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Supporting an Understanding of Mathematics Teacher Educators:Identifying Shared Beliefs and Ways of Enacting Their Craft

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ABSTRACT

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For many preservice elementary and secondary teachers, the first time they encounter the notion that mathematics can be taught in ways other than what they experienced is in their teacher preparation program. Therefore, the potential impact of the mathematics teacher educator on the understanding and development of preservice teachers is immense, especially as that impact is considered in connection to the various reform efforts related to mathematics education that have spread across the country for the past three decades. Given the scarcity of systematic research focused on mathematics teacher educators, the author sought to understand defining characteristics of that role. He looked specifically for beliefs that mathematics teacher educators have in common as well as shared ways of enacting their craft by interviewing 16 university professors from around the United States who support the learning and development of preservice and inservice mathematics teachers. The beliefs that emerged from this study related to mathematics teaching, the learning of mathematics teaching, and the teaching of mathematics teaching. Potential communities of practice were also identified, through which the enactment of mathematics teacher educators' craft could be better understood. The specific beliefs and ways of interacting are discussed in terms of how they can support development and reflection among current and prospective mathematics teacher educators, as well as how they can support continued systematic research of mathematics teacher education.

Keywords: mathematics teacher education, mathematics education, beliefs, community of practice

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Chapter 1: Introduction

Since the early 1980s, concern has been expressed that mathematics education in the United States is not encouraging students to reason mathematically (e.g., Boaler & Staples, 2008; Carpenter, Corbitt, Kepner, Linquist, & Reyes, 1981; Stigler & Hiebert, 2009). Recognizing these concerns, the National Council of Teachers of Mathematics (NCTM) called for a reform in mathematics education and released various standards (1989, 1991, 1995, 2000) that outlined a vision of mathematics pedagogy that valued mathematics as a discipline, while also valuing ways in which students think about mathematics (Ball, 2001). This approach to teaching, however, required teachers to teach in a way that was very different from how many of them had been taught (Schifter, 2001). The reform movement comprises the efforts to implement this vision.

As this reform has moved forward over the past three decades, it has prompted numerous research studies investigating various facets of mathematics education. While many studies consider how students think, develop, and learn (e.g., Carpenter, Franke, & Levi, 2003), others have focused on alternative methods of teaching (e.g., Simon, 1995). As a result, contemporary literature offers many insights regarding the work of students and teachers in a reform-based classroom. Building upon its standards documents and the research literature, the NCTM has operationalized its vision of reform-based teaching and learning in its recent publication *Principles to Actions: Ensuring the Mathematical Success for All* (2014). In this text NCTM describes “eight practices that provide a framework for strengthening the teaching and learning of mathematics” (NCTM, 2014, p. 9), five essential elements that should drive school mathematics programs, and a set of specific actions that can be taken by administrators, specialists, and teachers to promote teacher learning. In this way, systematic research surrounding mathematics education has supported a focused vision of how mathematics should be taught. One potential research topic in the context of the reform movement, however, has been largely ignored—research about mathematics teacher educators (MTEs; Chauvot, 2009). Therefore, little is known about the nature of MTEs' contribution to mathematics education reform or the specific beliefs, knowledge, and practices that empower MTEs to contribute fully to the field of mathematics education.

In light of these challenges the purpose of this study was to seek to understand more clearly who MTEs are. In seeking to define MTEs, this study investigated characteristics shared by MTEs, specifically attending to beliefs they held as well as ways they enacted their craft. I used the lens of community of practice (COP) as a way of gathering and interpreting data that identified, in a holistic way, the defining attributes of MTEs. Those data were also used to draw inferences regarding the underlying beliefs about mathematics teacher education held by MTEs. Identifying these beliefs offered a clearer definition of who MTEs are.

With this overall purpose in mind, the goals of this chapter are threefold. First, I share personal experiences that have led me to study MTEs and why I find defining MTEs to be a valuable consideration. Second, I discuss the importance of researching MTEs. Finally, I present the questions that guided this research and discuss challenges surrounding attempts to understand the role and characteristics of MTEs.

Personal Experience

While finishing an elementary mathematics license endorsement during my third year of teaching, I was asked to facilitate professional development sessions for the district in which I worked. I had very little experience teaching adults and teaching the teaching of mathematics, but I was excited about the prospect of trying something new. I was ready to sit down with someone at the district office who could act as my mentor, someone to give me guidance similar to what I received as a new teacher. What I found, however, was that there was no one to fill that role. This is not to say that no support was offered. There were always individuals with whom to collaborate or from whom to seek advice, but these individuals were always teachers who had been plucked out of their classroom just as I had been. Some had read a book or two about educating adults, but not one of us was truly expert in that area. The message I received became quite clear: “We like how you teach children, so somehow, we are not necessarily sure how, but somehow, help other teachers to do the same.” Such was my introduction into being a mathematics teacher educator.

For a few years I facilitated mathematics pedagogy professional development courses, and I assumed that being invited to return was a sign that I was doing things well. Though I felt confident in my position as a mathematics teacher, deep down I knew that as an MTE I was grasping at straws. I continually crashed up against my own erroneous assumptions (e.g., “This concept is easy, adults already know this, so it will not take more than a few minutes of review”), or surprised myself by how often I employed techniques with adults that I avoided when teaching children (e.g., direct instruction or lecturing). Being an elementary school teacher, even a successful one, was not sufficient preparation to being a successful MTE (Chauvot, 2009; Korthagen, Loughran, & Lunenberg, 2005; Tzur, 2001). I found myself in a strange position of loving the enterprise of teacher education, but still having no idea what it was or how I fit into that pursuit. It was largely because of this uncertainty that I pursued a master’s and then a doctoral degree. If I were going to continue to engage as an MTE, I wanted to know what it meant to be a MTE. I wanted to know if I believed and did the same things that other MTEs believed or did. I had already navigated these questions about classroom teaching. I wanted the same confidence as an MTE that I felt as a mathematics teacher.

Upon entering my doctoral program I was given ample opportunity to teach both preservice and inservice teachers, conduct research, and collaborate with expert MTEs. Along with teaching undergraduate methods courses, one of the major projects on which I worked involved a 2-year professional development program for all kindergarten through ninth-grade teachers in a local school district. This project gave me many occasions to observe and converse with those expert MTEs and reflect on who they are and what they do, giving me insights into their beliefs as well as the day-to-day nature of carrying out their craft. As a teacher of mathematics teachers, I was growing.

My classes and my mentors, however, regularly reminded me of the importance of doing research alongside the teaching. These reminders invited me to redefine what it meant to be an MTE. At the same time as I gained confidence in teaching teachers, I started to believe that part of being an MTE was conducting research. Once again, I struggled to navigate my questions regarding who MTEs are, what they do, and what beliefs they hold. Being a researcher seemingly involved an entirely new set of practices and beliefs that I had not considered. My lack of development led to various questions. Was I a teacher of teachers or a researcher? Which role would I prefer in situations where I had a choice?

During the first year of my doctoral program, as I was struggling with these questions, I attended the annual conference of the Association of Mathematics Teacher Educators (AMTE) in Fort Worth, Texas. I had had very few experiences that have been as professionally enriching as that first AMTE conference. I saw, for the first time, a large community of individuals engaged in the very practice I was making my own. I saw how natural the interplay of the teaching of teaching and researching was, and that I did not have to choose to emphasize one over the other. I was beginning to form a picture of what MTEs are and what they believe. For the first time I caught a glimpse of MTEs as a functioning global community, dedicated to what I was trying to become.

Since that initial conference experience, I have focused my efforts on engaging with, contributing to, and simply becoming a part of this community of MTEs. Despite my efforts, I remain unsure as to whether or not I am a full-fledged member of this community or if I remain on the periphery, legitimately engaging in actions of the community but not yet a part of it. In becoming an elementary school teacher, the path to membership in the community was clear. I took a series of courses, earned a degree, received a certification, and then gained employment. I never questioned if I legitimately belonged because I knew I had taken the requisite steps. Consequently, I wondered what it meant it be an MTE and what the defining beliefs and practices that characterized being an MTE. I also knew that apart from satisfying my own curiosity, systematic and ongoing research focusing on communities of MTEs has the potential of clarifying the role MTEs play in the aforementioned reform movement and strengthening the influence they have.

Researching MTEs as a Means of Improving Mathematics Education

As previously mentioned, the focus in research on students and teachers emphasizes the value of understanding those roles in reforming mathematics education, but the question remains: What role should MTEs play in improving mathematics education? While addressing this specific need, Jacobs (2003) argued in an AMTE lecture, a yearly lecture that was named in her honor, that

> the mathematics education of all . . . needs to prepare functionally literate individuals who understand (both quantitatively and spatially) the mathematical world in which they live and can use mathematics purposefully. . . .They need to know how to solve a

> problem, what a reasonable solution looks like, and how to use the ‘cranked out’ results to a problem. If there is any group of individuals who can lead and facilitate such a dramatic change it is Mathematics-Teacher-Educators. (p. 2)

In this one statement, Jacobs made clear the responsibility of MTEs in mathematics education reform. If MTEs are not simply participants in the reform movement, but leaders, understanding through research who they are by understanding what they believe, and what they do will provide many important insights.

Just as research is conducted about teachers as a means for guiding the improvement of teaching, researching MTEs can provide insights into improving mathematics teacher education. Though teacher education too often fails to take advantage of this parallel research tradition, that of researching teaching (Grossman & McDonald, 2008), recognizing connections between researching mathematics education and mathematics teacher education can help drive research about MTEs. For example, understanding the eight research-based *Mathematics Teaching Practices* from NCTM (2014), one may identify ways in which MTEs who are well versed in the literature may view mathematics teaching as well as potential practices they value when teaching mathematics teachers. Therefore, parallels from mathematics teaching that can clarify or direct research regarding MTEs and how MTEs can lead the way in mathematics education reform are raised throughout this study.

Defining Mathematics Teacher Educators

As MTEs have a critical role in understanding and contributing to the reform movement in mathematics education, it becomes imperative that who they are, what they do, and what they believe are well understood. Defining MTEs through systematic research lays a foundation for further study that will more fully empower MTEs in their leadership roles. As teacher educators, MTEs included, are often a neglected group in research studies (Korthagen et al., 2005), ambiguity persists regarding who they are. Lanier and Litle (1986) noted, ‘‘Teachers of teachers . . . are systematically overlooked in studies of teacher education. Even researchers are not exactly sure of who they are” (p. 528). If researchers cannot adequately identify who teacher educators are, then the target population of any teacher education study becomes blurred.

This ambiguity is apparent when one simply considers the label of MTE. Sztajn, Ball, and McMahon (2006) noted that “teacher developers comprise a vast array of professionals—teacher mentors, district leaders, mathematicians, faculty in schools of education, to name a few” (p. 150). Cochran-Smith (2003) extended the label of MTE to university professors who teach prospective teachers but do not view themselves as MTEs or contribute to the field of mathematics teacher education. It could also be argued that anyone who teaches, coaches, or mentors mathematics teachers will fall under this label (Strutchens, 2012).

The label of MTE can be applied to a broad range of individuals, which creates three sources of confusion regarding the label. First, there are those who support the learning and development of preservice or inservice mathematics teachers, as MTEs do, but are not formally recognized as such. These MTEs may include classroom teachers or district personnel who have opportunities to teach teachers, but may not do so as a full-time position. They may not view themselves as MTEs even when the work they do is similar to that done by fulltime MTEs. Second, there are individuals who are recognized as MTEs but do vastly different work from one another. An example of this category may be university professors, formally carrying the label of MTE, who have taken opposite stances on reform. In this way, one MTE may draw upon *Principles to Actions* and other reform-oriented documents to guide his/her work with preservice and inservice mathematics teachers, while another MTE may reject such documents outright, espousing more traditional approaches. Though the label they carry is the same, the work they do will be very different. Finally, the third source of confusion for the label would be those recognized as MTEs who work toward similar goals, but adopt different stances as to how to accomplish it. These MTEs may include university professors, some of whom work for reform, but one focuses on social justice while the other focuses on international comparisons. Their work may dovetail in various ways, but ultimately they will enact their craft in different ways. If researchers studied the first MTE, they might conclude that a typical MTE promotes reform in mathematics education, but does so through the lens of social justice. Such a conclusion would miss the potential complexity inherent in the label. As long as ambiguity persists in how the label of MTE is used, studying or interpreting studies that have been conducted involving this target population is difficult. In order to better research MTEs, a more holistic definition of who they are must first be established.

Research Questions

The purpose of this study was to define who MTEs are. To address this purpose I asked one overarching question: What are defining characteristics shared among MTEs? As that question was complex and multifaceted, I asked two subquestions meant to support answering the overarching question.

1. What beliefs about mathematics teacher education are commonly held by MTEs?
2. What are ways that MTEs commonly enact their craft?

Identifying Defining Characteristics of MTEs

In seeking even to define who an MTE is, there are various avenues one might take. There are many who argue that in seeking to define a group of individuals, one must look beyond a formal list of duties or practices that attempts to encapsulate all of the practices of that group.

In Gee's discussion of Discourses (the capital D is used to differentiate a group of individuals who have been enculturated within the same group from the same term used to denote basic communication, which receives the label *discourse*), Gee (2014) argued that

> to be a particular *who* and to pull off a particular *what* requires that we act, value, interact, and use language *in sync with* or *in coordination with* other people and with various objects ('props') in appropriate locations and at appropriate times. (p. 14)

He suggested, therefore, that to understand who MTEs are requires more than identifying *what* they do. He argued that the definition exists in the interaction that occurs among people of that group and within specific contexts.

Brown and Duguid (1996) further described the challenge of defining people by what they do as they noted that practice is frequently disconnected from learning and innovation. Learning is often given priority and considered a necessary precursor to practice. Such an approach, they argued, "obscure[s] intricacies of that practice" and that "without a clear understanding of those intricacies and the role they play, the practice itself cannot be well understood, engendered (through training), or enhanced (through innovation)" (p. 42). Thus, any approach to understanding MTEs that perpetuates the disconnected view of learning and practice will inevitably fail to capture an understanding of the intricacies of what it means to be part of that group. Defining MTEs, therefore, ought to involve the union of learning and practice, either by investigating how an individual becomes an MTE or even how experienced MTEs continue to learn and change their practice.

As interaction within a context and accounting for the interconnectedness of learning and practice are emphasized, trying to define MTEs by creating documents to outline specific protocols regarding what they should be or what they should do will offer few insights. As one comes to understand how those practitioners negotiate their day-to-day challenges and learn and enact their craft, one can describe more clearly who that practitioner is. For example, a text that explicitly describes what topics an MTE should teach in a methods course and how those topics should be taught may fail to capture the complexity of how an MTE may assess and respond to the needs of the students on any given day. Defining groups through lists of protocols creates systematic abstractions of the practice of a profession that may oversimplify the work that is actually carried out (Wenger, 1998b). This oversimplification results whenever one ignores how actual practice involves frequently negotiating abstract guidelines or protocols and situated demands (Brown & Duguid, 1996; Suchman, 1987; Wenger, 1998a). There is a parallel here that is common in mathematics education. Procedures for solving problems have been formalized into algorithms that students can learn and replicate. As one comes to understand the five strands of mathematical proficiency (National Research Council, 2001), that person recognizes that a student's understanding is manifest not just in adherence to a prescribed procedure, but in the situated ability to reason, adapt, and draw on conceptual understanding. Just as observing a student's use of an algorithm is insufficient in fully assessing mathematical understanding, defining MTEs by formalized practices is insufficient in fully understanding who they are.

Summary

This study seeks to contribute to the developing understanding of mathematics teacher education by more clearly defining who MTEs are. Just as systematic research of mathematics education has culminated in a document such as *Principles to Actions,* which supports mathematics teachers in better understanding productive beliefs and how to carry out their craft, systematic and ongoing research into the field of mathematics teacher education has the potential to give similar direction to MTEs. By defining more clearly who MTEs are by identifying characteristics and beliefs they share, understandings can be gained that contribute to broader principles or actions that characterize how MTEs enact their craft.

Chapter 2: Theoretical Framework and Review of the Literature

Rather than seeking to understand who MTEs are via a listing of their various responsibilities, the purpose of this study was to understand them in holistic ways. The research questions pointed specifically to beliefs MTEs commonly hold as well as common ways MTEs enact their craft. In order to explain how I went about answering those questions, in this chapter I (a) describe a lens used to help guide the research process as well as interpret some of the results, (b) present a theoretical framework for investigating beliefs, and (c) explore related research literature.

Community of Practice Lens

Studying a group that was not clearly defined poses a very real challenge. *Communities of Practice* (Wenger, 1998b) provided a more holistic lens to support efforts in defining MTEs. The purpose of this study was not to prove that MTEs always function in communities or belong to one large community of practice. Rather than trying to prove that MTEs function in COPs, considering MTEs through the COP lens ensured that the questions asked and the analysis performed sought to understand MTEs, through the beliefs they held and the ways they enacted their craft, in a more holistic, situated manner.

Given the potential of this lens, it is not surprising that COPs have provided a useful framework in previous studies to investigate issues in mathematics education (Baek & Barab, 2005; Cobb, McClain, de Silva Lamberg, & Dean, 2003; Cwikla, 2007; Graven, 2004; McGraw, Arbaugh, Lynch, & Brown, 2003). Though other researchers have discussed communities in various ways or adopted COPs in their own studies, Wenger championed the notion of considering groups of individuals as communities of practice so his work was the primary source for describing attributes of a COP.

For a group of people to be identified as a COP, three traits must be present: (a) *mutual engagement*, (b) *a joint enterprise*, and (c) *a shared repertoire* (Wenger, 1998b). These three traits, along with the notion of boundaries, a secondary characteristic of COPs, characterize the lens that was used and are discussed in the following sections.

Mutual engagement. The first trait that must be present for a group to be a COP is that the members of the group must consistently work together. Members of the community must interact regularly and show interest in the work of others in order to be mutually engaged. Simply joining the same organization as or working in close proximity with other people is not enough to guarantee mutual engagement.

Examples of mutual engagement in mathematics teacher education are plentiful. When colleagues make a habit of stopping by one another's office to discuss a class they are teaching or to collaborate on a study, they are mutually engaged. When MTEs frequently exchange emails or contribute to the same blogs or online forum discussions, they are mutually engaged. This communication does not have to lead to any specific goal to be considered mutual engagement, but it should become routine. It is these varied methods of communication that keep communities working together, so investigating them among MTEs should shed light on how they are mutually engaged.

Wenger (1998b) noted that being mutually engaged does not mean that members are entirely homogeneous. It is the practice that brings them together, despite varying life experiences or worldviews that they bring to the practice. The diversity creates daily opportunities for the community to negotiate new meanings, but as long as they remain mutually engaged, the community stays intact.

As mutual engagement is a defining trait of a COP, understanding how MTEs work together and what brings them together provides a helpful way of thinking how they enact their craft and yields meaningful insights into interactions from which beliefs can be inferred. Understanding how they communicate, how they include members in practices that matter to the group, and what they do to stay cohesive will illuminate how members of the profession are mutually engaged.

Joint enterprise. While the first trait specifies that members of a COP work together, pursuing a joint enterprise means that they work together to achieve specific purposes. Without mutual engagement, a community cannot be working in a joint enterprise. Though this enterprise may include a stated goal, formal goals do not capture the full nature of joint enterprises. In fact, unstated goals may be more important to the group than stated goals are. It is this joint enterprise that defines what the group is about. For example, MTEs from a university may all be seeking to provide quality education for students, but if different members hold to different learning theories (e.g., behaviorism and constructivism) then the goal of quality education would be carried out in very different ways, suggesting that the different faculty members are not actually working toward the same goal. The stated goal may be the same for both groups, but the fact that they view that goal in different ways suggests that they will pursue it differently, resulting in different joint enterprises. In some cases, an unstated goal of a COP may be to subvert the work of another. Working together within the same university with the same overarching goal does not ensure that groups will interpret that goal the same way. The specific goals that each group works toward (e.g., providing a quality education that focuses on pedagogy which embraces constructivist philosophies) define their joint enterprise.

The overall goal that a COP works toward is often complex and its members negotiate it daily in a situated manner. The responses of different members on different days will vary. Their responses, whether similar or dissimilar, will always be interconnected and in that way, their enterprise is unique and specific to the community pursuing it. Such a view does not ignore the cultural, historical, and societal expectations placed on the community, but it recognizes that it is still the members of the community that have to make sense of those other expectations in light of their pursuits. On a day-to-day basis, it is the members of the community that negotiate what that enterprise is and how it will be conceived.

Having specific goals to work toward in a COP creates a situation in which individual members have different responsibilities to carry out, and are, therefore, accountable to one another. Referencing these relationships of accountability, Wenger (1998b) noted,

> These relations of accountability include what matters and what does not, what is important and why it is important, what to do and not to do, what to pay attention to and what to ignore, what to talk about and what to leave unsaid, what to justify and what to take for granted, what to display and what to withhold, when actions and artifacts are good enough and when they need improvement or refinement. (p. 81)

Thus, understanding how MTEs are accountable to one another is an important aspect of understanding their joint enterprise, and consequently, their functioning as a COP. It is because this joint enterprise encapsulates the goals of the community, understanding that joint enterprise yields invaluable insights into what that community believes and how members of that community enact their craft.

