Heteromorphic to Homeomorphic Shape Match Conversion Toward Fully Automated Mesh Morphing to Match Manufactured Geometry

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Heteromorphic to Homeomorphic Shape Match Conversion Toward Fully Automated Mesh Morphing to Match Manufactured Geometry

Robert Ivan Yorgason

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

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The modern engineering design process includes computer software packages that require approximations to be made when representing geometries. These approximations lead to inherent discrepancies between the design geometry of a part or assembly and the corresponding manufactured geometry. Further approximations are made during the analysis portion of the design process. Manufacturing defects can also occur, which increase the discrepancies between the design and manufactured geometry. These approximations combined with manufacturing defects lead to discrepancies which, for high precision parts, such as jet engine compressor blades, can affect the modal analysis results.

In order to account for the manufacturing defects during analysis, mesh morphing is used to morph a structural finite element analysis mesh to match the geometry of compressor blades with simulated manufacturing defects. The mesh morphing process is improved by providing a novel method to convert heteromorphic shape matching within Sculptor to homeomorphic shape matching. This novel method is automated using Java and the NX API. The heteromorphic to homeomorphic conversion method is determined to be valid due to its post-mesh morphing maximum deviations being on the same order as the post-mesh morphing maximum deviations of the ideal homeomorphic case.

The usefulness of the automated heteromorphic to homeomorphic conversion method is demonstrated by simulating manufacturing defects on the pressure surface of a compressor blade model, morphing a structural finite element analysis mesh to match the geometry of compressor blades with simulated manufacturing defects, performing a modal analysis, and making observations on the effect of the simulated manufacturing defects on the modal characteristics of the compressor blade.
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NOMENCLATURE

$R^3$  Three-dimensional real coordinate space having the standard Euclidean structure

$(S, T, U)$  Parametric subspace of $R^3$ defined by the associated control volume

$i$  Index of a control point of a control volume in the $S^{th}$ direction of the $(S, T, U)$ parametric space

$j$  Index of a control point of a control volume in the $T^{th}$ direction of the $(S, T, U)$ parametric space

$k$  Index of a control point of a control volume in the $U^{th}$ direction of the $(S, T, U)$ parametric space

$l$  Number of control planes of a control volume in the $S^{th}$ direction of the $(S, T, U)$ parametric space minus 1

$m$  Number of control planes of a control volume in the $T^{th}$ direction of the $(S, T, U)$ parametric space minus 1

$n$  Number of control planes of a control volume in the $U^{th}$ direction of the $(S, T, U)$ parametric space minus 1

$P_{ijk}$  $(i, j, k)^{th}$ point of a control volume
CHAPTER 1. INTRODUCTION

In this chapter the problem statement is introduced and described, followed by the description of the research objectives and scope of this research. Finally, an outline is provided for the contents and organization of this thesis.

1.1 Problem Statement

The general engineering product cycle contains two major processes: design and manufacturing. Within the design process, geometry, materials, and processes are approximated or idealized in Computer Aided Design (CAD), Computer Aided Manufacturing (CAM), Finite Element Analysis (FEA), and Computational Fluid Dynamics (CFD). In contrast to the design process, the geometry, materials and processes within the manufacturing process replace the idealizations and approximations with their actual physical counterparts. These idealizations and approximations used in the design process lead to inherent discrepancies between the design geometry of a part or assembly and the corresponding manufactured geometry. The design geometry, already an approximation of the actual manufactured geometry, is approximated even further during simulation and analysis. This is done through the pre-processing step of meshing. The simulation and analysis results from the design process often do not match the actual results of the manufactured product because of the mesh approximations used in analysis and because of the discrepancies between design and actual manufactured geometry [1].

In order to determine simulation results representative of the actual manufactured geometries, an approximation to the manufactured geometry must be made and analyzed. There are options to accomplish this approximation. In the past, the most viable option was to create a CAD model from manufactured data and then create a mesh from the CAD model to approximate the CAD model and allow for analysis [2]. This method is very time-consuming and not very accurate [2]. As it becomes increasingly important to accurately predict analysis results of tightly
toleranced manufactured parts, industry is looking at mesh morphing to match manufactured geometry. Mesh morphing is the ability to move the nodes of a mesh based on CAD geometry to match manufactured geometry [2]. However, mesh morphing is still a mostly manual process which limits the ability to analyze every manufactured part that is produced. This limitation reduces the ability to produce analysis results of each manufactured part and reduces the ability to study the areas of the manufactured parts that are most sensitive to defects.

Turbomachinery parts (Figure 1.1), such as compressor blades, are high precision manufactured parts. The aforementioned limitations on mesh morphing do not allow for a realistic study to be performed on compressor blade manufacturing defects. This is due to the lengthy time that would need to be invested to manually morph the mesh of the CAD geometry to match each manufactured geometry. The problem is to improve the mesh morphing process by removing the necessary manual inputs required from the lack of a one-to-one relationship during mesh morphing to match manufactured geometry.

1.2 Research Objectives and Scope

The primary objectives of this thesis include developing a novel method, automating the method, and applying the method to an engineering problem. The novel method developed is a
method for converting many-to-one and one-to-many shape matching to one-to-one shape matching. This method is automated using the Java programming language combined with the NX Application Programming Interface (API) in the NetBeans Integrated Development Environment (IDE). The automated method is applied to an engineering problem to demonstrate the usefulness of the automated method.

To validate the success of these objectives, a compressor blade model is used and the results of the automated method are evaluated to demonstrate the how accurately the method performed. The scope of this research is limited to automation of many-to-one and one-to-many shape matching to one-to-one shape matching conversion of compressor blade geometries with small perturbation defects. Manufacturing bending defects are not included in this research. Other turbomachinery parts, such as fan blades, turbine blades, etc. are not included in this research. The only geometry included in this research is the portion of the compressor blade above the fillet to the tip. The defects studied in this research are limited to the pressure and suction surfaces of the compressor blade.

The unique and novel contribution that this research provides is a novel method for converting many-to-one and one-to-many to one-to-one shape matching that has been automated to be of practical use in the study of the effects of manufacturing defects on the modal characteristics of compressor blades.

1.3 Thesis Outline

Chapter 2 presents a review of relevant literature for the main topics discussed in this thesis, such as manufacturing defects, mesh morphing, and shape matching. Chapter 3 presents a detailed description of the methodology and implementation of the automated conversion of many-to-one and one-to-many shape matching to one-to-one shape matching and the process for determining the areas of the compressor blade surface that are most sensitive to manufacturing defects. Chapter 4 details the validation process and validation results the automated method from Chapter 3 on compressor blade surfaces with various defects. The compressor blade surfaces come from academic models with simulated manufacturing defects to provide a proof of concept for the automated method. Chapter 5 contains the demonstration of the automated method in Chapter 3 on a compressor blade model to observe the effects of manufacturing defects on the modal characteristics of
the aforementioned compressor blade. Chapter 6 provides conclusions from the results presented in Chapter 4, and it proposes enhancements to the automated method to make it applicable to a wider range of turbomachinery components and manufacturing defects.
CHAPTER 2. LITERATURE REVIEW

This chapter presents the relevant background information to the research contained in this thesis. The chapter begins with a discussion of shape matching to provide mathematical background into mesh morphing and the areas of mesh morphing that need improvement. Current approaches to accomplish mesh morphing in academia and industry are then presented to establish the present state of mesh morphing. Manufacturing defects are discussed next to provide background for the need of mesh morphing to match manufactured geometry. Finally, modal analysis is defined to describe the modal characteristics that will be compared for the compressor blades with manufacturing defects in Chapter 5 of this thesis.

2.1 Shape Matching

In this thesis, shape matching is in reference to the process of deforming or morphing an un-deformed shape to match the topology of a deformed version of the original shape. This section provides a brief background on shape matching, including Free-Form Deformation, Arbitrary Shape Deformation, the difference between heteromorphic and homeomorphic shape matching, Shape Registration, and Isogeometric Analysis.

2.1.1 Free-Form Deformation

Free-Form Deformation (FFD) is a technique developed by Sederberg and Parry for the parameterization and deformation of shapes [4]. The parameterization used in FFD is a mapping of $R^3$ Cartesian space $(X, Y, Z)$ to an $R^3$ parametric space $(S, T, U)$. This parametric space is defined by a control volume that consists of control points placed on a parallelepiped region. The lines between the control points serve to show the connectivity of the control points. The control points form a tensor product trivariate Bernstein polynomial such that any point $X$ has $(s, t, u)$ coordinates calculated in Equations 2.1 through 2.3 where $0 \leq s \leq 1$, $0 \leq t \leq 1$, and $0 \leq u \leq 1$. 
\[ s = \frac{T \times U \cdot (X - X_0)}{T \times U \cdot S} \quad (2.1) \]
\[ t = \frac{S \times U \cdot (X - X_0)}{S \times U \cdot T} \quad (2.2) \]
\[ u = \frac{S \times T \cdot (X - X_0)}{S \times T \cdot U} \quad (2.3) \]

The Cartesian coordinates of the control points are defined in Equation 2.4.

\[ P_{ijk} = X_0 + \frac{i}{l}S + \frac{j}{m}T + \frac{k}{n}U \quad (2.4) \]

The control points, \( P_{ijk} \), form \( l+1 \) planes in the \( S \) direction, \( m+1 \) planes in the \( T \) direction, and \( n+1 \) planes in the \( U \) direction. The Cartesian positions of the deformed nodes are determined by evaluating the vector valued trivariate Bernstein polynomial in Equation 2.5.

\[ X_{\text{FFD}} = \sum_{i=0}^{l} \left( \begin{array}{c} l \\ i \end{array} \right) (1-s)^{l-i} \left[ \sum_{j=0}^{m} \left( \begin{array}{c} m \\ j \end{array} \right) (1-t)^{m-j} \left[ \sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) (1-u)^{n-k} u^k P_{ijk} \right] \right] \quad (2.5) \]

Any geometry that is to be deformed must be located within the control volume. The positions of the nodes that define the geometry within the control volume are mapped from Cartesian space to the parametric space of the control volume. The parametric position of each of the nodes is constant throughout the deformation process. When the defining parametric space is deformed in Cartesian space, the nodes must also deform in Cartesian space in order to not deform in the defined parametric space.

