2014-11-01

The Effect of Students' Mathematical Beliefs on Knowledge Transfer

Kristen Adams
Brigham Young University - Provo

The Effect of Students' Mathematical Beliefs on Knowledge Transfer

Kristen Adams

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

Daniel Siebert, Chair
Keith R. Leatham
Steven R. Williams

Department of Mathematics Education

Brigham Young University

November 2014

ABSTRACT

The Effect of Students' Mathematical Beliefs on Knowledge Transfer

Kristen Adams
Department of Mathematics Education, BYU
Masters of Arts

Students learn many different concepts throughout their mathematical careers. In order to be successful in mathematics, students should be able to transfer these mathematical concepts learned in one situation and apply them to a new situation. There are many factors that might affect how students transfer knowledge; however research has focused mostly upon cognitive factors, even though affective factors might also exhibit a strong influence. This study examined how students' mathematical beliefs, specifically beliefs about mathematics education and self-efficacy, affect the transfer process. Data were collected from three middle school students. These data were collected through in-class observations, students' written work, and student interviews. The analysis of this data has shown that students' beliefs about mathematics education can affect specific steps in the transfer process as well as the transfer process in general. The data also showed that self-efficacy can influence the transfer process, but that this influence is mediated by the students' other mathematical beliefs.

Keywords: actor-oriented, affective factors, mathematical beliefs, self-efficacy, transfer

ACKNOWLEDGMENTS

I would like to thank my friends and family who encouraged and supported me throughout my research and the writing process. I would especially like to thank my advisor for his continual support and patience. I am grateful that he encouraged me to believe in myself and to never give up.

TABLE OF CONTENTS

LIST OF FIGURES

Chapter 1: Introduction

In mathematics classrooms every day, students learn valuable concepts that help them to be successful in their mathematics careers throughout their lives. This success includes success in the current classes they are taking, success in future mathematics classes, and success outside the context of schooling. Teachers hope that students will be able to take the important mathematical concepts that they have learned and apply them to future situations. This mathematical transfer of knowledge is vital to their future success. Without the ability to transfer knowledge, students will be less able to use previously learned information to help them be successful in their future mathematical situations.

The transfer of mathematical knowledge is applicable in a variety of situations. Transfer of knowledge is often defined as the application of knowledge learned in one situation to a different situation (Lobato & Siebert, 2002). The transfer of knowledge can happen within the mathematics classroom when students are asked to solve unfamiliar problems. Students can also transfer mathematical knowledge while working in a subject other than mathematics, such as physics. In addition, students can transfer mathematical knowledge when they take knowledge learned in their mathematics classes and apply it to everyday activities.

Transfer has been the focus of many studies throughout the past. Throughout early traditional transfer research, participants were given two different problems to solve that were similar from the researchers' point of view (Lave, 1988). Researchers selected these problems so that they differed in surface features, but had the same "deep structure." If the participants' performance improved from the first problem to the second problem, then transfer was said to have occurred (Lobato, 2008b). Researchers were disappointed to find that most participants did not exhibit transfer, but rather attended to surface features of the transfer problem instead of recognizing the deep structure similarities between the two problems (Lave, 1988).

There have been many arguments against the traditional view of transfer. One significant argument against the traditional view of transfer was presented by Lave (1988). Lave criticized past studies and experiments that had been done on transfer. She described many different transfer studies that had shown little or no evidence of students transferring knowledge across situations. Because of the lack of transfer shown in these experiments, she concluded that individuals seldom use transfer mechanisms in everyday life. One of Lave's arguments against the traditional transfer perspective is the notion that transfer happens across unrelated situations. Lave states that in learning there is the "implicit assumption that each context of activity exists in virtual isolation from each other" (p. 40). However, much of what transpires across situations and settings is related in various ways. The school setting has been considered decontextualized by some researchers; however, Lave sees no difference between the school setting and other situations of learning. According to Lave, a student's knowledge is contextualized within the situation that the knowledge is gained.

Another argument that Lave had against the traditional perspective of transfer is that the focus of many transfer experiments is the quest for the right answer to a problem. Students are given two versions of the same story or problem and are asked to solve the problem. Lave (1988) states that the students are asked to solve these problems based on the information given by the researchers using algorithms or formal reasoning to match a correct or ideal answer. Because of this, a considerable effort goes into matching transfer expectations by the student and researcher. Rather than solving the transfer problem, the unintended focus of these tasks is a matching game between the researcher and the student, where the student is expected to match their answer with what the researcher considers the correct answer. This is problematic because the students become more focused on getting the correct answer rather than constructing helpful similarities between past situations and the current situation.

In response to Lave's critique of past transfer studies, many researchers approached transfer from different perspectives in order to show that transfer exists and to measure transfer that was going undetected. In order to show that transfer exists, Bransford & Schwartz (1999) suggested that researchers reconceive transfer in terms of preparation for future learning. By teaching students the necessary skills to solve unfamiliar problems in future situations, Bransford and Schwartz found additional evidence of transfer. The researchers describe a study where students were being introduced to the concept of standard deviation. The students were given two different sets of numbers and were shown that the average of both sets of numbers were the same. The students were then asked to describe how the set of numbers were different. The students were asked to develop and refine a method that would describe the difference between any given set of numbers. The researchers then developed this method into a formula for standard deviation. The researchers were able to show that the students were using prior knowledge to learn about a new mathematical concept. Lobato & Siebert (2002) found that if transfer is measured from the student's perspective rather than the researcher's perspective, it becomes evident that students are often transferring their knowledge between situations. When approaching transfer from the student's perspective, evidence of transfer is found by looking for any influence from previous activities and by examining how students perceive situations as similar. Using this approach, Lobato has been successful in documenting specific instances of transfer as students attempted to use their understanding of slope and rates of change to solve real-world problems involving slope. Approaching transfer from this perspective gives researchers a greater opportunity to find evidence of transfer (Lobato, 2008b; Lobato, Ellis, & Munoz, 2003; Lobato & Siebert, 2002). Greeno, Moore, & Smith (1993) took a different approach when looking for evidence of transfer. They looked at transfer as participating in different activities in different situations. Greeno et al. examined how participating in one activity in one situation effects their participation in another activity in another situation. By looking at transfer from this perspective, they found evidence of transfer in the patterns of participation of the students.

The vast majority of research on the transfer of knowledge has focused on cognitive factors, which has led to many studies of student understanding. For example, Lobato and Siebert (2002) focused on understanding in their study of the quantitative reasoning that is involved when students are trying to make sense of a typical transfer situation. Boaler (1993) focused on developing students' understanding in context. For instance, Boaler (1993) found that when students learn mathematics in everyday contexts, they are able to transfer that knowledge to their everyday lives more easily. Such research has led to insight into how to improve transfer.

Despite the progress in research on transfer, researchers have overlooked some of the important factors that influence transfer. In particular, researchers have largely neglected to examine the effect of affective factors despite the fact that they may have an influence on transfer. Pugh and Bergin (2006) stated that there is a significant lack of research on the affective factors that can influences a student's ability to transfer knowledge. Researchers still need to identify particular affective factors that influence transfer and understand how those factors can influence transfer. In order to improve transfer research, researchers need to look at those affective factors that can influence transfer in addition to the cognitive factors.

Research on affect in mathematics problem solving provides some insight into how affect might influence transfer. Students' abilities to solve problems is often undermined by their beliefs about mathematics (McLeod, 1994; Schoenfeld, 1989). For example, Schoenfeld (1989) noted that some students assume that if they cannot solve a problem within the first 5 minutes they work on it, then they will never be able to solve it. Such a belief could keep students from persevering on mathematics problems, which often require longer than 5 minutes to solve. While the findings from the research on affect in problem solving are applicable, they are not specific in addressing the process of forming similarities between situations and transferring knowledge from one situation to another. Thus, research that specifically explicates the relationship between affect and transfer is needed.

In summary, there is a need for additional research in the area of transfer, particularly on how affective factors promote or inhibit the transfer of knowledge. In this study, I will be looking at how particular affective factors influence transfer. The findings from this study can provide researchers and teachers with insights into how beliefs influence the way students transfer knowledge.

Chapter 2: Literature Review and Theoretical Framework

There are many factors that can affect a student's ability to transfer knowledge. These factors can be cognitive or affective as discussed earlier. Throughout this section, I will be developing an argument to show that mathematical beliefs can affect students' ability to transfer knowledge. In this chapter, the concept of transfer will first be discussed. I examine some definitions of transfer and studies that have been done that can be related to the purposes of my study, which is to examine how students' mathematical beliefs affect the students' ability to transfer knowledge. Next, I discuss the different types of students' mathematical beliefs and how those beliefs affect their learning. Finally, I discuss the connection between students' mathematical beliefs and their ability to transfer knowledge.

When discussing learning and transfer, one may notice many similarities between these two ideas. Some of these similarities can make it seem like there is an overlap in the meaning of these terms. However, for the purpose of this thesis, I will be using these two terms in different ways. Learning is the act of acquiring knowledge. Acquired knowledge is mathematical knowledge that the student has previously gained. Transfer is taking acquired knowledge and applying it to new situations. While the application of acquired knowledge may lead to additional learning, the focus of my study is on the process by which students apply acquired knowledge, and not on examining or measuring the additional knowledge students might construct as a result of engaging in this transfer process. Throughout the study I will be looking at how students use their acquired knowledge to make sense of new situations. I will be using the terms learned knowledge and acquired knowledge synonymously.

Perspectives on Transfer

While there are many different ways of defining transfer, there are also some commonalities between these definitions. The general definition of transfer is the application of knowledge gained in one situation to another situation (Lobato & Siebert, 2002). However, each researcher adjusts this definition slightly to fit the needs of their study. For example, Evans (1999) stated that the transfer of knowledge refers to the use of ideas and knowledge learned in one context to another. The new situation where the student applies this knowledge can be in the school setting or in everyday activities. Perkins & Salomon (1988) stated that transfer occurs when something learned in one situation helps in a different situation. From these examples, we can see that even though each researcher slightly changed the definition to fit their needs, the underlying definition of transfer is similar.

One of the common assumptions underlying different definitions of transfer is that people cannot transfer knowledge unless they have formed some kind of similarity between the current situation and past situations. Thorndike, as described by Lobato (2006), claimed that transfer occurs to the extent to which original learning and transfer situations share common elements. The common elements can be considered the similarities that students are constructing between situations. Boaler (1993) stated that many researchers who have studied transfer believe that in order for students to transfer knowledge, the student must be able to recognize that the requirements of the current task are represented in previous learning. In order to do this, the students must be able to identify the similarities between the current task and previously learned concepts. Much of the research done on transfer makes similar statements or implies that students must be able to recognize previously learned concepts and ideas in the current task (Bransford, Brown, & Cocking, 1999).

While research suggests that transfer requires the construction of similarities between situations, there is a disagreement on the perspective used to measure transfer. One perspective is the traditional transfer perspective. In a traditional study of transfer, researchers generate transfer tasks that they perceive share underlying features with the initial learning task, such as a common solution approach, but have different surface features, such as different word problem contexts (Lobato, 2008b). When performance improves between the learning task and the transfer task, researchers say that the student has accurately transferred knowledge. This approach is problematic because the researchers are assuming that everyone is constructing the same similarities when solving the problems. There is an assumption within the traditional view of transfer that knowledge is said to transfer only when it is judged to be used correctly in a new situation (Lobato & Siebert, 2002). The researcher or teacher would determine how the students would use the information in order for it to be considered correct.

Using the traditional view of transfer can severely limit the amount of transfer researchers observe. For example, research participants who use acquired knowledge different from what the researcher used, or use the same knowledge as the researcher but not in the same way, would not be counted as having transferred knowledge, despite having met the requirements of the definition of transfer Thus, the traditional view is likely to lead to the recognition of only a very narrow kind of transfer, namely transfer that exactly matches the researchers' thinking. Because of the limitations of the traditional view of transfer, there was a need to come up with a different viewpoint from which to observe transfer.

From an actor-oriented transfer perspective, transfer occurs when the student sees one situation as being similar to one that they have already thought about (Lobato & Siebert, 2002). This type of transfer is defined as the personal construction of relations between situations. This means that the similarities that exist between situations are not objective; the similarities are not assumed to exist within the contexts of the problems. The similarities that are perceived exist because the student constructed them as similar. The similarities that one student may construct could be different from another student's because of the different ways that they construct these similarities. Because of this, one can find multiple similarities between two situations which lead to multiple ways of transferring knowledge. The evidence of transfer will look different depending on which similarities that the student perceives. This means that the researchers need to be open to evidence that students might construct unanticipated similarities.

Using an actor-oriented perspective can help researchers fill in the gaps that are left when using a traditional perspective of transfer. In the traditional view of transfer, students are supposed to reproduce the relations between the tasks that the researcher sees as important (Lobato, 2008a; Lobato & Siebert, 2002). That doesn't always happen because the tasks are perceived differently by each student and they may not see the same similarities that the teacher or another student can see (Lobato & Siebert, 2002). An actor-oriented transfer will acknowledge the relations of similarity that students construct.

The focus of the actor-oriented perspective is not whether the answer is right or wrong but on the similarities that the student constructs while transferring knowledge. In an actor-oriented perspective, evidence for transfer is found by examining a given activity for any indication of influence from previous activities by looking at the similarities that students build (Lobato, 2008a). There is not a correct way to construct similarities between situations; students should not be expected to make the same connections between contexts that researchers make. However, the ultimate goal of researchers is to have the students construct similarities that are close to the similarities that researchers see. By eventually constructing similarities that are close to that of researchers, the student will be better able to construct viable solutions to the mathematics problems they are solving.

There are many advantages in using an actor-oriented transfer perspective. One advantage is that it helps to guard against the false conclusion that transfer does not exist (Lobato, 2008b). Because of the many limitations that the traditional transfer perspective brings, some researchers have come to the conclusion that transfer does not happen (Lave, 1988). Another advantage of using the actor-oriented transfer perspective is that it helps researchers become aware of how students make sense of transfer situations (Lobato, 2008b). Researchers will see similarities that they may or may not have expected the students to construct. This perspective helps the researcher build a model of the students' thinking. By viewing the similarities that students construct, researchers can use that information to help revise their curriculum and make changes to their teaching practices (Lobato, 2008b). Because of these two advantages, I will be using the actor-oriented transfer perspective.

