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Mathematical Telling in the Context of Teacher Interventions with Collaborative Groups

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ABSTRACT

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Teacher telling is underrepresented in the mathematics education reform literature and deserves additional scrutiny. This case study examined a skilled teacher's telling practices during a university mathematics content course for pre-service elementary teachers. I identified telling practices through discourse analysis, attending to the presence of mathematics and the contribution of new structure or ideas from the teacher. The teacher utilized seven unique types of mathematical telling while supporting collaborative group work on tasks. The study identified subtle telling, implicit telling, and explicit telling. The results suggest that mathematical telling is an integral part of the teacher's role in inquiry-based instruction and should not be overlooked. Researchers can use the telling types to identify and describe telling practices more transparently. Practitioners can more consciously incorporate and discriminate between telling practices.

Keywords: collaborative groups, discourse, inquiry, mathematical telling, tasks, teacher interventions, teacher's role

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Chapter 1: Introduction

Changes in the nature and structure of mathematics classrooms recommended by reform advocates (National Council of Teachers of Mathematics, 2000) have turned researchers' attention to the new demands placed on teachers. For decades, classroom practice in American schools was quite uniform (Hiebert, 2003). Lessons were centered on teacher demonstrations of the new material including several example problems. Students were expected to listen, take notes, and follow the teacher's instructions on their assigned problems. In contrast, a central feature of reform instruction is student-centered inquiry into mathematics through the use of high-level tasks (Stein, Smith, Henningsen, & Silver, 2009). Teachers also expect students to participate in mathematical discourse communities where those students share and critique one another's thoughts and assume primary responsibility for explanation and justification of their mathematics (Lampert, 1990). These changes to the classroom structure have led researchers to re-examine the role of the teacher in meeting the demands of reform instruction (Clarke, 1997; Goos, 2004; Simon, 1995; Smith, 1996; Staples, 2007). This work has consistently identified the teacher as a facilitator of classroom mathematical activity and discourse, in contrast with a more traditional identity of explainer and demonstrator of concepts and procedures.

The above restructuring of mathematics pedagogy has led to an increase in the complexity of classroom discourse. One complex issue that has been identified as a major concern is the lack of attention to the ways a teacher conveys new mathematical information to students within a reform classroom (Lobato, Clarke, & Ellis, 2005). The predominant emphasis on the teacher's role as a facilitator of classroom discourse has resulted in confusion regarding whether teachers should ever "tell" or speak assertively about mathematics (Chazan & Ball, 1999; Lobato et al., 2005; Philipp, 1995).

The present research study seeks to contribute to a more nuanced understanding of inquiry-based instruction by considering the role of telling in inquiry. On the one hand, inquiry supplants teacher-centered telling practices such as lecture and demonstration with student-centered activities in an effort to magnify the student's voice, engage students in difficult thinking, and immerse students in authentic mathematical practices. On the other hand, more recent work has recognized the absurdity of never telling anything and has recommended alternative forms of teacher telling (Baxter & Williams, 2010; Chazan & Ball, 1999; Lobato et al., 2005). The question of what role telling should play in inquiry settings and what that telling should look like is a delicate one and warrants much more investigation from a variety of angles and research settings.

The choice for a teacher to tell or not tell is a dilemma that has emerged as a result of considering the implications of constructivism. Under the traditional transmission metaphor of teaching, the choice to tell was not as problematic. The teacher's mathematical performances for students were viewed as an efficient and natural way to provide students the information they would need in order to practice and commit to memory the material (Smith, 1996). Constructivism dismisses the notion that knowledge can be transported through speech or symbols to children's minds and instead emphasizes the subjective process of meaning making by individuals with unique mental schemas (von Glasersfeld, 1983). This does not mean that students cannot understand what the teacher is saying or that all telling is incomprehensible; it only suggests that speech and symbols may be interpreted very differently by individuals depending on their prior experiences and patterns of thinking. Overwhelming evidence has suggested that students' experiences in traditional teacher-centered classrooms do not typically lead to conceptual understanding, effective mathematical communication, or robust problem solving skills (Gregg, 1995; Hiebert, 2003; National Research Council, 2001; Schoenfeld, 1988). Therefore, inquiry-based practice seeks to immerse students in doing and talking about mathematics to provide more natural or accessible experiences from which students construct the intended mathematical concepts. With students centered in the processes of inquiry, teacher telling can be seen as a threat to student autonomy and responsibility for sense making. As a result, teachers who wish to intervene when student work appears unproductive or inconsistent with the teacher's learning goals face a tension commonly referred to as the telling dilemma.

While the telling dilemma can surface in many instructional settings, this study will explore telling within the context of teacher interventions with small groups and individuals as opposed to whole-class discussions. The first reason for this choice is that there is a great need to understand how teachers help students succeed on tasks in preparation for a whole-class discussion where students share their contributions. Many of the students' learning needs and obstacles surface as they work in their groups on the assigned task. For inquiry to be successful, a teacher must help students overcome these obstacles and sustain rich mathematical activity. Part of the teacher's support involves guiding students mathematically, and the literature has given insufficient attention to the sorts of telling practices teachers implement as they guide students during tasks. A second reason is that the literature on whole-class discussions has portrayed the teacher primarily as an orchestrator and facilitator of student contributions (Simon, 1995; Staples, 2007; Stein, Engle, Smith, & Hughes, 2008), whereas the research on teacher interventions with collaborative groups working on tasks has been more inclined to recognize the tendency for teachers to go beyond a facilitative role and provide direct assistance with the content (Brodie, 2000; Chiu, 2004; Meloth & Deering, 1999; Webb, 2009). This point is elaborated further in chapter two.

In considering the relationship between teacher telling and student learning, a well-known teaching experiment involving the tutoring of a single student, Ann, is particularly insightful in demonstrating how certain ways of telling might productively influence student constructions of mathematical ideas (A. G. Thompson & Thompson, 1996; P. W. Thompson & Thompson, 1994). The researchers designed an experiment to investigate how a teacher, Bill, might influence one of his student's (Ann's) understandings of rate and motion through one-on-one interactions around pre-planned tasks. The tasks used a simple computer application involving trips taken by a turtle and rabbit. The experiment took place an hour each day for four days. When Bill was unsuccessful helping Ann solve a task at the conclusion of the second day (leading to severe frustration in both Bill and Ann), he invited one of the researchers, Pat, to take over the beginning of the following session. Pat took substantial initiative as he assisted Ann in understanding the underlying concepts, and when Bill resumed teaching Ann she proceeded through the remaining tasks successfully.

A discussion of the different approaches used by Bill and Pat reveals important aspects of teacher interactions with students about mathematics. Despite Bill's struggles, he was committed to helping Ann solve the problems herself rather than showing her how to do it and inviting her to imitate his steps. He encouraged Ann to reason through the problems, using copious questions to try and guide her or point her in the right direction. Yet, Bill was unable to assist Ann in moving beyond a very difficult conceptual roadblock. There are several factors that seem to explain Bill's lack of success (P. W. Thompson & Thompson, 1994). Bill's language consisted primarily of numbers and calculations rather than the relationships between quantities of motion. His primary objective was moving through the specific task questions rather than cultivating a particular way of understanding the situation. When Ann's responses were inconsistent with

Bill's expectations, he failed to elicit and help her restructure her thinking. Bill's and Ann's implicit conceptualizations of the mathematics of the situation were so different from one another that Bill's verbal interactions with Ann led to severe miscommunications.

Whereas Ann's teacher Bill seemed to lack a clear direction for what he wished her to understand, the researcher Pat had articulated very explicit ways of thinking about the concepts of rate and motion that he intended for Ann to learn (A. G. Thompson & Thompson, 1996). As Pat worked with Ann he used a direct approach. Pat did not pose a written task for Ann and try to guide her to solve it primarily on her own; instead, he took considerable initiative in the conversation to encourage Ann to think about motion in specific ways. Pat used explanations, representations, hand gestures, and questions to convey the important conceptual ideas he wanted Ann to consider. His questions were a continual diagnostic tool to ensure Ann was assimilating the ideas meaningfully. The key to Pat's success had to do with his orientation toward the concepts and relationships of the situation, unlike Bill's focus on the calculations of the task. Pat was more aware of the mental images Ann was likely using while thinking about rates, and exploited this knowledge to introduce ideas that would lead her to construct new ways of conceptualizing situations of motion.

This singular teaching experiment with Ann demonstrated the compatibility of explicit teacher talk about mathematics and student construction of mathematical concepts. It also illustrated how the nature of the mathematical content expressed by the teacher (rather than just the level of explicitness) holds a great deal of explanatory power regarding the efficacy of mathematical negotiations of meaning between teachers and students. The stark contrast in Ann's thinking in response to Bill's telling practices and Pat's suggests that the issue is far more complex than simply deciding whether to tell or not tell. A diverse range of telling practices exists, some practices being more compatible with inquiry than others. Little is known regarding the ways teachers use mathematical telling practices effectively in inquiry-based settings. A greater understanding of the ways successful teachers convey mathematical information while maintaining high levels of student involvement and responsibility for learning will improve teachers' ability to make informed choices as they manage the telling dilemma.

Chapter 2: Framework

This chapter discusses the topics relevant to a study of the occurrence of mathematical telling within inquiry-based instruction in the context of teacher interventions with groups and individuals working on high-level tasks. First, the background of telling in mathematics education is presented followed by the definition of mathematical telling used in the present study. Second, the literature on teacher interventions with groups and individuals is reviewed with special attention to the relevance of mathematical telling to this research context.

The History of Telling

Traditionally, teaching by telling grew naturally out of a transmission metaphor of learning. The assumption was that teachers possess knowledge and to impart the knowledge to students, it needs to be explained and presented clearly. Students act as sponges that absorb information so long as they pay attention and perform adequate practice. Smith (1996) articulated the core set of beliefs shared by teachers in this paradigm—the mathematical content is seen as “a fixed set of facts and procedures” (p. 390), teaching mathematics entails demonstrating procedures step-by-step, and learning mathematics requires listening and practicing so as to be able to recall the correct procedures at the appropriate times. These beliefs teachers hold about mathematical knowledge and teaching can be deeply engrained since they tend to be mutually compatible, provide the teacher a sense of efficacy in performing a well-defined role, and allow one to cope with negative outcomes such as poor student motivation or performance in such a way that beliefs are reinforced rather than reconsidered (Gregg, 1995; Smith, 1996).

As a result of dissatisfaction with the traditional telling model of instruction, reformers in mathematics education turned to the epistemology of constructivism to rethink and restructure mathematics pedagogy. Constructivism (von Glasersfeld, 1983) redefined knowledge and learning to problematize the transmission metaphor of communication. In constructivism, knowledge is construed as the product of efforts by individuals to organize and explain their experiences rather than coming to know some external and objective idea. Consequently, words and symbols do not contain meaning in their own right. Rather, individuals interpret the words, symbols, and phenomena around them based on an accumulation of experiences that lead to the formation of an interpretive schema for making sense of one's lived reality. Efforts to reconcile mathematics pedagogy with the epistemological tenets of constructivism have led to the rejection of the assumption that simply telling students condensed and formalized mathematical information leads to meaningful comprehension. Researchers have explored ways of designing mathematical tasks informed by student thinking that prompt students to restructure their thinking in discipline-appropriate ways (Simon, 1995; Stein et al., 2009).

As the reform mathematics community abandoned telling-based teaching in the traditional sense, focus turned to new patterns of teacher discourse needed to enact the demanding role of eliciting and building on students' mathematical thinking. While none of the emerging descriptions of the teacher's role in the literature resemble the former teacher-centered practices of explanation and demonstration of solution procedures, most do portray the teacher as an assertive facilitator or participant. Teachers are expected to coordinate the sequence of activities, manage the sharing of student contributions (Staples, 2007; Stein et al., 2008), maintain students' focus on important and relevant mathematical ideas (Sherin, 2002; Wood, 1998), negotiate classroom mathematical practices and social norms to be taken as shared (Cobb, Wood, & Yackel, 1993), and foster student participation in a community of practice (Goos, 2004; Lampert, 1990).

While the above characterizations of teacher practice provide a refreshing contrast to the days of teacher demonstration and student imitation of formal mathematical procedures, researchers have recognized a limitation in the literature regarding the absence of discussions of the role of telling within inquiry-based settings (Baxter & Williams, 2010; Chazan & Ball, 1999; Lobato et al., 2005; Philipp, 1995). The reluctance to acknowledge and emphasize the types of telling that skilled teachers employ likely stems from the underlying goal to privilege student autonomy above teacher authority. However, rather than simply avoid framing teacher practice in terms of telling altogether, a few researchers have expanded or reformulated the meaning of telling to account for those instances in which teachers purposefully choose to tell. These approaches to telling are described in detail in order to build on their work.

Smith (1996) hypothesized that forms of judicious telling may increase teachers' sense of efficacy and promote meaningful student learning. Because teachers mediate between the particular mathematical ideas emerging in the local community and the generally accepted practices of the mathematical discipline as a whole, teachers are encouraged to introduce "useful terminology, ways of representing mathematical ideas, and counterexamples to student conjectures" (p. 397). Smith argued this kind of timely provision of mathematical information enables rather than disables students and is quite different from the general telling model of teaching.

Baxter and Williams (2010) provided evidence of telling by investigating the use of two forms of scaffolding in middle school classrooms that emphasized learning mathematics through meaningful discourse. The first type of scaffolding is known as social scaffolding and refers to those teaching acts that support students' adoption of appropriate discourse norms regulating their mathematical participation. The second type is analytic scaffolding, which refers to the supports offered by materials, the teacher, or other students, that more directly influence mathematical understanding. “Analytic scaffolding might include physical manipulatives, models, metaphors, representations, explanations, or justifications…” (Baxter & Williams, 2010, p. 11). The authors noted that teaching acts in general are not limited to one form of scaffolding or the other, and that analytic scaffolding is constructed in part by the discourse that emerges from social scaffolding. One way teachers in the study managed the dilemma of telling was to be more direct in providing social scaffolding while leaving the creation of analytic scaffolding to tasks and students. This is consistent with other studies in which teachers were shown to take on a more directive role primarily while negotiating sociomathematical norms, curtailing the teacher’s need to explain the mathematics directly (Cobb et al., 1993; Rittenhouse, 1998). However, Baxter and Williams (2010) also noted that when the task or the social structure of the classroom failed to elicit the intended mathematics from students, teachers intervened with some form of telling. Teachers offered alternative solution strategies or brief explanations and summarized ideas from lengthy discussions. Although teachers risked privileging their own strategies or ideas, the authors argued “such telling can be woven skillfully into an overall tapestry of student discourse and work on significant mathematics in ways that successfully implement the vision of reform” (p. 25).

Chazan and Ball (1999) argued that the term “telling” is insufficiently precise and expanded its meaning to include not only the provision of mathematical information but also the assertive moves a teacher regularly makes while facilitating meaningful mathematical discussions. They noted that the telling denigrated in reform documents is generally limited to evaluating a student’s answer as right or wrong and giving correct answers. Their expanded view of telling included acts such as attaching conventional terminology to students’ language, returning an issue to the floor by replaying a student comment or reminding students of a previous conclusion, requesting clarification, controlling the focus of a discussion, giving simple instructions (e.g., “come to the board”), requesting comments, and pressing students for their reactions or thinking (p. 2). Chazan and Ball used data from their own teaching experiences to illustrate the complex and context-dependent choices teachers face as they encounter situations where student arguments lose control or where students prematurely reach consensus on an incorrect mathematical idea. Both teachers chose to insert mathematical ideas in various ways to enhance their students’ mathematical activity. The authors concluded that additional research would need to make subtler distinctions between the kinds of telling teachers use as well as investigate the sense that students make of these acts.