Figure 2.1 shows a control volume defined by 27 control points containing a stereolithography (STL) body of a sphere on the left. On the right, the sphere is deformed by deforming the containing control volume. The FFD occurs as the center top control point is moved upward in Cartesian space resulting in the nodes on the sphere deforming to maintain their position in the parametric space.

While FFD is a powerful tool for shape deformation, there are disadvantages. The most crippling disadvantage is that FFD can only produce parallelepiped volume. [5]. Perry [5] presents
Figure 2.1: Free-Form Deformation Example of a Sphere.

Figure 2.2 and describes that the aforementioned disadvantage proves problematic for a ‘Y’ shaped object because of the inability to independently deform the different branches of the ‘Y’ as is shown in Figure 2.2b. If the branches are to be deformed independently, a single ‘Y’ shaped volumed is necessary. The closest that FFD can get is one control volume per branch of the ‘Y’. However, continuity is lost across the interfaces of the three different control volumes. Figure 2.2 displays the undeformed ‘Y’ geometry surrounded by an ASD volume on the left and the deformed ‘Y’ geometry surrounded by a deformed ASD volume on the right.

2.1.2 Arbitrary Shape Deformation

Arbitrary Shape Deformation (ASD) is similar to FFD, in that it also employs a control volume with control points to parameterize and deform the contained geometry. The main advantage of ASD over FFD is that it allows for control volume shapes that are not constrained to only being parallelepiped volumes [5] [6] [7]. Instead of being composed of a set of regular surfaces (FFD), ASD volumes are composed of irregular surfaces that define a set of tangent plane con-
tinuous Bezier volumes. This allows an ASD volume to be a generalization of a B-spline volume with the ability to more accurately model geometries with irregular topologies. The problematic ‘Y’ geometry’s branches can independently be deformed when modeled with an ASD volume, as shown in Figure 2.2 [5].

2.1.3 Homeomorphic vs. Heteromorphic

A simple example of both 2D homeomorphic and heteromorphic shape matching is shown in Figure 2.3. For this illustration, the colored dots are the defining nodes of the 2D shape and the lines connecting the nodes define the boundary of the 2D shape. As can be seen in Figure 2.3, homeomorphic shape matching refers to shape matching where both the source shape and the target shape have the same number of nodes. In particular, homeomorphic shape matching defines a one-to-one mapping between the source nodes and the target nodes. Therefore, the target location of the source nodes is already known in homomorphic shape matching.

The 2D heteromorphic shape matching example in Figure 2.3 shows the target shape with more nodes than the source shape. This is not necessarily always the case for heteromorphic shape matching. Heteromorphic shape matching only refers to the fact that there is not a one-to-one mapping between the source nodes and the target nodes. Heteromorphic shape matching is often unavoidable when the source and target nodes have different origins, such as different software packages in the case of the source nodes coming from a FEA software package and the
target nodes coming from a 3-dimensional geometric scanning software package. This lack of one-to-one mapping between source and target nodes greatly increases the difficulty of the shape matching process because the target locations of the source nodes are not known. There is currently no way of automatically determining exactly which source node corresponds to which target node in heteromorphic shape matching. The different mesh morphing methods presented in Section 2.2 attempt to address this problem in various ways.

2.1.4 Shape Registration

Shape registration is a form of shape matching that deals with alignment of a source shape to a target shape [8]. The homeomorphic and heteromorphic shape matching cases occur in shape registration, just as they do in mesh morphing. Chang and Zwicker developed an automated method for the registration of articulated shapes with missing data [8]. The missing data between source and the target shapes is what made the shape registration heteromorphic. Shape registration is most commonly used in computer graphics to handle the alignment of scanned shapes during shape motion. The target shape’s motion is essentially a deformation of the source shape. In that frame of mind, shape registration is shape matching to a target with deformations or defects. The method developed by Chang and Zwicker is an unsupervised shape registration method without any input templates [8]. That being said, they take advantage of an assumption that the motion or deformation results from only a few rigidly moving parts [8]. This reliance required by their method disables this method when applied to shape matching to match manufactured geometry where the deformations are not rigid and the moving parts are as much as equal to the number of points defining the target shape.
Another approach to accomplish shape registration is a "variational and statistical" method that preserves shape topology and has the ability to register shapes of 2, 3, or more dimensions [9]. In this method, a hierarchical approach is used to match the source to the target, such that global deformations and alignments are handled first to accomplish much of the shape matching, with only the final steps involving local shape matching. The local deformation is completed using the FFD technology presented in Section 2.1.1 [5], [9]. This method is limited in that its reliability is greatly reduced when the source and/or target shapes are defined by point clouds, which are the representations used in mesh morphing to match manufactured geometry.

2.1.5 Isogeometric Analysis

Another technology used to handle mesh updates due to changes in geometry is Isogeometric Analysis (IGA). IGA works by having the CAD geometry defined by the same basis functions that also define the finite elements used in finite element analysis [10], [11]. Hughes first presented IGA and its relationship to CAD and finite elements in 2005. This technology is attractive because a design change made to the CAD file does not require that a new finite element mesh be modified or re-created because the mesh is the CAD file. This method automatically performs mesh morphing due to the fact that the mesh morphs exactly when the CAD geometry is changed [10]. However, it does not address mesh morphing to match manufactured geometry. In the case of mesh morphing to match manufactured geometry, the engineer is attempting to move a mesh to match scanned geometry. Even if the mesh was equivalent to the CAD geometry, neither the definition of the CAD geometry, nor the definition of the mesh are related to the scanned data target shape.

2.2 Mesh Morphing

Mesh morphing is simply the moving of a mesh which involves translating at least one of the nodes in the mesh. Mesh morphing is commonly used to change an existing mesh to match a geometry change due to either design changes or a manufacturing defect [12]. It is a technology that has evolved from purely manual processes to current trends that are leaning towards full automation [2]. Brief descriptions of Siemens NX 9.0 (NX), HyperMorph, Air Force Research Laboratory (AFRL) MORPH, and Sculptor as mesh morphing options are provided below.
2.2.1 Siemens NX

NX is a CAD software package that also includes simulation and analysis capabilities. In NX 9.0, Siemens introduced some mesh morphing capabilities. These capabilities include modifying node locations relative to updated CAD and updating a geometry-based mesh without remeshing. These are accomplished through automatic and manual mechanisms.

Figure 2.4: Mesh Morphing Example from NX 9.0 (Courtesy of Siemens).

Figure 2.4 shows an example of the automatic mesh morphing of a cylindrical tube (original geometry on the right, deformed geometry on the left) to match a deformed CAD geometry. The automatic mechanism is as follows:

- Nodes are either mapped or already associated to CAD geometry.
- The nodes are moved to the new topological locations as long as the geometrical topology does not change.
- Any unmapped nodes (heteromorphic) are moved based upon the displacement of neighboring mapped nodes.
- Interior nodes (in the case of solid volume meshes) are smoothed and the node association to the geometry is maintained.
In the case that a change in the CAD introduces a change in topology, the automatic mechanism does not complete the mesh morphing to match the updated geometry. Following the automatic mechanism performed on a change in topology, manual actions must be taken by the user to complete the mesh morphing. This is done by manually moving the remaining nodes to their desired positions.

2.2.2 HyperMorph

HyperMorph is a mesh manipulation tool within HyperMesh (a product in the Altair HyperWorks family of products) that allows users to deform meshes while keeping mesh distortion at a minimum. Some key capabilities of the HyperMorph tool are:

- Deform mesh without remeshing
- Conform existing mesh to match new geometry
- Automatically store shape changes into design variables for optimization studies
- Create new surfaces directly from morphed mesh shapes
- Preserve element quality

Figure 2.5: Mesh Morphing Example from HyperMorph (Courtesy of Altair).

Figure 2.5 shows an example of mesh morphing being performed using the HyperMorph tool in HyperMesh with the red arrows showing the direction of the mesh deformation. From left to right, in Figure 2.5, the hole is repositioned relative to the new edges of the geometry, increased
in diameter, and rotated while becoming more elliptical. HyperMorph mesh deformation is a manual process in which the user selects a point or node on the existing mesh and then selects the corresponding geometrical point on the deformed geometry. The element quality of the existing mesh is preserved as the nodes are manually moved to their new locations.

2.2.3 AFRL MORPH Algorithm

The AFRL MORPH algorithm [2] was developed to “finely control the density and quality of the mesh for a population of parts without having to regenerate CAD models and the associated volumetric grids” [2]. Figure 2.6 is an example of intersections between the node normals on the mesh with the surface of scanned geometry. Figure 2.6 is the starting point for the MORPH algorithm. The basic steps to the MORPH algorithm are as follows:

- Compute FEM surface node normal vectors (Figure 2.6).
- Calculate node displacement vectors using a score-based alignment method using finite element model (FEM) to tessellated scan data (TSD) distance and alignment.

![Figure 2.6: Potential Node Normal Intersections at Blade Leading Edge [2].](image)
• Distribute local node displacements among adjacent nodes using a radial basis function (RBF, a real-valued function which only depends on the distance from an origin).

• Sum the RBF displacement vectors for all nodes and update node positions. Limit maximum node movement by a fraction of the global average element edge length.

• Check updated element shape. Limit node movements that would corrupt surface elements (e.g. aspect ratio and skew).

• Iterate through steps 1-4 and modify the RBF based on the rate of change of node positions. Run until FEM to TSD distance converges.

• Place surface mid-side nodes of quadratic elements between edge nodes. Compute mid-side node normal vectors based on the average of the edge node vectors and move those nodes directly along the normal vectors to the TSD.

• Update the positions of internal nodes using the iterative spring analogy (ISA).

2.2.4 Optimal Solutions Sculptor

One of the most fully automated examples of commercially available mesh morphing technologies exists in the Sculptor family of products by Optimal Solutions. All mesh morphing done in Sculptor is done using ASD which is similar to FFD [4]. Both ASD and FFD involve placing the mesh to be morphed within a control volume (defined by the blue, red, and green lines in Figure 2.7). The positions of the mesh nodes contained within the control volume are mapped to the control volume space. When the control points (green and yellow points in Figure 2.7) are moved, the mesh nodes contained within the control volume are also moved relative to their mapped position values. When mesh morphing occurs in Sculptor, nodes are not moved individually. Because of the effect of the control volume, the resulting morphed mesh is always a smooth continuous mesh. Sculptor uses optimization routines (GRG, Latin Hypercube, etc.) to achieve the desired morph.

