2014-12-01

Language Use in Mathematics Textbooks Written in English and Spanish

Kailie Ann Bertoch
Brigham Young University - Provo

Language Use in Mathematics Textbooks Written in English and Spanish

Kailie Bertoch

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

Blake Peterson, Chair
Daniel Siebert
Dawn Teuscher

Department of Mathematics Education

Brigham Young University

December 2014

ABSTRACT

Language Use in Mathematics Textbooks Written in English and Spanish

Kailie Bertoch
Department of Mathematics Education, BYU
Master of Arts

Research has suggested that although mathematics is a universal language, the notation, algorithms, and language in which it is taught varies from country to country. The literature has addressed the challenges associated with understanding the language of mathematics in English. Variable is a topic of mathematics where the literature has documented students' difficulty understanding the language of mathematics. Part of that difficulty is attributed to the fact that mathematics and English each have their own register and at times the differences in these registers conflict. This study examined the introduction and use of variable in the middle school textbooks of three English speaking countries and three Spanish speaking countries. The results of this study are that textbooks rarely make any attempt to use bridging language, and do not explain to students how they are using variable when they use it. The results of this study also show that the language used to talk about variable is similar English (Spanish) speaking countries.

Keywords: bridging registry, English registry, mathematics registry, variable

ACKNOWLEDGEMENTS

I would have never been able to finish my thesis without the support of my committee, my husband, and my family.

I would like to express my appreciation for my committee, especially my advisor Dr. Peterson, for his constant feedback on my writing and help answering my many questions through the course of this study. I'm also so thankful to my husband for all of his support and encouragement in pursuit of my master's degree. Lastly, I would like to thank my family for always teaching me the importance of education and pursuing my goals.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: RATIONALE

One of the defining experiences of my childhood was attending school for two years in Chile. Before that opportunity I had struggled with particular aspects of mathematics, and often required extra tutoring outside of school. The mathematics instruction in Chile seemed very different to me, as well as the vocabulary that they used to talk about specific aspects of mathematics. In the United States I was introduced to the term "intercept" when defining the place a line crossed the x or y axis. However, in Chile, they used the term "interseccion," which can be translated to intersection to define the same thing. Using the word intersection seemed easier to understand because it's closely related to how the word is used every day and I did not have to learn a new word, I could take the ideas and the definition I had of intersection and apply it to something mathematical. This idea of special mathematical language versus everyday language, and the ways that languages describe mathematics became of particular interest to me as I entered my graduate studies. My experience together with research in the field has contributed to my desire to study differences between Spanish and English languages in mathematics textbooks.

The NCTM principles highlight curriculum as part of a high-quality mathematics instructional program (National Council of Teachers of Mathematics, 2000). According to NCTM (2000), "a curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well-articulated across the grades." Curriculum in schools is the set of courses and their content offered at a school. Under this definition curriculum includes the written instructional materials available for teaching, learning, and assessing students for a given course of study. For much of the world the primary instructional material used is the textbook (Haggarty & Pepin, 2002; Schmidt, Wang, & McKnight, 2005). However, a significant amount of research focuses on teaching including international observation studies like TIMSS (Patrick Gonzales et al., 2000; Schmidt & McKnight, 1998), and there is limited research on the textbook aspect of curriculum.

According to Olson (1980) there are over 250 million textbooks in use by elementary and high school students worldwide. During the course of an average student's education he/she will attempt to absorb a minimum of 32,000 pages of textbook pages. NCTM (2000) put forth the following attributes of quality textbooks: it is important for textbooks to be coherent, focused on mathematics, and well-articulated. According to Yan et al. (2006) research in the textbook area of mathematics education is still inadequate when compared with research in other areas of mathematics education, even though during the school day about 75 percent of a student's classroom time and 90 percent of the time an average student spends on homework will be centered around a textbook (Olson, 1980). International textbook studies in the field of mathematics have focused on Asian textbooks and compared problem samples, physical characteristics, and problem complexity (McCrory, 2006; Peterson, 2008; Yan & Lianghuou, 2006). They have failed to look at how different languages approach mathematics that may contribute to student learning. Since textbooks play such an important role in students' education worldwide, there is a greater need for textbooks studies and in particular studies that focus on the use of language in textbooks.

Textbooks provide an interesting look at what mathematics is offered to students, and how that mathematics is presented. Olson (1980) explains that texts have authority; they are viewed as society's version of valid knowledge. It is the student's responsibility to master the knowledge in the textbook. With this viewpoint, information in textbooks is taken as truth and students cannot disagree with the textbook. For the teacher, the textbook provides a blueprint for content coverage and instructional sequence, which may have contributed to the common phrase, "Teaching by the book." Mathematics textbooks in the United States follow a typical outline of defining key vocabulary, stating theorems, and describing procedures needed for the chapter, showing different examples of how to apply those definitions, theorems, and procedures, and lastly providing different problems for students to practice those procedures. Another common characteristic of textbooks in the United States is their sole focus on one mathematics topic. In many international countries, texts are specific for grade level and may cover several different topics, but in the United States secondary texts are specific for the topic in mathematics.

Textbooks also help position the student towards the mathematics in the classroom through means of language. In most textbooks the language used is complicated for students to understand and requires assistance from the teacher. Herbel-Eisenmann and Wagner (2007) state that by using first person pronouns the textbook author is indicating personal involvement with the mathematics and by using second person pronouns the author is connecting the reader to the mathematics directly.

For many, learning mathematics is inseparable from learning a new language (Schleppegrell, 2007). Students have to be taught the meaning of mathematical symbolism, visual representations, as well as learn how to interpret word problems. Schleppegrell (2007) explains that mathematics symbolism was developed as a way to help express mathematical ideas that written language could not. All aspects of the mathematics language, written language, mathematical symbolism, visual representation and oral language, work together to help the teacher and student construct meaning and interact when discussing mathematics. In fact, researchers and educators have revealed many challenges associated with understanding the language of mathematics. One of the reasons is that unlike other subjects, Mathematics has its own register that includes unique vocabulary, syntax, semantic properties, and discourse (Kersaint, Thompson, & Petkova, 2013). Halliday (1978) defines register as the language we speak or write in certain situations. For a student, the way they use certain words will have a different meaning in the mathematics classroom than the same word might have in an English classroom. A unique characteristic of the mathematics register is the number of terms it contains that have been "borrowed" from everyday English (Pimm, 1987). These words include degree, relation, power, radical, complete, integrate, product, mean, real, imaginary, rational, and natural. In order to fully understand mathematics, students need to be able to navigate between both the English Language register and mathematics language register.

Most research regarding confusing or complex mathematical language has been conducted with the English language, and with the focus of helping English language learners better understand mathematics. Research is limited on whether or not these difficulties with mathematics language can be attributed to language or culture. Haggarty & Pepin (2002) suggest that learners in different countries are taught different mathematics, and have the opportunity to learn different mathematics. It is a mistake to assume that mathematics concepts and symbols are culture free. Mathematics is often considered a universal language; however the notation, algorithms, and approaches used for teaching and learning mathematics vary depending on the country. The idea of register, discussed more later, is powerful in that every language would have its own natural language register as well mathematics language register. There are many examples where a word in the mathematics register conflicts with the meaning in the English language register, but what we do not know is if those exist in other languages. The idea of register can be used to examine other international textbooks to see if they have a way of describing a mathematical idea that would not cause a student to have to shift meanings between registers or if the textbooks are written in a way to help students move from their common language register to the mathematics register more easily.

As research on language (Halliday, 1978; Kersaint et al., 2013; Pimm, 1987; Schleppegrell, 2007) and textbooks (McCrory, 2006; Olson, 1980; Peterson, 2008; Yan & Lianghuou, 2006) as well as my own experiences have indicated, textbooks from different countries may provide a different way of talking about mathematics. This may be attributed to the fact that textbooks in the United States present the mathematics with terms that are only available in the mathematics register, or terms that have different meanings in the mathematics register than they do in the English register. It is the purpose of this study to explore the ways textbooks from various English speaking and Spanish speaking countries present the concepts related to variable. Looking at the issue of textbook language has the potential to help teachers find better and more accessible ways of defining mathematical terms, and better words and phrases when describing the mathematics so that the mathematics is accessible to more students. Since textbooks from various English speaking and Spanish speaking countries will be used, this study also has the potential to examine if the language used to describe the concepts of variable is common across different English/Spanish speaking countries.

CHAPTER 2: THEORETICAL FRAMEWORK

This section is an overview of the stance taken during this research that will guide the methodology and analysis. This study will focus on the interplay between language and mathematics in the context of textbooks. In order to study language and how it describes specific mathematics concepts, the topic of variable in textbooks will be studied. Variable is beneficial to study because of its importance in the K-12 curriculum, as well as the prevalence of language difficulties surrounding it. Textbooks provide accessible data to examine language conflicts with this subject on an international level to help answer the research questions. In order to study mathematics and language, this section will outline previous research on language, as well as research on students understanding of variable. The research questions for this study will also be outlined.

Language in Learning Mathematics

Language is the primary vehicle for learning, instruction, and overall intellectual development (Kersaint et al., 2013). The role that language plays in everyday instruction cannot be underestimated. Teachers teach through language, textbooks convey mathematical knowledge through language, students build understanding through language, and students are assessed through language. This section will outline some different views on mathematics as a language, and student difficulties with mathematical language.

Mathematics as a language

Multiple researchers have claimed that mathematics is a language, and that it shares characteristics like most modern languages (Usiskin, 1996; Kersaint et al. 2013). Mathematics is both oral and written, and can be formal and informal. Just like with everyday language mathematics has the primary purpose of communicating concepts. Usiskin (1996) identifies mathematics as a written, oral, pictorial, and foreign language all of which contribute to his argument that mathematics is its own language. As a written language, the symbols of mathematics, like the letters in other languages, form the written language of mathematics. Everyday language is filled with the language of mathematics; examples include ordinal numbers, telephone numbers, addresses, and monetary values. The oral language of mathematics is important for the understanding of mathematical concepts. Without the oral language of mathematics the student would not be able to internalize the language or connect it with other ideas. The third component of mathematical language is its pictorial form. The pictorial forms of mathematical language include coordinate graphs of functions and relations, Venn diagrams, circle graphs etc. The pictorial forms of mathematics are a powerful aid to understanding the language of mathematics because they provide a different way of communicating the language. The last topic encompassed in Usiskin's (1996) definition of mathematics as language, is its foreign nature. For most students mathematics is a foreign language because it is spoken almost entirely at school and not at the home. As a result, learning mathematics can be likened to learning a foreign language, meaning that the earlier students are exposed to certain ideas the more likely they are to be successful. According to Usiskin (1996) since mathematics is a language and student difficulties can be attributed to our treatment of mathematics as a dead language, students need to be immersed in the language of mathematics in order to learn mathematics fluently.

A difficulty of treating mathematics as its own language is that mathematics does not originate as a spoken language, and people don't have a tendency to "speak mathematically." This gives rise to Pimm's (1987) idea, which will be used for the purpose of this study, that mathematics as a language is a metaphor. Mathematics can be compared to a language because they both have many of the same attributes, like borrowing words from other languages, but mathematics is not a language like English or French that is used every day. To explain the relationship between mathematics and language, Pimm introduces the idea of a register.

English registry vs. mathematics registry

Pimm (1987) draws on the work of Halliday (1978) who defines a register as, "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express those meanings (pg. 65)." The idea of register clarifies the distinction between everyday language and mathematical language. This idea leads to multiple different registers for communication such as a language of the school register, mathematics register, English register, church register etc. According to Halliday (1978) people can adopt a particular register on certain occasions and not on others. It is not just specific technical terms, but also certain phrases and even ways of argument that make up a register.

According to Pimm (1987) part of learning mathematics means to learn to speak like a mathematician, which is acquiring the mathematics register. A particular feature of the mathematics classroom is the widespread use of specialist terms like quadrilateral, parallelogram, and hypotenuse among others. Another particular feature of the mathematics register is the number of terms that it has borrowed from everyday English. Examples include face, degree, relation, power, radical, complete, integrate, legs, product, moment, mean, real, imaginary, rational, and natural. This feature of the mathematics register is not unique, but as Pimm (1987) suggests it is worth noting that in mathematics it is not just nouns and verbs to which the borrowing applies but also a wider range of grammatical constructions. Pimm (1987) states that the ability to express mathematical ideas in everyday natural language leads to the development of the mathematics register so that discourse about mathematical ideas, objects and processes can take place. Halliday (1978) also explains that the mathematics register came about in part through its use of composite words and expressions, such as if and only if, square root, absolute value, greatest common divisor etc. The language of mathematics can be confusing to students who learn these certain words in the English language register but have to apply precise definitions to them in the mathematics register. The difficulty students have with understanding definitions, and the language of mathematics can be attributed to the conflict that exists between the mathematics and English language register. One common example is the use of the word similar. In an English language register similar might take on the meaning of having common characteristics, while in a mathematics register similar is commonly used to describe geometric shapes that do not differ in shape but in size or position.

Herbel-Eisenmann (2002) describes what she calls classroom generated language (CGL), and transitional mathematical language (TML) as forms of bridging language. CGL is language that the teacher and students created during classroom discussions. In her study, she found this to be used when students first started to talk about the slope of a line and would use words such as "slanty" to talk about how steep a line was. She found that TML was used when students were asked to explore various contexts, such as looking at graphs and tables of values associated with certain situations. Herbel-Eisenmann states that TML is related either to a process or a location. She gives the example of a student looking at a table of values and pointing out that the y-values add a certain value as the student moved vertical down the table, when the x-values increased by 1. The student did not call this the "slope" but rather used a phrase like "add 3" or "goes up by 3." According to Herbel-Eisenmann TML occurred whenever the student or teacher contributions were stripped of the context.

Difficulties with Language. Since there are obvious differences between the meaning of words in the English language register and the mathematics register, the result is large alterations in the intended meaning for students. Although mathematics is considered by most to be a universal language, the idea of register suggests otherwise. For students in the United States, most mathematics classes are conducted in a mixture of the two registers. Adams (2003) explains that in order to make communication clearer, the reader has to develop meaningful, correct, and applicable definitions of mathematical terminology. These definitions, whether formal or informal, allow students to appropriately apply the mathematics vocabulary they encounter when reading mathematics. If a student is not using the appropriate register when thinking of certain words they will misunderstand or misinterpret that piece of the mathematics. Macgregor (2002) highlights these difficulties with the use of the English word "more". In English speaking communities children learn the word "more" to mean again or extra at a young age. This differs from the use of more in the mathematics classroom. In the mathematics register children have to recognize "more" as part of the comparative "more than". The words "more than" indicate that there are two quantities, one larger than the other, a meaning that is not typically found in the English register. Schleppegrell (2007) claims that it is easier for students to learn vocabulary that is mathematical than to learn the technical meanings for words that students already know in other contexts.

Student difficulties with language are furthered by other potential pitfalls as identified by Thompson and Rubenstein (2000). In an attempt to sensitize teachers to potential language problems students may encounter in the mathematics classroom, Thompson and Rubenstein identify multiple areas of difficulty. These categories include not only the idea of register but how some words have more than one mathematical meaning like square and range; how modifiers change the mathematical meanings in important ways like the difference between root and square root; and how a single English word may translate into Spanish or another language in two different ways, like round (redondear) as in “round off”, or round (redondo) as in circular. These potential pitfalls are of particular interest in this research study, since a comparison of differences between mathematical language in English and Spanish is to be explored.

Student difficulties with written language have also been well documented. Adams (2003) explains that reading and understanding the written language of mathematics is a multifaceted task because the student is challenged to acquire comprehension and mathematical understanding with fluency and proficiency through the reading of numerals and symbols in addition to words. It is important to know students’ abilities with their native language because mathematics texts may include complex language aspects that may confuse students’ understanding of mathematics. Abedi & Lord (2001) modified the language aspects of mathematics tests, while keeping the mathematics and the mathematics terminology the same. The linguistic features that were modified included (1) unfamiliar or infrequent words were changed, (2) passive verb forms were changed to active, (3) long nominals were shortened, (4) conditional clauses were replaced with separate sentences, (5) relative clauses were removed, (6) complex question phrases were changed to simple question words, and (7) abstract or impersonal presentations were made more concrete. The data from the Abedi & Lord study showed that unfamiliar or infrequent vocabulary and passive voice constructions affect the comprehension of students.

Schleppegrell (2007) also discusses difficult aspects of the language used in mathematics classrooms. Aside from the already established ideas of the conflict that exists between the mathematics register and the English register, Schleppegrell further highlights this difficulty by explaining that students also need to learn the language patterns associated with words with conflicting meanings and how they construct concepts in mathematics. Similar to Abedi and Lord (2010), Schleppegrell (2007) explains how complex grammar affects students learning of the mathematics. This complex grammar includes the use of long, dense noun phrases such as "the volume of a rectangular prism with sides 8, 10, and 12 cm." In order to solve problems, students have to make sense of these noun phrases. These phrases are typically used in relational processes that are constructed in clauses with be and have. The verbs be and have construct different kinds of relational processes that can be identified as either an attributive process or an identifying process, both of which are common to mathematics. An attributive process constructs information about membership in a class, like saying "Three and four are factors of twelve." Whereas an identifying process on the other hand, constructs relationships of identity and equality like, "A prime number is a number that can only be divided by one and itself." A key distinction among the two types of processes is that attributive clauses classify objects and events, while identifying clauses define technical terms and provide a bridge between technical and less technical ways of presenting knowledge in mathematics by enabling two formulations to be presented as equivalent. This is an important piece of language to consider for this study because in Spanish the verb "is" has two different forms, construing different meaning relationships that may help or hinder the teaching and learning of specific mathematics concepts.

Since most math textbooks will be in the mathematics registry, the question can be asked, Are there reasonable ways to write portions of the textbook in a bridging registry and what would that look like? Herbel-Eisenmann (2002) gives examples of phrases that are more complex and lie in the mathematics registry and how the same idea could be described in a bridging registry. Her example is with the idea of slope, and key language that is used in the mathematics registry is the coefficient, the slope, and rise/run. To describe the same ideas in the bridging registry textbooks could use language like constant pattern, what it goes up (or down) by, add __ each time, increasing by __ each time, as x goes up by 1, y goes up by __, the number times x, the "m", slantiness, and stair steps. The language in the bridging registry would be what Van Dormolen (1986) considers a non-kernel because it is a more explanatory method of describing a particular aspect of mathematics.

Mathematical Concept

The next topic of importance to this study is the particular concept of mathematics that will be investigated. Since the topic of variable is prevalent in the K-12 curriculum I have chosen to look at the different ways language is used to describe the various ideas related to variable. This section will outline the different ways students understand variable.

Variable

The concept of variable is typically introduced to students in pre-algebra with a basic definition that does not adequately define it. According to Bell (1951):

> "to state fully what a variable is would take a book. And the outcome might be a feeling of discouragement, for our attempts to understand what variable means would lead us into a morass of doubt concerning the meaning of the fundamental concepts of mathematics (pg. 101)."

The definition of the word variable has evolved over time. In a textbook from the 1950's a variable was defined as a changing number (Z. Usiskin, 1988). Nowadays the definition has changed so that many students think all variables are letters that stand for numbers. However the concept of variable itself is multifaceted and not well defined for students. The following list of common examples of the uses of variable was developed by Usiskin (1988):

Example	Common Name
$A = LW$	A Formula
$40 = 5x$	An equation to solve
$sinx = cosx * tanx$	An Identity
$1 = n * (1/n)$	A property
$y = kx$	A function of direct variation

Figure 1. Common uses of variable according to Usiskin

In each of the above examples the way the variable behaves is different. The uses of variable describe relationships of knowns, unknowns, arguments of functions, general numbers, and variable itself. In Geometry class, letters often represent points like in the example triangle ABC and in logic the letters p and q often stand for propositions. This is confusing for students who hold the popular view that a variable is just a letter that stands for a number. Variable can also be used from a statistics perspective and another possible definition for this use of variable is, “A variable is a quantity that assumes one of multiple possible values and measures an attribute of a person, place, thing, or idea.” This perspective of variable captures the idea that “height” or “weight” are variables in the sense that they would vary from person to person in statistical study. The mistake that many textbooks make when defining variable is trying to fit the idea of variable into a single conception because it oversimplifies the idea and in turn distorts the students’ view of variable and algebra.

Like the concept “number sense,” a sound understanding of numbers and their nature and the nature of their operations, Arcavi (1994) argues that such a thing exists in algebra and coined the phrase “symbol sense.” Symbol sense includes but is not limited to an understanding of how and when symbols can and should be used, an ability to “read” symbols, and sensing the different roles symbols can play in different contexts. The idea of symbol sense fits closely with students’ trouble with variable because of the different ways variables are used. Students need to understand when they can use variables, they need to be able to manipulate the variable as something abstract but then also read the variable with meaning attached, and understand that depending on the question the role of the variable might be different.

Research Questions

Since there are obvious situations where the mathematics register and the English register are at odds, it is important to see if that conflict occurs consistently in the English language as well as in other languages. Using Pimm’s (1987) idea of mathematics as a language is a metaphor will be used for this study. His idea uses register to examine inconsistencies between English language and mathematical language, and it would be insightful to see if these inconsistencies exist in other English speaking countries, as well as in Spanish. The topic of variable is full of language difficulties for students. It would be beneficial to see the different ways language is used across countries that could potentially impact students understanding of those topics.

