2013-07-02

Types of Questions that Comprise a Teacher's Questioning Discourse in a Conceptually-Oriented Classroom

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A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

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July 2013

ABSTRACT

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This study examines teacher questioning with the purpose of identifying what types of mathematical questions are being modeled by the teacher. Teacher questioning is important because it is the major source of mathematical questioning discourse from which students can learn and copy. Teacher mathematical questioning discourse in a conceptually-oriented classroom is important to study because it is helpful to promote student understanding and may be useful for students to adopt in their own mathematical questioning discourse. This study focuses on the types of questions that comprise the mathematical questioning discourse of a university teacher in a conceptually-oriented mathematics classroom for preservice elementary teachers. I present a categorization of the types of questions, an explanation of the different categories and subcategories of questions, and an analysis and count of the teacher's use of the questions. This list of question types can be used (1) by conceptually-oriented teachers to explicitly teach the important mathematical questions students should be asking during mathematical activity, (2) by teachers who wish to change their instruction to be more conceptually-oriented, and (3) by researchers to understand and improve teachers' and students' mathematical questioning.

Keywords: conceptually-oriented, discourse, problem solving, questioning discourse, mathematical discourse, mathematical questions

ACKNOWLEDGEMENTS

I would like to thank all those who have offered me support and confidence during my work on this research. I would especially like to thank my friends and family who supported me through this process. I would also like to thank my advisor for his exceptional effort, support, and help.

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CHAPTER 1: RATIONALE

Students' understanding of and participation in mathematical discourse is a legitimate goal of mathematics instruction for two reasons. First, it is important for developing mathematical understanding and learning mathematical content. Skemp (1978) defined mathematical understanding as knowing what to do and why. Students must be able to interpret the mathematical discourse of the classroom in order to know what to do as well as why to do it. If students do not understand the discourse, then they cannot understand others' explanations of what to do and why. Second, fluency in mathematical discourse is an important part of students' mathematics learning, and is a legitimate goal of mathematics instruction. Martin and Herrera (2007) defined mathematical discourse by saying that "the discourse of a classroom—ways of representing, thinking, talking, agreeing, and disagreeing—is central to what and how students learn about mathematics" (p. 46). They also stated that how math is learned affects what is learned. Understanding and participating in mathematical discourse is necessary for mathematical proficiency. The National Research Council (2001) said that being able to communicate about mathematics is an important component of mathematical proficiency. Communication falls under their strand of proficiency termed adaptive reasoning, or the capacity for logical thought, reflection, explanation, and justification. Students must be able to both understand the mathematical discourse of the classroom as well as participate in it in order to be proficient in adaptive reasoning. Thus, discourse is both a vehicle for learning and a goal of learning.

An important part of the mathematical discourse students should be learning is the part that includes the questions one asks when engaged in mathematical activity. This mathematical discourse of questioning is important to master for two reasons. First, questioning is important because it gives students a way to participate in and elicit mathematical conversations with others. By participating in and engaging others in mathematical discourse through questioning, students are given the opportunity to become proficient participants of the mathematics community and its conversations. Second, questioning is important because it allows students to participate in and continue the mathematical conversation with themselves. Richards (1991) stated that

> Our own conversation serves as an aid in posing and solving problems. Our ability to continue the conversation gives us the power to engage [in] mathematical issues. We first learn to continue the conversation by ourselves by participating in conversation with others. (Richards, 1991)

He asserted that being able to engage internally in mathematical discourse or conversation, which is done in part through self-questioning, helps students be able to solve mathematical problems. Consequently, competency in asking mathematical questions is essential for students to be able to participate in mathematical activity.

Students do not come into the mathematics classroom proficient in the necessary mathematical questioning discourse at the beginning of their mathematics education. Students participate in multiple discourses—their home discourse, their school discourse, their American teenager discourse in their social group (Gee, 1996)—but most likely young students do not already have a mathematical questioning discourse. The mathematics questioning discourse is specialized and is not innate for students; thus students must learn this specialized discourse.

Teacher questioning discourse is the major source of discourse available to students to learn from and copy. Gee (1996) posited that students learn a particular discourse by observing and interacting with people who are modeling that discourse. Because a large part of students' exposure to mathematical discourse occurs in the mathematics classroom, it follows that students will acquire the vast majority of their proficiency in asking mathematical questions by observing and copying the mathematical questioning of the classroom members whom they judge to be more mathematically expert than themselves. Furthermore, because the teacher is typically recognized as being the disciplinary expert in the class, students' mathematical questioning will largely depend upon the types of questions the teacher asks and sanctions in the classroom. Even when students attempt to model the questioning of students they perceive as being more expert than themselves, it is likely that these expert students are modeling the questioning discourse they have observed from their teachers. Thus, the teacher's mathematical questioning discourse is still the main source for the development of students' mathematical questioning discourse. Students can copy and adopt the mathematical questioning discourse of their teachers in an effort to become proficient in mathematical questioning discourse.

There are multiple types of mathematical discourse and some may be more beneficial than others. One particularly promising mathematical discourse is conceptually-oriented mathematical discourse. Thompson, Philipp, Thompson, and Boyd (1994) defined a conceptual orientation to teaching as one that focuses attention away from simply applying procedures and toward a rich conception of the situations, ideas, and relationships among the mathematics. There are particular ways of talking about mathematics when one has this orientation. In particular, interlocutors focus on the meaning of quantities, the relationships between quantities, the meanings of operations and the reasons why those operations are appropriate, and the meanings for the results of operations. Often comments are phrased in terms of how one is thinking or reasoning. I will refer to this type of discourse as conceptually-oriented discourse.

It is valuable to look at teacher questioning discourse in a conceptually-oriented classroom because it might address at least two different issues. First, because the teacher asks questions that focus on students' conception of the mathematics and connections between mathematical ideas, the questioning discourse used by teachers with conceptually-oriented discourse is valuable in helping students attain mathematical understanding. Second, teachers with a conceptually-oriented mathematical discourse could be participating in a questioning discourse, one that utilizes questions that are focused on understanding the mathematical ideas and connections between them, that could be particularly helpful if copied by students. It might be possible that the same teacher questions can be asked by the students to themselves, thus helping provide a framework of questions for students to use to continue the mathematical conversation internally and improve their participation in mathematical activity. Teacher questioning discourse in a conceptually-oriented classroom can be studied to hopefully provide insight into these two issues. By understanding the types of questions that a teacher utilizes in a conceptually-oriented classroom, we can gain a clearer picture of the types of questions available in the classroom to help students attain mathematical understanding as well as be used as a model for a student's own mathematical questioning discourse.

Little is known about the teacher mathematical questioning discourse in a conceptually-oriented classroom. Past researchers have studied discourse (Gee, 1996, 1999; Hiebert & Wearne, 1993), types of mathematical discourse (Thompson et al., 1994), mathematical teacher questioning discourse (Hiebert & Grouws, 2007; Rittenhouse, 1998), as well as purposes and types of mathematical teacher questioning discourse (Franke, Turrou, & Webb, 2011; Hiebert & Wearne, 1993; Kawanaka & Stigler, 1999; Moyer & Milewicz, 2002; Sahin & Kulm, 2008; Teuscher, Moore, Marfai, Tallman, & Carson, 2010; Wood, 1998), but none with the focus of studying the types of mathematical questions teachers ask in a conceptually-oriented classroom, the very kinds of questions that students might appropriate for their own mathematical questioning discourse. So my research question is as follows: What are the types of questions that comprise the mathematical questioning discourse of a teacher in a conceptually-oriented classroom? By understanding more about the teacher's mathematical questioning discourse in a conceptually-oriented classroom, we can better understand the types of questions to which students are being exposed. Being explicitly aware of these types of questions can help teachers draw students' attention to the types of questions students should be asking themselves and their classmates while engaged in mathematical activity. Also, teachers may not be voicing all of the important types of mathematical questions for engaging in mathematical practice, and an explicit awareness of what questions are being asked can be the first step to identifying these missing question types.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

This chapter will focus on the past research related to this study as well as how that research fits in with this study and the perspectives taken. First, I will focus on discourse and define a mathematical questioning discourse. Then I will discuss how others have categorized types of mathematical discourse and what is still lacking in the categorization of mathematical discourse. Finally, I will specifically describe teacher questioning discourse, how it has been categorized into purposes and types, and what is lacking in the categorized types of teacher's questioning discourse that will be the focus of this study.

Discourse

Although I am looking at mathematical questioning discourse, it is important to attend to discourse in general. One of the most prominent researchers in social linguistics is Gee. Gee (1996) defined "big D" Discourse as a combination of what one says, how it is said, and by whom. He said that Discourses are "ways of behaving, interacting, valuing, thinking, believing, speaking, and often reading and writing that are accepted as instantiations of particular roles (or 'types of people') by specific groups of people" (p. viii). That is, Discourse is so much more than just the words that someone speaks; it is also everything that goes into communicating who and what one is to a specific group of people. Gee (1999) distinguished between "big D" and "little d" discourse. He defined "little d" discourse as "any instance of language-in-use or stretch of spoken or written language" (p. 205). That is, "little d" discourse only looks at the words or utterances, spoken or written, and does not focus on the identity that is being communicated to specific people based on that language.

Mathematics education researchers are using the term discourse in two distinct ways, much like Gee (1996; 1999). Martin and Herrera (2007) and Rittenhouse (1998) referred to discourse in the “big D” sense of the word. Martin and Herrera defined discourse as ways of representing, thinking, talking, agreeing, and disagreeing about mathematics (Martin & Herrera, 2007). Martin and Herrera’s definition of discourse, specifically their references to ways of representing, thinking, and talking about mathematics, aligns directly with Gee’s (1996) definition of Discourse, particularly his references to ways of behaving, interacting, thinking, speaking, and reading. Martin and Herrera’s ways of agreeing and disagreeing about mathematics also fit implicitly with Gee’s valuing and believing. Similarly, Rittenhouse (1998) defined discourse as the particular way in which language, thoughts, and actions are used by members of particular groups, or a particular mathematics classroom. Rittenhouse focused on any type of communication, more than just talk, and how that communication is used by particular groups, like Gee. In contrast, Cobb, Wood, and Yackel (1993) and Wood (1998) referred to discourse as Gee (1999) did when he described “little d” discourse. They defined discourse as the talk or dialogue in which one engages, which is compatible to Gee’s definition of “little d” discourse. I similarly discuss discourse in this paper and make reference to its two distinctions, that of “big D” and “little d”.

In this study, unlike Gee (1996; 1999), I am focusing on a particular content area, so instead of focusing on discourse, I will focus on mathematical discourse. To define mathematical Discourse, I combine Gee’s (1996) definition of “big D” Discourse with Martin and Herrera’s (2007) definition of discourse to say that *mathematical Discourse* is ways of representing, thinking, talking, agreeing, and disagreeing about mathematics. I will use a “big D” in defining mathematical Discourse because it includes ways of thinking, talking, valuing, and representing ideas or acting while participating in mathematical activity, like was defined in “big D” Discourse. I will define *mathematical discourse* as language, spoken or written, regarding mathematics. I will use Gee's conventions of distinguishing between "big D" Discourse and "little d" discourse by using a capital or lower case d.

