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Teachers' Curricular Reasoning and MKT in the Context of Algebra and Statistics

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ABSTRACT

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This study investigates 7th grade teachers' curricular reasoning and MKT in algebra and statistics. Although the use of curriculum materials and MKT both influence the quality of mathematics instruction, no relationship between teachers' curricular reasoning and MKT was found. Further, teachers' curricular reasoning across algebra and statistics was very similar despite differences in these fields and differences in MKT. A model for thinking about the knowledge needed to provide high quality instruction is proposed by considering the role of curricular reasoning and MKT in instruction.

Keywords: MKT, curricular reasoning, mathematics teaching

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Chapter 1: Rationale

The construct of mathematical knowledge for teaching (MKT) is a popular research topic and provides valuable insight with respect to the inputs and outputs of math education. In particular, MKT helps describe the set of characteristics teachers bring to the classroom, and MKT is positively associated with student achievement (Hill, Rowan, & Ball, 2005). At the same time, researchers readily acknowledge that student achievement is not influenced by how well teachers perform on measures of MKT; rather, student achievement is influenced by what knowledgeable teachers do in the classroom (Hiebert, 2013; Thames & Ball, 2013). Thus, it would be valuable to precisely describe the instruction of knowledgeable teachers that mediates the inputs and outputs of mathematics education (Hiebert et al., 2005).

Any effort to describe the instruction of knowledgeable teachers is complicated by numerous factors which concurrently affect instruction. Hill et al. (2008) illustrate how the quality of instruction can converge or diverge with MKT. Hill et al. present two cases effectively illustrating how the quality of instruction can diverge from MKT. Noelle has relatively high MKT and a negative view of her mandated curriculum. While high MKT seems to help Noelle understand and process students' mathematical thinking, her negative view of the curriculum leads her to use supplemental materials that lack an ultimate purpose. As a result, the quality of Noelle's instruction is less than predicted from her MKT. Conversely, Rebecca has relatively low MKT and strong pedagogical skills that support her procedural view of mathematics. Although Rebecca frequently makes mathematical errors, her strong pedagogical skills minimize the impact of these errors and raise the overall quality of her instruction to a level higher than expected by her MKT. Because the quality of instruction varies based on many factors including MKT it is difficult to describe the instruction of knowledgeable teachers.

I seek to gain insight into the instruction of knowledgeable teachers by comparing and contrasting teachers' MKT with the way they reason about using curriculum materials (Roth McDuffie & Mather, 2009). There are certainly benefits and drawbacks associated with comparing and contrasting MKT with teachers' curricular reasoning rather than instruction. The benefits of studying curricular reasoning are primarily related to eliminating factors that influence instruction to highlight how teachers approach instruction. While curriculum materials do not determine how a lesson is implemented, a large part of teachers' work is to develop a lesson from curriculum materials to achieve some mathematical goal (Breyfogle, Roth McDuffie, & Wohlhuter, 2010). Thus, variation in the use of curriculum materials is a useful way to understand potential causes of variations in instruction. However, eliminating factors that influence instruction by focusing on the use of curriculum materials includes eliminating the opportunity to observe interactions between teachers and students. By relinquishing this opportunity it remains unknown how teachers enact their curricular reasoning as they interact with students. Although there is no simple correspondence between instruction and curricular reasoning, describing knowledgeable teachers' curricular reasoning could provide researchers with a framework for observing knowledgeable teachers' instruction. By establishing such a framework, future observations in the complex classroom environment could be more productive in terms of describing knowledgeable teachers' instruction. To summarize, benefits of focusing on curricular reasoning in research are related to understanding how teachers' approach mathematics instruction, and drawbacks to this focus are related to losing information with respect to classroom interactions. So, comparing and contrasting MKT with curricular reasoning can reveal qualitative differences in the approach knowledgeable teachers take to mathematics instruction.

I have chosen the setting of algebra and statistics to compare and contrast teachers' MKT with their curricular reasoning. Using multiple content areas of mathematics is intended to address a methodological issue related to conceptions of instruction: curricular reasoning naturally varies among individuals, and as a result variation in curricular reasoning might more rightly be attributed to noise across individuals than to variation in MKT. By comparing and contrasting MKT with curricular reasoning in multiple content areas the potential for noise is reduced. To illustrate, suppose a particular teachers' curricular reasoning is qualitatively different in algebra and statistics. A stronger argument can be made that this difference is due to variation in MKT than if a similar difference were observed across teachers. Choosing the content areas of algebra and statistics is based on the evidence that teachers' knowledge in statistics is generally lower than in other content areas (Groth, 2007; Jocobbe & Horton, 2010; Shaughnessy, 2007). A stark difference in MKT across algebra and statistics will provide more opportunity to attribute variations in teachers' curricular reasoning to variation in MKT. Further, with the recent adoption of Common Core State Standards in mathematics and an increased focus on statistics at some levels it is timely to consider how teachers might approach teaching algebra compared to statistics.

Chapter 2: Framework

To define a framework I will first discuss the setting of this study. Next, I present MKT and curricular reasoning as the major constructs considered in this study. After presenting these constructs I establish research questions and a corresponding hypothesis.

Algebra and Statistics

Throughout this study I use the setting of algebra and statistics. As algebra is a multi-faceted topic, I restrict my focus on algebra to ratio and proportion because of how prominent these concepts are throughout middle school and advanced mathematics. To clarify my view of statistics, it is useful to consider a distinction between probability and statistics. A useful way to distinguish between probability and statistics lies in what information is given. In probability parameters that govern a population (e.g. the mean) are known and the objective is to determine what data are expected to be observed. Card games are a good example of probability because characteristics of the cards in the deck are known and it is natural to wonder what hand players expect to be dealt. In statistics data are known and the objective is to estimate population parameters that produce the observed data. As an example, consider recording the time in seconds between customers arriving at a restaurant in an effort to determine the rate at which customers arrive. This question is statistical because the rate parameter is estimated using data. In essence, statistics begins with data.

Also, it is relevant to specify in what way I view algebra and statistics as distinct fields. Cobb and Moore (1997) present a distinction between these fields by examining the use of context. In algebra, or mathematics in general, the context can be considered unnecessary and used to motivate the abstraction of a pure mathematical idea. On the other hand, in statistics the context is vital for interpretation and meaning. Consider, as an illustration of the distinction between algebra and statistics, using a data set comprised of wait time for eruptions of Old Faithful (Shaughnessy, 2007). These data can certainly be used to compare and contrast various measures of center. However, the value of such a calculation might be entirely determined by the context. In the case of Old Faithful, wait times for eruptions follow a short-long pattern which produces a bi-modal distribution. Given this context, calculating any measure of center yields surprisingly little information. Perhaps a conditional mean (i.e. given the previous wait time is 89 minutes, the next eruption is expected to occur in 50 minutes) is more appropriate from a statistical point of view. I accept both uses of the Old Faithful data set as pertinent to students' education in mathematics. At the same time, I claim comparing measures of center is an algebraic use of the context, and relying on the context to choose a fitting statistic to report is a statistical use of the context.

Mathematical Knowledge for Teaching

My framework for mathematical knowledge for teaching is consistent with the MKT construct presented by Ball, Thames, and Phelps (2008). My reason for focusing solely on this construct for mathematical knowledge for teaching rests on its correlation with higher student achievement (Hill et al., 2005). In the simplest terms, whatever knowledge base MKT actually models is positively correlated with higher student achievement and as a consequence I am interested in the instruction of teachers who have higher MKT.

Two characteristics of MKT provide structure for my expectation about the interaction between MKT and conceptions of instruction. First, within the MKT construct mere pedagogical knowledge is not sufficient for teaching mathematics: pedagogical knowledge must be grounded in content to be useful in teaching (Shulman, 1986). MKT domains such as knowledge of content and students and knowledge of content and teaching emphasize the prominence of mathematical knowledge in pedagogy. For example, a teacher draws on knowledge of content and teaching when they determine how to pursue particular student contributions to develop mathematical ideas during a whole class discussion (Ball et al., 2008).

Second, teachers utilize pure mathematical knowledge that is separate from mathematical knowledge used by adults in other fields. This separate mathematical knowledge is referred to as specialized content knowledge in order to distinguish it from common content knowledge that might be used by any mathematically knowledgeable adult. Contrasting examples is useful for illustrating the difference between specialized and common content knowledge. In the act of teaching a teacher might need to evaluate the reliability of a novel algorithm for multiplying multi-digit numbers. This task can be accomplished with mathematical content knowledge, but mathematically knowledgeable, non-teacher adults are likely unprepared to efficiently make such an evaluation (Hill, Schilling, & Ball, 2004). Thus, evaluating the reliability of novel algorithms is an example of specialized content knowledge. This task stands in contrast to applying a mathematical algorithm which requires common content knowledge because mathematically knowledgeable adults apply algorithms whether they are teachers or work in other fields.

One concern about statistics arises from adopting this framework for MKT. Given that statistics is a distinct field from mathematics (Cobb & Moore, 1997; Franklin et al., 2007; Shaughnessy, 2007) some might wonder if MKT can adequately describe the knowledge needed for teaching statistics. Groth (2007, 2013) hypothesizes about the knowledge necessary to teach statistics and how this knowledge might develop. He argues for the distinction between mathematics and statistics, but also acknowledges likely structural similarity across the knowledge needed to teach statistics and the knowledge needed to teach mathematics. Because of this similarity, Groth blends the structure of MKT with the framework for statistical investigation proposed by Franklin et al. (2007) to present his hypothesis of statistical knowledge for teaching. For example, Groth classifies anticipating student difficulties distinguishing between random sampling and random assignment as knowledge of content and students related to collecting data during statistical investigation (Groth, 2007, 2013). Other researchers have been complementary of Groth's work (Jocobbe & Horton, 2010). To be clear, I am not using Groth's hypothesized statistical knowledge for teaching as part of the framework for this study; the reason for referring to Groth's work is only to demonstrate that the structure of MKT is a very reasonable way to describe statistical knowledge for teaching.

Curricular Reasoning

Curricular reasoning is a form of pedagogical reasoning that retains a focus on given curriculum materials (Roth McDuffie & Mather, 2009). Curricular reasoning focuses on how teachers support students' learning from specific materials rather than using materials as a starting point or in some other way. Teachers can engage in curricular reasoning as they plan, implement, or reflect on instruction by considering how curriculum materials are used to support student learning. For this study I focus on how teachers plan for instruction in terms of learning goals, modifying given materials, and thinking through teaching strategies to support student learning.

My view of curricular reasoning is further shaped by Hiebert and Grouws' (2007) proposal that establishing connections between mathematical ideas and allowing students to struggle with important mathematical ideas are valuable for developing conceptual understanding. Two frameworks in the literature shape how I view these ways of supporting conceptual understanding. The first framework addresses connections between mathematical ideas. Silverman and Thompson (2008) argue that teachers must transform personally powerful mathematical understandings to pedagogically powerful mathematical understandings. These authors discuss mathematical understandings in terms of Key Developmental Understandings (KDUs)—conceptual learning goals that enable students to see mathematics differently and cannot be achieved through explanation or demonstration (Simon, 2006). One KDU Simon suggests is the ability to see a ratio as a quantity that measures a multiplicative relationship. With this KDU students are able to understand related mathematics such as slope and probability differently than if they were to attend to the additive difference between two quantities. Teachers' personal mathematical understanding might include KDUs, but pedagogically powerful understanding requires being aware of "(1) how [KDUs] could empower their students' learning of related ideas; (2) actions a teacher might take to support students' development of [KDUs] and reasons those actions might work" (Silverman & Thompson, 2008, p. 502). While particular KDUs are difficult to identify (Simon, 2006), the essence of the construct is to recognize that some mathematical ideas are profitable in terms of understanding related ideas. As teachers utilize possible KDUs in their learning goals and teaching strategies they provide opportunities to make connections between mathematical ideas explicit. As I analyze teachers' curricular reasoning I will describe to what extent teachers' use possible KDUs to guide their goals and other pedagogical decisions.

Notably, for Silverman and Thompson pedagogically powerful understandings and MKT are synonymous. However, their conceptualization of MKT is fundamentally different from that of Ball, Thames, and Phelps (2008). As such, the structure of my framework could provide interesting comparisons across these two conceptualizations of MKT and help solidify what knowledge is needed to teach mathematics. For example, strong positive correlation between pedagogically powerful understandings and the construct of MKT presented by Ball, Thames, and Phelps might motivate a refined framework for the knowledge needed to teach mathematics that incorporates ideas from both conceptualizations of MKT.

The second framework is related to how students struggle with important mathematics. Stein, Grover, and Henningson (1996) document the implementation of mathematical tasks with respect to the level of cognitive demand. A mathematical task is defined as a classroom activity used to focus attention on a mathematical topic. The cognitive demand of a task can range from complex, non-routine thinking to memorization. The framework used by these authors submits that the level of cognitive demand required by a mathematical task can decline or be maintained throughout implementation partly by the teachers' learning goals and teaching strategies. For example, a task may be intended to establish connections among various methods of solving a problem, but the teachers' goals and strategies can allow decline in cognitive demand to simply rehearsing various solution methods without attending to connections among them. When teachers are able to maintain high levels of cognitive demand they allow their students to struggle with important mathematics. In my framework I distinguish between patterns of curricular reasoning partly based on the extent to which learning goals and teaching strategies are likely to maintain high levels of cognitive demand.

Given that cognitive demand is a major component of my framework for curricular reasoning, it is appropriate to consider where curricular reasoning fits into The Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2009). The cognitive demand of a task can be evaluated at various stages of The Mathematical Tasks Framework, and curricular reasoning is situated between tasks as they appear in curricular materials and tasks as they are set up by teachers. Thus, curricular reasoning has potential to inform how the cognitive demand of a task might decline before a lesson begins.

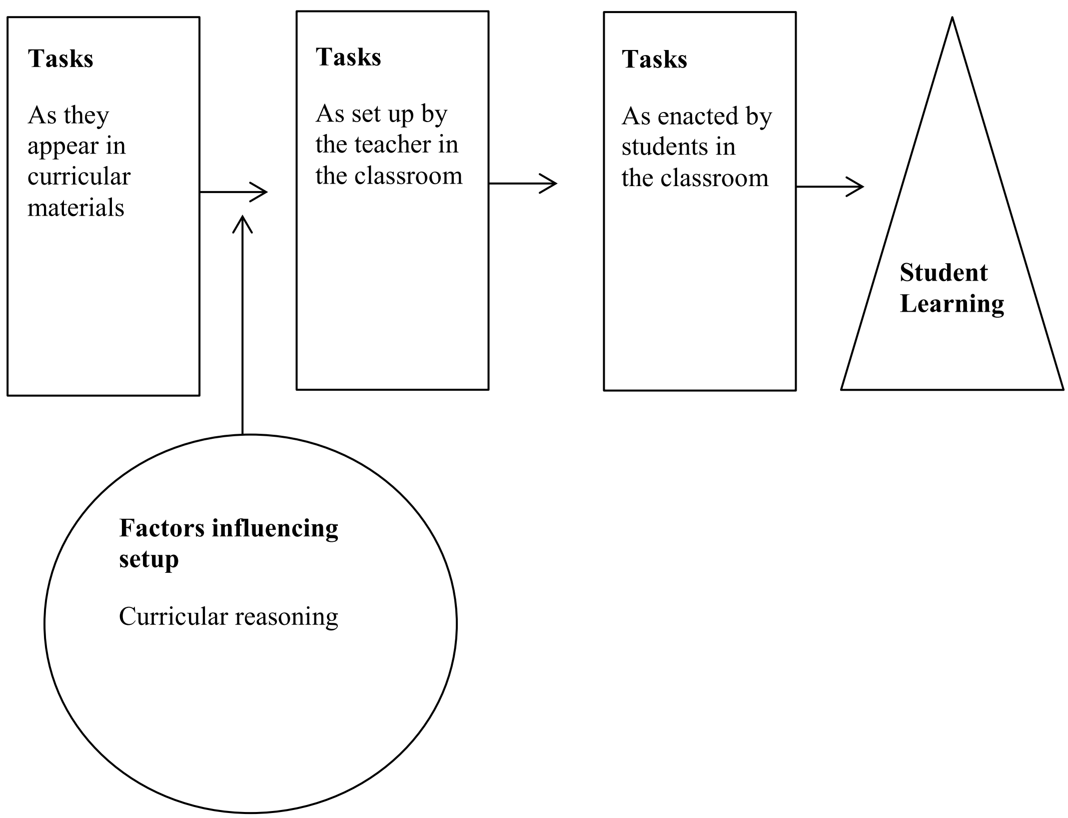

Figure 1. The Mathematical Tasks Framework modified from Stein, Grover and Henningsen (1996).

Finally, my framework for curricular reasoning considers how teachers hold their reasoning. Namely, teachers can hold their reasoning as belief or knowledge, and I find it beneficial to investigate how teachers hold their reasoning in order to learn about teachers' readiness to engage in worthwhile dialogue about curricular reasoning. Philipp's (2007) literature review on belief and affect is insightful regarding these constructs as well as their relationship to each other. Philipp suggests belief and knowledge can productively be considered ways of holding general notions. In terms of my framework, a teacher might know a particular way of reasoning about curriculum is effective or they might believe it is effective. For Philipp, the difference between holding a notion as knowledge or belief lies in the teachers' willingness to accept alternative notions as viable. The extent to which teachers are unwilling to recognize alternative notions as viable indicates the extent to which they hold their notions as knowledge. It is important to emphasize that the truth or validity of curricular reasoning is irrelevant with respect to whether a teacher holds their reasoning as knowledge or belief. Thus, teachers can reason about curriculum similarly, but some teachers can hold the reasoning as belief and other teachers can hold the reasoning as knowledge. I argue that teachers are better positioned to improve their curricular reasoning if they hold their reasoning as belief. Further, it is possible for a specific teacher to hold some reasoning as knowledge and other reasoning as belief.

