MU-MP Match* show a trend that, in general, the more the mathematical understanding that Mrs. Hunt went for in her move was related to the mathematical point from the instance of student thinking, the more likely it was to be aligned with the legitimacy principle. About half of all moves either had an inferable mathematical understanding (*core*, *peripheral*, *other*) or it was clear in those moves that the teacher was not going for anything mathematical (*not applicable*) (see *Figure 10*). Of these, *core* and *peripheral* moves were both aligned about 60% of the time, *other* moves were aligned about 40% of the time, and *not applicable* codes were aligned about 12% of the time (see *Figure 11*).

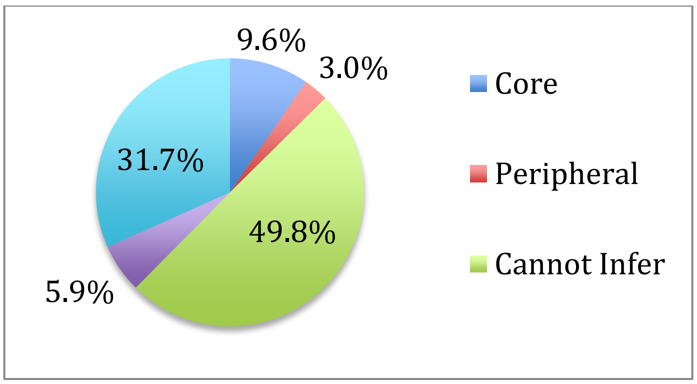

Figure 10: Move MU-MP Match codes for all moves.

Move MU-MP Match	Aligned		Misaligned		Neutral		**Total for Each Move MU-MP Match Code Frequency**
	Frequency	%	Frequency	%	Frequency	%	
Core	15	58	9	35	2	8	26
Peripheral	5	63	3	38	0	0	8
Other	6	38	9	56	1	6	16
Cannot Infer	104	77	9	7	22	16	135
Not Applicable	10	12	70	81	6	7	86

Figure 11: Move MU-MP Match codes with alignment to the legitimacy principle.

However, the most aligned type of move was when the mathematical understanding that the teacher was going for during that move was unable to be inferred by the move alone. The *cannot infer* code, which makes up about half of all move codes, was aligned for 77% of all moves (see *Figure 10* and *Figure 11*). The fact that moves with this code are more frequently aligned with the legitimacy principle than *core* or *peripheral* coded moves is explained in that when Mrs. Hunt makes clear what mathematical understanding she is going for in her response to the student thinking she is failing to honor the student thinking since she herself is preemptively jumping to the mathematical point herself without productively using the student thinking to get there.

Conclusion for the Move Level of Analysis

For the move level of analysis we found that Mrs. Hunt's practice was rarely aligned with the collaboration or sense making principles. I found that the only instances of alignment for the collaboration principle are when either the whole class or another student was invited to consider the instance of student thinking by self-monitoring their understanding, evaluating correctness, talking to each other, explaining a specific part of a student solution, or repeating or rewording the ideas from the instance of student thinking. The sense making principle is likewise mostly misaligned at the move level. The only alignment for the sense making principle was associated with the *same student*, *other student(s)*, and *whole class* actors when they were invited to make sense of the mathematics through sense-making moves, such as *justify, evaluate*, *elaborate*, and occasionally *repeat* or *literal* moves.

The mathematics and legitimacy principles are more aligned than not. Alignment with the mathematics principle was associated with more explicit use of student words or actions in the teacher response, and is almost completely correlated with *core* codes for *SM Ideas*. The legitimacy principle is closely related to alignment with the mathematics principle, but also is associated with moves where the teacher does not immediately make clear what mathematical understanding she is moving towards.

Alignment at Instance Level of Analysis

While data from the move level of analysis can help us understand what different parts of Mrs. Hunt's practice looks like at a small-grained level, some of the individual moves may be given too much weight given their place in Mrs. Hunt's larger practice. This is mostly due to the fact that while some instances of student thinking only have a single move in the teacher response, others may have up to four. Furthermore, in these multi-move instances, some moves (such as a repeat) may be less significant to the teacher's overall practice while others may be more significant (such as a justify). As such, data from the instance level of the principle analysis can illuminate aspects the teacher's practice that may have been obscured at the move level.

Instance Alignment for All Principles

The results of the principle analysis for the instance level are displayed in *Figure 12*, along with the principle analysis results for the move level to allow for comparison. Significantly, while the mathematics principle and the legitimacy principle are still more frequently aligned than not at the instance level, the difference between aligned and misaligned in both have shrunk significantly compared to the principle analysis results at the move level. By contrast, alignment with the sense making principle and the collaboration principle changed very little from the move level to the instance level. These principle analysis results at the instance level support previous conclusions about the teacher's practice at the move level, with some amendments. These results give further evidence that the teacher's practice is rarely aligned with the sense making principle or the collaboration principle, and though it still supports that the teacher's practice is more aligned than misaligned with the mathematics and legitimacy principles, the difference is only slight—and in fact, the combination of *neutral* and *misaligned* instances show that lack of alignment is more common for these two principles than alignment is at the instance level. These changes in alignment can be explained by the nature of Mrs. Hunt's multi-move responses.

Principle Analysis at the Move Level

	Mathematics Principle		Legitimacy Principle		Sense Making Principle		Collaboration Principle	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Aligned	156	58	139	51	16	6	22	8
Misaligned	92	34	99	37	231	85	228	84
Neutral	23	8	33	12	24	9	21	8
Total	**271**		**271**		**271**		**271**	

Principle Analysis at the Instance Level

	Mathematics Principle		Legitimacy Principle		Sense Making Principle		Collaboration Principle	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Aligned	100	49	93	45	10	5	20	10
Misaligned	84	41	88	43	172	83	167	81
Neutral	22	11	25	12	24	12	19	9
Total	**206**		**206**		**206**		**206**	

Figure 12: Principle analysis results at the move and instance levels.

Instance alignment as affected by multi-move responses. About one-fourth of the teacher responses (52/206) are multi-move responses. At the instance level, the principle analysis considers the combined moves in order to determine the extent to which the teacher's practice aligns with each core principle. As such, some moves have greater effect on the coding of the entire instance than others.

The consolidation of multiple *move* codes into a single *instance* code keeps the less significant moves from bearing too much weight. The most obvious and prevalent of these less significant moves are *repeat* moves. Repeats make up 36% of all moves in multi-move responses, and have very little impact on the overall effect of the response except to make the overall instance explicit with the student ideas and actions. For example, after a student reported the volume of a pyramid, Mrs. Hunt repeated the student answer and then asked for thumbs up or down if students got the same answers or not. If the teacher had not repeated the student answer before asking for agreement, the overall effect of the move would have been the same. Since all *repeat* moves are aligned with the mathematical and legitimacy principles, the consolidation of these *repeat* moves in multi-move codes helps to account for a drop in the alignment with these two principles.

Looking at *actor* codes can also help us make sense of the changes in the principle-alignment tables from the move level to the instance level. When we look at actor, we find that 40% of the multi-move responses <u>only</u> have moves with the actor of *teacher*, and the remaining 60% each has at least one move with the actor of *teacher* in combination with another actor. These 40% of all responses with only the teacher as the actor are clearly misaligned with the sense making and collaboration principles for all moves, due to them not allowing students to consider each other's thinking or be involved in the building of new mathematical ideas. These multiple misalignment codes at the move level for a single instance would be condensed into a single misaligned code for both the sense making principle and the collaboration principle for the instance level, whereas at the move level there would be at least twice as many misaligned codes for these multi-move responses. For the instances that have at least one non-teacher actor code, the *instance* codes for principle alignment consistently aligns with the move that has the non-teacher *actor* code.

Once these multi-move responses are condensed into single instances, the "weight" of the moves within these instances become better balanced with the 75% of all instances that are single-move instances, where before the single-move instances only accounted for 57% of all moves. For example, we can recognize now that all dismisses and adjourns—which are single-move responses to student thinking—make up a clear 33% of all responses and account for over three-fourths of all responses that are misaligned to the mathematics principle and the legitimacy principle and for about 40% of all responses misaligned to the sense making principle and the collaboration principle. This type of information tells us a lot about the teacher's practice, since she dismisses or adjourns a full third of all observed instances of student thinking that arise during whole-class discussion.

With these multi-moves condensed into instance codes, I can now consider simultaneous alignment for all principles at the instance level. As I am interested in how Mrs. Hunt's practice aligns with the four core principles, it makes sense to consider alignment through the effect of entire responses to instances of student thinking over individual moves within these responses, as students in the class would experience these teacher responses holistically instead of as individual moves.

Simultaneous Alignment at the Instance Level

In this section, I specifically present results that will help me answer two of my research questions. First of all, these results show how often Mrs. Hunt's practice was simultaneously aligned with all four core principles, and in what context such simultaneous alignment took place. Secondly, this section contains results that demonstrate whether Mrs. Hunt's practice was aligned when it "should be," such as around a MOST.

To begin, the top four rows of *Figure 13* show that it is much more rare for Mrs. Hunt's practice to be more aligned to the four principles than not at the instance level (or when the teacher's response to an instance has at least 2 *aligned* codes and has more *aligned* than *misaligned* codes). In fact, as seen in the "% of all Instances" column, only 10% of all instances have teacher responses that are more aligned with the four principles than not. In contrast, the bottom row shows that 38% of all teacher responses to instances are completely misaligned with all four principles.

Response Alignment with Principles	Total Instances		Number of MOSTs	% of Instances that are MOSTs	% of all MOSTs
	Count	% of all Instances			
4 Aligned	5	2	5	100	15
3 Aligned, 1 Neutral	5	2	2	40	6
3 Aligned, 1 Misaligned	13	6	4	31	12
2 Aligned, 1 Neutral, 1 Misaligned	1	0	1	100	3
2 Aligned, 2 Misaligned	65	32	6	9	18
1 Aligned, 3 Misaligned	14	7	3	21	9
1 Aligned, 1 Neutral, 2 Misaligned	3	1	0	0	0
4 Neutral	19	9	5	26	15
2 Neutral, 2 Misaligned	2	1	1	50	3
1 Neutral, 3 Misaligned	1	0	0	0	0
4 Misaligned	78	38	6	8	18

Figure 13: Simultaneous instance alignment with core principles.

