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Covariance Modeling and Space-Time Coding for MIMO systems

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Covariance Modeling and Space-Time Coding for MIMO Systems

Farnaz Karimdady Sharifabad

A dissertation submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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ABSTRACT

Covariance Modeling and Space-Time Coding for MIMO Systems

Farnaz Karimdady Sharifabad
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Doctor of Philosophy

The full spatial covariance matrix of the multiple input multiple-output (MIMO) channel is an important quantity in channel modeling, communication system signal processing, and performance analysis, and therefore this matrix forms the heart of the research outlined in this dissertation. The work begins with an investigation of a generalized framework for computing the full MIMO spatial covariance based on the power angular spectrum (PAS) of the multipath field and the transmit and receive antenna element radiation patterns. For the case of uniform linear arrays and when the PAS clusters satisfy uniform, truncated Gaussian, or truncated Laplacian distributions, a series expansion is used to allow analytic evaluation of the required integrals in the formulation. The study also demonstrates the validity of some simplifying assumptions used to reduce the complexity of the covariance computation by applying the technique to ray tracing data as well as considers an analysis of the convergence properties of the series when computed using a finite number of terms.

The insights and tools obtained from this covariance analysis are then used to develop a general approach for constructing MIMO transmit and receive beamforming vectors based on the full spatial covariance. While transmit and receive beamforming for the MIMO channel is a well-studied topic, when the transmit precoding is based on channel covariance information, developing near-optimal transmit and receive beamformers when the receiver is constrained to use linear processing remains an unsolved problem. This iterative beamforming algorithm presented here can accommodate different types of available channel information and receiver capabilities as well as either a sum power constraint or a per-antenna power constraint. While the latter is more realistic, construction of the optimal transmit precoder is less understood for this constraint. Simulation results based on measured channels demonstrate that the approach generates beamformer solutions whose performance rivals that achieved for an optimal nonlinear receiver architecture.

Keywords: MIMO systems, correlation, array signal processing, beam steering, time-varying channels, cooperative systems, radio propagation
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Farnaz Karimdady Sharifabad.
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Chapter 1

Introduction

The full spatial covariance matrix of the multiple-input multiple-output (MIMO) channel is an important quantity that is used in channel modeling, communication system signal processing, and performance analysis. This work formulates a closed-form expression for this covariance matrix based on the average multipath propagation characteristics and the radiation patterns of the antennas in the transmit and receive arrays. Simplifications to the formulation are established for cases where the propagation can be represented using discrete plane waves or where the power angular spectrum can be modeled using clusters that satisfy uniform, truncated Gaussian, or truncated Laplacian distributions. Numerical studies demonstrate application of the formulation as well as its accuracy under different scenarios.

Using the insights and tools obtained from this covariance analysis we develop a general approach for constructing MIMO transmit and receive beamforming vectors based on either full spatial covariance or channel state information. While transmit and receive beamforming for the MIMO channel is a well-studied topic, developing near-optimal transmit and receive beamformers for some cases particularly when the transmit precoding is based on channel covariance information and the receiver is constrained to use linear processing remains an unsolved problem. This work considers these scenarios and presents a generalized beamforming algorithm that can accommodate different receiver capabilities with only channel covariance information available at transmitter. Simulation results based on modeled and measured channels demonstrate that the approach generates beamformer solutions whose performance rivals that achieved with an optimal nonlinear receiver architecture.

While the developed beamforming technique is useful, there are situations that cannot be accommodated by this approach. Therefore, we present a more general iterative beam-
forming algorithm that can address certain situations for which other existing algorithms cannot specify effective beamformers. This iterative algorithm can establish beamformers for a wide range of scenarios by accommodating nonlinear and linear receivers, channel state information (CSI) or channel covariance information (CDI) at the transmitter and receiver, and both sum-power and per-antenna power constraints. Simulation results based on modeled and measured channels demonstrate that when the approach is applied to scenarios that can be accommodated by other algorithms, it generates beamformer solutions whose performance matches that of these existing methods. The results further show that when the algorithm is applied for reduced receiver capabilities or the more stringent per-antenna power constraint, the performance rivals that achieved for an optimal non-linear receiver or the more flexible sum-power constraint.

Finally, this work explores the potential benefits of cooperative MIMO signaling from three coherent base stations to a single four-antenna mobile station using channel measurements in an urban macrocellular environment at 2.66 GHz. The analysis compares the performance when the cooperative base stations have no information about the channels to that achieved when they can cooperatively apply the proposed transmit beamforming. In the latter case, the transmit power is constrained such that either the total power or the power on each base station antenna is limited. The results demonstrate that significant improvement is possible using the cooperative MIMO signaling, with the performance depending on the capabilities of the system.

1.1 New Contributions

1. In most practical scenarios, the channel covariance is constructed using a sample mean of channel estimates over a fixed observation window. However, channel variation often arises from mobility of the nodes or the scatterers in the environment. The variation in the key multipath features resulting from the mobility implies that the channel statistics are not wide-sense stationary, making it difficult to define the window over which the covariance should be estimated. Thus, having an analytical approach for estimating the covariance based on understanding of the multipath features over a localized region is a valuable tool both for system analysis and channel modeling.
This motivates our formulation of a closed-form expression for the full MIMO spatial covariance based on the power angular spectrum (PAS) of the multipath field and the transmit and receive element radiation patterns.

2. A convenient method for computing the one-sided covariance matrices using a series expansion when the PAS clusters satisfy uniform, truncated Gaussian, or truncated Laplacian distribution has recently been proposed [1]. While the full spatial covariance can be obtained using a Kronecker product of these one-sided covariance matrices when the PAS at the receive array is independent of that at the transmit array, real channels do not satisfy this separability assumption. Therefore, we present a generalization of this approach for simplifications of the formulation of the full spatial covariance where the propagation can be represented using these particular PAS distributions without any unrealistic assumptions of separability.

3. Recent work considers the problem of finding transmit and receive MIMO beamformer considering either CSI or CDI at the transmitter with either a sum power constraint or a per-antenna power constraint mainly for non-linear receive beamformer [2–8]. However, these algorithms cannot accommodate the wide range of system capabilities that might be considered when implementing a practical MIMO system. We therefore provide a general iterative beamforming algorithm that can accommodate different types of channel information (CSI or CDI) available at the transmitter, different receiver capabilities (linear or non-linear), and different power constraints (sum power constraint or per-antenna power constraint).

4. We further investigate the effect of using the proposed transmit and receive beamforming in the case of cooperative MIMO signaling from three coherent base stations to a single four-antenna mobile station using channel measurements in an urban macro-cellular environment under two different power constraint schemes where the transmit power is constrained such that either the total power or the power on each base station antenna is limited.
1.2 Organization of the Dissertation

Following this brief introduction providing the motivation of the main topic of this dissertation, fundamental background information regarding MIMO systems and the channel spatial covariance are presented in Chapter 2. Additionally, this chapter describes the basis for the system model considered throughout this dissertation.

Chapter 3 proposes a general analytical approach for full MIMO spatial covariance construction that involves an integration of the PAS of the multipath field and the transmit and receive element radiation patterns. Our proposed method does not consider the conventional assumption of prior work, i.e., the assumption of independence of the PAS at the receive and transmit arrays which in turn allows computation of the full covariance matrix using a Kronecker product of the covariance matrices at each end of the link. The presentation includes a thorough analysis of the applicability of common assumptions regarding the multipath field by assessing the impact of these assumptions on the covariance matrix using fields generated from ray-tracing data. Furthermore, for the case of uniform linear arrays and when the PAS clusters satisfy uniform, truncated Gaussian, or truncated Laplacian distributions, a series expansion is used to allow analytic evaluation of the required integrals in the formulation [1]. The study concludes with an analysis of the convergence properties of the series when computed using a finite number of terms.

Chapters 4 and 5 develop a general framework for constructing transmit and receive beamformers for a point-to-point MIMO link that accommodates different types of channel information (CSI or CDI) available at the transmitter, different receiver capabilities, and different power constraints. Chapter 4 focuses mainly on the specific case when the precoding is based on CDI and the technique uses the full spatial covariance matrix as opposed to the one-sided transmit covariance used in prior work [2,3] so that it can properly accommodate the receiver architecture. However, Chapter 5 presents a more general iterative beamforming algorithm that can accommodate different power constraints as well. Simulations based on a simple channel model as well as on experimental MIMO channel data reveal that the approach is highly effective and is able to generate transmit precoders whose performance when coupled with a linear beamforming receiver rivals that achieved with a system whose receiver allows optimal successive decoding.
Chapter 6 explores the potential benefits of cooperative MIMO signaling in an urban macrocellular environment from three coherent base stations to a single four-antenna mobile station. Chapter 6 also includes a comparison of the performance when the cooperative base stations have no information about the channels with that achieved when they can cooperatively apply transmit beamforming already proposed in previous chapters. In the latter case, the transmit power is constrained such that either the total power or the power on each base station antenna is limited. Further simulations are applied to demonstrate that significant improvement is possible using cooperative MIMO signaling, with the performance depending on the capabilities of the system. Finally, Chapter 7 summarizes the conclusions of this dissertation.
Chapter 2

Background

In this chapter, we present the fundamentals of multiple-antenna systems and provide a system model useful throughout this dissertation. The discussion then turns to the MIMO channel spatial covariance, including an introduction to the Kronecker approximation for representing the full spatial covariance.

2.1 MIMO

Multiple-input multiple-output (MIMO) technology has the potential of improving the achievable rate of wireless communication systems significantly. However, the obtainable benefits of MIMO can vary depending on the communication environment. The two main schemes for enhancing the performance of wireless systems using multiple antenna systems are spatial multiplexing and diversity. Using spatial multiplexing, the system can achieve higher data rates without the need for extra bandwidth by sending a unique data stream on each of the parallel sub-channels in a MIMO channel. In contrast, a diversity system redundantly sends the data stream across the parallel sub-channels to provide robustness against fading. It is important to emphasize, however, that while diversity does not explicitly increase the achievable rate, because it offers error coding across the spatial channels, it allows application of higher-rate temporal codes.

2.2 System Model

Consider a point-to-point MIMO communication link consisting of $N_t$ transmit antennas and $N_r$ receive antennas. As illustrated in Figure 2.0, the $N_r \times 1$ received signal vector is given by

$$y = Hx + \eta,$$  \hspace{1cm} (2.1)
Figure 2.1: System Model of a system with $N_t \times N_t$ MIMO channel with $K$ data streams, where $1 \leq k \leq K$.

where $H$ is an $N_t \times N_t$ channel matrix and $\eta$ is an $N_t \times 1$ noise vector whose entries are independent identically distributed zero-mean complex Gaussian random variables with variance $\sigma^2$. We assume that the system communicates $K \leq \min(N_t, N_r)$ data streams represented by the $K \times 1$ data vector $x_0$ with covariance $E\{x_0x_0^\dagger\} = I$, where $E\{\cdot\}$ is the expectation, $\{\cdot\}^\dagger$ represents the conjugate transpose, and $I$ is the identity matrix. The transmit vector $x$ is then formed through precoding with the $N_t \times K$ transmit beamforming matrix $B$ ($x = Bx_0$).

The receiver applies an $N_r \times 1$ unit-length beamforming vector $w_k$ when detecting the $k$th element of $x_0$, or $y_{0k} = w_k^\dagger y_k$ where $y_k$ represents a possibly modified version of the received vector $y$. For example, if the receiver uses only linear beamforming, then $y_k = y$. However, if the receiver also uses nonlinear successive interference cancellation (SIC), then $y_k$ represents $y$ modified such that the contributions of the previously detected symbols are removed. For notational simplicity, we let $w_k$ represent the $k$th column of the $N_r \times K$ matrix $W$ and $b_k$ represent the $k$th column of $B$.

The main focus of this work is to determine the optimal beamforming matrices ($B$ and $W$) based on the available channel information at the transmitter and receiver for different cases with different considerations. One of the main considerations is the power constraint scheme at the transmitter, where we consider two different power constraint schemes: sum power constraint and per-antenna power constraint. In the case of a sum power constraint, we assume that the average transmit power is $P$, or $E\{x^\dagger x\} = E\{\text{tr}(xx^\dagger)\} = P$, where
tr(·) denotes the trace. This results in \( \text{tr} (\mathbf{B} \mathbf{B}^\dagger) = P \). On the other hand, for a per-antenna power constraint we assume that the average power transmitted by the \( i \)th transmit antenna is \( \mathbf{E} \{ \mathbf{x}_i \mathbf{x}_i^\dagger \} = \mathbf{P} \), where \( \mathbf{x}_i \) and \( \hat{\mathbf{b}}_i \) represent the \( i \)th element of the vector \( \mathbf{x} \) and \( i \)th row of the matrix \( \mathbf{B} \), respectively. Consequently, we have \( \hat{\mathbf{b}}_i \hat{\mathbf{b}}_i^\dagger = P_i \) for \( 1 \leq i \leq N_t \).

Finally, any system must consider the type of channel information available at the transmitter. Conventionally, channel state information is assumed to be known at both the transmitter and receiver, which requires frequent feedback events of the channel information from the receiver to the transmitter. Recent work has also considered using channel distribution information, often in the form of spatial covariance matrices, since it is typically valid for a longer time period, reducing the required frequency of feedback events. Thus, we provide a brief information on the MIMO spatial covariance and its existing modeling in the following sections, and Chapter 3 describes the spatial covariance modeling in more detail as a main element of this work.

### 2.3 MIMO Spatial Covariance

The full spatial covariance matrix of the MIMO channel is an important quantity that captures both the antenna array geometry and the multipath spatial structure. Considering the system model presented in the Section 2.2, the \( N_r \times N_t \) matrix \( \mathbf{H} \) in (2.1) contains the complex channel transfer coefficients that are commonly treated as zero-mean complex random variables. Given this system model, the spatial covariance of the channel matrix \( \mathbf{H} \) can be constructed using

\[
\mathbf{R} = \mathbf{E} \{ \text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^\dagger \},
\]

where vec(·) denotes columnwise stacking of the matrix into a column vector. Naturally, in many cases, the expectation is approximated using a sample mean over a set of channel matrices observed over a time period, which assumes that the channel physical properties (such as the departure and arrival angles and complex gains of multipath components) remain constant over this estimation window. However, if the channel variation arises from moving communication nodes or scatterers, these channel properties vary in time, and therefore the estimation window must be chosen carefully. By examining (2.2), it can be observed that the
eigenvectors and eigenvalues of the spatial covariance can illustrate important characteristics of the MIMO channel, including the average power of the sub-channels. This explains why the spatial covariance is such a critical quantity in the analysis of multiple antenna systems.

2.4 Kronecker Channel Modeling

The common analytical approach for channel spatial covariance construction takes advantage of the power angular spectrum (PAS), which represents the average power in the propagating field as a function of angle [9]. The covariance constructed based on the Kronecker model can be represented as [10]

\[
R = \frac{1}{\text{tr} (R_r)} R_t^T \otimes R_r,
\]

(2.3)

where \( \otimes \) represents Kronecker product and \( R_r \) and \( R_t \) are defined as

\[
R_r = \frac{1}{N_t} E \{ HH^\dagger \},
\]

(2.4)

\[
R_t = \frac{1}{N_r} E \{ H^\dagger H \}.
\]

(2.5)

Figure 2.1 shows an image of a representative clustered PAS modeled, which means that the multipath energy is concentrated in a few regions in the transmit angle - receive angle space. However, the Kronecker model for the covariance assumes that the PAS at the receiver array is independent of that at the transmit array [11,12]. This gives rise to the PAS model for the same environment shown in Figure 2.2. Comparing Figures 2.1 and 2.2 demonstrates why the Kronecker model cannot precisely represent actual channels, motivating development of simple yet effective models that properly capture the physics of the multipath channel.
Figure 2.2: Representative clustered PAS showing three clusters satisfying a Laplacian distribution in both transmit and receive angles.