Shared repertoire. As members of a community are mutually engaged in a joint enterprise, they create shared ways of enacting their craft that facilitate their work. This shared enactment may include formalized routines, shared vocabulary, or products or documents that can be used by the entire community. Such creations constitute a shared repertoire and demonstrate a history of mutual engagement. The products or procedures may be formalized from extended practice, but they can also be used to further negotiate meaning. Because a shared repertoire emerges as communities are mutually engaged in a joint enterprise, that shared repertoire will be unique and specific to that community. Some communities may seek to adopt the formalized routines or vocabulary from another community, but the members will have to go through a process of making those things their own.

An example of how a shared repertoire can develop may occur if MTEs view the strategy of engaging teachers in lesson study as a valuable aspect of their practice. Routinely engaging in this activity may yield a recognized set of steps that other MTEs can follow. Once those steps have been established, Wenger would use the word *reified* to indicate how the process had been formalized enough to become an object with its own meaning, they can then be discussed, analyzed, updated, or manipulated in many other ways. In this way, the shared repertoire may be not only an artifact of a history of engagement, but it can serve to further the negotiation of meaning that occurs within a community.

Boundaries. While these three defining features effectively explain what happens within a community that leads to the creation of a COP, they do not give information regarding what happens at the periphery or just outside of that community. The notion of boundaries recognizes that in any COP, there are individuals and even other communities that engage in similar work but are not fully part of that community. How another community tries to adopt or make sense of the products or routines of another, as discussed in the previous section, how a COP interacts with outside individuals or other communities (these interactions are known as *boundary* *encounters*), or how an individual gains entry into a COP all give shape to the original COP and how it functions.

In any given community there are those who interact with the community without being full members of it. Wenger (1998b) recognized these interactions in his discussion of boundary encounters. Though his treatment of boundary encounters focused on the importance and challenge of different communities working together, the notion of boundaries also provides further means of defining a community. As long as a question remains as to who belongs to a specific community, membership can better be understood by identifying those who do not. Though many communities share similar goals and ways of interacting as MTEs, such as general teacher educators, mathematics teachers, or even mathematicians, belonging to a similar COP is not sufficient to being in an MTE COP.

In addition to providing definition for a COP, probing the boundaries also helps to clarify how an individual gains membership in the community. To describe ways in which individuals begin to gain admission to a community, Lave and Wenger (1991) introduced the concept of *legitimate peripheral participation*. Considering legitimate peripheral participation as a means of engaging with a COP hinges on two main ideas: peripherality and legitimacy (Wenger, 1998b). Peripherality suggests that an individual is allowed to engage in the practice of a community but without the full expectations or accountability, "including lessened intensity, lessened risk, special assistance, lessened cost of error, close supervision, or lessened production pressures" (Wenger, 1998b, p. 100).

Getting to engage in the practices of a community will not be helpful, however, if the members of the community do not view that individual as a legitimate future member. Though prospective teachers may be engaged in peripheral practices, they will have a difficulty learning and becoming part of the COP if the teachers of the community view them as incompetent. In a traditional master/apprentice relationship, the endorsement of the master is critical for the future acceptance of the apprentice as a legitimate member of the practice. Similarly, the belief from the community that an individual can grow to become a full member will influence the way the community interacts with that individual, ensuring further participation and learning (Wenger, 1998b). This view of legitimacy becomes crucial as the potential member makes mistakes or violates norms of the community. Without legitimacy, such missteps may be grounds for removal; with legitimacy, they become opportunities to learn. In this way, membership in a community is partially determined by the acceptance from the community. Individuals cannot be mutually engaged in a joint enterprise with members of a community that do not want to support them. Such decisions occur at the boundaries of COPs.

COP lens summary. As previously stated, the goal of this study was not to investigate the occurrences of COPs among MTEs or the formation and maintenance of COPs. By understanding the different components of COPs and utilizing them as a lens, I was able to seek for data that valued not only what MTEs did and believed, but also how they interacted on a day-to-day basis.

Theoretical Framework for Investigating Beliefs of MTEs

A strong benefit of studying MTEs through a COP lens is that it allows the investigation into beliefs they hold about mathematics teacher education in an authentic context. The COP lens supports an understanding of how different MTEs may hold the same label but still differ in their beliefs. In this way, the very data that informs how MTEs enact their craft can also be used to draw inferences regarding the beliefs that guide that community. The remainder of this section will (a) investigate the link between beliefs and COPs, (b) describe benefits of researching beliefs, (c) define how the construct of beliefs is conceptualized in this study, (d) disentangle this construct from similar constructs, (e) list questions to guide an investigation of beliefs, and finally, (f) establish a framework for identifying potentially relevant beliefs of MTEs.

Connecting beliefs and COPs. Though beliefs are not explicitly part of the defining features of COPs, Wenger (1998b) made a connection when he stated

> We wish to cause learning, to take charge of it, direct it, accelerate it, demand it, or even simply stop getting in the way of it. In any case, we want to do something about it. Therefore, our perspectives on learning matter: what we think about learning influences where we recognize learning, as well as what we do when we decide that we must do something about it—as individuals, as communities, and as organizations. (p. 9)

According to this view, one's perspectives on learning may be just as important, if not more so, as the practices of the community to which one belongs. The perspectives, may, in fact, play a key role in becoming a member of a community in the first place. To further emphasize the role of perspectives within a COP, Wenger (1998b) goes on to state that "a perspective is not a recipe; it does not tell you just what to do. Rather, it acts as a guide about what to pay attention to, what difficulties to expect, and how to approach problems" (p. 9). By claiming that perspectives influence what one does, what topics should be considered important, what issues should be considered problematic, and how to deal with those problems, Wenger connects these specific actions to underlying beliefs. A COP, therefore, is defined as much by the perspectives of the COP as by the practices it carries out.

It should be noted that in the text cited, Wenger used the term *perspective* synonymously with the term *belief*. In fact, right after the aforementioned quotation, he continued by listing four examples to emphasize his point, all of which began with the statement "If we believe . . ."

(Wenger, 1998b, pp. 9-10). Even Wenger's explanation of what a perspective does for an individual closely matches definitions of beliefs that claim that beliefs provide a lens through which a person interprets the world (Ambrose, Philipp, Chauvot, & Clement, 2003; He & Levin, 2008; Pajares, 1992; Philipp, 2007), and function as dispositions toward action (Cooney, Shealy, & Arvold, 1998; Rokeach, 1968).

Because beliefs cannot be observed directly, they must be inferred from what individuals say or do (Ambrose, 2004; Philipp, 2007; Philipp, Clement, Thanheiser, Schappelle, & Sowder, 2003). As the COP lens encourages seeking out rich, situated insights into what individuals do or say from which inferences regarding beliefs may be made, it forms an important part of this study.

Benefits of understanding beliefs. In depth studies of beliefs carry many benefits, including making teachers and researchers more aware of the various factors that influence education, improving novice preparation, and supporting mathematics teachers and MTEs as they seek to change their beliefs. Though beliefs have not been the focus of much research relating to MTEs, identifying benefits of studying beliefs in mathematics education supports studying beliefs of MTEs. In reviewing beliefs research in mathematics education, Thompson (1992) recognized that research studies shifted from viewing teachers in behavioral terms where basic skills or competencies were paramount, to viewing teachers as rational beings where norms, practices, and meanings received greater attention (see also Cobb, 1988). Along with this shift came the realization that no model of teaching and learning mathematics is complete without attention to the beliefs of the teacher. By including beliefs in research, the social, cultural, and historical influences that teachers encounter on a day-to-day basis became apparent. Thompson (1992) further noted that "research on teachers' beliefs has made clearer to us that no simple model of teaching and learning can be used to account for teachers' and students' actions in the classroom” (p. 142).

In her synthesis of beliefs research, Thompson (1992) noted that identifying beliefs of mathematics teachers has led to changes in teacher development programs. She recognized that studying beliefs would not create a checklist of necessary changes, but rather, it would raise questions for MTEs to consider and study further.

Ultimately, one of the greatest benefits of studying beliefs is that it generates a means of helping students, teachers, or teacher educators understand how to change any beliefs that are possibly hindering their learning (Philipp, 2007; Thompson, 1992). As teachers enter their college experience with already defined sets of beliefs regarding education (Nespor, 1987; Pajares, 1992), one challenge of teacher educators is to help students confront beliefs they hold that oppose contemporary research about learning and teaching practices. Because connections exist between beliefs and action, improvement in teacher practices cannot be fully realized without also shifting beliefs. Philipp (2007) hypothesized that the most meaningful change that teachers will experience occurs when beliefs and practices change simultaneously. Changing beliefs is difficult work (Pajares, 1992), yet increased research on beliefs can lead to improved reflection and more substantial shifts (Philipp, 2007).

Just as researching beliefs has led to broader understanding regarding the complexity of mathematics teaching, so too can it illuminate the complexities of being an MTE. Whether the interest is improving mathematics teacher education efforts, or supporting MTEs as they shift their beliefs, comprehensive studies of the beliefs held by MTEs can support those efforts.

Conceptualizing beliefs. One of the challenges facing any beliefs research is that because of its ubiquitous nature, it is not uncommon for studies to be carried out in which the authors never define how they conceptualize the beliefs (Philipp, 2007). Pajares (1992) argued that such an approach has created problematic definitions, unclear conceptualizations, and general misunderstandings. In an attempt to avoid such challenges, I discuss beliefs in terms of two defining characteristics and two organizational features that are regularly present in research studies. The two defining characteristics, *lenses for interpretation* and *dispositions toward action*, describe ways in which beliefs influence how individuals interact with their surroundings. The two organizational features, *organized in systems, context-dependence,* describe ways in which beliefs are held by individuals. All four of these characteristics and features contributed to the way in which the beliefs construct was defined in this study.

Lenses to interpret the world. The first feature that helps to define the beliefs construct is that beliefs provide a lens through which a person interprets the world. How individuals interpret their experience, their role in any given situation, and what constitutes appropriate behaviors is all filtered through their beliefs. This lens can be seen in a classroom, where the learning of students will be filtered through their system of beliefs (Brenner, 2006; Philipp, 2007) while teachers simultaneously filters the experience through theirs. In this way, students who believe that it is their responsibility to follow along quietly and obediently in a lesson may not ask questions even if they do not understand the concepts. The teacher, meanwhile, may interpret the lack of disruption as a sign of learning. Both groups may continue in that fashion until something happens that forces one or both to confront problems with their beliefs, such as an assessment (Pajares, 1992). It is not just the actions of the teacher that will determine learning (Müller & Liebsch, 2008). Beliefs of both the teacher and the student inevitably play a part. Thus, a deeper understanding of a person's beliefs will shed light on how that person interprets the world, and subsequently, how to better facilitate learning.

Dispositions toward action. The second characteristic of beliefs is that they can be conceptualized as dispositions toward action. Though a multitude of studies have sought to identify high-quality teaching strategies, beliefs research provides a way to understand why teachers may teach in a certain way in the first place (Donche & Van Petegem, 2011; Van Petegem & Donche, 2008). In other words, how one teaches is a direct reflection of what that person believes, because the beliefs themselves dispose the teacher to act in a certain way. Whether one considers the student, the teacher, or the MTE, how those individuals interpret their situations and what motivates them to act are crucial in understanding education, making high-quality research on beliefs of utmost importance.

Belief systems. Thompson (1992) discussed the organization of individuals' beliefs in her synthesis of beliefs research, and drew heavily from Green (1971). She attended specifically to three aspects of belief systems in Green's work. First, beliefs are organized in a quasi-logical structure, with some beliefs acting as *primary beliefs* while others act as *derivative beliefs*. For example, an MTE may hold as a primary belief that student discourse facilitates learning. This belief may lead to other beliefs such as, teachers must thoroughly plan questions for each task, or small-group work should occur during each task.

Second, individuals hold beliefs with varying levels of intensity, leading to *central beliefs* and *peripheral beliefs*. Central beliefs have often developed earlier than peripheral beliefs (Nespor, 1987; Nisbett & Ross, 1980; Pajares, 1992; Rokeach, 1968) and are more difficult to change (Ambrose, 2004; Philipp, 2007). It should be noted that just because a conceptualization is held as a primary belief, that does not mean that such belief is also central. It is possible that the derivative belief occupies a place of greater conviction. Though the peripheral beliefs are more fragile and susceptible to change, prospective teachers often have well-established beliefs regarding education before they enter college (Nespor, 1987; Nisbett & Ross, 1980; Pajares, 1992; Rokeach, 1968), as can be seen in how common it is that preservice teachers (PSTs) are resistant to change (Donche & Van Petegem, 2011; He & Levin, 2008). This resistance suggests that PSTs develop central beliefs about teaching before ever entering a teacher education program.

Though knowledge and beliefs both involve true/false dichotomies, one of their differentiating features is that beliefs are not held in the same all-or-nothing way in which knowledge is held (Brenner, 2006; Philipp, 2007; Thompson, 1992). Not only does the specific belief of an individual influence his/her perception and actions, but so does the degree to which or intensity with which the individual believes. As beliefs are organized as being central or peripheral, one not only must consider what the belief is, but the intensity with which it is held. Because of this attribute, it is not uncommon for beliefs to be studied or measured along continua that range from ways in which an individual may hold a belief firmly to ways in which an individual may hold it weakly or not at all (Donche & Van Petegem, 2011; Müller & Liebsch, 2008; Samuelowicz & Bain, 2001).

The third characteristic of belief systems is that beliefs are organized in clusters that are isolated from other clusters. Though beliefs are intertwined within a cluster, it is possible for individuals to possess conflicting beliefs because of the isolation between clusters (Leatham, 2006; Philipp, 2007; Thompson, 1992). This may be why beliefs are self-perpetuating despite conflicting evidence (Pajares, 1992).

A specific way of clustering beliefs relevant to the organization of belief systems is the to differentiate between domain-specific and domain-general beliefs (Müller & Liebsch, 2008). This difference suggests that individuals may have beliefs about learning that can be applied across domains, while also having beliefs regarding learning that pertain to a single domain. Buehl, Alexander, and Murphy (2002) found that undergraduate students held domain-specific epistemological beliefs when considering mathematics and history. They also found, however, that a moderate relationship between the instruments used suggested a degree of domain-generality. In other words, though students had beliefs that were unique to the learning of mathematics and history, there were also connections that supported an argument for an overarching set of general beliefs about learning. This organizational feature suggests that though there is value in studying the more general beliefs of students, teachers, and teacher educators, studying their domain-specific beliefs, such as the domain of mathematics teacher education, is also imperative.

Context-dependence. As discussed previously, one of the challenges in studying beliefs is that people seem to possess and act on conflicting beliefs. Though part of that discrepancy may be explained by the isolated nature of the aforementioned clusters, another reason for the discrepancy is that beliefs are enacted within a context. Beliefs are not universal and context-free (Ambrose, 2004; Cooney et al., 1998; Leatham, 2006; Philipp, 2007). Though teachers may carry as a primary belief that student discourse better facilitates learning, they may conceptualize that belief differently if they are teaching second grade geometry than if they are teaching algebra to adults. They may conceptualize the belief differently when working with English Learners or with students with special needs. Given the complex nature of beliefs, one cannot separate them completely from context. This complexity creates a challenge in studying or measuring beliefs because many approaches, such as the use of Likert-style items, ignore the connection to context in the search for universal beliefs (Ambrose, Clement, Philipp, & Chauvot, 2004; Ambrose et al., 2003; Philipp et al., 2003).

Beliefs construct defined. Given the distinctions and attributes investigated, I define beliefs by using the definition from Philipp (2007) with one addition included in the brackets. Beliefs are

> psychologically held understandings, premises, or propositions [organized in systems with distinct clusters that vary according to context] about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's *view* of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes (p. 259).

Disentangling constructs. Even when working from a definition of beliefs, there still remain similar constructs that can easily be confused with beliefs. If significant overlap exists between the conceptualization of the beliefs construct and another construct, any findings about beliefs would still be fraught with ambiguity and confusion as those findings may justifiably refer to some other construct. Such lack of clarity would not be new to educational research. For example, Murray (2007) asserted that the construct *disposition* is currently meaningless in the teacher education literature because the construct has not been sufficiently disentangled from other constructs. In the hope of avoiding that particular pitfall in approaching the beliefs construct, I sought to separate the beliefs construct from other constructs similar to it, namely knowledge, values, and attitudes. These three constructs are specifically addressed because of the ease with which they can be entangled with beliefs (see discussion of beliefs construct in various studies from Philipp, 2007).

Beliefs and knowledge. At first glance, the difference between knowing something and believing something seems apparent. I know that 2 + 2 = 4, but I believe that mathematics is more enjoyable than science. While knowledge is often linked to facts or truth (Pajares, 1992; Thompson, 1992), belief is often considered more of an opinion or affect. This initial differentiation is less clear, however, as one considers which ideas gain the prominence of being defined as fact. The history of science is replete with examples of ideas that were taken as fact, supported by empirical evidence, only later to be discredited and supplanted by an incommensurate paradigm (Kuhn, 1996). How does one define knowledge when the facts upon which it rests may be disproved? The definition for knowledge, therefore, must be broadened to include contemporary certainty without relying wholly on objective truth (Clement, 1999). To address the problematic link between knowledge and facts, Thompson (1992) uses the term *consentuality*. In her treatment of the differences between knowledge and beliefs, a conception can justifiably be labeled knowledge if there is general agreement that that idea is true. In this way, something does not have to be objectively true to be considered knowledge, but rather, there exists a high level of consentuality. While consentuality is a quality of knowledge, it is not necessary for beliefs.

Similarly, suggesting that beliefs are expressions of opinion, supported by affect rather than empirical evidence, is problematic. For example, when a person expresses deeply rooted religious beliefs, they may assert that they know particular religious ideas to be true (e.g., "I know there is a God"). Such assertions may be based on their life experiences, verified by others, and held with deep conviction. For such an individual, the line between belief and knowledge may be thin indeed. Given the proximity beliefs and knowledge for any given individual, it is not surprising that beliefs can be viewed as a subset of knowledge (Nisbett & Ross, 1980) or knowledge as a subset of beliefs (Leatham, 2006; Rokeach, 1968). In regard to this challenge, Thompson (1992) referenced the *conviction* with which an individual holds an idea. The deeper the conviction, the more the idea approaches knowledge for that person. In this way, a religious idea may be held as knowledge for one person, even though it may not have the broad consentuality of other ideas. Conversely, a child's lack of conviction that 2 + 2 = 4 may yield an idea held more as a belief than as knowledge, despite the high consentuality surrounding that idea.

Based on these challenges, and acknowledging that disagreements regarding the definitions of knowledge and beliefs may continue long after this study, I adopted the definition of knowledge reported in Philipp (2007). Knowledge comprises "beliefs held with certainty or justified true belief. What is knowledge for one person may be belief for another, depending upon whether one holds the conception as beyond question" (p. 259). This definition accounts for the consentuality and conviction attributes that Thompson (1992) described. It also suggests that the difference between knowledge and belief is not dependent on the idea itself, but rather, how it is held by the individual.

Beliefs and values. Part of the reason the constructs of beliefs and values can so easily become entangled is that we tend to use the word believe in various ways. For example, though "I believe in the power of student discourse" and "I believe that increasing student discourse improves learning," may sound similar, there are subtle distinctions. Bishop, Seah, and Chin (2003) identify two of these subtleties that differentiate beliefs from values. First, beliefs are associated with a true/false dichotomy, while values tend to adopt a desirable/undesirable dichotomy. In the examples above, believing in the power of student discourse is a value statement because it suggests the general desirability of discourse, while specifically stating that increasing discourse will improve learning is a statement others may directly agree or disagree with, suggesting a true/false dichotomy.

Whether or not a person agrees often rests with the second feature Bishop et al. (2003) identified. Beliefs, they argue, are more context dependent than are values. Though other people may share the general value that student discourse is important, whether or not they believe that increasing it will benefit learning is entirely dependent on the subject, the students, and other pedagogical decisions the teacher is making. Philipp (2007) suggested that because values are more internalized and less context dependent, they are more difficult to change. He also simplified the distinction between these two constructs when he stated that, "a belief *that* is about beliefs, but a belief *in* is about values" (p. 265). Given the close proximity of these constructs, leveraging one's discussion of values as a means of inferring beliefs can be useful. For example, if an MTE discusses the value of fieldwork, discussing that topic in greater depth is likely to elicit a belief.

Beliefs and attitudes. Both beliefs and attitudes have affective and cognitive components (Philipp, 2007), making them susceptible to entanglement. The possibility for entanglement can be seen more clearly as one considers how attitudes may influence perception and behavior, and are driven by context, features consistent with aforementioned definitions of beliefs. Pajares (1992) suggested that when "clusters of beliefs are organized around an object or situation and predisposed to action, this holistic organization becomes an attitude" (p. 314). A challenge with this definition, however, is that it leaves the two constructs entangled. When beliefs are already viewed as being organized in systems, this definition begs a couple of questions. Are there clusters of beliefs that are not attitudes, or are all clusters attitudes? If beliefs are naturally organized into clusters, are there beliefs that are not attitudes? Though it may be useful to view attitudes in a broader, more holistic way, more has to be added to this definition to allow for greater disentanglement.