Despite the low frequency of simultaneous alignment, when Mrs. Hunt's responses are considered by what types of student thinking she is responding to interesting facts arise. MOSTs, as described in my Theoretical Framework, are instances when a teacher's practice *should* align with all four principles, as MOSTs are opportunities to build. Although only 2% of all instances have teacher responses that are simultaneously aligned with all principles, the two right columns in *Figure 13* show that all five of these responses follow MOSTs. In fact, the first four rows of the far right column of *Figure 13* show that over a third of all responses to MOSTs are more aligned than not. This is significant, since it means that MOSTs comprise 52% of all instances that are more aligned than not, despite making up only 16% of all instances. In contrast, non-MOSTs—which account for 84% of all instances —make up only 48% of all instances that are more aligned with the four principles than not. This means that about a third of all teacher responses to MOSTs are more aligned than not to the four principles, wherein only about 6% of non-MOST instances have response that are more aligned than not to the four core principles.

These findings provide strong evidence that even though Mrs. Hunt was not familiar with the MOST framework, she responded in a way that was generally more aligned with the core principles when responding to MOSTs. Her practice thus contains responses that are consistent with the MOST frameworks that theorize that teachers *should* build around MOSTs. Moreover, this data provides evidence that while the teacher's practice may more commonly be misaligned to more principles than be aligned with more principles, her practice is more likely to be aligned with the building practice when responding to MOSTs.

Principle Analysis at the Cluster Level

While the results at the move and instance level are productive for identifying specific types of teacher responses that are associated with alignment or misalignment, it is necessary to look to the larger grain sizes of data analysis in order to make sense of how these instances of individual alignment or misalignment are related to alignment or misalignment in Mrs. Hunt's larger practice. Furthermore, while we have recognized how the teacher's practice is more likely to be aligned to the four core principles immediately following a MOST, these smaller grain sizes of analysis do not consider how the teacher's larger practice may stay aligned or become misaligned after the immediately alignment. To further answer the questions of how Mrs. Hunt's overall practice is aligned with the four core principles, we now consider alignment of her practice around clusters of instances of student thinking and her associated responses that are unified by a common theme or discussion point. As discussed in the data analysis section, to be aligned with a given principle the entire cluster need not be consistently aligned, or *end* with an *aligned* instance. Rather, the cluster should have an overall effect of alignment with the principle in order to be coded as *aligned*. As with at the move and instance level, I begin with a presentation of the overall principle analysis data at the cluster level. I then report on the findings for each principle individually before considering simultaneous alignment.

Individual Alignment for All Principles

Figure 14 displays the frequency and percentage alignment around each of the four core principles at the move, instance, and cluster levels. Interestingly, there is little change with alignment from the instance level to the cluster level, with the alignment of the mathematics and legitimacy principles remaining slightly below half for all instances and clusters. The sense making and collaboration principles have a slight rise in percentage from the instance to the cluster levels, but still remain low, with less than 15% of all clusters being aligned. The largest changes appear for the neutral and misaligned results for the mathematics and legitimacy principle, with only about a third of all clusters misaligned, down from over 40% of all instances, and *neutral* codes rising to a little over 40%, up from around 35% for both the mathematics and the legitimacy principles. One significant result that is immediately apparent is that with only two clusters that are aligned with the sense making principle there are *at most* two clusters that might possibly be simultaneously aligned to all of the principles. I will discuss this result along with how the principle alignment correlates across principles after discussion of each principle individually.

Principle Analysis at the Move Level

	Mathematics Principle		Legitimacy Principle		Sense Making Principle		Collaboration Principle	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Aligned	156	58	139	51	16	6	22	8
Misaligned	92	34	99	37	231	85	228	84
Neutral	23	8	33	12	24	9	21	8
Total	**271**		**271**		**271**		**271**	

Principle Analysis at the Instance Level

	Mathematics Principle		Legitimacy Principle		Sense Making Principle		Collaboration Principle	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Aligned	100	49	93	45	10	5	20	10
Misaligned	84	41	88	43	172	83	167	81
Neutral	22	11	25	12	24	12	19	9
Total	**206**		**206**		**206**		**206**	

Principle Analysis at the Cluster Level

	Mathematics Principle		Legitimacy Principle		Sense Making Principle		Collaboration Principle	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Aligned	15	48	14	45	2	6	4	13
Misaligned	10	32	10	32	25	81	24	77
Neutral	6	19	7	23	4	13	3	10
Total	**31**		**31**		**31**		**31**	

Figure 14: Principle analysis results at the move, instance, and cluster levels.

Again, these results contain evidence that I can use to address my research question of how the teacher's practice aligns with the core principles. The teacher's practice is strongly misaligned with the sense making principle and the collaboration principle, and is aligned to the sense making and the legitimacy principle for about half of all clusters. This evidence helps to more accurately capture the teacher's practice along with the smaller grain-sizes, since they are fairly consistent with each other even when other critical details such as the context of each move that has been analyzed, as was done in the cluster analysis.

While these results continue to answer my research question, further questions arise at this level of analysis. As I move forward, I present the results for cluster alignment with each individual principle in order to make sense of how the teacher's responses to individual instances are related to her larger practice at the cluster level. After discussing alignment of the four principles individually, I then report on simultaneous alignment with all four principles and whether Mrs. Hunt's practice within the clusters is aligned when it should be (around MOSTs).

Cluster Alignment with the Collaboration Principle

While there is slightly more alignment to the collaboration principle at the cluster level than at the smaller grain sizes, Mrs. Hunt's practice is still rarely aligned to the collaboration principle (*Figure 14*), and such alignment only takes place in very specific situations. The 10% of clusters that were coded neutral take place in clusters with conflicting evidence, and the remaining 75% that are coded *misaligned* are such largely due to a structure where the conversation flow moved between students and the teacher instead of having the students collaborate with each other.

The slight rise in alignment at the cluster level compared to the instance or move levels, which had a combined *neutral* and *misaligned* of almost 90% of all instances (*Figure 14*), is largely because often a single invitation from the teacher for students to consider each other's thinking will carry over to multiple instances that might otherwise count as neutral or misaligned. For example, at one point, Mrs. Hunt explicitly invites students to ask questions of a student at the board after she explained her solution to a problem. The students then exchange multiple instances of student thinking that are coded as *non-specific allow* moves as they collaborate around the student's thinking. While these *non-specific allow* moves were coded as *neutrals* because of the lack of explicit teacher direction, in the larger cluster analysis we see that the teacher created space in this cluster for students to collaborate. Specifically, the initial teacher response where Mrs. Hunt explicitly invited all students to ask questions of the student at the board carries over through the multiple instances and makes this cluster *aligned* to the collaboration principle.

On the other hand, sometimes a single *aligned* code at the instance level can be overwhelmed by other teacher responses that make the whole cluster turn misaligned for the collaboration principle. For example, in a similar situation, the teacher had a student share his solution at the board, but then went up to the board and asked a series of mostly *literal* questions that had the students interacting largely with the teacher's immediate ideas as she worked to correct the student's work. While the ideas that were being discussed globally may have belonged to the student who shared his solution initially, the instance-by-instance conversation was very much around the teacher's specific ideas and only connected back to the student thinking by the teacher herself. This example illustrates how while some clusters may begin with a promising potential for alignment, Mrs. Hunt's hesitance to let the students lead the conversation around the student thinking makes it so even though it may seem that the students are having the chance to discuss each other's mathematical thinking, often the teacher is doing the work around the student thinking with the students more as passive passengers along for the ride.

A close look at the data shows that every cluster that is aligned to the collaboration principle falls into one of two categories. Half of these *aligned* clusters are with students at the board showing their solutions to problems and the teacher opening it up to the whole class for questions and discussion around the student solution. The other half of the aligned clusters are where students are making sense of each part of the formula for surface area of a sphere and building off of each other's ideas to put the whole formula together. For example, during cluster I 34-39 the students were trying to pull together the ideas from their exploration of surface area into a rule or formula. During this cluster, multiple students added ideas or chimed in with agreement or disagreement as the ideas built on each other. In this situation the students were talking to each other about each other's ideas as they put together the formula for surface area, with the teacher acting almost only as a soundboard to reflect the student thinking to the rest of the class for consideration. Both situations described here show the teacher making space and encouraging the students to collaborate towards a common understanding or idea.

The few clusters that are neutral to the collaboration principle have conflicting evidence that leads to a mixed alignment around the principle. Half of these instances have the students discussing the meaning of a specific part of a student's solution that the teacher asked the meaning of, but has them talking past each other and dismissing each other's ideas instance-by-instance. This discussion of a similar mathematical idea but without actually responding to each other's ideas provides conflicting evidence and a *neutral* alignment with the collaboration principle. The other half of *neutral* coded clusters have instances where the students have the opportunity to discuss each other's ideas—usually through non-specific allows—but are cut off before they are able to get to the mathematics of the instance itself.

The remainder (and majority) of clusters are misaligned with the collaboration principle because the teacher does not invite students to consider each other's thinking explicitly or implicitly, or the teacher creates space for students to do so and no one rises to the occasion. This result is mostly because the flow within these clusters of the conversation goes from teacher to student to teacher and so forth, with very little student-student interaction of ideas.

Cluster Alignment with the Sense Making Principle

At the cluster level, alignment with the sense making principle is closely related to alignment with the collaboration principle. In fact, it is only when the collaboration principle is aligned that the sense making principle ever becomes either neutral or aligned. Half of all clusters that are aligned with the collaboration principles are aligned with the sense making principle as well. This tells us that in Mrs. Hunt's whole-class discussion, sense making only takes place at the cluster level when the teacher is having the students collaborate with each other. The other half of clusters that are aligned with the collaboration principle but not with the sense making principle are coded as *neutral* for the sense making principle because of contradicting evidence, such as an overt focus on correctness of the answer by consensus rather than mathematical reasoning.

So we see that with only 6% of all clusters aligned to the sense making principle (*Figure 14*), the cluster level of analysis supports that Mrs. Hunt's practice rarely has the students making sense of mathematics except in where the teacher has students walk through creating and making sense of mathematical algorithms. Although the teacher is introducing the ideas in this lesson in a conceptual manner, primarily it is her doing the sense making involved to present these understandings rather than allowing the students the wrestle and come to these conclusions themselves. This pattern in Mrs. Hunt's practice leads to an illusion of students coming up with these ideas themselves when they really are not, with the students passively following the teacher's lead as in cluster I 34-39 above.

The 13% of clusters that are coded *neutral* for the sense making principle are comprised of clusters which provide contradicting evidence for alignment or have the teacher making no clear direction about how students should be interacting with the mathematics at the moment, as in the multiple *allows* in cluster I 58-61, where the teacher has students talking about each other's thinking but does not specify anything for them to be making sense of.