Figure 2.3: Representative clustered PAS based on Kronecker model showing three clusters satisfying a Laplacian distribution in both transmit and receive angles.
Chapter 3

Closed-Form Evaluation of the MIMO Channel Spatial Covariance

The spatial covariance of the multiple-input multiple-output (MIMO) channel is a critical quantity used in channel modeling, system performance assessment, and even generation of transmit precoding strategies [4,13]. In most practical scenarios, this covariance is constructed using a sample mean of channel estimates over a fixed observation window. However, channel variation often arises from mobility of the nodes or the scatterers in the environment. The resulting variation in the key multipath features implies that the channel statistics are not wide-sense stationary, making it difficult to define the window over which the covariance should be estimated [14]. While there are techniques for estimating stationarity intervals for such data, having an analytical approach for estimating the covariance based on understanding of the multipath features over a localized region is a valuable tool both for system analysis and channel modeling.

The typical analytical approach for channel spatial covariance construction involves an integration of the power angular spectrum (PAS) – representing the average power in the propagating field as a function of angle – and the radiation patterns of the communicating antennas [9]. However, this approach has only been developed generally for the situation where the PAS at the receive array is independent of that at the transmit array, allowing computation of covariance matrices at each end of the link that can be combined into the full channel covariance using a Kronecker matrix product [11,12]. A very convenient method for computing these one-sided covariance matrices using a series expansion for specific PAS distributions has recently been proposed [1]. However, real channels do not satisfy the separability assumption [10,15,16], motivating generalization of the approach for the full spatial covariance.
This work formulates the equations for the full MIMO spatial covariance based on the PAS of the multipath field and the transmit and receive element radiation patterns. The equations are applied to multipath fields generated from ray-tracing data to test the suitability of key assumptions commonly used to simplify covariance computation. Furthermore, for the case of uniform linear arrays and when the PAS clusters satisfy uniform, truncated Gaussian, or truncated Laplacian distributions, a series expansion is used to allow analytic evaluation of the required integrals in the formulation [1]. The study concludes with an analysis of the convergence properties of the series when computed using a finite number of terms.

### 3.1 MIMO Spatial Covariance Construction

As mentioned in Chapter 2, assuming separability of the covariance at the transmitter and receiver allows the one-sided covariance matrices to be computed from the PAS at the node and the radiation patterns of the antennas in the array. It is the objective of this work to extend this approach to the case of the full covariance matrix, thereby avoiding the difficulties associated with the separability assumption, and to analyze the behavior of such a formulation for practical scenarios.

To begin, we formulate the covariance based on a full vector (polarization) representation of the electromagnetic propagation and antenna responses. We assume that the transmitter and receiver each reside in independent coordinate frames and define the propagation direction of the field using spherical coordinates with $\theta$ and $\phi$ respectively representing elevation and azimuth angles. We use the compact notation $\Omega = (\theta, \phi)$ to represent these angles in the following.

Given this notation, let the complex gain of the propagating field be represented as

$$
\beta(\Omega_r, \Omega_t) = \begin{bmatrix}
\beta_{\theta\theta}(\Omega_r, \Omega_t) & \beta_{\theta\phi}(\Omega_r, \Omega_t) \\
\beta_{\phi\theta}(\Omega_r, \Omega_t) & \beta_{\phi\phi}(\Omega_r, \Omega_t)
\end{bmatrix},
$$

(3.1)

where each matrix entry $\beta_{uv}(\Omega_t, \Omega_r)$ represents the complex gain for a multipath component departing at the angle $\Omega_t$ with polarization $v \in [\theta, \phi]$ and arriving at the angle $\Omega_r$ with
polarization $u \in [\theta, \phi]$ [17, 18]. We recognize that $\beta_{uv}(\Omega_r, \Omega_t)$ is in general a continuous function of angle, such as might be obtained using a plane wave spectral decomposition of a propagating electric field. To further simplify the notation, from this point onward we use the shorthand $\Omega = (\Omega_r, \Omega_t)$.

In general, the vector radiation pattern for the $n$th transmit antenna can be written as $s_{t,n}(\Omega_t) = \hat{\theta} s_{t\theta,n}(\Omega_t) + \hat{\phi} s_{t\phi,n}(\Omega_t)$, where the notation $\vec{s}$ indicates that the pattern contains electromagnetic polarizations with $\hat{\theta}$ and $\hat{\phi}$ representing the unit vectors in the $\theta$ and $\phi$ directions, respectively. However, to ease the notation in the formulation, we instead write this vector radiation pattern using matrix-vector notation as $s_{t,n}(\Omega_t) = [s_{t\theta,n}(\Omega_t), s_{t\phi,n}(\Omega_t)]^T$, where $\{\cdot\}^T$ indicates a transpose. If the radiation pattern of the $m$th receive antenna is $s_{r,m}(\Omega_r) = [s_{r\theta,m}(\Omega_r), s_{r\phi,m}(\Omega_r)]^T$, then the channel matrix element for these antennas is given by

$$H_{mn} = \int s_{r,m}^T(\Omega_r) \beta(\Omega) s_{t,n}(\Omega_t) d\Omega$$
$$= \int s_{mn}^T(\Omega) b(\Omega) d\Omega,$$

where

$$b(\Omega) = \text{vec} [\beta^T(\Omega)],$$
$$s_{mn}(\Omega) = s_{r,m}(\Omega_r) \otimes s_{t,n}(\Omega_t),$$

and $\otimes$ indicates a Kronecker product. We note that this is simply a polarimetric extension of the classical double-directional model commonly used in MIMO system representations [17].

Given these representations of the propagating field and the antenna patterns, our goal is to determine the covariance with elements $R_{mn,pq} = \mathbb{E} \{ H_{mn} H_{pq}^* \}$, where $\{\cdot\}^*$ represents the conjugate. Using (3.3), this covariance matrix element becomes

$$R_{mn,pq} = \int \int s_{mn}^T(\Omega) P(\Omega, \Omega') s_{pq}^* (\Omega') d\Omega d\Omega'$$

$$P(\Omega, \Omega') = \mathbb{E} \{ b(\Omega) b^*(\Omega') \}. $$

13
We note the similarity between this expression for the full covariance—which is a generalization of the Directional Model discussed in [19]—and the classic expression for the one-sided covariance introduced for antenna diversity characterizations [9]. Equation (3.6) is a general expression that can be used to compute the covariance based on knowledge of the average propagating field characteristics and the array radiation properties.

While (3.6) provides a general form applicable to any scenario, application of a common assumption significantly simplifies the computation. Specifically, we assume that the field arriving from (or departing into) one angle is uncorrelated with that arriving from (or departing into) another [9], or

$$P(\Omega, \Omega') = B(\Omega)\delta(\Omega - \Omega'),$$  

(3.8)

where $B(\Omega) = E\{b(\Omega)b(\Omega)\}$ represents the PAS of the propagating electromagnetic field. With this simplification, the covariance matrix element can be written as

$$\hat{R}_{mn,pq} = \int s_{mn}(\Omega)B(\Omega)s_{pq}^*(\Omega) d\Omega.$$  

(3.9)

This can be explicitly expanded as

$$\hat{R}_{mn,pq} = \sum_{\substack{u,v,u',v' \in [\theta, \phi]}} \xi^{(uv,u'v')}_{mn,pq},$$  

(3.10)

$$\xi^{(uv,u'v')}_{mn,pq} = \int s_{ru,m}(\Omega_t)s_{ru',p}^*(\Omega_t)B_{uv,u'v'}(\Omega)s_{tv,n}(\Omega_t)s_{tv',q}^*(\Omega_t) d\Omega,$$  

(3.11)

where $B_{uv,u'v'} = E\{\beta_{uv}\beta_{u'v'}^*\}$ represents the corresponding element of matrix $B$. This makes it clear that each covariance matrix element has contributions from all combinations of transmit and receive polarizations. If we further assume that the field propagating in one polarization is uncorrelated with that propagating in the other, (3.10) further simplifies to

$$\hat{R}_{mn,pq} = \sum_{u \in [\theta, \phi]} \sum_{v \in [\theta, \phi]} \xi^{(uv,uv)}_{mn,pq}. $$  

(3.12)
The following two sections demonstrate application of these equations to practical scenarios and explore the implications of different covariance computation techniques and simplifying assumptions.

### 3.1.1 Discrete Multipaths

In many practical channel models, the multipath propagation is represented as a set of discrete plane waves, with the $\ell$th wave defined by the angle of departure $\Omega_{t,\ell}$, the angle of arrival $\Omega_{r,\ell}$, and the complex gain $\gamma_\ell$. The discussion surrounding (3.12) suggests that if we understand how to create the contribution to the covariance matrix from one of the possible combinations of polarizations, we can extend this understanding to construct the contributions from the remaining polarization combinations. Therefore, we focus on a single electromagnetic polarization in the following, which means that the vector $b(\Omega)$ becomes the scalar $b(\Omega)$. Naturally, if the antennas themselves are predominantly polarized in a single dimension, then this formulation is complete.

Under these simplifications, we have

$$b(\Omega) = \sum_\ell \gamma_\ell \delta(\Omega - \Omega_\ell),$$  \hspace{1cm} (3.13)

which leads to the simplification of (3.6) given by

$$R_{mn,pq} = E \left\{ \sum_\ell \sum_{\ell'} \gamma_\ell \gamma_{\ell'}^* s_{mn}(\Omega_\ell) s_{pq}^*(\Omega_{\ell'}) \right\},$$  \hspace{1cm} (3.14)

where we emphasize that $s_{mn}(\Omega)$ is the scalar version of (3.5). Again assuming that the field arriving from (or departing into) one angle is uncorrelated with that arriving from (or departing into) another leads to the form

$$\hat{R}_{mn,pq} = E \left\{ \sum_\ell |\gamma_\ell|^2 s_{mn}(\Omega_\ell) s_{pq}^*(\Omega_\ell) \right\}.$$  \hspace{1cm} (3.15)

The resulting covariance depends upon the expectation used in (3.15). One reasonable assumption is that the covariance is defined as the nodes move around in a limited area. In
such a circumstance, we expect the departure and arrival angles $\Omega_{t,\ell}$ and $\Omega_{r,\ell}$ as well as the magnitude of the gain $\gamma_{\ell}$ for each multipath to remain relatively constant. However, the phase of each multipath, represented by the phase of $\gamma_{\ell}$, can be assumed to vary uniformly over the range $[0, 2\pi)$. In this case, the covariance simplifies to the form

$$\tilde{R}_{mn,pq} = \sum_{\ell} |\gamma_{\ell}|^2 s_{mn}(\Omega_{\ell}) s_{pq}^*(\Omega_{\ell}).$$  \hspace{1cm} (3.16)$$

3.1.2 Covariance Construction Based on the Kronecker Model

As discussed in Chapter 2, the covariance constructed based on the Kronecker model can be represented as

$$\mathbf{R} = \frac{1}{\text{tr}(\mathbf{R}_t)} \mathbf{R}_t^T \otimes \mathbf{R}_r.$$ \hspace{1cm} (3.17)$$

$\mathbf{R}_t$ and $\mathbf{R}_r$ are defined in the integral form as

$$\mathbf{R}_{t,mp} = \frac{1}{N_t} \sum_{i=1}^{N_t} \int \int s_{mi}^T(\Omega) \mathbf{P}(\Omega, \Omega') s_{pi}^*(\Omega') \, d\Omega \, d\Omega',$$ \hspace{1cm} (3.18)$$

$$\mathbf{R}_{t,qn} = \frac{1}{N_t} \sum_{i=1}^{N_t} \int \int s_{in}^T(\Omega) \mathbf{P}(\Omega, \Omega') s_{iq}^*(\Omega') \, d\Omega \, d\Omega'. $$ \hspace{1cm} (3.19)$$

Again under the assumption in (3.8), these expressions become

$$\mathbf{R}_{t,mp} = \frac{1}{N_t} \sum_{i=1}^{N_t} \int s_{mi}^T(\Omega) \mathbf{B}(\Omega) s_{pi}^*(\Omega) \, d\Omega,$$ \hspace{1cm} (3.20)$$

$$\mathbf{R}_{t,qn} = \frac{1}{N_t} \sum_{i=1}^{N_t} \int s_{in}^T(\Omega) \mathbf{B}(\Omega) s_{iq}^*(\Omega) \, d\Omega.$$ \hspace{1cm} (3.21)$$
Following the approach leading to (3.10), these can be explicitly expanded as

\[
R_{r,mp} = \sum_{u,v,u',v' \in [\theta, \phi]} \xi_{m,p}^{(uv,u'v')} \tag{3.22}
\]

\[
\xi_{m,p}^{(uv,u'v')} = \frac{1}{N_t} \sum_{i=1}^{N_t} \int s_{ru,m}(\Omega_t)s_{ru',p}^*(\Omega_s)B_{uv,u'v'}(\Omega)\sigma_{tv,i}^*(\Omega_t)\sigma_{tv,i}(\Omega_t) d\Omega, \tag{3.23}
\]

\[
R_{t,qn} = \sum_{u,v,u',v' \in [\theta, \phi]} \xi_{q,n}^{(uv,u'v')} \tag{3.24}
\]

\[
\xi_{q,n}^{(uv,u'v')} = \frac{1}{N_r} \sum_{i=1}^{N_r} \int s_{ru,i}(\Omega_t)s_{ru',i}^*(\Omega_s)B_{uv,u'v'}(\Omega)\sigma_{tv,n}(\Omega_t)\sigma_{tv,n}^*(\Omega_t) d\Omega. \tag{3.25}
\]

### 3.1.3 Numerical Analysis

With this simplified formulation, we are prepared to demonstrate application of the formulation to covariance construction and to explore the suitability of some of the assumptions used to simplify the covariance computation. Naturally, the formulations outlined require understanding of the multipath physical properties, and we therefore use a ray-tracing approach to model the multipath characteristics. Specifically, we use an urban environment where the transmitter remains at a single location while the receiver moves along a straight line. The ray tracing implementation takes into account multiple (up to 10) reflections from walls, single diffractions from building corners, multiple diffractions from rooftops, and combinations of a single diffraction and multiple reflections. The accuracy of the ray tracing program has been validated extensively, with details available in [20,21].

In this calculation, the building walls are assumed flat and smooth with relative permittivity of \( \epsilon_r = 3 \) and conductivity of \( \sigma = 0.0001 \text{ S/m} \). When a reflection occurs, the reflection coefficient is calculated based on the ray incidence angle and the wall parameters. For diffracted field calculations, the uniform theory of diffraction (UTD) [22] is employed. Since the walls are not perfect electric conductor (PEC), a heuristic modification [23] of UTD is adopted for the calculation of diffraction coefficients. The departure angle, arrival angle, and complex gain of each multipath component between the transmitter and receiver are sampled at an interval of \( \lambda/10 \) as the receiver moves, where \( \lambda \) denotes the free-space
Figure 3.1: 3D scene of the ray-tracing dataset, whose corresponding simulation results are presented in Figure 3.2(a)-(c).

wavelength at the simulation frequency of 2.5 GHz. The propagation is confined to the horizontal plane so that we only consider the angles $\phi_t$ and $\phi_r$.