Framework for identifying potentially relevant beliefs. Given this definition of beliefs and their organization, it would now be appropriate to ask which types of beliefs are most relevant to acting as an MTE. Though MTEs may be united in a belief that dogs are preferable pets to cats or that Disneyland provides a more enjoyable family experience than Six Flags, such beliefs are unlikely to direct their work as MTEs. Thus, though beliefs may be commonly held, that alone is insufficient in identifying the relevant beliefs that guide the work of MTEs and contribute to their ability to function as a community. Turning to research from the broader field of mathematics education yields potential beliefs that may be relevant to MTEs.

The framework (see Figure 1) that guided the investigation of beliefs drew upon the notion of beliefs varying in intensity, and added the notion of grouping and of levels as described by Tzur (2001). I introduce groupings as a way of thinking about beliefs among a group, in lieu of clusters, which describe how individuals organize their beliefs. As I have already described varying intensity of beliefs, I only briefly discuss how they influence the framework. I then describe levels in greater detail and establish how that idea strengthens the framework. I end this section by describing the topics within the groupings and at the various levels of the framework.

Organizing beliefs by groupings, recognizing varying intensity. While the notion of clusters helps to make sense of how an individual organizes beliefs, their purpose is not to describe how beliefs are organized as they are shared among different people. As the framework was used to consider beliefs as they are shared among a group of people, it became valuable to not think of beliefs in terms of clusters, but simply as they may be related to one another. De Corte, Op't Eynde, and Verschaffel (2002) noted that beliefs about the nature of mathematics, the learning of mathematics, and the teaching of mathematics are frequently researched together, suggesting what I have termed, a *grouping*. I refer to this grouping as the *Beliefs about Mathematics Grouping*. Organizing beliefs by groupings simply provided a way for me as the researcher to be cognizant of the different types of beliefs that might emerge. For example, recognizing the connectedness of beliefs about the nature of mathematics, the learning of mathematics, and the teaching of mathematics, it is reasonable to assume that as the content changes from mathematics to the teaching of mathematics there exists a new grouping with three types of beliefs, namely beliefs about the nature of mathematics teaching, the learning of mathematics teaching, and the teaching of mathematics teaching. I refer to this grouping as the *Beliefs about Mathematics Teaching Grouping*. While the Beliefs about Mathematics Grouping has been the focus of many studies, one goal of this study is to understand more fully the types of beliefs that exist in the Beliefs about Mathematics Teaching Grouping. Beliefs are organized within the framework in continua as a means of clarifying how beliefs about the same topic can be manifest in different ways because they can be held with different levels of intensity.

Organizing beliefs by levels. Though beliefs are organized in groupings, theory also suggests that a hierarchy may exist within those groupings. Such a hierarchy suggests that moving from one type of belief to another involves building upon the beliefs at one level to develop the beliefs for another. In seeking to understand the largely unresearched domain of mathematics teacher education more clearly, Tzur (2001) shared the results of a self-study that tracked his progress through various stages of becoming an MTE. He recognized that through each stage he had different issues and activities upon which to reflect. Though this approach was not specific to beliefs, his findings not only illuminated potential issues around which beliefs are generated but also how topics or concerns were relevant at each level. Three of the levels that

Tzur (2001) identified were learning math, learning math teaching, and learning the teaching of mathematics teaching. A level, he argues,

> is considered higher in that the reflective process engenders a conceptual reorganization of practices used at the lower level(s). Thus, each higher level focus embodies the lower level foci; it encompasses new, explicitly integrated ways of thinking of what at the lower level was used implicitly and/or locally. (p. 272)

Given this perspective, to understand the beliefs of MTEs, one must also consider the beliefs of an individual as a student of mathematics and as a teacher of mathematics. This connection between levels occurs because participation and reification at one level will provide reified objects that can be the focus for further participation at other levels, (For more information about the interplay of participation and reification, see Wenger, 1998b). Thus, not only are groupings useful in understanding beliefs, but also recognizing that at different levels, beliefs will take on different forms and be used in unique ways.

While the notion of MTEs functioning amidst various levels can also be seen in Zaslavsky and Leikin (2004) and Chauvot (2009), the emphasis in Tzur (2001) on reflection and developing perspectives provided a framework that was easily adapted to studying beliefs. Combining the organizational notions of groupings and levels, the framework for this study considers the topics of mathematics and mathematics teaching as groupings, and that within each grouping the nature, learning, and teaching of that topic are hierarchically organized. It should be noted that the highest level of the first grouping, beliefs about the teaching of mathematics, is also the lowest level of the second grouping, beliefs about the nature of mathematics teaching. As such, this level will serve as a bridge, linking and influencing beliefs from both groupings. As the levels in the Beliefs about Mathematics Grouping have received repeated attention in theoretical and research articles, each level of that grouping is described in subsequent sections.. The complete framework is then be presented, recognizing that though little research has been performed about the Beliefs about Mathematics Teaching Grouping, the same types of questions reasonably guide the exploration of the levels within that grouping.

Beliefs about the nature of mathematics. At this level, beliefs about the qualities or characteristics of mathematics are of interest. These qualities may include beliefs about which activities constitute performing math, or how various mathematical domains are related (Ernest, 1989). Such beliefs not only influence a student's perception of what math is and how it should be learned, but they also influence a teacher's choice of content and pedagogy (Ambrose et al., 2004). Beliefs about mathematics also influence an MTE's beliefs about the ways math should be taught, and consequently likely influence how mathematics teaching should be taught.

Beliefs about the learning of mathematics. At the next level of the framework one holds beliefs about how mathematics can and should be learned. At this level, one expresses beliefs about problem solving, reasoning mathematically, and communicating understanding (National Research Council, 2001) and how these specific mathematical processes may be learned or may enhance learning. Beliefs about the connections between mathematical and non-mathematical experiences pertain to this level as well (National Council of Teachers of Mathematics, 2014). In addition to beliefs about the process of learning mathematics, beliefs about evidence that constitutes the successful learning of mathematics can also be found at this level.

Beliefs about the teaching of mathematics. At the next level up, beliefs about what actions best promote the learning of mathematics become the focus. As a mathematics teacher reflects, beliefs regarding pedagogy, materials, differentiation, or equity may all arise (Sullivan & Mousley, 1994). These beliefs address how to help students learn as well as how to intervene if students are struggling. At this level individuals may express beliefs about the general goals of mathematics education (Ambrose, 2004).

Mathematics Teaching Grouping. Finally, at this grouping in the framework, the specific beliefs of MTEs become the focus. While the beliefs of the previous grouping certainly inform beliefs at this grouping, within this grouping an individual expresses beliefs about the types of actions and goals related to mathematics teaching, how one learns to teach mathematics, and the actions taken by MTEs to ensure that the content is learned.

The framework summarizing the types of beliefs that exist at each level and within each grouping is presented in Figure 1.

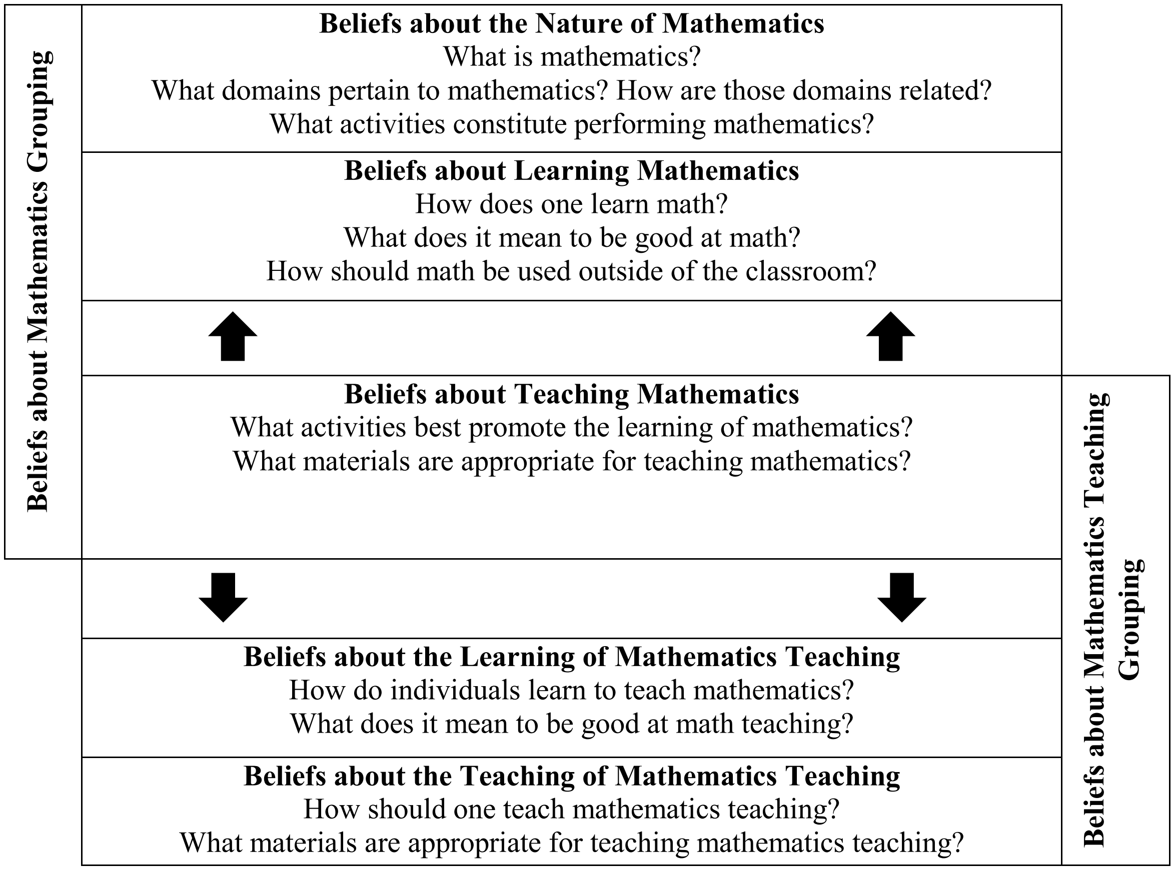

Figure 1. Beliefs framework.

Review of the Literature

Having established a theoretical rationale for studying MTEs through the lens of COPs, attending specifically to their beliefs about mathematics teacher education, the purpose of this section is to review the results of studies in which beliefs were of central importance in an education setting. Doing so provides specific examples of types of beliefs that are commonly held in mathematics education. These specific examples illuminated the nature of the evidence I received in this current study and directed what I looked for. In this section I review studies in which the results clarify the types of beliefs held in mathematics education.

It should be noted that because research regarding MTEs is relatively new (Lovin et al., 2012), many of the research results shared in this chapter focus on teachers or students. It is a goal of this study to access a parallel research tradition to teacher education, namely research on teaching (Grossman & McDonald, 2008), so as to better hypothesize what may be reasonably expected within mathematics teacher education.

For the remainder of this chapter, I share research results that highlight beliefs from the three levels of the Beliefs about Mathematics Grouping: namely, beliefs about the nature of mathematics, beliefs about the learning of mathematics, and beliefs about the teaching of mathematics. I then share results that highlight beliefs from the Beliefs about Mathematics Teaching Grouping. Based on these findings, the framework presented in Figure 1 will be updated to reflect specific insights from research studies.

Beliefs about mathematics grouping. As mentioned previously, the layers within this grouping are beliefs about the nature of mathematics, the learning of mathematics, and the teaching of mathematics. While these levels may be thought of as discreet categories, Tzur (2001) emphasized that levels depend on and build upon one another. As such, a degree of overlap is expected between these levels. Recognizing this overlap, I may categorize a research result as pertaining to the nature of mathematics that someone else may justifiably argue pertains to the learning of mathematics. Categorizing these beliefs, as has been done in numerous articles, is meant to recognize that though there may be overlap, the levels and groupings are distinct enough to justify discussing them separately.

Beliefs about the nature of mathematics. While beliefs about the nature of mathematics have been the topic of many research studies as well as theoretical papers, they typically address the specific beliefs that are held as well as who holds those beliefs. By identifying the beliefs that students, mathematics teachers, and MTEs held about the nature of mathematics from different research papers, I could better anticipate the types of beliefs MTEs in this current study had at this level.

Student beliefs about the nature of mathematics. Studies that addressed student perceptions of mathematics tend to focus on two different aspects of the nature of mathematics. Some focused on what activities students perceive as performing mathematics, while others seek to understand qualities attributed to mathematics. As to the first focus, students often equated computation with mathematics (Frank, 1988). Kouba and McDonald (1986) found that first grade students identified computational situations as mathematics 90% of the time, while situations involving geometric shapes were only identified as mathematics 50% of the time. As geometry does not have the same computational emphasis in first grade, that domain was not always believed to be part of mathematics.

While addressing the more general qualities that pertain to mathematics, some studies found that students' beliefs largely align with the nature of the instruction students received (De Corte et al., 2002; Greeno, 1991; Lampert, 1990). As students from many studies were the recipients of traditional mathematics instruction, it was not surprising that they viewed mathematics as mechanistic (Garofalo, 1989). Students frequently perceived mathematics as an objective discipline that one can master (Schoenfeld, 1989). Studies further suggested that students believed that mathematics problems should be solved in just one way (Garofalo, 1989), by applying the correct procedure (Schoenfeld, 1987). Because of these beliefs, students saw no need to check answers for reasonableness as long as they had carried out the proper steps (Garofalo, 1989). Coupled with these characteristics, however, was the student belief that mathematics is useful, though beliefs varied concerning specific uses (Kloosterman, Raymond, & Emenaker, 1996).

Mathematics teachers' beliefs about the nature of mathematics. When teaching mathematics, the way in which teachers conceptualized the domain undoubtedly influenced how they taught the subject. Ernest (1989) suggested that there are essentially three types of beliefs that teachers have, which can be organized hierarchically. He stated,

> First of all, there is the instrumentalist view that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts.
>
> Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created.
>
> Thirdly, there is the problem solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of enquiry and coming to know, not a finished product, for its results remain open to revision. (p. 2)

While the instrumentalist view was readily identified among students, the belief that mathematics is simply a collection of rules and procedures that one must memorize, has also been documented among PSTs (Benbow, 1993) and inservice teachers (Foss & Kleinsasser, 1996; Nisbet & Warren, 2000). This view, accompanied by traditional teaching practices (Gregg, 1995; Mewborn, 2001), was prevalent in many studies, though the beliefs widely varied in their intensity (Peterson, Fennema, Carpenter, & Loef, 1989; Schmidt & Kennedy, 1990).

Placing these views in a hierarchy, moving from viewing mathematics as a set of rules, to the Platonist view that mathematics represents an external reality that needs to be discovered, to finally the constructivist view where “Cognition serves the subject's organization of the experiential world, not the discovery of an ontological reality” (Von Glasersfeld, 1990, p. 23), is mirrored in Simon, Tzur, Heinz, Kinzel, and Smith (2000) as they investigate perspectives of inservice and preservice teachers. In their investigation of teachers in transition they found that teachers rarely jump directly from the traditional viewpoint of mathematics that permeated their own educational experience to a more cognitive view. They found that as teachers transitioned from a traditional perspective, characterized by the viewpoint of mathematics as being independent of the knower and emphasizing facts, rules, and procedures, they moved toward a *perception-based* perspective first. This perception-based perspective still maintained the Platonist view that mathematics represents an external reality, but that students come to understand that reality through inquiry and discovery.

The perspective they were seeking to help teachers develop was a *conception-based* perspective. They characterized this view in terms of the nature of mathematics by stating that "Mathematics is created through human activity. Humans have no access to a mathematics that is independent of their ways of knowing" (Simon et al., 2000, p. 584). Mathematics, therefore does not reflect an external reality, but rather, individual and cultural interactions within their world.

This same continuum of beliefs about the nature of mathematics, ranging from an instrumentalist view or traditional perspective to a dynamic view or conception-based perspective, was present in studies designed to measure beliefs that mathematics teachers hold. While creating an instrument to measure various beliefs of elementary mathematics teachers Ambrose et al. (2004) sought to measure the intensity with which teachers believed that "Mathematics, including school mathematics, is a web of interrelated concepts and procedures" (p. 4), suggesting the dynamic perspective (Ernest, 1989). The rubric for this instrument scores the aforementioned traditional beliefs the lowest (IMAP Web-Based Beliefs Survey Manual, 2003), particularly the view that mathematics consists of unrelated but utilitarian rules and facts, supporting the hierarchical structure in Simon et al. (2000).

MTEs' beliefs about the nature of mathematics. Given the sparse nature of contemporary research into the beliefs of MTEs, it was not surprising that little had been written about MTEs general view of mathematics. One exception was found in the belief shared among six MTEs as expressed in Lovin et al. (2012) that mathematics is problematic. In explaining what they meant by mathematics being problematic, they expressed their belief that

> assumptions in mathematics can change but also that mathematics is a human creation, constrained by an individual's current knowledge. Mathematics is often viewed in general society as a cut-and-dried or right-or-wrong discipline. We want students to develop a view of mathematics more aligned with the larger field of mathematics – a view in which mathematics emerges when ideas are discussed, argued, and revised. . . – and not the typical classroom view of mathematics. (p. 59)

While studies focused on beliefs held by MTEs generally addressed beliefs about the teaching and learning of mathematics or of mathematics teaching, as is explored in the following sections, the cited study highlights how beliefs about the very nature of mathematics also infuse the work of MTEs. As such, the results from studies about students, teachers, and MTEs will be included in the framework, which will serve as a guide for investigating beliefs of MTEs in this current study.

Beliefs about the learning of mathematics. Whether the target of a study was students, mathematics teachers, or mathematics teacher educators, beliefs expressed about the learning of mathematics could typically be categorized as pertaining to beliefs about how mathematics was or should be learned, or beliefs about what constituted the successful learning of mathematics.

How mathematics is or should be learned. When expressing beliefs about how one learns mathematics, students, teachers, and teacher educators expressed a variety of beliefs. Some addressed specific cognitive processes students used, while others addressed the types of activities in which students should engage to learn best. A common belief was that those who could learn mathematics were those who were genetically inclined to do so (Bracey, 1999; Devlin, 2000). A conflicting belief also seen among students was that mathematics could be learned by anyone if the individual worked hard to learn it (Kloosterman et al., 1996; Schoenfeld, 1989). This belief was even found to be a positive predictor of success on standardized tests (House, 2006). The hard work that students believed they must carry out, however, was to memorize the procedures that were shown to them (Schoenfeld, 1989). In this way, students adopted a passive belief about learning, in that they believed that the teacher would transmit the ideas to them, which they could then memorize (Frank, 1988). This belief about passive learning was further evident in the student held belief that only geniuses could discover mathematical principles (Schoenfeld, 1987).

Teachers' and MTEs' beliefs about how the learning of mathematics occurred displayed beliefs all along a continuum. One end of the continuum was seen as PSTs expressed beliefs that not all children would be good at mathematics and that logic and not intuition were necessary when learning math (Frank, 1990). Even the belief that learning is a passive process in which students receive knowledge was echoed by some teachers (Desforges & Cockburn, 1987; Simon et al., 2000).

At the other end of the continuum were the beliefs held by teachers that mathematical learning occurred best when concepts were learned before procedures, when children's informal mathematical knowledge was utilized, and when multiple pathways to learning were embraced (Ambrose, 2004). Philipp et al. (2003) listed different beliefs that teachers ought to possess in regard to the learning of mathematics. They sought to measure how firmly teachers believe that

> Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures. . . and, if students learn mathematical concepts before they learn standard algorithms, they are more likely to understand the algorithms when they learn them. If they learn the algorithms first, they are less likely ever to learn the concepts. (Philipp et al., 2003, p. 36)

Beliefs that further typified this end of the continuum were the beliefs that "mathematics learning is a process in which mathematical objects and relationships are constructed by the learner on the basis of her current knowledge and experiences; that is, the objects and relationships are reinvented, not discovered" (Simon et al., 2000, p. 593). Learning involves transforming one's ways of knowing and acting. As expressed by this belief, learning mathematics is not simply a process of obtaining and connecting new ideas, but also transforming and reordering existing ideas.

Beliefs held by MTEs also add to beliefs about how mathematics should be learned that fall at this end of the continuum. Lovin et al. (2012) found that they, as MTEs, shared the belief that mathematical understanding was generated through sense making. While surveying MTEs about what constituted quality mathematics instruction, Sullivan and Mousley (1994) found that MTEs expressed the belief that mathematical learning occurs when students investigated open-ended questions, especially when those questions were personally relevant to students, and when students were actively involved in problem solving. MTEs in this study also expressed that engaging in discourse was necessary for student learning.

Beliefs about successful mathematics learning. For students, having successfully learned math was simply a matter of being able to find correct answers (Frank, 1988) in very little time (Schoenfeld, 1987, 1989). Even PSTs expressed the belief that learning is seen through the speed and neatness of the students' work (Civil, 1990). Though similar beliefs were expressed by teachers, Philipp et al. (2003) were interested in the degree to which teachers believed that student learning should demonstrate flexibility and depth. Russell (2000) argued that this successful learning was manifest in mathematical fluency, characterized by accuracy, efficiency, and flexibility. Though information regarding MTEs is sparse, Lovin et al. (2012) argued that learning is enhanced as learners engaged collaboratively, learning the mathematics as a community. These beliefs, once again, formed a continuum of beliefs about what types of evidence constitute the successful learning of mathematics.