In Sculptor, there are two kinds of mesh morphing: homeomorphic and heteromorphic. Heteromorphic mesh morphing, in Sculptor, requires what are called “key point pairs” in order for the mesh to be morphed to the target. Essentially, the key point pairs constitute a portion of the
heteromorphic mesh morphing that is actually homeomorphic. Therefore, there is no need for key
point pairs in homeomorphic mesh morphing.

Table 2.1: Summary of Most Capable Mesh Morphing Technologies

<table>
<thead>
<tr>
<th>Technology</th>
<th>Automated Homeomorphic</th>
<th>Automated Heteromorphic</th>
<th>Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>NX 9</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>HyperMorph</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>AFRL MORPH</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Sculptor</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

In summary, NX, HyperMorph, AFRL MORPH, and Sculptor all contain the ability to
morph meshes to match manufactured data in varying degrees, however, each current algorithm
involves a significant amount of manual input. Existing methods to morph meshes are not auto-
mated. While Sculptor approaches the full automation of mesh morphing, an automated procedure
to determine the necessary key point pairs must be developed to complete the automation by con-
verting the heteromorphic mesh morphing into homeomorphic mesh morphing. Due to the lack
of availability of the MORPH algorithm and the lack of an automated method to perform home-
omorphic shape matching for NX 9.0 and HyperMorph, Sculptor was selected (Table 2.1) as the
platform for the research presented in this thesis.
2.3 Manufacturing Defects

Manufacturing processes are constantly being improved as technology advances and competition drives companies toward more reliable products. Even with constant improvement, manufacturing processes are not perfect. Imperfect manufacturing processes are sometimes acceptable, depending on how much they affect the performance of a given product. Mesh morphing to match manufactured geometry is generally employed because of the existence of manufacturing defects.

In the case of a jet engine, imperfect manufacturing can greatly affect its performance because there are many components that are very sensitive to manufacturing defects. Hulme [13] reports that compressor blades, specifically, can experience High Cycle Fatigue (HCF) failure due to unintentional mistuning, which is caused by manufacturing tolerances. Unintentional mistuning occurs when the manufactured blade does not match the CAD model perfectly, even though it often still satisfies manufacturing tolerances.

![Figure 2.8: 1st Stage Turbine HCF Failure](image)

If HCF is not prevented or detected early, it often leads to blade-out or blade liberation, both meaning that the blade affected by HCF is no longer physically connected or constrained to the jet engine. Blade liberation or blade-out can often be catastrophic for a jet engine. An image
of the 1st stage turbine from Aviation Investigation Report A13C0150 is shown in Figure 2.8. In this report, it was determined that one of the leading causes of the aircraft’s crash was the failure of the 1st stage turbine due to HCF.

Failures due to manufacturing defects, such as unintentional mistuning, can be prevented if they can accurately be predicted. These accurate predictions can come as a result of analyzing the manufactured geometry in addition to the analysis of the CAD geometry. Mesh morphing (Section 2.2), which uses shape matching (Section 2.1), matches the mesh from the CAD geometry to the manufactured geometry. This enables the analysis of the manufactured geometry in addition to the analysis of the CAD geometry which, as mentioned previously, would help prevent failures due to manufacturing defects.

2.4 Modal Analysis

One of the important analyses performed when designing compressor blades is a modal analysis. Modal analysis is the structural analysis of the dynamic characteristics of a part or component under the influence of vibrational loading. The objective of a modal analysis, when modeled with finite elements, is to determine the natural mode shapes and frequencies of those natural mode shapes of a structure under the influence of free vibration. While the main results of modal analyses are the natural mode shapes and frequencies of a structure, the complete set of modal characteristics that are considered in this research are as follows:

- Modal Frequency
- Maximum and Minimum X, Y, and Z Displacements
- Maximum and Minimum Displacement Magnitudes
- Maximum Max and Min Principle Stresses
- Minimum Max and Min Principle Stresses
- Maximum and Minimum Max Shear Stresses
- Maximum and Minimum Von-Mises Stresses
An example of these determined mode shapes from a modal analysis on a compressor blade model using the Siemens NX Nastran structural eigenvalue solver is displayed in Figure 2.9. The root of the blade (the bottom edge of the blade in the orientation shown) has a fixed constraint attached to its surface fixing all 6 degrees of freedom. Figure 2.9a displays the displacement magnitudes of a compressor blade under free vibration in its first mode which is referred to as first bending (1 Bending). Figure 2.9b shows the displacement magnitudes of a compressor blade under free vibration in its second mode which is referred to as first torsion (1 Torsion). The third mode, displayed in Figure 2.9c, is a higher, more complex mode which includes both bending and torsion. Figure 2.9c is actually the tenth mode resulting from the modal analysis performed on a compressor blade.

![Figure 2.9: Natural Mode Shape Plots of Displacement Magnitude.](image)

The natural mode shapes and frequencies determined from modal analyses are used to ensure that the part or component being analyzed does not experience the determined natural frequencies during use. This use of the natural modes and frequencies determined during modal analyses is referred to as blade tuning. A set of blades on an integrated blade rotor (IBR, bladed disk, or blisk) can also be tuned together to improve the overall modal characteristics of the IBR. Mistuning is when an individual blade’s geometry on an IBR is different from the other blades’ geometry. This can be done intentionally for the improvement of the overall modal characteristics of the IBR, but often occurs unintentionally due to manufacturing defects. The mistuning in research presented in this thesis will only refer to unintentional mistuning due to manufacturing defects.
defects. The modal characteristics of compressor blades can be sensitive to even the slightest of defects. When compressor blades are designed, their modal characteristics are predicted through analyses that cannot account for the defects that occur during manufacturing.
CHAPTER 3. METHOD

This chapter presents the details of the heteromorphic to homeomorphic conversion method, beginning with an overview of the method and how it fits into the general engineering design process. This chapter presents the different steps in the method and the automation status of each step.

3.1 Overview

Current engineering design processes often include mesh morphing to prevent the necessity of re-meshing when performing analyses of manufactured parts. Figure 3.1 shows an example of the engineering design process with the implementation of mesh morphing. The yellow-colored objects in Figure 3.1 represent files that are generated from the blue-colored processes. It is important to note that the mesh morphing process is a post-manufacturing process that is part of the quality process in Figure 3.1 and requires the manufactured geometry to be scanned before the mesh can be morphed. The process of mesh morphing is nearly always a heteromorphic process when the mesh is being morphed to match a manufactured part because the sampling used when scanning geometry is not related to the node positioning during the meshing process. A novel method for the conversion of heteromorphic mesh morphing to homeomorphic mesh morphing is presented and automated in order to automate the mesh morphing process.

3.2 Heteromorphic to Homeomorphic

In Figures 3.2 and 3.3, the blue-colored objects are main processes and the yellow-colored objects are subprocesses or substeps of the blue-colored main processes. As can be seen when comparing the normal heteromorphic process in Figure 3.2 to the homeomorphic process with the heteromorphic to homeomorphic conversion in Figure 3.3 there are several differences. The most noticeable is that there is a Heteromorphic to Homeomorphic conversion process added at the
beginning of the process outlined in Figure 3.3. While this addition is the most important difference to note between the two figures, its addition produces other necessary changes in the Shape Match Optimization process. The most important of these necessary changes are the subtraction of the Manually Select Key Point Pairs substep and the addition of the Save Design Set, Restore + Save ASD Volume, Import Mesh, Import Saved ASD Volume, and Apply Design Set substeps. While it may seem that the homeomorphic process with the heteromorphic to homeomorphic conversion would be slower because of the positive substep differential when moving from the process in Figure 3.2 to the process in Figure 3.3, it is, in fact, significantly faster. It is faster because the added substeps in Figure 3.3 are all single clicks handled automatically by Sculptor, while the substep subtracted from Figure 3.2 is a tedious manual process that requires a human to visually inspect the input geometries and decide which points from the source geometry should be paired up with select points from the target geometry.

Figure 3.1: Detailed Engineering Design Process with Mesh Morphing.
The heteromorphic to homeomorphic mesh morphing conversion uses NX’s API, while the actual mesh morphing is done using Sculptor. A diagram detailing the mesh morphing process with the heteromorphic to homeomorphic conversion is displayed in Figure 3.3 and the main steps are:

1. Import CAD and Scan STLs
2. Section STL Bodies
3. Find Leading and Trailing Edge Points
4. Divide Section Curves
5. Put Point Sets on Divided Sections
6. Export CAD and Scan Point Clouds

The result of the heteromorphic to homeomorphic conversion is two point cloud files that form a one-to-one set of data between the CAD STL and the scan data STL. The resulting point
cloud files are independent of both the CAD STL and the scan data STL. This independence means that they are neither one-to-one with the CAD STL nor the scan data STL. The following figures in Sections 3.2.1 through 3.2.6 follow a theme of CAD model geometry colored in blue and scan data geometry colored in red.

### 3.2.1 Import CAD and Scan STLs

The mesh that is to be morphed has a source CAD model from which the mesh was formed. An STL approximation of that CAD model is made in NX and exported. The scan data of the
manufactured part is triangulated and converted to STL format in a process prior to the mesh morphing. The triangulation of the scan data and conversion to STL format is common practice in industry for analysis of manufactured parts. The STLs of the CAD model and the scan data are assumed to be accessible as inputs to the heteromorphic to homeomorphic process. They are also aligned to the same orientation. To begin the process, both STLs are imported into NX using the NX API. This is done with an existing NX session open. An image of the imported STLs can be seen in Figure 3.4. The basic dimensions of the CAD model are: 4.0 inch max chord, 12.0 inch span, and the scan data STL differs from the CAD model by a 5 degree twist along the root-to-tip axis.

Figure 3.4: CAD and Scan Data STL Bodies Imported into NX.
3.2.2 Section STL Bodies

Both of the STL bodies are sectioned using the Section Curve feature in NX’s API. In order to determine the number of sections and spacing between each section, the geometry extremes of the STLs must be determined. When the STLs are imported into NX, they are parsed and the tips and roots of the bodies are determined based on an orientation along the z-axis. The STL points with the highest z-value are considered to be at the tip and those with the lowest z-value are considered to be located at the root. Once these geometric extremities of the bodies are determined, the spacing between sections is calculated based on a desired number of sections. Each body is sectioned from root to tip. An example of the sections is shown in Figure 3.5.