With the remainder of the clusters misaligned because the teacher is either doing the sense making for the students or no sense makings seems to be happening at all, we can conclude that at the cluster level Mrs. Hunt's practice is rarely aligned with the sense making principle, and that alignment is found in very specific places where the students are invited to be involved in the creation of specific mathematics through collaboration.

Cluster Alignment with the Mathematics Principle

About half of all clusters were aligned with the mathematics principle (see *Figure 14*). These aligned clusters have the student thinking being used both locally (instance-by-instance) and globally (over the whole cluster) such that it is the student mathematics—not the teacher's—that is driving the conversation. Take for example cluster I 99-122, where students are engaged in finding the total surface area of a complex 3-dimensional shape. The student thinking in this cluster was used globally since the cluster is centered around an instance of mathematical thinking where a student brought up what sections of the shape they still needed to find the area of to get the total surface area. Student thinking was also use locally in this cluster, as multiple students chimed in with different ideas and explanations about how to find that part of the area. Throughout this cluster, the teacher's responses to the student thinking serve only to facilitate the rise and combination of student mathematical ideas without introducing any main ideas herself.

Interestingly, while the percentage of clusters that are *aligned* is very close to the 49% of *aligned* responses at the instance level, the neutral code at the cluster level is almost twice as common as the *neutral* code at the instance level (*Figure 14*). The large amount of *neutral* codes is largely because the *neutral* code at the cluster level was largely assigned because of conflicting evidence. For example, the cluster around I 80-86 was coded *neutral* because Mrs. Hunt followed some student ideas that fit where she wanted to go, but completely ignored others that did not seem to follow the direction she wanted the class to go. On the other hand, cluster I 135-141 was coded *neutral* because Mrs. Hunt used student thinking in the moment, but globally the conversation itself was driven by teacher ideas rather than student ideas. Finally, cluster I 176-187 had Mrs. Hunt allowing the student ideas to drive the conversation at the beginning, but the teacher slowly took control until the student ideas were limited to checking agreement with the teacher's ideas. Thus, the *neutral* codes are credited to a general inconsistency in whether student mathematical thinking is allowed to stay at the front of the whole class conversation throughout a single cluster.

The misalignment for the mathematics principle throughout the clusters is caused by the teacher taking control of the mathematical flow of the conversation. For example, in cluster I 46-48, the teacher asked a literal question about the answer to a math problem and then proceeded to explain the reasoning behind the resultant student thinking, despite the fact that she had no way of knowing whether her explanation actually fit the student solution. Similarly, in I 49-51, it was the teacher's ideas that led the students to make connections between mathematical concepts. In cluster I 87-98, however, while the teacher uses student thinking for momentum moving forward, the fact that each step is being provided by the teacher herself makes the cluster overall misaligned to the mathematics principle. In this case, while the student ideas may be at the forefront in a limited, local way, since the teacher's ideas control the flow of the entire cluster, the cluster is thus misaligned. These combinations all capture the same basic situation where the teacher's ideas have greater prominence in the class discussion than the students over the cluster.

So it is that clusters with alignment to the mathematics principle have student ideas taking a more prominent place both locally and globally, thus influencing the direction of the cluster rather than just providing steps between two teacher ideas the teacher is trying to pull together. On the other hand, most misalignment to the mathematics principle is caused by too much control by the teacher, so that even while she allows student ideas to help move the conversation forward, it is the teacher's ideas more than the students that are framing the direction of the conversation and the development of mathematical ideas.

Cluster Alignment with the Legitimacy Principle

As seen in *Figure 14*, a little less than half of all clusters are aligned to this principle. Similar to at the move and instance levels, much of the results for the legitimacy principle can be explained with the results from the mathematics principle. In fact, the legitimacy principle matches exactly with the mathematics principle for 81% (25/31) of all clusters, since having student thinking at the forefront of the class discussion is one way of honoring student thinking. The 6 exceptions to this correlation can be explained by two main types of situations, both of which I will discuss below.

The first situation is where the mathematics principle is more aligned than the legitimacy principle, which takes place in half of the clusters that do not have the mathematics and legitimacy principle coded exactly the same. This result was generally caused because the teacher put a student's idea at the forefront of the cluster, but either only to correct it or to attribute the mathematical ideas to a source other than the student. Otherwise, a cluster was misaligned to the legitimacy principle because it was prematurely dismissed in such a way that the mathematics itself did not seem to be valued as legitimate. For example, in for cluster I 169-172 a student took a couple minutes to neatly show her work at the board and then explained her strategy. The following discussion around the student mathematics was a brief check of the correctness of her final answer by asking if the class got the same answer (with mixed results). While the student thinking was clearly at the front of the class discussion—making it *aligned* to the mathematics principle—there was no discussion around the main body of the student thinking that was presented at the board. All discussion is focused on the answer more than the solution or strategy, thus dismissing the bulk of the student mathematical ideas and making the cluster misaligned with the legitimacy principle.

The second situation where the mathematics and legitimacy principles do not match codes is when the legitimacy principle is more aligned than the mathematics principle—when Mrs. Hunt honored student thinking by treating it as legitimate and using it in the larger conversation, but did not allow the student thinking to remain at the forefront of the conversation. For example, in cluster I 49-51, a student asked the teacher if surface area had squared or cubed units. What followed was a conversation where the teacher walked the students through an explanation that she provided to answer the student's question. While the teacher's ideas are at the forefront during this explanation (making the cluster misaligned to the mathematics principle), the student thinking from the original question is certainly honored as the teacher responded to the student's query (making it aligned with the legitimacy principle). Similarly, in cluster I 46-48, the teacher took it upon herself to explain the solution she assumes students used in order to get a correct answer. While she is treating the mathematics as legitimate by taking the time to explain it, her mathematical ideas overtake the student's original ideas—making it so the alignment with the mathematical principle was less aligned than the legitimacy principle.

Overall, then, a better alignment with the mathematics principle than the legitimacy principle in a cluster stems from a teacher's overall aim to correct the student, credit the student thinking to a source other than the student, or to focus on an answer and more-or-less dismiss the more significant part of the student mathematical thinking. On the other hand, when Mrs. Hunt took over larger explanation of a student idea then the legitimacy principle ended up with better alignment than the mathematics principle.

Simultaneous Alignment at the Cluster Level

With an understanding of what parts of the teacher's cluster-level practice resulted in alignment or misalignment with the individual principles, we now again turn to the question of simultaneous alignment, how often it happens, and if it happens when it should according to the framework for the four core principles of quality mathematics instructions, or around MOSTs.

Figure 15 presents the breakdown of all 31 clusters and the extent to which the teacher's practice was simultaneously aligned with multiple principles. A few particular results stand out from this figure. Firstly, more than a fifth of all clusters are misaligned for all principles, while only 6% of all clusters are aligned to all four of the principles. Furthermore, only 12% are more aligned than not (having at least 2 *aligned* codes and more *aligned* than *misaligned* codes), as seen in the first four rows. Only two clusters (6%) have simultaneous alignment across all principles.

Response Alignment with Principles	Instances		Cluster	
	Count	% of all Instances	Count	% of all Clusters
4 Aligned	5	2	2	6
3 Aligned, 1 Neutral	5	2	1	3
3 Aligned, 1 Misaligned	13	6	-	-
2 Aligned, 1 Neutral, 1 Misaligned	1	0	1	3
2 Aligned, 2 Misaligned	65	32	9	29
1 Aligned, 3 Misaligned	14	7	4	13
1 Aligned, 1 Neutral, 2 Misaligned	3	1	-	-
4 Neutral	19	9	2	6
2 Neutral, 2 Misaligned	-	-	3	10
2 Neutral, 2 Misaligned	2	1	3	10
1 Neutral, 3 Misaligned	1	0	1	3
4 Misaligned	78	38	7	23

Figure 15: Simultaneous cluster alignment with core principles.

As I progress through this section, I first present the results of the analysis of these two instances of simultaneous alignment. Next, I report on lost opportunities—specifically on clusters that contained teacher responses with complete simultaneous alignment at the instance level but that were not simultaneously aligned for the whole cluster.

Clusters with simultaneous alignment with all principles. Of the two clusters with complete simultaneous alignment, one contains a teacher response that had complete simultaneous alignment at the instance level while the other does not. However, both of clusters contain an instance of student thinking that qualifies as a MOST, meaning for 2 out of the 33 MOSTs (about 6%) the teacher responds at least at the cluster level in a way that is appropriate considering the MOST and core principle frameworks. I begin with a report on cluster I 52-57 and how it is coded with complete simultaneous alignment at the cluster level despite inconsistent alignment at the move and instance levels throughout the cluster, and then contrast it with cluster I 34-39, which contains simultaneous alignment at the instance level.

Figure 16 displays cluster I 52-57, along with codes from both the TR coding framework and the MOST analytic framework. The figure presents each instance in the cluster, followed by the MOST code any instance of student thinking received, the *actor* and *move* codes for any move in a teacher response, and the alignment coding at the smallest-grain level (move).

	Dialogue	MOST Code	Teacher Move	Principle Alignment			
				Math	Legit	Sense	Collab
Mrs. Hunt	Kay, formula for the surface area of a sphere?		(Teacher prompt)				
I 52	Is it 4 squared pi—?	Cannot Infer					
Mrs. Hunt	Surface area of a sphere equals . . .		Teacher-Dismiss	M	M	M	M
I 53	4 pi r squared.	MOST					
Mrs. Hunt (move 1)	4 pi r squared.		Teacher-Repeat	A	A	M	M
Mrs. Hunt (move 2)	Thumbs up if you agree with Arleen, thumbs down if you disagree.		Whole Class-Evaluate	A	A	M	A
I 54	[Students put their thumbs up.]	Central					
Mrs. Hunt (move 1)	Kay. Looks like everyone agrees with her.		Teacher-Repeat	A	A	M	M
Mrs. Hunt	What does the 4 represent?		(Teacher prompt)				
I 55	(multiple students) "The number of . . ." "Great circles."	Central					
Mrs. Hunt (move 1)	The number of great circles.		Teacher-Repeat	A	A	M	M
I 56	Yeah.	Central	Teacher-Dismiss	M	M	M	M
Mrs. Hunt	And tell me the rest from there.		(Teacher prompt)				
I 57	(2 students): The rest of it's the area of the circle.	Central					
Mrs. Hunt (move 1)	The rest of it is the area of 1 circle.		Teacher-Repeat	A	A	M	M
Mrs. Hunt (move 2)	So we have 4 times the area of each of those great circles, so we have the surface area of the whole sphere.		Teacher-Connect	A	A	M	M

Figure 16: Coding for cluster I 52-57.