We assume that the transmitter and receiver each have a uniform linear array (ULA) consisting of $N_t = N_r = 4$ vertically-oriented dipoles, and we therefore consider only the
vertically-polarized components of the propagating electric field obtained from the ray-tracing simulations. The resulting scalar radiation patterns for the $n$th transmit ($1 \leq n \leq N_t$) and $m$th receive ($1 \leq m \leq N_r$) antennas placed along the $y$-axis are respectively

\begin{align*}
s_{t,n}(\phi_t) &= \exp[jk(n - 1)d_t \sin \phi_t], \\
s_{r,m}(\phi_r) &= \exp[jk(m - 1)d_r \sin \phi_r],
\end{align*}

where $k = 2\pi/\lambda$, $d_t = d_r = \lambda/2$ represents the spacing between array elements.

Because (3.14) and (3.15) both require explicit evaluation of an expectation, we approximate the expectation using a sample mean taken using the samples of the multipath field over a window of fixed length. In modeling studies, it might be appropriate to compute the covariance at a point in time using samples of the data observed over a window that is centered at the evaluation point. Alternately, in systems that might use the covariance for signal processing, the covariance must be estimated strictly from observations that occur
Figure 3.3: 3D scene of the three ray-tracing datasets, whose corresponding simulation results are presented in Figures 3.4, 3.5, and 3.6.

before the evaluation point. We refer to these two approaches for calculating the covariance as central and backward windowing, respectively.
In the following analyses, we quantify the difference between two covariance matrices using the correlation matrix distance (CMD) computed using [24]

$$CMD = 1 - \frac{\text{tr}(YZ)}{\|Y\|_F \|Z\|_F},$$  

(3.28)

where $\| \cdot \|_F$ and tr(·) represent the Frobenius norm and the trace, respectively, and the matrices $Y$ and $Z$ are covariances computed from (3.14)-(3.16).

We first consider the approximation made in the formulation of $\hat{R}$ in (3.15) that fields propagating in different departure/arrival angles are uncorrelated. For this analysis, the ray-tracing data is computed for the three-dimensional (3D) scene in Figure 3.0, where the line shows the movement of the receiver. Figure 3.2(a) shows the CMD between $Y = R$ from (3.14) and $Z = \hat{R}$ from (3.15) as a function of the window size used to estimate the expectations. Note that since both quantities use a sample mean for covariance estimation,
there is no practical distinction between central and backward windowing for this comparison. This result demonstrates that while the assumption of decorrelation is relatively inaccurate for small window sizes for this data, its validity increases for large window sizes. However, the window size required for this assumption to be valid in this environment is relatively large. In this specific scenario, the receiver moves along narrow streets (see Figure 3.0), meaning that multipaths within an angular range likely share common scatterers and therefore are more likely to be correlated. When averaging over a longer window, however, the impact of regions where this correlation is weaker becomes more pronounced. These observations suggest that the assumption that signals propagating at different angles are uncorrelated should be validated before its application in the environment of interest, particularly if high accuracy is desired.

Figure 3.2(b) shows similar results for $Y = \hat{R}$ from (3.15) and $Z = \tilde{R}$ from (3.16), thereby analyzing the impact of the assumption that only the phase of each multipath com-
ponent varies over the covariance estimation window. We emphasize for clarity that only (3.15) requires explicit evaluation of the expectation. The results for both central and backward windowing demonstrate that the assumption is valid for small windows but becomes decreasingly accurate with increasing window size, particularly for backward windowing. These results are intuitive, as application of (3.15) over large windows includes variation in the multipath angles and complex gains while application of (3.16) assumes variation of only the phases. The impact of this increases with backward windowing, since in this case the earliest points used in the sample mean for (3.15) are a significant distance away from the point of covariance evaluation.

Finally, Figure 3.2(c) plots the CMD between $\mathbf{Y} = \mathbf{R}$ from (3.14) and $\mathbf{Z} = \tilde{\mathbf{R}}$ from (3.16), therefore combining the two effects highlighted in the plots in Figure 3.2(a)-(b). Comparison with the other plots reveals that the impact of the assumption of decorrelation
with propagation angle dominates the error for small windows, while the impact of variation in the multipath angles and gains dominates the error for large windows.

The vertical line in Figure 3.2(c) shows the shadowing correlation length defined as the distance moved by the receiver at which the autocorrelation coefficient of the channel coefficient magnitude drops to a value of 0.5 (averaged over all receive antennas) [25, 26]. This distance provides some indication of how far the receiver must move for the large-scale shadowing of the channel to change appreciably. This is to a degree linked to the rate of change of the multipath structure. The fact that the CMD is approximately at its lowest point (for central windowing) when the window length is near the shadowing correlation length confirms this concept.

Figure 3.2 shows three additional scenarios for which the ray-tracing simulation is used to generate MIMO channel and covariance matrices. Figures 3.4(c), 3.5(c), and 3.6(c) plot the similar CMD between $Y = R$ from (3.14) and $Z = \tilde{R}$ from (3.16) as a function of window size for the three paths followed by the receiver. In Figure 3.4, since Rx 1 moves near
Figure 3.8: CMD using (3.28) between the covariances from (a) (3.14) and (3.15), (b) (3.15) and (3.16), and (c) (3.14) and (3.16) as a function of the covariance estimation window size for ray-tracing line of sight dataset 1.

A long building wall, the multipaths dominantly propagate parallel to the receiver motion path, and therefore the covariance is insensitive to window length. The shadowing correlation length is larger for this channel than fits on the plot axis and therefore is not shown. For Rx 2 (Figure 3.5), despite the fact that the shadowing correlation length (not shown) is approximately 600 wavelengths, the multipath angular characteristics clearly change more rapidly than the large-scale fading, resulting in the high sensitivity shown. This demonstrates that shadowing represents only one key characteristic of the multipath that may or may not be related to others (such as path angles or relative gains). For Rx 3 (Figure 3.6), the rate of change in the multipath structure apparently is similar to that of the large-scale fading since the CMD begins to increase dramatically for windows beyond the shadowing correlation length.

Figures 3.8 and 3.10 plot the same results for the two line of sight scenarios demonstrated in Figures 3.6 and 3.8, respectively. In Figures 3.8(a) and 3.10(a), the assumption
of decorrelation is relatively inaccurate for small window sizes since multipaths within an angular range likely share common scatterers and therefore are more likely to be correlated. When averaging over a longer window, however, the impact of regions where this correlation is weaker becomes more pronounced. In Figures 3.8(b) and 3.10(b), despite the fact that the shadowing correlation lengths are approximately 600 and 1000 wavelengths, respectively, the multipath angular characteristics clearly change more rapidly than the large-scale fading, resulting in the high sensitivity shown.

**Figure 3.9:** 3D scene of the three ray-tracing datasets, whose corresponding simulation results are presented in Figure 3.10.
Figure 3.10: CMD using (3.28) between the covariances from (a) (3.14) and (3.15), (b) (3.15) and (3.16), and (c) (3.14) and (3.16) as a function of the covariance estimation window size for ray-tracing line of sight dataset 2.

3.2 Clustered PAS

Section 3.1.1 not only demonstrates application of the covariance equations formulated in Section 3.1 to a practical scenario, but also helps to validate the assumption of uncorrelated scattering. With this as background, we now turn to simplified computation of the covariance for a specific scenario.

Experimental analysis and electromagnetic modeling have demonstrated that in many environments, the multipaths are clustered in angle, with the angular variation of the PAS for each cluster described by a common distribution function [18,27–32]. For such a multi-cluster PAS description and under the assumptions of 1) propagation confined to the horizontal plane and 2) linear antenna arrays whose patterns are isotropic within this horizontal plane, we can use a series expansion to analytically compute the integral in (3.9).
3.2.1 Existing Method for One-sided Covariance Construction

The method provided in [1] computes the one-sided covariance matrices appropriate for either the transmitter or receiver. However, to use the results in [1] to compute the covariance for a full MIMO system, one must combine the one-sided covariance matrices using, for example, a Kronecker product, an approach known to have deficiencies [10]. The advantage of the formulation that follows is that it demonstrates how to use the series expansions to compute the full covariance without resorting to this Kronecker form. Having such a series formulation for the full covariance provides a simple mechanism for computing covariance matrices that can be used in studies of MIMO systems.

The one-sided covariance for the receive array is given as [1]

\[
\xi_{m,p}^{(uv,u'v')} = \sum_{\ell=1}^{N_c} \alpha_{\ell} \left[ R_{XX} + jR_{XY} \right],
\]

where \( C(\phi) \) represents the functional form of the PAS and \( d_{r,mp} \) represents the distance between the \( m \)th and \( p \)th receive elements, with \( d_{r,nq} \) having an identical definition for the transmit array. The following sections provide series forms for these covariance matrix elements based on different functional forms for \( C(\phi) \).

**Uniform PAS**

We first consider the case where the cluster satisfies a uniform PAS centered at the angle \( \phi_0 \) and with width \( 2\Delta\phi \) described using [27]

\[
C(\phi) = u(\phi - \phi_0 + \Delta\phi) - u(\phi - \phi_0 - \Delta\phi),
\]

(3.30)
where \( u(\cdot) \) represents the unit step function. The real and imaginary parts of the one-sided covariance in (3.29) become

\[
R_{XX} = 2J_0(kd)\Delta \phi + 4 \sum_{\nu=1}^{\infty} \frac{J_{2\nu}(kd)}{2\nu} \cos(2\nu \phi_0) \sin(2\nu \Delta \phi),
\]

\[
R_{XY} = 4 \sum_{\nu=1}^{\infty} \frac{J_{2\nu+1}(kd)}{2\nu+1} \sin[(2\nu+1)\phi_0] \sin[(2\nu+1)\Delta \phi].
\]

(3.31)

**Truncated Gaussian PAS**

Experimental analysis has shown that clusters in many environments satisfy a Gaussian distribution [28, 29]. Because the PAS must be periodic in azimuth while the Gaussian function is non-zero over an infinite angular range, the function must be truncated. We add a degree of flexibility by allowing the truncation interval to be parametrically defined, leading to the representation

\[
C(\phi) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ \frac{(\phi - \phi_0)^2}{2\sigma^2} \right] \times [u(\phi - \phi_0 + \Delta \phi) - u(\phi - \phi_0 - \Delta \phi)].
\]

(3.32)

For this case, the real and imaginary parts of one-sided covariance in (3.29) become

\[
R_{XX} = J_0(kd) \text{erf} \left( \frac{\Delta \phi}{\sigma \sqrt{2}} \right) + \sum_{\nu=1}^{\infty} J_{2\nu}(kd) \cos(2\nu \phi_0) \exp \left( -2\sigma^2 \nu^2 \right)
\]

\[
\times \text{Real} \left[ \text{erf} \left( \frac{\Delta \phi - j2\nu \sigma^2}{\sigma \sqrt{2}} \right) - \text{erf} \left( \frac{-\Delta \phi - j2\nu \sigma^2}{\sigma \sqrt{2}} \right) \right],
\]

\[
R_{XY} = \sum_{\nu=1}^{\infty} J_{2\nu+1}(kd) \sin[(2\nu+1)\phi_0] \exp \left[ -2\sigma^2(\nu + \frac{1}{2})^2 \right]
\]

\[
\times \text{Real} \left[ \text{erf} \left( \frac{\Delta \phi - j(2\nu + 1) \sigma^2}{\sigma \sqrt{2}} \right) - \text{erf} \left( \frac{-\Delta \phi - j(2\nu + 1) \sigma^2}{\sigma \sqrt{2}} \right) \right],
\]

(3.33)

where \( \text{erf}(\cdot) \) is the error function, and \( \text{Real}(\cdot) \) indicates the real part.
Truncated Laplacian PAS

Finally, we consider the common scenario in which each cluster in the PAS satisfies a truncated Laplacian distribution given by [30,31]

\[
C(\phi) = \frac{1}{\sigma \sqrt{2}} \exp \left[ -\frac{\sqrt{2}|\phi - \phi_0|}{\sigma} \right] \times \left[ u(\phi - \phi_0 + \Delta \phi) - u(\phi - \phi_0 - \Delta \phi) \right].
\]  

(3.34)

For this case, the real and imaginary parts of one-sided covariance in (3.29) are given as

\[
R_{XX} = J_0(kd) \left[ 1 - \exp \left( \frac{\sqrt{2}\Delta \phi}{\sigma} \right) \right] + \frac{4}{\sigma \sqrt{2}} \sum_{\nu=1}^{\infty} \frac{J_{2\nu}(kd)}{(2\nu)^2 + \frac{2}{\sigma^2}} \cos(2\nu \phi_0) \\
\times \left\{ \sqrt{2} + e^{-\sqrt{2} \Delta \phi / \sigma} \left[ 2\nu \sin(2\nu \Delta \phi) - \frac{\sqrt{2}}{\sigma} \cos(2\nu \Delta \phi) \right] \right\},
\]

(3.35)

\[
R_{XY} = \frac{4}{\sigma \sqrt{2}} \sum_{\nu=1}^{\infty} \frac{J_{2\nu+1}(kd)}{(2\nu + 1)^2 + \frac{2}{\sigma^2}} \sin((2\nu + 1)\phi_0) \\
\times \left\{ \sqrt{2} + e^{-\sqrt{2} \Delta \phi / \sigma} \left[ (2\nu + 1) \sin((2\nu + 1)\Delta \phi) - \frac{\sqrt{2}}{\sigma} \cos((2\nu + 1)\Delta \phi) \right] \right\}.
\]

3.2.2 Full Covariance Construction

Equation (3.10) explicitly shows that each covariance matrix element consists of a sum of contributions \( \xi_{mn,pp}(uv,uv') \) each representing a different combination of polarizations at transmit and receive. We generally assume that the PAS response for each unique polarization combination has its own cluster description \( B_{uv,uv'} \). We therefore focus on the series expansion of a single term \( \xi_{mn,pp}(uv,uv') \), recognizing that we can compute the full covariance element by combining these contributions as in (3.10). If the PAS consists of \( N_c \) clusters, it can be expressed as

\[
B_{uv,uv'}(\Omega) = \sum_{\ell=1}^{N_c} \alpha_{\ell} C_{t,\ell}(\phi_t) C_{r,\ell}(\phi_r),
\]

(3.36)

where \( C_{t,\ell}(\phi_t) \) and \( C_{r,\ell}(\phi_r) \) represent the functional shape of the \( \ell \)th cluster in the transmit and receive angles, respectively.
The covariance contribution used in (3.10) becomes

\[ \xi^{(uv,u'v')}_{mn,pq} = \sum_{\ell=1}^{N_c} \alpha_{t,\ell} \mathcal{F}(d_{t,mp}, C_{t,\ell}) \mathcal{F}(d_{t,nq}, C_{t,\ell}), \]  

(3.37)

where \( \mathcal{F}(\cdot) \) represents a functional computed using

\[ \mathcal{F}(d, C) = \int \exp(jkd \sin \phi) C(\phi) d\phi. \]  

(3.38)

We next express the complex exponential in the integrand using a Fourier series representation, or

\[ \exp(jkd \sin \phi) = \sum_{\nu=-\infty}^{\infty} J_{\nu}(kd)e^{j\nu \phi}, \]  

(3.39)

where \( J_{\nu}(\cdot) \) represents the Bessel function of the first kind of order \( \nu \).