In order to provide some insight into the ideas mentioned in the previous paragraph, this study focuses on the following research questions: (1) In what registry are chapters related to variable in textbooks typically written? (2) Do textbooks use language that bridges the gap between the mathematics registry and the English language registry for students? (3) Are the uses of the English (Spanish) language to describe concepts such as variable common across different English (Spanish) speaking countries?

CHAPTER 3: LITERATURE REVIEW

In this chapter the existing literature on textbook comparisons, international textbook comparisons, literature on the language of textbooks, as well as the lack of literature on international language comparisons will be discussed. First, different textbook comparison studies regarding textbook structure will be presented. The next section will focus on international comparisons and international comparisons of textbooks since this study focuses on international textbooks. Lastly, a study on the ways textbooks position the student through language will be discussed.

Structural Analysis of Textbooks

This section discusses various comparison studies of textbooks including various ways of examining textbook as well as a look at an international comparison study of textbooks. First, the mechanisms by which McCrory (2006) critiques Mathematics for Elementary Teachers textbooks will be addressed. Next, a comparison by Peterson (2008) of textbook characteristics between Japanese and American textbooks will be presented.

Structural Analysis

McCrory (2006) examined Mathematics for Elementary Teachers textbooks by characterizing the books based on length, looking at the coherence, rigor, and claims. Since textbooks differ in their purpose and design, McCrory characterized textbooks between books that cover every topic that might be covered in a K-8 classroom as encyclopedic, books that are more narrative, books that emphasize particular aspects of mathematics, and shorter books that were written for a specific course or sequence of topics. Next, McCrory compared the mathematical coherence in the different textbooks. This category judges how well the different pieces of mathematics fit together in the chapter and throughout the whole textbook itself.

Another characteristic of the textbooks that was examined was the mathematical rigor. McCrory looked at the correctness, completeness, sparseness and elegance of the textbooks. In McCrory's study the rigor of the textbooks was found through looking at the definitions, since the definitions provided building blocks for the rest of the chapter or unit. The last aspect that McCrory used when characterizing the textbooks was the claims that the text made. Since their textbook study revolved around mathematics for prospective elementary teachers, the textbooks they used made explicit or implicit claims about the "right way" to teach mathematics. For McCrory (2006), these claims were not grounded in research because research cannot identify the "right way" to teach a topic of mathematics and thus hindered the texts reliability.
The study by McCrory is unique when compared to other studies presented later in this chapter. The research by McCrory (2006) is an example of a way to use the structure of textbooks to categorize textbooks. It gave an overview of different categories to consider when looking at the overall structure of textbooks, but does not examine much of the content of the textbooks or how the language of textbooks positions the mathematics in the textbooks.

International Structural Analysis Comparison. Peterson (2008) examined junior high school mathematics textbooks in Japan in comparison with junior high mathematics textbooks in the United States. In his study the general organization and structure based on the table of contents was examined in both the Japanese and American texts. According to Peterson (2008) there are six publishers who publish textbooks used for junior high school in Japan. Data is presented for all 6 Japanese textbooks. Six commonly used textbooks in the United States were also discussed in order to compare differences and similarities. Every textbook from the United States was significantly larger than any of the Japanese textbooks which could be attributed to the publishers attempting to write one textbook that satisfies each state's standards. Two contributing factors for the Japanese textbooks being so small is the scrutiny of the textbooks by the Japanese ministry of education and the parents' expectation that their students learn all of the content of the textbook. When examining the table of contents one major attribute of the U.S. textbooks was that they all have almost double the number of chapters than the Japanese texts. Another observation was that the curriculum in the United States is divided into the different subjects of pre-algebra, algebra, geometry, algebra II and precalculus. In Japan the curriculum is integrated, exemplified by the fact that two chapters out of 6 or 7 chapters in the seventh, eighth, and ninth grade textbooks are dedicated to geometry, which Japanese instructors use to help teach the concepts of linear and quadratic equations. A further observation about the textbooks is the careful use of language as students' progress through the different grades. The word linear in conjunction with equations is not even used until high school or college, and linear equations are referred to as first-degree equations in seventh grade. This pattern continues when students encounter quadratic functions, which are referred to as equations of the second degree.

Peterson (2008) provided an example of a comparison study between the structure of textbooks in the United States as well as the structure of textbooks in Japan. This research provides an example of how the structure of textbooks from different countries can be compared. It also introduces some interesting differences between the languages used to talk about mathematics in Japan versus how language is used to talk about mathematics in the United States. It is interesting to note that when comparing the two studies that focused on the structural analysis of textbooks, McCrory (2006) found that textbooks for prospective elementary teachers varied in their structures, whereas Peterson (2008) found that textbooks in the United States were similar across all categories that he examined, and the textbooks in Japan were similar across those same categories.

International Comparison Studies

This section provides some information on the major studies that have compared mathematics education internationally. First some highlights from the 1995 Third International Mathematics and Science Study (TIMSS) and 1999 TIMSS-R will be presented, as well as some things that can be learned from the TIMSS study.

TIMSS Study

The 1999 TIMSS study focused on the mathematics and science achievement of eighth-grade students in 38 participating nations, and is a follow up to the 1995 TIMSS studies. The 1995 TIMSS studies assessed the mathematics and science achievement of U.S. students in comparison to their peer internationally at three different grade levels. The 1995 results revealed that U.S. fourth grade students performed well in both mathematics and science, eighth grade students performed near the international average, and twelfth grade students performed below the international average and among the lowest of the TIMSS nations (P. Gonzales et al., 2000). The 1999 TIMSS results found that U.S eighth-graders exceeded the international average of the 38 participating nations in math and science. Specific to mathematics, U.S. eight graders outperformed their peers in 17 nations, performed similar to their peers in 6 nations and performed lower than their peers in 14 nations (P. Gonzales et al., 2000). The 1999 TIMSS study also noted differences in teaching and the curriculum. For example, eighty-six percent of U.S. eighth-grade students reported that they worked from worksheets or textbooks on their own almost always, which was higher than the international average of fifty-nine percent. A higher percentage of U.S. eighth-grade students reported that they could almost always begin their mathematics homework during class than their international peers as well.

Schmidt and McKnight (1998) claim that there are certain things educators can learn from the TIMSS study. The authors claim that the lowering of scores as grade levels increase is consistent with information gathered as part of the curriculum, textbook, and teaching data. The curriculum in the United States consistently covered more topics than did the curriculum in almost all the other TIMSS countries. The U.S. mathematics textbooks included more topics and were the largest (by page count and size) among the TIMSS countries. For example, in geometry the performance of U.S. students was much lower by eighth grade than it was in the fourth grade. Analysis of textbooks showed that U.S. treatment of geometry was comparatively weak after fourth grade.

The articles on the TIMSS studies are helpful in that they provide an idea of how the United States compares to other countries in mathematics, but also because they included curricular materials as part of their comparison. Authors such as Schmidt and McKnight (1998) seem to suggest a correlation between the scores of students on the TIMSS test to the content of their textbooks. This demonstrates a need to look at the content of textbooks from different countries to see how they compare.

International Textbook Comparison Studies

This aspect of textbook studies focuses on the content in the textbooks themselves. This section will highlight a comparison of textbooks internationally including the types of problems that students have access to in different countries, a comparison of international textbooks themselves and the types of conceptual questions they ask of students, as well as a comparison of problems that followed selected content in Chinese and U.S. textbooks.

Mathematics made available in textbooks. Haggarty and Pepin (2002) examined one chapter of geometry textbooks in lower secondary classrooms in England, France, and Germany. Haggarty and Pepin decided to focus on textbooks because they felt that a textbook reflects national curricular goals and therefore reflects national curricular traditions. Their study first investigated similarities and differences of mathematics textbooks, on the chapter of measuring angles, in three countries in order to understand how textbooks address the range of commonly taught topics in mathematics education. The study found that in French textbooks not only is the mathematics comprehensive, on the topic of measuring angles, but it is also presented in cognitively challenging situations. Measuring angles was discussed in three different sections of a 14-page chapter on angles. Characteristics of this chapter included the use of technical vocabulary, complex mathematical notation, and complete explanations before the student exercise section. Additionally students were asked to make conjectures and generalizations and there were connections made with other topics of mathematics. The chapter on measuring angles from Germany lasted 18 pages and included dense text and diagrams per page, technical vocabulary, a complete explanation preceding the student exercise section, and low-level application questions. The chapter on measuring angles from England is a 24-page chapter and included no explanation on how to use a protractor, low level questions, no notation for naming angles, only 21 opportunities for students to draw or measure angles, and only one question put in a context. Based on their findings, Haggarty and Pepin (2002) concluded that pupils across these three different countries have different opportunities to learn mathematics based on the content of the textbooks used.

The Haggerty and Pepin (2002) study is of interest because it compares one topic of mathematics from textbooks in each of the three countries. Their research demonstrated that textbooks vary in their presentation of mathematics in different countries, and suggests that students in different countries have different opportunities to learn mathematics. The study by Haggerty and Pepin made note of some of the language differences between the countries but did not expand on how the language helped present the mathematics in the chapter.

Problem Types. The study presented by Yan and Lianghuo (2006) compared how selected mathematics textbooks from mainland China and the United States represent various types of problems for classroom teaching and learning. For their study a problem was identified as a situation that requires a decision and/or answer, no matter if the solution is readily available or not to the student (Yan & Lianghuou, 2006). To examine their research question, seven classifications of problems were identified, including routine vs. non-routine, traditional vs. non-traditional, open-ended vs. closed ended, application vs. non-application, single-step vs. multiple-step, sufficient data vs. extraneous data or insufficient data, and problems in mathematical form vs. problems in verbal or visual form. The study found that the U.S. textbooks provided more problems than the Chinese textbooks, and that the U.S. textbooks place more emphasis on students' practice as evidenced by the higher ratio of exercise to text problems. Based on their study, the researchers drew the conclusions that the textbooks from both countries provided students with plenty of problems but the Chinese texts could do a better job varying the kinds of problems presented to students while the U.S. texts could provide more challenging problems in the textbooks.

Li (2000) used seventh grade mathematics textbooks from China and the United States to compare all relevant problems that followed the content presentation of addition and subtraction of integers. This included problems or problem components that had no accompanying solution, or in other words, the problems that the students are asked to practice. These were presented in sections labeled “check understanding,” “exercises,” or “application.” In order to analyze these problems Li (2000) developed a three dimensional framework consisting of (1) mathematical feature, (2) contextual feature, and (3) performance requirements. The results of this study by Li indicate that American textbook problems varied in problem requirements and emphasized conceptual understanding more than Chinese textbook problems; however Chinese textbooks included problems that required a higher level of mathematical content than American textbooks. The differences highlighted in Li’s study indicate that American textbooks emphasized students’ conceptual understanding while the Chinese textbooks do not. The study by Li is of note in relation to international testing since American students score dramatically lower than their Chinese peers. Li’s study provided a glimpse of the potential relationship between the expectations of students from textbooks and their performance in mathematics.

The studies by Li (2000), and Yan and Lianghuo (2006) are useful when considering a textbook study because of their international perspective and their methodology for analyzing the textbooks. In order to examine the content of the textbooks the researchers developed different categories for the problems, and then based on those categories identified similarities and differences between the textbooks. These studies are similar to this study, in that the content of textbooks will be analyzed, but this study will identify similarities and differences and categorize the language of textbooks.

Language of Textbooks

This aspect of textbook studies focuses on the language used in textbooks. This section will outline a study that was conducted on the language of textbooks and how it relates to students’ study of mathematics.

Eisenmann and Wagner (2007) studied the way the form of textbook language and accompanying images might position students in relation to mathematics, to their classmates, to their teachers and to their world outside the classroom. Of particular interest is the method by which the researchers studied the way the form of language might position students in relation to the mathematics. By focusing on personal pronouns and modality, the researchers were able to see who the text recognizes as the people associated with the mathematics. First person pronouns, like *I* and *we,* indicate an author's personal involvement with the mathematics. The use of the second person pronoun *you*, also connects the reader to the mathematics because the textbook author is speaking to the reader directly. Eisenmann and Wagner (2007) found that in their sample of textbooks first person pronouns were entirely absent and the second person pronoun *you* was used in either (1) you + a verb or (2) an innominate object + an inanimate verb + you. The second way the pronoun you was used suggests an "absolutist image" of mathematics, portraying mathematical activity as something that can occur on its own, without humans.

Eisenmann and Wagner (2007) provide one possible method for analyzing how the language of mathematics textbooks can influence the presentation of mathematics. Their study introduced some interesting aspects of language, such as pronouns, that can affect how students may react to the mathematics they are presented. While their study did focus on language and how it positioned students toward the mathematics it did not show any similarities or differences between textbooks, or incorporate an international perspective.

Conclusions

These studies suggest various way of performing a textbook analysis, as well as different results from the different methods of analysis. Research has captured differences between textbook structure, international textbooks, problem types, and how language is used in textbooks. However, research is limited on how language presents mathematical ideas in the United States, and abroad. In order to expand research on the documented differences between everyday language and the language of mathematics in textbooks this study focused on the following research questions: (1) In what registry are chapters related to variable in textbooks typically written? (2) Do textbooks use language that bridges the gap between the mathematics registry and the English language registry for students? (3) Are the uses of the English (Spanish) language to describe concepts such as variable common across different English (Spanish) speaking countries?

CHAPTER 4: METHODS

This chapter outlines the methods of data collection and analysis for this study. I describe the setting and documents of the study, the type of data collected and how it was collected, and how the data was analyzed.

Documents

This study focused on analyzing textbooks from three different English speaking countries (Australia, Canada, United States) and three different Spanish speaking countries (Mexico, Colombia, Chile). Textbooks from each of the countries were picked based on information from educators in the countries and research an investigation of the most common publishers. In order to conduct the study on the topics of variable, textbooks were purchased from all six countries at the seventh, eighth, and ninth grade level to ensure these topics would be discussed at some point. Analyzing these textbooks was helpful in answering my research question because, as discussed in my rationale, textbooks are the primary instructional tool in the classroom and children are exposed to textbooks in each of these countries. By looking at the different countries I was able to determine if the language used is consistent in the three English speaking countries and the three Spanish speaking countries or if there were subtle differences in language use between the countries. To continue with the study it is important to understand a brief history on the education systems in each of the countries in order to see how the textbooks fit in the education system as a whole. I will first discuss the education settings of the three English speaking countries and then the three Spanish speaking countries that were a part of the study.

Australia

Similar to the education system in the United States, schooling in Australia starts with a kindergarten or a preparatory year followed by 12 years of primary and secondary school. Australia has a national curriculum framework to ensure high academic standards across the country, meaning that regardless of whether a school is part of the Government, Catholic or independent systems, they are required to adhere to the same curriculum frameworks of their state or territory (Citizenship, 2013). Implementation of the Australian curriculum is the responsibility of states and territories. In New South Wales (NSW), curriculum is delivered via syllabi approved by the Minister for Education. The Board of Studies in NSW develops syllabi for use by all NSW schools (Government, 2013). Public schools are free to the public, however the schools may charge for attendance fees, stationery, textbooks, uniforms, and school camps.

Australia has begun implementing The Australian Curriculum which seeks to establish goals and knowledge that every Australian student should have access to. According to the ACARA (Australian Curriculum Assessment and Reporting Authority) the Australian curriculum for mathematics seeks to provide students with essential mathematical skills in number and algebra, measurement and geometry, and statistics and probability. It does not appear that the government or the ACARA have published a textbook that schools are mandated to use (ACARA, 2013).

The textbooks from Australia that were used for this study were published by Macmillan Education Australia. The textbook series claims to have been revised to address all the content descriptions and achievement standards contained in the Mathematics Australian curriculum.

Canada

Public education is provided free to all Canadians who meet various age and residence requirements. Each province and territory has departments/ministries responsible for education, headed by a minister who is almost always an elected member of the legislature and appointed to the position by the government leader of the jurisdiction. The ministries and departments provide educational, administrative, and financial management and school support functions, and they define both the educational services to be provided and the policy and legislative frameworks.

Local governance of education is usually entrusted to school boards, school districts, school divisions, or district education councils, similar to the United States. Their members are elected by public ballot. The power delegated to the local authorities is at the discretion of the provincial and territorial governments and generally consists of the operation and administration (including financial) of the group of schools within their board or division. The delegated local authorities are also responsible for, curriculum implementation, personnel, enrolment of students, and initiation of proposals for new construction or other major capital expenditures. Normally, all publicly funded schools are under the authority of their local district school board. These school boards follow a common curriculum set up by the province in which the board resides. Like in the United States the school districts decide the curriculum that the schools will follow (The Council of Ministers of Education).

There are significant differences between standard-setting in Canada and the United States. In Canada, standards are increasingly required by policy-makers to define public expectations for student performance in programs or institutions. Rather than making "high-stakes" decisions about individual students and their life chances, such as Grade 12 exit exams, provincial ministries are setting standards for making judgments about systemic rather than individual performance. The performance standards, along with scoring rubrics, results and exemplars of student performance are subsequently held up for emulation, and not yet generally used in direct application to determine individual student marks or to make program placement decisions (Hunter).

The textbooks from Canada for this study were the MathLinks books for grades seven, eight, and nine. The books are published by McGraw-Hill Ryerson Limited, one of the leading publishers in Canada, which is a subsidiary of the McGraw-Hill companies.

United States

Education in the United States is mainly provided by the public sector, with control and funding coming from three levels: local, state, and federal, in that order. Unlike most counties the U.S. department of education (comparable to a ministry of education) does not develop curriculum, set requirements for enrollment, determine state education standards, or develop testing to measure whether or not states are meeting their education standards. Those responsibilities are handled by the various states and districts as well as by public and private organizations. School districts are usually separate from other local offices, with separate officials and budgets. Educational standards and standardized testing decisions are usually made by state governments and as a result curriculum in the United States varies widely from district to district. In many schools in the United States, the curriculum taught in public schools is influenced by the textbooks used by the teachers. In some states, textbooks are selected for all students at the state level (Education, 2010).

In the United States textbooks publishers publish what they think will sell. In many cases textbook publishers will publish textbooks that correspond with as many of the different state standards as possible so they don't have to publish individual textbooks for each state. The selection of textbooks varies from district to district as there is no national curriculum. Some states within the United States come up with an accepted list of textbooks from which each school district can choose.

The textbooks that were used for this study were the Glencoe textbooks series by McGraw-Hill. This study focused on the pre-algebra, and algebra textbooks from this series. Two textbooks were used from the United States because there is not a standard sequence that every student follows from grades seven to grade 9. In other words students vary in the course they take in certain grades. These textbooks represent a commonly used textbook series in the United States and claim to meet the standards from various states. The textbooks also claim to have complete alignment with the Common Core State Standards. These textbooks were chosen because it is a broadly used textbook series and they are similar to textbooks that are common in school classrooms across the United States.

Chile

In Chile the Ministerio de Educacion (Ministry of Education) is the highest authority. They define educational policies, guide and supervise their application, ratify and officially prescribe norms and general plans for the sector, and supervise the correct administration and management of education. Chile is organized into 13 regions, and within each of the 13 regions a Secretaria Regional Ministerial de Educacion (Regional Secretariat) is charged with the planning, supervision, and evaluation of education within their region. Primary education (Educacion General Basica) is divided into two cycles. The first cycle includes grades 1-4, where students meet five days a week for five hours a day. The curriculum in this cycle is only a flexible guide for the teacher, with the exception that Spanish and mathematics must be assigned a minimum of five weekly lessons. The second cycle of primary education in Chile includes grades 5-8. The curriculum corresponding to these grades consists of 30 hour-long weekly lessons in the 5th and 6th grades, and 33 hour-long weekly lessons in the 7th and 8th grades. They include the same subjects and activities as the first cycle, plus a second foreign language in 7th and 8th grades. At least six hours must be assigned to Spanish, five to mathematics, and four to history and geography (Longo, 1992).

Chilean textbooks must be approved by the minister of education. The entire country of Chile uses the same textbook, and schools are even subject to drop in visits by workers from the ministry of education's office to ensure that the schools are implementing and using the correct textbooks.

The textbooks used for this study were the educacion basica texts from grades seven and eight as well as the educacion media primero textbook for grade 9. All textbooks have a seal of approval from the consejo superior de educacion (Higher Council for Education). The textbooks are published by Santillana, one of the main educational publishers in South America.

Colombia

Education in Colombia is under the jurisdiction of the Ministerio de Educación Nacional (Ministry of National Education), which sets the standards for both public and private education at all levels. The Columbian government finances 85% of elementary education, 60% of secondary education and 40% of higher education. Education in Colombia is divided into the following categories: (1) Preschool Education, Pre-kindergarten and kindergarten (Educación Preescolar), (2) Elementary Education, Grades 1-5 (Educación Primaria), (3) Secondary Education, Grades 6-11 (Educación Media), and (4) Higher Education, University studies (Educación Superior) (Services, 2004). In Colombia textbooks are not issued by the school or put on loan by the school, meaning that people must buy their own books (Site, 2008).

The textbooks that were used from Colombia were published by Proyecto Se for grades seven, eight, and nine. The textbook series claims to be one that is typical in Colombian schools. The textbook series also includes a list of Colombian standards that are covered.

Mexico

In Mexico, basic education is normally divided in three steps: primary school (*primaria*), comprising grades 1-6; junior high school (*media*), comprising grades 7-12; and higher education (*superior*). In 1992, the Secretariat of Public Education officially increased compulsory education from completion of primary school (grade six) to completion of lower secondary school (grade nine). Since 1993, the Secretaria de Educacion Publica (SEP) has gradually been devolving educational authority to the 31 state ministries of education, and now plays a role that is more supervisory than regulatory in nature. However, the SEP continues to directly administer basic education (primary and lower secondary education) and teacher education in the Federal District of Mexico City. In addition, the SEP continues to distribute free textbooks to primary and lower secondary schools throughout the nation, and the states are obliged to teach the curriculum set out by the SEP (Rolwing, 2006). Since the late 1950s, Mexico's National Commission of Free Textbooks has printed millions of books that are mandatory for both private and public schools (Licon, 2013).

