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Model Predictive Control and Estimation of Managed Pressure Drilling using a Real-Time High Fidelity Flow Model

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Abstract

When drilling an oil or gas well, well pressures may be controlled using a technology called managed pressure drilling. This technology often relies on model predictive control schemes; however, practical limitations have generally led to the use of simplified controller models that do not optimally handle certain perturbations in the physical system. The present work reports on the first implementation of a highly accurate system model that has been adapted for real-time use in a controller. This real-time high-fidelity model approximates the results of offline high-fidelity models without requiring operation by model experts. The effectiveness of the model is demonstrated through simulation studies of controller behavior under various drilling conditions, including an evaluation of the impact of sparse downhole feedback measurements.

Keywords: managed pressure drilling, drilling automation, pressure control, physics-based drilling flow model, nonlinear model predictive control

1. Introduction

Control of well pressures during oil and gas drilling operations is a safety-critical process. If the pressure of drilling fluid in the well is allowed to get too

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low, fluids residing in the surrounding rock formation can enter the wellbore during the drilling process, creating a potentially hazardous condition. Likewise, if the drilling fluid pressure is too high, the formation wall may fracture, causing costly fluid loss to the surrounding rock and perhaps creating a secondary incursion of formation fluid into the wellbore. Some wells in offshore environments or depleted reservoirs are particularly difficult to drill from this perspective, as the margin between formation fluid pressure (pore pressure) and fracture pressure can be quite narrow. Sometimes, pressure fluctuations caused simply by rapid movement of drilling components in and out of the well is sufficient to exceed safety limits.

One technology that offers highly responsive control over wellbore pressures during the drilling process is managed pressure drilling (MPD). This technology uses dynamically adjusted surface equipment, including a choke valve and multiple fluid pumps, to keep pressures within desired limits. Though a variety of controllers have been used to control MPD equipment [1, 2, 3, 4], model predictive control (MPC) is particularly well suited to this application.

MPC has found favor in a variety of industries for several reasons, including its capability of handling multiple inputs and outputs, and its ability to handle input and state constraints [5, 6, 7]. In MPD applications, researchers have exploited these capabilities and have developed MPC systems that control pressure, flow, and rate of penetration (ROP) simultaneously [8, 9], control pressure at two different locations in the well [10], and handle substantial heave motion on drill rigs subject to ocean surface disturbances [11, 12].

Understandably, a fundamental key to controller effectiveness is the accuracy of the model upon which the MPC is based; mismatches between the model and the physical system can lead to suboptimal control. Nevertheless, a simplified model is often accepted for use in the MPC in order to address practical limitations relating to such things as incomplete understanding of the physical process, limited computational power, or inadequate availability of expertise for control system maintenance, often trading optimal process control for usability [13]. Indeed, in many MPC applications, models include simplifications of some type,

such as reduction in model order or linearization of significant nonlinear processes (see [9, 10, 14, 15, 16, 17, 18, 19]). The simplest of these models may be termed gray box models, which incorporate linear transfer function models and nonlinearity blocks to describe relationships between manipulated variables and system behavior. These models have been derived empirically and may take the form of a nonlinear Hammerstein-Wiener model, for example [20, 21, 22, 23]. To improve this type of model, Patan [24] proposes using artificial neural networks to empirically build system models off-line. Other models (“low-fidelity flow models” or LFMs) are based on the underlying physics of the process, and capture the primary dynamics of the process [25], but exclude more complex but nonetheless relevant physical effects in the name of simplification.

Because such models necessarily omit certain physical effects, the compromised accuracy of these reduced order models is inevitable; studies suggest improved results could be achieved with more accurate models [8, 12]. Model errors due to structural model mismatch become more prominent during transient periods caused by operational changes. For example, Pedersen, et.al. [26], note the criticality of improved models when controlling pressures in the well during large process changes. During such changes a physical process will often pass through multiple nonlinear modes that are not well tracked by simpler models, e.g., passing through different flow regimes.

Another approach to addressing practical computing issues includes segregating optimization activities into off-line and on-line portions where optimal control solutions for various states are pre-computed and made available to simplified on-line routines [27, 28]. Such approaches deal effectively with limited computing facility but still inevitably detach the run-time model from some important aspects of the underlying physics of the process. In the oil and gas drilling industry, process variations due to geologic idiosyncrasies, previous production of a reservoir, etc. abound, suggesting substantial benefits can be secured with control systems integrated with reliable physics-based models [29].

Highly accurate physics-based models of the well drilling dynamic processes do exist. Such models are typically very detailed in order to capture complex

66 flow interactions. Unfortunately, these comprehensive models, known as “high-
67 fidelity flow models” (HFMs), require a large number of input variables and
68 are difficult to maintain in a real-time environment. Control systems based
69 on such models would need to be managed by subject experts. Thus, many
70 MPD automation research studies have stayed away from these more complex
71 and resource-hungry models, as suggested above. In an effort to reduce the
72 computational burden of using this type of model, Eaton et al. [30] developed
73 a method to switch between a simple linear empirical model, an LFM, and an
74 HFM. While this approach helps to compensate for the limitations of each type
75 of model, it is still dependent on an HFM, which can be problematic to use in
76 a real-time control system.

77 The present study takes a different approach, where a newly-developed real-
78 time high fidelity flow model (RT-HFM) is employed for the first time, and is
79 applied to bottomhole pressure (BHP) control. This model employs novel sim-
80 plifications tailored to the specific drilling application while maintaining critical
81 parts of an HFM. This allows the model to overcome resource difficulties expe-
82 rienced with other high fidelity models in real-time control applications, while
83 still providing for better control during the varied conditions encountered while
84 drilling. Figure 1 graphically depicts where the RT-HFM is positioned with re-
85 spect to other models described above. As shown, the RT-HFM approaches the
86 high fidelity end of the spectrum in terms of detail and complexity of equations.

