The Influence of Superhydrophobicity on Laminar Jet Impingement and Turbulent Flow in a Channel with Walls Exhibiting Riblets

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The Influence of Superhydrophobicity on Laminar Jet Impingement and Turbulent Flow in a Channel with Walls Exhibiting Riblets

Joseph F. Prince

A dissertation submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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ABSTRACT

The Influence of Superhydrophobicity on Laminar Jet Impingement and Turbulent Flow in a Channel with Walls Exhibiting Riblets

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Doctor of Philosophy

The object of this work is to explore the influence superhydrophobic (SH) surfaces exert in laminar jet impingement and when they are combined with riblets in turbulent channel flow. A SH surface generates an apparent slip due to the combination of micropatterning and chemical hydrophobicity. Because of surface tension, water does not enter the cavities between the features, increasing the contact angle of a water droplet on the surface and reducing the liquid-solid contact area.

An analysis based on the integral momentum approach of Karman and Pohlhausen is presented that predicts jet impingement behavior on SH surfaces. The model is first applied to the scenario where the slip at the surface is isotropic and a downstream depth is imposed such that a circular hydraulic jump occurs. The model predicts the thin film parameters downstream of the jet and the radial location of the hydraulic jump. An increase in the hydraulic jump radius occurs as slip increases, momentum of the jet increases, or the downstream depth decreases. Modifications to the model are made for the scenario where the slip at the surface varies azimuthally, as would be the case for a surface patterned with microribs. The average behavior is similar, although now an elliptically shaped jump forms with the major axis aligned parallel to the rib/cavity structures. The ellipse eccentricity increases as the slip increases, the jet momentum increases, or the downstream depth decreases. Where there is no downstream depth imposed on SH surfaces, the thin film breaks up into droplets instead of forming a hydraulic jump. Further changes are made to the model to incorporate this behavior for isotropic and anisotropic surfaces resulting in circular and elliptically shaped breakups respectively.

This work also explores SH surfaces with riblets in turbulent channel flow. Pressure drop measurements across surfaces exhibiting superhydrophobicity, riblets, and surfaces with both drag reducing mechanisms are presented. The SH surface reduces drag because the effective surface area is reduced and riblets are able to reduce drag by dampening the spanwise turbulence. Photolithography was used to fabricate all surface types. An aluminum channel with a control and a test section was used for testing. Pressure transducers recorded the pressure drop across smooth silicon wafers and patterned test surfaces simultaneously allowing for computation of the friction factors.

Keywords: superhydrophobic, riblets, turbulent, jet impingement, laminar, hydraulic jump, breakup, slip, Joseph Prince
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CHAPTER 1. INTRODUCTION

1.1 Motivation

Superhydrophobic surfaces have recently gained attention because they radically alter liquid flow behavior. They yield apparent slip at the surface due to microroughness such as posts or microribs combined with a hydrophobic coating. This slip is what gives superhydrophobic surfaces the ability to reduce drag in both laminar and turbulent flows. Liquid jet impingement has emerged as a novel application of superhydrophobic surfaces. When a liquid jet impinges on a superhydrophobic surface, it forms a thin film just downstream of the jet. Because the film is extremely thin, the influence slip exerts on the flow dynamics is readily apparent. Using superhydrophobic surfaces in jet impingement applications is also desirable because these surfaces are self-cleaning which can reduce fouling and result in predictable heat transfer rates over long periods of time.

Superhydrophobic surfaces have also been proposed as a means to reduce friction in turbulent flow. Turbulent flow is prevalent in a large number of industrial applications and because wall shear stress is extremely high in turbulent flow, any friction drag reduction that can be realized can save enormous amounts of energy. Riblets are another technique shown to reduce friction in turbulent flows. Riblets were introduced over 30 years ago as a way to reduce shear stress in turbulent flow by reducing the spanwise turbulent fluctuations. To the author’s knowledge, no one has yet considered the benefits of combining surfaces exhibiting superhydrophobicity and riblets.

The focus of this work is twofold. An analytical model is presented capable of predicting laminar jet impingement behavior on superhydrophobic surfaces where the patterning is isotropic or anisotropic. Additionally, this work experimentally explores the drag reduction that can be achieved across superhydrophobic surfaces, riblets, and the combination of the two drag reducing mechanisms. This is accomplished through the acquisition of pressure drop measurements in a confined turbulent channel flow.
1.2 Background

1.2.1 Superhydrophobic Surfaces

Superhydrophobic surfaces can be found in nature. Lotus and other leaves exhibit a self-cleaning property called the Lotus effect. It has been shown that it is the combination of microstructuring and an epicuticular wax that renders these leaves superhydrophobic and causes dirt and other contamination to be cleaned off by rolling water droplets [1]. Microfabrication techniques have made it possible to mimic nature by generating microstructures of the same size as the structures on the lotus leaf. Posts and ribs on the order of microns can be fabricated using standard photolithographic techniques combined with deep reactive ion etching (DRIE) to generate highly controlled features [2]. This allows for systematic study of various geometry so that the key parameters influencing the superhydrophobicity of the surface can be identified and optimized. An SEM of a surface with microposts is shown in Figure 1.1.

![SEM of microposts fabricated with DRIE.](image)

Figure 1.1: SEM of microposts fabricated with DRIE.

One important parameter in the characterization of superhydrophobic surfaces is the contact angle a droplet of water forms with the substrate as shown in Figure 1.2. When the contact angle, $\phi$, is less than 90° the surface is defined as hydrophilic. If $90^\circ < \phi < 150^\circ$ the surface is hydrophobic, and if $\phi > 150^\circ$ the surface is superhydrophobic as chemical hydrophobicity must be combined
Figure 1.2: Water droplet contact angles on a surface that is hydrophilic, hydrophobic, and superhydrophobic.

with microstructuring to achieve these contact angles [3]. The contact angle achieved is dependent on surface energy considerations where a droplet’s shape always minimizes the thermodynamic energy of the system [4]. The Young equation [5]

\[ \gamma_{SL} + \gamma_{LV}\cos \phi = \gamma_{SV} \] (1.1)

predicts the contact angle of a droplet on a surface where the solid-liquid interfacial tension is \( \gamma_{SL} \), the liquid-vapor interfacial tension is \( \gamma_{LV} \), and \( \gamma_{SV} \) is the interfacial tension between the solid and vapor phases. Here, \( \phi \) is the contact angle that forms between the droplet and the surface to satisfy thermodynamic equilibrium.

When microroughness exists on a surface the apparent contact angle formed by a droplet on the surface is different than for a smooth surface even with the same chemical hydrophobicity or hydrophilicity. When the surface chemistry is hydrophilic and there is microroughness, the apparent contact angle a water droplet forms on the surface is predicted by Wenzel’s model [4]. The right panel of Figure 1.3 shows this scenario where the surface is wetted and water enters the cavities between the microribs. This state is often referred to as the Wenzel state and is predicted by Equation 1.2.

\[ \cos \phi^* = r \cos \phi \] (1.2)

Here, \( \phi^* \) is the apparent contact angle, \( \phi \) is the Young contact angle, and \( r \) is the roughness ratio.
Figure 1.3: Droplet in the non-wetting or Cassie-Baxter state in the left panel and droplet in the wetting or Wenzel state in the right panel. Microribs are not to scale.

or the ratio of the wetted area to the projected area. Cassie and Baxter [6] expanded the model proposed by Wenzel to include surfaces with chemical hydrophobicity that have microroughness. This scenario is shown in Figure 1.3 in the left panel where the droplet of water does not enter the cavities between features and is referred to as non-wetting or in the Cassie-Baxter state. Equation 1.3 models this behavior.

\[
\cos \phi^* = r_w f \cos \phi + f - 1
\]  

Here, \( r_w \) is the roughness ratio for the wetted area (which is 1 for non-wetted microribs), \( f \) is the ratio of the liquid-solid contact area to the projected liquid-solid contact area. It is clear that if \( f = 1 \) then the Cassie-Baxter model collapses to the Wenzel model and if \( f = 0 \) the apparent contact angle is 180°.

In reality, because no surface is perfectly uniform, there is a range of contact angles that can form from what is called the receding contact angle up to the advancing contact angle [7]. A receding contact angle is the angle a droplet forms with a surface just before the contact line moves inward, and the advancing contact angle is defined as the angle formed just before the contact line moves outward away from the droplet. Any contact angle in between is generally termed a static contact angle. This variation in the contact angles is noted for both smooth surfaces and surfaces with micropatterning.
When a superhydrophobic surface is covered with water and is in the Cassie-Baxter state, the cavities between microfeatures are filled with air. It is a balance between surface tension and the difference in pressure between the air in the cavities and the water that keeps the water from entering the cavities. If the pressure in the water becomes larger than the surface tension term, the water will enter the cavities and the surface will transition to the Wenzel state. This behavior is governed by the Laplace equation [8] given in Equation 1.4.

\[ \Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]  

(1.4)

\( R_1 \) and \( R_2 \) are the radii of curvature of the liquid-vapor interface, \( \sigma \) is the surface tension, and \( \Delta P \) is the pressure drop across the interface.

### 1.2.2 Jet Impingement and Hydraulic Jumps

When a liquid jet impinges on a smooth horizontal hydrophilic surface it spreads out in a thin film until a hydraulic jump forms. The hydraulic jump can be described by a sudden increase in the thin film thickness concomitant with a decrease in the radial velocity of the film as shown in Figure 1.4. The jump forms at a location where the surface tension, momentum, and hydrostatic terms are balanced. There are different types of hydraulic jumps formed depending on the recirculating vortices that form at the jump [9]. Figure 1.5 shows Type I and Type II jumps. A Type

![Image of hydraulic jump](image.png)

Figure 1.4: Hydraulic jump resulting from jet impingement.
I jump has only one recirculating vortex at the substrate just downstream of the jump. A Type II jump has two recirculating vortices where one is located at the jump near the free surface and where the other is just downstream of the jump close to the substrate. Type II jumps occur when the downstream depth is extremely large and there is little change in the film velocity.

![Type I and Type II hydraulic jumps](image)

Figure 1.5: Different hydraulic jump types based on the recirculating vortices in the vicinity of the jump.

When the surface is hydrophobic or superhydrophobic and a liquid jet impinges, two scenarios are possible. If no downstream depth is imposed the thin film will break up into droplets because the surface tension forces become as large as the momentum forces and no hydraulic jump forms [10]. This breakup into droplets is shown in Figure 1.6. If, however, a downstream depth is imposed a hydraulic jump forms similar to the hydrophilic case.

![Image of thin film breakup](image)

Figure 1.6: Image of thin film breakup resulting from jet impingement on a superhydrophobic surface patterned with microribs where no downstream depth is imposed. The image was taken with a high speed camera at 500 FPS.
1.2.3 Drag Reducing Mechanism for Superhydrophobic Surfaces

If water is flowing over a superhydrophobic surface in the Cassie-Baxter state, the liquid-solid contact area will be substantially reduced due to the air in the cavities between features. While the shear stress at the liquid-solid contact interface is increased because it is supporting all the water, the shear stress at the liquid-gas interface is negligible and a net reduction in the average shear stress can be realized [11]. There is slip across the liquid-gas interface and there is no slip at the liquid-solid interface. However, averaging over the entire surface, there is an apparent or macroscale slip. The apparent slip velocity $u_s$ is the aggregate velocity at the surface and the slip length is the distance the velocity profile would have to be extrapolated into the wall to reach a value of 0 as depicted in the left panel of Figure 1.7. The slip length is a function of the cavity fraction $F_c$ and the module width $w_m$ [11]. The cavity fraction is simply the area of the liquid-gas interface divided by the total area of the surface. For microribs, $F_c = w_c / w_m$ where $w_c$ is the width of a cavity and $w_m = w_c + w_r$. Here $w_r$ is the width of a microrib. These parameters are shown graphically in the right panel of Figure 1.7. Superhydrophobic surfaces inherently generate both spanwise and streamwise slip. In turbulent flow the streamwise slip results in a drag reduction, but the spanwise slip enhances spanwise turbulence and causes an increase in drag. Because microribs

Figure 1.7: Schematic of flow over a superhydrophobic surface patterned with microribs depicting the slip length.
aligned parallel to the flow have more streamwise slip than spanwise slip, they decrease drag, but microribs aligned perpendicular to the flow increase the drag [12].

1.2.4 Drag Reducing Mechanism for Riblets

Riblets have been studied extensively over the last 30 years as a mechanism to reduce frictional drag in turbulent flows. They are typically an order of magnitude larger than microribs used in superhydrophobic surfaces and are on the order of the $h^+ \sim 8$ where $h^+ = h u_\tau / \nu$ is the height of the riblet normalized by the friction velocity, $u_\tau$, and the kinematic viscosity, $\nu$ [13, 14]. Here, the friction velocity is defined as $u_\tau = \sqrt{\tau_w / \rho}$ where $\tau_w$ is the shear stress at the wall and $\rho$ is the density. The kinematic viscosity is simply the dynamic viscosity, $\mu$, normalized by the density. The periodic spacing $s^+$ is typically twice as large as the height [14]. Liquid flowing past riblets wets the cavities in between the structures unlike superhydrophobic surfaces that are designed to be non-wetting. Figure 1.8 shows examples of some of the types of riblets that have been studied including blade, trapezoidal, and scalloped riblets.

![Figure 1.8: Different riblet geometries that have been tested.](image)

Riblets function by damping out spanwise turbulence and the number of vortex ejections from the viscous sublayer [15, 16]. Because turbulence is three-dimensional, a reduction in the spanwise turbulence reduces turbulence in all directions thereby reducing the drag at the surface in spite of the area increase due to the riblet structuring. The relative size of riblets scales with...
the kinematic viscosity and the friction velocity. In order to obtain a drag reduction, the $s^+$ value should be on the same order as the turbulent streaks such that there is a reduction in the number of vortices being ejected [15]. If riblets are significantly smaller or larger than this, an accompanying drag increase can occur. The optimal riblet has been found to be infinitely thin and with a height that is half the spacing. The typical reductions in the total frictional resistance that have been observed through the use of riblets are between 5-10% [14].

1.2.5 Motivation to Combine Superhydrophobicity with Riblets

There is strong motivation to combine superhydrophobic surfaces with riblets. The superhydrophobic surface reduces the effective liquid-solid contact area which can compensate for the increased area due to the riblet structuring. Additionally, the riblets damp out the enhanced spanwise turbulence generated by the superhydrophobic surface. Thus, it is very attractive to combine these mechanisms to determine if a greater drag reduction can be realized than by either superhydrophobicity or riblets alone.

1.3 Dissertation Organization

This dissertation is composed of five subsequent stand-alone chapters that can be or have been published individually. Each chapter has its own introduction, methodology, results, and conclusions. For this reason, the literature review will not be repeated here. A description of the content of these chapters follows.

Chapter 2 covers laminar jet impingement on superhydrophobic surfaces with isotropic slip where a downstream depth has been imposed and a hydraulic jump forms. An analysis based on momentum considerations is presented along with the results of the numerical solution. An expression that predicts the hydraulic jump radius as a function of slip and other relevant parameters is determined. In Chapter 3, the analytical model in Chapter 2 is expanded to include anisotropic slip as would be the case on microrib and cavity patterned superhydrophobic surfaces. This chapter is still only considering the case where a hydraulic jump forms downstream of the thin film. Results of this analysis showing the elliptical hydraulic jump are presented. Another expression predictive of the hydraulic jump radius for average slip is also developed. Chapter 4 describes the jet
impingement on superhydrophobic surfaces where no downstream depth is imposed and the thin film breaks up into droplets. The scenarios for both isotropic and anisotropic slip are considered. For both cases the breakup location is determined as a function of relevant parameters. For the anisotropic case, the elliptical shape of the breakup location is described.

In Chapter 5, the drag reduction in turbulent channel flow across superhydrophobic surfaces, surfaces with riblets, and surfaces exhibiting both drag reduction mechanisms is reported. The results are obtained in a closed channel flow using pressure drop measurements over a range of flow conditions. Chapter 6 presents the novel microfabrication method used to make surfaces with microribs and riblets using photolithographic techniques.

Finally conclusions of the entire work and recommendations for future work are given in Chapter 7. Appendix A contains the detailed steps to fabricate microribs, riblets and surfaces with both feature types. Appendix B gives details on the experimental setup and schematics of the channel design. Appendix C explains the uncertainty analysis of the experiments and Appendix D presents the Matlab code used in Chapters 2 through 4.
CHAPTER 2. ANALYSIS OF LAMINAR JET IMPINGEMENT AND HYDRAULIC JUMP ON A HORIZONTAL SURFACE WITH SLIP

This chapter was published in Physics of Fluids and has been modified here to be consistent with the stylistic requirements of this dissertation. It is reprinted with permission from Prince, J., Maynes, D., and Crockett, J., 2012. “Analysis of laminar jet impingement and hydraulic jump on a horizontal surface with slip.” Physics of Fluids, 24, p. 102103. Copyright 2012, American Institute of Physics

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2.2 Abstract
This paper explores the influence surface slip, uniform in all directions with constant slip length, exerts on the physics of laminar jet impingement on a flat horizontal surface. Slip exists on superhydrophobic surfaces, and due to the relatively thin film dynamics associated with the growth of the laminar jet after impingement, its influence on the fluid physics is significant. An analysis based on momentum considerations is presented that allows prediction of the relevant thin film parameters as a function of radial position from the impingement point, jet Reynolds number, and constant relative slip length of the surface. Further, the analysis allows determination of the hydraulic jump location in terms of laminar jet characteristics and imposed downstream liquid depth. The results reveal that at a given radial location, the boundary layer growth and thin film thickness decrease, while the surface velocity of the thin film increases with increasing slip at the surface. The departure from classical no-slip behavior is quantified over a range of realizable slip conditions. Increasing slip length also leads to formation of hydraulic jumps at increasing radial
location. An expression based on the results is presented that allows prediction of the hydraulic jump location as a function of the magnitude of the slip and all other influencing variables.

### 2.3 Introduction

When a liquid jet impinges on a smooth horizontal hydrophilic surface it spreads out as a thin film and slows until the film experiences a circular hydraulic jump as is illustrated in Fig. 2.1. Downstream from the sudden jump in fluid height the fluid moves at a slower radial velocity, following continuity. For small imposed downstream depths, Type I jumps (transition devoid of secondary flow in the downstream fluid) result [9, 17] and at higher depths a recirculating flow at the location of the jump (Type II) has been observed [17–22]. Analysis describing Type I jumps has previously been developed that includes the influence of gravity, viscosity, and surface tension on the jump radial location for a flat hydrophilic surface [9].

![Liquid jet impingement dynamics schematic.](image)

The dynamics of a hydraulic jump on a flat hydrophilic or completely submerged surface with the classical no-slip boundary condition at the surface has been studied extensively [9, 17, 21, 23–26]. An inviscid solution to the unidirectional, uniform (Type I) circular jump velocity problem was proposed by Birkhoff and Zarantonello [27], but is only of academic interest since viscous influences are critical in the overall process. Watson [23] developed a viscous model,
including boundary layer effects, for both laminar and turbulent jets which expresses the thin film thickness as a function of radial location and describes the jump radius as a function of the liquid jet velocity and diameter, downstream liquid depth, and fluid properties [23]. Bush and Aristoff later expanded the model to account for the influence of surface tension associated with the azimuthal curvature of the jump and abrupt increase in fluid height [9]. Existing experimental data show varying goodness of agreement to Watson’s model, with generally better agreement at larger jump radii [21]. Data for Type II jumps also compare well to a first order to Watson’s theory [17–22].

Superhydrophobic (SH) surfaces are common drag reducing surfaces and are fabricated with the combination of microscale surface patterning and a thin film hydrophobic coating. Provided the Cassie state is maintained [28], liquid will wet only the top of the microscale patterned features on the surface. Consequently, apparent velocity slip can exist at the tops of the surfaces due to the trapped air in the cavity regions, which increases with the relative size of the cavity regions [29,30]. Researchers have explored droplet rolling and impingement, laminar channel flows, and turbulent boundary layer flows over SH surfaces [11,28,31–41]. However, the dynamics of a liquid jet impinging on SH surfaces has received little prior attention and jet impingement on a horizontal surface with isotropic slip (hydrophobic or superhydrophobic) has not previously been analyzed. Thus, analysis of the jet impingement dynamics on surfaces exhibiting isotropic slip is the focus of this paper.

Recent papers by Dressaire et al. considered jet impingement on hydrophilic surfaces with varying microscale features, similar to what might exist on a SH surface [42,43]. The features included arrays of equally spaced round or square posts of radius, height, and spacing between the posts in the $100\mu m$ range. The cavities were flooded and the resultant hydraulic jumps exhibited polygonal shapes rather than the classical circular transition. The theory developed accounted for the effects of the flooded fluid through an estimated net slip velocity at the height of the features. To accomplish this the slip length was assumed to vary as a function of the thickness of the film so the equations could be solved using a similarity solution. However, the slip length on a SH surface is primarily a function of surface chemistry and geometry and is often a constant value [30,44,45].

When a liquid jet impinges on a smooth hydrophobic or patterned SH surface without an imposed downstream water depth, the physics differ radically from the hydrophilic case. A recent experimental study by Maynes et al. showed that in this scenario the radially spreading jet breaks
up into droplets that translate outward [10]. The SH surfaces explored exhibited surface patterning of alternating ribs and cavities. They also determined that this break-up transition occurs when the local Weber number (based on the local average fluid velocity and film thickness) is approximately unity. Further, the break-up location was observed to exhibit an elliptical shape, with the major axis aligned parallel to the rib/cavity direction. Another experimental investigation of jet impingement on SH surfaces (with isotropic patterning) was accomplished by Kibar et al. who measured the drag force on a SH wall due to a liquid jet impinging at an oblique angle [46]. In their work, the drag force was reduced by as much as 40% as the liquid-surface apparent contact angle increased from 145° to 167°.

While limited experimental work has addressed some aspects of the dynamics of liquid jets impinging on SH surfaces, to the authors’ knowledge, no theory has been presented to model the general phenomenon of jet impingement on a surface with apparent isotropic slip of constant slip length. Thus, this paper presents analysis describing liquid jet impingement on a surface with isotropic slip and an imposed downstream liquid depth. Further, analysis of the hydraulic jump location is also presented. The analytical model will first be presented after which results derived from the model will be presented and discussed. The influence of varying the magnitude of the slip length on the thin film dynamics and the resultant hydraulic jump are explored in the Results section. Finally, conclusions of the work will be given.

2.4 Analysis

2.4.1 Model Description

Consider a vertical liquid jet of speed $V$, viscosity $\nu$, volume flow rate $Q$, and radius $a$ impinging a horizontal surface that exhibits isotropic slip due to superhydrophobicity of the surface as illustrated in Fig. 2.1. The jet spreads out symmetrically in a thin film with decreasing thickness, $h$, as it moves radially outward. For Reynolds numbers ($Re = Q/\nu a$) smaller than $2.5 \times 10^4$ a laminar boundary layer of thickness $\delta$ develops from the impingement point and grows with increasing radial coordinate $r$ [23]. The boundary layer $\delta$ grows until it is equal to the height ($\delta = h$) of the thin film at $r = r_o$. As the boundary layer grows the liquid velocity outside of the boundary layer equals $V$. At $r > r_o$, the velocity at the air-liquid interface, $U(r)$, then begins to decrease with
increasing $r$ due to the viscous influence of the wall. At $z = 0$ an apparent slip velocity that varies with $r$ exists due to the superhydrophobic nature of the surface. It is the influence of this slip that is the primary focus of this paper.

Far from the impingement point two conditions are possible. The first is the classical circular hydraulic jump where the liquid depth increases suddenly with a concomitant decrease in liquid velocity. This is the condition illustrated in Fig. 2.1 and it will occur when the surface is hydrophilic or when a downstream depth, $H$, of sufficient height is imposed. The nature of this hydraulic jump varies depending on Reynolds number but we assume here that Type I jump behavior prevails [9]. We denote the radial location of the jump as $R_j$. The second possible condition occurs on hydrophobic surfaces where no downstream water depth is imposed [10]. Under these conditions the spreading film reaches a point where the thin film breaks up into droplets that continue to propagate outward. It is the hydraulic jump that is of interest in this paper.

The domain $r < r_o$, where $\delta < h$ and the velocity profile is developing, is referred to here as Region 1. The domain $r_o < r < R_j$, where the velocity profile is developed, is referred to as Region 2. In the classical paper by Watson, [23] he developed similarity solutions to the Navier-Stokes and continuity equations in both Regions 1 and 2. Dressaire et al. [43] also developed a similarity solution for a specific scenario where slip exists. However, as they pointed out, a similarity solution in the slip case is only possible when the ratio of the slip length to the local thin film height is a constant value. Since this constraint does not reflect the frequent scenario where the slip length maintains a constant value, we employ the integral approach of Karman and Pohlhausen to obtain an analytical solution [47, 48]. Shown later in this section is a comparison between the similarity solution of Watson [23] for the no-slip scenario and our result employing the integral methodology, with excellent agreement.

An integral momentum analysis of a control volume that extends from $z = 0$ to $h$ and is of radial thickness $\Delta r$ (see Fig. 2.1) relates the hydrostatic pressure gradient and local wall shear stress, $\tau_w$, to the spatial variation in the film’s radial momentum.

\[
\rho g h \frac{dh}{dr} + \mu \frac{du}{dz} \bigg|_{z=0} = -\rho \frac{1}{r} \frac{d}{dr} \left( r \int_0^h u^2 dz \right)
\]  

(2.1)
\(\rho\) and \(\mu\) are the liquid density and viscosity, respectively. In the worst case scenario explored the hydrostatic term exerts less than 0.4\% cumulative influence on the dynamics (height, surface velocity, etc.) of the thin film and is thus neglected hereafter for simplicity. A statement of conservation of mass for the same control volume is

\[
0 = \frac{d}{dr} \left( r \int_0^h u dz \right) \tag{2.2}
\]

At any \(r\) location the mass flow in the thin liquid film is equal to the incoming jet mass flow,

\[
\frac{a^2 V}{2r} = \int_0^h u dz \tag{2.3}
\]

We assume here a cubic velocity distribution of the form

\[
u = a + bz + cz^3 \tag{2.4}
\]

The coefficients \(a\), \(b\), and \(c\) are determined from boundary conditions in the \(z\) direction.

### 2.4.2 Region 1

In Region 1, \(\delta < h\) and at \(z > \delta\) viscous influences vanish. Thus the fluid velocity outside of the boundary layer is equal to the jet velocity. The boundary conditions necessary to determine the coefficients of Eq. 2.4 are: 1) a slip velocity at the plane of the surface, \(u(z = 0) = u_s\), 2) a constant velocity (equal to the jet velocity) at the edge of the boundary layer, \(u(z = \delta) = V\), and 3) vanishing shear stress at the edge of the boundary layer, \(\partial u / \partial z \bigg|_{z=\delta} = 0\). Applying these conditions results in the following expression for the velocity profile

\[
u = u_s + \frac{3}{2} (V - u_s) \frac{z}{\delta} - \frac{1}{2} (V - u_s) \frac{z^3}{\delta^3} \tag{2.5}
\]

The slip velocity, \(u_s\), in Eq. 2.5 can be expressed in terms of the so-called slip length by invoking Navier’s slip hypothesis [49],

\[
\lambda = \frac{\mu u_s}{\tau_w} \tag{2.6}
\]
Physically, $\lambda$ represents the wall-normal distance (into the wall) where the velocity, $u$, would vanish and is determined by the surface characteristics of the superhydrophobic or slipping surface. It has been shown by several investigators that for many superhydrophobic surface topologies and flow conditions the slip length, $\lambda$, may be assumed to be a constant value, whereas the slip velocity varies with the local wall shear stress. For this reason we choose to employ the slip length in all analysis considered here and recast the slip velocity in terms of it and the local wall shear stress as dictated by Eq. 2.6. The local shear stress is determined by its definition and may be expressed as

$$\tau_w = \mu \frac{du}{dz} \bigg|_{z=0} = \frac{3\mu (V - u_s)}{2\delta} \quad (2.7)$$

Substitution of (2.7) into (2.6) and after some manipulation the slip velocity may be expressed as

$$u_s = \lambda \frac{V}{\frac{2}{3} \delta + \lambda} \quad (2.8)$$

Thus, the velocity profile (Eq. 2.5) may be rewritten in terms of $\lambda$ as

$$u = \frac{V}{\frac{2}{3} \delta + \lambda} \left[ \lambda + z - \frac{\delta^3}{3\delta^2} \right] \quad (2.9)$$

Combining Eqs. 2.1 and 2.2 in Region 1 yields the classic integral momentum boundary layer equation for a constant freestream velocity and an axisymmetric surface.

$$\nu \frac{du}{dz} \bigg|_{z=0} = \left( \frac{d}{dr} + \frac{1}{r} \right) \int_0^\delta \left( Vu - u^2 \right) dz \quad (2.10)$$

Upon substitution of the velocity profile from (2.9) into (2.10) an equation for the boundary layer thickness as a function of radial position is obtained.

$$\frac{\pi \hat{r}}{Re} = \left( \frac{2}{3} \hat{\delta} + \hat{\lambda} \right) \frac{d}{d\hat{r}} \left[ \hat{r} \hat{\delta}^2 \left( \frac{13}{210} \hat{\delta} + \frac{1}{4} \hat{\lambda} \right) \left( \frac{2}{3} \hat{\delta} + \hat{\lambda} \right)^{-2} \right] \quad (2.11)$$

In the above expression all lengths have been normalized by the jet radius such that $\hat{\delta} = \delta / a$, $\hat{\lambda} = \lambda / a$, and $\hat{r} = r / a$. In the absence of slip ($\lambda = 0$), Eqs. 2.10 and 2.11 each reduce to their classical no-slip counterparts. However, due to the slip, a closed form solution to Eq. 2.11 does
not exist. Here it is solved numerically following a 4th order Runge Kutta approach to yield the normalized boundary layer thickness, \(\hat{\delta}\), as a function of \(\hat{\lambda}\), \(\hat{r}\), and \(Re\). Once the boundary layer thickness is known, the film thickness \(h\) can be determined by utilization of the conservation of mass principle (Eq. 2.3). Substitution of the assumed velocity profile (2.9) into (2.3) provides

\[
\hat{h} = \frac{h}{a} = \frac{1}{2\hat{r}} + \frac{\hat{\delta}^2}{4\left(\frac{2}{3}\hat{\delta} + \hat{\lambda}\right)}
\]

(2.12)

2.4.3 Region 2

Region 2 begins when the boundary layer has grown to the height of the film thickness and viscous influences extend from the wall to the top of the film. Now the velocity at the top of the film, \(U\), is a function of radial position and decreases with increasing \(r\). The same form of the assumed velocity profile (Eq. 2.4) is employed in this region, with the second velocity boundary condition now taking the form \(u = U\) at \(z = h\). Thus, the equation for the velocity in this region looks identical to Eq. 2.9 but with \(V\) replaced by \(U\) and \(\delta\) replaced by \(h\).

\[
u = \frac{U}{\frac{2}{3}h + \hat{\lambda}} \left[\hat{\lambda} + z - \frac{z^3}{3h^2}\right]
\]

(2.13)

Substitution of the above velocity distribution into the momentum equation (2.1) and conservation of mass (2.3), yields

\[
-\frac{\pi \hat{U} \hat{r}}{Re} = \left(\frac{2}{3}\hat{h} + \hat{\lambda}\right)
\]

\[
\frac{d}{d\hat{r}} \left[\hat{r} \hat{h} \hat{U}^2 \left(\frac{68}{315} \hat{h}^2 + \frac{5}{6} \hat{h} \hat{\lambda} + \hat{\lambda}^2\right) \left(\frac{2}{3}\hat{h} + \hat{\lambda}\right)^{-2}\right]
\]

(2.14)

and

\[
\frac{1}{2\hat{r}} = \hat{U} \hat{h} \left[\frac{5}{12} \hat{h} + \hat{\lambda}\right] \left[\frac{2}{3}\hat{h} + \hat{\lambda}\right]^{-1}
\]

(2.15)
respectively, where $\hat{U} = U/V$. Substitution of (2.15) into (2.14) yields an equation relating the normalized film thickness $\hat{h}$ as a function of $\hat{r}$.

$$
-\frac{2\pi}{Re} = \hat{h} \left( \frac{5}{12} \hat{h} + \hat{\lambda} \right)
$$

\[
\frac{d}{d\hat{r}} \left[ \hat{r}^{-1} \hat{h}^{-1} \left( \frac{68}{315} \hat{h}^2 + \frac{5}{6} \hat{h} \hat{\lambda} + \hat{\lambda}^2 \right) \left( \frac{5}{12} \hat{h} + \hat{\lambda} \right)^{-2} \right] 
\]

(2.16)

### 2.4.4 Numerical Approach

The general solution approach for a specified $Re$ and $\hat{\lambda}$ was as follows. First Eq. 2.11 was solved using a 4th order Runge Kutta scheme to yield $\hat{\delta}$ as a function of $\hat{r}$ with the initial condition being $\hat{\delta} = 0$ at $\hat{r} = 0$. Then, $\hat{h}$ was determined from Eq. 2.12. This was continued until $\hat{\delta} = \hat{h}$, which occurs at $\hat{r} = \hat{r}_0$ where Region 1 ends and $\hat{h} = \hat{h}_0$. Subsequently, Eq. 2.16 was solved using the same numerical scheme with the initial condition $\hat{h} = \hat{h}_0$ at $\hat{r} = \hat{r}_0$, providing $\hat{h}$ for all $\hat{r} > \hat{r}_0$. In Region 2 the surface velocity $\hat{U}$ was then determined directly at each $\hat{r}$ from Eq. 2.15.