To summarize, my framework for curricular reasoning consists of four components. First, I consider the use of context to provide insight into the distinction between pure mathematics and statistics. Second, I analyze learning goals and teaching strategies with respect to the likelihood they establish connections between mathematical ideas. I view these connections primarily in terms of KDUs which enable students to understand mathematical ideas as a consequence of their previous understanding (Silverman & Thompson, 2008). Third, I analyze learning goals and teaching strategies with respect to the likelihood they allow students to struggle with important mathematical ideas. I view the extent and nature of this struggle primarily in terms of cognitive demand. Fourth, teachers can hold their curricular reasoning as belief or knowledge.

Research Questions

With a framework for the major constructs of this study in place it is appropriate to establish the research questions:

1. What differences and similarities exist between teachers' MKT in algebra and statistics?
2. What differences and similarities exist between teachers' curricular reasoning in algebra and statistics?
3. What differences and similarities exist in curricular reasoning between teachers with higher and lower MKT?

The first two questions are largely precursors to the third question. The central purpose for this study is to contribute knowledge about what higher MKT teachers do in their instruction that is qualitatively different from teachers with lower MKT, and the first two questions provide a useful context for achieving this purpose. Describing knowledgeable teachers' teaching is difficult in part due to natural variation between teachers. This variation could be a result of a teacher characteristic such as beliefs, or it could result from contextual characteristics such as the curriculum teachers are required to use. Whatever accounts for variation between teachers, comparing individual teachers' curricular reasoning in two areas to their MKT within those areas has the potential to reveal how teachers with higher MKT approach instruction differently than teachers with lower MKT because other factors that could help explain variation across teachers are considered constant.

As previously mentioned, my expectation concerning the interaction between MKT and curricular reasoning is structured around two characteristics of MKT. Specifically, teachers with higher MKT have greater content specific pedagogical content knowledge and greater specialized content knowledge, and I expect these teachers to set goals and make plans that establish connections between mathematical ideas and maintain high levels of cognitive demand to a greater extent than teachers with lower MKT. For example, teachers with greater knowledge of content and students might plan specific ways to help students work through challenging aspects of mathematical tasks with prior knowledge whereas teachers with lower MKT might simply focus on getting answers. Building on students' prior knowledge tends to maintain high levels of cognitive demand, and focusing on getting answers is tends to decrease cognitive demand (Stein et al., 1996). Also, teachers with greater knowledge of content and teaching know how mathematical topics are related to each other and might more readily establish KDUs (Silverman & Thompson, 2008; Simon, 2006) as learning goals. To ascertain the reality of my expectation I will investigate whether teachers with lower MKT establish KDUs as learning goals and propose strategies that are likely to maintain high levels of cognitive demand to the same extent as teachers with higher MKT.

Chapter 3: Literature Review

Before reviewing the literature pertaining to this study I will briefly outline my purpose and the content for this section. My purpose is consistent with Simon's (2004) advice to use the literature review as an attempt to answer the research question with existing research. To facilitate accomplishing this purpose I separately address each research questions in the order they were originally presented. Attempting to answer the research questions requires a review of literature related to MKT and curricular reasoning specific to algebra and statistics. For clarity, the overarching purpose of my study is to learn how knowledgeable teachers might reason about curriculum in qualitatively different ways than less knowledgeable teachers—the content areas of algebra and statistics provide a setting to compare and contrast MKT with curricular reasoning.

Teachers' MKT in Algebra and Statistics

The construct of MKT has evolved in research over many years. While it is generally accepted that teachers need knowledge beyond the pure mathematics they teach, the extra knowledge teachers need and even the nature of that knowledge is surprisingly difficult to define. A foundational component of MKT is the essential nature of content knowledge. Shulman (1986) convincingly argues for content-specific pedagogical knowledge: teaching practices cannot be applied in the same form with equal effectiveness to every content area. Instead, teachers must infuse pedagogical knowledge with strong content knowledge in order to provide the best possible learning opportunities. Indeed, strong content knowledge leads to desirable results. By administering a one question mathematics content quiz to teachers Rowan (1997) establishes a small effect on student achievement; however, any effect with such limited data is remarkable and serves to illustrate the essential nature of teachers' content knowledge. At the same time, content knowledge alone is insufficient for effective teaching. Thompson and Thompson (1994) document one teacher's attempt to help a student develop conceptual understanding of rates. Notwithstanding strong knowledge regarding the content, the teacher is admittedly unable to help the student develop such understanding. To define the system of knowledge needed to effectively teach mathematics Ball, Lubienski, and Mewborn (2001) suggest using a practice-based approach; by reflecting on the practice of teaching mathematics researchers might identify what knowledge is used to teach mathematics. The construct of MKT resulting from this approach (Ball et al., 2008) is positively associated with student achievement (Hill et al., 2005). Such association between teacher knowledge and student achievement is rare (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012) and is my motivation for using Ball et al.'s MKT construct as my framework. Because of this association and other benefits knowledgeable teachers afford their students math education stakeholders advocate sustained opportunities for teachers to develop the complex system of knowledge needed to teach mathematics (Mewborn, 2003; Schoenfeld, 2002).

In light of the benefits associated with MKT it is concerning that teachers' MKT might be inadequate. Ma (1999) examines teachers' content knowledge as well as their ability to explain and represent important mathematical ideas. Her work is illustrative of how MKT domains of common content knowledge, specialized content knowledge, and knowledge of content and teaching are integral for high quality instruction. While Ma's work reveals that U.S. teachers' MKT in areas such as operations with fractions may be lower than desirable, teachers' MKT in statistics may be much lower than it is in other areas of mathematics. Jacobbe (2010) reports that some teachers lack thorough preparation in statistics, although these same teachers are otherwise well-regarded and mathematically knowledgeable. Other studies provide evidence that teachers' understanding related to measures of center is strikingly similar to students' understanding (Groth & Bergner, 2006; Jocobbe & Fernandes de Carvalho, 2011). Research on teacher knowledge in statistics is mostly limited to elementary grades, and comparisons to other areas of mathematics have not been established. At the same time, these researchers suggest that teachers' knowledge—particularly common content knowledge—in statistics is unusually low. Broadening the scope of research on teachers' MKT in statistics and establishing comparisons to MKT in other areas such as algebra could inform priorities in teacher preparation and professional development.

Comparing MKT across areas teachers are expected to teach is a difficult but intriguing task. The task is difficult in part because teacher knowledge is not readily quantifiable in absolute terms. Tests of teacher knowledge in various areas of mathematics are unreliable for comparison purposes due to relative scales. As a hypothetical example, testing might indicate that a teacher has above average MKT in statistics and average MKT in ratios and proportions; however, the teacher is only above average or average in relation to other teachers' performance on the same test and no statement can be made in terms of absolute knowledge in either area. Another option for quantifying teacher knowledge in absolute terms is to use surrogate variables such as the number of college courses taken or the number of years of experience teaching. Unfortunately, such variables are not valid measures of MKT (Ball et al., 2001). For now I am resigned to admitting that measuring teachers' MKT within particular content areas in absolute terms for convenient comparison is not practicable. Comparing MKT across various content areas of mathematics is nonetheless an intriguing task because teachers' MKT in particular areas could provide insight regarding student achievement. At the present, it is timely to include statistics in such comparisons because expectations for teaching statistics have sharply increased for many teachers with the adoption of Common Core State Standards in Mathematics. Thus, it is natural to wonder about teachers' knowledge for teaching statistics compared to knowledge for teaching in other areas including algebra.

While research has yet to empirically address comparisons of MKT across algebra and statistics, theoretical differences do exist. Groth (2007, 2013) along with McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) respectively present frameworks for statistical and algebraic knowledge for teaching. Both sets of authors use the framework for MKT presented by Ball et al. (2008) mixed with an additional framework for the purpose of developing a construct that is more specific to an area of mathematics. Groth uses the GAISE (Franklin et al., 2007) framework for problem solving in statistics to outline how teachers use their knowledge to teach statistics. McCrory, et al. (2012) elaborate how specific teaching practices enable teachers to use their knowledge and make algebra accessible to students. In many ways the theoretical differences between knowledge for teaching statistics and knowledge for teaching algebra reflect the difference between mathematics and statistics (Cobb & Moore, 1997). In statistics context provides meaning, and Groth's (2007, 2013) framework emphasizes how teachers use their knowledge to help students solve problems in context. In mathematics context obscures structure, and McCrory et al.'s (2012) framework illustrates how specific teaching practices can be utilized to teach pure mathematical ideas as coherent and comprehendible. These theoretical differences in knowledge for teaching statistics and algebra merit further research. As a portion of my research I will check for evidence that these theoretical differences in knowledge for teaching exist empirically in teachers' knowledge.

Teachers' Curricular Reasoning in Algebra and Statistics

Often the construct of curricular reasoning is used to provide insight into how teachers implement standards-based curriculum (Breyfogle et al., 2010; Breyfogle, Wohlhuter, & Roth McDuffie, 2012; Roth McDuffie & Mather, 2006, 2009). As teachers consider how curriculum materials can be utilized in response to student needs rather than as prescribed lessons, instructional practice and student achievement improve (e.g. Roth McDuffie & Mather, 2006). One issue not addressed in previous research is possible variation in curricular reasoning across mathematical content areas. Within my framework, variation in curricular reasoning across content areas seems likely because the nature of learning goals and effective teaching strategies varies across content areas. For example, in statistics a teacher might tend to focus lessons on data collection without a meaningful learning goal, but the same teacher might establish learning goals that can be considered KDUs (Simon, 2006) in algebra lessons. For this reason I claim content, in addition to beliefs and other factors, has the potential to alter teachers' curricular reasoning.

Jacobs and Morita (2002) use lesson scripts to demonstrate a distinction between Japanese and American teachers' ideas about effective pedagogy. Although these authors do not utilize the construct of curricular reasoning, lesson scripts are related to planning and the results of the study are intriguing with respect to how teachers hold their plans. On one hand, Japanese teachers' scripts reveal their preference for giving students ample time to develop mathematical ideas independently. On the other hand, American teachers' scripts show their preference for lessons to begin with review and end with practice. These results lead me to wonder what plans teachers hold as either knowledge or belief. Further, I wonder what causes plans to be held as either knowledge or belief. In Jacobs and Morita's study the difference can likely be explained as cultural. A portion of this study is dedicated to researching to what extent content might alter how teachers hold their curricular reasoning.

To be brief, curricular reasoning is a relatively new construct, and many facets of this construct remain unexplored. To date, research does not compare curricular reasoning across content areas of mathematics such as algebra and statistics. Exploring curricular reasoning in more detail has potential to develop understanding about instruction teachers provide for their students, how instruction differs across content areas of mathematics, and how instruction might improve.

Knowledgeable Teachers' Curricular Reasoning

Although it is sensible to believe that more knowledgeable teachers reason about curriculum differently, the nature of such a difference remains unexplored. I argue it is possible that knowledgeable mathematics teachers' curricular reasoning carries explanatory power regarding the instruction knowledgeable teachers provide for their students. Research shows how the relationship between teachers' MKT and their instruction is mediated in myriad ways; some of these ways include teachers' use of resources, beliefs, and experience in professional development (Hill, Blunk, et al., 2008). Because many of these mediating variables are components of teachers' curricular reasoning, I argue that exploring curricular reasoning can contribute to describing the instruction of knowledgeable teachers as called for by other researchers (Hiebert et al., 2005; Hill et al., 2005).

It seems profitable to study knowledgeable teachers' curricular reasoning in order to explain divergence of mathematical quality of instruction from MKT. Returning to the example of Noelle (Hill, Blunk, et al., 2008) will illustrate how identifying knowledgeable teachers' curricular reasoning is worthwhile. Noelle is a relatively knowledgeable teacher who has a negative view of her mandated curriculum; however, it is not clear why Noelle views the curriculum negatively. In practice, she selects mathematically rich tasks for students to work on, but she does not bring closure and purpose to these tasks. This lack of direction might indicate ambiguity in Noelle's goals and thus reflect her curricular reasoning. At the same time, the fact that Noelle parts from the mandated curriculum could indicate, for example, she has learning goals that are not aligned with the given curriculum, and as a result Noelle has a lack of curricular trust (Breyfogle et al., 2010). Her instruction may suffer more from an inability to properly develop curriculum materials around her goals than from curricular reasoning consistent with low MKT. In fact, Noelle's curricular reasoning might be consistent with other knowledgeable teachers, but her lack of curricular trust inhibits her ability to provide high quality instruction. Perhaps only knowledgeable teachers with appropriate resources are able to provide the quality of instruction expected of knowledgeable teachers, and Noelle's lower quality instruction might be best explained by circumstances out of her control. Describing the learning goals and teaching strategies that make up knowledgeable teachers' curricular reasoning might help establish an environment that more effectively utilizes knowledgeable teachers' potential to provide high quality instruction.

One study somewhat related to knowledgeable teachers' curricular reasoning compares expert and novice teachers' agendas (Leinhardt, 1988). Although the expert and novice labels are inconsistent with my framework for MKT, at least there is some evidence that expert teachers reason about curriculum in a qualitatively different way than novice teachers. In Leinhardt's (1988) study, both expert and novice teachers establish a procedural learning goal to produce equivalent fractions from a given fraction. This learning goal cannot reasonably be considered a

KDU because performing the required procedure does not empower students understanding of related ideas. Also, this learning goal does not provide an opportunity for students to struggle with important mathematics because it does not require a high level of cognitive demand. However, the teaching strategies employed by the expert and novice teachers do reveal some potential differences in these teachers' curricular reasoning. The expert teachers connect the skill of producing equivalent fractions to related mathematical ideas and concepts such as multiplying by one, but the novice teachers rely on rehearsing arbitrary steps in the procedure. While there is no evidence that these expert teachers focus on helping students acquire new KDUs, these expert teachers help students apply previously acquired KDUs to master a new idea.

Altogether, there is consensus that research is needed on how knowledgeable teachers approach instruction. Researchers are justifiably unsatisfied with identifying correlations between inputs and outputs of the mathematics education process (Hiebert et al., 2005; Hill et al., 2005). Instead, researchers insist the instruction of knowledgeable teachers be described in an effort to improve mathematics instruction overall (Hiebert, 2013; Thames & Ball, 2013). Suggestions for defining this instruction include examining relationships between teacher knowledge and uses of text, decisions about goals or priorities, and lesson planning. Further, such research within specific content areas such as statistics is needed (Shaughnessy, 2007). My ambition in carrying out this study is to help meet this need.

Chapter 4: Methods

Participants

I invited 7th grade teachers to participate in this study. Of 17 teachers invited to participate, I was able to collect a full set of data from 10 teachers. My primary purpose for selecting 7th grade teachers is related to increased expectations for teaching statistics. Previously, teachers in my sample have taught a significant amount of algebra, some probability concepts, and limited statistics. With the adoption of CCSS-M these teachers are now expected to teach a large amount of statistics content including topics related to variation and random sampling. As these teachers are expected to teach more statistics, it is timely to explore both their knowledge for teaching and curricular reasoning in this area of mathematics.

This study was carried out in three segments with each segment addressing one research question. In this section I describe the data collection and analysis for each segment. These descriptions follow the order in which the research questions were originally presented.

Teachers' MKT in Algebra and Statistics

Data. I measured teachers' MKT in algebra and statistics with instruments developed by the Learning Mathematics for Teaching (LMT) project (Hill et al., 2004). To reiterate, I used this test of MKT because it is correlated with higher student achievement (Hill et al., 2005). It is implausible that higher student achievement is caused by teachers doing well on the MKT assessment. Instead, I presume teachers with higher scores on the assessment provide instruction that is somehow qualitatively different than teachers who score lower on this MKT assessment, and the purpose of my study is to explore potential differences and similarities in teachers' curricular reasoning. Hence, the first step in my study was to identify teachers who score higher on this MKT assessment.

The LMT assessments used were specific to the domains of algebra and statistics and include questions to assess various domains of MKT. These domains of MKT include common content knowledge; specialized content knowledge; knowledge of content and students; and knowledge of content and teaching (Ball et al., 2008). For example, the following question from the statistics MKT test addresses common content knowledge:

> During a lesson on probability, Mr. Haigh's class was tossing a fair coin and recording the results on the blackboard. For the first 8 tosses, the outcome was "tails." What is the probability that the next toss will be "heads"?

Although this question takes place in the context of teaching a class, the knowledge required to select the correct response is common to anyone who uses probability and statistics. Another question from the statistics MKT test addresses specialized content knowledge:

> Alejandro rolled a fair six-sided die 10 times and kept track of the outcomes. He then averaged the outcomes and got 3.7. Which is a correct interpretation of Alejandro's findings?

Correctly answering this question requires specialized content knowledge because the ability to identify correct interpretations of student work is not needed outside of teaching. The algebra test has similar questions, and separately these assessments establish teacher's MKT in algebra and statistics that are strong or weak in relation to other teachers.