Figure 1: Spectrum of model fidelity in MPD automation

87 In the sections following, this RT-HFM model is described in detail, in con-
 88 text with LFM and HFM models. Its implementation into a MPC with a MHE
 89 (moving horizon estimator) is then detailed. Finally, results of testing this new
 90 control system are presented. This testing simulates an array of conditions en-
 91 countered while drilling oil and gas wells, including “normal” drilling ahead,
 92 making a pipe connection to change the length of the drill string, and displac-
 93 ing drilling fluid of one density with that of another density. The effectiveness
 94 of control based on the RT-HFM is further demonstrated by considering both
 95 the case where down-hole sensor data is available to provide BHP feedback to
 96 the model, and the case where sensor feedback is interrupted or severely lim-
 97 ited. This latter case is meaningful in the oil and gas environment due to the
 98 extreme conditions encountered and the wide diversity of drilling operations,
 99 both of which impact the quality or availability of sensor data.

100 **2. Real-Time High Fidelity Flow Model**

101 This section provides an overview of the RT-HFM and how it compares
 102 to other models used for control system development. First, a schematic and
 103 description of MPD gives context for the dynamic relationships that are modeled
 104 to create a predictive controller. Next, a description is given of three models of
 105 differing complexity, including the RT-HFM. The RT-HFM model is presented
 106 to contrast the obstacles facing both low and high fidelity models.

107 *2.1. Basic Flow Circuit and Related Parameters*

108 To construct an oil or gas well, a drill bit is attached to a long assembly of
 109 tubular components (drill string) that provide thrust and rotation to the bit. As
 110 it penetrates the rock formation, the drill bit creates a hole that is larger than
 111 the diameter of the drill string, thereby providing an annular space between
 112 the formation and the drill string. Drilling fluid occupies this annulus and also
 113 the inside of the drill string, and serves to maintain a pressure against the
 114 formation that controls the flow of fluids out of the formation. When actively

115 drilling, mud pumps located at the surface circulate the drilling fluid through
116 the drill string to the bottom of the well, where the fluid entrains cuttings and
117 carries them back to the surface through the annulus. In modeling this system,
118 the cross sectional areas of the annulus and the drill string bore, the flow rate of
119 the drilling fluid, and certain physical properties of the drilling fluid, entrained
120 cuttings, and entrained formation fluids are important parameters.

121 An MPD system additionally contains a choke valve and an auxiliary charge
122 pump, both located at the top of the well on the annulus side of the flow cir-
123 cuit. By manipulating the flow rate of the mud pumps and the opening of the
124 choke valve, the backpressure in the fluid may be regulated, which provides for
125 fine control of the BHP while drilling fluid is flowing. When mud pumps are
126 not generating sufficient backpressure, e.g., when the fluid is not flowing, the
127 auxiliary charge pump may be employed to increase annulus pressure. Figure 2
128 shows a simplified schematic of a typical MPD system. In the present study,
129 pressure is managed by manipulating back pressure alone. Though other tech-
130 niques such as manipulation of annular fluid level, mud density, etc can be part
131 of an MPD system, they are not studied in this paper. The MPC manipulates
132 two variables, the mud pump flow and the choke pressure. We assume cascading
133 control of back pressure pump flow and the choke valve opening based on the
134 choke pressure.

Figure 2: MPD Schematic

135 2.2. Low Fidelity Flow Model Equations

The low-fidelity flow model (LFM) developed by Kaasa et al. [25] simplifies the drilling operation into two control volumes and introduces the two physical parameters bulk modulus (β) and effective mud density (M). References [9, 14, 15, 16, 17, 18] are the MPC applications employing the LFM. The LFM equations and variable descriptions are shown in Equations 1 to 4 and Table 1.

$$\dot{p}_p = \frac{\beta_d}{V_d}(q_p - q_{bit}) \quad (1)$$

$$\dot{p}_c = \frac{\beta_a}{V_a}(q_{bit} + q_{back} - q_c + q_{res}) \quad (2)$$

$$\dot{q}_{bit} = \frac{1}{M}(p_p - f_d q_{bit}^2 + \rho_d g_c h_{bit} - p_{bit}) \quad (3)$$

$$p_{bit} = p_c + \rho_a f_a h_{bit} q_{bit}^2 + \rho_a g_c h_{bit} \quad (4)$$

Table 1: Summary of parameters used in LFM

Parameter	Description	Unit
p_p, p_c, p_{bit}	Pressure at mud pump, choke valve, and bottomhole	bar
q_p, q_c, q_{bit}	Volumetric flow rate at mud pump, choke valve, and bottomhole	m^3/min
q_{res}	Volumetric flow rate of reservoir gas influx	m^3/min
β_a, β_d	Bulk modulus of the fluid in annulus and drill string	bar
M	Effective density per unit length	$kgm^{-4}10e^{-5}$
f_a, f_d	Friction coefficients of annulus and drill string	s^2m^{-6}
V_a, V_d	Volumes of annulus and drill string	m^3
h_{bit}	Well depth	m

2.3. High Fidelity Flow Model Equations

The high fidelity flow model (HFM) was developed before the LFM, and has been used extensively for improving understanding of drilling operation hydraulics, to both assist the well design process and to provide real-time advisory assistance for many field applications [31]. However, the HFM equations are much more comprehensive than other models and therefore the model must be configured, monitored and tuned by an expert. Even then, model simulations that are free from numerical instability cannot be fully assured [32]. Due to these factors, such models are not being employed for real-time drilling automation applications, including MPC, which requires fast calculation speed and computational robustness. The governing equations of HFM are presented in [29] and are shown in Equations 5 to 12 and Table 2.