The approach outlined was benchmarked by comparing results of $\hat{\delta}$, $\hat{h}$, and $\hat{U}$ with results of Watson for the classic no-slip scenario [23]. This comparison is shown in Fig. 2.2 and the data illustrates two points. First, the assumed cubic velocity profile yields excellent agreement with the similarity approach of Watson for all parameters. The largest deviation exists for $\hat{h}$, yet this difference is still reasonable even at its maximum. Thus, it is appropriate to employ the assumed cubic velocity profiles to model the scenario with slip. Second, the agreement reveals that the step size of the integration scheme is sufficiently small so that numerical error is negligible. A discussion of the departure from this classical behavior for varying degrees of slip is contained in the Results section of the paper.

### 2.4.5 Hydraulic Jump Location

An expression to estimate the location of the hydraulic jump is now developed. Following the approach of Bush and Aristoff [9], an integral momentum balance between gravitational and surface tension forces and momentum fluxes into and out of a control volume of vanishing radial
width at the radius of the jump location, $R_j$, gives the following expression

$$
\frac{1}{2} \rho g (H^2 - h_j^2) + \frac{\sigma (H - h_j)}{R_j} = \rho \int_0^{h_j} u^2 \, dz - \rho U_1^2 H \tag{2.17}
$$

$H$ is the downstream water depth, $U_1$ is the downstream water velocity which is assumed to be uniform, $h_j$ is the thin film thickness just prior to the jump at $r = R_j$, and $\sigma$ is the liquid surface tension. In his original analysis Watson [23] neglected the surface tension force while Bush and Aristoff [9] showed that it exerts a small, but not completely negligible, influence on predicting the overall jump location. Thus we include the surface tension term here as well. We consider only strong jumps, such that $H \gg h_j$ as is frequently the case and thus the first two terms of Eq. 2.17 are simplified. In determining the radial jump location the maximum error introduced in the solution is 0.3% due to neglecting $h_j^2$ in the hydrostatic force term and 3% due to neglecting $h_j$ in the surface tension force term. This assumption greatly simplifies the subsequent analysis and since the worst case impact is small it is maintained throughout. The conservation of mass.

Figure 2.2: Film surface velocity, $\hat{U}$, film thickness, $\hat{h}$, and boundary layer growth, $\hat{\delta}$, as a function of the radial coordinate. Dashed line is Watson’s solution and solid line is the solution described here. The scaling is as in Watson [23].
expression (Eq. 2.3) evaluated just downstream of the jump yields

\[ Q = 2\pi R_j U_1 H \]  

(2.18)

Solving (2.18) for \( U_1 \) and substituting the result and the velocity profile of (2.13) into (2.17) gives an equation relating \( \hat{R}_j \) in terms of \( \hat{h}_j \) and \( \hat{U}_j \), which are both functions of radial position as described above.

\[
\frac{gH^2}{2} + \frac{\sigma H}{\rho R_j} + \frac{Q^2}{4\pi^2 R_j^2 H} = \\
h_j U_j^2 \left[ \left( \frac{68}{315} \hat{h}_j^2 + \frac{5}{6} \hat{h}_j \hat{\lambda} + \hat{\lambda}^2 \right) \left( \frac{2}{3} \hat{h}_j + \hat{\lambda} \right)^{-2} \right]  
\]  

(2.19)

\( U_j \) is the surface velocity of the film immediately prior to the jump and can be eliminated from (2.19) by use of (2.15). The result is an expression relating \( \hat{R}_j \) in terms of \( \hat{h}_j \) and constant specified values.

\[
\frac{ga^5 \hat{H}^2 \hat{R}_j}{Q^2} \left( 1 + \frac{2}{Bo \hat{H} \hat{R}_j} \right) + \frac{1}{2\pi^2 \hat{H} \hat{R}_j} = \\
\frac{1}{2\pi^2 \hat{R}_j \hat{h}_j} \left[ \left( \frac{68}{315} \hat{h}_j^2 + \frac{5}{6} \hat{h}_j \hat{\lambda} + \hat{\lambda}^2 \right) \left( \frac{5}{12} \hat{h}_j + \hat{\lambda} \right)^{-2} \right]  
\]  

(2.20)

\( Bo \) is the Bond number, \( Bo = \rho ga^2 / \sigma \). In practice \( \hat{R}_j \) was determined as follows. The normalized downstream depth, \( \hat{H} \), Reynolds number, Bond number, \( \hat{\lambda} \), and \( ga^5 / Q^2 \) were specified. Water was assumed to be the working fluid. \( \hat{h}(\hat{r}) \) was then determined as explained above. Subsequently, Eq. 2.20 was evaluated at varying \( \hat{R}_j = \hat{r} \) and corresponding \( \hat{h}(\hat{r}) \) until the left and right sides of the equation exactly balance. The corresponding \( \hat{r} \) then represents the radial location of the jump.
2.5 Results

2.5.1 Thin Film Dynamics

The models of the previous section have been exercised for \( Re \) ranging from \( 3 \times 10^3 \) to \( 2 \times 10^4 \) and for \( \hat{\lambda} \) ranging from 0 (no-slip scenario) to 0.2. When considering dynamics of the spreading thin film, results for \( \hat{\lambda} \to \infty \) (inviscid) are also shown as limiting case values. Typical slip lengths on SH surfaces can reach as high as 60 \( \mu \)m [32–34, 50, 51]. For such a surface, the value \( \hat{\lambda} = 0.2 \) would be realized with a liquid jet radius of 0.3 mm (diameter of 0.6 mm). This is a thin jet and probably represents a realistic lower limit on jet size. Thus, the range of \( \hat{\lambda} \) explored here spans the experimental range of realizable values.

First we discuss a detailed description of the flow-field and its defining characteristics. Specifically the boundary layer growth within the thin film, the variation of the thin film thickness, and the free surface velocity of the film as it spreads are discussed with respect to varying \( \hat{\lambda} \) and \( \Re \).

Figure 2.3 details the basic dynamics of the thin film flow at \( \Re = 6 \times 10^3 \) with varying radial coordinate, \( \hat{r} \). Shown are the free surface velocity, \( \hat{U} = U/V \) (top panel), the slip velocity at the SH surface, \( \hat{u}_s = u_s/V \) (middle panel), and the thicknesses of the thin film, \( \hat{h} \), and boundary layer, \( \hat{\delta} \), in Region 1 (bottom panel). Results are shown for \( \hat{\lambda} = 0.05, 0.1, 0.15, \) and 0.2. Also shown are results for the no-slip scenario as first presented by Watson (\( \hat{\lambda} = 0 \)) and the inviscid flow scenario (\( \hat{\lambda} \to \infty \)). The general shape of the \( \hat{U} \), \( \hat{h} \), and \( \hat{\delta} \) curves for all finite \( \hat{\lambda} \) are qualitatively similar to the no-slip scenario behavior over the entire \( \hat{r} \) domain considered. In Region 1, the boundary layer growth region, \( \hat{\delta} \) is rising from 0 at the stagnation point towards the height of the film. Here the velocity outside the boundary layer is equal to the uniform jet velocity \( V \) and the overall thickness \( \hat{h} \) is decreasing rapidly with increasing \( \hat{r} \). In Region 2, the thickness of the film continues to decrease until a local minimum is reached and then it increases, whereas \( \hat{U} \) decreases monotonically with increasing \( \hat{r} \).

The influence of slip on \( \hat{h} \), \( \hat{\delta} \), and \( \hat{U} \) is significant and of course now \( \hat{u}_s \) is non-zero. The boundary layer grows more slowly as the slip length increases and the \( \hat{h} = \hat{\delta} \) condition is attained at increasing distance from the jet as the relative magnitude of the slip length increases. For all cases this occurs before the minimum film thickness is reached and well before a hydraulic jump.
Figure 2.3: Film surface velocity, $\hat{U}$ (top panel), apparent surface slip velocity, $\hat{u}_s$ (middle panel), and film thickness, $\hat{h}$, and boundary layer growth, $\hat{\delta}$ (bottom panel), as a function of the radial coordinate. The Reynolds number is $6 \times 10^3$ for all scenarios and results are shown for $\hat{\lambda} = 0, 0.05, 0.1, 0.15, 0.2$ and $\to \infty$. 
will form. The thickness of the thin film also decreases more rapidly for all \( \hat{\lambda} \) than for the no-slip case and at \( \hat{\lambda} = 0.2 \), the \( \hat{h} \) vs. \( \hat{r} \) behavior is much closer to the limiting case corresponding to \( \hat{\lambda} \to \infty \) than it is to the no-slip case.

There is also an increase in the momentum flux within the thin film due to wall slip, and thus at a given radial location the free surface velocity also increases with increasing slip. The apparent slip velocity is a maximum (approaching 1) as \( \hat{r} \to 0 \) and the boundary layer thickness vanishes. The \( \hat{u}_s \) vs. \( \hat{r} \) behavior exhibits inflectional behavior at the point of transition between Regions 1 and 2 where the free stream velocity transitions from a constant value to one that decreases with increasing radius.

At higher Reynolds numbers the influence of slip has more pronounced effects. Figure 2.4 presents the same variables as in Fig. 2.3, but here the Reynolds number varies from \( Re = 3 \times 10^3 - 1.5 \times 10^4 \) while the slip length is held constant at \( \hat{\lambda} = 0.2 \). The same qualitative behavior with varying \( \hat{r} \) are observed for \( \hat{h} \), \( \hat{\delta} \), \( \hat{u}_s \), and \( \hat{U} \). In general, an increase in Reynolds number leads to the same alteration in the dimensionless film dynamical variables as an increase in slip length. Increases in either \( \hat{\lambda} \) or \( Re \) decrease the influence of viscosity and the overall behavior tends towards the inviscid scenario. The variability due to Reynolds number (over this range) is more significant than changing the slip length, as the overall variation in the curves due to Reynolds number differences is more exaggerated. Also note the difference in boundary layer growth among the varying Reynolds numbers is slightly more significant. Watson [23] showed that in the absence of slip, \( \hat{U} \) exhibits similarity for all \( Re \), where the behavior of \( \hat{U} \) as a function of \( \hat{r} Re^{-1/3} \) collapses to a single curve. Similarly, \( \hat{h} Re^{1/3} \) and \( \hat{\delta} Re^{1/3} \) each collapse to respective curves when plotted vs. \( \hat{r} Re^{-1/3} \) for all \( Re \). Such a collapse does not exist in the presence of slip of a constant slip length due to the interaction between wall slip and Reynolds number variation.

Figure 2.5 displays the minimum height of the thin film (top panel) and its radial location of occurrence (bottom panel) each as functions of slip length. Here each line represents a different Reynolds number. The minimum height of the thin film decreases as either Reynolds number or slip length increase. As the effects of viscosity are minimized, the thin film contains more momentum within a smaller film thickness. With increasing Re, the \( h_{min} \) curves are closer together and appear to be leveling off as the slip length increases. A limit is never reached since in the limiting case \( h_{min} \to 0 \) for \( \lambda \to 0 \). In addition, the location of the minimum height, \( \hat{r}_{min} \), is pushed
Figure 2.4: Film surface velocity, $\hat{U}$ (top panel), apparent surface slip velocity, $\hat{u}_s$ (middle panel), and film thickness, $\hat{h}$, and boundary layer growth, $\hat{\delta}$ (bottom panel), as a function of the radial coordinate. The relative slip length is $\hat{\lambda} = 0.2$ for all scenarios and the Reynolds number varies from $Re = 3 \times 10^3 - 1.5 \times 10^4$. 
Figure 2.5: Minimum thin film height as a function of slip length (top panel) and the location of the minimum height (bottom panel) for varying Reynolds number.

farther radially outward as effects of viscosity decrease (Reynolds number or slip length increase). The increase in $\hat{r}_{\min}$ with increasing $\hat{\lambda}$ is greater for larger Reynolds number. At $Re = 3 \times 10^3$, $\hat{r}_{\min}$ increases from 6.3 at $\hat{\lambda} = 0$ to 10.8 at $\hat{\lambda} = 0.2$, a 70% increase. However at $Re = 1.5 \times 10^4$, $\hat{r}_{\min}$ increases over 100% from 10.9 to 22.8 over the same $\hat{\lambda}$ range. The combined effect of increasing slip and Reynolds number to increase the momentum flux within the thin film leads to similar curvature between the minimum thin film height and its corresponding radial location for any slip situation.

The coefficient of friction, $C_f = \tau_w / 0.5 \rho V^2$ provides a measure of the relative wall shear stress. Using (2.6), $Re = Q / a \nu$, and normalizing the velocity and slip length by $V$ and $a$ respectively, $C_f$ can be rewritten as

$$C_f = \frac{2 \pi \hat{u}_s / \hat{\lambda}}{Re}$$

(2.21)
$C_f$ is shown as a function of $r$ for varying slip lengths in Fig. 2.6, where the Reynolds number is held constant at $6 \times 10^3$. Similarly, $C_f$ is shown as a function of $r$ for varying Reynolds numbers in Fig. 2.7, where $\lambda$ is held constant at 0.2. The coefficient of friction decreases with increasing radius for all slip and no-slip situations. However, in the case of no-slip, in Region 1 the boundary layer grows more quickly and the thin film becomes fully developed more quickly. Thus the coefficient of friction initially drops more quickly than in the slip cases. The point of inflection in the figures again represents the location where the thin film transitions from Region 1 to Region 2. In Region 2 the film thickness continues to develop and eventually thickens with an accompanying decrease in the wall shear stress. However, in the slip cases a smaller thin film height and faster free surface velocity dominate, leading to a slower decrease in the friction coefficient. Consequently, as the flow develops through Region 2 the skin friction coefficient for flows with greater slip is larger than for the no-slip case. This behavior prevails for all Reynolds numbers explored.

![Figure 2.6: Coefficient of friction, $C_f$, as a function of radial location within the thin film for varying slip lengths at Reynolds number $6 \times 10^3$.](image)

The effects of Reynolds number on the coefficient of friction at fixed $\lambda$ show similar ties to the discussion above. Again, at the largest Reynolds numbers ($1.5 \times 10^4$) the boundary layer grows more slowly and therefore the drop in $C_f$ with increasing $r$ is significantly less than for $Re = 3 \times 10^3$ where the boundary layer grows quickly and the corresponding wall shear quickly decreases. This phenomenon is enhanced in Region 2. However, when slip is present, $h$ grows more slowly and
the shear exhibits a slower decrease. The inflection point where the transition from Region 1 to 2 occurs is visible, but is not as distinct with increasing \( \hat{\lambda} \). The \( C_f \) vs. \( \hat{r} \) curves follow the same trend as the \( \hat{u}_s \) curves of Figs. 2.3 and 2.4. Here, however, at larger slip \( \hat{\lambda} \) dominates as \( \hat{r} \) increases (\( \hat{u}_s/\hat{\lambda} \) where \( \hat{u}_s \) decreases and \( \hat{\lambda} \) stays constant) due to the thinner film at increased slip. The result is a larger shear due to a thinner boundary layer and greater free-surface velocity.

### 2.5.2 Hydraulic Jump

The radial location of the hydraulic jump and how it is influenced by surface slip is now discussed. The hydraulic jump occurs where there is a local balance between the momentum of the thin film and the gravitational and surface tension forces exerted by the imposed water depth, as shown in Eq. 2.20. This radial location is a function of the jet Reynolds number, Bond number, Froude number, downstream depth, \( \hat{H} \), and the relative slip length, \( \hat{\lambda} \).

Figure 2.8 illustrates the influence of slip and downstream depth on the jump radius. \( \hat{R}_j \) is plotted as a function of \( \hat{\lambda} \) for \( \hat{H} \) values of 2, 5, 8, and 11 and \( Re = 6 \times 10^3 \) for all cases. For all of the data of Fig. 2.8, the jump location is in Region 2 of the spreading thin film. The data confirm the well established fact that for \( \hat{\lambda} = 0 \) the radius of the jump increases as the immersed depth decreases. This behavior prevails for all \( \hat{\lambda} \) and occurs because the gravitational force opposing the motion of
the thin film decreases with decreasing depth. The data also show that the jump radius increases with increasing slip length for constant \( \hat{H} \). Interestingly, the influence of slip is more pronounced for lower imposed depths, \( \hat{H} \). At \( \hat{H} = 11 \) the jump radius increases from nominally 6.5 at \( \hat{\lambda} = 0 \) to 8.5 at \( \hat{\lambda} = 0.2 \), a change of 30%. At a depth of \( \hat{H} = 2 \), however, the increase is approximately 60% (20 to 31) for the same \( \hat{\lambda} \) range. This occurs because at lower \( \hat{H} \) the radial location of the jump is greater because the gravitational and surface tension forces are smaller. Consequently, there is greater surface area over which reduced shear stress acts on the thin film, and thus the influence of slip is amplified at decreasing \( \hat{H} \).

The jump radius is also a function of Reynolds number, as seen in Fig. 2.9, where \( \hat{R}_j \) is plotted vs. \( Re \) for \( \hat{\lambda} = 0, 0.05, 0.1, \) and 0.2, with \( \hat{H} = 3 \) for all cases. These data reveal the well known fact that the radius of the jump increases with increasing Reynolds number. Further, the data show that the influence of slip is greater at increasing \( Re \). At \( Re = 3 \times 10^3 \), \( \hat{R}_j \) varies from 9 (at \( \hat{\lambda} = 0 \)) to 11 (at \( \hat{\lambda} = 0.2 \)), an increase of 22%. At \( Re = 1.5 \times 10^4 \), however, \( \hat{R}_j \) increases from approximately 36.2 (at \( \hat{\lambda} = 0 \)) to 55.8 (at \( \hat{\lambda} = 0.2 \)), an increase of 54%. As \( Re \) increases, and for the same slip and imposed downstream depth, the thin film height at a given radial location decreases. Further, the velocity at the surface increases, and for the cases where \( \hat{\lambda} \neq 0 \), the apparent slip velocity increases. Thus the momentum flux within the thin film increases with increasing \( \hat{\lambda} \), and this effect is more pronounced at larger \( Re \).
For the classical no-slip surface, Bush and Aristoff [9] showed that the radial location of the hydraulic jump is satisfied by the equation

\[
g a^5 H^2 j \left(1 + \frac{2}{Bo H j}\right) + \frac{1}{2 \pi^2 H j} = 0.01676 \left[\frac{R_j^3}{Re} g(\hat{\lambda}) + 0.1826\right]^{-1}
\]

where \(g(\hat{\lambda}) = 1\). We will show below that for surfaces with slip Eq. 2.22 may also be employed where the function \(g(\hat{\lambda})\) has been determined from our present results. For the no-slip surface, Eq. 2.22 contains the effect of all parameters that exercise influence on the hydraulic jump location. Shown in Fig. 2.10 is the left hand side of Eq. 2.22 plotted as a function of \(R_j^3/Re\), for \(\hat{\lambda} = 0, 0.05, 0.1, \) and \(0.2\). The figure illustrates similarity in the shape of the curves for all \(\hat{\lambda}\), with increasing \(\hat{\lambda}\) resulting in a shifting of the curve to the right. Thus, greater slip leads to an increase in the hydraulic jump radius. The similarity in the shape of the curves suggests a modification of the \(R_j^3/Re\) parameter to include functional dependence on \(\hat{\lambda}\) of the form \(g(\hat{\lambda})\). Analysis of the data of Fig. 2.10 and data for many other values of \(\hat{\lambda}\) suggests that collapse of the similar
curves of Fig. 2.10 occurs when
\[
g(\hat{\lambda}) \approx \left[ 1 + 5.4\hat{\lambda} + 73\hat{\lambda}^2 \right]^{-1}
\]  

(2.23)

Shown in Fig. 2.11 are the same data of Fig. 2.10, but where the left hand side of Eq. 2.22 is now plotted as a function of \( \left( \hat{R}_j^3/Re \right) g(\hat{\lambda}) \). For all \( \hat{\lambda} \) shown agreement is within 6%. Similar goodness exists for all values of \( \hat{\lambda} \), although only 0, 0.05, 0.1, and 0.2 are shown in the figure.

![Figure 2.10: Left hand side of Eq. 2.22 as a function of \( \hat{R}_j^3/Re \) for \( \hat{\lambda} = 0, 0.05, 0.1, \) and 0.2.](image)

Given the coupled nature of \( \hat{\lambda} \) and \( Re \), and the necessity of solution to the thin film dynamics using the integral analysis approach described above, the collapse of the curves for all \( \hat{\lambda} \) is remarkable. The significance of Fig. 2.11 and Eq. 2.23 is that it allows prediction of the hydraulic jump radius for any slip length and any combination of governing parameters. In practice, the \( \left( \hat{R}_j^3/Re \right) \) term in Eq. 2.22 is replaced with \( \left( \hat{R}_j^3/Re \right) g(\hat{\lambda}) \) and then \( \hat{R}_j^3 \) is determined for the situation of interest.

As a conclusion to this section, we now briefly discuss the average wall shear stress exerted on the surface in the thin film region. This average skin friction coefficient is computed by spatially averaging Eq. 2.21 from the impingement point out to the radial location of the hydraulic jump, which includes only the thin film region. This is shown in Fig. 2.12 for the scenario where \( \hat{H} = 3 \) and the average wall shear stress, \( \bar{C}_f \), is plotted as a function of \( Re \) for \( \hat{\lambda} = 0, 0.05, 0.1, \) and 0.2. The data reveal a power-law relation between \( \bar{C}_f \) and \( Re \) for all scenarios with the average shear stress
Figure 2.11: Left hand side of Eq. 2.22 as a function of \( \frac{\hat{R}_j}{Re} \) \( g(\hat{\lambda}) \) for \( \hat{\lambda} = 0, 0.05, 0.1, \) and 0.2.

decreasing with increasing \( \hat{\lambda} \). Since the average shear stress on the surface is less for the surfaces with slip, the momentum of the thin film is greater and the jump radius is larger, as was seen in Fig. 2.8. Similar behavior is observed with varying \( \hat{H} \), where the magnitude of \( \bar{C}_f \) for all \( \hat{\lambda} \) increases with increasing \( \hat{H} \), since the radial location of the hydraulic jump is smaller as \( \hat{H} \) increases.

Figure 2.12: Average skin friction coefficient from the jet stagnation point out to the radial location of the hydraulic jump as a function of \( Re \). Results are shown for \( \hat{\lambda} = 0, 0.05, 0.1, \) and 0.2, with \( \hat{H} = 3 \) for all cases.
2.6 Conclusion

The interest in surfaces exhibiting slip has grown in recent years with the advent and study of superhydrophobic surfaces. The flow dynamics of a laminar jet impinging on a superhydrophobic surface in the case of an imposed downstream depth has received little prior attention and has been explored in this paper. A model has been developed based on the integral approach of Karman and Pohlhausen to describe the behavior when the slip length is constant and isotropic. The continuity and momentum equations have been solved numerically assuming a cubic velocity profile allowing for the prediction of both the thin film parameters as well as the location of the hydraulic jump given an imposed downstream depth. The introduction of slip at the surface has the following effects on the flow:

- The boundary layer grows more slowly
- The height of the thin film decreases more rapidly
- The minimum height of the thin film decreases
- The momentum flux in the thin film increases
- The coefficient of friction is smaller initially and decreases more slowly with increasing radius
- The hydraulic jump radius increases.

The influence of the slip length is also exaggerated when the Reynolds number is larger and when the downstream depth is smaller. Similarity has been found in the family of curves for different slip lengths when plotted using the normalization of Bush and Aristoff [9]. An empirical modification to the parameters allows for the collapse of these curves. This is significant because the hydraulic jump radius can now be predicted directly for a given slip length and other applicable parameters.

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CHAPTER 3. ANALYSIS OF LAMINAR JET IMPINGEMENT AND HYDRAULIC JUMP ON A HORIZONTAL SURFACE WITH ANISOTROPIC SLIP

This chapter has been submitted in Physics of Fluids and has been modified here to be consistent with the stylistic requirements of this dissertation.

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3.2 Abstract

We present an analysis that describes the dynamics of laminar jet impingement on horizontal surfaces with anisotropic slip. For the scenario considered the slip length varies as a function of the azimuthal coordinate and describes superhydrophobic surfaces micropatterned with alternating ribs and cavities. The thin film dynamics are modeled by a radial momentum analysis for a given jet Reynolds number and specified slip length. Due to the thin film dynamics that prevail, the influence of slip on the entire flow field is significant and yields behavior that deviates notably from classical behavior. In an average sense the thin film dynamics exhibit similarities to behavior that exists for a surface with isotropic slip. However, there are also important deviations from this behavior caused by the azimuthally varying slip. These deviations become more pronounced at higher Reynolds numbers and at greater slip lengths. The analysis also allows determination of the azimuthally varying radial location of the hydraulic jump that forms due to an imposed downstream depth. The deviations from the no slip case and from the scenario of isotropic slip are determined over a range of jet Reynolds numbers and realistic slip length values. The results show that for all cases the hydraulic jump is elliptical, with eccentricity increasing as the Reynolds number or slip length increases, or as the downstream depth decreases. The radial location of the hydraulic jump...
is greatest in the direction of greatest slip (parallel to the microribs), while it is a minimum in the
direction transverse to the rib/cavity structures. The model results for the hydraulic jump radial
position are compared to experimental measurements with good agreement.

3.3 Introduction

This paper addresses the laminar flow dynamics of a liquid jet impinging normally on a
superhydrophobic (SH) surface with anisotropic surface patterning. Superhydrophobic surfaces
are fabricated with a combination of microscale surface patterning and a thin film hydrophobic
coating. The dynamics of liquids interacting with SH surfaces have been explored by several
investigators for a wide variety of liquid flow scenarios including impinging droplets or liquid jets
and laminar and turbulent channel flows [11, 28, 31–41]. However, liquid jet impingement on SH
surfaces has received relatively little prior attention. For liquid water in contact with a SH surface,
provided the Cassie state is maintained [28], liquid will wet only the top of the microscale patterned
features on the surface. Consequently, apparent slip can prevail at the tops of the surfaces due to
the trapped air in the cavity regions. In general, the slip increases with the relative size of the cavity
regions [29, 30]. If the surface patterning is random or the same in all directions, isotropic slip at
the surface will prevail, while if the surface patterning consists of alternating ribs and cavities as
considered in this paper, anisotropic slip exists. The overall slip is generally characterized by the
slip length, $\lambda$, defined to be the wall normal distance (into the wall) where the magnitude of the
aggregate velocity profile vanishes.

When a liquid jet impinges on a smooth horizontal hydrophilic surface with a classical
no-slip boundary condition it spreads out as a thin film and slows until the film experiences a
hydraulic jump as illustrated in the top panel of Fig. 3.1. For the classical surface the shape of
the hydraulic jump is circular and this scenario has been explored analytically and experimentally
by several investigators [9, 17–24, 26]. Watson developed an analytical model that expresses the
thin film thickness as a function of radial location and describes the jump radius as a function of
the liquid jet velocity and diameter, downstream liquid depth, and fluid properties [23]. Bush and
Aristoff expanded the model to account for the influence of surface tension associated with the
azimuthal curvature of the jump and abrupt increase in fluid height [9]. Existing experimental data
show varying goodness of agreement to Watson’s model, with generally better agreement at larger
jump radii [21]. In a recent paper, we extended the analysis of Watson to account for isotropic slip of constant slip length at the surface [52]. Such slip can prevail on SH surfaces with isotropic surface patterning. This study showed that increasing the slip length results in increased average velocity in the thin film, a thinner film thickness, a smaller local wall skin friction coefficient, and an increase in the radial location of the hydraulic jump that forms downstream. Further, the results allowed formulation of a model to predict the hydraulic jump location as a function of the characteristic surface slip length.

Two recent papers by Dressaire et al. also considered a related phenomenon [42,43]. They considered jet impingement on wetting hydrophilic surfaces with varying microscale post features,
similar to what might exist on a SH surface, where they modeled the slip-length with a varying value. However, in this study, the cavities were flooded and the superhydrophobic state did not exist.

When a liquid jet impinges on a smooth hydrophobic or patterned SH surface without an imposed downstream water depth, the physics differ radically from the hydrophilic case described above. A recent experimental study by Maynes et al. showed that in this scenario the radially spreading jet breaks up into droplets, which then translate outward [10]. The SH surfaces explored exhibited anisotropic surface patterning consisting of alternating ribs and cavities of nominal widths in the 4 and 34 micron ranges, respectively. The break-up location was observed to exhibit an elliptical shape, with the major axis of the elliptical shape aligned parallel to the rib/cavity direction. Presumably the elliptical shape resulted from anisotropic slip resulting from the alternating rib/cavity surface patterning.

It has been shown previously by several investigators considering laminar liquid flow in channels with superhydrophobic walls that the slip length along the direction of the ribs/cavities is nominally double the slip length in the transverse direction [53, 54]. The dynamics of a liquid jet impinging on SH surfaces with anisotropic slip has not been analyzed previously, although it is clear that the dynamics will differ from the isotropic slip scenario. Detailed analysis of the jet impingement dynamics on SH surfaces exhibiting anisotropic slip is the focus of this paper. The bottom panel of Fig. 3.1 illustrates qualitatively the shape of the hydraulic jump that is expected for the scenario of interest. Note that the rib and cavity features illustrated in the figure would be nominally two orders of magnitude smaller than what is shown, and hundreds of rib/cavity structures would exist inside the hydraulic jump.

While limited experimental work has addressed some aspects of the dynamics of liquid jets impinging on SH surfaces with anisotropic slip resulting from alternating rib/cavity surface patterning, no analytical framework has been presented to model this general phenomenon. Thus, the focus of this paper is to present analysis of liquid jet impingement on a surface with anisotropic slip with a slip-length that varies in the azimuthal direction. Further, analysis of the shape and location of the hydraulic jump will be presented. Results from the model are also compared to experimental data. The analytical model will be developed in Sec. 3.4 and Sec. 3.5 provides
results from the analysis where the influence anisotropic slip exerts on the thin film dynamics and the resultant hydraulic jump are quantified. Finally, conclusions of the work will be given.

3.4 Analysis

3.4.1 Model Description

Consider a vertical liquid jet of speed $V$, volume flow rate $Q$, viscosity $\nu$ and radius $a$ impinging on a horizontal surface that exhibits anisotropic slip due to the rib and cavity structure producing the superhydrophobicity of the surface as illustrated in Fig. 3.1. The jet spreads out asymmetrically in a thin film with decreasing thickness, $h$, as it moves radially outward. For Reynolds numbers ($Re = Q/a\nu$) smaller than nominally $2.5 \times 10^4$, a laminar boundary layer of thickness $\delta$ develops from the impingement point and grows with increasing radial coordinate $r$ [23], as can be seen in the top panel, upper left inset of Fig. 3.1. The boundary layer thickness $\delta$ is also a function of $\theta$, the azimuthal coordinate, due to the anisotropy of the surface slip and it grows until it is equal to the height ($\delta(r, \theta) = h(r)$) of the thin film at $r = r_o(\theta)$. As the boundary layer grows the liquid velocity outside of the boundary layer equals the constant jet velocity $V$. At $r > r_o$, the velocity at the air-liquid interface, $U(r, \theta)$, then begins to decrease with increasing $r$ due to the viscous influence of the wall. At $z = 0$, where $z$ is the coordinate normal to the surface, an apparent slip velocity that varies with $r$ and $\theta$ exists due to the superhydrophobic nature of the surface. It is the anisotropic influence of this slip that is the primary focus of this paper. Portions of the following theoretical development will resemble somewhat that presented by Prince et al. [52] but significant differences exist due to the anisotropy of the surface, and thus the entire analysis is included here for clarity.

