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A Design Approach to Fully Compliant Multistable Mechanisms Employing a Single Bistable Mechanism

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Abstract

A fully compliant multistable mechanism is a monolithic structure that is capable of staying at multiple positions without power input, and has many applications including switches, valves, positioners. However, it is difficult to design such a mechanism because of the complexities of the multistable behavior, the practical stress limits and the buckling constraints. This paper discusses the design approach for fully compliant multistable mechanisms which employs a single bistable mechanism and several end-effectors connected in series. The force-displacement characteristics of the end-effectors are derived using the pseudo-rigid-body model. The design approach to the fully compliant multistable mechanism is provided to determine the design parameters for the flexible segments considering the critical buckling load and the bending strength. Two design examples are presented to demonstrate the feasibility of the approach.

Keywords: Fully Compliant, Multistable Mechanism, Pseudo-rigid-body Model
A Design Approach to Fully Compliant Multistable Mechanisms Employing a Single Bistable Mechanism

Introduction

A compliant multistable mechanism possesses multiple distinct stable equilibrium positions within its range of motion (Howell, 2001). At each stable equilibrium position the mechanism can maintain stability without power input and tends to return to the same position if exposed to a small disturbance (Masters and Howell, 2003). Due to these characteristics, compliant multistable mechanisms have been used in switches, valves, relays, crash sensors, and positioners (Hyman and Mehregany, 1998; Jensen, Howell, and Salmon, 1999; Howell, 2001; Gomm, Howell, and Selfridge, 2002; Masters and Howell, 2003; Hwang, Shim, and Lee, 2003; Qiu, Lang, and Slocum, 2004; Oberhammer et al., 2006; Luharuka and Hesketh, 2007; Sönmez, 2007; Sönmez and Tutum, 2008; Gerson et al., 2012; Chen, Zhang, and Li, 2013; Wang, Chen, and Pham, 2014). Compliant multistable mechanisms can be partially compliant (some motion achieved through traditional hinges or slides) or fully compliant (all motion achieved through deflection of flexible components) (Midha, Norton, and Howell, 1994). Compared with partially compliant multistable mechanisms, the fully compliant counterparts exhibit advantages such as: elimination of friction, free of assembly and maintenance, increased efficiency, and ability to be monolithically fabricated. However, designing a fully compliant multistable mechanism is nontrivial because of the nonlinearities in both the multistable characteristics and the large deflections of the flexible segments.

Compliant multistable mechanisms have been the subject of much research. Qiu, Lang, and Slocum (2004) proposed a fully compliant bistable mechanism utilizing buckling behaviors of flexible beams. Masters and Howell (2003) and Wilcox and Howell (2005a,b) devised fully compliant bistable mechanisms by using tensural pivots which are subject primarily to bending and tension loads. Jensen and Howell (2003, 2004) studied compliant mechanisms whose pseudo-rigid-body models are four-bar mechanisms and developed
theories to identify compliant bistable mechanisms. Pendleton and Jensen (2007, 2008) designed a tristable compliant mechanism based on a symmetric four-bar Grashof mechanism. Chen et al. proposed a tristable mechanism which incorporate flexible elements experiencing combined tension and bending (Chen, Wilcox, and Howell, 2009), a fully compliant tristable mechanism configuration employing orthogonal compliant mechanisms (Chen et al., 2010), a tristable mechanism consisting of two prestrained Young bistable mechanisms (Chen and Du, 2013), and a synthesis method of multistable compliant mechanisms employing a single bistable mechanism (Chen, Gou, and Zhang, 2011). Han et al. (2007) demonstrated a quadristable compliant mechanism by orthogonally connecting one buckling-beam bistable mechanism to another. Oh and Kota (2009) developed an approach for synthesizing compliant multistable mechanisms with more than three stable states by connecting multiple bistable mechanisms of different load thresholds in series. Li and Wang (2015) proposed a fluidic origami exhibiting multistable behaviors. Fang, Wang, and Li (2017) analyzed the asymmetric energy barrier of multistable stacked-origami to achieve a static mechanical diode effect. Zirbel et al. (2016) presented analytical and numerical models to create bistable mechanisms in unused materials for space applications. Kochmann and Bertoldi (2017) reviewed the state-of-the-art in utilizing instabilities in solids and structures to achieve beneficial effective performance. Bende et al. (2018) studied the origins of multistability of linked conical frusta using experiment and a mechanical model, they found that tuning geometrical parameters can provide axial bistability. Loukaides, Lewis, and Bowen (2019) demonstrated designing and manufacturing a bistable shell exploiting the residual thermal stress through an analytical example, numerical simulations and a physical demonstrator. Multiple approaches have been proposed for analyzing compliant mechanisms, including the pseudo-rigid-body model (Howell, 2001; Chase Jr et al., 2011; Jensen and Howell, 2003), the matrix method (Lobontiu et al., 2013; Lobontiu and Garcia, 2004), topology optimization (Joo and Kota, 2004) including for micro (Toone et al., 2014; Baichapur et al., 2014; Khan and
Ananthasuresh, 2014; Chen and Du, 2013; Chen, Wilcox, and Howell, 2009; Gerson et al., 2012) and macro (Yellowhorse and Howell, 2018; Chen, Zhang, and Li, 2013) systems.

In our previous work Chen, Gou, and Zhang (2011), a general partially compliant multistable mechanism configuration was proposed, which employs a single bistable mechanism and several link-slider modules. By replacing each link-slider module (shown in Figure 1(a)) in this configuration by a compliant end-effector (shown in Figure 1(b)), a general configuration of the fully compliant multistable mechanisms is obtained, as shown in Figure 2. For designing this kind of fully compliant mechanism, the major difficulty lies in the tradeoff between the stiffness and flexibility of the flexible segments in the end-effectors. The end-effectors must be flexible enough so that they could easily deform to provide motion and produce little returning force at deflected shapes; on the other hand, the end-effectors must be stiff enough to switch the bistable mechanism between its two stable equilibrium positions. Additionally, the energy stored in flexible segments acts to return the device to its undeflected position and causes high stresses, which should be considered in the design of fully compliant mechanisms.