$$\frac{\partial}{\partial t}(A\alpha_m\rho_m) = -\frac{\partial}{\partial s}(A\alpha_mv_m\rho_m) + A\dot{m}_{g,m} \quad (5)$$

$$\frac{\partial}{\partial t}(A\alpha_g\rho_g) = -\frac{\partial}{\partial s}(A\alpha_gv_g\rho_g) - A\dot{m}_g + q_{fg} \quad (6)$$

$$\frac{\partial}{\partial t}(A\alpha_mx_{dg,m}\rho_m) = -\frac{\partial}{\partial s}(A\alpha_mv_mx_{dg,m}\rho_m) - A\dot{m}_{g,m} \quad (7)$$

$$\frac{\partial}{\partial t}(A\alpha_{fo}\rho_{fo}) = -\frac{\partial}{\partial s}(A\alpha_{fo}v_{fo}\rho_{fo}) + A\dot{m}_{g,fo} + q_{dg} + q_{fo} \quad (8)$$

$$\frac{\partial}{\partial t}(A\alpha_{fo}x_{dg,fo}\rho_{fo}) = -\frac{\partial}{\partial s}(A\alpha_{fo}v_{fo}x_{dg,fo}\rho_{fo}) + A\dot{m}_{g,fo} + q_{dg} \quad (9)$$

$$\frac{\partial}{\partial t}(A\alpha_{fw}\rho_{fw}) = -\frac{\partial}{\partial s}(A\alpha_{fw}v_{fw}\rho_{fw}) + q_{fw} \quad (10)$$

$$\frac{\partial}{\partial t}(A\alpha_c\rho_c) = -\frac{\partial}{\partial s}(A\alpha_c v_c\rho_c) + q_c \quad (11)$$

$$\begin{aligned} & \frac{\partial}{\partial t}[A(\alpha_m\rho_mv_m^2 + \alpha_g\rho_gv_g^2 + \alpha_{fo}\rho_{fo}v_{fo}^2 + \alpha_{fw}\rho_{fw}v_{fw}^2 + \alpha_c\rho_cv_c^c)] \\ & + \frac{\partial}{\partial s}[A(\alpha_m\rho_mv_m^2 + \alpha_g\rho_gv_g^2 + \alpha_{fo}\rho_{fo}v_{fo}^2 + \alpha_{fw}\rho_{fw}v_{fw}^2 + \alpha_c\rho_cv_c^c)] \\ & = -\frac{\partial(Ap)}{\partial s} - A\left(\frac{\partial p}{\partial s}\right)_{fric} \\ & + A[\alpha_m\rho_mv_m + \alpha_g\rho_gv_g + \alpha_{fo}\rho_{fo}v_{fo} + \alpha_{fw}\rho_{fw}v_{fw} + \alpha_c\rho_cv_c]g\cos\theta \end{aligned} \quad (12)$$

Table 2: Summary of parameters used in HFM

Parameter	Description	Unit
A	Flow line cross sectional area	m^2
α_a	Volume fraction of a	
v_a	Volume of a	m^3
q_a	Volumetric flow rate of a	m^3/s
ρ_a	Density of a	kgm^{-3}
$x_{a,b}$	Mass fraction of a in b	
$fric$	Frictional pressure loss	bar
m, g, fo, fw, c	Drilling mud, gas, formation oil, formation water, and formation cuttings	
$\dot{m}_{g,n}, \dot{m}_{g,fo}$	Rates of gas dissolution in drilling mud and formation oil	kg/s

2.4. Modifications of HFM for Real-time Use

Recently, work on a RT-HFM was announced, as part of an effort to decrease the computational cost of the HFM while still maintaining high model accuracy [32]. Since that time, a working model has been developed, and initial application testing has been completed as will be described below. The calculations in both the RT-HFM and HFM are based on a discretization of the well volume, and employ a numerical solver for the mass and momentum conservation equations (Equations 5 to 12) that govern the physics of the well. Mass transport

156 is calculated using the finite difference method on a one-dimensional grid, with
157 analytical solutions used to account for radial dependencies.

158 Like the HFM, the RT-HFM is a dynamic model that accurately simulates
159 significant fluid characteristics. For example, mass transport calculations in the
160 RT-HFM respect conservation of mass per component of the fluid, and respect
161 conservation of total momentum locally. Thus, the model accurately represents
162 dynamic effects such as compression of the fluid propagating along the well. To
163 reduce numerical iterations and the chance of model instability, the accuracy of
164 the model during the transient phases is relaxed.

165 Calculation complexity is also reduced in the RT-HFM by keeping the tem-
166 perature profile fixed, as has already been described in the previous paper [32].
167 Fluid property sub-models are also simplified to enable rapid and stable com-
168 putation. For example, in the sub-model that describes rheological behavior,
169 the HFM fits rheological property data to a non-linear Herschel Bulkley model,
170 while the RT-HFM utilizes the linear Bingham Plastic model, expressed as $\tau =$
171 $\tau_0 + \mu_\infty \gamma$, where τ is the shear stress and γ is the shear rate. In this Bingham
172 model, the rheological properties of the fluid are determined by the yield stress
173 τ_0 and the plastic viscosity μ_∞ . The model calculates plastic viscosity and yield
174 stress using a best fit to all available rheological data which means that cal-
175 culations normally match much better at low RPM values than the standard
176 procedure that uses only 300 and 600 RPM readings to find plastic viscosity
177 and yield stress. Compared to Herschel-Bulkley, the Bingham equation has an
178 accurate explicit solution for laminar flow, and the calculation is faster and more
179 robust for estimating the frictional pressure loss. Still, calculations in the RT-
180 HFM are relatively sophisticated, and include, among other features, pressure-
181 and temperature-dependent fluid properties, with density either from published
182 correlations or from input tables of laboratory data. Rheological behavior data
183 can be given in tabular form for different combinations of pressure and temper-
184 ature, in which case interpolation is used to get rheological behavior parameters
185 at the actual temperature in each grid box.

186 The RT-HFM is therefore much faster and robust than the HFM, and at the

187 same time more accurate than existing lower order models. Geometrical changes
 188 in the drill string and wellbore, e.g., a tapered drill string, are accommodated
 189 by the model. However, some other specific conditions are not accounted in
 190 the RT-HFM, e.g., a mud flow diversion through an underreamer, and pressure
 191 loss through a mud motor. Although the frictional pressure loss by the drill
 192 string rotation is included in the RT-HFM, the drill string vibration effects on
 193 the BHP and heave motion of the offshore rig are not considered in this study.
 194 However, such effects can be simulated in the model by inputting bit depth
 195 changes, thereby allowing calculation of pressure surge and swab effects. The
 196 difference in steady-state BHP output from the two models, as determined by
 197 simulations using various combinations of input variables, is shown in Figure 3.
 198 As shown, at the upper limits of flow and choke pressure considered in the
 199 present study, the difference in pressure prediction amounts to less than 2.5
 200 *bar*. At the point of primary interest in these simulations, which is in an 8-1/2
 201 inch section (see Table 8), this difference appears to be reasonable for effective
 202 model-based control.