The Transfer Process

In order to understand the affect that mathematical beliefs have on a student's ability to transfer knowledge, we must understand the transfer process that the student moves through while solving mathematics problems. According to Holyoak (1984, 2012), a student is able to transfer information between situations if the two situations are analogous or share a common pattern of relationships. Identifying this common pattern requires the student to make a comparison of the situations. Holyoak (1984, 2012) refers to the two different situations as the source and the target. Holyoak (2012) acknowledges that the source is not limited to one's recall of a previous situation, but can consist of any structured knowledge from long-term memory. The target is the unfamiliar situation that the student is currently working with and trying to solve. Because the source is typically more familiar or understood than the target, the student transfers knowledge from source to target by first mapping as many elements as possible from the source onto elements from the target, and then inferring relationships, properties, or structures in the target based on the corresponding, known relationships, properties, or structures in the source. For example, if a student is asked to find the slant of a set of stairs, she may refer back to the more familiar concept of slope to help find the slant of the stairs. In this situation, the source is the student's conception of slope and the target is the slant of the set of stairs.

Holyoak (1984, 2012) presented four steps that the student moves through during transfer. I will briefly mention those steps here and go into further detail about them below. Holyoak explained that the first step of using analogical reasoning to transfer knowledge is to create a mental representation of the (target) problem. The second step is to select a potential source for transfer from long term memory. The third step is to establish a set of correspondences that align the elements of the source and target, and then make inferences about the target based on this correspondence. The fourth step is to construct an abstract schema for a related group of situations. Holyoak noted that this fourth step does not necessarily occur for every instance of transfer.

While considering these four steps, I realized that Holyoak's third step actually combines two important activities: creating a correspondence between source and target; and transferring relationships, properties, and/or structures from source to target to solve the problem. As a result, I am taking the third step in Holyoak's process and splitting it into two separate steps. I will call these steps the Constructing Step and the Applying Step, respectively. Also, because the purpose of this research is to examine how beliefs affect the transfer of knowledge, I will not be attending to Holyoak's fourth step, which consists of constructing schema for future problem solving. These modifications to Holyoak's framework result in four steps, which I will refer to as the Transfer Process for the rest of this thesis.

I now illustrate the four steps of the transfer process with an example. Suppose a student is given a problem where she is told that the slant of a set of stairs must fall within a certain set of measurements in order to meet construction guidelines. The student is then given the measurement of the riser and the tread of a specific set of stairs and is asked to determine if those stairs meet guidelines. As the student listens to the description of the problem, she forms an initial mental representation, or understanding, of the target problem. This is the first step in the transfer process, which I will call the Understanding Step. The student may engage in additional activity during this step, such as vocally asking questions about the meaning of specific parts of the problem, perhaps about the meaning of slant, riser, or tread. These vocalizations provide evidence that the student is engaged in the Understanding Step. However, in many cases, the Understanding step is not directly observable by the researcher, because the student creates a mental representation while silently reading or listening to the problem.

Once the student has formed a mental representation of the problem, she moves to the second step of the transfer process, which I will call the Searching Step. In this step, the student searches through potential sources that could be a candidate to map onto the mental representation of the target problem. In the example mentioned above, after the student has created a mental representation of the problem, the student may select the concept of slope as a potential source for a problem involving steepness. In order to identify a potential source, it is necessary to make a few tentative mappings from elements of the source to elements of the target. In our example, the student may map steepness of stairs to the steepness of a line in the concept of slope. Similar to the Understanding Step, this step is also not often observed by the researcher. In fact, the researcher may not see evidence of these first two steps until the student starts engaging in the next step in the transfer process.

In the third step of the transfer process, which I will call the Constructing Step, the student takes the source identified in the Searching Step and tries to establish a more complete mapping or correspondence between the target and the source. By doing this, the student is attempting to fit the problem situation to the potential source. If the student is able to construct a mapping from source to target in a way that maps all of the perceived important elements of the source to all the perceived important elements of the target, then the student has constructed a viable similarity between source and target. In the stair problem mentioned earlier, the student might relate rise and run in her conception of slope to riser (the vertical board that covers the hole in between steps) and tread (the horizontal board on which people step) in the stair situation. Furthermore, because she has been able to match all of the important elements of slope to all of what she sees as the important elements of the stair problem, she might conclude that the source and target are similar enough to allow her to proceed to the next step. If the student is unable to satisfactorily match important elements of the potential source to the target, the student returns to the Searching Step and continues the search for another potential source.

The fourth step of the transfer process is the step that was separated from the third step in Holyoak's (1984, 2012) Analogical Reasoning Process. During this step of the transfer process, which I will call the Applying Step, the student takes the similarity constructed during the Constructing Step and applies it to the target problem by transferring relationships, properties or structures from the source to the target to find the solution. In the example mentioned above, once the student has constructed the similarity of using the concept of rise over run, the student will apply this similarity to the target problem. The student will then use the appropriate numbers to substitute into the rise over run formula for slope and then evaluate this expression to produce the slant of the stairs and conclude whether or not the slant of the stairs falls into the desired range for slant. If the similarity that was constructed does not help the student solve the target problem, most likely because the student cannot perform the necessary computations or because the answer does not seem reasonable to the student, she will return to a prior step or give up. Her decision of which step to return to will depend upon which step she feels is most likely where she went wrong.

Students' Mathematical Beliefs

Many researchers have tried to write a clear definition of mathematics beliefs; however, there is a large diversity in the terms used to describe students' mathematical beliefs. For example, Schoenfeld (1989) states that mathematical beliefs are the ways that students' conceptions about mathematics shape the ways that they engage in mathematical activities. Kloosterman (1996) defined mathematics beliefs as the personal assumptions from which students make decisions about the actions they will undertake. These beliefs influence the students' actions in the classroom. In another definition, mathematics beliefs refer to the students' feelings about mathematics, aspects of the classroom, or about themselves as learners of mathematics (Reyes, 1984). These are just three examples of the different definitions of students' mathematical beliefs; many more exist.

Because of the diversity in the definitions of students' mathematical beliefs, it would be useful to create a framework that captures the many different beliefs that researchers have identified. These different definitions of beliefs were discussed by Op`t Eynde, Corte & Verschaffel (2003). They summarized the works of many researchers on the topic of students' mathematical beliefs in order to come up with one comprehensive definition. The authors' definition of mathematics-related beliefs is the "implicit or explicit conceptions that students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context" (Op't Eynde, et al., 2003). These beliefs, along with prior knowledge, can determine their mathematical learning and problem solving in class (Op't Eynde, et al., 2003). Included within this framework of beliefs are three different types of mathematics beliefs that are held by students. These beliefs are beliefs about mathematics education, beliefs about the self, and beliefs about the social context (Op't Eynde, et al., 2003).

The first type of mathematics-related beliefs discussed is the students' belief about mathematics education. This refers to the beliefs that students have about the nature of mathematics (Op't Eynde, et al., 2003). For example, this would include students' beliefs that mathematics is difficult or based on a set of rules. It also contains the students' perception of the usefulness of mathematics outside the classroom.

The second type of belief that Op`t Eynde, et al.(2003) present is the students' belief about the self. Within this category they include four subcategories, which are self-efficacy beliefs, control beliefs, task-value beliefs, and goal orientation beliefs. Self-efficacy beliefs focus on the students' perceived level of success and failure in learning mathematics (Pajares & Miller, 1994). An example of this would be a student who is confident that he or she understands the difficult material within a specific mathematics unit (Op't Eynde, et al., 2003). Control beliefs are the student's beliefs that he or she can accomplish a certain task. An example is the belief that if they study in appropriate ways, then they will be able to learn the material (Op't Eynde, et al., 2003). Task-value beliefs focus on the reasons why a student engages in learning and problem solving. An example of a task-value belief is the belief that it is important to learn the material in a mathematics class (Op't Eynde, et al., 2003). This would also include the belief that the student knows why it is important to learn mathematics. Goal orientation beliefs include the reasons why a student tries to succeed in a course. An example of a goal orientation belief is a student who believes the ultimate goal in a mathematics course is trying to understand the content as thoroughly as possible versus a student who believes the ultimate goal is to get a passing grade even if he or she doesn't understand the course material (Op't Eynde, et al., 2003).

The third type of belief included in the definitions of students' mathematics-related beliefs are the beliefs about the social context of the classroom (Op't Eynde, et al., 2003). These beliefs refer to the students' views and perceptions of the classroom norms that guide teacher and student behavior in that specific classroom. This type of belief includes the perceived social norms that exist in the classroom. This would include the students' beliefs about the role of the teacher and the role of the student in the mathematics classroom. The beliefs about aspects of classroom culture that relate to mathematics activity fall into this category. An example of a belief about the social context would be the belief about what counts as an acceptable answer or explanation in their classroom.

Students' Beliefs About Mathematics Education and Beliefs About the Self

As mentioned above, there are three categories of mathematical beliefs that students may hold (Op't Eynde, et al., 2003). Within these three categories are many different subcategories of beliefs. Because it is beyond the scope of a master's thesis to examine all of these mathematical beliefs, I have chosen to focus on two specific types of mathematical beliefs. These beliefs are the students' beliefs about mathematics education and the students' beliefs about the self in relation to doing mathematics. I chose these two types of beliefs because I felt that these beliefs could have an effect on transfer, as I detail below.

Students' beliefs about mathematics education can reflect their opinion on what mathematics is like and how the student approaches mathematical problems and tasks (Op't Eynde, et al., 2003). For example, students often believe that mathematics problems only have one right answer and that there is only one way to solve those problems (Schoenfeld, 1992). Some students believe that the only way to be successful in mathematics is to memorize the procedures and apply them mechanically and without conceptual understanding. Often students believe that mathematics problems should be able to be solved in five minutes or less. Another belief that students hold is that the mathematics learned in school has nothing to do with the mathematics used in the real world.

Students' beliefs about mathematics can affect the way in which they approach mathematics problems (Op't Eynde, et al., 2003), and thus affect the transfer process that occurs during problem solving. For example, a student who views mathematics as a body of true facts and procedures to learn and apply will approach mathematics differently than the student who sees the subject grounded in problem solving and meaning making. The student who believes that mathematics is about memorizing procedures or formulas may begin the solution process by looking for a procedure or formula that will help them find the solution. In contrast, the student who believes that mathematics is about solving problem and sense making may begin the solution process by identifying important quantities and relationships between quantities in the problem, and then devise multiple ways to solve the problem. Because students' mathematical beliefs can affect the problem solving process, it seems reasonable to conclude that students' mathematical beliefs might also indirectly affect the transfer process, which takes place during problem solving. However, this indirect influence of mathematics beliefs on transfer has not been studied in mathematics education research.

Students' beliefs about mathematics may also have significant direct effect on how a student transfers knowledge. Just as student's beliefs about mathematics may affect the way that the student approaches a mathematics problem, the student's beliefs may also affect the similarities that the student sees between the source and the target problem. For example, a student who believes that mathematics is based on a set of memorized rules and procedures may use a different source to map onto the target problem when compared with a student who believes that mathematics is something that can be understood and makes sense. Although there has been research into the cognitive factors that can affect a student's ability to transfer knowledge (Bransford & Schwartz, 1999; Lobato, 2008b; Lobato & Siebert, 2002), there is a significant lack of research on the effect of mathematical beliefs on the transfer of knowledge (Pugh & Bergin, 2006). In fact, a thorough examination of the mathematics education literature yielded no studies concerning the effect of student's mathematical belief on the transfer process.

Another type of belief that is likely to influence transfer is self-efficacy, a subset of students' mathematical beliefs about the self (Op't Eynde, et al., 2003). Self-efficacy is a student's belief about his or her own ability to successfully perform a given task (Hackett & Betz, 1989). A student's belief about his or her own abilities is a major determinate of whether a student will attempt a task and how much effort he or she will put into the task. Students' self-efficacy beliefs can be an indicator of their mathematical performance (Pajares & Miller, 1994). Self-efficacy can predict performance levels better than anxiety levels and previous math experience (Pajares & Graham, 1999). Students who have a higher level of self-efficacy show greater persistence while solving difficult problems (Pajares & Graham, 1999). Those students who show a greater persistence while solving difficult problems may work harder to find a source to map onto the target problem while working through the transfer process. These same students may also be more willing to cycle through the transfer process again when the initial source they used did not map to the target problem adequately. This persistence may give a better result at the end of the transfer process.

Research has already shown that students' level of self-efficacy is one of the affective factors that can have an effect on their ability to transfer knowledge. For example, Dweck (1986) found that when students are asked to examine their own abilities and efforts and to set goals based on these self-efficacy beliefs, those students with higher self-efficacy beliefs will perform higher in situations involving transfer. In this study, students in an eighth-grade science class were asked to set goals during a specific unit. These students either set performance goals, which focused on their ability, or learning goals, which focused on their effort. Students were then tested for transfer of knowledge between concepts that were taught and concepts that had not been taught but were still related to the topic that had been learned. Students were not tested on whether they saw the similarities that the teachers believed were correct in order to link the two situations. Rather they were tested on their ability to see any type of similarities between two unfamiliar situations and to use those similarities to solve the given problems. Those students who set performance goals had confidence in their own abilities to solve difficult problems. Students who set performance goals attained higher scores on the transfer tests, produced more work on their transfer tests, and showed more attempts to apply what they had learned to the unfamiliar problems. This study shows one effect that self-efficacy may have when finding similarities between situations. However, this study did not show how the performance goals affected specific steps of the transfer process or how the performance goals affected the transfer process as a whole. A careful examination of mathematics education research revealed no other studies about the effect of mathematical beliefs on the transfer process. Therefore this current study will contribute to the research done in the field of mathematics education.

As mentioned earlier, beliefs about mathematics and self can affect how students approach mathematical situations and how students solve mathematical problems. Because of this, I propose that these beliefs can directly affect transfer. There is a lack of research in the area of how students' mathematical beliefs affect transfer (Pugh & Bergin, 2006). Any factor that significantly affects the transfer process needs to be studied carefully. The research question that I will answer in this study is: How do students' mathematical beliefs and self-efficacy directly affect the way they engage in the transfer process while solving mathematics problems?

Chapter 3: Methodology

The purpose of this chapter is to describe the research methods that I used during my study. I first describe the students that participated in the study and where the study took place. Next, I discuss the type of data that was collected during the study and include how the data was organized, managed, and analyzed throughout the study.