Far from the impingement point two conditions are possible. The first occurs on hydrophobic surfaces where no downstream water depth is imposed [10]. Under these conditions the spreading film reaches a point where the thin film breaks up into droplets that continue to propagate outward. The second possible condition is a hydraulic jump where the liquid depth increases suddenly with a concomitant decrease in liquid velocity. This is the condition illustrated in Fig. 3.1 and it will occur when the surface is hydrophilic or when a downstream depth, $H$, of sufficient height is imposed. The nature of this hydraulic jump varies depending on Reynolds number but
we assume here that Type I jump behavior prevails such that the flow after the jump is uniform and unidirectional [9]. We denote the radial location of the jump as $R_j$ and note it is a function of $\theta$.

The domain $r < r_o$, where $\delta < h$ and the velocity profile is developing, is referred to here as Region 1. The domain $r_o < r < R_j$, where the velocity profile is developed, but before the hydraulic jump occurs, is referred to as Region 2. In the classical paper by Watson, [23] he developed similarity solutions to the momentum and continuity equations in both Regions 1 and 2. Dressaire et al. [43] also developed a similarity solution for a specific scenario where slip exists. However, as they pointed out, a similarity solution in the slip case is only possible when the ratio of the slip length to the local thin film height is a constant value. Since this constraint does not reflect the more frequent scenario where the slip length maintains a constant value, we employ the integral approach of Karman and Pohlhausen to obtain an analytical solution [47, 48].

In order to close the problem, we assume that the thin film height or thickness, while varying in the primary spreading direction ($r$), does not vary with $\theta$. The implication of this assumption is that the gravitational force maintains a uniform height in $\theta$. Physical observations from laboratory experiments suggest that this is an excellent assumption and doing so allows solution to the problem.

An integral momentum analysis of a control volume that extends from $z = 0$ to $h$ and is of radial thickness $\Delta r$ (see Fig. 3.1) relates the hydrostatic pressure gradient and local wall shear stress, $\tau_w$, to the spatial variation in the film’s radial momentum. This yields an expression identical to Eq. 1 in Prince et al. [52] with the only difference being the necessary application of the equation over $\theta$.

\[
\rho g h \frac{dh}{dr} + \mu \frac{du}{dz} \bigg|_{z=0} = -\rho \frac{1}{r} \frac{d}{dr} \left( r \int_0^h u^2 dz \right) 
\]  

(3.1)

Here $\rho$ and $\mu$ are the liquid density and viscosity, respectively and $u$ is the local radial velocity in the thin film which varies with $r$, $z$, and $\theta$. In the worst case scenario explored the hydrostatic term exerts less than 0.4% cumulative influence on the dynamics (height, surface velocity, etc.) of the thin film and is thus neglected hereafter for simplicity. In Eq. (3.1), redistribution of radial momentum due to tangential velocity in the thin film has been neglected. This term is expressed as $\partial/\partial \theta \left( \int_0^h \rho u \nu_{\theta} dz \right)/r$, where $\nu_{\theta}$ is the tangential velocity. Compared to the radial momentum term (right hand side of Eq. 3.1) this term is always small for small $\lambda$ and small $Re$. For larger
and $\lambda$, this term, while always smaller than the radial momentum term due to $u$, can become large enough to exert some modest influence. At the end of this section we characterize the relative influence this term.

A statement of conservation of mass for the same control volume over a quadrant is

$$0 = \frac{d}{dr} \left( r \int_0^{\frac{\pi}{2}} \int_0^h u dz d\theta \right)$$

(3.2)

At any radial location the mass flow in the thin liquid film is equal to the incoming jet mass flow,

$$\frac{\pi a^2 V}{4r} = \int_0^{\frac{\pi}{2}} \int_0^h u dz d\theta$$

(3.3)

We assume here a cubic velocity distribution of the form

$$u = a + bz + cz^3$$

(3.4)

The coefficients $a$, $b$, and $c$ are determined from boundary conditions in the $z$ direction and vary with $\theta$.

3.4.2 Region 1

In Region 1, $\delta < h$ and at $z > \delta$ viscous influences vanish. Thus the fluid velocity outside of the boundary layer is equal to the jet velocity. The boundary conditions necessary to determine the coefficients of Eq. 3.4 are: 1) a slip velocity at the plane of the surface, $u(z = 0, \theta) = u_s(\theta)$, 2) a constant velocity (equal to the jet velocity) at the edge of the boundary layer, $u(z = \delta, \theta) = V$, and 3) vanishing shear stress at the edge of the boundary layer, $\partial u/\partial z|_{z=\delta,\theta} = 0$. Applying these conditions results in the following expression for the velocity profile

$$u = u_s + \frac{3}{2} (V - u_s) \frac{z}{\delta} - \frac{1}{2} (V - u_s) \frac{z^3}{\delta^3}$$

(3.5)
The slip velocity, $u_s$, in Eq. 3.5 can be expressed in terms of the so-called slip length by invoking Navier’s slip hypothesis [49].

$$\lambda = \frac{\mu u_s}{\tau_w}$$

(3.6)

Physically, $\lambda$ represents the wall-normal distance (into the wall) where the velocity, $u$, would vanish and is determined by the surface characteristics of the superhydrophobic or slipping surface. For laminar shear flow over a rib/cavity structured surface it has been shown that the slip length can be estimated as a function of cavity fraction, $F_c$, and flow direction relative to the ribs and cavities. When the flow is parallel to the ribs and cavities the slip length may be expressed as [55]

$$\lambda_0 = \frac{w_m}{\pi} \ln \left( \sec \left( \frac{\pi F_c}{2} \right) \right)$$

(3.7)

Here $w_m$ is the total width of the rib and cavity, $w_r + w_c$, and the cavity fraction is the width of a cavity relative to the total width of the rib and cavity, $F_c = w_c/w_m$.

Using these and expressions that follow, the relationship between slip velocity and shear can be specified for each direction, whether parallel (subscript 0 denoting $\theta = 0^\circ$) or perpendicular (subscript 90 denoting $\theta = 90^\circ$) to the rib/cavity structure. Here we follow the work of Crowdy [55] where for the rib/cavity structured superhydrophobic surface the slip length in the $\theta = 0^\circ$ direction relative to the $\theta = 90^\circ$ is [55]

$$\lambda_0 = 2\lambda_{90}$$

(3.8)

$M$ is defined as

$$M = \begin{pmatrix} \lambda_0/\mu & 0 \\ 0 & \lambda_{90}/\mu \end{pmatrix}$$

(3.9)

and the slip velocity is defined as

$$\begin{pmatrix} u_{s0} \\ u_{s90} \end{pmatrix} = M \begin{pmatrix} \tau_{w0} \\ \tau_{w90} \end{pmatrix}$$

(3.10)

Because the slip length is greatest in the direction parallel to the microribs, the shear stress will be least in this same direction. The resultant slip velocity as a function of $\theta$ (see Fig. 3.1) has been
estimated by Crowdy [55] as

\[ u_s(\theta) = \frac{\tau_w(\theta) \lambda_0}{\mu} \sqrt{\cos^2 \theta + \left(\frac{\sin \theta}{2}\right)^2} \]  \hspace{1cm} (3.11)

It has been shown by several investigators that for many superhydrophobic surface topologies and flow conditions the slip length, \( \lambda \), may be assumed to be a constant value, whereas the slip velocity varies with the local wall shear stress. For this reason we choose to employ the slip length in all analysis considered here and recast the slip velocity in terms of it and the local wall shear stress as dictated by Eq. 3.6. The local shear stress is determined by its definition and may be expressed as

\[ \tau_w = \mu \left. \frac{du}{dz} \right|_{z=0} = \frac{3\mu (V - u_s)}{2\delta} \]  \hspace{1cm} (3.12)

Substitution of (3.12) into (3.6) and after some manipulation the slip velocity, as a function of \( \theta \), may be expressed as

\[ u_s = \frac{\lambda V}{\frac{2}{3} \delta + \lambda} \]  \hspace{1cm} (3.13)

Thus, the velocity profile (Eq. 3.5) may be rewritten in terms of \( \lambda \) as

\[ u = \frac{V}{\frac{2}{3} \delta + \lambda} \left[ \lambda + z - \frac{z^3}{3\delta^2} \right] \]  \hspace{1cm} (3.14)

where \( \lambda \) is calculated through substitution of (3.11) into (3.6)

\[ \lambda(\theta) = \lambda_0 \sqrt{\cos^2 \theta + \left(\frac{\sin \theta}{2}\right)^2} \]  \hspace{1cm} (3.15)

The slip length variation as a function of \( \theta \) is shown in Fig. 3.2. Note the asymmetry in the shape of \( \lambda \), with the maximum slip length occurring parallel to the rib/cavity structures at \( \theta = 0^\circ \).

Combining Eqs. 3.1 and 3.2 in Region 1 yields the integral momentum boundary layer equation which holds at each \( \theta \),

\[ \nu \left. \frac{du}{dz} \right|_{z=0} = \left( \frac{d}{dr} + \frac{1}{r} \right) \int_0^\delta (Vu - u^2) \, dz \]  \hspace{1cm} (3.16)
Upon substitution of the velocity profile from (3.14) into (3.16) an equation for the boundary layer thickness as a function of radial position is obtained and is given by Eq. 3.17. Here \( \hat{\delta} \) is a function of \( \hat{r} \) and \( \theta \).

\[
\frac{\pi \hat{r}}{Re} = \left( \frac{2}{3} \hat{\delta} + \hat{\lambda} \right) \frac{d}{d\hat{r}} \left[ \hat{r} \hat{\delta}^2 \left( \frac{13}{210} \hat{\delta} + \frac{1}{4} \hat{\lambda} \right) \left( \frac{2}{3} \hat{\delta} + \hat{\lambda} \right)^{-2} \right]
\]  

(3.17)

In Eq. 3.17 all lengths have been normalized by the jet radius such that \( \hat{\delta} = \delta / a \), \( \hat{\lambda} = \lambda / a \), and \( \hat{r} = r / a \). In the absence of slip (\( \hat{\lambda} = 0 \)), Eqs. 3.16 and 3.17 each reduce to their classical no-slip counterparts. However, due to the slip, a closed form solution to Eq. 3.17 does not exist. Here it is solved numerically following a 4th order Runge Kutta approach to yield the normalized boundary layer thickness, \( \hat{\delta} \), as a function of \( \hat{\lambda}(\theta) \), \( \hat{r} \), and \( Re \). Once the boundary layer thickness is known, the film thickness \( h \), assumed constant in \( \theta \), can be determined by utilization of the conservation of mass principle (Eq. 3.3). Substitution of the assumed velocity profile (3.14) into (3.3) provides

\[
\hat{h} = \frac{1}{2\hat{r}} + \frac{1}{2\pi} \int_0^{\pi/2} \frac{\delta^2}{\frac{2}{3} \hat{\delta} + \hat{\lambda}} d\theta
\]  

(3.18)
3.4.3 Region 2

Region 2 begins when the boundary layer has grown to the height of the thin film thickness and viscous influences extend from the wall to the top of the film. Now the velocity at the top of the film, $U$, is a function of radial position and decreases with increasing $r$. The same form of the assumed velocity profile (Eq. 3.4) is employed in this region, with the second velocity boundary condition now taking the form $u = U$ at $z = h$. Thus, the equation for the velocity in this region looks identical to Eq. 3.14 but with $V$ replaced by $U$ and $\delta$ replaced by $h$.

$$u = \frac{U}{\frac{2}{3}h + \lambda} \left[ \lambda + z - \frac{z^3}{3h^2} \right]$$

(3.19)

Substitution of the above velocity distribution into the momentum equation (3.1) and conservation of mass (3.3), yields

$$- \frac{\pi \hat{U} \hat{r}}{Re} = \left( \frac{2}{3} \hat{h} + \hat{\lambda} \right) \frac{d}{d\hat{r}} \left[ \hat{r} \hat{h} \hat{U}^2 \left( \frac{68}{315} \hat{h}^2 + \frac{5}{6} \hat{h} \hat{\lambda} + \hat{\lambda}^2 \right) \left( \frac{2}{3} \hat{h} + \hat{\lambda} \right)^{-2} \right]$$

(3.20)

and

$$\frac{1}{4\hat{r}} = \frac{1}{\pi} \int_0^\pi \hat{U} \hat{h} \left( \frac{5}{12} \hat{h} + \hat{\lambda} \right) \left( \frac{2}{3} \hat{h} + \hat{\lambda} \right)^{-1} d\theta$$

(3.21)

respectively, where $\hat{U} = U/V$. In (3.20) and (3.21) $\hat{U}$ varies with $\hat{r}$ and $\theta$ and $\hat{h}$ varies with $\hat{r}$.

3.4.4 Approach

The general solution approach for a specified $Re$ and $\hat{\lambda}(\theta)$ was as follows. First Eq. 3.17 was solved at approximately 40 equally spaced $\theta$ values from $0^\circ$ to $90^\circ$ using a $4^{th}$ order Runge Kutta scheme to yield $\hat{\delta}$ as a function of $\hat{r}$ and $\theta$ with the initial condition being $\hat{\delta} = 0$ at $\hat{r} = 0$. Subsequently, $\hat{h}$ was determined by solving Eq. 3.18. This was continued until $\hat{\delta} = \hat{h}$, which occurs at $\hat{r} = \hat{r}_0$ where Region 1 ends and $\hat{h} = \hat{h}_0$. Since slip varies with $\theta$, the solution yields a different value of $\hat{r}_0$ for each $\theta$. Subsequently, as Region 2 begins, Eqs. 3.20 and 3.21 were solved using the same numerical scheme with the initial condition $\hat{h} = \hat{h}_0$ at $\hat{r} = \hat{r}_0$. This provides $\hat{h}(\hat{r})$ and $\hat{U}(\hat{r}, \theta)$ for all $\hat{r} > \hat{r}_0$. When applied to the scenario of isotropic slip with specified $\hat{\lambda}$ or no slip
cases ($\hat{\lambda} = 0$), this approach collapses to the solutions found by Prince et al. [52] and Watson [23], respectively.

The parameter ranges explored in this paper were determined as follows. Reynolds number values were considered from nominally $3 \times 10^3$ up to nominally $2.2 \times 10^4$, which spans the regime where laminar flow dynamics are anticipated. The anisotropic slip considered corresponds to rib/cavity patterned superhydrophobic surfaces with slip variation as described in Sec. 3.4.2. The range of realistic values of normalized slip along the rib direction $\hat{\lambda}_0$ is determined as follows. Previous investigators have shown that in laminar flows along rib/cavity structured surfaces, slip lengths as large as $40 - 50$ microns are realizable. A lower limit on the diameter of an impinging water jet that may be encountered in most applications is nominally $500$ microns ($0.5$ mm). Assuming a surface that yields the largest slip and this lower limit of jet diameter yields a normalized slip value of $\hat{\lambda}_0 = 0.2$. Thus, here we consider values of $\hat{\lambda}_0$ up to $0.2$, although more generally the jet diameter will be larger and values of $\hat{\lambda}_0$ would more likely range up to nominally $0.1$.

### 3.4.5 Hydraulic Jump Location

An expression to estimate the location of the hydraulic jump is now developed. Following the approach of Bush and Aristoff [9] at a given $\theta$, a balance between gravitational and surface tension forces and the momentum fluxes into and out of a control volume of vanishing radial width at the radius of the jump location, $R_j$, yields the following expression

$$
\frac{1}{2} \rho g \left( H^2 - h_j^2 \right) + \frac{\sigma (H - h_j)}{R_j} = \rho \int_{0}^{h_j} u^2 dz - \rho U_1^2 H
$$

(3.22)

$H$ is the downstream water depth (uniform in $\theta$), $h_j$ is the thin film thickness just prior to the jump at $r = R_j$, $\sigma$ is the liquid surface tension and $U_1$ is the downstream water velocity which is assumed to be uniform over the jump height but varying with $\theta$. In his original analysis Watson [23] neglected the surface tension force while Bush and Aristoff [9] showed that it exerts a small, but not completely negligible, influence on predicting the overall jump location. Thus we include the surface tension term here as well.

We consider only strong jumps, such that $H >> h_j$ as is frequently the case and thus the first two terms of Eq. 3.22 are simplified. In determining the radial jump location the maximum
error introduced in the solution is 0.3% due to neglecting $h_j^2$ in the hydrostatic force term and 3% due to neglecting $h_j$ in the surface tension force term. This assumption greatly simplifies the subsequent analysis and since the worst case impact is small it is maintained throughout.

The conservation of mass expression (Eq. 3.3) evaluated for the control volume of vanishing radial width at the radius of the jump location, $R_j$, at a given $\theta$ yields

$$U_1 = \frac{1}{H} \int_0^h u dz$$

(3.23)

where $U_1$ is the velocity downstream of the jump. Substituting (3.23) and the velocity profile of (3.19) into (3.22) gives an equation relating $R_j$ in terms of $h_j$ and $U_j$, where $U_j$ is the free surface velocity of the thin film immediately prior to the jump. Here, $U_j$ and $R_j$ are both functions of radial and angular position.

$$\frac{gH^2}{2} + \frac{\sigma H}{\rho R_j} + H \left[ \frac{U_j h_j}{H} \left( \frac{5}{12} h_j + \lambda \right) \left( \frac{2}{3} h + \lambda \right)^{-1} \right]^2 = h_j U_j^2 \left[ \left( \frac{68}{315} h_j^2 + \frac{5}{6} h_j \lambda + \lambda^2 \right) \left( \frac{2}{3} h_j + \lambda \right)^{-2} \right]$$

(3.24)

Eq. 3.24 can be rewritten in nondimensional form and rearranged to compare with the isotropic case [52]. However, for this scenario $\hat{U}_j$ cannot be easily estimated due to its variation with $\theta$.

$$\frac{a^5 g \hat{H}^2 \hat{R}_j}{Q^2} \left( 1 + \frac{1}{B_0 \hat{R}_j \hat{H}} \right) + \frac{\hat{H} \hat{R}_j}{\pi^2} \left[ \frac{\hat{U}_j \hat{h}_j}{\hat{H}} \left( \frac{5}{12} \hat{h}_j + \hat{\lambda} \right) \left( \frac{2}{3} \hat{h}_j + \hat{\lambda} \right)^{-1} \right]^2 = \frac{\hat{h}_j \hat{U}_j^2 \hat{R}_j}{\pi^2} \left[ \left( \frac{68}{315} \hat{h}_j^2 + \frac{5}{6} \hat{h}_j \hat{\lambda} + \hat{\lambda}^2 \right) \left( \frac{2}{3} \hat{h}_j + \hat{\lambda} \right)^{-2} \right]$$

(3.25)

In Eq. 3.25, $Bo$ is the Bond number, $Bo = \rho g a^2 / \sigma$. In practice $\hat{R}_j$ was determined as follows. The normalized downstream depth, $\hat{H}$, Reynolds number, Bond number, $\hat{\lambda}$, and $ga^5/Q^2$ were specified, where water was assumed to be the working fluid. $\hat{U}(\hat{r}, \theta)$ and $\hat{h}(\hat{r})$ in the thin film were then determined as explained in Sec. 3.4.3. Subsequently, for each direction of $\theta$, Eq. 3.25 was evaluated at varying $\hat{r}$ values until the equation was satisfied exactly. It is at this location that the hydraulic jump occurs and $\hat{R}_j = \hat{r}$.

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3.5 Results

3.5.1 Thin Film Dynamics

We first discuss the dynamics of the spreading thin film. The thin film dynamics have previously been described by Prince et al. [52] for the isotropic slip scenario and this general behavior applies for the present scenario as well. As slip increases the boundary layer grows more slowly, the height of the thin film decreases more rapidly, the momentum flux in the thin film increases, and the hydraulic jump radius increases. These general trends exist at each value of \( \theta \), where at \( \theta = 0^\circ \) (the direction parallel to the ribs and cavities) the greatest slip prevails. The present results show that at the \( \theta \) location corresponding to the average slip (occurring at \( \sim 47^\circ \)), where using Eq. 3.15 with \( \theta = 47^\circ \) we can define \( \hat{\lambda}_{47} = 0.77\hat{\lambda}_0 \) the behavior is nearly exactly that observed when isotropic slip exists. However, in the present case (anisotropic slip), deviation from this behavior at all other \( \theta \) positions exists to varying degrees. This deviation is shown and discussed in this section.

First we discuss a detailed description of the thin film dynamics and its defining characteristics. Specifically the boundary layer growth within the thin film, the variation of the thin film thickness, and the free surface velocity of the film as it spreads are discussed with respect to varying \( \hat{\lambda}_0 \) and \( Re \). Figure 3.3 shows these properties as a function of the normalized radial coordinate for various \( \theta \) values for the scenario of \( Re = 10^4 \) and \( \hat{\lambda}_0 = 0.1 \) (\( \hat{\lambda}_{90} = 0.05 \)). This value of \( \hat{\lambda}_0 \) represents an upper limit on realizable normalized slip lengths for the jet impingement process and is useful in giving general insight into the influence of the anisotropic slip on the dynamics. The images of Fig. 3.3 show the thin film surface velocity (top), the apparent surface slip velocity (middle), and the boundary layer thickness and thin film height as a function of the radial coordinate (bottom). Results are shown at the maximum slip length (\( \theta = 0^\circ \)), at the minimum slip length (\( \theta = 90^\circ \)), and at the average slip length (\( \theta = 47^\circ \)). Results for the isotropic slip scenario at the average slip value (\( \hat{\lambda}_{47} = 0.77\hat{\lambda}_0 \)) are included. Also shown are the results for the no-slip scenario as described by Watson [23].

Boundary layer behavior exists up to \( \hat{r} \approx 6 - 7 \) (Region 1). In this region the velocity at the top of the thin film remains constant at the speed of the incoming jet for all \( \theta \) locations. The boundary layer thickness (denoted by the dashed lines) is increasing with increasing \( \hat{r} \), while the
Figure 3.3: Film surface velocity, $\hat{U}$, average slip velocity over the ribs, $\hat{u}_s$, film thickness, $\hat{h}$, and boundary layer growth, $\delta$, as a function of the radial coordinate for varying $\theta$ directions. Shown for $\hat{\lambda}_0 = 0.1$ and $Re = 10^4$.

thickness of the thin film decreases rapidly. While the boundary layer is smaller at all $\theta$ than for the no-slip scenario, it exhibits only small variation with $\theta$. The slip velocity, $\hat{u}_s$ is a maximum at the stagnation point where the boundary layer thickness vanishes and then decreases with increasing $\hat{r}$. The slip velocity shows significant dependence on $\theta$, with increasing $\theta$ resulting in smaller $\hat{u}_s$. At the transition from Region 1 to Region 2 the $\hat{u}_s$ versus $\hat{r}$ curve exhibits an inflection.

In Region 2, both $\hat{U}$ and $\hat{u}_s$ are a maximum at $\theta = 0^\circ$ and each decreases for increasing $\theta$ (decreasing slip-length). Each of these also decreases with increasing $\hat{r}$ and at all values of $\hat{r}$, $\hat{U}$ for the anisotropic slip scenario is greater than for the no-slip case. Results for all variables shown in
Fig. 3.3, corresponding to the average slip location $\theta = 47^\circ$, agree nearly exactly with the behavior that exists for isotropic slip at this same average value. It is noteworthy that the $\hat{U}$ and $\hat{u}_s$ behavior for the $\theta = 0^\circ$ direction is much closer to the behavior that exists at $\theta = 47^\circ$ than the behavior along the $\theta = 90^\circ$ direction. The implication of this is that the velocity in the thin film exhibits greater deviation as the liquid is flowing transverse to the rib structures than exists in the direction along the ribs.

The height of the thin film is significantly greater for the no-slip case than for the scenarios with either isotropic or anisotropic slip. As explained previously, in order to close the solution $\hat{h}$ does not vary with $\theta$ and thus is the same at all $\theta$ in the case of anisotropic slip. As was shown previously [52], at a fixed $Re$, increasing slip yields greater departure from the no-slip behavior for all of the variables shown in Fig. 3.3.

As discussed above, general behavior of the thin film for the anisotropic slip scenario exhibits similarity with the isotropic slip scenario. Thus it is useful to quantify the difference that prevails relative to this previously solved scenario [52]. To do this we define two important ratios, $U/U_{47}$ and $u_s/u_{s47}$. $U_{47}$ and $u_{s47}$ are the thin film surface velocity and slip velocity at the approximate location of average slip ($\theta = 47^\circ$), which also corresponds to the behavior that prevails when isotropic slip exists with this imposed average slip length. These ratios thus provide the departure that prevails in the anisotropic slip scenario from the isotropic slip case. Figure 3.4 provides these ratios at $\theta = 0^\circ$ (parallel to the rib direction) and at $\theta = 90^\circ$ (transverse to the rib direction), which correspond to the locations of maximum and minimum slip. It is important to note that these two ratios ($U/U_{47}$ and $u_s/u_{s47}$) yield identical values at a specified $\theta$ and $\hat{r}$ and for a given $Re$ and $\hat{\lambda}_0$.

The left panel of Fig. 3.4 illustrates deviation from the isotropic scenario for varying slip length with $Re$ held constant at $10^4$ and the right panel characterizes the deviation at a fixed slip length of $\hat{\lambda}_0 = 0.05$ and for $Re$ ranging from $6\times10^3$ – $1.8\times10^4$. The data of Fig. 3.4 illustrates several important points. First, consistent with the discussion relative to Fig. 3.3, the ratios $U/U_{47}$ and $u_s/u_{s47}$ are always greater than one in the longitudinal direction ($\theta = 0^\circ$) and they are always less than one in the transverse direction ($\theta = 90^\circ$), due to the slip-length being a maximum at $\theta = 0^\circ$ and a minimum at $\theta = 90^\circ$. Second, these ratios exhibit greater departure from unity as $\hat{r}$ increases. The implication is that there is greater $\theta$ variation in the thin film surface velocity and slip velocity with increasing radial location from the center of the impinging jet. As will be
Figure 3.4: $U/U_{47}$ and $u_\theta/u_{s47}$ as a function of $\hat{r}$ along $\theta = 0^\circ$ (solid lines - parallel to the rib structures) and $\theta = 90^\circ$ (dashed lines - transverse to the ribs). In the left panel $Re = 10^4$ and $\hat{\lambda}_0$ is varied from 0.0125 to 0.2, while in the right panel $\hat{\lambda}_0 = 0.05$ and $Re$ is varied from $6 \times 10^3$ to $1.8 \times 10^4$.

discussed further below, this has important implications for the shape of the hydraulic jumps that form. Third, as the Reynolds number increases, the deviation from the isotropic scenario decreases. This behavior prevails for all slip lengths explored and the implication is that for slower jet speeds the thin film will exhibit larger variability in the film surface and slip velocities with varying $\theta$.

Lastly, the $U/U_{47}$ and $u_\theta/u_{s47}$ data show an interesting trend with varying $\hat{\lambda}_0$. As $\hat{\lambda}_0$ increases at a fixed $Re$ and position, initially these ratios show increased departure from unity (the isotropic scenario). However, above a certain value of $\hat{\lambda}_0$, the deviation from the isotropic case actually starts to decrease. For the $Re = 6 \times 10^3$ scenario illustrated in the left panel of Fig. 3.4, the greatest departure is at $\hat{\lambda}_0 = 0.1$ and at $\hat{\lambda}_0 = 0.2$ the deviation from unity is smaller. Similar behavior was observed for all $Re$ explored.

The assumption of negligible $r$-momentum due to velocity in the $\theta$ direction, $v_\theta$, is now discussed. An estimate of the magnitude of $v_\theta$ can be made using conservation of mass in each
cell, such that Eq. 3.2 becomes

\[ 0 = \int_0^h \frac{1}{r} \frac{\partial}{\partial r} (ru) \, dz + \int_0^h \frac{1}{r} \frac{\partial}{\partial \theta} (v_\theta) \, dz \]  \tag{3.26} \]

We define \( \bar{v}_\theta \) as the average tangential velocity at a given \( r \) and \( \theta \) position and estimate its value. A vector plot of \( \bar{v}_\theta \) and \( \bar{u} \) (average radial velocity) is shown in Fig. 3.5 for \( \hat{H} = 8 \) and \( \hat{\lambda}_0 = 0.05 \). The solid line is the location of the hydraulic jump. The left panel displays the vectors when \( Re = 6 \times 10^3 \). The variation in vector direction as the thin film spreads is small but noticeable. In the right panel \( Re = 2.2 \times 10^4 \). Here the variation in vectors is larger and the flow direction turns toward the longitudinal axes. Note that the maximum \( \bar{v}_\theta \) occurs near \( \theta = 45^\circ \) and increases relative to \( \bar{u} \) with increasing radius. These plots demonstrate a first-order estimate of the magnitude of \( \bar{v}_\theta \) within the thin film which would be present for the analysis accomplished here.

![Figure 3.5: Vector representation of the average \( r \) and \( \theta \) velocities up to the hydraulic jump. The vector in the upper right corner of each panel represents the incoming jet velocity, \( V \). In both panels \( \hat{H} = 8 \) and \( \hat{\lambda}_0 = 0.05 \). In the left panel \( Re = 6 \times 10^3 \) and in the right panel \( Re = 2.2 \times 10^4 \).](image)

The importance of \( v_\theta \) is in its contribution to momentum in the \( r \)-direction. With the estimation of \( \bar{v}_\theta \) from (3.26) an approximation of the \( r \)-momentum due to velocity in the \( \theta \) direction can be defined as \( P_{\theta r} = \frac{\partial}{\partial \theta} \left( \int_0^h \rho u v_\theta \, dz \right) / r \). Using \( \bar{u} \) and \( \bar{v}_\theta \) this becomes \( P_{\theta r} \sim \frac{\partial}{\partial \theta} (\rho \bar{u} \bar{v}_\theta) / r \). To estimate the cumulative effect of neglecting this term it is computed over the entire thin film region up to the hydraulic jump, such that \( \tilde{P}_{\theta r} = \int_0^{R_j} P_{\theta r} \, dr / R_j \). This value is compared with the \( r \)-momentum due to velocity in the \( r \)-direction, \( P_{rr} \), which is the right hand side of (3.1) integrated.
over the thin film as well, such that \( \bar{P}_{rr} = \int_0^R P_{rr} dr / R_j \). The ratio \( \bar{P}_{\theta r} / \bar{P}_{rr} \) is shown in Fig. 3.6 as a function of \( \hat{\lambda}_0 \) for \( Re = 6 \times 10^3 \), \( 1.4 \times 10^4 \), and \( 2.2 \times 10^4 \) for a constant \( \hat{H} = 8 \). The relative influence of \( v_\theta \) increases with increasing \( Re \) and \( \hat{\lambda}_0 \). Yet even at a relatively large \( Re \) of \( 2.2 \times 10^4 \) and slip length the estimated momentum due to \( v_\theta \) is only 12.5\% of the \( r \)-momentum due to \( u \). At a lower \( Re \) of \( 6 \times 10^3 \) and easily realizable value of \( \hat{\lambda}_0 = 0.025 \) the ratio is only 5\%. Note the effects of slip on the ratio are negligible for higher slip values, and this occurs sooner for lower \( Re \). Thus the effects of neglecting \( v_\theta \) are most pronounced (greater than 10\%) at \( Re > 2 \times 10^4 \) and \( \hat{\lambda}_0 > 0.075 \).

Figure 3.6: Ratio of \( r \)-momentum due to \( \theta \) velocity, \( P_{\theta r} \), to \( r \)-momentum due to radial velocity, \( P_{rr} \). Ratio is plotted as a function of \( \hat{\lambda}_0 \) for three Reynolds numbers at a constant \( \hat{H} = 8 \).

### 3.5.2 Hydraulic Jump Shape

When the jet impingement process occurs at an immersed surface with an imposed depth of liquid, a hydraulic jump will occur. The shape and location of this jump are directly impacted by the magnitude and variability of momentum in the thin film discussed in the previous section. For a rib/cavity structured superhydrophobic surface, as considered here, anisotropic slip prevails and the hydraulic jump shape deviates from the classical circular jump. The left panel of Fig. 3.7 shows the shape of the hydraulic jump (a top-down view) for longitudinal slip lengths, \( \hat{\lambda}_0 \), varying from 0 to 0.1 at a fixed Reynolds number of \( Re = 10^4 \) and the right panel shows the same view for
a fixed longitudinal slip length at $\hat{\lambda}_0 = 0.05$ ($\hat{\lambda}_{90} = 0.025$) and for Reynolds numbers ranging from $6 \times 10^3$ to $1.8 \times 10^4$. In both panels $\hat{H} = 8$. For the no-slip scenario the jump is of course circular. However, the resultant shape in the presence of anisotropic slip is a nominal elliptical shape with the major axis of the shape being aligned parallel to the rib/cavity structures ($\theta = 0^\circ$). As the slip length increases, the jump radius generally increases as well. This is evident at all $\theta$, although the increase is greater along the rib direction ($\theta = 0^\circ$) than it is in the transverse direction. In a similar manner, as the Reynolds number increases the radial location of the jump increases as well, with the increase again greater in the longitudinal direction. An increase in either $Re$ or $\hat{\lambda}$ results in an increased net momentum flux in the thin film, which leads directly to the increased jump radius. However the unbalanced slip in the longitudinal and transverse directions yield more eccentric jumps as the slip length or Reynolds number increase.