Interestingly, despite having multiple moves that are have complete simultaneous misalignment due to teacher-dismisses, and not even a single move in a teacher response to an instance of student thinking where the students are invited to make sense of mathematics, this cluster is aligned to all four core principles at the cluster level. I consider each principle individually to explain the simultaneous alignment for the entire cluster.

First, despite some brief misalignment with the mathematics principle due to the dismiss of the incomplete student thinking in I 52 and I 56, all other teacher responses are *core* to the *SM Ideas* of the preceding instance of student thinking and thus are aligned to the mathematics principle. It is important to note that select instances of teacher-dismisses are not necessarily "bad" despite the temporarily misalignment. I 56, for instance, could easily be argued to not *need* a teacher response, and thus can be implicitly dismissed just for the conversation to carry on. This result fits with what we noted in the Data Analysis Chapter: that misalignment does not necessarily mean the teacher has does something wrong, and can serve a larger picture of alignment with core principles as the teacher orchestrates a discussion around student ideas. So despite the *move* misalignment at I 52 and I 56, throughout the whole of the conversation student thinking is centered globally around a student idea (surface area of a sphere equals 4 pi r squared from I 53) and locally around the student ideas in each non-dismissed instance. Thus, the whole cluster is aligned with the mathematics principle.

Because Mrs. Hunt allows student thinking to be at the forefront of the conversation, this cluster is likewise aligned with the legitimacy principle. The treatment of the student thinking in the teacher responses as she allows students to think about other student thinking and to use their own words to explain mathematical ideas is an excellent example of honoring student thinking and recognizing student mathematical ideas as legitimate.

The sense making principle, as noted before, is an interesting one in this cluster, since it is not aligned for a single move in any teacher response. However, this is when the context of the cluster that might not be included in a teacher response becomes important. Specifically, the teacher prompts before I 55 and I 57 are clear invitations for sense making as the teacher asks the students to make sense of and then articulate the meaning of different parts of the equation to find the surface area of a sphere. Because these prompts are not included in any teacher response to a previous instance, they are not coded by the TR Coding, but still must be taken into account for the affect of the teacher's larger practice throughout the cluster. Because the whole of this cluster has the teacher drawing out an equation from the students and then pressing the class to make sense of that equation, this cluster is aligned with the sense making principle even without any specific alignment with the sense making principle in any teacher response.

Similarly, although the collaboration principle is only aligned at the move level in the second move of the teacher response for I 53, the affect of the teacher prompts as she asks the students to consider the student thinking in I 53 throughout this cluster, as well as the prompt for them to add to each other's thinking in the teacher prompt before I 57, has the students explicitly considering each other's thinking throughout this cluster.

Because of the larger effect of such significant teacher moves, this cluster ends up with complete simultaneous alignment despite the prevalence of multiple individual moves and responses with high misalignment. Overall, throughout this cluster Mrs. Hunt's practice around using student thinking is such that students are engaged in sense making as they collaborate with each other's mathematical ideas. Furthermore, the results within this cluster emphasize the importance of context in the larger-grained principle alignment analysis. Since TR Codes only code the teacher moves *in the responses* to instances of student thinking, sometimes key evidence could be left out about the teacher's larger practice around the use of student thinking without looking beyond what is coded.

Figure 17 presents cluster I 34-39, the other cluster with simultaneous alignment. Here, the students are just beginning to put together the formula for surface area for the first time after completing the orange peel task (to explore how many great circles can be covered by the surface area). From the teacher response to I 34 to the teacher response to I 39 all of Mrs. Hunt's moves are aligned with the mathematics and legitimacy principles except for I 36 and I 39. These two instances are dismissed to pursue more productive student comments that came after and before them, respectively. Like with the dismisses in cluster I 52-57, it does not feel like the teacher *needed* to do anything with the student agreements and disagreements in I 36 and I 39. With similar argument as the previous cluster, then, the data easily shows that this cluster should be aligned with the mathematics principle and the legitimacy principle as students use their own legitimate ideas to create mathematics from their experience.

	Dialogue	MOST Code	Teacher Move	Principle Alignment			
				Math	Legit	Sense	Collab
Mrs. Hunt	Kay, so what does this mean? What did we just find?		Teacher Prompt				
I 34	You multiply the area times 4.	Cannot Infer					
Mrs. Hunt	You multiply what?		Same Student-Clarify	A	A	M	M
I 35	The base times height times 4.	Cannot Infer					
Mrs. Hunt	Base times height times 4?		Same Student-Clarify	A	A	M	M
I 36	No.	Cannot Infer	(Teacher-Dismiss)	M	M	M	M
I 37	You multiply the area of the great circle times 4.	SM					
Mrs. Hunt	The area of the great circle times 4?		Same Student-Clarify	A	A	M	M
I 38	To find the surface area of the whole circle.	MOST					
I 39	Yeah.	Central	(Teacher Dismiss)	M	M	M	M
Mrs. Hunt (move 1)	So the surface of the orange filled up 4 great circles.		Teacher-Elaborate	A	A	M	M
Mrs. Hunt (move 2)	So . . . If we know the area of one great circle . . .		Whole Class-Repeat	A	A	A	A

Figure 17: Coding for cluster I 34-39.

The sense making of this cluster comes from two important places. Firstly, the teacher prompt at the beginning, while vague, pushes students to pull together the mathematical ideas out of the activity they just finished. Although the *same student-clarify* moves do not specifically push students to make sense of mathematics, these clarifies, in the context of following the teacher's question as she pushes the student to be more specific about what he discovered from the task, these teacher moves push that student towards making sense of the mathematics at hand. Secondly, while most of the sense making in I 34-38 is being done by a single student, the teacher then turns to the whole class and invites them to join in with the sense making to repeat the ideas that one student had just put together. Since during this cluster all students are then being invited to make sense of the mathematics at hand, this cluster is aligned with the sense making principle.

Finally, although a large part of the cluster is misaligned with the collaboration principle –since the teacher's responses are directed back to a single student—the invitation for a whole class repeat in response to I 38 is a request for the whole class to think about everything that their peer had just put together and rephrase it for themselves. Since this teacher response explicitly brings the whole class into making sense of the mathematics around the student thinking from I 34 to I 38, this cluster is also aligned with the collaboration principle.

It is particularly interesting to note how the completely aligned move at the end of this cluster affected the overall alignment of the cluster. While alignment or misalignment just at the end of a cluster does not mean that the whole cluster will match that coding, a fully-aligned teacher response at the end of a cluster has a strong affect to bring the whole class into what was happening with just a single student throughout. Unfortunately, the alignment from the end of this cluster does not last. A student's response to the request for a teacher repeat brings up circumference, and the teacher launches into a side conversation into the difference between the meanings of circumference and area. This move changes the topic away from the main ideas in cluster I 34-39 to something else, and it takes a while to get back to it. This cluster thus demonstrates that even alignment during a cluster does not necessarily mean that the core principles were aligned long enough for building to actually completely occur, but just that the core principles were simultaneously aligned for a portion of the discussion.

From these two clusters we find that very different move-level coding can result in cluster-level simultaneous alignment. However, we can see some aspects of the two clusters that are similar. In both clusters Mrs. Hunt is pushing students to create a mathematical formula, and to conceptually justify the meaning of that formula as it is created. In both cases the teacher is pushing the students to build off of each other's thinking as they wrestled with very specific part of the mathematics. Her actions of having the students do the creation of mathematics as a class community is key in both of these situations for simultaneous alignment.

Lost opportunities for simultaneous alignment at the cluster level. The fact that cluster I 34-39 is simultaneously aligned for all four core principles and includes one of the 5 instances that were aligned across all four principles (see *Figure 15*) raises the question of why the other 4 instances that were aligned at the instance level do not lead to alignment within their clusters. I will present key examples of these clusters and consider the evidence of why the teacher's alignment at a single instance was not enough to make the practice across the whole cluster aligned with all four principles.

Firstly, both cluster I 27-29 and cluster I 80-86 contain a teacher response with complete simultaneous alignment at the instance level but were coded as *neutral* across all four principles at the cluster level. Cluster I 27-29's neutrality is due to the fact that while the teacher started strongly by tossing a student's incorrect idea (a MOST) to the class for their consideration, she does not allow the student question to be resolved before she allows another student to change the topic (see *Figure 18*). The teacher never comes back to this topic, and the whole idea of whether a smaller or larger orange would have a different proportion to its great circle and surface area is left completely unanswered or— worse—left with the students thinking that their incorrect idea was actually correct. Even though there is brief alignment across all principles, and it feels like the teacher's moves were all appropriate until she allowed the topic to be changed, the fact that the student ideas were left behind so abruptly and not revisited has the same effect as a general *whole class-allow* throughout the whole cluster. This cluster is thus an example of how the teacher's practice may start out aligned with the four core principles, but can end up faltering because of lack of specificity during the discussion.

	Dialogue	MOST Code	Teacher Move	Principle Alignment			
				Math	Legit	Sense	Colla b
Mrs. Hunt	Kay, so what might give us a little bit of—like, what can make this differ?		(Teacher prompt)				
I 27	Like smaller oranges.	MOST					
Mrs. Hunt	Would a smaller orange make a difference?		Whole Class-Evaluate	A	A	A	A
I 28	(Chorus) Yeah.	MOST	Whole Class-Allow	N	N	N	N
I 28 line 2	Like the size of an orange.	MOST	Whole Class-Allow	N	N	N	N
I 29	Because of the, the surface area.	MOST					
Mrs. Hunt	Raul?		Other Student-Allow	N	N	N	N

Figure 18: Coding for cluster I 27-29.

Cluster I 80-86's neutrality, despite beginning with complete simultaneous alignment at the instance level, is due to inconsistency across all four principles. The cluster began with a student at the board, describing his solution to a complex surface area problem, which was largely correct but had some errors. Mrs. Hunt then aligned her practice at the instance level with all principles when she asked the class to consider the meaning of part of his solution. However, the teacher's ideas (not the student's) are what guided this conversation globally, with the teacher taking control of larger sense making while students were limited to very localized sense making without the opportunity to make sense of what is going on with the solution overall. Moreover, the teacher does not press for complete answers to her questions. Her request for the meaning of a part of the student's solution is followed by only instances of student thinking with uninferable mathematics before the teacher moves on to ask about what the another part of the solution meant. While she received a partial answer from a student, she took over further explanation from the students. The conflicting evidence of which ideas are at the forefront of student thinking, whether the student thinking is being honored or not, whether students are being asked to make sense of the mathematics or if the teacher is doing the sense making, and whether students are considering each other's ideas or the teacher's makes this cluster neutral across all four principles.