Beliefs about the teaching of mathematics. Given the diverse range of beliefs held about the learning of mathematics, it is not surprising that beliefs about the teaching of mathematics also differ. As some teachers held the belief that learning mathematics was simply an act of memorizing procedures and when to use them, beliefs that followed regarding the teaching of mathematics were that it involved offering frequent opportunities for students to memorize and practice (Foss & Kleinsasser, 1996; Lappan & Even, 1989; Southwell & Khamis, 1992; Wood & Floden, 1990). A similar belief was found as teachers equated "teaching as explaining" with their nurturing belief that teaching is caring (Ambrose, 2004, p. 97). PSTs did find, however, that simply presenting content was insufficient for deep understanding and that students needed significant time to process, representing a necessary shift in beliefs for many (Ambrose, 2004). When discussing beliefs about teaching, mathematics teachers and MTEs tended to discuss general approaches to teaching, specific practices to employ, or tools to use.

Research results in which beliefs about general approaches to teaching were expressed included various ideas. Among these was the belief that children think differently about mathematics than schooled adults (Ambrose, 2004). MTEs expressed beliefs that quality teaching involves attending to the varied needs of students and providing opportunities for all to feel success (Sullivan & Mousley, 1994). In the same survey from Sullivan and Mousley (1994), MTEs expressed beliefs that teaching should involve a clear purpose and attend to the prior knowledge of students.

In addition to beliefs about the general approaches to teaching, research also pointed to beliefs about more specific practices teachers should employ. These included beliefs teachers held about the need for teacher modeling followed by whole class discussions (Anderson & Piazza, 1996), grouping prior to instruction (Grossman & Stodolsky, 1995), or the importance of open-ended tasks (Cooney et al., 1998). MTEs added the belief that teachers should facilitate learning by presenting tasks that encouraged deep thought and independence (Sullivan & Mousley, 1994).

When speaking of their beliefs regarding tools, mathematics educators as well as MTEs expressed beliefs about the importance of including appropriate materials. These materials included the use of manipulatives (Anderson & Piazza, 1996), and technology (Cooney et al., 1998). While MTEs expressed the belief that appropriate materials should be used (Sullivan & Mousley, 1994), they did not specify what those materials should be.

Beliefs about mathematics teaching grouping. As can be seen from the various citations from the previous sections, the Beliefs about Mathematics Grouping has been the focus of many studies. One of the reasons for performing this current study was to better identify the beliefs that are held by MTEs within the Beliefs about Mathematics Teaching Grouping. Of the studies surveyed for this chapter, few explicitly addressed beliefs of MTEs, and of those that did, only Lovin et al. (2012) addressed beliefs specifically about this grouping. In this study, the results from six self-studies from different MTEs were compiled in search of common beliefs.

The only four beliefs that were at least partially held by all participants were "(1) mathematics is problematic and generated through sense-making; (2) a community of learners enhances learning; (3) mathematics teacher educators need to be explicitly aware of the learner in different contexts; and (4) teaching is complex at all levels" (p. 58). While the first belief specifically addresses the nature of mathematics, and the second could be applied to the learning of mathematics or the learning of mathematics teaching, the last two beliefs specifically addressed mathematics teacher education.

Though they did not explicitly focus on MTE beliefs, studies such as Chauvot (2009), Tzur (2001), and Van Zoest, Moore, and Stockero (2006) in which the knowledge or practices of MTEs was the focus provided findings from which beliefs could be inferred. The belief that MTEs needed to be aware of content and pedagogy at different levels, specifically pertaining to students in a school classroom as well as preservice and inservice teachers, was visible both in Chauvot and in Tzur. Chauvot added to that the belief that MTEs need to be aware of the various demands that could influence curriculum choices, such as accreditation demands or recommendations by experts in the field. Van Zoest et al. focused specifically on the expectations of PSTs and added the belief that MTEs should be aware of and knowledgeable about how to navigate PSTs' desire to hear about real life classroom experiences and for the teaching to be directly modeled to them. Though rather preliminary, these beliefs held by MTEs gave some insights into beliefs that could be expected in carrying out this study.

Framework revisited. In this chapter I presented a framework designed to help identify the types of beliefs that would likely influence MTEs. The framework contained a series of guiding questions rather than specific beliefs that could be expected (see Figure 1). The studies referenced in this chapter helped to give greater shape and definition to that framework by adding specific beliefs within the various levels that can support further beliefs research (see Figure 2). As many beliefs expressed in these studies fall along a range or hierarchy, as suggested in Simon et al. (2000), they were placed in the framework in continua, which were represented by placing specific beliefs at either side of a horizontal arrow. Information for these continua was drawn from individuals in different roles. Mathematics students, teachers and teacher educators reflect on the nature of mathematics and the learning of mathematics (Tzur, 2001), so the views of all three are included in the framework. It is typically mathematics teachers and MTEs that reflect on the teaching of mathematics, so their beliefs contribute to the beliefs about teaching mathematics section. It should be noted that the continua in the framework are not meant to capture the full range of possible beliefs one could hold within each of those categories. Placing these beliefs along a continuum demonstrates how a belief may be held with different levels of intensity. Though the resulting framework, updated through research studies, offers greater detail than the version presented in earlier in the chapter, and served to guide data gathering and analysis for this study about MTEs, it is still a framework in development. Therefore, it is revisited again in Chapter 4.

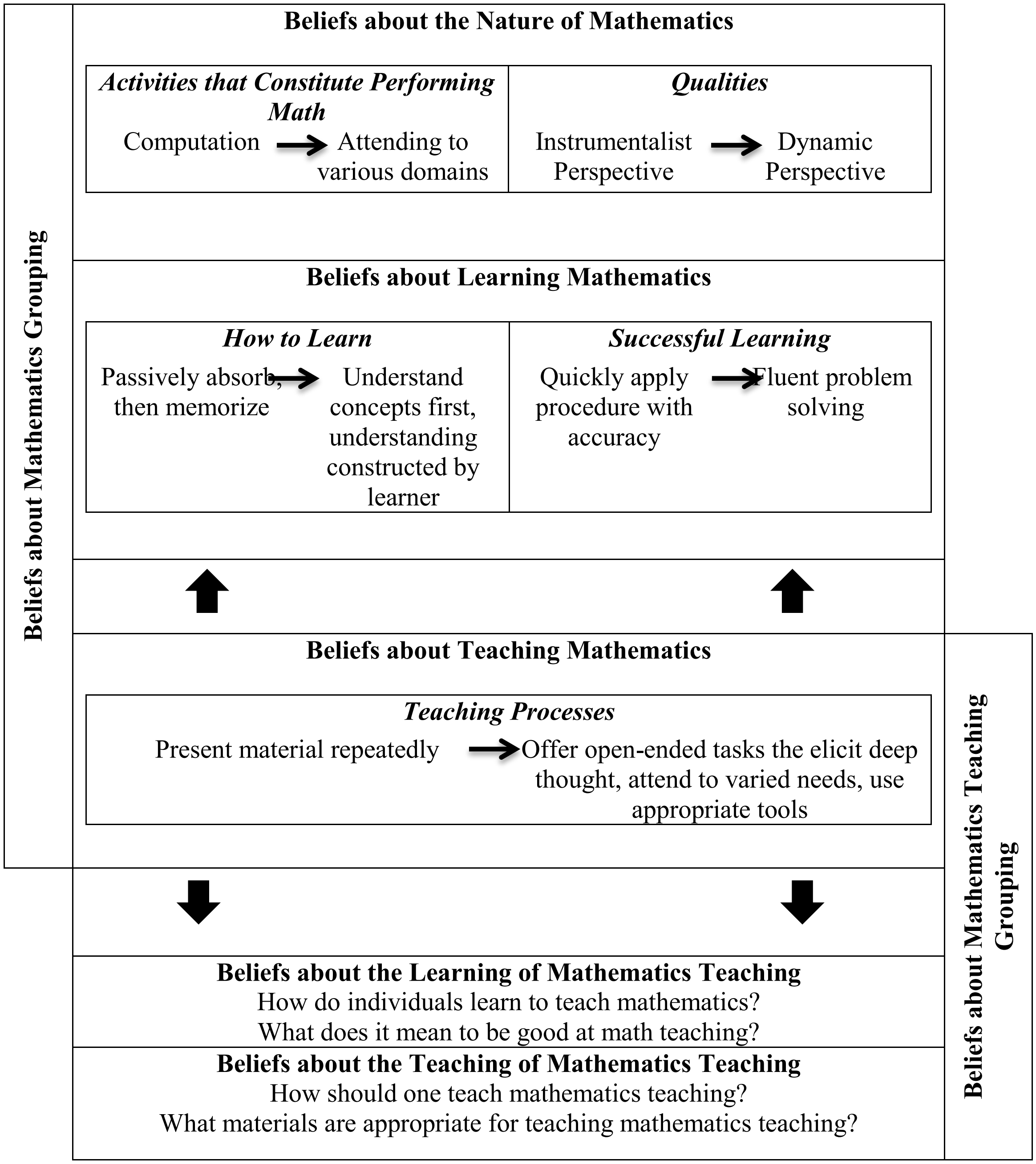

Figure 2. Beliefs framework enhanced through contemporary literature.

Summary

In seeking to answer the overarching research question, "What are defining characteristics shared among MTEs?" and the accompanying sub-questions addressing beliefs and the enactment of craft, a COP lens and a beliefs framework were investigated in this chapter.

The insights connected to the COP lens provided two benefits for this study. The first was to inform what kind of data could be gathered to improve my ability to infer beliefs, as discussed in Chapter 3, and to identify ways in which MTEs enact their craft that are not related to Beliefs about Mathematics Teaching. For example, identifying common groups with whom MTEs seek to be mutually engaged gave information about how MTEs enacted their craft beyond the insights gained about beliefs.

The framework was updated and served as an organizational tool to make sense of data gathered from MTEs. Though the data that were gathered as part of this study were informed by the COP lens, this framework stood independent of that lens. By that I mean, the beliefs that are placed within the framework do not need to be beliefs that are held within a specific community. The framework provides a way of thinking about beliefs generally, and those beliefs can be held by individuals whether or not they function within COPs. While both the COP lens and the beliefs framework provided guidance for gathering data and thinking about results, the purpose of this study was not prove that MTEs function in COPs or that they hold specific beliefs. In other words, the lens and the framework gave structure to the investigation and were not constructs in and of themselves to be tested.

Chapter 3: Method

In Chapter 1 I shared a claim from Jacobs (2003) that MTEs should lead the way in mathematics education reform. While that role may belong to MTEs, it is difficult to detail the specifics of how that role will be carried out given the little that is known about who MTEs are, what they do, and what they believe. As such, the goal of this study is to better describe MTEs by examining groups of MTEs through a lens of COPs and specifically identifying beliefs they hold specific to the Beliefs about Mathematics Teaching Grouping from the framework (see Figure 2), as well as other ways in which they enact their practice. While the previous chapters established a theoretical foundation for performing this study and drew from contemporary research to guide this current inquiry, this chapter will focus specifically on how the study will be carried out. I will first describe who the participants for the study will be and how they will be sampled. I will then describe the format by which the data will be gathered. Finally, the system for analyzing the data and the standards for ensuring justifiable conclusions will be shared.

Participants

While the population of this study consists of MTEs, it is because that label is somewhat nebulous that this study was conducted. Therefore, individuals who are employed as full-time university faculty, whose principle responsibilities involve educating preservice or inservice mathematics teachers, comprised the sample for this study. The purpose for this selection was to keep the target population as broad as possible while still making it feasible to perform the research. If all individuals who work with PSTs in mathematics were considered for this study, including mentor teachers that work with PSTs in the field, school or district level curriculum specialists, or classroom teachers that facilitate mathematics professional development then it would have been impractical to identify and then sample from the entire population. Limiting the population to university faculty kept the population broad but manageable for a study of this scope.

Even with this restriction, however, there were many differences in how MTEs function in university roles that were considered when choosing participants. In thoroughly drawing from this sample, the following differences were planned for: those who teach content courses and those who teach methods, those who teach elementary teachers and those who teach secondary, those housed in education departments as well as those in mathematics or specific mathematics education departments, and those who teach at universities where teaching is of primary concern and those who teach at universities where conducting research is an important responsibility, a distinction that will be referenced by addressing universities as either teaching-oriented or research-oriented universities.

Mathematics teacher educators were invited from a set of universities chosen purposefully (Patton, 1990). In choosing the universities from which to invite the participants, a list of potential universities was generated as evidence that a preexisting community structure was found among the MTEs that work there. The specific evidence of community structure that was used in creating this initial list was joint participation in national conference presentations. Though the purpose of this study was not to identify MTE COPs and describe how they function, there is still benefit in investigating ways in which MTEs are or are not mutually engaged in joint enterprises, producing shared repertoire. Therefore, a university was added to the list if at least two faculty members from that university had collaborated on a presentation between the years of 2011-2014 either for the Association of Mathematics Teacher Educator (AMTE) national conference, the National Council for Teachers of Mathematics (NCTM) national conference including the research pre-session, or the American Educational Research Association (AERA) national conference. The presentations had to have some aspect of mathematics education or mathematics teacher education as the focus to be included on the list. Multiple collaborative presentations from a university were tracked. Searching through programs from each conference yielded 202 universities with faculty that participated collaboratively in at least one presentation, and 853 collaborative presentations given during that time period at those conferences.

These universities were sorted into two groups, according to the research focus of the university. Universities receiving the Carnegie Classification of *Research University: Very High Research Activity* or *Research University: High Research Activity* were categorized as research-oriented universities. The rest were classified as teaching-oriented universities. This separation was not to suggest that faculty at research-oriented universities were uninterested in teaching or that faculty from teaching-oriented universities did not perform research. Rather, it was a decision made in recognition of the fact that an MTE at a research-oriented university may experience a different set of responsibilities or expectations than an MTE at a university where research is not a principle focus.

Once that separation was made, the list was divided into four sections, each representing a geographic quadrant of the United States, namely a west, midwest, south, and northeast quadrant. The purpose of this division was simply to ensure that the universities were not chosen from a single state or region. Once the universities were sorted into these four quadrants, they were arranged according to the quantity of collaborative presentations made at the aforementioned conferences, with the universities that made the most at the top of the list. MTEs from the top four universities in each quadrant, in both teaching-oriented and research oriented universities, were contacted. The first 16 MTEs to agree to be interviewed were chosen as participants. I checked the characteristics of the participants to see if there was an even mix of roles and responsibilities. Participant characteristics are summarized in Table 1.

Table 1

Summary of Participant Characteristics

	University Orientation		Department		Grade Band Focus			
Region	Teaching	Research	Mathematics	Education	Elementary	Secondary	Both	Total
Midwest	3	3	4	2	3	2	1	6
Northeast	2	0	2	0	1	1	0	2
South	2	4	3	3	2	3	1	6
West	1	1	2	0	0	1	1	2
Total	8	8	11	5	6	7	3	16

The greatest discrepancy among the 16 willing participants was that six came from the south and midwest quadrants each, while only two from the northeast and the west each. While not an even spread, many of the participants expressed having taught or attended graduate school in different parts of the country, suggesting that the perspectives offered would represent a broader outlook, and not regional specifics. There were also more than twice as many MTEs from mathematics departments than from education departments, but that difference mirrored the population from which the sample was taken. Universities generally had far more MTEs in their mathematics departments than their education departments. I was satisfied that there was an even mix of perspectives, so data collection began.

Data Collection

This qualitative study utilized a general interview guide approach (Gall, Gall, & Borg, 2006). In this design, semi-structured interviews created consistency across interviews while still allowing the interviewer to probe for greater depth whenever it was deemed appropriate. The questions for the interview protocol (see Appendix A) were created to specifically elicit data to answer the research questions and were guided by the relevant literature in Chapters 1 and 2. The first six questions were designed to yield data from which beliefs could be inferred. Both the

COP lens and the framework were consulted in the creation of these questions. The COP lens helped to make sure the questions got participants talking about their practice generally, rather than simply asking, “How do you believe PSTs should learn mathematics teaching?” By discussing their path to becoming an MTE along with attributes of their programs, goals of their classes, specific actions they took, insights could be gained regarding beliefs they held. The framework was used to make sure there was a mixture from the different groupings. For example, the second question asked MTEs what they wanted their students to come to understand about mathematics, the learning of mathematics, and the teaching of mathematics. It was included as an attempt to gain insights into beliefs MTEs held within the Beliefs about Mathematics Grouping. The combination of the beliefs framework and the COP lens informed the questions meant to elicit data to answer the first research sub-question regarding commonly held MTE beliefs.

Though participants did discuss interactions and collaborations in response to those first six questions, it was the remaining five that were designed to elicit data regarding ways in which they were or were not mutually engaged with others. The responses to these questions gave insights into how MTEs interacted with others as they sought to enacted their craft. For example, in responding to Question 9, participants gave insights into how ways they sought mutual engagement. In responding to Question 10, about professional readings, they gave insights into the extent to which they used or made sense of some of the shared repertoire of other COPs. The COP lens became an important tool in crafting questions to answer the second research sub-question, giving insights into how MTEs enact their craft.

The questions went through various stages of refinement. An early protocol was used to conduct small interviews with two members of my dissertation committee. As the questions asked were too general (e.g., "How would you define mathematics?") to elicit the type of data needed for this study, the questions were revised. This revised set of questions was presented to my dissertation committee, who identified potential weaknesses that could diminish the quality of the data gathered. They noted specifically that some of the questions were not specific to mathematics teacher education and could be confusing. The questions were once again revised. This revised protocol was used in the first interview, which was treated as a pilot interview for the study. After performing the interview, I reviewed the audio recording to assess whether or not the interviewee discussed, in depth, various aspects of being an MTE. The only change that was made to the protocol was to add a follow-up question in Question 5. The follow up question, "What is the role of the university in supporting the development of mathematics teachers?" was added to avoid Question 5 relating only to semantic differences between the different roles of those who work with PSTs. Satisfied that the questions accomplished what they were designed to do, based on the depth of the responses and the connections to some of the literature I had reviewed, the responses from the first interview were included in the data set and the slightly revised interview protocol was followed throughout. All interviews were conducted over videoconference or by telephone. Audio recordings of each interview were taken and then transcribed to form the data set for this study.

Analysis

Data from these interviews were analyzed and interpreted using thematic analysis (Braun & Clarke, 2006). Like many forms of qualitative data analysis, thematic analysis involved an iterative process of revisiting the data in increasingly refined ways. The carrying out of a thematic analysis, according to Braun and Clarke (2006), involves six phases. These are

1. familiarizing yourself with your data,

2. generating initial codes,
3. searching for themes,
4. reviewing themes,
5. defining and naming themes, and
6. producing the report. (p. 87)

I will describe the data analysis as it was completed and relate that to the phases of thematic analysis. The analysis was carried out among individual interviews first, and then across the entire data set.

Individual interviews. Data analysis began as soon as the first interview was started. While conducting and transcribing each interview I kept a notepad on hand in which I made memos about thoughts or questions I had in regard to participant responses. These memos formed a journal that was used throughout the analysis process as a way of maintaining a consistent vision, of identifying potential connections and inconsistencies, and of listing questions that could be part of a follow-up discussion that I carried out with interviewees. The follow-up conversation served as a way of clarifying any ideas I did not understand during the interview, to probe for greater insights, and to member check ways in which the participants' thinking would be represented in the study. Member checking is discussed further in the *Standards for Rigorous Qualitative Research* section. In this way I familiarized myself with the data, accomplishing the first phase of the process.

I then took the transcript of the first interview and, with the assistance of my dissertation chair, began the process of open coding. I kept a coding journal in which I began a list of the codes that I had used for that initial interview. We went through the entire first interview together, discussing the codes and identifying preliminary connections that might serve as later themes or sub-themes. This discussion helped to clarify what the different codes and connections were, but they also showed that both of us saw the content in a similar way. At no point in this process did we disagree as to how something should be coded. In this way, we began the process of the second phase, generating initial codes.

I then began the next portion of the analysis on my own. As part of the third phase, searching for themes, I took the codes from the interview and began bringing together similar codes according to the content of the code. I then reviewed these combinations with my dissertation chair as a way of auditing my own thinking to ensure consistency and to reduce subjectivity. In our discussion, we agreed that three codes should be grouped differently.

We began the fourth phase, reviewing the theme, together as we read through the statements to which the codes were attached in each combination to determine how well those statements actually fit together. Though this phase opened conversations about differences between beliefs and knowledge, as well as mathematics education and mathematics teacher education, none of the statements were moved to other combinations. Names of combinations were created and descriptive statements about each combination, or interview-specific theme, were collaboratively generated.

Going through this process with the first interview provided a system by which the rest of the interviews were analyzed individually. This first interview also provided a thematic map with potential ways of combining statements that was used in the analysis of subsequent interviews (See Appendix B). This thematic map offered structure for future interviews, but did not serve as *a priori* categorizations that had to be followed. The thematic map was adjusted as necessary for each interview. After going through these four phases, my dissertation chair reviewed the codes, the statements, and the themes for each interview analyzed to audit the work to determine the reasonableness of the preliminary findings, the consistency with which the codes and themes were used, and the avoidance of personal bias. Each analysis was approved. During this process, a second committee member was consulted in regard to the soundness of the overall process. He did not review the specific results of the analysis, but determined that the procedure being followed was sound.