Figure 3: Difference between steady state BHP output from HFM and RT-HFM to identify model mismatch at varying conditions

203 3. System Configurations

204 Control system architecture for MPD automation varies according to the
205 availability of BHP data. Two potential architectures are portrayed in Fig-
206 ures 4 and 5, including a semi-closed loop configuration and a full-closed loop
207 configuration. These figures display block diagrams of the signal chains for
208 each architecture, where blocks labeled MPC represent a controller, MHE la-
209 beled blocks represent an estimator, and system models (HFM, RT-HFM) are
210 shown in each appropriate block. Dashed lines represent the input signals of the
211 controller that are computed from hydraulic models as opposed to solid lines
212 represent the measured value from the rig. If BHP measurements are available,
213 a solid line between the rig and the controller and/or estimator represents the
214 path of measured data. Note that in these figures, the rig represents the equip-
215 ment involved in the drilling process, most specifically including mud pumps,
216 a choke valve, a charge pump feeding into the wells annulus, and automatic
217 controls for each of these pieces of equipment.

218 Figure 4 represents a *semi-closed loop* configuration, which is a common
219 configuration in MPD automation research when downhole data is sparse or
220 unavailable. To provide this missing feedback data, a MHE is introduced in
221 this configuration as a “soft” sensor to estimate the BHP from surface mea-
222 surements. While MHE is the estimation method presented in this paper, other
223 useful estimation methods are available. The MHE minimizes model mismatch
224 by dynamic optimization of certain unmeasured drilling parameters, such as
225 mud density and friction factor. Furthermore, the model may be occasionally
226 updated or tuned by periodic downhole measurements to increase accuracy.

Figure 4: Semi-Closed loop control configuration - Bottomhole pressure is estimated by Moving horizon estimator

227 If real-time BHP measurements are available, a *full-closed loop* configuration
 228 may be employed. This type of system is presented in Figure 5. Note that, even
 229 though bottom hole measurements are available in real time, an estimator is still
 230 present in this configuration, since it is needed to estimate unmeasured drilling
 231 parameters such as mud density or friction factor. Because actual BHP measure-
 232 ments are used in the MHE calculation, the resulting estimated quantities are
 233 more reliable than those of the semi-closed loop case in Figure 4. The RT-HFM
 234 is continuously updated with the estimated parameters to increase the accuracy
 235 and reliability of the controller. We note here that, while various telemetry
 236 systems can supply real time BHP measurements, varying quality and quantity
 237 of real-time data are available from these different schemes. Some downhole
 238 data is subject to transmission delays on the order of at least a few seconds
 239 and delivery of the data may be subject to deprioritization depending on what
 240 other data is occupying the limited transmission channel. Other data ceases to
 241 flow when mud pumps are shut off or when pipe connections are made. These
 242 factors introduce uncertainty into the control process and place more reliance on
 243 model-based estimates. High-speed telemetry provided by wired drill pipe over-
 244 comes many of these limitations and provides more timely and abundant data
 245 for full-closed loop control [33]. Although some relatively short latency of data
 246 transmission can be introduced to the high-speed telemetry system, which is
 247 dependent on system configuration, we assume in the case study that the whole

high-speed data transmission system operates in an idealized manner and we ignore this latency.

Figure 5: Full-Closed loop control configuration - Bottomhole pressure is directly measured and transmitted by bottomhole sensor and high speed telemetry. Friction factors are estimated for MPC model calibration

4. Model Based Control and Estimation

MPC calculations predict the future behavior of the process by evaluating the process model (in this case the RT-HFM) at the current time step. The MPC determines the sum of squared error between the setpoint trajectory and model prediction values throughout the prediction horizon and performs the control calculation by solving the quadratic programming (QP) objective function to find the optimal sequences of process inputs (MVs). The first value of the MV sequence is applied to the process and repeats the entire cycle for every time step.

The MHE algorithm shares this main concept with MPC. It calculates the unknown parameters in the model by solving the QP objective function that mainly includes the model errors. The objective function of MHE refers to the past data of measurements and model results while the MPC refers to the future model prediction and setpoint. Convergence proofs for this method of optimizing control are well-established and are not repeated here [34, 35].

265 The objective functions associated with MHE and MPC are shown in Equa-
 266 tions (13) and (19), and the parameter descriptions are shown in Table 3 and 4,
 267 respectively. The model equations used in MPC and MHE simulations (appear-
 268 ing in Equations (13) and (19) as f , g , and h) are the governing equations of
 269 RT-HFM which are discussed in section 2.3 and 2.4.

$$\begin{aligned}
 \min_{\Delta \mathbf{P}} \quad & \Phi = (\mathbf{Y}_p - \mathbf{Y}_m)^T \mathbf{W} (\mathbf{Y}_p - \mathbf{Y}_m) + \Delta \mathbf{P}^T \mathbf{V} \Delta \mathbf{P} \\
 s.t. \quad & 0 = f(\dot{x}, x, y, p, d, u) \\
 & 0 = g(x, y, p, d, u) \\
 & 0 \leq h(x, y, p, d, u)
 \end{aligned} \tag{13}$$

270 where $\mathbf{Y}_p$ and $\mathbf{Y}_m$ are the column vectors for measurement values and the model
 271 output values from the time step $(k-1)$ to $(k-N)$. $\Delta \mathbf{P}$ is the column vector
 272 for movement of the parameter adjustment for the past estimation horizon (N) ,
 273 from the time step $(k-1)$ to $(k-N)$. h and i denote the number of SVs
 274 (State Variables) and EPs (Estimated Parameters), respectively. The MHE
 275 configuration for this study has two SVs and two EPs shown in Table 5 .