![Figure 3.7](image)

Figure 3.7: Hydraulic jump location as a function of the $x(\theta = 0^\circ)$ and $y(\theta = 90^\circ)$ locations where $\hat{H} = 8$. In the left panel $\hat{\lambda}_0$ is varied from 0 to 0.1 for a fixed value of $Re$ at $10^4$, and in the right panel $Re$ is varied from $6 \times 10^3$ to $1.8 \times 10^4$ while $\hat{\lambda}_0 = 0.05$. 
3.5.3 Hydraulic Jump Location

The radial location of the hydraulic jump and how it is influenced by surface slip is now further discussed. The hydraulic jump occurs where there is a local balance between the momentum of the thin film and the gravitational and surface tension forces exerted by the imposed water depth, as described in Eq. 3.25. This radial location is a function of the jet Reynolds number, Bond number, $ga^5/Q^2$, downstream depth, $\hat{H}$, and the relative slip length, $\hat{\lambda}$.

Figure 3.8 shows the hydraulic jump location, $\hat{R}_j$, as a function of $\theta$ for $\hat{H} = 8$ and varying slip lengths and a fixed Reynolds number (left panel) and for varying Reynolds number and a fixed slip length (right panel). The left panel includes the no-slip case, which yields a constant value and provides a reference to which the scenarios with slip can be compared. Generally $\hat{R}_j$ increases with increasing slip length or Reynolds number. The only exception to this is at relatively small slip lengths where $\hat{R}_j$ actually decreases modestly in the vicinity of $\theta = 90^\circ$. Generally, as slip increases the increased value of $\hat{R}_j$ along the rib direction is much greater than the increase in $\hat{R}_j$ along the transverse direction. The primary physical reason why this occurs is that in the thin film there is local redistribution of mass flow from the $\theta = 90^\circ$ direction towards the $\theta = 0^\circ$ direction, resulting in greater momentum in the thin film along the rib direction. This naturally pushes the jump location outward. Note also that the shapes of the $\hat{R}_j$ vs. $\theta$ curves in the left panel of Fig. 3.8 exhibit similarities, except the no-slip scenario.

Figure 3.8: Hydraulic jump radius, $\hat{R}_j$, as a function of $\theta$. In the left panel $\hat{\lambda}_0$ varies from 0 to 0.1 for $Re = 10^4$, and in the right panel $Re$ varies from $6\times10^3$ to $1.8\times10^4$, while $\hat{\lambda}_0 = 0.05$. In both panels $\hat{H} = 8$. 
The right panel of Fig. 3.8 illustrates two points related to the Reynolds number influence on the shape of the hydraulic jump. First, at low $Re$ the disparity in jump radius between the two primary spread directions is very small. At low $Re$ the jump radius is relatively small and consequently the integrated influence of slip on the thin film momentum remains small. However, as $Re$ increases, with a concomitant increase in $\hat{R}_j$, the redistribution of mass flow in the thin film becomes more pronounced. Consequently, increasing $Re$ leads to greater eccentricity in the jump shape with $\hat{R}_j$ along the ribs being much greater than in the transverse direction.

Bush and Aristoff [9] developed the following expression that predicts the jump radius of a classical hydraulic jump on a hydrophilic surface as a function of relevant parameters where slip is not considered.

$$\frac{g a^5 \hat{H}^2 \hat{R}_j}{Q^2} \left(1 + \frac{2}{Bo\hat{H}\hat{R}_j}\right) + \frac{1}{2\pi^2 \hat{H} \hat{R}_j} = 0.01676 \left[\left(\frac{\hat{R}_j^3}{Re}\right) + 0.1826\right]^{-1} \quad (3.27)$$

Shown in Fig. 3.9 are the terms from the left hand side of Eq. 3.27 plotted as a function of $\hat{R}_j^3/Re$, as was first done by Bush and Aristoff [9] and allows influence from all parameters except the slip to be described by a single curve. A curve is shown for the classical no-slip scenario which matches that presented previously by Bush and Aristoff [9]. A curve is also shown corresponding to the $\theta = 47^\circ$ position for values of $\hat{\lambda}_0$ ranging from 0.025 to 0.2. Results are also shown for $\hat{\lambda}_0 = 0.1$ and along $\theta = 0^\circ$ and $\theta = 90^\circ$. These results reveal that increasing slip yields curves that are similar to the no-slip behavior, but that are shifted towards the right (i.e., larger jump radius). Further, the data show that, for a given $\hat{\lambda}_0$ value, along the $\theta = 0^\circ$ direction the curve is shifted further to the right and along the $\theta = 90^\circ$ direction the data lie to the left of the line corresponding to the average slip location ($\theta = 47^\circ$).

For the scenario of isotropic slip, Prince et al. [52] proposed a modification to Eq. 3.27 to include slip. They include a function, $g(\hat{\lambda})$, that modified the expression of Bush and Aristoff [9] to allow for a collapse of curves with different slip length values. The right hand side of Eq. 3.27 with this $g(\hat{\lambda})$ modifier becomes

$$0.01676 \left[\left(\frac{\hat{R}_j^3}{Re}\right)g(\hat{\lambda}) + 0.1826\right]^{-1} \quad (3.28)$$
Figure 3.9: Left hand side of Eq. 3.27 as a function of $\hat{R}_3/Re$ for $\hat{\lambda}_0$ ranging from 0 to 0.2 along the direction of average slip length ($\theta = 47^\circ$). Results are also shown along $\theta = 0^\circ$ and $\theta = 90^\circ$ at $\hat{\lambda}_0 = 0.1$.

where the function proposed by Prince et al. [52] is

$$g(\hat{\lambda}) \approx \left[1 + 5.4\hat{\lambda} + 73\hat{\lambda}^2\right]^{-1}$$

(3.29)

for the case of isotropic slip. A similar collapse can be accomplished along the direction of average slip ($\theta = 47^\circ$) using a function of the same form

$$g(\hat{\lambda}_{47}) \approx \left[1 + 4.2\hat{\lambda}_{47} + 32\hat{\lambda}_{47}^2\right]^{-1}$$

(3.30)

where $\hat{\lambda}_{47}$ is the average slip length. These values vary from the isotropic case due to the varying surface tension term as the radius of curvature of the ellipse varies with $\theta$.

Figure 3.10 shows the terms on the left hand side of (3.27) as a function of $(\hat{R}_3/Re)g(\hat{\lambda})$ for varying slip lengths as shown in the figure legend. Data are shown along the direction of average slip ($\theta = 47^\circ$), along the ribs ($\theta = 0^\circ$), and in the transverse direction ($\theta = 90^\circ$). The figure shows that along the $\theta = 47^\circ$ direction the curves for all slip values have collapsed to a single curve by the introduction of $g(\hat{\lambda}_{47})$. Along the $\theta = 0^\circ$ and $\theta = 90^\circ$ directions the data depart from this curve with the curves lying above the line corresponding to the $\theta = 0^\circ$ direction. The utility of Fig. 3.10, Eq. 3.27, and Eq. 3.30 are that they allow prediction of the radial location of the hydraulic jump.
Figure 3.10: Left hand side of Eq. 3.27 as a function of \( \hat{R}_j^{3}/Re \) for \( \hat{\lambda}_0 \) ranging from 0 to 0.2 along the direction of average slip length \( (\theta = 47^\circ) \), along the rib direction \( (\theta = 0^\circ) \), and along the transverse direction \( (\theta = 90^\circ) \).

along the ribs, transverse to the ribs, and at \( \theta = 47^\circ \) for any laminar flow jet impingement scenario on a superhydrophobic surface comprised of alternating ribs and cavities.

3.5.4 Hydraulic Jump Radius Ratio

The ratio \( R_{j0}/R_{j90} \) gives a quantitative measure of the eccentricity of the jump where we define \( R_{j0} \) as the hydraulic jump radius in the longitudinal direction \( (\theta = 0^\circ) \) and \( R_{j90} \) as the jump radius in the transverse direction \( (\theta = 90^\circ) \). For the no slip and isotropic slip scenarios, the jump ratio will be equal to one. Figure 3.11 shows the hydraulic jump ratio as a function of normalized slip length \( \hat{\lambda}_0 \). The left panel shows \( R_{j0}/R_{j90} \) for a range of downstream depths \( \hat{H} \) all at a constant Reynolds number of \( 10^4 \), and the right panel shows \( R_{j0}/R_{j90} \) for a range of Reynolds numbers plotted at a constant downstream depth of \( \hat{H} = 8 \). In both panels of Fig. 3.11 the jump ratio is greater than unity when slip is present and increases with increasing \( \hat{\lambda}_0 \). The rate of increase significantly diminishes after about \( \hat{\lambda}_0 = 0.05 \), although it does continue to increase slightly for all \( \hat{\lambda}_0 \) explored. This behavior is expected because as the slip length becomes large the influence of friction begins to decrease and when taken to the extreme case of infinite slip, the jump shape would revert to being circular since the flow would be essentially inviscid.
Figure 3.11: Hydraulic jump ratio, $R_{j0}/R_{j90}$, as a function of normalized slip length, $\hat{\lambda}_0$. The left panel shows results for four different normalized downstream depths at a constant $Re = 10^4$ and the right panel shows results for four different Reynolds numbers at a constant downstream depth of $\hat{H} = 8$.

It can also be noted in the left panel that as the downstream depth $\hat{H}$ increases, the jump ratio decreases. The implication of this is that at larger $\hat{H}$ the actual jump radius is smaller and thus there is less thin film area over which the slip can exert influence. In the right panel, the jump ratio increases with increasing Reynolds number values. This is to be expected because at larger Reynolds numbers the jump radius moves outward increasing the area where slip can exert influence. In summary, both larger slip lengths and Reynolds numbers result in larger jump radius ratios, while larger downstream depths cause the jump radius ratio to be smaller.

### 3.5.5 Comparison to Experimental Results

Johnson [56] performed jet impingement experiments on superhydrophobic surfaces patterned with microribs. He used a pressurized plenum to supply a steady flow of deionized water through a nozzle of radius $r = 0.6\text{mm}$. The jet emerging from the nozzle impinged normally onto an adjustable horizontal plate surrounded by a reservoir. This configuration allowed for control of the imposed downstream depth. Test surfaces were attached to the plate and viewed with two CCD cameras set up orthogonally relative to each other to allow for simultaneous measurement of the hydraulic jump radius in both the longitudinal and transverse directions.
Both smooth and superhydrophobic surfaces patterned with microribs were tested. A variety of cavity fractions were tested. These surfaces were manufactured using standard photolithographic methods. A thin layer of chromium was deposited to enhance the adhesion of the subsequent Teflon coating that rendered the surfaces superhydrophobic.

Figure 3.12 shows the hydraulic jump ratio as a function of the normalized downstream depth $\hat{H}$ for three different Reynolds numbers for $F_c = 0.8$. This corresponds to $\hat{\lambda}_0 = 0.025$. The experimental data could only be acquired over specific $\hat{H}$ ranges for each $\hat{\lambda}$ and Re combination. If $\hat{H}$ was too low then the surface tension force pulled the water downstream of the hydraulic jump off of the test plate and the impingement process changed to an unimmersed scenario. If $\hat{H}$ was too high, however, the jump would collapse inward onto the incoming jet. The data from Johnson’s experimental work are compared to the results of the analytical model presented in this paper in Fig. 3.12 where $R_{j0}/R_{j90}$ is plotted as a function of $\hat{H}$ for three Reynolds numbers. The experimental results follow the same trends as the model, with $R_{j0}/R_{j90}$ increasing for increasing Reynolds number. Further, the ratio $R_{j0}/R_{j90}$ decreases with increasing $\hat{H}$. However, in all cases the experimental data exhibit a slightly higher jump ratio than the model predicts. The experimental data show better agreement with the analytical model at lower Reynolds numbers, but all agreement
is very good. At $Re = 1.15 \times 10^4$ the average deviation between the experimental data and the analytical model is 2.9% and at $Re = 2.14 \times 10^4$ the average deviation is 5.8%.

### 3.6 Conclusion

The laminar flow dynamics of a liquid jet impinging on a surface with anisotropic slip where there is an imposed downstream depth has been modeled. The mass and momentum equations have been solved numerically following the approach of Karman and Pohlhausen [47, 48]. It has been assumed that the height of the thin film does not vary azimuthally and that the slip length varies azimuthally as predicted by Crowdy [55]. The thin film dynamics are still predicted well by the work of Prince et al. [52] for isotropic slip, and in the direction of average slip length the thin film behavior nearly exactly matches the isotropic scenario behavior. As the Reynolds number or the slip length increases, the deviation of the anisotropic scenario from the isotropic case increases for all other azimuthal directions, becoming the most pronounced in the flow directions along and transverse to the rib/cavity structures.

The shape of the hydraulic jumps formed on a surface with anisotropic slip are all elliptical with the eccentricity increasing as the slip length and Reynolds number increase or as the downstream depth decreases. A collapse similar to that found by Prince et al. [52] for the isotropic slip case was found for the anisotropic slip scenario along the direction of the average slip length. This allows the hydraulic jump location to be predicted as a function of the relevant flow parameters. While in the longitudinal and transverse directions the hydraulic jump data does not collapse, the radial location of the hydraulic jump has been found for all slip lengths explored. This data allows for prediction of the hydraulic jump radii in the longitudinal and transverse directions for any practically realizable slip lengths.

### 3.7 Acknowledgments

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CHAPTER 4. ON JET IMPINGEMENT AND THIN FILM BREAKUP ON A HORIZONTAL SUPERHYDROPHOBIC SURFACE

This chapter is to be submitted to Physics of Fluids, but has been modified here to match the stylistic requirements of this dissertation.

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4.2 Abstract
When a vertical laminar jet impinges on a horizontal surface it will spread out in a thin film on the surface. If the surface is hydrophobic, and a downstream depth is not maintained the film will breakup into droplets. We present an analysis that describes the location where break up occurs. All surfaces explored are hydrophobic or superhydrophobic (SH), where the SH surfaces exhibit an apparent slip at the surface. The influence of slip on the entire flow field is significant and yields behavior that deviates notably from classical behavior for a smooth hydrophilic surface where a hydraulic jump would form. Instead, break up into droplets occurs where the jet’s outward radial momentum is balanced by the inward surface tension force of the advancing film. For hydrophobic surfaces, or SH surfaces with random micropatterning, the slip on the surface is uniform in all directions and droplet breakup occurs in a circular pattern. When alternating rib/cavity microstructures are used to create a SH surface the slip varies as a function of the azimuthal coordinate, and thus the breakup location also varies with azimuth, and here the result is an elliptically shaped breakup. The thin film dynamics are modeled by a radial momentum analysis for a given jet Weber number and specified slip length. We then determine the location of breakup for multiple surfaces over a range of jet Weber numbers and realistic slip length values. Results show
the breakup radius increases with increasing Weber number and slip length. The eccentricity of the breakup ellipse for the rib/cavity superhydrophobic structures increases with increasing Weber number and slip length as well. The model results for the breakup radial position are compared to experimental measurements with good agreement.

4.3 Introduction

This paper addresses thin film breakup into droplets when a liquid jet impinges normally on a hydrophobic surface or superhydrophobic (SH) surface with slip. Superhydrophobic surfaces are fabricated with a combination of microscale surface patterning and a hydrophobic coating. These surfaces are of importance due to the apparent slip that prevails at the tops of the microstructured features when the Cassie-Baxter state (where air is trapped in the cavities between structures) is maintained [28]. In general, the slip increases with increasing cavity fraction [29, 30], $F_c$, defined as the relative size of the cavity regions. For a rib and cavity structured surface $F_c = w_c / (w_c + w_r)$ where $w_c$ is the width of a cavity and $w_r$ is the width of a rib. If the surface patterning is the same in all directions, or random, isotropic slip at the surface will prevail, while if the surface patterning consists of alternating ribs and cavities, anisotropic slip exists. The dynamics of liquids interacting with SH surfaces have been explored by several investigators for a wide variety of liquid flow scenarios including liquid jets, impinging and rolling droplets, and laminar and turbulent channel flows [11, 28, 31–41]. Thus far relatively little attention has been devoted to liquid jet impingement on SH surfaces. Superhydrophobic surfaces exhibiting isotropic and anisotropic slip scenarios are considered in this paper. The overall slip is generally characterized by the slip length, $\lambda$, defined to be the wall normal distance (into the wall) where the magnitude of the aggregate velocity profile vanishes.

When a liquid jet impinges on a smooth horizontal hydrophilic surface, a classical no-slip boundary condition exists. Due to this frictional resistance the fluid spreads out as a thin film and slows until the film experiences a hydraulic jump. For a smooth horizontal surface the shape of the hydraulic jump is circular and this scenario has been explored by multiple authors both analytically and experimentally [9, 17–24, 26]. Watson developed an analytical model to express the thin film thickness, including the boundary layer, as a function of radial location. He also described the jump radius as a function of the liquid jet velocity and diameter, downstream liquid depth, and
fluid properties [23]. Existing experimental data show varying goodness of agreement to Watson’s model, with generally better agreement at larger jump radii [21] where surface tension due to the azimuthal curvature of the jump is negligible. Bush and Aristoff later expanded the model to account for the influence of surface tension [9].

In recent papers, we extended the analysis of Bush and Aristoff to account for isotropic slip of constant slip length [52] and anisotropic slip with azimuthally varying slip length [57] at the surface. Such slip can prevail on SH surfaces with isotropic and anisotropic surface patterning respectively. When a downstream depth is imposed, a hydraulic jump forms that deviates from the classical behavior. These previous studies revealed that increasing slip length results in increased average velocity in the thin film, a thinner film thickness, a smaller local wall skin friction coefficient, an increase in the radial location of the hydraulic jump that forms downstream, and in the anisotropic case, an elliptically shaped jump. Further, the results allowed formulation of a model to predict the hydraulic jump location as a function of the characteristic surface slip length and azimuthal direction.

Recent papers by Dressaire et al. also considered a related phenomena [42, 43]. They considered jet impingement on hydrophilic surfaces in the Wenzel state (water wets the cavities) with varying microscale post features with cavity scales on the order of 100 microns. These feature sizes are similar to what might exist on a SH surface and they modeled the slip-length with a varying value. Hydraulic jumps were observed to exhibit variable polygonal shapes dependent on the post structure, where directions of greatest slip (most cavity surface area) corresponded to vertices of the polygon shapes. However, in this study, the cavities were flooded and the Cassie-Baxter state did not exist.

When a downstream depth is not present, and a liquid jet impinges on a smooth hydrophobic or patterned SH surface, the physics differ radically from the hydrophilic case described above. An experimental study by Maynes et al. showed that in this scenario the radially spreading jet will have one of three possible breakup transitions depending on the initial jet Weber number, $We = \rho V^2 a/\sigma$ where $\rho$ is the fluid density, $V$ is the incoming jet velocity, $a$ is the radius of the incoming jet, and $\sigma$ is the surface tension [10]. For $We < 500$ at the breakup location the depth increases abruptly and a thin water filament sets up around the periphery, as shown in the left panel of Fig. 4.1. For $500 \leq We \leq 750$ the water filament begins to break up into droplets, but they
Figure 4.1: Images of liquid jet impingement on a SH surface with the thin film breaking up into droplets. The frame rate was 500 fps and in this scenario the surface was SH with the rib/cavity structures aligned along the horizontal axis. In the left panel $We = 135$ and in the right panel $We = 930$.

are generally large, bulbous drops. At $We > 750$ the thin film breaks up into droplets around the perimeter at the transition point as shown in the right panel of Fig. 4.1. Both smooth hydrophobic surfaces and structured SH surfaces were explored in this previous work. The SH surfaces had anisotropic surface patterning consisting of alternating ribs and cavities of nominal widths in the 4 and 34 micron ranges, respectively. As with the hydraulic jump on an anisotropic surface, the transition was observed to exhibit an elliptical shape, with the major axis of the elliptical shape aligned parallel to the rib/cavity direction. The transition occurred nominally where the local Weber number, defined in terms of the local average velocity and thickness of the thin film ($We_l = \rho \bar{u}^2 h / \sigma$ where $\bar{u}$ is the average thin film velocity and $h$ is the height), was equal to unity.

While limited experimental work has addressed some aspects of the thin film breakup transition after a liquid jet impinges on a smooth hydrophobic or rib/cavity patterned SH surface with anisotropic slip, no analytical framework has been presented to model this general phenomenon. Thus, the focus of this paper is to present analysis of thin film breakup radius on smooth hydrophobic surfaces and patterned SH surfaces with both isotropic and anisotropic slip with a slip-length that varies in the azimuthal direction. Breakup is estimated to occur when the surface tension force balances the momentum present in the radially expanding thin film. The analytical model will be developed in Sec. 4.4. Section 4.5 provides results from the analysis where the influence of contact angle and slip, both isotropic and anisotropic, on the thin film dynamics and the resultant thin film
breakup are quantified. Results from the model are also compared to experimental data. Finally, conclusions of the work will be given.

4.4 Analysis

4.4.1 Model Description

The model of the radially expanding thin film on the surface follows the work of Prince et al. [52,57] and is summarized here for clarity. We consider a vertical liquid jet of speed $V$, volume flow rate $Q$, viscosity $\nu$ and radius $a$ impinging a horizontal surface that exhibits hydrophobicity (smooth surface) or superhydrophobicity (random patterned surface or rib/cavity structured) such that the contact angle ($\phi$) of the advancing thin film is greater than 150° as illustrated in Fig. 4.2. Note the scale of the post features illustrated in the figure inset; thousands of post structures would exist under the thin film region. The jet spreads out in a thin film with decreasing thickness, $h$, as it moves radially outward. For Reynolds numbers ($Re = Q/a\nu$) smaller than nominally $2.5 \times 10^4$, a laminar boundary layer of thickness $\delta$ develops from the impingement point and grows with increasing radial coordinate $r$ [23]. The boundary layer $\delta$ is also a function of $\theta$, the azimuthal coordinate, for the rib/cavity structured surface due to the anisotropy of the surface slip. The boundary layer grows until it is equal to the height ($\delta(r, \theta) = h(r)$) of the thin film at $r = r_o(\theta)$. The velocity outside of the boundary layer is equal to the constant jet velocity as the boundary layer grows and at $r > r_o$ the velocity at the air-liquid interface, $U(r, \theta)$, begins to decrease with increasing $r$ due to the viscous influence of the wall. For the superhydrophobic scenario at $z = 0$, where $z$ is the coordinate normal to the surface, an apparent slip velocity, $u_s$, that varies with $r$ and $\theta$ (in the anisotropic case) exists. Also at the leading edge of the thin film (the breakup location) the curvature due to the surface tension is quantified by the advancing contact angle of the liquid as illustrated in Fig. 4.2. It is the influence of the contact angle and slip on the breakup location that is the primary focus of this paper.

The parameter ranges explored in this paper include Weber numbers ranging from nominally 100 to 1000. For water and a jet diameter of 0.6 mm these correspond to Reynolds numbers from nominally $6.5 \times 10^3$ up to nominally $2.2 \times 10^4$, which spans the majority of the regime where laminar flow dynamics are anticipated. The anisotropic slip scenario corresponds to rib/cavity pat-
terned superhydrophobic surfaces with slip variation in \( \theta \). Slip lengths as large as 40 – 50 microns in laminar flows along rib/cavity structured surfaces have been realized by previous investigators. A lower limit on the diameter of an applicable impinging water jet is nominally 500 microns (0.5 mm). Assuming a surface that yields both the largest slip and the lower limit of jet diameter yields a normalized slip value of \( \hat{\lambda}_0 = 0.2 \). Thus, here we consider values of \( \hat{\lambda} \) ranging from the no-slip scenario, \( \hat{\lambda} = 0 \) up to 0.2, although more generally the jet diameter will be larger and values of \( \hat{\lambda} \) would more likely range up to nominally 0.1.

Figure 4.2: Schematic illustration of liquid jet impingement on a SH surface with the thin film breaking up into droplets. The top panel shows the side view and the bottom panel shows a top view. Inset is an SEM of microposts with 8\( \mu \)m diameter.

After jet impingement and thin film development, two conditions are possible. The first possible condition occurs on hydrophilic surfaces or hydrophobic and SH surfaces where a downstream depth, \( H \), of sufficient height is imposed. Here a hydraulic jump will exist, where the liquid depth increases suddenly with a concomitant decrease in liquid velocity. The second condition occurs on hydrophobic and SH surfaces where a very low downstream water depth is imposed [10]. Under these conditions the spreading film reaches a point where the surface tension in the thin
film balances the film momentum and causes it to break up into droplets that continue to propagate outward. We denote the radial location at breakup as $R_b$ and note it is a function of $\theta$ when the SH surface structure is anisotropic.

Region 1 is defined as the domain in the thin film where the velocity profile is developing, where $r < r_o$ and $\delta < h$. Region 2 is the domain $r_o < r < R_b$, where the velocity profile is developed, but before breakup of the film occurs. Similarity solutions to the momentum and continuity equations in both Regions 1 and 2 were developed by Watson for the no-slip scenario [23]. Dessaire et al. [43] also developed a similarity solution for the special case where the ratio of the slip length to the local thin film height is a constant value. Since a more common scenario is a constant slip length, Prince et al. [52, 57] employed the integral approach of Karman and Pohlhausen to obtain an analytical solution for this scenario [47, 48]. To close the problem, the assumption of a uniform thin film height with varying $\theta$ was made and experimental observation confirms this is a good assumption.

The solution in both Regions 1 and 2 consists of conserving mass and momentum from the jet nozzle exit through the thin film. A statement of conservation of mass for a control volume extending from $z = 0$ to $h$ of radial thickness $\Delta r$ over a quadrant is

$$0 = \frac{d}{dr} \left( r \int_0^\theta \int_0^h u dz d\theta \right)$$

(4.1)

where $u$ is the local radial velocity in the thin film which varies with $r$, $z$, and $\theta$. At any radial location the mass flow in the thin liquid film is equal to the incoming jet mass flow,

$$\frac{\pi a^2 V}{4r} = \int_0^\theta \int_0^h u dz d\theta$$

(4.2)

A momentum balance for the same control volume that includes forces due to the local wall shear stress, the hydrostatic pressure gradient, and the spatial variation in the film’s radial momentum yields

$$\rho gh \frac{dh}{dr} + \mu \frac{du}{dz} \bigg|_{z=0} = -\rho \frac{1}{r} \frac{d}{dr} \left( r \int_0^h u^2 dz \right)$$

(4.3)

Here $\rho$ and $\mu$ are the liquid density and viscosity, respectively. In the anisotropic case, Eq. 4.3 is evaluated at increments in the azimuthal direction of $\Delta \theta = 2.3^\circ$. The hydrostatic term was found
to exert less than 0.4% cumulative influence on the dynamics (height, surface velocity, etc.) of
the thin film and was thus neglected. Also in the anisotropic case, Eq. 4.3 has neglected the
redistribution of radial momentum due to tangential velocity in the thin film. This term was found
to be negligible for the range of parameters explored in this paper [57].

A cubic velocity distribution within the boundary layer (Region 1) or entire thin film (Re-
region 2) is assumed of the form

\[ u = a + bz + cz^3 \] (4.4)

The coefficients \( a, b, \) and \( c \) are determined from boundary conditions in the \( z \) direction and vary
with \( \theta \) for the anisotropic slip scenario. In Region 1 this profile models behavior within the bound-
dary layer and in Region 2 it is valid over the entire height of the thin film.

In Region 1 the boundary conditions necessary to determine the coefficients of Eq. 4.4 are;
1) a slip velocity at the plane of the surface, \( u(z = 0, \theta) = u_s(\theta) \), 2) a constant velocity (equal to the
jet velocity) at the edge of the boundary layer, \( u(z = \delta, \theta) = V \), and 3) vanishing shear stress at the
edge of the boundary layer, \( \partial u / \partial z |_{z=\delta, \theta} = 0 \). Applying these conditions results in the following
expression for the velocity profile

\[ u = u_s + \frac{3}{2} \left( V - u_s \right) \frac{z}{\delta} - \frac{1}{2} \left( V - u_s \right) \frac{z^3}{\delta^3} \] (4.5)

The slip velocity, \( u_s \), in Eq. 4.5 can be expressed in terms of the slip-length by invoking Navier’s
slip hypothesis [49],

\[ \lambda = \frac{\mu u_s}{\tau_w} \] (4.6)

Physically, \( \lambda \) represents the wall-normal distance (into the wall) where the velocity, \( u \), would
vanish and is determined by the surface characteristics of the superhydrophobic or slipping sur-
face. As noted earlier the upper limit for physically realizable slip lengths for jet impingement
is approximately \( \hat{\lambda} = \lambda / a = 0.2 \). Thus we explore slip length values from the no slip condition
\( \hat{\lambda} = 0 \) to a maximum value of 0.2. For rib/cavity structured SH surfaces the slip length paral-
lel to the microribs, \( \lambda(\theta = 0^\circ) = \lambda_0 \), is approximately twice that perpendicular to the microribs,
\( \lambda(\theta = 90^\circ) = \lambda_0 \), and the variation of \( \lambda \) through \( \theta \) is defined by Crowdy as [55]

\[
\lambda(\theta) = \lambda_0 \sqrt{\cos^2 \theta + \left(\frac{\sin \theta}{2}\right)^2} \quad (4.7)
\]

The boundary conditions in Region 2 take the following form: 1) slip velocity at the surface \( u(z = 0, \theta) = u_s(\theta) \), 2) free surface velocity at the top of the thin film \( u(z = h, \theta) = U(r, \theta) \), and 3) vanishing shear stress at the top of the thin film \( \partial u/\partial z|_{z=h,\theta} = 0 \). Thus, the equation with these boundary conditions for the velocity in this region yields

\[
u = \frac{U}{2h + \lambda} \left[ \lambda + z - \frac{z^3}{3h^2} \right] \quad (4.8)
\]

Where \( U \) varies with \( r \) and, in the anisotropic slip scenario, with \( \theta \) as well. The velocity profiles (4.5) and (4.8) are each substituted into (4.1) - (4.3) resulting in a system of equations for both Region 1 and Region 2 individually. They are solved via a 4th order Runge-Kutta scheme through Region 1 and then subsequently through Region 2. The validity of this method was previously benchmarked by Prince et al. [52, 57].

### 4.4.2 Thin Film Breakup Location

The thin film breaks up into droplets where the momentum forces in the thin film have decreased to the same order as the surface tension forces. An expression to estimate the location where the thin film breakup occurs is now developed. At the edge of the thin film, just before breakup, the fluid is advancing with a contact angle, \( \phi \). The pressure jump across the advancing film is defined by Laplace’s equation

\[
\Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \quad (4.9)
\]

where \( R_1 \) and \( R_2 \) are the two radii of curvature at the surface. \( R_1 \) is the radial curvature around the thin film at breakup, \( R_b \). \( R_2 \) is the radius of curvature of the film due to contact with the surface as shown in Fig. 4.2. Here we can approximate \( R_2 \) assuming the advancing film front has a uniform
radius of curvature. Using geometrical considerations we find a relationship among $R_2$, $h$, and $\phi$

$$R_2 = \frac{h}{1 + \cos(\pi - \phi)} \quad (4.10)$$

Laplace’s equation (4.9) can be simplified using (4.10) and assuming that at the location of breakup $R_b > R_2$ such that $1/R_b$ is negligible. The pressure jump across the edge of the spreading thin film front is thus

$$\Delta P = \frac{\sigma}{h} (1 + \cos(\pi - \phi)) \quad (4.11)$$

An integral momentum balance at the radial location of breakup yields

$$\Delta Phr\Delta\theta = \rho \int_0^h u^2 r \Delta\theta dz \quad (4.12)$$

In (4.12) the momentum flux of the droplets leaving the film are neglected similar to classical momentum analysis of a hydraulic jump where the momentum of the downstream liquid is negligible. Substituting the value of $\Delta P$ from (4.11) and the velocity profile of (4.8) into (4.12) and integrating yields the expression

$$\frac{1 + \cos(\pi - \phi)}{We} = \frac{\hat{U}^2}{\left(\hat{\lambda} + 2/3\hat{h}\right)^2} \left[\hat{\lambda}^2 + \frac{5}{6} \hat{\lambda} \hat{h}^2 + \frac{68}{315} \hat{h}^3\right] \quad (4.13)$$

where $\hat{h} = h/a$ and $\hat{U} = U/V$. The right hand side of (4.13) is calculated at each radius, as both $\hat{U}$ and $\hat{h}$ are functions of radial position, and when it is equal to the left hand side there is a balance between surface tension force at the thin film front and momentum in the thin film. This is where breakup is expected to occur and this radius is defined as $R_b$.