This study used textbooks published by Santillana for grades secondary one, two, and three. Santillana is a leading publisher in South America; however, these particular textbooks are printed in Mexico and are used in a sequential order for students in grades seven, eight, and nine.

Data Analysis

In order to answer the research questions a three part analysis was conducted across all of the textbooks. I will first describe each of the three passes through data and the purpose of each. After all three passes are presented the results from a language verification study will be presented to verify the author's Spanish speaking ability.

Pass 1

Part of pass 1 included identifying the places in the text where the idea of variable was discussed. Since the textbooks for this study range from grades 7-9 most have at least one chapter where the ideas of variable were defined. As part of the first pass I used part of Van Dormolen's textual analysis of the content of mathematics textbooks. The first thing that I did was identify the *kernels* within the topics of variable. Van Dormolen (1986) defines a *kernel* as a general expression that has to be learned as knowledge. He explains that *kernels* form the "hard core of mathematics," and include theorems, rules, definitions, methods, and conventions. At the same time textbooks also have *not kernels* which are the problems, exercises, examples, explanations, etc. that serve to make the "hard core mathematics" understandable, or to show how the mathematics can be applied. Van Dormolen (1986) explains that texts should have a global perspective; this means that there should be a logical progression in the sense that students need to know some kernels in order to be able to understand others. A textbook should connect the knowledge the students have learned with the knowledge they will learn in the future.

For the analysis the first step was to identify the kernels within the subjects of variable throughout the textbook up through the chapter where a discussion of linear equations occurred. If the textbook breaks up solving equations the data collection stopped after one section of the solving equations unit. The purpose of looking at multiple chapters was to see how the textbooks introduced the critical concept of variable and how they used it in later chapters. To do so, I looked at the explicit definitions of variable in the chapter. Any theorems that were a focus of the chapter were also identified as kernels. After identifying the kernels, I identified the key nouns or nominal phrases that made up the kernel. Using the nouns and noun phrases that I found, I put together a lexical chain (refer to Table 1 or Table 2) compiling instances where each noun phrase was referred to implicitly or explicitly. An instance referred to the sentence surrounding the explicit use of a noun phrase or the implicit use of a noun phrase.

Since the use of a noun phrase can either be explicit or implicit I defined both explicit and implicit as a guideline for identifying instances that used each noun phrase. An explicit reference occurred when the textbook stated the nouns/noun phrases directly. Meanwhile, to identify the implicit references to the nouns/noun phrases I read the sections of the textbook that were focused on variable and looked for instances where a synonym to the noun/noun phrase was used but the noun/noun phrase itself was not explicitly used. Additionally, to identify an implicit reference I also looked for places where the textbook used a letter with no explanation or explicit reference to the nouns/noun phrase. This occurred in places where the textbook used the letter in an example of a formula or expression. In the cases where a letter was used in an example if the sentence surrounding the letter or use of the letter was referring to the noun phrase without explicitly stating the noun phrase then the instance was coded as implicit. The question that helped guide the coding of implicit instances was: Is this sentence talking about variable/letter/number (whichever noun/noun phrase) here without stating it directly? For example under the constant kernel the noun phrase *decreases value* was identified. The instance, “Ask yourself 15 less what number makes 5? Or 15 minus 5 equals what number?” was identified as an implicit reference to decreases value because the portion that reads “15 less what number” is a synonym to “what number *decreased in value* by 15.”

The last part of my Pass 1 analysis was to use the lexical chain to track how the language was used in textbooks. I used the chart as a means to determine if the language used to describe a particular aspect of mathematics was in the English language registry, the mathematics registry or a bridging registry. My unit of analysis was the instances I identified that used the nouns/noun phrases found in the kernel. After the *kernels* were identified, and the important noun phrases identified from the kernel, I used a method described by Herbel-Eisenmann and Otten (2011) to see how words are put in relationship with each other. To do so Herbel-Eisenmann and Otten (2011) created lexical chains, allowing the researchers to trace the main mathematical idea through an entire string of text. A lexical chain is created by using the central ideas of the text, identified primarily through the nouns, and noun phrases. In creating the lexical chain, the string of instances for noun phrases in the table allows the researcher to see how the words that reference a specific mathematical idea shifted and changed, or if the word, or noun phrase is consistently used within the same registry (B. A. Herbel-Eisenmann & Otten, 2011). Once the lexical chain was created using the *kernels* from the first step in the analysis, I was able to see how the words used to describe specific mathematics topics transition throughout the text and if the words used were a part of the mathematics register or the bridging language register. The lexical chains were useful in determining the registry of a word because words have meaning only in relationship to other words and the lexical chain organized specific meanings a word taken over time.

If a word were constantly being used in relationship to solely mathematical vocabulary or ideas it fell in the *mathematics registry*. When the textbook presented a mathematical definition in symbolic form without any attempt to explain to students what was happening, this was coded as the mathematics registry. When the textbook presented a definition to students but did not explain or try to have students think about the concepts surrounding that definition, that definition (or sentence) was coded as the mathematics registry. When the textbook mainly used words and phrases that made sense in everyday English, it was coded as English registry. Similarly if the instances used most words and phrases that made sense in Spanish, it was coded as Spanish registry.

On the other hand, sentences that attempted to conceptually explain to students the ideas behind mathematical definitions were coded as a *bridging registry*. Multiple criterion were used to determine if an instance used a bridging language. Instances that substituted an English word to replace a math word will be coded as bridging registry, this was referred to as the replacement criterion. For the purpose of this study a mathematics word or term is one that has been formally defined in mathematics. Another criteria for bridging registry was if the textbooks explain what a particular noun/noun phrase means in mathematics using a particular context unique to the textbook such as examples that the textbook might be using. This criterion was labeled as the specific context criterion. The last criterion that was used to determine bridging registry was whether or not the textbook intended that the students reflect on the mathematics by asking them a question about the conceptual underpinnings of the mathematics or why the mathematics works the way that it does. The last criterion was labeled as the reflection criterion.

If multiple kernels from one textbook series had the same nouns/noun phrases, then I analyzed a specific noun phrase and decided which of the kernels that noun/noun phrase fell under. An analysis was done on the noun phrases to determine which kernel the instance best fit under. To do this, all of the instances that referred to the shared noun phrase were gathered and then matched to the kernel they best fit. For each instance the questions that guided this analysis were: How is the textbook using the letter/number? Or what is the textbook trying to describe with this statement? The answers to these questions were then matched with the kernels from which the noun phrases were drawn. If the answer to the above questions satisfies multiple kernels then the instance in question fit under both kernels. This happened when the instance had multiple noun phrases that fit with different kernels.

Kernel: In algebra, variables are symbols used to represent unspecified numbers or values. Any letter may be uses as a variable (pg. 6).

Definition				
	Explicit Nouns or Noun Phrases within the Kernel			
	Variable	**Symbols**	**Unspecified numbers/unknowns**	**Letters**
Explicit Uses	A term of an expression may be a number, a variable, or a product or quotient of numbers and variables. (pg 5)		Variables can represent quantities that are known and quantities that are unknown. (pg 6)	The term that contains x or other letters is sometimes referred to as the variable term. (pg 5)
	The term that contains x or other letters is sometimes referred to as the variable term. (pg 5)			
	A term that does not have a variable is a constant term. (pg 5)			

Figure 2. Explicit Uses of the Explicit Noun Phrases within the Kernel

Definition				
	Explicit Nouns or Noun Phrases within the Kernel			
	Variable	**Symbols**	**Unspecified numbers/unknowns**	**Letters**
Implicit Reference to idea or phrase			a number t more than 6 (pg 6)	a number t more than 6 (pg 6)
			the sum of a number and 14 (pg 7)	
			7 more than 11 times a number (pg 7)	
			two fifths of the square of a number j	two fifths of the square of a number j (pg 7)
			6 less than a number t (pg 7)	6 less than a number t (pg 7)
			1 minus the quotient of r and 7 (pg 7)	1 minus the quotient of r and 7 (pg 7)
			n cubed increased by 5 (pg 7)	n cubed increased by 5 (pg 7)
			If x is the number of cell phone minutes used above the plan amount and y is the number of megabytes of data used above the plane amount, interpret the following expressions. (pg 7)	
			Write an expression that describes the number of points scored on touchdowns T and points after touchdowns p by one team in a game. (pg 8)	
			The yards of fabric needed to make curtains is 3 times the lengthc of a window in inches, divided by 36. Write an expression that represents the yards of fabric needed in terms of the length of the window, L. (pg 9)	

Figure 3. Implicit Uses of the Explicit Noun Phrases within the Kernel

After I compiled the lexical chain and identified whether an instance in a kernel was in a bridging language registry or the mathematics registry, I was able to paint a picture of each textbooks series because I had information about the registry typical of each textbook series. This included information about how many times the kernel was referenced throughout the chapter and the number of non-kernels.

Pass 2

Once the lexical chains for each of the textbooks in the six countries were created and the registry analysis completed, I began working on part two of the analysis. For this stage in the analysis I compared and contrasted what I found in the textbooks from the English speaking countries. This analysis was used to describe what uses of the English language were common across textbooks from all three English speaking countries. Of course, the unique features of a textbook from a given country were also highlighted in this part of the analysis. A similar analysis of comparing and contrasting was also done across the textbooks from the Spanish speaking countries. During this part of my analysis I was able to note commonalities that all the English speaking textbooks had from looking at the charts associated with the sections on variable. Some things I noted were how often each textbook used each registry, if most of their instance uses were explicit or implicit, and the differences in the occurrence of the different definitions.

Pass 3

After the English textbooks were compared with each other and the Spanish textbooks were compared with each other, part three of the study compared the English textbooks with the Spanish textbooks. A second part of pass 3 looked at all of the bridging registry instances for commonalities. The last portion of pass 3 looked at how the textbook chapters that were analyzed talked about variable in regards to common uses of variable as defined by Usiskin (1988).

In comparing the English and Spanish textbooks I first looked for the differences and similarities between the kernels written in English and those written in Spanish. I then noted the frequency of explicit vs. implicit instances in the different languages. Finally I looked at the frequency of instances in the different registries. This analysis allowed me to answer questions such as: Was variable commonly addressed explicitly or implicitly in the different languages? Did books written in one language make greater use of bridging language than those written in the other language?

During pass three I also looked at the registry in which the English and Spanish textbooks were written with an eye toward any common language use. For example, if Spanish textbooks used the word intercept to talk about where a line crosses the x or y axis but made no attempt to help a student connect the word intercept with language they are already familiar with this would be an example of no bridging registry being used in the Spanish textbooks. However, if the textbooks used the word intersection, which has a similar meaning in the mathematics register as it does in the English Language register, this would be an example of the textbook using a bridging language to connect new ideas with what the student already knows. This allowed me to get some general differences and similarities between all six of the countries textbooks before I compared the bridging language instances or how the textbooks used variable.

Next, I compiled a master list of all of the unique bridging registry examples and used this to categorize the types of bridging language that was used in the textbooks. Questions that were asked when looking at the individual instances were: Did this instance have anything in common with any other instances? Were there any similarities between the actual words in any of the instances? Were there any similarities in structure between any of the instances? Using those as a guideline I was be able to group instances based on common structure, language use, as well as whether or not the instance was tied to a specific example in the textbook.

Lastly, to see how the textbooks talked about variable in the chapters that were analyzed I used a matching technique between variable instances. The first step in this process was to compile a master list of instances that were potentially talking about variable. To do this the noun phrases *letter, variable,* and *pronumeral* were used. All of the explicit and implicit instances to these noun phrases were compiled into a master list and kept under the country to which they belonged. Then, each instance was matched to one of Usiskin's categories of variable use which are (1) an equation to solve, (2) an identity, (3) a property, (4) an equation of a function of direct variation, or (5) a formula. The matching process used Usiskin's categories and descriptions of each category to decide how the instance was using variable. Since it was possible that some countries used variable in a way that was not talked about by Usiskin I separated any such instances that did not fit Usiskin's model and categorized them by looking for similarities and differences amongst them. I was then be able to talk about how the textbook might be using variable that was different from Usiskin's framework.

Spanish Language Verification

In order to verify my knowledge of Spanish I conducted pass 1 of the study on the use of the word "slope[1]" as outlined using a Spanish textbook and then gave the same sections of text to a mathematics education graduate student who is a native Spanish speaker to conduct pass 1 of

---

[1] This study had originally done background research to study variable and slope which was why slope was examined for the Spanish language verification. However the topic of slope was dropped to make the study more manageable.

the study This native speaker had an understanding of registry analysis from their linguistics class, and from conducting a study on registry use themselves.

The native speaker was given a copy of the explicit noun phrases and asked to conduct a registry analysis on the instances provided. Their analysis differed from my analysis by only 7 out of 124 instances, meaning that we had 94.4% agreement. The native speaker coded 4 instances as a bridging registry, while I coded them as the mathematics registry. They coded 3 instances as the mathematics registry, while I coded them as a bridging registry. Table 1 summarizes these findings.

Table 1

Registry Analysis of Explicit Instances

	Number of Instances	Percentage of Total
Researcher and Native Speaker agreed	117	94.4%
Native Speaker coded as Bridging, and Researcher coded as Math	4	3.2%
Native Speaker coded as Math, and Researcher coded as Bridging	3	2.4%

Additionally, the native speaker was asked to go through designated sections of the textbook and look for implicit references to the following noun phrases: directamente proporcionales (directly proportional), constante de proporcionalidad k (proportionality constant k), funcion (function), funcion lineal (linear function), dominio (domain), recorrido (range), puntos (points), recta (line), origen (origin), and plano cartesiano (Cartesian plane). The following table summarizes these findings:

Table 2

Spanish Language verification for Implicit References

	Number of Instances	Percentage of Total
Researcher and Native Speaker (common instances)	22	59%
Native Speaker Only	7	19%
Kailie Only	8	22%

By examining the number of instances that just the native speaker had, it was determined that 6 of these instances were categorized under the noun “puntos” (points). I asked the native speaker why he had put those instances under there and the native speaker thought that any reference to a point constituted a reference to an input and output and thus a function. I, on the other hand, thought of a point as referencing a specific place on a graph or Cartesian plane in the form (A, B). Therefore the discrepancy with 6 of the 7 native speaker only instances can be attributed to a difference in our mathematical understanding of the word “point” instead of difference in our understanding of the Spanish language. Similarly, under the category of “constante de proporcionalidad k” (proportionality constant k) I had found three implicit references to this idea that the native speaker had not. When we discussed why this was I realized that it had to do with our understanding of the relationship between slope and the proportionality constant k and not with the Spanish language. Under the categories of both “funcion” (function) and “funcion lineal” (linear function) there were four instances that I had found to be implicit references (2 instances each for function and linear function) that the native speaker had not. When we discussed this difference I realized that this could be attributed to the fact that we had different understandings of what an implicit reference was. For example, if an instance referred said the word “funcion” instead of “function lineal” I had coded this as an implicit instance because the instance did not directly state “funcion lineal,” whereas the native speaker did not code it as an implicit reference because “funcion” is part of the noun phrase “funcion lineal” and so he felt it was an explicit reference. After examining a reference the native speaker had included as an implicit reference that I had not, I felt that it was an oversight on my part. After I reviewed it I recognized that it should be an implicit reference, the same things was felt by the last instance I had found to be implicit that the native speaker had not.

The above difference can therefore be attributed to different understandings of the mathematics, clarifying what constitutes an implicit reference, and two oversights on the part of both myself and the native speaker. However none of the differences were from different translations of the Spanish language, or misunderstandings of what the instances meant. Since this study is concerned with the language aspect of the textbooks, the mathematical differences as well as the misunderstanding of what constituted an implicit reference were not significant to this research. Overall the native speaker and I had 92% agreement on the implicit references for the kernel, showing that my language abilities are acceptable to understand and analyze Spanish speaking textbooks. If I take into consideration the two oversights that were made the following chart summarizes the findings of the language verification study:

Table 3

Language Verification Study without instances whose differences were attributed to different mathematical understandings or an unclear meaning of implicit reference

	Number of Instances	Percentage of Total
Researcher and Native Speaker (common instances)	22	92%
Native Speaker Only	1	4%
Kailie Only	1	4%

CHAPTER 5: RESULTS AND ANALYSIS

The following section highlights the results from each countries textbook series. First, the results for the textbooks from English speaking countries will be presented followed by the results for the textbooks from the Spanish speaking countries. For each textbook series used in the study I will provide a brief overview of the general structure of the textbook in order to help me paint a picture of what the textbook series is like overall, and to provide information to compare textbooks across languages. The first country presented under each language category will contain examples and explanations of the bridging registry instances, the sections covering the other countries will follow with just the date presented.

Australia

The Australian textbook series follows an integrated curriculum across the MathsWorld 7, MathsWorld 8, and MathsWorld 9 textbooks. The textbooks are similar in structure in that all three textbooks have chapters on Algebra, linear equations, and linear relationships that were used for this study. The books are similar in length with the MathsWorld 7 having 784 pages, the MathsWorld 8 having 704 pages, and the MathsWorld 9 having 727 pages. The general format of each chapter is that each chapter introduces a problem, works through that problem in detail and introduces key terms as it goes. Different examples are presented with solutions, as well as other definitions in between. After the introduction of the new material, worked examples, and definitions the textbook has similar exercises for students to work on.

Although the textbooks may be similar in their structure the MathsWorld 7 textbook differs in the chapter on solving equations. In the Table of contents of the textbook it shows that there will be nine sections for this chapter, covering pages 369-417. However once I began my data collection from this textbook I noticed that some of the pages did not match up, there were gaps, and some chapters duplicated. This occurs with pages 376-385, where section 8.3: *Solving Equations: arithmetic strategies* should be. This also occurs with pages 400-409 where section 8.7: *Solving equations: doing the same thing to both sides* should be. It appears that duplicates of Sections 8.2 and 8.6 were inserted where sections 8.3 and 8.7, respectively, should be.

After introducing variable, the Mathsworld 7 textbook has chapters on Geometry and space, integers, and solving equations before introducing the idea of graphing linear relationships. The MathsWorld 8 textbook reviews variable and then has a chapter on percentages and financial mathematics before the chapter on solving equations. Similarly, the MathsWorld 9 introduces and reviews what a variable is and then has chapters on linear equations, probability, and coordinate geometry before discussing linear and non-linear relationships.

All of the kernels for this study were found in the main text of the textbooks, and are summarized in the following chart:

Kernel	Nouns/Noun Phrases in the Kernel
A *pronumeral* is a letter that stands for a number.	Pronumeral, letter, number
A *variable* is a pronumeral that can have a variety of values.	Variable, pronumeral, and pronumeral that can have a variety of values
A *term* may be a number, a single pronumeral or a pronumeral multiplied by a number.	Term, number, pronumeral, pronumeral multiplied by a number
An *expression* is made up of terms.	Expression, terms
To *substitute* means to replace a pronumeral with a particular number.	Substitute, pronumeral, and particular number

Figure 4. Summary of kernels and nouns/noun phrases from the Australian textbook Series

MathsWorld 7

For the first kernel (pronumeral) the MathsWorld 7 had 63 explicit references, and 44 implicit references. For example, an explicit reference to the noun "number" from the first kernel was, "When a letter stands for a number it is called a pronumeral (Vincent et al., 2014, p. 210)," and an implicit example for the noun "number" is "Using algebra, we can write this any order rule for addition as: $a + b = b + a$ (Vincent et al., 2014, p. 210)." This example was an implicit reference to a property of addition, which is why it can be considered an implicit reference to the noun "number." Of the 63 explicit references, 6 instances used a bridging registry (9.5%) and 57 instances used a mathematics registry (90.5%). From the implicit references, 5 of the 44 instances used a bridging registry (11.7%) and 39 instances used a mathematics registry (88.3%). An example of this bridging registry was, "When we backtrack with numbers, we must undo what has been done to the input number (Vincent et al., 2014, p. 385).” This explicit example showed the textbook trying to help students understand applying inverse operations to equations through the synonym of backtracking and used the replacement criterion.

The second kernel (variable) had 9 explicit references and 10 implicit references in the MathsWorld 7 textbook. None of the nine explicit instances used a bridging registry, which means that 100% of the instances used the mathematics registry. Two of the 10 implicit references used a bridging registry (20%), while the other eight instances used the mathematics registry (80%). An example of the bridging registry that was used for this kernel was, “If there are 2 lots of *m* toothpicks, we can write this as *m*+*m*. However we can also write this as 2*\m* (Vincent et al., 2014, p. 211).” This was an example of a bridging registry because the textbook attempted to describe what “2*m*” means in terms of grouping or “lots” instead of just translating the term into “2 times m.” Here the textbook used the specific context criterion and the replacement criterion.

The third kernel (term) had 19 explicit references to the noun or noun phrases within it, and 16 implicit references. Of the 19 explicit references zero were used a bridging registry, and 100% of the instances used the mathematics registry. Zero of the 16 implicit references used a bridging registry and 100% of the implicit references used the mathematics registry.