This study was completed using ethnographic methods. An ethnographic study focuses on the description and interpretation of a cultural or social group or system (Creswell, 1998). The researcher examines observable and learned patterns of behavior, customs, and ways of life of those who are being studied. This includes a prolonged observation of the group where the researcher is immersed in the lives of the participant. Data is collected through participant observation or through one-on-one interviews with the members of the group. Lobato (2008a) states that studying transfer from an actor-oriented perspective should be done using ethnographic methods. Evidence for transfer from this perspective is found by examining how students view situations as similar and how students use those similarities to solve problems. This evidence is found by observing students in the classroom and by conducting interviews, which is how I collected data throughout my study. Because collecting evidence of students' mathematical beliefs, learning, and transfer requires a researcher to build a model of the student's thinking and understanding, ethnographic methods allow the researcher to observe and interview the student in order to build this model.

Setting and Participants

I chose to conduct my study in a junior high classroom for several reasons. First, I believe that conducting my study in a junior high classroom gave interesting results that helped to answer my research question. When students reach junior high, mathematics can turn from a concrete subject where students can see the applications into an abstract subject where it is harder to see the applications. This can lead to a change in students' mathematical beliefs (Pajares & Graham, 1999). Because of this, their beliefs can have a greater influence on how students transfer knowledge, when compared with elementary or high school level mathematics. For example, mathematical beliefs can affect the way students approach their mathematics class (Reyes, 1984). If the students see the usefulness in learning mathematics, they may put forth a greater effort to learn with understanding which can influence how they transfer knowledge. Second, I chose this environment because it is where I have the most experience. In order to answer my research question, I needed to be able to anticipate answers and make sense of the actions of the students I interviewed. Because of the time I have spent working in a junior high classroom, I have experienced anticipating answers and reactions with my own students.

I conducted the study in the school where I currently teach; however I did not use my own students. I observed another teacher's classroom and four of her students. I chose to observe another classroom rather than my own for two reasons. First, I observed another classroom in order to bring more objectivity into my analysis. I was able to bring more objectivity into the analysis because they are not my students. I did not feel the need to protect them from scrutiny during analysis or defend their lack of understanding. I also did not feel like it was my fault, as the teacher, when they were unable to solve certain problems. I was able to put aside personal feelings and analyze the data that I had collected. Second, by observing another classroom, the students were more comfortable talking with a teacher other than their own. By doing this, the students know that the results of the mathematical tasks I asked them to complete did not affect their grade or their teacher's opinion of them.

The junior high school where I conducted this study is located in a suburban neighborhood. The population of the school is approximately 2000 students. The school is on an A/B day block schedule, which means that each class is held every other day for 75 minutes. The teacher of this eighth grade mathematics class had been teaching mathematics for 10 years. In the specific class that I observed, there were 36 students. The classroom was organized so that the students worked in groups of four. The members of the group remained the same throughout the entire unit. The students in this classroom were encouraged to share their thoughts and ideas with the class. The students would work through the problems with members of their group to find methods or solutions. The teacher encouraged the different groups to share their methods or solutions with the entire class, which would produce enlightening mathematical discussions. Rather than being an entirely teacher led classroom, this classroom encouraged student discussion and involvement.

In order to get data that was beneficial to answering my research question, I had the teacher choose four students who were willing to participate in classroom discussions, were willing to talk about the methods they use to solve specific mathematics problems, and were of different mathematical abilities. These students were of different mathematical abilities in order to see how their beliefs affect their ability to transfer knowledge. Students with differing levels of mathematical ability may demonstrate different beliefs while solving mathematics problems. By selecting students who perform at different mathematical levels, I was able to find a wide range of mathematical beliefs. Since the students demonstrated a wide range of mathematical beliefs, I was better able to examine how differing beliefs affected the students' ability to transfer knowledge. I specifically asked for the teacher to recommend two students who consistently received As, one student who consistently received Bs, and one student who consistently received Cs. It was important that the teacher selected students who had demonstrated good work habits in their mathematics class. Students with good work habits were more likely to work hard throughout the data collection process which would produce sufficient data to answer the research question. After the data had been collected, I realized that one of the students I had observed and interviewed was not going to be a good subject to analyze. This student was very quiet during the classroom observations and did not contribute to the group discussions very often. During the interviews, this student did not expand on his answers about his mathematical beliefs, despite promptings from the researcher. This student was also not able to express his thinking while reasoning through mathematics questions. Because of this, I decided to focus on the other three students instead. The other three students were the A, B, and C students, so I believed that the data gathered would be sufficient to answer the research question.

I will now describe the three students who were involved in the study. The names of the students are all pseudonyms. Melissa was an 8th grade student who consistently received A's in her mathematics class. Melissa consistently participated in group discussions during class. She also willingly helped other students understand concepts. Melissa was always willing to present her mathematical ideas in front of the class. Lucy was an 8th grade student who consistently received B's in her mathematics class. Lucy often participated in the group discussions during class. She often helped others, but she also asked those in her group to help her when she had questions. She often volunteered to present her mathematical ideas in front of the class. Jennifer was an 8th grade student who consistently received C's in her mathematics class. Jennifer would occasionally participate in classroom discussions, but was able to express her thinking when prompted. She was willing to work with other students. Jennifer occasionally presented her mathematical ideas in front of the class. During the interviews, all three students were able to clearly express their ideas and mathematical reasoning.

Data Sources

There are many types of data that were collected throughout this study, including classroom observations, student interviews, field notes, and students' written work. I collected evidence in a variety of ways because each type of data provided different insights into how students' mathematical beliefs directly affect their ability to transfer knowledge. I used the classroom observations to see what mathematical ideas the students might be drawing on as sources for transfer, the student interviews to investigate how students' beliefs affected transfer, the field notes to remind myself of student behavior in the classroom and to make notes of additional questions to ask the students during the interviews, and the students' written work to examine if the students consistently used specific methods to solve specific types of problems. I collected data for the length of the entire linearity unit, which was approximately four weeks. The linearity unit was chosen purposefully, because of the many different contexts in which the students would explore the concept of slope. It naturally offered many opportunities to observe students engaging in the transfer process.

Classroom Observations

The first way that I collected data to help answer my research question was through classroom observations. The classroom observations took place every day the class was held throughout the linearity unit and were video recorded. The video camera was focused on the teacher for whole group discussions and on the four students participating in the study during group work. The recordings lasted the entire length of the lesson. This video camera captured their interactions with the teacher and other students in whole group discussions or small group work. The video camera also captured a record of the mathematical content that the students had learned during class and might draw upon during the interviews.

There are several reasons why I observed the students in the classroom. I observed the classes to see what type of mathematical thinking and ideas the students might have had available to them in the interviews to use as a source for transfer. I was also able to confirm that the tasks the students were asked to complete during the student interviews were appropriate for measuring transfer. For example, it was important to make sure that the students had learned the necessary content during class, but had not worked on problems exactly like the problems that I asked during the interviews. I also used the classroom observations to look for what seemed to me to be inconsistencies in students' mathematical beliefs. Even though I asked the students about their mathematical beliefs throughout the interviews, the students' beliefs are not always interpretable from the students' verbal descriptions alone. Perceived inconsistencies in students' beliefs may also happen because the observer did not understand what the student was trying to express when discussing his or her beliefs. This means that the researcher's perception of students' beliefs gained during the interview process may need to be checked against additional evidence from the classroom. Students will often share their beliefs about mathematics with other students in the classroom. By listening to their conversations with other students and observing their behavior, I gained more insight into their beliefs. If my perception of the students' behavior was inconsistent with my interpretation of what they stated in the interview, I asked additional questions about those beliefs with the student in a subsequent interview.

Student Interviews

Each student was interviewed three times throughout the unit. These interviews took place at the beginning of the linearity unit, in the middle of the unit, and at the end of the unit. I chose to do three interviews throughout the study in order to examine how the students' beliefs affected their transfer throughout the unit and to examine how the students' beliefs might change throughout the unit. Each interview took place after school, in my classroom, lasted for 30-45 minutes, and was video recorded. Any interview longer than 30-45 minutes may have been too long and tiring for the student, which would have caused a lack of focus and may have affected the responses given by the student. Each interview was transcribed before analysis began. There were no other students in the classroom during the interviews. Each student was provided with a pencil, a calculator, blank paper to show their work and write the answer on, graph paper, and a ruler. The students were told that if they needed any other materials, they could ask for it; however, none of the students asked for any other type of material.

Each interview consisted of a series of questions to gain better understanding of the students' thinking. The questions that I asked the students are located in the Appendix. The first type of questions that were asked during each interview were questions about the student's general mathematical beliefs. For example, I asked the students to describe what mathematics is and what it means to be doing mathematics. I asked the students how they felt about mathematics and what motivates them to do mathematics.

The second type of questions that were asked during each interview were specific questions about their mathematical beliefs, including beliefs about mathematics as a subject and self-efficacy. For example, I asked each student how they knew if they were good at mathematics, how they tell if their answer is right or wrong, what they would do if they did not understand the material, or when they would use mathematics outside of the classroom. I also asked the students about their own perceived ability to solve mathematics problems and their confidence levels while solving mathematics problems.

The third type of questions that were asked during the interviews were mathematics questions that were used to assess the students' ability to transfer knowledge. These questions covered the same mathematical topics that were covered in class; however, the student had not seen the questions before the interviews. These transfer questions were mathematics questions that I used to specifically examine the students' abilities to construct similarities and apply those similarities; the transfer questions were not a different format or a different type when compared to the types of problems the students saw in the classroom or on their homework. They were, however, often situated in a real world context different from the ones the students investigated in class. The questions covered the topics of rate of change, slope, y-intercept, writing linear equations, and graphing linear equations.

The last type of questions that were asked during the interview were follow-up questions after the student was done solving a mathematical problem. Some of these questions included questions about the student's confidence in solving the problem and if this confidence level changed throughout the problem. The students were asked if they felt that their answer was correct and how they could tell if their answer was right or wrong. The students were also asked follow-up question regarding behavior in the classroom that seemed to contradict what seemed to me to be the beliefs they stated during past interviews. For example, one student stated that she was good at doing mathematics, but I noticed that she occasionally did not complete the entire homework assignment or did not turn in a homework assignment at all. During the next interview, I was able to follow-up with her on this topic.

Field Notes

The third way that I gathered data was by taking field notes during the classroom observation periods. Although I recorded the teacher and group discussions during the lesson, this did not always capture everything that was going on in the classroom. Field notes are the traditional method of recording observational and interview data in ethnography (Hammersley & Atkinson, 2007). Field notes are an important place to write down specific observations and impressions gained during the observation process. The field notes helped me to catalog the learning activities in the classroom and the content of the unit. These notes also helped me to make notes of other events or behaviors in the classroom that I needed to ask the students about during upcoming interviews. I rewrote my field notes into a more coherent flow after classroom observation so that they would be better understood. Because I was teaching during the remainder of the day, the transcription of these field notes happened immediately after the school day was finished.

Students' Written Work

The fourth way that I gathered data was collecting the students' written work. The written work included class notes, assignments, and assessments. I used the students' written work to examine if the students consistently used the same method to solve problems similar to those problems worked on during the interviews. I also used the students' written work to examine if the same beliefs were evident while the students worked on problems that were not being observed by another person. For example, when examining how Jennifer wrote linear equations, I used her written work to examine if the same beliefs that were evident during the interviews were also evident while she was working on her homework. Depending on the problems and questions given, their written work was also a good way to measure their ability to use knowledge learned in one situation and apply that knowledge to a new situation. All students involved in this study were encouraged to show all their solution methods on their written work. Because the students showed all their written work and solution methods, I was better able to identify situations where learning had taken place by examining the methods they used to solve mathematical problems.

Inconsistencies in Student Beliefs

When students are asked to articulate their beliefs, there may be instances where their responses are not seen as consistent by the researcher when compared to their actions in the classroom. Leatham (2006) described a situation where two teachers' beliefs seemed inconsistent from the viewpoint of the researcher. Leatham introduced the term "sensible system," which refers to the organization of beliefs and the implications for those inferring those beliefs and their organization. This means that if the teacher's beliefs do not seem to make sense with their actions, then the researcher needs to develop a different model of the teacher's beliefs in which it does make sense with their actions, rather than assuming that the teacher is inconsistent. Although my study will be focusing on students' mathematical beliefs, I believe that Leatham's findings are applicable in my case. When asking students about their beliefs, the researcher cannot always expect them to know the answer or be able to articulate their beliefs into an answer that makes sense to the researcher. Individuals organize their beliefs into systems that make sense to them with respect to their other beliefs.

Students also organize their beliefs in ways that make sense to them and fit in with the other beliefs they have (Leatham, 2006). Researchers may see inconsistencies between the different ways that students manifest their beliefs. However, just as in an actor-oriented transfer perspective, students may construct a belief system that the researcher did not anticipate. Each student will organize their beliefs in different ways and the researcher needs to understand the systems that the student has built before saying their beliefs are inconsistent. The ways that students describe their mathematical beliefs and their actions in the classroom may be consistent from the students' point of view. Because of this, the researcher needs to try to construct a model in which the students' beliefs can be seen as consistent.

Beliefs seen as contradictory from a researcher's perspective are not likely seen as contradictory to the one holding these beliefs (Leatham, 2006). Certain beliefs could also take priority over other beliefs. Students may believe one thing in a specific instance and the opposite in a different situation, which could seem inconsistent from the researcher's viewpoint. When a student acts in a way that seems inconsistent with their beliefs, the researcher needs to look deeper and gather more data. There may have been a misunderstanding on the part of the researcher or another belief may have taken precedence in that situation.

If I viewed the students' beliefs as inconsistent, it was most likely because I did not understand the beliefs that the student originally expressed. I addressed those different beliefs by gathering more data and included questions about those differences in the next interview with the student. I asked the students about the beliefs that they expressed in the interview and the contradictions that I noticed. By doing this I was able to gain a better understanding of each student's mathematical beliefs and collect more precise data for my analysis.