The remaining two teacher responses that had complete simultaneous at the instance level are included in the same cluster—cluster I 129-134 (see *Figure 19*). Yet despite having two teacher responses with complete alignment, this cluster ends up being aligned only with the mathematics and legitimacy principles, with misalignment for the collaboration and sense making principles. Although the cluster begins with the teacher inviting the class to consider the correctness of contradicting answers in I 130 and to justify their reasoning, the teacher's practice shifts to inviting students to answer literal questions that do not require sense making, and to considering the teacher's ideas rather than those of the other students. Overall, then, because of the briefness of the alignment with the sense making and collaboration principles compared to the misalignment in the larger part of the teacher's practice, this cluster ends up being misaligned with these two principles.

	Dialogue	MOST Code	Teacher Move	Principle Alignment			
				Math	Legit	Sense	Collab
Mrs. Hunt	If we break a sphere into little pieces, we can take the volume of each of those pieces . . .		(Teacher Prompt)				
I 129	And multiply by the whole number?	Cannot Infer					
Mrs. Hunt (move 1)	And multiply by the whole number . . .		Teacher-Repeat	A	A	M	M
Mrs. Hunt (move 2)	. . . if they're the same or add them all together if they're different.		Teacher-Elaborate	M	M	M	M
Mrs. Hunt	What does this [points to the pyramid shape on the board] look like?		(Teacher Prompt)				
I 130	Students say "Pyramid" and "Triangle."	MOST					
Mrs. Hunt	Triangle or pyramid?		Whole Class-Evaluate	A	A	A	A
I 131	(chorus) Pyramid.	MOST					
Mrs. Hunt (move 1)	A pyramid.		Teacher-Repeat	A	A	M	M
Mrs. Hunt (move 2)	Why a pyramid?		Whole class-Justify	A	A	A	A
I 132	Because it has a rectangular base.	MOST					
I 132 (line 2)	Yeah, the base.	Cannot Infer	Teacher-Dismiss	M	M	M	M
Mrs. Hunt (to I 132 line 1)	It has a . . . almost rectangular base. It's not quite flat, but it gets pretty close to a pyramid.		Teacher-Correct	N	N	M	M
I 133	And the lateral sides meet at a point.	Central					
Mrs. Hunt (move 1)	And the lateral sides meet at a point.		Teacher-Repeat	A	A	M	M
Mrs. Hunt (move 2)	Good.		Teacher-Evaluate	A	A	M	M
Mrs. Hunt (move 3)	What do we call that point?		Whole Class-Literal	A	A	M	A
I 134	The vertex.	Central					
Mrs. Hunt	The vertex.		Teacher-Repeat	A	A	M	M

Figure 19: Coding for cluster I 129-134.

So it is that in Mrs. Hunt's practice at the cluster level, alignment across all four principles is tied to alignment both globally and locally with each principle. This simultaneous alignment occurs only twice in this lesson, with both times centered around students collectively working on a problem and adding to each other's idea for a final solution. While other clusters have potential or even may begin with this same sort of alignment, ultimately the teacher ends up taking up too much control of the conversation and limiting the students to filling in gaps that help to push the teacher's ideas forward as they participate for engagement more than sense making or active learning themselves.

Analysis of Teacher's Overall Practice

The evidence provided at all grain levels supports the same conclusions about how Mrs. Hunt's practice aligns with the four core principles. At the smallest grain size, the move level, the teacher's practice appears somewhat aligned with the mathematics and legitimacy principles, but very misaligned with the sense making and collaboration principles. This analysis of Mrs. Hunt's practice is consistent with the instance level, only with somewhat less alignment with the mathematics and legitimacy principles due to the combination of multi-move instances into a single code. Moreover, alignment with all of the principles at the same time is extremely rare. At the instance level, only about 2% of all instances have responses where the teacher is aligned with all four of the core principles, which is slightly up at the cluster level to 6%. These results along with the recognition that almost 50% of all clusters are not aligned with a single principle, present a picture of the teacher's practice of responding to student thinking during a whole-class discussion as being misaligned with the collaboration and sense-making principles, fairly aligned with the mathematics and legitimacy principles, and rarely aligned simultaneously with all four core principles of quality mathematics instruction.

Although Mrs. Hunt frequently makes requests for student's thinking to be a part of whole-class discussion, and she uses student thinking to move the conversation along, the students are not generally involved in more global sense making that extends beyond a single instance and are usually not involved in considering each other's thinking. While Mrs. Hunt may have good alignment with all of the principles occasionally, this practice often falters as she takes more control over the student thinking, often dismissing student ideas even when they are compelling and interesting questions in order to pursue her own plan forward.

CHAPTER SIX: DISCUSSION

As described in my literature review, previous studies surrounding teacher response to student thinking, the quality of teacher responses, and the correlation between the two have focused on many of the same aspects as I have in my research. These shared aspects include how the teacher's response attended to the ideas in the instance of student thinking (Doerr, 2006; Lineback, 2015) and what moves the teacher uses in the response (Bellack et al., 1966; Doerr, 2006; Scherrer & Stein, 2013; Lineback, 2015). One of the main factors that I focus on differently than these previous studies is the separation of actor and move, which includes the explicit consideration of who is being invited to participate in the class around each instance of student thinking. Because my explicit consideration of actor is an addition to previous research, I organize the discussion of the results around the *actor* codes. This understanding of the effect of the actor in the teacher response can help fill that gap in the previous literature around teacher responses, as well as add further illumination into the best practices for quality mathematics instruction through the use of student thinking in whole class discussion.

I order the discussion of the actors from most to least common in Mrs. Hunt's practice, beginning with *teacher*, then *whole class*, then *same student.* Because there were only two instances of *other student* moves and both of them felt like *whole class-allows* in that the teacher called on a specific student to make a comment with no specific direction for that comment, I forego the separate discussion of *other student* and include any results for *other student* with the discussion about the *whole class.* For each actor, I synthesize the results around the actor to discuss alignment or misalignment with the principles, and make connections to previous research. This discussion around the four actors first revolves around the *move* and *instance* levels, after which I will discuss how the *actor* affects the alignment on the larger scales of cluster or teacher practice.

The Effect of Actor of *Teacher* on Alignment

The alignment with the core principles when the actor is *teacher* at the move level is more straightforward than the other actors. Having the actor of *teacher* is often associated with automatic misalignment with certain principles, and so with moves with the actor of *teacher*, the only consideration for alignment may be the actor itself. For example, the *teacher* actor is consistently misaligned with the collaboration principle, since no students are invited to consider the student thinking. Consequentially, with the actor of *teacher*, no students are invited to participate in any sense making around an instance of student thinking—thus leading to misalignment with the sense making principle as well. While teacher may be involved in teaching mathematical ideas in a conceptual way, it is only she that is actively involved in sense making, with the students the audience to her sense making.

These instances where there is no alignment with the sense making principle nevertheless have potential for at least the teacher to be involved with sense making when the moves are *clarify, elaborate, justify, connect, evaluate,* or *correct*. These moves make up teacher responses that had strong potential and opportunity for the students to make sense of the mathematics, along with a need and a space, as evidenced by the teacher taking the time to carry out these moves herself around the student thinking. However, the opportunity for alignment with the sense making principle was lost because the teacher did not enact a practice that aligns with the collaboration principle.

While the sense making principle and the collaboration principle are necessarily misaligned with the actor of *teacher,* the mathematics and legitimacy principles have the possibility of being aligned depending on different aspects of the teacher's practice. In fact, alignment with the sense making principle and the legitimacy principles are not uncommon due to Mrs. Hunt's habit of repeating student thinking before moving forward to another move. While this teacher move is not very productive on its own with its automatic misalignment with the sense making principle and the collaboration principle, it is a strong launching place to make explicit the student thinking at hand before making another move—and possibly with another actor with potential to make the whole teacher response aligned to more principles.

This teacher practice of repeating before responding with another move may fulfill Zoest et al.'s (2016) first building sub-practice, which is to make the object of consideration from student thinking clear before students are invited to grapple with it. So it is that while a move with the *teacher* actor may not be the most productive type of move for simultaneous alignment of the four principles, it can be productive if followed by moves that cause alignment with the other principles.

Furthermore, *dismisses, adjourns*, and single-move response *repeats* can still be important moves that can only have the actor of *teacher* that are very important to make space for following more productive lines of inquiry. While these moves are less aligned to the core principles in and of themselves—with *dismiss* and *adjourn* being completely misaligned across all principles—these teacher responses are still necessary as the teacher maneuvers through all of the student thinking in the class. So while *teacher* moves might be misaligned in the moment, in a larger cluster they are vital to helping the overall orchestration of the class.

One last consideration to touch on with the actor of *teacher* is what we learned from alignment with the mathematics and legitimacy principles when one is aligned and one is not. Generally, if the teacher's practice is aligned with the mathematics principle then it is also aligned with the legitimacy principle, unless she is evaluating or correcting the student thinking in such a way so as to virtually dismiss the student ideas—something that is common in Mrs. Hunt's practice (in fact, *evaluate* is the teacher's top move after repeats and dismisses/adjourns). This alignment with the mathematics principle without alignment to the legitimacy is due to the fact that the teacher is focusing at least briefly on the student thinking, but often the correcting ends up treating the student mathematics as something less legitimate.

On the other hand, if the teacher's response is aligned with the legitimacy principle then her response is also aligned with the mathematics principle unless she is interpreting student thinking even when it could not be inferred, such as when she evaluates or validates student ideas that were uninferable. These situations are rare, and only occur when the mathematics from the student thinking either has more than one interpretation or is vague enough to not have any clear interpretation at all. By evaluating or validating these instances the teacher honors student thinking, at least shallowly, but does not have the student thinking at the front of the classroom since the student's ideas are so unclear. The danger of this practice is that not only may the student thinking be unclear, but it is possible that by correcting or validating an uninferable comment the teacher is communicating an incorrect idea to a student who may have inferred the student thinking differently—including the student who initially created the instance of student thinking. This effect may be because the teacher was expecting a specific answer—such as when she asks why a student changed their answer from 4.4π to 14.13 by multiplying 4.4 by π. The student responded, "Because you can't mix a whole number and pi together." The mathematics in this instance is uninferable because of the vagueness of the term "mix," yet the teacher responds by applauding and praising this student thinking.