The fourth kernel (expression) had 33 explicit references. This kernel had only two nouns from the kernel compared to the other kernels which had three or four nouns or noun phrases identified. Zero of the 33 instances were in the bridging registry or the English language registry, meaning that 100% of the instances were in the mathematics registry. This kernel also had 29 implicit instances, and again 100% of the instances used the mathematics registry.

The last kernel identified for this textbook series (substitute) was explicitly referred to 31 times. Of those 31 instances zero were found to be in the bridging registry, which means that all 31 instances were in the mathematics registry (100%). This kernel was also implicitly referred to 28 times, and 2 of those instances were in the bridging registry (7.1%). The remaining 26 instances were all in the mathematics registry which makes up 92.9% of the total instances. An example of the bridging registry used matches the examples and was the same example from the second kernel (variable) which was, "When $m = 2$ there are 3 lots of 2 toothpicks (Vincent et al., 2014, p. 214)." Again this example was a bridging instance because it used the specific context criterion and the replacement criterion.

In total the MathWorlds 7 referred to the noun phrases within all five kernels explicitly or implicitly a total of 282 times. The textbook never used an English language registry, used a bridging language for only 15 of the instances, which is only 5.3% of the entire time the textbook talked about variable or linear equations. This means that the textbook used the mathematics registry 94.7% of the time when talking about concepts related to variable. It is also worth noting that for all five kernels the textbook implicitly referred to the nouns or noun phrases significantly less than they explicitly referred to them.

MathsWorld 8

For the first kernel (pronumeral) the MathsWorld 8 had 28 explicit references, with two instances falling under a bridging language category. These instances of bridging registry are the same one that was found as both an explicit reference to pronumeral and number in the textbook. The instance was, "The word pronumeral comes from the Latin 'pro' meaning 'for' and 'numeralis' and 'number' (Vincent et al., 2014, p. 118)." This instance broke the word pronumeral apart so that students have a better idea of what pronumeral meant. This instance meets the requirements for the specific context criterion for bridging registry use. This kernel was also referred to implicitly 25 times, with 100% of those instances falling under the Mathematics registry. The second kernel (variable) was explicitly referred to 16 times, with 100% of the instances falling in the Mathematics registry. This kernel was also implicitly referred to 18 times and 2 of those instances used a bridging registry. The instance that used a bridging registry fell under two different noun phrases and was, "Consider the linear rule y=2x+4, as the x-values increase in equal steps of 1 unit, the y values increase in equal steps of 2 units (Vincent et al., 2014, p. 543). Here the textbook is trying to explain to students the idea of rate of change and how the *x* and *y* variables are related in the equation and used the ideas of steps to do so. This instance meets the requirements for the specific context criterion.

The third kernel (term) was explicitly referred to 45 times. All of those instances were found in the mathematics registry. The kernel was also implicitly referred to 21 times and again 100% of those instances were in the mathematics registry. The fourth kernel (expression) was also explicitly referred to 19 times and 100% of the instances were in the mathematics registry. The fourth kernel was also implicitly referred to 5 times and again all of those instances were in the mathematics registry. Lastly, the fifth kernel (substitute) had 6 explicit instances with 100% of those instances falling under the mathematics registry. This kernel also had 5 implicit references to substitute and again 100% of those instances were in the mathematics registry.

In total the MathsWorld 8 textbook referred to the kernels related to variable explicitly or implicitly 188 times. Out of those 188 times, two of the instances used a bridging registry. This means that in all the textbook used a bridging registry about 1%, and used a mathematics registry 99% of the time.

MathsWorld 9

For the first kernel (pronumeral) the MathsWorld 9 had 26 explicit instances, and 30 implicit instances. An example of an explicit instance was, “Equivalent expressions give the same result for every value of the pronumerals (Vincent et al., 2014, p. 99).” This textbook reviewed the same material that was covered in the MathsWorld 7 textbook but also assumes that the reader has had experience with the ideas as evidenced by the decrease in the number of explicit instances. For the first kernel there were zero instances that were in the bridging registry and zero instances in the English language registry for both the explicit and implicit references. This means that for the first kernel all of the instances related to pronumeral were 100% in the mathematics registry.

The second kernel (variable) had 8 explicit instances and 11 implicit instances. All of these instances fell in the mathematics registry.

The third kernel (term) had 46 explicit instances, and 9 implicit instances. This kernel had the most explicit instances of any of the kernels in the 9th grade textbook. An example of an explicit use of the noun “term” from the 9th grade textbook is, “Quadratic comes from a Latin word meaning square or four and refers to the squared term, x^2. Trinomial means that there are three terms (Vincent et al., 2014, p. 105).” This explicit reference was also the only explicit instance that used a bridging registry. This instance linked the idea of square with quadratic and suggests that both terms will work in place of the other (replacement criterion) and it uses a context unique to this textbook series (specific context criterion). Again, this kernel had one instance that used a bridging registry, meaning that the majority of the explicit and implicit references are in the mathematics registry.

The fourth kernel (expression) had 38 explicit instances and 21 implicit instances. Like the other kernels this kernel did not have any instances in the bridging or English registry for either the explicit or implicit instances. Lastly, the fifth kernel (substitute) had 31 explicit instances and 2 implicit instances. All of these instances were in the mathematics registry for both the explicit and implicit instances.

In total the MathWorlds 9 referred to the noun phrases within all five kernels explicitly or implicitly a total of 222 times. The textbook never used an English language registry or a bridging registry, but instead only used a Mathematics registry. This could suggest a push for students to be more familiar with the language of mathematics. It is also worth noting that for all five kernels the textbook implicitly referred to the nouns or noun phrases significantly less than they explicitly referred to them. The instances in the MathsWorld 9 textbook also have more mathematical symbolism. The instances included less descriptive language and more mathematical symbolism than the MathsWorld 7 textbook.

A summary of the findings, for all five kernels (pronumeral, variable, term, expression, substitute), is provided in Table 4 and Table 5, separated by explicit or implicit use.

Table 4

Instances of EXPLICIT use of the noun phrases within the kernels for pronumeral, variable, term, expression, substitute

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	151 instances	114 instances	148 instances
	97.42%	100%	99.3%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	4 instances	0 instances	1 instance
	2.583.33%	0%	.7%

Table 5

Instances of IMPLICIT use of the noun phrases within the kernels for pronumeral, variable, term, expression, substitute

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	118 instances	72 instances	73 instances
	92.9%	97.3%	100%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	9 instances	2 instances	0 instances
	7.1%	2.7%	0%

The Australian textbook series has many interesting characteristics. When looking at the summary charts above, it is evident that the textbooks refer to the kernels less in the 8th grade textbooks than they do in the 7th or 9th grade textbooks. This may suggest that the textbooks assume that the reader knows the ideas related to variable by the time they get to the 8th grade. Another interesting characteristic is that apart from 1 explicit instance in the 9th grade textbooks, and two implicit instances in the 8th grade textbook, the 7th grade textbook is the only textbook that attempts to use any bridging language both explicitly and implicitly when referring to kernels. This could also be due to the fact that the ideas related to variable are being introduced in the 7th grade textbook and therefore require more explanation, and when they are talked about in the later textbooks they are just reviewed in passing which may be why the instances appear more mathematical.

Canada

The Canadian textbooks follow an integrated curriculum across the Math 7, Math 8, and Math 9 textbooks. The textbooks are similar in structure to one another in that each textbook has a chapter dedicated to linear relationships and then a chapter dedicated to solving linear equations. The MathLinks 7 textbook is 509 pages in length, the MathLinks 8 textbook is 528 pages, and the MathLinks 9 textbook is the shortest at 506 pages in length. The general format of each chapter across textbooks is example problems for students to work through, definitions in the margins, and then problems for students to practice the new skills with. The problems students were asked to work through followed a typical structure of posing a problem to students and then asking them a series of questions related to that problem. These questions included making a table of values, making predictions about the next values, and then checking their predictions.

With regards to language analysis, all of the kernels used for this study were found in the margins of the Canadian textbooks. In fact, all of the kernels (except those summarizing main ideas) were found in the margins of the text and did not make up the bodies of the textbooks. Another interesting characteristic of these kernels was that they were presented as bulleted lists and not as complete sentences, which seems to be unique to Canadian textbooks. The kernels identified related to variable and their noun phrases can be summarized in the following chart:

Kernel	Nouns/Noun Phrases within the Kernel
Variable: a letter that represents an unknown number, for example, x, A, n (McAskill et al., 2007, p. 358)	variable, letter, unknown number, x, A, n
Constant: a number that does not change, increases or decreases the value of an expression (McAskill et al., 2007, p. 360)	number that does not change, increases value, decreases value, expression, constant and number
Expression: any single number or variable, or a combination of operations ($+, -, \times, \div$) involving numbers and variables, for example, $5, r, 8t, x + 9, 2y - 7$ (McAskill et al., 2007, p. 358).	number that does not change, increases value, decreases value, expression, constant and number. In the third kernel the noun phrases identified were expression, single number, single variable, and combination of operations involving numbers and variables

Figure 5. Summary of kernels and nouns/noun phrases from the Canadian textbook Series

MathLinks 7

For the first kernel (variable) the 7th grade Math textbook had 83 explicit instances, this means that the textbook referred to each noun phrase from the kernel explicitly 83 times. Of those 83 instances zero were found to be in the English language registry, and zero of the 83 were found to be written in a bridging registry. This means that 100% of the 7th grade textbook used a mathematics registry when explicitly talking about the key noun or noun phrases within the first kernel.

The 7th grade textbook had 47 implicit instances of each noun phrase from the first kernel. These instances occurred when the textbook did not explicitly refer to the noun phrase, but the use of the noun phrase was implied from the context of the sentence. Of those 47 instances, zero used an English language registry, and zero used a bridging registry which means that 100% of the implicit instances were written in a Mathematics registry.

For the second kernel (constant) the 7th grade Math textbook had 44 total explicit instances referring to noun or noun phrases in the kernel. Five out of those 44 instances used a bridging registry, and zero of the instances used an English language registry. Therefore, 11.36% of the explicit instances used a bridging registry, the rest used a mathematics registry. There were also 110 implicit instances of which three were found to be in the bridging registry, meaning that out of all the instances of the second kernel, 2.7% of them were written in a bridging registry, and zero were written in the English language registry.

For the third kernel (expression) the MathLinks 7 had 34 explicit references to the kernel, of which 100% of the instances fell in the mathematics registry. The expression kernel also had 41 implicit references. Five of those implicit references were found to be using a bridging registry.

In total the MathLinks 7 referred to the noun phrases within all three kernels explicitly or implicitly a total of 359 times. The textbook never used an English language registry, used a bridging language only 3.6% of the time, and used a mathematical registry 96.4% of the time when talking about concepts related to variable. It is also worth noting that for the first kernel the textbook implicitly referred to the nouns or noun phrases significantly less than they explicitly referred to them, however for the second kernel the textbook referred to the ideas implicitly almost 3 times more than they referred to the ideas explicitly. For the third kernel (expression) the textbook also referred to the nouns and noun phrases more implicitly than it did explicitly.

MathLinks 8

The first kernel (variable) was explicitly referred to in the 8th grade textbooks 62 times, of which one instance was found to be in a bridging registry, and zero instances were found to be in the English language registry. The 8th grade textbook expounded on the use of variable and solving equations more than the seventh grade textbook. The noun phrases in the first (variable) kernel were implicitly referred to a total of 63 times, and none of those instances were found to be in the English language registry or in a bridging registry. The second kernel (constant) was explicitly referred to a total of eight times, with two of those eight instances appearing as a bridging registry, and the rest appearing as the mathematics registry. The second kernel was implicitly referred to a total of 69 times, of which five instances were a bridging registry, and zero used an English language registry.

The third kernel (expression) was explicitly referred to a total of five times, with 100% of those instances belonging in the mathematics registry. The expression kernel was also implicitly referred to a total of 30 times. Seven of those 30 instances used a bridging registry, while the other 23 instances used the mathematics registry. This means that for the second kernel 23% of the MathLinks 8 book used a bridging registry when talking about expression implicitly.

In total the MathLinks 8 textbook explicitly and implicitly referred to the nouns from all three kernels a total of 237 times, and zero of those 237 instances were written in the English language registry. However, in total 15 instances were found to be using a bridging registry meaning that 6.3% of the MathLinks 8 textbook uses a bridging registry when talking about variable, and a mathematics registry 93.7% of the time when talking about variable. It is again worthwhile noting that the textbook referred to the second kernel and third kernel more times implicitly than explicitly.

MathLinks 9

The first kernel (variable) was explicitly referred to 24 times in the MathLinks 9 textbook, with one instance falling in a bridging registry, and zero of the instances falling in the English language registry. The first kernel was also implicitly referred a total of 15 times and again one instance used in a bridging registry, and zero used the English language registry. The language that is used to talk about mathematics seems to switch in the 9th grade textbook and becomes more mathematical, with much less explanation. Contrary to the other textbooks in this series there is littler explanation of what some of math terms, such as constant and interval, mean. The 9th grade textbook is also different because it has more instances related to linear equations than either the Math 7 or Math 8 textbook.

The second kernel (constant) was explicitly referred to a total of two times, and one of the instances was in a bridging registry or the English language registry. It was also implicitly referred to a total of 18 times and one of those instances used a bridging registry, however zero used an English language registry.

The third kernel (expression) was explicitly referred to a total of four times, with zero instances falling in the bridging registry or English language registry. This kernel was also implicitly referred to a total of 7 times and again none of the instances used a bridging registry. This means that 100% of the instances that referred to the "expression" kernel explicitly and implicitly were written in the Mathematics registry.

In total the MathLinks 9 textbook had the fewest instances across the Canadian textbook series with only 70 instances with four instances using a bridging registry. The MathLinks 9 textbook used the Mathematics registry 94.3% of the time, and used a bridging registry about 5.7% of the time. This textbook may have had so few instances in comparison with the other textbooks because the textbook is assuming that the reader is already familiar with ideas like variable, constant, and expression and is therefore trying to move on to more complex material such as equations.

A summary of the findings, for both kernels (variable and constant), is provided in the Table 6 and Table 7, separated by explicit or implicit use.

Table 6

Instances of EXPLICIT use of the noun phrases within the kernels for variable, constant and expression

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	156 instances 96.9%	72 instances 96%	28 instances 85.7%
English Language Registry	0 instances 0%	0 instances 0%	0 instances 0%
Bridging Registry	5 instances 3.1%%	3 instances 4%	2 instances 14.3%

Table 7

Instances of IMPLICIT use of the noun phrases within the kernels for variable, constant and expression

	7^{th} Grade Textbook	8^{th} Grade Textbook	9^{th} Grade Textbook
Mathematics registry	190 instances	150 instances	22 instances
	96%	92.6%	91.7%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	8 instances	12 instances	2 instances
	4%	7.4%	8.3%

In summary, it is interesting that the kernels in this series were found in the margins of the textbook since Van Dormolen (1986) describes a kernel as the key mathematics that the textbook is trying to convey. However, in these textbooks, the key mathematics is located in the margins of the textbook and not the main body. Another interesting finding is that the textbooks implicitly referred to ideas more than they explicitly referred to them. This could be problematic for students because the textbook has recently defined the ideas that the students are expected to use and now they are expected to use these ideas related to variable without explicit direction to do so. The pattern of implicitly referring to nouns or noun phrases within the kernel more than explicitly referring to them is found in all the textbooks. While the textbook might be making the assumption that the students are familiar with the nouns and noun phrases by now, after being introduced to them in the 7^{th} grade textbook, this still could be confusing for students as they are more often coming across the nouns and nouns phrases more implicitly than explicitly. Since the ninth grade textbook had the fewest instances across the Canadian textbook series, this may suggest a push to have older students use the official language of mathematics. Another possibility is that students are expected to know and understand these ideas as by now they would have used them for two years.

United States

The United States textbooks do not follow an integrated curriculum; instead they are separated by the subjects Pre-algebra and Algebra. The textbooks are similar in structure to one another in that each textbook has five units that are further divided into chapters and both textbooks cover expressions and equations and linear functions. The Pre-Algebra textbook is 933 pages in length, and the Algebra textbook is 821 pages in length. The general format of each chapter across textbooks is an introductory explanation page where new vocabulary is introduced by bolding and highlighting the terms that students should familiarize themselves with. After this introduction, there are examples with solutions for students to look at followed by examples without solutions for them to practice. The last part of each chapter is the "practice and problem solving" section where problems are presented for students to practice what the textbook had explained.

With regards to language analysis, all of the kernels used for this study were found in the main text of each chapter. The kernels used for this study were found in the Pre-Algebra textbook because that is where the topic of variable was first introduced. The kernels identified related to variable are summarized in the following chart:

Kernel	Nouns/Noun Phrases within the Kernel
A variable is a letter or symbol used to represent an unknown value (McGraw-Hill/Glencoe, 2012, p. 11)	variable, letter, symbol, and unknown value

An expression like 5m+6 is an algebraic expression because it contains at least one variable and at least one mathematical operation (McGraw-Hill/Glencoe, 2012, p. 11)	expression, variable, and mathematical operation
A term without a variable is called a constant (McGraw-Hill/Glencoe, 2012, p. 180)	term, variable, and constant

Figure 6. Summary of kernels and nouns/noun phrases from the United States textbook Series

Pre-Algebra

For the first kernel (variable) the Pre-Algebra textbook had 34 explicit instances, this means that the textbook referred to the noun phrases from the kernel directly 34 times. Of those 34 instances zero were found to be in the English language registry, and only one of the 34 was found to be written in a bridging registry, and the rest used a mathematics language registry. This means that about 2.9% of the Pre-Algebra textbook used a bridging registry when explicitly talking about the key noun or noun phrases within the kernel.

The Pre-Algebra textbook had 56 implicit instances of the nouns or noun phrases from the variable kernel. Of those 56 instances, zero used an English language registry, and one used a bridging registry which means that 98% of the implicit instances were written in a Mathematics registry.

For the second kernel (expression) the Pre-Algebra textbook had 18 total explicit instances referring to noun or noun phrases in the kernel. Zero out of those 18 instances used a bridging registry, and zero of the instances used an English language registry. Therefore, 100% of the explicit instances of “expression” used a mathematics registry. The Pre-Algebra book did not have any implicit references to the nouns/noun phrases from the second kernel. This could because the noun phrases for this kernel were algebraic expression, variable and mathematical operation which even when written symbolically would fall under explicit instances.

For the third kernel (constant) the Pre-Algebra book had 23 explicit instances, of which two used a bridging registry. Therefore, when discussing the expression kernel the Pre-Algebra textbook used a bridging registry 8.7% of the time, and a mathematics registry 91.3% of the time. The Pre-Algebra textbook also had 92 implicit references to the constant kernel. Of those 92 instances, four were coded as a bridging registry. The Pre-Algebra textbook had two implicit bridging registry references under the noun "term," the same implicit bridging registries can also be listed under the noun "constant. The Pre-Algebra textbook therefore uses a bridging registry 4.3% of the time, and a mathematics registry 95.7% of the time.

In total the Pre-Algebra book referred to the noun phrases within all three kernels explicitly or implicitly a total of 223 times. The textbook never used an English language registry, used a bridging language only 3.6% of the time, and used a mathematical registry 96.4% of the time when talking about concepts related to variable. It is also worth noting that for the first kernel the textbook implicitly referred to the nouns or noun phrases significantly more than they explicitly referred to them, however for the second kernel the textbook referred to the ideas explicitly more than they referred to the ideas implicitly. The third kernel was implicitly referred to almost four times as often as the kernel was referred to explicitly.

Algebra

The first kernel (variable) was explicitly referred to in the Algebra textbook 44 times, of which three instances were found to be in a bridging registry, and zero instances were found to be in the English language registry. The noun phrases in the first kernel were implicitly referred to a total of 37 times, and none of those instances were found to be in the English language registry or in a bridging registry. The second kernel (expression) was explicitly referred to a total of 22 times, with zero of those 22 instances appearing as a bridging registry, and zero appearing as the English language registry. This means that all of the explicit instances to the second kernel appeared in the mathematics registry. The second (expression) kernel was not implicitly referred to, which like the fist kernel could be because the noun/noun phrases "algebraic expression," "variable," and "mathematical operation" can only be referred to explicitly.

The third kernel (constant) was explicitly referred to a total of 25 times. Two of the 25 instances are a bridging registry like the example, "In the term y, the coefficient is 1 since $1 * y = y$ by the multiplicative identity property" (McGraw-Hill/Glencoe, 2012). Here the textbook is trying to connect students' understanding of the multiplicative identity property and being able to recognize that the coefficient of the variable "y" is one. The third kernel was also implicitly referred to total of 61 times, where none of the instances used a bridging registry.

In total the Algebra textbook explicitly and implicitly referred to the nouns from all three kernels a total of 189 times, and zero of those 189 instances were written in the English language registry. However, in total 7 instances were found to be using a bridging registry meaning that 3.7% of the Algebra textbook uses a bridging registry when talking about variable, and a mathematics registry 96.3% of the time when talking about variable.

A summary of the findings, for both kernels (variable and constant), is provided in the Table 8 and Table 9, separated by explicit or implicit use.