![Figure 3.5: Section Curve Results from NX Section Curve Feature.](image)

3.2.3 Find Leading and Trailing Edge Points

It is trivial to have the same number of points in each of the resulting point clouds from the heteromorphic to homeomorphic process. This can be done by merely using the NX Point Set feature to put a desired number of points on each section. Only having the same number of points, however, does not guarantee the desired homeomorphic property. The point clouds must also be one-to-one. Resulting points from each Point Set feature do not necessarily represent a one-to-one relationship because the points are spaced out evenly around the section curves and the section curve of the CAD STL is possibly not the same arc length as the section curve of the scan data STL due to the manufacturing defects. To solve this problem, two point sets are put on each section curve starting near the trailing edge and going toward the leading edge. The section curve must be divided in order to put the two aforementioned two point sets. In order to divide the section curve
near the leading and trailing edge, the leading and trailing edge of each section must be found. The steps to determine the leading and trailing edge points for each section are:

1. Put Point Set on Section Curve
2. Calculate Linear Regression Approximation
3. Rotate Point Set Points to Linear Regression Axis
4. Determine Leading and Trailing Edge Points

The first step is to discretize the section curve to a sufficient refinement in order to consistently locate the leading and trailing edge points. This is done using the NX API to put Point Set features on both of the section curves. In Figure 3.6, 400 points are placed on each of the section curves, spreading out the points equally by curve parameter. The number of points for linear regression was selected to provide a sufficient sampling to account for the curvature of the leading and trailing edges to determine the leading and trailing edge points with consistent positions along the section curves.

![Figure 3.6: Point Sets of 400 Points on Each Section Curve.](image)

From the 400 points in each point set, a least squares linear regression is performed on the Cartesian coordinates of the points. For simplification, the z-value of the entire line is equal to the average z-coordinate of every point of the point set that is being approximated. This produces a 3D best-fit line for each point set. Figure 3.7 shows an example of the resulting linear approximations to the point sets. The cyan-colored line corresponds to the point set on the blue-colored section curve and the green-colored line corresponds to the point set on the red-colored section curve.
Once each section curve point set has its own linear approximation, a mathematical representation of an orthonormal coordinate system is created for each linear approximation. The coordinate system for a given linear approximation is defined with the x-axis equal to the linear approximation, the z-axis normal to the plane defined by the section curve, and the y-axis the result of the cross-product of the x and z axes. For each point set, its points are individually transformed to the coordinate system created from the linear regression.

When the transformation is complete, each point has essentially been projected to the linear regression of the section curve point set, such that the transformed x-coordinate of each point represents how far along the linear regression each point is located. One at a time, the points’ transformed x-coordinates are compared to find the points with the minimum and maximum transformed x-coordinate values. Figure 3.8 shows a blue-colored arrow and a red-colored arrow for the blue-colored and red-colored section curves, respectively, to represent the transformed x-coordinate values for a given point per section curve. The origin of these arrows is where they intersect the y-z plane. In Figure 3.9, the resulting points with the minimum and maximum
transformed x-coordinate values are enclosed in blue and red-colored circles. The points with the minimum x-coordinate values represent the leading edge points and the points with the maximum x-coordinate values represent the trailing edge points.

![Figure 3.9: Determined Leading and Trailing Edge Points.](image)

It is noted that the calculated leading edge points do not actually match what would be the physical leading edge points for the section curves shown in Figure 3.9 because the linear regression does not pass through the leading edge point. However, it is sufficient to have the resulting points be consistently located in the same position along the section curve. This consistency is sufficient because the leading and trailing edge points are only found to provide two points on the CAD STL section curve and their corresponding points on the scan data STL section curve so that the two section curves can be divided in the same locations. This process consistently chooses the same location on the section curve for both the CAD STL section curves and the scan data STL section curves. The points found using this process will be referred to as the leading and trailing edge points, even though they do not necessarily physically represent the leading and trailing edges perfectly in this example.

### 3.2.4 Divide Section Curves

With the leading and trailing edge points determined for each section curve, the section curves can now be divided. Each section curve is divided using NX’s Divide Curve feature. The section curves are divided with the leading and trailing edge points as boundaries for the division. The result of the section curve division is shown in Figure 3.10. The labels, “Curve 1” and “Curve 2”, refer to the order in which the Divide Curve feature returns the list of divided curves. As can be seen in Figure 3.10, the order is not always consistent.
In order to force the order of the divided curve list returned from the Divide Curve feature to be consistent, the following steps are taken:

1. Place Points at the Parametric Midpoint of Each Divided Curve
2. Create Coordinate System for Measurement
3. Rotate Point Set Points to Coordinate System
4. Determine Divided Curves Order

Using the NX API’s Create Point functionality, points are placed at the parametric middle of each of the output curves from the Divide Curve feature. The coordinate system used for each set of divided curves is set up in a similar manner to the coordinate systems used in Section 3.2.3. In this case, the origin of the coordinate system is located at the leading edge point and the x-axis is the line connecting the leading edge point to the trailing edge point. The z-axis is the vector normal to the plane defined by the section curve and the y-axis is the result of the cross-product of the x and z axes.

For each section curve, both midpoints are transformed from the world coordinate system to the new coordinate system. The transformed y-coordinates of both points are compared and if the transformed y-coordinate value of the midpoint of "Curve 1" is greater than the transformed y-coordinate value of the midpoint of "Curve 2", then the divide curve result list is reversed. This ensures that the first curve in the returned list of divided curves from the Divide Curve feature is always the curve whose midpoint’s transformed y-coordinate is less than the other curve’s midpoint’s transformed y-coordinate.
Figures 3.11 and 3.12 show measurement of the transformed y-coordinates of the midpoints of the scan data STL divided section curve. As can be seen, the scan data STL divided section curve result is already in the correct order. However, as can be seen in Figures 3.13 and 3.14, the CAD STL divided section curve results are not in the correct order and must be reversed. The corrected order is shown in Figure 3.15.

3.2.5 Put Point Sets on Divided Sections

Once the section curves have been divided and reordered (if necessary), a point set is put on each individual curve. The number of points per point set is 50. This number was chosen to maintain a point cloud that is sufficiently refined to capture the manufacturing defects. A number of points lower than 50 might not capture the maximum deviation of the defect. To prevent duplicate
points, the end and start points for each point set are ignored. The leading and trailing edge points are used in place of the end and start points. Figure 3.16 shows an example of the resulting point sets placed on the divided section curves.

### 3.2.6 Export CAD and Scan Point Clouds

The processes described in Sections 3.2.3 through 3.2.5 are repeated for each of the section curves created in the process described in Section 3.2.2. The resulting geometry is shown in Figure 3.17.

The point sets created in Section 3.2.5 with end and start points replaced by the leading and trailing edge points form point clouds. These two point clouds (one for the CAD STL and one for the scan data STL) are shown in Figure 3.18 and are written to text files with the ‘.pc’ file extension. As a final step, all of the geometry generated in the processes described in Sections
3.2.3 through 3.2.5 is deleted completely from the NX session. The deletion of all of the geometry is done so that the entire heteromorphic to homeomorphic process can be run in a loop on multiple sets of data, one after another. Every step of the process described in Section 3.2 is automated in a Java application. The Java application contains calls to the NX API to automatically perform each step of the heteromorphic to homeomorphic conversion process. The table below (Table 3.1) provides a summary of the heteromorphic to homeomorphic conversion method by listing its steps and how each step is carried out.

Table 3.1: Brief Summary of the Heteromorphic to Homeomorphic Conversion Method.

<table>
<thead>
<tr>
<th>Heteromorphic to Homeomorphic Conversion Method</th>
<th>Realization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td></td>
</tr>
<tr>
<td>Import CAD and Scan STLs</td>
<td>Automated</td>
</tr>
<tr>
<td>Generate CAD STL</td>
<td>NX function</td>
</tr>
<tr>
<td>Generate Scan STL</td>
<td>External function</td>
</tr>
<tr>
<td>Process</td>
<td>Realization</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Import STLs</td>
<td>NX API</td>
</tr>
<tr>
<td>Section STL Bodies</td>
<td>Automated</td>
</tr>
<tr>
<td>Parse STL Files</td>
<td>Java</td>
</tr>
<tr>
<td>Determine Root &amp; Tip</td>
<td>Java</td>
</tr>
<tr>
<td>Determine Section Spacing</td>
<td>Java</td>
</tr>
<tr>
<td>Create Section Planes</td>
<td>NX API</td>
</tr>
<tr>
<td>Create Section Curves</td>
<td>NX API</td>
</tr>
<tr>
<td>Find Leading &amp; Trailing Edge Points</td>
<td>Automated</td>
</tr>
<tr>
<td>Put Point Set on Section Curve</td>
<td>NX API</td>
</tr>
<tr>
<td>Calculate Linear Regression</td>
<td>Java</td>
</tr>
<tr>
<td>Rotate Point Set Points</td>
<td>Java</td>
</tr>
<tr>
<td>Find Leading &amp; Trailing Points</td>
<td>Java</td>
</tr>
<tr>
<td>Divide Section Curves</td>
<td>Automated</td>
</tr>
<tr>
<td>Divide Curves</td>
<td>NX API</td>
</tr>
<tr>
<td>Place Midpoints</td>
<td>NX API</td>
</tr>
<tr>
<td>Create CSYS</td>
<td>Java</td>
</tr>
<tr>
<td>Rotate Points</td>
<td>Java</td>
</tr>
<tr>
<td>Reorder Divided Curves</td>
<td>Java</td>
</tr>
<tr>
<td>Put Point Sets on Divided Sections</td>
<td>Automated</td>
</tr>
<tr>
<td>Create Point Sets</td>
<td>NX API</td>
</tr>
<tr>
<td>Remove Duplicates</td>
<td>Java</td>
</tr>
<tr>
<td>Export CAD &amp; Scan Point Clouds</td>
<td>Automated</td>
</tr>
<tr>
<td>Repeat Above for Each Section</td>
<td>Java</td>
</tr>
<tr>
<td>Write Point Cloud Files</td>
<td>Java</td>
</tr>
</tbody>
</table>

End of Table 3.1
Figure 3.17: All Geometry Used to in Heteromorphic to Homeomorphic Process.
Figure 3.18: Resulting Point Clouds from Heteromorphic to Homeomorphic Process.
CHAPTER 4. VALIDATION PROCESS AND RESULTS

This chapter contains the process used to validate the method presented in Chapter 3 and the results of the validation. Before the validation process is presented in detail along with the validation results, the validation metric is presented and explained.