Data Analysis

In order to answer my research question, I analyzed the data collected in a variety of ways. I began the data analysis while I was still collecting data. Throughout the data collection period, I looked across the belief statements gathered from the observations and the interviews to see if there were any apparent inconsistencies in the belief statements that were made. While interviewing the student, any statement that the students made regarding mathematics learning or problem solving was identified as a belief statement. During the classroom observations, the belief statements were inferred from comments and behaviors observed. For example, during the classroom observations, I noticed that Melissa asked questions, either to her group or to the teacher, on a regular basis. From this observation, I inferred that Melissa believed that asking questions was an important part of learning mathematics. I was then able to ask the student about these belief statements during the interview to confirm that they were statements that the student believed to be true. If I observed any inconsistencies in the students' belief statements, I was also able to ask the student about the perceived inconsistencies during the next interview. This allowed me to gain a better understanding of the students' mathematical beliefs which led to a better analysis of the effect of those beliefs on transfer. At this point, I was not categorizing the belief statements. I was identifying the belief statements in order to help me gather a more accurate set of data.

After the data collection had been concluded, I went through the transcripts of the interviews to identify any belief statements that had not been identified previously. I used the definition of mathematical beliefs mentioned earlier to help identify the students' belief statements. After these belief statements had been identified, I sorted each belief statement that had been identified into the categories included in the definition of students' mathematical beliefs. These categories were the students' beliefs about mathematics education, the students' beliefs about the self, and beliefs about the social context of the classroom (Op't Eynde, et al., 2003). My study only focused on students' beliefs about mathematics education and students' beliefs about self-efficacy, because I was confident that they would influence the transfer process, as explained in the previous chapter. The statements that the students made about control beliefs, task-value beliefs, goal-orientation beliefs, and beliefs about the social context of the classroom were not considered in my analysis.

In the next step of the analysis, I took the belief statements that fell into the category of beliefs about mathematics education and self-efficacy beliefs and identified the beliefs that most often occurred during the three interviews. In order to identify which beliefs occurred most often, I took the belief statements from the previously sorted groups and condensed those groups into smaller groups where the belief statements were similar. For example, one student was asked to find the slope between two points. The researcher asked her to explain why her particular method worked. She stated, “It just has to be that way.” This statement suggested that she did not know why the formula worked, just that it does. At another point during the interview, she was asked to graph an equation. When asked why she graphed the y-intercept first, she stated, “It is just the point you do first.” This statement also implies that she is following a rule without knowing why it works. These belief statements, along with similar statements, were summarized under the belief that mathematics is based on a set of rules. The summarized belief statements that occurred the most for each student were the beliefs that I chose to focus on during the remainder of my analysis.

Next I examined the mathematical problems solved during the student interviews. The purpose of the mathematics questions in the first interview was to gain an understanding of the students’ mathematical understanding. The mathematics questions asked during the first interview focused on previously learned material that may have related to the current unit, such as proportionality. Since the topics of these questions were not part of the new concepts the students were learning in class, the solution methods the students used on the problems in the first interview were not considered when looking for transfer. However, I did use the answers that the students gave during the first interview that referred to their mathematical beliefs during my analysis. I used the mathematics questions answered during the second and third interviews to find evidence of the students' mathematical beliefs that I had previously identified. There were multiple instances during the second and third interviews that the students expressed the previously identified beliefs. Because of this, I focused on the beliefs that seemed to influence their solution process whole solving mathematics problems. For example, Melissa believed that it was important to ask clarifying questions. Because this belief was evident during almost every mathematical task during the second and third interviews, I chose to analyze this particular belief and its effect on transfer.

The next step of the data analysis was to identify where and how the students' mathematical beliefs affected the steps in the Transfer Process (Holyoak, 1984, 2012). In the previous steps I had identified the students' mathematical beliefs that were expressed during the interviews, grouped those beliefs according to the definition of students' mathematical beliefs mentioned earlier, summarized the students' beliefs into different belief statements, and identified the belief statements that occurred the most often. I then found where the students expressed these beliefs during the interview opinion questions and examined the mathematical tasks that they were solving for evidence of these beliefs. I took each of these mathematical tasks and broke them down into the steps that the students moved through while solving the problem. After the solution process had been broken down, I tried to identify each step in the transfer process in the steps that the students took while solving the problem (Holyoak, 1984, 2012). These steps were the Understanding Step, the Searching Step, the Constructing Step, and the Applying Step. For example, I looked for an indication that the student understood what the problem was asking. When the student was able to move past the initial reading of the problem and begin the solution process, I concluded that the student understood the problem enough to attempt to find the solution. I also looked for indication that the student had identified and constructed a similarity between the source and the target problem by looking for evidence of what source the student was using and what similarity the student constructed between the source and the target problem context. I then examined the solution process for evidence of the student applying the previously identified similarity.

In order to recognize how the student's mathematical beliefs were influencing a step in the transfer process, I examined the strength of the belief in the transfer process. I went through every problem that the student solved in the second and third interviews and looked for actions during the transfer process that were consistent with and showed evidence of the identified belief. I then conducted thought experiments to see how the student's movement through the transfer process would change if that belief was different or was not present at all. I was particularly interested in identifying instances when beliefs were not only congruent with ongoing behavior, but also seemed necessary to explain why the student acted the way she did during the transfer process. When multiple instances of this type were found across problems, I created written descriptions of how the beliefs seemed to be consistent with and to influence the transfer process. I then compared these written descriptions to look for larger patterns across beliefs and students. These patterns became the results of my study.

Chapter 4: Results

In this chapter, I describe the results found from the data analysis. Through the data analysis, I found that students' mathematical beliefs affect transfer in two different ways. First, I describe two contrasting beliefs that demonstrate how beliefs affect the general transfer process. Second, I describe two contrasting beliefs that demonstrate how beliefs affect a specific step in the transfer process, namely the Constructing Step. And third, I discuss how the students' mathematical beliefs influence their self-efficacy belief.

Students' Mathematical Beliefs

Checking Unexpected Answers

One belief that Melissa demonstrated throughout the observation and interview process was that it is important to check unexpected answers before deciding that those answers are wrong. This belief, which I will call the Checking Answers Belief, falls within the category of beliefs about mathematics education, under the subheading of beliefs about mathematical learning and problem solving (Op't Eynde, et al., 2003). In this section, I describe how prevalent the Checking Answers Belief was while Melissa was solving mathematical problems. I also give an example and then discuss how this particular belief influenced the general transfer process. I then discuss how this belief affected the general transfer process.

There were many instances during the interviews where Melissa demonstrated the Checking Answers Belief. During the third interview, Melissa was asked how a student can know if he or she got the right answer. She stated, "…I could check it…as many times as I wanted…" Melissa also demonstrated the Checking Answers Belief while solving mathematical problems. There were four different instances during the two interviews where Melissa got an answer that she was not expecting or that she considered to be "weird." During each of these instances, Melissa stopped solving the problem and checked her work. Two of these times that she checked her answer, she found mistakes that led to the unexpected answer. The other two times, she was able to reassure herself that she was solving the problem correctly. Because of these different instances, we can see that Melissa believed it is important to check answers before deciding that they are wrong.

To give a better sense of how the Checking Answers Belief influenced the transfer process, I present a more detailed example, which occurred during the fourth question on the third interview. By looking at the steps that Melissa moved through in the transfer process while solving this problem, one can see the effect of the Checking Answers Belief. On this question, Melissa was asked to solve the paint problem:

> Hector is mixing paint for his bedroom. For each gallon of white paint, he needs to mix in 1/3 gallon of blue paint in order to make the correct shade. What amount of blue paint will Hector need if he needs to mix it into 2.75 gallons of white paint?

Immediately after Melissa read the question aloud, she stated, "Okay, that one is easy." This indicated that she had already moved through the Understanding Step of the transfer process and formed a mental representation of the problem. She thought that she understood what the question was asking and had already found a potential source that could be related to the target problem. Although there is not physical or verbal evidence of her moving through the Searching Step of the Transfer Process, it can be assumed that she picked a candidate for a mapping because she was able to use that potential mapping during the Constructing Step.

While working through the Constructing Step of the transfer process, Melissa attempted to fit the potential source that she found during the Searching Step to the target problem. The potential mapping that she seemed to have formed is a relationship between her conception of proportions and the target problem. While working through the Constructing Step, Melissa identified the 1/3 as the unit rate and then used her knowledge about proportional reasoning to work through the Applying Step. Although she didn't specifically say that this was a proportional reasoning problem, she set up the problem using proportions. She wrote the following:

$$\frac{1}{1/3} = \frac{2.75}{x}$$

Figure 1. Melissa's work: Interview #3, Question #4.

Melissa used proportional relationships to relate the different quantities of paint to each other. The first ratio in the proportion related the 1 gallon of white paint to the 1/3 gallon of blue paint. The second ratio in the proportion related the 2.75 gallons of white paint to the unknown quantity of blue paint.

During the Applying Step, Melissa used the similarity that she constructed during the previous step and applied it to the problem. Using the previously identified unit rate and proportion, Melissa solved the proportion using cross multiplication. She got the answer 0.916 and then stated that the answer didn't sound right to her. Later in the interview, when asked about this problem, she said, "I got a little confused. I thought it was going to have a whole number, or part of a whole number." However, before deciding that her answer was wrong because it was not what she was expecting, she worked through her solution process once again. Melissa did this by completing a second cycle through the transfer process. This cycle through the transfer process was less precise than the first time through because she was only checking the answer that she had found previously. When beginning this second cycle, Melissa moved directly to the Searching Step and found the potential source of using approximate fractions and estimation to check her answer. Although there is no evidence of her working through the

Constructing Step with this second cycle, it can be assumed that her potential source fit to the problem because she was able to move through the Applying Step. During the Applying Step, she applied the source of equivalent fractions and estimation to check her answer. She did this by iterating the amount of blue paint needed for one gallon of white paint until she was able to approximate the correct answer. She said, "…for each 1 gallon…so there are 2 one gallon so it will be 2/6…I mean 2/3. So right there is 0.666 repeating. So then there is 0.75 which is 2/3 and a little bit more. That is right…never mind." She was able to check her answer using fraction equivalents and estimating what the answer would be approximately. In this situation, Melissa felt like her answer was wrong, but after using a second solution process, she convinced herself that the answer was actually correct. Because she felt her answer was correct, she did not feel compelled to abandon the similarity she had initially formed and start over.

In this situation, the Checking Answers Belief affected the general transfer process that Melissa moved through. When Melissa got an answer that she wasn't expecting, her belief encouraged her to check her answer. By doing this, Melissa was able to cycle through the entire transfer process again in order confirm that the original similarity that she had constructed during the Constructing Step was correct. Melissa did this by suspending her disbelief and choosing a second potential mapping that was closely related to the original mapping that she constructed. Because of this, the second mapping allowed her to estimate the answer so that she could evaluate the reasonableness of her first answer. Once this second cycle through the transfer process was completed, she had the necessary confidence to conclude that her initial similarity worked and that her answer was correct. After being reassured that the similarity was reasonable, she once again applied the similarity to the problem and found the solution again. Because Melissa believed that it was important to check unexpected answers, she was able to check her answer by cycling through the transfer process again using a closely related similarity. If she did not have this belief, she may have begun the entire transfer process again using an unrelated, potentially less helpful, similarity.

Unexpected Answers are Wrong

One of Lucy's beliefs was that if you get an unexpected answer while solving a problem, then the answer is wrong. This belief, which I will call the Wrong Answer Belief, falls into the category of beliefs about mathematics education (Op't Eynde, et al., 2003). In this section, I describe how prevalent the Wrong Answer Belief was while Lucy was solving mathematical problems. I also give an example to show how this particular belief influenced the general transfer process. I then discuss how this belief affected the general transfer process. Finally, I compare the two beliefs that Melissa and Lucy held.

While solving mathematics problems, Lucy seemed to believe that her answers should be whole numbers or convenient fractions, not non-terminating decimals or difficult fractions. Although Lucy didn't specifically state during her interviews that weird or unexpected answers are wrong, this belief was evident while she was solving problems. Throughout the interviews, there were two problems where Lucy got an answer that she was not expecting. During both of these instances, Lucy rejected the initial answer that she got and cycled through the transfer process again to find an answer that fit the form that she was expecting. These actions suggest that Lucy believed that unexpected answers are incorrect.

An example of when Lucy demonstrated the Wrong Answers Belief occurred during the third interview. By looking at the steps in the transfer process, the effect of the Wrong Answers Belief can be seen. On the same paint problem that Melissa had worked on, Lucy was asked to find the amount of blue paint that is needed to mix into 2.75 gallons of white paint. Recall that for each gallon of white paint, 1/3 gallon of blue paint is needed to make the correct shade. Like Melissa, the Understanding and Searching Steps seemed to have led Lucy to select the concept of proportions as the potential source for the paint problem.

During the Constructing Step, Lucy tried to fit the proportion equation to the current problem by inserting quantities into the formula. Similar to Melissa, Lucy began by setting up a proportion in order to find the amount of blue paint that would be needed to mix into 2.75 gallons of white paint. She wrote the following:

$$\frac{\frac{1}{3}\ \mathit{blue\ paint}}{1\ \mathit{white\ paint}} = \frac{x\ \mathit{blue\ paint}}{2.75\ \mathit{white\ paint}}$$

Figure 2. Lucy's work: Interview #3, Question #4.

Lucy seemed confident in the way she placed the numbers into this equation, and she moved quickly toward solving the equation. This suggests that she believed that the potential mapping that she selected during the Searching Step was an appropriate similarity to construct.

During the Applying Step, Lucy attempted to solve the proportion that she had set up while constructing the potential mapping. She began by stating that she would do "one divided by three" and said that is what she would multiply the 2.75 by in order to find the answer. "So I would do 2.75 times 0.333333…oh…so then…" After pausing for a while, the researcher asked her why she was hesitating. She stated, "I didn't think that was right…I knew that the before way that I did it wasn't right because I got this weird number." She tried again to find the right amount of blue paint needed for 2.75 gallons of white paint; however, this time she seemed to be using arbitrary operations to find the correct answer. She stated, "I think I would divide it…or times it…do something to figure it out." During her first cycle through the Transfer process, Lucy correctly solved the proportion that she set up during Constructing Step. However, she ended up rejecting this answer and the solution method as well because she "got this weird number." At this point, there is also the possibility that Lucy may not have realized that she had solved the problem. After she had rejected this answer, she may have attempted to cycle through the transfer process again to find a second potential mapping but was unable to do so. This seemed to leave Lucy no option but to move directly to the Applying Step of the transfer process and try to select what seemed like somewhat arbitrary operations to produce an answer that had the appropriate form.