A justifiable concern about using the LMT assessment to measure middle school teachers' MKT is the extent to which the content of the assessment aligns with learning standards teachers are expected to achieve. While the purpose of the LMT assessment is not to ensure teachers know particular pieces of content, it is interesting to know what content is used to determine to what extent teachers have a knowledge base for teaching. This concern is especially applicable to statistics because of recent, dramatic changes in learning standards at the middle school level. Although I am not authorized to publish unreleased LMT assessment items, I can report that the content of the items does, in my judgment, address each of the eight statistics and probability learning standards for Grade 7 in the Common Core State Standards. The most thoroughly covered standard in the LMT assessment is probability of compound events. Four of the 20 questions on the LMT assessment do not correspond to learning standards in the Common Core. These questions include content related to displaying data and fair games. LMT assessment items do not use Grade 7 CCSS-M content of either multi-step ratio and percent problems or graphing proportional relationships. Content from other Grade 7 CCSS-M ratio and proportion learning standards are thoroughly utilized in the LMT assessment, and the LMT assessment also contains several questions about inverse proportional relationship which are not contained in the Grade 7 CCSS-M learning standards.

The LMT assessment was developed using item response theory and the total score on the test denotes a relative level of MKT. According to item response theory each item on the assessment is defined by its difficulty and potential to discriminate between ability. The difficulty parameter, θ, refers to a number of standard deviations relative to the average test taker and denotes the probability of a correct answer equal to $\frac{1}{2}$. For example, $\theta = 1$ means that a test taker whose true MKT is one standard deviation above the average test taker answers the question correctly with probability equal to $\frac{1}{2}$. The discrimination parameter captures the items potential to separate test takers with true MKT greater than the difficulty parameter from test takers with true MKT lower than the difficulty parameter. Considering an ideal test item will illustrate the discrimination parameter's purpose. Ideally test takers with MKT less than the difficulty parameter answer the question correctly with probability equal to zero and test takers with true MKT greater than the difficulty parameter answer the question with probability equal to one. Such an item would be ideal because it would perfectly discriminate between relatively high and low MKT among all test takers. Of course, an ideal test item does not exist, but a discrimination parameter closer to one separates test takers with higher MKT from those with lower MKT more reliably. A test, therefore, consists of many items with varying levels of difficulty in an effort to discriminate among test takers' true MKT. A test discriminates among test takers' MKT more reliably if the individual items have a discrimination parameter closer to one.

For convenience in scoring, teachers completed unabridged assessments of MKT. Completing the full measures results in simple correspondence to IRT scaled scores.

Analysis. Analysis of the MKT assessment was very straightforward based on the design of the test. The total number of correct responses corresponds to a relative level of MKT in terms of standard deviations from the average test taker. It is important to note that the scaled IRT score was not based on the sample in my study. Rather, the results of the MKT assessment show my participants' relative MKT within a large sample of test takers (Hill et al., 2004). To analyze these data I totaled the number of correct responses and identifying teachers' relative MKT in algebra and statistics from published tables. This analysis established teachers' relative MKT in algebra and statistics but did not yield any comparison of absolute statistics MKT compared to absolute algebra MKT.

Teachers' Curricular Reasoning in Algebra and Statistics

Data. Data for evidence of teachers' curricular reasoning was gathered from a questionnaire and an interview. The purpose of the questionnaire was to surface ideas related to teachers' curricular reasoning. One strength of questionnaires is the descriptive information that can result (Munn & Drever, 1990). However, a drawback of questionnaires is the lack of opportunity to explain; Munn and Drever suggest asking, "Why?" to introduce some explanation, but they also admit questionnaire data are likely to be superficial relative to interview data. Lack of explanation is a problem for describing curricular reasoning because my framework requires finding out to what extent teachers know why certain mathematical ideas are important for students to learn and why certain teacher moves might support students in learning those ideas. Therefore, data from the questionnaire does not fully reveal teachers' curricular reasoning. Nonetheless questionnaire data is important in terms of triangulation and forming preliminary inferences about teachers' curricular reasoning. The purpose of the interview was to refine my description of teachers' curricular reasoning by expanding, clarifying, and testing my preliminary inferences. A great strength of interviewing is the opportunity to probe and clarify responses (Brenner, 2006).

In the remainder of this section I will describe the questionnaire and its development as well as my interview protocol. The questionnaire was structured around two mathematical tasks appropriate for 7th grade students—one algebra topic and one statistics topic. Teachers were directed to read the problem statement for each task and record their responses to the questions. Completing the task gives teachers a factual basis for opinion questions related to the task (Munn & Drever, 1990) and also gives teachers an opportunity to complete the task as learners (Roth McDuffie & Mather, 2009). After completing the task, teachers responded to a series of questions related to how they might use the task in their classroom. These questions and prompts were written to surface teachers' curricular reasoning including what mathematical ideas teachers would like their students to learn and how teachers might position students to learn these ideas.

To develop the questionnaire I conducted a pilot study with 7th grade teachers who were not included in the sample for the study. The pilot study was structured around a statistics task that involved inferring the percent of red marbles in a jar by drawing random samples from the jar. The pilot study attempted to surface teachers' curricular reasoning by asking teachers to write a goal for the lesson and consider modifications and teacher moves they might make to support students in achieving the goal. Writing a goal for the lesson provides data about the mathematical ideas teachers would like their students to learn. Considering modifications and teacher moves provides data about how teachers might position students to learn the mathematical idea.

Analysis of the pilot study questionnaire successfully revealed some of teachers' curricular reasoning. Two examples illustrate curricular reasoning the pilot study successfully surfaced; the first example relates to mathematical ideas the teachers would like their students to learn, and the second example relates to what teachers might do to position students to learn mathematical ideas. First, when directed to write a goal for the task one teacher wrote about making inferences. Another teacher's goal related to using the mode of a data set to predict outcomes. The former teacher's goal is statistical, and the latter teacher's goal is probabilistic. Both goals provide insight into what mathematical ideas these teachers would like their students to learn. Second, in response to the question about what teacher moves might support the students in achieving the goal one teacher described students sharing their results by writing them on the board. Another teacher responded that they would provide repetition with similar problems. Both of these teacher moves are administrative in nature: the statements are related to what students will do more than what they will think.

Analysis of the pilot study questionnaire also surfaced some shortcomings of the questionnaire with respect to revealing teachers' curricular reasoning. In general, data from the questionnaire seemed to provide an incomplete description of teachers' curricular reasoning. Specifically, it was not apparent how teachers imagine suggested modifications and teaching strategies might support students in learning the proposed mathematical ideas. For example, one teacher wrote a goal to compare theoretical and experimental probability and stated they would modify the activity by making it more visual. The questionnaire did not prompt the teacher to explain the relationship between these statements. If participants were asked to explain why the suggested move or modification would support the goal the questionnaire may have formed a better description of teachers' curricular reasoning. To summarize, in my framework for curricular reasoning it is necessary to infer why the participants think a certain mathematical idea is important or why participants think a certain modification or teacher move will support students in learning the mathematical idea, but this inference cannot be made without sufficient opportunity to explain. Thus, descriptions of curricular reasoning were severely limited in the pilot study due to lack of opportunity to explain.

Based on pilot study results the questionnaire was revised in two ways to form a better preliminary description of teachers' curricular reasoning. Importantly, both of these revisions make the data I collected from the questionnaire more complete with respect to my framework for curricular reasoning. First, the revised questionnaire prompts teachers to explain more frequently. In particular, teachers are asked to explain the mathematical benefits of achieving the learning goal and how suggested modifications and teacher moves support students in achieving the learning goal. These explanations are important for making inferences about teachers' curricular reasoning because they are evidence of the extent to which teachers' mathematical understandings are pedagogically powerful (Silverman & Thompson, 2008). In the pilot study the teachers were directed to consider modifications and teacher moves that might support their chosen goal, but in the responses the relationship between modifications or moves and goals was mostly unclear. For example, one question in the pilot study asked what modifications the teacher would make to the task to meet the learning goal. A follow-up question included on the revised questionnaire asks teachers to explain why the modification supports the goal. With such prompts for explanation I believe teachers have more opportunity to explain why teaching strategies establish connections between mathematical ideas.

Second, the revised questionnaire uses tasks with a high level of cognitive demand (Stein et al., 1996). Tasks with high cognitive demand are needed to provide perspective on teachers' conceptions of instruction. For example, a teacher might propose a modification that causes some challenging aspects of the task to become routine. Conversely, a teacher might propose a modification to the task that builds on students' prior understanding and enhances their capacity to be successful in the task. I considered such proposals to respectively contribute to decline and maintenance of cognitive demand in a task (Stein et al., 1996).

After administering the questionnaire and performing initial analysis I collected further data related to teachers' curricular reasoning in an interview. My initial analysis of the questionnaire involved noting responses I found to be unclear, surprising or otherwise interesting. I developed questions to guide the interview from these notes. This structure for the interview is in line with the deductive approach to interviews which Brenner (2006) explains is useful for testing theories. The interview served three purposes. My first purpose in the interview was to clarify data collected from the questionnaire. Explanations in a questionnaire are typically brief and raise questions from researchers (Munn & Drever, 1990). By conducting an open-ended interview I was able to confirm or disconfirm my impressions from the questionnaire about teachers' curricular reasoning.

The next purpose of the interview was to collect data regarding how teachers hold their curricular reasoning. Philipp (2007) explains that notions can be held either as knowledge or belief. Philipp's use of the word belief is somewhat atypical in this case. When an individual holds notions as belief they are open to other approaches, but when an individual holds notions as knowledge they do not accept alternative approaches as viable or sensible. During the interview I presented participants with curricular reasoning that was counter to the goals and strategies proposed on the questionnaire, and I asked teachers to share their thoughts on applying the alternative curricular reasoning to their classroom. For example, research suggests that U.S. teachers might prefer to begin a lesson with review and end with practice (Jacobs & Morita, 2002). Suggesting lessons to teachers that do not incorporate these activities might determine if teachers seem to know lessons should begin with review and end with practice or to what extent they just believe these activities should be included.

My last purpose for conducting interviews was to elicit curricular reasoning that potentially conflicts with my initial description of teachers' curricular reasoning. Finding points of conflict in data will allow me to vet my description of teachers' conceptions of instruction and identify aspects of teachers' conceptions that are most fundamental. I attempted to elicit conflicting data by presenting teachers with a circumstance that might cause their reasoning to change. Research shows that teachers' beliefs about student needs, classroom environment, or other concerns associated with general education often take precedence over teachers' beliefs about mathematics and mathematics teaching (Leatham, 2006; Sztajn, 2003). As a result, considering such issues might alter teachers' curricular reasoning in significant ways. For example, a teacher might demonstrate curricular reasoning that is likely to maintain a high level of cognitive demand, but when faced with time constraints the teacher might introduce strategies that reduce the level of cognitive demand in an effort to ensure students have the information they need to complete the scheduled homework assignment. In such a case I would infer that the teachers holds their curricular reasoning as belief, and the teacher also seems to hold a more central belief that students need to be thoroughly prepared to complete the scheduled homework assignment.

In summary, I collected questionnaire and interview data to form a description of teachers' curricular reasoning. The questionnaire was designed to surface data related to teachers' pedagogically powerful mathematical understandings and their propensity for planning to support high levels of cognitive demand in a mathematical task. Data from the interview expands and clarifies as well as conflicts with data collected in the questionnaire. Consequently, data from the interview indicates to what extent teachers' hold their curricular reasoning as belief or knowledge. These data form the empirical basis for my description of teachers' curricular reasoning.

Analysis. I analyzed data from the questionnaire and interview using codes developed from the literature and aligned with my framework. To begin the analysis, I identified instances of curricular reasoning in the data. Instances of curricular reasoning were considered to be one of the following four types (Breyfogle et al., 2010; Roth McDuffie & Mather, 2009): (1) a goal for instruction, (2) an alteration to the curriculum materials as written, (3) a learning trajectory for students, and (4) a teaching strategy for working with students. The unit of analysis for an instance was an idea. Using ideas as the unit of analysis was useful because it captured the context of statements sufficiently well, and it was necessary to capture the context of statements in order to assign codes to the instances. For example, I found it difficult to code a teaching strategy if the goal of the lesson was unknown, so I expanded the unit of analysis to an idea. In the example the instance would be coded as a teaching strategy, and the goal is considered as a reason for selecting the strategy.

After identifying instances of curricular reasoning I coded the data for several categories of relevance to my framework. These categories were cognitive demand, belief or knowledge, KDUs and use of context. Codes for cognitive demand came largely from previous research (Henningsen & Stein, 1997). Many codes associated with maintaining and undermining the cognitive demand of a task during classroom instruction were useful for analyzing teachers' curricular reasoning. For example, when a teacher proposed a teaching strategy for dealing with a particular misconception it was useful to consider to what extent the strategy might providing scaffolding for the student or shift the students' focus to correct answers. Some codes associated with maintaining and undermining during classroom instruction, such as the amount of time spent on a task, were not able to be observed in teachers' curricular reasoning. These codes were omitted from the analysis.

My coding scheme for holding curricular reasoning as belief and knowledge was taken entirely from Philipp's (2007) advice. That is, if a teacher views another pattern of curricular reasoning as sensible, I consider the teacher to hold their curricular reasoning as belief. The inverse of this statement describes knowledge. Instances of curricular reasoning often did not consider another potential pattern of curricular reasoning, so it was often impossible to determine if the curricular reasoning in an instance was held as belief or knowledge. For these instances the code was simply left blank.

To code for KDUs I considered two ideas mentioned by Silverman and Thompson (2008). First, teachers might demonstrate understanding of how KDUs empower students to learn related mathematical ideas, so I looked for evidence that teachers consider how mathematical ideas develop over time. Second, teachers might describe teaching strategies that are useful for helping students develop KDUs, so I looked for evidence that teachers select strategies for the purpose of developing meaningful understanding of mathematical ideas rather than, for example, just helping students find a solution.

The final category of codes in my analysis related to how teachers used the real-world context of curriculum materials. This category contained three codes including two that highlight a proposed distinction between algebra and statistics (Cobb & Moore, 1997). First, the context could be used algebraically. For the use of context to be considered algebraic two conditions needed to be met. The teacher needs to reason about teaching new mathematical ideas with the context and particular features of the context need to be considered unimportant with respect to the new mathematical idea. For example, a teacher might use students' intuition about the ratio of candies in a bowl to help students understanding scaling quantities in a proportional relationship. In the end, it does not matter whether the quantities are Jolly Ranchers and Jawbreakers or buttons and shirts. In either situation the scaling procedure is the same when the quantities exist in a proportional relationship. Such a use of the context was considered algebraic.

Next, the context could be used in a statistical way. In statistics the context is vital for meaning and interpretation (Cobb & Moore, 1997). In statistics there is no guarantee that the calculations performed with one set of data will be informative for another set of data because the meaning depends on the context. Cobb and Moore specifically promote using exploratory data analysis to begin working with data. To engage in exploratory data analysis students might create dot plots, stem and leaf plots or histograms to represent the data and look for interesting patterns that might be important for the context. Using such analysis to link the data to the context was one characteristic of curricular reasoning that I considered to have a statistical use of context. To decide if teachers used the context statistically, it was useful to wonder if the same work could be done in class without any context at all. For example, using a set of numbers to explore how outliers affect various measures of center can be done without any context. If the set of numbers happen to represent something of interest to the students, it is entirely incidental and the use context is not statistical. On the other hand, if the context is used to determine whether calculating the median for a set of numbers is meaningful, the context is used statistically because it is central to the solution process.

Last, the context could be used as a setting for application. Teachers reasoned about using the context as a setting for application in two different ways. First, at times teachers did not expect students to learn any new mathematics by working on a problem. Instead, teachers might use a particular problem to apply or practice skills, such as calculating the mean, that have been learned in the past. Second, teachers also used the context as a way of checking for reasonable answers. For instance, students might find a speed that is obviously too slow for the context of the problem. In such cases, teachers might encourage students to notice the answer needs to be different because of the context. By doing so, the context is used to convince students that their original solution process must have some flaw.

My process for coding the data was typical of qualitative research. To begin, I coded all the data from the questionnaire and interview. After the initial pass I recoded all the data without referring to the first set of codes. At this point, I reconciled differences between the first and second passes of coding. Finally, I asked a colleague to code a portion of the data, and we assigned codes with satisfactory agreement.

After coding all the data I used quantitative and qualitative methods to consider the relative quality of teachers' curricular reasoning. The quantitative analysis was quite brief and consisted entirely of examining the frequency with which codes appeared. I chose to examine the frequencies in order to gain an initial sense for differences that might exist in the quality of curricular reasoning both across teachers and across content areas within the same teacher. After this cursory quantitative analysis, I used qualitative methods to confirm or disconfirm the results obtained in the quantitative analysis. During the qualitative analysis of the data I used memoing (Miles & Huberman, 1994) to understand more general patterns in teachers' curricular reasoning. For example, the quantitative analysis might reveal that teachers most often used the context as a setting for application. In such a case, memoing was useful because it helped me recognize distinct patterns in the ways different teachers reasoned about using the context as a setting for application. To address the relative quality of teachers' curricular reasoning I compared and contrasted the patterns in teachers' curricular reasoning that were establish previously. As I compared and contrasted these patterns I grouped teachers according to similarities across categories of codes related to curricular reasoning. After forming these groups I considered teachers to have higher quality curricular if the patterns in their curricular reasoning were more aligned with ideals in categories of codes. For example, with respect to cognitive demand, I considered a teacher to have higher quality curricular reasoning if their curricular reasoning established a pattern of maintaining rather than undermining the cognitive demand of curriculum materials as written.