This praise from the teacher may promote the idea that an approximate answer gained from the product of a rational and irrational number is preferable to the exact answer. It may also lead to misconceptions in the relationship between the 4.4 and the π or the meaning of 4.4π, since some students may understand the student thinking as something to do with combining like terms. One way or another, when the teacher responds in these ways such that the student thinking is honored but not understood, misconceptions are likely to arise since the student mathematics is not clearly at the forefront of the class discussion. This finding ties into Doerr's (2006) work about teachers interpreting student thinking. Her findings emphasize how important it is for a teacher to take the time and effort to listen and understanding student thinking in order to help students follow their own thinking to new learning rather than be guided along a hypothetical learning trajectory.

My research adds to this conclusion, emphasizing the importance of finding out what students are actually saying in order to make sure that mathematical concepts are communicated for other students to understand and to avoid promoting misconceptions caused when individuals are talking past each other. Clarifying such imprecision was the focus of Leatham et al. (2016), which noted that two main ramifications of not addressing imprecision in instances of student thinking are student confusion and parallel conversations—both of which were observed in Mrs. Hunt's class when imprecision in either the teacher's or a student's thinking was allowed to remain imprecise.

For the *teacher* actor, then, we find that alignment with the sense making and collaboration principles are impossible, and that alignment with the mathematics and legitimacy principles can be hampered by too much teacher control and authority or a lack of clarity in the use of student thinking. However, moves with the actor of *teacher* also have their place to help overall flow of the lesson and to help make explicit the specific student thinking that the teacher wants the class to consider in a given moment.

The Effect of Actor of *Whole Class* on Alignment

As noted in the results chapter, the second most common actor in Mrs. Hunt's class discussion was *whole class*, which has the potential to be simultaneously aligned with any or all principles. In fact, *whole class* actor moves are the only moves where we simultaneous alignment across all principles. This simultaneous alignment was only found with a handful of specific moves with the *whole class* actor: *justify, evaluate, repeat,* and *literal*. From these moves, only the *whole class-justify* move is consistently simultaneously aligned whenever it appears in a teacher response, with the class being invited to provide support or justification for another student's mathematical ideas. The other three move codes are only occasionally simultaneously aligned, for very specific types of each move. For example, the literals are specific literals that are directly centered in the main ideas from the instance of student thinking, such as when the teacher asks the class what a specific value in a solution a student has worked on the board means. Similarly, the repeat moves are specific types of repeats where the class is asked to put a student's idea in their own words. Thus we see that for *whole class –repeat* and *–literal* moves, when a teacher's response is focused on ideas that are closely related to the student ideas, then her practice is more likely to be simultaneously aligned with all four principles..

The efficacy of the *evaluate* move, on the other hand, is more closely related to the focus of what is being evaluated. If the teacher is simply requesting an evaluation of a student answer, such as with a thumbs up or a thumbs down, then there is no sense making being requested from the students. However, if the teacher makes an implicit request for an *evaluate* by repeating a student question and tossing it to the class for consideration, then the evaluate would be a move with perfect alignment across all four principles.

These *whole class* moves of *justify, evaluate, repeat,* and *literal* connect closely with certain moves that Lineback (2015) identified and categorized, which consist almost completely of what I would consider moves with a *whole class* actor. Most of these *move* codes would fit into Lineback's (2015) *consideration* or *elaboration* groups, which are the categories of moves that are defined similarly with at least two of the four core principles of quality mathematics instruction; the requirements are that student thinking is at the forefront of the class discussion, and that students be involved in sense making around each other's thinking.

Specifically, some of the moves I have described in this section would fit in her group *individual consideration,* which was defined as when the "[t]eacher asks the students what they think of another student's comment, whether another student's comment makes sense, whether they have a question or comment about a particular student's idea, or whether they agree with a particular student's idea" (Lineback, 2015, p. 438). This definition would include some cases of *whole class-evaluate* moves, where the teacher is more explicit about considering correctness or whether the solution makes sense. Another one of my most productive identified moves can fit in her description of an *inquiry* move within the *consideration* section: "Teacher broadcasts a student question for the rest of the class to discuss" (Lineback, 2015, p. 438). This description fits perfectly with the explanation I gave above of the *whole class-evaluate* where the teacher simply repeats the student question to the class for consideration. Similarly, my *justify* code is just like Lineback's justify code in her MOR group, meaning the teacher's response requests some sort of elaboration around the main idea of the student thinking. This consistent identification of similar types of teacher moves with an actor of the whole class between our two studies validates the findings and theories contained in both this thesis and Lineback's work: that *whole class* moves such as my *justify* and *evaluate* are moves that are aligned with the mathematics principle. Furthermore, while collaboration is not a key part of Lineback's framework, the language within these definitions themselves implies that Lineback considered that having students consider each other's thinking was key to her *inquiry* and *individual consideration moves.* This language ties into the consistent alignment that the corresponding moves in my own research have with the collaboration principle.

Another finding with the actor of *whole class* is that the teacher response was likely to be misaligned with the mathematics principle if the teacher responded to the student thinking in a way that may have been briefly core to the student ideas in the moment, but either ended up using the student ideas as a springboard to her own ideas or as a propellant to help advance her own ideas rather than focusing on the student's. This was common in Mrs. Hunt's practice, and often led to misalignment with the mathematics principle. From this finding we note that that it is important to have student mathematical ideas at the forefront of the conversation not just locally, but globally, with the student ideas steering and shaping the overall conversation. So it is that most misalignment to the mathematics principle is caused by too much control by the teacher, so that even while she allows student ideas to help move the conversation forward, it is the teacher's ideas more than the students that are framing the direction forward. When the teacher's practice is more aligned with the mathematics principle, however, student ideas take a prominent place both locally and globally, thus influencing the direction of the cluster rather than just providing steps between two teacher ideas the teacher is trying to pull together.

This discussion point is also consistent with some of Lineback's (2015) work, specifically with her *steppingstone* move. This move includes teacher responses that use student thinking just as I described, with the teacher asking for and then stringing together student thinking to keep her own ideas moving forward. Lineback notes that this is a *minimal* move, where the student ideas are seen as peripheral over the duration of the conversation.

For a *whole class* actor, then, we find that it is possible to have alignment with any, all, or none of the principles. Alignment is influenced by what the whole class is being invited to do around the student thinking, such as the potentially sense making moves of *justify, evaluate, repeat,* and *literal.* A key element of alignment is to recognize the role of the student ideas in alignment, such as whether students are focused on understanding versus correctness, or if the student thinking as an object to consider or only part of a puzzle the teacher is constructing. From the results we thus build strong evidence for the importance of inviting the *whole class* to be involved in considering about each other's thinking.

The Effect of Actor of *Same Student* on Alignment

When a move has an actor of *same student,* it has the potential to be aligned with all of the core principles except for the collaboration principle, which requires students to be invited to consider each other's thinking. It is sensible that alignment with both the mathematics and the legitimacy principles is common with *same student* moves, since the teacher is responding to the student thinking and going right back to the student who provided the ideas in the first place. Because of this correlation, the instances where misalignment occur are significant. These teacher responses are caused by the teacher taking control of the student ideas, such as when a student makes a comment and the teacher responds by asking if they really meant something else, as when a student said they needed to find the circumference to find the surface area of a sphere and the teacher asked, "Do you find the circumference, or would you find the area?" While her response goes back to the student, she is not really keeping the student thinking at the forefront of the conversation or honoring it as legitimate; her response is such that any student would reject their own original answer of circumference as incorrect even if that was, indeed, what they had thought and intended to say. The teacher's authoritative position itself can threaten the development of the student ideas.

Only a particular subset of moves with the *same student* actor are aligned with the sense making principle. In these cases, students make sense of their own mathematics mostly as the teacher pushes them to clarify, justify, elaborate on their ideas, or consider the correctness of their own thinking. The last move—when the students are asked to *evaluate* their own thinking—is often a sideways sort of teacher-correct, since by asking the students to do so implies that the student's thinking was incorrect in the first place, and can cause a misalignment with the mathematics principle or the legitimacy principle by replacing their idea with a correct one. The clarifies, justifies, and elaborates are requests for the student to communicate his or her thinking better or further, which requires that student to reconsider their own thinking. While this active sense making does not expand beyond the student in the given move, at least that student is given the opportunity to be involved in sense making around the mathematics.

Interestingly, the literature around teacher responses to student thinking in whole-class discussions hardly touches on the possibility or effects of the teacher returning to the same student. Instead, they focus more on responses that have the actor of teacher or multiple students. This lack of attention to *same student* actors may be because this sort of "aside" during a whole class discussion—even with the whole class as an audience—is not considered to be a part of whole-class conversation since the whole class is not involved. Still, as seen through my results, there are times during whole class discussions where teachers *should* go back to the same student, especially for clarification if the meaning of their comment was initially unable to be inferred. My research has thus begun to explore a part of whole-class discussion that seems to have been less researched than others.

Still, some research ties into my discussion points around the actor of *same student*. While she did not focus specifically on whole-class discussion, Doerr (2006) explored how teachers responded to student thinking throughout the duration of a task. Her results included multiple examples of a teacher responding to a single student in a way that I would consider to have the actor of *same student*. Doerr (2006) believed that teachers should "respond in ways that enable students to further develop their emerging models" (p. 267), and emphasized that moves to push students to justify and further elaborate on their ideas were vital to help students further develop their understanding of mathematics as they rejected, built, or refined mathematical models. Her conclusions with these moves are consistent with my findings that having students justify, clarify, evaluate, and elaborate around their own ideas are sense making activities. The key, we both note, has to do with allowing the students to focus on their own ideas without injecting the teacher's ideas into the mix to lead them to a conclusion while bypassing the sense making opportunity.

For an actor of *same student*, then, we find that alignment with the collaboration principle is impossible due to the exclusion of any other students in the class. Alignment with the mathematics principle and the legitimacy principle are common, since the teacher often is returning to the same student for more information or ideas around what was already said. However, because the conversation is only between the teacher and the student there is a possibility of the teacher taking an authoritative role even while requesting the student to respond, thus pushing student ideas to the side even while requesting the same student to consider their thinking further. While sense making may only take place with that single student, moves such as clarify, elaborate, justify, and evaluate are still productive to helping that individual student strengthen and refine mathematical understandings.