Lucy's belief that unexpected answers are wrong caused her to cycle through the transfer process again to select a different source each time she encountered an unexpected answer during the interviews. In the problem mentioned above, this belief caused her to abandon a correct mapping and cycle through the transfer process again to identify a completely different source. When Lucy was unable to identify a second potential source, she resorted to what appeared to be arbitrary actions to solve the problem. There is a possibility that Lucy could have realized that initial answer was correct if she had checked her work. She did not check any of her answers during the interviews; however, since she got answers that were of the form she was expecting, she may have assumed that these answers were correct. In the other problem where Lucy got an unexpected answer, she cycled through the transfer process to identify a different potential source, but could not find one and gave up. The answer that she got on this second problem was indeed wrong, but she was not able to back up her decision that the answer was wrong with a mathematical reason, only that the numeric form of the answer did not meet her expectations.

The mathematical beliefs held by Melissa and Lucy affected the transfer process that they each moved through. Initially it seemed as though the beliefs affected each student's transfer process in different ways, because Melissa was successful in all of her transfer attempts, and

Lucy was not. However, when examining the student's reaction to unexpected answers, one can see that the beliefs yielded similar actions by the students. When Melissa and Lucy's solution method produced unexpected results, both students questioned the similarity they had formed. Both students cycled through the transfer process again, and both seemed to search for an alternate source to map onto the problem. However, they seemed to differ in how strongly they rejected their initial similarity, which seemed to result in a difference in what content was available to them to draw upon as a source. Melissa did not immediately reject her initial similarity that she had constructed. Because she did not reject this similarity, she seemed to feel comfortable using a related source to create a second similarity, closely related to the initial similarity, to fit to the target. Lucy, on the other hand, seemed to reject not only the initial similarity that she had constructed, but also the source and the knowledge related to that source. This required her to search for a completely different potential source. This left her without alternative sources to draw upon, which in turn left her guessing about what operations to perform to solve the problem. Because Melissa was able to suspend her disbelief in her solution method, she was left with more resources on which to draw upon to construct a second, confirming similarity. These two situations show that the differences in beliefs can cause students to cycle through the transfer process in different ways and lead to substantially different outcomes during transfer.

Asking Clarifying Questions

One belief that Melissa exhibited multiple times throughout the classroom observations and the individual interviews was that it is important to ask clarifying questions about concepts or the current problem if something is not understood. This belief, which I call the Question Belief, falls under the subcategory of beliefs about mathematics education, specifically beliefs about mathematical learning and problem solving (Op't Eynde, et al., 2003). The Question Belief was evident from both the interview responses and the solutions to problems during the interviews. In this section, I describe how prevalent the Question Belief was while Melissa was solving mathematical problems. I also give an example to show how this particular belief influenced the transfer process. I then discuss how this belief affected the Constructing Step in the transfer process.

During the second interview, Melissa was asked what students can do in class to help them understand the concepts. Melissa stated that they could "think of more questions sometimes or try to think of a way to word a question…and then ask it." This suggests that Melissa believed that it is important to ask questions in order to gain a better understanding. She also demonstrated this belief while working through different mathematical tasks. During the two interviews where she was solving mathematical problems, Melissa asked clarifying questions during five tasks, asked procedural questions during two tasks, and did not ask clarifying or procedural questions during the remaining four tasks. Clarifying questions consist of questions that clear up confusion about what is being communicated in the problem. Procedural questions consist of questions that solicit help recalling formulas or steps of a procedure. Melissa gave correct answers on the four problems where she did not ask clarifying questions. This suggests that she was able to make sense of these problems by herself and did not need to ask clarifying questions.

An example of the Question Belief occurred during the fourth question on the second interview. By looking at the steps that Melissa moved through in the transfer process while solving this problem, one can see how the Question Belief affected the Constructing Step of the transfer process. On this problem, Melissa was asked to solve the stair problem:

According to carpenters' guidelines, the slant of the stairs must be between 45% and 60%. (Comparing the riser (vertical) to the tread (horizontal).) In a set of stairs, the riser is 8 inches and the tread is 11 inches. Do these stairs meet regulations?

At the beginning of this task, Melissa seemed to struggle to understand the problem and identify an appropriate source for transfer. In particular, she had a hard time understanding what the 45% and the 60% meant in relation to the problem. The first clarifying question she asked was if the "rise is 45% to 60%?" This question suggests that she had identified slope as a possible source for transfer during the Searching Step and had moved to the Constructing Step, but that she was struggling to map the quantity of rise in the source to an appropriate quantity in the target. In order to address her question, the researcher described to her what the term "riser" and "tread" meant when talking about stairs. Melissa then asked, "Is it saying that they have to be that far apart?" The researcher answered her by saying that when comparing the riser to the tread, the slant of the stairs must be between 45% and 60%. This answer seemed to help Melissa realize that the 45% and the 60% referred to the value of the slope, not the differences in length. She may have realized this by recognizing that the comparison of the rise and run of the stairs can be represented as a percent.

Melissa was then able to apply this similarity during the Applying Step of the transfer process. After this question was answered, Melissa said, "Okay. So right here we just need to really do…would be 8 divided by 11." She then put 8 divided by 11 into her calculator and said, "Umm…8 divided by 11…and I got 0.72." In order to determine if this set of stairs met the regulations, she changed the original 45% and 60% into decimals. She concluded that, "0.73 is over 0.6," so the stairs did not meet the regulations.

The Question Belief affected the Constructing Step of the transfer process. Because Melissa was able to ask specific clarifying questions about how the quantities related to the concept of slope, it is clear that she had already moved through the Understanding and Searching Steps when she began asking questions. Granted, Melissa's questions during the constructing step indicate that she had not formed a good understanding of the problem during the first step of the transfer process, but it seemed good enough to enable her to choose a productive source for transfer. By asking the clarifying questions during the Constructing Step, Melissa was able to strengthen her understanding of the target and match rise to the riser and run to the tread, thus confirming to herself that this problem did involve the concept of slope and that the similarity she had constructed was appropriate. Without asking clarifying questions, Melissa probably would not have been able to correctly map the source to the target problem.

Using Observer Perceived Arbitrariness

One of Jennifer's beliefs was that if she knew the form of the answer but didn't know how to solve the problem, then using arbitrary actions, or guessing, was an acceptable solution method. This belief, which I will call the Observed Perceived Arbitrariness Belief, or the OPA Belief, falls under the category of beliefs about mathematics education, and more specifically, beliefs about mathematical learning and problem solving (Op't Eynde, et al., 2003). In this section, I describe how prevalent the OPA Belief was while Jennifer was solving mathematical problems. I also give an example to show how this particular belief influenced the transfer process. I then discuss how this belief affected the Constructing Step of the transfer process. . And finally, I compare the two beliefs that Melissa and Jennifer held and their differing effect on the Constructing Step.

Observer perceived arbitrariness (OPA) is evident when students perform actions or operations that they are not able to justify mathematically and the observer perceives as being arbitrary. This belief was observable when Jennifer was solving a problem for which she knew the formula or form of the answer, but did not know which numbers to substitute for which variables in the formula or form. Jennifer's actions were never completely arbitrary, because they were guided by her anticipation of what form the answer should take; nevertheless, the way that she placed numbers into formulas or the way she decided what computations to use seemed arbitrary to me, because there appeared to be no repeated pattern across similar problems.

An example of the OPA Belief occurred during the fourth question on Jennifer's third interview. By looking at the steps that Jennifer moved through in the transfer process while solving this problem, one can see the effect of the OPA Belief on the Constructing Step. Recall the paint problem mentioned earlier:

> Hector is mixing paint for his bedroom. For each gallon of white paint, he needs to mix in 1/3 gallon of blue paint in order to make the correct shade. What amount of blue paint will Hector need if he needs to mix it into 2.75 gallons of white paint?

After answering this question, Jennifer was asked a series of follow-up questions. One follow-up question asked her to write a linear equation to represent the amount of blue paint needed for x gallons of white paint. Jennifer's subsequent answer of an equation in slope intercept suggests that Jennifer understood that the question was asking her to write a linear equation, which led her to choose the potential source of the slope-intercept form of the equation or y=mx+b.

During the Constructing Step, Jennifer seemed to haphazardly select numbers to substitute into the format of the linear equation, which was the source that she had selected. Jennifer selected 2/3 as the slope, which was actually the total amount of blue paint that should be mixed into the white paint, and not the unit rate of 1/3. She then selected 2.75 as the y-intercept, which was actually the amount of white paint that the blue paint is mixed into, and not 0, the correct y-intercept. This suggests that she knew the correct form for writing a linear equation, but did not know what numbers went into that equation. After Jennifer had selected values that seemed to fit the form of the linear equation, she moved to the Applying Step and inserted the numbers she had selected for the slope and y-intercept into the slope intercept form to produce the equation y=2.75+2/3x.

After examining the linear equations she wrote for the other problems posed in the previous interviews for patterns or reasons as to why she may have chosen to use these specific numbers, no patterns were evident. This suggests that Jennifer frequently used OPA to write equations. From the observer's perspective the number selection seems arbitrary. When asked why she was supposed to write equations in that form, she stated, "This is how someone told me...there is only one way to do it."

The OPA belief affected the Constructing Step of the transfer process. Jennifer was able to relate the target problem to the source that a linear equation takes the form of y=mx+b. She was able to select the correct form of the answer; however, she did not seem to know how to select the appropriate values for the slope and the y-intercept. Because she believed that if she knew the form of the answer but did not know how to solve the problem, using arbitrary actions was an appropriate solution method, she was able to select two values during the Constructing Step and produce an answer in the correct form. Furthermore, once she produced the answer, she had no need to go back and question the similarity she had formed because the answer was of the appropriate form.

By contrasting how Melissa and Jennifer solved problems, it is possible to see how differences in beliefs can affect a single step in the transfer process. Even though Melissa and Jennifer's mathematical beliefs led to different actions while solving the given problem, we can see that each belief affected the Constructing Step in the transfer process. When Melissa was unsure if the selected source could fit the target problem, she asked clarifying questions in order to better understand the problem. By asking the clarifying questions, she was able to adjust the similarities, if needed, during the Constructing Step so that the similarities would better fit to the problem. Jennifer's belief also affected the actions that she took during the Constructing Step. Even though she knew the final format of the answer, she seemed unable to correctly identify which numbers to fit to variables in the formulas. Because she believed that it was appropriate to arbitrarily select numbers from the problem to fit to the equation, she was still able to construct a similarity and finish solving the problem. Thus, while both girls' beliefs enabled them to successfully construct similarities, the difference in their beliefs caused variation in the correctness of their similarities.

Self-Efficacy Beliefs

Self-efficacy is a student's belief about his or her own ability to understand the material or successfully perform a given task (Hackett & Betz, 1989; Op't Eynde, et al., 2003). A student's level of self-efficacy is a major determinate in whether a student will attempt a task and how much effort he or she will put into the task. Each student's self-efficacy may be characterized by different statements or behaviors. All three students involved in the study showed high levels of self-efficacy as demonstrated by their beliefs in their own ability to solve mathematics problems, their willingness to attempt to solve the problems that were given to them, and their willingness to persist with a problem until they found a solution. However, I found that examining the students' self-efficacy independently from their mathematical beliefs did not explain how the students transfer knowledge. The results of this study show that the students' levels of self-efficacy may indicate certain behaviors that a student will demonstrate while solving a problem, such as those mentioned above, but without examining the influence of the students' mathematical beliefs on the transfer process, there is not enough information to predict how a student will transfer knowledge. In this study, since each student's self-efficacy was mediated by different mathematical beliefs, the effect of self-efficacy on the transfer process differed between students. In the following section, I describe how each student's level of self-efficacy and how their self-efficacy, combined with their mathematical beliefs, may have affected the transfer process while solving mathematical problems.

Jennifer

Jennifer demonstrated high self-efficacy throughout the linearity unit. Jennifer's high self-efficacy had one main characteristic: her belief in her own ability to "do" mathematics. When asked about her ability to do mathematics, Jennifer stated, "I am really good at math." She also stated, "I feel really confident…I can do [math] all by myself." Jennifer was then asked how she knows that she is good at mathematics. She said that she knows that she is good at mathematics if she gets A's on her papers. When asked how her mathematical abilities compared to the rest of the class, she said that she would be one of the higher people in the class because she can do mathematics. Jennifer's confidence in her own ability to do mathematics was evidenced in her willingness to attempt every problem, her inclination to quickly decide on a possible source and mapping from source to target, and her quickness in arriving at answers, which she unquestioningly accepted as correct.

Jennifer's self-efficacy seemed to have been influenced by her belief about what it meant to do mathematics. In particular she believed that doing mathematics is only a matter of selecting the correct rule or procedure to use, and then performing the required computations. Evidence of this belief can be found when examining statements that Jennifer made while solving mathematics problems. After Jennifer had solved a problem using a particular method, she was asked why she chose to use that method. Some of those answers included, "…[the method] is a rule…" or "…that is how you [solve the problem]…I can't explain [the method]." These statements seemed to indicate that Jennifer believed that mathematics is based on a set of rules or procedures that just need to be memorized. Jennifer's confidence in her own ability to recognize the type of problem, and in return select the proper procedure or method to use, allowed her to quickly solve mathematics problems. Each time she approached a problem she didn't hesitate in selecting a solution process and performing computations in order to get an answer. However, her speed and inability to explain the reasoning behind her solution suggested that she was more focused on producing an answer than understanding and reasoning through the problem.

Jennifer's belief that doing mathematics consists of identifying the type of problem and then selecting the correct rule or procedure to use, combined with her high sense of self-efficacy, may have influenced each step during the transfer process. An example of this type of thinking occurred during the second question in the second interview. Jennifer was presented with a set of points in a table:

x	1	4	10	13
y	2	7	17	22

Figure 3. Interview #2, Question #2.

As soon as she saw the problem, she exclaimed, “I like these!” and immediately began a process that seemed similar to finding the slope. This statement shows how her high self-efficacy seemed to allow her to proceed without questioning her classification of the problem type. This statement is also highly suggestive that Jennifer was focusing on identifying problem types instead of understanding the problem itself. When this problem was given to her, there were not any directions attached to the problem, just the table. Before I could give directions, she made the above statement and began solving the problem. Since the problem was presented in a table format, she assumed that this problem would require her to find the slope. This is also evidence of how her belief about doing mathematics, combined with her high self-efficacy, affected the Understanding Step of the transfer process. Jennifer felt that if she recognized the type of problem then she understood the problem enough to find an adequate solution.