While the contributions summarized above have primarily *expanded* telling to include facilitative and participatory roles that indirectly focus students on important mathematics, Lobato et al. (2005) *reformulated* telling to emphasize the productive ways teachers introduce novel mathematical ideas to students in reform settings. The imprecision of the term “telling” and the reform community’s general reluctance to acknowledge the ways teachers often introduce novel mathematical ideas to students (as opposed to merely orchestrating student contributions) led Lobato et al. to reformulate telling as “initiating.” Initiating is defined as “the set of teaching actions that serve the *function* of stimulating students’ mathematical constructions via the introduction of new mathematical ideas into a classroom conversation” (p. 110, emphasis in original). Initiating acts can take various forms, including statements, questions, imperatives, or even gestures. Teaching acts classified as initiating might include describing a new concept, summarizing student work, providing information to test ideas, introducing an alternative strategy for consideration, presenting a counterexample, using Socratic questioning to introduce a concept, or presenting a new representation (Lobato et al., 2005, pp. 110-111).

Although similar acts can be found in Smith (1996), Baxter and Williams (2010), Chazan and Ball (1999), and other reports, Lobato et al.'s (2005) purpose of reformulating telling as initiating was to reconsider the theoretical roots of telling so as to legitimize teaching acts that directly convey mathematical ideas within the constructivist school of thought. Their approach is holistic as opposed to fragmented. Teaching acts are considered in their broader context, and initiating is informed by and interacts with the teacher's elicitations of students' mathematical thinking (defined as "eliciting"). In addition, the function of an initiating act is determined by three major components: the teacher's intention in performing the act, the nature of the act itself, and the student's interpretation of the act's significance. The student's interpretation is particularly important to ensure the teacher does not talk past the student. Also, these speech acts should communicate mathematical meanings and concepts as opposed to rote procedural content in order to be genuinely considered initiating.

Collectively, the research described above has made two important contributions to understanding the role of telling in inquiry-based settings. First, it has challenged the tacit assumption that the teacher should never tell. Rather than simply avoiding the issue of telling altogether, the field is moving toward identifying what to tell, when, and how. Second, it has suggested some hypothetical telling acts that are seen as potentially compatible with inquiry and has differentiated between these acts and the traditional teacher-centered, lecture-based instructional model. Some of these telling acts have been demonstrated using insightful examples from teaching practice.

There are also two unaddressed limitations in the telling literature. First, there is little empirical evidence from which effective telling practices have been derived. With the exception of analytic scaffolding (Baxter & Williams, 2010), the recommendations for telling lack empirical backing and stem from thought experiments or the researchers' anecdotal experiences. While these are skillfully illustrated in well-chosen teaching episodes, it remains unknown the extent to which telling practices occur over time in a daily inquiry-based classroom. The present study seeks to address this limitation by systematically investigating a data sample for instances of telling. A second and related issue is that existing telling constructs aid only in recognizing telling but not classifying the variation in telling into specific, unique types. For example, analytic scaffolding (Baxter & Williams, 2010) and initiating (Lobato et al., 2005) are two constructs that help identify whether a teacher has told students important information but do not classify important differences and distinctions in this telling. In order to build on this work and define a construct with which to investigate the variations in mathematical telling implemented by inquiry-based teachers, the remainder of this section describes issues pertaining to the use of a framework for examining telling.

Identifying Telling in Discourse

Three main attributes of discourse could be considered when framing an investigation of telling. The first is grammatical form. The literature has demonstrated that the form of an utterance, such as whether it is imperative, declarative or interrogative, influences but does not clearly determine what information is eventually conveyed to students (Chazan & Ball, 1999; Lobato et al., 2005; Wood, 1998). Thus, speech is too complex to divide telling from not telling using only the grammatical form of the sentence.

The second attribute is function, which is essentially the perceived role or purpose of the speech by participants. Initiating and eliciting (Lobato et al., 2005) constitute one framework that depends heavily on function rather than form. In determining the function of a speech act, the "function triad" (Lobato et al., 2005, p. 115) includes the teacher's intentions, the nature of the action, and the students' interpretation. Thus, while function is a critical aspect of telling, it can be difficult to infer, especially when the three components of the function triad do not align.

A third attribute of discourse is its content, informed by the surrounding discourse and broader context. This approach is more interpretive than the first approach of classifying the grammatical form of the discourse because the researcher uses the contents of the discourse as evidence to build a coherent and defensible account of the data. The approach is less interpretive than investigating the complete function of the discourse, since the teachers' intentions and students' interpretations are not interrogated (for example, through interviews) beyond what is revealed from the content of the discourse itself. The present study attended to the content of the discourse rather than the grammatical form or perceived function by the participants. This choice was related to the eventual research question of the study. Because telling is poorly understood or at least poorly documented in inquiry contexts, I chose to examine how discourse itself exhibits characteristics with telling potential. It is unclear whether teachers and students are aware of and sensitized to the many ways telling might occur. It therefore seemed reasonable to look for evidence of telling in the content of the discourse itself rather than through the perspectives of the participants.

Framing Mathematical Telling

In developing a study to classify telling, it is necessary to articulate a set of boundaries determining what speech will be considered telling. This is not a straightforward task. Attention has already been called to the imprecision of the term "telling" (Chazan & Ball, 1999). Traditionally, declarative statements that seemed to transfer information from teacher to students were the most obvious form of telling. However, constructivism implies that transmission of knowledge from one mind to another is in fact impossible and nonsensical. Regardless of the type or quality of the speech acts, individuals have no direct access to the contents of another's thoughts. It is not difficult to find episodes in which students make little or no sense out of a teacher's explicit language. From this perspective, telling as it was historically conceived becomes impossible because no speech is capable of literally transmitting information.

To circumvent the obstacle that students have no direct access to the meanings and ideas expressed by others, we might alternatively conceive of telling as the set of all speech acts having the potential to stimulate new student thought via the introduction of information. While this definition is more compatible with constructivism, it is also problematic. All language is built as speakers make use of particular vocabulary, grammatical structure, vocal intonation, and other components in order to achieve some purpose. These components of speech may invoke an abundance of new thoughts regardless of how explicit or implicit they seem. For this reason, there is no such thing as neutral speech. Any group of words, sounds, or gestures may lead students to think about why these acts took place while other possibilities did not and to make inferences about the teacher's intentions and the meaning of his or her acts. As a consequence, we are forced to concede that *all* speech is telling because of its potential to stimulate new thought.

The preceding discussion should make it clear that efforts to definitively classify speech using a binary "telling or not telling" scheme are unlikely to be consistent or meaningful, because it is practically impossible to know the extent to which a given speech act has an influence on a student's thinking. Thus, in order to identify and analyze teacher discourse that most resembles what is intuitively thought of as telling, I use two characteristics to define a boundary around a particular subset of relevant speech acts: a) the presence of mathematics in the discourse, and b) the owner or source of the mathematical ideas expressed. The first characteristic is to focus the study on mathematical telling as opposed to telling in general. The second characteristic examines whether the mathematics of the teacher's discourse includes novel ideas inserted by the teacher (which are considered telling) or merely restatements and elicitations of student ideas (not considered telling). The characteristics narrow the research focus toward the telling acts that have historically been absent or unreported in discussions of inquiry-based practice.

The first of the two aforementioned characteristics is the presence of mathematical content. Research has already demonstrated that non-mathematical telling, such as communicating social norms and expectations, is used successfully by teachers in inquiry-based or discourse-based teaching (Baxter & Williams, 2010; Cobb et al., 1993). However, there is much less certainty regarding the extent to which teachers should communicate mathematical concepts or ideas directly. One agreed-upon exception is providing mathematical conventions, which cannot be discovered or invented purely by student reasoning. The extent to which teachers might tell other kinds of mathematical information remains largely unknown. Thus, this study intentionally investigates instances of telling in which the teacher makes a direct and clear reference to mathematics.

The second characteristic is the degree to which the teacher's utterances convey ideas originating from the teacher rather than a student. Because research demonstrates the benefits of building mathematical learning upon the thinking of students, teachers may depend extensively on students to provide appropriate explanations or make integral connections. In such settings, teachers sometimes limit their contributions to repeating ideas originating from other students. When a class arrives at an incorrect conclusion or confronts an insurmountable obstacle, it can seem as though the teacher must somehow enable a student to come to the rescue with a breakthrough, essentially offloading telling to students (Lobato et al., 2005). Utterances in which the teacher shares information originating from his or her own thinking are a stronger class of telling that teachers may hesitate to use. This study examines the instances where a teacher shares original thinking not expressed by students because less is known regarding the occurrence of this telling in inquiry-based instruction.

Together, these two characteristics of discourse outlined above form the basis for defining "mathematical telling" in the present study. Mathematical telling, then, refers to a teacher utterance that makes a clear reference to mathematical content and initiates ideas originating from the teacher. This conceptualization of mathematical telling is of interest because it constitutes a particular set of telling practices historically absent in the vocabulary of mathematics education reform.

Teacher Interventions with Groups and Individuals

This section addresses the choice of instructional context for an investigation into mathematical telling. Two primary instructional formats within inquiry-based instruction are whole-class discussion and collaborative work in small groups. Although mathematical telling is potentially a natural component of both of these settings, the literature on whole-class discussion has downplayed the teacher's mathematical contributions and has emphasized orchestrating and facilitating student participation, whereas the literature on teacher interventions during group work has acknowledged to some extent the teacher's need to provide original mathematical information when working with the details of student thinking. For this reason, teacher interventions with groups and individuals constitute a reasonable context for initial explorations into the types of mathematical telling teachers use in inquiry. The present study contributes to this body of literature by expanding the limited amount of empirical data for teacher interventions and by improving descriptions of mathematical intervention (telling) practices with finer detail and distinction.

The literature investigating teacher practices in whole-class contexts has focused overwhelmingly on orchestrating and facilitating student contributions as opposed to providing information through practices resembling mathematical telling. For example, Stein et al. (2008) proposed five practices—anticipating student responses, monitoring work on the task, selecting student solutions for public presentation, sequencing the presentations, and connecting them together. These practices facilitate the sharing of student work without commenting on potential teacher interventions during the task or novel mathematical contributions during the discussion. Similarly, the primary teaching tools described in Simon's (1995) teaching experiment involved assessing student thinking and responding with tasks meant to challenge the misconceptions in their thinking. Rather than explaining a concept or modeling his own thinking, the teacher depended on the tasks and on student contributions and insights to advance his learning goals. Staples (2007) specifically studied the teacher's role in supporting whole-class collaborative inquiry, and found three major components: supporting students in making contributions, establishing and monitoring a common ground, and guiding the mathematics. The first two components are clearly facilitative. The third component (guiding the mathematics) did involve some mathematical aid such as providing "food for thought" that "generally did not reshape the task in a significant way" (p. 188). Staples observed that the teacher helped students think about the problem rather than treating obstacles as a sign that they needed hints, next steps, or an explanation. Thus, even when teachers aided students in their cognitive mathematical work, their role has been described as facilitating student thought more than influencing it directly. Even though the literature on telling reviewed earlier (Baxter & Williams, 2010; Chazan & Ball, 1999; Lobato et al., 2005) has begun to challenge the assumption that teachers never tell within whole-class discussion, it remains unclear how often teachers prefer to use such telling in those discussions. For example, in a whole-class setting teachers may be able to rely on one of the many students in the class to express key contributions they themselves might have otherwise made.

In contrast, at least some of the research on teacher interventions with collaborative groups working on tasks has acknowledged the need for teachers to go beyond a facilitative role and provide direct assistance with the content (Brodie, 2000; Ding, Li, Piccolo, & Kulm, 2007; Meloth & Deering, 1999). Brodie (2000) noted that one teacher's focus only on the students' process of engagement, rather than the products of their collaboration, prevented her from identifying critical misconceptions and intervening directly to challenge their thinking, thus reinforcing the misconceptions. Ding et al. (2007) evaluated teacher interventions and found them to be of higher quality when teachers identified students' cognitive obstacles and directly guided or scaffolded their thinking. Meloth and Deering (1999) observed that regular explicit content help, such as providing information, using pointed questions, modeling thinking, or giving a brief explanation of a key concept, often enriched group discussions so long as it built on the group's ideas. Henningsen and Stein (1997) found that scaffolding, modeling high-level performance, and sustained pressure for explanation were regularly present when students maintained high-level mathematical activity on a task. Each of these studies has acknowledged the use of various teacher practices that act upon students' mathematical thinking as opposed to merely facilitating peer collaboration on the tasks. Due to their sometimes direct and explicit nature, many of these practices likely resemble mathematical telling.

Nevertheless, not all research supports the use of explicit, content-related teacher help. Cohen (1994) cautioned that excessive teacher interventions and an inability to delegate responsibility and authority to students may impede student collaboration and learning. Chiu (2004) examined students' on-task behavior and progress toward a correct problem solution and found that these usually declined when teachers gave what Chiu called higher levels of help content (such as solution procedures or commands on how to proceed). Dekker and Elshout-Mohr (2004) conducted a teaching experiment in which one classroom received only process help from the teacher and a second classroom received only product help. Student performance in the process help classroom improved more than in the product help classroom. Despite these results, Webb (2009) offered something of a reconciliation by making the following general observation:

> The results of these studies examining teacher interventions with small groups suggest that the distinctions between whether teachers provide help that focuses on the subject matter content (content-related help) or guidance about what collaborative processes groups should carry out (process-related help), and between whether teachers provide more-explicit versus less-explicit content help, may matter less than whether teachers first try to ascertain student thinking and the strategies they are using and then base their help on what they learn about students' strategies and thinking about the task. (p. 17)

Webb maintained that connecting teacher aid to student thinking is a major influence on intervention quality regardless of whether help content is implicit or explicit. Explicit help such as mathematical telling that respects and extends student work is therefore not incompatible with reform-oriented, inquiry-based classrooms where a primary objective is to elicit and build on the thinking of the students.

The primary conclusion from reviewing literature on whole-class and small-group contexts is that an initial investigation into a teacher's use of mathematical telling may be better suited to the small-group context. In general, this area illustrates a higher tendency for the teacher to inquire into and engage directly with the details of students' mathematical thinking, leading to a higher probability that practices like mathematical telling constitute an important teacher tool within the context of teacher interventions supporting student exploration of mathematical tasks.

A major concern regarding this body of research motivates the present study on mathematical telling and highlights its primary contribution to this area. The literature on teacher interventions with collaborative groups remains small and descriptions of teacher practice too general and imprecise. Studies have until now only distinguished whether the teacher gives product help (regarding the mathematics of the task) or process help (regarding the norms of collaboration), as well as whether mathematical help is a high or low level (Brodie, 2000; Chiu, 2004; Dekker & Elshout-Mohr, 2004; Meloth & Deering, 1999). These classifications are an early attempt to understand differences in teaching practice that affect student learning. However, their generality can lead to inappropriate conclusions. For example, the consolidation of all teacher interventions addressing mathematical content under the umbrella term of product help overlooks important variations within this category and obscures the incongruent findings from multiple studies (Brodie, 2000; Dekker & Elshout-Mohr, 2004; Ding et al., 2007; Meloth & Deering, 1999) either supporting or discrediting product help. The present research study adds to this body of literature by expanding empirical accounts of teacher interventions with collaborative groups in the literature and by developing more specific and detailed characterizations of mathematically assertive teacher practices referred to here as mathematical telling.

In summary, this study will systematically investigate the following research question: "What types of mathematical telling does a knowledgeable and experienced inquiry-based teacher use to support students while intervening with small groups and individuals working on mathematical tasks?"

Chapter 3: Methodology

This chapter outlines the methodology used in the study. First I describe the setting and participants of the case chosen for the study, and why they were appropriate to my research question. I describe the nature of the data used and the methods of analysis that were employed. Finally, I describe potential threats to validity and how I tried to address them in my methods.

