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Problems Faced by Reform Oriented Novice Mathematics Teachers Utilizing a Traditional Curriculum

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ABSTRACT

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Utilizing a Traditional Curriculum

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Task-based instruction has been a promising method of instruction advocated by mathematics education researchers over the past twenty years. However traditional curricula constitute a majority of the curricula utilized in the United States. The purpose of this study is to identify the problems reform oriented novice teachers face when utilizing a traditional curriculum to plan task-based lessons. In order to identify these problems three novice teachers' interactions with curricula were observed and characterized using the frameworks of past researchers. Through analysis of teachers' textbook interaction practices it was found that teachers struggled to plan task-based lessons due to issues encountered finding/constructing mathematical tasks, and due to problems associated with being naturally oriented toward procedures while utilizing a traditional curriculum.

Keywords: traditional curriculum, reform instruction, reform curriculum, lesson planning, novice mathematics teachers

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Chapter 1: Introduction

Task-based instruction centered on the National Council of Teachers of Mathematics (NCTM) teaching standards (NCTM, 1991) has been a particularly promising method of instruction advocated by researchers over the past twenty years. However the curricula used in United States has consistently remained traditional (Banilower, 2013). Therefore the following research is concerned with understanding the problems reform oriented novice teachers face when utilizing a traditional curriculum. It is important to note that the following document rests upon two fundamental assumptions. The first is that task-based instruction is worthwhile instruction, and the second is that curriculum affects the instruction of teachers, especially novice teachers. The following section is dedicated towards supporting the validity of these two assumptions and situating the purpose of the research study.

Task-based Instruction is Worthwhile Instruction

Task-based instruction is important for two vital reasons. First, it engages students in authentic mathematical activity and inquiry. One primary goal in mathematics education is to increase students' ability to solve meaningful non-routine mathematics problems (Lester, 2013). This cannot be done by simply teaching the rules, procedures, and terms of mathematics. Romberg (1994), using an analogy to mathematics, stated that when learning to play basketball it is important to remember that the most important part of becoming a good basketball player is through *playing the game*. Learning basketball strategy, practicing free throws, and rebounding are all important but not nearly as important in developing basketball aptitude as playing the game. Likewise, if we want students to learn to *do* mathematics they must be given the opportunity to play the game of mathematics by continually participating in meaningful mathematical problem solving (Romberg, 1994). Task-based instruction provides students with the opportunity to solve many worthwhile non-routine problems on a consistent basis.

The second reason task-based instruction is worthwhile instruction is because it develops powerful conceptual understandings of mathematical relationships. Conceptual understanding empowers students with the flexible thinking that is useful in solving a variety of problems. There have been numerous studies documenting the positive effects task-based instruction has on student mathematical understandings (Blanton & Kaput, 2005; Boaler & Staples, 2008; Lampert, 1990). These studies document how students build exceptional understandings of mathematical content as they engage in meaningful problems and have meaningful discussions about mathematics through a task-based lesson. Critics of task-based instruction have claimed that task-based instruction does not promote adequate fluency of mathematical procedures because students learning in a task-based environment are often not told what algorithms are best to utilize and have less time to practice procedures. However, researchers have found that on average students learning in task centered environments perform *at least as well* with regard to standardized tests as students learning in traditional environments (Franco, Sztajn, & Ortigao, 2007; Harwell, Post, Maeda, Davis, Cutler, Andersen, & Kahan, 2007; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008).

Curriculum Affects Instruction

Past researchers who have attempted to understand how teachers use curricula have seemingly agreed upon three general ways to categorize teacher curricula use (Lloyd, 2008; Remillard & Bryans, 2004): 1) some teachers choose to follow written curricula materials rigorously, 2) some teachers utilize a given curricula as a guide and make certain personalized adaptations for a given lesson, and 3) some teachers choose to use provided curricula materials minimally. Although there are teachers that fall into category three research has shown that a great deal of teachers fall into categories one and two (Remillard & Bryans 2004; Lloyd, 2008; Sherin & Drake, 2009). In particular, researchers have found that many novice teachers tend to follow curriculum materials strictly (Bush, 1986; Kuhs & Freeman, 1979; Remillard & Bryans, 2004). This is most likely due to the fact that novice teachers have few past teaching experiences to draw upon as they create lesson plans and engage in instruction. For this reason some have suggested that curricula *should* affect the instruction of novice teachers because there are many novice teachers who have not had time to sufficiently explore secondary mathematics content (Ball & Feiman-Nemser, 1988). Therefore, based upon the evidence above it is reasonable to claim that curriculum affects the instruction of a substantial number of teachers in the United States, and since novice teachers have been shown to be even more reliant upon curricula this study is concerned with observing the interactions between novice teachers and curricula.

The Problem

Reformers have attempted to aid teachers in the utilizing of task-based instruction by creating task-based curricula. However, traditional curricula still constitute a majority of the curricula utilized within the United States (Banilower, 2013). It is now possible for the pre-service or in-service teacher to learn a task-based approach towards teaching mathematics in their teacher education or professional development program but be required to utilize a traditional curriculum as a professional. The likelihood of this disconnect becomes especially apparent when one considers the common practice (Remillard, 2005) of school districts mandating that teachers utilize a common district wide curriculum. Thus it is reasonable to conclude that reform orientated novice teachers are likely to face problems when utilizing a traditional curriculum from a reform based theoretical perspective.

It is important to note that the problems reform oriented teachers face when utilizing a traditional curriculum are not inconsequential. Studies have shown that quality task-based instruction that enhances student achievement usually occurs when the curriculum a teacher is utilizing is also task-based (Tarr et al., 2008). Therefore it is reasonable to conclude that the problems teachers experience while utilizing a traditional curriculum effect student achievement. Thus, the purpose of the following research to identify the problems novice teachers face when attempting to implement task-based instruction while utilizing a traditional curriculum.

Chapter 2: Theoretical Framework

What is Reform Instruction?

Reform instruction has been given a variety of labels. The following research uses the term "task-based" because worthwhile mathematical tasks represent the central difference between traditional and reform curricula. However, the type of instruction based on the NCTM professional teaching standards (NCTM, 1991) has also been called standards based, inquiry based, reform orientated, discussion based, and nontraditional. All of these designations seem to refer to a *general type of instruction that focuses on building mathematical understanding of course content through student engagement with worthwhile mathematical tasks.* A worthwhile mathematical task is a problem that engages students intellectually and becomes a vehicle whereby students can have powerful mathematical discussions and make connections to previously understood course content (NCTM, 1991), and a "reform oriented" teacher is someone who is intent on trying to teach students through the utilization of worthwhile mathematical tasks and meaningful classroom discourse. The following is a list characteristics that research has shown to be important features of worthwhile mathematical tasks.

A worthwhile mathematical task:

- Is based on meaningful course content (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997)
- Requires a high level cognitive demand (Boaler & Staples, 2008; Stein, Smith, Henningsen, Silver, 2009)
- Allows students from a variety of backgrounds to have access to the task (NCTM, 1991)
- Requires students to justify their mathematical thinking (Boaler & Staples, 2008)
- Allows students to make connections between different representations (Smith & Stein,

2011)

- Provides students with the opportunity to make conjectures and form generalizations (NCTM, 1991)
- Provides a meaningful context wherein students can make sense of the mathematics (Stylianides, 2008)

What is a Reform Curriculum?

Generally speaking a reform curriculum is a curriculum intended to support task-based instruction. However, when I refer to reform curricula I am specifically referring to curricula based upon the NCTM Curriculum and Evaluation standards (NCTM, 1989). These types of curricula were developed in order to promote conceptual understanding and meaningful mathematical problem solving. I use the term curricula to mean the written curriculum or the "resources designed for the use of teachers and students during instruction" (Remillard, 2005, p. 213). I choose to adopt this particular definition because it seems to be the one meant by most researchers and teachers who use the term. The central identifying feature of a reform curriculum is that it contains worthwhile mathematical tasks that teachers can use as a focal point for class discussions. These tasks are sequenced in a meaningful way that promotes student connections of various mathematical ideas. In many instances a reform curriculum also contains information about the mathematical content embedded within a given task, and suggestions for how teachers can effectively have meaningful class discussions.

What is Traditional Instruction?

In order to better understand reform instruction researchers have articulated what typifies traditional mathematics instruction. Gregg (1995) describes this tradition as "the familiar routine of checking the answers to the previous day's homework, working some of these homework problems on the chalkboard, introducing new material, and assigning seatwork." (p. 443). Within the tradition of school mathematics the teacher or textbook is the source of mathematical authority (Gregg, 1995) and the learning of facts and procedures dominate what students learn (Lampert, 1990). Thus one of the central identifying features of a traditional curriculum is that it contains numerous examples of procedures and procedure based practice problems that students can engage in.

What is a Traditional Curriculum?

The bulk of mathematics curricula in the United States are traditional (Banilower, 2013). A traditional curriculum usually comes in the form of a textbook. Typically within each section of the textbook a mathematical idea is introduced and then is followed by a few examples about how to perform a certain type of routine procedure. Competency of procedures supposedly requires students to understand central mathematical ideas shown previously. However, studies have shown that children can demonstrate routine procedural competency without having conceptual understanding of the underlying mathematical ideas (Erlwanger 1973; Reys & Yang, 1998). At the end of a given section there are usually a series of practice problems students solve during class and then do for homework. These problems usually only measure procedural competency and tend to have very little resemblance to worthwhile mathematical tasks provided during reform instruction because they do not contain a useful context wherein students can grapple with powerful mathematical ideas, contain multiple ways to enter the task, require justification or explanation, or provide opportunities to look for patterns or make generalizations. In the cases where application problems do exist within traditional curriculum they usually exist at the end of a section after a student has already practiced a given procedure. In this case the student often simply uses the procedure he or she has just been doing upon the numbers written in the “application”. There is often little critical thinking required of the student to develop powerful conceptual understandings.

The Curriculum Enactment Process

At this point I have been able to designate what I understand reform and traditional curricula to be and designate what appears to constitute reform and traditional mathematics instruction. However in order to investigate the problems reform oriented teachers face when utilizing traditional curricula I must first establish a general framework that conveys how a written curriculum evolves to become classroom instruction. Past reform attempts in mathematics education have failed because curriculum developers disregarded the process a written curriculum goes through before it is enacted by teachers. For example in the 1950s and 1960s educators and researchers within the United States attempted to “implement” what was determined to be high quality instruction by distributing new “reform” curriculum materials to teachers. The expectation was that teachers’ instruction would conform to the type of instruction advocated by the curriculum developers. This period of reform is considered to have failed by many because curriculum developers ignored how teachers interact with, adapt, ignore, or implement reform materials (Bosse, 1995). Thus in order to avoid repeating unsuccessful history and implement successful reform there is a need to understand the process of how a written curriculum becomes classroom instruction.

There are four different types of curricula researchers refer to. The following section is dedicated towards articulating the four types of curricula and constructing an interconnecting framework between the various types of curricula (See figure below). Throughout this paper I use the term “curriculum” in reference to the written materials a teacher utilizes to construct instruction. However, before a textbook is written curriculum developers first identify the goals and aims of a given curriculum. This is called the *intended curriculum* (Herbel-Eisenmann, 2007) and for a reform curriculum the intended curriculum is usually based on the NCTM standards (NCTM, 2000). After goals are identified by curriculum developers a curriculum is then written typically in the form of a textbook. This is called the *written curriculum*. At this point a teacher uses a written curriculum to prepare for the mathematics lessons he or she will be teaching. I will refer to these plans as the *planned curriculum* (Gehrke, Knapp, Sirotnik, 1992). Finally, after the teacher has planned appropriately he or she enacts the planned curriculum during classroom instruction and creates what is known as the *enacted curriculum* (Ball & Cohen, 1996). The reason why separate names exist for each of these types of curricula is because research has shown that a curriculum changes as it moves from being the intended curricula to being the enacted curricula. For example Herbel-Eisenmann (2007) was able to identify how a standards-based written curriculum did not necessarily align entirely with the principles and standards advocated by NCTM, and various researchers have noted how written curricula is often vastly different then the curriculum enacted by teachers (Breyfogle, Wohlhuter, McDuffie, 2012).

Researchers have attempted to understand what happens to a curriculum as it moves from the intended curriculum to the enacted curriculum. For example many researchers over the past few decades have worked to describe what factors influence the construction of a planned curriculum from written materials (Gadanidis, Gadanidis, Schindler, 2003; McCutcheon, 1980; McCutcheon 1981) and what factors influence the construction of the enacted curriculum (Ball & Cohen, 1996; Lloyd, 2008; Remillard & Bryans, 2004). These researchers have all been concerned with being able to describe in some way how teachers interact with curricular materials, and have found that teachers are influenced by their resources, goals, and orientations

(Schoenfeld, 2011) as they construct the planned and enacted curriculum. Specifically the researchers above have discovered that teachers' curricular decisions during the planning and enactment process are affected by, students, beliefs about quality mathematics instruction, personal understandings of course content, school district leaders, coworkers, the social norms that exist in their classroom, prior teaching experiences, and the provided or chosen written curriculum. Therefore in examining how teachers interact with traditional curricula I will need to be aware of the factors that may influence how teachers approach curriculum materials.

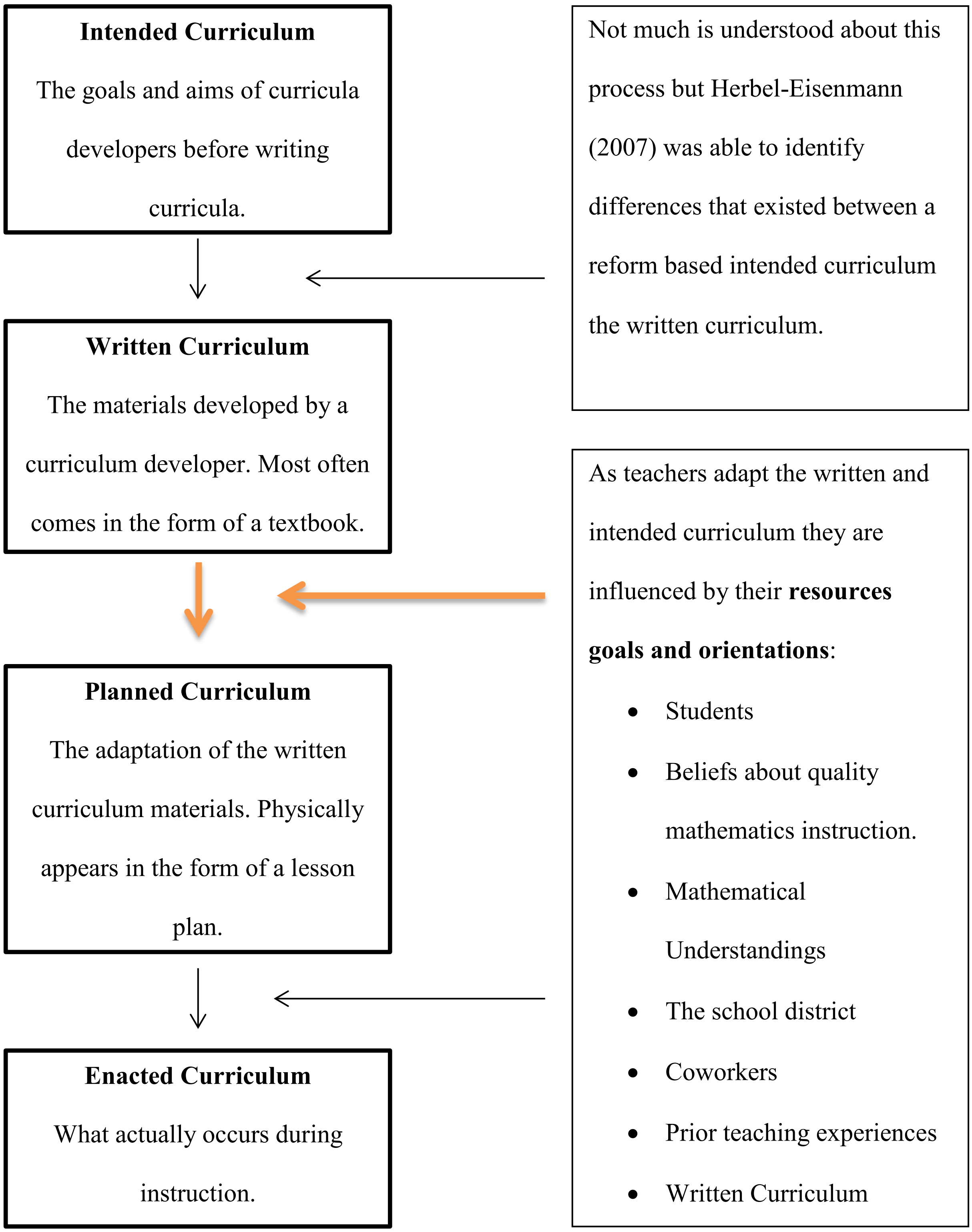

Figure 1. The Curriculum enactment process

At this point it is important to note that I will focus on the evolution of a curriculum as it moves from being a written curriculum to being a planned curriculum. I argue that the bulk of interaction between a teacher and a written curriculum occurs as they plan for lessons. Therefore I will be primarily concerned with analyzing the teacher construction of a planned curriculum. In doing this I believe my focus will be better situated to understand the problems novice reform oriented teachers face when utilizing a traditional curriculum.