This paper provides a systematic approach for the design of fully compliant multistable mechanisms, which enables designers to achieve fully compliant mechanisms with desired numbers of stable equilibria by utilizing a single bistable mechanism. The pseudo-rigid-body model (PRBM) is used to describe the displacements and force relationships of the end-effectors. A design approach is detailed, including determining configuration parameters according to the desired number of stable equilibrium positions, choosing a bistable mechanism, determining design parameters for the end-effectors with considering the bending stress and the critical buckling load, and calculating force-deflection characteristics or potential energy curves to verify the multistability. Two fully compliant multistable mechanisms with seven and eight stable equilibrium positions are presented to demonstrate the effectiveness of the design approach.
Models

One fully compliant bistable mechanism and several end-effectors are employed as building blocks in the general configuration in Figure 2. This section discusses the models of the two building blocks.

Fully Compliant Bistable Mechanism

The force-displacement characteristics of the fully compliant bistable mechanism is usually obtained by using finite element analysis (FEA) (Cherry, Howell, and Jensen, 2008) or chained beam constraint model (CBCM) (Chen and Ma, 2015; Ma and Chen, 2016, 2019). CBCM is a simple and efficient method for predicting the large deflection behavior of flexible beams. We built the finite element model and CBCM for a fully compliant bistable mechanism, whose design parameters (labeled in Figure 2) are listed in Table 1, and obtained its force-displacement characteristics.

Figure 3 compares the two force-displacement characteristics of the fully compliant bistable mechanism (with respect to coordinate system $u_1o_1v_1$ shown in Figure 2). The two curves are consistent, as shown in Figure 3. We use the results of the CBCM analysis (the solid line in Figure 3) to design the multistable mechanism.

As shown in Figure 3, the force-displacement characteristics is divided into three parts: the bistable behavior, the post-bistable behavior (Wilcox and Howell, 2005a), and the reverse behavior (Chen, Gou, and Zhang, 2011). The post-bistable behavior indicates that the shuttle of the bistable mechanism can be further deflected before failure when deflected past the second stable equilibrium position, and the reverse behavior means the shuttle is deflectable when pulled in the reverse direction from the as-fabricated position.

The whole force-displacement relationship for the bistable mechanism can be simply denoted as

$$F_b = f(z)$$ (1)

where $F_b$ is the output force of the bistable mechanism and $z$ is the displacement of the
shuttle from the as-fabricated position along the $u_1$-axis.

In Figure 3, $z_b$ is defined as the distance between the two stable equilibrium positions of the bistable mechanism. $F_s$ is defined as the critical force required to switch the bistable mechanism from the as-fabricated position to the second stable equilibrium position and $F_w$ is defined as the critical force that must be applied to return the mechanism from the second to the first stable equilibrium position. Their corresponding shuttle displacements (defined as $z_s$ and $z_w$) can be determined by analyzing the force-displacement characteristics of the bistable mechanism. If the bending stress in the flexible element ($\sigma = M_z w/(2I)$) exceeds the bending strength of the material, the bistable mechanism will lose its function. According to the solutions of CBCM, the effective motion range of the bistable mechanism can be determined and denoted as $[z_{\min}, z_{\max}]$.

**End-effector**

The configuration of the compliant multistable mechanism in Figure 2 includes several end-effectors connected in series. Each end-effector’s end-block (labeled as end-block $i$ for generality) is connected to the functioning-block of the previous end-effector (labeled as end-effector $i - 1$). The first end-effector is connected to the shuttle of the bistable mechanism and the shuttle is considered as functioning-block 0. The last functioning-block (labeled as functioning-block $n$) is the functioning-block of the multistable mechanism and has more than two stable equilibrium positions.

Because the deflections of the flexible segments in the end-effectors exceed the prediction capability of CBCM, pseudo-rigid-body model (PRBM) is used to analyze the behavior of the flexible segments in the end-effectors. The flexible segments in end-effectors are treated as small-length flexural pivots. Figure 4 shows a single linkage of the PRBM of an end-effector at a deformed position, whose end-block and functioning-block are actually the functioning-block of the previous end-effector and the end-block of the subsequent end-effector, respectively. A local coordinate system $u_i o_i v_i$ is established for each
end-effector, as shown in Figure 2. Each local coordinate system \( u_i o_i v_i \) rotates anticlockwise 90 degrees from the previous local coordinate system \( u_{i-1} o_{i-1} v_{i-1} \). The displacements of functioning-block \( i \) along the \( u_i \)- and \( v_i \)-axes (labeled as \( D_{ui} \) and \( D_{vi} \)) are calculated as

\[
\begin{align*}
D_{ui} &= L_i \left[ \cos \alpha_i - \cos(\Theta_i - \alpha_i) \right] \\
D_{vi} &= L_i \left[ \sin(\Theta_i - \alpha_i) + \sin \alpha_i \right]
\end{align*}
\]

where \( L_i \) is the length of the pseudo-rigid-body link, \( \Theta_i \) is the pseudo-rigid-body angle of the flexible segment on the fixed side of end-effector \( i \), and \( \alpha_i \) is the initial inclined angle of the flexible-rigid-flexible segment sets in parallel in end-effector \( i \) with respect to the \( u_i \)-axis.

As shown in Figure 4, end-block \( i \) (i.e. functioning-block \( i - 1 \)) moves along not only the \( u_i \)-axis but also the \( v_i \)-axis, and the displacements are actually \( D_{v(i-1)} \) and \( D_{u(i-1)} \), which can be calculated as:

\[
\begin{align*}
D_{u(i-1)} &= -L_i \left[ \sin(\Theta_i - \alpha_i) - \sin(\hat{\Theta}_i - \alpha_i) \right] \\
D_{v(i-1)} &= L_i \left[ 2 \cos \alpha_i - \cos(\hat{\Theta}_i - \alpha_i) - \cos(\Theta_i - \alpha_i) \right]
\end{align*}
\]

where \( \hat{\Theta}_i \) is the pseudo-rigid-body angle of the flexible segment attached to end-block \( i \), which is different from \( \Theta_i \) due to the motion of end-block \( i \) along the \( v_i \)-axis. For the shuttle, we have \( D_{u0} = 0 \) and \( D_{v0} = z \), therefore, \( \Theta_1 = \hat{\Theta}_1 \). Given \( z \), \( \Theta_i \) and \( \hat{\Theta}_i \) can be obtained by simultaneously solving Equations (2) and (3). One position of end-block \( i \) corresponds to two positions of functioning-block \( i \), of which one has \( D_{vi} < 0, \Theta < 0, \hat{\Theta} < 0 \), the other has \( D_{vi} > 0, \Theta > 0, \hat{\Theta} > 0 \).