Case Study

Because an empirical exploration of the various types of mathematical telling occurring within inquiry is relatively novel, a case study is well suited for the present study. Case studies allow for generative interplay between theory and data, resulting in highly detailed descriptions of the phenomenon (VanWynsberghe & Khan, 2007). Case studies usually involve small sample sizes, contextual detail, natural settings, specific boundaries, working hypotheses, multiple data sources, and extendibility (VanWynsberghe & Khan, 2007). The temporal and spatial boundaries around the case used in this study are described in the following section.

Setting and Participants

The case chosen occurred within a one-semester university mathematics content course for pre-service elementary education majors held in the fall of 2011. The course is the second of two such mathematical courses taken sequentially. The teacher was a Caucasian female faculty member of the university with a doctoral degree in mathematics education and two years of experience teaching this particular course. She had four additional years of experience teaching similar courses at another university. The class included around twenty predominantly white students, one of whom was male and the others female. The class met twice weekly for two hours each session. The classroom contained whiteboards on three walls, a projector and screen in front, a document camera, and cupboards with various manipulatives. Students sat in groups of size four to six at hexagonal tables spread throughout the room.

Data were collected from the first three curricular units, each consisting of approximately seven lessons. The first unit covered fractional reasoning using iterating and partitioning images (Siebert & Gaskin, 2006). The second unit involved making sense of fraction operations. The third unit covered basic probability and statistics concepts. It should be noted that although this classroom had some similarities with other inquiry-based mathematics classrooms in general, other characteristics make it unique. Specifically, the students were pre-service elementary teachers rather than just mathematics students, and they had previous experience with many of the topics but were experiencing them in a very novel way. As a result, it was not uncommon for the mathematical discourse of the classroom to include pedagogical considerations.

The course described above was selected for the case study because of its inquiry-based format and the teacher's frequent and successful use of mathematical tasks. The teacher is considered by her colleagues to be a skilled instructor whose practice exemplifies the characteristics advocated by mathematics education reformers. The teacher's preference for and comfort with inquiry-based teaching provided an optimal setting for examining the compatibility of mathematical telling and inquiry. The extensive use of group work on mathematical tasks proffered the opportunity to observe the teacher's verbal interventions with groups and individuals.

Unit of Analysis

In discussing case studies, VanWynsberghe and Khan (2007) distinguished between the case, which consists of everything within the specified boundaries of space and time, and the phenomenon of interest occurring within the case, which they referred to as the "unit of analysis." While the case I considered consisted of all classroom instruction in the course described above, the phenomenon of interest was the instantiation of mathematical telling during teacher interventions with groups and individuals as they worked on mathematical tasks. The collection and analysis of data were focused on identifying and understanding this phenomenon as it occurred within the chosen case. The methods used for this data collection and analysis are outlined in the following sections.

Data Collection

The data used in this study were collected during the fall of 2011 for another research project under the direction of a faculty member from the same university. The data collected during this time that were utilized in the present thesis included classroom video, field notes, transcriptions of selected portions of the class, and paper artifacts (task sheets, homework sheets, and reference handouts).

Classroom video data were collected concurrently with field notes and artifacts for approximately two months, spanning the three units described previously. The lessons were recorded in their entirety through an unobtrusive permanent camera installed at the rear center of the classroom. The direction of the camera lens can be adjusted through a control panel in one corner of the classroom, and a research assistant controlled the camera to follow the teacher throughout each lesson. Three hanging microphones at the front left, center, and right of the room captured sound from the room, and the teacher always wore a microphone that provided clear recordings of teacher speech. For the end of the second and the entire third unit, small table microphones were placed on two of the tables to enhance audio quality of group conversations at those tables. The research assistant at the control panel increased and decreased the various microphone pickup levels as needed in order to hear the conversations occurring between the teacher and students.

In each lesson, I (or occasionally another member of the research team) took detailed field notes to outline the unfolding activities and conversations. One column was used to record the events taking place, including who was speaking and the essence of what they said. A second column was used to record personal notes, reactions, or impressions as they occurred. The field notes provided a condensed sketch of all lessons and indexed the content of the video recordings.

The classroom video data from the first unit and the third unit were purposefully selected to be transcribed for analysis. The second unit was omitted to limit the data to a manageable size for careful and detailed analysis. The mathematical content of the second unit, fraction operations, was similar to and built on the topics from the first unit, and thus seemed less likely to reveal as much variation in mathematical telling practices as the third unit. The first unit on fractions was selected because many of the classroom norms were being established and negotiated toward the beginning of the semester. The third unit on probability occurred later in the semester when most of the classroom norms had stabilized and covered significantly different mathematical content. Thus, the first and third units comprised a purposeful sample for focal analysis. Each of these two units contained seven lessons for a total of fourteen lessons.

For each lesson, all portions containing group work or individual work were reviewed on the video, and all instances of teacher interventions in these settings were transcribed by undergraduate research assistants. An intervention was seen as any time a mathematical dialogue emerged between the teacher and at least one student, even if the teacher gave little to no explicit guidance. These transcriptions formed the basis for later coding and analysis. In order to understand roughly how much of the total instructional time was contained in these transcriptions, I reviewed time stamps recorded in the field notes and video transcriptions. Using the time stamps, I summed the approximate intervals of time the teacher allocated to group work for each of the fourteen lessons. Group work constituted roughly one third of the total instructional time.

As a final source of data, all paper documents administered by the teacher during the class were collected. These documents included classroom tasks, homework sheets, and handouts containing supplementary reference material. These artifacts were reviewed briefly and referred to as necessary while analyzing the classroom transcriptions to make sense of the dialogues occurring around those documents.

The data sources described above were considered sufficient for satisfactorily answering the research question, which was to understand the types of mathematical telling a knowledgeable and experienced inquiry-based teacher used to support students while intervening with small groups and individuals working on mathematical tasks. The transcriptions of teacher interventions with groups and individuals within the fourteen lessons constituted the primary data source for analyzing the mathematical telling implemented by the teacher while supporting task exploration. The additional data sources served as a resource to consult when making sense of the transcribed conversations. Specifically, the field notes provided a summary of the proceedings of each lesson, which were reviewed concurrently with the transcriptions to situate the analyses of the transcribed portions of data within their broader context of the entire lesson. The task sheets, homework, and informational handouts were used when needed to make sense of the transcribed conversations that made references to ideas from those documents.

Data Analysis

My analysis comprised three main objectives: 1) partition the data into sensible chunks of interrelated discourse, 2) develop precise codes for identifying teacher speech acts that exhibited the characteristics of mathematical telling as defined in the framework, and 3) classify the instances of mathematical telling by developing a coding scheme that distinguishes between unique types of mathematical telling. I began my initial analyses by looking only at the first two lessons. After I developed coding schemes from these two lessons, I began scaling up to the full fourteen lessons. The additional data led to modifications and refinements to the coding schemes until the codes began to fit all the data well. This work is detailed below.

Partitioning the Data into Conversations

The first objective was to partition the data into interrelated chunks of discourse to facilitate subsequent coding. Recall that the transcriptions included only the portions of each lesson in which the teacher yielded time to students to work individually or in groups. Within these transcriptions, I wanted to partition the data into "conversations" or chunks of interrelated discourse. Because my study focused on teacher telling practices, conversations among students only in absence of the teacher were not transcribed. Thus, the conversations necessarily included the teacher and at least one student. By looking at the data I identified indicators that one conversation had ended and a new one had begun. These indicators included conclusive discourse demarcating the conversation's end, a change in participants (such as when the teacher moved from one table to another, or from one subset of students at a table to another), or an abrupt change in topic such that the preceding discourse was no longer relevant for making sense of the ensuing discourse.

Coding for Mathematical Telling

The next phase required identifying mathematical telling by developing a consistent coding scheme that operationalized the theoretical construct of telling. The two aspects of discourse used were the presence of mathematical content and the contribution of something new by the teacher (as opposed to merely repeating a student idea). Each teacher turn in the conversation was examined and coded for the presence of these two aspects.

For the first aspect, the presence of mathematical content in the discourse, I began by reading each teacher turn and marking whether I felt mathematical content was present or absent. In many cases this choice was clear. There were, however, some utterances that were puzzling in determining whether they were appropriately included as mathematical for this study. I examined these utterances and constructed a definition that enabled me to make these decisions consistently.

A teacher utterance was defined to be mathematical and coded as “M” so long as the utterance conveyed mathematical language or ideas and was used from a mathematical *posture*. I use posture to mean the intended angle from which the utterance is to be construed. A statement has a mathematical posture if the practices and ways of thinking from the mathematical discipline are required to enact and make sense of the utterance. This stipulation of mathematical posture was developed in order to exclude statements that contained mathematical language but did not really invoke any of the mathematical ideas or meanings. For example, at one point the teacher asked the students to hand in a math autobiography assignment. Even though the word “math” was used, it was only a label for a homework assignment and did not appeal to any mathematical ideas or meanings. This kind of statement had a managerial posture rather than mathematical. Other similar examples are moderating group conversations by regulating speaker turns or instructing groups to write up their mathematical work on a white board to be shared. It is possible for an utterance to have more than one posture, but if a statement did not use a mathematical posture it was not coded as mathematics.

In determining the presence of mathematical content in the discourse, I considered the language used in the utterance itself as well as the relevant surrounding discourse that informed the meanings of that language. For example, pronouns that referred to prior mathematical discourse in the conversation were interpreted as mathematical because they appealed to the same meanings as the prior utterance. A consequence of this approach was that some teacher utterances were ambiguous. After a student shared mathematical thinking, the teacher sometimes said phrases such as "okay," "alright," "uh-huh," or "keep going." These phrases were sometimes evaluative of the students' mathematical ideas (and were therefore coded as "M") and sometimes were merely an acknowledgment of understanding or a prompt for the student to continue speaking (which were not coded as "M"). To the extent possible, surrounding discourse was used to help decide between these two possibilities. For example, if the student's prior utterance was a question seeking validation, and the student discourse following the teacher's phrase accepted it as validation, then the teacher utterance was coded "M." If the surrounding discourse was not sufficient to make a confident determination, these utterances were coded as "U" for uncertain. The code "U" was also given if the teacher utterance was interrupted or incomplete, causing the meaning to be ambiguous. Of all codes given at this stage, roughly 63 percent were mathematical ("M"), 33 percent were not mathematical, and 4 percent were uncertain ("U").

The second aspect of discourse used to identify instances of mathematical telling was the contribution of something new (mathematically) from the teacher. I only applied these codes to statements that were already coded as "M" from the process above. I began the code development by looking through initial data and seeking to identify contributions from the teacher versus repetitions of a student's ideas. At first I hypothesized that three levels would be necessary – new teacher idea, teacher adjustment or modification of a student idea, and reference to a student idea with no teacher contribution. I planned to include in my study all utterances introducing a new teacher idea (the first level) and exclude those that referred to student ideas with no contribution (the third level); however, the middle level of adjusting or modifying a student's idea contained more subtle teacher contributions. I pondered whether to include them in a study of telling as I examined some examples carefully. My framework illustrated that telling is not as straightforward as a binary "did tell" or "did not," and a major goal of the study was to understand the kinds of mathematical content of in a teacher's discourse that has clear potential to influence student thinking (whether or not the potential is realized). I therefore chose to include these more subtle contributions in the definition of mathematical telling. As a consequence of this decision, it was no longer necessary to distinguish between a teacher contribution that was wholly the teacher's versus a contribution that modified a student's idea. Any utterance with a mathematical contribution from the teacher was coded as "T" for "Teacher contribution." An utterance that did not contribute something new mathematically, such as requesting a student to say more about their mathematical thinking, was coded as "S" for "reference to Student ideas only" and was excluded from all subsequent coding analyses.

An example from the data illustrates the value of including all mathematical teacher contributions, including those that seem more subtle, in the definition of mathematical telling. In the second lesson, the students were using the images of partitioning and iterating to find fractional parts of discrete sets of objects. One question on a task asked students to find two-fifths of thirty. One group was talking with the teacher about their thinking on the problem. The students realized that to find the size of one-fifth, they needed to partition the thirty objects into five parts. The conversation continued with a student comment and teacher response as follows:

Student: And to find what two-fifths is you multiply that six by two?

Teacher: Uh-huh, you take two of those groups right?

At first glance this teacher response appears to be a mild rewording of the student's idea. However, two important features make this teacher utterance an important one for a study of telling. First is the evaluative nature of the response. Excessive evaluation is commonly believed to undermine some goals of inquiry-based instruction, such as relegating authority for justification to students. It therefore seemed important to consider teacher evaluation of student ideas as one form of mathematical telling in order to identify how frequently it occurs and what it looks like in inquiry settings. A second feature of the teacher's response is the intentional modification of the student's language from that of calculations ("multiply that six by two") to the imagery of groups ("take two of those groups"). Prior research has illustrated that teacher language influences the types of reasoning students employ during mathematical activity (A. G. Thompson, Philipp, Thompson, & Boyd, 1994). The teacher modeled, or privileged, one particular way of talking about mathematics. This was seen as a sufficiently important contribution to be considered telling.

Classifying Mathematical Telling into Types

After developing the coding scheme above to define and identify instances of mathematical telling (statements coded as "M" for mathematics and "T" for teacher contribution), the next stage of analysis was to search for key similarities and differences across the instances of mathematical telling. In this stage I worked to create and revise codes until I arrived at a stable and useful classification scheme of mathematical telling.

My early attempts at classifying the instances of mathematical telling focused on the mathematical concepts and ideas embedded in the discourse of the teacher. I attended to whether the teacher talked about mathematical quantities and their relationships, the givens or assumptions of the problem, the mathematical goal or objective of the activity, the use of tools or representations, justifications of a claim, conventions, etc. These efforts were deemed inadequate because I felt I was missing the essence of the telling instances as they related to the negotiated activity between teacher and student. For example, the teacher might focus student attention to the problem's givens in one instance to help the student begin problem solving and gain access to the question, and in another instance to point out a contradiction between the givens and a flawed student argument. Despite referring to the same mathematical fact or relationship that was provided in the problem's givens, the two telling acts are very different.

To address this concern I broadened my focus to both the mathematical content itself and its relationship to the student's ongoing mathematical activity. I defined a construct that I called the "discursive function for engaging in classroom mathematics" to explicate the aspects of discourse I used to group together similar telling practices and distinguish the groups from one another. The construct is used to examine enacted verbal discourse (without appeal to nonverbal intentions and interpretations) and determine the observable function of teacher utterances within a mathematics classroom context. The three aspects of the utterance examined are: (a) the mathematical content being expressed, particularly what step in the task or problem solving process it is and how resolved or unresolved it is, (b) the relevance of the telling act to the student's learning and ongoing classroom mathematical activity, and (c) the surrounding discourse of both students and teacher to provide additional evidence for answering (a) and (b) and to maintain overall coherence. This researcher construct of discursive function for engaging in classroom mathematics enables the researcher to create a plausible and coherent account of the data, by categorizing the telling practices implemented by the teacher according to their role in the enactment of classroom mathematics.

The grain size for attaching a code to a chunk of discourse was a "teacher telling act." As a general rule, a telling act was one teacher speaking turn. However, if a teacher's turn was incomplete when a student interrupted and the teacher continued to finish the turn afterward, and one utterance was not meaningful or interpretable without the other, then the two pieces were joined into a single telling act to maintain coherence. In addition, if within a speaking turn there was a change in discursive function for engaging in classroom mathematics, the turn was broken into separate telling acts and received separate codes. Separating the teacher discourse this way enabled me to attach only one code per teacher telling act. This was deemed more appropriate than double-coding longer strands of discourse because the instances examined did not seem to *simultaneously* enact multiple telling codes but rather *changed* from one code to another. This was true of all codes with one major exception, "validation." Validation proved to be a function that could occur on its own or as a consequence of several of the other codes, and was therefore the only code allowed to be double coded across the entire data set with other telling codes. This phenomenon is explained further in chapter four.