To compare the breakup location with the hydraulic jump location (if breakup were not to occur) we consider the integral momentum balance at the radial location of breakup found in (4.12) and compare it to the integral momentum balance at the radial location of a hydraulic jump as presented by Watson [23]. Equation 40 of Watson is shown here for the purpose of comparison

$$\frac{r_1 d^2 g a^2}{Q^2} + \frac{a^2}{2\pi^2 r_1 d} = \frac{2r_1 a^2}{Q^2} \int_0^h u^2 dz \quad (4.14)$$
where $r_1$ is the radial location of the hydraulic jump and $d$ is the downstream depth imposed on the thin film. The left hand side of Eq. 4.14 represents the gravitational forces of the downstream depth and the momentum of the fluid downstream of the jump. The right hand side represents the radial momentum in the thin film. The left and right hand sides of Eq. 4.12 are equal at the radial location of the hydraulic jump. In a similar manner we multiply (4.12) by $2\frac{ra^2}{\rho Q^2}$ to yield

$$\frac{\Delta P h 2ra^2}{\rho Q^2} = 2\frac{ra^2}{Q^2} \int_0^h u^2 dz$$

(4.15)

Now both equations (4.14 and 4.15) include momentum in the thin film in the same form on the right hand side of the expression. Watson showed that using a similitude solution the right hand side of Eq. 4.14 may be expressed as $0.01676 \left[ \frac{(r_1/a)^3}{Re} - 0.1826 \right]$. Such an analysis is not so simple here due to the slip at the surface and the variation in $\theta$ that exists in the velocity. However, we can assume the right hand side will still be a function of $(r/a)^3 Re^{-1}$, $\hat{\lambda}$, and $\theta$. Following this notation we rewrite the balance of forces at the breakup location using (4.11), rearranging (4.15), and nondimensionalizing to yield

$$\frac{1 + \cos(\pi - \phi)}{We} \hat{R}_b = f(\hat{R}_b^2 Re^{-1}, \hat{\lambda}, \theta)$$

(4.16)

Thus, we can display results for the breakup location relative to the same abscissa that was used by Watson [23], Bush and Aristoff [9], and Prince et al. [52, 57].

4.5 Results

4.5.1 Breakup on a Surface with Isotropic Slip

First we discuss the breakup radius on a surface with isotropic slip as a function of the contact angle $\phi$. On a surface with isotropic slip the breakup will occur in a circular pattern such that a single radius, $R_b$, defines the entire breakup location. Figure 4.3 provides the breakup radius for varying $\hat{\lambda}$ varies with $\phi$. Here the Reynolds number is held constant at $Re = 1.4x10^4$. As $\hat{\lambda}$ increases, $\hat{R}_b$ also increases, nearly uniformly. Thus as more momentum is present in the thin film, breakup will occur farther from the jet. Also note that as $\phi$ increases $\hat{R}_b$ decreases concomitant
with the increased surface tension pulling inward on the thin film. This variation in $\hat{R}_b$ decreases as $\phi$ increases and thus for higher contact angles small variations in $\phi$ may be neglected.

Figure 4.3: Normalized radial breakup location, $\hat{R}_b$ as a function of contact angle, $\phi$. $\hat{\lambda}$ varies from 0 to 0.2 and $Re = 1.4 \times 10^4$.

Figure 4.4 depicts the breakup radius as a function of Weber number on an isotropic slip surface. Here $\phi = 160^\circ$ and the slip on the surface is varied from the no slip condition, $\hat{\lambda} = 0$ to $\hat{\lambda} = 0.2$. For all slip lengths the radius where breakup occurs increases nearly linearly with increasing $We$. This is due to the relationship between $We$ and the momentum in the thin film, which must be balanced by the surface tension at breakup. Also, the breakup radius increases uniformly with increasing $\hat{\lambda}$, since this increase also results in an increase in momentum.

The relationship of the surface tension force as a function of the momentum in the thin film is shown in Fig. 4.5. The lines shown are for varying slip lengths and contact angles. The collapse of lines are for varying contact angles, however it is not exact due to the neglected gravitational term due to the thin film height. The solid lines correspond to the no-slip scenario. As $\hat{\lambda}$ increases the curves are shifted upward corresponding to an increase in $R_b$. The shape of these curves is very similar to those of the hydraulic jump location from Watson [23], Bush and Aristoff [9], and Prince et al. [52, 57].
Figure 4.4: Normalized radial breakup location, $\hat{R}_b$ as a function of $We$. Slip lengths vary from $\hat{\lambda} = 0$ to $0.2$. The contact angle is held constant at $\phi = 160^\circ$.

Figure 4.5: Ratio of momentum terms, correspond to plots of hydraulic jump locations.

We can collapse the data of Fig. 4.5 by accounting for the slip through a parameter defined as $g(\hat{\lambda})$, similar to the method of Prince et al. [52]. The best collapse of the data occurs for

$$g(\hat{\lambda}) = 132\hat{\lambda}^2 + 9.7\hat{\lambda} + 1 \quad (4.17)$$

When the $x$-axis is divided by this parameter the result is Fig. 4.6. Here all the data collapses to a single curve. Using this plot the breakup radius of the thin film for a jet of known $Re$ and a impinging on a surface of known $\phi$ and $\lambda$ can be estimated.
We compare the smooth surface results to those of Maynes et al. [10] in Fig. 4.7. Although in the experiments the jet diameter varied, the results showed no discernible trend with diameter and thus all the data are plotted here without deliniation. A smooth PDMS surface, \( \phi = 106^\circ \), and a smooth Teflon coated Silicon surface, \( \phi = 117^\circ \), were explored. Neither surface has slip, \( \hat{\lambda} = 0 \).

Figure 4.7 shows how the breakup radius varies with \( \text{We} \). The data reveal that over the entire \( \text{We} \) range explored \( \hat{R}_b \) increases with increasing \( \text{We} \) or decreasing \( \phi \). An increase in \( \text{We} \) corresponds to a relative increase in momentum or decrease in surface tension, and thus breakup occurs farther from the impingement point. As \( \phi \) decreases the influence of surface tension decreases, and thus the breakup radius increases. Very good agreement between the model and experimental data is found for these cases.

### 4.5.2 Anisotropic Surface Breakup

When the structure on the surface is anisotropic the slip on the surface is anisotropic and the resultant breakup is no longer circular. Here we assume a rib and cavity structure where the water sits on top of the ribs and air is trapped in the cavities, the Cassie-Baxter state. In this scenario the slip length along the rib/cavity structures (\( \theta = 0^\circ \)) is twice that transverse to the rib/cavity structures (\( \theta = 90^\circ \)) [55]. The contact angle varies from approximately \( \phi = 135^\circ \) to \( 145^\circ \), which results in a breakup location variation of 2 – 3% due to variation in \( \phi \) (see Fig. 4.3). Thus an average contact angle has been selected for all \( \theta \) and variation of \( \phi \) with \( \theta \) has been
neglected. The breakup shape for the rib/cavity structured surfaces is shown in Fig. 4.8. The x-coordinate corresponds to the direction parallel to the rib/cavity structures and for each of the panels $\phi = 160^\circ$. The left panel shows how the breakup location changes for varying slip length along the rib/cavity structures, $\hat{\lambda}_0$, for $We = 300$. It can be seen that as the slip length increases the shape of the breakup location becomes more elliptical and is farther from the impingement point, $(\hat{x}, \hat{y}) = (0, 0)$. The right panel shows the breakup shape as a function of Weber number for $\hat{\lambda}_0 = 0.05$, which corresponds to $F_c = 0.94$. Here it can be seen that as Weber number increases the shape becomes more elliptical and increases in all azimuthal directions.

The variation of $\hat{R}_b$ with $\theta$ is shown in Fig. 4.9. The left panel shows that for all slip lengths explored the breakup radius perpendicular to the rib/cavity structures, $\hat{R}_{b90}$, is smaller than the radius parallel to the rib cavity structures, $R_{b0}$. It also confirms the increased breakup radius at all locations for increasing $\hat{\lambda}$. The disparity in radius increases with increasing slip. Note for $\hat{\lambda}_0 = 0.05$ and 0.1 at $\theta = 90^\circ$ the breakup radius is just less than the no slip scenario. This is due to the redirection of fluid from the transverse to longitudinal direction.

The right panel of Fig. 4.9 reveals that for a given $We$ the breakup radius is greatest along the rib/cavity structures and least transverse to the structures. This variation increases with increasing $We$. This is due to the overall increase in $\hat{R}_b$ as $We$ increases.
Figure 4.8: Breakup shape for jet impingement on rib/cavity structured surfaces. The $x$-coordinate corresponds to the direction parallel to the rib/cavity structures, and the $y$-coordinate corresponds to the direction transverse. The jet impinges directly in the center. In both panels $\phi = 160^\circ$. In the left panel $We = 300$. In the right panel $\hat{\lambda}_0 = 0.05$.

Figure 4.9: Breakup location as a function of $\theta$ for jet impingement on rib/cavity structured surfaces. In both panels $\phi = 160^\circ$. In the left panel $We = 300$. In the right panel $\hat{\lambda}_0 = 0.05$. 

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The ratio of the breakup radius in the longitudinal to the transverse direction, $R_{b0}/R_{b90}$, is a measure of the eccentricity of the ellipse where breakup occurs. This is plotted as a function of $We$ for varying $\lambda$ in the left panel of Fig. 4.10 and as a function of $\lambda$ for varying $We$ in the right panel. The data in the left panel show that $R_{b0}/R_{b90}$ increases nearly linearly with $We$ for all $\lambda$ explored. As $We$ increases the breakup radius increases and the surface area over which the slip disparity has influence also increases. The right panel shows that over the lower range of $\lambda_0$ the ratio is increasing, however a limit of $R_{b0}/R_{b90}$ is approached as $\lambda$ increases. This occurs for lower $We$ at lower $\lambda_0$.

![Figure 4.10: Ratio of breakup radii, $R_{b0}/R_{b90}$, for anisotropic surface.](image)

We compare the SH rib/cavity model results with the experiments of Maynes et al. [10] who measured breakup radii for PDMS and Teflon coated Silicon surfaces structured with ribs and cavities of cavity fraction $F_c = 0.8$. This resulted in contact angles of $\phi_0 = 133^\circ$ (PDMS) and $140^\circ$ (Si) along the rib/cavity structures and $\phi_{90} = 142^\circ$ (PDMS) and $149^\circ$ (Si) transverse. The slip length in the longitudinal direction for both surfaces is $\lambda_0 = 0.05$. In the model we assume an average $\phi = 141^\circ$ as $\hat{R}_b$ is a weak function of $\phi$. Figure 4.11 shows $\hat{R}_{b0}$ and $\hat{R}_{b90}$ as a function of $We$ for the model and experiments of Maynes et al. [10]. Both radii increase with increasing $We$. $\hat{R}_{b0}$ is greater than $\hat{R}_{b90}$ for all $We$, concomitant with the direction of greatest slip. Note the variation in radii increases with $We$ as inertial effects increase relative to surface tension effects.
The model and experimental data show very good agreement. We expect the model to slightly underpredict $R_{b0}$ and slightly overpredict $R_{b90}$ since the r-momentum due to azimuthal velocity, which would increase the disparity in these directions, has been neglected.

Figure 4.11: Normalized breakup radius in both the longitudinal ($\theta = 0^\circ$) and transverse ($\theta = 90^\circ$) as a function of Weber number compared with experimental results. For the model an average for $\phi = 141^\circ$ was used. Experiments were performed on both PDMS and Teflon coated Si surfaces with rib/cavity structures of $F_c = 0.8$.

Figure 4.12 explores the variation of the normalized thin film area ($A/A_j$ where $A_j$ is the initial jet area, $\pi a^2$) with cavity fraction for varying $Re$. Here, $\phi$ is calculated as a function of $\lambda$ using the Cassie-Baxter relation, plotted in the right panel. For this scenario a value of $\phi = 110^\circ$ is assumed for a smooth surface. Note that $\phi$ and $\hat{\lambda}_0$ both increase with increasing $F_c$, but $\phi$ does so at a more constant rate. At high $F_c$, $\hat{\lambda}_0$ increases rapidly. It is interesting to note in the left panel that except at very high $F_c$, there is very little variation in thin film area with $F_c$. Thus the increase in momentum in the thin film due to the increase in slip with increasing $F_c$ is nearly exactly balanced by the concomitant increase in $\phi$ resulting in a greater surface tension force at the front of the thin film.

In Fig. 4.13 we present the area ratios as a function of $We$ and compare the results of the model with experiments presented by Maynes et al. [10]. Two $\phi$ values and their corresponding $F_c$ are shown. It is interesting to note that all values are very similar due to the increase in slip causing an increase in outward radial momentum, but the concomitant increase in contact angle causes an
Figure 4.12: The left panel shows the thin film area before breakup normalized by the initial jet area, $A_j$, as a function of $F_c$ (and thus $\hat{\lambda}$) for varying $We$. The right panel represents the relationship between slip length and contact angle for varying cavity fraction using the Cassie-Baxter relation.

increase in inward surface tension force such that a balance is reached. The thin film area increases nearly linearly with $We$. The comparison to experiments is very good.

Figure 4.13: Thin film area before breakup normalized by the initial jet area, $A_j$, for varying $\phi$ and $F_c$ (at $F_c = 0, \hat{\lambda} = 0$ and at $F_c = 0.8, \hat{\lambda} = 0.025$) with $We$. Lines correspond to the model presented here. Markers correspond to experimental results presented in Maynes et al. [10].
4.6 Conclusion

We have explored the radius of thin film breakup which occurs when a liquid jet impinges on a horizontal hydrophobic or superhydrophobic surface. For smooth or isotropic structured surfaces the breakup radius is uniform in the azimuthal coordinate and the breakup is circular. If the SH surface has an anisotropic surface structure, such as ribs and cavities as explored here, the resultant breakup radius is also asymmetric. For the rib/cavity structures the breakup takes on an elliptical shape where the major axis is aligned along the ribs and cavities and the minor axis is aligned transverse to the ribs and cavities. For all breakup scenarios explored an increase in thin film momentum due to increases in $We$ or $\hat{\lambda}$ results in an increase in $R_b$. Increases in forces due to surface tension (increases in $\phi$) result in a decrease in $R_b$. The variation with $We$ is nearly linear, whereas the variation with $\hat{\lambda}$ diminishes as $\hat{\lambda}$ increases. It was found that $\phi$ is relatively unimportant in $\hat{R}_b$, however it does balance with variations in $\hat{\lambda}$ when the Cassie-Baxter equation is applied. Thus the breakup radius is a greater function of $We$ as the $\hat{\lambda}$ increases.

The breakup radius in the isotropic scenarios can be estimated by matching the data to the curve shown in Fig. 4.6, where a function accounting for the effects of slip, $g(\lambda)$, is defined. This can also be accomplished for the anisotropic scenario by using the average slip length of the surface in Fig. 4.6 and estimating the radius ratio from Fig. 4.10. Comparisons of the model to experiments show excellent agreement.

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CHAPTER 5. PRESSURE DROP MEASUREMENTS IN TURBULENT CHANNEL FLOW OVER SUPERHYDROPHOBIC SURFACES WITH RIBLETS

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5.2 Abstract

In this paper we consider the combined drag reducing mechanisms of superhydrophobicity with riblets. Pressure drop measurements were acquired for turbulent flow in a channel with superhydrophobic walls, riblet walls, and walls with both drag reducing mechanisms. The superhydrophobic structuring was composed of alternating microribs (15 \( \mu \)m tall and 8 \( \mu \)m wide) and cavities (32 \( \mu \)m wide), aligned parallel to the flow. Superhydrophobic surfaces function to reduce drag by minimizing the effective liquid-solid contact area as water will not penetrate the cavities between microribs due to surface tension. The riblets were nominally 80 \( \mu \)m tall, 18 \( \mu \)m wide, spaced with a period of 160 \( \mu \)m and were also aligned parallel to the flow. Riblets function by damping out spanwise turbulent motions. Since turbulence is a three-dimensional phenomenon, this destruction of turbulent motions acts to reduce the average friction at the surface. Fabrication of the drag reducing surfaces was completed with photolithographic techniques on silicon wafers. The wafers were inserted into a channel consisting of a control section with smooth wafers and a test section with patterned wafers. In all cases, the test section walls were structured on top and bottom while the side walls were left smooth. The channel had a hydraulic diameter of 7.3 mm and an aspect ratio of 10:1. Tests were obtained over a Reynolds number range of \( 5 \times 10^3 \) to \( 1.5 \times 10^4 \). The superhydrophobic surfaces with riblets showed a maximum drag reduction of 7.0\% which was a higher reduction than either the surfaces patterned with riblets or the superhydrophobic surfaces.
5.3 Introduction

The object of this paper is to explore the drag reduction that can be obtained in turbulent flow over superhydrophobic surfaces with superimposed riblets. A superhydrophobic surface is defined as exhibiting high contact angles greater than $150^\circ$ when a water droplet is placed on the surface [3, 58]. This is possible through the combination of microroughness such as microribs and a hydrophobic coating. Because of surface tension, water on the surface will not penetrate the cavities unless a pressure greater than the Laplace pressure is applied [8]. An example of such a surface patterned with microribs is shown in Figure 5.1. When water flows over this surface, there will be no slip across the tops of the microribs but there will be slip across the cavities resulting in an apparent slip across the entire surface when considering the macroscale flow. Thus, there is a drag reduction because the effective area over which friction prevails has been reduced. In laminar flow there is a drag reduction if the ribs and cavities are aligned with or transverse to the streamwise flow direction. However, the drag reduction is greater when the ribs and cavities are aligned with the flow [54]. Superhydrophobic surfaces have been considered in many flow scenarios including both numerical and experimental work in both laminar and turbulent flow [2, 30, 34, 51, 53, 54, 59–63].

Figure 5.1: SEM image of microribs with 80% cavity fraction.
In turbulent flow, DNS simulations were performed by Min and Kim in which they imposed a specified slip length parallel to the flow direction and perpendicular to the flow direction [12]. They determined that while streamwise slip results in a drag reduction, spanwise slip results in a drag increase. Fukagata and Kasagi determined a theoretical prediction for turbulent channel flow based on the DNS results of Min and Kim [62]. Their work allows for the prediction across surfaces with slip parallel and transverse to the flow. Martell et al. also simulated turbulent channel flow using DNS [61]. Rather than imposing a slip length, Martell modeled the region above a microrib or micropost as no shear and the region above a cavity as a no slip region. The flow was modeled at a friction Reynolds number of $Re_{\tau} = 180$. Here $Re_{\tau} = u_{\tau}H/\nu$ where $u_{\tau}$ is the friction velocity, $H$ is half the channel height, and $\nu$ is the kinematic viscosity. The friction velocity is defined classically as $u_{\tau} = \sqrt{\tau_w/\rho}$ where $\tau_w$ is the shear stress at the wall and $\rho$ is the density. For the largest spacing employed, the results showed a drag reduction of nearly 40%.

Woolford et al. obtained PIV and pressure drop measurements for turbulent channel flow across superhydrophobic surfaces patterned with silicon microribs with a cavity fraction $F_c$ of 80% and obtained a drag reduction of 11% when the microribs were aligned parallel to the flow direction [2]. Here, $F_c = w_c/w$ where $w_c$ is the width of a cavity and $w$ is the combined width of a cavity and a microrib. When they aligned the microribs transverse to the flow, a modest drag increase occurred. Daniello et al. also reported drag reduction across superhydrophobic surfaces with similar patterning fabricated from PDMS but with a cavity fraction of 50% [51]. They used pressure drop measurements as well as PIV and were able to measure a drag reduction of 50%. Peguero and Breuer performed testing over superhydrophobic surfaces with a variety of different patterning techniques [64]. They used PIV to measure the drag in a turbulent channel flow and did not obtain any measurable drag reduction. Recently, Park et al. used microfabricated flexure beams to measure the drag reduction in a turbulent boundary layer flow [65]. They used a high-speed camera to optically record the displacement of a smooth plate and a superhydrophobic plate placed side-by-side in a water tunnel. They showed higher drag reduction for higher cavity fractions reaching a maximum of 70% skin friction reduction when $F_c = 95\%$.

It is apparent that there is some discrepancy among the results that have been obtained experimentally. Aljallis et al. recently published a study of the drag reduction achievable in boundary layer flows at high Reynolds numbers across superhydrophobic surfaces [66]. They dragged su-
perhydrophobic plates in a towing tank testing in transitional and turbulent flow from a $Re_L$ of $10^5$ up to $10^7$. While at low $Re_L$ they were able to obtain a drag reduction of $\sim 30\%$, at higher Reynolds numbers, the air was removed from the cavities between features and the drag reducing effect of the surfaces was lost due to high shear rates at the air-water interface. Samaha et al. also performed testing that showed a loss of drag reduction over time [67]. They shot continuous jets of water parallel to a submerged test plate while monitoring the air trapped in the cavities at various Reynolds numbers. They found that over time the air was removed from the cavities much more quickly at higher Reynolds numbers due to mass convection. While it is clear that superhydrophobic surfaces can reduce drag in turbulent flow, there is still much work to be done in determining what mechanisms and flow regimes cause the air in the cavities to be removed and drag reducing capabilities to be decreased.

Riblet surfaces have been studied for more than 30 years, both experimentally and computationally. They function by damping out spanwise turbulence [15, 16, 68] which is a completely different drag reducing mechanism as compared to superhydrophobic surfaces. They are also an order of magnitude larger than the superhydrophobic ribs/cavities shown in Figure 5.1. The riblets used in this study are shown in the SEM image in Figure 5.2. Computationally, riblets have been explored by a large number of researchers. Goldstein et al. [15] used an immersed boundary technique to model flow over riblets and over a surface with simulated spanwise damping. He was able to show that the two cases were very similar, justifying the assumption that riblets function by damping out spanwise oscillations. He showed a drag reduction of nominally $4\%$. Chu and Karniadakis [69] also used DNS to simulate V-groove riblets and rounded riblets over a Reynolds number range from 500 to 3500 for channel flow. Their laminar flow results showed no drag reduction, while the transitional and turbulent flow showed a drag reduction reaching a maximum of $6\%$ at a Reynolds number of 3500. The rounded riblets did not show a significant drag reduction compared to the V-groove riblets. Choi et al. [68] performed a DNS study using V-groove riblets at $s^+$ values of 20 and 40, achieving a drag reduction of 5-6%. $s^+$ is simply the periodic spacing of the riblets in wall units, defined classically as $s^+ = su_+/\nu$. They determined that changes in the velocity profiles and turbulent statistics were only significant in the inner region of the turbulent boundary layer and that transverse motions were again reduced by the riblets.
Walsh [13, 70] was one of the first to research riblets experimentally and used a drag balance to measure the friction reduction achievable with V-groove riblets. He tested a wide range of different spacings, $s$, between riblets and different riblet heights, $h$. He obtained a maximum reduction in friction of 8% where $h^+$ and $s^+$ were equal to nominally 10 and 17, respectively. Suzuki and Kasagi used particle tracking velocimetry (PTV) to explore the drag reduction mechanism of trapezoidal riblet surfaces [16]. They performed tests at an $s^+$ equal to 15 and 31. They showed that variation in the mean velocity profile is confined to the viscous sublayer and that the spanwise fluctuations are suppressed. Bechert et al. [14] later performed experiments using a Berlin oil channel with a precision shear stress balance testing a range of different geometries. They showed that an infinitely thin riblet, or a riblet with a thickness $t = 0$, is the most effective geometry and determined that the blade riblet should be spaced at nominally $s^+ = 15$ and the height should be half the spacing. They obtained a maximum drag reduction of 9.9%.

While researchers have looked at both superhydrophobic surfaces and riblet surfaces in turbulent flow, to the authors’ knowledge no one has considered a surface which combines both drag reducing mechanisms. It is attractive to combine these mechanisms because they are complimentary. While riblets increase the surface area, superhydrophobic surfaces decrease the effective area due to their slip at the air-water interface. Additionally, while superhydrophobic surfaces will
inherently have a component of the spanwise slip that results in enhanced spanwise turbulence, riblets function by damping out spanwise turbulence. It is the focus of this paper to determine the drag reduction that can be obtained in turbulent flow when the surface is both superhydrophobic and patterned with riblets. The following sections present the experimental method, results and discussion, and conclusions of this work.

5.4 Experimental Method

5.4.1 Microfabrication

Photolithographic processes were used to manufacture superhydrophobic surfaces, riblets, and surfaces with both drag reducing mechanisms. Both the microribs for the superhydrophobic surfaces and the riblets are aligned parallel to the streamwise flow direction. The superhydrophobic surfaces are fabricated using conventional methods. AZ®nLOF™2020, a photo-sensitive resist, is spun onto a silicon wafer and soft baked. A photomask is then used to selectively expose the resist with ultraviolet light. The regions that are exposed cross-link and become insoluble to the developer. The wafer is then post-exposure baked and developed in AZ®300 MIF. The photoresist serves as a mask while the silicon surface is etched using a DRIE process. Next, residual photoresist is removed using Nano-Strip™. The substrate is rendered chemically hydrophobic by vapor depositing 300 nm of chromium as an adhesion promoter and then by spinning on a 1:29 volumetric ratio of 6% Teflon® AF 1600 mixed in Perfluoro-compound FC-40 or FC-75. An SEM of a superhydrophobic surface with microribs is shown in Figure 5.1. Here, the microribs are 8 µm wide, 15 µm tall and spaced periodically every 40 µm resulting in a cavity fraction $F_c = 0.8$. As a way to ensure that a defect in one of the cavities does not cause failure along the entire length of the wafer, microridges that are 8 µm wide and the same height as the microribs are fabricated transverse to the flow in the spanwise direction and are periodically spaced every 2.5 mm similar to Muralidhar et al. [71].

The riblets are fabricated using SU-8 2075. The photoresist is spun onto the surface, selectively exposed to UV, post-exposure baked, and developed. A layer of aluminum 1 µm thick is vapor deposited onto the SU-8 to strengthen the riblet structures. The left panel of Figure 5.2 shows an SEM of the riblet microstructures with dimensions of 80 µm tall, 18 µm wide, and spaced pe-
riodically every $160 \, \mu m$. Using the optimum $s^+ = 15$ and this riblet spacing, the friction velocity can be calculated from the definition of a wall unit. The friction velocity combined with the definition of the friction factor can then be substituted into the Blasius correlation, $f = 0.316 / (Re^{0.25})$, for turbulent flow [72] to determine the $Re$ at which the optimum drag reduction occurs. For this riblet spacing of $s = 160 \, \mu m$, the optimum should be at $Re = 1.10 \times 10^4$.

The combined surface with both microribs and riblets is fabricated by following the process to make microribs up to the removal from the Nano-Strip™. Here, a different photomask is used where every fourth rib is $28 \, \mu m$ wide and is used as a platform for the riblet features. The riblet process is then followed to completion with some modification to allow for bubbles originating from the cavities between the microribs to rise to the surface of the photoresist before spinning. (Note: the full process is outlined in Chapter 6). Finally the Cr deposition and application of the Teflon® coating is completed rendering the surface chemically hydrophobic. This process results in a substrate that has a cavity fraction of 67.5% due to the $28 \, \mu m$ wide microribs and is shown in the SEM image in Figure 5.3.

![Figure 5.3: SEM image of riblets fabricated from SU-8 on top of microribs just before the application of Cr and Teflon®.](image)
5.4.2 Flow Loop

A recirculating flow loop with water was used to test the drag across the various surfaces and is illustrated in Figure 5.4. After leaving a centrifugal pump, the water passes through a heat exchanger to minimize temperature increase due to viscous heating. Then the flow enters a Coriolis flow meter where the temperature and mass flow rate are measured. Subsequently, the water enters a flow conditioner consisting of a section of glass spheres, an array of plastic tubes, and screens to remove any residual vorticity introduced at the pump. Following the conditioner, the flow passes through a fitting manufactured with wire EDM to make a smooth transition to a rectangular channel with an aspect ratio of 10:1 and a hydraulic diameter $D_h = 7.3$ mm as depicted in Figure 5.5. The hydraulic diameter is defined as $D_h = 4A/P_w$ where $A$ is the area of the channel cross-section and $P_w$ is the wetted perimeter. From the entrance of the channel to the first wafer in the channel there are 22 hydraulic diameters for flow development. There are 8 wafers placed end to end on both top and bottom of the channel for a total of 16 wafers. Each wafer is 8 cm long, 4.9 cm wide (with only 4 cm exposed to the flow), and 0.5 mm thick. The channel has been precision machined such that the surface of the wafers is flush with the channel both upstream and downstream of the wafers. The wafers are divided into two sections. The first section is a control section at the beginning of the first set of eight wafers (4 on top and 4 on bottom), where smooth wafers are always inserted. The second section, also comprised of eight wafers, is the test section where the patterned wafers are placed. Thus, each section is 4 wafers or $44D_h$ long. The first two wafers of each section give the flow a chance to develop in response to the surface type and are a total of $22D_h$ long. Over the subsequent two wafers of each section, 7 pressure taps have been placed through the spacer between the top and bottom of the channel as shown in Figure 5.5. The purpose of multiple pressure taps is to remove noise from the estimate of the pressure gradient. Two differential pressure transducers (model IDP10-A made by Viatran) are used to measure the pressure drop over both the control and the test section simultaneously after the work of Frohnapfel et al. [73].

Because high pressures will force the air out of the cavities of the superhydrophobic surfaces, tight pressure control is maintained throughout the channel. A third differential pressure transducer (model PX2300-1DI made by Omega) is attached to the channel through one port and the other is open to atmosphere to measure the static pressure in the channel. A four-bar mecha-
Figure 5.4: Schematic of flow loop used to acquire pressure drop measurements.

Figure 5.5: Schematic of channel with pressure taps outlining key geometric parameters.
nism allows the angle of the channel to be adjusted from 0° to 45°. An inclinometer is attached to the top of the channel that records the angle of the channel. The purpose of controlling the angle of the channel is to maintain a pressure gradient that is approximately zero throughout the channel. This is done by matching the hydrostatic pressure gradient to the frictional pressure gradient. Upon leaving the channel, the flow enters a reservoir that is open to atmosphere and can be adjusted to different elevations. Therefore, if the Reynolds number is increased then the angle of the channel can be increased so the hydrostatic pressure gradient again matches the frictional pressure gradient, and the reservoir can be lowered to maintain the same target static pressure.

Pressure drop measurements over taps 1-2, 1-3, 1-4, 1-5, 1-6, and 1-7 were performed at 1000 Hz for 20 seconds over each tap combination and at each Reynolds number. Here, the taps have been numbered from 1 to 7 in each section where 1 is the tap farthest upstream of the control or test section. Reynolds numbers ranging from $5 \times 10^3$ to $1.5 \times 10^4$ were tested in sequence from the low $Re$ up to the high $Re$ to avoid any pressure spikes that could affect the air layer of the superhydrophobic surfaces. The target pressure for the channel was set to 750 Pa which is far below the Laplace pressure of 1990 Pa for the microribs and above the Laplace pressure of 440 Pa for the riblet geometry when coated with Teflon®. Thus, water should enter the cavities between the riblets, but not between the microribs of the combined surface.

### 5.4.3 Friction Factor Determination and Channel Validation

The Darcy friction factor can be defined as [74]

$$ f = \frac{8 \tau_w}{\rho \bar{V}^2} \quad (5.1) $$

where $\bar{V}$ is the average velocity in the channel. It is a measure of the shear stress normalized by the density and the square of the average velocity and can be determined experimentally from the equation for the major head loss in turbulent pipe flow [72].

$$ f = \frac{2D_h(-dp/dx)}{\rho \bar{V}^2} \quad (5.2) $$
Here $dp/dx$ is the pressure gradient along the channel. The friction factor can be solved for experimentally by measuring the pressure gradient, measuring the channel dimensions to compute the hydraulic diameter, and using a measurement of the flow rate to calculate the average velocity. The pressure gradient is determined by measuring the differential pressure between each tap and the tap farthest upstream and then with knowledge of the location of each tap, a least squares line fit can be performed where the slope of the $p$ vs $x$ data is the pressure gradient. The right panel of Figure 5.6 shows the line fits used to compute a few of the pressure gradients for one of the control cases. For all calculations of the pressure gradient, the differential pressure measurements between taps 1-2 were discarded due to high instrument error because of the small distance in the streamwise direction. It is also important to note that the friction factor is an average of the shear stress in the channel. In the testing conducted here, the sidewalls of the channel are not patterned with riblets nor are they superhydrophobic and will modestly lower the drag reduction realized over the patterned surfaces as the area fraction of the sidewalls to the total channel area is 9.1%.