Table 8

Instances of EXPLICIT use of the noun phrases within the kernels for variable, expression and constant

	Pre-Algebra Textbook	Algebra Textbook
Mathematics registry	73 instances	87 instances
	97.3%	95.6%
English Language Registry	0 instances	0 instances
	0%	0%
Bridging Registry	2 instances	4 instances
	2.7%	4.4%

Table 9

Instances of IMPLICIT use of the noun phrases within the kernels for variable, expression and constant

	Pre-Algebra Textbook	Algebra Textbook
Mathematics registry	144 instances	98 instances
	97.3%	100%
English Language Registry	0 instances	0 instances
	0%	0%
Bridging Registry	4 instances	0 instances
	2.7%	0%

The United States textbook is unique in that it did not follow a single pattern of either explicitly or implicitly referring to nouns/noun phrases more often. For the variable, and expression kernels both the Pre-Algebra and Algebra books had more explicit references than implicit references. However, the constant term had more implicit references than explicit references for both the Pre-Algebra and Algebra books. This is interesting to note because if the textbook refers to the kernels implicitly more than explicitly, they might be making the assumption that the students are familiar with the nouns/noun phrases from the kernel and that they can recognize them even if they aren't being explicitly used. Another interesting feature of the United States textbooks is that besides the expression kernel, the Pre-Algebra textbooks have more instances overall than the Algebra textbooks. This could be because the Algebra textbook might be making the assumption that the students would have already had access to the topics presented in the Pre-Algebra textbook and therefore not go into as much detail leading to less instances.

Colombia

The Colombian textbooks follow an integrated curriculum across the Matematicas 7 (Math 7), Matematicas 8 (Math 8), and Matematicas 9 (Math 9) textbooks. The Math 7 textbook has 303 pages and 8 chapters. The chapter used for this study was chapter 4 which was titled, "Lenguaje algebraico. Funciones (Algebraic Language: Functions)." The Math 8 textbook has 328 pages, and 14 chapters. Two chapters were used for this study and they were, "Expresiones algebraicas (Algebraic expressions)" and "Ecuaciones desigualdades (Inequality equations)." Lastly, the Math 9 textbook had 368 pages and the chapter that was used for this study was, "Funciones (Functions)." These textbooks follow a format of giving a brief definition of a major concept, followed by different examples and alternative activities for students to complete. At the end of each chapter there is an "actividades" (activities) section that might serve as a homework section for students.

The language analysis for these textbooks used kernels from the main body of the textbook. The kernels for algebraic expression and unknown quantity were both found in the Math 7 textbook, however the kernel that was used for variable was found in the Math 8 textbook. The Math 7 textbook never defined the term variable, however they used term throughout the chapter that was analyzed. The noun phrases that were used are summarized in the following chart:

Kernel:	Noun Phrases:
En matematicas se utilizan las letras para expresar cantidades desconocidas; las letras de la expresion, pueden representar uno o varios numerous (In math letters are used to express unknown quantities; the letters of the expression may represent one or more numbers) (Orozco et al., 2012, p. 132)	Letras, cantidades desconocidas, numeros (letters, unknown quantity, number)
En algebra, la expresion "cantidad cualquiera" se puede representar escribiendo una letra, por ejemplo n. Esta letra corresponde a una variable (In algebra, the expression "any amount" can be represented by writing a letter, for example n. This letter corresponds to a variable) (Garcia et al., 2011, p. 54)	Expresion, cantidad cualquiera, letra, variable (expression, unknown quantity, letter, variable)

Una expresion algebraica es una combinacion de numeros y letras unidos por los signos de las operaciones aritmeticas: adicion, sustraccion, multiplicaion, division, y potenciacion (An algebraic expression is a combination of numbers and letters connected by the signs of arithmetic operations: addition, subtraction, multiplication, division, and empowerment) (Orozco et al., 2012, p. 134)	Expresion algebraica, numeros, letras, operaciones aritmeticas (algebraic expression, number, letter, arithmetic operations)

Figure 7. Summary of kernels and nouns/noun phrases from the Colombian textbook Series

Math 7

For the unknown quantity kernel the 7th grade Math textbook had the least explicit references of any of the textbooks with only 5 references. All of these references used the mathematics registry, meaning that zero instances used a bridging or the English language registry. The unknown quantity kernel also had the least amount of implicit references of any textbook so far with only 4 instances. Again, all of these instances used the mathematics registry. An example from the mathematics registry was, “El valor numerico de una expresion algebraica es el numero que se obtiene al sustituir las letras por numeros determinados y hacer las operaciones indicades” (Orozco et al., 2012, p. 135). This can be translated to, “The numerical value of an algebraic expression is the number that is obtained by substituting numbers for letters and doing the indicated operations.” This instance fell under the mathematics registry because it not only used a lot of mathematics vocabulary but described a mathematical process and does not suggest any deeper thinking by the student nor does it attempt to explain the concept in more common Spanish language terms.

For the variable kernel the Math 7 textbook has 19 explicit references to the kernel. All those 19 instances were found to be using the mathematics registry. An example of this registry use was, "Expresa con letras y signos matematicos la siguiente propiedad: la suma de la medida de los angulos internos de un triangulo es igual a 180" (Orozco et al., 2012, p. 133). This can be translated to, "Express with letters and mathematical signs the following property: the sum of the measure of the internal angles of a triangle is equal to 180." This can be considered the mathematics registry because it used only mathematical language and words; in other words, even the words that could potentially make sense in the Spanish language registry used the mathematics registry. The variable kernel was also implicitly referred to 11 times. All of these instances were in the mathematics registry, meaning that this kernel was only referred to using a bridging registry one time and it was an explicit reference.

Lastly, the expression kernel was explicitly referred to 21 times. None of those instances used a bridging registry. There were also 8 implicit references to the expression kernel and none of the instances used a bridging registry or the Spanish language registry, which means that 100% of the implicit references to the expression kernel used the Mathematics registry.

Math 8

For the Math 8 textbook the unknown quantity kernel was explicitly referred to four times with all of the instances falling in the mathematics registry. The unknown kernel was also implicitly referred to ten times and again all of these instances fell in the mathematics registry. One of the more interesting explicit instances that was in the mathematics registry was, "Las letras que suelen usarse como cantidades conocidas son *a, b, c, d, m*, y *n*, y las letras *x, y, z* usualmente representan incognitas" (Garcia et al., 2011, p. 237). This can be translated to, "the letters that are used for known quantities are a, b, c, d, m, and n, and the letters x, y, z usually represent unknowns." This was the only instance that a textbook has made an attempt to differentiate the way that different letters are used. This could be particularly important for students when it comes to linear equations.

The variable kernel was also explicitly referred to 29 times and all of these instances fell in the mathematics registry. The same kernel was implicitly referred to 17 times and again all of these instances were in the mathematics registry. An example from the implicit references to the variable kernel was, "Una ecuacion lineal con una incognita se puede expresar de la forma $Ax + B = C$ con *A, B, C* son elementos de los numeros reales y A no puede ser igual a 0," (Garcia et al., 2011, p. 231). This can be translated to, "A linear equation with one unknown can be expressed by $Ax + B = C$, where *A, B, C* are elements of the real numbers and *A* can't equal 0." This fell under the mathematics registry because it is a definition of a mathematics equation, and does not help the student make connections with language they already know. Nor does it try to have students understand things on a more conceptual level.

Lastly, the expression kernel was explicitly referred to 23 times and all but one of the instances were in the mathematics registry. The same kernel was implicitly referred to 26 times and again all of the instances were in the mathematics registry. An example of the explicit bridging reference to the noun phrase "expression algebraica" was, "El lenguaje algebraico utiliza las expresiones algebraicas para expresar de manera abreviada, con precision y claridad, una situacion o propiedad (Garcia et al., 2011, p. 56)." This translates to, "Algebraic language uses algebraic expression to express in abbreviated form, with precision and clarity, a situation or property. This is an example of a bridging registry because it helps students think about what an expression might be used for, and helps them realize that expressions help express word problems more succinctly.

Math 9

The Math 9 textbook did not have any explicit or implicit references to the unknown quantity kernel. This could be because the chapter that was applicable and studied for this study was titled “Funciones” (Functions) and the textbook series may assume that students have had enough experience with the phrase “unknown quantity” by the time they reach their study of functions. The 9th grade textbook had 3 explicit references to variable kernel, and 11 implicit references to the variable kernel. All of the instances, both explicit and implicit, use the mathematics registry. An example from the implicit references was, “La ecuacion general de la recta esta dada por $Ax + By + C = 0$, en donde *A, B, C* son numeros reales. A *y* B no pueden ser cero a la vez (Orozco et al., 2012, p. 62).” This can be translated to, “The general equation for a line is given by $Ax + By + C = 0$, where *A, B, C* are real numbers. *A* and *B* can’t be zero at the same time.” Here the textbook uses mathematical language to describe the general equation of a line, which makes it fall under the mathematics registry.

Lastly, the Math 9 textbook had two explicit references to the expression kernel, and five implicit references to the expression kernel. All of these instances used the Mathematics registry. An example is, “Para todo par de numeros reales a y b sobre la recta real, se establece alguna de las siguientes relaciones de orden...” (Orozco et al., 2012, p. 14). This explicit reference to the noun phrase “letras” in the expression kernel can be translated to, “For any pair of real numbers a and b on the real line, the following order relations are established….””

A summary of the findings for all three kernels (unknown quantity, variable, and expression) is provided in Table 10 and Table 11, separated by explicit or implicit use.

Table 10

Instances of EXPLICIT use of the noun phrases within the kernels for unknown quantity, variable, and expression

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	45 instances	55 instances	5 instances
	100%	98.2%	100%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	0 instances	1 instances	0 instances
	0%	1.8%	0%

Table 11

Instances of IMPLICIT use of the noun phrases within the kernels for unknown quantity, variable, and expression

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	23 instances	53 instances	16 instances
	100%	100%	100%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	0 instances	0 instances	0 instances
	0%	0%	0%

These tables summarize how the Colombian textbook series presents the idea of variable to students. One of the most interesting characteristics of this textbook series is that it did not attempt to use a bridging registry at all when describing ideas related to the concept of variable. All of the instances that were identified across the Math 7, Math 8, and Math 9 textbook used the Mathematics registry. This has been the first textbook series thus far to not have a single bridging language registry across the textbook series.

Mexico

The Mexican textbook series follows an integrated curriculum across the Matematics secundaria 1, secundaria 2, and secundaria 3 textbooks. The secondary math one textbook has 272 pages, five different “bloques” (blocks), and 37 chapters. The secondary math two textbook has 352 pages, five “bloques” (blocks), and 35 chapters. The secondary math three textbook has 319 pages, five “bloques” (blocks), and 30 chapters. The chapter used from the secondary one textbook for this study were “ecuaciones de primer grado” (equations of the first grade/ linear equations). The chapters used from the secondary two textbook were “operaciones con expresiones algebraicas” (operations with algebraic expressions), “soluciones de ecuaciones” (solutions of equations), “variacion conjunto entre variables” (co-variation between variables), “las funciones lineales y sus graficas” (linear functions and their graphs), “graficas lineales con m constant” (linear graphs with the constant m), and “graficas lineales con b constant” (linear graph with the constant b). The chapters used from the secondary three textbook are “factorizacion de expresiones” (factorization of expressions), “razon de cambio” (rate of change), “factorizacion” (factorization), “applicacion de funciones” (applications of functions).

The language analysis for these textbooks used kernels from the main body of the textbook as well as from the margins. This textbook series used mathematical terms in the main body and then defined them in the margins of the textbook. The kernels and noun phrases that were identified for this textbook series are summarized in the following chart:

All of the kernels from this textbook series were found in the secondary math 2 textbook. The noun phrases that were used are summarized in the figure 8.

Kernel:	Noun Phrases:
Incognita: En una ecuacion, cantidad o valor que no se conoce y que hay que averiguar. Generalmente se representa con una letra (Unknown: In an equation, the quantity of value that is not known and needs to be found out) (Escareno & Lopez, 2010, p. 174)	Ecuacion, cantidad/valor, letra, incognita (equation, quantity/value, letter, unknown)
Literales son letras utilizadas para representar numeros. Tambien so les conoce como variables (Literals are letters used to represent numbers. They are also known as variables) (Escareno & Lopez, 2010, p. 111)	Literales, letras, numeros, variables (literals, letters, numbers, variables)
Como recordaran, una expresion algebraica es la combinacion de numeros, letras, y signos de operacion (As you remember, an algebraic expression is the combination of numbers, letters and operation signs) (Gaisman et al., 2010, p. 30)	Expresion algebraica, numeros, letras, signos de operacion (algebraic expression, numbers, letters, operation signs)
Representa a cada uno de los elementos de un conjunto de datos. Cada elemento del conjunto es un valor de la variable (Variable:	Variable, elementos de un conjunto de datos (variable, elements of a data set)

Represents each element of a data set. Each element of the data set is a value of the variable) (Gaisman et al., 2010, p. 85)	

Figure 8. Summary of kernels and nouns/noun phrases from the Mexican textbook series

This textbook presented two different definitions for the word variable, one from a mathematical point of view and one from a statistics perspective. Both kernels were used in this textbook analysis.

Mathematics 7 (Secondary 1)

For the unknown kernel the secondary 1 textbook had 13 explicit instances, with one of the instances falling under a bridging registry. The unknown kernel for the secondary 1 textbook also had 2 implicit references and both of these instances used a Mathematics registry.

The variable kernel had 3 explicit references to the kernel, all of which were in the Mathematics registry. This kernel also had 5 implicit references to the kernel all of which used the Mathematics registry.

The expression kernel had 2 explicit references to the kernel as well as 3 implicit references to the kernel. All of the instances used the mathematics registry which means that 100% of the instances in the secondary one textbook used a mathematics registry when talking about ideas related to an expression.

Lastly, the statistics variable kernel had zero explicit references to the kernel, and only one implicit reference to the kernel. The implicit reference to this kernel used the mathematics registry.

Mathematics 8 (Secondary 2)

For the unknown kernel the secondary two textbook had 11 explicit references, all of which used the mathematics registry. This kernel also had 5 implicit references, and again all of these instances used the mathematics registry. The variable kernel had 24 explicit references to the kernel and all of these instances used the mathematics registry. The variable kernel also had 12 implicit instances and again all of these instances used the mathematics registry. The expression kernel had 20 explicit references and all of these instances used the mathematics registry. This kernel also had five implicit references to the kernel and all of the instances used the mathematics registry.

The kernel for the statistics definition of variable had seven explicit references, and ten implicit references. The instances for both the explicit and implicit references to this kernel used only the mathematics registry. This means that 100% of the instances related to the statistics definition of variable used the mathematics registry.

Mathematics 9 (Secondary 3)

The secondary three textbook had 12 explicit references to the unknown kernel. Two of those instances used a bridging registry (the same instance referenced two different noun phrases). The unknown kernel also had 4 implicit references and all of these instances used the mathematics registry.

This textbook also had 17 explicit references to the variable kernel, and three of those instances used a bridging registry. This kernel also had 7 implicit references to the kernel, all of which used the mathematics registry.

The expression kernel also had three explicit references to the kernel, and all three instances used the mathematics registry. This kernel also had four implicit references, and again all of those instances used the mathematics registry. The kernel for the statistics definition of variable had zero explicit references and ten implicit references. The ten implicit references to the kernel also all used the mathematics registry.

Table 12 and 13 summarize the findings for all four kernels; unknown, variable, expression, statistics definition for variable:

Table 12

Instances of EXPLICIT use of the noun phrases within the kernels for unknown, variable, expression, and statistics definition for variable

	Secondary 1	Secondary 2	Secondary 3
Mathematics registry	17 instances 94.44%	62 instances 100%	27 instances 84.37%
English Language Registry	0 instances 0%	0 instances 0%	0 instances 0%
Bridging Registry	1 instances 5.56%	0 instances 0%	5 instances 15.63%

Table 13

Instances of IMPLICIT use of the noun phrases within the kernels for unknown, variable, expression, and the statistics definition for variable

	Secondary 1	Secondary 2	Secondary 3
Mathematics registry	11 instances	32 instances	24 instances
	100%	100%	100%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	0 instances	0 instances	0 instances
	0%	0%	0%

These tables summarize how these Mexican textbooks present the idea of variable to students. It's interesting to note that none of the textbooks used any bridging language when they implicitly referred to a kernel. It's also interesting that all of the kernels came from secondary 2 textbook but some of the noun phrases are referred to in the secondary 1 textbook. This means that students are expected to know and be familiar with certain words or phrases before they are actually defined.

Chile

The Chilean textbook follows an integrated curriculum across the Matematica 7, Matematica 8, and Educacion Media 1 textbooks. The Matematica 7 textbook is divided into two textbooks and has 321 pages. The chapter that was used for this study came from the second half of the textbook, and was titled "Algebra y ecuaciones" (algebra and equations). The Matematica 8 textbook was also divided into two textbooks, and the chapter that was used for this study was titled "Funciones y proporcionalidad" (functions and proportionality). Lastly, the Educacion

Media 1 textbook has 343 pages, the chapter that was used for this study was titled “Funciones lineal y afin” (Linear and affine functions). All of the textbooks follow a similar format where each chapter starts with an “evaluacion diagnostica” (diagnostic evaluation), and then introduce various topics that will be discussed in the chapter. As part of the introduction the textbooks walk through an example with the students and then give a few more examples at the end of the introduction. They also list “practica” or practice problems for students at the end of each section of new material.

The Chilean textbooks were the only textbooks that did not define any of the ideas related to variable. Therefore the kernels that were used in both the Colombian and Mexican textbook series were used as the kernels for the Chilean textbooks. The common noun phrases from the Colombian textbook kernels and Mexican textbook kernels were used as the noun phrases for the Chilean textbooks. These noun phrases were for an unknown, unknown quantity, expression, and variable kernels. The following chart summarizes the noun phrases that were used for this textbook series:

Kernel:	Noun Phrases:
Incognita (unknown)	Incognita, cantidad/valor, letra, ecuacion (unknown, quantity/value, letter, equation)
Cantidades desconocidas (unknown quantity)	Letras, cantidades desconocidas, expresion, numeros (letters, unknown quantity, expression, number)
Expresion (expression)	Expresion algebraica, numeros, letras, operaciones artimeticas, signos de operacion

	(algebraic expression, number, letters, arithmetic operations, operation signs)
Variable (variable)	Letras, variables (letters, variables)

Figure 9. Summary of kernels and nouns/noun phrases used for the Chilean textbook series

Since there was overlap within the kernels with the noun phrases “letras” (letters) and “numeros,” (numbers) the instances for these noun phrases were subdivided into whether or not they best fit under the expression kernel, the variable kernel, the unknown quantity kernel, or the incognita kernel.

Matematica 7

For the incognita kernel the Matematica 7 textbook had 20 explicit instances and 21 implicit references to the kernel. All of these instances used the Mathematics registry, meaning that 100% of the incognita instances for the Matematica 7 textbook used the Mathematics registry. For the unknown quantity kernel the Matematica 7 textbook has three explicit references to the kernel and all of the instances were in the mathematics registry. The unknown quantity kernel was also implicitly referred to seven times, and again all of these instances used the mathematics registry.

The expression kernel was explicitly referred to five times, and all of these instances used the mathematics registry. This kernel was implicitly referred to two times with both instances falling in the mathematics registry. Lastly, the variable kernel has four explicit references, and 9 implicit references to the kernel. All of these instances used the mathematics registry.

Matematica 8

For the incognita kernel the Matematica 8 textbook had five explicit references and 12 implicit references to the kernel. All of the instances, both explicit and implicit, were in the mathematics registry. The unknown quantity kernel had one explicit reference and three implicit references and all four of these instances were in the mathematics registry.

The expression kernel only had one explicit instance and didn't have an implicit references to the kernel. This instance fell under the mathematics registry. Lastly, the variable kernel had four explicit references and eleven implicit references.

Matematica 9

For the incognita kernel the Matematica 9 textbook has 14 explicit references and three implicit references, and all of these references used the mathematics registry. The unknown quantity kernel had three explicit references and three implicit references to the kernel. All of these instances used the mathematics registry.

For the expression kernel the Matematica 9 textbook has four explicit references and one implicit reference. All of these instances fall under the mathematics registry. Lastly, for the variable kernel the Matematica 9 textbook had four explicit references and 6 implicit references. All of those instances used the mathematics registry. Tables 14 and 15 summarizes these findings:

Table 14

Instances of EXPLICIT use of the noun phrases within the kernels for incognita, unknown quantity, variable, and expression

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	32 instances	11 instances	25 instances
	100%	100%	100%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	0 instances	0 instances	0 instances
	0%	0%	0%

Table 15

Instances of IMPLICIT use of the noun phrases within the kernels for incognita, unknown quantity, variable, and expression

	7th Grade Textbook	8th Grade Textbook	9th Grade Textbook
Mathematics registry	39 instances	26 instances	13 instances
	100%	100%	100%
English Language	0 instances	0 instances	0 instances
Registry	0%	0%	0%
Bridging Registry	0 instances	0 instances	0 instances
	0%	0%	0%

These tables summarize how the Chilean textbooks present the ideas related to variable to students. The Chilean textbooks did not make any attempt to use any bridging language registry or any Spanish language registry. All of the instances across the Matematica 7, Matematica 8, and Matematica 9 textbooks used the Mathematics registry.

English Textbook Analysis

Looking across all of the English speaking textbooks and the Spanish speaking textbooks there were many similarities. This section will discuss the similarities between the English speaking textbooks and then the Spanish speaking textbooks. One of the first similarities was that the English textbooks share kernels. This means that the textbook series all defined the same concepts in mathematics. Another similarity was amongst the nouns/noun phrases across all of the kernels.