I developed an initial set of categories by looking at the first few lessons and inventing codes for each telling act according to their apparent discursive function for engaging in classroom mathematics. I came up with about seven codes. I then took each code and examined all of the telling utterances I had assigned that code in order to write theoretical descriptions or definitions of the essence of the code. When I felt the codes were developed enough to be tested, I asked a peer graduate student to apply the codes on one lesson to compare reliability. Our codes agreed on 70 percent of the telling acts. In addition, I performed a self-reliability check in which I coded the same lesson from scratch twice, about a week apart. Comparing the two documents yielded a 79 percent agreement. I used these two reliability checks with myself and with my peer to identify the codes that were most commonly conflated. I studied the cause of these conflations, considering the theoretical overlaps between codes. I then refined the coding scheme by making a number of changes. I dissolved one of the codes completely since it was poorly defined and overlapped with several other codes. I combined two codes whose boundary was unclear, forming a more general and encompassing code. I also identified a couple of new codes that I felt more accurately captured some of the more rare and puzzling telling instances. I added more details and examples to the definitions of the codes to tease apart the theoretical boundaries between them. The improved coding scheme again had seven codes.

I conducted another self-reliability check with this new coding scheme. I selected a longer lesson and coded it from scratch twice, three days apart. The percentage of total agreement was 76.5 percent. Much of the error came from a single code, the "validate" code. This code (mentioned above) is described with more detail in the results section and was used whenever the teacher validated the student's mathematics. Validation was unique in that it was the only code that could occur simultaneously with another code. It was also the most difficult to consistently identify given that validation often occurred in subtle ways. When I recomputed the reliability ignoring the validation code, agreement was 88.7 percent. Conflations between codes were analyzed and the definitions updated to increase future consistency. The self-reliability measures can be interpreted to suggest that the results of this study are generally useful to think about important categories and variations in teacher telling acts within inquiry, even though data from reality can antagonize theoretical constructs and small levels of uncertainty are usually inevitable in a qualitative study of this nature.

The resulting coding scheme, with the seven telling types, was applied to all fourteen lessons in my data set. When I finished the coding, I proceeded to analyze and report on the seven types. I looked at each telling type, one at a time, and read through all of the instances of that type of telling. My purpose was to understand the telling type thoroughly to provide accurate descriptions and illustrative examples in the results. As I engaged in this process, I recognized that within some of my seven telling types there were important variations that I wished to communicate in the results. Thus, I developed subcategories for a few types when necessary to capture the variations I was experiencing while reading through the data. An unforeseen consequence of this process was that the subcategories made the original categories even more clear and well-defined. The subcategories helped me understand some of the instances that bordered two telling types and code them consistently. The seven telling types and their subcategories are described in detail and illustrated with examples in chapter four.

Threats to Validity

There are four threats to validity that I address in this section. First is whether the choice to privilege the researcher's perspective on the data impacted the credibility of the results. Second is the reliability of the telling codes and framework. Third is whether the telling practices reported are appropriately designated as inquiry. Fourth is the ability to generalize this particular case study to the mathematics education field more generally.

The first issue relates to the choice to develop the researcher's account of the data rather than investigate the participants' interpretations of their experience. This choice risked attributing meanings to data that were inconsistent with the meanings constructed by the participants. However, the purpose of this study was not to investigate how participants experience teacher telling, but rather how teacher discourse within inquiry can exhibit the characteristics of telling (whether the teacher and students are aware of it or not). The identification and classification of telling in this study was driven by a purposeful theoretical framework in order to reveal the ways teacher discourse conveys mathematical information to students. The results are therefore the account from the researcher's perspective viewed through the lens of the theoretical framework. This is no more or less valid than the account as it might have been reconstructed from the teacher's or students' perspective—it is simply the story from a purposeful viewpoint meant to expand the field's understanding of what mathematical telling is and looks like in an inquiry-based setting.

The second issue is the reliability of the telling codes and framework. Earlier I discussed the peer and self reliability checks used to refine the telling codes. The resulting coding scheme was stable and relatively consistent. High reliability is always ideal; nevertheless, specific pieces of data can be difficult to classify and can challenge a particular framework. I used the discrepancies between coding rounds to develop and revise the problematic areas of the coding scheme. The result was a telling framework that illuminates the nature of telling. Although a minority of telling instances continued to challenge the boundaries between codes, the framework provided deeper insight into the phenomenon of teacher telling. Mewborn (2005) argued that although misfits in the coding process provide opportunities to refine a framework, it is more important for a framework to increase understanding of a general idea or phenomenon than to perfectly characterize a single instance in particular.

A third issue concerns the identification of this classroom as inquiry-based. The case study examined in this thesis exhibited the central, primary characteristics of inquiry including high-level tasks, group collaboration, and student-centered whole-class discussions. However, it could be argued that some of the specific components of the instruction (mathematical telling potentially being one of these) were deviant from the overall inquiry-based approach and are not appropriately designated as components of inquiry. This concern would be tenable if certain teacher practices like teacher telling created tension by establishing contradicting classroom norms. Throughout the classroom observations and data analyses, such contradicting norms were not apparent. The instruction was enacted as a clear and consistent approach to engage students in the process of mathematical inquiry. The examples of mathematical telling provided in chapter four provide evidence of how the telling practices fit within the overall inquiry setting. Because the overall instruction widely matched the characteristics of inquiry, the specific practices observed were considered to be component parts of a coherent inquiry-based instructional approach.

The fourth issue is the ability to generalize the study to the broader field of mathematics education. Because of the contextual circumstances particular to the case used, the findings may not directly transfer to other settings and contexts. The classroom setting bounding the case under investigation involved pre-service elementary education teachers who regularly made sense of the lesson content pedagogically as well as mathematically, which is not typically the case for primary and secondary mathematics students. The results were derived from one teacher’s practice and it is unknown how the telling framework would fit additional teachers. The results also may depend heavily on the particular mathematical curriculum of the course, over which the teacher had a great deal of flexibility and little pressure from external assessments and standards. In light of these particulars, the case was chosen so as to provide the highest probability of answering the research question regarding the instantiation of mathematical telling in inquiry-based settings. The teacher in this case study was highly capable and qualified in enacting inquiry-based instruction resembling the practices promoted by NCTM (2000, 2007) and others. Because discussions about the types of mathematical information told effectively by teachers in inquiry have been generally absent, the case purposefully explored the existence of mathematical telling within strong inquiry-based patterns of instruction. Thus, the sampling choice might be thought of as an "ideal-typical case" (LeCompte & Preissle, 1993) in that mathematics instruction in general does not tend to demonstrate this level of quality or fidelity to inquiry practice. Therefore, any findings about the instantiation of mathematical telling in this inquiry-based environment would have important implications regarding the future characterization and investigation of practices within other successful inquiry-based classrooms.

Chapter 4: Results

Using the means described in the previous chapter, my data analysis resulted in seven different types of mathematical telling: assess, interpret, qualify, clarify task, guide, disclose and validate. In this chapter I report in detail on each of the seven types. For each type of telling, I describe its defining characteristics, organize it into subcategories based on notable variations (when applicable), and analyze typical examples from the data. Although it is not possible to order the telling types definitively from least to most substantial, the order roughly follows a progression from what some might unwittingly view as innocuous telling to more candid interventions. Following the presentation of each telling type individually, I discuss the frequency of the telling types (see Figure 4) and discuss emergent considerations regarding telling.

Assess

The first type of mathematical telling is to *assess*. An assessment is a mathematical query posed by the teacher in order to ascertain what the student thinks or understands. In a broad sense, every question or task can elicit student thinking that is subsequently evaluated by the teacher and could informally be considered an assessment. However, the definition of assessment in this telling framework is more exclusive and not all assessments can be considered telling. The definition of assessment in this category reflects only "telling assessments" in which the teacher contributed something new mathematically. A telling assessment is a mathematically structured or constrained request rather than just a general elicitation of student thinking. For example, a question such as "How did you solve the problem?" is a general question without any imposition of additional mathematical structure and is therefore not a telling assessment. When assessments introduce a new mathematical constraint, structure or idea to the conversation they are ways of telling. A student responding to a teacher's telling assessment not only shares his or her own thinking but must either conform or object to the constraints embedded in the assessment. Another defining characteristic of assessment is that the teacher uses the question in order to become aware of the student's understanding rather than as a means of addressing student obstacles the teacher is already aware of. Thus, guidance questions that focus or lead students to overcome an already evident student issue or misconception go beyond an assessing function and are considered a different type of telling.

Assessments played two key roles that will be illustrated using examples from the data. The first role was to ascertain or reinforce student performance on an already familiar skill or concept. One assessment of this sort was used repeatedly by the teacher in a particular lesson from unit one. The students were learning how to identify and justify fractional quantities using two fraction images, partitioning and iterating (see Siebert & Gaskin, 2006). A partitioning image is used to justify a quantity as $1/n$ by arguing that when one whole is partitioned into n equal pieces, one of those pieces is the quantity $1/n$. An iterating image is used to argue that if a quantity that is iterated n times makes up one whole then that quantity is $1/n$. The written task students were engaged in allowed them the freedom to use either image for each problem given. As the teacher wandered among the tables, she routinely assessed the students to ensure they were labeling their own justifications with the correct image. For example, after hearing one group's discussion about dividing the whole (a set of twelve items) into four parts, the teacher interjected as follows.

Teacher: So, when you said, "The twelve in four parts," is that iterating or partitioning?

Student: Partitioning.

Teacher: Um-hmm. Yeah.

The students had not already explicitly labeled their explanation so the teacher verified they recognized their use of a partitioning image. Because these two images were major learning objectives of the course, the teacher frequently assessed her students' ability to recognize and produce justifications appealing to both images.

Because the concepts in this assessment had already been introduced to the students beforehand, one might question whether this assessment is appropriately labeled as telling. It is therefore helpful to discuss the telling characteristics of this example in detail. Contrast this assessment with a completely open student elicitation such as, "What are you thinking on this problem?" The assessment introduced concepts and constraints that were not in play before the teacher's interjection. First of all, it drew attention to the label or type of justification the students might have been appealing to, invoking any images or meanings the students associated with the words "iterating" and "partitioning." It is difficult to know what those students' associations of meaning were and to what extent they were already in the students' minds before that moment.

In addition, the structure of the question precludes the possibility that the student's verbalization appealed to an alternative image besides partitioning or iterating. This is not a trivial contribution, given that at another time one student constructed a solution that the teacher felt did not adequately reflect either image. In that instance, the question presented a square and asked the students to shade in one fourth and justify their response using partitioning or iterating. One student made four smaller squares inside the larger one. In a dialogue with the teacher, this student said, "Would it work that 4 small squares are equal to 1 large square?" This wording exhibited an alternative image in which the whole comprises its parts in a static state of equilibrium, obscuring the preliminary actions inherent in both a partitioning and an iterating image. The teacher acknowledged this formulation as mathematically correct but required the student to explicate an action image producing the fractional arrangement. This instance shows that when the teacher's assessments provided only two options for fraction images (iterating or partitioning), viable alternative images were excluded as possibilities. This illustrates that even the mildest constraints are not as neutral as is often supposed.

As further evidence of the telling content in this assessment, consider the additional embedded constraint that the student's solution process be only one of partitioning or iterating, not both. Additional instances from the data suggest that this is also not a trivial imposition of structure, since at various times several students voiced their concern that they had appealed to *both* images in a single justification. They would not likely have voiced this concern if the tasks and assessments had not constrained them to appeal to only one image per justification. Even though in practice it is very difficult to determine the precise influence any single telling act had on students, the considerations presented here illustrate the naivety in supposing that the mathematical ideas and constraints inherent in assessments of this nature are not telling and do not reveal important information about mathematics.

The second role of assessment was to test the depth of student understanding by complicating or extending the question or task students were engaged in. These assessments were not as routine and introduced significant mathematics the students had not yet considered. In the following example, the students were made to write a division word problem whose solution is three fourths that uses a measurement perspective (i.e., the problem's total amount and the group size are known and the number of groups, or fractional part of a group for non-integer quotients, must be calculated). After first eliciting the students' word problem, the teacher used two assessments. The first assessment corresponds to the first role discussed earlier (verification of a familiar concept or skill), while the second assessment corresponds to the second role of extending and complicating the mathematical question or task.

Teacher: Okay, okay what did you write?

Student 1: We wrote four bananas make a bushel. You ate one banana so there are only three remaining. What fraction of a bushel is left?

Teacher: Hmm, so what fraction of the bushel *is* left?

Student 2: Three-fourths.

Teacher: Okay, tell me why that is a measurement division problem and not just a three-fourths problem? Like 'cause I could look at that and I could think, well it's three-fourths because it takes four bananas to make a bushel and there are three of them. And, um so tell me why that is also a division problem. Why is that three divided by four?

The first assessment immediately following the students' word problem simply verified they had found the correct answer to their own problem. The second assessment, in contrast, increased the complexity of the task from constructing a division word problem to justifying its structure as truly being division and not just a fraction problem. The students had not recognized this potential conflation between fraction and division concepts. By making students aware of their interrelated structure and requiring a conceptual distinction between the operation of division and the fractional result, the assessment encouraged deeper reasoning about the unintuitive structure of measurement division with a rational quotient.

Interpret

The second type of mathematical telling is to *interpret* students' mathematical formulations. In order to clarify or characterize what a particular student's reasoning entails, the teacher rephrases the idea, summarizes it, generalizes it into more condensed form or infers unspoken parts of the student's thinking. Even though the meanings expressed by the teacher are attributed to the students, they are filtered through the teacher's framework of understanding and therefore may or may not align with what the student has actually said or thought. The teacher sometimes provides students an opportunity to affirm the accuracy of the interpretation. Because the teacher's interpretations extrapolate beyond the literal student expression, the contribution is an act of telling.

Consider this example from the data. The task required students to rewrite fractions into decimal form using a provided square grid with ten rows and ten columns forming one hundred smaller squares. One table was working on writing one-eighth as a decimal. The teacher stopped there and asked about their progress. They began to share the different ways they had approached the problem. The first student to share described the following strategy.

Student: The first way that I did it, I knew that eight could go into ten, and so that means I have two rows [of] ones left over and then I have to divide those two rows equally into these groups.

Teacher: Okay, 'cause you're trying to form the whole things into eight groups right?

The student's strategy was apparently to first see how many tenths, or complete rows of ten small squares in the grid, could be divided evenly into eight groups. This would have yielded one row per group, leaving two rows left over. The student would then decompose those two rows into twenty hundredths and divide them evenly into the eight groups. It is unclear whether the student was thinking about the grid as one whole and its parts as fractional amounts or the grid as one hundred ones due to her choice of language—she referred to the leftover hundredths as ones. She described her first action in terms of whole number division, eight going into ten, in such a way that it is unclear whether she saw the partitioning action in this division and its fractional meaning. In response to the student's verbalized strategy, the teacher interpreted the student's approach by inferring the underlying goal. Her rephrasing suggested the purpose of the student's operations was to divide the entire whole into eight groups and her language explicated the partitioning action behind the division. Even though this underlying purpose might be inferred from the student's work, the teacher's interpretation is considered telling because she summarized the student's approach in a broader and clearer way in order to inform the meanings behind the operations the student was performing on the quantities.

Qualify

The third type of mathematical telling is to *qualify* the mathematics. The teacher describes or qualifies a mathematical part of the conversation according to human experience such as student feelings, emotion, motivation, mathematical difficulty, interest, worth, subjectivity, humor, common sense or everyday experience. The vast number of ways a teacher can qualify mathematics makes this telling type rather diverse. The underlying purpose of these qualifications is generally to reassure students as they engage in mathematics by humanizing the experience. Because some students may see mathematics as boring, threatening, irrelevant, contrived, predetermined, difficult, tedious or unnatural, a teacher qualifies mathematics with attention to these issues to align classroom activity with common human needs and expectations.