Before inserting patterned wafers into the channel, tests were conducted with smooth wafers in both the control section and the test section. The purpose was to validate the design and method for determining the friction factor. The results of these tests are shown in the left panel of Figure 5.6 where the average friction factor over 6 tests is plotted for both sections of the channel. It is easy to see that there is reasonable agreement between the control and the test sections as would be expected as both have smooth surfaces. The largest deviation is 2.6% at a $Re = 1.5 \times 10^4$. The Blasius correlation [72] is also plotted which has been corrected by the effective diameter as shown by Jones [75]. There is fair agreement between the control data and the correlation where the average of both sections has a 5.4% difference from the Blasius correlation in the worst case, which is reasonable agreement for experimental data. It is not surprising to see this deviation as these are lower $Re$, and while the Blasius correlation has been corrected by the effective diameter, it intended is for circular cross-sections.

### 5.4.4 Uncertainty

The uncertainty in the friction factor, $f$, is the result of error in the estimate of the pressure gradient, density, flow rate, and channel dimensions. The uncertainty in the pressure gradient is due to error in the least squares regression of the pressure tap data (shown in Table 5.1), the
Figure 5.6: Friction factor data where smooth surfaces have been placed in the control and test sections (left panel) and quality of the line fit through the pressure data relative to the first tap over smooth surfaces (right panel).

Error in the tap placement (0.2%), error associated with variation in the tap geometry (0.5%), and instrument error in the pressure transducers (0.4%-3.5%). The error in the density is associated with the uncertainty in the temperature which is ±0.5°C resulting in a maximum error in density of 0.01%. Over the entire test range, the Coriolis flow meter has an uncertainty of 0.40%. Combining all these errors results in a total uncertainty ranging from a typical value of about 5% up to a maximum error of 14.8% for the worst case in combined surface testing. The high errors over the combined surfaces are a result of the large variation in pressure gradient from tap to tap along the channel for this scenario.

Table 5.1: Uncertainty of the slope of the linear regression fit of the pressure measurements.

<table>
<thead>
<tr>
<th>Case</th>
<th>%Error in $dp/dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>smooth</td>
<td>1.8-7.0</td>
</tr>
<tr>
<td>riblet</td>
<td>1.6-5.2</td>
</tr>
<tr>
<td>SHS</td>
<td>1.8-8.5</td>
</tr>
<tr>
<td>combined</td>
<td>4.9-14.9</td>
</tr>
</tbody>
</table>
5.5 Results and Discussion

Results for pressure drop measurements are presented for superhydrophobic surfaces, riblet surfaces, and surfaces with superhydrophobicity and riblets (combined surfaces). For each case, the control data obtained simultaneously over smooth surfaces are compared against the surface of interest. The left panel of Figure 5.7 shows the average friction factor acquired over 4 tests with riblet patterned surfaces in the test section. Here, the friction factor for the riblets is lower compared to the smooth surfaces over most of the range of Reynolds numbers. It can be noted that the friction factor is a strong function of the Reynolds number as is expected because the effective riblet size and spacing changes as the Reynolds number changes. At a Reynolds number of $5 \times 10^3$, the riblets have a higher friction factor as compared to the smooth surfaces, however, at all other $Re$ the friction factor is smaller for the riblets compared to the control. From $Re = 7 \times 10^3$ to $Re = 1.1 \times 10^4$ the deviation in the friction factor between the two surfaces grows, reaching a maximum percent difference of 3.0%. The difference between the friction factors then becomes smaller and is only 0.1% at a Reynolds number of $1.5 \times 10^4$. Thus, the optimum is obtained here at $Re = 1.1 \times 10^4$ and is the same as that predicted by using the Blasius correlation and the definition of a wall unit to estimate the optimal $Re$ for a given riblet spacing.

Figure 5.7: Average friction factor for riblets compared to smooth surfaces over a Reynolds number range of $5 \times 10^3$ to $1.5 \times 10^4$ (left panel) and % change in $f$ across riblets compared to experimental data of Bechert as a function of the riblet periodic spacing in wall units (right panel).
In the right panel of Figure 5.7 the percent change in $\bar{f}$ is plotted as a function of the periodic spacing of the riblets in wall units and the data are compared to results of Bechert et al. [14]. In general, the drag reduction for all the data is a strong function of $s^+$. The drag reduction increases as $s^+$ increases until a maximum reduction is reached at nominally $s^+ = 16$ and then decreases for larger $s^+$ values. The riblets used by Bechert featured a $t/s$ ratio of 0.02 and 0.04 and the riblets used in this study had a ratio of 0.1. For a $t/s = 0.1$ the optimum occurs at an $s^+ = 15$ where a reduction in drag of 3.0% was obtained. The optimum shifts modestly to higher $s^+$ values as $t/s$ becomes smaller reaching a maximum drag reduction of 9.9% at an $s^+ = 17$ where $t/s = 0.02$. It is clear from the plot that as $t/s$ is increased the drag reduction is lower although no functional relationship can be determined reliably without more data. This trend is consistent with the ideal riblet geometry where the riblet should be infinitely thin to obtain a maximum reduction in friction [14]. The drag reduction obtained in this study is consistent with the data obtained by Bechert when considering the higher $t/s$ value.

Next, the scenario for superhydrophobic surfaces with a $F_c = 0.80$ is compared to smooth surfaces in the control section. Figure 5.8 shows the average friction factor for these surfaces plotted as a function of the Reynolds number and averaged over 3 independent tests. The average

![Figure 5.8: Friction factor data for superhydrophobic surfaces with a cavity fraction of 0.8 compared with smooth surfaces.](image)
The friction factor for the superhydrophobic surface is lower than the control for all Reynolds numbers explored. The deviation is greatest at lower Reynolds numbers where a percent difference of 3.2% occurs at $Re = 5 \times 10^3$ and decreases as $Re$ is increased to a minimum of 0.7%. The data are acquired starting at a Reynolds number of $5 \times 10^3$ and is taken in sequence up to $1.5 \times 10^4$. It is possible that some air is being lost from the cavities in spite of the careful control exercised in maintaining the static pressure.

After these sets of data were taken, a test was performed to remove the air from the cavities to show that air was still present during the tests. While maintaining a Reynolds number of $1.5 \times 10^4$, the static pressure in the channel was dropped from the target pressure of 750 Pa to 1240 Pa vacuum pressure for 1 minute and the static pressure was then restored to its previous value of 750 Pa. The pressure drop was then measured again over the surfaces resulting in a 5.2% higher drag over the superhydrophobic surfaces relative to the control surfaces. It is noted that the friction factor for the smooth surface was not altered by this pressure change. It seems likely here that more air has been removed from the cavities and the surface patterning is acting more like microroughness than superhydrophobicity.

The left panel of Figure 5.9 shows the average friction factor behavior over 2 tests for surfaces exhibiting superhydrophobicity and riblets as compared to smooth surfaces. At all Reynolds

![Figure 5.9](image)

Figure 5.9: Friction factor as a function of Reynolds number for surfaces that are both superhydrophobic and have riblets (left panel) and quality of the line fit through the pressure data relative to the first tap for a combined surface (right panel)
numbers explored, the combined surfaces have a lower friction factor than the smooth surfaces. The highest deviation shown is 7.0% and takes place at a Reynolds number of $Re = 7.2 \times 10^3$. This is a larger reduction in drag than is seen for any of the other surfaces in spite of the combined surfaces having a lower $F_c = 67.5\%$ compared to the superhydrophobic surfaces. The deviation between the friction factors of the two surfaces decreases abruptly at a $Re = 9 \times 10^3$ and then gradually decreases up to $Re = 1.5 \times 10^4$ reaching a minimum percent change of 2.7%.

The large error bars are explained by the right panel of Figure 5.9 which shows the line fit through the $p$ vs $x$ data for one of the cases with combined surfaces. There is a modest degree of variation in the pressure resulting in a poorer line fit and this is responsible for the large error bars in the friction factor data for these surfaces. Further study is needed to explain the cause of the wide pressure variation with streamwise position for the combined surfaces.

In Figure 5.10 the average percent change in friction factor of the surfaces that are superhydrophobic with riblets and those that only have riblets are plotted as a function of $s^+$. The combined surfaces do not follow the same trends with the periodic spacing of the riblets that was demonstrated by the surfaces with riblets alone. They outperform the riblet surfaces at low $s^+$ but have a similar reduction in drag at higher $s^+$. It is possible that air is also being lost from the

![Figure 5.10: Drag reduction surfaces with riblets and across superhydrophobic surfaces with riblets as a function of the periodic spacing in wall units.](image)
cavities as seems to be the case for the superhydrophobic surfaces, and this may be responsible for the abrupt drop in the drag reduction that occurs. The loss of air does not seem to be caused by mass convection as the failure in performance is rapid, nor does it appear to be caused by a failure of the coating as this would probably be a gradual change. It seems most likely that high shear rates at the air-water interface are the cause, similar to the failure seen by Aljallis et al. [66].

Figure 5.11 plots the average percent change in the friction factors for all surfaces considered in this study. The case with smooth surfaces in both the control and test sections of the channel is also included for comparison. The smooth surfaces should show essentially no change in friction factor, however, they show a slight reduction in friction factor within the uncertainty of the measurements. The variation is minimal and is not a strong function of Reynolds number. The riblet data show strong dependence on \( Re \) as has been shown earlier and reach a maximum reduction in \( \bar{f} \) of nominally 3.0% at a \( Re = 1.1 \times 10^4 \). The superhydrophobic surfaces decrease in drag reduction as the \( Re \) increases from a maximum of 3.2% to 0.7%. The surfaces with combined drag reducing mechanisms show a large drag reduction of 6.9% and 7.0% for \( Re = 5 \times 10^3 \) and for \( Re = 7 \times 10^4 \) respectively and then the drag reduction diminishes for higher \( Re \). The combined surfaces do outperform the other surface types at all Reynolds numbers although not as much as...
anticipated at higher Reynolds numbers. It would seem that the maximum reduction in drag should have occurred at higher $Re$ near $Re = 1.1 \times 10^4$ similar to the optimum for the riblets. However, if air is being lost from the cavities this may have skewed the optimum.

5.6 Conclusions

Pressure drop measurements have been obtained in a turbulent channel flow over superhydrophobic surfaces, riblets surfaces, and surfaces exhibiting both superhydrophobicity and riblets. The superhydrophobic surfaces have been micropatterned with ribs using photolithography and then coated with Teflon® to render them superhydrophobic. The riblets have been fabricated from an ultra-thick photoresist and are an order of magnitude larger than the microribs used in the superhydrophobic surfaces. The combined surfaces with both feature types have been manufactured using the same processes with some modification. The friction factor has been obtained for each of the surface types using a channel that has a control section with smooth surfaces and a test section with the patterned surfaces. A pressure transducer has been attached to each section so that the friction factor can be calculated directly. Seven pressure taps have been placed in each section to measure the pressure gradient.

The riblet surfaces showed a drag reduction of 3.0% at a Reynolds number of $1.1 \times 10^4$ and showed less reduction at higher and lower Reynolds numbers as expected. The superhydrophobic surfaces showed a maximum of 3.2% drag reduction. During the tests, the drag reduction decreased as the $Re$ was increased most likely due to air leaving the cavities [66, 67]. The combined surfaces showed a maximum drag reduction of 7.0%. At low $Re$, they showed a higher drag reduction than either the riblets or the superhydrophobic surfaces.

5.7 Acknowledgments

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CHAPTER 6. PHOTOLITHOGRAPHIC FABRICATION OF A MULTILEVEL SURFACE WITH SUPERHYDROPHOBIC MICRORIBS AND SUPERIMPOSED RIBLETS

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6.2 Abstract

A multilevel manufacturing method has been developed to fabricate riblets on top of superhydrophobic microribs as a means to generate a drag reducing surface in turbulent flow. The method consists of dry etching a silicon wafer to generate microribs that are 15 µm tall and 8 µm wide where every fourth microrib is 28 µm wide. Riblets that are nominally 75 µm tall and 16 µm wide are then built on top of the wider microribs by spinning on an ultra-thick photoresist. Modifications are made to the spin process to allow for the bubbles of air originating from the cavities between the microribs to rise to the top of the photoresist. The post-exposure bake is also modified to minimize thermally induced stresses in the riblets. The riblets are fabricated repeatedly with good adhesion to the microribs. They have been tested in a rectangular channel with a 10 to 1 aspect ratio and shown to withstand turbulent flows up to a Reynolds number of $1.5 \times 10^4$ with minimal degradation.

6.3 Introduction

A multilevel microfabrication method is presented based on deep reactive ion etching (DRIE) and spin coating an ultra-thick photoresist to manufacture surfaces for turbulent drag reduction. For many applications it is desirable to fabricate features photolithographically on existing
micron-scale features. Some examples include radio-frequency sensors, ultrasound generators, and other cantilever sensors [76–78].

There currently exist three common methods for micro-machining multilevel structures [79]. The first method is to make some modifications to the spin coating process of an ultra-thick photoresist. This method requires no expensive equipment to be purchased and can be performed on standard clean room equipment [80]. The disadvantage to this method is that it requires long wait times and extremely slow ramping of the temperature during baking depending on the thickness of the resist. A second method is to use a spray coating process to apply the photoresist. This method is quite effective and uses much less photoresist as compared to traditional spin on coatings. The obvious drawback is the specialized spray on equipment that must be used [79]. The third method is to use an electroplating resist to coat the surface. The advantage of this method is that the photoresist is highly conformal especially around sharp corners, but unfortunately the process is limited to metal substrates [81].

The focus of this paper is to explore the first method of using an ultra-thick SU-8 coating on existing microstructures. Here, rather than using the SU-8 as a mask for further etching, the photoresist serves as the secondary structuring to manufacture friction reducing riblets on top of superhydrophobic microribs. It is beyond the scope of this paper to discuss the drag reduction obtainable with these surfaces as the emphasis here is placed on the manufacture and robustness of the features. The riblets are designed to be 16 µm wide and 80 µm tall with a periodic spacing of 160 µm. They reduce friction by damping out spanwise turbulence [14]. These SU-8 structures are fabricated on silicon microribs that are 15 µm tall and 28 µm wide that have been manufactured using DRIE. An SEM of this surface is shown in the left panel Figure 6.1. It can be noted that the 28 µm microribs are platforms on which the riblets are built. The right panel of Figure 6.1 shows an SEM image after the riblets have been built on the platforms. Between successive wide (28µm) ribs are three microribs of the same height but with a width of 8 µm. The microribs are spaced periodically every 40 µm and are combined with a hydrophobic coating to render the surface superhydrophobic. Recently, superhydrophobic surfaces have also been used in turbulent drag reduction [2,51]. Because the microribs are hydrophobic, surface tension prevents water from entering the cavities between features and water flowing over the surface has a reduced liquid-solid contact area [51].
The organization of the rest of this paper is as follows. The method used to fabricate these multilevel features is described with relevant process parameters. Results including SEMs of the surfaces and testing demonstrating the robustness of the features in a turbulent channel flow are then reported. Finally conclusions of the paper are presented.

6.4 Methodology

6.4.1 Fabrication

The steps to fabricate microribs on a silicon substrate are shown in Figure 6.2. The silicon wafer is spin coated with AZ®nLOF™2020 at 2750 RPM. The photoresist is to serve as a mask during the DRIE process. The wafer is soft baked at 110°C for 1 minute, exposed to 70 mJ/cm², post-exposure baked at 110°C for 1 minute, and developed in AZ®300 MIF for 1 minute [82]. Next the wafer is dry etched until the cavities between the microribs are 15 µm deep by alternating between etch and passivation steps using the gases SF₆ and O₂ for etching and C₄F₈ for passivation. After the dry etch, the wafer is placed in Nano-strip to remove the residual photoresist. An SEM of the microribs and cavities after completing these steps is shown in the left panel of Figure 6.1.

Once a substrate has been prepared with microribs, riblets are fabricated on top of the microribs following the general process depicted in Figure 6.3. OmniCoat is applied by a spin
coating process at nominally 2000 RPM until the coating has evaporated and is then baked on a hot plate at 200°C for 2 minutes. This coating is applied to enhance the adhesion of the subsequent SU-8 coating. To ensure good coverage over the side walls of the microribs, OmniCoat is applied two times. SU-8 2075 is dispensed onto the silicon substrate and spun at low RPM until the SU-8 reaches the edges of the wafer. At this point, many air bubbles exist under the resist because of the cavities between the microribs. In order to remove these bubbles, the wafer is placed in a sealed wafer carrier and put on a hotplate at 55°C for 2 hours. The elevated temperature reduces the viscosity of the photoresist accelerating the rate at which the bubbles rise to the surface, and the sealed container minimizes the amount of solvent evaporation. The carrier is then placed on a room temperature surface for 2 hours. After the bubbles rise to the surface, the wafer is spun according to Table 6.1. The final spin at 6100 RPM is to minimize the amount of edge bead that forms during the soft bake. The wafer is soft baked at 65°C and 95°C for 5 and 15 minutes respectively. During the soft bake, a substantial edge bead on the order of 25 µm forms around the edge of
Table 6.1: Program for spinning SU-8 after bubbles have been removed.

<table>
<thead>
<tr>
<th>Speed (RPM)</th>
<th>Time (s)</th>
<th>Acceleration (RPM/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>2100</td>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>6100</td>
<td>2</td>
<td>max</td>
</tr>
</tbody>
</table>

the wafer due to surface tension. After the wafer has cooled it can be spun at nominally 3000 RPM while a swab soaked in acetone is pressed against the edge of the wafer to remove the edge bead. This is a critical step as the edge bead will prevent the photomask from coming into close contact with the photoresist and the feature resolution will be diminished due to diffraction [77]. It should be noted that the cooling after the soft bake and after every other heating step needs to be very slow to minimize thermal stresses. If the photoresist has large thermal gradients, lift off of the features is more likely. After the edge bead is removed, the wafer is heated at 65°C for 5 minutes to remove any acetone that may have been absorbed by the resist to prevent the wafer from sticking to the photomask. The wafer is exposed with 650 mJ/cm² at a wavelength of 365 nm. If shorter exposures are used lift off of the riblets becomes a problem. The wafer is post-exposure baked at 45°C, 55°C, and 60°C for 5, 5, and 120 minutes, respectively [83]. A low post-exposure bake temperature is used to minimize the thermal stresses that are induced, and the bake time is increased to still have a sufficient amount of time for cross-linking at low temperature. The wafer is developed in MicroChem’s SU-8 developer for 12 minutes with continuous agitation. After which, it is hard baked at 60°C, 90°C, 120°C, and 165°C each for 5 minutes, and finally at 200°C for 30 minutes. Increasing the temperature in steps also helps to minimize the chance of riblet lift off.

A few steps are also performed that are not directly related to the multilevel manufacturing, but are important for this specific application. In order to further strengthen the SU-8 riblets, 1 μm of aluminum is vapor deposited at 1.5 nm/s. To render the surfaces chemically hydrophobic, a 300 nm thick layer of chromium is vapor deposited at 0.5 nm/s for adhesion promotion and a solution of Teflon® AF dissolved in FC-40 is spin coated at 1000 RPM for 20 seconds. A layer of chromium on the order of 10 times thicker than is usually used [84] for adhesion promotion is deposited because the high aspect ratio structures block a significant portion of the deposition as the wafers rotate in a multi-axis planetary in the thermal evaporator. The wafer is heated at
90°C, 120°C, and 165°C each for 5 minutes, and finally at 200°C for 20 minutes. This heats the amorphous Teflon® above the polymer transition glass temperature so that some reflow of the polymer is possible to achieve better adhesion to the chromium [85]. The coating process is performed twice following these same heating steps to achieve good coverage on the side walls of the microribs and riblets.

6.4.2 Setup for Testing Feature Robustness

A recirculating flow loop was used to test the drag reduction across these surfaces, but also functions as a test of the robustness of the multilevel structures. A centrifugal pump was used to circulate distilled water at room temperature through the flow loop. The flow rate was controlled with valves in conjunction with a Coriolis flow meter. A total of 8 test wafers were diced and placed end to end on the top and bottom of an aluminum channel with an aspect ratio of 10:1 and with a hydraulic diameter of \( D_h = 7.3 \text{ mm} \) where \( D_h = 4HW/(2W + 2H) \). Here, \( W \) is the width and \( H \) is the height of the channel. A Reynolds number range of \( Re = 5 \times 10^3 \) up to \( 1.5 \times 10^4 \) was tested for the surfaces. The Reynolds number is defined as \( Re = \bar{V}D_h/\nu \) where \( \bar{V} \) is the average velocity and \( \nu \) is the kinematic viscosity of water.

6.5 Results

Figure 6.4 shows an SEM image of a surface with both microribs and riblets just after the aluminum vapor deposition. The riblets have been built on top of the 28 µm wide microribs. The photomask for the riblets is designed with rectangular slots that are 8 µm wide where the chromium on the glass mask has been etched away. When the SU-8 is exposed, the UV light diffracts passing through the mask and then spreads out over a distance of 80 µm until it reaches the silicon substrate [77]. Thus the riblets are twice as wide as the slots in the mask as can be seen in the SEM image of Figure 6.4.

Determining the amount of time for the bubbles to rise to the surface of the photoresist required some iteration. Although one hour of heating at 55°C removed a substantial number of the bubbles, it did not allow enough time for all the bubbles to rise to the surface. Consequently, bubbles left on the surface after the spin process would pop during the soft bake. When this
occurred, valleys formed resulting in a nonuniform surface. Two hours of heating worked very well combined with 2 hours of waiting afterward for cooling. A long time for complete cooling was needed because the plastic wafer carrier is a good thermal insulator.

The left panel of Figure 6.5 shows the thickness of the SU-8 at two different spin speeds of 1800 and 2100 RPM. The right panel depicts the locations where the thickness measurements were sampled. “A” is located in the center of the wafer and “B” and “C” are located nominally 2 cm and 4 cm from the center, respectively. Four wafers were tested where two were spun at 1800 RPM and two at 2100 RPM. A profilometer was used to measure the thickness of the photoresist relative to the tops of the microribs at three locations on the wafer as denoted in Figure 6.5. In general, the thickness obtained was 22% less than that predicted by MicroChem in [86]. This is certainly not due to the temperature of the photoresist being higher than room temperature because of the extremely long cooling time applied after waiting for the bubbles to rise to the surface of the photoresist during heating. The MicroChem data does not include a spin up to 6100 RPM to minimize the edge bead that is formed and this discrepancy may be related to this difference. There
Figure 6.5: The thickness of the SU-8 as a function of the spin speed given for 3 different locations on each wafer.

There is scatter in the data at both rotation rates selected due most likely to differences in the amount of SU-8 that is dispensed on the wafer.

Figure 6.6 shows the uniformity of riblet height over 9 different surfaces that were fabricated. Here, all surfaces were spun at 2100 RPM. Once again, the heights of the riblets are reported relative to the tops of the microribs and were obtained with a profilometer. The riblet height was tested at the same three locations on the wafers. The height was fairly uniform both over the sur-
face of a single wafer and also over all 9 wafers. The sample standard deviation over all 27 height measurements was only 4 $\mu$m.

Figure 6.7 is an SEM taken of one of the wafers after testing in an aluminum channel to verify robustness of the surface features. The flow in the channel was turbulent and was slowly ramped up from a velocity of 0.7 m/s at $Re = 5 \times 10^3$ to a velocity as high as 1.8 m/s at a $Re = 1.5 \times 10^4$. This was performed 4 times where flow was passing over the surface for nominally 30 minutes for each iteration. The riblets failed over less than 1% of the 8 wafers tested, however, the aluminum coating has begun to detach from the surfaces as can be noted in Figure 6.7.

### 6.6 Conclusions

It has been determined that multilevel fabrication using DRIE and ultra-thick photoresist can be used to manufacture riblets on a superhydrophobic surface patterned with microribs. Some modifications to the spin process are needed to allow for bubbles to rise out of the cavities to the top of the photoresist before spinning. Additionally, the post-exposure bake was modified to
minimize the thermal stresses induced in the riblets by lowering the final bake temperature and increasing its duration. Riblets have been fabricated with good repeatability and little variation in the riblet height. The surfaces have been tested in a channel flow with water and shown to withstand velocities as high as 1.8 m/s in turbulent flow with minimal degradation.

6.7 Acknowledgments

This research was supported by the National Science Foundation (Grant CBET-1066356).
CHAPTER 7. CONCLUSIONS

Superhydrophobic surfaces have been proposed for many applications such as a means to reduce drag in laminar and turbulent flows, alter the behavior of jet and droplet impingement, and minimize fouling because of self-cleaning properties [32, 51, 52, 87]. Superhydrophobic surfaces show unique behavior because they have microroughness combined with chemical hydrophobicity yielding high water droplet contact angles and a reduced liquid-solid contact area due to surface tension effects.

This work explores two aspects of superhydrophobic surfaces. First, laminar jet impingement on superhydrophobic surfaces demonstrating an apparent slip has been studied through the development of an analytical model that can predict the flow parameters and behavior downstream of the impingement point. Second, superhydrophobic surfaces micropatterned with ribs have been combined with larger riblet features as a means to generate a drag reduction in a turbulent channel flow. A summary of the contributions this dissertation makes in each of these areas follows. Recommendations for further study are also provided.

7.1 Jet Impingement Model

7.1.1 Isotropic Slip with an Imposed Downstream Depth

An analytical model has been developed that predicts the flow dynamics of a liquid jet impinging on a superhydrophobic surface where the surface is micropatterned randomly and a downstream depth is imposed causing a circular hydraulic jump to form. The model employs the integral control volume approach of Karman and Pohlhausen [47, 48] by solving the continuity and momentum equations numerically. The relevant thin film parameters are predicted based on the jet Reynolds number, normalized slip length, and normalized radial distance from impingement.
Because of the existence of slip at the surface, the boundary layer develops slower, the thin film height decreases more quickly, and there is increased momentum in the thin film.

The model also predicts the hydraulic jump radius for the scenario with slip. As the downstream depth decreases, the Reynolds number increases, or the slip length increases, the hydraulic jump radius increases. By comparing the results of the hydraulic jump radii of this model to the work of Bush and Aristoff [9], a modification to the normalization they proposed causes a collapse of the curves with various slip values. This modification then allows for prediction of the hydraulic jump radius for any isotropic superhydrophobic surface with slip.

7.1.2 Anisotropic Slip with an Imposed Downstream Depth

By modifying the model used for the isotropic case, the analytical model was extended to encompass the scenario where the slip is anisotropic such as is the case for micropatterned surfaces with ribs. The slip length has been modeled according to the work of Crowdy [55] to vary with azimuth. In order to close the mass and momentum equations, a constant height is assumed in the azimuthal direction. The average thin film dynamics are similar to those for the isotropic model, but now variation from the isotropic case in the azimuthal direction exists, with the greatest deviation in the directions parallel and perpendicular to the ribs and cavities. As the slip length or the Reynolds number increases, the difference from the isotropic case increases.

Because the slip is anisotropic, the hydraulic jump that forms is elliptically shaped where the major axis of the ellipse is aligned with the rib/cavity structures. The hydraulic jump becomes more eccentric as the Reynolds number increases, the slip length increases, or the downstream depth decreases. A similar collapse of the hydraulic jump radii for different slip length values to that performed for the isotropic slip case is found along the direction of average slip length. The ratio of the hydraulic jump radius along the ribs and cavities to the radius transverse to the features is also determined over a practical range of slip lengths. Combining the collapse along the direction of average slip length with the ratio of the hydraulic jump radii allows for the prediction of the hydraulic jump radius along the axes parallel and perpendicular to the rib/cavity structures given a slip length and other relevant parameters. The model results for the ratio of the hydraulic jump radius parallel and perpendicular to the ribs and cavities is compared to experimental data with very good agreement.
7.1.3 Isotropic and Anisotropic Slip with no Imposed Downstream Depth

When a liquid jet impinges on a hydrophobic or superhydrophobic surface and no downstream depth is imposed, the thin film breaks up into droplets rather than forming a hydraulic jump. Further modifications to the analytical jet impingement model were made to predict the breakup radius where the momentum is balanced by the surface tension forces. The model was first exercised for hydrophobic surfaces without slip and superhydrophobic surfaces with isotropic slip where for both the breakup radius is circular. Increasing the slip length, increasing the Weber number, or decreasing the contact angle increased the breakup radius. The critical nondimensional parameters were determined allowing for a collapse of the breakup radius onto a single curve for different contact angles, slip lengths, and other important inputs.

For the superhydrophobic surface with anisotropic slip, the breakup radius is elliptically shaped where the major and minor axes are aligned parallel and perpendicular to the ribs and cavities respectively. Comparison of the breakup radius was made to experimental data with excellent agreement. Plots of the ratio of the breakup radius along and transverse to the ribs and cavities were generated showing that as the slip length or the Weber number are increased the breakup ratio increases. Combining these plots with the collapsed curve for breakup radius for isotropic slip allows for prediction of the breakup radius in the directions parallel and transverse to the ribs and cavities.

7.2 Pressure Drop Experiments

Pressure drop tests were performed over superhydrophobic surfaces patterned with microribs, riblet patterned surfaces, and surfaces with both drag reducing mechanisms (combined surfaces). A channel with a 10:1 aspect ratio was divided into a control and a test section with surfaces on the top and bottom of the channel in both sections. Seven pressure taps were machined in the spacers between the top and bottom plates of each section to allow for computation of the pressure gradient with a linear regression.

Photolithographic techniques were used to generate the microribs through dry etching of silicon wafers. These wafers were coated with Teflon™ to render them superhydrophobic. The riblets have been manufactured on silicon wafers with an ultra-thick photoresist (SU-8).
bined surfaces have been made with a combination of both techniques. The modification of the processes was necessary to enable removal of the air from the cavities between the microribs when applying the SU-8 and to minimize the thermal stresses in the resist during baking to prevent lift off. The riblets had little variation in height and were robust showing minimal structural failure over 8 surfaces that were tested in the channel.

The testing over the riblets showed the drag reduction was a strong function of Reynolds number, reaching a maximum value of 3.0% reduction. The superhydrophobic surfaces showed the largest reduction in drag at \( Re = 5 \times 10^3 \) where a reduction of 3.2% was observed. This reduction decreased as the Reynolds number increased. It is possible that this change in performance was caused by air leaving the cavities between successive ribs even with careful control of the pressure [66]. The combined surfaces showed the highest drag reduction at low \( Re \) from \( 5 \times 10^3 \) to \( 7 \times 10^3 \) reaching a maximum of 7.0%. The reduction then decreased as the Reynolds number increased. Again, it is likely that the surfaces are losing air as the \( Re \) is increased.

### 7.3 Recommendations for Further Study

Three recommendations for future study are made in the area of jet impingement on superhydrophobic surfaces. First the jet impingement model neglects angular momentum as well as radial momentum due to flow moving azimuthally. While the model still shows good agreement with the experimental data, there is some deviation and this could be related to these simplifications. Second, while some experimental work has been done with isotropic hydrophobic and anisotropic superhydrophobic surfaces where the thin film breaks up and some has been completed on anisotropic surfaces where the thin film experiences a hydraulic jump, to the authors knowledge there is no work on isotropic superhydrophobic surfaces where a hydraulic jump forms or where the thin film breaks up. A third area needing study is the possibility of both breakup and a hydraulic jump existing simultaneously on a surface. It is also conceivable that a jet impinging on superhydrophobic surface could break up into droplets and then form a hydraulic jump downstream of the breakup location if a downstream depth were imposed after the film had already broken up.

Further study in the pressure drop testing would be recommended in two areas. First regarding the superhydrophobic surfaces, it would be interesting to vary the module width of the
ribs and cavities and the depth of the cavities. The depth of the cavities may affect the amount of
time air remains if it is indeed being lost and changing the module width may affect the stability
of the air-water interface as well as change the Laplace pressure at which the air pockets would
fail [88]. A second area of interest would be using microposts with the riblets. Microposts have
the potential to have higher cavity fractions than is possible with microribs, however, the spanwise
slip is much larger which would result in increased spanwise turbulence. Thus, it may be easier to
see the influence the riblets have by creating riblets on a surface patterned with microposts.
REFERENCES


[58] Xue, Y., Chu, S., Lv, P., and Duan, H. “Importance of hierarchical structures in wetting stability on submersed superhydrophobic surfaces.” Langmuir. 84


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APPENDIX A. MICROFABRICATION PROCESSES

This appendix describes the microfabrication procedures used to make surfaces with microribs, riblets, and with both types of features. The instructions for operating the photolithography, metrology, and other cleanroom equipment are not outlined here as BYU’s Integrated Microfabrication Lab (IML) website at www.cleanroom.byu.edu contains this information. The specific procedures, settings, and parameters needed to fabricate these features will be the focus of this appendix.