The free-body diagram of links 1, 2 and 3 for end-effector \( i \) is shown in Figure 5. The
static moment equilibrium equations for links 1 and 3 can be expressed as

\[
\begin{align*}
4K_i \Theta_i & - (F_{ui} + F_{v(i+1)} \sin(\Theta_i - \alpha_i) - (F_{vi} - F_{u(i+1)} \cos(\Theta_i - \alpha_i) = 0 \\
4K_i \dot{\Theta}_i & - F_{ui} L_i \sin(\dot{\Theta}_i - \alpha_i) + F_{vi} L_i \cos(\dot{\Theta}_i - \alpha_i) = 0
\end{align*}
\]  

(4)

where \( K_i \) is the stiffness of the small-length flexural pivots in end-effector \( i \).

**Force-displacement Characteristics and Potential Energy**

When functioning-block \( n \) moves along the \( v_n \)-axis from the fabricated position to other stable equilibrium positions, the bistable mechanism switches between its two stable equilibrium positions. Given the displacement of functioning-block \( n \) (i.e. \( D_{vn} \)), the corresponding \( z \) can be calculated from Equation (2)-(3) and \( F_b \) can be determined from the force-displacement characteristics of the bistable mechanism, then the actuation force applied on functioning-block \( n \) (i.e. \( F_{u(n+1)} \)) can be calculated by solving Equation (4). Furthermore, the force-displacement characteristics (\( F_{u(n+1)} \) versus \( D_{vn} \)) can be achieved, from which we can know whether the mechanism possesses the required stable equilibrium positions or not.

The potential energy stored in the multistable mechanism is calculated by

\[
V = \int_0^{D_{vn}} F_{u(n+1)} dD_{vn}
\]

(5)

Thus the potential energy curve (\( V \) versus \( D_{vn} \)) of the multistable mechanism can be plotted. The minima of the curve represent the stable equilibrium positions of the mechanism.

**Design Approach**

To design a fully compliant multistable mechanism, we should determine the configuration parameters according to the number of the stable equilibrium positions
desired at first, then choose the bistable mechanism and determine the design parameters
for the end-effectors, and finally verify the stable equilibria by plotting the
force-displacement characteristics or the potential energy curve of the mechanism. The
approach is discussed in the following.

**Determination of Configuration Parameters**

The configuration of a fully compliant multistable mechanism is decided by three
parameters: the number of the end-effectors (labeled as \( n \)), the initial inclined angle of each
end-effector (\( \alpha_i \)) and the placement of the bistable mechanism (denoted as a flag \( p \), where \( p \)
equals 0 or 1).

When the initial inclined angle \( \alpha_i \) is larger than 0 °, each position of end-block \( i \) (i.e.
functioning-block \( i - 1 \)) corresponds to two positions of functioning-block \( i \); while for
\( \alpha_i = 0 \) °, functioning-block \( i \) loses one stable equilibrium position, which is similar to the
synthesis approach for \( l_i > a_i \) and \( l_i = a_i \) in Reference Chen, Gou, and Zhang (2011).
Therefore, \( 2^n + 1 \sim 2^{n+1} \) stable equilibrium positions can be achieved from the two stable
equilibrium positions of the bistable mechanism connected with \( n \) end-effectors in series.

In the general configuration, the bistable mechanism can be placed in two patterns at
the as-fabricated position. One is that the shuttle moves along the negative \( u_1 \)-axis to reach
the second stable equilibrium position, as shown in Figure 2, which is denoted as \( p = 1 \); the
other is that the shuttle moves along the positive \( u_1 \)-axis to reach the second stable
equilibrium position, which is denoted as \( p = 0 \). When \( p = 1 \), functioning-block 1 moves
along positive \( v_1 \)-axis to reach all its stable equilibrium positions, while when \( p = 0 \),
functioning-block 1 must move along both positive and negative \( v_1 \)-axis to reach all its
stable equilibrium positions, which contributes to reducing the maximum deflection and
bending stress in the flexible segments in end-effector 1, but leads to the possibility of
losing one stable equilibrium position when \( n > 1 \), because functioning-block 1 could not
reach the stable equilibrium position at the negative \( v_1 \)-axis if \( \alpha_2 \) is not large enough. For
instance, the mechanism with \( n = 2, \alpha_1 > 0, \alpha_2 = 0 \) has seven stable equilibrium positions for \( p = 1 \), as shown in Figure 6; or has five stable equilibrium positions for \( p = 0 \), as shown in Figure 7.

Table 2 presents a look-up table including the total number (labeled as \( N_m \)) of stable equilibrium positions of the fully compliant multistable mechanism configurations consisting of a bistable mechanism and one or two end-effectors, which can be used for a guide for designing fully compliant multistable mechanisms with the desired number of stable equilibrium positions.

Some configurations in Table 2 can be used as three types of multistable mechanisms with different \( N_m \) for different \( \alpha_i \). As shown in Figure 8 (the configuration with \( n = 1, p = 0, \alpha_1 > 0 \)), if \( \alpha_1 \) is too small to allow end-block 1 (i.e. the shuttle) to reach the second stable equilibrium position, this configuration has only two stable equilibrium positions (shown in Figure 8(a)); if \( \alpha_1 \) takes a precise value that causes the shuttle reaching the second stable equilibrium position when \( \Theta_1 = \hat{\Theta}_1 = \alpha_1 \) (shown as solid line in Figure 8(b)), the configuration has three stable equilibrium positions; while if \( \alpha_1 \) is large enough, the configuration has four stable equilibrium positions, as shown in Figure 8(c). This configuration is used as a quadristable mechanism (with four stable equilibrium positions) in practice, because the bistable and tristable mechanisms have simpler configurations.

In addition, there are multiple configurations in Table 2 that could provide a specific number of stable equilibrium positions. For instance, four configurations in Table 2 can be used as quadristable mechanisms, of which two configurations have simpler structures \((n = 1)\), as shown in Figure 9 and Figure 10. The configuration in Figure 10 \((p = 1)\) needs a larger initial inclined angle to reach all the stable equilibrium positions, which results in larger deformation and higher stress in the flexible segments, hence the configuration in Figure 9 \((p = 0)\) is the best choice. In Table 2, the configurations with “*” are recommended for their relatively small maximum of deformations of the flexible segments. The design approach discussed below are for these recommended configurations.
Determination of Bistable Mechanism

As the post-bistable behavior and reverse behavior of the bistable mechanism are both used in the fully compliant multistable mechanisms, the bistable mechanisms with larger motion ranges are preferred over those with smaller motion ranges. Considering that returning forces are produced in deflected end-effectors, the bistable mechanism should have large critical forces to hold the end-effectors at different stable equilibrium positions. Some suggestions about the dimensions of the fully bistable mechanism can be obtained from the literature Ma and Chen (2016); Dunning et al. (2012).