$$\begin{aligned}
 \mathbf{Y}_p = col\{ & y_{1_p}(k-1), y_{1_p}(k-2), \dots, y_{1_p}(k-N), \\
 & y_{2_p}(k-1), y_{2_p}(k-2), \dots, y_{2_p}(k-N), \\
 & \dots, \\
 & y_{h_p}(k-1), y_{h_p}(k-2), \dots, y_{h_p}(k-N) \}
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 \mathbf{Y}_m = col\{ & y_{1_m}(k-1), y_{1_m}(k-2), \dots, y_{1_m}(k-N), \\
 & y_{2_m}(k-1), y_{2_m}(k-2), \dots, y_{2_m}(k-N), \\
 & \dots, \\
 & y_{h_m}(k-1), y_{h_m}(k-2), \dots, y_{h_m}(k-N) \}
 \end{aligned} \tag{15}$$

$$\begin{aligned}
\Delta \mathbf{P} = \text{col}\{ & \Delta p_1(k-1), \Delta p_1(k-2), \dots, \Delta p_1(k-N), \\
& \Delta p_2(k-1), \Delta p_2(k-2), \dots, \Delta p_2(k-N), \\
& \dots, \\
& \Delta p_i(k-1), \Delta p_i(k-2), \dots, \Delta p_i(k-N), \}
\end{aligned} \tag{16}$$

where, $\Delta p(k-N) = p(k-N) - p(k-N+1)$ denotes the movement size of EP in each time step throughout the estimation horizon (N). $\mathbf{W}$ and $\mathbf{V}$ are the diagonal weighting matrices for multiple state variables and estimated parameters, as follows:

$$\begin{aligned}
\mathbf{W} = \text{diag}\{ & w_{y_1}(k-1), w_{y_1}(k-2), \dots, w_{y_1}(k-N), \\
& w_{y_2}(k-1), w_{y_2}(k-2), \dots, w_{y_2}(k-N), \\
& \dots, \\
& w_{y_h}(k-1), w_{y_h}(k-2), \dots, w_{y_h}(k-N) \}
\end{aligned} \tag{17}$$

$$\begin{aligned}
\mathbf{V} = \text{diag}\{ & v_{p_1}(k-1), v_{p_1}(k-2), \dots, v_{p_1}(k-N), \\
& v_{p_2}(k-1), v_{p_2}(k-2), \dots, v_{p_2}(k-N), \\
& \dots, \\
& v_{p_i}(k-1), v_{p_i}(k-2), \dots, v_{p_i}(k-N) \}
\end{aligned} \tag{18}$$

Table 3: Summary of parameters used in QP objective function for MHE

Parameter	Description
Φ	Objective function
h, i	Number of state variables(h) and estimated parameters(i)
N	Horizon length for MHE
k	Current time step
$\mathbf{Y}_p, \mathbf{Y}_m$	Measured CV (y_p) and model result of CV (y_m)
$\Delta \mathbf{P}$	Moves of estimated parameters
$\mathbf{W}, \mathbf{V}$	Weighting Matrices for state variables and parameters
u, x, p, d	Model inputs(u), states(x), parameters(p), and disturbance(d)
f, g, h	Model equation (f), output function (g), and inequality constraints (h)

$$\begin{aligned}
\min_{\Delta \mathbf{U}} \quad & \Phi = (\hat{\mathbf{Y}} - \mathbf{Y}_{ref})^T \mathbf{Q} (\hat{\mathbf{Y}} - \mathbf{Y}_{ref}) + \Delta \mathbf{U}^T \mathbf{R} \Delta \mathbf{U} \\
s.t. \quad & 0 = f(\dot{x}, x, y, p, d, u) \\
& 0 = g(x, y, p, d, u) \\
& 0 \leq h(x, y, p, d, u)
\end{aligned} \tag{19}$$

276 where $\hat{\mathbf{Y}}$ and $\mathbf{Y}_{ref}$ are the column vectors for model prediction values and the
 277 reference trajectory from the time step $(k+1)$ to $(k+P)$. $\Delta \mathbf{U}$ is the column
 278 vector for the control moves for the future control horizon (M), from the time
 279 step $(k+1)$ to $(k+M)$. m and l denote the number of CVs and MVs, respectively.
 280 The MPC configuration for this study has one CV and two MVs summarized
 281 in Table 5.

$$\begin{aligned}
\hat{\mathbf{Y}} = \text{col}\{ & \hat{y}_1(k+1), \hat{y}_1(k+2), \dots, \hat{y}_1(k+P), \\
& \hat{y}_2(k+1), \hat{y}_2(k+2), \dots, \hat{y}_2(k+P), \\
& \dots, \\
& \hat{y}_m(k+1), \hat{y}_m(k+2), \dots, \hat{y}_m(k+P)\}
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathbf{Y}_{ref} = \text{col}\{ & y_{1_{ref}}(k+1), y_{1_{ref}}(k+2), \dots, y_{1_{ref}}(k+P), \\
& y_{2_{ref}}(k+1), y_{2_{ref}}(k+2), \dots, y_{2_{ref}}(k+P), \\
& \dots, \\
& y_{m_{ref}}(k+1), y_{m_{ref}}(k+2), \dots, y_{m_{ref}}(k+P)\}
\end{aligned} \tag{21}$$

$$\begin{aligned}
\Delta \mathbf{U} = \text{col}\{ & \Delta u_1(k+1), \Delta u_1(k+2), \dots, \Delta u_1(k+M), \\
& \Delta u_2(k+1), \Delta u_2(k+2), \dots, \Delta u_2(k+M), \\
& \dots, \\
& \Delta u_l(k+1), \Delta u_l(k+2), \dots, \Delta u_l(k+M), \}
\end{aligned} \tag{22}$$

where, $\Delta u(k+M) = u(k+M) - u(k+M-1)$ denotes the movement size of MV in each time step throughout the control horizon (M). $\mathbf{Q}$ and $\mathbf{R}$ are the diagonal weighting matrices for multiple CVs and MVs, as follows:

$$\begin{aligned}
\mathbf{Q} = \text{diag}\{ & q_{y_1}(k+1), q_{y_1}(k+2), \dots, q_{y_1}(k+P), \\
& q_{y_2}(k+1), q_{y_2}(k+2), \dots, q_{y_2}(k+P), \\
& \dots, \\
& q_{y_m}(k+1), q_{y_m}(k+2), \dots, q_{y_m}(k+P)\}
\end{aligned} \tag{23}$$

$$\begin{aligned}
\mathbf{R} = \text{diag}\{ & r_{u_1}(k+1), r_{u_1}(k+2), \dots, r_{u_1}(k+M), \\
& r_{u_2}(k+1), r_{u_2}(k+2), \dots, r_{u_2}(k+M), \\
& \dots, \\
& r_{u_l}(k+1), r_{u_l}(k+2), \dots, r_{u_l}(k+M)\}
\end{aligned} \tag{24}$$