Chapter 3: Literature Review

As stated previously various researchers have attempted to understand how teachers interact with a curriculum. There are three purposes for the following section: 1) to articulate how previous research on teacher interaction with curricula has empowered me with a useful framework in exploring teacher interaction, 2) to describe how my study is different from other studies about teachers appropriation of curricula materials, and 3) to demonstrate the necessity of my study.

How do Teachers Interact with Curricula?

Sherin and Drake (2009) developed a framework for understanding how teachers interact with written reform curricula materials and stated that teachers interact with a given curriculum before, during, and after instruction in three different ways. The first is by *reading.* When different teachers read curricular materials they often read with different perspectives. For example Remillard (1999, 2000) identified two teachers who had significantly different ways of reading curricula materials. While one teacher focused on reading the activities the curricula provided another focused on reading the mathematical ideas imbedded in the activities. It is also important to note that not only can teachers read with different purposes but they can read with different attitudes about the curricula as well.

The second way teachers interact with curricula is through *evaluation*. Evaluation is the process of critiquing and interpreting curriculum materials. During this process a teacher uses his resources, goals, and orientations (Schoenfeld, 2011) about mathematics and mathematics instruction (See figure on p. 11) to interpret curriculum materials. For example a teacher evaluating a given portion of a curriculum may consider how prepared his or her students are for the mathematical concepts embedded in the lesson or how well aligned a task is with his or her beliefs about what constitutes a worthwhile mathematical task. Thus evaluation often represents a significant portion of the interaction which occurs between teacher and curriculum.

The last way teachers interact with curricula materials is through *adaptation*. Adaptation is the process of adjusting the provided curriculum to fit the needs of a specific classroom and the vision of a specific teacher. For example a teacher attempting to adapt a traditional mathematics problem of solving two algebraic equations with two unknowns may adapt the problem by constructing a context wherein the students can grapple with the embedded mathematical ideas. Although adaptation ranges from a disregard of the curriculum to a strict adherence to the curriculum, research has demonstrated that many teachers apply an adaption strategy between these extremes wherein curriculum materials are adjusted by adding to or replacing certain aspects of the prescribed written curriculum (Lloyd, 1999; Remillard & Bryans 2004).

In this study I will utilize Sherin and Drake's (2009) framework for examining teacher's curriculum usage. However, I am concerned with understanding how teachers with a reform perspective read, evaluate, and adapt a *traditional curriculum* instead of examining how teachers read, evaluate, and adapt a written *reform curriculum*. I am also concerned with examining teachers reading, evaluating, and adapting in greater detail than Sherin and Drake (2009). I am particularly interested in the evaluation portion of this framework since this is the portion that represents the detailed thinking processes teachers go through when interacting with a curriculum. For example Sherin and Drake (2009) in describing Beth's (a teacher from their study) evaluation and adaptation strategy stated that Beth "frequently created new materials, similar to the ones provided in the teacher's guide and student activity book, that she felt could serve as transitions between one activity and the next or between schoolwork and homework"

(Sherin & Drake, 2009, p. 476-477). In this particular example Sherin and Drake (2009) were able to describe how Beth adapted curricula materials and gave a general reason as to why she adapted in the manner in which she did. However, these descriptions lacked specificity. It was unclear *why* Beth altered the curriculum materials, *why* the curriculum materials, although similar to Beth's constructed activities, were determined to be insufficient, and *why* Beth felt that her newly constructed activities were better than the provided materials. Understanding Beth's evaluation process in greater detail would have provided meaningful insights into how she interacted with the curriculum. Therefore, my study is concerned with investigating teachers' evaluation strategies along with teachers reading and adapting strategies in a level of detail not addressed by Sherin and Drake (2009). In engaging in this level of detail I hope to be able to decipher which aspects of a traditional curriculum are problematic for teachers and why they are problematic. I also hope to be able to decipher how teachers' personal reading, evaluating, and adaptation strategies might make creating task-based lessons problematic. In making these detailed discoveries I hope to be able to develop understandings that will help future teachers who utilize traditional curricula with reform centered perspectives.

Curricular Reasoning

As teachers engage in the practice of evaluation they engage in curricular reasoning. "*Curricular reasoning* refers to the thinking processes that teachers engage in as they work with curriculum materials to plan, implement, and reflect on instruction" (Breyfogle, McDuffie, Wohlhuter, 2010, p. 308). Specifically, Breyfogle et al. (2010) identified three processes teachers engage in as they participate in curricular reasoning. First, teachers analyze curriculum materials from a learner's perspective. Quality curricular reasoning occurs when teachers are able to consider the unique perspectives of their students and are able to adjust their lessons accordingly.

This practice is largely a matter of anticipating student thinking and throughout this document I use the phrases "analyzed from the learners perspective" and "anticipated student thinking" interchangeably. The second process of curricular reasoning is the mapping of learning trajectories (Breyfogle et al., 2010). Mapping learning trajectories refers to teachers' ability to assess how mathematical understandings build over time and assess how various mathematical topics connect. When teachers participate in this process they are able to focus on essential mathematical ideas in lessons and devise and sequence appropriate learning activities that help students build upon previous understandings. The final process of curricular reasoning identified by Breyfogle et al. (2010) is the process of reflection and revision of previously implemented mathematics lessons and experiences. In engaging in this process teachers reflect on their own personal experiences with teaching and adjust current lesson plans accordingly.

The curricular reasoning framework is exceptionally useful in describing the detailed and complicated evaluation processes teachers engage in when lesson planning. I will use curricular reasoning as a detailed framework for describing how teachers evaluate curriculum since my overarching goal is to not only depict how reform minded teachers generally read, evaluate, and adapt traditional curriculum materials, but I am especially concerned with describing the detailed interaction between teacher and textbook that occurs within the process of evaluation. It will be useful to understand how teachers analyze curriculum materials from the learner's perspective, map learning trajectories, and reflect on past lessons while they engage in the practice of evaluation so that I can provide empirically based conjectures about the problems reform oriented teachers experience as they utilize traditional curriculum materials.

How do Teachers use Curricula Materials?

Studies have been able to describe in some detail how teachers use curriculum materials.

For example, Lloyd (2008) examined how one teacher, Anne, chose to appropriate two different reform curricula and examined why this particular teacher made the curricula decisions she did. Lloyd (2008) was able to describe Anne's general curricula use as adaptive and was able to describe how Anne read, evaluated, and adapted curricula materials. She also found that Anne's curricular decisions were influenced by her student teaching placement, resources, mathematical background, and characteristics of the reform curriculum materials. However, like Sherin and Drake (2009) Lloyd did not describe how features of the provided curricula influenced Anne's appropriation of curricula materials, and she did not describe why Anne made the curricula decisions she did in the level of detail that is necessary to make inferences about what was problematic for her when constructing lesson plans. For example in discussing Anne's appropriation of the two different curricula of *Everyday Mathematics* (EM) and *Mathematics Their Way* (MTW) Lloyd (2008) states, "Anne's MTW design work did not involve reading, evaluating, and marking up of lesson pages that characterized her design of the EM lessons. Instead, Anne acquired firsthand experience with Mrs. Roy's instructional approaches through a sort of tutorial, and adapted those approaches according to her goals for student learning" (p. 83). In this example Lloyd (2008) is able to describe Anne's curriculum interaction with MTW. However, it is unclear why Anne chose not to utilize this particular curriculum. Did Anne have reasons for favoring EM over MTW? If she did, what were those reasons? Why did Anne read, evaluate, and adapt the EM materials the way she did? By answering these questions about Anne's curricula interactions I would be able to start to understand what aspects of her interactions with cuuriculum were problematic.

Studies of traditional curriculum use. There have been various studies about how teachers use reform curricula materials but few studies examining teacher's traditional curricula usage. McDuffie and Mather (2006) were able to conduct a unique study in which they examined one reform minded teacher's appropriation of traditional curricular materials. In this study McDuffie worked with a teacher (Lerenz) to construct and implement reform oriented lessons. At the beginning of the year Lerenz, who advocated reform-based problem centered practices, implemented lessons traditionally because she had a prescriptive relationship with the curricula materials. However by the second half of the school year Lerenz came to understand that the curriculum materials provided should be tools to help her meet the learning objectives she had set in place for her students. This later view is consistent with what NCTM (2000) has determined to be a productive relationship with curriculum. By the end of the year Lerenz used her textbook minimally (only to follow the sequence of mathematical topics) and relied upon worthwhile mathematical tasks provided by McDuffie as she planned task-based lessons.

McDuffie and Mather's (2006) study is similar to my study in that both are concerned with understanding how reform minded teachers implement reform with traditional curricula. However there are some questions that still must be answered. Do most reform oriented teachers have a prescriptive view of traditional curricula? How do teachers who do not have researcher help (McDuffie) finding/constructing task-based lessons that align with the topics of a traditional textbook approach lesson construction? How were Lerenz's interactions with the traditional textbook at the beginning of the year different than a traditionally oriented teacher implementing traditional instruction? How do novice reform minded teachers who are more reliant on the mathematics provided by a curriculum utilize a traditional curriculum? My study is concerned with answering these types of questions. Specfically, the research questions my study is interested in answering are; 1) How do novice teachers with a reform oriented perspective *read*, *evaluate*, and *adapt* wtraditional curricula? 2) How do novice teachers with a reform oriented perspective use curricular reasoning to evaluate traditional curriculum materials? 3) Based on the answers to the first two questions, what is problematic for reform oriented novice teachers utilizing a traditional curriculum when planning task-based lessons? In answering these questions I hope to provide understanding that will help reformers devise solutions that will help reform oriented novice mathematics teachers concerned with utilizing traditional curricula.

Chapter 4: Methodology

In order to understand the problems reform oriented novice teachers face when utilizing a traditional curriculum I needed to observe reform oriented novice teachers as they planned lessons utilizing a traditional curriculum. These observations were my primary source of data. However, I needed to take certain precautions to ensure that the data I gathered was reliable and useful. The following section is dedicated towards articulating how I gathered and analyzed my data. I will describe why the methods I chose were reliable and useful mechanisms in answering my research questions.

Finding the Reform Oriented Teacher

The first step in carrying out my research proposal was to find reform oriented novice teachers. These novice teachers needed to have sufficient understanding of what constituted task-based instruction so they were dedicated towards its implementation. I also needed to ensure that the novice teachers I selected had similar ideas with regards to what constituted task-based instruction. Therefore to find these potentially similar minded reform oriented novice teachers I first obtained a list of previous students who had graduated with a degree in mathematics education within the last three years at Brigham Young University (BYU) and had taught secondary mathematics for at least one year. This list of graduates was narrowed as I identified which of the previous graduates taught mathematics within a close proximity to BYU. At this point I then circulated the narrowed list amongst the BYU mathematics education faculty and previous university supervisors of BYU student teachers. The faculty identified students they believed to "have the knowledge, ability, and desire to implement quality task-based instruction". From these recommendations I obtained a further narrowed list of candidates for my study.

There are four fundamental reasons I used the methodological choices explained above to locate reform oriented novice teachers. The first reason was because it allowed me to find teachers who had similar backgrounds with regards to task-based instruction. Students who graduated with a degree in mathematics education at BYU were required to take five mathematics education courses. One of the primary functions of each of these courses was to convey to preservice teachers that "students learn best when given regular opportunities to reason about and make sense of mathematics in an environment of high expectations and strong support." (BYU Mathematics Education Department Learning Outcomes). Thus students who major in mathematics education at BYU are taught to "understand how to design learning environments and mathematical experiences that engage all students in the exploration and development of mathematical ideas, as well as how to effectively foster these environments and orchestrate these experiences by promoting conceptual understanding, procedural fluency, and authentic mathematical practices." (BYU Mathematics Education Department Learning Outcomes). Students who participated in the mathematics education courses at BYU learned to design, choose, and assess mathematical tasks and became familiar with facilitating meaningful class discussions while utilizing worthwhile mathematical tasks. Therefore by selecting graduates from BYU I reasonably ensured that the participants in my study had similar backgrounds and understandings about what constitutes task-based instruction. This similarity helped ensure that the different planning practices utilized by teachers were not a result of varying backgrounds.

The second reason I chose the methods listed above was because it allowed me to find teachers with little teaching experience who were not first year teachers. I did not wish to observe first year teachers who were adjusting to the realities of teaching secondary mathematics and were adjusting to their newly acquired role as a teacher, but I still wanted to observe teachers who had little experience in the classroom since they were likely be more reliant on the provided textbook or other curricular resources in planning lessons. Therefore I chose teachers with one to three years of teaching experience.

The third reason I chose to survey BYU faculty to identify participants was because surveying and interviewing teachers about their understanding or utilization of task-based instruction has proven to be an unreliable mechanism for determining whether a teacher is committed to implementing task-based instruction. There have been various researchers who have interviewed or surveyed teachers and found that while teachers may explicitly advocate reform principles for teaching mathematics they do not implement these principles in their practice (Frykholm, 1999; McDuffie & Mather, 2006). In both of these studies teachers have claimed that utilizing a traditional textbook was one reason why their explicit reform based orientations about mathematics did not align with their teaching practice-an additional rationale for the importance of my study (Frykholm, 1999; McDuffie & Mather, 2006). By using the methods mentioned above I was able to reliably begin the process of finding reform oriented teachers since the BYU faculty were able to observe the potential participants as they expressed their evolving ideas about what constituted task-based instruction and were able to observe the potential participants as they attempted to implement such ideas in their teaching.

The final reason for selecting participants from the pool of BYU graduates teaching in the area is because it is a sample of convenience. I did not have the ability to travel across the United States to find reform oriented teachers, and it would not have been practical to randomly gather a sizeable group of teachers and subsequently extensively interview and observe each teacher to identify whether or not the teacher was reform oriented. However, interviews were a useful mechanism in validating my decisions regarding participant selection.

After I generated a list of reform oriented novice teachers I needed to select participants for my study. I began the selection process by first dividing my previously constructed list of potential participants into two groups. The first group consisted of teachers who taught in a school district that either allowed teachers to select their own curriculum or mandated the use of a specific reform curriculum. The second group consisted of teachers who taught in school districts that mandated the use of a traditional curriculum. I decided what constituted a traditional or reform curriculum by comparing the curriculum to the requirements of traditional and reform curricula listed above. I then utilized the BYU faculty recommendations and selected two novice reform minded teachers who were utilizing a traditional curriculum and one teacher who was using a reform based curriculum.

I chose to use the described manner of participant selection for two purposes. First, I chose to observe two novice teachers utilizing a traditional curriculum since traditional curriculum usage is what this study is centered on. Studying two participants allowed me to make reasonable conjectures and conclusions about the problems the reform oriented teachers faced when utilizing a traditional curriculum without burdening myself with an unreasonable amount of data to analyze. The second reason I chose to select teachers in the above manner is because I wanted to compare the practices of curricular engagement and reasoning used by the teachers utilizing a traditional curriculum with the practices used by a teacher using a reform curriculum. In making this comparison I better enabled myself to observe the differences between these two practices, and the contrast provided a more reliable identification of the problems reform oriented teachers face while utilizing a traditional curriculum.

Interviews

After I selected the participants for my study I then conducted interviews (see appendix) with each participant. These interviews had two purposes. The first was to once again ensure (as much as possible) that the teachers that I selected were reform minded and were intending to implement reform instruction in their practice. The second purpose of the interview was to develop a basic understanding of how the participants engaged in curricular reasoning as they read, evaluated, and adapted curriculum materials. Obtaining this simple information allowed me to have some basic insights and understandings prior to the observations of teacher planning. This enabled my field notes and questions to be significantly more insightful.