Determination of Design Parameters for End-effectors

The main design parameters for end-effectors include the initial inclined angles, the lengths, the stiffnesses and the in-plane thicknesses of the flexible segments.

**Length \( l \) and initial inclined angle \( \alpha \).** As shown in Figure 4, the length of the flexible segments \( l_i \) in end-effector \( i \) can be calculated as

\[
l_i = L_i - L_{r_i}
\]

where \( L_{r_i} \) is the length of the rigid segments in end-effector \( i \) and is decided by the dimensions of the mechanism.

When \( \alpha_i > 0 \), \( \alpha_i \) is determined from the motion range \([z_{\text{min}}, z_{\text{max}}]\) of the bistable mechanism as follows.

For \( n = 1 \), the shuttle reaches its minimum displacement along the \( u_1 \)-axis at the position where \( \Theta_1 = \alpha_1 \), as shown in Figure 9(c) or Figure 10(c). The minimum displacement of the shuttle is calculated as

\[
z \mid _{\Theta_1 = \alpha_1} = 2L_1 (\cos \alpha_1 - 1)
\]

\( z \mid _{\Theta_1 = \alpha_1} \) should be larger than \( z_{\text{min}} \). When \( p = 0 \), the bistable mechanism exhibits its
reverse behavior, and its minimum displacement is labeled as $z_r$ (as shown in Figure 9(c)); when $p = 1$, the bistable mechanism exhibits the post-bistable behavior, and its minimum displacement is labeled as $z_p$ (as shown in Figure 10(c)), thus $z_{\theta_1=\alpha_1}$ must be smaller than $z_b$ ($z_b$ is negative for $p = 1$) to ensure the shuttle reaching the second stable equilibrium position, that is

$$\begin{cases} z_{\min} < z_{\theta_1=\alpha_1} < 0, & p = 0 \\ z_{\min} < z_{\theta_1=\alpha_1} < z_b, & p = 1 \end{cases}$$

Solving equation (7) and inequality (8) for $\alpha_1$ yields

$$\begin{cases} 0 < \alpha_1 < \arccos(1 + \frac{z_{\min}}{2L_1}), & p = 0 \\ \arccos(1 + \frac{z_b}{2L_1}) < \alpha_1 < \arccos(1 + \frac{z_{\min}}{2L_1}), & p = 1 \end{cases}$$

For $n = 2$, $\alpha_1$ is determined as stated above, then the deformation at $\Theta_2 = \alpha_2$ is discussed to determine $\alpha_2$. Function-block 1 reaches its minimum displacement along the $u_2$-axis when $p = 0$, and the shuttle is pushed towards but is not required to jump to its second stable equilibrium position, as shown in Figure 11(a), hence $z$ must be smaller than $z_s$ at $\Theta_2 = \alpha_2$; when $p = 1$, the shuttle is pulled to its maximum displacement along the $u_1$-axis, then $z$ must be smaller than $z_{\max}$ at $\Theta_2 = \alpha_2$, as shown in Figure 11(b). That is

$$\begin{cases} 0 < z_{\theta_2=\alpha_2} < z_s, & p = 0 \\ 0 < z_{\theta_2=\alpha_2} < z_{\max}, & p = 1 \end{cases}$$

Then $\alpha_2$ could be determined by solving inequality (10).

Similarly, when $n > 2$, consider the deformations of the shuttle at $\Theta_1 = \alpha_1, \Theta_2 = \alpha_2, \ldots, \Theta_n = \alpha_n$ in order to determine $\alpha_1, \alpha_2, \ldots, \alpha_n$.

**Stiffness $K$.** The end-effectors in their deformed position produce a spring-back force to the bistable mechanism, which is output by end-block 1 in the $u_1$-direction and actually the reaction force to $F_{u_1}$. This spring-back force must be balanced by $F_b$ (provided
by the bistable mechanism) applied on end-block 1 to deflect the end-effectors at a specific position. If the end-effectors are too stiff, they can not be held at the stable equilibrium positions, whereas if the end-effectors are too flexible, they can not switch the bistable mechanism. Therefore, a trade-off is needed between flexibility and stiffness of the flexible segments in the end-effectors.

The ranges of the stiffnesses depend on the critical forces of the bistable mechanism, the buckling critical load of the flexible segments and the maximum actuation force. The stiffness of the flexible segments in end-effector 1 is determined at first.

For the configuration with \( p = 1 \), at the stable equilibrium position in Figure 10(b), \( F_{u1} \) must be smaller than \( F_w \) to prevent the shuttle from returning to the as-fabricated position, and two relative maximum stiffnesses (denoted as array \( K_w \)) can be solved by substituting \( F_{u2} = 0, F_{v2} = 0, F_{u1} = F_w \) and \( z = z_w \) into Equations (2)-(4). Similarly, at the fourth stable equilibrium position (shown in Figure 10(e)), \( F_{u1} \) must be smaller than \( F_s \), two relative maximum stiffness (denoted as array \( K_s \)) can be solved by substituting \( F_{u2} = 0, F_{v2} = 0, F_{u1} = F_s \) and \( z = z_s \) into Equations (2)-(4). For the configuration with \( p = 0 \), at the stable positions shown in Figure 9(a) and (e), the bistable mechanism is both at the second stable position, thus only \( F_{u1} < F_w \) at \( z = z_w \) needs to be considered. There will be two solutions to Equations (2)-(4), which are denoted as \( K_w \) (For generality, \( K_s = K_w \) is supposed for \( p = 0 \)), then the maximum stiffness of the flexible segments can be determined as

\[
K_{\text{max}} = \min\{K_s, K_w\}
\]  

(11)