The Relationship between MKT and the Quality of Curricular Reasoning

Data. For this segment of my study I did not collect any new data. Instead, I looked for patterns across results of the two previous segments in an effort to describe knowledgeable teachers' curricular reasoning. Because I did not collect new data during this phase of my research, it was crucial to collect a full set of data in the previous segments. To ensure I collected the data needed for analysis in this segment I used a data accounting sheet as suggested by (Miles & Huberman, 1994). The data accounting sheet is a table of participants and data sources needed for each research question and displays which sources of data are needed from which participants. After collecting all sources of data for teachers' MKT and curricular reasoning as well as performing analysis of these constructs I had a basis for describing knowledgeable teachers' curricular reasoning.

Analysis. To investigate a potential relationship between MKT and the quality of curricular reasoning I conducted two stages of analysis. The first stage was to compare the quality of individual teacher's curricular reasoning across content areas with MKT. This comparison was somewhat elusive because MKT is not measured in terms of absolute knowledge. Instead, MKT scores are relative to other test takers. Consequently, a teacher might have the same MKT score in algebra and statistics, but this does not imply the teacher has the same level of MKT in both content areas. The reason for this disparity is that the general population of teachers can have greater knowledge in one content area. To work around the lack of a MKT measurement in absolute terms, I considered pairs of teachers that shared a common score on the MKT assessment in one content area. By considering these pairs I was able to be confident the change in MKT across content areas was different for the teachers. Now, to get a sense of a relationship that might exist between MKT and the quality of curricular reasoning, I compared the change in the quality of curricular reasoning to the change in MKT across content areas.

For the second stage of this analysis I compared the quality of curricular reasoning across teachers for each content area. I established the null hypothesis that there is no difference in MKT among teachers with higher and lower quality curricular reasoning. I used a permutation test (Higgins, 2004) to test the alternative hypothesis that there is a difference in MKT among teachers with higher and lower quality curricular reasoning. A permutation test was useful in this case because it does not rely on a large sample size to be reliable. Instead, a permutation test builds up the distribution of a test statistic under the null hypothesis by shuffling the data across labels. Building up this distribution makes it possible to determine the probability of observing a test statistic at least as extreme as the test statistic calculated from the data. More simply, a permutation test yields a p-value by comparing the observed test statistics to the distribution of the test statistic under the null hypothesis. However, this p-value does come with one caveat. Because the p-value was obtained using Monte Carlo methods, it is necessary to account for Monte Carlo error. This error is often accounted for by constructing a 95% confidence interval for the p-value. For this analysis, I used R to shuffle MKT scores across the labels of higher and lower quality curricular reasoning 1 million times and computed the difference in mean MKT for the labels of higher and lower quality curricular reasoning each time. My rationale for shuffling the data in this way is that under the null hypothesis it does not matter which MKT scores are labeled with higher or lower quality curricular reasoning. After building up the distribution of the test statistic under the null hypothesis I obtained a 95% confidence interval for the true p-value of the test statistic computed from the observed data.

Chapter 5: Results

In this section I use the data to make three claims of relevance to my research questions. When fitting I address how the results answer the research questions. For each claim I state a generalization and describe examples from the data to support the generalization.

Individual Teacher's Curricular Reasoning in Algebra and Statistics

Individual teacher's curricular reasoning was largely constant across the areas of algebra and statistics. This claim constitutes my answer for the research question related to the similarities between teachers' curricular reasoning in algebra and statistics. In short, individual teachers had few differences in their curricular reasoning across algebra and statistics. To be clear, differences in curricular reasoning were present across teachers, but individual teacher's curricular reasoning was substantially similar across these two areas of mathematics. For example, Figure 2 highlights the frequency with which teachers' curricular reasoning maintained or undermined high levels of cognitive demand. In some cases the frequency for these codes within a content area was zero and no bar is present. Overall this figure shows that teachers maintained or undermined the cognitive demand with roughly the same frequency in algebra and statistics.

Qualitative evidence confirms that teachers' curricular reasoning was largely constant across algebra and statistics. Sharee reasoned about several strategies for maintaining cognitive demand. These strategies included providing scaffolding for students and pressing for explanation. In both algebra and statistics Sharee was aware that some students might have difficulty beginning to work on the problems. Sharee reasoned that she could help such students by brainstorming previously learned ideas such as drawing a picture, creating a table or considering algorithms that might be useful. Further, Sharee reasoned about pressing students to explain their answers. Several instances of curricular reasoning revealed Sharee's intention to have students "explain their decision."

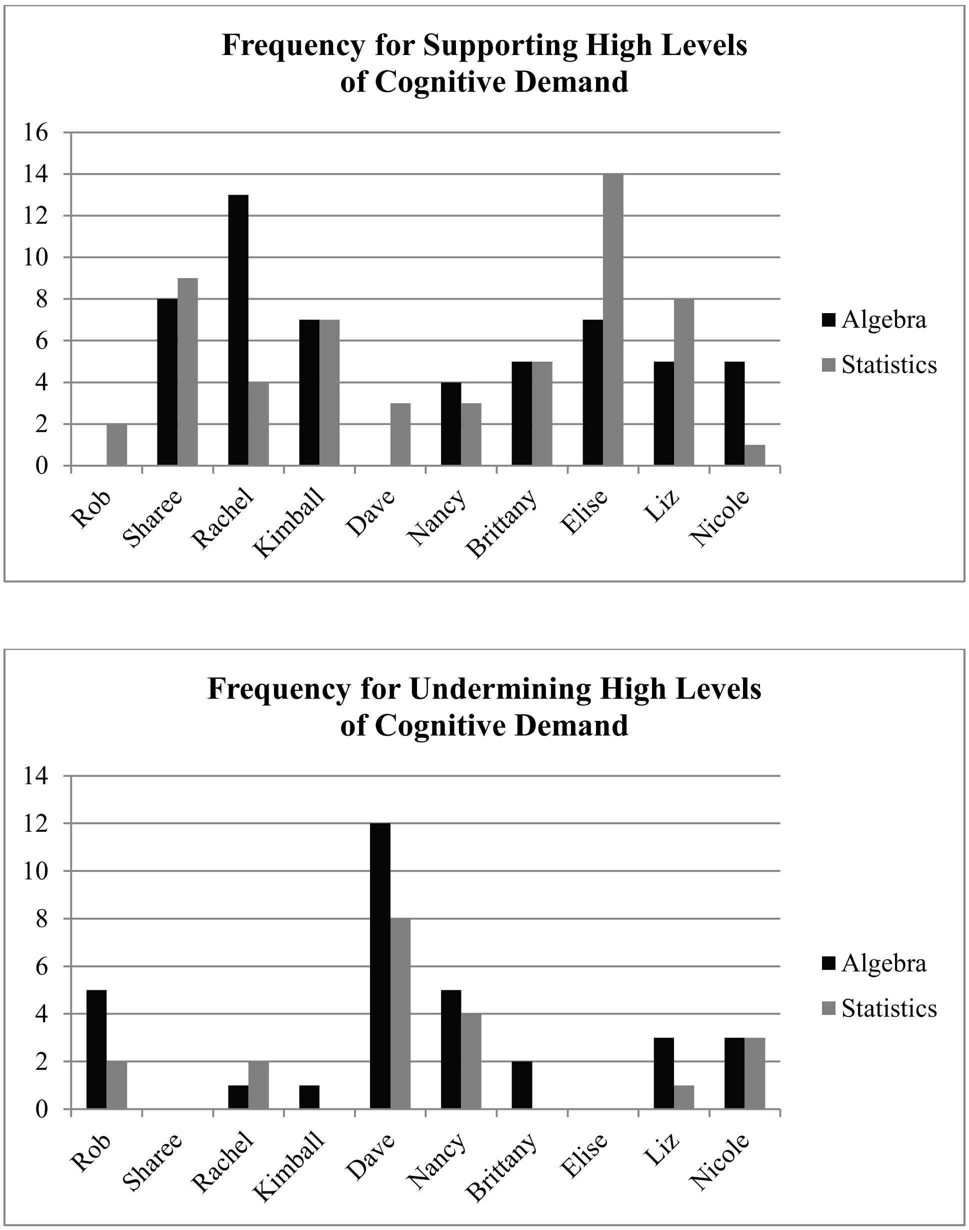

Figure 2. Frequency of codes related to cognitive demand of teachers' curricular reasoning

Even in cases where the frequencies in Figure 2 might suggest distinction between teachers' curricular reasoning in algebra and statistics, qualitative analysis showed many similarities. For example, Rachel's curricular reasoning had a higher frequency of codes for maintaining cognitive demand. At the same time, the qualitative nature of these codes was largely the same. Specifically, Rachel reasoned about building on students' prior knowledge. In algebra Rachel reasoned about helping students understand unit rates by making a connection to a conceptual understanding of division. Rachel said, "I think a lot of the time students would just think it's 200 feet and he has six seconds to get there and back and just divide it to get their answer. They don't realize we've got a total and we're making groups of one second. How many fit in each group? If they can get that basic understanding—that when you divide two different quantities you have a group of something not just a number." This statement shows Rachel reasoned about helping students understand rates by making a connection to prior knowledge of partitive division (Thompson & Thompson, 1994). Similarly, in statistics she reasoned about helping students interpret the meaning of previously learned methods. Moreover, Rachel reasoned about using the context algebraically regardless of the content of the problem. For both algebra and statistics problems Rachel used the context to develop ideas that apply in any context. For example, she reasoned about using data to help students understand the effect of outliers on measures of center. Outliers have the same effect on measures of center in any context, so Rachel used the statistical context algebraically because she planned to teach a new mathematical idea that is not affected by superficial features of the context.

Rachel and Sharee illustrate the claim that teachers' curricular reasoning across the content areas of algebra and statistics was mostly constant. The case of Rachel is helpful because the frequencies in Figure 2 suggest a possible difference in her curricular reasoning in algebra and statistics. Nonetheless, qualitative analysis showed that Rachel's curricular reasoning substantively the same across these areas of mathematics.

The Quality of Teachers' Curricular Reasoning

The quality of teachers' curricular reasoning varied across teachers. Almost all of the variation in curricular reasoning occurred in the categories of cognitive demand and use of context. Unfortunately, the categories of belief and KDU were not useful for finding distinctions in the quality of teachers' curricular reasoning because teachers almost uniformly held their curricular reasoning as belief and neglected to consider how mathematical ideas develop over time. Because cognitive demand and use of context were useful categories for characterizing the quality of curricular reasoning, frequencies for these categories along with a decision about the relative quality of teachers' curricular reasoning are displayed in Table 1. It is important to note that content areas of algebra and statistics are combined in Table 1 because of the previous claim that teachers' curricular reasoning was largely constant across the content areas. After some explanation of the table I will provide a qualitative description of some teachers' curricular reasoning in an effort to justify and illustrate the levels of quality of curricular reasoning.

For convenience in reading Table 1, a line is drawn to separate teachers with lower quality curricular reasoning from teachers with higher quality curricular reasoning. Within the groups of higher and lower quality curricular reasoning there were a couple teachers, Nancy and Elise, whose curricular reasoning was different enough to motivate some further distinction which is indicated by the '+' symbol. In other words, Nancy and Elise had lower and higher quality curricular reasoning respectively, but their curricular reasoning was also enough of an improvement compared to other members of each group to justify some formal distinction. For other teachers with the same level of quality of curricular reasoning the order in the table is insignificant. That is, one should not infer from Table 1 that Liz's curricular reasoning was of higher quality than Kimball's or vice versa.

Table 1
The Quality of Teachers' Curricular Reasoning Combined over Algebra and Statistics

	Cognitive Demand		Use of Context			Curricular
Teacher	Maintain	Undermine	Algebraic	Statistical	Application	Reasoning
Rob	2	7	0	1	4	Lower
Dave	3	20	1	0	4	Lower
Nicole	6	6	1	0	3	Lower
Nancy	7	9	1	0	5	Lower+
Sharee	17	0	5	0	1	Higher
Rachel	17	3	5	0	2	Higher
Kimball	14	1	7	0	3	Higher
Brittany	10	2	1	0	2	Higher
Liz	13	4	2	5	2	Higher
Elise	21	0	2	8	2	Higher+

I will use the examples of Elise and Dave to illustrate the difference between higher and lower quality curricular reasoning. Elise demonstrated the highest quality curricular reasoning of all teachers in the study. Table 1 indicates that Elise entirely avoided including ideas that undermine the cognitive demand of tasks as written in her instances of curricular reasoning. Further, Elise's curricular reasoning was full of strategies for maintaining the cognitive demand of tasks. Elise planned to provide scaffolding to help students make progress in their mathematical thinking. For instance, in the context of a statistics problem Elise reasoned that students could give more attention to patterns and concepts if she provided technology to perform computation. Elise also based her goals, plans and strategies for instruction on students' prior knowledge. For a problem involving scaling up the proportion of candies in a bowl Elise knew some students might begin to draw pictures of candy bowls containing the given number of candies. Elise reasoned that she could ask students how they know how many bowls to draw. By doing so, Elise intended to build on students' prior knowledge of multiplication as groups of a certain size to help them recognize the relationship between proportional quantities. Another distinguishing feature of Elise's curricular reasoning was the way she used the context to serve disciplinary objectives. Specifically, Elise expressed how essential the context is for statistical problems and she aimed to help students use the context to inform both the methods they choose and the way they interpret results. Also, for proportional reasoning problems Elise planned to use the context algebraically. Returning to the example of the candy bowl problem, Elise hoped students would be able to abstract a method for scaling quantities that could be applied regardless of the context. To summarize, I consider Elise's curricular reasoning to be higher than any other teacher in the sample because of her strategies for maintaining the cognitive demand of curriculum materials as written, the lack of evidence for undermining the cognitive demand of curriculum materials as written and her ability to use the context of the problems to distinguish between statistical and purely mathematical goals.

I considered Dave's curricular reasoning to be lower quality for two reasons. First, as the frequencies in the Table 1 indicate, Dave's reasoning often undermined the cognitive demand of curriculum materials as written. In instances where Dave's reasoning undermined the cognitive demand he often spoke about the challenge of having limited time to help all students achieve the goals for the lesson. In an effort to save time, Dave preferred to avoid attempting to surface multiple solution strategies or representations for a problem. In response to one problem that explicitly required multiple solution strategies Dave insisted, "Let's just learn it one way and apply it to the context." By insisting the focus be on a single approach to the problem Dave's reasoning undermined the cognitive demand of curriculum materials as written because he eliminated the opportunity to establish connections between representations and strategies as required by the curriculum materials as written. Second, the use of context in Dave's curricular reasoning did not align with disciplinary ideals. Rather than use the context to inform method and interpretation or to develop new mathematical ideas, Dave reasoned that students already knew any mathematics relevant to the curriculum materials and the challenge of the problem was to apply the mathematics within a context. While there are likely to be times when such application of mathematical procedures is appropriate, the overall quality of Dave's curricular reasoning was lowered by the lack of using the context in a way that is aligned with the disciplines of mathematics and statistics.

Reflecting on the two preceding examples highlights key differences in higher and lower quality curricular reasoning. In terms of cognitive demand, teachers with higher quality curricular reasoning presented a multitude of strategies for maintaining the cognitive demand of curriculum materials as written. These strategies included scaffolding, pressing for explanation and building on prior knowledge. Teachers with lower quality curricular reasoning more often undermined the cognitive demand of curriculum materials as written. Some ways teachers undermined the cognitive demand included not holding students accountable for doing mathematical work and shifting the focus of work to obtaining correct answers. In terms of use of context, teachers with higher quality curricular reasoning more often aligned the use of context with disciplinary ideals. In mathematics this ideal involves abstracting generally applicable processes from particular situations, and in statistics the ideal entails use the context as a vital reference point for making decisions about methods to use and interpretation of results. Such use of context stands in contrast to the way teachers with lower quality curricular reasoning used the context. Teachers exhibiting lower quality curricular reasoning more often reasoned about the context as a setting to apply previously learned mathematical ideas. While this use of context might be worthwhile, it is not well-aligned with disciplinary views on use of context.

For some teachers the distinction between higher and lower quality curricular reasoning was less straightforward. I categorized Liz's curricular reasoning as higher quality because her reasoning included multiple strategies for maintaining the cognitive demand of curriculum materials and her use of context was aligned with disciplinary ideals. Liz's strategies for maintaining cognitive demand included providing scaffolding and building on students' prior knowledge. One way Liz provided scaffolding was by suggesting ways to manage computation. For example, Liz predicted that students might be inconsistent with the order they write ratios as they solve problems involving proportional reasoning. That is, students might initially write a ratio as Quantity A to Quantity B and later unwittingly write the ratio as Quantity B to Quantity A. To combat this issue, Liz reasoned that she could suggest students label the ratios. Liz also established goals that were appropriate based on students' prior knowledge. For example, Liz was confident that students knew how to calculate mean, median and mode of a data set, so her curricular reasoning built on that knowledge by setting a goal for students to make decisions about which measure of center would be most appropriate for particular sets of data. At the same time, Liz's curricular reasoning contained some ideas that undermined the cognitive demand of curriculum materials. Some instances of Liz's curricular reasoning showed that she might shift the focus of a lesson to finding correct answers. To illustrate, in a proportional reasoning problem involving speed, Liz thought some students might divide seconds by feet, and she planned to "remind students to usually put distance over time." By offering this reminder Liz undermined the cognitive demand by introducing a rule of thumb rather pressing students to justify their work or make sense of the situation. Overall, the higher quality aspects of Liz's curricular reasoning were more influential for me than the lower quality aspects, and I categorized her curricular reasoning as higher quality.