4.1 Validation Metric

The measurement performed for the validation is that of the maximum deviation between the final design iteration of the shape match optimization and the target shape. The resulting maximum deviation from the heteromorphic to homeomorphic conversion method is compared to an ideal homeomorphic case. The homeomorphic case is considered ideal because there is never a one-to-one relationship between a mesh of CAD geometry and the corresponding scanned data of the manufactured geometry. Because the ideal homeomorphic case is never realized with actual mesh morphing to match manufacturing geometry, the heteromorphic to homeomorphic conversion method is considered a valid alternative to heteromorphic mesh morphing if the maximum deviations post-optimization are on the same order as the corresponding deviations of the ideal homeomorphic case.

4.2 Validation Process

A diagram of the validation process is provided in Figure 4.1. The yellow-colored objects are STL files, the green-colored objects are point cloud files, and the purple-colored objects are ASD volumes. The blue-colored objects are the subprocesses within the validation process and the two red-colored objects are the results of the validation process that are compared to determine the validity of the heteromorphic to homeomorphic conversion method. Arrows entering a subprocess originate from the inputs to the subprocess, while arrows originating at a subprocess end at the
result(s) of the subprocess. There are four STL files, 4 ASD volumes and two point cloud files used throughout this process. These files and volumes are defined as follows:

- **STL A**: STL of CAD file
- **STL B**: Deformed STL of CAD file
- **ASD 1**: ASD volume used to produce STL B
- **STL C**: Homeomorphic shape match result between STL A and STL B
- **PC A**: Point cloud file representing STL A
- **PC B**: Point cloud file representing STL B
- **ASD 2**: ASD volume used for both homeomorphic shape match processes
- **ASD 3**: ASD volume saved from homeomorphic shape match between PC A and PC B
- **STL D**: STL result from applying the design set in ASD 3 to STL A
- **ASD 4**: ASD volume used to compare STL D and STL B

The metrics used in this validation are the comparison of the post-optimization maximum deviations of STL C and STL D, the comparison of the pre- to post-optimization percent increases in maximum deviation between STL C and STL D, and the differences in maximum deviation between STL B and STL C compared to STL B and STL D. These metrics are included in the plots in Section 4.3.

The objective of the validation is to verify that the homeomorphic shape match of the point cloud files resulting from the heteromorphic to homeomorphic conversion method produces a resulting shape whose deviations are on the same order, if not smaller than the deviations from a standard homeomorphic shape match. Following the diagram in Figure 4.1, the outline of this validation process is provided below.

- Deform STL A with ASD 1
- Export Deformed STL A as STL B
Figure 4.1: Validation Process Flow.

- Homeomorphic Shape Match of STL A and STL B with ASD 2 to Produce STL C
- Convert Heteromorphic to Homeomorphic to Produce PC A and PC B
- Homeomorphic Shape Match of PC A and PC B with ASD 2 and ASD 3 to Produce STL D
- Compare STLs C & D with STL B with ASD 4

An image of STL A used for validation is shown in Figure 4.2. STL A is a representation of a compressor blade. The basic dimensions of this STL model are 2 inches root to tip, 1.8 inches maximum chord, and 0.0635 inches maximum thickness with a -1.0 degree to 1.0 degree twist linearly distributed from root to tip. The white-colored points shown in Figure 4.2 are the defining points of STL A.
The ASD volume (Figure 4.3) used to deform the source STL model (ASD 1) consists of 5 control points in the S-direction, 20 control points in the T-direction and 29 control points in the U-direction (the S, T, and U directions being the basis for the parametric space defined by the ASD volume). The number of control points were selected to provide a sufficiently refined grid with each set of 4 control points on the surface creating square-shaped control surface patches. The square-shaped patches defined by a set of 4 control points allow for better deformation control of the geometry located within the control volume. The green dots in Figure 4.3 are the control points of the control volume. To form the control volume, the exterior control points are first fit to the exterior surfaces of the source STL model with a boundary layer of 0.1 inches. This boundary layer is only the offset distance of the control points from the surfaces of the source STL. If the boundary layer were to be 0.0 inches, the control points would be placed exactly on the surfaces of the source STL and this placement would leave the control volume intersecting the surfaces of...
the source STL instead of completely enclosing the source STL body. The interior control points are defined as 3 points spaced evenly between the exterior control points.

As mentioned in Chapter 3, mesh morphing is almost always a heteromorphic process, but a homeomorphic process is needed to validate the heteromorphic to homeomorphic conversion method. In order to produce the needed homeomorphic process, a deformed version of STL A is needed. This deformed version of STL A is created by importing STL A into Sculptor, surrounding it with ASD 1 (Figure 4.3), freezing ASD 1, and deforming ASD 1 by moving its control points. An example of this deformation is shown in Figure 4.4. The yellow control points are the control points that were moved from their original positions. STL A deforms along with the deformed ASD 1. The resulting deformed STL model is STL B and has a one-to-one relationship with STL A. STL B becomes the target for STL A in a homeomorphic shape matching process.

The heteromorphic to homeomorphic conversion method is performed on STL A and STL B as if the two STL models did not already have a one-to-one relationship. The resulting point
clouds (PC A and PC B) are shown in Figure 4.5. The image on the left in Figure 4.5 is PC A and PC B is on the right with the defect highlighted in red.

A homeomorphic shape match optimization is performed on the STL A with the STL B as the target. The objective of the optimization is set to minimize the maximum deviation. The ASD volume used in the shape match optimization (ASD 2) is constructed in the same manner as ASD 1 with different numbers of control points. The number of control points is reduced to decrease the time required by Sculptor during the shape match optimization process and to create a realistic scenario, where the control volume used to perform the shape match optimization is independent of the deformation that produced the defects. This reduction, although more realistic, reduces the potential accuracy of the shape match optimization for both the ideal homeomorphic case and the heteromorphic to homeomorphic conversion method equally. ASD 2 consists of 5 control points in the S-direction, 13 control points in the T-direction, and 16 control points in the U-direction. The only control points that are used as design variables in the shape matching optimization are
the exterior control points. The interior control points of ASD 2 only exist to ensure a more localized deformation through the thickness of the source STL model when an exterior control point is deformed. The STL model resulting from this optimization is referred to as STL C. STL C is the STL against which the resulting STL from the heteromorphic to homeomorphic conversion method (STL D) is compared for the validation of the heteromorphic to homeomorphic conversion method.

A separate homeomorphic shape match optimization is performed on the PC A and PC B. This optimization’s objective is similarly set to minimize the maximum deviation. The resulting design set is stored in ASD 3 and applied to STL A to produce STL D. ASD 4 consists of 2 control points in the S-direction, 2 control points in the T-direction, and 2 control points in the U-direction. The definition of ASD 4 is not significant, as long as it completely envelopes STL B. The minimum number of control points that is permitted in each direction is two. ASD 4 is used to compare STL C and STL D with STL B, measuring the mean, maximum, and sum of the deviations of each node. An example of the measurement of the deviations between each white STL-defining point is shown in Figure 4.6. The maximum deviation between STL B and STL C is 0.0145 inches, while the maximum deviation between STL B and STL D is 0.0152 inches.
This validation process was repeated on 20 deformed STL models. Each deformed STL model had a single perturbation located on its surface with half of the deformed STL models containing perturbations on the pressure surface and the other half containing perturbations on the suction surface. The locations of the perturbations ranged from root to tip and trailing edge to leading edge of both the pressure and suction surfaces. The placement of these perturbations was random and is displayed in Figure 4.7 and Table 4.1. The size of the perturbations ranged from 0.024055 to 0.072948 inches in height (Table 4.1). The effective diameter of the perturbations ranged from 0.29 to 0.49 inches (Table 4.1). These ranges resulted from deforming ASD 1 to produce defects while attempting to maintain relative smoothness in Sculptor. All perturbations were raised from either the pressure or section surfaces and were hemispherical in shape.
4.3 Validation Results

The validation results include a comparison of the maximum deviations post-optimization of the ideal homeomorphic case and the heteromorphic to homeomorphic conversion. While the mean, maximum, and sum of the deviations of each node were measured, the maximum deviations will be the only measurements considered for validation of the heteromorphic to homeomorphic conversion method. The validation results are limited to the maximum deviations because the shape match optimization’s objective is set to minimizing only the maximum deviation.

4.3.1 Maximum Deviation Post-Optimization

The three data sets, Point Cloud Heteromorphic to Homeomorphic, STL Homeomorphic, and STL Heteromorphic to Homeomorphic, presented in Figure 4.8 represent the maximum deviations between PC A post-optimization & PC B, STL B & STL C, and STL B & STL D, respectively. These deviations are all measured distances between corresponding points from either the STLs or point clouds. Figure 4.8 shows that the maximum deviation between PC A post-optimization and PC B is smaller than the maximum deviation between STL B and STL C, ranging from a 1.8% decrease to a 48.3% decrease for about half of the tests (13/20), while the maximum deviation...
between STL B and STL D is smaller than the maximum deviation between STL B and STL C, ranging from a 1.45% decrease to a 47.55% decrease for only 3 out of the 20 tests. The maximum deviation between STL B and STL D is never smaller than the maximum deviation between PC A post-optimization and PC B.

**Test 14 Anomaly Explanation**

All but tests 12 and 14 demonstrated the heteromorphic to homeomorphic conversion method as a valid method to use in place of heteromorphic mesh morphing to match manufactured geometry. A closer look at Test 14 gives insight into this subject.

Figures 4.9 and 4.10 show the ideal homeomorphic shape matching on the left and the shape matching done on the heteromorphic to homeomorphic conversion method on the right for tests 14 and 1, respectively. In Figure 4.9, the maximum deviation of the defect is located very near to a control point and the STL point density is far less than that of the point cloud. This results in
the ideal homeomorphic case not being as constrained while having a very useful design variable for the optimization because the maximum deviation point is very near to a control point.

On the other hand, in Figure 4.10, the maximum deviation falls almost directly in between control point and the STL point density is very similar to the density of the point cloud on the right. This creates very similarly constrained shape match optimizations.

The relative location of the control point to the maximum deviation point and the similarity in point density between the STL and the point cloud file resulted in Test 1’s homeomorphic STLs producing a post-optimization maximum deviation of 0.027034 inches and the heteromorphic to homeomorphic case producing 0.028082 inches. However, Test 14’s homeomorphic case produced 0.003551 inches while the heteromorphic to homeomorphic conversion method only achieved a post-optimization maximum deviation of 0.015662 inches.