Analyzing the interviews collectively. Once each interview had been analyzed, and the process audited by my committee chair, I gathered together all of the interview-specific themes from each interview. The descriptions of the interview-specific themes functioned similarly in the collective analysis as the individual codes did for the interview-specific analysis. These descriptions ranged from a single sentence to a small paragraph, depending on the complexity of the theme they described. Similar to how the third phase, searching for themes, was carried out with the individual interviews, these interview-specific themes were grouped together based on similar ideas and topics being discussed. This was done in two stages as there were nearly 200 interview-specific themes. The first was to group the themes broadly according to what topic the theme referred to. For example, all statements about learning mathematics were placed together as were all statements about the teaching of mathematics. Once those broad groups were generated, the statements from within each group were categorized more specifically so as to discover overarching themes from across the entire data set. The interview-specific themes were grouped together not only if they discussed similar concepts, but if they directly opposed one another. As this process was well suited to identifying patterns and similarities in what the MTEs discussed, memos were used throughout this process to keep track of how the participants spoke about things differently or inconsistently. This process of systematically documenting differences or overt contradictions provided opportunities to prevent negative cases. The negative cases helped to better understand that the themes were complex viewpoints.

Just as with the individual interviews, once the themes were generated, as a way of revisiting the fourth phase, reviewing themes, I went back to the individual interviews from which the interview-specific themes came as well as the actual statements that originally formed those interview-specific themes as way of making sure that the themes did represent similar thinking across participants. This process did not yield drastic changes to the themes, rather, it provided another opportunity to take memos as a way of capturing some of the subtlety and different ways of thinking about a topic within each theme.

It should be noted that it was not necessary for every participant to discuss a topic for it to be considered a theme. Because the MTEs interviewed for this study came from a variety of circumstances, they did not all discuss the same topics. For example, Participant 15 routinely mentioned that his focus was teaching PSTs content. Though he had opinions regarding pedagogy, he left that aspect of the preparation to the college of education. The absence of his comments in any theme regarding pedagogy, therefore, did not indicate a lack of agreement, but simply, that he did not have much to say about that theme. For a collection of ideas to be discussed as a theme, it had to capture the thinking of at least 10 of the participants. The purpose of setting a limit to 10 participants was to ensure that the findings emphasized themes that were important to a large majority of the participants uninfluenced by my bias. For example, four participants discussed having elementary school teachers specialize in different ways. While I found their discussion interesting, I did not present it as a theme. Seven of the nine the themes shared surpassed the 10-person minimum.

The final two stages of the thematic analysis, defining and naming the themes and producing the report are carried out in the next chapter. Defining and naming the themes is a process more in-depth than simply paraphrasing what was talked about. It involves providing a thick description of similarities as well as any subtle differences. It is a time when any negative cases are presented. Therefore, the beliefs and ways of enacting one's craft that are discussed in Chapter 4 are the themes that emerged from this collective analysis of the interview data.

Standards for Rigorous Qualitative Research

An important consideration in any qualitative study is taking steps to ensure that the results shared do not simply reflect the bias of the researcher. These steps are carried out during the data gathering and the analysis. In order to ensure rigorous data collection and analysis in this study, specific efforts were taken to attend to the credibility, transferability, dependability, and confirmability of the results of this study (Williams, 2014).

Credibility refers to the extent to which the reader may believe the claims made by the researcher. In this study, credibility was maintained by triangulating statements across participants. The themes shared in Chapter 4 do not represent a single voice, but ideas as they emerged from at least 10 individuals. In other words, the themes shared do not represent specifically chosen ideas that fit my own worldview that I am trying to promote. Another way of enhancing credibility is through the ethical treatment of participants. While the nature of the investigation did not open participants up to exposing uncomfortable truths about themselves, I was quick to change the subject if any participant began to specifically demean the work of another colleague. In this way, the reader can be sure that the findings represent honest views of the participants, and not vulnerabilities that they might regret sharing. Another step taken to maintain the credibility of claims in this study was through progressive subjectivity checks. The audits made by my dissertation chair accomplished various things, some of which are described below. One of the goals of those audits was to check the overall reasonableness of the codes and themes. This was done to make sure that the results were trustworthy.

Though the goal of qualitative research is rarely to generalize findings to a wide population, it is important that the reader knows enough about those being studied to know where such results may reasonably transfer. Transferability was addressed by strictly adhering to the sampling procedure described. By interviewing MTEs from a variety of universities and departments, with different emphases and responsibilities, readers from different perspectives may find connections to their own experience. Transferability was also a goal in using thick descriptions of the themes in Chapter 4. The themes are largely described using the participants' own words. By maintaining this emic perspective, not only was the credibility enhanced, but so too was the readers' ability to decide to what degree the results could reasonably be transferred to themselves.

Dependability refers to the care and stability of the researcher in designing and conducting a study while confirmability refers to the extent that conclusions of a study could be reasonably made by another. These standards were addressed by having a dissertation committee review and approve the design for this study and by having a member of that committee audit field notes during and at the end of the study. The process followed by my dissertation committee chair and I has already been outlined. The result of that audit was greater confidence that the process was sound and consistent, and that the results could have been reached by other reasonable parties. A final step that was taken to ensure the confirmability of the results was to send a summary of the themes from each interview to the interviewee to provide a opportunity to agree with, correct, or elaborate on how his/her thinking had been represented, also known as member checking (Lincoln & Guba, 1985). Though various efforts were made to contact the participants a second time, only half responded to the follow-up request for member checking. Of those responses, six confirmed that the statements sent to them accurately described their thinking. Two asked me to change a few words in the statements so as to better reflect some of the nuances of the viewpoints. Through the member checking and the regular audits of the process itself, the overall dependability and confirmability of the results was strengthened.

Chapter 4: Findings

The overall research question that guided this study was, "What are defining characteristics shared among MTEs?" The analysis described in Chapter 3 yielded a variety of themes that contribute to answering that very question. In this chapter I share those themes. Because the goal of this study was to discover commonalities among MTEs, some of the nuance and complexity from individual interviews was not the primary focus. Though some of the individuality is lost when focusing on the common themes, efforts were made to thoroughly describe the differences in which participants discussed those themes. I begin by sharing the themes that directly relate to the sub-question, "What beliefs about mathematics teacher education are commonly held by MTEs?" and discuss how they relate to the beliefs framework from Chapter 2. I then share the themes that relate to the second research sub-question, "What are ways that MTEs commonly enact their craft?" These themes are discussed in relation to the COP lens.

Beliefs About Mathematics Teacher Education

I suggested in Chapter 2 that when considering the beliefs of MTEs, they would not only hold beliefs about the nature, learning, and teaching of mathematics, or the Beliefs about Mathematics Grouping but that they would also hold beliefs about the nature, learning, and teaching of mathematics teaching, or the Beliefs about Mathematics Teaching Grouping (see Figure 2). I was surprised to find that though there were questions specifically designed to elicit responses related to MTEs' beliefs about the nature of mathematics and the learning of mathematics, participants spent little time talking about those two categories of beliefs. As they responded to the question, "What do you hope your students (PSTs) will come to understand about mathematics?" most of the respondents did what Participant 3 did, and jumped almost immediately to how they hoped an understanding of mathematics could impact the future teaching of mathematics. Her response to this question was,

> My hope is that they will be pretty deeply familiar [with it], and because I work in K-5 with the elementary mathematics continuum, able to certainly solve problems across that grade band, but also to make connections between the mathematics and to understand these learning structures. How is it developing over time? What do we expect kids to know around place value and algorithm and second grade that is different than what we expect them to know in fourth grade?

While some insights can certainly be gained regarding her beliefs about the nature of mathematics, her response had more to do with the importance of understanding mathematics so as to better teach it. As I said, the lack of attention to the Beliefs about Mathematics Grouping surprised me. I then revisited the discussion of how teachers focus on different things at different levels in Tzur (2001). He recognized that at the level of MTE, the questions that guide the reflection have more to do with teaching mathematics and the learning of mathematics teaching. Certainly reflections about the nature of mathematics and the learning of mathematics still occur, but they may not be as central to an MTEs reflection. After revisiting that piece, I was less surprised that all of the themes relating to beliefs, identified beliefs within the Beliefs about Mathematics Teaching Grouping. Though the beliefs that emerged from this study related specifically to the Beliefs about Mathematics Teaching Grouping, in Chapter 5 I connect those beliefs to beliefs in the framework pertaining to the Beliefs about Mathematics Grouping. I make the argument that though participants did not routinely discuss, explicitly or implicitly, beliefs about the nature of or learning of mathematics, their beliefs about the nature of, the learning of, and the teaching of mathematics teaching give insights into their beliefs about the nature of and learning of mathematics.

The five themes that emerged to help answer the first research sub-question were,

1. Deep content knowledge enables PSTs to make better decisions as mathematics teachers.
2. Understanding students is a critical aspect of mathematics teaching.
3. PSTs should experience learning (whether mathematics content or pedagogy) using the same methods that they will be expected to use.
4. MTEs need to understand the contexts for which they are preparing PSTs.
5. MTEs have a vision of PSTs as reflective practitioners who learn and develop and push the field of mathematics education forward.

The first two are discussed as beliefs about mathematics teaching. The third is discussed as it relates to beliefs about the learning of mathematics teaching. The final two are discussed as beliefs about the teaching of mathematics teaching. The purpose of categorizing these beliefs in this way was not to prove that the original framework correctly captured the way in which MTEs hold beliefs about mathematics teaching, but rather, to utilize an organizational tool to facilitate discussion.

It should be noted that most of these beliefs specifically reference PSTs because they were often shared in a context where the participants discussed teaching courses to PSTs. While the beliefs are general enough to be applicable to inservice teachers (ISTs), the differences in how MTEs enacted their beliefs as they shifted from work with PSTs to ISTs was not specifically addressed in this study. It is reasonable, however, to assume that if an MTE believes that content knowledge supports mathematics teaching for PSTs, that the MTE would also believe that content knowledge is important for ISTs as well. Because of that connection, ISTs are still referenced in this chapter and the next.

Beliefs about mathematics teaching. As participants discussed what they hoped PSTs would come to understand about mathematics, about learning, and about teaching, and as they discussed strengths and weaknesses of their programs, MTEs routinely discussed the teaching of mathematics. In doing so, however, they did not simply list things that all mathematics teachers should do and or know. Rather, they discussed the importance of understanding mathematics content as well as students in a mathematics classroom as a way of empowering the teacher to make decisions that would support learning.

Deep content knowledge. Not surprisingly, there was a general consensus that mathematics teachers, and I include elementary school teachers who teach mathematics in that classification, need to understand mathematics content. MTEs did discuss this idea differently across the participants, however. One of the participants was content to simply say that PSTs needed to know the content, while the rest went on to describe different ways that knowing the content could empower mathematics teachers.

An example of simply arguing for mathematics understanding could be seen as Participant 15 said, "Well, one big thing is that I hope they can do the mathematics and that they believe they can do the mathematics." Participant 14 also expressed the simple belief that the elementary teachers needed to just learn math in stating, "They had to learn basic algebra, they had to learn basic geometry . . . so I was looking for if they understood the basic concepts of what we were testing; base numbers and different things." When pressed, few participants stopped there. Most went on to describe how that content knowledge could be used.

This belief took on different forms for different MTEs, but the root belief that understanding mathematics supported the teaching of mathematics was ever present. Participant 8 insisted that,

> We have to learn [to] break that material down, and how do you do that? That's what I try to show in my methods courses. Even in my content courses, how to break that material down so that even a 5-year old could understand that. And they can understand it if we break it down to their ability.

In other words, knowing mathematics allows the teacher to break it down in such a way that a child can understand it. Participant 2 discussed digging into mathematical practices when stating that it is important, "getting them to have a vision of how to implement mathematical practices in their lessons. And how to get students to really dig into mathematics, because most of them haven't had that experience in their own learning." Neither participant really discussed how they hoped the students would come to understand the mathematics, they were just concerned that if the mathematics teacher understood the content well enough, they could help students.

Some participants took that idea further as they discussed the teachers' ability to listen to and respond to students. In a way, deeply understanding mathematics content enables the teacher to be more student-centered. Participant 13 expressed that idea while asserting,

> I think what makes an expert teacher is someone who listens to the student and has the mathematical background to come up with some hypotheses about what the student is thinking, what is the next best question to ask, what would be a good task to help that student or a group of students or even the whole class. I guess it's really becoming more student-centered, more connected to a really strong mathematical knowledge.

Understanding mathematics, then, is not an end in and of itself. MTEs routinely viewed mathematical knowledge as a necessary tool for interacting with students.

Participant 4 as part of discussion of PSTs' learning mathematics stated,

> They should be looking to how kids are thinking and what resources they are bringing to bear in so doing, and rethinking what they were used to thinking as errors or misconceptions as resources for learning. And to listen closely to students and find best ways of reaching out and helping individual students come to understand content as something deeply meaningful.

Such understanding of content goes way beyond the accurate performance of algorithms or even the broader ability to problem solve. Understanding mathematics content empowers the teacher to value the meanings that a student brings into a mathematical experience. While learning mathematics content should be a goal in any mathematics teacher education program, these MTEs are determined that such knowledge is explored in a way that supports a teacher's ability to interact mathematically with a student.

When MTEs hold the belief that mathematics teachers need strong content knowledge to better engage as teachers, an ancillary belief emerged which more fully shapes the theme. In teaching mathematics content to preservice or inservice teachers, several MTEs believe it is necessary to be aware of the prior life experiences that these populations have had with mathematics. Those who taught elementary PSTs discussed views of mathematics they needed to overcome as did those who taught secondary PSTs. For those teaching elementary school teachers, they argued that MTEs need to be aware of the potential for negative views of mathematics. Participant 11 noted the obstacle, "Preservice elementary teachers—those are always harder because they have all this extra baggage with them. They have this negative attitude towards mathematics, they don’t feel that they are very good at mathematics.” Participant 6 demonstrated this belief when saying,

> With preservice elementary teachers, many of them have had negative experience in mathematics and so there’s this need to develop this sense of agency—that they’re capable of doing it and they want to do it. Many of them will come into my class dreading the idea of doing math.

A lack of capacity or desire among elementary PSTs to do mathematics can be a huge hurtle for MTEs. This attention to capacity was echoed in a statement by Participant 15 about elementary teachers when he said, “we really want to make sure that they can do the math. . . . There are some students that are incredibly math-phobic and they think they can’t do the math even though they’re perfectly capable.” This belief suggests that as MTEs seek to help their elementary PSTs understand the mathematics deeply enough to effectively teach, they must be sensitive to the potential negative affect that the students harbor.

In teaching secondary PSTs, MTEs argued the need to be aware of a very different phenomenon. Because secondary students often major in mathematics, they are students who have traditionally been successful in mathematics classrooms. While this may seem an advantage, MTEs saw it as a distinct disadvantage in trying to support secondary teachers’ understanding of mathematics in the way previously described. Participant 12 described this challenge.

> Obviously these kids have been through at least 14 years of mathematics courses they are very successful. They are very bright and intelligent kids. They are strong mathematically, they’ve got mostly A’s and B’s through all their undergraduate courses . . . At the beginning we try to . . . create a perturbation. So we move in. I use angle

> measure as my topic, angle measure and tree functions as something I know they aren't completely comfortable with. We give them some tasks that kind of make them realize, "Oh wait I don't know a lot about what is going on here. I don't know why the sign function looks the way it does," or "I don't know when we say we have an angle measure with 20 degrees, I don't really know what to make of that 20 degrees other than saying it has a measure of 20 degrees."

Without creating that sense of perturbation, secondary PSTs may not come to appreciate that the procedural understanding that had worked for them for so long may not be enough. This participant argued that secondary PSTs need to experience the realization that there are mathematical concepts they do not know, despite having been successful with that mathematics in previous experiences. Participant 1 said it a bit more bluntly in arguing that with secondary PSTs, MTEs had "to kind of knock them down a notch, right? Sometimes they think they know everything."

In summary, though some may simply argue that mathematics teachers need to know mathematics content, a typical MTE will believe that knowing mathematics content allows the teacher to interact mathematically with students, that a teacher with deeper mathematical understanding can be more student-centered. As MTEs referenced their own experience teaching mathematics to PSTs, they believed it was very important to be aware of the prior experience that PSTs may have had in order to know how to approach them mathematically.

Deep knowledge of students. In discussing the teaching of mathematics, most participants acknowledged that it was complex. To describe why the process was not so straightforward, Participant 5 shared the adage, "I don't teach math, I teach students math." The complexity, as they described it, was based on the belief that good mathematics teaching attends to the needs of students, and because those needs are varied and complex, so too is the teaching. Once again, this belief was stated in subtly different ways depending on who discussed it. The belief ranged from needing to understand a student's life experiences and background, to needing to understand the specific meanings surrounding mathematics that the student brings to the learning experience. In other words, some argued that it is important to understand students generally, while others argued that it is important to understand students in a specific mathematical experience.

Participant 3 exemplified the earlier view of this belief while stating,

> I think that one thing we are really focused on as math teachers here is . . . connecting to kids and to families and especially to diverse kids and families. So we spend a good bit of time in our preparation courses having our students shadow children or having them go out and investigate neighborhoods or interview kids. It's not just about the math but about their lives. We are really trying to help them see children as whole children not just as mathematical learners. I think that is a big focus of our teaching.

Such a vision of teaching extends far beyond basic lesson planning.

In addition to having a deep understanding of students' life experiences and backgrounds, a few participants placed the general needs and ability of the students above all else. Participant 8 argued, "I still go back to, it's meeting the needs of the learner regardless of the content you're teaching. If you relate to that learner, you're going to be effective." She continued by describing the instruction that aligned with such a belief. She said,

> I would expect to see students working in groups, 3-5. I would expect to see some in their desk, some out of their desk. Some standing, some sitting, on the computer, working in centers, working on hands-on projects, maybe not hands-on depending on the learning

> style of the student. I would expect to see students working in situations that best fit their background needs; so, learning, and ability, and emotional needs. Seeing that they should have given a skill set test so they know what the student should be working on. You may have 3 different lessons going on, but they still relate to the same content, they may be at different levels.

Her argument, then, is that student needs take precedence. While the understanding of content is not completely abandoned, there is a very clear sense that a good mathematics teacher should be constantly responsive to student needs. A classroom attending to learning, ability, and emotional needs is certain to be complex. While her argument that the mathematics teacher should attend to needs of the students was echoed by many participants, most did not elevate that need as far above content as she did. This participant pushed the notion of attending to student needs to an extreme that was not consistent with other participants.

Many of the participants discussed the belief that PSTs should understand the needs of students as it pertained to attending to the mathematical understanding that students bring to the experience. Participant 7 discussed how difficult it can be "figuring out how to engage with different students. How can I really build on the mathematics they know? How can I make sure I'm not taking a deficit perspective?" In this view, building on students' mathematical knowledge involves placing the students' mathematical understanding above the teacher's own conceptions of the content. Instruction shifts from one year to the next or even one class period to the next if an understanding of the content knowledge which students bring to the learning experience drives the teacher.

Participant 12 took this idea further in when acknowledging a preference for professional readings that express ideas such as,

> Hey, let's ignore really what we want students to know and just figure out how they think and then start trying to build up maybe what we want them to know, because how they think should transform what we might think we want them to know.

Such a view extends the belief that teachers should be aware of what understandings students bring to the learning experience by arguing that the students' formation of meanings become the very targets of instruction. Once again, an understanding of the student and how that student thinks about mathematics is believed to be a critical aspect of good mathematics instruction.

The way these participants discuss the need to understand and attend to the meanings that students bring into the learning experience demonstrates a clear link, and potential overlap with the last theme, where MTEs discussed how understanding mathematics content allows them to be more student-centered. The reason they were separated was to recognize that though a teacher may understand both mathematics content and student thinking within a given context, one of those may drive instruction more than the other. For example, Participant 15 argued, "I think pedagogy is important, but I think most teachers, if they're deficient in their teaching of mathematics are deficient because of a lack of content knowledge, not because of a lack of pedagogy knowledge." He routinely cited the need for the teacher to be driven by a deep understanding of content. The statement shared in the last paragraph from Participant 12 argued that the view should shift to allow the meanings that the students hold be the driving force for instruction. Taking either of these views to an extreme may place an MTE at the periphery of how MTEs hold this belief. On the one hand, a desire for deep content knowledge could supplant important pedagogical insights, and on the other, attending to students' needs could lead to ignoring how those students' specifically think about mathematics.

Though most participants acknowledged the importance of understanding students in some way as they discussed teaching mathematics, a couple of participants did so rather superficially. Participant 14 noted the importance of different student needs in stating that elementary teachers need to know how to,

> not make it too long for an elementary student. I would usually have them use as a guideline, for every year of age your student is, that's how much, how many minutes you have for their attention span, so for elementary students you had to really focus in on what you're trying to get across.

While estimates of attention span may help prospective teachers write practical lesson plans, such a view of attending to the needs of students does not quite capture the depth of other participants. This superficiality was another way in which MTEs may find themselves more in the periphery of how MTEs hold these beliefs.

Beliefs about learning mathematics teaching—experiencing the pedagogy. Although the MTEs talked about it differently, one of the most consistent beliefs relating to the learning of mathematics teaching was that prospective teachers should experience learning using the same methods they would be expected to carry out in the classroom. This belief surfaced among those who primarily taught content courses as well as those who focused more on pedagogy courses. Regardless of their focus, most saw that the link between content and pedagogy was crucial, and discussed their teaching as emphasizing both content and pedagogy at some point. In discussing topics linked to this particular belief, some MTEs described this type of experience strictly in terms of teaching mathematics content, others taught mathematics content and then took a step back to make explicit connections to the pedagogy, while others still discussed the struggle of modeling the pedagogy to teach pedagogy.