A.1 Microribs

The process for manufacturing high aspect ratio microribs is described in this section. The fabrication involves standard photolithographic processes using a negative photoresist combined with deep reactive ion etching (DRIE) to generate the structures. An SEM image of the microribs is shown in Figure A.1. The microribs are nominally 15 \( \mu \)m tall, periodically spaced at 40 \( \mu \)m, and are 8 \( \mu \)m wide. The steps are as follows:

1. Clean wafer with spinner using acetone to remove residual photoresist or other contaminates, followed by a rinse with isopropanol to remove the residual acetone.

2. Dehydration bake the wafer at 150\(^\circ\)C for 15 min in the clean oven 2 manufactured by Labline Instruments.

3. Spin on AZ®nLOFT\textsuperscript{TM}2020 at 2750 RPM for 60 seconds which will result in a thickness of nominally 2 \( \mu \)m. The AZ®nLOFT\textsuperscript{TM}2020 is a negative, i-Line photoresist (PR), most sensitive to the wavelength of light equal to 365 nm [89]. Additional information about the resist can be found on BYU’s IML website.

4. Soft bake at 110\(^\circ\)C on a hot plate for 1 min to expel the majority of the solvent in the resist.
5. Expose the wafer to 70 mJ/cm$^2$ using the Karl Suss Mask Aligner (MA 150 CC) and the desired photomask to begin the cross-linking process. The Karl Suss UV Intensity Meter (Model 1000) can be used to determine the exposure time based on the mercury lamp intensity. A typical time would be 10 seconds if the bulb is outputting 7 mW/cm$^2$.

6. Post exposure bake the wafer at 110$^\circ$C on a hot plate for 1 minute to continue the cross-linking process. This will render the PR insoluble to the developer everywhere cross-linking has taken place.

7. Develop in AZ®300 MIF for 1 minute to remove the PR that has not been cross-linked.

8. Dry etch the wafer in the inductively coupled plasma (ICP) of the STS. The recipe is outlined in Table A.2.

9. Remove cross-linked PR by placing wafer in Nano-strip composed of 90% sulfuric acid, 5% peroxymonosulfuric acid, < 1% hydrogen peroxide, and 5% water [90]. The solution should be heated to 90$^\circ$C until PR is removed. A typical time is 2 hours.

10. Vapor deposit 250 to 300 nm of Cr using the thermal evaporator. The purpose of this layer is to promote adhesion of the subsequent Teflon® coating. The process for operating the evaporator can be found on the IML website. It is important to maintain a deposition rate as
Table A.1: This is the recipe that should be followed to etch silicon microribs.

<table>
<thead>
<tr>
<th>ICP Parameters</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>Pump down</td>
<td>30 s</td>
</tr>
<tr>
<td></td>
<td>Stabilization</td>
<td>10 s</td>
</tr>
<tr>
<td></td>
<td>Process</td>
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</tr>
<tr>
<td></td>
<td>Pump out time</td>
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</tr>
<tr>
<td><strong>Advanced options</strong></td>
<td>Parameter switching</td>
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<tr>
<td></td>
<td>Parameter ramping</td>
<td>off</td>
</tr>
<tr>
<td><strong>Parameter switching</strong></td>
<td>Etch phase</td>
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<td></td>
<td>Passivation phase</td>
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<td>Overrun etch</td>
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</tr>
<tr>
<td></td>
<td>Overrun pass.</td>
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</tr>
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<td>Cycles</td>
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<td>Starts with</td>
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<tr>
<td></td>
<td>Ends with</td>
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</tr>
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<td><strong>Process step mode</strong></td>
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<td></td>
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<tr>
<td>C4F8 (Pass.)</td>
<td>Flow (sccm)</td>
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</tr>
<tr>
<td></td>
<td>Tol (%)</td>
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<tr>
<td>SF6 (Etch)</td>
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<td></td>
<td>Tol(%)</td>
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<td>O2 (Etch)</td>
<td>Flow (sccm)</td>
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<tr>
<td></td>
<td>Tol(%)</td>
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Table A.2: This is the recipe that should be followed to etch silicon microribs.

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<th>ICP Parameters Cont.</th>
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<td>Etch power 600 W</td>
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<tr>
<td></td>
<td>Platen gen. to 13.56MHz</td>
<td>Passivation power 600 W</td>
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<td></td>
<td>Matching Auto</td>
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<td></td>
<td></td>
<td>Load 50%</td>
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<td></td>
<td></td>
<td>Tune 50%</td>
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<tr>
<td></td>
<td></td>
<td>Tolerance 15%</td>
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<table>
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<tr>
<td>Parameters</td>
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</tr>
<tr>
<td></td>
<td>Tolerance 25%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max flow 40 sccm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min flow 10 sccm</td>
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</table>

<table>
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</thead>
<tbody>
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<td></td>
<td>Test time 30 s</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max leak rate 15 mTorr/min</td>
<td></td>
</tr>
</tbody>
</table>

close to 0.5 nm/s as possible, especially when opening the shutter to ensure good adhesion to the silicon wafer. It is critical that the planetary is rotating throughout the deposition process to ensure good coverage over the sidewalls of the microribs.

11. Spin coat the wafer with a Teflon® solution immediately upon removal from the thermal evaporator to limit contamination of the surface. A 6% solution of Teflon® AF 1600 should be mixed in a 1:29 volumetric ratio with FC-40 or FC-75. The wafer should be spun at 1000
RPM for 20 s and should be immediately placed on a hot plate at 90°C. It should be heated at 90°C for 5 min, 165°C for 5 min, and 330°C for 20 min. The temperatures selected are not arbitrary. 160°C is the glass temperature of the Teflon® AF 1600 and gives time for a slow change through the glass transition. 330°C is hot enough to allow for the Teflon® to flow without being hot enough to burn off the surface [85].

12. Dice the wafer using the Disco saw to a width of 49.2 mm and a length of 80.1 mm.

13. Repeat Step 11 to coat the wafer with a second layer of Teflon®.

A.2 Riblets

This section explains the steps to fabricate riblets out of a thick photoresist (SU-8 2075). Here the photoresist is not a protective mask, but will function as the actual microstructure. An SEM of these features is shown in Figure A.2 and are 80 µm tall, spaced periodically at 160 µm, and are 18 µm wide.

Figure A.2: SEM of silicon surface patterned with riblets out of SU-8 2075.

1. Clean and dehydration bake the wafer as outlined in Section A.1.
2. Spin on OmniCoat at 2000 RPM until the coating has evaporated.

3. Heat wafer on a hot plate at 200°C for 2 min.

4. Spin on SU-8 2075 by programming the spinner with the following steps:
   (a) 500 RPM for 5 s with an acceleration of 100 RPM/s
   (b) 2250 RPM for 30 s with an acceleration of 300 RPM/s
   (c) 6100 RPM for 2 s with the maximum acceleration available

5. Stop the spinner when the SU-8 reaches the edge of the wafer. Wait until the large bubbles have risen (5 min), reset the spinner, and start the spinner again.

6. Soft bake the wafer at 65°C for 5 min, ramp up to 95°C, and bake for 10 min. Leave on hot plate for 30 min for the wafer to return to room temperature.

7. Remove the edge bead (EBR) by using a swab soaked in acetone to lightly touch the edge of the wafer while it is spinning on the spinner at nominally 3000 RPM. The removal of the edge bead is important because it allows the mask to come into closer contact with the SU-8. This allows for better feature size control because it limits how far the light will spread before passing through the PR.

8. Heat the wafer at 65°C for 5 min and ramp slowly back to room temperature to remove any acetone that has been absorbed during the edge bead removal process. The purpose of this step is to prevent the wafer from sticking to the mask during exposure.

9. Expose the wafer with 650 mJ/cm² to cross-link the photoresist.

10. Post exposure bake the wafer at 45°C for 5 min, 55°C for 5 min, and 60°C for 60 min. Again, turn off the hot plate and wait for 30 min to allow the wafers to cool to room temperature on the hot plate. Thermal stresses are damaging to the features and can cause cracking or lift off from the silicon substrate.

11. Develop in MicroChem’s SU-8 developer until the surface does not generate a white film when rinsed in isopropanol [86]. This will generally take 10-14 min depending on the level of agitation.
12. Hard bake the surface by heating the wafer on a hot plate at 60°C, 90°C, 120°C, and 165°C each for 5 minutes. Then heat the wafer at 200°C for 30 min and let the hot plate cool to room temperature for at least 30 min on the hot plate. This process will cross-link all the photoresist on the wafer that may not yet be cross-linked.

13. Vapor deposit 1 µm of aluminum using the thermal evaporator. This layer will serve to strengthen the SU-8 and keep it attached to the surface during dicing and subsequent testing. The process for operating the machine can be found on the IML website. It is important to maintain a deposition rate as close to 1.5 nm/s as possible, especially when opening the shutter to ensure good adhesion to the silicon wafer. It is critical that the planetary rotate throughout the deposition process to get good coverage over the wafer.

14. Dice the wafer using the Disco saw to a width of 49.2 mm and a length of 80.1 mm. (Note: This process did not require spinning on a layer of SU-8 2025 to protect the riblets. However, this was required when building the riblets on the platforms for the combined surfaces described subsequently.)

A.3 Surface with Microribs and Riblets

The processes for manufacturing a surface with microribs and riblets are described in this section. The process for fabricating the microribs is identical to that used in manufacturing microribs alone and will not be duplicated here. The riblet process used in the combined surface manufacture is similar to the riblet fabrication process in the previous section, however, because many of the parameters have been adjusted, they will be repeated here with modifications. An SEM of a surface that has both microribs and riblets is shown in the left panel of Figure A.3.

1. Follow steps 1-9 outlined in A.1 using the mask with microribs and platforms. The riblets will be fabricated on the platforms. A surface with microribs and platforms is shown in the right panel of Figure A.3.

2. Clean and dehydration bake the wafer as outlined in A.1.

3. Spin on OmniCoat at 2000 RPM until the coating has evaporated.
4. Heat wafer on a hot plate at 200°C for 2 min.

5. Spin on SU-8 2075 by programming the spinner with the following steps:
   
   (a) 500 RPM for 5 s with an acceleration of 100 RPM/s
   (b) 2100 RPM for 30 s with an acceleration of 300 RPM/s
   (c) 6100 RPM for 2 s with the maximum acceleration available

6. Stop the spinner when the SU-8 reaches the edges of the wafer. Heat in a sealed wafer carrier for 2 hr on a hot plate set to 55°C. Remove carrier from hot plate and wait 2 hr. Reset the spinner, remove wafer from the carrier and place on spinner. Run the program again allowing it to run to completion.

7. Soft bake the wafer at 65°C for 10 min, ramp up to 95°C, and bake for 15 min. Wait 30 min for the hot plate to gradually return to room temperature.

8. Remove the edge bead (EBR) by using a swab soaked in acetone to lightly touch the edge of the wafer while it is spinning on the spinner at nominally 3000 RPM.

9. Heat the wafer at 65°C for 5 min and ramp slowly back to room temperature to remove any acetone that has been absorbed during the edge bead removal process.

Figure A.3: SEM of surface with microribs and riblets in the left panel and a surface before the riblets have been built on the wider microribs referred to as platforms.
10. Expose the wafer with 650 mJ/cm² to cross-link the photoresist.

11. Post exposure bake the wafer at 45°C for 5 min, 55°C for 5 min, and 60°C for 120 min. Again, turn off the hot plate and wait for 30 min to allow the wafers to cool to room temperature on the hot plate.

12. Develop in MicroChem’s SU-8 developer until the surface does not generate a white film when rinsed in isopropanol [86]. This will generally take 10-14 min depending on the level of agitation.

13. Hard bake the surface by heating the wafer on a hot plate at 60°C, 90°C, 120°C, and 165°C each for 5 minutes. Then heat the wafer at 200°C for 30 min and let the hot plate cool to room temperature for at least 30 min on the hot plate.

14. Vapor deposit 1 µm of aluminum using the thermal evaporator. It is important to maintain a deposition rate as close to 1.5 nm/s as possible especially when opening the shutter to ensure good adhesion to the silicon wafer. It is critical that the planetary rotate throughout the deposition process to get good coverage over the wafer.

15. Spin on SU-8 2025 to protect the riblets during dicing by programming the spinner with the following steps:

   (a) 500 RPM for 5 s with an acceleration of 100 RPM/s

   (b) 2000 RPM for 30 s with an acceleration of 300 RPM/s

16. Stop the spinner when the SU-8 reaches the edges of the wafer. Heat in a sealed wafer carrier for 1.5 hr on a hot plate set to 55°C. Immediately, reset the spinner and spin the wafer.

17. Soft bake the wafer at 55°C for 5 min, 65°C for 5 min, 75°C for 5 min, 85°C for 5 min and at 95°C for 10 min. Wait 30 min for the hot plate to gradually return to room temperature.

18. Dice the wafer using the Disco saw to a width of 49.2 mm and a length of 80.1 mm.

19. Immerse the wafer in acetone until all SU-8 2025 is removed. Check that all SU-8 is removed with the microscope before moving to the next step.
20. Vapor deposit 250 to 300 nm of Cr using the thermal evaporator. The purpose of this layer is to promote adhesion of the subsequent Teflon® coating. The process for operating the evaporator can be found on the BYU cleanroom website at www.cleanroom.byu.edu. It is important to maintain a deposition rate as close to 0.5 nm/s as possible, especially when opening the shutter to ensure good adhesion to the silicon wafer. It is critical that the planetary is rotating throughout the deposition process to enable good coverage over the sidewalls of the microribs.

21. Spin coat the wafer with a Teflon® solution immediately upon removal from the thermal evaporator to limit contamination of the surface. Teflon® AF1600 solution should be mixed in a 1:29 volumetric ratio with FC-40 or FC-75. The wafer should be spun at 1000 RPM for 20 s and should be immediately placed on a hot plate at 90°C. It should be heated at 60°C, 90°C, 120°C, and 165°C each for 5 min. Then heat the surface to 200°C for 20 min and wait 30 min for the wafers to return to room temperature before removal. The wafers cannot be heated above 200°C because it will destroy the SU-8 riblets.

22. Repeat the previous step so that the wafer is coated 2 times.

A.4 Alternative Manufacturing Method of Surface with Microribs and Riblets

This section contains an alternate way of manufacturing a surface with microribs and riblets. The method uses wet etching of silicon to fabricate trapezoidal riblets and then a deep reactive ion etch to make the microribs. A rendering of such a surface is shown in Figure A.4. Reflections off the sidewalls of the riblets prohibited effective manufacture of the microribs. This will be elaborated on in detail in this section. An overview of the process to manufacture the features is shown in Figure A.5. The steps to fabricate these features are as follows:

1. Clean and dehydration bake the wafer as outlined in A.1.

2. Grow oxide 2 μm thick in Bruce Furnace. Because the process takes an entire day, multiple wafers should be processed on each run. The first wafer placed in the quartz cassette should be a dummy wafer as the gas flow will be different past the first wafer. Program the furnace according to Table A.3.
Figure A.4: CAD rendering of surface with trapazoidal riblets combined with microribs.

Figure A.5: Process used to manufacture a surface with trapazoidal riblets combined with microribs.

Table A.3: These are the parameters that should be programmed into the Bruce Furnace to obtain a thickness of 2 µm.

<table>
<thead>
<tr>
<th>Oxide Growth Process</th>
<th>Time</th>
<th>Temperature °C</th>
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</tr>
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<td>Push</td>
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<td>850</td>
<td>O₂ dry, O₂</td>
</tr>
<tr>
<td>Stabilize</td>
<td>1 s</td>
<td>850</td>
<td>N₂</td>
</tr>
<tr>
<td>Ramp</td>
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<td>1125</td>
<td>N₂</td>
</tr>
<tr>
<td>Stabilize</td>
<td>20 min</td>
<td>1125</td>
<td>N₂</td>
</tr>
<tr>
<td>Dry O₂ Process</td>
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<td>N₂</td>
</tr>
<tr>
<td>Pull</td>
<td>12.28 hr</td>
<td>1125</td>
<td>O₂ wet</td>
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<tr>
<td>Expose w/ UV &amp; Develop</td>
<td>1 s</td>
<td>1125</td>
<td>N₂</td>
</tr>
<tr>
<td>RIE Etch using ANELVA</td>
<td>1.25 hr</td>
<td>850</td>
<td>N₂</td>
</tr>
<tr>
<td>Stabilize</td>
<td>1 s</td>
<td>850</td>
<td>N₂</td>
</tr>
<tr>
<td>Pull</td>
<td>4 min</td>
<td>850</td>
<td>N₂</td>
</tr>
</tbody>
</table>
3. Spin on OmniCoat at 2000 RPM until the coating has evaporated.

4. Heat wafer on a hot plate at 200°C for 2 min.

5. Spin on SU-8 2025 by programming the spinner with the following steps to achieve a thickness of 23 μm:
   (a) 500 RPM for 10 s with an acceleration of 100 RPM/s
   (b) 4000 RPM for 30 s with an acceleration of 300 RPM/s
   (c) 6100 RPM for 2 s with the maximum acceleration available

6. Soft bake the wafer at 65°C for 3 min, ramp up to 95°C, and bake for 6 min. The wafer needs to be cooled on a nonmetal surface.

7. Expose the wafer with 400 mW/cm² to cross-link the photoresist.

8. Post exposure bake the wafer at 55°C for 5 min, 65°C for 5 min, and 95°C for 5 min. Again, the wafer needs to be cooled on a nonmetal surface.

9. Develop in MicroChem’s SU-8 developer until the surface does not generate a white film when rinsed in isopropanol [86]. This will generally take 4 min depending on the level of agitation.

10. Etch away the silicon dioxide not protected by the SU-8 using a reactive ion etch (RIE) in the ANELVA. Set the O₂ gas flow to 0.9 sccm and the CF₄ rate to 18 sccm. The power should be set to 250 W and the pressure in the chamber should be maintained at 100 mTorr. Details for how to operate the ANELVA are located on the IML website. These settings will etch SiO₂ at 90 nm/min. The film thickness should be checked using the Filmetrics F20 film measurement system to verify the etch rate periodically. A typical etch time is about 20 min.

11. Remove residual photoresist by placing wafer in Nano-strip. The solution should be heated to 90°C until the PR is removed.

12. Heat a solution of 45% potassium hydroxide (KOH) on a hot plate to 90°C. (The hot plate should be set to 110°C.) Place a dummy piece of silicon in the KOH for 5 min to buffer the
solution. The silicon should etch at a rate of 1.4 \( \mu m/min \) and the silicon dioxide etch rate should be 19.7 \( \mu m/min \). Etch the surface until the riblet features are 80 \( \mu m \) tall.

13. Clean and dehydration bake the wafer as outlined in A.1.

14. Spin on OmniCoat at 2000 RPM until the coating has evaporated.

15. Heat wafer on a hot plate at 200°C for 2 min.

16. Spin on SU-8 2075 by programming the spinner with the following steps:

   (a) 500 RPM for 5 s with an acceleration of 100 RPM/s

   (b) 2200 RPM for 30 s with an acceleration of 300 RPM/s

   (c) 6100 RPM for 2 s with the maximum acceleration available

17. Steps 6-22 from section A.3 could then be followed exactly.

The problem with this method occurs during the exposure of the SU-8 2075. Because the riblets are wet etched at a 54.7° angle, the light that is incident on the riblet structures will reflect down across the silicon surface in between the riblets as shown in the top panel of Figure A.6. One way to avoid this is to spin on an anti-reflective coating (ARC) such that the light would not reflect off the silicon. Unfortunately, this is anticipated to be difficult because ARCs are sensitive to thickness and the ARC would need to be spun onto a surface with riblets that are 75-80 \( \mu m \) tall. Evidence that this is indeed the issue is shown in Figure A.6 in the bottom panel where the microribs have been rotated perpendicular to the riblet features. Here, the reflection off the riblet is inline with the microrib, and it can be noted that the microribs are not affected by the reflections off the sidewalls of the riblets. This problem with reflections off the side walls has also been well documented by Linder et al. [77].
Figure A.6: Reflections off sidewalls of the riblets into the valleys between riblets in the top panel and in the bottom panel microribs that have been aligned perpendicular to the riblets.
APPENDIX B. EXPERIMENTAL SETUP

This appendix contains the schematics of the channel design and specifications of the equipment used in the flow loop to measure the drag across riblets, superhydrophobic surfaces, and surfaces with both drag reducing mechanisms.

B.1 Flow Loop Equipment

Figure B.1 shows the flow loop used to take pressure drop measurements. All equipment is specified in Table B.1 according to the labels in Figure B.1.
Table B.1: Equipment used in flow loop.

<table>
<thead>
<tr>
<th>Label</th>
<th>Equipment</th>
<th>Model</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>centrifugal pump</td>
<td>TE-5.5C-MD</td>
<td>March Mfg.</td>
</tr>
<tr>
<td>B</td>
<td>1/3 hp motor</td>
<td>TQJ56A34F7A</td>
<td>Marathon Electric</td>
</tr>
<tr>
<td>C</td>
<td>heat exchanger</td>
<td>500 XS</td>
<td>The Heat-X-Exchanger Co.</td>
</tr>
<tr>
<td>D</td>
<td>centrifugal pump</td>
<td>AC-2CP-MD</td>
<td>March Mfg.</td>
</tr>
<tr>
<td>E</td>
<td>1/40 hp motor</td>
<td>0125-0052-1000</td>
<td>Fasco Ind.</td>
</tr>
<tr>
<td>F</td>
<td>Coriolis mass flow meter</td>
<td>CMF-DNHOAOAIC-H0000A</td>
<td>FCI</td>
</tr>
<tr>
<td>G</td>
<td>differential pressure XDCR</td>
<td>IDP10-A</td>
<td>Viatran</td>
</tr>
<tr>
<td>H</td>
<td>pressure XDCR</td>
<td>PX2300-1DI</td>
<td>Omega</td>
</tr>
<tr>
<td>I</td>
<td>inclinometer sensor</td>
<td>IS-2-30</td>
<td>Level Developments</td>
</tr>
</tbody>
</table>

B.2 Channel Schematics

This section contains the schematics of the channel design used in the manufacture of the aluminum channel.
Figure B.2: Schematic of top and bottom plates of the channel (view A).
Figure B.3: Schematic of top and bottom plates of the channel (view B).
Figure B.4: Schematic of top and bottom plates of the channel (view C).
Figure B.5: Schematic of spacer between the top and bottom plates (view A).
Figure B.6: Schematic of spacer between the top and bottom plates (view B).
Figure B.7: Schematic of spacer between the top and bottom plates (view C).
Figure B.8: Schematic of end cap attached to the entrance and exit of the channel.
Figure B.9: Schematic of wedge for inclinometer (view A).
Figure B.10: Schematic of wedge for inclinometer (view B).
Figure B.11: Schematic of the bar used to align the wedge for the inclinometer.
Figure B.12: Schematic of kinematic bar attached to the fixture.
Figure B.13: Schematic of kinematic bar (link 3).
Figure B.14: Schematic of kinematic bar (link 4).
Figure B.15: Schematic of fixture to support the channel.
APPENDIX C. UNCERTAINTY ANALYSIS

The uncertainty analysis used in the channel experiments to compute the error in the friction factor, $f$, and the Reynolds number, $Re$, are outlined in this appendix.

C.1 Friction Factor

The friction factor, $f$, is computed from the equation for major head loss in turbulent pipe flow as given in [72].

$$f = \frac{2(-dp/dx)D_h}{\rho \bar{V}^2} \quad (C.1)$$

Here, the hydraulic diameter, $D_h = 2HW/(H+W)$ where $H$ is the height of the channel cross-section and $W$ is the width, $dp/dx$ is the pressure gradient along the channel, $\rho$ is the density, and $\bar{V}$ is the average velocity. In order to relate the actual measurements to this equation, substitution for $D_h$ and for $\bar{V}$ is necessary. $\bar{V}$ can be related to the mass flow rate, $\dot{m}$, through continuity where $\dot{m} = \rho \bar{V} HW$ and $D_h$ can be related to $H$ and $W$ by its definition. These substitutions result in Equation C.2.

$$f = \frac{4(-dp/dx)\rho H^3 W^3}{\dot{m}^2 (H+W)} \quad (C.2)$$

The functional relationships found in Equation C.2 can be used to estimate the propagation of the errors [91] in each measurement resulting in Equation C.3.

$$\frac{\chi_f}{f} = \left[ \left( \frac{\chi_{dp/dx}}{dp/dx} \right)^2 + \left( \frac{\chi_{\rho}}{\rho} \right)^2 + \left( \frac{2\chi_{\dot{m}}}{\dot{m}} \right)^2 \right. \left. + \left( \frac{\chi_H(2H+3W)}{H^2+WH} \right)^2 + \left( \frac{\chi_W(2W+3H)}{W^2+WH} \right)^2 \right]^{1/2} \quad (C.3)$$

Here, $\chi$ is the uncertainty of the variable that is subscripted. If this equation is multiplied by 100 the percent error in $f$ is obtained.
Now that the error propagation of the different parameters has been determined, each of
the individual measurement errors can be found and substituted into Equation C.3. The uncertainty
in the pressure gradient is dominated by the error in the least squares linear regression through the
pressure tap data computed as outlined by Weisberg [92]. Table C.1 shows the range of error
introduced by the line fit for all the surface types. The error is the largest for the combined surfaces

<table>
<thead>
<tr>
<th>Case</th>
<th>% Error in $dp/dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>smooth</td>
<td>1.8-7.0</td>
</tr>
<tr>
<td>riblet</td>
<td>1.6-5.2</td>
</tr>
<tr>
<td>SHS</td>
<td>1.8-8.5</td>
</tr>
<tr>
<td>combined</td>
<td>4.9-14.9</td>
</tr>
</tbody>
</table>

due to large fluctuation in pressure along the channel for this case.

The instrument error of the pressure transducer is also significant in computing the pressure
gradient and is equal to $\pm 0.2\%$ of the full scale (FS) which results in error of 2.5 Pa. The error is the
highest when the pressure drop being measured is small. Because of this, the data between pressure
taps 1 and 2 was discarded because of high instrument error. The next highest instrument error
occurs when $Re = 5 \times 10^3$ and the pressure drop is measured over taps 1-3 resulting in a maximum
error of $\pm 5.7\%$. Here, $Re = \rho \bar{V} A / \nu$ where the cross-sectional area $A = HW$ and $\nu = \mu / \rho$ where
$\mu$ is the dynamic viscosity. The error reduces as the pressure drop is measured between taps that
are farther apart or when the Reynolds number is increased. For example when $Re = 7 \times 10^3$ and
when measuring between taps 1-3, the error is $\pm 2.8\%$ or for the best case when $Re = 1.5 \times 10^4$
and the pressure drop is recorded over taps 1-7, the error is $\pm 0.3\%$.

Because the pressure gradient includes the error over all tap combinations 1-3, 1-4, 1-5, 1-6,
and 1-7 at each Reynolds number, the instrument error can be pooled as shown in Equation C.4.

$$\chi_{cp} = \left[ \frac{1}{N} \sum_n \chi_{cn}^2 \right]^{1/2} \tag{C.4}$$
Here, $\chi_{cp}$ is the pooled instrument error, $\chi_{cn}$ is the instrument error for each pressure tap combination, and $N$ is the total number of pressure measurements. Pooling the instrument error results in uncertainty ranging from $\pm0.4\%$ to 3.5%.

The pressure taps themselves are also potential sources of error. The applicable biases in this scenario are the result of the geometry of the taps, burrs that may be extending out into the flow beyond the tap, and any chamfer that exists at the entrance of the tap [93–96]. These biases are caused by curvature introduced into the streamlines in the vicinity of the pressure tap and vortices that set up in the tap [94]. Equation C.5 estimates the true static pressure at the wall, $p_w$, that exists based on different biases, $\Pi$, related to the geometry.

$$p_w = p_{mw} - \Pi \tau_w$$  \hspace{1cm} (C.5)

Here, $p_{mw}$ is the measured wall pressure, $\Pi$ is a nondimensional parameter that quantifies the bias of the tap, and $\tau_w$ is the shear stress at the wall. The hole size $d_s$ is one parameter that affects the pressure reading and the error is predicted by McKeon and Smits [94]. For these experiments, $d_s^+$, defined as the normalized hole size, ranges from 41 up to 124. Here, $d_s$ is normalized by the friction velocity, $u_\tau$, and the kinematic viscosity where $u_\tau = \sqrt{\tau_w/\rho}$. For these values, $\Pi$ ranges from 0.2 to 0.4 and results in errors less than 1% of the static pressure. The relative tapping depth, $l_s/d_s$, also generates a bias [95] where $l_s$ is the depth of the tapping. For the taps used in this study, $l_s/d_s = 5.5$ resulting in errors that again are less than 1% of the static pressure. Both errors for hole size and relative tapping depth are positive errors and will raise the static pressure reading. However, because the geometry is the same for all taps and the pressure differential between two taps is the quantity being measured this bias error will be negligible.

The taps were all inspected with an SLR camera to verify that no burrs were present. An example of one of the taps is shown in Figure C.1. It is easy to see that a small chamfer exists around the rim of the tap that is nominally 20% of the hole diameter. Chue [96] has shown that errors as large as 1% can be detected due to a chamfer. Because all of the taps have chamfers, it is only the variation in chamfers that will affect differential pressure readings. It is estimated that in the worst case this would amount to half the error measured by Chue because of the variation in chamfer sizes and has been included in the error associated with the differential pressure readings.
The only other error associated with the uncertainty of the pressure gradient is the error in the location of the pressure taps. The uncertainty is ±0.2% when considering the error in the distance between any two consecutive taps which is 25 mm.

The error in $H$ is determined by combining the errors associated with each thickness measurement used in calculating $H$. The thickness of the cross-section of the flow is computed by determining the total thickness of the channel, $H_T$, when it is assembled and then subtracting the thickness of the top plate, $H_{tp}$, the thickness of the bottom plate, $H_{bp}$, the thickness of the wafers, $H_w$ and $H_{wb}$, and the thickness of the vacuum grease, $H_{vg}$, as shown Figure C.2. The dimensions are not to scale and have been modified here for clarity. A very thin layer of vacuum grease is used to adhere the wafers to the top and bottom plates for assembly of the channel. Equation C.6 shows how $H$ is calculated.

\[
H = H_T - (H_{tp} + H_{bp} + H_w + H_{wb} + 2H_{vg})
\]  

(C.6)

The total channel thickness was found over the region where pressure drop measurements were made in both the control and the test sections and was determined with 20 micrometer measurements for each section resulting in a total of 40 measurements. The thickness of the top and bottom plates was determined in the same manner. Four micrometer measurements were used for the wafer thickness of each wafer as the wafer thickness has negligible variation. The vacuum grease thick-
ness was estimated from the mean thickness found by inserting a wafer 20 times into the bottom plate and measuring the vacuum grease thickness each time by subtracting the plate and wafer thickness from the combined thickness of the plate, wafer, and vacuum grease. The micrometer used for the total channel thickness has an instrument error of 5.5 $\mu$m and the micrometer used for the plate and wafer thickness has an instrument error of 1.2 $\mu$m. Equation C.7 shows the computation of the total error in the channel height, $\chi_H$, determined by combining the instrument error, $\chi_c$, and the error in the estimate of the mean, $\chi_s$, for each thickness, $H_n$, listed in Equation C.6. The resolution error is contained within the error of the estimate of the mean.

$$\chi_H = \left[ \sum_n (\chi_c)_n^2 + (\chi_s)_n^2 \right]^{1/2} \quad (C.7)$$

Here, $\chi_s = t_{(df,P)} \bar{S}_H$ where $t$ is the t-statistic, $df$ is the number of degrees of freedom, $P$ is the probability taken to be 95%, and $\bar{S}_H$ is the standard deviation of the means of the height measurements. A typical value for the error in $H$ is $\pm 0.8\%$ and the worst case resulted in $\pm 1.0\%$.