The teacher in this study qualified mathematics in many ways. She downplayed student errors to minimize embarrassment, attached practical value to the activity or task, characterized the relative difficulty of mathematical ideas, acknowledged interesting contributions, described subjective aspects of their mathematical activity, joked with students using mathematics, and evaluated ideas as either awkward or natural according to informal daily reasoning. These acts aided in establishing a safe, enjoyable, inviting and motivating environment where the classwork felt worthwhile rather than contrived.

This first example illustrates how the teacher identified parts of a student's justification as being awkward. It is taken from the lesson in unit one on using partitioning and iterating images to name and justify fractional quantities. Students at one table were working on the question, "There are 20 students on the bus, but there are still 28 empty seats. What fraction of the seats are filled?" The numerical answer these students had chosen to use was 20/48, but they were having difficulty understanding how to define this quantity using a partitioning or iterating image and sought the teacher's help. After the teacher asked some guiding questions meant to elicit an appropriate justification, this dialogue followed. The first teacher turn was a prompt to structure the students' justification. The subsequent teacher turns contain teacher qualifications of the students' mathematics (marked in boldface for clarity).

Teacher: But, like, so I want you to, don't just, like, jump to an answer. Think, "How can I explain this using an iterating or partitioning image?" 'Kay, so why is a bus, one seat of a bus, one forty-eighth? Why is it one forty-eighth?

Student 1: Because a whole is equal to…

Student 2: Oh! If you divide all… Okay. So, if you divide all of the seats into forty-eight equal pieces.

Teacher: Okay. So, if you take the bus and divide it into, like forty-eight equal spots. **That's a little awkward isn't it**?

Student 2: Yeah.

Teacher: Um, for buses. I think…

Student 2: Or, if you take the one, and multiply it by forty-eight—

Teacher: Then you have the number of seats.

Student 2: Yes.

Teacher: Yeah. **I, I almost think iterating is less awkward in terms of buses, to think about. Because partitioning, it's like, we don't really partition the bus into forty-eight pieces.** I can think, I, one, one seat is one-forty-eighth of the bus because it takes forty-eight seats to make up the whole bus. Right? **And so, I think iterating flows more naturally there.**

The first idea proposed by Student 2 ("divide all of the seats into forty-eight equal pieces") appealed to a partitioning image in order to justify why one seat was 1/48 of all the seats. Although it is mathematically correct to partition a whole comprised of forty-eight discrete objects into forty-eight groups of size one, the teacher qualified this response as awkward based on the way students might think about the actual context of seats in a bus. The student then initiated an alternative justification using an iterating image. The teacher identified this second image as more natural for the context of buses. She considered it more common to think of a bus as having iterations or copies of one seat that fill the bus than beginning with the full bus and partitioning it into seats. This example illustrates that the teacher cared not only that students produce mathematically acceptable responses to the task but also that their work feel natural and realistic to them. This validated the kinds of reasoning students bring from everyday life and communicated beliefs about what mathematics is and how it is practiced. It is interesting that within the qualifications of her final turn the teacher also modeled a complete and correct response that drew from the ideas the students had contributed. This kind of solution modeling is another telling type discussed later in the "disclose" category.

The next example of qualification demonstrates how the teacher attached value to the mathematical task in order to motivate a skeptical student. The task had several questions about calculating probabilities. The student worked on a question that read, “What is the theoretical probability of drawing a red and a blue ball from a bag containing 2 red and 3 blue balls if you return the first ball to the bag before the second ball is drawn?” The class had not yet developed multiplicative and additive strategies for handling probabilities involving multiple independent events. They were calculating the probabilities by making lists of all the equally likely outcomes and determining the fractional part of the outcomes that answered the question. One student began to find the activity tedious and asked the teacher whether a shorter method exists. The teacher did not provide one but talked about looking for patterns in the solution’s organization, which led to this dialogue.

Student: So like, what’s the point of listing all the combinations? What’s it teaching us?

Teacher: What’s this teaching you? Well for a lot of—

Student: Like to be patient.

Teacher: No, no, no [laughs]. It’s not teaching you to be patient. It’s first of all helping us think about the situation, like what’s really involved here in the situation. We have these shortcuts. We can’t make meaning of the shortcuts if we don’t know what’s actually going on. And so I know you have these nice shortcuts like multiplication, but if the shortcuts don’t make sense in terms of the situation then they are meaningless. But writing out the combinations can help us see where the shortcuts are going to come in.

In order to respond to the student's complaint, the teacher qualified the mathematical value of the task. She affirmed that authentic mathematical activity involves making sense of situations, identifying patterns, and connecting those patterns to meaningful methods or shortcuts. To implement a shortcut with no understanding was not considered worthwhile mathematics for the teacher in this classroom. The teacher's response in this episode can be seen as an attempt to humanize the task by reinterpreting the objective of the student's mathematical activity in a way that privileges natural human inquiry and sense making over mechanical memorization and repetition.

Clarify Task

The fourth type of mathematical telling is to *clarify the task*. This occurs when the teacher clarifies the mathematics of the task, question, activity, or intended form of a response without addressing any parts of the actual answer or solution process. The teacher's utterance is meant to address student confusion regarding what the teacher intends for students to do or engage in so as to achieve the learning objective.

The teacher in this study chose to clarify the task at different times and in different ways. Sometimes the students voiced confusion and the teacher clarified the task for them, while other times the teacher initiated the clarification upon seeing evidence the students were not engaged in ways the teacher desired. The teacher varied in directing clarifications toward one student, one table or the entire class. Sometimes the teacher needed to clarify the task toward the beginning, while other times she clarified task components later on by responding to student work or even extending the task after students had finished.

There were two main roles the teacher accomplished as she clarified the task. The first was to provide clear instructions for how students were to engage in the intended learning activity. The teacher gave basic instructions for what to do on which problems. She clarified which parts she did and did not require students to explain their work. She determined the form of a solution, for example a diagram or written explanation. She also told students the resources they should either use or avoid, such as previously learned concepts or tools like calculators.

The following example illustrates the teacher clarifying basic instructions. The task was a worksheet from the probability and statistics unit with sets of data and questions that were meant to develop the concept of mean absolute deviation. The first question asked students to represent data sets about concert ticket prices on number lines. Because the students had already learned various ways of graphing data, they became uncertain about what the teacher expected them to do. As students worked in their groups, the following conversation occurred between the teacher and students from one table.

Student 1: So, are we supposed to be making a box-and-whisker plot?

Teacher: Nope. Just, just, uh, draw a dot to represent the data points. You're just looking—

Student 2: Oh… oh.

Teacher: You just want to be able to see the data points relative to each other.

Student 1: So, you would just [inaudible].

Student 2: We're marking the lowest and the highest, the median—

Teacher: No, just, like, draw all the data plots. Like, just draw a point for each one of them.

Student 2: Oh, draw the points on the line.

Teacher: Uh-hum. Just so you can see how they're spread apart. You want to be able to see all the data points. [The teacher then directs herself to the entire class]

> Yeah, do not draw a box-and-whisker plot. Just draw a dot for each of the data points. I just want you to be able to see the data points, and how they are, how they are positioned relative to each other.

Even though the data on this problem could have been graphed appropriately using a box-and-whiskers plot, the teacher directed students to make a dot plot in order to be able to see all of the data points and their relative positions in preparation for the rest of the task about mean absolute deviation. This choice to make the graphing instructions explicit likely limited some of the many opportunities for learning about data and statistics but enhanced other opportunities that concerned the teacher most at this point in the unit.

The second role of clarifying the task was to explain the mathematical ideas and meanings involved in the question or activity students were working on. In these instances, students were not as confused about the instructions of the task as they were the mathematical premises establishing the structure of the task. They were unable to access and make progress on the task because of obstacles in comprehending what was involved mathematically. In these cases, the teacher reread, rephrased, explained or added supporting information to the task.

The following example occurred during the same task from the previous example on mean absolute deviation. In order to make sense of this example I have included the exact wording of the introduction to the task:

> Patrons of a particular concert hall purchase concert tickets through different ticket venues. The manager of the concert hall is interested in how much regular concertgoers spend on tickets. She surveys a random sample of people who attended all six of the concert hall's summer series, and determines that the members of her sample paid, on

average, $45 per ticket. However, she thinks that the mean of the data does not tell the full story.

The task then provides the data for three concertgoers of the manager's sample. Each of these three concertgoers paid six prices for the six tickets of the summer series. Furthermore, each of these concertgoers spent an average of $45 per ticket. The question on this task pertinent to the upcoming example read as follows: "True or false? If the average price per ticket paid in the manager's data sample is $45, then the average price per ticket paid for each of the concertgoers in the sample will also be $45. Explain." Because this question as well as the context scenario is rather complicated, students were struggling to begin working on it. The following conversation took place with the teacher and one table.

Student 1: I don't even understand this.

Student 2: I think they're the sample manager stats.

Student 3: I think that's the sample we have up here.

Student 1: But what's the [inaudible]?

Teacher: What?

Student 1: We're just confused by the two different data samples?

Teacher: So, um, this one right here?

Student 1: Yeah.

Teacher: So, um, so what they did is they collected from all of the concertgoers who went to all six. And they said, "Oh, the average amount spent over all those concertgoers was forty-five dollars." And, if you notice, each of these, the average for each concertgoer was also forty-five dollars. So, it's saying if the average for, across the whole set, was forty-five dollars, does that mean that every single concertgoer will also have an average of forty-five dollars?

Student 1: Oh, each subset of data?

Teacher: Yeah, will each subset of data also have an average of forty-five dollars.

Student 2: To figure it out based on our profile?

Teacher: No, no. Don't… that, that one's separate from the pictures. That's just, like, um, in, in this, this right here, they all have an average of forty-five dollars. So, my question is, um. I mean, it might be contrived, or it might be that they would always have an average of forty-five dollars. Like, I just want you to think about, like, does the fact that everybody all together has an average of forty-five dollars mean that all of the six concert series subsets will have an average of forty-five dollars spent, too?

Student 3: If Concertgoer A had like all forty, but Concertgoer C was like all sixty. And if they wouldn't—

Teacher: Would it be. Yeah. [Teacher leaves as students continue working]

Because of the complexity of the scenario, the students struggled to become engaged with the mathematics of the question being asked. The teacher clarified the context of how the data was sampled and to what the present question referred. She also tried to focus the students on the key part of the question by asking whether the phenomenon the students had just observed within the data presented to them was a mathematical necessity or simply a contrived coincidence. This enabled students to access the problem and resume their inquiry into its solution, as seen by the counterexample being crafted by Student 3 at the end of the dialogue.

Guide

The fifth type of mathematical telling is to *guide*. The teacher gives guidance to students while they work on tasks, develop and revise solutions, construct justifications, discuss concepts, and reflect on strategies. The guidance provides additional mathematical structure or supporting information that students are meant to incorporate into their activity. The teacher may focus students toward a useful area, redirect their attention away from unproductive ways of thinking, use pointed questions to lead students in a particular direction, suggest hints and strategies, identify errors or justification deficiencies in student work, edit portions of student solutions or justifications, and give counter-arguments to problematic student propositions. A key characteristic of guidance is that it leaves some portion of the solution mathematically incomplete or unresolved, requiring the student to act on, incorporate, or resolve the guidance provided from the teacher.

Each instance of guidance exhibited one of the following four purposes: focus toward or away from an idea, lead students into productive ways of thinking, address reasoning errors, and give helpful hints and suggestions. These four purposes are found in Figure 1 along with some specific ways the teacher accomplished these purposes. These four purposes will be described in more detail and demonstrated using examples that follow.

Guide			
Focus	Lead	Address reasoning errors	Give hints, suggestions
Direct attention toward useful items	Pose questions or next steps	Identify contradiction or invalid assumption	Suggest a method
Encourage promising approaches	Organize justifications	Identify explanation deficiency	Estimate or bound the solution
Discourage poor approaches	Help students explain concepts	Give counter-arguments	Interpret the problem
Point to parallel examples/scenarios	Ask rhetorical questions	Workshop a student justification	State principles or guidelines
Remind of previous tasks and experiences	Help students make a conclusion/inference	Recognize and debate opposing viewpoints	Explicate criteria for an acceptable solution
		Locate or correct error	

Figure 1. Purposes of guidance.

Focus

The first purpose of guidance was to focus the students' attention toward or away from an idea. These instances of guidance generally told students where to look but did not explain what to conclude or infer as a result of focusing on that aspect. Consider this example from the first day of class. The task involved using Cuisenaire rods to answer questions about fractional quantities. One student who I'll call Laurel (all names are pseudonyms) was working on the problem, "If purple is 2, what is the value of black?" She had placed one purple next to a black and then lined up a row of seven of the smallest white blocks alongside them as shown in Figure 2. Laurel was unsure how to begin reasoning about this situation to come up with an answer and called for help from the teacher. The teacher elicited Laurel's initial thinking and provided focus as seen in this dialogue.

Teacher: How do you think, you have something started there, can you tell me what you have started there?

Laurel: Well, I just… The purple is four out of seven, the black is seven out of seven.

Teacher: Okay, so what information do you have here? So, again forget, forget that, that you were thinking for a minute, and let's just focus on the blocks. It says what? What is, what's the information you have?

Laurel: That this is two.

The student's description of the situation involved "out of" language that reflects a problematic image of fractions as one whole-number amount out of another whole-number set (Siebert & Gaskin, 2006). This sort of whole-number reasoning made it difficult for Laurel to compare and operate on fractional quantities. The teacher responded by focusing Laurel away from this type of unproductive reasoning and instead toward the manipulatives together with the problem's given information (that one purple block is the quantity two), a fact the student's reasoning completely neglected.

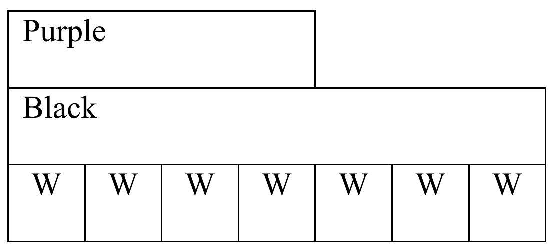

Figure 2. Laurel's arrangement of purple, black and white Cuisenaire rods.

Lead

The second purpose of guidance was to lead students into productive ways of thinking. When a problem or concept was too complex for the student to manage, the teacher would break down the problem into pieces or steps and lead the student to answer or consider one piece at a time. This served to organize the solution or explanation process. The term leading has developed negative connotations for some, especially when applied to situations like "funneling" where the teacher trivializes the task and undermines the student's problem solving process (Wood, 1998). The way the teacher in this study used guidance to lead students differed from those kinds of debilitating practices. The teacher did not trivialize the student's activity, but structured it by making one aspect salient and problematic and requiring the student to reason deeply about the meanings involved in order to resolve it. An example of leading will be given shortly in an excerpt in which leading as well as the third purpose of addressing reasoning errors occurred concurrently.

Address Reasoning Errors

The third purpose of guidance was to address errors in student reasoning. As students constructed solutions and justifications, the teacher identified contradictions, invalid assumptions, explanation deficiencies and invalid claims in their reasoning. She also gave counterarguments to flawed propositions, edited the wording of a justification, and pointed out opposing viewpoints among students to be debated and reconciled.