Table 4: Summary of parameters used in QP objective function for MPC

Parameter	Description
Φ	Objective function
m, l	Number of CVs(m) and MVs(l)
P, M	Prediction horizon(P), Control horizon(M)
k	Current time step
$\hat{Y}$	Predicted CV value of dynamic model
$\hat{Y}_{ref}$	Desired set point trajectory in the prediction horizon
ΔU	Control moves of MV in the control horizon
Q, R	Weighting Matrices for CVs and MVs
u, x, p, d	Model inputs(u), states(x), parameters(p), and disturbance(d)
f, g, h	Model equation (f), output function (g), and inequality constraints (h)

Table 5: Variable configuration of MPC and MHE

Case	MPC			MHE				
	CV	MV		DV	SV		EP	
	y_1	u_1	u_2		y_1	y_2	p_1	p_2
Normal drilling	p_{bit}	p_{choke}	q_p		spp	p_{bit}	f_a	f_d
Pipe connection	p_{bit}	p_{choke}		q_p	spp	p_{bit}	f_a	f_d
Density displacement	p_{bit}	p_{choke}	q_p	ρ_{mud}	spp	p_{bit}	f_a	f_d

282 The horizon lengths and weighting factors for both MPC and MHE are
 283 obtained by manual tuning based on operational preference and experience, and
 284 are reported in Tables 6 and 7. The single value of the weighting factor is used for
 285 the each elements of the diagonal weighting matrix. There are research studies
 286 that propose the methods of finding optimal weighting matrices [36, 37, 38].

Table 6: Horizon lengths and weighting factors for MPC

Case	MPC				
	P^*	M^*	q_{y_1}	r_{u_1}	r_{u_2}
Normal drilling	40	15	1000	100	0.1
Pipe connection	10	10	1000	100	0 (DV)
Density displacement	300	240	1000	100	0.05

* P and M represent a prediction and a control horizon, respectively.

Table 7: Horizon lengths and weighting factors for MHE

Case	MHE							
	N^*	semi			full			
		w_{y_1}	v_{p_1}	v_{p_2}	w_{y_1}	w_{y_2}	v_{p_1}	v_{p_2}
Normal drilling	25	1	500	200	1000	2000	2000	500
Pipe connection	30	1000	1000000	1	100	500	10000	10000
Density displacement	120	1	500	200	1	100	500	200

* N represents an estimation horizon.

287 5. Case Studies

288 In this section, the three MPD operating scenarios used to test the RT-HFM-
289 based controller are described. In each test case, the response of the well pressure
290 is simulated by an HFM. As mentioned previously, this HFM has been shown
291 through field experience to accurately represent field conditions in the tested
292 regime [31]. Referring to Figures 4 and 5, the HFM simulates those physical
293 processes denoted by the “Rig” block in the diagrams (including the physical
294 well being drilled). A vertical wellbore profile has been chosen to reproduce a
295 recent MPD operation in the North Sea [39], with model parameters as shown
296 in Table 8.

Table 8: Wellbore Conditions

Parameter	Value (AES)	Value (SI)
Well depth	12,349 ft	3,764 m
Riser inner diameter	9.66 in	0.25 m
Water depth	731.6 ft	223 m
Casing inner diameter	8.535 in	0.216 m
Casing depth	12,349 ft	3,764 m
Drill string average outer diameter	5.0 in	0.127 m
Pore pressure gradient	11.0 ppg	1,330 kg/m ³
Fracture pressure gradient	16.0 ppg	1,927 kg/m ³
Initial mud density	12.4 ppg	1,490 kg/m ³
Mud temperature	122 °F	50 °C

297 The performance of the controller for each scenario was observed for both
 298 semi- and full-closed loop configurations. In the drilling industry, the full-closed
 299 loop configuration is a less common option because a real-time feedback signal
 300 from the well bottom is frequently not available; the majority of drilling oper-
 301 ations presently use a semi-closed loop configuration, where sensor data may
 302 be provided at infrequent or irregular intervals. However, the full-closed loop
 303 configuration is at times available and the level of control achieved with this
 304 configuration represents a best-case scenario to which we can compare control
 305 achieved by semi-closed loop. Therefore, the purpose of this comparison is not
 306 to show that one configuration is superior to the other (full-closed loop con-
 307 trol is certainly the ideal case), but rather to determine whether the model of
 308 the system provided by the RT-HFM can enable the controller in a semi-closed
 309 loop configuration achieve a level of control similar to that in the full-closed
 310 loop configuration. The semi-closed loop configuration relies more on the model
 311 equations than the full-closed loop configuration in the absence of the BHP mea-
 312 surement; thus, by comparing the two, we are able to measure the effectiveness
 313 of the model. The performance comparisons between various controllers such as

PID *vs.* Hammerstein-Wiener MPC [20] and LFM MPC *vs.* HFM MPC [30] have been investigated in previous research.

Both semi-closed loop and full-closed loop control schemes were tested under the following scenarios:

1. Normal drilling (mud flow, drillstring rotation, formation penetration)
2. Pipe connection (cessation of mud flow and drillstring rotation)
3. Mud density displacement over a fixed period of time

Prior to each of the scenarios listed above, an initial model calibration step is performed. This step uses the MHE exclusively, and it is assumed that both BHP (p_{bit}) and SPP (standpipe pressure, spp) measurements are available during this period. In common practice, the BHP measurement is available from downhole sensors at various points in time for model calibration. The friction factors in the annulus and drillstring (f_a and f_d) are estimated based on both BHP and SPP by minimizing the differences between the model and the measured value. The estimated parameters (EPs), which are f_a and f_d , are updated in the MPC model to allow prediction with improved model accuracy.