Once the teachers in my study were interviewed I then reviewed the field notes I took during the interview process to ensure that I had selected three exceptional participants for my study. The criteria for an exceptional participant was that they were: committed to utilizing the chosen curriculum type, committed to implementing task-based instruction to the best of their ability, and committed to planning task-based lessons to the best of their ability prior to instruction. I then further ensured that these criteria were met by observing one lesson from each of these three teachers prior to conducting planning observations. When I determined that one of my three teachers was not an "exceptional participant" for my study I would then select a new participant from my faculty generated list and interviewed and observed the potential participant accordingly. It is important to note that this type of participant selection screening had major implications as to how teachers were asked to participate in my study. Initially teachers were only asked to participate in a study wherein I came to observe their classroom and asked them the interview questions provided in the appendix. Once this portion of my study was carried out and the three exceptional participants were identified I then asked teachers to participate in a second portion of my study wherein they were asked to engage in videotaped planning sessions. If I did not ask teachers on separate occasions to participate in different parts of my study and told teachers exactly what I was doing then teachers not determined to be "exceptional participants" may feel unfairly judged as teaching professionals.

In concluding this section it is important to note that I interviewed and observed seven potential participants as I attempted to identify exceptional participants. During the screening process it was found that some teachers had abandoned trying to teach in a task-based manner or were "secretly" utilizing a reform based curriculum even though their school district had adopted specific traditional curriculum. These interviews revealed some interesting insights into the various reasons why some teachers, who had been highly recommended by BYU faculty, chose to abandon task-based instruction. However because these did not really address my research questions, they were not analyzed as part of my study.

The Observations

The bulk of my data consisted of video observations of teachers while they were engaged in the process of planning. Before a particular observation took place I would set up a camera behind the shoulder of the participant. This allowed me to hear what they were saying as they planned and allowed me to later observe what parts of the curriculum they were physically referring to as they engaged in the planning process. Situating the camera like this provided valuable insight into how teachers *read* (Sherin & Drake, 2009) curriculum materials.

After the camera was situated I then asked the participant to explain what they were thinking as they planned for a particular day's lesson. It is important to note that in doing a pilot study prior to conducting the actual study I learned that simply telling teachers to "speak their thoughts when planning" did not adequately communicate what I expected teachers to do while their planning sessions were being video-taped. Therefore before the first observation I had the participants read the following transcript so that it was clear to teachers what behavior I expected them to engage in while planning:

> I am interested in observing how you plan for lessons. Meaning that I am interested in observing what you are actually thinking when planning a lesson. Therefore I ask you to express every thought that comes to mind when planning. For example if you are teaching students how to factor and are planning to have students practice factoring during instruction. I would like you to tell me why you are making this particular decision. There are a substantial number of reasons you could be making this particular planning decision. For example you could make this decision because you believe that students will be asked to factor in standardized assessments. You could be making this decision because that is what the textbook recommends. You could be making this decision because practicing problems similar to the homework problems is simply a norm that exists in your classroom. Therefore a typical response during a planning session may be as follows, "I am then going to have students practice factoring using problems number 1, 2, and 5 in the textbook because I think students only learn when they do things and because factoring is important to be able to solve equations." It is also important to note that I would like you to read out loud anything that you read from the textbook or any other sort of materials and realize that I may occasionally ask questions to understand why you are making particular planning decisions. I will not ever be doubting or judging what you plan or how you plan but I am only interested in understandings how you plan using a curriculum.

As teachers engaged in the oral planning process I made field notes about what I observed, asked questions for clarification, and wrote questions that I wanted to ask the participants after the planning session was concluded. These oral observations provided insight into how teachers *read* and *evaluated* (Sherin & Drake, 2009) curriculum materials and provided insight into how teachers engaged in *curricular reasoning* (Breyfogle et al., 2010). After a planning session was concluded I would then look at my field notes and ask the teacher any questions that I had about what was actually planned for in the lesson and why the teacher chose to make certain specific planning decisions. The post planning questions allowed me to ask questions I thought might be distracting to ask the participant during the planning process.

The Participants

Three teachers were selected to participate in the study. The following section provides background for each participant.

Susan. Susan was a second year teacher, teaching at a secondary charter school in a suburban area. She was videotaped as she planned six mathematics lessons using the reform based Pathways algebra curriculum to teach a secondary mathematics II class of 24 students. This was Susan's second time teaching this course with the Pathways curriculum (Carlson, O'Bryan, Joyner, 2012). The Pathways curriculum is organized into a series of tasks that are structured to enable student exploration of algebraic ideas, and the curriculum also contained a teacher edition that had notes to better prepare teachers who planned on using the Pathways curriculum. Lesson planning sessions took an average length of 25 minutes to complete. Two of the six lessons observed were procedurally based lessons that Susan constructed herself and four of the lessons observed were Pathways centered task-based lessons.

Peter. Peter was a second year teacher, teaching at a public high school in a suburban area. He was videotaped as he planned six mathematics lessons using a traditional calculus textbook (Finney, Demana, Waits, & Kennedy, 2012) to teach an AP calculus class of 25 students. This was Peter's first time teaching this course. The calculus textbook was organized in a traditional layout and emphasized examples and practice problems. Videotaped lesson planning sessions were approximately 60 minutes in length. These sessions only captured about half of a planning session since Peter actually spent between two and three hours planning for a given lesson. However, in three of the planning sessions I was able to observe Peter as he engaged in the first half of planning (reading the textbook and identifying the big ideas) and in the other three planning sessions I was able to observe Peter as he engaged in the second portion of his planning (planning what to do during a lesson). Therefore I was able to capture all of the varying planning practices Peter engaged in.

Lisa. Lisa is also a second year teacher, teaching at a public high school in a suburban area. She was videotaped as she planned two mathematics lessons for her secondary II classes and one mathematics lesson for her secondary III classes. This was her second time teaching each of these courses. She would use her traditional textbook (Carter, Cuevas, Day, 2012) along with resources she found online to plan lessons. The videotaped lesson planning sessions were approximately 45 minutes, but it was clear that Lisa would usually spend an additional thirty to forty minutes outside of these sessions making extra preparations. However, at the end of each planning session I questioned her about what additional planning would be carried out prior to lesson implementation. I also specifically questioned her about what her reading and curricular reasoning would look like in the remaining time she spent planning.

Data Analysis

In order to answer my research questions I needed to describe how my participants read, evaluated (curricular reasoning), and adapted curricula. Therefore the videotaped lesson planning sessions were coded using Studio Code (Sportstec, 1997-2015) software in the following manner:

Pass 1. Chunks of video segments were coded as instances of reading. Reading instances were identified when a teacher read or referenced any curricular materials in a planning session. These reading chunks were then subcoded using open coding. In engaging in open coding I would ask myself, “what kind of materials is this particular teacher reading”, “how is this particular teacher reading”, and “why is this particular teacher reading in this manner”. This enabled me to develop distinct categories that described teachers’ reading practices.

Pass 2. In my theoretical framework I defined the evaluation portion of Sherin & Drake’s (2009) framework as consisting of curricular reasoning (Breyfogle et al. 2010). Therefore in attempting to code how teachers’ evaluated curriculum I analyzed each planning session and coded for instances in which teachers analyzed from the learners perspective, reflected on past experiences, and mapped learning trajectories (the three aspects of curricular reasoning). Each of these curricular reasoning practices were coded in the following manner.

Analyzed materials from the learner’s perspective. I looked for instances in which the teacher took the position of his or her students and anticipated the types of thinking his or her students might engage in. For example after reading a question in her curriculum Susan stated, “So I think the point of this is what they’ll see is that as the— [the symbol “—“ indicates a pause in speaking] as you add more and more volume to the water the rate of change is going to get bigger and bigger”. Notice that in this example Susan anticipated what her students would see when presented with a particular question. All of these types of instances were coded as instances where the teacher was analyzing curricula from the learner’s perspective. After these instances were identified they were then subcoded by looking at how the teacher was anticipating student thinking and for what purpose a teacher was anticipating student thinking. The teachers various types of anticipation were categorized and analyzed by looking at patterns within the categorized subcodes.

Reflection. I looked for instances across the videotaped planning session in which a teacher reflected upon experiences in the past to inform his or her decisions during a lesson planning session. These instances were subcoded according to the type of reflection a teacher engaged in. For example teachers would often reflect upon what has occurred within the most recent lessons in order to better prepare for a lesson they are currently teaching. However teachers would also reflect upon how well a lesson was implemented last year in deciding how to teach that particular lesson this year. These various types of reflection were coded for each teacher. Once these codes were generated patterns within a specific type of reflection behavior were analyzed and a description of a teacher's various reflection practices was constructed.

Mapped Learning Trajectories. I looked for instances in which a teacher developed a learning progression for students. This means that I looked for instances in which a teacher sequenced various mathematical concepts, tasks, or questions. For example a teacher could be deciding the order certain topics should be covered in over several lessons or could be deciding the sequencing of certain questions within a task. Patterns amongst these instances were identified and analyzed.

Summary. After each participant's data were analyzed and patterns were identified a detailed description of each teachers planning processes was written (see results) to answer my first two research questions. Prior to being finalized the participants were given a written copy of their planning practices and asked if the outlined practices adequately described how they interact with curricula and create lessons. No participant communicated objections to the results obtained.

Cross-case analysis. After completing a summary of the curriculum interaction during lesson planning for each of the participants, I attempted to answer my third research question by making conjectures about the problems reform oriented novice mathematics teachers experience when utilizing a traditional curriculum. This was accomplished by careful reading through each aspect of Peter and Lisa's written results and continually asking "what seems problematic about this particular planning practice". When I identified something problematic I then checked to see if it was something that was problematic for both teachers utilizing a traditional curriculum, and I checked to see whether it was something that was problematic for Susan. In doing this I was able to use a cross-case analysis to create conjectures about the problems Peter and Lisa encountered creating task-based lessons.

Chapter 5: Results

Before providing a detailed analysis of the planning practices of my three participants it is first important to note two initial results that came from early analysis of data. The first result was that in analyzing the data I realized that it was not reasonable to think of curricular reasoning (Breyfogle et al. 2010) as only being the thinking practices which occur within the process of evaluation (Sherin & Drake, 2009). It is true that as the observed teachers engaged in curricular reasoning they were involved in the process of evaluation, but is it also true that as teachers engaged in curricular reasoning they were participants in the process of adaptation. For instance as teachers mapped learning trajectories they were often adapting or constructing the mathematical tasks they were going to use during instruction. Thus in analyzing and describing teachers' curricular reasoning I found that I was concurrently analyzing and describing teachers' evaluation and adaptation processes. Therefore, the results section contains the three case studies of the three different teachers and within those case studies I characterize a teachers reading and curricular reasoning processes. By doing so in a detailed manner I also provide an adequate description of how teachers evaluated and adapted curricular materials.

The second important result found in initially looking at the data was a useful coding strategy in making sense of how teachers map learning trajectories. In looking at various instances of mapping learning trajectories I found that there were three different types. First, teachers could map across lessons. This means that a teacher would engage in the process of deciding which topics to cover and what order to cover them in across a series of lessons. The second mapping practice was that teachers would map within a lesson, meaning that teachers would determine what tasks or discussions should take place in the lesson to facilitate student understanding. The third type of mapping occurred as teachers mapped within a particular question. When engaged in this practice teachers would identify how to lead students toward a specific idea in a specific question. These three types of mapping practices were used as subcodes when analyzing instances of mapping learning trajectories, and within each of the case studies all three types of mapping are characterized for a specific teacher.

The Case of Susan

Susan had a standard routine she engaged in when planning a task-based lesson for a particular day. She had all of her "lesson plans" from her previous year of teaching stored on a computer. The "lesson plans" were not stereotypical lesson plans that described what she was going to do and how she was going to do it, but they were tasks (structured like worksheets) that Susan adapted based upon her interaction with the Pathways curriculum the previous year. These tasks usually looked similar to the tasks found within the Pathways curriculum. At no point in time during the lesson planning sessions did she actually consult the Pathways curriculum but she always utilized the tasks she had adapted from Pathways curriculum the previous year to plan lessons. She said that as the years progressed she was likely to consult the Pathways curriculum less and less because, "it's been like two years. I have started to memorize the questions and the answers, and I know why they're there. And I would hope that if I didn't like something that I would change it immediately and not wait till next year to go back and change it." After providing this rationale she assured me that even though there were some minor changes to the Pathways materials that the tasks from which she planned lessons highly resembled the Pathways curriculum. Thus Susan's "reform curriculum" was not the Pathways textbook but was actually her modified version of the Pathways curriculum.

It is also important to note that in two of the lesson planning sessions I observed Susan did not engage in what she would refer to as typical instances of lesson planning. During both of these planning sessions she was concerned with constructing procedural based lessons that helped students practice and learn a specific type of procedure. Thus during the planning of these lessons she had entirely different planning norms which consisted of her searching online databases, past notes, and worksheets from other classes to find appropriate practice problems that her students could work on.

In the following sections I will describe how Susan read curriculum materials and engaged in curricular reasoning within each of these two types of planning instances.

Task-based reading.

Typical linear planning practice. Susan's typical planning practice occurred within four of the six lessons as she personally worked through her adapted mathematical tasks from the previous year from start to finish while personally engaging with the mathematics within the tasks. The reading and task work was the mechanism that prompted curricular reasoning. Thus a great majority of the instances of reading within Susan's six lessons where instances where she was reading and doing task work because that was the order in which the task work and readings were placed. When asked about this practice she said that she spent most of her time engaging in the mathematics of the mathematical task while linearly reading because "that's [the mathematics] the big thing, just like reminding myself the purpose of each of these questions, cause I have to have that purpose in mind otherwise I'll forget what to focus them [her students] in on." Going through the task linearly provided Susan with the opportunity to reacquaint herself with the purposes for each question in the task.

Skim by bold, pictures, and tables. When Susan would initially pull up last year's mathematical task or be working through mathematical tasks she would sometimes be immediately drawn to pictures, tables, or titles before engaging in a physical reading of text.

Thus, her evaluation of the curriculum was sometimes prompted by images rather than by linear reading. For example as Susan was explaining what she was doing before pulling up last year's task on her computer she stated, "I really liked the way that I started out exponentials last year, like it just went really well, so I'm gonna go back and try to and remind myself what I did." Once she pulled up the task she saw the picture on the first page of the task and said, "Oh yeah yeah yeah, K, I remember what I did." At this point she then proceeded to explain that the picture reminded her what she did last year that she liked so much. Notice the "reminder" or prompt to engage in reflection came from images and not from the written text. This type of reading is different from the type of reading that occurred during the linear reading of physical words but is important to note that this reading practice did not occur very often. It is likely this practice occurred infrequently because there were few pictures included within Susan's lesson plans last year.

Non-task-based reading. In two of Susan's six lessons she did not engage in her "typical" planning practice described above. Twice Susan decided to plan lessons that were procedural in nature. She would decide that students needed practice with a particular concept and would dedicate a day towards practicing a procedure. When planning these types of lessons she would search the Internet or her previous lesson plans for appropriate practice problems. Thus her curriculum was no longer the Pathways curriculum or even her modified version of the Pathways curriculum but became a collection of various resources such as Kuta Software (Kuta Software, 2015) and other online sites found in web searches. Her reading of curriculum materials became a process of looking for practice problems that she deemed appropriate. For example in one planning session Susan decided that she was going to give her students the opportunity to practice long division of polynomials. Therefore in planning she said, S: I am kinda just going to look through what I already have and try and find long division problems from last year…these are from my math II class but I'm just gonna use some of these… cause I know that these ones all like divide out nicely like without a remainder. So I am just going to use those ones and put them in here. I'm basically just gonna have a bunch of practice problems.

Notice that when Susan engaged in this particular reading practice she had clear goals about the desired characteristics of the practice problems. This occurred in all of these types of reading instances. Susan would always look for certain practice problems that "worked out nice" or that forced students to utilize a certain specific procedure. In the example above the procedure she wanted her students to practice was that the procedure of polynomial division that did not contain a remainder. Thus Susan read these curriculum materials with a specific purpose and was able to complete these two planning sessions exceptionally quickly (under 20 minutes each).

Curricular reasoning. In the following section I will describe how Susan engaged in curricular reasoning as she planned lessons. Specifically I will describe how she analyzed from a learners perspective, reflected on past experiences, and mapped learning trajectories. It is important to note that although I will describe these topics separately they often happened simultaneously and prompted one another.

Analyzing from the learner's perspective. Susan primarily engaged in two practices when she analyzed from the learner's perspective or anticipated student thinking. The first is that she anticipated specific accurate responses students might have when working on a specific problem or task. For example when planning for a lesson on the difference between linear and exponential relationships she stated,

S: Once it looks like everyone has got the line [a linear equation] and the curve [an exponential equation] graphed I'll bring them back together as a class, and we'll talk about the two different shapes. So I want to ask them like, "why is the school like the north one, why is that one a line". I hope I'll get like the response that it's because they are decreasing at a constant rate of twenty, so it's linear, so it's a line. And then I'll ask them and then why is the other school the exponential one, why is it like curving? And I hope to get the response that they understand that it's not decreasing at a constant rate because it's a percentage.