An example of the teacher using guidance to address reasoning errors (the third purpose of guidance) and then lead a student into productive thinking (the second purpose of guidance) is now provided. The excerpt is a continuation of the prior guidance example in which the student, Laurel, used Cuisenaire rods to construct fractional amounts from other amounts. Recall from Figure 2 the student had lined up individual white blocks along the purple and black blocks and was reasoning about the amounts inappropriately. The teacher had first tried focusing Laurel away from her method and toward the given information in the problem (that a purple block was 2). As the conversation progressed, Laurel continued to struggle and the teacher went beyond focusing to pointing out issues with Laurel's reasoning and leading her in more productive directions. Laurel's language repeatedly referred to the individual white blocks as ones. For example, when the teacher asked Laurel why she chose to line up white blocks, she responded, "Just because they were ones." This created tension between purple being given as two and seeing purple as composed of four ones. The teacher explicitly addressed this student's problem using guidance as seen in the following excerpt.

Teacher: So I'm going to tell you, you said two things that were contradictory to me. I want to see if you can reconcile it. We're given that purple is two—

Laurel: But I'm saying it's four.

Teacher: —but you're saying it's four.

Laurel: So it's four ones so four equals two. Four equals… [trails off]

The teacher clearly pointed out the contradiction in Laurel's argument. Even though the teacher had focused the student on the given information that purple is 2 at least three times by this point, the Laurel's image of fraction as a whole number of pieces out of a whole set was so strong that she was still unable to abandon her notion that purple was somehow 4. The above dialogue was immediately followed by the guidance presented below in which the teacher led the student with questions and steps to reason more appropriately.

Teacher: So if this [purple] is, if this is two, what's this? [picks up one white block]

Laurel: A half.

Teacher: Okay what, how did you get that? Why do you know it's a half? (This teacher turn was not coded as guidance)

Laurel: Because one half multiplied by four is two.

Teacher: Okay, so what would one be?

Laurel: One would be, [shows the amount of two whites with fingers]

Teacher: Yeah. Okay, so now use this to figure out what black is. So you know that these [whites] are not one.

Soon after this, Laurel correctly identified black as three and a half. The success of this final interchange can be attributed to the teacher's leading moves that organized and structured the student's solution process. The first leading question forcefully established the given assumption that purple is two as the premise from which to make a new deduction about the quantity that one white represents. This helped Laurel to stop thinking of purple as four and white as one, and was the pivotal turning point after which Laurel began to reason appropriately about the situation. After the teacher elicited Laurel's justification for deducing white as one half, the teacher led the student to next identify one whole and finally to quantify black. The excerpts given in this example demonstrate how the information and structure from various kinds of guidance can enable students to become successful on an inquiry-based task when deeply held misconceptions and problematic thought patterns are major impediments to appropriate engagement with the concepts.

Give Hints and Suggestions

The fourth and final purpose of guidance was to provide helpful hints and suggestions. This included suggestions for how to carry out a method, a helpful perspective or interpretation of a situation or context, bounds or constraints on the set of possible solutions, general principles about how to approach problems in mathematics, and criteria the teacher required for student solutions to be acceptable.

An example of guidance through hints and suggestions occurred on a task where students were given a context source for a hypothetical data set and were asked to identify it as categorical (qualitative) or quantitative and name types of graphs (bar chart, dot plot, histogram) that could be made with the data set. There were eight problems. One group became confused on the seventh problem and solicited the teacher's help. The problem read, "The number of grams of chocolate in the 12 best-selling candy bars in the United States." A student who I'll call Darla reported their answer that it was quantitative and that they wanted to use a bar chart. They probably asked for the teacher's help because they had learned that bar charts corresponded to qualitative data and not quantitative. A second student who I'll call Beth chimed in, "It's number of grams and that gives you a number," and they had associated the word "number" with quantitative data. The dialogue continued with two teacher hints in the form of general principles about mathematics:

Beth: But is it categorical, because it's, like, each candy bar has that?

Teacher: Uh-huh, so… Just because we say it's categorical doesn't mean there are no numbers involved. Um, 'cause we always have a count for each category, or we always have some value associated with each category. So, yeah.

Darla: It's just they switched the order on us, because before they would say the qualitative [part first]…

Teacher: Uh-huh, yeah. So, it, it actually has… it's like, you know, a lot of math problems, if you see numbers in a certain order, it doesn't mean you use those numbers in that order, right? And so, you have to think about the situation itself.

To answer the students' question, the teacher revealed hints in the form of helpful principles and guidelines. Because students often possess an oversimplified association between "qualitative" and "non-numerical," the teacher first clarified the principle that even when data is qualitative it involves either counts or values associated with the categories. Subsequently, Darla realized that the information in this problem was presented in a different order than the problems preceding it. Specifically, the previous problems' wordings were all similar to "the heights of 20 five-year-old children" or "the eye colors of students in your class" where the structure was [Independent variable used by students to classify problem] of [context item for a count or value]. The candy bar problem switched the order of this structure and confused the students. Because the students were attending to superficial aspects of the question, the teacher shared the helpful principle that solving mathematics problems requires investigating the mathematical meanings of the situation rather than picking out numbers and quantities based on their order in the question. This example corroborates the findings of others that even high quality mathematical tasks in written form can suffer from decline in worthwhile student mathematical activity during implementation, requiring interventions to realign student activity with practices that are valued (Henningsen & Stein, 1997).

Disclose

The sixth type of mathematical telling is to *disclose*. A disclosure is a communication of mathematical ideas in more complete and resolved form. Some examples of mathematical content in a disclosure are a solution component, an alternate solution, a justification, an explanation of an idea, a mathematical norm or convention, or a description of a concept. The teacher discloses this information for various reasons such as to help students construct solutions and justifications, answer student questions about the material, discuss complex concepts, and verify student answers to the task.

The choice of the word "disclosure" for this telling type is not meant to imply that only in this category was mathematical information disclosed. All of the telling types disclosed important mathematical information to students. Rather, the term is meant to emphasize the more resolved and finished form of the utterance, in contrast with telling types such as guidance in which a portion of the mathematics was purposefully unresolved to require student incorporation and resolution of the conveyed information. Because the disclosure telling type is the most complete and direct, it may appear the most similar to historical conceptions of telling as the transfer of facts and information. However, as was discussed in chapter two, this historical telling paradigm does not account for all of the rich ways that discourse can convey meaning and impose structure on student mathematical activity. In addition, the content and purposes of the disclosure telling type are drastically different from teacher discourse in traditional lecture and recitation teaching models.

The instances of disclosure in this study were sorted into four purposes: amplify student input, explain mathematical concepts, model appropriate reasoning, and provide mathematical norms and expectations. These four purposes are found in Figure 3 along with the specific ways the teacher accomplished these purposes. Each of these four purposes will be described at length below, followed by in depth examples illustrating their use in the data.

Disclose			
Amplify student input	Explain	Model reasoning	Norms & expectations
Elaborate on a student answer	Explain idea or concept	Justify a solution	Mathematical convention
Finish student thought	Distinguish concepts	Justify an algorithm	Notation
Rephrase and modify mathematical image	Clarify method	Correct flawed student work	Terminology
Validate a response and support/give reasons	Illustrate with an example	Provide solution component	Expectation for justifications
Synthesize input from multiple students	Contrast problem structures	Provide alternate solution	Form of the answer

Figure 3. Purposes of disclosure.

Amplify Student Input

The first purpose of disclosure was to amplify student input. After a student shared correct and appropriate reasoning, the teacher often added strength to the student's input by reiterating the student's ideas and using some of the student's language. These amplifications validated the student's input as mathematically correct as well as expanded or modified the mathematical ideas in significant ways. The teacher elaborated on the response or filled in additional details, reworded the student verbalization to be more complete or correct, finished the student's sentence with the anticipated mathematical thought, subtly modified the language to appeal to an alternative mathematical image, supported her validations by supplying a reason or underlying justification, or synthesized multiple contributions from a group or one student into a single correct and complete formulation.

The effect these amplifications had on student thinking is difficult to determine from verbalized discourse alone. The teacher amplified student input partly to establish more complete and mature ways of expressing the mathematical ideas, but also partly to acknowledge and accept the input before moving on to new questions or topics. Because the teacher's language borrowed from the student's language and echoed student ideas, a portion of the teacher's transformations were subtle and it was not always clear whether the student recognized or appreciated the new mathematical content contained in the teacher's utterance. However, other teacher amplifications contributed new mathematical ideas more plainly. This telling subcategory was similar to a construct known as revoicing (O'Conner & Michaels, 1993) in that the underlying positive validation and agreement accompanying the teacher's transformations allowed students to retain ownership of the teacher's amplifications of their ideas to a certain degree.

The following example shows the teacher amplifying a student idea to reinforce its validity and elaborate on its significance. The students were reflecting on their results of a probability task. They were first asked to predict the probability that a Hershey's Kiss (a small chocolate candy with a flat base and curved, cone-like shape) would land on its base with the point up when tossed. They then performed a 50-trial experiment. One group of students was surprised with their experimental result, 32 percent. The teacher entered a conversation with them. The first student, who I've called Laurel in earlier excerpts, summarized her conversation with another student, who I'll call Jasmine, when they were predicting values.

Laurel: We're thinking, 'cause the two things we talked about was first Jasmine was like it would be fifty-fifty 'cause there's two. Like one, there's two ways. But then I was kind of bratty and was like no, because—

Jasmine: Yeah you were.

Laurel: —no because there is like more than one way, like there's more than one way for it to fall on its side, but there's only one way for it to fall on its face. But I don't know if that's really right.

These two students were trying to deduce a theoretical probability using their perceptions of the situation. Jasmine exhibited a common misconception of counting possible outcomes without realizing they may not be equally likely. Laurel believed that there were more ways for the chocolate to fall on its side than its face, but was uncertain because the ways were not a discrete count like they would be for sides of a die, for example. The teacher responded that this thinking was interesting, and after a couple of other comments the dialogue closed with the teacher's amplification in this manner.

Laurel: Yeah, but I feel like there is more than one way it can fall.

Teacher: But, yeah, it's definitely more likely that it will fall this way 'cause there's, it's easier for it to fall that way than this way. Yeah that makes sense. That is

> actually really important, that like you can think there is two possibilities but it's still not one in two. Or one half probability.

The teacher strengthened the students' confidence in Laurel's reasoning that the chocolate was more likely to fall on its side than its base. She then stressed the importance of the students having gained this insight to overcome the common misconception that two distinct outcomes are always equally likely, one of the major learning objectives of this task. Even though Laurel had already voiced the basic mathematical ideas the teacher disclosed in this example, the teacher rephrased the ideas and validated them, making this contribution a teacher amplification of student input.

Explain

The second purpose of disclosure was to explain the mathematics. The teacher defined concepts, explained ideas, distinguished related concepts from one another, clarified the steps and processes involved in calculation and solution methods, illustrated concepts with typical or hypothetical examples, and contrasted the structure of related but different task problems. Many of these explanations were prompted by students asking the teacher to clarify an aspect after they had worked on the task for some time and identified an area of confusion. Other times the student felt insecure about an answer or conclusion and verified it with the teacher, who subsequently recognized and addressed a student misconception.

This next example shows the use of an explanation to encourage students to recognize and think about a key distinction between two statistical concepts. The task was meant to elicit prior student knowledge and understanding about summary statistics like mean, median and mode to describe data. There were four contexts provided for data sets (but no actual numbers for the data set) and students had to come up with a single numerical value that could describe and summarize such a data set. One group of students was discussing the context, “Shoe sizes in the women’s section of a shoe store.” They began to consider using range. They ran into issues about how exactly to describe the range. One student made up specifics for the shoe store scenario, starting from size 6 to size 12. She first calculated 12 minus 6 to be a range of 6, but then counted the number of sizes from 6 to 12 and got 7. She then asked the teacher if “range plus one” would work. The teacher threw it back to the group to decide whether 6 or 7 was best. Another student affirmed that the range is 7 since there are that many sizes. Perhaps to help students notice the difference between “number of sizes” and “range of sizes,” the teacher reminded them that there could also be half sizes of shoes. A student replied, “No. Not at my shoe store!” It is likely this student was avoiding an additional conceptual difficulty rather than voicing an actual preference for shoe store practices. The teacher did not want to allow the students to avoid thinking about this important aspect of range. She explained range explicitly in the following disclosures.

Teacher: You don’t offer half sizes!? So, okay, so it’s like... So, it sounds like, okay, it sounds like these are two different numbers. When you’re talking about the range, um, that’s saying I go… I, like, I encompass this amount. Whereas, if I say I offer fourteen sizes, that’s a count of the number of sizes you offer. And I think those are two different numbers.

Student 1: You’d have one number… [mostly inaudible]

Teacher: Well, a count is, a count is a, um, is a single number. Yeah, that’s a single number. That’s a single number. A range, like, my sizes range, between… um… the range of my sizes is six.

Student 2: Is the range the one number that's how many numbers are in between the smallest and the greatest?

Teacher: Yeah, it's, it's the distance between the smallest and the greatest. So, that's a six.

The teacher's first move in this excerpt was to explain the difference between the range of a data set and the number of unique data values, a nonstandard statistic that only makes sense with discrete data. The teacher's second turn appears to have sanctioned the use of this latter summary statistic for this shoe store problem. She returned to the shoe store scenario to explain how range is conventionally calculated, affirming it would be 6 rather than 7 for shoe sizes from 6 to 12, and that it is expressed as one number rather than as an interval of extreme values.

In response to these explanations and examples, Student 2 in the excerpt asked an interesting clarifying question about what range means conceptually. She used the language "how many numbers are in between the smallest and the greatest." It is not clear what the student intended by this. From one perspective, the question is nonsensical since there are always infinitely many numbers between any two numbers on a number line. It is possible that the misconception between range and count of unique data values lingered and caused the student to use this language. A second possibility is that the student was thinking of range as the difference between the largest and smallest values and used the words "how many numbers" to mean how many unit increments. The student may have also been thinking in a different way entirely. Regardless of the student's mental meanings and imagery at that moment, the teacher's final statement is an example of a teacher amplification of a student idea that involved a subtle but crucial modification of the mathematical image. The teacher responded with a positive, affirming "yeah" but then worded the definition of range differently. She replaced the language of "how many numbers" with the notion of the "distance" between the largest and smallest values, a more mathematically correct and universal image of range that works consistently across both discrete and continuous scenarios.

The teacher's disclosures above were consistent with the goals of inquiry even while revealing a significant amount of mathematics to the students. When students were reasoning idiosyncratically about range and other measurements of variation (and even conflating multiple measures), they were not going to somehow arrive at the conventional use of range on their own. The teacher explained what range is and contrasted it with the students' invented statistic of shoe-size counts. As a result of the teacher's disclosures, the students' inquiry into these concepts was pushed further than what would have likely occurred had the students answered this task question by simplifying their hypothetical context and avoiding emerging complications and difficulties.

Model Reasoning

The third purpose of disclosure was to model appropriate reasoning. As students constructed solutions, verified answers, and asked clarifying questions about the problems, the teacher sometimes disclosed all or part of a solution or justification as a model for the student to make sense of and emulate. The unabashed provision of solution components could at first seem in conflict with inquiry, in which students are meant to retain primary responsibility for problem solving and justification. For this reason, I carefully investigated the contexts of these disclosures. Among the four subtypes of disclosure, modeled reasoning was not uncommon, suggesting that mathematical disclosure via modeled reasoning was not an aberrance to this teacher's practice. The following examples demonstrate how modeled reasoning was implemented by this teacher.