The Effect of Actor on Alignment over Clusters

While there is not generally a single, consistent actor throughout all the moves in an entire cluster, a change of actor during a cluster can shift the alignment in that cluster due to the effects of the different actors as noted in the previous discussion sections. Because of this, the "main" actor of a cluster largely influences the overall alignment of the cluster with the core principles. Take, for example, a cluster that might have the "main actor" of *teacher*. While the mathematics and legitimacy principles may be aligned, like with a single *move* with the actor of *teacher*, there is no way for the cluster to be aligned with the sense making or collaboration principles. Similarly, with a cluster that has the teacher repeatedly going back to the same student, there may be alignment with the mathematics principle, the legitimacy principle, and the sense making principle, but there cannot be alignment with the collaboration principle unless the whole class is likewise invited to make sense of the student thinking at hand. As in the case with a single move, though, only a cluster with the main actor of *whole class* has the potential to be aligned with the core principles, and thus allow building to take place.

The shifting from one "main actor" in a cluster to another "main actor" is likewise often correlated with a shift of overall alignment or productivity in the cluster. For example, during Mrs. Hunt's class discussion there was a cluster that began with a student solution on the board and the teacher inviting the whole class to discuss it. After a few student-to-student exchanges, the teacher shifted the conversation to between herself and the *same student* who provided the solution as she tried to help the student notice a mistake in her work that the rest of the class had not noticed. When this strategy did not work, the teacher again shifted the actor to *teacher* as she told the student where the mistake was and how to fix it. While the individual moves in this cluster began as generally aligned to the core principles, this alignment shifted with the changing of actors until there was complete misalignment across all four principles with the *teacher* actor. Thus, while the conversation started more productive, it ended up misaligned with all principles as the actors switched from *whole class*, to *same student*, to *teacher*.

This alignment shift can also happen the other way, with the teacher beginning with tighter control over the conversation and then releasing more to the whole class for consideration and thus resulting in better alignment. However, this occurrence was rare in Mrs. Hunt's classroom, as clusters that began with more teacher control often remained in teacher control, even if *whole class* moves were used to keep the momentum of the conversation moving forward. Another type of situation was one where the teacher began by allowing the students to openly discuss a student idea with little teacher interference, only to let the conversation peter out because of lack of direction in the *whole class-allow* moves.

The consideration of these different types of cluster structures connects to other research, especially when we consider what kind of cluster structure might be most productive. As Zoest et al. (2016) noted as they theorized on the four sub-practices of building, a cluster of building should begin with (a) the object of consideration from an instance of student thinking being made clear, and then (b) having the teacher present that object in a situation for students to make sense of the ideas. The teacher would then (c) orchestrate the class discussion, and conclude with a (d) solidifying of the mathematical understandings that were built during the entire cluster. While the first sub-practice may need a *teacher* or *same student* actor for the clarification of the object of consideration, after the tossing of the idea to the students for consideration, the third sub-practice seems like there would be mostly *whole class* actors, with students talking to each other about each other's ideas and with the teacher as a facilitator. The fourth sub-practice, the conclusion, would likely still include a large proportion of *whole class* actors, but with a rise in the *teacher* actor as the teacher provided vocabulary or mathematics conventions for what the students had discovered. Thus, while a cluster that contained a whole building cycle would have need of occasional shifts in actors, the main actor should be the whole class for the majority of the conversation.

Finally, as at the move level, having the student ideas at the forefront of the conversation both locally and globally is vital to alignment at the cluster level. Even if the sole actor throughout a cluster is *whole class*, if the ideas the teacher is requesting from the class are limited to those that will only help move the discussion forward along the path that the teacher's ideas have framed, then misalignment across multiple or even all principles is likely. Using Wood's (1998) terminology, if the teacher's practice *funnels* the student mathematics towards a desired end, her practice is more likely to be misaligned to any principle than if her practice *focuses* the student ideas by listening and then guiding the students to help develop their own mathematical ideas.

CHAPTER SEVEN: IMPLICATIONS AND CONCLUSION

Within this chapter I will first discuss the implications of my thesis and then conclude by explicitly answering my research questions.

Implications

In this section I discuss the methodological, theoretical, and practical implications of my study. I discuss the implications of both the form and the results of my study in each of these categories.

First, the methodological implications include the recognition of the value of coordinating the coding schemes from the MOST Analytic Framework (Leatham et al., 2015), the TR Coding (Peterson et al., 2017), and the core principles of quality mathematics instruction (Van Zoest et al., 2016). The combination of these codes creates a powerful, multi-faceted lens that provides an opportunity to view, make sense of, and understand classroom discourse at a level of detail that allows us to get a better grasp on the diversity and complexity of teacher responses. The combination of being able to distinguish between what types of student thinking a teacher is responding to along with the detail in the teacher response (including the separation of the actor and moves and the consideration of what ideas are central to the teacher's response) opens a door to better make sense of how whole-class discussions are orchestrated in current teachers' classrooms—and, specifically, how whole-class discussions flow when more consistently aligned with the core principles. While it was not explicitly used as such in my research, the MOST Analytic Framework can be used to differentiate not only between MOSTs and non-MOSTs, but between other different types of student thinking. As such, the coordination of these coding schemes also opens the door to the understanding of how teachers adapt responses depending on the types of student thinking that they are responding to, as well as theorize how they *should* respond to student thinking in their classroom and *when* they should do it considering what student thinking is on the table.

The theoretical implications of my research adds to and fleshes out previous research around the moves and actors in a teacher's response that leads to more productive use to student thinking through building. I found that the more explicitly a teacher uses a student's actions or words in her response, the more likely her practice is to be aligned with the mathematics principle, which keeps student thinking at the forefront of the conversation. I found that alignment to the collaboration principle was impossible without turning specific student thinking over to the whole class or another student for consideration, and that, such as with Mrs. Hunt, too much teacher control or teacher-individual student conversation can prevent the rest of the class from becoming actively involved in the mathematics. I also found that the moves that were most closely associated with alignment with the four core principles of quality mathematics instruction were those requesting from the whole class for a justification, elaboration, evaluation, or rephrasing of another student's thinking. Knowledge of these specific aspects of teacher response helps to build the theory around productive use of student thinking in whole class discussions.

Finally, I have observed the practical implications of my research in my own work as an active secondary mathematics teacher. The recognition of the various aspects of both student thinking and teacher responses has given me the ability to notice and discuss my own responses to student thinking, as well as given me a stronger lens to make sense of student thinking in my own classroom. Specifically, there are four ways my work on this thesis has affected my own practice:

1) I have recognized the importance of having students consider each other's thinking beyond just correctness, which was often the full extent of how students interacted with

each other's thinking in Mrs. Hunt's room. Rather, I have recognized the importance of pushing students to make connections between solutions and for them to discuss and justify their solutions with underlying mathematical understandings. Since the only sense making moves I observed in my research were justifications, evaluates, whole class repeats, or specific types of whole class literals (e.g. "What does this number mean in context of their solution?"), I have recognized the need to specifically and explicitly state my expectations for my students when I want them to be involved with each other's thinking. This specificity helps avoid situations where a potentially productive conversation peters out after a vague invitation for students to ask questions or talk about each other's ideas.

2) Since my analysis of Mrs. Hunt's lesson allowed me to recognize that the only time building can happen is with whole class be invited to consider student thinking, my research has helped me recognize the need for me as a teacher to step aside and let the students talk to each other. This means proactively planning time in the lesson to make space for this conversation to take place so that I avoid cutting potentially productive conversations short in order to get done with a hypothetical learning trajectory I have in my mind beforehand.

3) With the recognition that truly honoring student thinking goes beyond just having the student feel like I cared about their participation, my research has helped me become more aware of how I am using the content of the student thinking beyond just recognition of its existence. True building as theorized by Van Zoest et al. (2016) has new and stronger mathematical understandings growing from the student ideas on the table. This, along with the practice I received from identifying the mathematical ideas and the

mathematical potential of the student thinking in the classroom, helps me adapt and shift my lesson towards the opportunities for growth as found within the student thinking that arises.

4) I have recognized that mistakes in student thinking are one of the most significant places for either fostering a classroom that honors student thinking or pushing students further into a passive role towards their own thinking. Creating a situation where students are allowed to clarify and elaborate on their own thinking, and where students can critique and correct each other’s work in a way that still recognizes the incorrect ideas as coming from a legitimate place is vital towards making students active agents in their own learning.

Conclusion

At the beginning of my research, I set out with a vague question about how teachers use student thinking in their classrooms during discussions, with the recognition that teachers often did not know how to respond to student thinking productively. I found that research has already delved deeply into understanding the complex and multi-faceted aspects of class discussions, including detailed coding schemes that made sense of how teachers respond to student thinking with different moves (Bellack et al., 1966; Scherrer & Stein, 2013; Lineback, 2015) and in relation to the ideas in the student thinking (Doerr, 2006). I found that some research had classified the productivity of teacher responses in different ways, including how certain responses could facilitate or block students’ opportunities to learn (Anderson, 2009), or the extent to which responses involved students in considering each other’s thinking (Lineback, 2015). I noticed that much of this research focused only on specific types of teacher responses, such as those that were directed to the whole class, and that the actor that was invited to consider the student thinking was often included in part of the definition for the move, if it was considered at all.

In order to add to this body of research, I coordinated three coding schemes: the MOST Analytic Framework (Leatham et al., 2015), the TR Coding (Peterson et al., 2017), and the four core principles of quality mathematics instruction (Van Zoest et al., 2016) in order to view *all* types of student thinking and multiple aspects of *all* types of teacher responses in whole class discussion. I then analyzed Mrs. Hunt's lesson and compiled the results with a focus on how Mrs. Hunt's practice was aligned with the four core principles, as well as what teacher practices were associated with more or less alignment.

Findings From Mrs. Hunt's Practice

I found that Mrs. Hunt's practice is aligned with the mathematics principle in a little less than half of the responses, with the legitimacy principle just a bit below that, meaning that for about half of all teacher responses the teacher keeps the student thinking at the forefront of the classroom discussion at least briefly. This keeping of the student thinking at the front is slightly higher than the teacher's alignment with honoring student mathematics, largely due to Mrs. Hunt's tendency to allow the student thinking to briefly at the front of the class discussion as she corrected something about it.