Notice in this example that Susan hoped that her students would give certain specific accurate responses. These types of instances represent a sizeable portion of all of the instances where Susan analyzed from the learners perspective, and I would conjecture that one reason she engaged in this activity is so that she could refamiliarize herself with problems in the curriculum and their associated mathematical ideas.

The second primary practice of anticipating student thinking occurred as Susan predicted specific *misconceptions* students might possibly have. For example one problem in Susan's curriculum asked students to graph $f(x) = (x+1)(x-3)^2$ and Susan anticipated student thinking in the following manner,

S: What I'm gonna have them do is mark the two x-intercepts that we found on there and then just have them do a rough sketch of what they think it might look like and my guess is that they're probably gonna draw a parabola because there's just the two intercepts so they're probably gonna think that it crosses twice and draw some sort of parabola, hopefully positive cause they see that its positive.

In this excerpt Susan was concerned with anticipating a specific student misconception. The misconception was that if a graph has two x-intercepts it must be a parabola. It is important to note that Susan did not always identify *specific* misconceptions; she sometimes simply anticipated that her students would not remember a specific topic previously covered and then planned appropriately. This practice of predicating student misconceptions was important and also comprised another sizeable portion of the instances of anticipating student thinking.

I have just described the primary practices Susan engaged in when she anticipated student thinking or analyzed from the learner's perspective. However there were two other secondary, less prevalent, types of anticipating student thinking that occurred consistently enough to merit being depicted in this characterization. Firstly, Susan engaged in the process of predicting how much time a task would take students to complete. This anticipation usually occurred at the end of the lesson planning process, and she generally would make this prediction based upon other previous instances of anticipating student thinking.

The second minor instance of anticipating student thinking occurred as Susan projected pedagogical issues she might experience based upon conjectured student work. For example one task in Susan's curriculum asked students to construct a graph using a provided unlabeled blank set of axes. Anticipating that students would place different quantities on different axes, Susan decided that she would tell students which quantities to place on which axis so that "everyone would be on the same page" for the discussion later. Notice that in this instance of anticipating student thinking Susan did not predict possible correct understandings or misconceptions but anticipated certain types of responses that may negatively impact discussion later. Once again this practice did not occur as often as the primary practices described above but stilled seemed relevant.

Reflection. Susan engaged in two different reflection practices. The first is that she reflected upon recently implemented lessons, usually the lesson most recently taught. During this reflection she focused on a specific concept, procedure, or idea that her students did or did not learn in a recently (within the current school year) implemented lesson. This thinking often informed her learning trajectories or anticipated student thinking (thus once again demonstrating the interconnectedness of all three components of curricular reasoning). Consider the following excerpt taken from one of Susan's planning sessions,

> S: So they [her students] haven't done this since we did quadratics like over a month ago so I'll probably give them like five minutes to work on this question and just tell them to talk to the person next to them and figure out what the x-intercepts are.

Notice that in this excerpt Susan identified a particular task as potentially problematic for her students due to reflection upon the last time her students saw this type of material. Once again there is a substantial connection between anticipating student thinking and reflection. In this instance Susan anticipated that students would have problems or issues but that anticipation is prompted by a specific type of reflection. This was common. Susan would reflect upon what her students did or did not do recently in order to better inform her curricular reasoning.

It is also worth noting that this particular reflection process was not usually a reflection upon specific student's thinking but was a reflection upon what she previously determined to be the classes "overall understandings" of a specific concept. For example consider the following conversation between the researcher and Susan,

> S: I know that for them cutting 9 in half is going to take them a little longer.
>
> R: Now when you say "them" are you thinking of the class as a whole or of specific students?
>
> S: The class as a whole. Like everyone's kinda on the same page. Because we go through the lessons like together so we're kinda all on the same page.

Notice that in this example Susan stated that she is not thinking of specific students because they are "all on the same page". Susan never in all the observed planning sessions once referenced a specific student when planning. Thus I would conjecture that Susan mostly focused on what the *class's* understandings or misunderstandings were when reflecting, rather than on what *specific students'* misconceptions were.

The second reflection practice Susan engaged in was that she reflected upon her first year of teaching. This practice occurred in two different ways. First, Susan reflected upon *specific* conceptions that students had last year. For example in discussing the different strategies students had last year when solving a problem she stated,

> S: So some interesting ideas that come out from this is that some students will times it by— like they'll multiply by .03 [writes "(1980)*.03"], and they'll then get the number of students that are added, and then they'll go back and add it to the original. But then there's always a couple students who will do, which is, this is awesome, they'll take 1980 times 1.03 (writes "(1980)(1.03)"]. Which does it all in one step, it gives you the hundred percent of this [points to the 1980] and adds on the three percent. So I always have one or two students who do it this way and I'll have them at this point go up to the board and explain why this works.

In this excerpt Susan demonstrated how she reflected upon a specific conception students had in reference to a specific question in the curriculum. This practice was common, and also refers to all of the instances in which Susan identified certain specific *misconceptions* students had the year prior.

The second way in which Susan reflected on her past year of teaching was by reflecting in a "general sense" without referencing any specifics. For example in one planning session

Susan stated that she was choosing to use her lesson plan from last year “because I really liked the way that I started out exponentials last year, like it just went really well. So I’m gonna go back and try and remember remind myself what I did, because I remember really liking it”. In this example Susan demonstrated a surprisingly common planning practice of reflecting on last year’s lesson without remembering or referencing actual experiences from the previous year. Thus actual specific experiences are not what solely characterize this particular practice of reflection for Susan, but general “feelings” of the past year also influenced her curricular reasoning and planning choices.

In conclusion Susan’s reflection practice can be summed as a combination of reflections on recently implemented lessons and lessons implemented last year. The only other reflective practice exhibited was when Susan reflected on her preservice teacher education courses. It was surprising that this type of reflection only occurred three times and would be interesting to study why this practice happened such an infrequent amount.

Mapping learning trajectories.

The purpose of this section is to characterize how Susan engaged in each of three mapping practices identified in the initial results.

Mapping across lessons. Susan usually stated that she liked the lesson sequence she used last year and basically utilized the same lesson sequence. Some reasons Susan stated that she used last year’s sequencing was because she wanted to emphasize the mathematical content that was relevant to students “everyday lives” and because she had a general feeling that last year’s sequencing worked well. However it is important to remember that Susan did adjust last year’s lesson plan sequence by adding the two newly constructed lessons that were more procedurally driven. She stated that she added these lessons due to feedback from students and state standardized assessment feedback. She especially emphasized the role of state standardized tests in her rationale for adding these types of lesson. For instance when asked why she was adding a procedurally centered lesson that taught students how to write quadratics expressions in vertex, standard, and factored form she stated,

> S: While my purpose is to prepare them for calculus and those ideas of change like the Sage test [the Utah State standardized assessment] is only going to be doing this stuff [points to her lesson plan]. So like there's kind of a give and take. Like, I'd love to focus on like the more conceptual— I think it's the more conceptual. Like the derivatives. This is a little bit less enjoyable, more procedural. But like this is what they're going to be tested on. So like we kind of go back and forth between what I want and what the test wants.

Mapping within the lesson. It is important to remember that Susan planned two different types of lessons, Pathways lessons (which she stated were her usual lessons) and procedural centered lessons. In both of the procedural lessons she planned a lesson she had not taught before, therefore she had to construct a new lesson. During this construction process Susan would choose which practice problems students should engage in. These problems were often chosen based upon the procedures she wanted her students to engage in, and upon the mathematical understanding her students would be coming into the classroom with. They were ordered so that the problems she felt were easier (decided what was easy based on reflection and anticipation of student thinking) came before more challenging problems. The goal was to give students some "nice hard practice" so they were prepared for "whatever they might encounter".

In planning the Pathways lessons Susan rarely planned the task or the sequencing of problems within the lesson. She trusted her past lesson plans and the Pathways curriculum.

Evidence of this fact was seen by observing that she usually used the exact same task as used in the previous year with minor edits, and by observing that Susan did not spend a great deal of time initially identifying and describing the overarching mathematical ideas embedded in a lesson prior to planning the lesson. She often discussed the purpose of a particular question but trusted, based on experience, that her curriculum was properly designed to bring out the mathematical ideas that were important. Therefore Susan did not usually engage in the practice of constructing an overarching learning trajectory for a class session. She trusted that the curriculum developers and her thought processes from the previous year had accomplished that adequately. She did however continually *validate* the learning trajectory provided by her curriculum throughout the course of the lesson. For example in the following excerpt Susan was mentally working through a mathematical task used in her previous year of teaching that was taken from the Pathways curriculum. She was going through the task problem-by-problem and engaged in the various aspects of curricular reasoning. The following is an example of her *validating* a provided learning trajectory.

> S: [Reads the problem in the task] "Use the graph of the function f that represents the *height of water in the bottle* in terms of *volume of the water in the bottle* to complete the following table" So— right now I'm just looking at the graph and the table and just trying to remind myself about what these numbers are going to look like so I can just kinda just know what to predict. So they are going to grow really fast and then really slow and then really fast. Ok so— The point of this table will be for them realize the relationship between the change of the change of y and the concavity. So I think hopefully there's a question that directs them in on that. [Reads the next question] "Describe *how changes in the average rate of change",* that's that column, "relate to the shape of the graph". Yeah

perfect. So there's a question that I used last year that focuses them in on the relationship between this column and the concavity.

Notice how Susan validated the given learning trajectory in the curriculum. She first read the task question, deciphered it purpose, and then checked to make sure that the identified purpose was achieved. This practice describes how she validated learning trajectories within a task in the Pathways curriculum.

Mapping in the Question. Susan was constantly engaged in the practice of planning a discussion or a series of questions to help her students better arrive at conceptual understanding of an idea given a specific problem. For example consider the following excerpt,

> S: So I remember— What did I do last year? Ok— so I want to have like a way to show them that taking the year 1 and times-ing by 1.04 two times is different than times-ing by 1.08. So I think the example I'm going to give is if I take 1000 dollars [writes "1000"] and I times by— ok— so like they're [her students] asking instead of just times-ing by 1.04 and times-ing by 1.04 [writes "100 × 1.04 × 1.04") to get from year 1 to year 2, if they can they just times by 1.08. And I usually point out like a counter example. Like what if I did 1000 times 3 times 3 (writes "1000 × 3 × 3") would that be the same as doing 1000 times 6 [writes "1000 × 6"], and usually they realize like "oh that's not the same cause this is 9 cause its exponential and that's different than just being double". So I'll probably need to point that out to a couple of them as I walk around to monitor from here to here.

In this excerpt Susan is clearly reflecting on last year's implementation of this lesson, but she is also considering a "small" learning trajectory to help her students gain understanding about why they could not use the number 1.08. In this case she specifically referenced the construction of a counter example to help remedy a misconception. This practice happened repeatedly (so often and in such variety it was difficult to accurately obtain the number of instances) as Susan planned Pathways lessons and happened seldom as she planned procedurally based lessons.

Summary. Susan was able to plan task-based lessons as she linearly worked through the mathematical tasks provided by her curriculum. She placed a significant amount of trust in the tasks and the sequencing of tasks she had used the previous year, and reflection on previously implemented lessons played an integral role in her curricular reasoning. She spent a great deal of time mapping learning trajectories within a question and was able to prepare for potential specific common conceptions or misconceptions her students might have by simply doing the mathematical tasks she would be giving to her students.

The Case of Peter

Peter utilized a typical traditional calculus textbook when he planned for a lesson. He used the sequencing found in the textbook fairly strictly, tried to cover the material found within a book section in a given class period, and usually provided his students with homework problems which were taken from the end of each textbook section.

When Peter planned for a particular lesson he engaged in two processes. He first began by identifying the “big ideas” found within a textbook section and then engaged in the process of planning a lesson using his textbook. Peter generally attempted to complete the process of identifying the big ideas before planning a lesson, however, he could at any time oscillate between these fundamental processes, and sometimes these processes appeared to happen concurrently. In the following section I will characterize how Peter read and engaged in curricular reasoning within both of these planning process.

Reading. Peter was constantly reading and referencing the textbook as he planned and prepared lessons. He described a typical planning session as follows, "This is what is typical, I look through and figure out what do I know, I go to the book, I think about— K what does it mean, how does the book present it, and how can I present that." Note that embedded within Peter's explanation were his two planning processes. He began by deciphering what certain things meant in the textbook and then determined how he might present those meanings. The following section provides a detailed characterization of Peter's reading practice within Peter's two fundamental planning processes.

Process 1: Identifying the "big ideas". As Peter attempted to identify the big mathematical ideas found within a given lesson he usually followed the pattern outlined below.

1) He skimmed the textbook.
2) He read and interpreted the introduction of a section in the text.
3) He attempted to understand the highlighted definitions, theorems, and equations found in a section.
4) He went over relevant examples.

The sequencing of these four practices was not always linear. Peter often oscillated between these practices as he attempted to identify and understand what the big mathematical ideas were in a given section. However, it is important to note that almost every instance of Peter's "big idea" reading can be placed into one of these four categories. In the following section I will construct a detailed description of each of these four practices.

Skimming the textbook. Before Peter would begin a rigorous examination of the textbook he usually skimmed the textbook first. This occurred because when Peter starting planning he was primarily concerned with identifying and writing down what he personally knew about a given topic, and in order to identify the topic(s) he needed to personally reflect on he would skim the textbook. For example the following conversation occurred between the researcher and Peter while Peter was skimming the textbook at the beginning of a planning session,

R: What are you looking at while you are skimming?

P: Uh, I am just looking for recognition. Do I recognize? So for example I know what the Fundamental Theorem [of calculus] is saying and so I am just looking through and just flipping through. Does that look familiar to me? [flipping through pages] Yes…I am just glancing through and saying 'students might have a problem with that' [pointing to an example in the textbook], "I don't feel confident with that' [pointing to another example]— So I am just getting a general idea of, ok so what do I really need to spend time on.

R: And you are getting at that idea by looking at titles?

P: Looking at titles, looking at these boldface, mostly these examples. Like the application of the theorem or what's the book providing.

Notice in this example that Peter's purpose in skimming was to identify what he "recognized". After each of these skimming sessions Peter placed everything he understood about a given topic on a piece of paper, almost creating a personally constructed concept image of a topic (Tall & Vinner, 1981). However this excerpt is exceptional because it not only described why Peter chose to skim initially but revealed how he skimmed. Peter skimmed by looking at pictures and bolded titles, but was especially mindful of the examples that the textbook was providing. This was true in almost all instances in which Peter skimmed. Thus while Susan seemed to skim by looking at pictures and images Peter's skimming was dictated by examples because examples are what painted a clear picture of what the applications were for a given topic.

Introductions. After Peter would skim a textbook section he would then begin the process of reading. Traditional textbooks often begin a section by attempting to introduce in some small way the origin of the concept found in the title of the lesson. While Peter may not have given careful attention to every part of the textbook section when reading he almost always read the introduction to see if it contained an intuitive way to present material. For example he stated "I started at the front just to look at how they are presenting the material to me, and then see if that logical flow will make sense for my students". Notice that Peter engaged in two activities when reading these introductions. First, he was "looking at how they are presenting the material to me". Meaning, that Peter checked to see how his concept image of a topic meshed with what the book was presenting. This was a common characteristic within all Peter's "big idea" reading practices. For example, in the excerpt below is what Peter explained to the researcher after he finished reading the introduction to the section on summations and its relationship with definite integrals,

> P: So this all sounds familiar to me, and so— but now I think at this moment I just feel like, ok, that makes sense but at the same time I'm like "I don't feel like I have the foundation conceptually for what this really means". Ok, so what I'm doing is just trying to figure out— what are they saying? Ok this is the statement I got from it. This is their kinda image of what a integral or why an integral is needed, so I'm just trying to connect that with my concept [points to his written concept image he had created when skimming and reflecting on his own ideas] —see if I can clarify anything.

In this excerpt Peter articulated that he was less comfortable with the mathematics found in this section and so the purpose of reading the introduction was to not only to see how his understanding meshed with the books explanations, but in this case the textbook explanation represented a possible way in which Peter came to understand the mathematics better.