In the following, we consider evaluation of the integral in (3.38) for three different forms of the PAS cluster \( C(\phi) \). Because the contribution of each cluster in (3.37) is separated into a portion that depends on the transmit behavior and another that depends on the receive behavior, the clusters do not need to satisfy the same distribution at both ends of the link. Therefore, we present results generically for different forms of \( C(\phi) \), recognizing that these results can be combined as appropriate using (3.37) to construct the covariance.

**Uniform PAS**

Considering the case where the cluster satisfies a uniform PAS centered at the angle \( \phi_0 \) and with width \( 2\Delta \phi \), the integral in (3.38) becomes

\[ \mathcal{F}(d, C) = 2 \sum_{\nu=-\infty}^{\infty} \frac{J_{\nu}(kd)}{\nu} e^{j\nu \phi_0} \sin(\nu \Delta \phi). \]  

(3.40)
Truncated Gaussian PAS

For the case of truncated Gaussian PAS, the functional in (3.38) becomes

\[ F(d, C) = \frac{1}{2} \sum_{\nu=-\infty}^{\infty} J_{\nu}(kd) \exp \left( j \nu \phi_0 - \frac{\sigma^2 \nu^2}{2} \right) \times \left[ G_{\nu}(\Delta \phi) - G_{\nu}(-\Delta \phi) \right], \]  

(3.41)

\[ G_{\nu}(\Delta \phi) = \text{erf} \left( \frac{\Delta \phi - j \nu \sigma^2}{\sigma \sqrt{2}} \right), \]  

(3.42)

where erf(·) is the error function.

Truncated Laplacian PAS

Finally, the functional in (3.38) for a truncated Laplacian PAS is given as

\[ F(d, C) = \sqrt{\frac{2}{\sigma}} \sum_{\nu=-\infty}^{\infty} J_{\nu}(kd) \frac{e^{j \nu \phi_0}}{\nu^2 + \frac{2}{\sigma^2}} \times \left[ \sqrt{\frac{2}{\sigma}} + e^{-\sqrt{2} \Delta \phi / \sigma} L_{\nu}(\Delta \phi) \right], \]  

(3.43)

\[ L_{\nu}(\Delta \phi) = \nu \sin(\nu \Delta \phi) - \frac{\sqrt{2}}{\sigma} \cos(\nu \Delta \phi). \]  

(3.44)

3.2.3 Numerical Analysis

While the series forms for computing the integral in (3.38) appear convenient, their use in practical computations requires their evaluation with a limited number of terms in the sum. We therefore here study the computational error associated with such practical evaluations for a single cluster. When estimating \( F(d, C) \) using the series in (3.40), (3.41), and (3.43), the sum is evaluated using an odd number of terms \((M)\) for \(- (M - 1)/2 \leq \nu \leq (M - 1)/2\). Alternatively, when computing \( F(d, C) \) numerically using (3.38), we use a Lobatto quadrature numerical integration that adapts to ensure accuracy better than \(10^{-6}\) [33]. For clarity, we refer to the value of \( F(d, C) \) computed using these two techniques as \( \mathcal{S} \) and \( \mathcal{I} \), respectively. The normalized error caused by using a finite number of series terms to compute \( \mathcal{S} \) is

\[ \varepsilon = \left| \frac{\mathcal{S} - \mathcal{I}}{\mathcal{I}} \right|, \]  

(3.45)
Figure 3.11: Normalized error $\varepsilon$ computed from (3.45) and averaged over the range of $\phi_0$ as a function of the number of terms $M$ used in the series evaluation for $kd = 10$ and $AS = \pi/9$ for all three cluster distributions.

When showing results for the different cluster shapes, it is useful to have a consistent mechanism for quantifying the “width” of the cluster shape in angle. We therefore use the cluster angle spread (AS) defined as

\[
AS = \sqrt{\frac{\int \phi^2 C(\phi) d\phi}{\int C(\phi) d\phi}}.
\] (3.46)

For a uniform cluster shape, we solve for the value of $\Delta \phi$ that achieves the specified value of AS. For a truncated Gaussian or Laplacian cluster shape, we use $\Delta \phi = \pi$ and solve for the value of $\sigma$ that achieves the specified value of AS.

Figure 3.10 plots the error $\varepsilon$ as a function of the number of sum terms $M$ for the three PAS distributions averaged over values of $\phi_0$ on $[0, 2\pi)$ for $kd = 10$ and $AS = \pi/9$. While the value $kd = 10$ may seem large, we emphasize that here $kd$ does not necessarily represent the spacing between adjacent antennas but rather the spacing between the two
antennas for which the covariance element is being computed, and therefore we have chosen something large that might accommodate the distance between the two outermost array elements (as larger $kd$ tends to require more terms, as discussed below). For this case, the error for all cluster shapes becomes negligible for $M > 30$. It is interesting that the behavior for the Gaussian cluster shape differs from that for the other shapes, with higher error when $M$ is small but a more rapid decrease in the error with increasing $M$. This is because the Gaussian shape is the smoothest of the three functions considered, and therefore its energy is most concentrated in the lower-order Fourier terms. Figure 3.10 also includes a plot of this average error when $I$ represents the complex exponential on the left hand side of (3.39) and $S$ represents its Fourier series computed using the same $M$ terms. This comparison shows that the accuracy of the series approximation of the functional matches that for the truncated Fourier series approximation of the radiation pattern.

Based on this analysis, we now define the Required Number of Terms $M_0$ as the minimum value of $M$ such that $\varepsilon \leq 10^{-4}$ (0.01%). Figures 3.11(a)-(c) plot $M_0$ as a function of the antenna element spacing $kd$, cluster central angle $\phi_0$, and cluster AS, respectively. For each plot, only one parameter is swept, while the others assume the values $kd = 10$, $\phi_0 = \pi/6$, and AS = $\pi/9$. Perhaps most striking is the observation from Figure 3.11(a) that $M_0$ increases basically linearly with $kd$. This is intuitive, as a larger value of $kd$ in (3.39) produces higher frequency variation of the complex exponential with angle, requiring more terms in the Fourier series representation. The behavior with $\phi_0$ is more complicated to analyze, but the basic trend is that the required number of terms increases if the variation of the complex exponential with angle is more rapid over the extent of the PAS cluster. For example, for $\phi$ near $\phi_0 = \pm \pi/2$, $\sin \phi$ varies relatively slowly, and therefore the number of terms shown in Figure 3.11(b) near these values are reduced. Finally, as the angle spread increases, the low-frequency Fourier terms become more important, resulting in the slight decrease in $M_0$ with AS shown in Figure 3.11(c). Once again, for the Gaussian shape, fewer terms are needed generally. Once the AS grows beyond a certain value in Figure 3.11(c), the value of $M_0$ for the Gaussian shape increases, as the discontinuity created by the truncation becomes more prominent as the AS (and therefore value of $\sigma$) increases. Most importantly,
for this scenario using $M_0 > 60$ appears to accommodate any practical parameter value with high accuracy.

We emphasize that a key motivation for this development is that it allows formulation of the full MIMO covariance matrix without resorting to a Kronecker product representation using the one-sided covariance matrices computed using this same Fourier series approach [1]. To explore the difference between these two approaches, we assume a single electromagnetic polarization ($\theta$) so that $\hat{R}_{mn,pq} = \xi_{mn,pq}^{(\theta,\theta)}$. For simplicity, the PAS clusters at the transmitter and receiver have the same functional shapes and parameters except for the cluster central angles $\phi_0$ that are realized as uniformly distributed random variables on $[0, 2\pi]$, and the multi-
Figure 3.13: CMD using (3.28) between the full covariance and the covariance based on Kronecker model for an $N_r \times N_t$ MIMO system as a function of (a) $M$, (b) $N_c$, and (c) AS for all three PAS cluster distributions. Unswept parameters assume the values of $kd_r = kd_t = 2$, $\text{AS}_r = \text{AS}_t = \pi/9$, $N_r = N_t = 4$, and $N_c = 4$.

cluster distributions use $\alpha_\ell = 1$ for all values of $\ell$. The $N_r \times N_t$ MIMO system uses ULAs with $d_{r,mp} = (m-p)d_r$ and $d_{t,nq} = (n-q)d_t$. Figure 3.12 shows the CMD from (3.28) with $Y$ computed from the series expansions in this work and $Z$ can be achieved using (2.3) where $R_r$ and $R_t$ are the one-sided covariance matrices computed as defined in (3.20) and (3.21) using the approach in [1]. These results show that under most circumstances, the covariance constructed using the Kronecker product differs significantly from the covariance constructed without this approximation. This confirms what has been demonstrated previously [10], and we particularly note confirmation of the observation that the Kronecker model becomes less
accurate for larger array sizes. This error stems from the fact that the Kronecker model effectively forces the PAS to have the form $B(\Omega) = B_r(\Omega_r)B_t(\Omega_t)$, which means that the receiver sees the same functional shape of arrival energy regardless of the departure angle of the transmitted energy and which is obviously true for the model in (3.36) only for a single cluster ($N_c = 1$). Given this observation, however, if all clusters have large angle spreads so that each becomes essentially uniform over the entire range of transmit and receive angles, then this separable PAS form is effectively satisfied and the Kronecker model becomes accurate, as shown in Figure 3.12(c).

3.3 Summary

This work formulates a closed-form expression for the full MIMO spatial covariance matrix based on knowledge of the propagation environment PAS and the antenna radiation patterns. A simplified representation for the case where the multipath can be modeled using discrete plane waves is used with ray-tracing data to explore the suitability of different simplifying assumptions. Finally, for the case where the transmitter and receiver have linear arrays of isotropic radiators and the PAS can be described using clusters described using uniform, truncated Gaussian, and truncated Laplacian distributions, the integrations used in the formulation can be computed analytically, leading to a series form for the covariance matrix elements. Exploration of this formulation shows that the number of required series terms to achieve high accuracy depends on the cluster properties.
Chapter 4

Covariance-Based MIMO Beamforming for Different Linear Receiver Architectures

4.1 Introduction

Communicating over the point-to-point multiple-input multiple-output (MIMO) channel has been a topic of considerable interest over the past decade and now represents a well-understood problem. Specifically, development of transmit precoders and receive architectures based on available channel state information (CSI) or partial CSI such as the channel distribution information (CDI), often in the form of spatial covariance matrices, has received considerable attention [4]. The use of CDI, particularly at the transmitter, has some appeal, as transmit precoding based on CDI typically is valid for a longer time period, reducing the required frequency of feedback events [2, 3, 13, 35–37].

While the topic of CDI-based precoding has achieved some maturity, one subtle point has not been adequately addressed in the literature. Specifically, developed algorithms construct a transmit beamforming matrix from CDI that, when coupled with an optimal receiver capable of beamforming and successive decoding, maximize the upper bound on the achievable average rate [2, 3]. In contrast, when accurate CSI is available to the transmitter and receiver, simple transmit and receive beamforming can be implemented to maximize the rate. What appears to be missing, therefore, is a framework that allows construction of a transmit precoder based on CDI that maximizes the rate when the receiver is also constrained to implement simple beamforming based on either CSI or CDI. It is noteworthy that existing CDI-based transmit beamforming techniques use only the covariance matrix at the transmitter rather than the full channel spatial covariance, suggesting that it might be difficult to modify them to allow construction of transmit precoders that consider the receiver capabilities.
This work uses the basic concepts recently developed for MIMO broadcast channel signaling based on CSI at the transmitter and receiver [38,39] to develop a general framework for constructing transmit and receive beamformers that accommodates different receiver capabilities in the case of available CDI information at the transmitter and CSI information at the receiver of the point-to-point MIMO link. Simulations based on a simple channel model as well as on experimental MIMO channel data reveal that the approach is highly effective and is able to generate transmit precoders based on CDI whose performance when coupled with a linear beamforming receiver rivals that achieved with a system whose receiver allows optimal successive decoding. The analysis highlights the impact of practical implementation details on the communication performance.

4.2 Point-to-Point MIMO Signaling

Assuming the system model presented in Section 2.2, in the case of a sum power constraint we assume that the average transmit power is $P$, or $\mathbb{E}\{\text{tr}(xx^\dagger)\} = P$. This is achieved if the precoding is performed using

$$x = \sqrt{\frac{P}{\text{tr}(BB^\dagger)}} Bx_0. \quad (4.1)$$

Under these assumptions and considering that the transmit precoding vector is specified, for an optimal but not necessarily linear receiver the achievable rate of the system is given as [40]

$$C_{\text{opt}} = \log \left| I + \frac{\alpha_B^2}{\sigma_0^2} HH^\dagger \right|, \quad (4.2)$$

where $|\cdot|$ is the matrix determinant. Note that if the transmitter has no knowledge of channel information, then $BB^\dagger = \frac{P}{N_t} I$ and (4.2) simplifies to the traditional capacity expression for an uninformed transmitter, or

$$C_{\text{UTx}} = \log \left| I + \frac{P}{N_t\sigma_0^2} HH^\dagger \right|. \quad (4.3)$$
Alternatively, if the transmitter and receiver possess perfect CSI, then the optimal transmit and receive beamforming matrices are the dominant right and left singular vectors of the channel matrix $H$, with the power for each stream determined from the Waterfilling solution [4].

### 4.2.1 Existing CDI Precoding Methods

The key focus of this work is construction of the transmit precoding matrix $B$ based on the channel spatial covariance for different assumptions about the capability of the receiver. We therefore consider existing methods for establishing these precoders. Because the channel spatial covariance represents an average channel description, we start the development by taking the expectation of (4.2) to obtain the average rate [13,39] under the assumption that the entries of the channel matrix $H$ are wide-sense stationary random variables. By Jensen’s inequality, this average rate can be expressed as [41–43]

$$\mathcal{C}_{\text{opt}} = E\{C_{\text{opt}}\} \leq \log \left| I + \frac{N_z \alpha^2_B}{\sigma^2_{\eta}} B^\dagger R_t B \right|, \quad (4.4)$$

where $R_t = E\{H^\dagger H\}/N_t$ is the transmit covariance matrix and in the development we have rearranged the order of matrices as allowed under this determinant form. The accuracy of using Jensen’s inequality to achieve this upper bound has been studied previously [41–43], with the finding that the accuracy depends on the level of element correlation. However, the tightness of (4.4) as a bound is not as important as whether or not the value of $B$ that maximizes the bound also approximately maximizes the expectation of (4.2).

If we now take the eigenvector decomposition $R_t = U_t \Psi_t U_t^\dagger$, where $U_t$ is the unitary matrix of eigenvectors and $\Psi_t$ is the diagonal matrix of real, non-negative eigenvalues, then the upper bound on the average capacity is maximized by choosing $B = U_t \Gamma$ [2] where $\Gamma$ is a real, non-negative, diagonal matrix specifying the power allocation for each beamforming vector in (column of) $B$. Two techniques exist for constructing the matrix $\Gamma$. The first and most straightforward approach is to use the Waterfilling solution to maximize the upper bound on the capacity represented in (4.4) [44]. The second approach uses an iterative solution for the power allocation assuming that the channel spatial covariance is separable [3].
To simplify later discussions and comparisons, we refer to these two techniques as CDI Waterfilling and CDI Kronecker, respectively.