For the configurations with \( p = 1 \), the flexible segments in end-effector 1 are compressed by the shuttle during the motion along the \( v_1 \)-axis from the as-fabricated position to the second stable equilibrium position shown in Figure 10(b). If the maximum compressive load exceeds the buckling critical load of the flexible segments in end-effector 1, end-effector 1 will fail to switch the bistable mechanism and jump directly to the fourth stable equilibrium position without reaching the second and third stable equilibrium.
positions. The buckling critical load of a fixed-fixed inclined beam is given as Zhang and Chen (2013)

\[ P_{cr} = \frac{4\pi^2 K_1}{l_1} \sin(\frac{\pi}{2} - \alpha_1) \]  \hspace{1cm} (12)

During the motion of the shuttle traveling from the as-fabricated position to the second stable equilibrium position, the maximum axial compression force acting on the flexible segments in end-effector 1 is \( F_s / \cos \alpha_1 \). Therefore, the following inequality is established

\[ \frac{4\pi^2 K_1 \sin(\frac{\pi}{2} - \alpha_1)}{l_1} > \frac{F_s}{2} \]  \hspace{1cm} (13)

Solving for \( K \) yields

\[ K_1 > \frac{F_s l_1}{8\pi^2 \cos \alpha_1} \]  \hspace{1cm} (14)

The value of the right-hand side of inequality (14) is defined as \( K_{cr} \).

For the configurations with \( p = 0 \), the flexible segments are compressed only during the motion from the as-fabricated position (shown in Figure 9(b)) to the third stable equilibrium position (shown in Figure 9(d)), while the bistable mechanism exhibits the reverse behavior and buckling would not cause losing the stable equilibrium position. For generality, \( K_{cr} = 0 \) is supposed when \( p = 0 \).

The flexible segments in end-effector 1 have the largest angular deflection in the fourth stable equilibrium position, where the maximum actuation force (labeled as \( F_{ac} \)) is needed to actuate functioning-block 1 to the previous stable equilibrium position. \( F_{ac} \) is transmitted by the end-effectors to \( F_{u1} \) to switch the shuttle to the second stable equilibrium position, therefore, \( F_{u1} \) must be larger than \( F_s \) at \( D_{v0} = z_s \) for \( p = 1 \) or be larger than \( F_w \) at \( D_{v0} = z_w \) for \( p = 0 \). By substituting \( F_{u(n+1)} = F_{ac} \), \( F_{v(n+1)} = 0 \), \( F_{u1} = F_s \) (or \( F_{u1} = F_w \)) and \( D_{v0} = z_s \) (or \( D_{v0} = z_w \)) into Equations (2)-(4), a relative minimum stiffness is obtained, denoted as \( K_{ac} \). Hence the minimum stiffness of the flexible segments
in end-effector 1 can be determined as

\[ K_{\text{min}} = \max\{K_{ac}, K_{cr}\} \quad (15) \]

No matter how many end-effectors are employed, functioning-block 1 has four stable equilibria at most, which are all discussed as above. For the configurations with \( n > 1 \), assume \( n = 1 \) and calculate \( K_{1\text{max}} \) and \( K_{1\text{min}} \) at first. After determining \( K_1 \), assume \( n = 2 \) and calculate \( K_{2\text{min}} \) and \( K_{2\text{max}} \) from Equations (11)-(15). \( K_3, \ldots, K_n \) can be determined by the same method.

If \( K_{i\text{max}} < K_{i\text{min}} \), reduce the initial inclined angle and recalculate \( K_{i\text{max}} \) and \( K_{i\text{min}} \) until \( K_{i\text{max}} > K_{i\text{min}} \).

**In-plane thickness \( h \).** The stiffness of the small-length flexural pivot is given as

\[ K = \frac{EI}{l} = \frac{Ebh^3}{12l} \quad (16) \]

where \( I \), \( h \) and \( b \) are the moment of inertia, the in-plane and the out-of-plane thickness of the small length flexural pivot, respectively. The in-plane thickness \( h \) affects the stiffness significantly. \( h \) can be rewritten as

\[ h = \sqrt[3]{\frac{12 Kl}{Eb}} \quad (17) \]

By substituting \( K = K_{i\text{max}} \) and \( K = K_{i\text{min}} \) into Equation (17), two extremum values of \( h \) are achieved. One is corresponding to \( K_{i\text{min}} \), which is the minimum in-plane thickness, denoted as \( h_{i\text{min}} \); the other is corresponding to \( K_{i\text{max}} \), denoted as \( h_{ikm} \).

For the flexible segments in the fully compliant multistable mechanisms, bending is the predominate loading condition. The maximum bending stress occurs at the middle of the flexible segment at the position of the maximum deflection, and can be calculated as

\[ \sigma_{\text{max}} = \frac{Eh_i\Theta_{i\text{max}}}{2l_i} \quad (18) \]
where $\Theta_{i,\text{max}}$ is the maximum angular deflection of the flexible segment, corresponding to the maximum $D_{vi}$. The maximum bending stress in the flexible segments must not exceed the bending strength of the material (denoted as $\sigma_b$), i.e., $\sigma_{\text{max}} < \sigma_b$. Therefore, the in-plane thickness of the flexible segments satisfies

$$h_i < \frac{2l_i \sigma_b}{E \Theta_{i,\text{max}}} \quad (19)$$

The value of the right-hand side of inequality (19) is defined as $h_{is}$. By comparing $h_{is}$ with $h_{ikm}$ calculated from $K_{i,\text{max}}$, the maximum in-plane thickness of the flexible segments in end-effector $i$ is determined as

$$h_{i,\text{max}} = \min\{h_{ikm}, h_{is}\} \quad (20)$$

If $h_{i,\text{max}} < h_{i,\text{min}}$, $l_i$ should be increased or $\alpha_i$ should be decreased to obtain smaller $\Theta_i$ and larger $h_{is}$.

The design flow chart is shown in Figure 12. Given all the design parameters and the motion path of the bistable mechanism, the actuation force $F_{u(n+1)}$ is calculated by using the design approach above, then the force-displacement characteristics and potential energy curves can be plotted to verify whether or not the mechanism has the desired stable equilibrium positions. A voice coil motor is one of many approaches for actuating the multistable mechanisms.

Note that a change in material properties (assuming they remain linear elastic) results in changes to the actuation force required to transition between stable equilibrium positions but does not change the location of the positions. Changes in geometry, especially those that change the relative stiffnesses, can result in changes in the required actuation forces, the location of the stable equilibrium positions, and whether or not it is multi-stable.
Design Examples

This section demonstrates the design processes of two fully compliant multistable mechanisms with seven and eight stable equilibrium positions, respectively. We assume that each design is fabricated from a polypropylene sheet of 6 mm thick \((b = 6 \text{ mm})\) with the Young’s modulus \(E = 1.4 \text{ GPa}\) and the bending strength \(\sigma_b = 65 \text{ MPa}\).