4.4 Discussion

While the results presented above are only a small portion of all the data that was collected, they are enough to evaluate the heteromorphic to homeomorphic conversion method’s capability to provide sufficiently accurate inputs to the shape match optimization which, in turn, result in a sufficiently accurate morphed mesh. The validation of this method is neither an exercise in the application of optimization techniques nor the fine tuning of optimization parameters. It is also neither an exercise in ASD volume generation nor the determination of the ideal ASD volume to use in an optimization. Therefore, it is acknowledged that, both by tuning the optimization parameters used in the shape match optimization and determining the ideal ASD volume for a given pair of source and target models, the optimization results can be improved. In order to minimize the effect of the optimization parameters and the ASD volumes on the differences between the individual results, the optimization parameters are held constant for all tests and the ASD volumes (ASD 1, ASD 2, ASD 3, and ASD 4) are identical for each of the tests.

4.4.1 Evaluation

As presented above, in Section 4.3, the PC A to PC B shape match optimization is better than the STL A to STL B shape optimization for about half of the tests in maximum deviation,
maximum deviation difference, and maximum deviation percent increase. The large range in deviation for the different tests is due to the relative position of the control points that define the ASD volumes to the positions of the STL-defining points and point cloud points. If a control point is directly above either a STL-defining point or a point cloud point that happens to also be located at the maximum deviation point of a defect, then the shape match optimization will reduce the maximum deviation difference more than if either the control point is not near the STL-defining point or point cloud point or the STL-defining point or point cloud point is not located at the maximum deviation point of a defect. The PC A to PC B shape match optimization design set applied to STL A, resulting in STL D, is only better than the STL A to STL B shape optimization 15% of all 20 tests in maximum deviation, maximum deviation difference, and maximum deviation percent increase. These results would lead one to conclude that, while the shape match optimization of PC A to PC B is comparable to the shape match optimization of STL A to STL B, the PC A to PC B shape match optimization design set applied to STL A, resulting in STL D, is not.

This conclusion is not unfounded, and yet, it is also not evidence enough to invalidate the heteromorphic to homeomorphic conversion method. The homeomorphic shape match optimization which is used as the standard that the heteromorphic to homeomorphic conversion method is compared to is more than ideal. It is unrealistic, in regards to mesh morphing to match manufactured geometry, because meshes never have a one-to-one relationship with the geometry scan files they are meant to match. Therefore, the heteromorphic to homeomorphic conversion method does not attempt to match the ideal homeomorphic shape match optimization perfectly. It only attempts to show that the homeomorphic shape match of the point cloud files resulting from the heteromorphic conversion method produces a resulting shape whose deviations are on the same order, if not smaller than the deviations from a standard homeomorphic shape match. All post-optimization maximum deviations (.007753 to .025101 inches) are on the same order as the deviations from a standard homeomorphic shape match (.003551 to .027034 inches) and 15% of the tests resulted in post-optimization maximum deviations smaller (1.45% to 47.55%) than the corresponding deviations from their standard homeomorphic shape match counterparts.

The objective of the heteromorphic to homeomorphic conversion method is to convert a pair of heteromorphic files to a pair of homeomorphic files that can be shape matched with the resulting design set applied to the source file of the pair of heteromorphic files, in essence bypassing
the required heteromorphic shape match optimization. This objective is achieved completely with the method described in Chapter 3. The conversion of the heteromorphic pair of files to a homeomorphic pair of files is not sufficient on its own. In addition to this conversion, the validation results presented in this chapter show that the heteromorphic to homeomorphic conversion method provides accurate inputs to the shape match optimization which, in turn, results in a sufficiently accurate morphed mesh. Therefore, the heteromorphic to homeomorphic conversion method is a valid method to use in place of heteromorphic shape match optimizations. The research objectives to present a novel method for converting heteromorphic to homeomorphic shape matching and automating that conversion method are accomplished by the heteromorphic to homeomorphic conversion method validated in this chapter.
Figure 4.8: Post-Optimization Maximum Deviations vs. Test Number.
Figure 4.9: Test 14 Defect (Ideal Homeomorphic on the Left and Point Cloud Conversion on the Right)

Figure 4.10: Test 1 Defect (Ideal Homeomorphic on the Left and Point Cloud Conversion on the Right)
CHAPTER 5. METHOD DEMONSTRATION

This chapter demonstrates the usefulness of the heteromorphic to homeomorphic conversion method in the ability to conduct engineering analysis without manufacturing many parts with defects. The steps necessary for this demonstration are presented, followed by observations from this demonstration of the heteromorphic to homeomorphic conversion method.

5.1 Method Demonstration Process

The steps presented below outline the three main portions of the process used to demonstrate the usefulness of the heteromorphic to homeomorphic conversion method.

• Simulate Manufacturing Defects

• Heteromorphic to Homeomorphic Conversion and Shape Match

• Modal Analysis

5.1.1 Simulation of Manufacturing Defects

In this demonstration of the heteromorphic to homeomorphic conversion method, actual manufactured geometry is not used. This is for multiple reasons. The main reason is that actual manufacturing defects are not located in a structured enough grid on the blade surfaces to be able to make observations concerning the relationship between the location of a defect on the modal characteristics of a compressor blade. Actual manufacturing defects may or may not be located more often in certain areas on the blade surfaces due to the manufacturing processes used to produce the blades.

In order to better control the location of the defects that are used in this demonstration, the manufacturing defects were simulated in a CAD system. The blades with defects could have
been meshed and analyzed without using mesh morphing or the conversion of heteromorphic to homeomorphic shape matching, but then the mesh on the blade without defects would not be the same mesh as the one created on the blade with a defect. The dissimilar mesh created on the blade with a defect could result in changes in modal characteristics from the difference in the number of elements in the mesh without even considering the effect of the defect. All of the simulated defects are spherical in shape and the center of the sphere is located on the surface of the blade. The defects are 0.05 inches in diameter with a 0.025 radius edge blend on the edge created when the defect is placed on the surface. Because the spherical defect centers are located on the surface of the blade, the height of the defect is equal to the radius of the sphere which, in this case, is 0.025 inches. These dimensions of the simulated manufacturing defects were provided via suggestions from engineers at Pratt & Whitney. A cross-section of a simulated manufacturing defect is shown in Figure 5.1. The red surface is the surface on the defect sphere and edgeblend, while the blue surface is the surface of the blade. The center of spherical defect is the white plus sign and the gray color represents the section cut into the blade. In this demonstration, only one manufacturing defect was permitted for each test. The compressor blade model used in this demonstration is the same blade as is used in Chapter 4 for validation of the heteromorphic to homeomorphic conversion method (STL A). A tool (Hailstorm) was developed using the NX API to automate the production of the simulated manufacturing defects.
The parameters for Hailstorm are shown in Figure 5.2. The ‘Select Face’ parameter allows the user to select a desired face on which the defects will be placed. The ‘U’ and ‘V’ are integers representing the number of defects desired in the parametric u and v directions, respectively, on the surface. The ‘U Start’, ‘U End’, ‘V Start’, and ‘V End’ are parametric percentages used to define start and end of the distribution of the simulated defects across the surface. The ‘Diameter’ is the diameter of the defect. All defects are assumed to be smooth in this demonstration and this relative smoothness is accomplished by placing an edge blend on the defect. The ‘Radius’ parameter is the radius of the edge blend that is applied to the defect. Finally, the ‘Boolean’ enumeration provides the user with the option of placing a dent-like defect by selecting ‘Subtract’ or a bump-like defect by selecting ‘Unite’.

Figure 5.3 shows the progression of the Hailstorm automation tool from the blade without any defects to the final simulated manufacturing defects. The intermediate image in Figure 5.3 shows the grid of simulated manufacturing defect locations and one existing defect. The final image on the right in Figure 5.3 shows an overlay of the STL bodies with manufacturing defects (in red) on top of the blade without any defects (blue).
Once the parameters are provided with values and the user has selected either ‘OK’ or ‘Apply’, the simulated manufacturing defects are placed on the selected surface. This is done by placing a Point Set Feature on the selected surface with the ‘U’, ‘V’, ‘U Start’, ‘U End’, ‘V Start’, and ‘V End’ parameters as shown in the center image in Figure 5.3. Once the Point Set Feature has been placed on the desired surface, a Sphere Feature is placed, having its center coincident with the first point from the Point Set Feature. The Sphere Feature’s diameter is defined as the ‘Diameter’ parameter. The Sphere Feature is either united to the blade or subtracted from the blade, depending on the value of the ‘Boolean’ parameter. The resulting edge from the unite/subtract of the Sphere Feature and the blade is then blended with the a constant circular blend radius defined by the ‘Radius’ parameter. Immediately after the sphere and edge blend pair are created, an STL body representation is exported and saved in the directory where this automation tool is stored.

Figure 5.3: The Hailstorm Process, Beginning to End.

Once, the STL body is exported for that particular sphere and edge blend pair, the Sphere Feature is edited, changing its location to having its center coincident with the next point in the Point Set Feature. The edge blend automatically updates with the edit of the Sphere Feature’s location because the edge blend is associated with the edge produced by the union/subtraction of the sphere with the blade. When the sphere and edge blend pair have updated to their new location, an STL body representation is, once again, exported and saved. This process of updating the location of the sphere and edge blend pair and exporting an STL body is repeated until an STL body repre-
sentation has been exported for each of the simulated manufacturing defects. Hailstorm concludes by deleting the Point Set Feature, Sphere Feature, and Edge Blend Feature, and importing all of the STL bodies back into the the NX Part as shown in the image on the right in Figure 5.3. The reimporting of the STL bodies is not necessary for the production of the simulated manufacturing defects. It is only included in Hailstorm to allow for visual inspection of the resulting simulated manufacturing defects by the user.

The STL files exported, using Hailstorm, contain the simulated manufacturing defects. The parameter values in use during the creation of the simulated manufacturing defects for this demonstration of the heteromorphic to homeomorphic conversion method match those shown in Figure 5.2. Therefore, 25 defects were studied ranging from 5% to 95% of the chord and 5% to 95% of the span of the blade. These ranges are selected to allow for the Sculptor’s shape matching algorithm to function properly. With 0% to 100% selected as a range for the simulated defects, Sculptor failed to match the defect properly. These 25 defects were placed on the pressure side of the blade.