I categorized Nancy's curricular reasoning as lower quality for two reasons. First, Nancy's curricular reasoning often undermined the cognitive demand of curriculum materials for students she considered to be struggling. For Nancy, struggling learners included students with persistent difficulty learning mathematical ideas. Such students may or may not have an IEP. While it is unclear exactly how a student comes to be seen as struggling in Nancy's class, this classification was consequential with respect to the maintenance of cognitive demand in Nancy's curricular reasoning. For example, a proportional reasoning problem required students to use two solution methods and then compare the methods. However, Nancy reasoned that she would remove this requirement for struggling students. Thus, Nancy did not expect struggling students to fulfill the cognitive demands of the curriculum materials as written. Second, the use of context in Nancy's curricular reasoning did not align with disciplinary ideals. For example, to justify the goal of computing measures of center from a data set Nancy stated, "I think they know the algorithm really well, but they may not understand how to apply it in context." I considered such instances of curricular reasoning to be evidence of Nancy's tendency to use the context as a setting for application.

Although Nancy's curricular reasoning often undermined the cognitive demand of materials as written for students she considered to be struggling, her strategies for maintaining the cognitive demand for other students improved the quality of her curricular reasoning. Specifically, Nancy's curricular reasoning contained useful strategies for maintaining the cognitive demand of curriculum materials for non-struggling students. In reference to the previously mentioned proportional reasoning problem, Nancy expressed the need to compare and contrast multiple solution strategies and representations. Also, she was attentive to building understanding from students' prior knowledge. Such strategies for maintain cognitive demand for non-struggling students were the reason I considered Nancy's curricular reasoning to be somewhat higher quality than other teachers with lower curricular reasoning.

Thus far, the examples used to illustrate the distinction between higher and lower quality curricular reasoning have not highlighted the fact that some teachers used the context algebraically regardless of the context. The examples of Elise and Liz showed how teachers might align the use of context with disciplinary ideals. On the other hand, the examples of Dave and Nancy showed how the context of a problem is used a setting for application of previously learned mathematics. In contrast, Sharee, Rachel and Kimball most often reasoned about using the context algebraically regardless of the content of the problem. To elaborate, whether the problem asked a mathematical or statistical question, these teachers reasoned about using the problem to develop a solution process that could apply in any situation. For example, one problem provided students with a table of wait times for eruptions of Old Faithful and asked students to draw inference about how long one would expect to wait for an eruption. Kimball reasoned about using this context algebraically. He stated, "The goal is to see the effects of outliers on measures of central tendency." Because Kimball's goal for the lesson would lead students to a conclusion that applies regardless of the context, I considered the use of context to be algebraic. In terms of quality of curricular reasoning, I considered using the context algebraically regardless of content to be higher quality than using the context as a setting for application and lower quality than aligning the use of context with disciplinary ideals. Thus, the use of context exemplified by Sharee, Rachel and Kimball served to both elevate and limit the overall quality of their curricular reasoning.

Lack of Relationship between MKT and the Quality of Curricular Reasoning

The data suggest that MKT is not associated with higher and lower quality curricular reasoning and is a direct answer to the third research question regarding the relationship between MKT and curricular reasoning. This claim is true both for individual teachers across content areas and across teachers regardless of content area. Regarding individual teachers, an increase or decrease in MKT across content areas was not associated with a change in the quality of curricular reasoning across the content areas. In fact, the individual teacher's quality of curricular reasoning across content areas was essentially unaffected by variation in MKT. Variation in MKT across content areas is somewhat elusive because MKT is measured in relative rather than absolute terms. Said differently, if the population of teachers has less absolute MKT in, say, statistics, a score of 0 for statistics MKT would indicate less absolute MKT than the same score for algebra MKT. Unfortunately, there is no scale in place to adjust for varying levels of absolute MKT in the population across content areas. The structure of Table 2 is an attempt to overcome this challenge by highlighting pairs of teachers that had the same level of MKT in one content area and a different level of MKT in the other area. Looking at these pairs of teachers provides an assurance that at least one of the teachers had a different level of absolute MKT in algebra and statistics. Now, with the assurance that at least one teacher in each pair had a varying level of MKT across content areas it is useful to revisit the claim that teachers' curricular reasoning across content areas was more or less fixed. Combining these two results provides evidence that MKT did not have a strong relationship with curricular reasoning because variation in MKT was not associated with variation in curricular reasoning. For example, based on Table 2 Rob or Kimball (maybe both) had a different level of MKT across algebra and statistics, yet Rob's curricular reasoning across these content areas is largely constant and lower quality.

Similarly, Kimball's curricular reasoning across these content areas is largely constant and higher quality. Therefore, either Rob or Kimball (maybe both) demonstrate that MKT can vary across content areas with little, if any, variation in the quality of curricular reasoning. Similar reasoning can be applied to each of the pairs in Table 2 because all the teachers demonstrated largely constant curricular reasoning across algebra and statistics.

Table 2
Pairs of Teachers with the Same MKT in One Content Area

Teacher	Algebra MKT	Statistics MKT	Quality of Curricular Reasoning
Rob	1.26	1.09	Lower
Kimball		2.05	Higher
Nancy	1.67	1.09	Lower+
Brittany		1.61	Higher
Rob	1.26	1.09	Lower
Nancy	1.67		Lower+
Sharee	0.71	1.61	Higher
Brittany	1.67		Higher

Variation in MKT across teachers also had no association with the quality of curricular reasoning. A permutation test (Higgins, 2004) for the hypothesis that MKT scores in algebra and statistics are not equal among teachers with higher and lower quality curricular reasoning was not significant at the $\alpha = 0.1$ level. Technically speaking, the permutation test yielded a 95% confidence interval for the p-value which did not cover any significant p-values. Therefore, the data in this study do not suggest there is a difference in MKT among teachers with higher and lower quality curricular reasoning.

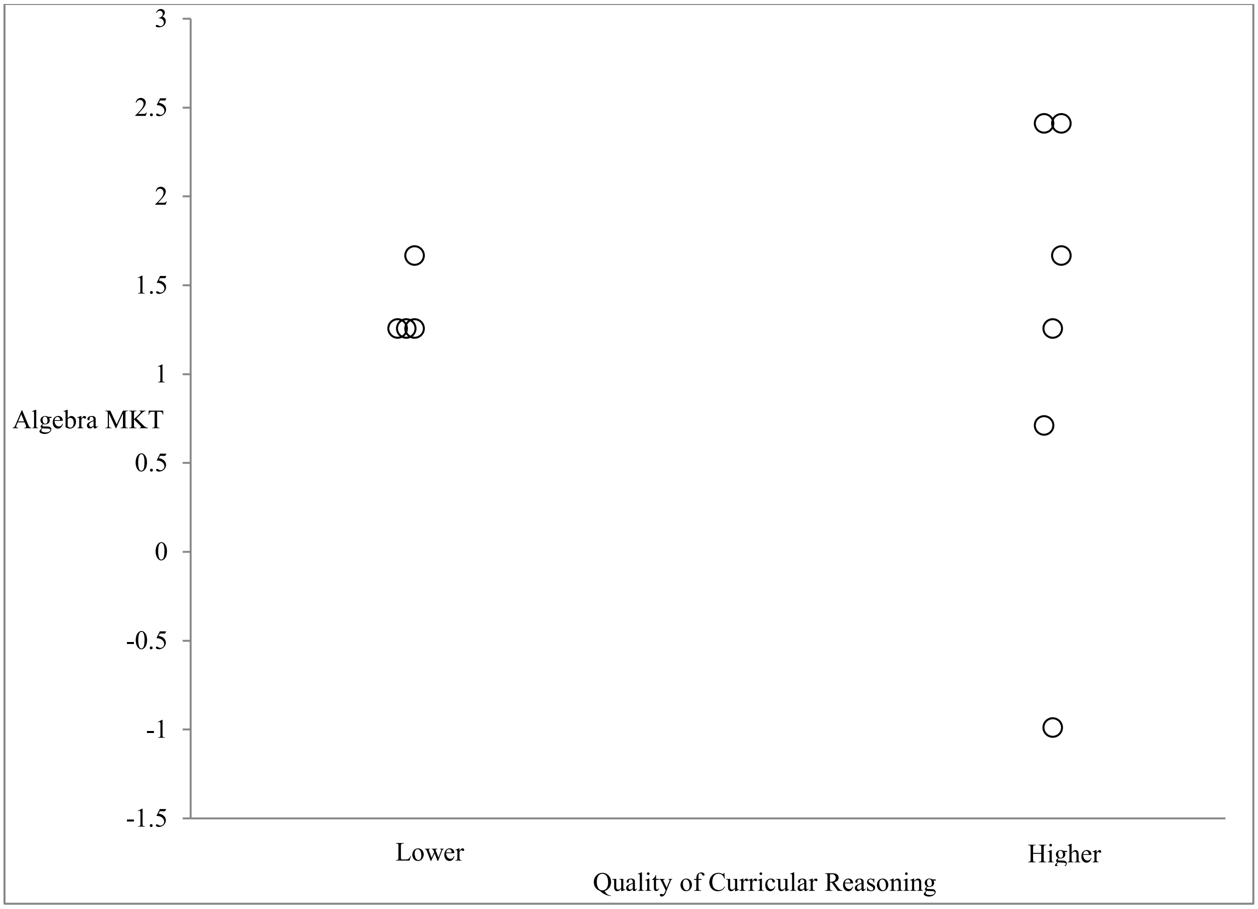

Figure 3. Algebra MKT and the quality of curricular reasoning.

Higher curricular reasoning was distributed across the range of MKT scores. This distribution was true for both algebra and statistics as shown in Figure 3 and Figure 4. In both algebra and statistics teachers with MKT less than one standard deviation above the mean and more than two standard deviations above the mean had higher quality curricular reasoning. For MKT between one and two standard deviations above the mean, there was a mix of teachers with higher and lower quality curricular reasoning. Due to this mix, the data support the claim that MKT and the quality of curricular reasoning have little relationship across teachers.

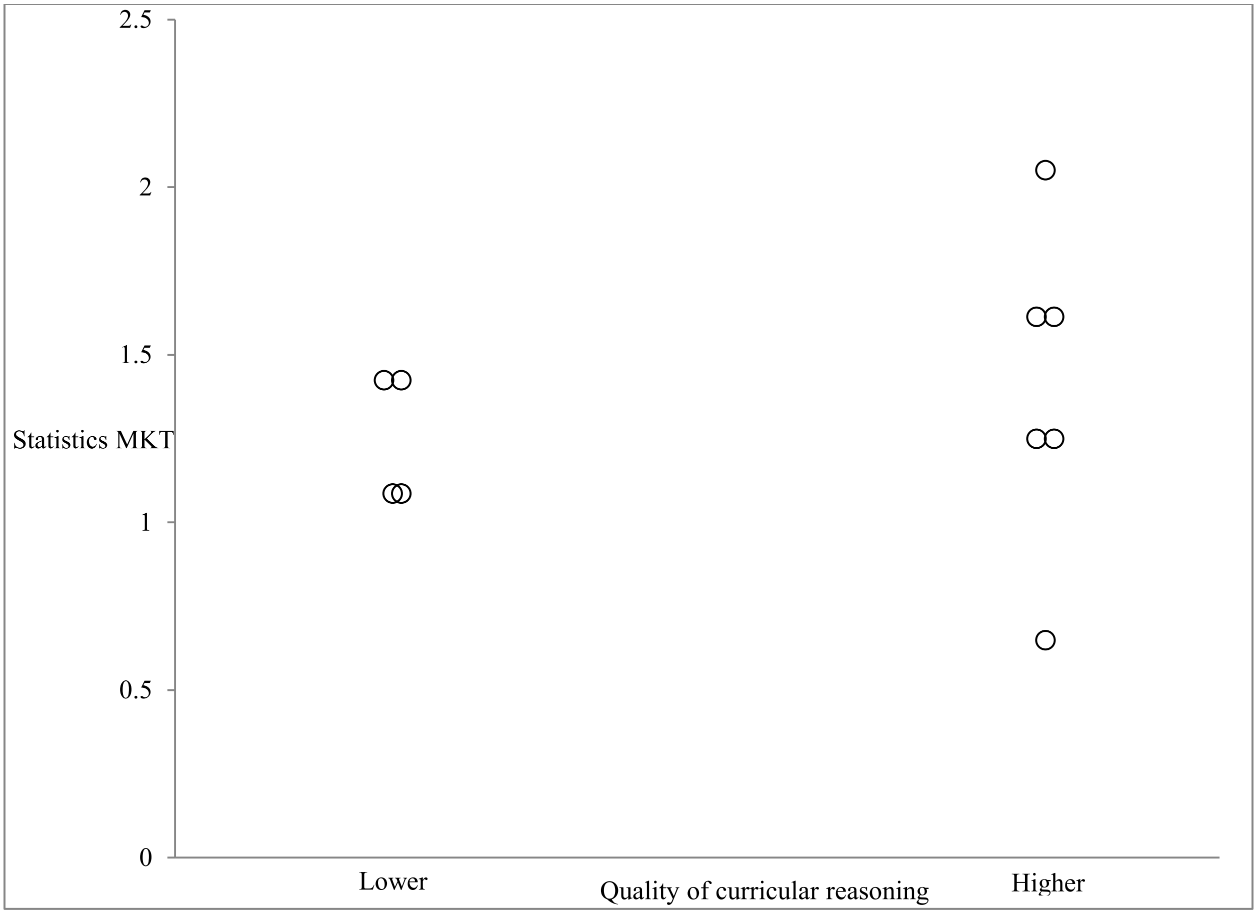

Figure 4. Statistics MKT and the quality of curricular reasoning

The scatterplot in Figure 5 illustrates how the quality of curricular reasoning was distributed across levels of MKT in algebra and statistics. As a note, ten teachers participated in this study, but Figure 3 only displays eight distinct points. The reason for this discrepancy is the fact that two pairs of teachers had the same MKT in algebra and statistics as well as the same quality of curricular reasoning. For this reason, it is helpful to know that the markers with a thicker line actually represent two teachers. Also, for simplicity Figure 5 only distinguishes between higher and lower quality curricular reasoning rather than including the levels of lower+ and higher + as in Table 1. If these levels were represented in Figure 5, distinct markers at approximately (1.3, 1.6) and (1.4, 2.5) would be needed to represent lower+ and higher+ respectively.

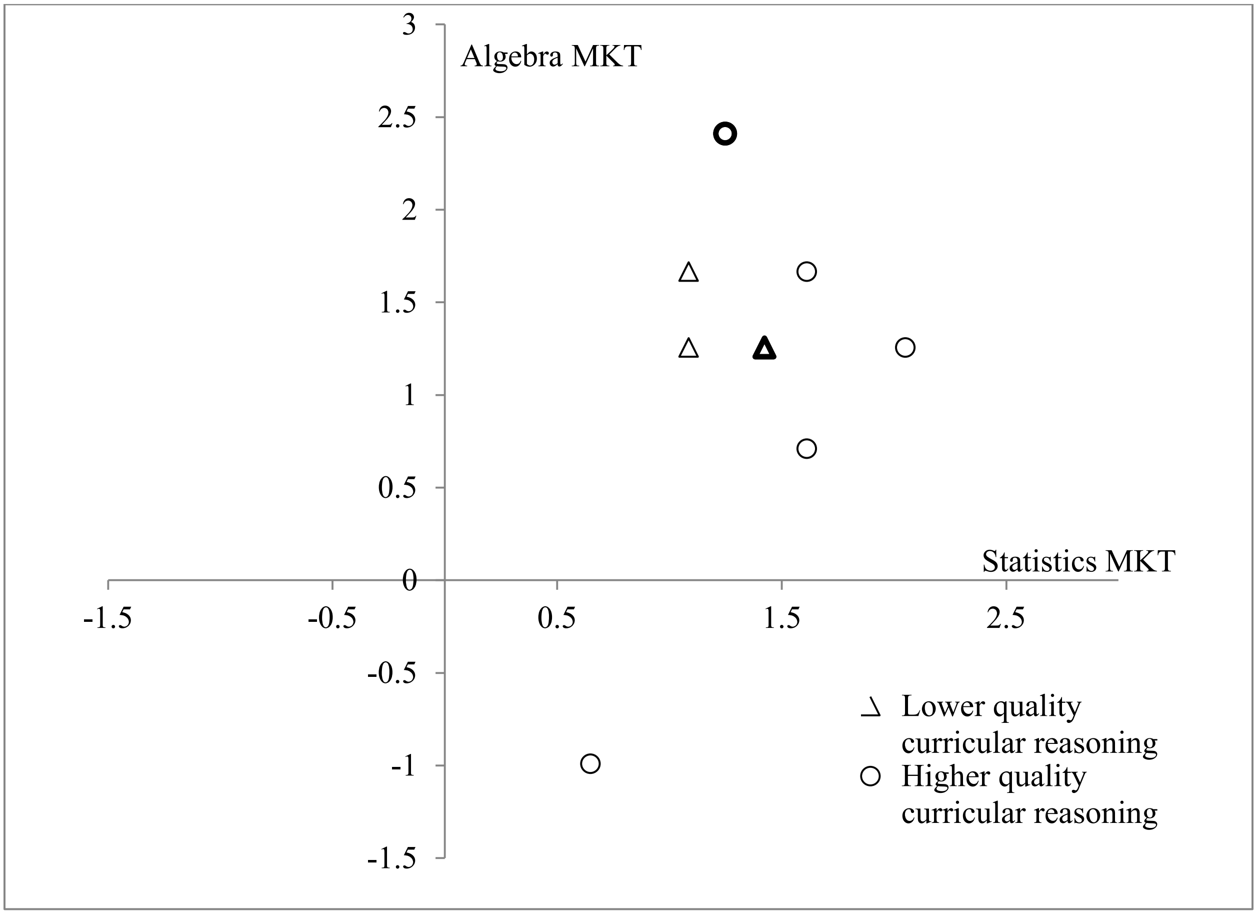

Figure 5. Scatterplot of Teachers' MKT and the Quality of Curricular Reasoning

There also does not appear to be a strong relationship between curricular reasoning and MKT when statistics and algebra MKT are considered in tandem. As seen in Figure 5, the space between one and two standard deviations above average MKT in algebra and statistics contains a mix of higher and lower quality curricular reasoning. At the same time, teachers with MKT less than one standard deviation above the mean or more than two standards deviations above the mean had higher quality curricular reasoning. Although lower quality curricular reasoning appears to form a cluster in Figure 5, the distribution of higher quality curricular reasoning across the range of MKT seems to support the claim that the relationship between curricular reasoning and MKT is weak.