Fully Compliant Octostable Mechanism

The fully compliant octostable mechanism has eight stable equilibrium positions \((N_m = 8)\). For simplicity, we assume \(K_1 = K_2 = K\) and \(h_1 = h_2 = h\). The process of designing is as following:

**Step 1** The configuration parameters \(n = 2, p = 1, \alpha_1 > 0 \, ^\circ\) and \(\alpha_2 > 0 \, ^\circ\) are obtained according to Table 2.

**Step 2** The design parameters of the bistable mechanism are the same as those shown in Table 1, except \(N_p = 3\). The two critical forces of the bistable mechanism are \(F_s = 14.68 \text{ N}\) and \(F_w = -14.42 \text{ N}\), whose corresponding displacements of the shuttle are \(z_s = -2.2 \text{ mm}\) and \(z_w = -8.1 \text{ mm}\). The minimum and maximum displacements of the shuttle are \(z_{\text{min}} = -11.3 \text{ mm}\) and \(z_{\text{max}} = 3.0 \text{ mm}\).

**Step 3** The lengths of the pseudo-rigid-body links and the lengths of the rigid segments in the two end-effectors are given as \(L_1 = 68 \text{ mm}, L_2 = 70 \text{ mm}, Lr_1 = 62 \text{ mm}\) and \(Lr_2 = 64 \text{ mm}\). The lengths of the flexible segments are determined as \(l_1 = 6 \text{ mm}\) and \(l_2 = 6 \text{ mm}\) using Equation (6). \(20.96 \, ^\circ < \alpha_1 < 23.52 \, ^\circ\) is obtained using Equation (9), according to which \(\alpha_1 = 22 \, ^\circ\) is determined eventually. \(\alpha_2 < 12.75 \, ^\circ\) is calculated by substituting \(D_{u0} = 0, \Theta_2 = \alpha_2\) and \(D_{v0} = z_{\text{max}}\) into Equation (2)-(3) and \(\alpha_2 = 7 \, ^\circ\) is used in this work.

**Step 4** According to Equation (4), given \(F_{u3} = 0\) and \(F_{v3} = 0\), the stiffness of the flexible segments is expressed as

\[
K = \frac{F_{u1}L_1L_2 \sin(\Theta_1 - \alpha_1) \sin(\Theta_2 + \Theta_2 - 2\alpha_2)}{2L_1[\Theta_2 \cos(\Theta_1 + \Theta_2 - \alpha_1 - \alpha_2) + \Theta_2 \cos(\Theta_1 - \Theta_2 - \alpha_1 + \alpha_2)] + 4\Theta_1L_2 \sin(\Theta_2 + \Theta_2 - 2\alpha_2)}
\]
Four groups of pseudo-rigid-body angles corresponding to $z_s$ and $z_w$ can be calculated from Equation (2)-(3), by substituting them and $F_{u1} = \{F_s, F_w\}$ into Equation (21), we obtain $K_s = \{0.063, 0.071\} \text{N} \cdot \text{m-rad}^{-1}$ and $K_w = \{0.047, 0.067\} \text{N} \cdot \text{m-rad}^{-1}$. Thus $K_{\text{max}} = 0.047 \text{N} \cdot \text{m-rad}^{-1}$ is determined from Equation (11).

According to Equation (4), given $F_{u3} = F_{ac}$ and $F_{v3} = 0$, $K_{ac}$ can be expressed as

$$K_{ac} = \frac{-L_1 L_2 F_{ac}[\cos(\Theta_1 - \Theta_2 + \Theta_2 - \alpha_1) + \cos(\Theta_1 + \Theta_2 + \Theta_2 - \alpha_1 - 2\alpha_2)]}{8L_1[\Theta_2 \cos(\Theta_1 + \Theta_2 - \alpha_1 - \alpha_2) + \Theta_2 \cos(\Theta_1 - \Theta_2 - \alpha_1 + \alpha_2)] + 16L_2 \Theta_1 \sin(\Theta_2 + \Theta_2 - 2\alpha_2)}$$

$$- \frac{L_1 L_2 F_{u1}[\cos(\Theta_1 + \Theta_2 + \Theta_2 - \alpha_1 - 2\alpha_2) - \cos(\Theta_1 - \Theta_2 - \Theta_2 - \alpha_1 + 2\alpha_2)]}{4L_1[\Theta_2 \cos(\Theta_1 + \Theta_2 - \alpha_1 - \alpha_2) + \Theta_2 \cos(\Theta_1 - \Theta_2 - \alpha_1 + \alpha_2)] + 8L_2 \Theta_1 \sin(\Theta_2 + \Theta_2 - 2\alpha_2)}$$

(22)

The maximum actuation force provided by the actuator is given as 35 N. By substituting $F_{ac} = 35 \text{ N}$, $F_{u1} = F_s$ and the pseudo-rigid-body angles corresponding to $z_s$ into Equation (22), we obtain $K_{ac} = \{0.0085, 0.0084\} \text{N} \cdot \text{m-rad}^{-1}$. $K_{cr} = 0.0012 \text{N} \cdot \text{m-rad}^{-1}$ is achieved from Equations (14). Therefore, $K_{\text{min}} = 0.0085 \text{N} \cdot \text{m-rad}^{-1}$ is determined from Equation (15).

**Step 5** $h_{km} = 0.73 \text{ mm}$ and $h_{\text{min}} = 0.42 \text{ mm}$ are calculated from Equation (17). End-effector 2 reaches its maximum deflection angle when $z = 0$, and $\Theta_2_{\text{max}} = 1.013 \text{ rad}$ is calculated from Equations (2)-(3). Substituting $\Theta_2_{\text{max}}$ and $\sigma_b$ into Equation (19) results in $h_s = 0.55 \text{ mm}$. Then the maximum in-plane thickness of the flexible segment is determined from Equation (20) as $h_{\text{max}} = 0.55 \text{ mm}$. $h = 0.5 \text{ mm}$ is used in this work. The design parameters for the two end-effectors are listed in Table 3.