Specifically, if we assume that the receive statistics of the channel coefficients do not depend on the transmit statistics, then we can write $R = R_t^T \otimes R_t / \text{tr}(R_t)$, $R_t = E\{HH^\dagger\}/N_t$ is the receive covariance matrix, and $\{\cdot\}^T$ represents the transpose. Under this model and with the assumptions that $R_t = I$ and the receiver has perfect CSI, the optimal power allocation $\Psi_i = \Gamma_i^2$ for the $i$th beamforming vector ($i$th column of $B = \Gamma_i U_t$) can be computed using the procedure in [3].

\[
\Psi_i = \frac{\Psi_i F_i(\Psi)}{\sum_{j=1}^{N_t} \Psi_j F_j(\Psi)},
\]

\[
F_i(\Psi) \triangleq E \left\{ \frac{\Psi_{t,i} z_i^\dagger Q_i^{-1} z_i}{1 + \Psi_i \Psi_{t,i} z_i^\dagger Q_i^{-1} z_i} \right\},
\]

\[
Q_i \triangleq \sigma_n^2 I + \sum_{j=1}^{N_t} \Psi_j \Psi_{t,j} z_j z_j^\dagger,
\]

where $z_i$ is an $N_t \times 1$ vector containing zero-mean, unit-variance complex Gaussian random variables.

4.3 Iterative Beamforming (IBF) Framework

As mentioned, if both nodes possess perfect CSI, then linear beamforming at the transmitter and receiver achieves the optimal capacity. Alternatively, if sub-optimal transmit precoding (including no precoding) is used, then the achievable capacity is reduced and the receiver must combine linear minimum mean squared error (MMSE) beamforming with successive interference cancellation (SIC) to realize the achievable rate in (4.2) [40,45,46]. The prior techniques developed for CDI-based precoding construct the transmit beamformers under the assumption of MMSE-SIC at the receiver, and they therefore are inappropriate for use when the receiver uses only linear beamforming based on CSI or CDI.

This observation motivates development of an alternate framework for construction of transmit and receive beamforming matrices based on the receiver capabilities in the case of available CDI at transmitter and CSI at receive CSI (CDIT, CSIR). As the approach is
based on the regularized channel inversion algorithm developed for the MIMO broadcast channel \[38,39\], we refer to it as \textit{Iterative Beamforming Framework} (IBF).

In developing this framework, we must add to our system model that the receiver applies the $N_r \times K$ receive beamforming matrix $W$ to the received signal, or $y_0 = W^\dagger y$. We take the expectation of achievable rate \[38\] and apply Jensen’s inequality to obtain \[13\]

$$\overline{C} = \mathbb{E}\left\{C\right\} \leq \bar{C} = \sum_{k=1}^{K} \log (1 + \mathbb{E}\{\text{SINR}_k\}),$$  

(4.8)

$$\text{SINR}_k = \frac{|w_k^\dagger H b_k|^2}{\sigma_n^2 w_k^\dagger w_k/\alpha_B^2 + \sum_{i \in L_k} |w_k^\dagger H b_i|^2},$$  

(4.9)

where $b_k$ and $w_k$ respectively represent the $k$th columns of $B$ and $W$ and (4.9) represents the signal-to-interference-plus-noise ratio (SINR) for the $k$th data stream. As discussed in Appendix A.3, we further make the approximation $\mathbb{E}\{\text{SINR}_k\} \approx \mathbb{E}\{n_k\}/\mathbb{E}\{d_k\} = \bar{n}_k/\bar{d}_k$, where $n_k$ and $d_k$ respectively represent the numerator and denominator of (4.9). This approximation results in

$$\bar{C} = \sum_{k=1}^{K} \log \left(1 + \frac{\bar{n}_k}{\bar{d}_k}\right),$$  

(4.10)

$$\bar{n}_k = b_k^\dagger A_k b_k,$$  

(4.11)

$$\bar{d}_k = \sigma_n^2 w_k^\dagger w_k/\alpha_B^2 + \sum_{i \in L_k} b_i^\dagger A_k b_i,$$  

(4.12)

where $A_k = \mathbb{E}\left\{H^\dagger w_k w_k^\dagger H\right\}$.

Because the second term in (4.12) represents the interference, the set $L_k$ of integers used in the sum depends on the capability of the receiver. Specifically, if only linear receive beamforming is allowed, the $k$th stream experiences interference from all other streams, or $L_k = \{1 \leq i \leq K, i \neq k\}$. If, on the other hand, the receiver uses SIC, then we assume that the $k$th stream experiences interference only from streams that have not yet been detected. For simplicity in notation, we assume that the receiver detects streams in numerical order of their indices so that $L_k = \{k < i \leq K\}$. 

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4.3.1 CDIT, CSIR

One practical scenario is to construct the transmit beamformer based on CDI but the receive beamformer based on CSI. This allows reduction in the frequency of feedback of the beamforming matrix $\mathbf{B}$ to the transmitter but recognizes that the receiver can more easily maintain accurate CSI. Our objective is to find the beamforming matrix $\mathbf{B}$ that maximize the upper bound rate given in (4.10). To begin, we take the gradient of (4.10) with respect to $b_j^*$. This operation leads to

$$
\frac{\partial \hat{C}}{\partial b_j^*} = \sum_{k=1}^{K} \frac{\bar{n}_k^{(j)}}{d_k} - \frac{\bar{n}_k}{d_k} \left[ \frac{\bar{n}_k^{(j)} + \bar{d}_k^{(j)}}{\bar{n}_k + \bar{d}_k} \right],
$$

where

$$
n_k^{(j)} = \frac{\partial \bar{n}_k}{\partial b_j^*} = \begin{cases} A_j b_j & j = k \\ 0 & j \neq k, \end{cases}$$

and

$$
d_k^{(j)} = \frac{\partial \bar{d}_k}{\partial b_j^*} = \begin{cases} (\sigma^2 \eta/P) b_j & j \notin \mathcal{L}_k \\ (\sigma^2 \eta/P) b_j + A_k b_j & j \in \mathcal{L}_k. \end{cases}$$

In this analysis, we have used the fact that the receive beamformer weights are normalized such that $\mathbf{w}_k^\dagger \mathbf{w}_k = 1$. Substitution of (4.14) and (4.15) into (4.13) leads to

$$
\frac{\partial \hat{C}}{\partial b_j^*} = \frac{1}{d_j} A_j b_j - G_j b_j,
$$

and

$$
G_j = \sum_{k=1}^{K} \frac{\bar{n}_k}{d_k (\bar{n}_k + d_k)} \left[ \frac{\sigma^2 \eta}{P} \mathbf{I} + \xi_k^{(j)} A_k \right],
$$

where

$$
\xi_k^{(j)} = \begin{cases} 1 & j \in \hat{\mathcal{L}}_k \\ 0 & j \notin \hat{\mathcal{L}}_k, \end{cases}
$$

43
where $\hat{L}_k = \{1 \leq i \leq K\}$ for receive beamforming and $\hat{L}_k = \{k \leq i \leq K\}$ for receive beamforming with SIC.

Given this form for the gradient, we can set (4.16) to zero with the goal of solving for the beamforming vector $b_j$. However, we recognize that the beamforming matrix $B$ appears explicitly within $\bar{n}_k$ and $\bar{d}_k$, and therefore directly solving for $b_j$ is difficult. Instead, we can rearrange the resulting equation into the form

$$b_j = \frac{1}{d_j} G_j^{-1} A_j b_j,$$

which can be solved iteratively to obtain a solution for $b_j$, $1 \leq j \leq K$. It is important to notice that this iterative computation of $b_j$ depends on the value of receive beamformers $W$ through $A$, and we therefore consider two different approaches to determine $W$ in each iteration.

**Channel Covariance Approach**

One approach to determine $W$ is to assume a single value of $W$ over the time period during which the transmit beamformer $B$ based on CDI is assumed valid. Thus, a similar procedure can be applied to determine the receive beamformers $W$ in each iteration. However, here we evaluate $\bar{n}_k$ and $\bar{d}_k$ as

$$\bar{n}_k = w_k^\dagger \tilde{A}_k w_k,$$

$$\bar{d}_k = \sigma^2 q w_k^\dagger w_k / \alpha^2 B + \sum_{i \in \mathcal{L}_k} w_i^\dagger \tilde{A}_i w_k,$$

where $\tilde{A}_k = \mathbb{E}\left\{H b_k b_k^\dagger H^\dagger\right\}$. Specifically, taking the gradient of (4.10) with respect to $w_j^*$ and setting the result equal to zero leads to the equation

$$\tilde{G}_j w_j = \tilde{A}_j w_j,$$
where

\[
\tilde{G}_j = \frac{\tilde{n}_j}{\bar{n}_j + d_j} \left[ \frac{\sigma^2_\eta}{\sigma^2_B} I + \sum_{k \in \mathcal{L}_j} \tilde{A}_k \right].
\] (4.23)

Straightforward solution of (4.22) leads to the iterative form

\[
w_j = \tilde{G}_j^{-1} \tilde{A}_j w_j.
\] (4.24)

Appendix A.4 shows how the matrices \(A_k\) and \(\tilde{A}_k\) can be computed based on knowledge of the beamforming matrices \(B\) and \(W\) and the full covariance matrix \(R\). Finally, examination of these equations reveals that (4.19) depends on \(W\) and (4.24) depends on \(B\). This implies that in practice, we must first initialize \(B\) and \(W\) and then perform an iterative computation in which we construct \(B\) using (4.19) and \(W\) using (4.24) once per iteration until both beamformer matrices have converged [39].

After calculation of \(B\), the receiver considers the available CSI information, and for each estimated channel \(H\), a new value of \(W\) can be obtained considering the achieved value of \(B\) and MMSE (SIC) receiver as

\[
w_j = \left[ \frac{\sigma^2_\eta}{\sigma^2_B} I + \sum_{k \in \mathcal{L}_j} H_{b_k} b_k^\dagger H_k^\dagger \right]^{-1} H_{b_j}.
\] (4.25)

**Sample Mean Approach**

In practice, the receiver computes a new value of \(W\) (using (4.25) for example) each time it estimates a new channel matrix \(H\), and therefore it is arguably more accurate to estimate \(A_k\) using a sample mean with the true values of \(H\) and \(W\) instead of full covariance matrix. Thus, this approach takes advantage of CSI information even in the calculation of \(B\), while the channel covariance approach only considers CDI information to construct \(B\). Naturally, the sample mean approach is more computationally costly, since we must compute this sample mean at each step in the iterative computation. An analysis of the performance impact of using these two different approaches for computation of \(A_k\) appears in the simulation results shown later.
It is important to emphasize here the enabling feature of the generalized framework for beamformer computation in this scenario. As mentioned previously, transmit precoders based on the eigenvectors of $\mathbf{R}_t$ are designed under the assumption that the receiver uses MMSE beamforming with SIC, and application of these precoders with a simple MMSE receiver provides poor performance (an observation we base on our own simulations). In contrast, implementation of the proposed IBF algorithm under the assumption of MMSE beamforming at the receiver allows establishment of a precoder appropriate for this scenario.

### 4.3.2 CDIT, CDIR

This technique for constructing the transmit and receive beamforming matrices is easily extended to the case where these beamformers should be constructed based on CDI at both the transmitter and receiver. In this case, we use the value of $\mathbf{W}$ obtained from the Channel Covariance Approach for the receive beamformers rather than simply use $\mathbf{W}$ to construct the transmit beamformers.

### 4.3.3 Beamformer Initialization

One important detail concerning the implementation of the generalized beamforming algorithm is the initialization of the beamforming matrices $\mathbf{B}$ and $\mathbf{W}$. One approach is to randomly initialize these matrices, usually as unitary matrices obtained from the singular value decomposition of a random matrix. Recognizing that convergence of the iterative equations to the beamformers that achieve the global optimum rate is sensitive to the initialization of these matrices, we should run the procedure with at least 20 random initializations and choose the result that achieves the highest rate. A more sophisticated approach can lead to essentially the same results. Specifically, for the practical case of CDIT and CSIR, we use the initializations $\mathbf{B} = \mathbf{U}_t$ (eigenvectors of $\mathbf{R}_t$) and $\mathbf{W} = \mathbf{I}$. We demonstrate the effectiveness of this practical initialization in the computational examples to follow.
4.4 Results

4.4.1 Simulation Approach

The performance of the beamforming algorithms detailed in this work are demonstrated here through a sequence of simulations based on realistic MIMO channels. In all cases, the MIMO channel matrix $\mathbf{H}$ is generated and normalized so that

$$\|\mathbf{H}\|_F^2 = N_r N_t,$$  \hspace{1cm} (4.26)

where $\| \cdot \|_F$ represents the Frobenius norm. With this normalization, the ratio $P/\sigma_n^2$ represents the single-input single-output (SISO) signal-to-noise ratio (SNR) [47].

Most of the analysis is performed with MIMO channels obtained from an experimental measurement campaign. In these experiments, the transmitter and receiver each have linear arrays of monopoles with half-wavelength element spacing. The system sends probing signals at a center frequency of 2.45 GHz and uses the signals to estimate the MIMO channel coefficients. The simulations use an adjacent subset of antennas in the transmit and receive array to generate the $N_r \times N_t$ channel matrix. The transmitter is held stationary while the receiver moves at a constant velocity of approximately 30 cm/s, and channels are sampled at an interval of 2.5 ms corresponding to a distance moved by the receiver of 0.0062 $\lambda$, where $\lambda$ is the wavelength at the signal carrier frequency. All measurements are taken within an open area between several buildings on the Brigham Young University campus with the transmitter and receiver positioned on either side of a dense stand of trees. Details concerning the measurement system used to collect the channel matrices can be found in [48,49].

While the experimental data is highly valuable in the analysis, its use does not allow systematic evaluation of the effect of one key channel characteristic, namely the channel directivity or equivalently multipath richness, on the performance. More specifically, in a point-to-point channel, we expect that if the multipath propagating signal consists of a relatively small number of components, then transmit beamforming can be effective in focusing the radiated power into the directions with the highest gain to the receiver. We therefore use a two-ring channel model to analyze the impact of multipath richness on the performance of different transmit precoding techniques. In this model, the transmitter and
receiver each lie within distinct circles of radius 40 \( \lambda \). A fixed number of scatterers lie on each circle, with the angle of each scatterer location being specified as a uniform random variable on \([0, 2\pi)\). Scatterers on the transmit and receive circles are paired, and each propagation path has an angle of departure and angle of arrival defined by the locations of each scatterer pair. Each multipath is also given a complex gain specified as a zero-mean, unit-variance circularly symmetric complex Gaussian random variable. The \( N_t \)-element linear transmit array remains stationary at the center of its circle while the \( N_r \)-element linear receive array moves along a straight line within its circle. Normalized channel matrices are easily computed in this two-dimensional propagation model using established techniques [47].

Because transmit precoding is most effective for low SNR environments, the simulations use a total transmit power of \( P = 1 \) and noise variance \( \sigma^2 = 1 \) for an SNR of 0 dB. Unless otherwise specified, \( N_t = N_r = K = 4 \). The covariance is estimated using a sample mean to approximate \( \mathbf{R} = E \{ \text{vec} (\mathbf{H}) \text{vec} (\mathbf{H})^\dagger \} \) over a specified window size, and

![Figure 4.1: Capacity averaged over all experimentally-obtained channel matrices for CSIT, CSIR and computed using the Waterfilling and iterative algorithms as a function of the number of antennas in the transmit and receive arrays (\( N_r = N_t \)).](image-url)
this covariance is used to construct the transmit beamforming matrix $\mathbf{B}$ that is fed back to the transmitter at a regular feedback interval. In this way, the beamforming matrix is constructed from one set of channels and then applied to communicate over a different set of channels. Unless otherwise indicated, the estimation window and feedback intervals used are $1\lambda$ and $2.5\lambda$, respectively. Both feedback interval and window size are considered as number of sample multiply by sample interval of $0.001\lambda$.