The second activity Peter engaged in when reading introductions is reflecting on how well his students would understand a topic if presented with this particular introduction sequence. Once again when reading an introduction Peter stated, "I started at the front just to look at how they are presenting the material to me, and then see if that logical flow will make sense for my students". Notice Peter reflected on his students understanding and his own understanding concurrently. It is important to note that for Peter there was often a grey area between what his personal concept images of certain mathematical ideas were and what the mathematical understandings he believed his students possessed. This practice of simultaneous thinking for himself and for his students will be described in more detail latter. The point is that Peter used the learning trajectory of the introductions of sections to reflect upon his own understandings and to assess the appropriateness of a particular learning trajectory for his students.

Interpreting Definitions. While reading through the textbook Peter spent a vast amount of time conceptually interpreting the highlighted definitions, theorems, and equations found in the textbook section. These highlighted portions often resembled the title of the lesson. For example in a textbook section on linearization Peter was provided the equation $L(x) = f(a) + f'(a)(x - a)$, and in summing up his reading practice the following conversation occurred:

> P: I got to here (pointed to $L(x) = f(a) + f'(a)(x - a)$) looking at linearization, I was asking, ok what is this, where does this come from? What does that mean? Because up here [pointed to text above the equation] it didn't describe anything above and all of the sudden they just give a definition. And so then I spent some time looking at okay so

where does this come from. [He then explains the mathematics he learned on his own associated with the linearization equation].

R: So did you figure all of this out on your own?

P: Yes, I looked online; typing it in, everything just gave that [pointed to the linearization equation]. No explanation at all.

It is important to note that the process of "making sense" of a theorem looked different depending on the textbook section and its associated mathematics; many times Peter would work out example problems that contained those big ideas, sometimes he looked online, sometimes he consulted with a coworker, and sometimes he read other calculus textbooks. However amongst this diversity of approaches there appeared to be at least one common theme as Peter attempted to interpret theorems in particular. Peter always read the associated proof of a theorem in hopes that it would reveal the conceptual underpinnings of a theorem, and was always verbally frustrated if the proof was not provided. There could have been other common practices Peter utilized within this practice but more data would need to be gathered.

Looking at examples. Peter often focused on the examples provided within the text. He did this for two reasons. The first is because he was struggling to understand a particular mathematical topic and he believed that if he saw the concept being used he would better understand the associated ideas. For example when planning a lesson on integration he stated, "so right now what I'm gonna be working on, I just want to work through some problems and make sure and solidify my understanding of it." [He then worked through some of the examples given in the textbook]

The second reason Peter looked at examples is because he believed that examples helped to clarify why the big idea he had previously identified was important. For example he stated, "And then I looked at their examples, I wanted to see so— we understand the idea so why is this important? Why do we even want to understand what is happening? Why do we use this?" The examples often provided Peter with the possible "applications" for the concepts being used.

Process 2: Planning the lesson. Peter's reading practices looked different when he was planning for what he was going to do during class time than when he was identifying the big ideas in a textbook section. Specifically Peter used the textbook quite a bit less when he was actually planning, and would usually only utilize the textbook as a reference. The following excerpt highlights the various ways Peter utilized the textbook as a reference,

> R: How would you describe the role the textbook had in your lesson today?
>
> P: So, I would describe it— more of a check for me. I don't have an outline saying, "this is exactly what you need to teach". And I guess that's what the textbook is. All I am told, I wish this was different, is that this is what you need to teach…This is more of a check to make sure that one, what I was stating and how I was thinking was accurate, I feel comfortable with it already but I was using more of a check. And then I also use it as a way of K how do I— Cause I feel like my conceptual view of this is very informal and the book is very formal, and so I feel comfortable with the informal part by the formal uh such as the theorem here, that alone, I need the book to help me make sure I can connect the informal with the informal— And so this is helping me in a sense keeping the rigor in my terminology use correct. And so that's why I say that the textbook is more of a check for me to see if what— It's a check for me to see if the informal thinking really lines up with the formal thinking. And if not then I need to change it.

This excerpt once again demonstrates that one of Peter's primary purposes when reading was to identify the big ideas given in a section and to reconcile those ideas with his own “informal” understandings. However, this excerpt also described how Peter used the textbook as a reference in a few ways that appeared to be consistent across all of his lessons when contrasted with the rest of the data obtained. First Peter used the textbook as a reference that told him what general topics he should be teaching. Thus, his lesson plans usually linearly followed the sequencing found within the textbook. He chooses to go from section to section in a linear manner and generally chose to introduce the big ideas and examples found in the textbook linearly. The second way in which Peter used the textbook was as reference or “check” to make sure that the words and terminology he used had acceptance in the mathematical community. Thus the textbook was often a vocabulary reference or source that told Peter how he “should” present certain ideas and what notation he should use.

The final way in which Peter read the textbook as a reference was that he looked at the homework problems at the end of the section to determine what his students would need to do later. As he planned he sometimes chose which content to cover by looking at the types of problems the textbook would ask students to do in the homework. This was not a particularly common practice but seemed to be a relevant practice in a few of the lessons observed.

Curricular reasoning.

Analyzing from the learner’s perspective.

Process 1: Identifying the Big Ideas. In every lesson planning session observed, Peter spent a significant amount of time grappling with mathematics as he attempted to understand the big mathematical ideas found within a textbook section. He often had several concerns, misconceptions, and questions that arose as he utilized the reading strategies described above. As concerns arose Peter would analyze from the learners perspective and anticipate student thinking *based on his own concerns*. For example after reading the traditional definition of a definite integral Peter had the following conversation with the researcher,

> P: So I am not going to use that definition— no probably not.
>
> R: Why did you say that?
>
> P: Well thinking about— Maybe it's because I really didn't go in depth into this. But— Just the thing that they're saying that f is integrable without really defining what this means and then I [the variable used to represent the definite integral] is a definite integral without defining what an integral really means. And there is nothing previous [flips back through the textbook section] to really specify what an integral means and so we're using this definition— Unless the whole definition is that there is some value I. Which if you just give that to students— or if you just— Just to me. I'm looking at that and I don't really know what that means! So, the whole meaning of this is lost and I feel like my students will get lost in the whole "what does each part really mean".

Notice that in this example Peter is both thinking of his own concerns and the concerns of his students simultaneously. He assumed that if he was struggling to understand something that his students are likely to experience those same issues, and therefore anticipated student thinking based upon his own concerns. I would argue that many teachers engage in this practice but in observing Peter plan lessons this practice seemed especially prevalent. As he identified big ideas he often placed his concerns or questions on a sheet of paper and those questions are often what he drew upon when he engaged in the process of planning a lesson. It is important to note that it was difficult to quantify how prevalent this practice was because Peter spent a majority of his time grappling with mathematical issues and was not always explicitly clear if he was thinking of just his own concerns or was simultaneously thinking of his students' potential concerns. However, it is important to recognize that this is an essential characteristic of how Peter anticipated student thinking,

Process 2: Planning the lesson. As Peter planned for specific lessons he would exhibit planning practices similar to Susan. Specifically he identified various specific questions, concerns, and conceptions students might have within the implementation of a lesson. For example in planning a lesson on the "u-substitution" method for solving integrals Peter stated the following,

> P: Probably what I'll do is give them this question [writes $\int (x^2 + 1)^2 (2x) dx$] and I will ask them to integrate it. Now one view I already know that they're gonna try, or some might try, is they'll just multiply everything out. If they multiply everything out then yes it will come to be a simple case of do the inverse of a power rule idea. And so that's one view students will have.

In this example Peter planned to ask his students a specific question (integrate this function) and expected to receive a specific mathematical answer (students will multiply and use the power rule of integration). This was Peter's primary method of analyzing from the learners perspective.

In conclusion Peter's analyzing from the learner's perspective can be summed as a product of anticipating student conceptions based on personal conceptions while identifying the big mathematical ideas, and anticipating specific student conceptions in response to constructed/adapted questions created while planning. It is important to note that often the former type of anticipation dictated and determined the latter type of anticipation.

Reflection. In analyzing Peter's planning sessions exceptionally few instances of reflection were identified. It is important to realize that there might have been more implicit cases of reflection during the observations but few were explicitly made. In almost all of the instances identified Peter reflected upon recently implemented lessons and identified a mathematical conception his students would bring into the classroom. For example, in a lesson planning session Peter said the following,

> P: How do my students view the derivative? What— what gives them— what's their conceptual understanding of what a derivative is? A lot of them will probably say well it's the rate of change or the instantaneous rate of change of the function. And so— If I— The only value that they've seen with derivatives is if I was to look at something for how the rate of change is.

In this excerpt Peter anticipated what his students might say if he asked them what a derivative is. He then articulated a student response and justified his anticipated response by reflecting on past lessons and stating that his students have only seen certain types of problems that require a specific conception of derivative. This excerpt demonstrates Peter's reflection practice and also once again demonstrates how interconnected the practices of analyzing from the learners perspective and reflection are.

Mapping learning trajectories.

Mapping across lessons. There were few instances in which Peter chose to consider the layout of several lessons in his textbook. Generally he trusted that his textbook would provide a logical intuitive sequencing of the topics in the calculus curriculum. However I did have the opportunity to observe one particularly significant instance in which Peter reconsidered the overall structure of various lessons. In one lesson planning session I watched Peter as he began planning with the first section of the textbook chapter on integration. This chapter sequenced integration lessons in a traditional fashion in which summations were covered initially, followed by a treatment of definite integrals, followed by a treatment of indefinite integrals. When Peter first started planning he skimmed the textbook and wrote down everything he knew about integrals. He knew that integrals were used to take antiderivatives and he knew that they were used to calculate the area underneath a curve. However as Peter read the textbook section on summations and eventually the definition of a definite integral he struggled to understand the big ideas associated with definite integration (a summation of two quantities being multiplied together). Since he was confused he decided to consult two other calculus textbooks. The first textbook he consulted contained the exact same sequencing as his primary textbook and he experienced similar confusions. However in the second textbook the authors chose to cover the topic of indefinite integrals prior to covering definite integrals. This sequencing resonated better with Peter since his own personal concept image of integration was antiderivative centered. In summing up this experience he stated,

> P: I would have gone with this way [discussion the meaning of definite integrals first], that was fine with me. But the problem is going from here [a section about summations] to looking into an integral I felt that there was a jump that I wasn't ready for. And the book didn't really provide a good connection for me so that's why I went to other books to figure out ok how could I make that connection myself and then which way would be best for my students. And sadly I think "the way I think that's how they're going to think" [mocking himself].

There are a few important things to note about this instance of mapping learning trajectories across lessons. First, it is important to note that this instance is rare. At no other time in the data gathered did Peter reconsider the sequencing of the presented material. Second, even when Peter diverted from the sequencing he still felt the need to follow the sequencing of a textbook. It appeared that he was only confident about changing the sequencing of lessons as long as another textbook followed that particular sequencing. Finally, it is important to note how Peter makes learning trajectory decisions. In this particular case Peter decided what he was going to teach based upon his personal conception of mathematics. He decided that since an antiderivative approach to integrals made the most sense to him than it is likely that an antiderivative approach would make most sense to his students (although he was not particularly fond of that reasoning). This leads to an important characteristic of Peter's planning practice. Peter plans in a manner that resonates best with his personal conceptions of mathematics while also attempting to respect student thinking.

Mapping within a lesson. Peter does not have direct access to worthwhile mathematical tasks. Therefore he spends a significant amount of time determining what learning activities he wanted his students to engage in and determining the sequencing of those activities in a given lesson. The goal of this section is to describe how Peter used his textbook to plan the sequencing of class activities and discussions. I will also describe why Peter tended to choose a particular sequencing.

As stated above, Peter initially planned lessons by reading the textbook and attempting to identify certain big ideas. During this process he would list out his personalized conception of those big ideas on a sheet of paper. At this point he would then often attempt to devise a task wherein those big ideas could be "discovered" by his students. The following is an example of Peter stating what further work he needed to accomplish before teaching a lesson on the Fundamental Theorem of Calculus,

> P: I need to figure out like a context that will actually bring out these ideas, or some kind of question or task so that way we could start discussing about the second fundamental theorem…I am going to think about it a little more, try and look through some of these

problems [examples and homework problems], pick out a problem that I can adapt, and then build a task out of that that will lead to establishing the ideas of the fundamental theorems.

R: Ok. And when you say a task, what do you mean by a task?

P: A task can be either just a question, for example I could give them— like almost like— one of the example questions. Ok what if I— If I took the integration of this [points to function] and then took the derivative of it what would it be? Something like that or It doesn't necessarily have to be a full-blown like contextual "here is a company doing this". It can just be a question that addresses these ideas, and so what I'm looking for is if there is something like that that'd be awesome but I don't think I'll have the time to actually find it. But if I can, I think it would be neat for them to actually see why you might want both of these theorems and the application of them. Where will we see that? And so I need to find some kinda task or question that could address that. If not then I need to find in the book a question that I could use to get these ideas out and then we can look for the application later.

These descriptions of Peter's planning processes described how Peter attempted to construct a learning trajectory within a lesson. He mentioned that it would be ideal or "awesome" to find some "contextual" centered worthwhile mathematical task but stated that he didn't have the time to actually find one. (However, it is worth mentioning that Peter never tried to take the time to look outside of his textbook for tasks in all of the six lesson planning sessions observed.) Peter then stated that if could not find a context embedded task that he would then adapt a problem from the book to pull out the big ideas. This was a common practice of Peter's. He often used the examples found in the textbook as problems students could engage with or as focal points for a class discussion.

It is important to note that often instead of immediately adapting an example from the book in task construction Peter would attempt to construct an "ideal" worthwhile mathematical task but then would abandon that practice in the interest of time (he often had already spent 2-3 hours planning at this point). For example in one lesson planning session the textbook used the notion of being locally linear at a point as a conceptual underpinning for discussing linearization but as Peter attempted to plan the lesson he stated, "Sadly I can't think of a task that will get them from locally linear to here [points to the linearization equation], and so now it's going to be more of a 'guided lecture', ew I hate that word, on what is happening" Notice that in this example Peter could not think of a worthwhile mathematical task and was compelled to construct a "guided lecture" based learning trajectory. This occurred in almost all of the lesson planning sessions observed. Thus, Peter's lesson plans usually were prepared "guided lectures" and these lectures were often centered on utilizing examples found in the book to pull out essential mathematical ideas.

It is now important to characterize *how* Peter constructed his guided lectures or adapted tasks from the textbook. Peter's guided lectures were based upon how he understood or recently relearned the mathematics in a textbook section. Evidence of this fact can be seen in recalling how he anticipated student thinking and planned. Peter often anticipated student thinking by considering his own mathematical concerns and he planned by taking his notes that he constructed while identifying and grappling with the big mathematical ideas in a section. He would then use those notes to create a learning trajectory for his students. This trajectory often resembled the learning trajectory of the textbook but Peter would often remove some aspects of an example or homework problem so that students were compelled to choose how they thought the problem should be solved.

It important to note that the more comfortable Peter was with the mathematics in a textbook section the more Peter departed from the learning trajectory of the textbook. For example in the section on the “u-substitution” method for solving integrals, Peter created his own lesson based on his personal conception of integration, which varied significantly from the textbook. In this instance the textbook became a resource that insured that Peter covered all the material he was supposed to cover and became a resource that ensured his personal conceptions and practices was accepted in the mathematical community.

There was one final important practice Peter often engaged in as he planned a learning trajectory within a lesson. After he completed the preliminary planning of a lesson Peter then looked over the homework problems in a section to see if what he would be teaching his students was adequate preparation for the homework problems. For example while Peter was engaging in this practice he stated,

> P: So I’m just skimming here. Just here— looking at types of problems just thinking of my students. Can they do it?
>
> R: Can they do it based on what?
>
> P: What I’ve seen in the past class and also lets say best scenario this [his lesson plan] clicks and so I’m just looking at it and seeing if any misconceptions might come out on these problems.

At the end of a planning session Peter often skimmed the homework problems or examples and asked himself if his lesson would prepare his students for the types of problems provided in textbook. He also seemed to use this time to check if he had covered all of the material in the section. Thus the homework problems often represented what students should be able to do as a result of the lesson. In a way they often represented an assessment that Peter was teaching toward.

Mapping within the question. Peter was often so concerned with developing a worthwhile learning trajectory within the lesson that he did not spend a significant amount of time considering the various issues within a single problem and how he might guide students within a problem. This did not mean that Peter did not engage in solving various problems. Peter actually spent a great deal of time doing problems, but as he solved problems he was often concerned with the underlying mathematics, how one problem naturally leads to another type of problem, or he was just concerned with listing possible misconceptions without devising responses to those misconceptions. For example when planning a lesson on linearization he stated,

> P: So then what we'll do is example 1— yeah I'll adapt it a little bit [writes example 1 on his planning sheet]— Then I'll do another example for them to try— [writes example 2], K once we have established that we will do one more just to make sure its solidified [writes example 4], alright so then after this whole linearization business that'll probably be it for linearization. I'll need to just quickly summarize how and why linearization in detail.