In all of the interview problems, Jennifer did not seem to have any problems selecting potential mappings during the Searching Step of the transfer process. This may have been because her Understanding Step consisted mainly of recognizing clues, often superficial ones that she had become sensitized to during instruction and homework that she could use to identify the source (i.e., the problem type and its associated procedure). In the problem above, for example, the table format triggered Jennifer's identification of the problem as a slope problem, which she knew was solved by using the slope formula for points.

It is difficult to find evidence of Jennifer moving through the Constructing Step of the transfer process. For each problem that she solved, there was no indication of Jennifer testing the mapping between the potential source and the target problem that she found during the previous step. The Constructing Step is an important step in the transfer process. For example, when Melissa asked questions during the Constructing Step, she was able to select a source that better fit to the problem, thus leading her to get the correct solution. If Jennifer had tested the source that she selected during the Searching Step, she may have been more likely to understand why the source works and may have been more likely to get a correct solution. Jennifer may have thought that it was not necessary to understand the mathematical concepts involved in the problem because she was able to get an answer that fit the correct format, which, in this situation, was the format of a fraction that included the differences between the x- and y-coordinates. Since the answer met the form that Jennifer was expecting, she was able to accept the source that had been constructed without considering the mathematical implication of the potential source. Because of this it appeared as though Jennifer spent little time on the Constructing Step.

Recall Jennifer's previously identified OPA Belief. Once Jennifer picked a procedure to perform, it often appeared that she was selecting numbers at random to substitute into the procedure during the Constructing Step. For example, when Jennifer was asked to graph an equation, she believed that the line had to go through the x-axis and the y-axis and had to include a rise over run portion. Evidence of this fact can be seen when examining each problem where she was required to graph. When graphing, Jennifer always had a point on the x-axis and a point on the y-axis and used rise over run to graph the line. It appeared as though Jennifer selected these numbers arbitrarily from the problem and placed them on the graph. Each time she graphed a problem, she included the three details mentioned above; however, there was no observable pattern in how she selected the numbers to use. Because of her lack of understanding about the procedure for graphing a linear equation, she likely was unable to check if the numbers she used to graph made sense. Or perhaps, she did not think she needed to check her solution because she thought that she had previously identified the problem type and the corresponding procedure correctly.

Looking back at the slope problem, once Jennifer had moved to the Applying Step of the transfer process, she began by finding the differences between the x-coordinates and the y-coordinates. She then formed fractions by putting the differences into a fraction, with the x-coordinates over the y-coordinates. She then said, "…then you just reduce that one down to…so they are all 3 over 5…and then you do that…" This statement fits with Jennifer's belief that mathematics is about finding the correct procedure. We can see that she is confident about which procedure to use to find the slope. However, at the end of the problem, she is unable to describe why that is the correct procedure to use. Following this statement, Jennifer took the original answer of 3/5 and inverted the fraction to get 5/3 as her answer. When the researcher asked why she inverted the answer, she stated, "That is how I learned how to do it…she just told us to do it that way." This statement is a confirmation that Jennifer believed that memorized rules and procedures are a large component of what it means to do mathematics and that she did not need to know the mathematical explanation behind the rule in order to be successful. This interaction also shows that because Jennifer believed that successfully solving a mathematics problem only includes identifying the type of problem and then finding the correct procedure or formula to use, she believed in her ability to do mathematics because she was always able to produce an answer, regardless of its correctness.

Lucy

Lucy also demonstrated high self-efficacy throughout this unit. Lucy's high self-efficacy had one important characteristic. The main characteristic of Lucy's self-efficacy was her belief in her own ability to do mathematics problems. When asked what it meant to do mathematics, Lucy explained that doing mathematics meant that she can get the correct answer to the problem and understand the process she went through to get that answer. When asked about her belief in her own ability to do mathematics, she stated, "I am pretty confident." As a result, Lucy's belief in her own ability to solve mathematics problems led to persistence when solving difficult problems. While working through the mathematics problems given in the interviews, Lucy attempted to solve every problem given to her. Because Lucy felt like some of her answers were wrong, she was required to move through the transfer process multiple times before finding an adequate solution. She was unable to find a solution to two problems, but did not give up on those problems until she had at least identified 3 potential sources or had worked on the problem for at least five minutes. This behavior demonstrates persistence on her part.

Lucy's self-efficacy seemed to have been influenced by her belief that her answers should make sense within the context of the mathematical situation. When asked how she knew if an answer was correct, she said that the answer is correct if it makes sense to her. She also stated that she believed that her answers should make sense to her when the problem is finished. This belief can be seen when Lucy got an answer that did not make sense within the mathematical situation. If the answer did not make sense to her, she concluded that the answer was wrong. Instead of checking an answer that she believed to be incorrect, Lucy would give up on that answer, and often the source that produced it, and begin the transfer process again. When asked how she knew that the answer was incorrect, she would say that the answer did not seem right. This may indicate that she based her decision on the correctness of the answer on whether the answer made sense to her.

The influence on transfer of Lucy's general belief in her ability to do mathematics was often mediated by her more specific belief about her ability to sense the correctness of a solution. To illustrate how this occurred, I describe Lucy's attempt to solve the Weight Problem. The problem is as follows:

> Steve is going on a diet to lose weight. After 2 weeks on the diet, Steve weighs 260 pounds. After 9 weeks on the diet, Steve weighs 211 pounds. What is the slope and y-intercept in this situation? How much will Steve weigh after 5 weeks?

While working on this problem, Lucy attempted to form three different potential mappings. The first potential mapping that she formed used the concept of unit rate as the source. When she first tried to find out how much weight the man lost, she assumed the problem was proportional, meaning that the initial weight was zero pounds, and tried to find the unit rate. She found that the man would have lost 130 pounds in one week, which she determined to be incorrect because it did not seem right to her. This caused her to go back to the second step in the transfer process and search for a different potential mapping between the target and the source. The next potential mapping that she used was between the proportion equation and the target problem. When she set the two ratios into a proportion and attempted to solve the problem by using scale factors, she found that there was not a common scale factor to use. She sensed that this shouldn't have happened, and thus abandoned this potential similarity and searched for a new one. The next potential mapping that she formed was between her conception of linear equations and graphs. The potential source that she was mapping to was the concept of the y-intercept. She was able to associate the y-intercept with the beginning weight of the man since she learned that the y-intercept was where graphs began. While attempting to find the beginning weight of the man, she found the sum of the weight of the man after two weeks and nine weeks on the diet, getting 471 pounds. By adding 260 pounds and 211 pounds, Lucy believed that she could get back to the beginning weight of the man. After thinking about that answer, she stated, "…no, that's not right," and then gave up on the problem. Note that in all three of her attempts to construct a similarity, she obtained an answer that did not make sense to her, which caused her to reject the similarity she had constructed and cycle through the transfer process again. In fact, Lucy never accepted any similarity she constructed during the interviews that resulted in an answer that did not make sense to her.

The above example illustrates both how Lucy's self-efficacy affected the transfer process, and how this effect was mediated by one of her beliefs about mathematics. Like Jennifer, Lucy's strong self-efficacy enabled her to try every problem in the interviews. Furthermore, when Lucy experienced difficulties, her strong self-efficacy seemed to support perseverance, as demonstrated by her three cycles through the transfer process in the example above. However, unlike Jennifer, Lucy occasionally cycled through the transfer process multiple times on the same problem. This major difference between the way Jennifer and Lucy engaged in the transfer process cannot be explained by their self-efficacy alone. Jennifer believed that solving a problem was only a matter of determining the problem type and selecting and performing the correct procedure or formula. This belief did not encourage Jennifer to check if her answer made sense within the context of the problem. Because of this, Jennifer only moved through the transfer process once for each problem. In contrast, Lucy believed that her answer needed to make sense within the context of the problem, and she felt compelled to move through the transfer process again if her answer did not make sense to her. Because of their different beliefs about mathematics, Jennifer and Lucy seemed to move differently through the transfer process.

Melissa

Melissa also demonstrated high self-efficacy throughout the linearity unit. When asked about her ability to do mathematics, she stated, “I am good at it and it is fun.” She also stated that if she understood the concepts and got the right answers on previous problems, then she would be confident in her abilities to solve future mathematical problems. Melissa’s self-efficacy seemed to enable her to attempt and find a solution to every problem given. Melissa's self-efficacy also seemed to afford her a strong sense of perseverance when solving difficult problems. Although Melissa had the easiest time out of the three students finding a solution to each problem, there were times when Melissa doubted her answer. Her perseverance seemed to allow her to cycle through the transfer process again when the answer did not make sense to her.

The effect of Melissa's self-efficacy on the transfer process seemed to have been mediated by two of her mathematical beliefs. First, she believed that she should make sense of the mathematics problem and the solution to the problem. When asked what it meant to make sense of mathematics problems, she said that mathematics problems made sense when she understood the problem and could get the correct answer. When Melissa was asked what it meant to do mathematics, she gave a similar answer. She said that it meant she should be able to understand the solution process and to get most of the answers correct. Melissa's actions while solving problems in the interviews provided further evidence of these beliefs in that she seemed to try to understand every problem she encountered and solve every problem in a way that made sense to her mathematically. Second, she believed that she should ask clarifying questions if the problem did not make sense to her, a belief that I documented earlier in the results section. Because Melissa believed that she could ask questions to clarify a problem that did not make sense to her, she was able to narrow down the potential sources that could be mapped onto the target problem and construct the mappings in ways that made sense to her mathematically. These questions allowed her to either select a different potential source or to move through the transfer process successfully.

The influence of Melissa's self-efficacy beliefs on transfer were affected by her mathematical beliefs of seeking understanding and asking clarifying questions. This can be seen during the Stair Problem mentioned earlier. When she reached the Constructing Step in the transfer process, she realized that she did not fully understand the problem. Melissa noticed this because she believed that she needed to make sense of the problem before moving on in the transfer process. Instead of giving up, her self-efficacy seemed to enable her to persevere and made it possible for her to apply the mathematical beliefs that she held, specifically the belief that it is important to ask clarifying questions. As described earlier, Melissa then asked a series of questions that allowed her to construct a useful similarity and solve the problem. Melissa believed in her own ability to make sense of the problem, which allowed her to ask questions so that she could determine how to construct the similarity correctly. Melissa was able to combine her self-efficacy beliefs with her mathematical belief and ask clarifying questions that would help her to make sense of the situation. The responses to these questions helped to resolve her confusion and allowed her to successfully move through the transfer process.

When comparing Melissa to Lucy and Jennifer, their high levels of self-efficacy seemed to result in them sharing some of the same characteristics as they engaged in the transfer process. All three girls exhibited willingness to try all of the problems and confidence strong enough to enable them to persevere during difficult problems. However, there were significant differences in how the three girls engaged in the transfer process, differences that could not be explained by their self-efficacy beliefs alone. Melissa and Jennifer, for example, seemed to be engaging in different ways of making sense of the problem and constructing a similarity. Melissa seemed to always attempt to make sense of the quantities and the relationships between them in a problem, while Jennifer seemed intent on identifying the problem type. During construction of the similarity, Melissa tried to construct a mapping that made sense mathematically, while Jennifer focused on inserting numbers into the slots of the formulas. These major differences are not a result of self-efficacy beliefs, but rather how the self-efficacy beliefs were mediated by the girls' mathematical beliefs.

This finding of self-efficacy mediated by mathematical beliefs can also be seen by contrasting Melissa and Lucy's engagement in the transfer process. When the problem did not make sense to Melissa mathematically, her mathematical belief allowed her to ask clarifying questions to move toward an answer that made sense to her mathematically. These questions enabled Melissa to confirm the potential source she had chosen and construct a meaningful mapping between source and target. In contrast, Lucy did not ask questions when she struggled. Consequently, she failed to receive valuable feedback early in the transfer process that could have supported her selection of an appropriate source and the construction of a useful similarity. Also, Lucy believed that the answers should make sense within the context of the mathematical situation, but did not seem overly concerned that her solution made sense mathematically. It seemed to be that as long as the answers were reasonable for what the problem was asking then Lucy believed them to be correct. When the answers did not seem reasonable for the situation, she would try to work through the transfer process again rather than reexamine her understanding of the problem and the similarity she had constructed. Each time she did this she would choose a different source to map the target problem onto. This sometimes led to an exhaustion of sources with which to construct a similarity, as well as the rejection of a source that might have proved useful. Once again, these differences are attributable to the differences in mathematical beliefs and not self-efficacy.

Discussion

The research question that was presented before the research began was, How do students' mathematical beliefs influence how the students engage in the transfer process? In this study, I found how students' mathematical beliefs can have an impact on what they do during the transfer process. The mathematical beliefs that the three students held were analyzed in conjunction with the transfer process that each student moved through. In this section, I will discuss some of the mathematical beliefs that each student held and their effect on the transfer process. I will also discuss the effects of self-efficacy on the transfer process. Next, I will discuss the link between student's mathematical beliefs and self-efficacy and their effect on the transfer process. And finally, I will discuss the differing levels that each student was able to transfer knowledge on.

The different mathematical beliefs that each student held allowed the students to move through the transfer process in different ways. Some mathematical beliefs had a positive effect on the transfer process. For example, Melissa believed that it was important to ask clarifying questions and to check the answer to each problem before concluding that the answer was right or wrong. Because she held these beliefs, Melissa was able to demonstrate more flexibility when transferring knowledge. These beliefs allowed her to stop during the transfer process and adjust similarities as needed. The Asking Questions Belief mainly affected the first three steps of the transfer process. This belief seemed to allow Melissa to increase her understanding of the target question. Her better understanding of the problem enabled her to select appropriate sources during the Searching Step and to construct appropriate mappings from these sources to the target problems during the Constructing Step. By asking questions Melissa was also able to cycle through the transfer process again if she found that the potential source did not map well onto the target problem. The Checking Answers Belief had a similar effect on the transfer process. This belief mainly affected the third and fourth steps of the transfer process. If Melissa checked her answer while constructing the similarity or applying the similarity to the target problem and found that her answer was correct, she was able to continue with the transfer process. However, if she found that her answer was wrong, this belief, combined with her belief that mathematics should make sense, allowed her to cycle through the transfer process again. This allowed her to once again work through the previous steps of the transfer process to determine where the mistake was made. An analysis of Melissa's successful use of transfer to solve problems demonstrates how mathematical beliefs can support a mathematics learner as she engages in the transfer process.