Kernel and Noun Analysis

First, all of the English speaking countries had a kernel about variable. Table 16 summarizes the number of instances for the variable from each textbook series:

Table 16

Comparison of the number of EXPLICIT and IMPLICIT instances for the variable kernel across Canadian, Australian and American textbooks

Textbook Series:	Number of Explicit Instances:	Number of Implicit Instances
United States	78	93
Canada	169	125
Australia	33	39

Looking at the chart the Canadian textbook series stands out with significantly more instances, both explicitly and implicitly, to the variable kernel. It was also the only textbook series that had more explicit instances than implicit instances, as the United States and Canada both had more implicit instances than explicit instances. It is also interesting that Canada had so many more instances than the United States even though their definitions of *variable* were very similar. Canada defined variable as, “Variable: a letter that represents an unknown number, for example, *x*, A, *n,"* (McAskill et al., 2007, p. 358). The United States defined variable as, "A variable is a letter or symbol used to represent an unknown value," (McGraw-Hill/Glencoe, 2012, p. 11). The differences in the number of instances could be attributed to the fact that Canada's definition included *x*, A, *n* as nouns and therefore would have instances under each of those.

All three English speaking textbooks also defined "expression," and all three textbooks defined it differently. Canada defined expression as, "Expression: any single number or variable, or a combination of operations (+, −, ×, ÷) involving numbers and variables, for example, 5, r, 8t, x+9, 2y-7," (McAskill et al., 2007, p. 358); Australia defined expression as, "An expression is made up of terms" (Vincent et al., 2014, p. 217), and the United States defined expression as, "An expression like 5m+6 is an algebraic expression because it contains at least one variable and at least one mathematical operation" (McGraw-Hill/Glencoe, 2012, p. 11). Table 17 summarizes the number of explicit and implicit instances for the expression kernels.

Table 17

Comparison of the number of EXPLICIT and IMPLICIT instances for the expression kernel across Australian, Canadian and American textbooks

Textbook:	Number of Explicit Instances:	Number of Implicit Instances:
United States	40	0
Canada	43	78
Australia	90	55

This time it is Australia who had the most instances under the expression kernel. This is interesting because their definition only contained two nouns; *expression* and *terms.* In order to understand their definition of expression, a student would also need to understand how they defined *term.* The Australian textbook series defined *term* before *expression* as, "A term may be a number, a single pronumeral or a pronumeral multiplied by a number" (Vincent et al., 2014, p. 217). Another interesting finding of this chart was that the United States was the only textbook series to not have any implicit references to the expression kernel. This tells us that the textbook only ever explicitly referred to expression.

Looking across the various kernels from the three English speaking countries revealed that there were many differences across all of the textbooks. For example besides *variable* and *expression* the Canadian and United States textbooks also defined *constant,* whereas the Australian textbooks defined kernels related to pronumeral, term, and substitute. The Australian textbooks were also unique in that their kernels all built on one another and used mathematical vocabulary from the previous kernel. For example, the textbooks defined term as "a term may be a number, a single pronumeral or a pronumeral multiplied by a number." This definition used the previous definition for *pronumeral,* as well as the next definition the Australian textbooks defined, *expression*, which was defined as, "an expression is made up of terms." This definition used the previous definition of *terms.* Van Dormolen (1986) stated that textbooks should build on themselves and the Australian textbook does a good job of incorporating previous definitions and building up as the textbook progresses. However, this means that students have to be familiar with more mathematical vocabulary because the definition builds on the previous mathematical definitions than if the textbooks were to use a definition that didn't rely on terms from the previous definition.

There is also a lot of overlap in the nouns/noun phrases that these textbook series chose to use when they defined their kernels. These three textbook series all share the nouns *variable, letter, and expression*. Canada used the above nouns when defining *variable and expression* in their textbook series, Australia used them when defining *variable, pronumeral, and expression,* and the United States used them when defining *variable, expression, and constant.* Table 18 summarizes the number of total instances (across kernels) for the noun phrase *variable* across each textbook series.

Table 18

Number of EXPLICIT and IMPLICIT instances for the noun phrase "variable" from each textbook series and their registry

Textbook Series	Explicit Instances		Implicit Instances	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
United States	57	3	26	0
Canada	58	1	42	1
Australia	16	0	18	2

Looking across the different textbook series there were different observations to be made. One such observation was that the United States and Canada were very similar in the number of explicit mathematics registry instances as well as explicit bridging registry instances. Australia had far fewer explicit mathematics registry instances than the United States or Canada. This could be because Australia only used the noun *variable* in one of their kernels versus Canada who used it in two of their kernels and the United States who used it in three of their kernels. It could also be because Australia used the idea of pronumeral when defining variable, and it is the only English speaking textbook to introduce the noun *pronumeral*. This means that in order for students to understand the meaning of variable they first have to understand the meaning of pronumeral. The Australian textbook series defined pronumeral as, "a letter that stands for a number.” In the MathsWorld 8 (Australia) textbook the authors break the word “pronumeral” apart in an attempt to get students to understand its meaning by saying that, “The word pronumeral comes from the Latin 'pro' meaning 'for' and 'numeralis' and 'number'” (Vincent et al., 2014, p. 118). The idea of having to understand the meaning of pronumeral in order to understand the meaning variable is more complicated for students. That being said, Usiskin (1988) states that the meaning of variable is multifaceted and the Australian textbook series was the only English speaking textbook to make an attempt and breaking the idea of variable apart and keeping it separate from a pronumeral. By doing so the Australian textbooks have tried to make a distinction between “variable” and what Usiskin calls and “an equation to solve.”

Another interesting observation is that the United States and Canada both used a bridging registry when talking about variable, but Australia does not. An example from the Canadian textbook series of a bridging language was, “Isolate the variable means to get the variable by itself on one side of the equation” (McAskill et al., 2008, p. 373). The United States had a similar bridging registry explicit instance with the example, “It is then helpful to use the distributive property to isolate the variable for which you are solving” (McGraw-Hill/Glencoe, 2012, p. 126). Both of these instances were coded as explicit bridging registry because they were both using a word, isolate, that is used in the English language register and applying the same meaning to a mathematical setting. This was an example of the replacement criterion as stated in the methodology. The instance from Canada was a stronger example of bridging registry because they tied the meaning of *isolate* to mathematics, whereas the example from the United States expected students to already be able to draw on their own meaning of *isolate.*

When looking at the implicit mathematics registry instances for the noun *variable*, Canada and the United States did not share as many commonalities. In fact, it was difficult to find anything in common with the number of mathematics registry instances each country has. This time, the United States and Australia were closer in number with the United States having 26 instances and Australia with 18, however that is still a difference of 8 instances. In this category Canada had the most with 42 instances. There was a similarity with Australia and Canada because they each had one implicit reference to a bridging registry, while the United States did not have any. Canada's implicit bridging registry instance to the noun *variable* was, "The number of squares, s, increases by 3 for each figure number, n. Multiplying the figure number, n, by 3 results in 2 more than the number of squares. Therefore, subtracting 2 from 3n equals the number of squares, s" (McAskill et al., 2009, p. 212). This can be compared to Australia's instance which was, "If there are 2 lots of m toothpicks, we can write this as $m + m$. However we can also write this as $2 \times m$"(Vincent et al., 2014, p. 211). Both of these instances dealt with equations. In the Canadian example the textbook described the effect that subtracting 2 had on the pattern they are trying to find, while the Australian example linked the word "2 lots" with "2 groups" and thus twice as many or as much of an item. In this case a "lot" was a common used term in the English language but not a mathematical term that was commonly used. Both of these instances used the specific context criterion that was defined in the methodology.

Next, all textbooks shared the noun *letter*, the Canadian and United States textbook series used *letter* for their *variable* kernel and the Australian textbook series used it for their *pronumeral* kernel. Table 19 summarizes the number of instances each textbook series had for the noun *letter*.

Table 19

Number of EXPLICIT and IMPLICIT instances for the noun phrase "letter" from each textbook and their registry

Textbook	EXPLICIT INSTANCES		IMPLICIT INSTANCES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
US Pre-Algebra	4	0	9	0
US Algebra	3	0	10	0
MathLinks 7 (Canada)	5	0	16	0
MathLinks 8 (Canada)	0	0	24	1
MathLinks 9 (Canada)	0	0	2	0
MathsWorld 7 (Australia)	5	2	20	1
MathsWorld 8 (Australia)	1	0	8	0
MathsWorld 9 (Australia)	2	0	17	0

One thing that each textbook series had in common, was that across all of the textbooks every textbook had more implicit mathematics registry instances for the word letter than explicit mathematics registry instances for the word letter. It was also interesting to note that each textbook series did not have many instances of the noun *letter* compared to the noun *variable*. This could be because the textbooks do not explicitly state the word *letter* as often as they do *variable* because students already recognize that the textbooks were using a letter. When the number of instances from each country is totaled, as shown in Table 20, Australia and Canada had very similar counts.

Table 20

Number of EXPLICIT and IMPLICIT instances for the noun phrase "letter" from each textbook series and their registry

Textbook Series	EXPLICIT INSTANCES		IMPLICIT INSTANCES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
United States	7	0	19	0
Canada	5	0	42	1
Australia	8	2	45	1

This shows that the Canada and Australia had very similar counts for explicit references to the Mathematics registry as well as implicit references to the Mathematics registry, and they had the same count for implicit bridging references to the noun *letter.* For the Canadian textbooks the implicit bridging registry occurred when the textbook stated, "Whenever possible choose variables that are meaningful. For example, h for height and t for temperature" (McAskill et al., 2008, p. 356). For the Australian textbook series it occurred in the example, "3m means 3 lots of m toothpicks" (Vincent et al., 2014, p. 314). These examples were similar because both used a letter to stand for something specific, and both were implicit references because the textbook did not explicitly state the word *letter*. These examples were also examples of bridging registry because the Canadian textbook tried to get students to remember that the letters they pick, when used in the context of a problem, stand for something specific; whereas the Australian textbook was not only reminding students that m stands for the number of toothpicks but they bring in the word *lot* as another way of saying *group* or *3 multiplied by* m, both were instances that used the specific context criterion.

Another observation that can be made was that the Australian textbook was the only one that made any attempt to use a bridging registry when explicitly referring to *letter*. The instance occurred with the example, “She chose the letter m to represent the number of toothpicks in the cup” (Vincent et al., 2014, p. 210). Here, the textbook explicitly stated the word *letter* which was why it was an explicit reference and the example used the idea that the students can pick any letter but that letter is linked to something specific for the problem that they are working on. The Australian textbook also had an instance in the mathematics registry that was linked to the above example and it stated, “We could use any letter instead of m to represent the number of toothpicks” (Vincent et al., 2014, p. 213). This example was similar to mathematics registry instances in both the Canadian and United States textbooks. The Canadian example said, “Any letter can be used as a variable to represent a number or amount (McAskill et al., 2007, p. 361), and the United States example says, “Any letter can be used as a variable” (McGraw-Hill/Glencoe, 2012, p. 5). All of these examples said that the letter picked for the variable does not matter. This is important to take note of because neither of these textbook series addressed the issue of how certain letters mean something mathematically specific. When the textbook stated that any letter can be used as a variable this can lead to confusion about how letters such as $m\ and\ b$ have specific meanings in the equation $y = mx + b$, or the letter i is used for imaginary number, or even the letter *e* is used for Euler's number, and are not often used as variable letters.

Lastly, all three of the textbook series used the noun *expression* at some point in one of their kernels. The Canadian textbook series used *expression* in their *constant* and *expression* kernels, while Australia and the United States only used *expression* in their *expression* kernel. Table 21 summarizes the number of instances each textbook series had for the noun *expression*.

Table 21

Number of EXPLICIT and IMPLICIT instances for the noun phrase "expression" from each textbook series and their registry

Textbook	Explicit Reference		Implicit Reference	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
United States	11	0	0	0
Canada	55	0	21	0
Australia	76	0	26	0

From the above chart it is clear that the United States had the fewest explicit and implicit references to the noun *expression,* while Australia had the most explicit and implicit instances. Canada and Australia had a similar number of implicit instances, but vary more so with their explicit references. Another interesting finding is that neither country had a single bridging registry instance for *expression*.

Other Findings

The following section discusses any interesting language aspects that were found in each of the textbook series as well as summarizes how each textbook talks about the meaning of the different letters in $y = mx + b$.

An interesting language finding occurred in the Australian textbooks when they used the phrase "backtracking" instead of "inverse operations." The language used was interesting because the idea of "backtracking" means to move backwards which is essentially what is happening to an equation when applying inverse operations. The language of "backtracking" already implies what needs to happen to the equation versus the language of "apply inverse operations" which would require students to understand the meaning of "inverse operations." This was an important finding because this example of bridging language clearly used language from the English registry in the Mathematics setting. This seems like something that could be easily adapted in other English speaking countries to help students understand the meaning of "applying inverse operations." It was surprising that this clear example of bridging registry came from the Australian textbook series, since this series also provided students with another word that would only makes sense in the mathematics registry, pronumeral.

The Australian textbook series was the only English speaking textbook series that made a distinction between variable and pronumeral. They defined a variable as, "A variable is a pronumeral that can have a variety of values," and a pronumeral as, "A pronumeral is a letter that stands for a number" (Vincent et al., 2014, p. 217). This is the only textbook series in this study that attempted to distinguish a variable as an unknown versus a variable that can vary. However, even though the textbook made the distinction between these two uses of variable there was still no attempt to clarify how the different letters in the equation $y = mx + b$ are used. The textbook did not make any effort to distinguish the role of the letters "m" and "b" versus the role of the letters "x" and "y." It was not made clear to students how the uses of these letters are different.

Like the Australian textbooks, all three Canadian textbooks talked about variable and linear equations, however none of them presented the general form of the equation of a line – some variation of $y = mx + b$. The textbook, after defining a variable as a letter that represents an unknown number, did not distinguish between the different letters, how they are used, and whether or not all are considered variables. Even though they discussed what a "constant" was, the textbook made no effort to distinguish the letter b in the above equation from the other letters. Similarly, all three textbooks had chapters dedicated to solving linear equations of different forms such as $ax + b = c, a/x + b = c, a(x + b) = c$ etc, but they never distinguished how the letters a and x, for example, get used in those equations. The textbooks introduced the section with a title such as, "Solving one-step equations: $x + a = b$" (McAskill et al., 2007), and then presented a problem designed to get students exploring the math themselves. Different examples were then given of different ways the students could potentially solve equations of that form such as $j + 4 = 12, 15 - y = 5, 17 = d - 9$ (McAskill et al., 2007) and then the students were asked to solve problems of a similar nature. This was interesting because students were not directly instructed of the uses for x, a, and b that were given in the equation in the heading, however it was implied that two of the letters will be replaced with numbers and only one letter will remain. The main idea conveyed in these textbooks was the idea that a variable is used to represent an unknown number, and the textbooks very rarely use it otherwise.

Lastly, the textbooks from the United States, despite being the longest in length, did not have significantly more instances than the textbooks from Australia or Canada. Both textbooks had chapters on the slope intercept form of a line $y = mx + b$, however they did not distinguish between the uses of the different letters in the equation. The Pre-Algebra stated that, "An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is in slope-intercept form" (McGraw-Hill/Glencoe, 2012, p. 439). This definition makes no reference to how the uses of x or y (variables) in the equation differ from the uses of m or b (parameters). In the Algebra textbook, the slope intercept form of the line was not introduced in the first discussion on linear equations and therefore did not get coded. Instead, the Algebra book first introduced the standard form of a line and said, "In this equation, C is called a constant, or a number. Ax and By are variable terms" (McGraw-Hill/Glencoe, 2012, p. 180).

It was interesting that the Pre-Algebra book introduced the slope intercept of a line first, and dedicated a whole chapter to it but only made mention of the standard form of a line in the margins of the text. It seems like the textbooks should be consistent with the material they are emphasizing and in what order they do so. It was interesting that the textbook defined a variable as "A variable is a letter or symbol used to represent an unknown value," yet they don't talk about whether or not the different letters in $y = mx + b$ or $Ax + By = C$ are all variables. Even though the textbooks talked about a constant they did not distinguish the letters $b, or\ C$ in the above equations from the other letters.

It is clear that no English language textbook used in this study made any attempt to define or address the uses of each of the letters in the equation $y = mx + b$. This issue is not even a problem about language, but about mathematics. The textbooks did not present a mathematical definition for the different uses that these letters have. The question that would arise for students then is, "Are all of these letters variables?"

Spanish Textbook Analysis

Looking across all of the Spanish speaking textbooks there were many similarities. The following section will discuss similarities between the kernels as well as the noun phrases that the textbooks from Mexico, Colombia, and Chile used.

Kernel and Noun Analysis

The kernel analysis for the Spanish textbooks varied from that of the English textbooks. The Chilean textbooks did not define a kernel related to *variable* or *unknown* therefore the analysis for that textbook series was done based off of the common nouns and noun phrases between the Colombian and Mexican textbook series. Since the Chilean textbooks never defined any kernels related *variable,* this part of the analysis focused on common kernels between the Mexican and Colombian textbook series.

First, both the Mexican and Colombian textbook series had a kernel about *variable* and *expression.* Table 22 summarizes the number of instances from each textbook series for the *variable* kernel:

Table 22

Number of EXPLICIT and IMPLICIT instances for the Variable kernel

Textbook Series:	Number of Explicit Instances:	Number of Implicit Instances
Colombia	51	39
Mexico	51	44

It is interesting to note that the textbook series from both countries had more explicit instances than implicit instances. Another aspect of the *variable* kernel that was unique to the Mexican textbook series was that they defined variable in two different ways. They said that (1) “Variable: Literales son letras utilizadas para representar numeros. Tambien so les conoce como variables (Variable: Literals are letter used to represent numbers. They are also known as variables)" (Escareno & Lopez, 2010, p. 111), or (2) "Variable: Representa a cada uno de los elementos de un conjunto de datos. Cada elemento del conjunto es un valor de la variable (Variable: Represents each element of a data set. Each element of the set is a value of the variable)" (Gaisman et al., 2010, p. 85). The second variable kernel used more of a statistics perspective when it defined variable. Looking at the above table revealed that, had the Mexican textbook series only had one *variable* kernel, the Colombian textbook series would have had more explicit and implicit references than the Mexican textbook series.

Both textbook series also defined expression. The Mexican textbook series said that, "Expression: Como recordaran, una expresion algebraica es la combinacion de numeros, letras, y signos de operacion" (Gaisman, M. T., et al. 2010, p. 30), which was translated to, "Expression: As you may remember, an algebraic expression is a combination of numbers, letters, and signs of operation." The Colombian textbooks defined expression as, "Expresion Algebraica: Una expresion algebraica es una combinacion de numeros y letras unidos por los signos de las operaciones aritmeticas: adicion, sustraccion, multiplicaion, division, y potenciacion" (Orozco, L. S. A., et al. 2012, p. 134). This translated to, "Algebraic Expression: An algebraic expression is a combination of numbers and letters connected by the signs of arithmetic operations. Addition, subtraction, multiplication, division, and empowerment"

Table 23

Number of EXPLICIT and IMPLICIT instances for the Expression kernel

Textbook Series:	Number of Explicit Instances:	Number of Implicit Instances
Colombia	46	39
Mexico	25	12

From the chart above it was clear that both the Colombian and Mexican textbooks had more explicit instances than implicit instances. Another finding was that the Colombian textbook had more explicit and implicit instances than the Mexican textbooks. It is interesting that the Colombian textbook had more when both definitions were similar. Both kernels share the noun phrases *expresion algebraica (algebraic expression), numeros (numbers), letras (letters), and operaciones aritmeticas/signos de operacion (arithmetic operations/ signs of operation).* Since both textbook series had the same noun phrases this shows that the Colombian textbook referred to these noun phrases more, which suggested that they gave students the opportunity to become more familiar with the noun phrases and the ideas that made up the expression kernel.

The differences between the kernels from these textbooks was that the Mexican textbook had an *incognita* kernel and the Colombian textbook had a *cantidades desconocidas* kernel. This translated to an *unknown* kernel and an *unknown quantity* kernel. The *unknown* kernel said that, "En una ecuacion, cantidad o valor que no se conoce y que hay que averiguar. Generalmente se representa con una letra," This translated to, "Unknown: In an equation, the quantity or value that is not known and needs to be found out. It is usually represented by a letter (Escareno & Lopez, 2010, p. 174)." The *unknown quantity* kernel said that, "En matematicas se utilizan las letras para expresar cantidades desconocidas; las letras de la expresion, pueden representar uno o varios numerous (Orozco et al., 2012, p. 132)." This translated to, "In math letters are used to express unknown quantities; the letters of the expression may represent one or more numbers. The *unknown* kernel stated that the *unknown* was a specific quantity, whereas the *unknown quantity* kernel stated that the letters represented one or more numbers.

There was also a lot of overlap in the nouns/noun phrases that these textbooks series chose to use when they defined their kernels. The three textbook series all shared the nouns *letra* *(letter), numeros (numbers), expresion algebraica (algebraic expression), and variable (variable).* Mexico used the above nouns when defining *incognita (unknown), variable (variable), expresion (expression), and variable (statistics perspective).* Colombia used the nouns when defining *cantidades desconocidas (unknown quantities), variable (variable), and expresion algebraica (algebraic expression).*

The first noun that all three textbooks had in common was *letra.* Mexico used the noun phrase *letra (letter)* in the *incognita (unknown)* and *expresion* (expression) kernels, Colombia used *letra (letter)* in the *cantidades desconocidas (unknown quantities), variable (variable), and expresion algebraica (algebraic expression).* Lastly, the Chilean textbook used the noun *letra (letter)* in the *incognita (unknown), cantidades desconocidas (unknown quantities), expresion (expression), and variable (variable)* kernel. Table 24 summarized the number of total instances (across kernels) for the noun phrase *letra* (letter) across each textbook series.