**Step 6** The minimum and maximum displacements of the shuttle, i.e. $z_p$ and $z_r$ (shown in Figure 10(c) and Figure 11(b)), can be calculated from Equation (2)-(3). When functioning-block 2 moves from the fabricated position to the stable equilibrium position of the maximum $D_{c2}$, the shuttle moves from 0 to $z_r$ at first, then from $z_r$ to $z_p$, finally from $z_p$ to 0; when functioning-block 2 moves from the fabricated position to the stable equilibrium position of the minimum $D_{c2}$, the shuttle moves from 0 to $z_p$ then from $z_p$ to 0. Given the motion path of the shuttle as $0 \rightarrow z_p \rightarrow 0 \rightarrow z_r \rightarrow 0 \rightarrow z_p \rightarrow 0$, the corresponding output forces of the bistable mechanism ($F_b$) can be obtained by linear interpolation using the
output-force-displacement characteristics of the bistable mechanism, and the corresponding pseudo-rigid-body angles $\Theta_1, \Theta_2, \hat{\Theta}_2$ and the displacements $D_{v2}$ can be calculated from Equation (2)-(3). According to Equation (4), the actuation force $F_{u3}$ can be expressed as

\[
F_{u3} = -\frac{4KL_1[\Theta_2 \cos(\Theta_1 + \hat{\Theta}_2 - \alpha_1 - \alpha_2) + \hat{\Theta}_2 \cos(\Theta_1 - \Theta_2 - \alpha_1 + \alpha_2)]}{L_1 L_2 \cos(\Theta_2 - \alpha_2) \cos(\Theta_1 + \hat{\Theta}_2 - \alpha_1 - \alpha_2)} - \frac{4KL_1 L_2 \sin(\Theta_2 + \hat{\Theta}_2 - 2\alpha_2)[4K\Theta_1 - F_{u1} L_1 \sin(\Theta_1 - \alpha_1)]}{L_1 L_2 \cos(\Theta_2 - \alpha_2) \cos(\Theta_1 + \hat{\Theta}_2 - \alpha_1 - \alpha_2)}
\] (23)

By substituting $F_{u1} = F_b$ and the pseudo-rigid-body angles into Equation (23), the output forces ($-F_{u3}$) of the octostable mechanism can be calculated, and the corresponding potential energy $V$ can be calculated from Equation (5).

The output-force-displacement characteristics ($-F_{u3}$ versus $D_{v2}$) is shown in Figure 13 and the potential energy curve ($V$ versus $D_{v2}$) is shown in Figure 14. It can be seen that there are 15 zero points in the output-force-displacement characteristics. Eight zero points are corresponding to local minima (labeled as "$S_1", "S_2", \cdots, "S_8"$) and seven zero points are corresponding to local maxima (labeled as "$U_1", "U_2", \cdots, "U_7"$) in the potential energy curve, respectively, which indicates that the mechanism has eight stable equilibrium positions as expected.

A finite element model of the octostable mechanism is built and analyzed in the commercial software ANSYS. The simulation results show the eight stable equilibrium positions. The fully compliant octostable mechanism is machined out of polypropylene sheet using the dimensions shown in Tables 1 and Table 3. Figure 15 demonstrates eight distinct stable equilibrium positions of the prototype. The predicted, simulated and measured values for the eight stable equilibrium positions of functioning-block 2 are listed in Table 4. To compare with the measured $D_{v2}$, the predicted $D_{v2}$ in the stable equilibrium positions should be read from the force-displacement characteristics or the potential energy curve rather than be calculated from Equation (2)-(3) using $D_{u0} = 0$ and $D_{v0} = z_b$, because the spring-back forces produced by the deflected end-effectors deviate the shuttle slightly
from its stable equilibrium positions.

**Fully Compliant Septistable Mechanism**

The fully compliant septistable mechanism \((N_m = 7)\) has the configuration of \(n = 2, p = 1, \alpha_1 > 0^\circ, \alpha_2 = 0^\circ\) according to Table 2. The design parameters of the bistable mechanism are the same as which used in the fully compliant octostable mechanism. The lengths of the pseudo-rigid-body links and the lengths of the rigid segments in the two end-effectors are given as \(L_1 = 68\) mm, \(L_2 = 66\) mm, \(L_{r1} = 62\) mm and \(L_{r2} = 60\) mm, \(F_{ac} = 35\) N, then the design parameters for the two end-effectors are obtained by the same method used in the design for the fully compliant octostable mechanism, which are listed in Table 5.

When functioning-block 2 moves from the fabricated position to the stable equilibrium position of the maximum or the minimum \(Dv_2\) along the \(v_2\)-axis, the shuttle moves from 0 to \(z_p\) then from \(z_p\) to 0. Given the motion path of the bistable mechanism as \(0 \rightarrow z_p \rightarrow 0 \rightarrow z_p \rightarrow 0\), where \(Dv_2\) varies from the minimum to the maximum, corresponding \(F_b\) can be obtained by linear interpolation using the output-force-displacement characteristics of the bistable mechanism, furthermore, the displacements \(Dv_2\), the output forces \(-F_{u3}\), and the potential energy \(V\) can be calculated from Equation (2)-(3), (23) and (5).

The output-force-displacement characteristics of the septistable mechanism \((-F_{u3} \text{ versus } Dv_2\) are plotted in Figure 16, and the potential energy curve \((V \text{ versus } Dv_2)\) are plotted in Figure 17. There are 13 zero points in Figure 16, represent 13 equilibrium positions of functioning-block 2, six of which are unstable (labeled as "\(U_1^\prime\)", "\(U_2^\prime\)", \(\cdots\), "\(U_6^\prime\)"), corresponding to the local maxima of the potential energy curve in Figure 17, and seven of which are stable (labeled as "\(S_1^\prime\)", "\(S_2^\prime\)", \(\cdots\), "\(S_7^\prime\)"), corresponding to the local minima of the potential energy curve.

It can be seen from Figure 16 and Figure 17 that the absolute forces and energy are
not symmetric about the as-fabricated position, because end-block 2 (i.e. functioning-block 1) moves along not only the \(u_2\)-axis but also the \(v_2\)-axis during the motion, which causes the increase of the critical force required to switch functioning-block 2 from the sixth to seventh stable equilibrium position.

Using the dimensions shown in Tables 1 and 5, a fully compliant septistable mechanism is machined out of polypropylene sheet and shown in Figure 18. As expected, the prototype successfully demonstrates seven stable equilibrium positions. The predicted and measured displacements of functioning-block 2 are listed in Table 6.