Notice that Peter at this time was planning a sequence of examples. It is important to note that Peter did not plan implementing these examples as they are seen in the textbook but intended to adapt them to be part of his "guided lecture" or intended that his students would practice these problems. I actually have little idea as to how Peter utilized these examples because he did not detail his plans at this time. However it is clear that he did not explicitly state specific concerns students might have when engaged in these specific problems and did not plan how those examples would be discussed. Therefore it seems that Peter spent significantly less time mapping learning trajectories within questions.

Summary. Peter's lengthy planning practices can be summed as a compilation of identifying conceptual ideas and planning what to do during a lesson. He would identify conceptual ideas by focusing on introductions, definitions, theorems, and examples. When planning lessons he would trust the sequencing of mathematical topics across lessons and would try to construct a mathematical task within a lesson, however, in the interest of time he would usually plan a "guided lecture" for his students.

The Case of Lisa

Lisa's mathematics department had constructed a list of topics that teachers should cover when teaching secondary mathematics II or III. Within each topic the mathematics department had created a list of "I can" statements that represented what students should be able to do when finished participating in a given lesson. For example one topic Lisa was supposed to cover was the topic of Circular Trigonometry. Within this topic some of the learning targets were, "I can determine the amplitude and period of the sine and cosine functions" and "I can identify the domain of range of the sine and cosine functions". The sequencing of these procedurally centered learning targets was based upon the district adopted traditional textbook, but it is important to note that Lisa rarely chose to utilize the textbook. She felt that her lessons were best constructed by using a combination of the learning targets, her personal intuitions, and the resources available on the Internet. Therefore Lisa planned lessons by initially consulting several learning targets and then would try to find tasks online that helped build student understanding of the procedures associated with certain learning targets. In this case Lisa's curriculum was the learning targets, her textbook, and any resources she consulted on the Internet. If she were not able to find a task online she constructed her own.

In reflecting upon the above description of Lisa's planning norms the argument could be made that Lisa is not using a traditional curriculum since she is not using the textbook. However, I would like to reiterate that the purpose of the study is use data to hypothesize some of the problems reform oriented teachers face when utilizing a traditional curriculum. Lisa has been provided with a traditional curriculum but has the desire to implement task-based instruction. Her planning norms could represent the norms of many teachers throughout the United States. In fact Ball and Cohen (1996) pointed out that some teachers have claimed that good teachers should choose not to follow or use a curriculum. Thus Lisa's choice could represent the choices of many and thus I feel that her problems and concerns should be represented when discussing the problems reform minded teachers experience when utilizing a traditional curriculum. Therefore the purpose of the following section is to describe how Lisa reads curriculum and engages in curricular reasoning.

Reading. Lisa was mostly reading and utilizing online materials as she tried to plan task-based mathematics lessons. Specifically, she had two different reading practices in which she engaged as she searched online materials. The following section describes each of these practices.

Searching for Tasks. Lisa's primary reason in using the Internet was to find worthwhile mathematical tasks that she could implement in the classroom. Consider the following excerpt,

> L: The book has a lot of problems that are using combinations and permutations but I'm gonna look to see if I can find anything anywhere else that would give me— kinda— a better start than making something up myself. Which is easier if you find something. But— I don't think I will.
>
> R: What do you mean by better start?

L: Well right now— Really some kind of task or problem or— that I like— That we could use either off this idea or start so that it makes them start thinking about this without just jumping right in and here's what this is and this is what we're going to do and then here's some problems that let you practice that. So some kind of, like to the class period, some kind of problem or launch essentially that would get them thinking about this without being like, "Here's what we are going to do. Think about it like this. This is great!" And then do some problems like that— And I never really know how to find them. Sometimes I can't.

Notice in this excerpt that Lisa was using the Internet because she hoped that she would find a task that allowed her students to think and learn about the conceptual underpinnings of the procedures associated with combinations and permutations. This reading practice occurred in almost every planning session.

There are two other common important characteristics of Lisa's planning that should be identified from the above excerpt. The first is that Lisa would consistently state across planning sessions that the reason she would use the Internet was because it was "easier" than creating a task herself. Lisa was often stating that she did not have the time to construct her own tasks and she often said that her personally constructed tasks were susceptible to unforeseen problems. The second important characteristic to note in Lisa's online search for tasks was that she was not confident that she would find tasks online. Lisa would often state while or before searching online that she did not think she would actually find a task. This attitude seemed to be based on prior experience with unsuccessful searches for mathematical tasks.

"Traditional" websites. The second reason Lisa used the Internet was that she was interested in discovering how certain websites explained how to solve routine practice problems.

For example, Lisa stated the following as she searched across online websites in planning for a lesson on trigonometric functions,

> L: I try when I do problems that give them contexts that they're kinda coming up with things I try and look at what— how they would see it presented normally, like if they were to read the book, which they don't do. But if they were to look at the book or if they were to ask someone else for help what would someone present that was just gonna to give them formulas that they had memorized or whatever. Like what is the amplitude defined as? Or what is— a shift, where does a shift show up, how is it explained in the book? You know?
>
> R: Got it
>
> L: So that we have that kind of connection as well when we're going through this kind of context problem that they're kinda coming up with all of those things. Or they can see the connection between a period, what is a period— in this context. So— that's when I do what some of them will do when they don't pay attention [searches on the internet "amplitude of trig function"]
>
> R: Ok
>
> L: And I have found that I go back to things like the book or searching things like this online because I have started— Well I've learned from the students through what we've gone— Like last year— Thinking about these things, comparing them to different like a context as opposed to just defining on the graph what's an amplitude. And so I go back to see "ok if you were just going to look for a formal definition what would it tell you"— so that I'm making that clear in their mind when they see it in different ways. If that makes sense?

R: Yes

L: So that's why I go back and look because sometimes I think about it differently or define it differently because I think that will help them understand or they will point out things in a context that maybe you wouldn't if you were just defining what it was and telling them to find that.

There are a few essential portions of this excerpt I would like to highlight that provide insight into how Lisa consistently read online materials. First, it is important to notice Lisa viewed the online procedural based websites the same way she viewed her provided textbook. In this excerpt she almost used the terms interchangeably, and it actually seemed like she placed more importance on the online procedural websites than on the textbook because her students would not read the textbook but were more likely to search the Internet for solutions to problems. It is also important to note that in comparing her limited reading of the textbook to her reading of online procedurally centered websites that she read her textbook and the online resources in the exact the manner and for the exact same reasons described in this excerpt above. Lisa utilized the textbook and procedurally based websites so that she knew what her students would be provided when they asked for help in solving problems and so that she could study how to connect a generalized procedure to the context of the task. She did not want her students to not be able to generalize across contexts.

In conclusion of this section I would like to describe *how* Lisa read the textbook and online procedurally centered problems. In reading she seemed to focus on the examples and the types of procedural problems that will be asked of students. She did not read the material around the problems. She assumed that, likely based on past experience, that this information was not helpful in helping her meet her goals.

Curricular reasoning.

Analyzing from the learner's perspective. There were two different ways in which Lisa analyzed curriculum materials from the learner's perspective: 1) She anticipated the types of thinking and experiences students would have outside the classroom in doing homework after class, and 2) She would anticipate specific mathematical conceptions students would have in response to specific mathematical questions. Both of these practices will be described in this section below.

Anticipating outside the classroom. Lisa continually anticipated the experiences her students might have outside the classroom after a class session would be complete. This practice was described in the reading section above. When reading procedurally centered online material and the provided textbook, Lisa understood that these were resources students would use when looking for help at home. She would then reason and construct in class learning trajectories with this perspective in mind. For example, she often was concerned with connecting mathematical tasks to formal procedures because students would be told and would likely experience formal procedures online or in the textbook after a class session was complete. This is a unique type of anticipation of student thinking. It is one in which Lisa anticipated the probable struggles or concerns of her students *after* a class session rather than *during* a class session.

Anticipating specific mathematical conceptions. Similar to Susan and Peter, Lisa would often anticipate specific mathematical conceptions, ideas, or strategies students might have in reaction to a specific mathematics problem. For example in planning a lesson on permutations and combinations Lisa saw that the textbook and online resources simply presented students with various examples of traditional permutation and combination problems. These resources would then demonstrate how to solve these problems using traditional formulas. In reading these problems Lisa anticipated what her students might do if given traditional permutation and combination problems without being shown examples or being given formulas. She guessed that her students would simply try to list out all the possible outcomes in a specific problem, possibly using tree diagrams. These types of anticipation instances represented the majority of the instances of analyzing curriculum materials from a learner's perspective.

It is important to note that most instances of anticipating specific mathematical student thinking occurred concurrently with some type of reflection. Lisa usually justified her anticipated student thinking by reflecting on recently taught lessons to students or lessons taught the previous year and she felt uncomfortable anticipating student responses without having past experiences to draw upon. For example when considering which permutation and combination problems to give students in a lesson Lisa stated, "So to be totally honest when I give one of these I can guess what they'll say but I don't— but not very well. So if I taught it before I kinda use like "I saw previous". Thus it seemed that anticipating student thinking did not have as significant of a role in Lisa's curricular reasoning in comparison to her reflective practices.

Reflection. Lisa had three different reflection practices: 1) She reflected on lessons implemented the previous year, 2) she reflected on lessons given recently, and 3) she reflected on her teacher education experiences. Each of these reflective practices will be described below.

Previous years. It could be argued that Lisa's most influential resource in planning lessons were her experiences from teaching the previous year. Lisa was always referencing her experience or inexperience when planning to teach a particular topic. Consider the following dialogue between the researcher and Lisa,

R: How often are you able to pull from like something you did in the past?

L: Um, a lot.

R: A lot?

L: Yeah. Last year was not as much because I had Math one, two, and three and I hadn't necessarily taught every topic. And so a lot of times I'd just look at it from like "here's how I would do it", I mean thinking back of when I learned that topic. You know. I was given the formula and whatever and then just did the math. So I would do it and then try and find either tasks or things that'd help explain the connections, even proofs. I'll look up proofs as to why formulas are the way they are. And sometimes I still do that. But a lot of times now that I'm teaching— pretty much everything I'm teaching I've taught before. And sometimes I draw on things that I taught before but it didn't work, and so I know those connections weren't made or that students didn't understand that concept as well. And so I kind of start over and look for things that will help teach it better. If that makes sense? But a lot of times now— Pretty much whenever I teach sometimes I think about how I taught it before and what was good and not good, what connections they made and didn't make, or what I didn't like or what I did.

There are many important characteristics of Lisa's reflection practice that are depicted in the above dialogue. However, it is first interesting to note that based upon the above response, Lisa's planning practice in her first year of teaching seemed to have been similar to Peter's planning practices. She approached mathematical ideas from her own conceptions or how she had learned things as a student and often consulted proofs and definitions in the textbook when planning lessons. However the above dialogue most importantly depicted the nature and significance of Lisa's habit of reflecting on past teaching experiences when planning. Notice that Lisa stated when planning a lesson she always initially reflected upon last year's experiences. In doing this she thought about whether the appropriate connections between procedures and ideas were made and used that thinking to approach this year's lesson. This practice was consistent across all of Lisa's planning sessions, and demonstrated the extreme value Lisa placed upon her past experiences. Thus Lisa's reflection on past experiences was an essential aspect of her curricular reasoning and often was the starting location for planning.

This year. Lisa would often reflect upon recently implemented lessons in deciding what she was going to plan. This reflection happened in two different ways. First, Lisa would reflect on the mathematical conceptions her students would be bringing into the classroom based upon previously implemented lessons. For example when attempting to find a task on permutations and combinations she stated,

> L: I'm also kind of thinking in terms of where my students are coming from where we just talked. We just talked about about probabilities—and we talked about like independent and dependent events. And we talked about like probability statements. —So if I jump back into a question about probability, which this helps us see [points to an online curriculum] to a— They also did like tree diagrams, Venn Diagrams. So all of those skills they have so I'm also looking for something that's like that's where their mind will go, like right now for probability. And so if there's some kind of— any of those things that connect and then we can see "ok this is what we're doing every time, this is kinda what's going to be happening."

Notice that Lisa was reflecting on past lessons so that she knew "where her students are coming from" and was attempting to select a task that respected those mathematical orientations. Specifically Lisa stated that she knew, based on past lessons, where her students' minds would gravitate and therefore wanted to select/use a task that used that thinking. Therefore Lisa was constantly reflecting on past lessons with the current mathematical conceptions of her students in mind as evaluated the quality of a worthwhile mathematical task.

Along with considering her students mathematical perspectives when planning lessons Lisa also reflected upon the norms and pedagogical strategies she had been using throughout the current school year. For example, when planning for a review of previously covered material Lisa determined that she should have the students work on the review in groups because there were times in which this pedagogical strategy worked well in lessons implemented earlier in the year. In another instance Lisa planned to have her students' rate themselves regarding their comfort level with solving different types of problems. This pedagogical strategy was used because it was a norm that had been implemented over the course of the school year and because it seemed to have adequately helped students self-assess their knowledge. Thus when Lisa planned she often reflected on her norms and past pedagogical decisions when deciding what to do during a lesson.

Teacher education reflection. Unlike Susan or Peter, Lisa often reflected upon her teacher education experiences when planning task-based lessons. These reflection instances all occurred as Lisa attempted to find/construct a task to teach a particular lesson. For instance, when planning a lesson on circular trigonometry Lisa reflected on her experiences at a district mathematics meeting, and at this meeting it was mentioned one could use the context of a Ferris wheel to teach about trigonometric functions. Therefore Lisa was able to search specifically online for a Ferris wheel task. Another example occurred when Lisa was attempting to create a task for a permutation and combination lesson. She remembered that in her student teaching experience the cooperating teacher used an ice cream context to delineate the difference between permutations and combinations. An ice cream bowl was used to represent a combination (order doesn't matter) and an ice cream cone was used to represent a permutation (order does matter).

Therefore Lisa reflected on her teacher education experiences often in the planning sessions observed and this better enabled her to create/find mathematical tasks.

Mapping learning trajectories.

Mapping across lessons. Lisa followed a curriculum sequence that had been created by her mathematics department. This sequence of topics was a sequence of "learning targets" (see above) and was based on the traditional sequencing of the district textbook. In describing how Lisa determined what to teach for a given series of lessons she stated,

> L: So I kind of use this [points to the learning target sequencing created by the mathematics department] as a kind of a reference point so that I'm at like the same point, or teaching the same types of things as the rest of the department. And so that's what I look for. Although usually I just kinda take these in general, sometimes I do them in a different order. So I look at that and then I look at what I did— how I taught it last year. —And then just kinda try— usually I can remember how it went and adjust, or know that it didn't go well at all and start over.

Notice that Lisa follows the sequencing created by the mathematics department generally so that she can ensure that she is aligned with the mathematics department. However she mentions that she sometimes does not follow the same order as the designed layout. There was one particular instance I would like to describe that may highlight why and when Lisa chooses to change the ordering of topics she does. In one planning session observed, Lisa was attempting to create a task to begin her lessons on curricular trigonometry. She wanted to find or construct something that gave meaning to the procedural learning targets found in the curricular trigonometry section. She thought of using the context of a Ferris wheel but had difficulty identifying how to use the Ferris wheel context to give meaning to certain procedures. For example, what would a period change represent in the context of a Ferris wheel task? Therefore Lisa looked online for a Ferris wheel task. She found a task but it did not fit the progression being used by the district in fact it did not cover all the material the district was asking her to cover. Thus in adopting the task she inadvertently decided to change the ordering of the curriculum. Thus it is likely that Lisa's practice of adopting found mathematics tasks represents a primary reason why Lisa may not rigorously follow the sequencing constructed by her mathematics department.

Mapping in a lesson. Lisa spent the majority of her time determining the tasks she wanted her student to engage in during a lesson. In each planning session observed she began by identifying her learning targets and the traditional ways to teach a procedure-based lesson. She would then attempt to find a task that helped students develop the conceptual underpinnings associated with certain mathematical procedures. These tasks were found by reflecting on teacher education experiences and by searching online. Tasks were then evaluated by reflecting on recently implementing lessons, and on how well she thought a task would develop the conceptual underpinnings associated with a learning target. The sections above have characterized these typical planning practices with support from data. However, it has not been described what Lisa did when she could not find a mathematical task online that she could adapt to her needs. In the following dialogue Lisa explains what she usually did if she could not find a mathematical task online or in reflection,

R: What happens if you can't find that something? What do you do?