5.1.2 Heteromorphic to Homeomorphic Conversion and Shape Match

As described in Chapter 3, the heteromorphic to homeomorphic conversion method is constructed such that it can be run in a batch mode. This batch mode is utilized to convert the simulated manufacturing defect STL files and the STL A file from a heteromorphic relationship to a homeomorphic relationship. The parameters for the heteromorphic to homeomorphic conversion method used in converting the simulated manufacturing defect STL files are as follows:

- 400 points for linear regression of each section
- 101 sections
- 100 points per divided section

The number of points for linear regression was selected to provide a sufficient sampling to account for the curvature of the leading and trailing edges to determine the leading and trailing edge points with consistent positions along the section curves. The number of sections and number of points per section were selected jointly to provide a sufficiently refined mesh in the point cloud.
to capture the simulated manufacturing defect. These parameters resulted in point cloud files each containing 19,998 points. The ASD volume used in Sculptor’s shape match optimization of the point cloud files is the same as the ASD volume used for shape matching in Chapter 4 (ASD 2). The resulting design set from the shape match optimization is applied to the NX Nastran simulation file. This simulation file and its setup are defined in detail in the following section, Section 5.1.3

5.1.3 Modal Analysis

NX’s Advanced Simulation environment is used for the modal analysis setup and solution. The solver used is NX’s Nastran linear structural solver with the solution type being SEMODES 103. The only subcase in the simulation is the Eigenvalue Method, using Lanczos and the eigenvalue method. No loads were applied to the model. The complete root of the blade was constrained to be fixed in all degrees of freedom.

The mesh used in the modal analysis (shown in Figure 5.4) contains 194238 Hex8 elements with 113488 nodes. The mesh is swept from tip to root of the blade. This mesh is constructed using the ‘3D Swept Mesh’ functionality in Advanced Simulation with the following parameters:

- Element Type: CHEXA(8)
- Source Element Size: 0.03 inches
- Attempt Quad Only: Off - Allow Triangles

This finite element model is solved to produce the first 10 modes as results. The 10 modes of concern for this modal analysis are shown in Figure 5.5. Mode 1 is in the upper left and the modes increase in number to the right and down. The 1 Bending mode is Mode 1 and 1 Torsion is Mode 2. The other modes displayed in Figure 5.5 (Modes 3-10) are more complex modes that occur at higher frequencies than 1 Bending and 1 Torsion. The colors shown on the modal images are representative of the displacement of the blade without any defects during a modal analysis. The modes shown in Figure 5.5 and all of the figures in Appendix C show that the displacements at the root of the blade for all 25 tests and all 10 modes were 0 inches. The magnitudes are not shown because the images in Figure 5.5 are only displayed to show what 10 modes are being considered. A report is generated after the solution is generated in order to output the resulting stresses and
displacements. This model is set up on the CAD STL and exported. The exported simulation’s nodes are morphed according to the design set produced in Section 5.1.2 and a morphed simulation is exported from Sculptor. Each of the 25 morphed simulations are then imported back into NX’s Advanced Simulation, solved, and reports are generated for the 25 solved simulations. The post processing of the results contained in these reports and the discussion of the results are contained in the following section, Section 5.2.

5.2 Method Demonstration Observations

This section outlines the post processing of the analysis results of the 25 blades with simulated manufacturing defects in order to make observations regarding their locational effect on
the modal characteristics of the compressor blade. A discussion is included as to interpret the observations made.

5.2.1 Post Processing

The reports generated from the simulation results in Section 5.1 are in .html format when exported from NX’s Advanced Simulation. The simulation reports generate multiple sets of data for each of the 10 modes being analyzed which are used as metrics when comparing the 25 tests to the blade without defects, including:

- Modal Frequency
- Maximum and Minimum X, Y, and Z Displacements
- Maximum and Minimum Displacement Magnitudes
- Maximum Max and Min Principle Stresses
- Minimum Max and Min Principle Stresses
- Maximum and Minimum Max Shear Stresses
- Maximum and Minimum Von-Mises Stresses
While, this data does not include the stress and displacement values for every element and node in the simulation, it does account for the modal frequencies, maximum displacements, and maximum stresses, whose values are of most importance as a result of a modal analysis. In order to better analyze these results, they data is copied into Excel from the .html webpage.

In order to make observations concerning the locational effect of a defect on the modal analysis results, the data was analyzed in the frame of the magnitude of the percent error in relation to the simulation results of the blade without any defects. This error is the difference between the simulation results from a blade with a defect and the simulation results from the blade without any defects. These simulation results are considered to be the nominal values to which all 25 simulated manufacturing defect cases are compared.

The percent errors are calculated in Excel and output to .txt files to be plotted. Excel doesn’t quite possess the capability to plot the contour plots desired, therefore Python is used in the PyCharm IDE to plot contour plots of the magnitudes of the percent errors. Contour plots are used to be able to visualize the position of the defect and its percent error magnitude for a given metric.

Figures 5.6 through 5.10 display the resulting contour plots of the percent error magnitude of the modal frequencies for all 25 test across all 10 modes. The y-axis of each contour plot is the position of each defect in percent span (5% to 95%, root-to-tip length). The x-axis of each contour plot is the position of each defect in percent chord (5% to 95%). The color of each contour plot is the magnitude of the percent error of the modal frequencies caused by each of the 25 tests relative to the labeled colorbar. It is important to note that Figures 5.6 through 5.11, all of the figures in Appendix A, and all of the figures in Appendix B only plot 5% to 95% of the chord and 5% to 95% of the span (root-to-tip). Each of these figures contain all 25 defects plotted along with their individual effect on the modal characteristic specified in the specific plot. For example, Figure A.13a in Appendix A does not show that the maximum z-displacement for mode 5 occurred at the location 95% of the chord and 95% of the span. Instead, Figure A.13a in Appendix A shows that a defect located at 95% of the chord and 95% of the span caused a greater change in the maximum z-displacement of mode 5 when compared to the mode 5 maximum z-displacement of the blade without any defects than any other of the 25 defect locations. A red color located at the bottom of
one of these figures does not indicate an error is located at the bottom of one of these figures. It indicates that a defect located at the bottom of one of these figures caused an error.

All modes except mode 9 contain the maximum percent error magnitudes 5% from the boundaries of the blade’s pressure surface. These nine modes, in fact, contain their maximum percent error magnitudes at or within 5% of their respective edges. In 70% of the modes, the maximum percent error magnitudes were due to simulated manufacturing defects located at or within 5% of the root of the blade.

![Mode 1 Contour Plot](image1.png)

(a) Mode 1

![Mode 2 Contour Plot](image2.png)

(b) Mode 2

Figure 5.6: Magnitude of Frequency Percent Error Contour Plots

While there are 60 other figures similar to Figures 5.6 through 5.10 that show different metrics, Figures 5.6 through 5.10 are most illustrative of the effect of the location of the defects in relation to magnitude of percent error. Similar results were found in all of the stress contour plots. The total number of contour plots generated (129) resulted from the 17 modal characteristic metrics for each of the 10 modes. The minimum x, minimum y, minimum z, and minimum magnitude displacement metrics were not plotted because their percent errors were undefined due to the nominal value being zero. This reduced the number of plots from 170 to 130. The maximum y displacement for mode 7 also contained a zero value resulting in an undefined percent error, so that plot was removed, reducing the number of plots from 130 to 129. To illustrate the similarity of the results displayed in Figures 5.6 through 5.10 with the other 60 figures in Appendix A, the
maximum percent error magnitudes from each of the 129 plotted percent error magnitudes were counted based on their locations on the pressure surface of the blade and plotted in Figure 5.11 as counts of maximum percent error magnitudes vs. defect position on the pressure surface. For example, Figure 5.8a adds one count to the 50% chord, 95% span location in Figure 5.11, while Figure 5.7b adds one count to the 5% chord, 5% span location. Appendix B contains 13 contour plots that constitute a breakdown of Figure 5.11 based on the individual modal characteristics in order to observe how a specific modal characteristic is affected by the position of a defect across all
10 modes. Figure 5.11 agrees with Figures 5.6 through 5.10 in that the majority (46%) of the maximum percent error magnitudes across all 129 plots were caused by defects located at the root of the blade. Defects located at the leading edge caused 34% of the maximum percent error magnitudes. Defects located at the trailing edge resulted in 30% of the maximum percent error magnitudes. Defects located at the tip caused 25% of the maximum percent error magnitudes. These percents do not add up to 100% because the corner locations are double counted for the edges that share a corner. Defects located at the edges caused 82% of the maximum percent error magnitudes across
all 129 plots, while defects located on the interior surface resulted in only 18% of the maximum percent error magnitudes.

![Maximum Percent Error Magnitude Count](image)

**Figure 5.11:** Total Count of Maximum Percent Error Magnitudes vs. Pressure Surface Defect Position.

### 5.2.2 Discussion

From Figures 5.6 through 5.10, it is observed that, with 90% of the modes being affected most greatly by defects near the edges of the blade, the compressor blade’s modal characteristic frequency changed the most when a defect was located 5% from an edge of the blade. These edges include the root, tip, leading edge, and trailing edge. This is in all likelihood due to the fact that the edge defects increase the mass distribution towards the edges of the blade combined with the
fact that during a modal analysis, the majority of the displacement (which induces stresses) occurs at the free edges. It is interesting to note that 70% of the maximum percent error magnitudes occurred when defects were located 5% from the root. Although the root was fixed in all degrees of freedom, which results in the least amount of displacement at 5% of the span from the root and zero displacement at the root, defects located near the root generally affected the modal frequencies more than the defects located near the free edges. This is likely due to the fact that a change in geometry near the constrained end of the blade modifies the stiffness distribution, and, in turn, changes the shape and frequency of the modes.

This objective of this chapter was to demonstrate the usefulness of the heteromorphic to homeomorphic conversion method. This was accomplished by using the method to make observations related to the existence of a locational effect of pressure-side manufacturing defects on the modal characteristics of a compressor blade. Based on the results in Figures 5.6 through 5.10, there does exist a locational effect of pressure-side manufacturing defects on the modal characteristics of a compressor blade. This locational effect is such, that defects located near the extremities of the blade affect the modal frequencies more than anywhere else on the blade and the extremity of the blade that is most troublesome is the constrained root of the blade.
CHAPTER 6. CONCLUSIONS AND FUTURE WORK

This chapter concludes the research presented in this thesis with conclusions related to the heteromorphic to homeomorphic conversion method and its application to determining the presence of a locational effect of pressure-side manufacturing defects on the modal characteristics of a compressor blade. Areas of future work are also offered as recommendations as to where this research could proceed and other approaches to solving the heteromorphic mesh morphing problem.