Table 24

Number of instances for "letra" for each textbook

Textbook	EXPLICIT INSTANCES		IMPLICIT INSTANCES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
Math 7 (Colombia)	15	0	4	0
Math 8 (Colombia)	8	0	33	0
Math 9 (Colombia)	1	0	8	0
Math 7 (Mexico)	4	0	0	0
Math 8 (Mexico)	3	0	2	0
Math 9 (Mexico)	1	0	8	0
Math 7 (Chile)	8	0	15	0
Math 8 (Chile)	0	0	12	0
Math 9 (Chile)	1	0	4	0

Looking at the chart all of the Math 7 textbooks had the most explicit mathematics registry instances than the other textbooks from their series. Another observation was that in the

Colombian and Mexican textbooks as the grade increased the number of explicit instances decreased and the number of implicit instances increased. When all of the instances are totaled the Colombian textbook series had the most explicit and implicit instances out of all the Spanish textbooks used for this study. All of the textbooks had a similar instance explicitly referring to *letra* in their 7th grade textbook. The Chilean textbook said, "El lenguaje algebraico nos permite representar una informacion dada mediante operaciones con numeros y letras" (Casals, M. d. P. B., et al. 2009, p. 172). The Mexican textbook said, "Incognita: En una ecuacion, cantidad o valor que no se conoce y que hay que averiguar. Generalmente se representa con una letra" (Escareno, F. and O. L. Lopez 2012, p. 174), and the Colombian textbook said, "En matematicas se utilizan las letras para expresar cantidades desconocidas; las letras de la expresion, pueden representar uno o varios numeros" (Orozco, L. S. A., et al. 2012, p. 132). These can be respectively translated to, "The algebraic language permits us to represent given information by operations with numbers and letters," and "Unknown: In an equation, a value or quantity that is not known and needs to be found out. It is usually represented by a letter," and "In mathematics they use letters to express unknown quantities; the letters of the expression can represent one or various numbers." All of these instances usef the mathematics registry and talked about how letters are used to represent unknown numbers, or information in algebra. This means that one characteristic of Spanish textbooks is to talk about how letters are used to represent information in algebra.

Next, all three textbook also used the noun *numeros (numbers)* at some point in their kernels. The Mexican textbooks used *numeros (numbers)* in their *variable and expression* kernels, the Colombian textbooks used it in their *cantidades desconocidas (unknown quantities) and expresion algebraica (algebraic expression)*, and the Chilean textbook used *numeros* *(numbers)* in their *cantidades desconocidas (unknown quantities) and expresion (expression)* kernels. Table 25 summarized the number of instances in the mathematics registry and bridging registry for the noun *numeros (numbers)* across each textbook series*:*

Table 25

Number of instances for "numeros" in each textbook series

Textbook	EXPLICIT INSTANCES		IMPLICIT INSTANCES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
Colombia	21	0	0	0
Mexico	17	1	2	0
Chile	6	0	6	0

Like with the *letra (letter)* noun the chart shows that the Colombian textbook had the most explicit instances compared to the other Spanish speaking textbooks. With this noun phrase the Colombian and Mexican textbooks were most similar with their total explicit mathematics registry count. The only bridging registry instance occurred with the 9th grade Mexican textbook when it said, “En situaciones como la del recorrido de Marta con la bicleta, en las que una variable esta relacionada con otra de manera lineal, la razon de cambio, es decir, el numero que indica la manera en que cambia una de las variables (la distancia) cuando la otra cambia (el tiempo), es siempre el mismo” (Gaisman, M. T., et al. 2010, p. 58). This translates to, “In situations such as the route of Marta with the Bike Path, in which one variable is related to another in a linear fashion, the rate of change, i.e. the number that indicates how changing one of the variables (distance) when the other changes (time) is always the same.” This instance used the bridging registry because it tied the idea of “rate of change” to a real life situation (specific context criterion) and tried to explain to students the relationship each variable had in the context of the story problem with the idea of "rate of change."

There were far fewer implicit instances with the noun *number* than with the noun *letra*. This could be because it is hard for textbooks to implicitly refer to *number*. One example from the 8th grade Mexican textbook series was, "Terminos independientes. Son los terminos de una expresion algebraica que no contienen variables (Gaisman, M. T., et al. 2010, p. 31)." This translated to, "Independent terms: Are the terms of an algebraic expression that doesn't contain variables." This was an implicit reference to number because the textbook described the fact that numbers can be independent terms.

Next, all textbooks shared the noun *expresion algebraica (algebraic expression)*. Mexico used *expresion algebraica (algebraic expression)* in their *expresion (expression)* kernel, Colombia used it for their *expresion algebraica (algebraic expression)* kernel, and Chile used it for their *expresion (expression)* kernel. Table 26 summarized the number of instances for the noun *expression algebraica* (algebraic expression) across the different countries:

Table 26

Number of instances for "expresion algebraica" in each textbook series

Textbook	EXPLICIT INSTANCES		IMPLICIT INSTANCES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
Colombia	13	0	1	0
Mexico	12	0	0	0
Chile	4	0	0	0

The noun phrase *expresion algebraica* had fewer instances than the *numeros* noun. This noun phrase did not have any instances that fell under the bridging registry category for either explicit or implicit references. The only implicit instance came from the 8th grade Colombian textbook series when it said, "Coeficiente: Es el numero real que aparece en cada termino. Por ejemplo: $\sqrt{2}x$ el coeficiente es $\sqrt{2}, \frac{7x^2}{2y}$ el coeficiente es $\frac{7}{2}, \frac{ax^3\sqrt{y}}{5bw^6}$ el coeficiente es $\frac{1}{5}$ (Coefficient: Is the real number that appears in each term. For example: $\sqrt{2}x$ the coefficient ss $\sqrt{2}, \frac{7x^2}{2y}$ the coefficient is $\frac{7}{2}, \frac{ax^3\sqrt{y}}{5bw^6}$ the coefficient is $\frac{1}{5}$" (Garcia, W. F. E., et al. 2011, p. 56). Here the textbook focused on teaching students what a coefficient was but they did not reinforce the idea that each of these examples were also examples of an algebraic expression.

All of the Spanish textbook series had fewer implicit instances of the *expresion algebraica* noun phrase than either the *letra* or the *numeros* noun. This could also be due to the fact that these textbook series introduced the idea of an *expresion algebraica* whereas they did not introduce the ideas of *numeros* or *letra.*

The last noun, *variable,* was used in the Mexican textbooks under the *variable* kernel and the *statistics variable* kernel. *Variable* was also used in the Colombian and Chilean textbooks under the *variable* kernel.

Table 27

Number of instances for "variable" in each textbook series

Textbook	EXPLICIT INSTANCES		IMPLICIT INSTANCES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
Colombia	19	0	8	0
Mexico	32	2	6	0
Chile	9	0	13	0

Table 27 shows that the Mexican textbook had the most explicit mathematics and bridging registry instances, however it had the fewest implicit mathematics registry instances. This could be due to the fact that this textbook series was the only one of the three countries to have two *variable* kernels. This could also be because the textbook was careful to explicitly use *variable* and make it clear to students when it was referring to the noun *variable.* The Chilean textbook had the most implicit instances whereas the Colombian and Mexican textbook had similar number of references to the noun *variable.* This is important to take note of because the Mexican textbook series had two *variable* kernels but did not have more implicit references than the Colombian or Chilean textbooks.

The Spanish textbook series only had two bridging registry instances and both of them were in the Mexican 9th grade textbook series. These instances occurred in the Mexican 9th grade textbook series when it said, “En situaciones como la del recorrido de Marta con la bicleta, en las que una variable esta relacionada con otra de manera lineal, la razon de cambio, es decir, el numero que indica la manera en que cambia una de las variables (la distancia) cuando la otra cambia (el tiempo), es siempre el mismo,” and “Para ecuaciones no lineales, es decir, ecuaciones en las que la variable esta elevada a una potencia mayor o igual a dos, encontramos que puede haber varios valores que satisfagan la ecuacion, es decir, varias soluciones (Gaisman, M. T., et al. 2010, p. 58)." These respectively translated to, "In situations such as the route of Marta with the Bike Path, in which one variable is related to another in a linear fashion, the rate of change, i.e., the number that indicates how changing one of the variables (distance) when the other changes (time) is always the same," and "For nonlinear equations, meaning, equations in which the variable is raised to a power greater than or equal to two, we find that there may be several values which satisfy the equation, which means, various solutions.

These instances used a bridging registry because both make an attempt to clarify and put in different words ideas such as *rate of change* and ideas related to *nonlinear equations*. In both instances the statements such as *i.e., and meaning* followed by clarifying statements so the reader could make sense of a more complex idea. They are also used context specific to each of their textbook series.

Other Findings

The following section summarized how each textbook series talked about the meaning of the different letters in $y = mx + b$, as well as any interesting language aspects that were found in each of the textbook series.

The Chilean textbook series presented the slope intercept form a line,$y = mx + b$ however it did not differentiate how the different letters were used. This textbook series was also unique in that it did not define variable or expression, or unknown at all. However, the textbook series still explicitly and implicitly referred to the nouns and noun phrases that were typically associated with these terms. This would be confusing to students because they are expected to understand what a variable is when they have not even been presented with a formal definition.

Also according to Van Dormolen (1986) textbooks are supposed to build upon one another, and this textbook series did not exhibit that characteristic.

The Chilean textbook series also presented an interesting idea related to language. In the Math 9 textbook there was an instance that referenced the *incognita* kernel that stated, “En la funcion lineal $f(x) = mx$ el coeficiente m recibe el nombre de coeficiente de direccion o pendiente, ya que su valor determina la inclinacion de la recta respecto del eje X (Pena et al., 2010, p. 121).” This translated to, “In the linear function $f(x) = mx$ the coefficient m receieves the name of coefficient of direction or slope, because its value determines the inclination of the line from the X axis.” This instance revealed that the Chilean textbooks used the name *coefficient of direction* to describe what the letter m stands for in the above equation. This seems like a useful idea for students to become familiar with since the sign of the slope determines which direction the line will follow. The textbook uses the *coefficient of direction* or *slope* interchangeably which gives students the opportunity to become familiar with both names and helps them make connections with the meaning of each name. For example with the word slope students can associate the idea that something is not level or slants. This coupled with the idea of the *coefficient of direction* gives students a more complete definition of what the coefficient m is.

One of the most interesting things about the Colombian textbook series was that not only did the textbook present both the slope intercept form a line, $y = mx + b$ and the standard form of a line, $Ax + By + C = 0$, but this textbook series also made the first attempts thus far in differentiating the ways letters are used in the above equations. This occurred with the instance “Las letras que suelen usarse como cantidades conocidas son *a, b, c, d, m*, y *n*, y las letras *x, y, z* usualmente representan incognitas (Garcia et al., 2011, p. 237).” This can be translated to, “the letters that are used for known quantities are a, b, c, d, m, and n, and the letters x, y, z usually represent unknowns." Although this instance does not explain why some letters are used for known quantities, and other are used for unknowns, it was the only textbook series to even make an attempt to address the different uses of letters in equations.

The Colombian textbook series was also unique in that the textbook series did not define variable until the Math 8 book, but explicitly used variable in the Math 7 textbook. This would be confusing to students because they are expected to understand what a variable is when they have not even been presented with a formal definition.

The Mexican textbook series presented the general form of a line as $y = mx + b$ however the textbook series never explained how the uses of these "variables" differ from one another. They do mention that m is the pendiente (slope) and b is the constante (constant), but they don't explain why x and y are often referred to as variables and $m \text{ and } b$ are given specific values. The textbook also did not make any attempt to clearly state the difference between an unknown and a variable.

The Mexican textbooks were also unique because they defined *variable* in two different ways. So far, this has been the only textbook to use the word *variable* from a statistics perspective. This also highlights the fact that we use *variable* in multiple ways and for students it can be difficult to keep track of the definition they should be applying to each situation.

One example of different language from the 8th grade Mexican textbook series is, "Terminos independientes. Son los terminos de una expresion algebraica que no contienen variables" (Gaisman, M. T., et al. 2010, p. 31). This instance translated to, "Independent terms: Terms of an algebraic expression that do not contain variables." This is comparable to a *constant term* in English. The idea of calling a *constant* an *independent term* would be helpful to students because they could associate the idea of "independent" with "independence from a variable." This would help them to become familiar with the idea that constants are not affected by the variable.

A second example of an interesting language finding from the Mexican textbook series came from the example, "La siguiente ecuacion representa la situacion 3x+2=11, Si restan 2 de ambos miembros, se altera la igualidad (Escareno & Lopez, 2010, p. 139)?" This can be translated to, "The following equation represents the situation 3x+2=11, if you subtract two from both members, is the equality altered?" Instead of using "terms of an equation" the statement used "members" and the question that is asked gets students thinking about keeping the equations equal and the operations they perform to solve the equation. Using *member* could be helpful to link the idea that on either side of the equation those terms are members of an expression.

Bridging Language Analysis

The following section discusses some of the major findings from the research of this study and answers the following research questions: (1) In what registry are chapters related to variable in textbooks typically written? (2) Do textbooks use language that bridges the gap between the mathematics registry and the English language registry for students? The third research question was answered in the previous section when common nouns/noun phrases and kernels will analyzed as well as any common instances that were found across textbooks. This section is divided into two sections: a section discussing various findings with the bridging language, which answers the first two questions; and a section discussing the different uses of variable, which is a finding of the research not directly related to the research questions.

Based on the previous results it is evident that textbooks do not do a good job of using bridging language for the different uses of letter in their mathematics textbooks. This finding is true for both English and Spanish textbooks used for this study. Table 28 summarizes the findings across all six textbook series for the percentage of the analyzed portions of the textbooks that used the mathematics registry or the bridging registry.

Table 28

Total number of bridging registry instances in each textbook series

Textbook	EXPLICIT AND IMPLICIT INSTANCES		PERCENTAGES	
	Mathematics registry	Bridging Registry	Mathematics registry	Bridging Registry
Colombia	197	1	99.5%	.5%
Mexico	148	6	96.1%	3.9%
Chile	146	0	100%	0%
United States	413	12	97.2%	2.8%
Canada	637	33	95%	5%
Australia	715	18	97.5%	2.5%

This table tells us that while neither language does a very good job at using bridging language in their textbooks, overall the English speaking textbooks do better at using bridging language than the Spanish textbooks. It is clear that there is a lack of bridging language about the different uses of letter in the textbook series studied for these six different countries. This next portion explores how the bridging language is used in these textbooks.

Recall that instances were categorized as involving bridging language if they met one or more of the following criteria: (1) the instance substituted an English word to replace a math word (replacement criterion), (2) the instance explains the meaning behind a particular noun/noun phrase using a unique context (specific context criterion), and (3) the instance intended that the students reflect on the mathematics (reflection criterion). Further analysis of the bridging registry instances was done to look for any common ways the textbooks might have used bridging language in their texts. This analysis revealed that the bridging registry instances found in the textbooks could be categorized into three different categories. They were as follows: (1) Specific examples, (2) Reflection statements, (3) Explicitly telling, and (4) Clarifying statements.

Specific Examples

The bridging language instances that fell under the category of *specific examples* illustrated the use of a letter through a specific example from the textbook. The textbook authors attempted to link the idea that the letter they used stood for something specific in the context of the problem. They explained particular noun/noun phrases by using a particular real world problem unique to the textbook. For example, the Australian textbook series used an example in their 7th grade textbook that dealt with an unknown number of toothpicks in a cup. Two examples of bridging registry referred to this scenario: “If there are 2 lots of m toothpicks, we can write this as $m + m$. However we can also write this as $2 * m$” (Vincent et al., 2014, p. 211), and “When $m = 4$ there are 3 lots of 4 toothpicks” (Vincent et al., 2014, p. 214). Another example occurred in the Canadian textbook series when it said, “The number of squares, s increases by 3 for each figure number, n. Multiplying the figure number n, by 3 results in 2 more than the number of squares. Therefore subtracting 2 from $3n$ equals the number of squares, s” (McAskill et al., 2009, p. 212). The Mexican textbook series also had another example of a word problem that used bridging language. This instance was translated[2] to, “In situations such as the route of Marta with the bike path, in which one variable is related to another in a linear fashion, the rate of change, i.e., the number that indicates how changing one of the variables

[2] Since the Spanish textbook instances have been previously translated, they will only appear in English from this section forward.

(distance) when the other changes (time) is always the same" (Gaisman et al., 2010, p. 58, *translated by author*). All of these examples discussed *letter* in the context of a word problem, which provided students a way to link the use of letter to something outside of the abstract mathematical meaning.

Reflection Statements

Another type of bridging language that was used involved asking students to reflect and think about a question posed by the authors. As stated in the methodology, instances were coded as bridging language if the instance asked students to reflect on the mathematics by asking them a question about the conceptual underpinnings of the mathematics. Many of these instances occurred in the Canadian textbook and took the form of, "Ask yourself: What number less 9 makes 17 or what number results from adding 17 and 9?" (McAskill et al., 2008, p. 396), an example from the United States was, "Rewriting a subtraction expression using addition will help you identify the terms of an expression" (McGraw-Hill/Glencoe, 2012, p. 181). This was an example of a reflection statement because it suggested that the textbook intends to have students reflect on the relationship that the terms have with another. Students can think about why it makes sense that a subtraction expression can be rewritten as an addition expression. The only Spanish example of a reflection statement came from the Mexican textbook series and was translated to, "The following equation represents the situation $3x + 2 = 11$, if you subtract 2 from both members is the equality altered" (Escareno & Lopez, 2010, p. 139, *translated by author*)? These examples all asked students to reflect somehow and to think more conceptually about the mathematics and why the mathematics works which made them examples of a bridging language.

Clarifying Statements

Lastly, the instances of bridging language used clarifying statements. Clarifying statements occurred when the authors explicitly tell how a strategy works or describe the meaning of some aspect of mathematics. They also appeared when the textbook immediately followed a mathematical statement with some type of clarification. In all cases, there was an attempt by the authors to clarify or explain the meaning of a mathematical word or idea. One indicator of examples that fell in this category were linking phrases such as "that is to say" or "because" which combined the first clause with the explicatory clause. There was an example of this type of bridging instance in all of the English speaking textbooks, but the only examples of this type of bridging language in the Spanish language were found in the Mexican and Colombian textbook series. For example, "The expression $2(x + 4)$ and $2x + 8$ are equivalent expression because no matter what the value of x is, these expression have the same value" (McGraw-Hill/Glencoe, 2012, p. 174). Another example of this category was "A four step strategy exists for setting up and solving worded problems with algebra. It can be summarised as follows. Translate the words into algebra. Decide on the unknown and give it a pronumeral, then formulate an equation using this pronumeral" (Vincent et al., 2014, p. 417). This was an example of bridging registry because it connected the idea of pronumeral and unknown together. The use of the word pronumeral was specific to the Australian textbook series and this instance suggested that pronumerals were used for unknown which satisfied the specific context criterion in the methodology. An example from the Colombian textbook series was, "Algebraic language uses algebraic expression to express in abbreviated form, with precision and clarity, a situation or property" (Garcia et al., 2011, p. 56). These instances were all examples of clarifying statements because the instances stated something like "equivalent expressions" and then described what that was, or they told the students some sort of strategy for solving mathematical problems.

These examples are all similar in how they presented information to students in that they either explicitly told them how a strategy works or they described (told) the meaning of some aspect of mathematics. This category of bridging language allows the students to become more familiar with complex mathematical vocabulary by using synonyms that they already understand.

Other Observations about Bridging Language

Table 29 summarizes the breakdown of the number of instances in the different categories of bridging language across all six countries:

Table 29

Number of each type of bridging language instances

Specific Examples	Reflection Statements	Clarifying Statements
6	23	14

It is clear from the table that there were significantly more examples of *reflection statements*. Most of these examples occurred in the Canadian textbooks and take the form of, "Ask yourself…"

Explanation of Letter Use Analysis

As stated in the theoretical framework for this study, Usiskin (1988) identified five common contexts in which variables are used. They are (1) a formula, (2) an equation to solve, (3) an identity, (4) a property, and (5) a function of direct variation. While doing this analysis, I considered the possibility that the textbooks may have uses of variables or contexts in which variables are used that do not fit into one of Usiskin's five categories. However, all of the analyzed instances that talked about letters fit into one of the categories of Usiskin's framework. Although Usiskin says that each of these cases are different uses of variable, I am going to talk about Usiskin's framework as five common uses of letter to represent numbers. Using letter instead of variable will help avoid the pitfall of the reader thinking that I am only talking about variables as representing changing quantities.

Figure 10 shows which countries' analyzed textbooks use a letter in the introductory chapters on variable and linear equations in the ways defined by Usiskin followed by a discussion on what these common uses of letters looked like in the textbook.