L: Sometimes I make it up.

R: So you create something?

L: So I create something. Yeah. Sometimes I create things and they work. Sometimes I create things and they don't and so I just have to kinda of tell them how to do these things. Other times I can see just kinda abstractly where the connections are. And so we'll just use— like it will be less— I don't know how to say this— It will be less— I mean connecting to like context but connection within the math. You know? Like we talk about factoring. You know all the kids when they see a quadratic to factor they're like "what multiplies to this and adds to get this right". So every time they say that just the connection between the multiplication of the binomials to, I don't know, x squared minus three x minus 4 [writes $x^2 - 3x - 4$]. —So like for factoring we talked about a context. We talked about quilt squares and using MVP [The Mathematics Vision Project, a reform curriculum created for teachers in Utah] this year but— but anytime now we're kinda pulling out of the context and they need to be able to just factor that without drawing things forever right. And so just making these connections between "well why is this [points to $(x^2 - 3x - 4)$] this [points to $(x - 4)(x - 1)$]? Why are we doing this? When we factor we have factors of four that means two and two because they multiply together. These two multiply together. And so if you multiply these out [points to $(x - 4)(x + 1)$] and then we get practice to FOIL or whatever— that gets that. And so these are equivalent. Not like magically this [points to $(x^2 - 3x - 4)$] becomes this [points to $(x - 4)(x + 1)$] but they are the same thing just written different ways right. And so even things like that sometimes I can't find context to connect it to or even really tasks that they can discover things, but when I— I try and find series of questions at least that have some kind of mathematical connections.

In this excerpt Lisa mentions important practices she engaged in when she could not devise or find a mathematical task. When Lisa could not find a mathematical task she would create her own task. These tasks sometimes would not work and she had to simply tell her students how to "do things". This could have meant she had to tell them: how to do the procedures associated with the task, the mathematical connections within the task, or the way to property engage in the task. Either way, it seems that Lisa's constructed tasks were not always as ideal as she would have liked them to be.

It is also important to note what Lisa's constructed tasks looked like. According the above excerpt, Lisa's constructed tasks that allowed her students make mathematical connections between procedures or abstract mathematical ideas. Her new tasks may not have been context centered or discovery oriented but they were built so that students use their intuitive understanding of numbers in combination with an understanding of previous mathematical procedures or ideas to give meaning to new mathematical procedures. Thus, when Lisa did not have a task, her created tasks focused students on making sense between mathematical procedures and pure mathematical ideas.

There is one final aspect of Lisa's task creation I would like to discuss. It has been noted throughout this case study that Lisa repeatedly stated that she avoided creating her own tasks because it was easier to find a task and because her tasks didn't always work. However it is important to state why Lisa's constructed tasks "did not work". In asking Lisa about what she would have done if she had not found the Ferris wheel task she said,

> L: So this one if I hadn't found this— I probably would have created one. And sometimes that's horrible. I created one the other day that was similar to this and the question made no sense to them. And so I like to find something where I can read it and be like "oh this is what I got out of it" instead of creating it where I totally know what I want them to get out from my head but that's not how they read it and I realize it as soon as I hand it out

and they read it and are like “what are you talking about”. Right. And so that’s what I would do is create one, but I’d go over it three or four times myself before giving it to them Friday where I can try and flesh out maybe things that don’t make any sense or that are labeled wrong or whatever.

Notice that in this excerpt Lisa was able to describe why task creation is problematic for her. She articulated that she often has a difficult time creating questions that make sense to students and it is easier when she is given a task because then she is assuming the role of a student when reading it.

Mapping in a problem. There were few instances in which I was able to observe Lisa plan a learning trajectory for a specific question. Lisa spent most of her time determining what tasks and questions she was going to provide her students with. This does not mean that Lisa does not plan these types of learning trajectories. It simply means it was not a matter of emphasis in the planning sessions observed. However it is worth noting that at the end of each observation Lisa said she planned to work through problems and tasks chosen for her students. It is possible that she was engaging in this practice during this time.

Summary. Lisa rarely consulted the textbook when planning task-based lessons. She would read and utilize materials that she found online and would attempt to follow the sequencing of topics found in learning targets created by her mathematics department. She would often attempt to find tasks online, and reflection on her student teaching experience seemed to play an integral role in helping her to find or construct mathematical tasks.

Chapter 6: Cross-Case Analysis

It seems that the most obvious problems reform oriented teachers face as they utilize a traditional curriculum is that they do not have the time to plan task-based lessons. This problem was the one most often articulated as I directly asked teachers what they felt were the problems they faced when planning task-based lessons. Peter would spend over two hours planning a task-based lesson. This enormous amount of time could have originated from the fact that Peter was interacting with a curriculum for the first time or from the fact that Peter was teaching a class which contained more advanced mathematical topics. However, it still seemed like Peter's time spent lesson planning was lengthened due to issues encountered utilizing a traditional textbook.Lisa would also spend at least an hour planning a task-based lesson, and it is worth noting that in conversation both teachers admitted that they did not always have the time to plan quality task-based lessons. In contrast, Susan usually spent under 30 minutes planning for a day's lesson and her lessons were usually task centered. The identification of this problem is not ground breaking. A lack of time is a seemingly obvious problem reform oriented traditional curriculum user's face, and the pertinence of this problem could have been quickly revealed without conducting this study. What this study does do is that it helps answer *why* teachers do not have time to plan reform based lessons. What is it specifically about using a traditional curriculum that is time consuming and problematic for reform oriented teachers? The purpose of the following section is to use the results obtained in the previous section to make conjectures about the problems reform oriented novice teachers face when utilizing a traditional curriculum, and these conjectures should offer meaningful details as to why the construction of task-based lessons is time consuming.

Textbooks Focus on Procedures

As Peter and Lisa engaged in the practices of identifying conceptual ideas and planning task-based lessons, they encountered issues that seemed to occur as result of the procedural nature of the traditional textbook they were utilizing. The purpose of the following section is to describe these problems. Specifically I will identify why the procedural nature of the textbook made it difficult for Peter and Lisa to identify conceptual ideas and to engage in the process of planning task-based lessons.

Thinking "backwards" to identify conceptual ideas. In looking at the case of Peter it seems that he spent a sizeable amount of time identifying the "big mathematical ideas" found in a textbook section. This means that he spent a significant amount of time attempting to understand the mathematics in the textbook and attempting to identify what he believed to be the important mathematical concepts embedded within a textbook section. This is natural considering that this was Peter's first time teaching calculus, but in analyzing the case of Peter and considering the case of Lisa it seemed that certain aspects of utilizing a traditional textbook made this process especially challenging.

Peter often struggled to identify the important mathematical concepts embedded within the textbook. He engaged in few reflection experiences and was therefore compelled to read the textbook to obtain personal conceptual meanings associated with a mathematical idea. In reading the textbook Peter often would often try to focus on ideas found in theorems and definitions, and then would attempt to pull apart the conceptual underpinnings of these ideas. This practice seemed to be problematic because he seemed to be engaged in thought processes that required him to be constantly thinking "backwards" about conceptually based mathematical ideas. Meaning, that instead of being able to use personal understandings of mathematical ideas to develop conceptual understanding and make sense of the rules, procedures, and theorems of mathematics Peter was presented with rules, procedures, and theorems first, and was then compelled to consider how these seemingly isolated pieces of mathematics made sense within his own understanding of mathematics.

Task-based instruction is often valued because it helps build conceptual understanding of mathematics by utilizing tasks that appeal to students natural intuitions. Worthwhile mathematical tasks allow students *and teachers* to construct meanings associated with a mathematical idea that are based upon how they understand mathematics. During task-based instruction students are provided with a task that has multiple entry points for a variety of students to begin to access an important conceptual mathematical idea (NCTM, 2014). These "conceptual" ideas are complex and are different from memorized mathematical procedures because they are connected to how a student personally understands mathematics. For example in learning the concept of linearization a student or teacher could memorize the formula of $L(x) = f(a) + f'(a)(x - a)$ and remember that given a value of x close to a this formula provides an approximation for the value of $f(x)$. However, a student or teacher could also learn that one way to approximate $f(x)$ is by first finding the equation of the tangent line at some close point $\left(a, f(a)\right)$ where a is near the desired value of x. The student or teacher could then possibly use his or her intuitive understanding of local linearity and realize that by inputting x into the equation of the tangent line one receives a reasonable approximation for $f(x)$. This example demonstrates at least two different ways to view linearization. One is as a formula that when provided with specific inputs one can determine the corresponding outputs. Others have entitled this type of understanding as "instrumental understanding" (Skemp, 1976). Instrumental understanding is a type of understanding that refers to knowing mathematical rules and when it is appropriate to use them.

The second way to view linearization is connected to how a person understands other mathematical ideas. In the above example the concept of linearization could be a conceptually developed from a student's intuitive notion of local linearity, and given this type of understanding a student could adequately explain *why* his or her method of approximation worked. Some have called this type of understanding "relational understanding" (Skemp, 1976). Relational understanding is connected to how a student understands other mathematical ideas, and it is often valued because it is a type of understanding enables a student to explain why certain rules and procedures correctly calculate a desired answer. Worthwhile mathematical tasks are tailored to develop relational or conceptual understandings because they are designed so that students can connect intuitive understandings to new mathematical concepts. For example a worthwhile linearization task might begin with problems that require students to grapple with the notion of local linearity, and then may be constructed so that students use that notion to develop relational understanding associated with the procedure of linear approximation.

Peter attempted to develop personal relational understanding so he could later help students develop relational understandings. He would initially start the process of developing personal relational understanding by skimming the textbook and writing down his own understandings associated with the ideas in the textbook. However, Peter only had the opportunity to relearn or refamiliarize himself with calculus concepts by looking at theorems, definitions and examples. Arguably, learning through tasks would have better provided Peter with an opportunity to develop relational understanding from his own personal intuitions and understandings. However Peter had to utilize an alternative route in attempting to obtain relational understanding. As he read his calculus textbook he was provided with definitions, theorems, and examples and then was forced to consider how his personal understandings of mathematics connected to the formalisms of the book. This type of thinking required Peter to be regularly thinking “backwards” while identifying relational ideas. Instead of being able to use his natural intuitions to solve a task and thus develop conceptual mathematical understandings Peter was looking at formulas and then had to figure out why those formulas made sense to him. For example in planning the linearization lesson Peter was only provided with the linearization equation and instrumental explanations associated with the equation. Peter then had difficulty identifying why the equation worked because he did not have the opportunity to develop the conceptual underpinnings associated with the linearization equation before being introduced to the equation. Lisa also seemed to have this problem but it was not as pronounced. She would initially spend time reviewing her procedurally centered learning targets which were based on her traditional textbook, and would review the procedures students would be introduced to when looking at traditional websites. She would then also work “backwards” as she considered the relational underpinnings of a rule or procedure she had read in the textbook, online, or in the learning targets. In contrast Susan rarely spent time deciphering the important relational mathematical ideas or grappling with mathematics, but was able to engage in an efficient process of validating what the important mathematical ideas were in a section as she read through a provided mathematical task. Working through the mathematical tasks in her curriculum provided her with a valuable opportunity to refamiliarize herself with course content from her own personal mathematical conceptions and enabled her to easily identify what the important relational mathematical ideas were in a lesson. Thus Peter and Lisa were both required to engage in a less efficient “backwards” process of identifying important relational ideas while Susan was able to engage in an efficient method for identifying important relational ideas.

Oriented toward procedures while lesson planning. It was difficult and time consuming for Peter and Lisa to construct task-based lessons because the traditional curriculum they were using oriented them toward mathematical procedures during the planning process. In the case of Peter he was always trying to construct a lesson that focused on building relational understanding (Skemp, 1976). However the titles of the sections in the book, the homework problems for students, and the examples Peter used to familiarize himself with course content were all procedurally centered and would thus lead toward an instrumental understanding (Skemp, 1976). The homework problems given at the end of the section and the example problems rarely asked students to describe relationships and explain their thinking but usually required procedures. Thus in utilizing these resources Peter naturally oriented himself during planning sessions towards procedures, meaning, that Peter's focus was still heavily centered on procedures when planning lessons even though he was concerned with constructing lessons that developed relational understanding (Skemp, 1976). For example he often reviewed what the students would be asked to do for homework when deciding whether he covered all the course material. This occurred with Lisa as well. Lisa's lessons were not specifically focused on building understanding of an idea or concept within the Utah Core Standards but she was oriented toward procedurally centered learning targets and then would participate in the process of constructing/finding task-based lessons that respected those learning targets and developed the conceptual underpinnings associated with the procedures within the learning targets. In contrast Susan's curriculum constantly caused her to consider worthwhile mathematical tasks that were constructed to develop understanding centered on relationships between mathematical ideas and students' natural intuitions. These tasks were not procedurally centered. For example Susan's task on concavity required students to consider how the rate of change between two quantities changes, rather than focusing on a procedure for identifying concavity that is based on the shape of a graph or some other type of ad hoc rule. Thus in using task-based lessons Susan felt that she needed to add some procedural lessons for her students throughout the course of the school year.

It is important to note that I am not stating that Lisa's and Peter's lesson plans were inadequate or too procedurally focused. I am simply stating that when teachers seek to construct conceptually oriented lessons but use textbooks that orient them toward procedures, difficulties arise. For example after finding the Ferris wheel task Lisa had a difficult time determining how to use the task because should could not figure out how certain procedures could be tied back to a Ferris wheel context. For instance, she could not decide how to construct a problem where the period would change on a Ferris wheel because a Ferris wheel always rotates 360 degrees. Being oriented toward procedures makes it difficult to focus on mathematical relationships. Thus one problem reform oriented teachers might face when utilizing a traditional curriculum is that they are naturally oriented toward doing procedures, and will likely experience the difficulties associated with time constraints as they focuses on the conceptual underpinnings associated with these procedures.

Obtaining Worthwhile Mathematical Tasks

Since Peter and Lisa were reform oriented teachers they were concerned with utilizing worthwhile mathematical tasks in their lessons. However, Peter and Lisa experienced a variety of problems as they attempted to find or construct mathematical tasks. Thus the purpose of this section is to identify problems that were associated with finding or constructing mathematical tasks.

Finding mathematical tasks. Lisa often stated that she had difficulty finding mathematical tasks. The materials she would find online were often highly procedural in nature.

Therefore one problem that seems to exist is that there seems to be a lack of public resources for teachers to draw from when creating task-based lessons, or there seems to be a lack of knowledge of where to find task-based resources, or it is difficult to find tasks that match a specific learning target. However, it is important to remember that Lisa still generally had successes in finding and creating mathematical tasks because she had meaningful task-based student teaching experiences to reflect on when considering a task for a lesson. Thus another related problem that may exist for some teachers is that they have not been provided with enough task-based experiences before entering the classroom. This may have been a problem for Peter. He would often state that he chose to do a "guided lecture" instead of a mathematical task because he could not think of a context or conceptual approach to start from when planning a lesson. It's possible that he would have not had as much trouble finding/constructing mathematical tasks if he could draw on past experiences learning or teaching calculus in a task-based environment. For Lisa it seemed to be easier to find tasks than for Peter because she had task-based experiences as a student teacher and in professional development that were related to the course content she was teaching.

In contrasting Peter and Lisa's teacher education experiences it is important to remember that Peter is teaching a calculus course while Lisa is teaching a standard secondary mathematics course. It is not likely that Lisa and Peter had significantly different teacher education experiences, especially considering that they graduated at the same time from BYU with the same college major. It may just be that the teacher education program at BYU does not focus on exposing students to calculus tasks since novice teachers are unlikely to teach calculus in their first few years of teaching. However, this fact does not make Peter's lack of reflection experiences less problematic. His experiences in comparison with Lisa's clearly indicate the value of having meaningful teacher education and student teaching experiences that expose teachers to variety of worthwhile mathematical tasks that align with the course content they will be teaching. Thus I would once again argue that it possible that many reform oriented novice teachers experience serious disadvantages due to not being able to participate in teacher education or student teaching experiences that were task centered.

Constructing mathematical tasks. In providing reform oriented teachers with traditional curricula to teach lessons, teachers are implicitly being asked to be curriculum developers. Both Lisa and Peter were constantly engaged in the process of mapping learning trajectories and developing mathematical tasks within a lesson, while Susan was not as focused in this practice because she was simply using the task she had used last year that was based upon the Pathways curriculum. In observing Peter and Lisa, I was able to identify certain problems teachers experience when constructing mathematical tasks.