Some MTEs purely used mathematics content lessons, expecting the pedagogy to transfer. This can be seen in Participant 9's description of methods instruction.

> Well, the easiest part, if there is an easy part to that, is teaching a math lesson. That's what I like to do best in a methods kind of class, and I've had students complain, especially in an elementary math methods class, that I wasn't teaching them how to teach, that I was teaching too much math.

While he was not alone using mathematics content lessons as a vehicle to model pedagogy, most participants went on to describe specific ways in which they could make more explicit connections to the pedagogy.

An example of how one would teach content and then step back to discuss pedagogy was seen as Participant 4 referred to the process as "switching hats."

> Switching hats means that those students working with their groups, the student teachers, they work on some problem, but then, whether or not they're done working, I'll come and say, "Okay [*clap, clap*], switch hats" and then we do this gesture [*mimics removing a hat and putting on a new one*], they know what that means. It means take off your student problem solver hat and put on your teacher researcher hat and lets talk about what happened here.

PSTs got the chance to explore mathematical problems as students, and then switch roles and discuss them as teachers. They could talk about how specific aspects of the pedagogy impacted them as learners. The message throughout the course is that what is done on campus can and should be carried over to the classroom. Projects were given to support PSTs as they sought to make those connections to pedagogy. Participant 12 applied the same model to support PST learning about pedagogy and reflected about the design of his entire university course.

> [W]e . . . have strategic reflection points during the semester where we talk about how we have designed the course. We open it up to them, "What are you seeing from us in terms of implementation and what we are doing?"

In addition to using mathematics content as a way of exploring pedagogy, some participants discussed the challenge of trying to model pedagogy while teaching pedagogy. In other words, rather than using mathematical tasks to teach pedagogy, pedagogy became the very topic to be investigated.

Participant 13 discussed how her courses, regardless of whether the topic was mathematics content or mathematics pedagogy, followed a similar format.

> I would want the students to be talking. I would want them to come prepared with some stimulus, either a task they've worked on or an article they've read, and that they're talking. Then, maybe depending on the size of the group, or maybe what happened yesterday they might start in small groups, but everyone needs to be actively involved. And then hopefully the instructor is guiding that discussion and sharing bodies of research or particular research that is relevant, particular experiences that they've seen in the classroom.

In this description she recognizes that the stimulus is what drives the thinking and the conversation. That stimulus could be a meaningful mathematics problem or a pedagogical issue.

Adding a final perspective to how MTEs support the learning of mathematics teaching by implementing pedagogical instruction that utilizes the same methods they espouse, was Participant 16. His perspective was one that viewed learning not as an accumulation of facts, but as a process of becoming a legitimate participant in the activity. Ways of thinking, of reasoning, of communicating, and of critiquing were valuable goals for any instruction. As that is how he hoped mathematics to be taught, he utilized that same vision to support PSTs' learning of how to teach mathematics. He described a project that he felt exemplified that vision.

> One that's probably a pretty clear example of what I'm talking about is when I work on the issue of technology. Obviously it's a big issue right now, so it's like, "Okay, so here's this big issue in math education. What's the role of devices? What's the role of calculators? What's the role of apps?" all these things. So instead of me saying, "Here's the good, here's the bad, and here's the criteria you should use for distinguishing the good from the bad" and kind of putting that mental apparatus into their head, instead of that I've actually, for a few years now, done a lesson where we have a full debate. . . . I actually have a four round debate structure. To me I think that actually matches with my philosophy of, "Let's bring thoughtful arguments, but then let's also critique those." At the end we can reflect on some of the key themes and then as a community we can come to some sort of resolution on it, not that it all gets resolved in a single class period, but you kind of know what I mean. So it's more about, "Let's engage in dialogue and discourse about this issue in a thoughtful way and then see what ideas can stand up to scrutiny and which ideas actually are kind of easy to critique and push aside."

He later discussed the challenge of knowing which topics are most important for a methods course, but he certainly had a clear belief that the learning of the teaching of mathematics is supported by experiencing not only content, but also pedagogy, using the same approaches to teaching that one would hope the teachers would use.

Throughout this discussion regarding beliefs tied to the learning of mathematics teaching, the consistency across participants has been mentioned. It should be noted that there were two participants who took a decidedly different view of learning math teaching. Though the message was the same with both, Participant 8 stated it most clearly,

> Well, I think one of the things that I practice and preach is that you either have the gift of teaching or you don't. I can teach you the how to teach the content, but I can't teach you how to deliver because it is so individualized.

Whether it was discussing teaching as a gift or referring to good teachers as naturals, the underlying message is the same—pedagogy cannot be taught. In a theme where some of the most consistency across participants was found, this belief stood in stark contrast.

Beliefs about the teaching of mathematics teaching. In addition to discussing how PSTs should learn mathematics teaching, MTEs in this study discussed two beliefs that inform how they carry out their practice of teaching mathematics teaching. They believed that when teaching PSTs about teaching mathematics MTEs should understand the contexts of the schools for which they are preparing PSTs, and that MTEs should view PSTs as reflective practitioners who continually improve.

Understanding contexts. This first belief, that MTEs should understand the contexts in which PSTs will eventually teach, ranged from simply understanding the practical implications of MTE instruction in a K-12 setting to recognizing the school contexts so as to empower PSTs to be agents of change.

In regard to understanding practical implications, Participant 5, while discussing how beneficial lab schools could be in keeping him connected to school contexts, made the comment, "Again, that was the power of having my wife be a first grade teacher. I'd run something by her and she'd go, 'You are crazy!'" The potential of impractical implications in teaching mathematics teaching was also apparent in Participant 2's frustration with how other MTEs' products failed to keep K-12 students in mind.

> I also often sit at conferences and read papers that have come out and think, "How does this really work where the rubber hits the road with that classroom teacher?" I think my perspective is valuable and important and sometimes I don't know we do such a great job thinking about how the things we're thinking about have an impact on kids in K-12 classrooms.

Participant 3 echoed the need for keeping an eye on K-12 implications while talking about how helpful it is for MTEs who teach elementary PSTs to have a deep understanding of elementary contexts.

> So one of the big things for me is understanding the elementary context. Not just in an intellectual way but a really embodied, visceral way. And those connections with other subject areas and just what it is like to set up a day in that way and to have intense relationships with kids, which is different than if you see them 50 minutes a day. So that is a big thing for me.

According to this belief it is not enough simply to consider oneself as knowledgeable in the teaching of elementary mathematics teaching without a clear picture of how the expectations of elementary school teachers may differ from those of secondary teachers. She goes on to discuss how the ability to make sense of what it is like in a K-12 context becomes even more critical when sending PSTs into diverse schools.

> I taught primarily in schools that serve poor and minority kids and that rounds me and focuses me in a way that I think if I had experiences in other kinds of schools I wouldn't be able to draw on those the same way.

An underlying belief expressed by these MTEs when considering the teaching of mathematics teaching is that it needs to be done in a way that prepares PSTs to engage in the contexts in which they will work.

While Participant 4 echoed the need to understand school contexts, he extended this idea to explain how that understanding would help PSTs to better change those contexts.

> So it's really important for me that [the university classroom] will be a unique place, a safe place; a haven where we can think in new ways about the whole educational process, and feel comfortable, both in the tricky place of critiquing the very system we are about to become part of and in developing within ourselves a sort of rebellious . . . a responsible rebelliousness where we are both overground and underground. Rather than choosing between being part of the system or being subversive. Rather, finding a place of functioning within the system even as we understand its limitations and struggle to bring in the good stuff, into lessons that are driven by curriculum and tests. So I'm really looking for them not to be either/or, because you lose both ways. If you fall into the system of just perpetuating everything that you and I don't believe in, but if you just become this radical constructivist modeling crazy stuff then there will be zero retention after half a year or year or maximum 2 years because you'll just fall off because you cannot do what you believe in. . . . These are student teachers who haven't even taught a day in their lives. It's probably impossible to thrust them into this sort of future prospective time machine . . . of saying, "Here you are in the future and here are the things that you are struggling with," when the very vocabulary you are expressing for pitching that scenario is not yet at their fingertips. So we need to equip them for this time machine, for going forward so that they can be in a place where they can hear us saying,

> "We understand both worlds and so should you. You will only survive if you figure out how to make these good things happen within the system." It's only from within the system that the system will change. So that's what we're trying to do.

While such a view supports the belief that MTEs need to "understand both worlds," it is not simply to make sure the PST feels comfortable and can fit it. It is so the PST can fit in, and then create change. This belief suggests that only by understanding the school context can the MTE empower PSTs in that way.

In summary, a typical MTE believes it is important for MTEs to understand school contexts. In fact, while discussing traits they look for in a high-quality MTE hire, many of the participants who came from a teaching background expressed a desire for the potential new hire to have a background in K-12 contexts. Every time that desire was expressed, however, it was followed by a discussion about how an MTE could gain that understanding in alternative ways. Participant 13 described trying to gain that understanding as she transitioned from teaching secondary courses to elementary courses, and realizing that all of her life experience had been at the secondary level.

> Then, of course, as I did all that I recognized that I've never taught elementary, so I've gone back and took one year and I taught third grade, this was 5 years ago, and they gave me a course release so that I could go be a third grade math teacher for a year. So I've made efforts to try to beef up my own experience. So even though I had K-12 teaching experience, that was a limitation in me, so I tried to make up for that.

The overall belief was clear, in teaching the teaching of mathematics, MTEs need to maintain a clear vision of the contexts into which they were sending the PSTs.

Viewing PSTs as reflective beings. To understand MTEs beliefs about the teaching of mathematics teaching, it is helpful to keep in mind the belief they held about the teaching of mathematics. Teaching, they argued, was more about understanding and responding to the needs of students and the mathematical understanding they bring to a learning experience rather than carrying out a prescribed set of teaching moves. With this perspective in mind, it was not surprising that most of their discussion surrounding how to teach teaching involved maintaining a vision of PSTs as future teachers who would need tools to reflect and learn, because it would be impossible to prepare them for every teaching scenario they might face. This goal did not suggest that one could not identify sound pedagogical principles. In fact, most discussed the importance of steering PSTs away from traditional mathematics instruction. The belief was simply that PSTs needed to learn how to choose among a variety of sound pedagogical approaches to meet the needs of the students they are teaching. This belief was discussed in terms of specific projects to help PSTs to be more reflective, of ways of connecting PSTs to resources they can use in the future to progress, and finally, of ways to help PSTs be comfortable with and trusting in who they are as teachers.

Participant 7 expressed this belief generally in saying,

> so I was at another institution where I taught methods courses before. It's similar to the content, right? It's like, you're never going to know everything but be open to new ideas . . . One of the most important things that I want people to understand about teaching and learning is that you're never going to know everything so don't even try, and don't feel bad about it . . . Just be confident enough to say, "I don't know, let's figure it out together" or "I don't know, let me think about it and we'll talk about it tomorrow." So I just feel like more teachers need to do that. I think that would be helpful.

The challenge that most MTEs recognized while having that goal is that when one wants PSTs to feel confident with not knowing everything, it makes it challenging for the MTE to know what topics or content to actually teach. Participant 4 described a research project aimed at creating an environment in which PSTs can practice investigating their own implementation of mathematics instruction.

> It’s a research paper in which they are researching some aspect of teaching and learning that emerges for them as interesting as they analyze the video data of their own implementation of the problem of the lesson plan in the classroom. And we don’t tell them in advance what their research problem should be, rather we tell them to go out and teach as best they can and we tell them what kind of data to collect, and we tell them to go back and look at the video three times and then to come and tell us things that they found interesting. Together, in consultation with the course instructor and TAs, we customize, individually, for each one of the 40 people, during office hours and through communications online. We customize for each person a research problem that they then go investigate and elaborate through the readings that they had had in the course as well as any new readings that we give them in order to come up with some insight, or some sort of assertion that they can back with the qualitative analyses of their work. . . . I’m looking to see that they are having new insights into student thinking and into their own practice as teachers. I’m looking to see reflection, at the end, after the conclusions they are supposed to take a step back and reflect on the whole process.

Teaching mathematics teaching, in this mode, is not so much about a PSTs ability to implement any objectively correct practices, but rather, to become more reflective. Participant 2 shared how she adjusted an assignment in a methods course to elicit the same sort of reflective demeanor.

> So they had to videotape themselves and reflect on it and write about it a little bit. When I first got the assignment it was an established assignment that they had to do, so I had to give it. So I gave the assignment out and got back these reflections that said, "Oh, I'm doing so great and everything is wonderful!" And then I went out and supervised student teachers and they were writing the same things and I kept thinking, "No! What you're doing is not wonderful, and you're not seeing that. And this assignment is not making you aware of that." And so we have adapted the assignment and we have gathered the work from the early versions of it, and it's gone through a couple of revisions, and we're comparing the ways the students are writing about their own work. And we're getting them, first of all we're getting them to say, "Okay, maybe I'm not doing such a great job." . . . I don't want them to be really negative about their own work, but at the same time you really want them to recognize that you do have work to do and you do have room to grow and learn. The biggest change that we made was that we had students transcribe part of their lesson. And then they had to categorize the statements that they made, according to a framework. The students started then recognizing that what they're doing in their classroom wasn't really matching up with productive discourse that research says we know is productive. And so then they got a little more reflective about what they were doing in their classroom. So just being able to get them to think more critically about the work they are doing is a step in the right direction.

While not all MTEs could describe specific assignments or projects that were designed to elicit a sense of continued learning, reflection, and improvement, some addressed this belief in a different way. Because they viewed reflection and improvement to be important, they wanted to connect PSTs to resources that would allow them to engage reflectively with the broader field of mathematics education. Participant 11 focused mostly on joining a professional organization, rather than fishing for lessons online.

> I know that they can just find activities online but let's find really meaningful activities . . . So I just want them to know, "Here are some resources. You should become a member of NCTM. I don't care what grade level you are teaching you should be a NCTM member." Because you are all going to be teaching mathematics and there are some great resources for them.

Access to a professional organization, she believes, provides PSTs ways of continuing to think and grow. Contributing to this notion that PSTs cannot learn everything about teaching from a university methods course, Participant 16 said,

> Another thing that I, definitely when I'm designing the course, something that is on my mind is, I try to take all the different sources of support that the preservice teachers might have when they become teachers and I try to have each of them show up in the course some way.

The sources he mentioned were having PSTs attend a professional conference, read practitioner articles, do a book review, and critique online resources. All of this was meant to empower PSTs to become legitimate participants in the field of mathematics education, continually learning and improving. Once again, this focus of teaching teachers was not about picking up specific teaching templates, but empowering PSTs to keep learning.

Participant 10 showed concern that when teaching is approached from an evaluative perspective, that PSTs are not given the tools to grow. She described it in this way.

> I see them as people who already have a lot of knowledge that they need to be teachers. . . . I'm thinking more of . . . helping them to help themselves and to be able to help

> themselves when I am gone. And to be able to see them as colleagues where they are now and as they leave my influence. I . . . see that as more empowering for them. I think a lot of times teacher educators are more evaluative in nature. "Well, you're not doing this right or this right or this right." "So if you fix those three things than you are going to be great." I don't think that empowers them very well to take on their own future job and be able to grow as a professional. I see my job more as to teach basic principles and then draw things out of them. And the things I draw out of them are the things that match with their philosophy and their knowledge and their experience, usually in the classroom, with their kids. They know their kids better than I do and their situation better than I do and so this allows them to draw on principles and draw on their own knowledge rather than me coming in with some kind of structure that is imposed on them.

This view supports the argument that teaching mathematics teaching is not about teaching a set of techniques, but coming to understand who one is as a teacher. It values the knowledge and experiences that the PSTs bring to the learning experience, and it empowers them to be the ones to push their field forward. Participant 5 builds on this by referencing the need for MTEs to trust PSTs.

> So there's really, there's an element . . . of trust. Trusting them as mathematicians, trusting them as teachers. We try to come in with a very humble stance of, "We've gotten to know you, but we don't know exactly what's the best way for you to teach. We certainly don't know your students the way you know your students and what's best for them. And in some cases you're using a curriculum that we're not familiar with. There's all sorts of things we don't know. Let us help you to better understand your own interaction with that."

While it may be surprising that MTEs do not articulate a list of topics one needs to know in order to teach mathematics, this overall view of teaching the teaching of mathematics mirrors beliefs MTEs hold about teaching mathematics. Just as the students' conceptions were placed before the content at that level, so too was the PSTs' understandings placed before the specific content of a mathematics methods course. How PSTs think, interact, and reflect on their own practice is believed to be far more important than their ability to list ways one should teach mathematics. Perhaps it should not be surprising that MTEs resist approaching teaching mathematics as an algorithm to be memorized, but rather, as a set of practices that empowers.

Revisiting the beliefs framework. In recognition of the beliefs that emerged through the various interviews with MTEs, the Beliefs Framework was updated. Even though the statements from the participants suggested that MTEs hold the beliefs in different ways, the beliefs are not placed on a continuum in the same way other beliefs are. Those continua showed how a belief may be different if held with different levels of intensity. The intensity with which MTEs held these beliefs was not a focus of this study. These beliefs from this study represent only those views of a small percentage of MTEs. More work will have to be done to understand the extent to which such beliefs are held and how those beliefs differ among a wider group of MTEs. All of the bullet points in the Beliefs about Mathematics Teaching Grouping were added for Figure 3. While these changes to the overall framework are minimal, by placing the beliefs that emerged from this study on the same framework that contained beliefs in the Beliefs about Mathematics Grouping, certain connects became more apparent which are discussed further in Chapter 5.

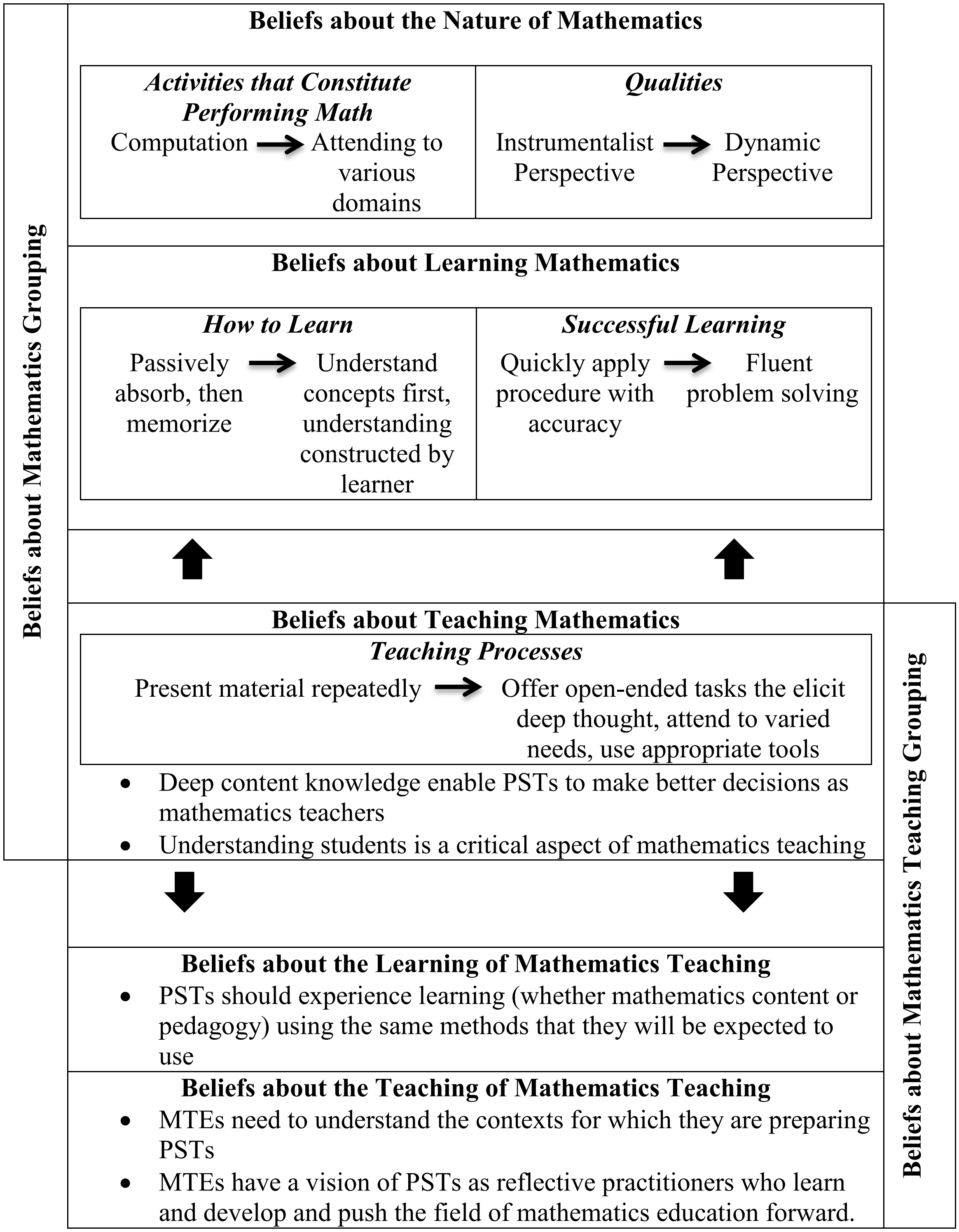

Figure 3. Beliefs framework including beliefs of MTEs.