After the model calibration period, the three control scenarios use the MHE and MPC in parallel for real-time estimation and control. For these three control scenarios, the variable configurations in the MPC are changed based on the scenarios, while the MHE uses the same variable configurations for all scenarios (Table 5). The detailed description of each scenario is shown in the following subsections

5.1. Normal Drilling

BHP control provides the ability to drill in narrow pressure profile wells in addition to optimizing the ROP [40]. Thus, the controller’s ability to respond appropriately (e.g., quickly but with a minimum of overshoot) to setpoint changes is important when drilling within these tight pressure limits. To quantify the controller’s ability to track setpoint changes, the “normal” drilling scenario introduces three different step changes in setpoint.

343 In a normal drilling scenario both the choke pressure (p_{choke}) and drilling
 344 fluid flow (q_p) can be manipulated by the controller. Once key unmeasured
 345 variables, including system friction factors (f_a , f_d), are initially calibrated in
 346 the model by the model calibration procedure, the MPC algorithm adjusts the
 347 manipulated variables to minimize the difference between the set point and the
 348 calculated BHP (in semi-closed loop) or measured BHP (in full-closed loop)
 349 across the prediction horizon. This accurately drives the BHP to the set point
 350 while complying with any user-provided constraints, such as the maximum rate
 351 of change for the manipulated variables such as choke pressure (p_{choke}) and mud
 352 flowrate (q_p).

353 5.2. Pipe Connection

354 During a pipe connection procedure, the normal control processes must be
 355 modified to accommodate the addition of more pipe. As the drill bit deepens
 356 during the drilling process additional pipe lengths are periodically added to the
 357 drill string. The addition of pipe is typically required every one to three hours
 358 but is dependent on the pipe stand length and the ROP [41]. During this pipe
 359 connection it is necessary to ramp the drilling fluid flow rate to zero, attach
 360 the new pipe length, and then bring the flow rate back up to normal conditions
 361 again. As the mud flow rate is brought to zero the controller then relies solely on
 362 the choke pressure until the pipe connection is complete. However, because pipe
 363 connection is a planned event and the ramp rate is known in advance, the ramp
 364 rate can be passed into the MPC acting as a measured disturbance variable
 365 (DV) so that the changing mud flow can be considered in the BHP predictions
 366 of the MPC and improve control accuracy during the pipe connection period.

367 5.3. Mud Density Displacement

368 At the end of the MPD operation it is optimal to shut off the choke valve.
 369 To reduce reliance on the choke pressure in the control scheme, higher density
 370 mud is fed to the bottom hole. This higher density mud serves as a less accurate
 371 substitute for the choke valve because it exerts a higher pressure on the open

hole so that the choke manifold can be released and disengaged from BHP management. In this scenario, the mud density changes over a period, thus allowing for a slow opening of the choke valve. As the density changes, the controller accounts for this change in system dynamics as it manages the BHP. This scenario differs from the pipe connection scenario in that the controller adjusts to rely solely on the drilling fluid flow, whereas for pipe connection the drilling fluid flow must be ramped down. Throughout the period of density transition and choke valve ramp up, the estimator is relied upon to provide accurate friction factors that keep the model accurate despite the changing conditions.

6. Results and Discussions

This section presents the results of the case studies. The control performances of both semi-closed loop and full-closed loop configurations are shown and quantified using ISE (integral of squared error) index shown in Table 9.

6.1. Normal Drilling

Figures 6 and 7 display the results of the normal drilling case study for the semi-closed loop and full-closed loop configurations, respectively. Although the results appear to be nearly equivalent, the two cases are important to compare and contrast. Figure 6 represents the case where downhole measurements are unavailable; therefore, there is an expected offset between the model-predicted values and the measured values. In an actual drilling process without any bottomhole measurements, there would likely be an even more substantial offset between the true and estimated BHP. Large sources of potential error include unknown temperature profiles along the drillstring annulus and non-Newtonian fluid properties that are influenced by high pressures and varying temperatures. In Figure 7 the bottomhole conditions are directly measured and transmitted in real-time to the surface. This allows for offset-free control, which is the most desirable condition. However, even with downhole sensors and near-instantaneous

400 feedback, there are likely to be periods of time when the telemetry system is not
 401 available or when there is sensor error. Highlighting both case studies shows
 402 that MPC can be used with or without the bottomhole sensors with an accurate
 403 predictive model that is calibrated to the drilling process. The suitability of a
 404 semi-closed loop approach using real-time model-based control as opposed to a
 405 full closed loop approach depends upon the tolerance of the specific application
 406 to the presence of an offset.

407 In the semi-closed loop case the BHP is calculated by the MHE, minimizing
 408 the difference between measured and modeled SPP. It is assumed that bottom
 409 hole conditions are communicated to the surface at infrequent intervals during
 410 the drilling process. This is compared to the full-closed loop conditions where
 411 BHP measurements are regular and frequent. The model parameters (f_a and f_d)
 412 are calibrated during the model calibration period. This calibration is observed
 413 in Figures 6 and 7 between 0 and 200 seconds. The model calibration period
 414 is shown in Figure 8 separately in magnified time scale. The MPC controller
 415 is activated after 200 seconds. The control movements of the MVs (p_{choke} ,
 416 q_p) are observed in Figures 6c and 7c. Figures 6b and 7b show the estimated
 417 friction factors (f_a and f_d). The BHP control is more effective in a full-closed
 418 loop scenario as the controller is able to use actual bottomhole conditions to
 419 inform the control moves. However, it is of interest that the improvement in
 420 control performance observed in the full-closed loop case is minor and that the
 421 full-closed loop scenario brought the BHP about 0.5 *bar* closer to the setpoint
 422 as compared to the semi-closed loop scenario. While this is not an exhaustive
 423 survey across a multitude of well conditions, it is indicative of the value and
 424 effectiveness of using a semi-closed loop system that employs a RT-HFM. As
 425 can be seen in Figures 6 and 7, very similar control moves were made, and the
 426 resulting BHP measurements differed by less than one *bar*.