One purpose of several examples of modeled reasoning was to curtail time spent by students in an activity that was unlikely to afford a proportionally worthwhile learning benefit. This was especially true when the teacher provided a solution component or corrected student errors. In one lesson on using a one hundred square grid to convert fractions into decimal form, the teacher circulated and monitored student progress in the groups. Occasionally as students shared their solutions she would recognize a calculation flaw, especially on complex problems like converting 1/7 into a decimal. One pedagogical choice might have been to hint that an error existed and perhaps which step it was on. However, in this case the teacher chose to simply disclose the flawed step and incorrect calculation for the students. Clearly it was important for students to correct the errors, since one group mistakenly came to the conclusion that the decimal repeated far sooner than it really did, for example. The teacher chose to disclose the correct numerical calculations to productively manage the students' activity, enabling them to avoid having to hunt for a calculation error and instead focus on justifying the general process of dividing the whole into fractional amounts by using successive partitions of powers of ten. This choice was a contextual one based on her judgment of potential learning opportunities of the situation.

Another example is repeated from earlier where it was included in the qualify telling type. The students were using partitioning and iterating images to name and justify fractional quantities. The question was, "There are 20 students on the bus, but there are still 28 empty seats. What fraction of the seats are filled?" The students attempted a partitioning justification for their answer of 20/48 by dividing the seats up into 48 equal pieces. In the prior analysis I illustrated how the teacher attended to the alignment of the justifications with everyday reasoning about the context of buses. In the present analysis, I focus on how the teacher amplified student input (the first purpose of disclosure discussed earlier) and modeled justifications for the students.

Student 2: Oh! If you divide all… Okay. So, if you divide all of the seats into forty-eight equal pieces.

Teacher: Okay. So, if you take the bus and divide it into, like forty-eight equal spots. That's a little awkward isn't it?

Student 2: Yeah.

Teacher: Um, for buses. I think—

Student 2: Or, if you take the one, and multiply it by forty-eight—

Teacher: Then you have the number of seats.

Student 2: Yes.

Teacher: Yeah. I, I almost think iterating is less awkward in terms of buses, to think about. Because partitioning, it's like, we don't really partition the bus into forty-eight pieces. I can think, I, one, one seat is one-forty-eighth of the bus because it takes forty-eight seats to make up the whole bus. Right? And so, I think iterating flows more naturally there.

There are a number of interesting disclosures in this excerpt. After Student 2 first formulated a partitioning justification, the teacher amplified it while rephrasing a couple of key parts. Where the student referred to the whole as "all the seats," the teacher used the word "bus" for the whole to enable her to envision it prior to being partitioned. She made equivalent modifications on other problem contexts, such as when a student talked about dividing 36 inches into 36 pieces and the teacher substituted "a yard" for 36 inches. Another image modification was to change "pieces" to "spots" to align the partitioning action with the context. Even with this improved alignment, the teacher conceded the justification was awkward. Student 2 then ventured a new justification appealing to an iterating image when she said "take the one, and multiply it by forty-eight." The student may or may not have finished her own sentence in the same way as the teacher's addition, "then you have the number of seats," but the student consented. The teacher's final move was to model the reformulated iterating justification for the quantity 1/48. One function of the teacher's telling in this excerpt seems to have been to enact the role of a co-participant (see Rittenhouse, 1998) in the activity, potentially modeling the kinds of brainstorming, evaluation and revision that occur with mathematical composition. Many of the tasks in this class required students to not only solve problems but additionally to organize a mathematical justification into clear and compelling prose, a high-level skill requiring both mathematical and linguistic finesse. Because the teacher chose to model mathematical reasoning during high-level tasks such as this one, it is worth investigating whether tasks cultivating more ambitious mathematical practices and learning objectives are frequently accompanied by an increased need for modeling by an expert participant like the teacher. It was evident in this class that the teacher did not model justifications for the students to copy down and memorize word for word but instead helped them make sense of and reconstruct the justifications using their own understanding.

Provide Norms and Expectations

The fourth and final purpose of disclosure was to provide mathematical norms and local teacher expectations. The teacher articulated common mathematical conventions and practices, notation choices and meanings, terminology for concepts, justification expectations specific to this class, and appropriate forms of the answer (such as whether a fraction needed to be simplified or not). Mathematical content of this sort has been endorsed as appropriate to tell students when they would be unable to discover this type of knowledge by reasoning idiosyncratically (Philipp, 1995; Smith, 1996). Even so, the teacher's disclosure of this sort of information influenced students in ways not unlike the other telling types in that the information privileged certain ways of thinking shared by the mathematical discipline and structured the students' mathematical activity.

In the following example, a student brought up a general question she had about the appropriate way to express an answer. The task she referred to involved calculating the theoretical probability of drawing marbles from a bag with certain outcomes. For one of the questions, there were 20 possible outcomes, 12 of which satisfied the desired conditions, so the student had a probability of 12/20. The student's conversation with the teacher went as follows.

Student: Okay, and I have a question about yesterday, because I wasn't here. When you do things like twelve over twenty, can you simplify that to three over five, or would you rather it be twelve over twenty?

Teacher: Oh, I'm okay with either.

Student: But, is there a correct way to do it?

Teacher: Is there a correct way to do it?

Student: Do you know what I'm saying? Like with statistics?

Teacher: Well, they're both the same fraction.

Student: I know. But with statistics, is it, like, better to leave it, like, obviously four out of five sounds better than eight out of ten, and…

Teacher: Yeah, I mean, you would probably simp… um, usually, um, a lot of times we report them in terms of percentages, or, um, or decimals, 'cause that way you can make them always consistent with any other that you find.

The significant aspect of this conversation is partly the mathematical content told (disciplinary norms and practices) but more so the way the participants reasoned and made sense of the situation. The student's initial question about how the teacher expected the answers to be presented initially seemed to be a simple and trivial question for the teacher to address. The teacher, consistent with the mathematical community in general, viewed the responses as mathematically equivalent and expressed no preference. It is very surprising that the student, knowing that either response was acceptable in terms of satisfying the teacher, persisted on whether there was a "correct" way to do it, appealing specifically to one particular branch of mathematics—statistics. This is a legitimate question since there are sometimes contextual or disciplinary preferences for expressing numbers in certain ways despite being mathematically equivalent. For example, perhaps a reason for not simplifying the fraction is to conserve the meanings of the numerator and denominator in reference to the context's discrete outcomes and possibilities. When the teacher recognized the student's concern, she brought up the possible disciplinary preferences of using percentages or decimals to maintain comparability. This example illustrates that the teacher needs to be sensitive to student thinking regarding mathematical norms and conventions just as a teacher is sensitive to how students engage with and make sense of mathematical concepts and solution methods.

Validate

The seventh and final type of mathematical telling is to *validate*. The teacher either validates or invalidates a student expression of mathematics. This telling type is unique in that it does not convey any new mathematical ideas or structures other than whether a particular student utterance is correct or incorrect. By defining validation this way, it became necessary to treat this code differently than the others. For the first six telling types, an instance of mathematical telling was categorized as only one of the six. In contrast, validation could occur as the only function of a telling utterance or as an additional consequence of one of the other six telling functions. The term validate was chosen to convey a meaning more specific than the general idea of "evaluation," which determines worth based on many standards beyond just mathematical correctness. For example, when the teacher tells a student to "forget what you were thinking for a minute," the teacher has evaluated the thinking as being presently unhelpful but has not conveyed a sense of right or wrong. Such statements were not considered validations.

In practice it was difficult to definitively identify all instances of validation due to its complexity and subtlety in language. Explicit words such as "right" or "wrong" were clear indicators. However, some words functioned as either a mathematical validation or a conversational acknowledgment of hearing and comprehending student speech or even marking a turn transition from student to teacher, depending on the context. Such words included "yeah, okay, uh-huh" and "alright," among others. Other teacher language had no such words at all but appeared to function as a validation in the discourse simply by repeating student language with positive emphasis or negative skepticism. Beyond these difficulties in interpreting spoken discourse, there are subtle conversational structures and norms that likely function to validate or invalidate student work, sometimes unintentionally. For example, a teacher may pause after hearing a student remark, leading the student to believe something is wrong.

Because of these complexities surrounding validation, a strategy was devised so as to be able to contribute a meaningful analysis of validation using the data sources and methodology of this study. The field of mathematics education has emphasized students taking authority for justification and evaluation of mathematics instead of external sources like textbooks, answer keys and teachers, and so it seemed reasonable to code only for the most clear and obvious cases of validation to determine whether such telling occurred at all in this classroom. The frequency of validation was therefore at least as much as what is reported here, with additional subtle instances being likely. The criteria used to identify clear, defensible instances of validation required looking at the content of the teacher's discourse itself as well as the surrounding student discourse. In the teacher's discourse, I looked for markers of validation like using explicit (dis)agreement words, repeating student utterances with positive emphasis or negative skepticism, or correcting a student response by providing the appropriate answer. I then looked at the surrounding discourse for confirming or disconfirming evidence of a validation. I first ensured that the supposed validation aligned with the validity of the student's reasoning. For example, if the teacher used ambiguous agreement words such as "uh-huh" and the student's response was incomplete or incorrect, I erred on the side of caution and interpreted these as conversational acknowledgments rather than clear cases of validation. The student's reaction to the validation was also considered. Confirming evidence included when a student acknowledged the validation, concluded the conversation (being satisfied with the teacher's response to his or her answer), moved on to the next part, or adjusted his or her reasoning (after an invalidation). If a student response was inconsistent with a teacher validation having occurred, then the teacher utterance was not coded as a clear instance of validation.

The teacher in this study validated students' mathematics in several key forms. The most common form was an expression of agreement. Beyond using basic agreement words, the teacher also sometimes restated, rephrased, or completed the student's mathematical verbalization. At times she provided an explanation or justification of the student's idea she was validating. She occasionally praised the student's thinking or gave encouragement. The teacher also invalidated students' mathematics. She did this by negating their response, providing a counterargument, correcting an error by providing the intended answer, or challenging a response by repeating it with a questioning tone. The teacher also critiqued student ideas by identifying what was either worthwhile or deficient in the student's thinking. The nature of student thinking preceding teacher validations varied throughout the data. Students shared brief answers, approximations, solutions, methods, explanations, concerns, questions, clarifications, uses of notation and representations, and examples. Most student expressions were verbal but a few were written (such as pictures or explanations on their task sheets).

The following example of validation is typical of the teacher's practice and illustrates the use of an invalidation through a questioning tone, and then a validation with a simple agreement. This example occurred at the conclusion of a conversation that has been partially reported and analyzed earlier in the guidance section. The task was from the first day of class where the students used Cuisenaire rods to begin with one fractional quantity and deduce another. Recall that a student named Laurel struggled deeply on the problem, "If purple is two, what is the value of black?" The teacher used various types of guidance to move Laurel away from unproductive thinking and toward appropriate reasoning on the problem. As reported earlier, Laurel finally managed to identify a white block as the quantity one-half rather than one. It remained to appeal to her block arrangement (see Figure 2) to deduce the quantity of black. The conversation finished as follows:

Teacher: Okay, so now use this to figure out what black is. So you know that these [whites] are not one.

Laurel: So it's one and one half.

Teacher: So the black is one and a half?

Laurel: Oh, no. So the black would be, like, three and a half.

Teacher: Yeah, does that make sense?

Laurel: Yes. It does make sense.

Laurel first answered that black was one and one half, probably because the black is one and one half *more* than purple. The teacher restated Laurel's own conclusion, but in the form of a question. Laurel's immediate response, "Oh, no," indicates that the teacher's utterance was an invalidation of her answer. Laurel corrected her answer to account for the full length of black, and the teacher validated it this time as correct. The teacher's "yeah" was not ambiguous in this context since she asked if the response made sense, a question she usually asked after the student gave a correct response.

Validation frequently occurred as a consequence of the other six telling types as well as on its own. Validation was common with disclosure and guidance, was somewhat infrequent with clarify task, and was very rare with the qualify, interpret, and assess telling types. Within the disclosure telling type, many of the validations accompanied an amplification of student input. Within guidance, many validations were used to address reasoning errors. Validations also occurred with the other purposes of disclosure and guidance, however. Within the clarify task telling type, the teacher tended to validate or invalidate the students' interpretations of the task instructions. The concurrence of validation with the remaining telling types was too infrequent to report any patterns.

The following is an example of validation occurring concurrently with guidance (addressing reasoning errors). One group was trying to write a word problem whose solution is three divided by four from a measurement perspective. After some discussion one student ventured an idea.

Student: Well I mean like, so like, if you had like you said, there are… There's like 48 donuts and 36 of those donuts are all, or like, not 36. Twelve of those donuts are all eaten, how many are left?

Teacher: Uh, huh but then that sounds like, that sounds like 48 minus 36 to me, is what that sounds like.

The teacher invalidated the student's response by critiquing the inappropriate structure of the word problem. The critique communicated mathematical information beyond the fact that the student's word problem was incorrect, causing it to merit the guidance code in addition to an invalidation. The validation and guidance qualities are embedded in the same string of teacher discourse, illustrating the unique way validation is capable of concurring with other telling types. Parenthetically, observe how the affirmative words "uh, huh" functioned as an acknowledgment of the student's response but clearly did not validate the student's mathematics in this example.

Frequency of the Telling Types

The purpose of this section is to discuss the pervasiveness of the mathematical telling types reported in this chapter. It is well to recall that this study examined only teacher interventions with groups and individuals as they worked on collaborative tasks and excluded all examination of whole-class discussion. This context comprised about one third of the total instructional time. Thus, this framework for mathematical telling was only one component of the teacher's broader role to enact inquiry-based instruction. The frequency of the types of mathematical telling will be calculated only in relation to the setting in which they occurred (teacher help during explorations of the task) rather than the larger classroom experience as a whole.

To reliably quantify the occurrence of the mathematical telling, I used the partitions of data referred to in chapter three as conversations rather than speaker turns. This approach yielded two advantages. The first was that because many of the teacher's turns were interrelated and ongoing attempts to achieve a particular purpose or goal, discrete counts of telling codes would potentially overestimate the presence of some telling types and underestimate others. For example, an assessment was usually accomplished in a single turn, while instances of guidance or disclosure regularly extended multiple turns with interruptions and negotiations of meaning before accomplishing their purpose. I felt that using simple code counts might inappropriately exaggerate the telling acts that extended multiple turns but served the same purpose. Conversations provided a useful alternative. Each conversation began and evolved according to the perceived needs of one student or group. In a given conversation, the teacher used whichever telling types were deemed appropriate to address these needs. Each telling type that occurred at least once in a given conversation was counted as contributing to the resolution of those student needs. This provided a reliable way to calculate the presence of each of the telling types without being undermined by the complexities that occur as speakers alternate turns. The second advantage to using conversations was that they provided some sense of the regularity of the telling types across multiple conversations. In order to be considered frequent, a telling type would need to occur persistently throughout multiple conversations involving a variety of students and situations rather than occur multiple times in a few long but perhaps anomalous conversations.

The calculated results for the frequency of each telling type are in Figure 4. The first code "M" is shown on the graph as 100 percent to indicate that this was the whole from which all of the percentages were calculated. It does not mean that all of the conversations between the teacher and small groups or individuals referred to mathematics. The percentages of the individual telling types have a sum larger than 100 percent because most conversations contained multiple telling types.

There are at least two interesting features of the graph in Figure 4. The first is the remarkably high percentage for "MT," 96 percent. This suggests that when the teacher referred to mathematics, she nearly always used mathematical telling sometime during the conversation as opposed to merely facilitating collaborative norms or innocently eliciting their thinking. The second interesting feature is the unforeseen trend that telling types with more mathematical information and constraints occurred more frequently than those with less. This is especially remarkable given that the literature in mathematics education has downplayed the role of highly-constrained, explicit communicative acts.