Chapter 6: Discussion

In this section I will first briefly summarize the results of this study. After this summary I will discuss what these results add to previous research as well as limitations of the study and suggestions for further research.

I found several important results related to teachers' curricular reasoning and the relationship between curricular reasoning and MKT. First, the qualities of maintaining or undermining the cognitive demand of curriculum materials as written and the use of context were useful for distinguishing between teachers' curricular reasoning. At the same time, the degree to which teachers held their curricular reasoning as belief was not useful for distinguishing between teachers' curricular reasoning because all teachers almost universally held their curricular reasoning as belief. Second, individual teachers typically reasoned about using curriculum materials the same regardless of context. For example, if a teacher reasoned to use the context as a setting for application in for algebra, in general the teacher also reasoned to use the context as a setting for application in statistics. Next, curricular reasoning varied widely across teachers. Last, variation in curricular reasoning did not have a strong relationship with MKT. The weakness of this relationship was manifest in two ways. First, individual teacher's MKT varied across content areas, but individual teacher's curricular reasoning was largely constant across content areas. Second, variation in curricular reasoning across teachers could not be explained by variation in MKT because teachers with higher MKT had lower quality curricular reasoning and vice versa.

I will now discuss how these results fit within the existing body of research. To begin, it is striking that all teachers in this sample held their curricular reasoning almost exclusively as belief. As a reminder, my framework for belief specifies that individuals hold a notion as belief if they are willing to accept alternative notions as reasonable (Philipp, 2007). Within this framework the interpretation of the result is that teachers generally found alternative patterns of curricular reasoning to be sensible which is similar to Jacobs and Morita's (2002) finding that American teachers were willing to accept multiple ideal scripts for mathematics lessons. However, it is unclear what affect this result has on classroom practice. Further research is needed to understand how well teachers are able to enact alternative scripts or curricular reasoning they find sensible.

Next, the evidence in this study indicates a weak relationship between curricular reasoning and MKT. Based on results from studies that observed instruction (Hill, Blunk, et al., 2008; Sleep & Eskelson, 2012), MKT seems to be critical during the enactment of a lesson. Specifically, some characteristics of a lesson such as the use of precise language and the frequency of mathematical errors appear to be associated with MKT. At the same time, it is counterintuitive that curricular reasoning and MKT may vary separately because the construct of MKT includes domains that seem useful for planning mathematics instruction. Nonetheless, there are multiple reasons why empirical evidence would support a weak relationship between MKT and curricular reasoning. First, the LMT assessment (Hill et al., 2004) of MKT might be ill-suited to measure the MKT teachers utilize for planning a lesson. For example, knowledge of content and students may play an important role in planning effective instruction. However, developing test items to measure knowledge of content and students separate from other domains of MKT is an unresolved challenge (Hill, Ball, & Schilling, 2008). Next, the sample in this study may limit the ability to find a relationship between MKT and curricular reasoning. Teachers in this sample generally had average to above average MKT, and a relationship between MKT and curricular reasoning could be more apparent if the sample included teachers with lower MKT. Also, the sample in this study was small and could lack sufficient power to detect the relationship between MKT and curricular reasoning. Further, the sample in this study was drawn from a suburban area. A sample of teachers from another (e.g. urban) context might yield different results. Finally, MKT and curricular reasoning may in fact be weakly related constructs. This is plausible because curricular reasoning is broader than MKT. MKT encompasses the mathematical knowledge teachers use to perform their work, but teachers draw on a wider array of knowledge to make actual decisions about instruction. This array includes ideas about student needs, community expectations and personal preferences. Thus, teachers might be aware that their curriculum materials can support high levels of cognitive demand based on their MKT, but teachers may choose to undermine the cognitive demand of the curriculum materials as written if they feel pressure from their department to cover a list of topics in order to assign a specific set of homework problems. In conclusion, the evidence from this study indicates a weak relationship between MKT and curricular reasoning, but there are many avenues for future research to follow to garner more evidence to either support or refute this initial claim.

Given the results of this study and others (Hill, Blunk, et al., 2008; Sleep & Eskelson, 2012; Stein & Kaufman, 2010) I propose a model for how curricular reasoning and MKT might be situated in the Mathematical Tasks Framework (Stein et al., 1996) and use this model to offer an explanation for some perplexing cases of instruction. I used the construct of curricular reasoning as a way to capture the goals and plans teachers make as they anticipate using curriculum materials to provide instruction, so in Figure 4 I placed curricular reasoning between tasks as they appear in curricular materials and tasks as they are set up by the teacher in the classroom. Because I did not find a strong relationship between curricular reasoning and MKT and other researchers have documented an association between MKT and the quality of instruction (Hill, Blunk, et al., 2008; Sleep & Eskelson, 2012), I consider MKT to influence the space between tasks as set up by the teacher and tasks as enacted by students. Previous research also contains examples of resources apart from MKT that teachers use to support the quality of instruction during the enactment of a lesson. One such resource is the supports built into the curriculum materials (Hill, Blunk, et al., 2008; Stein & Kaufman, 2010). In these cases teachers adhere to curriculum materials in a way that increases the quality of instruction beyond the expectation given the teacher's MKT. The list in Figure 4 of factors influencing implementation is not meant to be comprehensive. Rather, the list is intended to provide examples of resources teachers may draw on during the implementation of a lesson that are relevant to this study. These factors are relevant because of their utility for offering an explanation for perplexing cases of instruction.

Perplexing cases of instruction can be categorized as a mismatch between MKT and the quality of instruction, and the model in Figure 4 can help provide insight into why teachers with higher MKT can provide lower quality instruction and why teachers with lower MKT can provide higher quality instruction. In the former case, teachers' curricular reasoning might compromise demanding aspects of curriculum materials prior to setting up the lesson in the classroom. By doing so, teachers reduce their opportunities to use higher MKT to support higher quality instruction. Sleep and Eskelson (2012) present the case of Marie which appears to fit in the category of lower quality curricular reasoning and higher MKT. Marie's MKT is near the 90th percentile, but her use of curriculum materials is suspect. Some aspects of Marie's instruction were consistent with her higher MKT. Specifically, she used precise mathematical language and avoided mathematical errors during instruction. However, Marie's curricular reasoning, especially her goals for instruction, undermined the potential of her curriculum materials. Although Marie's curriculum materials provide opportunity for students to make connections among multiple representations and engage in non-algorithmic thinking, Marie used the materials as an application of previously learned procedures and made an effort to focus students' attention on one way of solving the problem. In the end, Marie's curricular reasoning appears to be detrimental to the overall quality of her instruction.

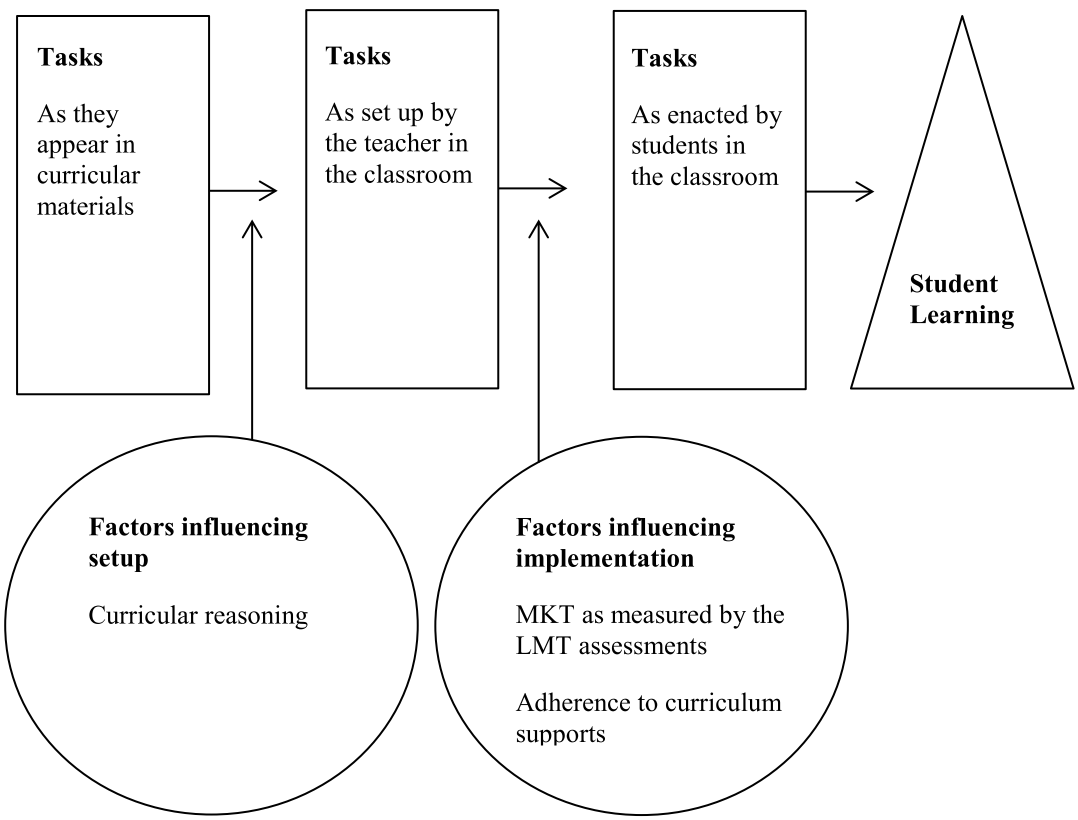

Figure 6. The Mathematical Tasks Framework adapted from Stein, Grover and Henningsen (1996) to emphasize some factors that could influence the quality of instruction.

In the latter case of perplexing instruction, teachers' curricular reasoning might preserve demanding aspects of curriculum materials through setting up the lesson in the classroom.

Moreover, these teachers might make plans to supplement their lower MKT during the enactment of the lesson, and one way to supplement lower MKT might be with earnest attention to curriculum supports. This was presumably the case for a group of teachers implementing cognitively demanding curriculum materials (Stein & Kaufman, 2010). In this study the authors found that MKT was not associated with teachers' ability to enact lessons that maintain high levels of cognitive demand, but the authors did find that using the teacher support materials in the curriculum was correlated with faithful enactment of the materials. The support materials largely focused on the important mathematical ideas to teach with each lesson, and it's plausible to think that these materials helped increase the quality of teachers' curricular reasoning because teachers generally hold their curricular reasoning as belief. Given the lack of an association between MKT and cognitively demanding curriculum implementation, it stands to reason that a group of teachers in this study were able to provide high quality instruction, at least in terms of cognitive demand, by adopting high quality curricular reasoning and in spite of their lower MKT.

These explanations for perplexing cases of instruction rely on the result in this study that the quality of curricular reasoning varies in large part separately from MKT. Inasmuch as these explanations are viable, there are potential implications for teacher education and professional development. Presently there is a heavy focus in teacher education and professional development on improving teachers' MKT. This focus is justified to the extent that it improves the quality of instruction. However, curricular reasoning may be a piece of the other part of the story that Sleep and Eskelson (2012) referred to. That is, teachers with lower quality curricular reasoning are likely to reduce the quality of instruction even if they have higher MKT and high quality curriculum materials. Further research can clarify how teacher education and professional development should balance increasing MKT with increasing the quality of curricular reasoning in order to optimize teaching quality and student learning.

Conclusion

The findings of this research study indicate that individual teachers' curricular reasoning in algebra and statistics is substantially similar. Regardless of the content of curriculum materials teachers' goals and plans for instruction were largely constant. That is, teachers' curricular reasoning supported similar levels of cognitive demand and used the context in similar ways for curriculum materials in both algebra and statistics. Also, the degree to which the quality of teachers' curricular reasoning was unrelated to MKT is surprising. The lack of a relationship between these constructs may be due to limitations in the study, but there do appear to be aspects of curricular reasoning that are distinct from MKT. This finding has implications for teacher education because it seems some level of quality in teachers' curricular reasoning is necessary to support high quality instruction.

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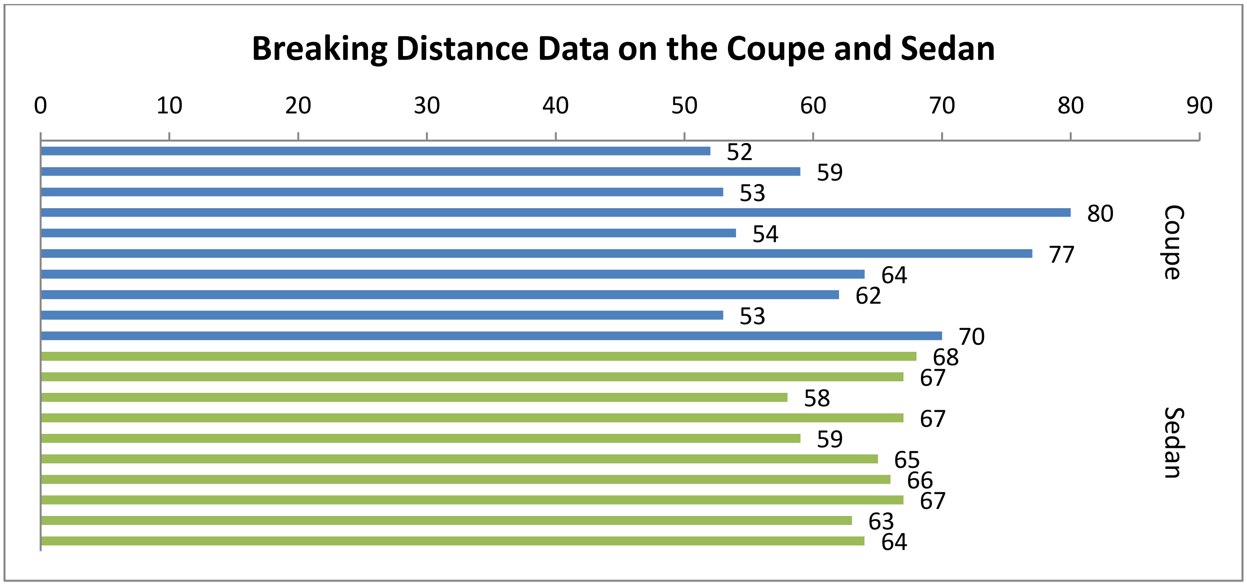
Appendix A

This appendix contains an abbreviated copy of the questionnaire I gave to teachers. This copy includes each of the four problems teachers used as the basis of their responses and the set of 8 prompts teachers responded to for each student problem

Student Problem #1



Which model of car has the better braking distance? Justify your choice.

This problem is adapted from McClain and Cobb (2001).

Student Problem #2

The data in the following table represent wait times (in minutes) between eruptions of Old Faithful over three consecutive days.

Day 1	51	82	58	81	49	92	50	88	62	93	56	89	51	79	58	82	52	88
Day 2	86	78	71	77	76	94	75	50	83	82	72	77	75	65	79	72	78	77
Day 3	65	89	49	88	51	789	85	65	75	77	69	92	68	87	61	81	55	93

Work in groups to analyze the data and make a decision on how long you would expect to wait for an eruption of Old Faithful.

This problem is adapted from Shaughnessy (2007).