6.1 Conclusions

A novel method to convert heteromorphic shape matching to homeomorphic shape matching for a compressor blade was developed and automated. The heteromorphic to homeomorphic conversion method was completely successful in converting a pair of heteromorphic files to a pair of homeomorphic files that can be shape matched with the resulting design set applied to the source file of the pair of heteromorphic files, in essence bypassing the required heteromorphic shape match optimization. In addition to this conversion, the heteromorphic to homeomorphic conversion method provided accurate inputs to the shape match optimization which, in turn, resulted in a sufficiently accurate morphed mesh with deviations on the same order as the ideal homeomorphic shape match optimization 85% of the time and deviations between 1.45% and 47.55% smaller in 15% of the tests. Therefore, the heteromorphic to homeomorphic conversion method is a valid method to use in place of heteromorphic shape match optimizations that results in a sufficiently accurate morphed mesh. This valid method was demonstrated to be useful in the ability to conduct engineering analysis without manufacturing many parts with defects.

6.2 Future Work

The heteromorphic to homeomorphic conversion method in this research was limited to the geometry of a compressor blade only. The most obvious step, in future work, would be to
generalize this conversion method to be capable of handling any geometry. The ideal route is to modify this method so that it can first convert other turbomachinery geometries such as turbine blades, fan blades, and eventually complete IBRs.

Another path of future work for the conversion of heteromorphic shape matching to homeomorphic shape matching is to look at modifying the scanning paths of the machines and softwares that handle the geometric scanning of the manufactured parts. If, for example, one could modify the path of the scanning device to sample points on the manufactured geometry that correspond to the points of a given mesh from the CAD file, then the resulting scan would already have the desired one-to-one, or homeomorphic, properties that the heteromorphic to homeomorphic conversion method produces.

Lastly, the current heteromorphic to homeomorphic conversion method presented in this thesis could be integrated directly into Sculptor for a more seamless mesh morphing experience for the user. This would remove the extra steps in shape matching that are required additions when adding the heteromorphic to homeomorphic conversion method to the shape matching process because they would become internal steps to Sculptor handled automatically.

Lastly, the heteromorphic to homeomorphic conversion method could be integrated into an automated work flow, which would allow for real time analysis on actual manufactured parts. This would be biased to the manufacturing defects common to the manufacturing processes in use, but it would be specific to the industry using the tool.
REFERENCES


APPENDIX A. PERCENT ERROR MAGNITUDE CONTOUR PLOTS

(a) Mode 1

(b) Mode 2

Figure A.1: Magnitude of Maximum X-Displacement Percent Error Contour Plots

(a) Mode 3

(b) Mode 4

Figure A.2: Magnitude of Maximum X-Displacement Percent Error Contour Plots
Figure A.3: Magnitude of Maximum X-Displacement Percent Error Contour Plots

Figure A.4: Magnitude of Maximum X-Displacement Percent Error Contour Plots
Figure A.5: Magnitude of Maximum X-Displacement Percent Error Contour Plots

Figure A.6: Magnitude of Maximum Y-Displacement Percent Error Contour Plots
Figure A.7: Magnitude of Maximum Y-Displacement Percent Error Contour Plots

Figure A.8: Magnitude of Maximum Y-Displacement Percent Error Contour Plots
Figure A.9: Magnitude of Maximum Y-Displacement Percent Error Contour Plots

(a) Mode 8

Figure A.10: Magnitude of Maximum Y-Displacement Percent Error Contour Plots

(a) Mode 9
(b) Mode 10
Figure A.11: Magnitude of Maximum Z-Displacement Percent Error Contour Plots

Figure A.12: Magnitude of Maximum Z-Displacement Percent Error Contour Plots
Figure A.13: Magnitude of Maximum Z-Displacement Percent Error Contour Plots

Figure A.14: Magnitude of Maximum Z-Displacement Percent Error Contour Plots
Figure A.15: Magnitude of Maximum Z-Displacement Percent Error Contour Plots

Figure A.16: Magnitude of Maximum Displacement Magnitude Percent Error Contour Plots
Figure A.17: Magnitude of Maximum Displacement Magnitude Percent Error Contour Plots

(a) Mode 3
(b) Mode 4

Figure A.18: Magnitude of Maximum Displacement Magnitude Percent Error Contour Plots

(a) Mode 5
(b) Mode 6
Figure A.19: Magnitude of Maximum Displacement Magnitude Percent Error Contour Plots

Figure A.20: Magnitude of Maximum Displacement Magnitude Percent Error Contour Plots
Figure A.21: Magnitude of Maximum Von-Mises Stress Percent Error Contour Plots

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Figure A.24: Magnitude of Maximum Von-Mises Stress Percent Error Contour Plots
Figure A.25: Magnitude of Maximum Von-Mises Stress Percent Error Contour Plots

Figure A.26: Magnitude of Minimum Von-Mises Stress Percent Error Contour Plots
Figure A.27: Magnitude of Minimum Von-Mises Stress Percent Error Contour Plots

Figure A.28: Magnitude of Minimum Von-Mises Stress Percent Error Contour Plots
Figure A.29: Magnitude of Minimum Von-Mises Stress Percent Error Contour Plots

(a) Mode 7  
(b) Mode 8

Figure A.30: Magnitude of Minimum Von-Mises Stress Percent Error Contour Plots

(a) Mode 9  
(b) Mode 10
Figure A.31: Magnitude of Maximum Min Principle Stress Percent Error Contour Plots

Figure A.32: Magnitude of Maximum Min Principle Stress Percent Error Contour Plots
Figure A.33: Magnitude of Maximum Min Principle Stress Percent Error Contour Plots

(a) Mode 5

(b) Mode 6

Figure A.34: Magnitude of Maximum Min Principle Stress Percent Error Contour Plots

(a) Mode 7

(b) Mode 8
Figure A.35: Magnitude of Maximum Min Principle Stress Percent Error Contour Plots

Figure A.36: Magnitude of Minimum Min Principle Stress Percent Error Contour Plots
Figure A.37: Magnitude of Minimum Min Principle Stress Percent Error Contour Plots

Figure A.38: Magnitude of Minimum Min Principle Stress Percent Error Contour Plots
Figure A.39: Magnitude of Minimum Min Principle Stress Percent Error Contour Plots

Figure A.40: Magnitude of Minimum Min Principle Stress Percent Error Contour Plots
Figure A.41: Magnitude of Maximum Max Principle Stress Percent Error Contour Plots

Figure A.42: Magnitude of Maximum Max Principle Stress Percent Error Contour Plots
Figure A.43: Magnitude of Maximum Max Principle Stress Percent Error Contour Plots

(a) Mode 5

(b) Mode 6

Figure A.44: Magnitude of Maximum Max Principle Stress Percent Error Contour Plots

(a) Mode 7

(b) Mode 8
Figure A.45: Magnitude of Maximum Max Principle Stress Percent Error Contour Plots

(a) Mode 9

(b) Mode 10

Figure A.46: Magnitude of Minimum Max Principle Stress Percent Error Contour Plots

(a) Mode 1

(b) Mode 2
Figure A.47: Magnitude of Minimum Max Principle Stress Percent Error Contour Plots

Figure A.48: Magnitude of Minimum Max Principle Stress Percent Error Contour Plots
Figure A.49: Magnitude of Minimum Max Principle Stress Percent Error Contour Plots

Figure A.50: Magnitude of Minimum Max Principle Stress Percent Error Contour Plots
Figure A.51: Magnitude of Maximum Max Shear Stress Percent Error Contour Plots

Figure A.52: Magnitude of Maximum Max Shear Stress Percent Error Contour Plots
Figure A.53: Magnitude of Maximum Max Shear Stress Percent Error Contour Plots

Figure A.54: Magnitude of Maximum Max Shear Stress Percent Error Contour Plots
Figure A.55: Magnitude of Maximum Max Shear Stress Percent Error Contour Plots

Figure A.56: Magnitude of Minimum Max Shear Stress Percent Error Contour Plots
Figure A.57: Magnitude of Minimum Max Shear Stress Percent Error Contour Plots

Figure A.58: Magnitude of Minimum Max Shear Stress Percent Error Contour Plots
Figure A.59: Magnitude of Minimum Max Shear Stress Percent Error Contour Plots

(a) Mode 7

(b) Mode 8

Figure A.60: Magnitude of Minimum Max Shear Stress Percent Error Contour Plots

(a) Mode 9

(b) Mode 10
APPENDIX B. PERCENT ERROR MAGNITUDE COUNT CONTOUR PLOTS

Figure B.1: Total Count of Frequency Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.2: Total Count of Maximum X-Displacement Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.3: Total Count of Maximum Y-Displacement Percent Error Magnitudes vs. Pressure Surface Defect Position.
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Figure B.5: Total Count of Maximum Displacement Magnitude Percent Error Magnitudes vs. Pressure Surface Defect Position.
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Figure B.7: Total Count of Minimum Von Mises Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.8: Total Count of Maximum Max Principle Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.9: Total Count of Minimum Max Principle Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.10: Total Count of Maximum Min Principle Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.11: Total Count of Minimum Min Principle Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.12: Total Count of Maximum Max Shear Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
Figure B.13: Total Count of Minimum Max Shear Stress Percent Error Magnitudes vs. Pressure Surface Defect Position.
APPENDIX C. MODAL ANALYSIS MODES FOR SIMULATED DEFECTS

Figure C.1: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 1.

Figure C.2: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 2.
Figure C.3: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 3.

Figure C.4: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 4.
Figure C.5: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 5.

Figure C.6: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 6.
Figure C.7: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 7.

Figure C.8: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 8.
Figure C.9: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 9.

Figure C.10: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 10.
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Figure C.12: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 12.
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Figure C.14: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 14.
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Figure C.16: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 16.
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Figure C.20: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 20.
Figure C.21: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 21.

Figure C.22: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 22.
Figure C.23: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 23.

Figure C.24: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 24.
Figure C.25: The 10 Modes Resulting from a Modal Analysis of Simulated Defect 25.