Calculationally and Conceptually-Oriented Mathematical Discourse

Many have talked about the importance of mathematical Discourse. Hiebert and Wearne (1993) claimed that the opinion that classroom discourse influences learning is uncontroversial. Martin and Herrera (2007) stated that the mathematics Discourse in the classroom, specifically the way students participate in the Discourse, is an important part of students' learning in the mathematics classroom. However, as Martin and Herrera asserted, just engaging in mathematical Discourse is not enough. There are multiple types of mathematical Discourse and it is the type of mathematical Discourse that really defines the experience that teachers and students have with the mathematics. Thus, more than just the importance and influence of mathematics Discourse in general, we want to look specifically at the types of mathematical Discourse that occur in the classroom.

Some researchers have examined mathematical Discourse and categorized it into different types. One prominent framework described two types of mathematical orientations that one can utilize in the mathematics classroom. Thompson et al. (1994) described a calculational and conceptual orientation to teaching.

A calculational orientation to teaching was defined by Thompson et al. (1994) as being driven by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results. There is an emphasis on identifying and performing procedures and a tendency to speak exclusively in the language of numbers and numerical operations. The mathematical understanding that is associated with a calculational orientation is what Skemp (1978) termed instrumental understanding, or the knowledge of the rules or procedures without the understanding of why they work, when to use them, and why they are important. I am extending Thompson's definition of a calculational orientation to Discourse, and I define *calculationally-oriented Discourse* as the particular ways of representing, thinking, talking, agreeing, and disagreeing about mathematics when one has a calculational orientation. For example, Discourse regarding the numerical result to a computation, disagreement over the correct answer to a problem, and the statement of the proper procedure to attain a solution are all instances of calculationally-oriented Discourse.

Thompson et al. (1994) defined a conceptual orientation to teaching as one that focuses students' attention away from simply applying procedures and toward a rich conception of the situations, ideas, and relationships among the mathematics. Teachers with a conceptual orientation have the expectation and encourage students to be intellectually engaged in tasks and activities that make them active participants in the construction of mathematical meaning in the classroom. Teachers with a conceptual orientation work towards what Skemp (1978) defined as relational understanding, or the understanding of what to do and why. I am extending Thompson et al.'s description of a conceptual orientation to Discourse, and I define *conceptually-oriented Discourse* as the Discourse when one has a conceptual orientation. For example, Discourse regarding the explanation of one's reasoning, the sufficiency or correctness of an explanation, and the connection between a mathematical concept and its multiple representations are all instances of conceptually-oriented Discourse.

The type of mathematical Discourse one engages in affects not only how mathematics is learned but what mathematics is learned (Martin & Herrera, 2007). Students who are in a calculationally-oriented classroom and engage in calculationally-oriented Discourse work towards instrumental understanding of what to do, while students in a conceptually-oriented classroom who engage in conceptually-oriented Discourse work to attain the relational understanding of what to do and the why behind it (Skemp, 1978; Thompson et al., 1994). Because we want students to not only know what to do but also why they do what they do, plus have an understanding of the connections between the mathematical ideas and concepts, I have focused my study on the questioning on classrooms where the dominant mathematical Discourse is conceptually oriented. I refer to these classrooms as conceptually-oriented mathematics classrooms.

Mathematical Questioning Discourse

A mathematical questioning Discourse is a special subpart of a mathematical Discourse. *A mathematical questioning Discourse* consists of the questions that are asked while using mathematical Discourse to engage in mathematical activity. Although it does not comprise an entire Discourse because one cannot communicate fully using only questions regarding mathematics, it is situated within a mathematical Discourse. Because mathematical questions include or make reference to how one represents, thinks, talks, agrees, and disagrees about ideas in mathematics, it is appropriate to use a "big D" when referring to this component of mathematical Discourse. Mathematical questioning Discourse can be participated in by both the teacher and the students, although their manner and level to which they participate may differ. Because mathematical questioning Discourse is embedded in a mathematical Discourse, questioning Discourses are heavily influenced by the particular mathematical Discourses in which they are situated.

Thompson et al. (1994) provided a list of questions that a teacher with a conceptual orientation might ask. The examples provided focused on questioning the meaning of the numbers in the problem or the significance of the result, such as, "To what does (this number) refer in the situation we're dealing with? [or] What did this calculation give you (in regard to the situation as you currently understand it)?" (p. 86). These example questions highlighted the teacher's focus on a rich conception of the situations and relationships among the mathematics. Figure 1 provides a list of example questions a conceptually-oriented teacher might ask.

- "(This number) is a number of what?"
- "To what does (this number) refer in the situation we're dealing with?"
- "What did this calculation give you (in regard to the situation as you currently understand it)?"
- "Who agrees with [that student]'s reasoning?"
- "Did anyone think of the problem differently?"
- "Can you explain your reasoning?"

*Figure 1.*Mathematical Questions a Conceptually-oriented Teacher Might Ask. Adapted from Thompson, "Calculational and conceptual orientations in teaching mathematics," by A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A., 1994, *Professional development for teachers of mathematics*, p. 86. Copyright 1994 by National Council of Teachers of Mathematics.

Thompson et al.'s (1994) list of questions that a conceptually-oriented teacher might ask was meant to suggest specific questions teachers could ask during discussions to change their teaching practice. It is unlikely the authors intended it to be an exhaustive list of the types of mathematical questions a conceptually-oriented teacher might ask, particularly since their list seems to be derived from reflections on their own teaching and not based on actual classroom data. There are no empirical studies that identify the types of questions a teacher or student with a conceptual orientation would use. It seems that there are questions that also would fall under the conceptually-oriented category, but that are not found in Thompson et al.'s list, such as Polya's (1945) questions to help in the problem solving process, such as, "'Do you know a related problem?' [or] 'Can you check the argument?'" (pp. xvi-xvii). A more complete list of example questions for conceptually-oriented Discourse is needed if researchers are to understand the types of questions that students are being exposed to in a conceptually-oriented classroom.

Similarly, Thompson et al.'s (1994) list does not include a categorization of the types of mathematical questions a conceptually-oriented teacher might ask, nor have other researchers suggested a categorization for the types of mathematical questions that are asked during conceptually-oriented instruction. However, a meaningful categorization of these questions seems possible and helpful. For example, surely the question of "(This number) is a number of what?" is different in purpose and form than that of "Who agrees with [that student's] reasoning?" Based purely on form, the first question is asking for a fill-in-the-blank identification of a quantity, while the second question is asking for an expression of an opinion. Also, one question's purpose is to focus the students' attention on the meaning of a number in an equation or procedure, while the second question engages students in justification and argument of a mathematical idea. Categorizing these teacher questions would be helpful in order to distinguish more clearly between types of questions that could be adopted and used by students in their own problem solving. Thus, a categorization of questions could be very helpful in teaching students a conceptually-oriented questioning Discourse.

Teacher Mathematical Questioning Discourse

Many researchers in mathematics education have, though, already studied teacher mathematical questioning Discourse and have differentiated between different types of questions (Boaler & Brodie, 2004; Hiebert & Wearne, 1993; Moyer & Milewicz, 2002; Sahin & Kulm, 2008; Teuscher et al., 2010). The main focus of studying teacher questioning Discourse has been to identify teachers' skillful use of questions—that is, skillful in eliciting student thinking or engaging students in developing mathematical understanding, versus less skillful use of questions in order to improve pedagogy. There is a clear divide in the frameworks as to what constitutes skillful questioning and what questions are less than skillful. First, I will summarize what have been valued as teachers' less than skillful use of questions. Then I will summarize the teachers' questioning that is viewed as skillful and different frameworks' descriptions and categorizations of skillful questioning. I will then argue why the work done separating skillful from less than skillful questioning is not enough.

Less Than Skillful Questioning

Different researchers have outlined types of teacher mathematical questioning Discourse which can be classified as less than skillful. One prominent and well-used framework for categorizing teacher questions into different types was developed by Sahin and Kulm (2008). Sahin and Kulm performed a case study on two sixth grade teachers' questioning and found that the teachers used three different types of questions: probing, guiding, and factual questions, the latter two types of questions being those the researchers considered to be less skillful. *Guiding* questions prompted students to fill in the missing information the teacher suggested about problems and derivations of mathematical concepts and procedures in order to lead students to use particular mathematical concepts and procedures to solve problems. These questions did not require students to participate in any mathematical activity besides basic computations and procedures as the teacher was the one directing the solving of the problem. *Factual* questions checked students' recall of specific mathematical facts or procedures in order to assess basic information before moving forward. These questions required only recall of mathematics facts or procedures from the students and did not require any exploration or additional thinking beyond what students had already done. Other researchers who have studied teacher questioning also noted these two types of questions (Franke et al., 2011; Kawanaka & Stigler, 1999; Redfield & Rousseau, 1981; Teuscher et al., 2010; Wood, 1998) that are most often categorized as a less than skillful use of questions by a teacher.

Skillful Questioning

Researchers have outlined other questions that are viewed as part of skillful questioning. However, a lot of frameworks have not been very specific about what skillful questioning looks like and have often lumped all skillful questioning into one category of questions.

Skillful questioning not well defined. Many researchers describe skillful questioning and create one type of question that comprises all those questions utilized in skillful questioning. Moyer and Milewicz (2002) defined *skillful questioning* as listening to student responses and ideas to construct a specific probe for more information about the students' answers. For example, if looking at a student's correct drawing of one-third of a circle a teacher could ask, "How did you figure that out? How did you know you had to put two lines to make three parts?" (p. 308). Also, many researchers used the term *probing* questions to denote all those questions that are used in skillful questioning. Sahin and Kulm (2008) defined *probing* questions as questions asking for clarification, justification, or explanation to extend students' knowledge. Many others in the mathematics education field have similarly categorized skillful teacher questions into a single category (Franke et al., 2011; Kawanaka & Stigler, 1999; Redfield & Rousseau, 1981; Teuscher et al., 2010; Wood, 1998). Franke et al. (2011) and Teuscher et al. (2010) termed this single category of questions as probing questions, Kawanaka and Stigler (1999) as describe/explain questions, Redfield and Rousseau (1981) as higher cognitive questions, and Wood (1998) as a pattern of discourse called funneling.

In addition to all skillful questioning being grouped into one category of questions, a second problem with these frameworks is that most were developed from data involving teachers who might not have had a conceptual orientation. Several researchers have noted the need for further and more descriptive frameworks for teacher questioning Discourse (Hiebert & Grouws, 2007; Hoster, 2006). Hiebert and Wearne (1993) asserted that the majority of research regarding teacher questioning Discourse has been done in classrooms where the Discourse is focused on the acquisition of written computation algorithms, or what I call calculationally-oriented Discourse, and not on classrooms with the focus on student expression of ideas and connections and reflections on the mathematics, or what I term conceptually-oriented Discourse.

Skillful questioning more defined. Other researchers have given greater insight into what skillful teacher mathematical questioning Discourse might look like. These studies also focused on teachers that were conceptually-oriented. An examination of these frameworks suggests that teachers, particularly conceptually-oriented teachers, are asking mathematical questions that model the types of questions students might ask themselves while engaged in mathematical activity.