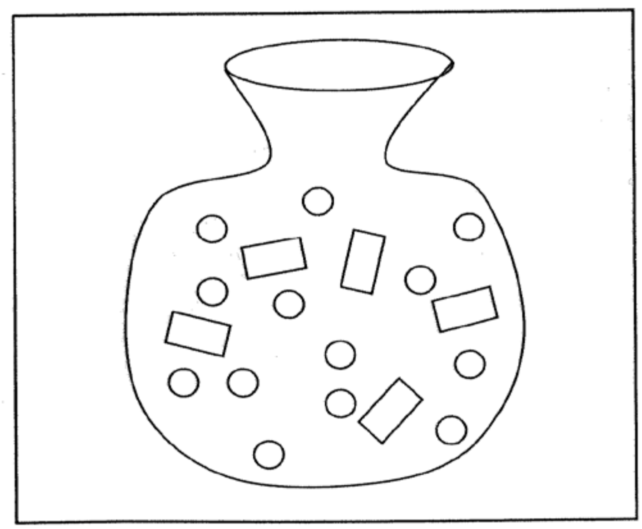
Student Problem #3

A rabbit travels 100 feet from its home then immediately turns around and travels back. At what speed does the rabbit need to travel in order to complete the trip in 6 seconds? Draw a picture to represent your solution.

This problem is adapted from Thompson and Thompson (1994).

Student Problem #4

The candy jar shown in the picture contains Jolly Ranchers (the rectangles) and Jawbreakers (the circles). Please use this candy jar to respond to the prompts.



Suppose you have a larger candy jar with the same ratio of Jolly Ranchers to Jawbreakers as shown in the candy jar in the picture. If the jar contains 100 Jolly Ranchers, how many Jawbreakers are in the jar?

Now, solve the problem with a different approach.

What is the relationship between the two different approaches you used to solve the problem?

This problem is adapted from Smith, Silver, Stein, Boston, Henningsen, and Hillen (2005).

1. Please read through the statement of Student Problem #4. Do the problem and respond to any prompts as if you were a student.

2. In practice this problem could be used to develop a variety of learning goals. After completing the problem, what is a primary mathematical learning goal?

3. How is this mathematical learning goal connected to other mathematics students will learn?

4. What do you suppose your students already know about the mathematical learning goal?

5. In what ways will you adapt, supplement, or omit portions of the problem to meet your students' needs and the mathematical learning goal?

6. Explain your reasoning for making the changes listed above.

7. As with most lessons, it is unlikely that implementing this problem in a classroom will go as planned. Describe one trouble spot (e.g. student frustration, misconceptions, unanticipated student thinking) you foresee occurring during the lesson.

8. Explain how you will help students overcome the trouble spot identified above to achieve the mathematical learning goal.

Appendix B

This appendix contains the general protocol for the interviews I conducted with my participants. The specific questions were guided by each teacher's responses to the questionnaire.

Follow-up on the questionnaire

SP #1

1.

SP #2

2.

SP #3

3.

SP #4

4.

Present alternate patterns of curricular reasoning

1.
2.

Present common teaching predicaments

1.
2.

Appendix C

This appendix contains a table of examples from the data for each of the codes I used during the analysis. This coding formed the basis for distinguishing between higher and lower quality curricular reasoning.

	Code	Example	Rationale
Factors associated with maintaining cognitive demand	Builds on prior knowledge	Develop fundamental understanding of creating rates from a context. Many of the students will look at his as a division problem. The total (200 ft) is divided into 6 groups (or 6 seconds), and they figure out the total number of feet in one group/second.	In this example the teacher builds on a conceptual understanding of operations to help students understand the middle school mathematics topic of finding rates.
	Scaffolding	Explain "wait time" because the students might not know what wait time is. Some students may initially start drawing the given jar multiple times until they had 100 Jolly Ranchers. I would ask students how they know how many times to draw the jar. Ask if there is another way they could represent the same idea without drawing, possibly getting to repeated addition or multiplication. This would lead to scale factor.	In both of these examples the teachers helps students make progress in working on the problem without reducing the difficulty of the problem.
	High-level performance modeled	If this task was used to get different approaches out, I might allow students to skip the different approach if they could only think of one method and have them go back and work it another way after some different approaches had been shared.	The teacher expresses an intention to share exemplary solutions with the class.
	Sustained pressure for explanation and meaning	I would ask questions like the following: If you want to emphasize that Old Faithful is predictable, what statistical measure would you use and why? If you were putting up a sign to tell people how long they have to wait to see Old Faithful erupt, what would you say and why? If you were a park ranger telling	The teacher is not satisfied with an answer to the question. Instead, she plans to press students to consider the meaning of their answer and justify different answers in different situations.

		some visitors who had just missed the eruption when to come so they won't miss the next eruption, what would you tell them? Why? I would want students to not only calculate statistical measures but be able to justify why they chose a particular measure.	
	Teacher draws conceptual connections	If you had your line that says, "this is 100 feet in 3 seconds." Well, if we're looking for one second, let's break it up. Okay, so this is one second. If this is a third of this chuck, how far did he go in this one second?	The teacher is describing how they might represent finding a unit rate using a number line. This representation is conceptual because it relies on understanding multiplication/division in terms of groups of a certain size. Also, the teacher provides this explanation—not the student.
Factors associated with decline in cognitive demand	Inappropriateness of the task	Turns out 7th graders don't know how to round very well, and it's something I fight with all year. I bust their tail at the beginning of the year, I don't let up in the middle of the year and by the end of the year I get a lot of the kids to be able to round. Students know how to simplify fractions, and the goal is for students to be able to write ratios in simplest form to find a unit rate.	In both of these examples the teacher makes an elementary mathematics topic a focus of the lesson. In the second example it is unclear if the teacher is aware that simplifying a ratio to simplest form does not always lead to a unit rate.
	Lack of accountability	Let's just learn it one way and apply it to the context.	This statement was made in response to a prompt in the curriculum materials to solve the problem in multiple ways and then compare the solution strategies. This statement indicates the teacher does not plan to hold students accountable to doing this mathematical work
	Challenges become non-problems	Ask a less subjective question than 'better.' It focuses the number of valid responses.	The question of 'better' would provide an opportunity for students to understand how variance plays a role in describing the distribution of data, but the teacher only requires students to compute various statistics.

	Focus shifts to correct answer	Students might put seconds over feet, so remind student to usually put $\frac{Distance}{Time}$.	Rather than help students develop a conceptual understanding of finding a rate, this teacher plans to provide a template for calculating speeds.
Held as	Knowledge or belief (K/B)	A weakness in my teaching, I've always taught proportions with the cross-product. Proportions have always been a conundrum to me because there's a method that if students get really good at they can solve a proportional problem really slick every time if they recognize that it's proportional, but yet it doesn't really help them understand what proportional means when they use that. People talk about context and word problems and all this stuff, and I'm okay with a limited amount, but, I mean, they just learned it. They're not ready to put it into work. I feel they're movin' too fast in general. I'd rather move faster and just learn the math and then, at a delayed pace, start integrating into context what they've learned a while ago. I would like there to be an offset. I don't want to eliminate context, don't get me wrong. I just want to offset the timing of it.	The first paragraph of text is an example of belief because the teacher presents two ways for teaching proportions that he finds sensible. On the one hand, cross-products are useful for finding answers. On the other hand, other strategies for working with proportions build greater understanding. The second paragraph of text is an example holding curricular reasoning as knowledge because the teacher does not find it sensible to teach mathematics within a context. The teacher seems to 'know' students would be better off if they were comfortable with pure mathematical ideas before they solve problems in a context.
Key Developmenta l Understanding	Empowered learning	Thinking about average speeds connects later on with tangent lines. Unit rates lead to constants of proportionality, scale factor, solving proportions, scaling up and down, percent problems, etc.	These teachers identify topics students might understand as a consequence of understanding an idea in middle school math.

	Eye on the horizon	If they could think of this as an area model where it's rate times time and then start dividing it up into pieces they could do that, but most kids don't think of distance equals rate times time as an area model. … When you get into calculus you gotta' be able to think about the area under the curve and you would divide it up. A lot of what I push in 7th grade is what is an equals sign. They think at the end of the problem I put, "equals 25." What is the importance of that equals sign? So that when I get into the inequalities they get that it's not just equals two. That the equals sign in a proportion, it's not just the middle.	Each of these is an example of 'eye on the horizon' because the teacher makes an effort to expose students to ways of thinking about a mathematical topic that will be useful in future mathematics. In other words, the teacher considers ways of knowing mathematics that are consequential beyond the scope of a grade level.
Use of Context	Algebraic	Maybe have the students predict the effects of outliers. Would the mean, median or mode change the most if there was a sedan with 85?	This is an example of using the context algebraically because the result is an idea that applies to any context. Specifically, outliers affect statistics the same regardless of the context.
	Statistical	Students may get caught up on the 'invalid' data value. Ask why 789 could not be a possible value. Ask what we should do with it, how might that affect your answer? Without the context you don't know whether it an impossible piece of data or not. If we don't have the context, we can't say, "this is a bad piece of data because…"	This is an example of using the context statistically because the teacher recognizes the central role of the context in the solution process.

	Setting for application or interpretation	Does it make sense the rabbit is traveling about one-third of a foot every second, and you think in six seconds they're going to cover 100 feet? I think they know the algorithm really well, but may not understand how to apply it in context.	The first statement is an example of using the context as a setting for interpretation. The teacher plans to encourage students to recognize incorrect answers by reflecting of practical outcomes in the context. The second statement is an example of using the context as a setting for application. The teacher presumes that students are familiar with a pure mathematical idea but is unsure whether or not students can use the idea in a context.

Appendix D

This appendix contains an abridged version of the rationale, framework, literature review and methods. This version of these sections is intended to be used for a publication along with the results, discussion and conclusion that appear in the main body of the thesis.

Rationale

The construct of mathematical knowledge for teaching (MKT) is a popular research topic and provides valuable insight with respect to the inputs and outputs of math education. In particular, MKT helps describe the set of characteristics teachers bring to the classroom, and MKT is positively correlated with student achievement (Hill et al., 2005). At the same time, researchers readily acknowledge that student achievement is not influenced by how well teachers perform on measures of MKT; rather, student achievement is influenced by what knowledgeable teachers do in the classroom (Hiebert, 2013; Thames & Ball, 2013). Thus, it would be valuable to precisely describe the instruction of knowledgeable teachers that mediates the inputs and outputs of mathematics education (Hiebert et al., 2005).

Any effort to describe the instruction of knowledgeable teachers is complicated by numerous factors which concurrently affect instruction. Hill et al. (2008) illustrate how the quality of instruction can converge or diverge with MKT. Hill et al. present two cases which effectively illustrating how the quality of instruction can diverge from MKT. Noelle has relatively high MKT and a negative view of her mandated curriculum. While high MKT seems to help Noelle understand and process students' mathematical thinking, her negative view of the curriculum leads her to use supplemental materials that lack an ultimate purpose. As a result, the quality of Noelle's instruction is less than predicted from her MKT. Conversely, Rebecca has relatively low MKT and strong pedagogical skills that support her procedural view of mathematics. Although Rebecca frequently makes mathematical errors, her strong pedagogical skills and adherence to the textbook minimize the impact of these errors and raise the overall quality of her instruction to a level higher than expected by her MKT. Because the quality of instruction varies based on many factors including MKT it is difficult to describe the instruction of knowledgeable teachers.

I seek to gain insight into the instruction of knowledgeable teachers by comparing and contrasting teachers' MKT with the way they reason about using curriculum materials (Roth McDuffie & Mather, 2009). Although focusing on teachers' reasoning about curriculum materials eliminates the opportunity to observe instruction, there are certainly benefits s associated with comparing and contrasting teachers' MKT with their curricular reasoning. The benefits of studying curricular reasoning are primarily related to stepping away from the complex environment of instruction to highlight how teachers prepare for instruction. While curriculum materials do not determine how a lesson is implemented, a large part of teachers' work is to develop a lesson from curriculum materials to achieve some mathematical goal (Breyfogle et al., 2010). Thus, variation in the use of curriculum materials is a useful way to understand potential causes of variation in instruction.

Framework

To define a framework I will first discuss the setting of this study. Next, I present MKT and curricular reasoning as the major constructs considered in this study. After establishing this framework I present the research questions I will address in this study.

Algebra and Statistics

Throughout this study I use the setting of algebra and statistics. With the recent adoption of CCSS in mathematics and an increased focus on statistics at some levels it is timely to consider how teachers might approach teaching algebra compared to statistics. As algebra is a multi-faceted topic, I restrict my focus on algebra to ratio and proportion because of how prominent these concepts are throughout middle school and advanced mathematics. To clarify my view of statistics, it is useful to consider a distinction between probability and statistics. A useful way to distinguish between probability and statistics lies in what information is given. In probability parameters that govern a population (e.g. the mean) are known and the objective is to determine what data are expected to be observed. Card games are a good example of probability because characteristics of the cards in the deck are known and it is natural to wonder what hand players expect to be dealt. In statistics data are known and the objective is to estimate population parameters that produce the observed data. As an example, consider recording the time in seconds between customers arriving at a restaurant in an effort to determine the rate at which customers arrive. This question is statistical because the rate parameter is estimated using data. In essence, statistics begins with data.

Mathematical Knowledge for Teaching

My framework for mathematical knowledge for teaching is consistent with the MKT construct presented by Ball, Thames, and Phelps (2008). My reason for focusing solely on this construct for mathematical knowledge for teaching rests on its correlation with higher student achievement (Hill et al., 2005). In the simplest terms, MKT is positively correlated with higher student achievement and as a consequence I am interested in the instruction of teachers who have higher MKT.

Curricular Reasoning

Curricular reasoning is a form of pedagogical reasoning that retains a focus on given curriculum materials (Roth McDuffie & Mather, 2009). Curricular reasoning focuses on how teachers support students' learning from specific materials rather than using materials as a starting point or in some other way. Teachers can engage in curricular reasoning as they plan, implement, or reflect on instruction by considering how curriculum materials are used to support student learning. For this study I focus on how teachers plan for instruction in terms of learning goals, modifying given materials, and thinking through teaching strategies to support student learning.

My framework for curricular reasoning is further shaped by Hiebert and Grouws' (2007) proposal that establishing connections between mathematical ideas and allowing students to struggle with important mathematical ideas are valuable for developing conceptual understanding. Two frameworks in the literature shape how I view these ways of supporting conceptual understanding. The first framework addresses connections between mathematical ideas. Silverman and Thompson (2008) argue that teachers must transform personally powerful mathematical understandings to pedagogically powerful mathematical understandings. These authors discuss mathematical understandings in terms of Key Developmental Understandings (KDUs)—conceptual learning goals that enable students to see mathematics differently and cannot be achieved through explanation or demonstration (Simon, 2006). One KDU Simon suggests is the ability to see a ratio as a quantity that measures a multiplicative relationship. With this KDU students are able to understand related mathematics such as slope and probability differently than if they were to attend to the additive difference between two quantities. Teachers' personal mathematical understanding might include KDUs, but pedagogically powerful understanding requires being aware of "(1) how [KDUs] could empower their students' learning of related ideas; (2) actions a teacher might take to support students' development of [KDUs] and reasons those actions might work" (Silverman & Thompson, 2008, p. 502). While particular KDUs are difficult to identify (Simon, 2006), the essence of the construct is to recognize that some mathematical ideas are profitable in terms of understanding related ideas. As teachers utilize possible KDUs in their learning goals and teaching strategies they provide opportunities to make connections between mathematical ideas explicit. As I analyze teachers' curricular reasoning I will describe to what extent teachers' use possible KDUs to guide their goals and other pedagogical decisions.

The second framework is related to how students struggle with important mathematics. Stein, Grover, and Henningson (1996) document the implementation of mathematical tasks with respect to the level of cognitive demand. A mathematical task is defined as a classroom activity used to focus attention on a mathematical topic. The cognitive demand of a task can range from complex, non-routine thinking to memorization. The framework used by these authors submits that the level of cognitive demand required by a mathematical task can decline or be maintained throughout implementation partly by the teachers' learning goals and teaching strategies. For example, a task may be intended to establish connections among various methods of solving a problem, but the teachers' goals and strategies can allow decline in cognitive demand to simply rehearsing various solution methods without attending to connections among them. When teachers are able to maintain high levels of cognitive demand they allow their students to struggle with important mathematics. In my framework I distinguish between patterns of curricular reasoning partly based on the extent to which learning goals and teaching strategies are likely to maintain high levels of cognitive demand.

Finally, my framework for curricular reasoning considers how teachers hold their reasoning. Namely, teachers can hold their reasoning as belief or knowledge, and I find it beneficial to investigate how teachers hold their reasoning in order to learn about teachers' readiness to engage in worthwhile dialogue about curricular reasoning. Philipp's (2007) literature review on belief and affect is insightful regarding these constructs as well as their relationship to each other. Philipp suggests belief and knowledge can productively be considered ways of holding general notions. In terms of my framework, a teacher might know a particular way of reasoning about curriculum is effective or they might believe it is effective. For Philipp, the difference between holding a notion as knowledge or belief lies in the teachers' willingness to accept alternative notions as viable. The extent to which teachers are unwilling to recognize alternative notions as viable indicates the extent to which they hold their notions as knowledge. It is important to emphasize that the truth or validity of curricular reasoning is irrelevant with respect to whether a teacher holds their reasoning as knowledge or belief. Thus, teachers can reason about curriculum similarly, but some teachers can hold the reasoning as belief and other teachers can hold the reasoning as knowledge. I argue that teachers are better positioned to improve their curricular reasoning if they hold their reasoning as belief. Further, it is possible for a specific teacher to hold some reasoning as knowledge and other reasoning as belief.