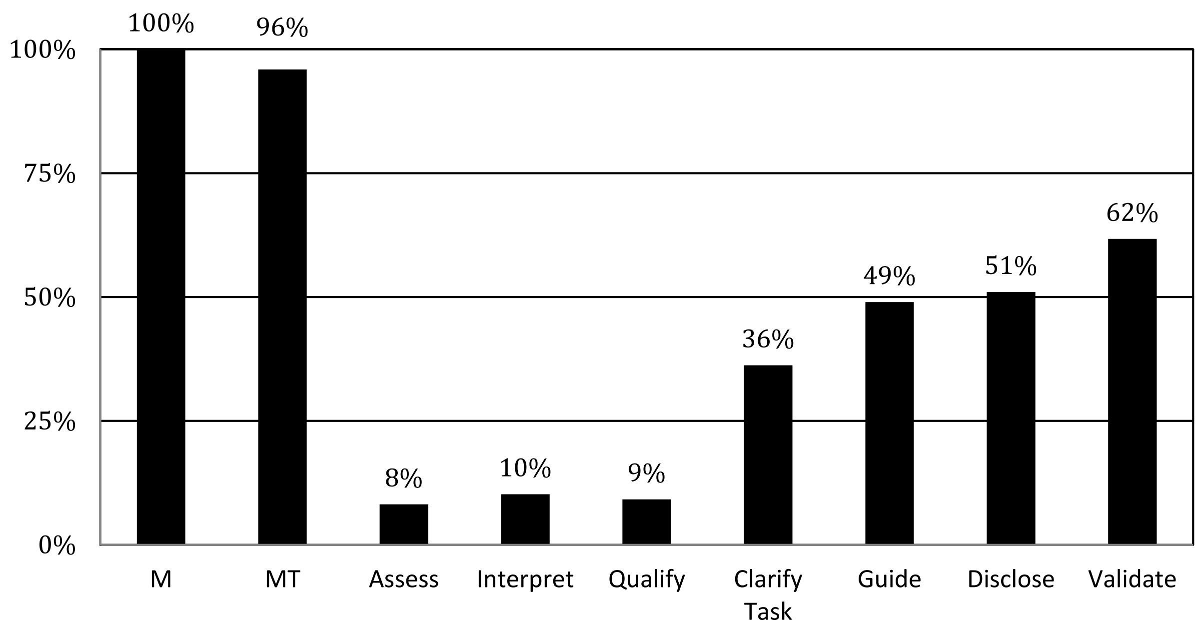

Figure 4. Telling type frequency by the percentage of conversations in which each type occurred at least once. M refers to the 196 conversations containing at least one reference to mathematics and constitutes the whole from which all other percentages were calculated. MT refers to conversations that included any instance of one of the seven types of mathematical telling.

Discussion

There are several findings that follow from the analyses in this chapter. First, the frequency of mathematical telling in this study contrasts sharply with depictions of inquiry in the literature. This teacher did not limit herself to regulating student collaborative processes only as some have recommended (Cohen, 1994; Dekker & Elshout-Mohr, 2004). Most of her conversations with students involved mathematics, and Figure 4 showed that 96 percent of these mathematical conversations involved at least one type of mathematical telling. The frequency of this telling is unique from other studies where the teacher's role was portrayed almost exclusively as eliciting and orchestrating student contributions (Ball, 1993; Goos, 2004; Lampert, 1990; Sherin, 2002; Simon, 1995; Staples, 2007; Stein et al., 2008; Wood, 1998). Although this study did not show whether the rate of telling is comparable for other inquiry-based teachers in other classrooms, the pervasiveness of the mathematical telling presented in this chapter leads one to question whether the presence of telling generally in inquiry-based instruction has been severely underestimated.

Three other findings have to do with broad characteristics of telling spanning multiple telling types from the mathematical telling framework. The first of these three is that telling occurred in subtle ways. By scrutinizing each teacher speech act for visible contributions of mathematical information, the study revealed subtle ways the teacher told. The instances of telling that tended to be more subtle were when the teacher assessed, interpreted student input, qualified the mathematics, guided students by focusing or leading with questions that withheld key information, and amplified student input by rephrasing a student conclusion. The glaring omission of certain information in such telling practices can eclipse their subtle contributions and leave the impression that no telling has occurred. The excerpts presented in this chapter demonstrate that these mathematical contributions generally are not trivial and have important potential to influence student thought, whether or not this potential is realized by the student.

The second finding about telling is that the teacher used implicit forms of communicating information to productively influence students in ways that have not been clearly articulated in prior literature. The telling types that I am referring to are guidance and, to a lesser extent, clarifications of the task. These telling acts might be misconstrued as innocent or harmless either because they refer only to the mathematics of the task and not the solution or because they convey ideas implicitly and require the student to infer the intended mathematical message. Some examples of this implicit guidance include when the teacher focused student attention without explicating what she intended for them to see, or when she led a student to arrive at a conclusion through a question rather than explicit directions. Several of the excerpts from earlier in this chapter illustrated that when the students were confused about the task's setup or solution process, the teacher's task clarifications or guidance played an obvious role in moving those students forward. The changes in student progress and thinking in response to these telling acts preclude one from dismissing them as innocent and harmless.

These clear articulations of indirect telling and its influence have been remarkably absent from prior literature because such telling was unrecognized or reported in ambiguous terms. For example, Chazan and Ball (1999) curiously avoided discussing novel teacher contributions of mathematics, even in indirect ways, and instead emphasized the incorporation and management of ideas originating from students as an expanded form of telling. Staples (2007) examined many facets of the teacher's role and claimed that at times the teacher guided students with "food for thought" that "generally did not reshape the task in a significant way" (p. 188). Staples observed that the teacher helped students think about the problem rather than treating obstacles as a sign that they needed hints, next steps, or an explanation. Such ambiguous language failed to articulate what this "food for thought" consisted of and how it helped students think more productively about the problem. The claim that the help did not reshape the task in a significant way diminished the appreciable influence of the teacher's help, reducing its explanatory power and therefore its identifiable worth in the classroom. Other studies have used terms that potentially masked the telling that was going on, like "press" (Webb et al., 2009) and "scaffolding" (Anghileri, 2006; Henningsen & Stein, 1997). Even though Webb et al. (2009) attended to the influence that high press had on students' explanations, the term "press" connotes that the information was somehow extracted from students without designating any influence as telling. The scaffolding metaphor has sometimes been applied to mathematically assertive teacher help that is considered effective without accounting for its telling function; when used this way, scaffolding is essentially a euphemism for telling. These examples show how accounts of implicit telling in the literature differ significantly from the carefully articulated accounts of mathematical telling in the present study.

The third finding regarding telling is that explicit telling was a tool utilized by this teacher within inquiry. Explicit telling includes the telling types that are dense with mathematical information such as validation, disclosure, and some instances of guidance (mainly addressing reasoning errors or providing hints and suggestions). The literature has been very reticent to point out and describe instances of this sort of telling. In contrast, the teacher in this case study consistently explicated students' reasoning errors, gave hints or suggestions, explained concepts, modeled solutions, and validated or invalidated students' mathematics. Both disclosures and validations occurred in over half of the teacher's mathematical conversations with students. Although the effects of these telling acts were not measured, the following hypotheses may partially explain their purposes. The teacher's disclosures did not seem to prematurely end students' problem solving activity by giving away answers. Instead, they seemed to be intended as models for students to reason deeply about and make sense of. The validation may have provided students reassurance to encourage risk-taking and exploratory thinking. Validation may have helped students calibrate their own evaluations of mathematics with the criteria established by the mathematical discipline. It may have also set clear expectations for how student work would be graded on assignments and tests to establish trust and fairness.

The fact that mathematical telling occurred with high frequency in subtle, implicit, and explicit ways in inquiry were unique findings from this study. Conclusions and implications around these findings are discussed in chapter five.

Chapter 5: Conclusion

The restructuring of mathematics pedagogy from traditional lecture toward student-centered inquiry has prompted investigations into the teacher's role. A sensitive component of this role is how teachers "tell" or convey important mathematical information to their students at appropriate times and in appropriate ways. Although the literature has sanctioned certain forms of telling, it is missing empirical evidence illustrating variations and key characteristics of telling. This qualitative study examined one case of high-quality task-based instruction to generate descriptive categories of the teacher's mathematical telling practices. Mathematical telling was defined as teacher discourse that both referred to mathematics and introduced new mathematical structure or constraint to the student. The analyses revealed seven types of mathematical telling. The teacher utilized these telling types skillfully while supporting students working on tasks in collaborative groups.

Contributions

The results of this study provide empirical evidence that mathematical telling is compatible with inquiry-based instruction. The telling of mathematical information was an integrated part of this teacher's overall instructional approach. Descriptions and analyses of typical excerpts from the classroom support the conclusion that mathematical telling is compatible with inquiry and is likely an integral part of student-centered instruction. This validates earlier hypotheses that certain ways of telling are a part of the teacher's role (Baxter & Williams, 2010; Chazan & Ball, 1999; Lobato et al., 2005; Smith, 1996) but also greatly broadens the range of telling practices viewed as productive or important. This is especially true of the more direct and explicit telling types.

The seven types of mathematical telling, along with their subcategories, constitute a framework for understanding more deeply how telling can be recognized in discourse and what that telling looks like. This framework answered calls for subtler distinctions between the kinds of telling teachers use (Chazan & Ball, 1999) and the kinds of interventions used to support collaborative groupwork (Webb, 2009). The prior telling-related frameworks of analytic and social scaffolding (Baxter & Williams, 2010) and initiating and eliciting (Lobato et al., 2005) established a theoretical basis for productive telling in non-traditional settings. The present study extended these frameworks by using empirical data to not just identify but also classify discursive telling with unprecedented distinction and precision. The new framework targeted the context of teacher interventions with collaborative groups and characterized these interventions more carefully than prior constructs. Specifically, the framework classified variations in teacher help far beyond the umbrella term of product help (Dekker & Elshout-Mohr, 2004) or Chiu's (2004) numerical scale rating help content from low to high.

The methodology of the study introduced a useful approach to study the construct of telling. Much of the literature that discussed telling used the term in a loose, intuitive sort of way (Baxter & Williams, 2010; Chazan & Ball, 1999). In contrast, Lobato et al. (2005) reformulated the theoretical roots of telling, and their work heavily influenced the present study. Their reformulation as "initiating" rejected telling as a simple binary construct (did tell or did not) based on grammatical form and treated it instead as a spectrum of teacher acts with potential to initiate novel ideas. This initiating potential is derived in part from the discourse's mathematical content and structure. The main point of departure between Lobato et al.'s approach and the approach used in the present study was the consideration of function. Lobato et al. defined telling as a function of discourse based on the perceived role by participants. It would therefore be necessary to examine the intentions of the teacher and interpretations of the student and reconcile these when they do not align. Because it is complex in practice to investigate teacher intentions and student interpretations for every speech act, the present study attended only to the visible content of the discourse, informed by its surrounding context, to create an explanatory account of the events. This proved a reliable and practical alternative for identifying discourse that exhibited characteristics of telling.

Implications

Theoretical characterizations of inquiry-based instruction must account for the telling acts witnessed in this study. Inquiry is not unguided discovery learning with a hands-off teacher; instead, inquiry makes use of scaffolding and structure to immerse students in ambitious learning activity aligned to particular learning objectives (Hmelo-Silver, Duncan, & Chinn, 2007). Mathematical telling is an important part of this structure. The high incidence of telling in this study suggests that researchers and teachers should not overlook or ignore the teacher's mathematical telling practices with students. Failure to account for these practices creates unrealistic portraits of inquiry and could mislead those who are trying to implement it. Acknowledging mathematical telling as a component of inquiry will help teachers be informed as they take advantage of discursive patterns that strongly support students' mathematical thinking.

The seven specific types of mathematical telling are useful for researchers and practitioners. Researchers can use the framework in future studies to identify and describe teacher telling practices with improved precision. Researchers should no longer overlook subtle contributions of information, downplay the presence or influence of implicit mathematical help, and ignore explicit telling acts that occur. The field can use the framework to be more descriptive and transparent about teacher telling practices, improving subsequent efforts to characterize the teacher's role and correlate specific telling practices with student learning. Practicing teachers can use the framework to help them recognize their own use of telling practices. Teachers with reservations about interacting mathematically with students can use the telling framework as a potential model for incorporating supportive mathematical interventions. Teachers often need to intervene when student activity declines (Henningsen & Stein, 1997), and teachers need not confine themselves to an extreme, overly conservative approach for fear of undermining inquiry.

A final implication is that telling practices should be viewed as one of many available instructional tools for creating structure and managing student activity. Because inquiry-based instruction makes use of structure to align student activity to worthwhile mathematical learning objectives, telling is a natural component of inquiry alongside the launching of tasks. From this perspective, the mathematical constraints embedded either within the task or within the teacher's telling practices become somewhat comparable, if not interchangeable. Regardless of whether the teacher imposes structure beforehand in the written form of the task or afterward in the verbal form of telling, the constraints all work toward the primary objective of managing productive student activity. This implication is analogous to an earlier observation made by others; just as questions can tell and statements can induce further questions (Lobato et al., 2005; Wood, 1998), mathematical tasks can tell and their solutions can induce further tasks or lines of inquiry. This perspective mitigates misconceptions about the need to avoid certain forms of discourse just because they refer to mathematical ideas from either the task or its solution. The perspective defines a learning space in which the mathematical task and its solution are relative to each other as they are negotiated between teacher and student.

Limitations and Directions for Future Research

The first limitation of this study is the small sample size typical of a case study. The case examined one teacher from one classroom during part of one semester. The small sample size was necessary to avoid data overload and conduct in-depth qualitative analyses. Although it is likely that the resulting forms of mathematical telling are shared by other inquiry-based teachers, this study does not show how widespread or exhaustive they are. Additional research on mathematical telling practices is needed to corroborate, improve, or challenge the results of this study based on results in other classrooms.

A second limitation is that the study examined only the telling *potential* of discourse by limiting analyses to the expressed content of the discourse itself. No methods were employed to ascertain how the participants experienced these telling acts beyond what was visible in the discourse. A benefit to this approach is that the resulting telling framework was not undermined by the complexities of the participants' highly subjective interpretations of their experiences. However, the drawback is that this study cannot link the observed telling practices with the teacher's instructional goals or the students' learning outcomes. Future research would need to investigate teachers' awareness of telling in their practice, their purposes for and evaluations of that telling, students' awareness of and reactions to teacher telling, and causal relationships between teacher telling and student learning and engagement.

A third limitation is that the study examined mathematical telling only in the context of teacher interventions with collaborative groups. The study's objective was to generate a detailed and descriptive qualitative framework for telling, and the context of collaborative groups appeared most fruitful for this purpose based on literature reviews. Future research can investigate mathematical telling practices within whole-class settings and compare and contrast them with the practices in collaborative groups. Research can also investigate how mathematical telling practices and other teaching practices interact as integrated parts of the whole teaching environment, instead of studying telling practices exclusively.

The final limitation is that the results from this study must not be misinterpreted to justify the indiscriminate use of telling. Not all telling acts are effective or desirable, and telling may be appropriate in some cases but not others. The results from this study explain what did happen in one case of high-quality instruction, and not necessarily what should happen. These results were likely influenced by the demographics and maturity of the participants, the established collaborative norms of this particular classroom, and the mathematical content of the class. Care should be exercised in using the results to prescribe practices for teachers in differing contexts. For example, it is not necessarily true that the relative frequencies of a teacher's telling practices should resemble those observed in Figure 4.

Summary

Descriptions of the teacher's role in meeting the new demands of inquiry-based instruction have insufficiently attended to the ways teachers tell mathematical information to their students. This study systematically investigated telling in discourse to increase comprehension of mathematical telling practices used by a skilled inquiry-based teacher. The detailed framework of seven telling types and their subcategories increased understanding of telling and expanded the repertoire of appropriate moves a teacher might make while intervening with collaborative groups. The findings included the recognition and characterization of subtle telling, implicit telling, and explicit telling. The data excerpts illustrated the compatibility of mathematical telling with the goals of inquiry. These results suggest future characterizations of inquiry should be more transparent and precise when discussing teacher telling practices.

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