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

(c) Choke pressure (MV1) and Mud flowrate (MV2)

Figure 6: Control performance for normal drilling scenario (Semi-closed loop)

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

(c) Choke pressure (MV1) and Mud flowrate (MV2)

Figure 7: Control performance for normal drilling scenario (Full-closed loop)

427 Figure 8 presents the results of the MHE for the initial model calibration
428 of the annulus and drillstring friction factors. The MHE first estimates the
429 annulus friction factor (f_a) at 150 seconds by minimizing the difference between
430 the RT-HFM BHP and the HFM BHP. At 400 seconds the MHE finds the drill
431 string friction factor (f_d) by minimizing the difference between the RT-HFM
432 SPP and the HFM SPP. Once both friction factors are estimated, the model
433 is ready to be used effectively for control. After this initial model calibration
434 step, the MPC controller is turned on with the updated friction factors for the
435 subsequent control scenarios. While the controller turns on, the friction factors
436 are continuously adjusted by the MHE in parallel with MPC in real-time.

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

Figure 8: Model calibration using MHE

6.2. Pipe Connection

The pipe connection procedure results are shown in Figures 9 and 10. During the pipe connection procedure, mud flowrate moves through a wider operational range from the range of normal drilling to a zero flow rate which produces greater model mismatch than the normal drilling scenario. According to the settings and models of this study, the primary factor influencing the model mismatch is the mud flowrate because the friction factor of the drill string and annulus is only important when the mud is flowing. At the beginning of the case study,

445 the model calibration proceeds until the MPC control starts at 400 seconds.
 446 From 600 seconds, mud flowrate starts ramping down and stays at the zero
 447 flowrate for about 300 seconds until the new segment of a drill string is added.
 448 The mud flow rate is then ramped back up to the normal drilling range (1000
 449 l/min). As the mud flow ramps down, the BHP is maintained by compensatory
 450 moves in the choke pressure. Because it is a planned change in the mud flow,
 451 the ramp down is communicated to the predictive controller and the controller
 452 determines effective moves to maintain a steady bit pressure through the pipe
 453 connection procedure. The MHE continuously adjusts the annulus and drill
 454 string friction factors to match the BHP or SPP, depending on the control mode.
 455 Figures 9 and 10 show the results of the semi-closed loop and full-closed loop
 456 control mode, respectively. Although the mud flowrate is varied greatly, both
 457 control modes show acceptable control performance maintaining the BHP within
 458 ± 1 bar deviation. However, the semi-closed loop mode shows slightly worse
 459 control performance than the full-closed loop mode after mud flowrate ramps up
 460 completely because the friction factors are not stabilized their original values.
 461 The objective of MHE to minimize model and process discrepancies is satisfied
 462 for the semi-closed loop mode by minimizing the SPP difference between the
 463 HFM and RT-HFM instead of BHP difference in full-closed loop. Thus, the 0.5
 464 bar gap between ‘Simulated BHP (HFM)’ and ‘Estimated BHP (RT-HFM)’ is
 465 observed in Figure 9a but not shown in Figure 10a.

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

(c) Choke pressure (MV) and Mud flowrate (DV)

Figure 9: Control performance for pipe connection scenario (Semi-closed loop)

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

(c) Choke pressure (MV) and Mud flowrate (DV)

Figure 10: Control performance for pipe connection scenario (Full-closed loop)

466 6.3. *Mud Density Displacement*

467 The results in Figures 11 and 12 demonstrate the impact of changes in
468 drilling fluid density on the estimated friction factor and the resultant changes in
469 the choke pressure and drilling fluid flow. The differences between a semi-closed
470 and full-closed loop mud density displacement scenario are also displayed. The
471 drilling fluid density change is observed at 0.5 hours as the density is stepped
472 up from 1.5 S.G. to 1.6 S.G. The choke valve is subsequently ramped down
473 over a 2.5 hour period between 0.5 and 3 hours. It is observed that because of
474 the aforementioned system changes, the drilling fluid flow slowly decreases to
475 further counteract the increase in density. This indicates that fully opening the
476 choke valve does not provide enough pressure relief and the controller selectively
477 relies on the drilling fluid flow to maintain control. Additionally, the standpipe
478 pressure drops off substantially but the BHP stays high as a result of the higher
479 density drilling fluid. In the full-closed loop configuration, the RT-HFM matches
480 the simulated real world conditions more closely, which improves the quality of
481 BHP control provided by the MPC controller.

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

(c) Choke pressure (MV) and Mud flowrate (DV)

Figure 11: Control performance for mud density displacement scenario (Semi-closed loop)

(a) Bottomhole pressure (CV) and Standpipe pressure

(b) Friction factors in annulus and drill string

(c) Choke pressure (MV) and Mud flowrate (DV)

Figure 12: Control performance for mud density displacement scenario (Full-closed loop)

Table 9: Control performance comparison in ISE index

Case	<i>ISE</i>	
	Semi-closed loop	full-closed loop
Normal drilling	762.86	756.35
Pipe connection	53.64	17.14
Density displacement	64.62	58.39

7. Conclusions and Future Work

This study represents the first implementation of a high-fidelity grade physics-based model in a real-time control application using MPC. In the simulations reported herein, the model-based control schemes achieved tight controller performance that successfully maintained the BHP to within one *bar* of the set point during normal drilling, pipe connection, and mud density displacement operations. Although the results are obtained from the simulation environment under the noise-free condition, this responsive model performance validates our hypothesis that high-fidelity physics-based models can be successfully embedded in real-time control systems, and suggests the suitability of this model for use under real MPD conditions, including wells with tight pressure margins.

Moreover, the highly detailed model embedded in the estimator and controller provides advantages when operating in the relatively harsh oilfield drilling environment, since it allows for periods of sparse feedback communication that is characteristic of the environment, with minimum performance degradation. Our simulation studies have shown that in all three operational scenarios, the difference in controller response between a full-closed loop control scheme and a semi-closed loop is marginal. Future work should include optimization of tuning parameters to maximize controller performance, and testing on a broader suite of field conditions which include the measurement noise and unmeasured disturbances.

This work also lays the foundation for further work with more complex

504 systems and control scenarios that leverage the power of MPC.

505 For example, although pressure and vibration control methods are evolving
506 separately in the drilling industry, these should be integrated into a single
507 research topic in the future as the technologies mature. A recent example of
508 such multivariable control investigates ROP optimization combined with BHP
509 control based on fundamental flow models [42]. In this work, the mutual effects
510 of the drill string dynamics and hydraulics are considered. This topic should be
511 revisited in view of the advances made on both research fronts.

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