Research Questions

With a framework for the major constructs of this study in place it is appropriate to establish the research questions:

1. What is teachers' MKT in algebra and statistics?
2. What differences and similarities exist between the quality of teachers' curricular reasoning in algebra and statistics?
3. To what extent is MKT related to the quality of teachers' curricular reasoning in algebra and statistics?

The first two questions are largely precursors to the third question. The central purpose for this study is to contribute to knowledge about what teachers with higher MKT do in their instruction that is qualitatively different from teachers with lower MKT by examining MKT and the use of text as suggested in previous research (Hill et al., 2005).

Literature Review

Before reviewing the literature pertaining to this study I will briefly outline my purpose and the content for this section. My purpose aligns with Simon's (2004) advice to use the literature review as an attempt to answer the research questions with existing research. To facilitate accomplishing this purpose I separately address each research questions in the order they were originally presented.

Teachers' MKT in Algebra and Statistics

It is concerning that many teachers' MKT might be inadequate. Ma (1999) examines teachers' content knowledge as well as their ability to explain and represent important mathematical ideas. Her work is illustrative of how MKT domains of common content knowledge, specialized content knowledge, and knowledge of content and teaching are integral for high quality instruction. While Ma's work reveals that U.S. teachers' MKT in areas such as operations with fractions may be lower than desirable, teachers' MKT in statistics may be much lower than it is in other areas of mathematics. Jacobbe (2010) reports that some teachers lack thorough preparation in statistics, although these same teachers are otherwise well-regarded and mathematically knowledgeable. Other studies provide evidence that teachers' understanding related to measures of center is strikingly similar to students' understanding (Groth & Bergner, 2006; Jocobbe & Fernandes de Carvalho, 2011). Research on teacher knowledge in statistics is mostly limited to elementary grades, and comparisons to other areas of mathematics have not been established. At the same time, these researchers suggest that teachers' knowledge—

particularly common content knowledge—in statistics is unusually low. Broadening the scope of research on teachers' MKT in statistics and establishing comparisons to MKT in other areas such as algebra could inform priorities in teacher preparation and professional development.

Teachers' Curricular Reasoning in Algebra and Statistics

The construct of curricular reasoning is often used to provide insight into how teachers implement standards-based curriculum (Breyfogle et al., 2010; Breyfogle et al., 2012; Roth McDuffie & Mather, 2006, 2009). As teachers consider how curriculum materials can be utilized in response to student needs rather than as prescribed lessons, instructional practice and student achievement improve (e.g. Roth McDuffie & Mather, 2006). One issue not addressed in previous research is possible variation in curricular reasoning across mathematical content areas. Within my framework, variation in curricular reasoning across content areas seems likely because the nature of learning goals and effective teaching strategies varies across content areas. For example, in statistics a teacher might tend to focus lessons on data collection without a meaningful learning goal, but the same teacher might establish learning goals that can be considered KDUs (Simon, 2006) in algebra lessons. For this reason I claim content, in addition to beliefs and other factors, has the potential to alter teachers' curricular reasoning.

To be brief, curricular reasoning is a relatively new construct, and many facets of this construct remain unexplored. To date, research does not compare curricular reasoning across content areas of mathematics such as algebra and statistics. Exploring curricular reasoning in more detail has potential to develop understanding about instruction teachers provide for their students, how instruction differs across content areas of mathematics, and how instruction might improve.

Knowledgeable Teachers' Curricular Reasoning

Although it is sensible to believe that more knowledgeable teachers reason about curriculum differently, the nature of such a difference remains unexplored. I argue it is possible that knowledgeable mathematics teachers' curricular reasoning carries explanatory power regarding the instruction knowledgeable teachers provide for their students. Research shows how the relationship between teachers' MKT and their instruction is mediated in myriad ways; some of these ways include teachers' use of resources, beliefs, and experience in professional development (Hill, Blunk, et al., 2008). Because many of these mediating variables are components of teachers' curricular reasoning, I argue that exploring curricular reasoning can contribute to describing the instruction of knowledgeable teachers as called for by other researchers (Hiebert et al., 2005; Hill et al., 2005).

Altogether, there is consensus that research is needed on how knowledgeable teachers approach instruction. Researchers are justifiably unsatisfied with identifying correlations between inputs and outputs of the mathematics education process (Hiebert et al., 2005; Hill et al., 2005). Instead, researchers insist the instruction of knowledgeable teachers be described in an effort to improve mathematics instruction overall (Hiebert, 2013; Thames & Ball, 2013). Suggestions for defining this instruction include examining relationships between teacher knowledge and uses of text, decisions about goals or priorities, and lesson planning (Hill et al., 2005). Further, such research within specific content areas such as statistics is needed (Shaughnessy, 2007). My ambition in carrying out this study is to help meet this need.

Methods

Participants

I invited 7th grade teachers to participate in this study. Of 17 teachers invited to participate, I was able to collect a full set of data from 10 teachers. My primary purpose for selecting 7th grade teachers is related to increased expectations for teaching statistics. Previously, teachers in my sample have taught a significant amount of algebra, some probability concepts, and limited statistics. With the adoption of CCSS-M these teachers are now expected to teach a large amount of statistics content including topics related to variation and random sampling. As these teachers are expected to teach more statistics, it is timely to explore both their knowledge for teaching and curricular reasoning in this area of mathematics.

This study was carried out in three segments with each segment addressing one research question. In this section I describe the data collection and analysis for each segment. These descriptions follow the order in which the research questions were originally presented.

Teachers' MKT in Algebra and Statistics

I measured teachers' MKT in algebra and statistics with instruments developed by the Learning Mathematics for Teaching (LMT) project (Hill et al., 2004). To reiterate, I used this test of MKT because it is correlated with higher student achievement (Hill et al., 2005). It is implausible that higher student achievement is caused by teachers doing well on the MKT assessment. Instead, I presume teachers with higher scores on the assessment provide instruction that is somehow qualitatively different than teachers who score lower on this MKT assessment, and the purpose of my study is to explore differences and similarities in teachers' curricular reasoning. Hence, the first step in my study was to identify teachers who score higher on this MKT assessment.

Analysis of the MKT assessment was very straightforward based on the design of the test. The total number of correct responses corresponds to a relative level of MKT in terms of standard deviations from the average test taker. It is important to note that the scaled IRT score was not based on the sample in my study. Rather, the results of the MKT assessment show my participants' relative MKT within a large sample of test takers (Hill et al., 2004). To analyze these data I totaled the number of correct responses and identifying teachers' relative MKT in algebra and statistics from published tables. This analysis established teachers' relative MKT in algebra and statistics but did not yield any comparison of absolute MKT in statistics compared to absolute MKT in algebra.

Teachers' Curricular Reasoning in Algebra and Statistics

Data for evidence of teachers' curricular reasoning was gathered from a questionnaire and an interview. The purpose of the questionnaire was to surface ideas related to teachers' curricular reasoning. One strength of questionnaires is the descriptive information that can result (Munn & Drever, 1990). However, a drawback of questionnaires is the lack of opportunity to explain. Nonetheless questionnaire data is important in terms of triangulation and forming preliminary descriptions of teachers' curricular reasoning. The purpose of the interview was to refine my description of teachers' curricular reasoning by expanding, clarifying, and testing my preliminary inferences. A great strength of interviewing is the opportunity to probe and clarify responses (Brenner, 2006).

The questionnaire was structured around two mathematical tasks appropriate for 7th grade students—one algebra topic and one statistics topic. Teachers were directed to read the problem statement for each task and record their responses to the questions. Completing the task gives teachers a factual basis for opinion questions related to the task (Munn & Drever, 1990) and also gives teachers an opportunity to complete the task as learners (Roth McDuffie & Mather, 2009). After completing the task, teachers responded to a series of questions related to how they might use the task in their classroom. These questions and prompts were written to surface teachers' curricular reasoning including what mathematical ideas teachers would like their students to learn and how teachers might position students to learn these ideas.

After administering the questionnaire and performing initial analysis I collected further data related to teachers' curricular reasoning in an interview. My initial analysis of the questionnaire involved noting responses I found to be unclear, surprising or otherwise interesting. From these notes I developed questions to guide the interview. Brenner (2006) explains that a deductive approach to open-ended interviews is useful for testing theories, so I used this approach to test my description of teachers' curricular reasoning. While the protocol for each interview was guided by responses on the questionnaire, the general structure and outline for each interview was constant.

I analyzed data from the questionnaire and interview using codes developed from the literature and aligned with my framework. To begin the analysis, I identified instances of curricular reasoning in the data. Instances of curricular reasoning were considered to be one of the following four types (Breyfogle et al., 2010; Roth McDuffie & Mather, 2009): (1) a goal for instruction, (2) an alteration to the curriculum materials as written, (3) a learning trajectory for students, and (4) a teaching strategy for working with students. The unit of analysis for an instance was an idea. Using ideas as the unit of analysis was useful because it captured the context of statements sufficiently well, and it was necessary to capture the context of statements in order to assign codes to the instances. For example, I found it difficult to determine whether a teaching strategy maintains the cognitive demand of a lesson or not if the goal of the lesson is unknown, so I expanded the unit of analysis to an idea. In the example the instance would be coded as a teaching strategy, and the goal is considered to be a reason for selecting the strategy.

After identifying instances of curricular reasoning I coded the data for several categories of relevance to my framework. These categories were cognitive demand, belief or knowledge, KDUs and use of context. Codes for cognitive demand came largely from previous research (Henningsen & Stein, 1997). Many codes associated with maintaining and undermining the cognitive demand of a task during classroom instruction were useful for analyzing teachers' curricular reasoning. For example, when a teacher proposed a teaching strategy for dealing with a particular misconception it was useful to consider to what extent the strategy might providing scaffolding for the student or shift the students' focus to arriving at the correct answer. Some codes associated with maintaining and undermining during classroom instruction, such as the amount of time spent on a task, were not able to be observed in teachers' curricular reasoning. These codes were omitted from the analysis.

My coding scheme for belief and knowledge was taken entirely from Philipp's (2007) advice. That is, if a teacher sees another pattern of curricular reasoning as sensible, I consider the teacher to hold their curricular reasoning as belief. The inverse of this statement describes knowledge. Instances of curricular reasoning often did not consider another potential pattern of curricular reasoning, so it was often impossible to determine if the curricular reasoning in an instance was held as belief or knowledge. For these instances the code was simply left blank.

To code for KDUs I considered two ideas mentioned by Silverman and Thompson (2008). First, teachers might demonstrate understanding of how KDUs empower students to learn related mathematical ideas, so I looked for evidence that teachers consider how mathematical ideas develop over time. Second, teachers might describe teaching strategies that are useful for helping students develop KDUs, so I looked for evidence that teachers select strategies for the purpose of developing meaningful understanding of mathematical ideas rather than, for example, just helping students find a solution.

The final category of codes in my analysis related to how teachers used the real-world context of curriculum materials. This category contained three codes including two that highlight a proposed distinction between algebra and statistics (Cobb & Moore, 1997). First, the context could be used algebraically. For the use of context to be considered algebraic two conditions needed to be met. The teacher needs to reason about teaching new mathematical ideas with the context and particular features of the context need to be considered unimportant with respect to the new mathematical idea. For example, a teacher might use students' intuition about the ratio of candies in a bowl to help students understanding scaling quantities in a proportional relationship. In the end, it does not matter whether the quantities are Jolly Ranchers and Jawbreakers or buttons and shirts. In either situation the scaling procedure is the same when the quantities exist in a proportional relationship. Such a use of the context was considered algebraic.

Next, the context could be used in a statistical way. In statistics the context is vital for meaning and interpretation (Cobb & Moore, 1997). In statistics there is no guarantee that the calculations performed with one set of data will be informative for another set of data because the meaning depends on the context. Cobb and Moore specifically promote using exploratory data analysis to begin working with data. To engage in exploratory data analysis students might create dot plots, stem and leaf plots or histograms to represent the data and look for interesting patterns that might be important for the context. Using such analysis to link the data to the context was one characteristic of curricular reasoning that I considered to have a statistical use of context. To decide if teachers used the context statistically, it was useful to wonder if the same work could be done in class without any context at all. For example, using a set of numbers to explore how outliers affect various measures of center can be done without any context. If the set of numbers happen to represent something of interest to the students, it is entirely incidental and the use context is not statistical. On the other hand, if the context is used to determine whether calculating the median for a set of numbers is meaningful, the context is used statistically because it is central to the solution process.

Last, the context could be used as a setting for application. Teachers reasoned about using the context as a setting for application in two different ways. First, at times teachers did not expect students to learn any new mathematics by working on a problem. Instead, teachers might use a particular problem to review or practice skills, such as calculating the mean, that have been learned in the past. Second, teachers also used the context as a way of checking for reasonable answers. For instance, students might find a speed that is obviously too slow for the context of the problem. In such cases, teachers might encourage students to notice the answer needs to be different because of the context. In the end, the context does not provide insight into the correct solution process. The context is only used to convince students that their original solution process must have some flaw.

My process for coding the data was rather typical. To begin, I coded all the data from the questionnaire and interview. After the initial pass I recoded all the data without referring to the first set of codes. At this point, I reconciled differences between the first and second passes of coding. Finally, I asked a colleague to code a portion of the data, and we discussed how codes were assigned with satisfactory agreement.

After coding all the data I used quantitative and qualitative methods to consider the relative quality of teachers' curricular reasoning. The quantitative analysis was quite brief and consisted entirely of examining the frequency with which codes appeared. I chose to examine the frequencies in order to gain an initial sense for differences that might exist in the quality of curricular reasoning both across teachers and across content areas within the same teacher. After this cursory quantitative analysis, I used qualitative methods to confirm or disconfirm the results obtained in the quantitative analysis. During the qualitative analysis of the data I used memoing (Miles & Huberman, 1994) to understand more general patterns in teachers' curricular reasoning. For example, the quantitative analysis might reveal that teachers most often used the context as a setting for application. In such a case, memoing was useful because it helped me recognize distinct patterns in the ways different teachers reasoned about using the context as a setting for application. To address the relative quality of teachers' curricular reasoning I compared and contrasted the patterns in teachers' curricular reasoning that were establish previously. As I compared and contrasted these patterns I grouped teachers according to similarities across categories of codes related to curricular reasoning. After forming these groups I considered teachers to have higher quality curricular if the patterns in their curricular reasoning were more aligned with ideals in categories of codes. For example, with respect to cognitive demand, I considered a teacher to have higher quality curricular reasoning if their curricular reasoning established a pattern of maintaining rather than undermining the cognitive demand of curriculum materials as written.

Knowledgeable Teachers' Curricular Reasoning

For this segment of my study I did not collect any new data. Instead, I looked for patterns across results of the two previous segments in an effort to find any relationship between MKT and the quality of curricular reasoning. Because I did not collect new data during this phase of my research, it was crucial to collect a full set of data in the previous segments. To ensure I collected the data needed for analysis in this segment I used a data accounting sheet as suggested by (Miles & Huberman, 1994). The data accounting sheet is a table of participants and data sources needed for each research question and displays which sources of data are needed from which participants. After collecting all sources of data for teachers' MKT and curricular reasoning as well as performing analysis of these constructs I had a basis for describing knowledgeable teachers' curricular reasoning.

To investigate a potential relationship between MKT and the quality of curricular reasoning I conducted two stages of analysis. The first stage was to compare the quality of individual teacher's curricular reasoning across content areas with MKT. This comparison was somewhat elusive because MKT is not measured in terms of absolute knowledge. Instead, MKT scores are relative to other test takers. Consequently, a teacher might have the same MKT score in algebra and statistics, but this does not imply the teacher has the same level of MKT in both content areas. The reason for this disparity is that the general population of teachers can have greater knowledge in one content area. To work around the lack of a MKT measurement in absolute terms, I considered pairs of teachers that shared a common score on the MKT assessment in one content area. By considering these pairs I was able to be confident the change in MKT across content areas was different for the teachers. Now, to get a sense of a relationship that might exist between MKT and the quality of curricular reasoning, I compared the change in the quality of curricular reasoning to the change in MKT across content areas.

For the second stage of this analysis I compared the quality of curricular reasoning across teachers for each content area. I established the null hypothesis that there is no difference in MKT among teachers with higher and lower quality curricular reasoning. I used a permutation test (Higgins, 2004) to test the alternative hypothesis that there is a difference in MKT among teachers with higher and lower quality curricular reasoning. A permutation test was useful in this case because it does not rely on a large sample size to be reliable. Instead, a permutation test builds up the distribution of a test statistic under the null hypothesis by shuffling the data across labels. Building up this distribution makes it possible to determine the probability of observing a test statistic at least as extreme as the test statistic calculated from the data. More simply, a permutation test yields a p-value by comparing the observed test statistics to the distribution of the test statistic under the null hypothesis. However, this p-value does come with one caveat. Because the p-value was obtained using Monte Carlo methods, it is necessary to account for Monte Carlo error. This error is often accounted for by constructing a 95% confidence interval for the p-value. For this analysis, I used R to shuffle MKT scores across the labels of higher and lower quality curricular reasoning 1 million times and computed the difference in mean MKT for the labels of higher and lower quality curricular reasoning each time. My rationale for shuffling the data in this way is that under the null hypothesis it does not matter which MKT scores are labeled with higher or lower quality curricular reasoning. After building up the distribution of the test statistic under the null hypothesis I obtained a 95% confidence interval for the true p-value of the test statistic computed from the observed data.