Hiebert and Wearne (1993) categorized teacher questions into four general categories, three of the four of which can be considered those which constitute skillful questioning. The first category that is part of skillful questioning is *describe strategy* questions that ask students to tell how they solved the problem or another way to do it. The second is the g*enerate problems* category, or questions that ask students to create a story or problem to match the situation or given constraints. The final category is the *examine underlying features* category that includes asking students to explain why a procedure was chosen or why it works as well as the nature of a problem or strategy. These categories do break down and more specifically describe the category of skillful questioning that might be used by conceptually-oriented teachers, but they do not cover all question types a teacher with a conceptual orientation would use according to Thompson et al. (1994). For example, Thompson et al.'s questions regarding the meaning of different quantities or calculations in reference to the situation like, "To what does (this number) refer in the situation we're dealing with?" would not fit in any of Hiebert and Wearne's categories. More categories or types of questions need to be created in order to classify each type of question that a teacher might ask or that a student might be able to ask themselves in engaging in mathematical activity.

Boaler and Brodie (2004) presented a categorization that more clearly defines different types of questions used when one engages in skillful questioning. They created 9 different categories or types of questions, 7 of which are those that a teacher might ask that would be categorized as skillful types of questions. *Inserting terminology* questions are those that enable correct mathematical language to be used once the mathematical ideas are under discussion. *Exploring mathematical meanings and/or relationships* are questions that point to underlying mathematical relationships and meanings. They make links between mathematical ideas and representations. *Probing or getting students to explain their thinking* questions are questions that ask students to articulate, elaborate, or clarify ideas. *Linking and applying* questions point to the relationships among mathematical ideas and mathematics and other areas of study or life. *Extending thinking* questions extend the situation under discussion to other situations where similar ideas may be used. *Orienting and focusing* questions help students to focus on key elements or aspects of the situation in order to enable problem solving. *Establishing context* questions talk about issues outside of mathematics in order to enable links to be made with mathematics (p.776).

Boaler and Brodie's (2004) categorization seemed to be the most descriptive and specific in terms of skillful questioning and how each category of questions can be used by the teacher to promote mathematical activity. However, this categorization is also incomplete. This categorization of teacher questioning Discourse does not include those questions of a teacher presenting a task to students for mathematical exploration. Because of this study's focus on improving pedagogy and the resulting categorization of types of questions that teachers use, it makes sense that Boaler and Brodie's framework lacks the types of questions where students are asked to explore the mathematics. Further, no frameworks have identified these types of questions.

Shift of focus needed. The focus of past frameworks has been on changing and improving teachers' practice in the nature of the Discourse in the classroom. No one has tried to identify or categorize the mathematical questions that teachers ask that could be used by students as part of their own mathematical questioning Discourse. The focus needs to be on teacher mathematical questioning Discourse through the lens of student appropriation, or what questions are available in the teacher questioning Discourse for students to adopt. With this focus, a framework could both inform teachers on how to improve their practice as well as inform teachers what questions they could model that students could appropriate. Because the focus in the past has always been on improving pedagogy for the teachers, there is an obvious category missing from existing frameworks—that of exploration in the mathematics. Recall that no categorization of teacher questioning Discourse includes those questions of a teacher presenting a task to students for mathematical exploration because of the focus on those types of questions to improve pedagogy. But if there is one entire category or type of question that is missing from the previous frameworks, what other types of mathematical questions are missing from these frameworks? The fact that no framework includes a type of question specifically about the exploration in the mathematics suggests the possibility that there are key question types that are missing from the existing frameworks.

In summary, there is a need for further research on the types of mathematics questions a teacher with a conceptually-oriented Discourse uses. Many have studied teacher questioning Discourse, but the perspective used to examine teachers' questions has been pedagogical, basing the categorization on what type of learning the questions might invoke or reveal. No studies have examined teachers' questions with the specific focus of understanding the types of mathematical questions that teachers ask. While Thompson et al. (1994) provided a list of questions that a teacher with a conceptually-oriented Discourse might ask, this list needs to be expanded as well as categorized to better understand the types of mathematical questions being modeled by the teacher. And though Boaler and Brodie (2004) presented a more complete categorization of questions used by a conceptually-oriented teacher, their categorization is also incomplete in describing the mathematical questions of the teacher. So my research question is as follows: "What are the types of questions that comprise the mathematical questioning Discourse of a teacher in a conceptually-oriented classroom?"

CHAPTER 3: METHODOLOGY

This chapter outlines the methods of data collection and analysis for this study. I describe the setting and participants of the study, the types of data collected and how it was collected, how the data was managed and analyzed, and how the results emerged from the data.

Setting

The setting for this study is the course Concepts of Mathematics, a mathematics course for preservice elementary teachers at Brigham Young University. The course met 2 days a week for 2 hours per session for 15 weeks. This course is required in the elementary education sequence, and it is typically taken during the sophomore or junior year. Students enrolled in this course are expected to have taken a college algebra course, or equivalent, as a prerequisite. This course is focused on the conceptual understanding of fractions, probability and statistics, and early algebra. One particular section of this mathematics course was the setting for this study.

The study was performed on data collected during the fall 2011 semester. A series of 15 two-hour-long class periods were studied to understand better the types of questions that comprise the mathematical questioning Discourse of a teacher in a conceptually-oriented classroom. The first 8 class periods were the class's first unit, which was on fractions, and the other 7 class periods were the class's third unit, which was on probability and statistics. I wanted to examine the first unit since I anticipated that the teacher would focus on modeling skilled mathematical questioning Discourse right at the beginning of the semester as it was the first time that students would have had her, specifically, as a teacher and a model for the Discourse of that particular classroom. I wanted to look at complete units—the entire fraction unit and the entire probability and statistics unit—so that I could examine the full range of questions that are asked at different points in a unit. I also wanted to examine class sessions from at least two separate units to ensure that I looked at mathematical questioning Discourse spanning a range of mathematical topics, since the topic may or may not affect the questioning Discourse of the teacher. Thus, I studied all the class sessions from the first unit and the third unit—units focusing on very different mathematical content. I felt that 15 two-hour class periods from these two units were likely enough to get a sense of the types of questions that comprise the teacher's mathematical questioning Discourse.

Participant

The participant for this study will be referred to as Carla, the teacher of the Concepts of Mathematics course. Carla is a university professor with a bachelor's, master's, and PhD in mathematics education. Her classroom was the setting for the study because of her emphasis on conceptually-oriented mathematics and mathematical Discourse. We know that Carla has a conceptually-oriented classroom because of her task-based curriculum that focused on students developing a rich understanding of the important mathematical concepts, and the situations, ideas, and relationships among the mathematics related to the topics of study. For example, in the first unit, Carla's students developed two different meanings for fractions based on iterating and partitioning; connected these meanings to fractions as quotients, ratios, and decimals; and learned why the algorithm for simplifying fractions works.

Data Collection

All classes were videotaped, which included both whole class discussions and small group discussions. The teacher wore a wireless microphone that captured everything the teacher said, and field notes were taken. All video was transcribed and field notes were typed on a computer as they were being generated. The class discussions were especially useful to be able to analyze the teacher questioning Discourse that took place on a classroom level. The small group discussions took place at the students' 6-person tables. The teacher walked around the classroom during the small group work and also participated in these small group discussions. Data was also collected in the form of classroom work. The teacher often passed out worksheets or tasks for the students to work on. These worksheets and tasks comprise a written form of teacher questioning Discourse that was also important to study.

Data Analysis

I first looked at the transcript from the second lesson to begin my data analysis. I wanted to first look at only one transcript of data in order to begin my analysis and establish my coding scheme before moving on to the rest of the data. I hypothesized that the teacher may have spent more time modeling skillful questioning at the beginning of the semester, so I wanted to analyze one of the first days of class. I chose to first analyze the second class session of the first unit to ensure that I did not miss too many types of questions because of any time the teacher may have spent during the first day's class session establishing classroom norms and discussing the syllabus.

My unit of analysis was a question that the teacher asked as well as the necessary surround to understand what the question was about or what it was asking the students to do. For example, if the teacher asked, "Why?" enough surround discourse was also needed to be examined to understand "Why what?" Or, if the teacher asked, "What do you think?" enough surround was needed to understand what topic or issue she was asking about. I also included in my analysis questions that were embedded in sentence. That is, I included sentences that were not in the form of a question, but that had a question embedded within. For example, I included such sentences as, "Talk with your tables about why that answer makes sense." The phrase, "why that answer makes sense" could have been said in the form, "Why does that answer make sense?" The teacher still was asking students to participate in mathematical activity but the request was embedded in a sentence. Thus, I went through the transcript from the second lesson and extracted all the questions and embedded questions as well as the needed surround to understand what the question was about or asking students to do.

The questions and surround from the first analyzed transcript were then coded and analyzed using a method of external and internal codes (Knuth 2002). External codes are codes used and adapted from the literature; internal codes are codes I created when I found that none of the external codes directly matched with the type of question I was encountering. I used a number of external codes from researchers such as Sahin and Kulm (2008), Boaler and Brodie (2004), Moyer and Milewicz (2002), Hiebert and Wearne (1993), and Stein and Smith (2011). For example, I used the external code of "Factual" question from Sahin and Kulm (2008) and utilized their same description to designate Factual questions from other mathematical questions. However, since I was interested in discovering what types of questions comprise the mathematical questioning Discourse of a teacher in a conceptually-oriented classroom, and most or all of these researchers were not specifically focused on a teacher with conceptually-oriented Discourse, I needed to also create additional internal codes or categories of question types during data analysis. The literature has only focused on teacher's questioning Discourse to inform teachers how to choose or use skillful questioning in their own teaching, but no one has created categories to analyze and categorize the types of questions that teachers use that could be adopted by students in their own mathematical questioning Discourse and conversations with others. Since no one has categorized teacher questioning Discourse in this way, I created additional internal codes to add to the external categories already developed by researchers. Questions that did not involve mathematics were grouped into a single category termed pedagogical questions, and were not counted as part of the mathematical teacher questioning Discourse. After coding the first transcript, I attempted to create a categorization and form as many categories of types of questions as I felt were needed to describe Carla's mathematical questioning Discourse. From there, I began to move on to code data from the next two chosen transcripts, the fourth class session (the middle of the first unit), and the eighth class session (the review day from unit one).

As I coded, I used a coding program called TAMS Transcript Analyzer. This program allowed me to code a question by the click of a button as well as color code each code in an easy-to-use manner for organization. Also, when I searched for a code, I could control how much of the surround I saw, a feature very helpful in determining the nature or purpose of each question. This program was very helpful in the coding and analysis of my data.

At the end of coding the three transcripts, I still did not feel comfortable with my coding scheme. Upon further reflection, I noticed that the existing category schemes for mathematical questions in the field were focused on the form of the answer to the question, i.e., what product was being requested by the question. For example, the product could be a result of a computation, an explanation of reasoning, or an expression of opinion. Most of these categorizations had the purpose and emphasis of helping students to think more deeply—which supports the researchers' use of and focus on the form of the answer. And if the category schemes addressed function at all, it was the pedagogical function on which they focused (Boaler & Brodie, 2004; Hiebert & Wearne, 1993; Moyer & Milewicz, 2002; Sahin & Kulm, 2008; Stein and Smith, 2011). By *pedagogical function*, I mean the purpose of the question that relates to the instructional goals or ideals of a teacher. Because these category schemes were developed for teachers' use, it is not surprising that their categorization was based on the pedagogical function of questions.