	United States	Canada	Australia	Colombia	Mexico	Chile
A Formula	X			X		X
An Equation to Solve	X	X	X	X	X	X
An Identity						
A Property	X	X	X		X	
A Function of Direct Variation	X	X	X	X	X	

Figure 10. Types of letter use in each textbook series

A Formula

The example that Usiskin (1988) used for the use of a letter in a formula is $A = LW$. Usiskin (1988) states that when letters are used in a formula like with the example, $A = LW$, A, L and W stand for the quantities area, length, and width and are "knowns." He goes on to say that with the example above we are describing how the three quantities are related and there is not the feel of an unknown because we are not solving the equation for anything. Following this as a guideline an analysis was done on the instances in all of the textbooks to see if any used a letter as part of a formula and if there was any attempt to use bridging language to explain how the use of letter in this context is different than in other contexts. In the United States textbook series some instances that fit under the category of using a letter in the context of a formula were, "$P = 2l + 2w$ or $P = 2(l + w)$ (McGraw-Hill/Glencoe, 2012, p. 223)," "$A = lw$ (McGraw-Hill/Glencoe, 2012, p. 224)," and "$A = \frac{1}{2}bh$ (McGraw-Hill/Glencoe, 2012, p. 224)." Some translated examples from the Spanish speaking textbooks were, "The perimeter, P, of the rectangle is the sum of the measures of its sides then: $P = 2x + 2y$ (Orozco et al., 2012, p. 134)," and "The density of a body is the ratio between the mass and volume, $Density = m/V$ (Orozco et al., 2012, p. 134). Based on the analysis the countries that used a letter as part of formula were the United States, Colombia, and Chile. All of the instances under this category referred to a specific formula like perimeter, area, and density.

Even though three of the textbooks used a letter in a formula none of them explained to students how that relationship was different than something of the form $5x = 10$. To a student, those equations might look the same and they might not realize that the use of a letter in a formula where several of the letters represent "knowns" differs from the use of letter in an equation to solve where the letter represents an "unknown".

An Equation to Solve

Of all the categories Usiskin (1988) listed this was the most widely featured category for the sections of textbook analyzed for this study. Usiskin gives the example that an equation to solve takes the form of $40 = 5x$ and states that in these examples we are treating x or whichever letter is in use as an unknown and that when solving these kinds of problems the letter is either an unknown or a constant. Examples from the textbooks used for this study take the form of, “The first step in translating verbal phrases into algebraic expression is to choose a variable and a quantity for the variable to represent” (McGraw-Hill/Glencoe, 2012, p. 11), “If the pronumeral represents the input number, we can write $2 * n + 3 = 11$ or simply, $2n + 3 = 11$” (Vincent et al., 2014, p. 373), and “Solve the equation $x + 4 = 24$” (Orozco et al., 2012, p. 136, translated by author). The instances for this category varied in that some were solely an equation for students to solve and others were explanations of picking a letter to stand for an unknown. The countries to use a letter in an equation to solve were the United States, Canada, Australia, Mexico, Colombia, and Chile.

All of the analyzed textbooks used letter as part of an equation to solve, but they did not explain how the use of letter was different than their definition of variable because it did not vary. One country that attempted to rectify this issue was Australia. Australia introduced the term *pronumeral* to define the use of a letter as an unknown, and they defined a variable as a pronumeral that varies. None of the textbooks explained to students how an equation like $6x + 4 = 16$, where x is an unknown, is different than the use of a letter in an equation of the form $y = 4x + 3$, where x and y can take on many different values.

A Property

This category included examples of different properties of numbers. Here the letter is used to generalize a pattern of numbers. Using a letter as part of a property allows relationships between numbers to be described mathematically. Usiskin’s example of a letter being used in a property is $1 = n * \left(\frac{1}{n}\right)$ in which n can take on any value except 0 and the equation is still true. The different textbook examples of this use of letter in a property included, “Symbols: For any numbers a and b, $a + b = b + a$. For any numbers a and b, $a * b = b * a$ (McGraw-Hill/Glencoe, 2012, p. 18),” “For any number a, $a * 0 = 0 * a = 0$ (McGraw-Hill/Glencoe, 2012, p. 19)," and "Distributive property. Is the property of number and algebraic expression that combines the product and the sum. It can be represented with the given three numbers a, b, c in $a(b + c) = ab + ac$ (Gaisman et al., 2012, p. 34, translated by author). The most common property used in the textbooks was the distributive property. The countries that used letter in a property were the United States, Canada, Australia, and Mexico and the uses across all countries was similar.

When the textbooks used examples of a letter in a property, they did not explain to students that the letter is generalizing an arithmetic pattern, and the letter identifies an instance of the pattern. \ However, students were not told about this use of letter and might get confused that they are not supposed to solve for a value for the letter. This could be a place where textbooks use bridging language through specific examples, or reflection statements to help students understand this use of letter. Textbooks could highlight the fact that a "property" could be thought of as, "a quality or a characteristic of the number operations" This might help students know that the property describes a characteristic of the numbers and is not something that they should be solving for, or is not an example of letter being used as a formula.

A Function of Direct Variation

The last example from Usiskin's list of common uses for letters is a function of direct variation which looks like $y = kx$. The examples that were found in the textbooks were, "Consider the equation $y = 2x + 3$ (McGraw-Hill/Glencoe, 2012, p. 400)," "The relationship is linear and can be expressed as $(x, x - 2)$ or $y = x - 2$," and "Consider the linear rule $y = 2x + 4$ as the x-values increase in equal steps of 1 unit, the y values increase in equal steps of 2 units (Vincent et al., 2014, p. 543)." These instances are all similar to the example from Usiskin in that they are equations in which the letters represent values that co-vary.

This last common use of letter was used in all the textbooks but Chile. The example that Usiskin uses is $y = kx$ where x is an argument of a function, y the value, and k a constant. He states that only with this use of letter is there an actual feel of variability; yet none of the textbook chapters analyzed make any attempt to link the idea that a variable "varies" in a linear equation. They expected students to understand that y is dependent on x and that x can take on any value we want but they did not explain how that is different than something of the form $6x + 12 = 36$. Students would be confused and wonder why x cannot be anything we want in the example above if textbooks are saying that a variable is a letter that takes on various numbers.

Other Findings with Letter

Another interesting aspect of the use of letter that was found in these textbooks was that they all, at some point, discussed the equation $y = mx + b$. This has the form of an equation of direct variation but the textbooks also did not differentiate the uses of letters in these equations. It is not clear to the reader how the uses of these letters is different. The textbooks never explained that in the above equation x and y are variables and m and b are parameters. When the textbooks began using numbers in the above equation they use numbers in place of m and b, like in the example, "Consider the equation $y = 2x + 3$ (McGraw-Hill/Glencoe, 2012, p. 400)," but do not clarify why m and b are different than x and y. Another example of this was when the Canadian textbook instructed students to, "Solve one step equations: $x + a = b$ (McAskil et al., 2007, p. 395)." The textbook than proceeded to have examples of equations for students to solve like, "The situation can be modelled by the equation $d + 4 = 11$ (McAskill et al., 2007, p. 396)." From this it was understood that students were to infer that the x is an unknown but a and b will be replaced with known numbers. The only textbook that attempted to make the distinction is the Colombian textbook when it said, “The letters that are often used as known quantities are a, b, c, d, m, n, and the letter x, y, z usually represent unknowns (Garcia et al., 2011, p. 237).”

Overall none of the textbooks talked about how they used letters when they use them. When textbooks began explaining algebraic concepts with letters they should incorporate more bridging language to help students understand the different ways that letters get used in algebra, and that not all of the uses constitute the use of a variable. By not doing so students may become confused and not realize what role the letter has in the mathematics they are discussing. The fact that the textbooks did not adequately discuss the different ways they are using letters can contribute to students thinking that when they see a letter it is a variable when, in fact, it may be a known or an unknown and not vary at all. Even worse, was the fact that the textbooks did not explain the difference between uses of letters in the equations $A = LW$ and $y = kx$. Where the formula has some knowns and some unknowns but the equation of direct variation has a known parameter, k, and two variables that actually do vary. The different uses letter has is a place where authors or teachers can use bridging language and ask students to reflect on the values that x can take in the two equations $x + 3 = 4$ and $y = 3x + 4$, or help make students make sense of what x means using a particular context. All of the textbooks followed a similar pattern of defining a variable as a letter that stands for a number and then using it in different examples, like the categories above, but never explained how they are using letter. They use the word variable in places where the values that the letter can take on do not “vary.” This leaves students wondering what the real definition of a variable is and how letters are used in different ways.

CHAPTER 6: CONCLUSIONS

This study discussed the type of registry that textbooks used to present topics related to variable. The kernels in each textbook series were identified and then the nouns/noun phrases that made up those kernels were used to compile lexical chains to identify instances of language related to each kernel. Each instance was then analyzed to decide what registry it was using. The findings of this research were that (1) chapters that introduced variable used for this study rarely made any attempt to use bridging language when describing concepts related to variable, (2) the language used to talk about variable in textbooks was similar across the three English speaking countries and the three Spanish speaking countries, and (3) the chapters that were used for this study did not clearly define or describe how they used variable in a given example or situation. The first section will discuss the limitations of the study and the second section will discuss the implications of this study for textbooks.

Limitations

There are limitations to this study. The data from this study came from textbooks series from three English speaking countries and three Spanish speaking countries but not every textbook in these countries was analyzed; similarly a textbook series from every English/Spanish speaking country was not used. Thus the conclusions cannot be drawn beyond the sections of the text that were analyzed but do provide insight into issues that may exist in other textbooks.

Another limitation was that specific chapters from each textbook were analyzed and as a result the whole textbooks themselves were never analyzed. The data that was found and collected came from specific chapters meaning that there could be more examples of bridging language that fit under a category not found in this study. However, since this study was interested in looking at the use of bridging language to talk about variable, it is reasonable to believe that the most likely locations for such language to appear is in the chapters that introduced variable up through the first chapter on linear equations and not the whole textbook.

Implications for Textbooks

The different categories of bridging language and findings on the use of variable in textbooks that were used in this study can be used by textbook writers in at least two ways. The first is that textbook writers can look at the different categories of bridging language that were found in this study and make changes to questions or examples in their textbooks so that they could incorporate more bridging language. Each category of bridging language that was identified could potentially be useful to textbook writers. Reflection statements would be helpful because if the textbooks wanted to incorporate more bridging language they could ask questions that get students thinking about the mathematics. Clarifying statements would be beneficial to textbook authors because it shows examples of textbooks using mathematical terms but then using other language to help students understand what the textbook meant by those terms. Specific examples would be helpful when the textbook wants to connect the mathematics to a real world example and thus to language in the English or Spanish registry. The second is that textbook writers could make more of an effort to clearly explain to students the different ways they are using letters to represent numbers.

First, textbook writers can use the different bridging language categories to help them incorporate more bridging language in their textbooks. The categories of bridging language that were found in this study could help textbook writers modify questions, examples, and definitions so that they are accessible to more students. The two most popular bridging registry categories for the textbooks used in this study were reflection statements and clarifying statements. These could be easily incorporated into other textbooks to get students thinking more about the mathematics. Textbooks could add clarifying statements after they use mathematical language that might be difficult for students to understand like in the examples above, or they could ask students to reflect on why the mathematics work. By doing so, the textbooks would be using language that connects something in mathematics with something the student is already familiar with. Incorporating more examples of bridging language, like the examples found in this study, will help students because it will allow students to make sense of mathematics concepts in the context of language in the bridging or English (Spanish) registry that is more familiar to them.

Second, this research found that across the English and Spanish languages, variable is not defined clearly. This study could be used by textbook writers to help rectify the previously mentioned problem. This study used Usiskin’s (1988) research on the common uses of variable as a guideline and textbooks could do the same. They could explicitly state how they are using variable and why using a variable in that particular way for that situation makes sense. The findings from this study can also be used by textbook writers to improve the language and how variable is used in textbooks. This study presents a model of identifying bridging registry instances related to variable that could be refined and applied to other topics in mathematics. It could also be applied to reform mathematics textbooks to see how the language varies from that of typical mathematics textbooks.

This study found that there is lack of explanation of the different uses of letter and a lack of bridging language present in the textbooks studied. A possible solution to both of these weaknesses could be done simultaneously. This would be for textbooks to incorporate some of the different categories of bridging language as they address or explain the different uses of letters. For example, textbooks could ask students reflection statements about the different role the letter x plays in the equations $y = 3x + 4$ and $x + 3 = 4$. Textbooks could also use specific examples in their work to help students understand that when a letter is used in a formula it stands for a specific quantity. Textbooks could also work to incorporate more bridging language to explain the idea of a letter being used as a variable. To do this textbooks could use synonyms such as “changing” or “different values.” If students are not familiar with what *variable* means or the idea of something varying, textbooks could use “a letter that can change as students are more likely familiar with the idea of change. Another word that would be helpful would be “different values” because then students could link the idea that a variable can take on different values and does not have to be just one value. Another example of a phrase that would help students understand what a variable is would be “able to vary." This would be helpful to students because they would be able to link that a variable is something that can vary.

Summary

How language is used in textbooks is important to study. Textbooks are a primary instructional tool available to students in school. This study informs textbook writers’ categories of bridging language that they could use when describing concepts such as variable to students. This would help students have a better understanding of the mathematics because they would be linking vocabulary the students are already familiar to mathematical vocabulary. This not only suggests a greater need for textbooks to use bridging language but a greater need for textbooks to make it clear to students how they are using letters to represent numbers as each new use is introduced. In these textbooks it is not clear to students how the use of x in an equation like $3x + 6 = 12$ is different than the use of x in an equation like $y = 3x + 6$ or how m and x are different in the equation $y = mx + b$. By making it clear to students how they are using letters to represent numbers, and by using more bridging language when clarifying those uses of letters, we can improve mathematics textbooks in both languages.

It is interesting to note that regardless of whether the textbooks came from countries with a national curriculum or a locally determined curriculum, there were great similarities in the way they used bridging language and the way the talk about the concept of variable. Thus, a solution to these issues by a change to the locus of control of curriculum will not likely yield results.

Based on the analysis of the portions of six textbooks in this study, the answers to the research questions for this study seem to indicate that textbooks are unlikely to use much language in the bridging registry when they introduce the use of letters in algebra and the concept of variable. This study shows that textbooks are weak in two areas: bridging language and explaining to students the different roles letters have in algebra. I would propose that the introduction of letters in algebra would be a place for textbooks to incorporate more bridging language and not rely solely on the mathematics registry. Since the problem spans both the English and Spanish language textbooks, this suggests that both languages could improve their discussion of the use of letters and incorporate more bridging language to help students understand the different ways they are using letter.

References

Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education, 14*(3), 219-234.

ACARA (2013). *Rationale*. from http://www.australiancurriculum.edu.au/Mathematics/Rationale.

Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786-795.

Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics, 14*(3), 24-35.

Bell, E. T., Bell, E. T., Mathematician, W., Bell, E. T., & Mathémacien, E. (1951). *Mathematics: Queen and servant of science*: McGraw-Hill New York.

Casals, M. d. P. B., et al. (2009). *Matematica 7, Santillana Bicentenario.* Santiago, Chile, Santillana.

Casals, M. d. P. B., et al. (2009). *Matematica 8, Santillana Bicentenario.* Santiago, Chile, Santillana.

Citizenship, D. o. I. a. (2013). "What is the Australian Education System?". Retrieved August 8, 2013, 2013, from http://www.australia.idp.com/getting_started/study_in_australia.aspx.

Education, U. S. D. o. (2010). Overview of the U.S. department of education. from http://www2.ed.gov/about/overview/focus/what.pdf

Escareno, F. and O. L. Lopez (2012). *Matematicas 1*. Mexico, Editorial Trillas.

Gaisman, M. T., et al. (2010). *Matematicas: Secundaria 2*. Mexico, Editorial Santillana.

Gaisman, M. T., et al. (2010). *Matematicas: Secundaria 3*. Mexico, Editorial Santillana.

Gonzales, P., Calsyn, C., Jocelyn, L., Mak, K., Kastberg, D., Arafeh, S., . . . Tsen, W. (2000). *Pursuing Excellence: Comparisons of International Eighth-grade Mathematics and Science Achievement from a US Perspective, 1995 and 1999: Initial Findings from the Third International Mathematics and Science Study-Repeat*: National Center for Education Statistics.

Government, N. S. W. (2013). Educational Resources: Board of Studies NSW. Retrieved August 8, 2013, 2013

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal, 28*, 567-590.

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpation of language meaning*. Baltimore, MD: University Park Press.

Herbel-Eisenmann, B. A. (2002). Using Student Contributions and Multiple Representations To Develop Mathematical Language. *Mathematics Teaching in the Middle School, 8*, 100-105.

Herbel-Eisenmann, B., & Wagner, D. (2007). A framework for uncovering the way a textbook may position the mathematics learner. *For the learning of mathematics, 27*(2), 8-14.

Herbel-Eisenmann, B. A., & Otten, S. (2011). Mapping Mathematics in Classroom Discourse*. *Journal for Research in Mathematics Education, 42*(5), 451-485.

Hunter, D. *Defining educational standards and determining their reasonableness.* University of Regina. Retrieved from http://www.saskschoolboards.ca/old/ResearchAndDevelopment/ResearchReports/EvaluationAndReporting/99-07.htm#ssic

Kersaint, G., Thompson, D. R., & Petkova, M. (2013). *Teaching mathematics to English language learners* (2nd ed.). New York, NY: Routledge.

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, 297-312.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education, 31*, 234-241.

Licon, A. G. (2013). Mexico heads back to school with flawed textbooks. from http://news.yahoo.com/mexico-heads-back-school-flawed-textbooks-071724793.html

Lobato, J., & Thanheiser, E. (2002). Developing understanding of ratio and measure as a foundation for slope. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 162-175). Reston, VA: National Council of Teachers of Mathematics.

Longo, A. L. (1992). An updated view of the education system of Chile. *WENR*.

Luz Stella Alfonso Orozco, L. P. Z. L., Johanna Andrea Fuentes Diaz, Nelson Enrique Rodriguez, Marlady Bogota Torres, Oscar Yesid Marino Beltran (2012). *Matematicas 9: Redes de aprendizaje para la vida*. Bogota, D.C, Proyecto Se.

Luz Stella Alfonso Orozco, L. P. Z. L., Johanna Andrea Fuentes Diaz, Nelson Enrique Rodriguez, Marlady Bogota Torres, Oscar Yesid Marino Beltran (2012). *Matematicas 7: Redes de aprendizaje para la vida*. Bogota, D.C, Proyecto Se.

MacGregor, M. (2002). Using words to explain mathematical ideas. *Australian Journal of Language and Literacy, The, 25*(1), 78.

McAskill, B., et al. (2007). *MathLinks 7*. Canada, McGraw-Hill Ryerson.

McAskill, B., et al. (2008). *MathLinks 8*. Canada, McGraw-Hill Ryerson.

McAskill, B., et al. (2009). *MathLinks 9*. Canada, McGraw-Hill Ryerson.

McCrory, R. (2006). Mathematicians and mathematics textbooks for prospective elementary teachers. *Notices of the AMS, 53*(1), 20-29.

McGraw-Hill/Glencoe (2012). *Pre-Algebra.* USA, McGraw-Hill Education.

McGraw-Hill/Glencoe (2012). *Algebra*. USA, McGraw-Hill/Glencoe.

Moschkovich, J., Schoenfeld, A. H., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema & T. P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 69-100). Hillsdale, NJ: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics. (2000). *Executive summary: Principles and standards for school mathematics*. Reston, VA: Author.

Olson, D. R. (1980). On the language and authority of textbooks. *Journal of Communication, 30*(1), 186-196.

Pena, A. B., et al. (2010). *Matematica 1, Proyecto Bicentenario*. Santiago, Chile, Santillana.

Peterson, B. E. (2008). A look at Japanese junior high school mathematics textbooks. In Z. Usiskin & E. Willmore (Eds.), *Mathematics curriculum in pacific rim countries-China, Japan, Korea, and Singapore* (pp. 209-231). Charlotte, North Carolina: Information Age Publishing.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*: Routledge & Kegan Paul London.

Rolwing, K. (2006). Education in Mexico. *World Education News and Reviews, 19*(3).

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139-159.

Schmidt, W. H., & McKnight, C. C. (1998). What can we really learn from TIMSS? *Science, 282*, 1830-1831.

Schmidt, W. H., Wang, H. C., & McKnight, C. C. (2005). Curriculum coherence: An examination of US mathematics and science content standards from an international perspective. *Journal of Curriculum Studies, 37*(5), 525-559.

Services, W. E. (2004). Colombia Education Overview. from http://www.wes.org/ca/wedb/colombia/coedov.htm

Site, C. I. (2008). Education. from http://www.colombiainfo.org/en-us/colombia/education.aspx

Stump, S. L. (2001). High school precalculus students' understanding of slope as measure. *School Science and Mathematics, 101*(2), 81-89.

The Council of Ministers of Education, C. Education in Canada: An Overview. Retrieved October 12, 2013, from http://www.cmec.ca/299/Education-in-Canada-An-Overview/index.html#03

Thompson, D. R., & Rubenstein, R. N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *The Mathematics Teacher*, 568-574.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. *The ideas of algebra, K-12, 8*, 19.

Usiskin, Z. (1996). Mathematics as a language. *Communication in mathematics, K-12 and beyond, 86*.

Vincent, J., et al. (2014). *MathsWorld 7: Australian curriculum edition*. Australia, Macmillan Publishers.

Vincent, J., et al. (2014). *MathsWorld 8: Australian curriculum edition*. Australia, Macmillan Publishers.

Vincent, J., et al. (2014). *MathsWorld 9: Australian curriculum edition*. Australia, Macmillan Publishers.

William Fernando Estrada Garcia, V. M. G., Francisco Eduardo Romero, Luz Helena Silva Calderon (2011). Norma matematicas para pensar 8. Bogota, D.C., Colombia, Editora Geminis Ltda.

Yan, Z., & Lianghuou, F. (2006). Focus on the representation of problem types in intented curriculum: A comparison of selected mathematics textbooks from mainland China and the United States. *International Journal of Science and Mathematics Education, 4*, 609-626.