Ways of Enacting Their Craft

In seeking to better understand who MTEs are, studying beliefs provided insights into the cognitive structures that MTEs hold in regard to their work. In Chapter 2 beliefs were said to be dispositions toward action. Each of the beliefs mentioned, therefore, could also be discussed as ways in which MTEs enact their craft. The goal of this section is not simply to repeat the prior themes in a different context. Rather, it is to share themes that describe how MTEs can be understood through ways in which they go about their work on a day-to-day basis. Their general ways of being and their interactions, give continued insights into who MTEs are. As the goal of this study was not to find out whether or not MTEs functioned in COPs, I make no such claims. The COP lens was simply utilized to a way of discussing the various interactions. In this section I share themes that address the general traits of MTEs, and three ways in which MTEs avoid isolation by pursuing mutual engagement. Those three interactions occur within the university, with partnering local schools, and within the broader field. It was through these interaction themes that it became clear that though MTEs chose to interact in different ways with different groups, they viewed their work as inherently collaborative.

General traits of MTEs. While it may seem odd to discuss general traits as part of a broader discussion of how MTEs enact their craft, responses of participants indicated it was precisely the broader ways of being that they believed to be important. When asked about what they looked for in a potential hire, MTEs responded with lists of traits, often not specific to mathematics teacher education. For example, Participant 6 noted, “I look for a collaborator. I look for open-mindedness. . . . I look for opinionated. I like people who have strong opinions, but I still like them to be open to other people’s thinking.” Collaborating, being open-minded—these are ways of being. Though some did note the importance of being knowledgeable in the literature of the profession, most jumped specifically to traits that support an understanding of how an MTE should carry out his/her craft. Certainly, given the context of the question, many may have taken it as a given that the individuals being hypothetically interviewed for an MTE position have content knowledge and knowledge of the literature, it was still surprising the consistency with which they listed ways of thinking and interacting as being important traits they desired. Being inquisitive, reflective, and open to new learning were routinely cited as important qualities. Being flexible, dedicated, and motivated showed up on people's lists as well.

Through these lists it quickly became apparent that who an MTE is was far more important than what, specifically, the MTE knew. This did not mean that each hiring experience was such a unique experience that desirable traits could not be identified, it simply suggested ways in which MTEs interacted with others or engaged in their practice were at least as important as things the MTE knew. Participant 9 expressed that the knowledge an MTE might need can be learned while being an MTE. He noted, "I said before, everyone can learn. They can learn to be a math teacher educator too and you can get better at it if you're smart, and capable, and adaptable, and motivated. That's what I look for." Though these traits are not unique to being an MTE, it was interesting that MTEs identify quality in a colleague by who that person is, rather than what the MTE knows.

Being mutually engaged within the university. As MTEs described their work and the strengths and weakness of their programs, they routinely discussed the variety of individuals with whom they had the opportunity to interact on a day-to-day basis. These interactions suggested the possibility of potentially beneficial COPs. One such COP is the one that can form among the various individuals at a university that work with PSTs. PSTs pass through math departments and colleges of education. They have tenured faculty teaching their classes as well as non-tenure track, and graduate teaching aides. When these different personnel are mutually engaged in a joint enterprise, producing and using a shared repertoire, there are some tangible benefits. Participant 10 described a collaboration she enjoyed,

> One of the things I really love about [my university] is that its one of the few math departments where the math and the math educators listen to each other and like each other and learn from each other. So I think in our program the professors that are teaching the math classes that the perspective teachers take are doing some really innovative things with their teaching.

For this participant, being mutually engaged as mathematicians and mathematics educators helped define how she participated in the preparation of PSTs. Working together directly informed how the MTEs in that department carried out their responsibilities.

Participant 2 lamented losing the interactions that she had experienced at a former university where faculty communicated across departments. She noted,

> The former program I was in was very connected. So I knew what they were getting in their college of [education] courses and what they were getting in their content courses, and I knew how everything fit together. And everyone talked to everybody else. There was a lot of communication throughout the program.

As an MTE she valued having a shared vision across departments, and consistent engagement with her colleagues provided that. Having benefitted from that collaboration at her previous university, she recognized its absence at her current university.

Sometimes those interactions arose out of need to support the graduate teaching assistants as they taught the courses. Participant 15 described one such scenario,

> And there's a TA training that takes place just before the semester starts, well . . . in the last 2-3 weeks of the semester the new TAs who are going to teach [the course] are going to current [course] sections and they can see how this is done. . . . Then [the coordinating professor] videotapes . . . new TAs. . . . He has a library full of short sample lessons, either mock teaching, like to a group of TAs or from the actual classroom. Where you take video of actual situations, and they're bound to 10-15 minutes sections so we see how to cover one particular topic. How to teach it well or not so well. Generally at these weekly meetings we watch one of these videos and we critique it, what was good, what was not so good, what could improve, that sort of thing.

Being mutually engaged through regular meetings, with the goal of identifying high-quality teaching, supported novice MTEs. While it was likely easier to teach a course in isolation, specific steps were taken to make sure everyone was on the same page.

Unfortunately, such positive interactions are not always the norm among MTEs. As most of the MTEs in this study described their relationships within their university, it was to acknowledge the lack of meaningful engagement. Participant 6 noted the divide between mathematicians and mathematics educators.

> I think it would be really nice if mathematics educators and mathematicians could work closer together at the academic level on their discussions about teaching. I just realized that when I talk about teaching, the only people I talk to are the mathematics educators and the mathematics teacher educators. I don't talk to the mathematicians. There's probably only one mathematician I talk to. I think probably more open dialogue in that regard could help the prospective teachers in the mathematicians' classes.

In that statement she recognized a stark lack of mutual engagement between mathematicians and mathematics educators. She suggested that greater engagement could result in better experiences for the PSTs in the mathematicians' classes. In that very statement she also revealed one of the obstacles to supporting meaningful interactions. She viewed the benefits as unidirectional. There was no acknowledgement that increased engagement with a mathematician would also support her own teaching.

This chasm between mathematicians and mathematics educators showed up in a variety of ways. Who teaches the content courses? In which department should courses be housed? How could upper level math courses be connected more clearly to the mathematics that secondary teachers teach? These were all obstacles that helped to characterize the work of MTEs. For many, part of enacting one's craft involved facing disputes within the university. Indeed, even though Participant 15 described careful attention to teaching assistants, his interaction with the college of education was far less amicable. He said,

> after another two to three semesters, I started taking control of these courses. This is where the hostile takeover came. I basically took away control of these courses from my colleague who I didn't think was doing a good job setting the syllabus.

In seeking to understand how MTEs engage on a day-to-day basis as they enact their craft, this difference in how they interact with their university colleagues is interesting to note. Some suggested that being an MTE involved engaging interactions among colleagues within their university while others saw those interactions simply as a source of conflict that they must face.

Being mutually engaged with partnering schools. When asked whether or not they considered cooperating teachers as MTEs, all but one responded that they did, or that they did as long as the cooperating teacher carried out his/her responsibility to the PST well. The reason they gave was that they recognized that cooperating teachers worked as part of a larger, systematic effort to educate PSTs. It was precisely because MTEs viewed cooperating teachers or district mathematics specialists as contributing components to the learning of development of mathematics teachers that they viewed interacting with local schools as an important part of what they did as MTEs. When many of the participants discussed partnerships with schools, however, they did so in a similar manner as Participant 7, who spoke of a university/school partnership as an ideal that was hard to achieve.

> [I]t would be great if every district had a partnering university that had formally trained mathematics educators that could train [district mathematics specialists]. . . . It's kind of like you have your master mathematics educator and then you have your practicing mathematics educator. . . . If you were partnered with the university you could then help those master teachers, or whatever you want to call them.

While she recognized potential benefits of having high quality interactions with school districts, it was not an interaction she had achieved.

In seeking to create this sort of engagement, Participant 13 was part of a partnering program, aimed at improving the field experience of PSTs. She described the experience as,

> one of the highlights in my career. . . . We had a small group of graduate students. One of our goals was really to facilitate learning among everyone in student teaching. So we took that field experience time and tried to make it much less about the mentor teacher telling the student teacher what to do, and the university teacher supervising and making sure nothing caught fire or something. We really set up these triads. . .so the practicing teacher and the student teacher and the university teacher were all learners and were all

> teachers. I felt like it worked really well for a few years. It worked well when it was small. I learned a lot. The teachers really became close colleagues and the student teachers really benefited because they felt like they were part of the team.

She found that partnering with cooperating teachers required purposefully dismantling any sense of hierarchy and allowing university faculty, cooperating teachers, and PSTs work on the same level. Though they worked toward better support for student teachers, a product of that endeavor was close relationships and meaningful learning among all. Though hard to achieve, this MTE viewed that sort of interaction as an important part of what she does.

Participant 8 described attempts in which she is currently engaged to establish a mutually beneficial partnership between schools and the university. She described a class for PSTs that was held part-time at a local elementary school and was connected to professional development efforts being carried out at the school. She described it as,

> [a] class [that is] fused with an inservice training, in which they're in the schools Monday, Tuesday, where they teach mathematics lessons and they're there all day for inservice training. And then they're in the classroom, on campus, on Wednesdays and Thursdays, in which we can talk about what they learned in the classroom. We can apply that and do more things within the classroom to show them experiences, methods, and so forth.

Though simply having a situation that encourages regular meetings is not enough to ensure meaningful engagement as a community, it is worth noting that by combining PST fieldwork with inservice work, this participant has created a situation in which the members of the program could mutually benefit and have similar desired outcomes, namely, to improve as teachers.

Similar to other MTEs, this participant sought for ways to increase interactions with local schools to better enact her role as an MTE.

It was interesting to note that though most MTEs viewed high quality interactions with local schools as being an important part of how they prepare PSTs, many had not found ways of capitalizing on that interaction. It seemed that engaging with local schools or simply desiring to characterized ways in which MTEs in this study discussed their craft.

Being mutually engaged within the broader field. The one area in which MTEs consistently expressed regularly interacting with others as part of enacting their craft was with colleagues in their field. I say *their* field because not everyone interviewed primarily identified as an MTE. Though they recognized that they functioned as MTEs, some viewed themselves more as educational researchers who also happened to teach mathematics teachers, rather than MTEs who studied and researched mathematics teacher education. In fact, when asked about connecting with other MTEs, Participant 4 said, “I don’t.” The reason was simply that that was not where he spent his time. He did regularly interact with other researchers, just not necessarily those primarily focused on the teaching and learning of mathematics teaching. Whether they were other MTEs, mathematics educators, or researchers with similar or differing interests, most MTEs in this study viewed interacting regularly with others as an integral part of their work.

While many discussed the importance of professional conferences, they viewed those conferences as a more formal interaction. Presenting at conferences did not epitomize their day-to-day work. Participant 6 expressed that idea as she described how she contacted and had discussions with various people.

> So I think those are the things that I do to have conversations with people. Different people in different settings, just to stay on top of things. Those are very immediate things.

> Those don't include the obvious answer of going to a conference, because obviously things happen at a conference. I would describe those [interactions] as my daily, my more daily kinds of activities that happen all the time. . . . I think [conferences are] more formal. I think it's not the same as the stuff I do on a daily basis.

This viewpoint was particularly surprising given that conference presentations were used as a way of identifying universities for this study. While conferences were important, there were other day-to-day interactions that gave insights into how MTEs enact their craft.

Participant 3 discussed the importance of engaging in mathematics education communities as well as early childhood communities. She went described the importance of inviting in a diverse set of views as she claimed that without these interactions,

> my work would be much less smart. I learn a lot from the other people I work with. And even in a building, even though you do have diverse perspectives, you get so you can think the same way. "This is how we teach our courses, this is how we do our research. This is the right way." And so when you work with someone who is in a different context, that can open it up for you a little better.

While this statement is not enough to ensure that she is fully engaged in a variety of communities, she does support the need for reaching outside of the university for diverse perspectives.

Participant 12 echoed this need for diverse perspectives as he discussed the importance of reaching out to a broader field.

> I have quite a few . . . what I call frolleauges [friends and colleagues], [who] aren't necessarily doing research in my area but I keep up to date with them through email or Skype or Google Plus. Just random conversations where we kind of share what we are

> doing and that sort of stuff. Even though our fields and research [do not] overlap at all. I try to stay up to date and one person who comes to mind. . . . I talk with him all the time and he will challenge me with my work in ways that people within my area of research don't. Because people within my area of research, we don't talk past each other, but we operate on a level where we can say things and know what each other somewhat we mean. Where with [this friend] it's not that case. He doesn't have the same terminology as me and so we get this nice little push back and push forward thing. "What do you mean that doesn't make sense? I'm not buying that sort of thing." So I think through interactions like that you learn a lot. Not only about them and what they do but also even more about what you try to do and where to put things, which I think is always fun.

It is precisely these "random conversations" that inform his broader practice. It is not enough to just converse with individuals who think the same way that he does. He noted that individuals who share research interests may have the same terminology or ways of thinking about a topic. This connection occurs when individuals work together enough to create a shared repertoire. What is interesting about this comment and the previous comment was that they found the shared repertoire to be potentially limiting. As MTEs they did not want their interactions with others to always be characterized by similar viewpoints and agreement. They wanted their thinking to be challenged.

Participant 5 also valued the pushback that can come while engaging with a broader field. His preferred method was social media. By blogging, he had an opportunity to reflect on his craft. He explained,

> I think that Brian Cambourne in literacy instruction talks about the importance of talking our way to understanding or writing our way to understanding. So some of the process of

> just writing stuff . . . helps me, because I'm thinking about my audience and I'm thinking as I'm writing these, "Wait a minute, is this really what I mean? Am I really being clear about what that process is? If so, what do I need to do about that?" One of the things is pushback. One of the things that I've learned from literacy instruction is this idea of gradual release of responsibility. I thought, "This is great! This idea that I'm going to model for them, then we're going to work on it, and then I'll set them free." And I was a novice at it, so it seemed like a very linear sort of thing for me. So I put it out there and people were like, "I don't like that. What if kids aren't ready for this?" So it really got me thinking about [how I could refine my understanding about this topic].

He viewed the process of putting ideas out to be critiqued and receiving feedback to be integral to his work as an MTE. Similar to being a mathematics teacher, there was a constant need for reflection and new learning. Mutual engagement with the broader field of mathematics teacher education provided that opportunity.

In addition to expressing how much they valued working groups like this, MTEs also expressed how much work it is to maintain them. Participant 7 noted,

> You have to work at maintaining those relationships. You have to check in. If you haven't heard from someone in awhile, you check in. If you read something that reminds you of someone, you send it to them. It's all part of the work.

Participant 16 found that the immediacy of Twitter supported maintaining those connections.

> So I follow math [education] folks [on Twitter]. I follow teacher educators that will tweet about stuff that they're doing with preservice teachers. And they have blogs and stuff, but the tweets will reference the blogs. And I will put some things about, "Doing this in class today" or "Talked about this cool article." So in Twitter I'm somewhat a part of a

> community of math teacher educators. And I definitely do follow links and I've used things that I became aware of through Twitter.

Participant 9, on the other hand, appreciated the face-to-face relationships.

> [Colleagues would] all come over to my house and we'd sit around and have a few beers and those kinds of things. It's really, really important. You can't necessarily do that over Skype.

In seeking to understand how an MTE enacts his/her craft it is important to note that they valued reaching beyond their university to regularly interact with colleagues. For many, they reached beyond those who had similar ideas and invited diverse views and welcomed pushback on their thinking. Engaging with others is what pushed them to improve. In a basic sense, they did not do their work in isolation.

Summary

Over the course of this chapter I shared nine themes that emerged from the data analysis that supported answering the research questions. The first five themes connected directly to the Beliefs Framework that has been used as an organizational structure to help make sense of the themes as they emerged. These first five themes gave insights into what MTEs believe. The COP lens was most directly influential in discussing the last four themes, as they gave insights into ways in which MTEs enact their craft. Recognizing ways in which MTEs sought interaction as part of their responsibilities as MTEs related to the second research subquestion.

Chapter 5: Discussion of Findings

In response to the overarching question, "What are defining characteristics shared among MTEs?", the themes that surfaced from this study help to give shape to what a typical MTE is like. Though there are certainly exceptions, MTEs are dedicated to learning and developing their craft. They are open to different ways in which others may view mathematics education or mathematics teacher education. MTEs are likely to believe that (a) deep content knowledge enables PSTs to make better decisions as mathematics teachers, (b) an understanding of students is a critical aspect of mathematics teaching, (c) PSTs should experience learning using the same methods that they will be expected to use, (d) MTEs need to understand the contexts for which they are preparing PSTs, and (e) MTEs should have a vision of PSTs as reflective practitioners who learn and develop and push the field of mathematics education forward. Typical MTEs in this study rarely work in isolation. While such a finding is not surprising given the emphasis that was placed on collaborative conference presentations as part of the sampling procedure, it is interesting to note the interactions that they view as important to the work they do. They form communities among other faculty within their university, with teachers and administrators at local schools, and/or with other professionals who work with preservice and inservice teachers as a way of supporting engagement in their craft.

Connections Among the Themes

When these themes were considered together, two connections became apparent and further shaped an understanding of who MTEs are. The first is that mathematics content knowledge and pedagogical knowledge are inherently intertwined. The second is that learning at all levels is a dynamic process of problem solving and reflection.

Content and pedagogy inherently intertwined. Although the sampling process used for this study called for distinguishing among professors who taught content courses, pedagogy courses, or both, it was difficult to survey precisely which types of courses the participants taught. The difficulty was not due to participants' inability to list whether their course was designed to focus on content or pedagogy, but rather, because whenever they discussed teaching content, it was with an eye toward pedagogy, or when they taught pedagogy, it was linked to their exploration of mathematics content. When one considers the beliefs that *deep content knowledge enables PSTs to make better decisions as mathematics teachers*, and *PSTs should experience learning using the same methods that they will be expected to use*, then the connectedness of content and pedagogy becomes more apparent. Even the most ardent proponent of learning content recognized that mathematics teachers need different mathematical knowledge than mathematicians. The way one needs to know mathematics as a teacher is intrinsically tied to the decisions one makes as a teacher.

It was this connectedness that influenced how Participant 7 taught her content courses. She explained,

> We talk about problem solving, but it's more like I'm modeling what I want you do to in your classroom, but I don't think that's good enough. Even though I say it, I don't know if they really get it. . . . Basically I'm just teaching the way I want you to teach in the classroom.

At first glance this statement is simply a replication of the many statements in which MTEs described teaching pedagogy by using the same methods the MTEs want PSTs to use. This participant, however, is describing work done in a content course. In this way, learning mathematics content alone is not the sole goal of this particular content course. It is precisely because MTEs want PSTs to learn content deeply enough to support decision making while teaching that pedagogical principles arise, even in content courses.

The reverse was also true as MTEs described their methods courses. Nearly every participant who taught methods focused courses used mathematical investigations as a means of discussing pedagogy. When PSTs experienced mathematics content lessons in methods courses, the purpose was not simply to model what a lesson would look like. MTEs discussed taking a step back and conversing about their experience as learners. For PSTs to have that conversation, they would have to have actually learned mathematics content or had their understanding of mathematics challenged in some way. Otherwise, those pedagogical discussions could only address things they noticed the teacher doing or the students doing. Exploring mathematics content served pedagogical discussions. Very few participants discussed the idea of exploring pedagogical issues in a way that was disconnected from mathematics content.

This connection between content and pedagogy not only surfaced as belief themes were considered together, but it was also apparent in subtle ways as participants discussed their programs. While describing his mathematics education program, Participant 16 casually noted that, "We actually have three methods courses for mathematics. So I teach the first one and then the second two are a little more content focused than mine." The notion that content and pedagogy are learned alongside one another was interwoven through various discussions.

This connection between content and pedagogy is not unique to this study. Ball, Thames, and Phelps (2008) described the *Mathematical Knowledge for Teaching* (MKT) that mathematics teachers need to possess as one that is unique to teaching. Mathematics teachers need to understand mathematics content so as to

> anticipate what students are likely to think and what they will find confusing. When choosing an example, teachers need to predict what students will find interesting and motivating. . . . Teachers must also be able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language. (p. 401)

In other words, the content knowledge that mathematics teachers need to have is directly connected to how those teachers will support student learning. The need to have an interwoven understanding of content and pedagogy is present in many of the Mathematics Teaching Practices from NCTM (2014). Whether teachers are "[e]stablish[ing] mathematics goals to focus learning" or "[e]licit[ing] and us[ing] evidence of student thinking" (p. 10), they need to not only understand the mathematics, but also various ways in helping students engage with the mathematics. Possessing separate mathematics content knowledge and general pedagogical knowledge does not supporting teaching mathematics in the way NCTM (2014) promotes.

Because the connection between content and pedagogy exists in the literature, it may seem like an obvious link to an experienced MTE. When this link is considered with another theme that emerged from this study, however, a potential danger arises. MTEs viewed interacting with other faculty members within their university as part of enacting their craft. Many characterized that interaction, however, as one of conflict. When that conflict existed, it often centered around who is qualified to teach which courses or in which departments should courses be housed. When those topics are at the center of the conflict, and viewed as the primary obstacles to overcome in supporting the learning and understanding of PTSs, content and pedagogy are treated as separate entities.