However, my purpose was different. I wanted to know, primarily for the students' sake, what questions comprise the mathematical questioning Discourse of a teacher with a conceptually-oriented Discourse in order to help students know what types of questions they should ask themselves in mathematical practice. My focus was to study teacher questioning Discourse through the lens of student appropriation. Thus, I realized that a focus on pedagogical purpose, or one that relates to the instructional goals or ideals of the teacher, does not answer or address my purpose as clearly since a pedagogical function is unique to the interest of a teacher. A student would rarely have a purpose that would be the same as a teacher in asking a question. Knowing the form of the answer to the question similarly does not provide much insight into the types of questions that students should ask in order to become proficient in the Discourse of questioning, especially considering that students might not be able to identify what form of the answer they are looking for before deciding what type of question to ask. If students always knew the form of the answer to the question, they might not be in need of a framework to guide their mathematical questioning Discourse. So using pedagogical functions and forms of the answer to categorize question types did not fit well with my research question. I had not anticipated this problem before my analysis, however, because there was no framework for categorization of questions types that I could find that sorted by anything other than form of the answer or pedagogical function.

A second problem I encountered as I was coding was that I often struggled to identify which category of mathematical questions each question should pertain to. I had difficulty matching the questions with both the form of the answer as well as the pedagogical purpose outlined in the description of the question type. For example, Factual questions "check students' recall of specific mathematical facts or procedures *in order to assess basic information before moving on*" (Sahin & Kulm, 2008). If one only looks at questions that check student's recall of facts, that could be done in a probing way, or in a way that extend students' knowledge, or in a way to check student understanding, or in a way to bring something to light or make public an (previous) idea. Some of these purposes, though, do not seem to fall well under the Factual category. For example, in one instance, the teacher asked the question, "What is 1/5 of that bar?" This question seemed to be one that checked the students' recall of specific mathematical facts or procedures, the form of the answer for a Factual question. But after asking the question, the conversation turned to an extended discussion about what 1/5 of the bar was. Thus, the question did not seem to have the pedagogical function of assessing basic information before moving on, which is the function associated with the external code of Factual question (Sahin & Kulm, 2008), so I felt I could not identify it as a Factual question. Also, the question, "So are there any questions about what we talked about last time?" assessed basic information before moving on, but did not check students' recall of specific mathematical facts. So I struggled to match up each question with the description, particularly descriptions that included a pedagogical function that did not always seem to correlate with the form of the answer of the question.

As I reflected on how I dealt with questions that did not seem to fit well with my external and internal codes, I realized that I often based my final decision on what I assumed to be the mathematical function of each question. I defined *mathematical function* as the purpose of the mathematical activity that one is being asked to engage in. An example of how I considered mathematical function is evident in the previous question, "What is 1/5 of that bar?" I considered what the question was asking of the students. That is, I considered what part of mathematical practice students were being asked to engage in. The question from the example above was used at the beginning of a task and the teacher seemed to be asking this question to get students to establish relevant information to use in the task. This led me to categorize it as a Factual question, despite the extended conversation that followed. I similarly resorted to attempting to identify the mathematical function of many of the questions to help me decide in which category each question should fit. While mathematical function seemed to help somewhat in making coding decisions, the decisions nonetheless were not entirely satisfactory.

Because of these difficulties with the first coding scheme, I decided to create a new coding scheme that sorted first on the mathematical function of the mathematical questions. By attending to mathematical function of questions, I could solve my first problem of creating a more useful categorization to answer my research question. Students can more easily identify what part of the mathematical practice they want to engage in and match that with the type of questions that they can and should ask themselves. So then teachers can utilize this categorization based on mathematical function in order to inform them how to help students become proficient in the mathematical Discourse of questioning. Focusing first on mathematical function aids in the study of teacher questioning Discourse by more readily informing teachers of the types of questions that engage students in mathematical activity so the teachers can explicitly teach students to recognize and utilize certain types of questions based on the type or purpose of the mathematical activity being engaged in. By focusing on mathematical function, I also solved my second problem by more easily categorizing each question because I could more easily identify what part of mathematical practice students were being asked to engage in. Thus, a shift to categorization based on mathematical function solved both my problems that I had encountered when I focused on form and pedagogical function.

As I created new codes for the types of questions used by a teacher in a conceptually-oriented classroom, I tried to identify the mathematical function of each question to define the categories for the questions. I could not rely on any external codes (Knuth, 2002) because none existed with the criteria of a focus on the mathematical function of the question. I thus performed a thought experiment, partially thinking of the questions I had seen in the transcripts I had coded, and partially looking at my previous list of codes and associated example questions based on the pedagogical function of questions to create an initial new list of codes that were based on the mathematical function of the question. I used the example questions that I had associated with each pedagogical function or form of the answer of the question to give me ideas for what the mathematical function of those types of questions might be. My list was incomplete, however, which became apparent especially as I began to use the list to recode the first few transcripts of data. When I encountered questions for which I could not find an appropriate category, I created new categories.

Some of the questions that the teacher asked, however, did not seem to have a mathematical function. When there was a question that had no apparent mathematical function, I categorized it as a pedagogical question and disregarded it in my analysis of the teacher's mathematical questioning Discourse. I worried, though, that I might be missing the mathematical function or purpose of a question because I did not have access to what the teacher was thinking. To compensate for the lack of access to the teacher's thinking, I concluded that there was no mathematical purpose only if the question did not result in student mathematical activity and if the teacher did not try to renegotiate the meaning of the question toward mathematics.

In order to determine the mathematical function of a question, I utilized the surrounding dialogue. The surround was critical to sorting through the questions and coding them. I would first make a hypothesis of the code of the question based on the question itself—the form of the question. Then I would use the surrounding discourse to test if my hypothesis seemed reasonable. I would look to see if the hypothesized mathematical function fit with the student responses and search for evidence that the teacher was satisfied with the students' interpretation of the mathematical function of the question. If the teacher seemed to attempt to renegotiate the students' interpretation of the mathematical function of the question, I looked for the evidence in the subsequent dialogue for when both the students and the teacher seemed to no longer contest the mathematical function of the question, and then coded the original question with this uncontested mathematical function. It is true that without actually accessing the teacher's thinking about each question she asked, I cannot be sure about the teacher's intended mathematical function of her questions. Nevertheless, the results I obtained from my method of coding the mathematical function were consistent with the question and surrounding discourse and seemed to have explanatory power in categorizing the questions that comprised Carla's mathematical questioning Discourse.

When I had created a list of types of questions without any need to create further categories as I continued to code, I realized that in each category, there seemed to be important differences between questions. So I sought to identify possible subcodes, or ways to further divide the questions in each category into subcategories. I first thought of creating subcategories based on the pedagogical function since many other researchers had also focused on this aspect of mathematical questions. In identifying the pedagogical function component of the categories of question types, I considered the idea of, "What is she (the teacher) trying to accomplish with this question?" Was she generating discussion, making the problem easier, setting up background information, getting the students to conjecture a possible solution, or asking students to justify their previous answer? In identifying what part of the mathematical process students were being asked to engage in, I would try to identify what purpose that would serve for her, as the teacher. As I considered each mathematical function and looked at the questions that had been coded with a particular mathematical function, I identified that the vast majority of the questions with a certain mathematical function seemed to share a common pedagogical function – suggesting to me that turning to pedagogical function as a secondary filter to code the mathematical questions was not useful.

Next, I turned to researchers' other focus—the form of the answer of questions—to create subcategories to my categories of mathematical function. As I considered the questions related to each mathematical function, it seemed to be a logical pathway to next break each category into subcategories based on the form of the answer to the question. I would ask questions such as, "What product results from the mathematical process the students are being asked to engage in?" This led to subcategories for many of the categories that existed in my coding scheme. For example, for the Accessing Relevant Information category of question, I created a subcategory for each type of relevant information that could be accessed. A few subcategories, for instance, are Accessing a Past Idea (bringing up an old idea), Accessing a Numeric or Computational Result (identifying or performing a computation), or Accessing Context (talking about issues outside of mathematics that can/will be used in the mathematics problem). Each of these subcategories for Accessing Relevant Information has the same mathematical and pedagogical functions; they simply have a different form based on the result of the question, or in this case, the type of information that is being accessed.

Upon further reflection, however, some of my categories of questions seemed to have only one possible form of an answer, but also seemed to have important differences between questions within the category. For example, for the Analyzing the Explanation category of questions, each answer was in the form of an analysis, normally a “yes” or “no” accompanied sometimes with a short justification or explanation of the analysis. There seemed to be a logical pathway, however, to further separate these questions within the Analyzing the Explanation category into subcategories based on what type of mathematical object the teacher was asking the students to engage with. For instance, what the teacher asked the students to analyze became the criteria to create subcategories within the Analyzing the Explanation category. Thus, the subcategories were based on the form of the answer, or if the form of the answer was all the same for a category, subcategories were created based on the type of mathematical object the teacher was asking the students to engage with.

Finally, I examined my categories and subcategories to make families of codes. From my coding, I had a number of categories based on a common mathematical function with different subcategories for the different forms of the answer to the question. I realized that some of the questions had mathematical functions that were related or very similar to the mathematical functions of other categories of questions. Where this was the case, I was able to combine these categories and create one larger category and the added type of question as a subcategory of a different form of an answer. After doing any possible refining and combining of my categories of questions into families, I had 5 different families of codes, or 5 different categories of mathematical functions for questions. I then proceeded to code the rest of my data. Lastly, I counted the number of times each question type appeared. I present these results next.

CHAPTER 4: RESULTS

In this chapter I describe the 5 different question categories and their accompanying mathematical functions that comprise the mathematical questioning Discourse of a teacher with a conceptually-oriented Discourse as defined in this study. The first 3 categories I discuss are presented in the order in which the questions from that category typically appeared in the lesson. Questions from the last two categories I present were often spread throughout the lesson and occurred with relatively the same frequency. Thus, these last two categories are discussed in the alphabetical order of their category titles.

In order to describe each category, I include the following information for each of the 5 main categories: first, I describe the mathematical function, or what mathematical processes the students were being asked to engage in as a result of the question. Second, I describe the primary pedagogical function of that question type. Sometimes the pedagogical function was very similar to the mathematical function, and other times, the two functions differed greatly. Recall that my analysis revealed that the vast majority of questions in each category shared a common pedagogical function. Thus, there is only one pedagogical function listed for each category. Third, I name and describe the subcategories for each main category of questions. These main categories were further divided into 21 subcategories. Recall that these subcategories were based on the form of the answer to the question or what type of mathematical object students were being asked to engage with. I will present each subcategory according to frequency—the first subcategory presented being the most commonly used in the data. Fourth, I provide example questions from the data for each subcategory to provide guidance and clarification of what a question of the prescribed type would look like. Lastly, I present a table that summarizes all of this information for each category.

Accessing Relevant Information Category

The first category of types of questions that comprise the mathematical questioning Discourse of a teacher in a conceptually-oriented classroom is Accessing Relevant Information. These questions occurred most frequently during the Launch phase, or the introductory phase of a task. The mathematical function of these types of questions is to bring to light, gather, privilege, or sanction relevant information to use in the problem solving process. This is where the teacher can bring to light knowledge and ideas that students can or should use in the problem solving process or even sanction knowledge as knowledge able to be used without having to explain why it is correct. An example question of this type is, “What did we talk about yesterday?” The pedagogical function of these questions is to set the stage for the students to be successful in the problem solving process. Responses to these questions were often short and required no new work, analysis, or evaluation. The subcategories were based on the form of the answer to the question, and are Accessing a Past Class Experience or Past Answer, Accessing a Numeric or Computational Result, Accessing a Past Idea, Accessing Context, and Accessing a Past Procedure. These all are questions that bring to light information to use in the problem solving process, an important first step in problem solving (Polya, 1945) and helpful in conceptually-oriented instruction.