### 4.4.2 CSIT, CSIR

To begin, we provide a simple demonstration that the iterative approach for beamformer computation converges to an optimal solution. Specifically, for each of the experimentally-obtained channel matrices, we compute the capacity using the Waterfilling solution and based on beamformers obtained with the iterative algorithm assuming CSIT and CSIR (without SIC, as this is unnecessary for this case). Figure 4.0 shows the rate averaged over all channel realizations obtained using these two methods as a function of the number of antenna
elements in the transmit and receive arrays. As can be seen, both methods achieve the same performance, although it is interesting to note that the actual beamforming solutions are not always the same.

4.4.3 CDIT, CSIR

We next compare the performance of the IBF beamforming technique with that of the previously-reported methods for beamforming based on CDIT [3, 41, 44]. To make the comparison fair, the IBF algorithm is implemented assuming MMSE-SIC at the receiver. Figure 4.1 plots the rate averaged over the ring model realizations as a function of the number of multipaths included in the model for the three transmit beamforming algorithms. As can be seen, as the directivity of the propagation channel decreases (more paths), transmit beamforming becomes less effective, resulting in a decrease in the rate. More importantly, however, all three algorithms achieve essentially the same rate despite the fact that the IBF beamforming algorithm involves the extra approximation $E \{ \text{SINR}_k \} \approx E \{ n_k \}/E \{ d_k \}$.

Figure 4.3: Average rate achieved using different transmit precoders for CDIT, CSIR (MMSE-SIC) as a function of the covariance estimation window size for the experimental data.
Finally, the considerable difference between the rate for an uninformed transmitter and that for CDI-based beamforming illustrates the improvement offered by transmit precoding, although the benefit is reduced as the channel directivity decreases.

Figure 4.2 plots the same results based on the experimental data as a function of the window size used to estimate the covariance. This data shows that in this case the results for the different precoders provide slightly different results, particularly for larger window sizes. The small degradation observed for the iterative beamforming likely stems from decreased accuracy in the approximation \( \text{E}\{\text{SINR}_k\} \approx \text{E}\{n_k\}/\text{E}\{d_k\} \) with increasing window size and error incurred by constructing the matrix \( \mathbf{A}_k \) from the channel spatial covariance, as discussed in Section 4.3.1.

Figure 4.3 plots the average rate achieved for the IBF beamforming algorithm assuming random initialization and initialization with the eigenvectors of \( \mathbf{R}_t \) as a function of the interval at which the transmit beamforming vectors are fed back. The results are obtained
from the experimental data. As can be seen, the two initialization techniques achieve the same results. Since the algorithm must be run multiple times when the initialization is random, initializing with the eigenvectors clearly achieves significant reduction in algorithmic computational cost without degrading performance. The results in Figure 4.3 also show that as the interval between transmit beamformer feedback increases, the performance decreases. Since this experimental data is not wide-sense stationary, the covariance changes with the receiver motion, and therefore reduced feedback frequency results in use of increasingly outdated transmit beamforming vectors.

Figure 4.3 further compares the performance achieved when the receiver uses MMSE-SIC to that obtained using MMSE beamforming alone, where we emphasize that the latter case cannot be accommodated by the other algorithms for computing transmit beamformers based on CDIT. While there is a reduction in achievable rate for the sub-optimal receiver, the difference between the two cases is relatively small, demonstrating that the IBF beam-
forming algorithm allows generation of transmit precoding vectors that achieve near-optimal performance with a simple MMSE receiver. Figure 4.4 plots the performance under the same scenario but as a function of the covariance estimation window size, comparing the results obtained when calculating $A_k$ from the channel spatial covariance and from a sample mean, as discussed in Section 4.3.1. While the latter is slightly more accurate, its overall impact on performance is relatively small.

Figure 4.5 plots the performance as a function of the number of transmit antennas $N_t$ when the number of receive antennas is $N_r = 4$. When $N_t \leq N_r$, the number of allowable data streams communicated grows with the size of the transmit array, which accounts for the nearly linear growth in rate observed. However, when $N_t > N_r$ such that the number of data streams is limited by the number of receive antennas, we observe that the rate continues to increase with $N_t$. This occurs because the extra control over the transmit beamforming enabled by the larger transmit array allows the precoding to better exploit

Figure 4.6: Average rate achieved using IBF beamforming for CDIT, CSIR (MMSE-SIC and MMSE) as a function of the number of transmit antennas $N_t$ for $N_r = 4$ receive antennas using the experimental data.
the multipath propagation. Also, while Figure 4.5 reinforces the fact that the MMSE-SIC receiver outperforms the MMSE receiver by a small margin, the results also show that when the transmitter has increased control (larger $N_t$), the performance benefit of the optimal receiver also increases.

4.4.4 CDIT, CDIR

Finally, we consider application of the technique assuming that both the transmit and receive beamforming vectors are computed using CDI. Note that this represents another situation to which other algorithms for computing transmit beamformers based on CDIT cannot be applied. In this analysis, the transmitter and receiver beamforming vectors are updated at the feedback interval. Figure 4.6 plots the average rate as a function of the covariance estimation window size for different feedback intervals. For small estimation window size, the CDI essentially contains the information of the CSI, and the performance
is high particularly for frequently updated beamformers (small feedback interval). When the feedback interval is large, however, the performance is dominated by the fact that the beamformers are outdated and therefore becomes insensitive to the covariance estimation.

Figure 4.7 compares the performance resulting from receive beamformers based on CDI and CSI using MMSE as a function of the feedback interval. Naturally, using CSI at the receiver provides notably improved performance. The third curve in this plot shows the behavior when the receive beamformers based on CSIR are updated only at the feedback interval. This curve reveals that even with stale CSI, the performance obtained using receive beamformers based on CSIR is as good or often better than that achieved using beamformers based on CDIR.

4.5 Summary

This work proposes an iterative algorithm that specifies transmit and receive beamformers based on different capabilities of the receiver in the case of available CDI informa-

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**Figure 4.8:** Average rate achieved using IBF beamforming for CDIT, CDIR and CDIT, CSIR as a function of the feedback interval for the experimental data.
tion in the transmitter and CSI information in the receiver. Simulations using the method in realistic MIMO channels show that its performance matches that obtained with existing algorithms when used in conjunction with optimal receiver architectures but that it allows specification of near-optimal transmit precoding for simple MMSE receivers. The simulations further demonstrate the impact of different implementation details and parameters on communication performance.
Chapter 5

Iterative Beamforming for Point-to-Point MIMO Communication

5.1 Introduction

In a point-to-point multiple-input multiple-output (MIMO) system in which both the transmitter and the receiver have accurate channel state information (CSI), optimal throughput can be realized using simple linear beamforming at both nodes [4]. However, if the transmitter does not possess accurate CSI, the receiver must resort to nonlinear successive decoding in addition to beamforming to maximize the throughput. These observations highlight the critical link between the transmit precoder and the receiver capabilities. Because most studies of transmit precoding focus on maximization of the mutual information, they inherently assume a receiver capable of nonlinear detection. For example, under the assumption that the total transmit power is constrained and the transmit precoding is based on channel distribution information (CDI) [2, 3], transmit precoding algorithms that maximize the rate assuming a linear receiver do not exist. Chapter 4 mainly considers this specific case and finds the optimal beamforming for a linear receiver architecture.

While in Chapter 4 the problem of finding optimal beamforming has been solved considering only a sum power constraint at the transmitter, this chapter reconsiders the problem to accommodate other power constraints. In the case of the per-antenna power constraint that limits the average power transmitted from each individual antenna, all previously developed approaches assume the transmitter has perfect CSI and that the receiver is capable of nonlinear processing [5, 6, 50–52]. In this case, no algorithms exist for generating the rate-optimal transmit precoding vectors under a per-antenna power constraint based on 1) transmit CSI with a linear receiver or 2) transmit CDI for any type of receiver. Table 5.1 presents a summary of the availability of precoding algorithms as a function of these different assumptions.
Table 5.1: Existing transmit precoding algorithms

<table>
<thead>
<tr>
<th>Transmitter Power Constraint</th>
<th>Receiver Architecture</th>
<th>Channel Information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CSI</td>
</tr>
<tr>
<td>Sum</td>
<td>Linear</td>
<td>[4]</td>
</tr>
<tr>
<td></td>
<td>Nonlinear</td>
<td>[7,8]</td>
</tr>
<tr>
<td>Per-Antenna</td>
<td>Linear</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>Nonlinear</td>
<td>[5,6]</td>
</tr>
</tbody>
</table>

The objective of this work is to adapt the basic concepts recently developed for MIMO broadcast channel signaling [38,39] to develop a general framework for constructing transmit and receive beamformers for a point-to-point MIMO link that accommodates different types of channel information (CSI or CDI) available at the transmitter, different receiver capabilities, and different power constraints. When the precoding is based on CDI, the technique uses the full spatial covariance matrix as opposed to the one-sided transmit covariance used in prior work [2,3] so that it can properly accommodate the receiver architecture. Simulations based on a simple channel model as well as on experimental MIMO channel data reveal that the approach is highly effective and is able to generate transmit precoders whose performance when coupled with a linear beamforming receiver rivals that achieved with a system whose receiver allows optimal successive decoding.

5.2 Point-to-Point MIMO Signaling

5.2.1 Power Constraints

Assuming the system model presented in Section 2.2 and with $\mathbf{B}$ specified, for an optimal receiver the achievable system rate is [40]

$$C_{\text{opt}} = \log \left| I + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{B}^\dagger \mathbf{H}^\dagger \right|,$$

(5.1)

where $| \cdot |$ is the matrix determinant. If the transmitter has no knowledge of the channel information, then $K = N_t$ and $\mathbf{B}^\dagger = (P/N_t) \mathbf{I}$ where $P$ is the total transmit power, and (5.1) becomes the capacity for an uninformed transmitter. When developing an algorithm for
constructing the transmit precoding matrix $\mathbf{B}$ that maximizes the achievable communication rate, we must consider constraints on the transmit power. Most work in MIMO signaling assumes a sum-power constraint (SPC) that limits the total power transmitted from all antennas and that can be written as $\mathbb{E}\{\mathbf{x}^\dagger\mathbf{x}\} = \mathbb{E}\{\text{tr}(\mathbf{x}\mathbf{x}^\dagger)\} = \text{tr}(\mathbf{BB}^\dagger) = P$.

While the SPC is common, in most radios the output power of each antenna is constrained by the power amplifier (PA) capabilities. If the SPC is used, then each PA must be able to accommodate the total power $P$ despite the fact that on average, it will only have to accommodate its proportional share (typically $P/N_t$). Therefore, algorithms have been developed to accommodate a per-antenna power constraint (PAPC) where the average power transmitted from each antenna can be uniquely specified \[5,6,50–52\]. If $P_i$ represents the average power radiated from the $i$th transmit antenna, we have $\mathbb{E}\{x_i x_i^\dagger\} = \mathbb{E}\{\hat{\mathbf{b}}_i^\dagger \mathbf{x}_0 \mathbf{x}_0^\dagger \hat{\mathbf{b}}_i\} = \hat{\mathbf{b}}_i \hat{\mathbf{b}}_i^\dagger = P_i$, where $x_i$ and $\hat{\mathbf{b}}_i$ represent the $i$th element of $\mathbf{x}$ and $i$th row of $\mathbf{B}$, respectively.

5.2.2 Existing Precoding Methods

Our objective is to develop a framework for generating transmit precoding vectors under the assumptions in Table 5.1 where currently no algorithms exist. In preparation for this development, it is instructive to first review existing algorithms for two of the scenarios listed. Here and throughout this work, CSI used for precoding or decoding is assumed to be perfect.

CDI-Based Precoding with SPC

Two existing algorithms for finding the optimal beamformers based on CDI available at transmitter with SPC in the presence of non-linear receiver, namely CDI Kronecker and CDI Waterfilling are discussed in detail in Section 4.2.1. As mentioned in Chapter 4, the former assumes the channel matrix entries are wide-sense stationary random variables and take the expectation of (5.1) \[13,39\], by Jensen’s inequality this average rate becomes

$$
\overline{C}_{\text{opt}} = \mathbb{E}\{C_{\text{opt}}\} \leq \log \left| I + \frac{N_t}{\sigma^2} \mathbf{B}^\dagger \mathbf{R}_t \mathbf{B}^\dagger \right|
$$

(5.2)
CSI-Based Precoding with PAPC

Once again, past work on transmit precoding with a PAPC maximizes the mutual information, which inherently assumes the availability of nonlinear detection at the receiver. Furthermore, only CSI-based precoding has appeared in the literature. While much of the work has been focused on the multiuser MIMO downlink channel [50–52], a recent algorithm has been proposed for the point-to-point MIMO channel considered here [5,6]. This framework formulates an iterative approach that finds the transmit beamformers to maximize the capacity, with the algorithm referred to as the Drop-Rank method, the algorithm of which is presented in Table 5.2 [6]. However, there does not appear to be a straightforward mechanism for extending this to linear receivers or to CDI-based precoding.

5.3 Iterative Beamforming

When choosing transmit precoders to maximize the achievable rate in (5.1) or its upper bound in (5.2), in most cases the receiver must combine linear minimum mean squared error (MMSE) beamforming with nonlinear SIC to realize the rate [40,45,46]. Because the prior work for CDI-based precoding under a SPC or CSI-based precoding under a PAPC focuses on capacity maximization, the transmit precoders are derived under the assumption of MMSE-SIC at the receiver, and they therefore are inappropriate for use when the receiver uses only linear beamforming based on CSI (or CDI).

This observation motivates development of an alternate framework for constructing transmit and receive beamforming matrices based on the available channel information, receiver capabilities, and power constraint. The iterative beamforming (IBF) approach proposed in this work is based on an algorithm developed for CSI-based beamforming in the MIMO broadcast channel [53]. In developing this framework, we incorporate the receive beamforming into the expression for the achievable rate to obtain [38]

\[
C = \sum_{k=1}^{K} \log (1 + \text{SINR}_k),
\]

\[
\text{SINR}_k = \frac{|w_k^\dagger H b_k|^2}{\sigma_n^2 + \sum_{i \in L_k} |w_i^\dagger H b_i|^2} = \frac{n_k}{d_k},
\]
Table 5.2: Drop-Rank Algorithm: Iterative search for $Q = BB^\dagger$ when $N_t \leq N_r$

**Drop-Rank Algorithm**

Iterative Phase:
- Initialize $\tilde{D}$ diagonal $> 0$, $P$ diagonal $> 0$, $\epsilon > 0$.
- Initialize $\Delta = 1 + \epsilon$. (loop terminating variable)
- $K = V_H \Sigma_n V_H^\dagger$ and $\tilde{K} = K^{-1}$ where
  - $V_H$ is a $N_t \times N_t$ unitary matrix of right singular vectors of $H$
  - $\Sigma_n$ is a $N_t \times N_t$ diagonal matrix containing the real singular values of $H$.