In contrast, an analysis of Lucy's and Jennifer's mistakes in the interviews illustrates how mathematical beliefs can negatively affect students during the transfer process, particularly during steps three and four. Lucy's belief that unexpected answers are wrong affected both of these steps. When Lucy found an unexpected answer during the Constructing or the Applying Step, she often tried to find a different potential mapping to use to construct her similarity. Sometimes this led her to abandon a similarity or source that could have actually resulted in a correct solution. If after a couple of attempts she was unable to construct a similarity that led to the answers she expected, she would either abandon the transfer process altogether and engage in what seemed like arbitrary operations to produce an answer that fit her expectations, or give up on the problem. Jennifer believed that using arbitrary actions was an appropriate solution method as long as her answer matched the expected form of the answer. This belief allowed Jennifer to move through the Constructing Step of the transfer process faster than the other students. Once Jennifer had chosen the right procedure or formula to solve the problem, the OPA Belief allowed her to construct the similarity by simply substituting the given numbers into the procedure or method selected during the previous step. This belief did not require her to make mathematical sense of the chosen similarity. It seemed as though she did not need to test the similarities that were selected. As long as the answer fit the expected format, the similarity fit the problem.

The results of this study suggest that students' self-efficacy also affects their transfer of knowledge. In this study, all three students had high levels of self-efficacy. Not surprisingly, all three students acted in ways that were consistent with the findings of previous research on self-efficacy. Researchers have found that a student's belief about his or her own abilities to perform specific mathematical tasks is a major determinate of whether a student will attempt a task and how much effort he or she will put into the task (Hackett & Betz, 1989; Pajares & Graham, 1999). Confirmation of this statement can be seen in this study. Each student attempted every task placed before them. Furthermore, despite the fact that one student was not able to completely solve two of the problems, each student demonstrated significant perseverance while attempting to solve the problems given during the three interviews. While not conclusive, these findings are consistent with the claim that self-efficacy influences whether or not a student engages with a problem long enough to participate in the transfer process.

Although high self-efficacy seems to explain some of the commonalities, it does not explain the differences exhibited by students as they engaged in the transfer process. Instead, the effect of students' self-efficacy beliefs on the transfer process seemed to be mediated by other beliefs. The students' self-efficacy allowed the students to start the transfer process and to persevere through the transfer process when the problem became more difficult. However, other beliefs about mathematics or innate abilities seemed to determine how the students engaged in the transfer process. For Melissa, these beliefs supported a conceptual approach to constructing similarities. Lucy's and Jennifer's beliefs, in contrast, sometimes derailed the transfer process or allowed a superficial approach to constructing similarities. Thus, this study suggests that self-efficacy beliefs influence whether the student tries to engage in the transfer process, but does not completely explain how they engage in the transfer process.

When comparing the effect of Melissa, Lucy, and Jennifer's mathematical beliefs on their ability to transfer knowledge, it seemed as though these beliefs allowed them to move through the transfer process at different levels. Melissa's mathematical beliefs allowed her to develop a conceptual understanding of the problems and the mathematics involved in solving those problems. She was able to ask questions and check her answers so that, not only did the problem make sense to her, but the mathematics that she performed also made sense to her. Her beliefs allowed her to construct similarities that fit the problem. From the researcher's perspective, this showed that Melissa was moving through the transfer process on deep, conceptual level. I will call this type of transfer Conceptual Transfer. Jennifer's mathematical belief allowed her to move through the transfer process without necessarily developing and using conceptual understanding. When Jennifer was working through the steps of the transfer process, she was only trying to identify which memorized procedure or formula would be the most useful to get an answer that appeared to be right, based on the specific problem type. From the researcher's perspective, this showed that Jennifer was moving through the transfer process on a shallow level, when compared with Melissa. I will call this type of transfer Procedural Transfer. Initially, Lucy's mathematical beliefs seemed to motivate her to attempt to understand the problem and to construct a meaningful similarity. This enabled her to solve many of the problems correctly. However, each time she got an answer that seemed to be incorrect, she rejected that similarity. It seemed as though Lucy's ultimate goal was to get an answer that appeared to be right. While her problem solving during interviews suggested that she valued conceptual understanding, it was a means to an end. And if it failed to produce an answer that

Lucy expected then she would turn to arbitrary computations to produce the answer. From the researcher's perspective, this showed that Lucy was moving through the transfer process on a level between Melissa and Jennifer. I will call this type of transfer Answer-Oriented Transfer. She was still trying to develop understanding and select similarities based on that understanding, but when that failed, she did not hesitate to abandon conceptual understanding and try to produce the answer a different way.

Chapter 5: Conclusion

In past transfer studies, researchers have focused on the cognitive factors that affect the transfer process. However, there are other factors that may affect the transfer process, specifically affective factors. This research study focused on how students' mathematical beliefs, specifically beliefs about mathematics education and self-efficacy, affected their ability to transfer knowledge. Four students participated in this research study, although only the data from three of the students was analyzed. Data was gathered from classroom observations, student work, and student interviews. In order to analyze the data, I first identified the students' mathematical beliefs and categorized the beliefs according to the definition mentioned earlier (Op't Eynde, et al., 2003). I then examined the beliefs that occurred most often and studied the influence of these beliefs, specifically on the transfer process. The findings of this study showed that students' beliefs about mathematics education had an effect on the general transfer process as a whole and on specific steps in the transfer process. The findings of this study also showed that influence of the students' self-efficacy's on the transfer process is mediated by their other mathematics beliefs.

Contributions

The findings of this study extend the research on transfer, specifically mathematical transfer. Previous studies (Dweck, 1986; Pugh & Bergin, 2006) have shown how affective factors affect problem solving, suggesting they might also affect the transfer process. Self-efficacy studies have shown that self-efficacy can affect transfer using pre- and post-tests. However, no one has tried to trace the effect of self-efficacy while students are engaged in the transfer process. The current study showed how specific beliefs affected both the whole transfer process and specific steps of the transfer process. Pugh and Bergin (2006) stated that there is a need for more research into affective factors that can affect a student's ability to transfer knowledge. The current study has answered that call by documenting two affective factors that affect transfer. My findings suggest that the influence of mathematical beliefs on transfer is a promising area of research and affirms Pugh and Bergin's claim that this research is important for better understanding how students transfer knowledge.

The findings of this study also extend the research on self-efficacy. The findings of this study confirmed the research done by Pajares & Miller (1994) where they stated that students with higher levels of self-efficacy demonstrate a greater persistence when solving difficult problems. However, this study also adds to self-efficacy research by finding that its influence on transfer can be mediated by the students' other mathematical beliefs. In a previous study, the researchers examined how self-efficacy affected mathematical performance at the beginning and the end of the school year (Pajares & Graham, 1999). The current study examined specific steps in the transfer process where self-efficacy had an influence, rather than only looking at how self-efficacy affected the beginning and the end of the transfer process. By identifying where self-efficacy has the most influence in the transfer process, researchers can begin to examine how to improve the transfer process by studying how to improve self-efficacy during those specific steps.

Implications

Implications for Teachers

As a result of the findings in this research study, mathematics teachers need to be more aware of their students' mathematical beliefs in order to help their students transfer knowledge more effectively. Teachers should attempt to foster positive beliefs about mathematics education in their students. There are a variety of ways that teachers can use the findings of this study to help encourage the development of positive mathematical beliefs. Teachers should give their students problems that have non-traditional answers, specifically answers that are uncommon fractions or decimals. By doing this, students can learn that not all answers to mathematical problems are whole numbers. Perhaps by doing this, students can learn that an answer that they may consider to be weird is not necessarily wrong. Teachers should teach their students the importance of checking answers, both those that look right and those that seem weird. This will allow students to develop better mathematical reasoning and help to improve the construction of similarities. Teachers should also encourage students to ask clarifying questions, thus improving their understanding of the problem and their ability to construct similarities. Teachers should not only teach their students mathematics, but specifically focus on teaching their students what mathematics is about. As students develop a more positive set of mathematical beliefs and develop a deeper understanding of what doing mathematics means, students may be better able to transfer knowledge.

Implications for Research

Researchers studying the topic of knowledge transfer need to be aware that only looking at student knowledge is not enough to determine what affects a student's ability to transfer knowledge or how to improve the transfer of knowledge. In order to identify ways that students can improve their ability to transfer, researchers must also study the effect that students' mathematical beliefs have on transfer. This study only focused on two types of mathematical beliefs, and yet found that these affected the transfer process. There are many more types of mathematical beliefs that students might be drawing upon as they transfer knowledge. This suggests that mathematical beliefs may have an even stronger impact the transfer process than shown in this study.

The findings in this study suggest that self-efficacy seemed to be associated with the level of effort students were willing to invest in the transfer process, while mathematical beliefs were much more likely to affect the similarities that the students constructed. As a result, it may be the case that researchers studying perseverance during the transfer process might want to focus their research on self-efficacy, while researchers who wish to study the type of similarities and the quality of similarities constructed might want to focus their research on students' mathematical beliefs.

Limitations

One limitation of this study is that only two affective factors were examined throughout the course of the research, specifically student beliefs about mathematics education and self-efficacy. Just as the students in this study did not hold the same types of mathematical beliefs, this research does not assume that all students will hold the same type of beliefs. However, the findings in this research does suggest that, because these two types of beliefs did affect the transfer process for the students in this study, there may be other mathematical beliefs that students hold that can also influence the transfer process. These different beliefs may not affect the transfer process in the same way as those beliefs mentioned in this study, but they may still affect the transfer process. It is important to examine any belief that may have the possibility of affecting the transfer process. There are many more affective factors that could possibly affect how students transfer knowledge. More research needs to be done in order to study the effect of other types of mathematical beliefs identified by Op`t Eynde, Corte & Verschaffel (2003) to find if and how those mathematical beliefs affect the transfer process.

There were also limited generality in the population that was studied throughout the research. First, the population was limited to four students, although only data from three students was analyzed. This is a small population and may not represent all populations. However, the results from this study suggest that because the student's mathematical beliefs did affect the transfer process, researchers could expect to find that the mathematical beliefs of a larger population of students could also affect the transfer process. However, the students from the larger population may have different types of mathematical beliefs that affect the transfer process. Second, the three students analyzed during this study were all female. Although similar results are expected, researchers may want to consider studying groups of containing both male and female students. And third, the students studied during this research came from a classroom that developed problem solving skills and encouraged sharing ideas with the group. Students coming from a more tradition classroom may experience different mathematical beliefs that can affect their ability to transfer knowledge. Although these students may not be affected by the same mathematical beliefs that were identified during this study, the results suggest that there may be other types of mathematical beliefs that fall within the categories of beliefs about mathematics education and beliefs about the self that can affect students who have learned mathematics from a more traditional background. Researchers focusing on this topic should select students from a wide variety of instructional backgrounds to identify the types of mathematical beliefs that these students hold and how those beliefs affect the transfer process.

The ability to transfer knowledge is a necessary skill for students to use while solving mathematics problems. As mentioned earlier, there are a variety of different factors that can affect the transfer process. One factor that can influence the transfer process is the affective factors, specifically students' mathematical beliefs. The findings of this study show that mathematical beliefs can affect how students construct similarities and move through the transfer process. If we can better understand and attend to students' beliefs about mathematics, we can begin to improve how students transfer knowledge between two situations. This will allow students to improve in their ability to find and construct similarities between mathematical situations and form connections between different mathematical situations.

References

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more "real?". *For the Learning of Mathematics, 13*(2), 12-17.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (1999). *How people learn: Brain, mind, experience, and school*. Washington D.C.: National Academy Press.

Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of research in education, 24*, 61-100.

Creswell, J. W. (1998). *Qualitiative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA: SAGE Publications.

Dweck, C. (1986). Motivational processes affecting learning. *American psychologist, 41*(10), 1040-1048.

Evans, J. (1999). Building bridges: Reflections on the problem of transfer of learning in mathematics. *Educational Studies in Mathematics, 39*(1), 23-44.

Greeno, J. G., Moore, J. L., & Smith, D. R. (1993). Transfer of situated learning. In D. K. Detterman & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, cognition, and instruction* (pp. 99-167). Westport, CT: Ablex Publishing.

Hackett, G., & Betz, N. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal for Research in Mathematics Education, 20*(3), 261-273.

Hammersley, M., & Atkinson, P. (2007). *Ethnography: Principles in practice* (3rd ed.). New York: Routledge.

Holyoak, K. J. (1984). Analogical thinking and human intelligence. *Advances in the psychology of human intelligence, 2*, 199-230.

Holyoak, K. J. (2012). Analogy and relational reasoning *The Oxford handbook of thinking and reasoning* (pp. 234-259).

Kloosterman, P., Raymond, A., & Emenaker, C. (1996). Students' beliefs about mathematics: A three-year study. *The Elementary School Journal, 97*(1), 39-56.

Lave, J. (1988). *Cognition in Practice*. New York, NY: Cambridge University Press.

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*, 91-102.

Lobato, J. (2008a). Research methods for alternative approaches to transfer: Implications for design experiments *Design research in education* (pp. 167-194). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Lobato, J. (2008b). When students don't apply the knowledge you think they have, rethink your assumptions about transfer. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 289-304): Mathematical Association of America.

Lobato, J., Ellis, A. B., & Munoz, R. (2003). How "focusing phenomena" in the instructional environment afford students' generalizations. *Mathematical Thinking and Learning, 5*(1), 1-36.

Lobato, J., & Siebert, D. (2002). Quantitative reasoning in a reconceived view of transfer. *The Journal of Mathematical Behavior, 21*(1), 87-116.

McLeod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to the present. *Journal for Research in Mathematics Education, 25*(6), 637-647.

Op't Eynde, P., Corte, E., & Verschaffel, L. (2003). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13-37). Netherlands: Kluwer Academic Publishers.

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology, 24*, 124-139.

Pajares, F., & Miller, M. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*(2), 193-203.

Perkins, D., & Salomon, G. (1988). Teaching for transfer. *Educational Leadership, 46*(1), 22-32.

Pugh, K. J., & Bergin, D. A. (2006). Motivational influences on transfer. *Educational Psychologist, 41*(3), 147-160.

Reyes, L. H. (1984). Affective variables and mathematics education. *The Elementary School Journal, 84*(5), 558-581.