The first subcategory, the one that appeared most frequently in the teachers’ Discourse, is Accessing a Past Class Experience or Past Answer. These questions asked students to recall and share past experiences from class activities or answers that had not been reflected on yet. The students took a short amount of time to answer these questions as they were simply reciting or voicing an experience from earlier in that class session or another class session. An example of this type of question is when the teacher was asking students to recall their experience from class of rolling two dice and said, “Did anybody roll a 7?” Also, students were asked their experience from a class activity through the question, “Who found that it was hard to roll a 2?” Students were able to answer quickly with their experiences from class activities. Another group of questions included in this subcategory are those when students were asked to recall or voice an answer they had already worked out at a previous time in the class. A question of this sort is something like, “What did you get for number 3?” Questions from this subcategory did not require any further work, analysis, or computation by students, but asked merely for students to voice an experience or answer they have already had or worked for in order to contribute to the discussion or exploration of a new task.

The second most frequently asked questions comprise the subcategory of Accessing a Numeric or Computational Result. This type of question occurred when the teacher asked students to identify the result of or perform a simple computation. For example, when the teacher asked “What is 2/3 of the group?” she asked them to perform the computation of 2/3 of the group in the given situation. This computation was assumed to be somewhat trivial and required a short amount of time for the students to compute. These questions could take a bit longer for students to respond to because of the need to perform a computation, but since these questions required a computation that was not new or difficult for students, these questions did not require extensive effort or thought on the part of the students and still took a relatively short time for students to answer. These questions were also asked before the main task in the lesson was given. By asking this question, the teacher seemed to be gathering and privileging needed information for the problem at hand to facilitate student problem solving. A second example of this type of question is when the teacher asked, “What is 50% of that number?” The teacher asked this question when she was preparing students to begin to explore a probability task. The question privileged information to be used in the task as the quantity of 50% of the number of students in the task was relevant. This mathematical computation took so little effort because of its simplicity that it could be the case that many students had the answer to this computation memorized. Though the answer may be derived from recalling a memorized computational fact, the question asked that students perform a numerical computation.

The third most frequently asked questions form the subcategory of Accessing a Past Idea. These were the questions asked by the teacher when she asked the students to recall and/or recite mathematical concepts or facts from a previous day or time in class. Examples of these types of questions are, “What did we talk about yesterday?”, “What were the elements to a good explanation that we talked about last time?”, and “How did we define [that idea] before?” These questions sanctioned previous knowledge as taken as shared and able to be used in the current situation. The questions also privileged information as they brought students’ attention to particular ideas or knowledge that could be used in the current problem solving situation.

The fourth subcategory is Accessing Context. This subcategory of questions includes those questions when the teacher asked students to talk about real world contexts, personal experiences, or knowledge outside of mathematics that can or would be used in the mathematics problem. An example from this subcategory is, “How many of you have played Settlers of Cataan?” In this instance, the teacher asked this question about a game because she planned to use the strategy and rules to help make sense of the mathematics problem at hand. Another example of this type of question was when the teacher was preparing to use certain information for a probability task and asked the students, “How many brothers and sisters do you all have?” After collecting this data, the teacher asked the students to find the mean of the collected data and then the probability of a student having a particular number of siblings.

The fifth subcategory is Accessing a Past Procedure. Of the 21 subcategories identified in this study, this type of question was asked the least. These questions occurred when the teacher asked students to identify a process that was already known to them in order to use that procedure in a subsequent problem, and did not require students to prove or explain the process. For example, the teacher asked, “What do you do to find out what it’s equal to?” in reference to an equation with one unknown variable. She asked students the process to solve for the unknown variable, but it was a process that the students knew well and did not need to learn, prove, or even explain. The teacher also asked, “How do you do long division?” This question was asked about a process, long division, with which the students already were familiar and most students responded easily with the intended procedure. The teacher then asked students to use their experience with the long division algorithm to make sense of fractional remainders. Students did not need to discover nor explain this procedure nor prove why it works; they simply used it in a subsequent problem. This type of question sanctions and privileges information by asking students to recall a previously learned or proved procedure. This and the other subcategories of the Accessing Relevant Information category can be found in Figure 2.

Category: Accessing Relevant Information
Mathematical Function: Bringing to light, gathering, privileging, or sanctioning relevant information to use in the problem solving process
Pedagogical Function: Set the stage for students to be successful in the problem solving process

Subcategories

Form of the Answer [Code]	Description	Example Questions
Accessing a Past Class Experience/ Past Answer [AccessExperience]	Recalling and sharing past class experiences or answers that have not been reflected on yet	"Did anyone roll a 7?" "What did you get for number 3?" "Who found that it was hard to roll a 2?"
Accessing a Numeric or Computational Result [AccessAnswer]	Identifying or performing a computation	"What is 2/3 of the group?" "What is 50% of that number?"
Accessing a Past Idea [AccessIdea]	Bringing up old ideas from class	"What did we talk about yesterday?" "How did we define [that idea] before?" "What were the elements to a good explanation that we talked about last time?"
Accessing Context [AccessContext]	Talking about real world contexts, personal experiences, or knowledge outside of mathematics that can/will be used in the mathematics problem	"How many of you have played Settlers of Cataan?" "How many brothers and sisters do you all have?" "How do meteorologists decide the chance of rain?"
Accessing a Past Procedure [AccessProcedure]	Identifying a known process	"How do you do long division?" "What do you do to find what it's equal to?"

Figure 2. Accessing Relevant Information Category Description.

Exploring the Mathematics Category

The second category is Exploring the Mathematics. The mathematical function of this category of questions is to engage students in investigating or explaining something beyond what they already know to create new understandings of processes or justifications, or to create modifications of existing processes or justifications. These questions generally occurred after those pertaining to the Accessing Relevant Information category, and at the start of the students' exploration of a task or problem. These questions were often followed by extended conversation in groups or silent work by the students as they developed new insights. An example question from this category is, “Why does that get you the right answer?” This question fits this category because it asked students to create a justification for the functionality of a method or process, an idea they had not previously considered or justified, as seen in the extensive time students took to consider and formulate their responses to the question as well as the nature of the responses. Time is one factor used to determine whether a question is coded as part of this category. The code of Exploring the Mathematics only applies when students are given time to consider the response and are not expected to answer right away. Note that the majority of these questions led to a long exploration, normally in groups, where students took 15 or more minutes to explore the answer. The pedagogical function of this category of questions is similar—it is pressing students to conjecture or explore something mathematically new to them to get them to progress to a higher level of mathematical thought or understanding. The subcategories, or forms of the answer to these questions, are Exploring to Construct a Justification, Exploring to Construct a Process, Exploring to Modify a Process, and Exploring to Modify a Justification. These are questions that often launch students on a mathematical task (Stein & Smith, 2011), a common part of conceptually-oriented instruction.

The first subcategory, the one that appeared most frequently in the teachers’ Discourse, is Exploring to Construct a Justification. These questions engaged students in creating or understanding a justification for or explanation of a mathematical concept beyond what they had previously done or known. An example question from this subcategory is when the teacher presented two fractional quantities and stated that they were equivalent and asked, “How can we reconcile the idea that [this fraction] has to be equal to [that fraction]?” This question asked students to create a justification for why the two quantities were equivalent—a concept that students had not previously tried to justify. Another example of this type of question is when the teacher discussed with students a particular procedure and asked students, "Why does that get you the right answer?" Students may have used the procedure before, but they were asked to create or understand the justification for why the procedure worked.

The second most frequently asked questions comprised the subcategory of Exploring to Construct a Process. These questions engaged students in constructing or understanding a mathematical process. An example question of this type is, "How can you solve this?" This question was in reference to finding a way to choose a representative sample of 60 crates of eggs to see how many were cracked. Constructing a representative sample was a process that the students were unfamiliar with or beyond what they had considered previously and a process that many or most students constructed for the first time. A second question that fits in this subcategory is when the teacher asked, "How can you determine if you're right?" The teacher asked students to consider their answer to a problem with finding equivalent fractions and asked students to think deeper and discover a way to check the answer they had already found. Another example question from the data was, "Can you think of a different way to do this problem?" This question engaged students in creating a different strategy for solving a particular problem on which they had already worked.

The third subcategory is Exploring to Modify a Process. These questions engaged students in modifying an existing process so that it could be used in a different mathematical situation. An example of this type of question is when the teacher was asking students to consider two fractional quantities and asked, "Would this work with other numbers? Try doing the same thing." This subcategory differs from the preceding subcategory of Exploring to

Construct a Process because the teacher asks students to engage in taking a process that has been explored or created and modifying it to use in a different situation. Another example question from this subcategory is when the students finished constructing a process to solve a particular problem and the teacher asked, “What about in this problem?” The teacher asked students to take the procedure they had explored and constructed in one problem and apply it to a similar problem.

The fourth and final subcategory is Exploring to Modify a Justification. These questions engaged students in modifying an existing justification to explain or justify a different mathematical situation. An example of this type of question is, “Will that always give me something that works?” This question was asked by the teacher after the students justified why they could use a picture to simplify or reduce their specific fraction. She asked them to modify their existing justification to explain or justify the idea of simplifying any fraction. Another example of a question from this subcategory is when the teacher asked about two related or equivalent fractions in general terms and asked, “How can you explain that [this] is always equal to [that]?” Students were asked to take their justification for a specific example and apply it to a justification for all fractions of that type. This and the other subcategories of the Exploring the Mathematics category can be found in Figure 3.

Category: Exploring the Mathematics
Mathematical Function: Engaging students in investigating or explaining something beyond what they already know to create new understanding of processes, justifications, or modifications of existing processes or justifications
Pedagogical Function: Pressing students to think beyond what they have previously done or thought to get students to progress to a higher level of mathematical thought or understanding

Subcategories

Form of the Answer [Code]	Description	Example Questions
Exploring to Construct a Justification [ExploreJustify]	Engaging students in creating or understanding a justification for or explanation of a mathematical concept	"How can we reconcile the idea that [this] has to be equal to [that]?" "Why does that get you the right answer?"
Exploring to Construct a Process [ExploreProcess] [ExploreStrategy]	Engaging students in constructing or understanding a mathematical process	"How can you determine if you're right?" "Can you think of a different way to do this problem?" "How can you solve this?"
Exploring to Modify a Process [ExtendProcess]	Engaging students in modifying an existing process so it can be used in a different mathematical situation	"Would this work with [other numbers]?" "What about in this example? Try doing the same thing"
Exploring to Modify a Justification [ExtendJustify]	Engaging students in modifying an existing justification to explain or justify a different mathematical situation	"Will that always give me something that works?" "How can you explain that [this] is always equal to [that]?"

Figure 3. Exploring the Mathematics Category Description.