Iterative Phase: while ($\Delta > \epsilon$) do
- Form $F = K \tilde{D} \tilde{K}^\dagger - I_n$
- Perform the eigenvalue decomposition $F = U_F \Lambda_F U_F^\dagger$.
- Let $k$ be the number of non-positive eigenvalues of $F$.
- Form $S = -U_F^k \Lambda_F^k U_F^k \dagger$ where
  - $\Lambda_F^k$ is the $k \times k$ diagonal matrix of all the $k$ non-positive eigenvalues of $F$
  - $U_F^k$ consists of the corresponding $k$ eigenvectors.
- Form $Z = \tilde{K} S \tilde{K}^\dagger$.
- Form $Q = \tilde{D} - \tilde{K} \tilde{K}^\dagger + Z$.
- Form $\tilde{D} = \tilde{D} + P - \text{diag}(Q)$.
- Compute $\Delta = |\text{tr} (D[Q - P])|$.

where (5.4) represents the signal-to-interference-plus-noise ratio (SINR) for the $k$th data stream.

Because the second term in the denominator of (5.4) represents the interference, the set $\mathcal{L}_k$ of integers used in the sum depends on the capability of the receiver. Specifically, if only linear receive beamforming is allowed, the $k$th stream experiences interference from all other streams, or $\mathcal{L}_k = \{1 \leq i \leq K, i \neq k\}$. If, on the other hand, the receiver uses SIC, then we assume that the $k$th stream experiences interference only from streams that have not yet been detected. For simplicity in notation, we assume that the receiver detects streams in numerical order of their indices so that $\mathcal{L}_k = \{k < i \leq K\}$. Observations regarding the impact of this ordering are provided in Section 5.3.4. Throughout the development, we use
the notation CSIT and CSIR (CDIT and CDIR) to indicate that CSI (CDI) is available at
the transmitter and receiver, respectively.

5.3.1 CSIT, CSIR

The IBF algorithm assuming precoding based on CSIT is derived by maximizing a
cost function consisting of the rate in (5.3) and a Lagrange multiplier term incorporating
the power constraint. The detailed mathematical developments for the SPC and the PAPC
are provided in Appendices A.1 and A.2, respectively. For both power constraints, the \( j \)
transmit beamformer obtained from this derivation is written as

\[
\mathbf{b}_j = \frac{1}{d_j} \mathbf{G}_j^{-1} \mathbf{A}_j \mathbf{b}_j, \tag{5.5}
\]

\[
\mathbf{G}_j = \Lambda'_f + \sum_{j \in \hat{L}_k} \frac{n_k}{d_k(n_k + d_k)} \mathbf{A}_k, \tag{5.6}
\]

\[
\mathbf{A}_j = \mathbf{H}^\dagger \mathbf{w}_j \mathbf{w}_j^\dagger \mathbf{H}, \tag{5.7}
\]

where \( \Lambda'_f \) is a diagonal matrix that is defined for the SPC and PAPC in (A.13) and (A.22),
respectively. The summation in (5.6) is over all values of \( k \) such that the fixed index \( j \) is
within the set \( \hat{L}_k \), where \( \hat{L}_k = L_k \cup k \). Since \( \mathbf{b}_j \) appears explicitly on both sides of (5.5) as well
as within \( \mathbf{G}_j \), (5.5) can be solved iteratively until convergence. Since the Lagrange multiplier
formulation does not guarantee enforcement of the power constraint, in each iteration the
transmit beamformers are normalized to explicitly enforce the constraint. We also notice
that the achieved algorithm for the SPC is basically similar to the beamforming algorithm
developed in Section 4.3.

While an iterative formulation for computing the receive beamformers is also possible,
Appendix A.1 shows that the resulting equation is equivalent to the MMSE beamformer
(with or without SIC). Therefore, the receive beamformer for the \( j \)th data stream can be
constructed using

\[
\mathbf{w}_j = \left[ \sigma_n^2 \mathbf{I} + \sum_{k \in \mathcal{L}_j} \mathbf{H} \mathbf{b}_k \mathbf{b}_k^\dagger \mathbf{H}^\dagger \right]^{-1} \mathbf{H} \mathbf{b}_j. \tag{5.8}
\]
Finally, examination of these equations reveals that (5.5) depends on \( W \) and (5.8) depends on \( B \). This implies that in practice, we must first initialize \( B \) and then perform an iterative computation in which we construct \( W \) using (5.8) and \( B \) using (5.5) once per iteration until both beamformer matrices have converged [39]. For this work, algorithm convergence is achieved when the value of the computed rate changes by less than 0.01% between iterations.

### 5.3.2 CDIT, CSIR

This technique for constructing the transmit and receive beamforming matrices can easily be extended to the case where the transmit beamformers are constructed based on CDI. We take the expectation of the rate in (5.3) over the randomly varying channel \( H \) and apply Jensen’s inequality to obtain [13]

\[
\bar{C} = \mathbb{E}\{C\} \leq \sum_{k=1}^{K} \log(1 + \mathbb{E}\{\text{SINR}_k\}).
\]  

As discussed in Appendix A.3, we also make the approximation \( \mathbb{E}\{\text{SINR}_k\} \approx \mathbb{E}\{n_k\}/\mathbb{E}\{d_k\} = \bar{n}_k/\bar{d}_k \). We can now perform the procedure of Appendices A.1 and A.2 using \( \bar{n}_k \) and \( \bar{d}_k \) in place of \( n_k \) and \( d_k \), respectively. This leads to the same iterative equation (5.5) but with the replacement

\[
\mathbf{A}_j \leftarrow \mathbb{E}\left\{\mathbf{H}^\dagger \mathbf{w}_j \mathbf{w}_j^\dagger \mathbf{H}\right\}.
\]  

When implementing this algorithm, however, we must carefully consider approximation of the expectation in \( \mathbf{A}_j \). If we assume a single value of \( \mathbf{W} \) is valid over the time window during which the transmit beamformer \( \mathbf{B} \) based on CDI is assumed valid, \( \mathbf{A}_j \) can be computed based on knowledge of the beamforming matrix \( \mathbf{W} \) and the full covariance matrix \( \mathbf{R} \), as illustrated in Appendix A.4. In practice, the receiver computes a new value of \( \mathbf{W} \) each time it estimates a new channel matrix \( \mathbf{H} \), and therefore it is arguably more accurate to estimate \( \mathbf{A}_k \) using a sample mean with the true values of \( \mathbf{H} \) and \( \mathbf{W} \) over the time window. Naturally, this latter approach is more computationally costly, since we must compute this sample mean at each step in the iterative computation. Our analysis with the
experimental data discussed in Section 4.4 shows that the accuracy improvement associated with this more costly approach provides very little performance improvement.

5.3.3 CDIT, CDIR

The IBF framework can also specify receive beamformers based on CDI by using the iterative computation for $w_j$ represented in (A.17)-(A.19) and making the substitution

$$\tilde{A}_j \leftarrow E\left\{H_b b_j^H H^H\right\}.$$  

This is a problem that has been considered from an information theoretic standpoint, but since practical techniques for coherent symbol demodulation without CSI at the receiver remains an unsolved problem, we simply point out the development for completeness and already present some simulation results considering this scenario in Section 4.4.

5.3.4 Beamformer Initialization and SIC Ordering

One approach for initializing $B$ in the IBF algorithm is to use unitary singular vector matrices computed from random matrix realizations. Recognizing that convergence of the iterative equations to the beamformers that achieve the global optimum rate is sensitive to the initialization of these matrices, we run the procedure with at least 20 random initializations and choose the result that achieves the highest rate. Alternatively, if using IBF with CSIT or CDIT, we can initialize $B$ as the right singular vectors of the channel matrix $H$ or the eigenvectors of the transmit covariance matrix $R_t$, respectively. Our computations have demonstrated that this approach leads to the same rate as that achieved using the random initialization but with reduced computational burden.

This initialization discussion also relates to the specification of the order in which symbols are detected when implementing SIC at the receiver. Specifically, because we have assumed detection in the order of the stream index, achieving the optimal rate means that the iterative algorithm orders the beamforming vectors in the optimal way. The algorithm’s ability to achieve this ordering depends on the initialization. Since optimal ordering typically means that the stream with the highest SINR should be detected first, we have found that the proper ordering of the initializing unitary matrix corresponds to the case where the singular values (or eigenvalues) are placed in decreasing order.
5.4 Results

5.4.1 Simulation Approach

The performance of the IBF algorithm is demonstrated here through simulations. In all cases, the MIMO channel matrix $H$ is generated and normalized so that

$$\|H\|_F^2 = N_r N_t,$$

(5.11)

where $\|\cdot\|_F$ represents the Frobenius norm. With this normalization, the ratio $P/\sigma_n^2$ represents the single-input single-output (SISO) signal-to-noise ratio (SNR) [47].

Most of the analysis is performed with MIMO channels measured at a carrier frequency of 2.45 GHz for linear transmit and receive arrays of monopoles with half-wavelength element spacing. The transmitter is held stationary while the receiver moves at a constant velocity of approximately 30 cm/s, and channels are sampled at an interval of 2.5 ms corresponding to a distance moved by the receiver of 0.0062 $\lambda$, where $\lambda$ is the wavelength at the carrier frequency. All measurements are taken within an open area between several buildings on the Brigham Young University campus with the transmitter and receiver positioned on either side of a dense stand of trees. Details concerning the measurement system used to collect the channel matrices can be found in [48,49]. The simulations use an adjacent subset of antennas in the transmit and receive array to generate the $N_r \times N_t$ channel matrix.

Use of this experimental data does not facilitate systematic evaluation of the effect of the channel directivity, or equivalently multipath richness, on algorithm performance. We therefore also use a two-ring channel model to analyze the impact of multipath richness on the performance of different transmit precoding techniques. In this model, the transmitter and receiver each lie within distinct circles of radius 40 $\lambda$. A fixed number of scatterers lie on each circle, with the angle of each scatterer location being specified as a uniform random variable on $[0, 2\pi)$. Scatterers on the transmit and receive circles are paired so that each propagation path has a single departure and arrival angle as well as a complex gain specified as a zero-mean, unit-variance circularly symmetric complex Gaussian random variable. The linear transmit array remains stationary at the center of its circle while the linear receive
array moves along a straight line within its circle. Normalized channel matrices are easily computed in this two-dimensional propagation model using established techniques [47].

Because transmit precoding is most effective at low SNR environments, unless otherwise specified the simulations use a transmit power of $P = 1$ and noise variance $\sigma^2_\eta = 1$ for an SNR of 0 dB, with other default parameters being $N_t = N_r = K = 4$. When the PAPC is applied, we use $P_t = P/N_t$ for all transmit antennas. The covariance is estimated using a sample mean to approximate $\mathbf{R}$ over a specified window size (in terms of receiver motion) of $1\lambda$, and this covariance is used to construct the transmit beamforming matrix $\mathbf{B}$ that is fed back to the transmitter at a regular feedback interval of $2.5\lambda$.

5.4.2 CSIT, CSIR

To begin, for each of the experimentally-obtained channel matrices, we compute the capacity using the Waterfilling solution [4] and based on beamformers obtained with the IBF
algorithm with the SPC (based on simple MMSE beamforming at the receiver) assuming CSIT and CSIR. As shown in Figure 4.0 in chapter 4, the rates obtained using these two methods over a variety of system parameters are indistinguishable. This demonstrates that for this simple case, the IBF algorithm provides the expected result.

While this simple demonstration is encouraging, it is more interesting to explore the IBF algorithm performance for the PAPC. Figure 5.0 shows the rate averaged over the experimental channels as a function of the number of antenna elements in the transmit and receive arrays for the IBF algorithm with a PAPC and the previously-published Drop-Rank algorithm [5,6] with both MMSE and MMSE-SIC receivers. For the MMSE-SIC receiver, the performance of the two algorithms is almost identical. However, when an MMSE receiver is used, the performance of the Drop-Rank precoder degrades significantly while that of the IBF precoder remains almost unchanged. This is not surprising, since the Drop-Rank algorithm is designed for a nonlinear receiver, and therefore using it with a linear receiver is
inconsistent with its design goals. The key point, therefore, is that the IBF precoder design *can* accommodate reduced receiver capabilities and, at least for this case, do so with little performance degradation.

Figures 5.1 and 5.2 demonstrate the achieved rates for different techniques as a function of $\alpha$, where before normalization, the experimental channel matrix is multiplied by matrix $L$ as

$$H = H L = H \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (5.12)$$

This represents the scenario when the sub-channels between receiver antenna elements and one of transmit antenna element is stronger than the sub-channels from the rest of transmit antenna elements. As can be seen, by increasing the value of $\alpha$ the effectiveness of trans-
mit beamforming with the PAPC decreases while IBF with the SPC is still effective. The overall performance of algorithms with per-antenna power constraint is similar to what mentioned for Figure 5.0; however, Drop-Rank algorithm is numerically more stable in extreme constraints on per-antenna power.

Because IBF requires an iterative computation, it is important to consider the convergence of the algorithm. Figure 5.3 demonstrates the rate achieved by IBF algorithm as a function of number of random initializations compared to the rate obtained by initialization with channel singular vectors for both power constraints and two different array sizes. As it
Figure 5.5: The fractional difference $\xi(n)$ between the converged rate and the rate at each iteration index $n$ in the IBF algorithm for CSIT, CSIR with an MMSE-SIC receiver averaged over all experimentally-obtained channel matrices for different algorithm initializations.

can be seen in Figure 5.3, IBF with SPC is capable of converging to the optimal rate even with very few number of random initializations, while IBF with PAPC using MMSE-SIC receiver requires in average 20 random initializations, where in both cases initialization with channel singular vectors can achieve the same rate. For IBP with PAPC using MMSE receiver, larger number of random initializations is needed and channel singular vectors is not as effective in large arrays size of $N_r = N_t = 8$.

For each iteration, we compute the fractional difference

$$
\xi(n) = \frac{C_{\text{final}} - C^{(n)}}{C_{\text{final}}},
$$

where $C^{(n)}$ and $C_{\text{final}}$ denote the rate at the $n$th iteration and at final convergence, respectively. Figure 5.4 plots this convergence metric as a function of the iteration index $n$ for both power constraints and for two different array sizes. The algorithm is initialized with the
Figure 5.6: The cumulative distribution function (CDF) of the number of iterations required in the IBF algorithm to converge for CSIT, CSIR with MMSE and MMSE-SIC receivers with considering initialization with channel singular vectors averaged over all experimentally-obtained channel matrices.

channel singular vectors and 20 randomly-generated vectors in the top and bottom plots, respectively. As can be seen initialization with the channel singular vectors is highly effective, even with the PAPC. Furthermore, regardless of the initialization, the iteration converges relatively quickly.

For similar scenarios, the cumulative distribution function (CDF) of the number of iterations required in the IBF algorithm to converge is illustrated in Figures 5.5 and 5.6 for initialization with channel singular vectors and initialization with random vectors, respectively. Comparison of Figure 5.5 to Figure 5.6 confirms that the fast convergence of IBF with initialization of the channel singular vectors compared to IBF with random vectors initialization. Figure 5.5 also illustrates the number of iterations required in Drop-Rank algorithm, which is comparable to the number of iterations required in IBF with random initializations.
5.4.3 CDIT, CSIR

We next compare the performance of the IBF algorithm with that of the previously-reported methods for beamforming based on CDIT [3,44]. Figure 5.7 plots the rate averaged over the ring model realizations as a function of the number of multipaths included in the model for the three transmit beamforming algorithms assuming MMSE-SIC at the receiver. As can be seen, the performance of IBF with the SPC matches that achieved with the previously-developed CDI-based precoding algorithms (that also use the SPC). Since CDI Waterfilling and IBF both maximize the upper bound on the rate while CDI Kronecker maximizes the rate, their nearly identical performance suggests that maximizing the upper bound yields acceptable results. Furthermore, the considerable difference between the rate for an uninformed transmitter and that for CDI-based beamforming illustrates the improvement
Figure 5.8: Average rate achieved using different transmit precoders for CDIT, CSIR (MMSE-SIC) as a function of the number of paths in the two-ring channel model.

offered by transmit precoding, although the benefit is reduced as the channel directivity decreases (more paths).