For example, when the number of mathematics courses a professor has taken becomes the primary metric for determining if that professor is qualified to teach mathematics to secondary

PSTs, then the connection to pedagogy might be overlooked. Many of the MTEs who prepare secondary teachers lamented that the content courses their students were taking had no practical applications to teaching. They wanted the mathematicians who taught the content courses to be more cognizant of the fact that not all of the students in those classes were studying to be mathematicians or engineers. Many students in those classes were studying to be teachers and needed more explicit connections to teaching as they learned the mathematics content. As MTEs continue the interactions with other faculty members, this connection between content knowledge and pedagogical knowledge supports the argument that conversations should shift from which professors or which departments are best suited to teach which topics, and focus more on how PSTs can have connected content and pedagogical experiences as part of a broader program.

Learning involves problem solving and reflection. I was initially surprised by how infrequently MTEs in this study discussed beliefs about the nature of mathematics and beliefs about the learning of mathematics. As I considered these belief themes in a more connected way, however, a consistent vision of learning emerged. Specifically, when considering the beliefs that *an understanding of students is a critical aspect of mathematics teaching*, *MTEs need to understand the contexts for which they are preparing PSTs*, and *MTEs should have a vision of PSTs as reflective practitioners who learn and develop and push the field of mathematics education forward,* a common thread can be seen. Each of these beliefs can be understood as extensions of beliefs about the nature of, the learning of, and the teaching of mathematics that were explored in the literature discussed in Chapter 2.

At one end of the beliefs about the nature of mathematics continuum was the dynamic view, which argued that mathematics was not static. Rather, it was a growing body of knowledge, created as a means of making sense of real world phenomena. Such a view of mathematics supported a problem solving view of learning mathematics. In this view students of mathematics were expected to draw upon life experiences to construct mathematical meanings, attempt problem solving strategies, present their thinking, and critique their thinking and that of other students. Though the participants did not routinely discuss beliefs at these levels, the beliefs that did emerge demonstrated a similarly dynamic view of content and a problem solving approach to learning mathematics teaching.

MTEs in this study believed that mathematics teaching was characterized by understanding mathematics deeply enough to respond to the mathematical needs of students, as well as understanding the various needs of students. In other words, mathematics teaching, as a content area, is a growing and changing body of knowledge that conforms to the real world contexts in which it is used. It is not static knowledge that can be distilled and memorized. This view of teaching echoed the complexity that Lovin et al. (2012) described when discussing the belief that "teaching is complex at all levels" (p. 58). They argued,

> [t]eaching is complex because it is perplexing work with human beings in which there is no guarantee that what worked with one group of learners will work again with a different group. Teaching is complex because one cannot simply follow a list of steps to teach. (p. 59)

Thus a dynamic view of mathematics teaching suggests it is complex and should responsive to the varied needs of students. It is because the needs and the mathematical meanings of the students change that the authentic contexts in which the content, mathematics teaching, is to be used, vary. Reflecting on the dynamic view of the nature of mathematics, and what a similar view among MTEs would be like about mathematics teaching as content, helps to explain why

MTEs in this study were so concerned with understanding the needs of the students and PSTs. The beliefs expressed about mathematics teaching suggest that the MTEs in this study likely held dynamic views of mathematics content.

Reflecting on the problem solving view of learning mathematics also helps to explain some of the beliefs that emerged from this study. The idea that there is not one correct way of teaching mathematics echoes beliefs found in the literature that there is not one specific procedure that students must use as they engage in problem solving as a means of constructing or transforming their understanding of a problem. Just as students may go about solving a problem using different mathematical meanings and life experiences, so too will mathematics teachers draw on different life experiences and understandings to support student learning, a meaningful problem in which mathematics teachers are engaged daily. Though differing strategies are valued and promoted in a problem solving classroom, the difference in the strategies is not the final goal. The mathematical soundness of those strategies should be discussed and evaluated. In this way, there is not specific right way of solving a mathematics problem, though there are incorrect ways. When one considers that view of learning mathematics, it is not surprising that many MTEs viewed believed that PSTs would benefit from engaging in the problem solving practices of trying out a teaching strategy, gathering data, and evaluating its utility. Viewing PSTs as reflective practitioners is simply another way of discussing PTSs as problem solvers.

Finally, connecting beliefs from this study with beliefs about the nature of mathematics and the learning of mathematics also helps to clarify why MTEs routinely developed networks of colleagues who could challenge their thinking and why many MTEs argued that they wanted their new hires to be open and inquisitive. If the complexity of mathematics teaching cannot be distilled to a few bullet points or to an algorithm that can replicated in every situation, then an

MTE should be open to different ways of handling novel situations and to respect that another MTE may have an equally valid, but distinct, approach. Working in isolation presumes that one already has the necessary knowledge or expertise to carry out one's craft, or that conferences and journal articles are sufficient to challenge one's thinking. When one views the teaching and learning of mathematics teaching as a problem solving endeavor, however, then the regular ability to present one's ideas, however unpolished, and receive feedback or pushback from others is simply a part of a broader, collaborative problem solving effort. The problems they are engaged in solving are those of an MTE—how one learns or should teach mathematics teaching.

Conclusions and Implications

The existence of shared beliefs and ways of enacting one's craft that surfaced among the MTEs in this study support the conclusion that though mathematics teacher education has connections to other fields, such as mathematics education and teacher education, it also possesses differentiating features that can be explored and understood. I echo the final statement made by Lovin et al. (2012) that "the knowledge required to teach teachers is indeed a specialized knowledge domain that warrants continued investigation" (p. 66). A few of the participants in this study expressly argued that the mathematical content knowledge that they needed as an MTE was not vastly different than the mathematical content knowledge that they needed as mathematics teacher, and yet, they also argued that being a good mathematics teacher did not necessarily ensure that one would be a good MTE. This specialized knowledge domain relates more specifically to learning and teaching of mathematics teaching, and specific enactments of the beliefs that emerged from this study help to characterize this domain. For example, knowing how to effectively instill a problem solving view of mathematics teaching that supports the development of reflective PSTs is a type of knowledge that is unique to MTEs.

In addition to helping to characterize the knowledge domain MTEs possess, the results of this study have some clear potential benefits. Attention to the findings of this study can empower MTEs to be more reflective in their practice, encourage MTEs to develop various COPs, and support MTE preparation programs to consider goals and practices.

Empowering MTEs to reflect. Most participants sought to be reflective and inquisitive and to empower PSTs to be the same. A major benefit of this study is that it provides themes upon which MTEs can reflect. Especially for MTEs just entering the field, the beliefs themes present opportunities to ask questions. Where do I stand on the learning of mathematics teaching? Do I see it as a process of engaging in mathematics content and hoping that methods will be learned, or do I use the pedagogy I am trying to teach to investigate pedagogical issues? Do I think that it is even possible to learn how to teach? Teachers at any level may believe that everyone teaches the same way they teach, as Participant 13 expressed. These belief themes can help an MTE reflect on potential, different approaches.

Though Participant 13 described the excitement in learning that others did teach differently, she did not discuss the potential dangers of never finding out that there are other ways of enacting one’s craft. If MTEs work in isolation and never open themselves up to diverse ways of supporting PSTs’ learning of mathematics teaching, they may begin to believe that their approach is the best way of teaching mathematics teachers. With that conviction, and the belief that other MTEs do things the same way, they may become less open to new ideas, a trait directly opposed to what most MTEs in this study sought after in a colleague. By identifying commonly held beliefs as well as the different ways in which MTEs discussed those beliefs, MTEs can not only evaluate whether or not they hold that belief, but if so, in what ways and to what extent. As beliefs were described as being dispositions to action, any reflection on one’s beliefs would inevitably support reflection on ways in which an MTE could enact those beliefs. Clearly identifying one's beliefs invites the individual to evaluate the ways in which those beliefs are present in a syllabus, textbook selection, class assignments or projects, assessments, or general pedagogical approaches. If that MTE further recognizes that there are different ways of enacting the same belief, the MTE could feel empowered to try different things, to gather worthwhile data, and to assess their effectiveness. Having access to these various belief themes provides MTEs with more focused ways to reflect on their own craft.

Encouraging MTEs to avoid isolation. MTEs in this study acknowledged a number of interactions that characterized their work. There were examples in the study of strong collaborations across departments at the university and among universities and partnering schools. A desire to improve collaboration among departments or collaborating schools may resonate with some readers. They may recognize, for example, that their mathematics education program suffers from treating mathematics content and pedagogy as entirely separate rather than intertwined, and that such a weakness could be addressed through improved interactions between departments or between faculty members. Other readers may recognize that a lack of collaboration with partnering schools is negatively impacting the fieldwork experiences of their PSTs, so they may target that interaction as one that needs improvement. As those who are engaged in such efforts continue to publicize their successes and struggles, it empowers the rest of the field to find ways of navigating the obstacles and forming meaningful interactions. This study found that a typical MTE rarely works alone. They recognized that they were members of a much larger systematic effort to support the learning and development of preservice and inservice mathematics teachers.

In addition to seeing how some MTEs struggled with creating meaningful interactions in their immediate surroundings, results also showed how beneficial it could be for MTEs to have networks of professionals, many of whom may have different backgrounds and viewpoints, with whom to share ideas. Conferences and publications were important to typical MTEs, but who they were as MTEs was better characterized by the less formal, day-to-day interactions, including those supported by technology. The desire to seek out dissenting voices is an interesting goal in light of the COP lens described in Chapter 2. While the notion of mutual engagement helps to explain why MTEs desire greater immediacy in their communication than can be achieved through conference attendance, it does not explain why some MTEs purposefully chose to interact with individuals who will challenge their thinking. Though it is critical that communities navigate tensions in order to benefit all members, the COP literature did not discuss benefits of purposefully creating tensions as a way of supporting the growth of each member.

Creating networks where pushback and tensions would arise was specifically identified as a way in which many MTEs are mutually engaged with others. For example, the pushback one might receive from Twitter followers or in responses to blog posts was viewed as a critical component to their interaction. It is these day-to-day interactions that shape the MTE. Working in isolation, or simply collaborating within the same building is not enough to push the field forward. As MTEs reflect on their interactions, they may find, as some participants in this study confessed, that they tend to always interact with the same people. Over time they adopt a shared vocabulary that facilitates their interaction and solidifies their views on certain topics. This may contribute to the previously mentioned danger of believing that one has all the answers simply because the insulated nature of their communication fails to bring in views that challenge their worldview. Results from this study may encourage some readers to broaden their professional network to interact with individuals with whom they previously did not interact.

Supporting MTE development efforts. Whether they are formal, graduate programs, state certifications for mathematics specialists, or simply district run workshops, there are various efforts across the nation to provide specialized preparation for those who work with mathematics teachers. One specific effort, sponsored by the AMTE is the Service, Teaching, and Research (STaR) program. This program accepts 30 early career MTEs every year to create a strong network, meant to support not only the development of novice MTEs, but also the general field of mathematics teacher education. The StaR program provides means by which early MTEs share resources, connect with others in the field, including through Twitter feeds and personal blogs, and access a bank of resources to support teaching, research, and service efforts. The results from this study not only support the need for such a program, as it facilitates early MTEs attempts to avoid isolation, but they contribute to the growing knowledge bank surrounding mathematics teacher education that programs like StaR are trying to build.

In addition to university based MTEs, questions may arise surrounding other individuals who work in the effort of supporting the learning and development of mathematics teachers. In a joint statement the AMTE, Association of State Supervisors of Mathematics (ASSM), the National Council of Supervisors of Mathematics (NCSM) and the NCTM (2013) recommended the use of elementary mathematics specialists (EMSs) and that programs that prepare such specialists should specifically focus on mathematics content and pedagogical knowledge, and leadership knowledge and skills. When participants from this study were asked whether or not they viewed EMSs, cooperating teachers, or other district employees that worked specifically with mathematics teachers as MTEs, all but two said that they did, though nearly half added the caveat that the only earned the label if they carried out the work of an MTE in a similar manner as university based MTEs.

As MTEs in this study were willing to recognize non-university professional who educate mathematics teachers as MTEs, as well as the desire many expressed for non-university MTEs to do things in a similar manner as university MTEs, it could be useful for EMSs and other non-university MTEs to consider their practice in light of the results from this study. Though this study did not specifically include MTEs outside of a university, the results of this study can provide topics to guide discussions on what specific aspects of those EMS preparation programs may specifically entail. For example, an EMS preparation program that includes foci on content and pedagogical knowledge may find guidance on how to achieve those goals by discussing different ways in which MTEs from this study transitioned between exploring content and pedagogical problems. After reflecting upon the results of this study, an EMS preparation program may choose to include a component that helps connect the EMSs to a broader community of MTEs. This study identifies a series of beliefs held by MTEs as well as general ways of being and interacting that are valued by MTEs. Though the conclusions of this study are not meant to transfer to every MTE in every potential role, they can support discussions held in regard to MTE development programs.

Limitations

Given the ambiguity of identifying who an MTE is and what an MTE does, identifying a target population was a bit problematic. Though there was a core group that seemed obvious, individuals who primarily work in teacher preparation programs teaching mathematics teachers, there were others who also could conceivably be included in the target population. There were different ways in which the target population could have been identified and from which a sample could have been drawn. Each approach has strengths, which, in turn, highlight limitations in the way data were collected for this study. Arguments could be made that the sample was too broad, too narrow, or too small. In addition to limitations connected to the sample, the lack of prolonged engagement and the utilization of a single data source also limit the ways in which the results can be interpreted.

Scope of the sample. One of the ways to handle the problem of the ambiguity of the target population could have been to restrict the sample to those who primarily function as MTEs. This restriction could be done by only inviting those who identify as MTEs, such as members of AMTE, to participate. Such an approach would have helped to ensure that participants had reflected specifically on their role as an MTE and potentially engaged in research about mathematics teacher education. It could be argued that with such a sampling the responses received may have been more reflective or more representative of the specific field of mathematics teacher education. While receiving data of that sort could certainly be valuable, the choice to broaden the sampling to include any university faculty that taught mathematics content or pedagogy to inservice or preservice teachers was done out of recognition of the vast number of teachers that pass through such classrooms. Just because a person does not identify primarily as an MTE does not mean that that person cannot provide meaningful insights that are representative of the broader field. While there may be utility to reserving the label of MTE to only those who identify as such and perform research specific to mathematics teacher education, I side with the majority of participants in this study who argued that in considering mathematics teacher education, one should have an eye to the larger effort and the different roles individuals carry out. By broadening the sample, a more diverse set of voices was welcomed, but so too was the potential for interviews where the participant may not have reflected much on his/her role as an MTE.

At the other end of the sampling conversation could be those that argue that cooperating teachers or district specialists engage as legitimate MTEs on a daily basis and were not included in this study. The results, therefore, cannot be viewed as representative of a wide variety of MTEs. The goal for sampling in this study was to broaden the scope as much as was feasible. The reason for excluding these MTEs was mostly for practical concerns. Identifying individuals in those roles throughout the country, recognizing the various ways in which schools and districts employ teachers as teacher leaders or specialists, became untenable. While theirs is a valuable voice to consider, it was not included in this study.

Size of the sample. As is often the case in qualitative research, the goal for this study was not to generate a representative sample of MTEs from which results could be generalized to all MTEs. Results from this study are specific to this study. Care was taken to elicit the perspectives of a variety of MTEs in different positions and locations. By so doing the reader may carefully consider the results as well as specific statements of the participants to determine the degree to which the results transfer. Though having a larger sample would increase confidence in the generalizability of the findings, it would also decrease the depth at which the experience of MTEs could be investigated.

Prolonged engagement. One of the ways in which a qualitative researcher can support the credibility of any claims is to study a specific phenomenon over a long period of time. The single interview and a follow up contact is not enough to argue that prolonged engagement occurred. Rather than studying a specific phenomenon over a long period of time, I chose to study a wide variety of individuals. I sought consistency across people rather than consistency across time. As the purpose of this study was to identify defining characters shared among MTEs, it made more sense to interview a wider variety of participants than it did to spend more time with just a few. The results, therefore, should be understood to reflect commonalities across individuals, but that any one of those individuals may enact those beliefs in different ways and at different times.

Multiple data sources. In seeking to find consistency over multiple individuals, another component of qualitative research that was not included was the use of multiple data sources from which beliefs could be inferred. Though wonderful insights could have been gained by comparing interview results with specific instantiations of those enacted beliefs, either through observations of the participants teaching or through other interactions, including a variety of voices from all over the country was chosen instead. Just as interviewing more people was chosen over prolonged engagement, choosing to interview more people was preferred over gathering varied data from fewer individuals. This approach allowed for conclusions to be made that were typical across a range of individuals, but it could not detail specific ways in which participants acted upon those beliefs in different contexts or even how important those beliefs were to the participants.

Recommendations

Though the results of this study provide insights on which MTEs may reflect, an important take away from the entire experience is simply that more research should be done in connection to the individuals who prepare mathematics teachers. Even the responses of the participants supported this conclusion. In discussing methods courses for secondary teachers, Participant 2 lamented the lack of systematic research to support MTEs' ability to make certain decisions. She stated,

> I think that in the field of math [education, our knowledge is] very unstructured and not necessarily built in scholarly ways at this time. I think that that is something that we as math educators need to work on, is how do we learn from each other's practice? How do we take what other people have done and think about it in a different way and try to replicate it and build on that? And right now the literature in math [education] doesn't do that very well. Especially when you look at methods, it's very disconnected and disjointed. And the other challenge we have is how do we know whether or not what we're doing is effective? So while I want to learn from other people, I want to know that if I'm going to take what they're doing that it's going to be effective and I need to know in what context they used it. So how was their context different from mine? And how am I going to learn from that. There's still a lot of work to be done in that.

Participant 1 shared a similar concern.

> And then there's the hallmarks of instruction that we know are going to make a difference. I don't know that as a community we've done a lot, or that we've done a good job of identifying what that is and putting it out there to be studied. Maybe we're still new enough that it's not feasible, that we're not ready to put it out there to be studied. So instead we've got pockets of people who are examining these different things so that one day, hopefully, it will all come together.

Though such concerns were not prevalent enough to form a theme for this study, it was interesting to note that some MTEs felt that one of the reasons it was hard to answer certain questions was because the field had not systematically investigated the work of MTEs. After engaging in this study I see four specific areas in which results from this study could support further exploration.

1. As this study was more exploratory, additional research should be conducted to determine how well these findings generalize across a wider sample of MTEs.
2. While participants discussed challenges and successes surrounding their various interactions, further research should be conducted in which greater understanding of those working networks could be gained. How are they formed? What specific benefits do they offer? Do virtual networks differ from physical networks?
3. This study gathered data from individuals in different departments and positions and locations. It looked at what individuals in these various settings had in common. Though differences in the way they thought were identified, there was no attempt at looking at specific differences. For example, did MTEs who prepared elementary PSTs hold beliefs differently than MTEs who prepared secondary PSTs?
4. Building on the findings of this study, similar work could be done with MTEs who do not work in universities. Comparing how the groups function could yield valuable insights into how to prepare and support MTEs for the roles they may carry out.

Final Thoughts

I began this process by outlining my own efforts to become a mathematics teacher educator, complete with perceived successes and obvious shortcomings. I questioned the role that MTEs hold in the broader reform movement that mathematics education is experiencing. After conversing with a variety of MTEs from different universities and in different roles I have a more complete picture of what it means to be an MTE. I had numerous opportunities to contemplate others' viewpoints in light of my own efforts and even used some the participants' ideas to design my own instruction. Though this exercise was invaluable in my own learning and growth as an MTE, it is my hope that this work will support the reflection and development of other MTEs and contribute to the growing knowledge within the field of mathematics teacher education.

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APPENDIX A: INTERVIEW PROTOCOL

1. What brought you to mathematics teacher education? Were there experiences or lessons you learned along the way that have really shaped you, or that you would hope other novice MTEs would experience?
2. What do you hope your students will leave your classes understanding about: Mathematics; teaching, learning mathematics; being a professional teachers; etc. Why do you value those specific traits? What are some things you do to get them there? How do you know when they've come to know/understand those things?
3. Tell me about your mathematics education program. What do you see as its strengths? In what ways does it best support the learning and development of mathematics teachers? What are some parts of your program that you are still working on? Why do you view that as a necessary improvement?
4. If you were to walk into a classroom where an amazing MTE was having a lesson with preservice or inservice teachers, what are some things you would expect to see/hear?
5. What makes someone an MTE? Would you consider ______ an MTE? (Provide examples of individuals who do similar work as MTEs but may not identify as one, e.g., mentor/cooperating teachers, district math facilitators, coaches, etc.). What is the role of the university in supporting the development of mathematics teachers?
6. What are some core principles that you hold as an MTE?
7. If you were on a hiring committee looking to hire a novice MTE, what are some qualities you would look for that would indicate a potentially smart hire? What are some areas you would make sure to target in your mentoring to help that individual progress?

8. Turning to your place in the larger math education community, what do you see as your place/contribution in the larger community?
9. How do you stay connected with what's going on the discipline? With colleagues across the country/world?
10. What are the most important things for you to read or keep up on for your work as an MTE?
11. If you complete control at this university and could make any changes you wanted, what is one thing would you change? (Same question about our field, or MTEs in our field)

APPENDIX B: THEMATIC MAPS DERIVED FROM THE FIRST INTERVIEW