Explaining One's Thinking Category

The third category is Explaining One's Thinking. The mathematical function of this category is to ask students for articulation, elaboration, or clarification of concepts or strategies to get ideas out to the group so they can be verified or discussed. Questions of this type ask students to share or clarify the insights they have already gained. These questions most often occur after students have engaged in a task or solved a particular problem and occur after the first two categories of questions have been asked in the lesson. Students take almost no time before a response to this type of question as they do not need to do any further work or thinking, but only articulate thinking or strategies they have already considered or worked with. An example question of this type is, “How did you solve that problem?” The pedagogical function is to ask students to articulate, elaborate, or clarify ideas or strategies to determine their level of understanding, cause reflection by the class on students’ reasoning, or bring relevant ideas to the discussion so that students can learn from one another. The two subcategories, or forms of the answer to these questions, are Explaining One’s Own Thinking, and Explaining Other’s Thinking. This is an important category of questions to help the teacher, other students, and the student, himself, be aware of his own reasoning. It differs from the previous category because it involves students in explanations of the processes they have already constructed, the thinking they have just engaged in, or the thinking of others that they have just heard. It does not require additional work or exploration by the students to respond. These questions aid in classroom discussion and student understanding and are an important part of conceptually-oriented instruction (Thompson et al., 1994).

The first subcategory, the one that appeared most frequently in the teachers’ Discourse, is Explaining One’s Own Thinking. These questions asked students to articulate, elaborate, or clarify their own process, solution method, or explanation of reasoning. An example of this type of question is when the teacher asked a student in reference to a solution, “How did you know to divide the fraction into 16 pieces?” This question falls into this category because the student was expected to answer immediately and share her reasoning for a process she had already performed. Also, when the teacher asked a student in a class discussion, “How did you solve that problem?” The teacher asked the student to articulate her process or solution method to a problem she had already solved, and the student responded immediately. This type of question is often seen in written assignments given to students when they are asked to answer a question and then requested to "Explain [their] reasoning."

The second subcategory is Explaining Other's Thinking. These questions ask students to articulate, elaborate, or clarify a mathematical concept or another person's thinking that is not their own. An example question is, "Can you explain what she just said?" Another example is when the teacher would ask the class after one student's explanation, "Who can put what she just said in your own words?" This question asked students to explain an idea from another student, but did not take extra time for students to explore or investigate before responding. These types of questions also occur when the teacher asks students a question about a mathematical concept that they do not need or take time to explore or investigate. An example of this was when the teacher asked, "Can you explain, using a partitioning perspective, why the answer is 3/4?" This question asked students to explain the answer using a particular method or perspective that was not the students' own. The student had done problems like this before, however, and was able to respond immediately, thus making this question an Explain Others' Thinking type of question. This and the other subcategory of the Explaining One's Thinking category can be found in Figure 4.

Category: Explaining One's Thinking
Mathematical Function: Asking for articulation, elaboration, or clarification of concepts or strategies to get ideas out to the group so they can be verified or discussed
Pedagogical Function: Asking students to articulate, elaborate, or clarify ideas* or strategies to determine their level of understanding, cause reflection by the class on students' reasoning, or bring relevant ideas to the discussion

Subcategories

Form of the Answer [Code]	Description	Example Questions
Explaining One's Own Thinking [ExplainReasoning]	Asking for articulation, elaboration, or clarification of one's own process, solution method, or explanation of reasoning	"How did you get 1/5?" "How did you know to [process]?" "What did you do to solve that?" "Explain your reasoning."
Explaining Other's Thinking [ExplainConcept]	Asking for articulation, elaboration, or clarification of a mathematical concept or another person's thinking that is not their own	"Can you explain what she just said?" "Who can put what she just said in your own words?" "Can you explain, using [a specific method] why the answer is [the answer]?"

Figure 4. Explaining One's Thinking Category Description. *Adapted from "The importance, nature and impact of teacher questions," by Boaler, J., & Brodie, K., 2004, Paper presented at the Proceedings of the 26th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education, p.776. Copyright 2004.

Analyzing Explanations Category

The fourth category is Analyzing Explanations. The mathematical function of this category is to ask students to analyze and form an opinion about the correctness or sufficiency of a statement or explanation or to conjecture what another person is thinking. An example of this type of question is, "What did you like about that explanation?" This question asked students to analyze and express their opinion about the explanation of another person. These questions most often occurred after the exploration of a task or a problem in the class and after different students had presented their various solutions or ideas. Questions in this category do not require students to investigate a mathematical concept deeply beyond which they have done in the past, but analyze and express an opinion of a statement or explanation. Note that in contrast to Exploring

Mathematics questions, students responded to questions in this category almost immediately in a class discussion, or worked for at most 1-2 minutes in groups before responding. The pedagogical function of this category is to get students to participate in the mathematical conversations of the classroom and participate in mathematical argumentation. Also, this category of questions asked students to engage in the mathematical activity of analysis and did not require an explanation for their analysis, although students sometimes included a brief explanation as part of their response. Each response to the Analyzing Explanations category had the same form of an answer: an analysis of normally a "yes" or "no" accompanied sometimes with a short justification or explanation of the analysis. Because the form of answer to these questions did not differ, the subcategories are based on the type of mathematical object the teacher was asking the students to analyze. The subcategories are as follows: Analyzing the Validity, Analyzing the Sufficiency of an Explanation, Analyzing Personal Understanding, and Analyzing to Form Conjectures about Others' Thinking. This is an important category because it involves students in conversations about the mathematics and promotes a rich conception of the ideas (Thompson et al., 1994).

The first subcategory, the one that appeared most frequently in the teachers' Discourse, is the Analyzing the Validity subcategory. These questions asked students to express or form an opinion of the validity of an argument or an idea. An example of this type of question is when the teacher asked the class, "Is that right?" in reference to a students' explanation of why two fractions were equivalent. The class responded in the affirmative and a couple of students raised their hands to explain or comment further on their analysis on the validity of the explanation. Another example question is when the teacher asked, "Who agrees with [that student's] reasoning?" The students were able to express an analysis of another student's explanation by expressing an opinion of agreement or disagreement with the particular explanation. This category of questions included, as well, statements that asserted an opinion, followed, by the question, “Right?” often a pause, and often a student response to the short question. An example of this is when the teacher said, “So those fractions are the same, then, right?” The teacher then paused and students responded in the affirmative to this particular question.

The second subcategory is Analyzing the Sufficiency of an Explanation. These questions asked students to express or form an opinion about the clarity and completeness of an explanation rather than its correctness. An example question is when the teacher asked students, “Is that a clear explanation?” Students responded “yes” or “no” and a few offered some explanation as to why they held the opinion that they did. Another example is “Is there anything missing from that explanation?” Sometimes questions in this subcategory required more than a simple yes or no answer. For example, the question, “What did you like about that explanation?” elicited brief, multi-word responses from the students. Yet students were still analyzing the sufficiency of the explanation as they responded to the question.

The third subcategory is Analyzing Personal Understanding. The Analyzing Personal Understanding questions asked for students to assess their understanding of an explanation of a particular concept, process, justification, or representation. Sometimes these questions would be short in length and in response, such as, “Do you understand?” or “Ok?” Other instances of these questions similarly asked for an analysis of one’s personal understanding of a concept, but were longer questions. An example of a longer question of this type is, “After hearing those explanations and going over the classwork, do you feel ok to move on to something else?” This question similarly asked students to assess their own understanding of a concept.

The fourth and final subcategory is Analyzing to Form Conjectures about Others' Thinking. These questions asked students to conjecture or speculate what others' thought processes or explanations were or would be. This occurred when the teacher presented a student answer or explanation after which she would ask, "What do you think that child was thinking?" Another example is when the teacher asked, "How would you expect [this student] to solve each task?" Each of these questions asked students to consider what they knew about a particular person and that person's understanding, and analyze and conjecture what that person might do or think about a particular problem. These questions generally require a bit longer of a response from students, but request an analysis of another's understanding or thinking. This and the other subcategories of the Analyzing Explanations category can be found in Figure 5.

Category: Analyzing Explanations
Mathematical Function: Asking students to analyze and form an opinion about the correctness or sufficiency of a statement or explanation or to conjecture what another person is thinking
Pedagogical Function: Getting students to participate in the mathematical conversations of the classroom and participate in mathematical argumentation

Subcategories

Type of Mathematical Object [Code]	Description	Example Questions
Analyzing the Validity [AnalyzeValidity]	Asking students to express or form an opinion of the validity of an argument or idea	"Who agrees with [that student's] reasoning?" "Is that right?" "Do you think it's possible?" "Right?" *[followed by a pause and student response]*
Analyzing the Sufficiency of an Explanation [AnalyzeIdea]	Asking students to express or form an opinion about the clarity and completeness of an explanation	"Is that a clear explanation?" "Is there anything missing from that explanation?" "What did you like about that explanation?"
Analyzing Personal Understanding [Understand]	Asking students to self-reflect, analyze, and form their opinion of their own understanding of an explanation of a concept, process, justification, or representation	"Does that make sense?" "Ok?" "Do you understand that explanation?" "Do you feel ok to move on?"
Analyzing to Form Conjectures about Others' Thinking [AnalyzeOthers]	Asking students to make sense of others' ideas or speculate what others' ideas or thought processes were	"What do you think that child was thinking?" "How would you expect [a student] to solve each task?"

Figure 5. Analyzing Explanations Category Description.

Linking and Applying Category

The fifth category of questions is Linking and Applying. The mathematical function of this category is to make connections between mathematical concepts; their meanings; their representations; related terminology, strategies, and mathematical content; other school subjects; and real life contexts and experiences. These questions occurred throughout the course of the lesson. An example of this type of question is, “What does 1/5 mean?” This is an example of a question from this category because it asks students to take the numerical representation of the fraction 1/5 and link that to the meaning of the fraction one-fifth. This category differs from the preceding because it engages students in the mathematical activity of making connections and defining the connections regarding the mathematics rather than analyzing explanations of the mathematics. Note that as opposed to the Exploring Mathematics category of questions, students responded to these questions almost immediately in a class discussion, or worked for at most 1-2 minutes in groups before responding. The pedagogical function is to point to relationships among mathematical ideas, their representations and meanings, and life situations to help students make connections and see mathematics as a whole and not as disjoint parts. Each response to the Linking and Analyzing category had the same form of an answer, a connection. Thus, the subcategories are based on the type of mathematical object the teacher was asking the students to engage with. The subcategories for the Linking and Applying category are as follows: Linking and Applying Mathematical Meanings, Linking Strategies, Linking and Applying Topics in Mathematics, Linking and Applying Mathematics to Life, and Linking and Applying Terminology. This category of questions aids in helping students make the necessary connections needed for understanding (Thompson et al., 1994).

The subcategory that appeared most frequently in the teachers’ Discourse is Linking and Applying Mathematical Meanings. These questions asked students to point to underlying mathematical relationships between mathematical concepts, their meanings, and their representations. An example of this type of question is the question given above: “What does 1/5 mean?” A second example of this type of question is when the teacher asked a student in reference to a picture on the board, “Where do you see 1/5 in this picture?” The student was asked to link the numerical representation of 1/5 to its pictorial representation. A third example is the question, “Can you draw a picture to represent this situation?” This question asked students to take the mathematical concept or situation at hand and connect it to its pictorial representation.

The second subcategory is Linking Strategies. These questions ask students to identify the relationships between different strategies or solution processes for the same mathematical topic, often through comparison. Two examples of this question type are, “How are these strategies the same?” and “What was different between those two strategies?” Through comparison, students became aware of which processes or ideas were common to both solution methods.