It can be seen from Table 4 and Table 6 that the pseudo-rigid-body model predicted the stable equilibrium positions. A main contribution to the differences is the imprecision in machining the prototype. Another factor is that the linear-elastic model was assumed while polypropylene has some nonlinear material characteristics and is susceptible to stress relaxation, and sections modeled as rigid are not perfectly rigid. In addition, tiny out-of-plane displacement in the measuring process has also been identified as a contributing factor to the differences between the measured and predicted displacements.

**Conclusion**

Based upon the general partially compliant configuration in Reference Chen, Gou, and Zhang (2011), this paper presented a design approach to fully compliant multistable mechanisms that consist of a single bistable mechanism and several end-effectors connected in series. The chained beam constraint model is used to accurately describe the force-displacement characteristics of the fully compliant bistable mechanism. The equations of the pseudo-rigid-body model of the end-effector are provided to calculate the displacements and forces when the end-block moves both in horizontal and vertical directions. The detailed design approach including choosing a configuration and bistable mechanism, determining the parameters for the end-effectors considering the buckling and bending stress, and verifying the multistability using the force-deflection characteristics
and potential energy curve is provided. Two fully compliant multistable mechanisms demonstrate the feasibility of the design approach.

Energy storage characteristics of the flexible segments offer the multistable mechanisms more than two distinct and repeatable stable states, and it requires no power input to keep the mechanism at each stable state. The fully compliant multistable mechanisms are able to be monolithically fabricated, and they return to their as-fabricated position without any disassembly when not in operation. Owing to these advantages, the fully compliant multistable mechanism can be useful in valves in fluidic applications, switches and positioners in optical applications.

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References


Hwang, Il Han, Yu Seok Shim, and Jong Hyun Lee. 2003. “Modeling and experimental
characterization of the chevron-type bi-stable microactuator.” *Journal of Micromechanics and Microengineering* 13 (6): 948.


Table 1

*Design parameters for a fully compliant bistable mechanism. \( N_p \) is the number of the bistable structure sets in parallel.*

<table>
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<tr>
<th>( E )</th>
<th>( N_p )</th>
<th>( b )</th>
<th>( L_{b1} )</th>
<th>( L_{b2} )</th>
<th>( L_{b3} )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
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<td>Initial inclined angles of end-effectors</td>
<td>Total number of stable equilibrium positions ($N_m$)</td>
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Table 3

*Design parameters for the end-effectors of a fully compliant octostable mechanism.*

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<th>$b$</th>
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<th>$L_2$</th>
<th>$l$</th>
<th>$h$</th>
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<td>62 mm</td>
<td>64 mm</td>
<td>6 mm</td>
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<td>7 °</td>
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Table 4

The predicted, simulated and measured values for the stable equilibrium positions of the fully compliant octostable mechanism. All units are in mm.

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Table 5

*Design parameters for the end-effectors of the fully compliant septistable mechanism.*

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<td>0.5 mm</td>
<td>22°</td>
<td>0°</td>
</tr>
</tbody>
</table>
Table 6

The predicted, simulated and measured values for the stable equilibrium positions of the fully compliant septistable mechanism. All units are in mm.

<table>
<thead>
<tr>
<th>Stable position</th>
<th>Predicted $D_{u2}$</th>
<th>Simulated $D_{u2}$</th>
<th>Measured $D_{u2}$</th>
<th>Predicted $D_{v2}$</th>
<th>Simulated $D_{v2}$</th>
<th>Measured $D_{v2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2nd</td>
<td>7.1</td>
<td>6.8</td>
<td>5.4</td>
<td>-29.7</td>
<td>-30.5</td>
<td>-28.1</td>
</tr>
<tr>
<td>3rd</td>
<td>14.0</td>
<td>14.0</td>
<td>18.7</td>
<td>-40.7</td>
<td>-42.6</td>
<td>-45.2</td>
</tr>
<tr>
<td>4th</td>
<td>25.3</td>
<td>25.8</td>
<td>25.6</td>
<td>-51.9</td>
<td>-55.1</td>
<td>-53.9</td>
</tr>
<tr>
<td>5th</td>
<td>9.5</td>
<td>9.0</td>
<td>8.9</td>
<td>34.1</td>
<td>34.8</td>
<td>31.1</td>
</tr>
<tr>
<td>6th</td>
<td>18.0</td>
<td>18.3</td>
<td>21.1</td>
<td>45.3</td>
<td>48.0</td>
<td>48.1</td>
</tr>
<tr>
<td>7th</td>
<td>25.4</td>
<td>27.4</td>
<td>27.2</td>
<td>52.0</td>
<td>55.8</td>
<td>54.8</td>
</tr>
</tbody>
</table>
Figure 1. A link-slider module and its fully compliant counterpart: end-effector.
Figure 2. A general configuration of a fully compliant multistable mechanism.
Figure 3. The force-displacement characteristics of a fully compliant bistable mechanism.
Figure 4. PRBM of end-effector $i$ at its as-fabricated position (dashed line) and deformed position (solid line), respectively.
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Figure 7. A schematic of a fully compliant quinquestable mechanism ($p = 0$).
Figure 8. A schematic of fully compliant multistable mechanisms with the same configuration but different stable equilibrium positions (shown as solid lines and dashed lines) for different initial inclined angle $\alpha$. The dotted lines in (a) shows a unstable equilibrium position where $\Theta_1 = \alpha_1, z < z_s$. 
Figure 9. A schematic of a fully compliant quadristable mechanism \((p = 0)\) illustrated in its four stable equilibrium positions and a middle unstable equilibrium position. The previous equilibrium position is shown as dashed lines.
Figure 10. A schematic of a fully compliant quadristable mechanism \((p = 1)\) illustrated in its four stable equilibrium positions and a middle unstable equilibrium position. The previous equilibrium position is shown as dashed lines.
Figure 11. A schematic of the deformations of fully compliant multistable mechanisms at $\Theta_2 = \alpha_2$. The as-fabricated position is shown as dashed lines.
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Figure 13. The force-displacement characteristics of the fully compliant octostable mechanism.
Figure 14. The potential energy curve of the fully compliant octostable mechanism.
Figure 15. A fully compliant octostable mechanism with eight stable equilibrium positions.
Figure 16. The output-force-displacement characteristics of the fully compliant septistable mechanism.
Figure 17. The potential energy curve of the fully compliant septistable mechanism.
Figure 18. A fully compliant septistable mechanism with seven stable equilibrium positions.