The findings around the mathematics and legitimacy principles are similar at the cluster level to what they were at the move and instance levels. About half of all clusters have student ideas at the forefront of those discussions, and about the same portion of clusters honor the student thinking. While some of the *misaligned* codes from the instance-level analysis may be considered less significant due to their correlation with the dismissal of instances that, perhaps, *should* be dismissed for the teacher to pursue a more productive instance of student thinking, the fact that the percentages remain consistent at the cluster level is telling. We can conclude that Mrs. Hunt's practice is aligned with the mathematics and legitimacy principles both locally and globally for about half of the whole-class discussion, leaving her practice only somewhat aligned with these principles.

Mrs. Hunt's practice around the sense making principle, however, is strongly misaligned on all levels, with about 95% of all moves, instances, and clusters misaligned with the sense making principle. We do on rare occasion see the teacher push an individual student to make sense of her own ideas, or invite the whole class to make sense of another student's solution at the board. However, these opportunities for consideration rarely extend beyond checking for agreement or understanding. This results with infrequent sense making throughout the class, as the student interaction often seems limited to discussion of answers rather than the mathematics behind the answer. Furthermore, these sense-making opportunities are often cut short by the teacher taking more control over the development of ideas. This high level of teacher control takes the sense making out of the hands of the students and puts the teacher in the role of handing the mathematical understandings to the students rather than allowing them to figure it out on their own. To conclude, then, while Mrs. Hunt does seem to have a goal for conceptual learning of the mathematics, she remains the authority of the mathematics and rarely allows the students themselves to be actively involved in making sense of the mathematics. As such, her practice is found to be misaligned with the sense making principle.

Similarly, more than 85% of all moves, instances, and clusters are misaligned with the collaboration principle, meaning that for the majority of the time students are not invited to explicitly consider each other's thinking. The opportunities students do have to consider each other's thinking is when students are again invited to share ideas at the front of the classroom and the teacher asks the class for questions or evaluation, and again are often short-lived and moved on from after agreement of a correct answer—or the teacher takes control to help correct an incorrect answer. I can then conclude that Mrs. Hunt's practice is misaligned with the collaboration principle, with the students of the class rarely being invited to explicitly consider each other's thinking either locally or globally.

Considering when the alignment of the principles coincides, I note that less than 3% of all instances and only about 6% of all clusters are simultaneously aligned with all four principles. Furthermore, such occasions are often cut short, either because the teacher allows another student to change the direction of the conversation to something else or because she takes too much control and cuts off the building. Thus, while Mrs. Hunt's practice may be somewhat aligned with the mathematics and legitimacy principles and very misaligned with the sense making and collaboration principles, with consideration of all four principles at once her practice is also misaligned with the four core principles in general.

Still, despite the mixed alignment with the mathematics and legitimacy principle, and the low alignment with the sense making and collaboration principles, I would still consider Mrs. Hunt to be a teacher with a practice that is more aligned with these principles than most of the teachers observed from the collection of teachers from which she was selected. Furthermore, it must be considered that while we do not know the "ideal" proportion of alignment for a teacher's practice that uses student thinking in the most effective way possible, it is wrong to think that 100% of all teacher responses to student thinking *should* be aligned with all four core principles. However, I theorize that the mathematics and legitimacy principles should be higher at the move and instance levels than we found in Mrs. Hunt's practice, as I believe that any time a teacher responds to student thinking they should treat the student mathematics as legitimate and student mathematics should clearly drive the mathematics conversation. I also theorize that the sense making and collaboration principles should have higher alignment at the move and instance levels, with more of the sense making being taken from the teacher and granted to the students.

At the cluster level, however, I theorize that an ideal mathematics classroom would ideally have 100% alignment for both the mathematics and legitimacy principles. This is because while the teacher may need to sort through a variety of student thinking and thus have misalignment due to dismisses at the move or instance levels, throughout a cluster of student thinking there seems to be no reason for the teacher not to have the student thinking to be at the forefront of the conversation and for the student mathematical thinking not to be treated as legitimate. While it is more difficult to make the same claim of 100% alignment with collaboration and sense making at the cluster level without further consideration, I certainly theorize that it should nonetheless be much higher than it was found to be in Mrs. Hunt's classroom.

Findings Concerning Principle Alignment

What do we learn about teacher responses from considering when they are more or less aligned with the core principles? As previously noted, MOSTs are instances that have been identified where alignment across all four principles *should* happen. It seems that Mrs. Hunt was able to naturally intuit that MOSTs should be treated differently, as they were more aligned with the principles than non-MOST instances and were the only types of instances of student thinking that had teacher responses with simultaneous alignment across all four principles. With this result in mind, this last section of my discussion summarizes the overall practices that we found from the results that led to alignment with each principle individually. As such, in response to a

MOST the most productive teacher response would be such that it would be aligned with all four principles both in the instance and in the larger cluster around the MOST.

In order to be aligned with the mathematics principle, my results show that a teacher's moves need to make clear what student thinking is on the table—either explicitly or by allowing students to clarify themselves. This clarity can help students recognize what they are supposed to discuss. To align their larger practice with the mathematics principle, teachers need to use the student ideas not just locally, but in a global sense to help student thinking drive the conversation rather than the teachers' thinking.

In order for their practice to be aligned with the legitimacy principle, teachers need to respond in a way that not only acknowledges the instance of student thinking, but acknowledges and takes advantage of the *content* of the student thinking as well. Especially when teachers are trying to correct a student idea, teachers should honor the student thinking as coming from a legitimate place by allowing other students to be leaders in making sense of the mistake and helping explain and correct it. To align their larger practice with the legitimacy principle, teachers should use the student thinking and the mathematical content of the student thinking, thus going beyond acknowledging or validating student thinking.

In order for a teacher's practice to be aligned with the sense making principle, my results show that a teacher must engage students in sense making activities such as justifying, elaborating, evaluating, and correcting each other's thinking. The teacher's responses need to 1) turn the ideas back to either another student or to the whole class and 2) make sure that she is clear about what sense making activities students should be involved with around the student thinking. To align their larger practice with the sense making principle, teachers should focus on pushing the sense making back to the students rather than only allowing students to be observers as the teachers engage in sense making.

In order for teachers' practices to be aligned with the collaboration principle, teachers need to turn the mathematics from student thinking over to the whole class—or at least another student—for conversation and consideration. To align with the collaboration principle, this practice of turning the student ideas back to the students should take place more than just directly after a single teacher response. Rather, teachers need to plan to put the ideas to the students and give them time and space to consider each other's thinking without taking control after a single whole class move.

Summary

Through my research I found that I was able to conclude how well a teacher's practice aligned with the four core principles both individually and all at the same time. I was able to find parts of her practice that got in the way of alignment with the four core principles of quality mathematics instruction—such as too much teacher control in making sense of and correcting student mathematics—and also that facilitated alignment to the principles—such as turning student ideas over to the class for justification or evaluation. My research provided a new combined framework of multiple frameworks to be able to understand productive classroom discourse and what teachers can do to more productively respond to student thinking.

REFERENCES

Anderson, K. T. (2009). Applying positioning theory to the analysis of classroom interactions: Mediating micro-identities, macro-kinds, and ideologies of knowing. *Linguistics and Education*, *20*(4), 291-310. doi:10.1016/j.linged.2009.08.001

Bellack, A. A., Kliebard, H. M., Hyman, R. T., & Smith, F. L. (1966). *The language of the classroom.* New York: Teachers College Press.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the Learning of Mathematics, 7*(3), 2-8. doi: 10.2307/40247900

Davies, N., & Walker, K. (2005). Learning to notice: One aspect of teachers' content knowledge in the numeracy classrooms. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory research and practice (Proceedings of the 28th Annual Conference of the Mathematics Education Research Group of Australasia, Melbourne*) (pp. 273-280). Sydney, Australia: Mathematics Education Research Group of Australasia.

Dieterle, J. M. (2010). Social construction in the philosophy of mathematics: A critical evaluation of Juilian Cole's theory. *Philosophia Mathematica, 18*, 311-328. doi: 10.1093/philmat/nkq007

Doerr, H. M. (2006). Teachers' ways of listening and responding to students' emerging mathematical models. *Zentralblatt fur Didaktik der Mathmatik, 38*, 255-268. doi: 10.1007/BF02652809

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*, 403-434. doi: 10.2307/749875

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63.

Leatham, K. R., Van Zoest, L. R., Stockero, S. L., Peterson, B. E. (2014). Teachers' perceptions of productive use of student mathematical thinking. In P. Liljedahl, S. Oesterle, C. Nicol, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36*, (Vol. 4, pp. 73-80). Vancouver, Canada: PME.

Leatham, K. R., Peterson, B. E., Stockero, S. L, & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education, 46*, 88-124.

Leatham, K. R., Peterson, B. E., Merrill, L., Van Zoest, L. R., & Stockero, S. L. (2016). Imprecision in classroom mathematics discourse. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1236–1243). Tucson, AZ: University of Arizona.

Lineback, J. E. (2015). The redirection: An indicator of how teachers respond to student thinking. *Journal of the Learning Sciences, 24*, 419-460. doi: 10.1080/10508406.2014.930707

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge, MA: Harvard University Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (2007). *Mathematics teaching today: Improving practice, improving student learning* (2nd ed.). Reston, VA: Author.

National Council of Teachers of Mathematics (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author.

Peterson, B. E., & Leatham, K. R. (2009). Learning to use students' mathematical thinking to orchestrate a class discussion. In L. Knott (Ed.)*, The role of mathematics discourse in producing leaders of discourse* (pp. 99-129). Charlotte, NC: Information Age.

Peterson, B. E., Van Zoest, L. R., Rougée, A. O. T., Bernard, R. G., Leatham, K. R., & Stockero, S. L. (2017). *Teacher response coding scheme codebook*. Unpublished manuscript.

Scherrer, J., & Stein, M. K. (2013). Effects of a coding intervention on what teachers learn to notice during whole-group discussion. *Journal of Mathematics Teacher Education, 16*, 105-124. doi: 10.1007/s10857-012-9207-2

Sherin, M. (2002). A balancing act: Developing a discourse community in a mathematics classroom*. Journal of Mathematics Teacher Education 5*, 205-233.

SportsTec. (1997-2015). Studiocode [Computer program]. Camarillo, CA: Vitigal Pty Limited.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teachers College Press.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: National Council of Teachers of Mathematics.

Van Zoest, L. R., Peterson, B. E., Leatham, K. R., & Stockero, S. L. (2016). Conceptualizing the teaching practice of building on student mathematical thinking. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1281–1288). Tucson, AZ: University of Arizona.