The third subcategory is Linking and Applying Topics in Mathematics. These questions ask students to recognize and define relationships among mathematical topics. An example question is, “Where else have we used this?” This question was in reference to different definitions of division that students connected to a unit in previous mathematics class. A second example of this type of question is, “How is this similar to the partitioning definition?” In both of these cases, students were asked to identify connections between topics that they may have previously been unaware of.

The fourth subcategory is Linking and Applying Mathematics to Life. These questions ask students to recognize and define relationships between mathematical topics and non-mathematical topics or situations. For example, a question Carla used of this type was, “Where in real life do we see statistics in use?” A second example is, “What’s a statistic that you see in sports?” In both of these cases, students were asked to name real world contexts or objects that were related to the mathematical topic they were learning about.

The fifth and final subcategory is Linking and Applying Terminology. These questions ask students to recognize and define relationships between mathematical concepts and the correct mathematical language once the concepts are under discussion. An example question of this type is, “What is this called?” These questions link the correct mathematical language with the corresponding concepts. A second example of this type of question is, “How would we write this correctly?” This and the other subcategories of the Linking and Applying category can be found in Figure 6.

Category: Linking and Applying
Mathematical Function: Making links between mathematical concepts and their meanings, their representations, related terminology, strategies, and mathematics and other areas of study/life
Pedagogical Function: Pointing to relationships among mathematical concepts, their representations and meanings, and life situations to help students make connections and see mathematics as a whole and not as disjoint parts

Subcategories

Type of Mathematical Object [Code]	Description	Example Questions
Linking and Applying Mathematical Meanings [LinkMeaning]	Pointing to underlying mathematical relationships between mathematical concepts, their meanings, and their representations	"What does 1/5 mean?" "Where do you see 1/5 in this picture?" "Can you draw a picture to represent this situation?"
Linking Strategies [LinkStrategies]	Recognizing relationships among different strategies or solution processes for the same mathematical topic	"How are these strategies the same?" "What was different between those two strategies?"
Linking and Applying Topics in Mathematics [LinkMath]	Recognizing and defining relationships among mathematical topics	"How is this similar to the partitioning definition?" "Where else have we used this?"
Linking and Applying Mathematics to Life [LinkLife]	Recognizing and defining relationships between mathematical ideas non-mathematical ideas or situations	"Where in real life do we see statistics in use?" "What's a statistic that you see in sports?"
Linking and Applying Terminology [Terminology]	Recognizing and defining relationships between mathematical concepts and the correct mathematical language once concepts are under discussion	"What is this called?" "How would we write this correctly?"

Figure 6. Linking and Applying Category Description.

Questions That Did Not Fall into One of the Five Categories

There were some questions that Carla asked that did not have a corresponding mathematical function, and thus did not fit into one of the five categories above. The questions were coded Managing Classroom Functions. This group of questions included teacher activities such as managing homework ("Did I pass out an extra?"), managing turn taking (calling on a student by name), classroom management ("Can you please quiet down so we can all hear [student's] explanation?"), and pleasantries between teacher and students ("Did you have a nice weekend?"). Because there is no obvious mathematical function apparent for all of these questions, they are not included in the 5 categories of questions describing mathematical questioning Discourse.

Counts of the Types of Questions

After coding all of the transcripts and tasks from the first and third units of the class data, I determined the count of how often each code occurred. Table 1 shows the counts of each category and subcategory of questions from the written tasks in the class (in-class worksheets and homework problems), the transcripts of classroom data, and the sum total of both. The occurrence of each question type ranged from 14 to 506 times in all the transcripts and tasks together. Thus, some question types occurred much more frequently than others in the mathematical questioning Discourse of the teacher, Carla. The least common question type was Accessing a Past Procedure with the most common being Linking and Applying Mathematical Meanings. Some categories, altogether, were more or less common than the other categories. The Explaining One's Thinking category of questions was the least common to be used by the teacher while the Linking and Applying category of questions was most common and occurred a total of 726 times in the transcripts and tasks analyzed.

Table 1

Counts of Codes for Each Subcategory of Question Found in the Data

Code	Tasks	Transcripts	Total	Total of Category
Accessing Relevant Information				
Accessing a Past Experience or Past Answer	0	198	198	
Accessing a Numeric or Computational Result	16	71	87	
Accessing a Past Idea	0	63	63	
Accessing Context	3	37	40	
Accessing a Past Procedure	0	14	14	402
Exploring the Mathematics				
Exploring to Construct a Justification	82	260	342	
Exploring to Construct a Process	23	80	103	
Exploring to Modify a Process	19	17	36	
Exploring to Modify a Justification	0	24	24	481
Explaining One's Thinking				
Explaining One's Own Thinking	61	87	148	
Explaining Other's Thinking	19	17	36	184
Analyzing Explanations				
Analyzing the Validity	45	229	274	
Analyzing the Sufficiency of an Explanation	31	209	240	
Analyzing Personal Understanding	0	126	126	
Analyzing to Form Conjectures about Others' Thinking	6	49	55	695
Linking and Applying				
Linking and Applying Mathematical Meanings	143	363	506	
Linking Strategies	1	79	80	
Linking and Applying Topics in Mathematics	5	58	63	
Linking and Applying Mathematics to Life	1	47	48	
Linking and Applying Terminology	0	29	29	726
Managing Classroom Functions	0	323	323	

Discussion

It is likely that Carla did not have the goal of modeling skillful mathematical questioning Discourse for her students with each question that she asked. Rather, she was probably more focused on the mathematical or pedagogical function associated with the questions. However, the questions were part of the mathematical questions available for students to adopt, regardless. In this section, I discuss issues related to the frequency of different question types, the appearance of two question types that do not appear in other frameworks for teacher questioning, and the possibility of an additional three subcategories of types of questions that did not appear in the data but are nonetheless important to engaging in mathematical activity.

Frequency of Questions

Some question types occurred much more frequently than others in the mathematical questioning Discourse of the teacher. So then, one could ask what the frequency of each type of question tells us about students' exposure to these questions and if students had enough examples of each type of question from which to learn. Unfortunately, the research literature does not provide a sense for how many examples of a question type are necessary for students to begin to adopt that question type into their own questioning Discourse. However, it seems reasonable that regular exposure to a question type, say at least once per class session, might be necessary for students to adopt a question type, particularly if the goal is for students to continue using that question type over an extended period of time. Given that the data from this study were taken from 15 class session, question types that occurred less than 30 times, 2 times per 2 hour class session, may not have been modeled enough for students to adopt. Based on frequency counts, students had significant exposure to each major category of questions as outlined in this framework, experiencing each question type at least 180 times. It would not be surprising if this level of exposure was high enough to enable many students to use these questions in their mathematical questioning Discourse. However, some subcategories of questions may not have been modeled enough for students to adopt; the types of question of Accessing a Past Procedure, Exploring to Modify a Justification, and Linking and Applying Terminology were only used by the teacher 14, 24, and 29 times, respectively, over the course of the 15 class sessions, and thus may not have been sufficiently modeled.

Of the five major categories, questions from the Explaining One's Thinking category appeared less frequently in the data than other question types. This seems odd since the classroom was conceptually-oriented and one where students were expected to share their thinking and work towards a rich conception of the mathematics. However, it could be that as the culture of the class became more established, the teacher needed to remind students to explain the reasoning behind their answers less and less often. This was confirmed in the data – as the semester progressed, there was a decrease in the times the Explaining One's Thinking questions were asked in the classroom data studied – from 8 instances in the first lesson of the first unit, 4 instances by the middle lesson, and 0 instances in the last lesson, with the other lessons following that same general trend.

Further, one would expect that questions stimulating the discussion about the meanings of quantities and their representations to be among the most common in a conceptually-oriented classroom with the focus on students' rich conception of the situations, ideas, and relationships among the mathematics. As expected, the Linking and Applying category of questions were the most frequently asked by the teacher in this conceptually-oriented classroom with the subcategory of Linking and Applying Mathematical Meanings to be the most common within that category. The second most frequently asked category of questions in this conceptually-oriented classroom was Analyzing Explanations. This makes sense in a conceptually-oriented classroom, where students are encouraged understand mathematics, because requests to analyze explanations require them to make sense of the ongoing mathematical conversation.

In general, there were many more instances of most question types in class discourse than in the written assignments. This is not surprising since the amount of spoken discourse in the class far exceeded the amount of written discourse found in handouts and worksheets. Even so, some categories of questions appeared almost entirely in class discourse. For example, 383 of the 402 instances of Accessing Relevant Information questions occurred in class; only 19 appeared in the written assignments. Further, the subcategories of Accessing a Past Idea, Accessing a Past Procedure, and Accessing a Past Experience or Past Answer never appeared in written assignments at all. The questions associated with the Accessing Relevant Information category, however, are those meant to bring to light, gather, privilege, or sanction relevant information to be used in the problem solving process and are most commonly associated with the Launch phase of a lesson done before the exploration of a task. The questions during the Launch phase, or questions from the Accessing Relevant Information category, make sense to be asked in class when a task is introduced so the teacher could help set the stage for students to be successful in the problem solving process. Sometimes students worked on new tasks on their written assignments, as well, but there were very few instances of Accessing Relevant Information questions, or questions launching the task on a written assignment. It could be, then, that the teacher made an effort to model the types of questions that should be asked during the Launch phase of a task during class discussions through the use of Accessing Relevant Information questions, but then allowed for students to practice and participate in that Launch phase of a task or problem on their own in the written assignments.

In contrast, other types of questions were asked more often in written assignments than in class despite the fact that the amount of spoken discourse far exceeded the amount of written text. For example, the questions pertaining to Exploring to Modify a Process occurred more in the written assignments. The questions from the Exploring the Mathematics category are associated with the Explore phase of a lesson or task where students are exploring the mathematics. Thus, it could be that these questions occurred in class but most often in the written form of a task because that is the most common place where students were asked to modify processes explored in class or from earlier in the task itself. Because this subcategory of questions includes those questions modifying a previously explored process, it makes sense that follow up questions about modifying an existing process and applying it to new situations would be done most frequently in homework or written assignments following the class discussion.

Two New Types of Questions

This categorization of types of questions identifies two types of questions used by teachers in a conceptually-oriented classroom not identified in previous mathematics education literature (e.g., Boaler & Brodie, 2004; Hiebert & Wearne, 1993; Moyer & Milewicz, 2002; Sahin & Kulm, 2008; Teuscher et al., 2010). These two new types of questions are Exploring the Mathematics and Analyzing Explanations. I will first discuss why these types of questions did not occur in past frameworks. I will then talk about both categories suggested by this framework as those used by a conceptually-oriented teacher.

Past studies that produced frameworks for teacher questioning have had a different focus or intent than the one in this study. The focus of past studies has been on changing and improving teachers' ability to elicit student participation and foster conceptually-oriented discussions in the classroom, and not on what types of mathematical questions that teachers model for the students that students could use as part of their own mathematical questioning Discourse. Also, few of the past studies examined teacher Discourse in conceptually-oriented classrooms as this study did.