Lisa often stated that she preferred finding mathematical tasks to creating mathematical tasks because it was “easier”. When she found mathematical tasks she could take the role of a student when examining the task, but when creating a mathematical task she would sometimes construct questions that did not make sense to students. Therefore a problem that seems to exist for reform minded teachers is that it is difficult to create tasks because it is genuinely difficult to anticipate student thinking. For example when Peter was planning lessons his anticipation of student thinking and general lesson construction was often a product of his own conceptions and concerns; however, it was very possible that Peter’s concerns were entirely different than his students’ concerns. In contrast reform curricula are often based on research about student conceptions, and often experience trial implementations before being given to teachers. Therefore one problem novice reform minded teachers’ might experience when constructing mathematical tasks is that they may have difficulties genuinely anticipating student reactions to their created learning trajectories within a lesson because they may have no support for doing so. This may result in the unsuccessful implementation of mathematical tasks.

Other Problems Observed

I have described the problems Peter and Lisa faced as they worked with the procedural aspects of a traditional curriculum and as they attempted to obtain a mathematical task to use in their lessons. The problems which occurred during these practices seemed to represent the most significant problems faced by reform oriented teachers utilizing a traditional curriculum. However, in looking at the data I was able to identity some other, perhaps less pertinent, problematic aspects of utilizing a traditional curriculum.

Lack of misconception knowledge. Utilizing a traditional textbook to identify important ideas seemed to be problematic because there were few instances in which the textbook identified common misconceptions students (or teachers) might have when exploring mathematical ideas. This caused Peter to experience certain common misconceptions without aid when grappling with conceptual ideas in planning sessions. For instance in one lesson planning session Peter struggled to identify the difference between definite and indefinite integrals, and had a difficult time understanding the connection between Riemann Sums and definite integration, both of which are common student misconceptions. It is likely that the textbook would have been more helpful if it had been written to help remedy these misconceptions rather than just articulate the associated mathematics (Ball & Cohen, 1996). In contrast reform curricula are often research based, field tested, and therefore are often built to remedy misconceptions. Specifically, the Pathways curriculum used by Susan had teacher notes which included specific examples of student misconceptions. Susan stated she utilized these teacher notes during her first year of teaching. Susan's experience with these teacher notes likely contributed to her ability to anticipate student thinking effectively and map learning trajectories within a specific question. Thus a likely problem that reform minded teachers face when utilizing a traditional curriculum is that they are not provided with possible misconceptions and pitfalls students face with grappling with certain ideas, and that could lead to more time consuming planning sessions as they experience and explore those pitfalls during their planning time.

Assumptions about Textbooks. Peter and Lisa both negatively referenced their textbook and often made assumptions about what is in the textbook without specifically examining the textbook. For example, Lisa often only read the examples in the textbook and seemed to assume that the other portions of the textbook did not have additional insights to offer her or her students as she constructed a task-based lesson. It is important that in this case Lisa may be entirely accurate or her conjectures may be adequately based on prior experience. However the negative attitude exhibited by Lisa and Peter did not demonstrate a healthy relationship with the textbook, and I feel that problems could result because of this poor relationship. Thus I would conjecture that reform oriented teachers might be making the planning of task-based instruction more problematic by making assumptions about the potential helpfulness of a textbook or resource without evidence. This conjecture would need to be examined and validated with further research.

Conclusion

I have identified reasons why reform oriented teachers utilizing a traditional curriculum may take more time to plan task-based lessons. However there are consequences to not having enough time to properly plan a task-based lesson. In a sense, the immediate problems faced by Lisa and Peter when planning lessons created additional problems. For instance, in the observations of lesson planning, I saw that Susan was able to spend a significant amount of time anticipating student thinking and creating a learning trajectory within a specific question. This seemed to be a useful practice in helping her prepare to address specific student misconceptions. In contrast Peter and Lisa rarely spent time mapping learning trajectories within a specific question. Not creating these learning trajectories could prevent these teachers from having positive task-based experiences in the classroom even when they might have found or constructed a worthwhile mathematical task. Not having time to create learning trajectories within a question is just one example of a practice teachers may not be able to engage in as a result of the other problems identified above. I would conjecture that there are many of these other types of problems for specific teachers who are trying to implement reform instruction while utilizing a traditional textbook.

Chapter 7: Conclusion

The purpose of this study was to identify some of the problems novice reform oriented teachers might face when utilizing a traditional curriculum. A framework for examining teacher's interactions with curricula was constructed utilizing the frameworks of Sherin and Drake (2009) and Breyfogle et al. (2010). Specifically I identified how three teachers read, evaluated, and adapted a curriculum and engaged in curricular reasoning as they planned mathematics lessons. I was then able to make certain conjectures about the problems novice reform oriented teachers face when utilizing a traditional curriculum. The purpose of the following section is to discuss certain implications and limitations of my research.

Implications

In the following section I would like to describe the implications associated with each problem identified in the results section above and discuss some possible implications of the framework I constructed to characterize how teachers interact with curricula.

Textbooks focus on procedures.

Thinking "backwards" to identify conceptual ideas. One problem identified was that Peter and Lisa would be compelled to engage in "backwards" thinking as they attempted to identify the conceptual ideas embedded in a mathematics lesson. It was observed that Lisa and Peter often had to develop relational understanding from procedures rather than being given the opportunity to develop relational understanding from tasks and personal understandings. This process seemed to be less efficient. Ideally, this problem would be remedied by providing teacher notes in the textbooks that focus teachers on essential mathematical ideas (Ball & Cohen, 1996; Kim & Atanga, 2014; Remillard & Reinke, 2012). Specifically, researchers have studied which types of teacher-help texts better prepare teachers for lesson enactment (Remillard &

Reinke, 2012). However, given unchanging nature of traditional curriculum materials it is unlikely that traditional textbooks will adopt such practices. Therefore I believe other less drastic changes need to be considered as well.

In reflecting on the data it seems that one way a traditional curriculum could better help teachers gain access to conceptual ideas in a textbook is by providing proofs in the textbook that are more oriented toward building relational understanding (Skemp, 1976). Peter and Lisa both referenced the importance of proofs when attempting to access underlying concepts. In fact Peter was always significantly frustrated when a proof was not provided in association with a theorem. However in observing Lisa and Peter I never saw an instance in which consulting a textbook proof seemed to be exceptionally helpful. It could be that the teachers only valued proofs because they may have been the only source of conceptual information to which the teachers had access in the traditional textbooks. Thus while proofs may be a valuable mechanism in helping teachers' access a relational understanding of concepts associated with mathematical procedures and ideas I believe that their formal and often unintuitive nature could be improved upon. Thus one possible area of future research would be to explore how teachers make sense of mathematical proofs and identify what aspects of mathematical proofs in textbooks can be improved upon to better provide teachers with access to relational understanding within a textbook section.

Oriented toward procedures while lesson planning. It was found as the teachers utilized a traditional curriculum they were naturally oriented toward mathematical procedures during the planning process in comparison to being oriented to conceptual relationships. In the data this problem originated from the procedural nature of Peter's textbook and the procedural nature of Lisa's learning targets. In remedying these issues it may be difficult to change the nature of a traditional textbook but it is possible for districts and mathematics departments who are concerned with implementing quality task-based instruction to ensure that school district learning targets are conceptually focused rather than procedurally focused. This may help teachers find and construct mathematical tasks that better fit the nature of provided learning targets.

Obtaining worthwhile mathematical tasks.

Finding mathematical tasks. Three potential problems were identified that were associated with the issues reform oriented teachers experience when attempting to find worthwhile mathematical tasks. First, it was found that there may not be enough public resources to help teachers who are trying to find worthwhile mathematical tasks. If this is true then I believe that a focus in the mathematics education community should be to promote the public availability of worthwhile mathematical tasks. It may also be helpful for NCTM or state education offices' to create a bank of mathematical tasks that have been created and refined by teachers so that all teachers do not constantly have to create their own tasks.

A second potential problem identified is that there may just be a lack of knowledge amongst teachers of where to find mathematical tasks. Once again the acquisition of this knowledge could be a focus of NCTM and state offices. District leaders, math coaches, and department chairs could be influential in helping teachers acquire this type of knowledge as well, but I would conjecture that it is unlikely that the leaders in districts that do not utilize a reform curriculum are reform minded.

The third potential problem identified above was that it may be possible that teachers do not have enough past teacher education task-based experiences to draw upon when attempting to plan task-based lessons. Thus a focus of teacher education might be to continually provide teachers with mathematical tasks or ways of finding them. Preservice teachers should have the opportunity to work with teachers who utilize tasks while doing their student teaching and should be exposed to many tasks while being trained. In addition inservice teachers should be regularly exposed to useful tasks during professional development. It is likely that professional development would need to originate from state offices due to the unlikelihood that a teacher working in a school district that mandates the use of a traditional curriculum would have the resources or desire to support reform instruction. I would also claim that it would be helpful for perspective mathematics teachers to learn in a task-based manner in their mathematics classes as well as in their education classes so that they are exposed to more mathematical tasks and become more familiar with task-based learning. It is important to note that I realize that these goals for mathematics teacher education are far from being realized. However, I offer these suggestions based on my data of how perservice teacher education could improve to help teachers who are trying to implement tasks in their teaching.

Constructing mathematical tasks. When teachers attempt to create task-based lessons utilizing a traditional curriculum they are implicitly being asked to be curriculum developers. However, there are negative consequences which may occur when teachers take the role of a curriculum developer. For example, Lisa and Peter encountered problems constructing task-based lessons because they had difficulties genuinely anticipating student thinking. This problem does not seem to be limited to the experiences of Peter and Lisa. Ball & Cohen (1996) also found that teachers have issues constructing a curriculum because they have little knowledge about research on student conceptions. Therefore they state that curricula developers should not only develop curricula that focuses on teaching students, but curriculum developers should develop curricula that also teaches teachers about student conceptions and misconceptions with regards to specific topics. Ball and Cohen also state that this means that more research needs to be carried out in the education community about the nature of student conceptions so curriculum developers can use that knowledge to construct better curricula.

I believe my study validates the finding findings and claims of Ball and Cohen (1996). Novice teachers don't have the background to be curriculum developers. Their background is in enacting curriculum not writing curriculum. It seems that Peter and Lisa would have been better prepared to enact reform based instruction if they had access to research on student thinking. However I believe that curricula are not the only source teachers can use to access student misconceptions. Practitioner mathematics education journals continually focus on sharing research about student mathematical conceptions, and I believe that teachers could also access useful student conception information within these journals. Thus I believe that school district leaders, state education leaders, and professional developers should focus on utilizing these journals a source for teacher education.

Other problems observed.

Lack of misconception knowledge. Peter experienced many of the common misconceptions associated with certain calculus concepts when engaged in the process of identifying relational ideas. Once again it would be useful for traditional textbooks to provide information about common student misconceptions (Ball & Cohen, 1996). This practice would not only help teachers enact worthwhile mathematical tasks but may also help teachers avoid the time consuming process of experiencing certain common mathematical pitfalls during planning time, and could also help teachers better genuinely anticipate student thinking.

Assumptions about textbooks. The last problem I found that reform oriented teachers might experience when utilizing a traditional curriculum is that they may not be completely taking advantage of what a traditional textbook has to offer those trying to implement reform based lessons. Traditional textbooks can still be valuable resources even though they do not adequately support the teacher interested in implementing task-based lessons. They are mathematically precise, represent the voice of the mathematical community, and may offer a logically sound sequencing of mathematical ideas. Thus I believe it is important that in teacher education courses that professors and professional developers focus on teaching the value of productively utilizing provided resources. Sometimes the heat of the battle between traditional and reform mathematics instruction causes some to forget the worthwhile aspects of a particular curriculum.

The framework of curricula interaction. I have listed various implications and actions that could be taken as a result of the identification of problems that reform oriented novice teachers' face when utilizing a traditional curriculum. However, one implicit result of my study was that I developed a viable mechanism of characterizing how teachers interact with textbooks as they plan mathematics lessons. Specifically, I used the read, evaluate, and adapt framework of Sherin and Drake (2009) and then meshed it with the framework of curricular reasoning (Breyfogle et al. 2010), and I specifically used curricular reasoning to describe how teachers evaluated and adapted curricula. The combining of these two frameworks helped me construct adequate descriptions of teachers' curricula interaction, which were more detailed than studies which have been carried out previously. I was also able to contribute to the curricular reasoning framework byspecfically characterizing the process of mapping learning trajectories as a product of mapping across lessons, within lessons, and within questions. However it is important to note that I would argue that this conglomerated framework of textbook interaction is not a perfect framework for depicting *all* of practices teachers engage in as they utilize a textbook to construct a planned curriculum. Further work need to be carried out to develop a robust framework that encompasses all of the practices and processes a teacher engages in when constructing a planned curriculum. For example there were times in which teachers were personally doing mathematics problems when planning a lesson in order to familiarize themselves with course content. It seems like this practice should be considered a part of a teacher's curricular reasoning because the teacher is engaged in meaningful thinking with curricular materials. However, it is difficult to categorize this practice as a practice of analyzing from the learners perspective, reflection, or mapping learning trajectories because when teachers are working out mathematics problems on their own their thinking is not always student centered or pedagogically centered. Thus I believe that future study and focused research would need to be conducted to further develop a detailed framework that accurately depicts all of the processes and practices a teacher engages in when utilizing a textbook to construct a planned curriculum.

Although the framework I utilized was not perfect it was useful, and it may be worthwhile to consider using this framework and the associated analysis as a tool for helping teachers self-reflect on their planning practices. Teachers often receive feedback on their lessons as professionals observe them while they teach, and I would argue that teachers naturally reflect on lesson enactment. However teachers' rarely receive feedback on their planning practices. This framework and its associated analysis may be a useful tool for the teacher or preservice teacher who is concerned with reflecting and obtaining feedback on their planning practices and textbook use.

Limitations

It is important to remember that this study is a case study. I was only able to observe three teachers plan lessons and it would be necessary to conduct further research to learn how prevalent the problems identified in this research are. It is also important to note that in only observing one teacher utilize a reform curriculum I was not able to observe how a reform oriented teacher initially interacts with the mathematics found in a reform curriculum. Susan stated that she spent more time preparing lessons in her first year of teaching. It would be worthwhile to compare the struggles of teachers using both types of curricula who are experiencing a curriculum for the first time. However it is worth noting that both Susan and Lisa were in their second year of teaching with the same curriculum used in their first year of teaching, and that Lisa struggled a great deal more than Susan to identify and articulate underlying mathematical relationships. Thus while it may have been valuable to compare teachers who were both teaching a course for their first time it was still valuable to observe how much one teacher struggled in comparison to another within their second year of teaching.

It is also worth noting that the methods I used to conduct this study were difficult to carry out. Teachers were initially uncomfortable planning in front of a researcher. I would guess that this discomfort arose from the stress of not knowing how to create task-based lessons while utilizing a traditional curriculum, and arose from the general vulnerability associated with doing, exploring, and refamiliarizing one's self with mathematical ideas in front of another person. In conducting this type of research, care needs to be taken to ensure the participants feel comfortable and are willing to express their honest concerns when planning lessons.

Finally, it is essential to note that my study only characterizes the problems that occur as teachers construct the planned curriculum and not as they produce the enacted curriculum. Teachers are always making in the moment decisions when teaching and it is important to realize that there may be different problems reform oriented teachers experience as they try to enact lessons while using a traditional curriculum. A future area of research would be to specifically discover how a traditional curriculum effects the implementation of reform oriented lessons.

Conclusion

In conclusion in characterizing how reform oriented teachers read, evaluate, adapt, and engage in curricular reasoning I was able to identify some problems reform oriented novice teachers face when planning task-based lessons that were not seen by a teacher using a reform curriculum. If these problems are prevalent, actions from curriculum developers, teacher educations, district leaders, state leaders, national organizations, and teachers should be taken to remedy these problems. These actions will need to respect and consider the way in which a curriculum is utilized in planning sessions and in curricular enactment.

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Appendix

Interview Questions

1. How has your education at BYU influenced your teaching practice?
2. Would you say you are a strong proponent of task-based instruction? Why or why not?
3. How do you think students learn mathematics?
4. What is a worthwhile mathematical task?
5. Do you implement worthwhile mathematical tasks in your teaching practice? Why or why not?
6. Do you like the textbook you are using? Why or why not?
7. What would you change about the textbook you are using?
8. How would you describe how you typically plan for a day's lesson?
9. What are your greatest concerns when you a planning a lesson?
10. What would you say is the role the textbook has in your planning?
11. What are your greatest challenges in using the textbook to plan your lessons?