The error in the width of the cross-section of the flow, $\chi_W$, is determined similarly to $\chi_H$. The width of the channel cross-section is the distance between the left and right spacers as shown in Figure C.3. Here the dimensions are not to scale and have been exaggerated for clarity. $W$ is computed by measuring the location of the spacers relative to the top plate as described by Equation C.8.

$$W = W_{tp} - (W_{ls} + W_{rs}) - (W_a + W_b) \quad (C.8)$$
Here, \( W_{tp} \) is the width of the top plate, \( W_{ls} \) and \( W_{rs} \) are the left and right spacer widths, respectively, \( W_a \) and \( W_b \) are the deviations of the left and right spacers, respectively, where a positive value indicates the spacer is inset from the top plate. Because the channel assembles in the same configuration each time and because of the geometry of a depth micrometer, \( W_b \) must be calculated from the difference between \( W_c \) and \( W_d \) as shown in Figure C.3. 20 micrometer measurements are taken over the control section and the test section for a total of 40 measurements for each width listed in Equation C.8. The width measurements were recorded with 3 different micrometers with errors listed in Table C.2. The errors are combined following Equation C.9 as they were treated for the height of the channel.

\[
\chi_W = \left[ \sum \frac{(\chi_{c_n})^2}{n} + \frac{(\chi_{s_n})^2}{n} \right]^{1/2}
\]

Here, for the worst case, the error in \( W \) is 0.3%.

Table C.2: This table lists the error associated with each micrometer used in the computation of the width of the channel.

<table>
<thead>
<tr>
<th>Micrometer</th>
<th>Instrument Error</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>depth</td>
<td>±25 ( \mu )m</td>
<td>( W_a, W_b, W_c, W_d )</td>
</tr>
<tr>
<td>outside</td>
<td>±5.5 ( \mu )m</td>
<td>( W_{tp} )</td>
</tr>
<tr>
<td>outside</td>
<td>±1.2 ( \mu )m</td>
<td>( W_{ls}, W_{rs} )</td>
</tr>
</tbody>
</table>
The error in the density measurement originates from the error in the temperature where the Coriolis flow meter measures the temperature to ±0.5°C and from the error in the linear interpolation used to estimate the density. The error due to temperature error is ±0.01% and the error due to interpolation is negligible. The error in the mass flow rate is ±0.40% over the Reynolds number range explored.

When all of these errors are combined into Equation C.3, the error in \( f \) can be estimated. \( \chi_{dp/dx} \) dominates the error over the entire range of \( Re \). Figure C.4 shows the error plotted as a function of \( Re \) for the control section with smooth surfaces and the worst case with combined surfaces in the test section. The error for the control section for this test ranges from ±5.5%

down to ±4.1%. For the combined surfaces, the errors are much larger ranging from ±11.0% up to ±15.0%. The cause for the large error is the uncertainty in the linear regression due to large pressure variation along the channel for the combined surfaces.

The error due to the quality of the linear regression can be minimized through replication [91] where the uncertainty in \( dp/dx \) can be pooled as shown in Equation C.10

\[
\chi_{dp/dx} = \left[ \frac{1}{M} \sum_n \chi^2_{(dp/dx)n} \right]^{1/2} \quad (C.10)
\]

Figure C.4: Uncertainty in the friction factor plotted as a function of \( Re \) for smooth surfaces and for the worst case with combined surfaces.
Here, $\chi_{dp/dx}$ is the pooled error of the pressure gradient, $\chi_{(dp/dx)n}$ is error for each replicate, and $M$ is the number of replicates. For the uncertainty associated with the pressure gradient, the error of the means can be computed by dividing the error by the square root of the number of replicates. For all other errors, the errors can simply be pooled similar to Equation C.10. A pooled mean of $\bar{f}$ can also be computed. For 2 replicates, the error in $\bar{f}$ is plotted as a function of $Re$ in Figure C.5. It is clear that replication reduces the error substantially. The maximum error in $\bar{f}$ is now $\pm 4.8\%$ for the smooth surfaces and $\pm 10.8\%$ for the combined surfaces.

![Figure C.5: Error in $\bar{f}$ when multiple replicates have been performed for a smooth surface and for a surface with superhydrophobicity and riblets.](image)

### C.2 Reynolds Number

This section quantifies the error in the Reynolds number. The Reynolds number is computed from Equation C.11.

$$Re = \frac{\bar{V}D_b}{\nu} \quad \text{(C.11)}$$

With some substitution, the Reynolds number can be expressed in terms of the measured parameters as shown in Equation C.12.

$$Re = \frac{2\dot{m}}{\mu(H+W)} \quad \text{(C.12)}$$
\( \mu \) is defined as the dynamic viscosity and \( \mu = \nu \rho \). The propagation of error contributed by each measurement is shown in Equation C.13.

\[
\chi_{Re} = \left[ \left( \frac{\chi m}{m} \right)^2 + \left( \frac{\chi \mu s}{\mu} \right)^2 + \left( \frac{\chi H}{H + W} \right)^2 + \left( \frac{\chi W}{H + W} \right)^2 \right]^{1/2} \tag{C.13}
\]

The error in the viscosity was determined by perturbing the viscosity due to the error in the temperature. The curve fit [74] used to predict viscosity as a function of temperature is shown in Equation C.14

\[
\ln \frac{\mu}{\mu_o} = a + b \frac{T_o}{T} + c \left( \frac{T_o}{T} \right)^2 \tag{C.14}
\]

where \( T \) is the temperature, \( T_o \) and \( \mu_o \) are referenced to 293 K, and the constants \( a \), \( b \), and \( c \) are 9.41, -26.45, and 17.04, respectively. Over the range of temperatures tested, the worst error in viscosity was 2.0%.

The other errors associated with mass flow, channel height, and channel width have been described in Section C.1. After combining all the errors, the worst case error in \( Re \) is 2.04%.
APPENDIX D. MATLAB CODE

This appendix contains the Matlab code used for the analytical jet impingement model.

D.1 Jet Impingement on a Surface with Isotropic Slip with a Hydraulic Jump

jetsolver.m is used to solve the thin film region.

% This function solves the thin film region for isotropic case

% In this function variables are as follows:
% a=diameter (m)
% x=radial distance from impingement (m)
% d=boundary layer thickness (m)
% h=height of the thin film (m)
% kr1=location where transition happens from region 1 to region 2
% rho=density (kg/m^3)
% gamma=surface tension (N/m)
% nu=kinematic viscosity (m^2/s)
% g=gravity (m/s^2)
% Uinf=jet velocity (m/s)
% Q=flow rate (m^3/s)
% tmax=maximum number of divisions in theta direction
% L=vector of slip length values (m)
% dx=size of radial step (m)
% xlocfinal=max radial location for solving thin film (m)

function [x d h ut kr1]=jetsolver(rho,gamma,nu,g,Uinf,a,Q ...
    ,tmax,L,dx,xlocfinal)
xmax=floor(xlocfinal/dx*1.1)+100; %maximum vector index for x
th=linspace(0,pi/2,tmax); %theta values defined
dx=dx/10; % change dx for high resolution

%--------set up other needed parameters--------------------------
k=1; %counter
x=zeros(xmax,1);
dxo=dx; %original dx
d=zeros(xmax,tmax); %initialize d
ds(1,1:tmax)=d(k,1:tmax); %to enable passing single sets of values
xs=x(k); %current x location
h=zeros(xmax,1); %initialize h

%---------region 1----------------------------------------
flagnearendR1=0; %flag to change step near end of region 1

while h(k)>=min(d(k,:))
    if k>20&&flagnearendR1==0
        dx=dxo*10;
    end
    k=k+1;
    k1=1.5*nu*x(k-1).*(1./(d(k-1,1:tmax)+1.5.*L))./Uinf; %k1 for runge kutta
    eps=dx/5; % perturbation size
dxf=.5;
    kv=k1;
    options=optimset('Display','off','TolFun',1e-16,'MaxIter',600 ...
    ,'Algorithm','trust-region-reflective','TolX',1e-16,'MaxFunEvals',1200 ...
    ,'Diagnostics','off','DiffMaxChange',0.01,'DiffMinChange',1e-13 );
    f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do2=fsolve(f,[ds]+eps,options); %solve system of equations in sysqr1

k2=1.5*nu*(x(k-1)+.5*dx).*(1./(do2+1.5.*L))./Uinf; % k2 of runge kutta
kv=k2;
dxf=.5;
f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do3=fsolve(f,[ds]+eps,options); %solve system of equations in sysqr1

k3=1.5*nu*(x(k-1)+.5*dx).*(1./(do3+1.5.*L))./Uinf; %k3 runge kutta
kv=k3;
dxf=1;
f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do4=fsolve(f,[ds]+eps,options); %solve system of equations in sysqr1

k4=1.5*nu*(x(k-1)+dx).*(1./(do4+1.5.*L))./Uinf; % k4 of runge kutta
kv=1/6.*(k1+2*k2+2*k3+k4);
dxf=1;
f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do5=fsolve(f,[ds]+eps,options); %solve system of equations in sysqr1

d(k,1:tmax)=do5;
ds=d(k,1:tmax);
x(k)=x(k-1)+dx;
xs=x(k);
fdummy=3/8.*ds-9/16.*ds.*L./(ds+1.5.*L);
h(k)=a^2/(8*x(k))+2/pi.*simp(pi/2,fdummy,tmax); %solve for h using Simpson rule
if h(k)<=min(d(k,:))&&dx>dxo/100
    h(k)=h(k)*2;
k=k-1;

xs = x(k);
ds = d(k, 1:tmax);

dx = dx/10;
flagnearendR1 = 1;
end

dx = dxo*10; % change step size

%----------------region 12--------------------------

b = zeros(xmax, 1);
for j = 1:k
btemp = (find(h(j) < d(j, :)));
if isempty(btemp)
    b(j, :) = tmax + 1;
else
    b(j) = btemp(1);
end
end

xloctemp = find(b < tmax);
kr1 = k - 1;
k = xloctemp(1) - 1;

ut(xmax, 1:tmax) = Uinf;
ut(:, :) = Uinf;
uts(1, 1:tmax) = ut(k, 1:tmax);
xo = x(k);
hs = h(k);
xs=x(k);

while x(k)<xlocfinal
    k=k+1;
    ds=d(k,1:tmax);
    bs=b(k);

    k1=-1.5*nu.*ut(k-1,1:tmax).*x(k-1).*(1./(h(k-1)+1.5.*L)); %k1 of RK
    eps=hs/10;
    kv=k1;
    dxf=.5;
    options=optimset('Display','off','TolFun',1e-14,'MaxIter',300 ...
    ,'Algorithm','trust-region-reflective','TolX',1e-10,'MaxFunEvals',600 ...
    ,'Diagnostics','off','DiffMaxChange',0.01,'DiffMinChange',1e-13);
    f=@(hut)syseqr2(hut,L, xs,dx,kv,hs,uts,a,dxf,Uinf,bs,th,ds); %solve system
    if bs>0
        huto2=fsolve(f,[hs uts(bs:end)]+eps,options);
        huto2=[huto2(1) ut(k-1,1:bs-1) huto2(2:end)];
    else
        huto2=fsolve(f,[hs uts]+eps,options);
    end

    k2=-1.5*nu.*huto2(2:end).*x(k-1)+.5*dx.*1./(huto2(1)+1.5.*L)); % k2 of RK
    kv=k2;
    dxf=.5;
    f=@(hut)syseqr2(hut,L, xs,dx,kv,hs,uts,a,dxf,Uinf,bs,th,ds);
    if bs>0
        huto3=fsolve(f,[hs uts(bs:end)]+eps,options); %solve system
        huto3=[huto3(1) ut(k-1,1:bs-1) huto3(2:end)];
    else
        huto3=fsolve(f,[hs uts]+eps,options); %solve system
        huto3=[huto3(1) ut(k-1,1:bs-1) huto3(2:end)];
    end
huto3=fsolve(f,[hs uts]+eps,options); %solve system
end

k3=-1.5*nu.*huto3(2:end).*(x(k-1)+.5*dx).*(1./(huto3(1)+1.5.*L)); %k3 of RK
kv=k3;
dxf=1;
f=@(hut)syseqr2(hut,L,xs,dx,kv,hs,uts,a,dxf,Uinf,bs,th,ds);
if bs>0
    huto4=fsolve(f,[hs uts(bs:end)]+eps,options); %solve system
    huto4=[huto4(1) ut(k-1,1:bs-1) huto4(2:end)];
else
    huto4=fsolve(f,[hs uts]+eps,options); %solve system
end

k4=-1.5*nu.*huto4(2:end).*(x(k)+dx).*(1./(huto4(1)+1.5.*L)); %k4 of RK
kv=1/6.*(k1+2.*k2+2.*k3+k4);
dxf=1;
f=@(hut)syseqr2(hut,L,xs,dx,kv,hs,uts,a,dxf,Uinf,bs,th,ds);
if bs>0
    huto5=fsolve(f,[hs uts(bs:end)]+eps,options); %solve system
    huto5=[huto5(1) ut(k-1,1:bs-1) huto5(2:end)];
else
    huto5=fsolve(f,[hs uts]+eps,options); %solve system
end
huto5;
x(k)=x(k-1)+dx;
xs=x(k);
ut(k,1:tmax)=huto5(2:end);
h(k)=huto5(1);
hs=h(k);
uts=ut(k,1:tmax);
[k x(k)];
us(k,1:tmax)=1.5.*ut(k,1:tmax).*L./(h(k)+1.5.*L);
end

% variables to output
ut=ut(1:k,:);
x=x(1:k,:);
d=d(1:k,:);
h=h(1:k,:);

syseqr1.m is used in the solution of region 1 by fsolve in jetsolver.m.

% function for region 1 to be solved by fsolve
function out=syseqr1(do,ds,L,xs,dx,kv,a,dxf)
d=do;
ytemp=((xs).*(39/280.*ds+27/280.*ds.*(1.5.*L./(ds+1.5.*L))-
     33/140.*ds.*(1.5.*L./(ds+1.5.*L)).^2));
mom=abs([ytemp+kv.*dx.*dxf-(xs+dxf*dx).*(39/280.*d+
     27/280.*d.*(1.5.*L/(d+1.5.*L))-
     33/140.*d.*(1.5.*L/(d+1.5.*L)).^2]).*1e8);
out=[mom];

syseqr2.m is used in the solution of region 2 by fsolve in jetsolver.m

function out=syseqr2(hut,L,xs,dx,kv,hs,uts,a,dxf,Uinf,bs,th,ds)
h=hut(1);
ut=hut(2:end);
if bs>0
    ytemp=((xs).*uts(bs:end).^2.*hs.*(17/35+39/140.*(1.5.*L(bs:end)./(hs+1.5.*L(bs:end)))
     +33/140.*(1.5.*L(bs:end)./(hs+1.5.*L(bs:end))).^2));
    mom=[ytemp+kv(bs:end).*dx.*dxf-(xs+dxf*dx).*ut.^2.*h.*(17/35+...
39/140.*(1.5*L(bs:end)./(h+1.5*L(bs:end)))+...
33/140.*(1.5*L(bs:end)./(h+1.5*L(bs:end)).^2)];
n=length(L);
ba=pi/2;
f2=(ut./Uinf.*(5/8.*h+9/16.*h.*L(bs:end)./(h+1.5.*L(bs:end))));
f1=(h-3/8.*ds(1:bs-1)+9/16.*ds(1:bs-1).*L(1:bs-1)./(ds(1:bs-1)+1.5.*L(1:bs-1)));
inte=simp(ba,[f1 f2],n);
else

ytemp=((xs).*uts.^2.*hs.*(17/35+39/140.*L.*(1.5./(hs+1.5.*L))...
+33/140.*(1.5.*L./(hs+1.5.*L)).^2));
mom=[ytemp+kv.*dx.*dxf-(xs+dxf*dx).*ut.^2.*h.*(17/35+...
39/140.*(1.5.*L./(h+1.5.*L)))+...
33/140.*(1.5.*L./(h+1.5.*L)).^2)];
n=length(L);
ba=pi/2;
f=(ut./Uinf.*(5/8.*h+9/16.*h.*L./(h+1.5.*L)));
inte=simp(ba,f,n);
end
mass= [-pi*a^2/16+(xs+dxf*dx)*inte];
out=[mom mass ].*1e8 ;
simp.m is simpsons rule for integration

function out=simp(ba,f,n)
h=ba/(n-1);
if mod(n,2)==1
    out=h*(f(1)+4*sum(f(2:2:n-1))+2*sum(f(3:2:n-2))+f(end))/3;
elseif n<3
    out=h*(f(1)+f(2))/2;
else
    ntemp=n-3;
    out1=h*(f(1)+4*sum(f(2:2:ntemp-1))+2*sum(f(3:2:ntemp-2))... 
        +f(end-3))/3;
    out2=3*h*(f(end)+3*(f(end-1)+f(end-2))+f(end-3))/8;
    out=out1+out2;
end

watson.m is to solve thin film with Watson’s model

% a is radius
function [dw,hw,utw,krw]=watson(x,a,Rea,nu,Q,kr1,Uinf,xlocfinal)
c=1.402;
l=.567*(a)*Rea^(1/3);
dw=zeros(length(x),1);
hw=dw;
Ut=utw+Uinf;
k=1;

dw(k)=sqrt(pi*sqrt(3)*c^3/(pi-c*sqrt(3))*nu.*x(k).*a^2/Q);
hw(k)=(a)^2./(2.*x(k))+(1-(2*pi/(3*sqrt(3)*c^2))).*dw(k);

% r<ro
while hw(k)>dw(k)
    k=k+1;
    dw(k)=sqrt(pi*sqrt(3)*c^3/(pi-c*sqrt(3))*nu.*x(k).*a^2/Q);
hw(k)=(a)^2./(2.*x(k))+(1-(2*pi/(3*sqrt(3)*c^2))).*dw(k);
end

end
krw=k-1;
oxo=x(k);

% $r > ro$
while $x(k) < x_{loc_{final}}$

    $k = k+1;$

    $hw(k)=2\pi^2*nu.*(x(k).^3+l^3)/(3*sqrt(3)*Q.*x(k));$
    $utw(k)=27*c^2/(8*\pi^4).*Q^2./(nu.*(x(k).^3+1^3));$
end

hj.m calculates the hydraulic jump

function out=hj(h,ut,H,Ls,Uinf,a,ro,x,gamma,Q,nu)

g=9.81;

$F=a^4.*Uinf^2./(64.*x.^2.*H)+.5*g*H^2+gamma.*(H)./(ro.*x)...$
$-ut.^2.*(17/35.*h+39/140.*h.*1.5./(h./Ls+1.5)+...$
$33/140.*h.*(9/4)./(h./Ls+1.5).^2);$  
$Fb=x.*g.*H.*2.*a^2./Q^2.*(1+2./(ro.*g.*x.*(H-h)./gamma))...$
$a^2./(2*\pi^2.*x.*H)-.01676.*((x./a).^3.*(Q./(nu*a)).^-1 ...$
$+.1826).^-1;$

rjsi=find(min(abs(F))==abs(F));
xjs=x(rjsi);

min(abs(F));

out=xjs;
D.2 Jet Impingement on a Surface with Anisotropic Slip with a Hydraulic Jump

jetsolver.m is used to solve the thin film region but is different than that used for the isotropic case.

% This function solves the thin film region for anisotropic case

% In this function variables are as follows:
% a=diameter (m)
% x=radial distance from impingement (m)
% d=boundary layer thickness (m)
% h=height of the thin film (m)
% kr1=location where transition happens from region 1 to region 2
% rho=density (kg/m^3)
% gamma=surface tension (N/m)
% nu=kinematic viscosity (m^2/s)
% g=gravity (m/s^2)
% Uinf=jet velocity (m/s)
% Q=flow rate (m^3/s)
% tmax=maximum number of divisions in theta direction
% L=vector of slip length values (m)
% dx=size of radial step (m)
% xlocfinal=max radial location for solving thin film (m)
function [x d h ut kr1 b]=jetsolver(rho,gamma,nu,g,Uinf,a,Q ... ,tmax,L,dx,xlocfinal)

xmax=floor(xlocfinal/dx*1.1);
th=linspace(0,pi/2,tmax);
dx=dx/10;
%-------set up other needed parameters-------
k=1; %counter
x=zeros(xmax,1);
dxo=dx; %original dx
d=zeros(xmax,tmax);
ds(1,1:tmax)=d(k,1:tmax); %to enable passing single sets of values
xs=x(k); %"
h=zeros(xmax,1);

%----------------region 1----------------------

while h(k)>=min(d(k,:))

    if k>20
        dx=dxo*20;
    end

    k=k+1;

    k1=1.5*nu*x(k-1).*(1./(d(k-1,1:tmax)+1.5.*L))./Uinf;
    eps=dx/5;
    dx=.5;
    kv=k1;
    options=optimset('Display','off','TolFun',1e-16,'MaxIter',600 ... 
                 ,'Algorithm','trust-region-reflective','TolX',1e-16,'MaxFunEvals',1200 ... 
                 ,'Diagnostics','off','DiffMaxChange',0.01,'DiffMinChange',1e-13);
    f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
    do2=fsolve(f,[ds]+eps,options);

    k2=1.5*nu*(x(k-1)+.5*dx).*(1./(do2+1.5.*L))./Uinf;
    kv=k2;
    dx=.5;
f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do3=fsolve(f,[ds]+eps,options);

k3=1.5*nu*(x(k-1)+.5*dx).*(1./(do3+1.5.*L))./Uinf;
kv=k3;
dxf=1;
f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do4=fsolve(f,[ds]+eps,options);

k4=1.5*nu*(x(k-1)+dx).*(1./(do4+1.5.*L))./Uinf;
kv=1/6.*(k1+2*k2+2*k3+k4);
dxf=1;
f=@(do)syseqr1(do,ds,L,xs,dx,kv,a,dxf);
do5=fsolve(f,[ds]+eps,options);

d(k,1:tmax)=do5;
ds=d(k,1:tmax);
x(k)=x(k-1)+dx;
xs=x(k);
fdummy=3/8.*ds-9/16.*ds.*L./(ds+1.5.*L);
h(k)=a^2/(8*x(k))+2/pi.*simp(pi/2,fdummy,tmax);
[k x(k)];
end

%---------------region 12---------------------

b=zeros(xmax,1);
for j=1:k
btemp=(find(h(j)<d(j,:)));

if isempty(btemp)
    b(j,:)=tmax+1;
else
    b(j)=btemp(1);
end
end

xloctemp=find(b<tmax);
kr1=k;
k=xloctemp(1)-1;

ut(xmax,1:tmax)=Uinf;
ut(:, :)=Uinf;
uts(1,1:tmax)=ut(k,1:tmax);
xo=x(k);
hs=h(k);
xs=x(k);

while x(k)<xlocfinal
    k=k+1;
ds=d(k,1:tmax);
bs=b(k);

    k1=-1.5*nu.*ut(k-1,1:tmax).*x(k-1).*(1./(h(k-1)+1.5.*L));
    eps=hs/10;
kv=k1;
dxf=.5;
options=optimset('Display','off','TolFun',1e-14,'MaxIter',300 ... 
    ,'Algorithm','trust-region-reflective','TolX',1e-10,'MaxFunEvals',600 ... 
    ,'Diagnostics','off','DiffMaxChange',0.01,'DiffMinChange',1e-13 );
f=@(hut) syseqr2(hut, L, xs, dx, kv, hs, uts, a, dxf, Uinf, bs, th, ds);
if bs>0
    huto2=fsolve(f, [hs uts(bs:end)]+eps,options);
    huto2=[huto2(1) ut(k-1,1:bs-1) huto2(2:end)];
else
    huto2=fsolve(f, [hs uts]+eps,options);
end

k2=-1.5*nu.*huto2(2:end).*(x(k-1)+.5*dx).*(1./(huto2(1)+1.5.*L));
kv=k2;
dxf=.5;
f=@(hut) syseqr2(hut, L, xs, dx, kv, hs, uts, a, dxf, Uinf, bs, th, ds);
if bs>0
    huto3=fsolve(f, [hs uts(bs:end)]+eps,options);
    huto3=[huto3(1) ut(k-1,1:bs-1) huto3(2:end)];
else
    huto3=fsolve(f, [hs uts]+eps,options);
end

k3=-1.5*nu.*huto3(2:end).*(x(k-1)+.5*dx).*(1./(huto3(1)+1.5.*L));
kv=k3;
dxf=1;
f=@(hut) syseqr2(hut, L, xs, dx, kv, hs, uts, a, dxf, Uinf, bs, th, ds);
if bs>0
    huto4=fsolve(f, [hs uts(bs:end)]+eps,options);
    huto4=[huto4(1) ut(k-1,1:bs-1) huto4(2:end)];
else
    huto4=fsolve(f, [hs uts]+eps,options);
end
\[ k4 = -1.5 \times \nu \times huto4(2:end) \times (x(k-1)+dx) \times (1/(huto4(1)+1.5 \times L)); \]
\[ kv = 1/6 \times (k1+2 \times k2+2 \times k3+k4); \]
\[ dxf = 1; \]
\[ f = @(hut) syseqr2(hut, L, xs, dx, kv, hs, uts, a, dxf, Uinf, bs, th, ds); \]
\[ if \; bs > 0 \]
\[ huto5 = fsolve(f, [hs \; uts(-bs:end)] + \varepsilon, options); \]
\[ huto5 = [huto5(1) \; ut(k-1,1:bs-1) \; huto5(2:end)]; \]
\[ else \]
\[ huto5 = fsolve(f, [hs \; uts] + \varepsilon, options); \]
\[ end \]

\[ huto5; \]
\[ x(k) = x(k-1) + dx; \]
\[ xs = x(k); \]
\[ ut(k,1:tmax) = huto5(2:end); \]
\[ h(k) = huto5(1); \]
\[ hs = h(k); \]
\[ uts = ut(k,1:tmax); \]

\[ [k \; x(k)]; \]
\[ us(k,1:tmax) = 1.5 \times ut(k,1:tmax).L/(h(k)+1.5 \times L); \]

\[ end \]

\[ ut = ut(1:k,:); \]
\[ x = x(1:k,:); \]
\[ d = d(1:k,:); \]
\[ h = h(1:k,:); \]

hjth.m calculates the hydraulic jump with varying $\theta$
function out=hjth(h,ut,H,Ls,Uinf,a,ro,x,gamma,Q,nu,xc)

g=9.81;

F=H.*(ut./H.*(5/8.*h+9/16.*h./((h./Ls+.5))).^2+...
    .5*g*H^2+gamma.*(H)./(ro.*xc)...
    -ut.^2.*(17/35.*h+39/140.*h.*1.5./((h./Ls+1.5))+...
    33/140.*h.*(9/4./((h./Ls+1.5)).^2);

rjsi=find(min(abs(F))==abs(F));
xjs=x(rjsi);

min(abs(F));
out=xjs(end);

hjthrob.m is a more robust version of hjth.m

function out=hjthrob(h,ut,H,Ls,Uinf,a,ro,x,gamma,Q,nu,xc,jj,guess)

g=9.81;

F=H.*(ut./H.*(5/8.*h+9/16.*h./((h./Ls+1.5))).^2+...
    .5*g*H^2+gamma.*(H)./(ro.*xc)...
    -ut.^2.*(17/35.*h+39/140.*h.*1.5./((h./Ls+1.5))+...
    33/140.*h.*(9/4./((h./Ls+1.5)).^2);

F2=H.*(ut./H.*(5/8.*h+9/16.*h./((h./Ls+1.5))).^2+...
    .5*g*H^2+gamma.*(H)./(ro.*100)...
    -ut.^2.*(17/35.*h+39/140.*h.*1.5./((h./Ls+1.5))+...
    33/140.*h.*(9/4./((h./Ls+1.5)).^2);

if 0 %debug =1
plot(x,abs(F))
ylim([0 .00002])
xlabel(’r(m)’)  
ylabel(’Hydraulic jump eq’)  
xlim([0 .05])
print(’-dtiff’,’-r600’,[’aa’ num2str(jj) ’.tiff’])
end

if jj==1
  rjsi=find(min(abs(F))==abs(F));
xjs=x(rjsi);
  min(abs(F));
  out=xjs(end);
end
endif

if jj~=1
  minv=minima(abs(F));
  closemin=find(min(abs(x(minv)-guess))==abs(x(minv)-guess));
  out=x(minv(closemin));
end

stcorr.m solves for the hydraulic jump using hjth.m and hjthrob.m and surface tension iteration.

function [rj]=stcorr(tmax, h,ut,H,L,Uinf,a,rho,x,gamma,Q,nu,jk,th)
d debug=0;
  ^%-----------------------------
  %radius of curvature

%calculate jump radius without surface tension
  for jj=1:tmax
    rc=rcv(jj);
    
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\[ r_{j(k,j)} = h_{jth}(h, ut(:,jj), H(L(jj), Uinf, a*2, \rho, x, \gamma, Q, \nu, rc); \]
\[ r_{jo} = r_j; \]
end

\[ tt = 0; \text{ err} = 100; \text{ rj2} = r_j - r_j + 100; \]
while tt < 100

\[ tt = tt + 1; \]
\%force shape to match that of slip length
\[ \text{ado} = [1.6054147540e-06 \ 0.077878578251 \ 340.22348625 \ 0.3376716813\]’;
\[ \text{options} = \text{optimset}('\text{TolFun}', 1e-10, '\text{TolX}', 1e-10, '\text{MaxFunEvals}', 40000, '\text{MaxIter}', 10000); \]
\[ \text{ad} = \text{lsqnonlin}(@Lfit, \text{ado}, [], [], \text{options}, \text{th}, r_{j(k,:)}); \]
\[ \text{ad1} = \text{ad}; \]
\[ r_{j(k,:)} = (\text{ad}(1).*(\cos(\text{th})).^2 + \text{ad}(2).*(\sin(\text{th})/\text{ad}(3)).^2)^{\text{ad}(4)}; \]
\%
\%convert to cartesian
\[ \text{xx} = r_{j(k,:)}.*\cos(\text{th}); \]
\[ \text{yy} = r_{j(k,:)}.*\sin(\text{th}); \]
\[ \text{xxp} = -r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 0, \text{th}).*\sin(\text{th}) + \ldots \]
\[ r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 1, \text{th}).*\cos(\text{th}); \]
\[ \text{xxpp} = -2.*r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 1, \text{th}).*\sin(\text{th}) \ldots \]
\[ -r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 2, \text{th}).*\cos(\text{th}); \]
\[ \text{yypp} = r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 1, \text{th}).*\sin(\text{th}) + \ldots \]
\[ r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 0, \text{th}).*\cos(\text{th}); \]
\[ \text{yypp} = 2.*r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 2, \text{th}).*\sin(\text{th}) \ldots \]
\[ +r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 1, \text{th}).*\cos(\text{th}) \ldots \]
\[ -r_{j\_deriv}(\text{ad}(1), \text{ad}(2), \text{ad}(3), \text{ad}(4), 0, \text{th}).*\sin(\text{th}); \]

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kurv=(xxp.*yypp-yy.*xxpp)./(xxp.^2+yyp.^2).^1.5;

rjold=rj;
rcv=1./kurv;

%calc new rj with rad of curvature

guess=0;
for jj=1:tmax
    rc=rcv(jj);
    rj(jk,jj)=hjthrob(h,ut(:,jj),H,L(jj),Uinf,a*2,rho,x,gamma,Q,nu,rc,jj,guess);
    guess=rj(jk,jj);
end

mean(rcv)

aaa=.1;
rj=aaa.*rj+(1-aaa).*rjold;
figure(20)
title('rj cartesian')
plot(rj(jk,:)./a.*cos(th),rj(jk,:)./a.*sin(th),'.')
hold on
plot(-rj(jk,:)./a.*cos(th),rj(jk,:)./a.*sin(th))
plot(rj(jk,:)./a.*cos(th),-rj(jk,:)./a.*sin(th))
plot(-rj(jk,:)./a.*cos(th),-rj(jk,:)./a.*sin(th))
axis equal
drawnow

dend

D.3 Jet Impingement on a Surface where Breakup of the Thin Film Occurs

breakup.m is the only other code needed for the solution of the case with breakup and is to be used with jetsolver.m from the anistropic slip model.
% h is the thin film height
% uts is the free surface velocity
% Ls is the slip length (m)
% a is the radius
% ro is the density
% x is radial distance
% gamma is the surface tension
% Q is the flow rate
% nu is the kinematic viscosity
% phi is the contact angle

function out=breakup(h,uts,Ls,a,ro,x,gamma,Q,nu,phi)
  g=9.81;
  F=ro.*uts.^2./(2/3.*h+Ls).^2.*(Ls^2.*h+5/6.*Ls.*h.^2+68/315.*h.^3)-gamma.*(1+cos(pi-phi));
  rbsi=find(min(abs(F))==abs(F))
  rbs=x(rbsi);
  out=rbs;