The first type of question added by this framework is Exploring the Mathematics. Because of the past focus on improving pedagogy and eliciting student participation and Discourse, Exploring the Mathematics questions are missing from past frameworks. That is, no categorization of teacher questioning, (e.g., Boaler & Brodie, 2004; Hiebert & Wearne, 1993; Moyer & Milewicz, 2002; Sahin & Kulm, 2008; Teuscher et al., 2010), includes those questions teachers ask when presenting a task to students for mathematical exploration. Questions asking students to explore the mathematics generally result in students' successive individual or group work and do not generally aid in eliciting student participation in classroom discussion. It follows, then, that such questions are absent from past frameworks focused on pedagogy and questions to elicit student participation. However, the omission of this type of question from teacher questioning frameworks is highly problematic given the important role that mathematical exploration plays in reform-oriented classrooms. The framework proposed in this study acknowledges the important role that mathematical exploration questions play in conceptually-oriented classrooms.

This framework also introduces a second type of question new to the existing mathematics education literature—that of Analyzing an Explanation. These questions are an important aspect of conceptually-oriented teaching as they are questions that invite students to participate in the mathematical activity of analysis and evaluation and participate in mathematical conversations with others and themselves. No previous categorization of teacher questioning Discourse includes those questions of a teacher inviting students to engage in the mathematical activity of evaluation (e.g., Boaler & Brodie, 2004; Hiebert & Wearne, 1993; Moyer & Milewicz, 2002; Sahin & Kulm, 2008; Teuscher et al., 2010). Past frameworks were generally not based in a conceptually-oriented classroom, so it makes sense that none of the previous frameworks would have noted those questions asking students to form or express an opinion about the correctness or sufficiency of an explanation, or a conjecture of another person's thinking.

Noting that past frameworks have missed two types of questions used by conceptually-oriented teachers, the question remains if the categorization from this study is still missing a type or types of mathematical questions that a teacher in a conceptually-oriented classroom should use.

Missing Questions by Conceptually-Oriented Teachers

In order to examine if this framework lacks any type of important mathematical question that students would benefit from having the teacher model for them, I turn to a framework known for its focus on the types of questions students should ask themselves to be successful when engaging in the mathematical activity of problem solving, written by Polya (1945). He separated problem solving into 4 different phases and provided example questions that one should use when engaged in each phase of the problem solving process. The 4 phases of problem solving according to Polya's framework are as follows: Understanding the Problem, Devising a Plan, Carrying Out the Plan, and Looking Back. I will discuss each of Polya's phases and explain how each of his categories relates to mine.

Polya's (1945) first phase of problem solving is Understanding the Problem. He stated that students must become aware of what is the unknown, what are the data, what is the condition, what would a suitable figure look like, and what is the suitable notation. My first category of Accessing Relevant Information is similar to Polya's first phase of the problem solving process. It includes questions that help students gather all of the relevant information to help them to understand the problem itself as well as the background information needed to solve the problem. For example, a question like, "What do you do to find what that's equal to?" is one that could be used to understand the problem at hand, but is also part of the Accessing Relevant Information category of questions. My category of Linking and Applying is also related to this phase because it asks students to create or find connections between mathematical concepts and their figures as well as make connections between the mathematical concepts and the correct mathematical language to be used. For example, the question, "How would we write this correctly?" could be used to determine suitable notation in understanding the problem and is part of the Linking and Applying category.

Polya's (1945) second phase of problem solving is Devising a Plan. He stated that students must find the connection between the data and the unknown. They may be obliged to consider auxiliary problems if an immediate connection cannot be found. They should eventually obtain a solution plan to the problem through the use of questions such as, "Could you derive something useful from the data?", "Do you know a related problem?", or "Have you taken into account all essential notions involved in the problem?" (p. xvii). This phase is represented partially in my subcategory of Linking and Applying termed Linking and Applying Topics in Mathematics. This subcategory asks students to recognize relationships between mathematical ideas and can be exemplified in the example question, "Where else have we used this?" when the teacher was referencing a definition of division that the students then related to another unit in mathematics. However, there are no categories or subcategories in my framework where the students are asked to find the connection between the data and the unknown, consider an auxiliary problem, or obtain or create a solution plan. This phase of Polya's problem solving process is underrepresented in the data from Carla's conceptually-oriented classroom.

Polya's (1945) third phase of problem solving is Carrying out the Plan. This phase requires students to carry out their plan, check each step, see if the each step is correct, and see if they can prove each step. This phase is similarly spread across multiple categories and subcategories in my framework. The Exploring the Mathematics category in my categorization mostly describes the type of mathematical activity that would be involved in a student carrying out the plan and investigating the mathematics to form a justification or the construction of a process. An example question of Exploring the Mathematics that fits in with this phase of problem solving is, “Can you prove that this solution is correct?” The Analyzing the Validity of an Explanation subcategory includes questions like, “Is that right?” that could be used to check each step to see if it is correct. Also, Exploring to Construct a Justification subcategory or Explaining One’s Own Thinking include questions that would cause students to try to prove each step.

Polya’s (1945) fourth phase of problem solving is Looking Back. He stated that after solving a problem, a student must examine the solution obtained by checking the result and argument, seeing if the result can be derived differently, and seeing if the solution can be used in another problem. This last phase is seen in many of my categories. When students examine a solution and argument, they are using questions from the Analyzing an Explanation category, such as, “Does that solution or method work?” When they explore to see if the result can be derived differently, they are using the subcategory of questions of Exploring to Construct a Process, such as “Can you find a different way to do it?” And when students consider whether the solution can be used in another problem, they are using the subcategory of questions of Linking and Applying Topics in Mathematics, such as the question, “Can you use this method for some other problem?”

A comparison of Polya’s framework and the categories of questions from this study suggests that a conceptually-oriented teacher’s questioning Discourse may model many of the questions that students should ask themselves as they progress through solving a problem. This is important because it suggests that just by focusing on helping students develop relational understanding, conceptually-oriented teachers will likely model many of the mathematical question types students need to see, even if that is not an explicit goal of their instruction.

Despite how well the framework from this study compared with the types of questions asked in Polya's (1945) four phases of problem solving, we see that the second phase of his problem solving process, Devising a Plan, is underrepresented in the data from this study and the associated framework. In particular, those questions associated with finding the connection between the data and the unknown, considering auxiliary problems if an immediate connection cannot be found, and obtaining a plan of the solution were not found in any of the existing categories from my framework. To address this problem, additional subcategories of questions might be added to the framework from this study—two to the category of Linking and Applying and one to the category of Exploring the Mathematics. The first subcategory that could be added is Linking and Applying Information in the Problem. This is where students find the connection between the data and the unknown in the problem by asking a question like, "What is the relationship between the data and the unknown?" The second subcategory that could be added is Linking and Applying Similar Problems. This is where students consider auxiliary problems and how a related problem could help solve the existing problem. Example questions of this type are, "Do you know a related problem? [and] Could you use it?" (p. xvi). The third subcategory that could be added is Exploring to Create a Plan. This subcategory includes questions where students can investigate to create a plan to solve the problem at hand, such as in the question, "Could you solve part of the problem?" (p. xvii). Adding these questions types to the framework fills in the deficiencies identified when comparing the framework with Polya's second phase.

It seems that although conceptually-oriented teachers aim at modeling and promoting a rich understanding of ideas and conceptions of mathematics, the teachers may fail to sufficiently model good thinking in the form of Devising a Plan. However, with the teacher as the disciplinary authority and a logical source of mathematical questioning Discourse from which students can adopt a questions they can ask themselves, if the teacher does not properly model the thinking and questioning that goes into the Devising a Plan phase of problem solving, where else can students learn it? A teacher must model good mathematical questioning Discourse for students so that they have a model from which to adopt, but as seen in this study, teachers with a conceptually-oriented Discourse may be deficient in modeling the questioning that pertains to Polya's second step of problem solving, Devising a Plan.

CHAPTER 5: CONCLUSION

This study discussed the types of mathematical questioning Discourse of a teacher in a conceptually-oriented classroom. A framework was made with 5 different categories of questions and their accompanying mathematical functions, as well as 21 subcategories of these question types, that comprise the mathematical questioning Discourse of a teacher with a conceptually-oriented Discourse. Two categories of questions were added by this framework to the existing literature on the types of mathematical questions utilized by teachers in the classroom.

The following sections describe the implications of this study and directions for future research. The first section will identify the implications for mathematics teaching. The second section will discuss implications for research. The third section will address the limitations of this study and future directions for research.

Implications for Teaching

This categorization of questions for a teacher with a conceptually-oriented Discourse can be used by mathematics teachers in at least two ways. The list of questions can be used by conceptually-oriented teachers to explicitly teach the important mathematical questions students should be asking during mathematical activity, and by teachers who wish to change their instruction to be more conceptually-oriented.

First, this categorization can be used by conceptually-oriented teachers to teach mathematical questioning Discourse to their students. The list of questions provides teachers with specific categories or types of questions that they are likely already asking in their instruction, and that they can draw students' attention to in order that students can use the same types of questions when they are participating in mathematical activity. Teachers can explicitly teach awareness to students of the types of questions students should be asking themselves. I do not suggest that this categorization becomes a lesson or focus of instruction in itself. Rather, I suggest that teachers can model these types of questions for their students and explicitly point out and name these types of questions for their students.

Second, teachers can use this categorization in an effort to become more conceptually-oriented in their instruction. All five categories of questions engage students in mathematical activity that can lead to relational understanding. Because the mathematical and pedagogical functions are delineated for each category, teachers can select a form of mathematical activity or a particular pedagogical purpose, match the activity or purpose to a question category, and then refer to the subcategory descriptions and examples for support in creating questions that will help guide instruction to achieve the desired mathematical or pedagogical function. Thus, this categorization can be used as a catalyst for change in the instruction of teachers.

Implications for Research

This list of question types can be used by researchers to understand and improve teachers' and students' mathematical questioning. This framework identifies two types or categories of questions that have been missing in the past research on teacher questioning. The framework also suggests that conceptually-oriented teachers are missing some types of questions in their mathematical questioning Discourse, specifically those correlated with Polya's (1945) second phase of problem solving of Devising a Plan, which might be useful for students to see modeled.

Limitations and Future Directions

There are limitations to this study. The data from this study are from only one teacher in one of her classes. Clearly this study does not represent the types of questions that comprise the mathematical questioning Discourse of all conceptually-oriented teachers. There may well be more categories or subcategories that were not represented in the data analyzed, and thus, future studies of additional teachers and classes must be done to ensure that a complete list can be obtained for the types of questions that comprise the mathematical questioning Discourse of conceptually-oriented teachers.

Finally, this study focused on the questions of the teacher to see what questions were available for students to adopt but did not examine if students actually began to adopt the mathematical questioning Discourse of the teacher. Student appropriation was not examined in this study; rather, student appropriation was the lens through which I studied teacher questioning. Further research needs to be done to examine how students begin to adopt the mathematical questioning Discourse of their teacher and what process students engage in when they adopt the mathematical questioning Discourse of their teacher, if they adopt it at all.

Summary

Teacher mathematical questioning Discourse is important to study. Teacher mathematical questioning discourse is the discourse available to students to learn from and copy. This study informs teachers of the types of mathematical questioning Discourse they can utilize to provide useful modeling of questioning Discourse for their students as well as to become more conceptually oriented as a teacher. This study suggests a greater need for conceptually-oriented teachers to model the thinking and questioning related to the stage of problem solving that Polya (1945) termed Devising a Plan. By studying and utilizing this framework for the types of questions used by a teacher in a conceptually-oriented classroom, we can better understand how to improve both teachers' and students' mathematical questioning Discourse.

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