Schoenfeld, A. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for Research in Mathematics Education, 20*(4), 338-355.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Appendix

Interview Questions

Interview #1

Before the interview began, the students were informed that I would be asking them a series of questions about their beliefs and attitudes about mathematics. In order to make sure that the students understand the definitions of beliefs and attitudes, we took a few minutes to talk about what these words mean. I explained that when I talked about their beliefs about mathematics, I was asking about their opinion on certain aspects of mathematics. When I asked about their attitude about mathematics, I was asking about their feelings about certain aspects of mathematics. I let them know that if they don't understand what the question meant during any part of the interviews, I would rephrase it using different words to help them understand.

<u>Beliefs</u>

- How do you feel about school?
    - What is your favorite subject? Why?
    - What is your least favorite subject? Why?
- What is mathematics?
    - Who uses mathematics?
- Do you like mathematics?
    - Why or why not?
- What is your attitude about mathematics?
    - How do you feel about mathematics? Why?
    - Is it important to learn? Why or why not?
    - Is it useful? Why or why not?
    - Is mathematics useful outside of school? Why or why not?
    - In what situations would you use mathematics, in addition to school?
    - How do you think that your attitude about mathematics affects your ability to learn?
- When do you enjoy or feel good about mathematics?
    - When do you not like mathematics?
- Do you think everyone is capable of learning math? Why or why not?
- What motivates you to do mathematics?
    - Why does that motivate you?
- How confident are you about your own abilities to do math? Why?
    - How would you rank your mathematical abilities compared to the rest of your class?
    - What experiences have you had that lead you to think that way?
- How confident do you feel to help other students?
    - How would you help someone who didn't understand the material?
- How do you think your belief about your own mathematical abilities affects your ability to learn?
- What does it mean to be good at math?
    - How do you know if you are good at math?

    - How do you know if someone else is good at math?
    - What kinds of people are good at math?
- Why are people successful in math?
    - Why are people not successful in math?
- What is the most important part of math class?
- What is the best way to teach math?
    - Is math better learned in a group or individually? Why?
    - Is it better to listen to the teacher individually or discuss math as a group?
    - How long should the teacher stand in front of the room and be the "teacher?"
- What is the best way for you to learn mathematics?
    - Do you learn best listening or participating?
    - Does it ever help you to learn more mathematics by explaining math to others? Why or why not?
    - Does it ever help you learn more mathematics by having another student explain it to you? Why or why not?
    - Does everyone learn math the same way? Why or why not?
- How do you show the teacher you understand the material?
- What would you do if you didn't understand the lesson?
    - How would you show the teacher that you don't understand the material?
- How do you know if your answer is right or wrong?
    - What do you do if you get the answer right?
    - What do you do if you get the answer wrong?
- Are you ever unsure of yourself or the answer that you get?
    - What do you do when that happens?
- When you do mathematics problems do you usually feel like there is only one correct way to do the problem, or do you often feel like there are multiple correct ways?
    - Why do you think that?
- How long do you typically spend on a problem before giving up?
    - Does it depend on the type of problem?
    - Does it depend on whether you are working in class or at home?
- How would someone act in the classroom if they felt that mathematics is an important thing to learn?
- How would someone act in the classroom if they felt mathematics is not an important thing to learn?

<u>Initial Understanding Questions</u>

1. Two workers working for 9 hours together made 243 parts. One of the workers makes 13 parts an hour. If the worker maintains a steady pace all day, how many parts does the second worker make in an hour?

2. Suppose that 18 wheat crackers have 167 calories. How many calories are in 11 crackers?

3. If 1 inch on a map represents 195 miles, how far apart are two cities that are 2.125 inches apart on the map?

4. In class today, Amy, who weighs 160 lbs., found out that she would weigh 416 pounds on Jupiter. How much would Jess weigh, if her weight on Earth is 120 lbs.?
5. A shirt that usually sells for $18 has been marked down 30%. How much is the shirt after the discount?

Interview #2

<u>Mathematical Beliefs Questions</u>

- It is really important for my study that you answer these questions honestly. I won't tell your teacher or anyone else how you answer.
- What did you learn in your mathematics class today?
    - How would you explain it to a student who was absent?
- How confident do you feel about the things you learned in class today?
    - How confident do you feel about your understanding compared to the rest of your class?
    - If the teacher asked you to explain this concept in front of the class, what would you say?
- Do you think this topic is important in mathematics? Why or why not?
- Do you think this topic is important in your life? Why or why not?
- How can you use this topic outside of school?
    - What kind of situation would use this kind of mathematics outside of school?
- Why do you think your teacher asked you to learn this concept?
- Do you think that you are working hard in your math class?
- Are you trying hard to understand and do everything that you are supposed to do?
- Is there anything else that you could be doing to help you understand the concepts?
- Do you see any connections between what you are currently learning and things that you have learned in the past?
- Do you try to make connections between things that you have learned in the past and things that you are learning now?
- Do you see any overlap between things that you have learned in the past and the things you are learning now?
- Was everything new in this unit or were there ideas that you learned before?
- How do you keep the methods straight in your head? How do you remember how to find the slope when you are given points, table, or a graph?
- Can you use the same math to solve problems that look different? Can you use the same method to solve problems that look different? (Finding slope from ordered pairs and from a table.)
- Is there any part of math that doesn't make sense to you?
- Can you think of any real world examples that use slope?

<u>Mathematics Questions</u>

1. Find the slope between these two points: A(4, -6), B(-1, 1).
    - What does the slope mean?

- Can you change the points so that the slope is steeper?
- Can you change the points so that the slope is less steep?
- Are there other points on this line that have the same slope between them?
    - If yes, what are those points? How do you know?
    - If no, why not?
- Are there points on this line that do not have the same slope between them?
    - If yes, what are those points? How do you know?
    - If no, why not?

2. Find the slope of a road that rises 15 feet for every horizontal change of 90 feet.
    - How do you know that this is the slope?
    - What does that number mean in relation to the road?
    - Can you make a different road that has the same steepness?
        - How do you know this road is the same?
        - Can you make a different road that has the same steepness, but has different measurements than the other two roads? How many pairs can you make?
    - Can you make a different road that is steeper than the first road?
        - How do you know this road is steeper?
    - Can you make a different road that is less steep than the first road?
        - How do you know this road is less steep?

3. The Waterside Boating Dock charges a $45 rental fee for a boat in addition to charging $18 per hour for usage.
    - Does it make sense to talk about slope in this situation? If so, what is the slope in this situation? How do you know? If not, why not?
    - Does it make sense to talk about the y-intercept in this situation? If so, what is the y-intercept in this situation? How do you know? If not, why not?
    - Write an equation for the situation. Tell me what the equation means.
    - Graph the situation. Can you interpret the graph? (Tell me what it means.)
    - A family paid $180 to go boating. How many hours did they rent the boat? Explain your thinking.

4.

x	1	4	10	13
y	2	7	17	22

- Do these points form a line?
    - How do you know?
- (If they use a graph) Can you find out without graphing?
- Is there more than one slope on this line? (Between two different sets of points.)
- What does the slope mean?
- Can you change the numbers so that it is not linear?

5. According to carpenter's guidelines, the slant of the stairs must be between 45% and 60%. (Comparing the riser (vertical) to the tread (horizontal).)
    - In a set of stairs, the riser is 8 inches and the tread is 11 inches. Do these stairs meet the regulations? If yes, how do you know? If no, can you change the measurement of the riser so that it meets the regulations? Can you change the measurement of the tread so that it meets regulations? Can you change both measurements to that it meets regulations?
    - A construction worker needs to construct a set of stairs to a door that is 4 feet off the ground. How long can the base of the stairs be so that the regulations are met?
    - Construction workers are building a different set of stairs. There is a space that is 5 meters long where the horizontal part of the stairs must fit. Where can the door go so that it meets the regulations?

6. Lake City is expecting a large snow storm. Currently it is 40 degrees outside. The weatherman said that the temperature will drop 3 degrees per hour.
    - Does it make sense to talk about slope in this situation? If so, what is the slope in this situation? How do you know? If not, why not?
    - Does it make sense to talk about the y-intercept in this situation? If so, what is the y-intercept in this situation? How do you know? If not, why not?
    - Write an equation to represent the situation. Tell me what the equation means.
    - Graph the situation. Can you interpret the graph? (Tell me what it means.)
    - After a few hours, the temperature outside is 22 degrees. How many hours has the temperature been changing?

<u>Questions to ask during and immediately after the Mathematics Questions</u>

- How confident did you feel about your ability to solve this problem when you first read it?
    - Why do you feel that way?
- Did that confidence change as you started working on the problem?
    - Why do you think your confidence did or did not change?
- Do you think you solved the problem correctly?
    - Why or why not?
- Do you think that the math you used to solve this problem would be useful outside of school?
- Ask questions about student actions that occurred while they were solving the question, such as a confused look on their face, expressions of frustration, or a slouched posture.

<u>Questions I asked specific students after observing classroom behavior</u>

<u>Gemma</u>

- Why do you think you are good at math?
- Normally people like what they are good at, so why do you not like math?
- What are your grades in math?
- Why do you think you are getting those grades in math?
- Do you understand the things that you are learning in math?

- Do you ever have a hard time getting the homework done?
    - Do you worry about that?
- Sometimes my students don't do their homework. Do you know why students don't do their homework sometimes?

- Can you use the same math to solve problems that look different? (Finding slope from ordered pairs and from a table.)
- Is there any part of math that doesn't make sense to you?

Lauryn
- Do you like this unit as much, less than, or more than past units?
- Why don't you like graphing?
- Why do you think that there aren't real-world applications in this unit?

Interview #3

Mathematical Beliefs Questions
- What have you been learning in your mathematics class during this unit?
    - How would you explain these concepts to a student who has been missed a lot of this unit?
- How confident do you feel about the things you have learned the past few weeks?
    - How confident do you feel about your understanding compared to the rest of your class?
    - If the teacher asked you to explain this concept in front of the class, what would you say?
    - If you had the unit test today, how do you think you would do? Why?
- Do you think the topic of this unit is important in mathematics? Why or why not?
- Do you think the topic of this unit is important in your life? Why or why not?
- How can you use this topic outside of school?
    - What kind of situation would use this kind of mathematics outside of school?
- Why do you think your teacher asked you to learn the concepts from this unit?
- What connections do you see between the lessons in this unit?
    - What connections do you see between the lessons in this unit and the other lessons you have had recently?
- How have you used information that you have learned in the past in your current mathematics class?
    - What kind of past knowledge did you use throughout this unit?
    - When did you use this knowledge?
    - Do you think it is useful to use things that you have learned in the past? Why or why not?
- What did you think about this unit?
    - How is useful in your life?
- How has your attitude about math affected your ability to learn?

- How has your attitude about math affected your ability to remember things you have learned in the past?
- How do you think your confidence level in your own mathematical abilities has affected your ability to learn?
- How has your confidence level in your own mathematical abilities affected your ability to remember things you have learned in the past?

Mathematics Questions

1. Mary is planning a party. The cost for 15 people is \$195. The cost for 40 people is \$520.
    - Does it make sense to talk about slope in this situation? If so, what is the slope? How do you know? If not, why not?
        - What does the slope mean in this situation?
    - Does it make sense to talk about a y-intercept in this situation? If so, what is the y-intercept? How do you know? If not, why not?
        - What does the y-intercept mean in this situation?
    - Mary decides to invite 27 people to her party. How much will this cost her?
    - Write an equation to represent the cost of having a party for x people.
    - Graph the situation. Can you interpret the graph? (Tell me what it means.)
    - If the party cost \$455, how many people did Mary invite?

2. Beth is making chocolate chip cookies. Each cookie gets ¾ teaspoon of mini chocolate chips.
    - What amount of mini chocolate chips will Beth need to make two dozen cookies?
    - What amount of mini chocolate chips will Beth need to make x amount of cookies?
    - Graph the situation. Tell me what the graph means.
    - What is the slope? How do you know? What does it represent in this situation?
    - What is the y-intercept? How do you know? What does it represent in this situation?
    - What is the x-intercept? How do you know? What does it represent in this situation?

3. Steve is going on a diet to lose weight. After 2 weeks on the diet, Steve weighs 260 pounds. After 9 weeks on the diet, Steve weighs 211 pounds.
    - Does it make sense to talk about slope in this situation? If so, what is the slope in this situation? How do you know? If not, why not?
    - Does it make sense to talk about the y-intercept in this situation? If so, what is the y-intercept in this situation? How do you know? If not, why not?
    - How much will Steve weigh after 5 weeks?
    - How much will Steve weigh after x weeks? (Write an equation.)
    - Graph the situation. Can you interpret the graph? (Tell me what it means.)
    - If Steve weighs 190 pounds, how long has he been on his diet?

4. Hector is mixing paint for his bedroom. For each gallon of white paint, he needs to mix in 1/3 gallon of blue paint in order to make the correct shade.
    - What amount of blue paint will Hector need if he needs to mix it into 2.75 gallons of white pain? (Use 5 gallons if they can't answer the question.)
    - What amount of blue paint will Hector need if he needs to mix it into x gallons of paint?
    - Graph the situation. Tell me what the graph means.
    - What is the slope? How do you know? What does it represent in this situation?
    - What is the y-intercept? How do you know? What does it represent in this situation?
    - What is the x-intercept? How do you know? What does it represent in this situation?

5. For a science experiment, Jason measured the height of a plant every week. He noticed that after 4 weeks, the height of the plant was 15 centimeters. After 8 weeks, the height of the plant was 16 centimeters.
    - Write an equation for the situation.
    - What is the slope? What does the slope mean in the situation?
    - What is the y-intercept? What does the y-intercept mean in the situation?
    - Will you always be able to use this equation to find the height of the plant?

6. Amy and her mom measured the weight of her new dog. She recorded the information in the table. (Assume the growth is linear.)

Age (months)	Weight (pounds)
2	12
5	20

- Write an equation for the situation.
- What is the slope? What does the slope mean in the situation?
- What is the y-intercept? What does the y-intercept mean in the situation?
- Will you always be able to use this equation to find the weight of the dog?

<u>Questions to ask during and immediately after the Mathematics Questions</u>

- How confident did you feel about your ability to solve this problem when you first read it?
    - Why do you feel that way?
- Did that confidence change as you started working on the problem?
    - Why do you think your confidence did or did not change?
- Do you think you solved the problem correctly?
    - Why or why not?
- Do you think that the math you used to solve this problem would be useful outside of school?
- Ask questions about student actions that occurred while they were solving the question, such as a confused look on their face, expressions of frustration, or a slouched posture.