When the channel is highly directive (few paths), very specific transmit beamforming is required to fully exploit the channel. Since the PAPC limits the range of beamformers that can be achieved, IBF with the PAPC performs about the same as or slightly better than the uninformed transmitter. As the number of paths increases, however, the results show that it becomes easier to form beams that can take advantage of the propagation environment while still satisfying the PAPC, although these results still fall short of those achieved with the more flexible SPC.

Figure 5.8 shows the rate achieved using different techniques with the SPC as a function of SNR over the experimentally-obtained $4 \times 4$ MIMO channel matrices using MMSE-SIC and MMSE receivers. All results show that the effectiveness of transmit beamforming relative to uninformed transmission decreases with increasing SNR, which is the expected result. Once again, all three algorithms achieve nearly identical performance for a MMSE-SIC re-
receiver. However, when using a simple MMSE receiver, the performance of CDI Waterfilling and CDI Kronecker degrades significantly, while IBF is able to maintain competitive performance. While the performance of IBF with a MMSE receiver drops below that of uninformed transmission with a MMSE-SIC receiver, it is much higher than that of uniformed transmission with the MMSE receiver. Figure 5.9 plots similar results for the IBF algorithm with the PAPC using MMSE-SIC and MMSE receivers. As there are no existing algorithms for this case, only the uninformed transmitter performance is shown for comparison purposes. The results are quite similar to those for the SPC, both in trend and in achieved performance. It is interesting that for these experimental channels, the reduction in performance caused by using the PAPC is quite small.
Figure 5.10: Average rate achieved using different transmit precoders for CDIT, CSIR (MMSE-SIC and MMSE) with the PAPC as a function of SNR using experimentally-obtained channel matrices.

5.5 Summary

This work proposes an iterative algorithm that specifies transmit and receive beamformers based on different types of channel information, different capabilities of the receiver, and different power constraints, thereby offering beamforming solutions for specific situations where solutions have not yet been available. Simulations using the method in measured and modeled MIMO channels show that its performance matches that obtained with existing algorithms when used in conjunction with optimal receiver architectures but that it further allows construction of near-optimal transmit precoding for simple MMSE receivers. The simulations further demonstrate the impact of different system parameters on communication performance.
Chapter 6

Measurement-Based Performance Analysis of Cooperative MIMO Beamforming

While multiple-input multiple-output (MIMO) technology generally can improve the spectral efficiency of wireless communication, the improvements can be small in cellular communications where limited multipath angle spread is observed at the sectored and elevated base station (BS) and compact antenna spacing limits spatial diversity at the mobile station (MS) [54, 55]. One potential solution for overcoming the challenge of limited angle spread at the BS is to enable antennas from multiple BSs to coherently cooperate for MIMO processing [4]. Recent work on this topic using measured channels has demonstrated that while cooperative MIMO processing adds complexity to the system implementation, it can achieve significant performance gains [56–65].

Prior work on this topic has focused on a set of coherent measurements from three BS sites to a single MS in a macrocellular environment [65]. For a point-to-point link involving all three BS sites cooperating to communicate to the mobile, the BS cooperation results in an average increase in the communication data rate of 53% over that achieved by a single BS, with the increase exceeding 100% when all BSs observe similar channel gains to the MS. While these results are impressive, the analysis was incomplete as it did not carefully consider how different channel information available to the cooperative BSs might be used to improve performance.

This work therefore extends the study of this cooperative MIMO environment by applying different algorithms to simulate the impact of the cooperative processing by the BSs on the point-to-point MIMO link. Specifically, in this work, we consider the case where the BSs cooperate to downlink data to a single MS and compare performance when the BSs have no channel state information (CSI) to that when the BSs use available CSI to
apply transmit beamforming to establish the link. In the latter case, we constrain the power so that either the total power transmitted by the cooperative BSs is limited (sum-power constraint) or the power from each BS is limited (per-BS power constraint). The results show the significant improvement enabled when using the sum-power constraint as well as more limited improvement achieved when using the per-BS power constraint under low signal-to-noise ratio (SNR) conditions.

6.1 System Analysis

While the details of the system model and the propagation measurements used in this study are provided in [65], we summarize relevant descriptions here to ease the discussion of the results. Consider a point-to-point MIMO communication link consisting of $N_B$ cooperative BSs and a single MS with $N_r$ receive antennas. $\hat{H}[f]$ represents the $N_r \times N_B$ multi-BS (MIMO) channel matrix at the $f$th frequency, which is normalized so that the average of the channel power gains for the strongest BS-to-MS link is unity [65]

$$\|H[f]\|_F^2 = \frac{N_r}{\beta_p} \|\hat{H}[f]\|_F^2,$$

(6.1)

where

$$\beta_p = \max_i \beta_i = \max_i \frac{1}{N_t} \sum_{f=1}^{N_f} \|\hat{h}_i[f]\|_F^2,$$

(6.2)

$N_f$ and $\hat{h}_i[f]$ represent the number of frequencies and $i$th column of $\hat{H}[f]$, respectively.

For the purposes of this study, we will assume applying the studied beamforming algorithm (IBF) in chapter 5 with minimum mean squared error receiver with successive interference cancellation (MMSE-SIC) for channel state information known to both bases-stations and receiver(CSIT, CSIR). We now consider three different options for constructing $B$.

Uninformed Transmitter: If the cooperative BSs have no information about the channel (no CSI), then the best option is for each to transmit a unique data stream, each with identical average transmitter power. If the total average transmit power is $P$, this means $BB^\dagger = (P/N_B)I$. 

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**Tx Beamforming with SPC:** If the cooperative BSs have CSI, then they can apply beamforming to improve performance. We use an iterative beamforming (IBF) algorithm that constructs the transmit beamformer $\mathbf{B}$ based on the available CSI [66]. If the total average transmitted power $P$ is constrained using a sum-power constraint (SPC), the solution constrains $\text{tr} (\mathbf{B} \mathbf{B}^\dagger) = P$. Note that with this constraint, if the power transmitted from one BS is reduced, that power can be allocated to the other BSs. This notion of power sharing means simply that if one BS is turned down or even off, the other BSs may be able to increase their power without violating restrictions from regulatory agencies. The rate achieved using this solution is identical to that obtained using the Waterfilling capacity solution [4].

**Tx Beamforming with PBPC:** The IBF algorithm can also accommodate a per-BS power constraint (PBPC) that allows us to specify the average power transmitted from each BS antenna. Mathematically, we have $\hat{\mathbf{b}}_i \hat{\mathbf{b}}_i^\dagger = P_i$, $1 \leq i \leq N_B$, where $\hat{\mathbf{b}}_i$ represents the $i$th row of the matrix $\mathbf{B}$ and $P_i$ is the average power transmitted from the $i$th antenna.

### 6.2 Results

The urban macrocell environment in Kista (also called “Mobile Valley”), Sweden consists of three BS sites that emulate a realistic cellular deployment topology. At each BS, a single antenna mounted a few meters above the average rooftop level of approximately 25 m transmits a linearly-polarized ($45^\circ$ from vertical) signal. The main lobe of each antenna pattern is pointed downwards between $6^\circ$ and $8^\circ$ from horizontal and approximately towards the centroid of the triangle formed by all three BS sites. The MS consists of two dipole and two loop antennas in a square array with an inter-element spacing of approximately 30 cm, which is 2.6 wavelengths at the center frequency of 2.66 GHz.

Measurement of the channel between all three BS and four MS antennas is accomplished using the Ericsson channel sounder that is based on a prototype for LTE [67] but with a custom frame structure and rate [65]. A single transmit unit generates orthogonal frequency-division multiplexing (OFDM) channel sounding symbols that are distributed to the antennas at the three BS sites using RF-over-fiber equipment. To avoid problems with non-orthogonality of the OFDM symbols due to the MS mobility, the transmissions from these three BS antennas are time multiplexed at the symbol level.
Table 6.1: Specifications for the Ericsson channel sounder

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>2.66 GHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>19.4 MHz</td>
</tr>
<tr>
<td>Frequency bins</td>
<td>432</td>
</tr>
<tr>
<td>Transmit power</td>
<td>36 dBm</td>
</tr>
<tr>
<td>Channel acquisition rate</td>
<td>190 channels/sec</td>
</tr>
<tr>
<td>Number of BS</td>
<td>3</td>
</tr>
<tr>
<td>BS antenna</td>
<td>1 Kathrein (18 dBi 45° polarized)</td>
</tr>
<tr>
<td>MS antenna</td>
<td>2 dipoles +2 magnetic dipoles</td>
</tr>
</tbody>
</table>

The MS uses four parallel receiver chains to simultaneously down-convert the signals from the four receive antennas. Disciplined rubidium clocks (Stanford Research Systems, PRS10) at the transmitter and receiver provide a highly accurate synchronization (Allan standard deviation less than $10^{-12}$) between the BS and the MS. Based on this timing reference, error in the measured propagation distance over all routes is less than 1 m. The resulting system generates a full $4 \times 3$ MIMO channel matrix at a rate of 1500 observations per second (based on 0.667 ms probing frames), but because of bandwidth limitations between the system and the storage medium, the observations are stored at a rate of 190 channels per second, providing high spatial resolution given the maximum van speed of 30 km/hr. All of the parameters used in the measurements are provided in Table 6.1.

The measurements consist of data from two different routes each requiring approximately 9 minutes of measurement time. The routes include regions of line-of-sight (LOS), obstructed line-of-sight (OLOS), and non-line-of-sight (NLOS) propagation. The position data for each channel sample is logged using a GPS receiver. Figure 6.0 illustrates the two routes along with markers indicating the distance traveled along each route and the positions of the base stations.

Unless otherwise specified, we perform the analysis assuming an SNR of 0 dB, as transmit beamforming is most beneficial at low SNR. When the PBPC is used, all BSs transmit the same power. Furthermore, to make performance comparisons, we often match
Figure 6.1: Location of BSs and routes 1 (---) and 2 (•) traveled by the MS. Distances in meters from the starting points are indicated by circles and diamonds for routes 1 and 2, respectively.

the MS to the BS that results in the highest received power, a pairing that requires some level of cooperation among the BSs. However, when the term cooperative BS is used, it explicitly refers to the case where the BSs jointly and coherently participate in the communication to the MS.

Figures 6.1(a) and (b) illustrate the rate achieved using cooperative BS communication as a function of position along route 1 of Figure 6.0 at an SNR of 0 dB and 10 dB, respectively, for the three different transmit beamforming approaches considered. The performance for the best BS-to-MS link is also included for comparison. These results demonstrate that cooperative communication with uninformed transmitters can significantly improve the rate compared to that achieved using the best BS-to-MS link. When examining the channel gain $\beta_i$ from each BS antenna averaged over all receive antennas on the MS shown in Figure 6.1(c), it is clear that cooperation is beneficial in regions where there is no single dominant BS-to-MS line, such as is the case for positions between 900 and 1200 m.
Applying transmit beamforming (IBF) can provide significant additional improvement in performance. Figures 6.1(a)-(b) show that if the beamforming is coupled with the SPC, the algorithm can allocate power to the BSs that contribute most to the communication, leading to significantly improved performance for either SNR. If, however, the power from each BS is constrained (PBPC), performance improvement occurs mainly for low SNR environments. Because the three BSs each transmit equal power, IBF with PBPC is very similar to uninformed transmitter communication, but with the added capability of allowing the BSs to cooperate in forming beams.

**Figure 6.2:** Average rate achieved using cooperative communication as well as the best BS-to-MS link for an SNR of (a) 0 dB and (b) 10 dB and (c) average channel gain $\beta_i$ between each BS and the MS as a function of position along route 1.
Figure 6.3: Two dimensional pdf of the geometry factor and the average rate achieved using (a) cooperative BS with uninformed transmitter (b) IBF with the SPC (c) IBF with the PBPC normalized by the average rate of the strongest BS-to-MS link for all channels in routes 1 and 2 for an SNR of 0 dB.

Figure 6.2 plots two dimensional probability density functions (pdfs) of the geometry factor – defined as the ratio of the power received on the strongest BS-to-MS link to the sum of the powers received on the other two links – and the average rate achieved using cooperative transmit beamforming normalized by that achieved for the best BS-to-MS link ($C_{BL}$) for the concatenated data from both routes. The subscripts U, S, and P respectively indicate the rate for the uninformed transmitter, IBF with the SPC, and IBF with the PBPC. These results demonstrate that cooperative communication with or without transmit beamforming can provide significant improvement when all three BSs have similar channel gains (small value of the geometry factor). However, for large geometry factors, which indicates the presence of one or two dominant BS-to-MS links, only IBF with SPC provides benefit because of its ability to allocate power to the dominant link(s).
Given these observations, we define the Low Geometry Factor (LGF) as the ratio of the power received on the weakest BS-to-MS link to the sum of the powers received on the other two links. We then apply a simple selection algorithm where this weakest BS-to-MS link is allocated power only if the LGF is larger than a specified LGF Threshold (LGFT). Figure 6.3(a) plots the average rate achieved using IBF with the PBPC as a function of this LGFT. The top horizontal axis shows the Weakest BS Activity, which simply indicates the percent of time the weakest BS is transmitting for the corresponding LGFT value specified on the bottom horizontal axis. For the solid curve, the power allocated to each BS is always the same, so that when the weakest BS is active, the total transmitted power increases.
Figure 6.5: Average rate achieved using uninformed transmitter as a function of the low geometry factor threshold level for all channels in routes 1 and 2 for an SNR of 0 dB.

Therefore, as the LGFT increases, it is more likely that we turn off radiation from the weakest BS, which means that we lose this contribution (as small as it may be) and therefore the performance decreases. The bulk of the benefit occurs at an LGFT of approximately $-13$ dB (weakest BS active 20% of the time), which means that if the LGF is below this value, there is little point in using the weakest BS for communication.

In contrast, for the dashed curve in Figure 6.3(a), the total transmitted power is held constant, which means that if the weakest BS is activated, it takes power from the other two. Therefore, unless the weakest BS has reasonable gain so that it can contribute significantly to the communication, it is better to deactivate it so that the power can be assigned to the other
BSs. As a result, the performance tends to increase with LGFT, although it does slightly decrease once the LGFT passes a certain value (approximately $-8$ dB which corresponds to a weakest BS activity of 2.5%).

In Figure 6.3(b), similar two power allocation policies are considered with the only difference that in addition to the weakest BS, if the ratio of the power received on the second weakest BS to the power received by the strongest BS is less than LGFT, power is only allocated to the strongest BS. The performance is similar to Figure 6.3(a), where for the solid curve the main benefit happens at an LGFT of approximately $-13$ dB and for the dashed curve the best result is obtained for LGFT of approximately $-8$ dB. Figure 6.4 consider the same scenarios represented in Figure 6.3; however, it illustrates the average rate achieved using uninformed transmitter instead of the achieved using IBF algorithm. The overall performance matches the performance in Figure 6.3.
Figure 6.7: Two dimensional pdf of the geometry factor and the average rate achieved using (a) cooperative BS with uninformed transmitter (b) IBF with the SPC (c) IBF with the PBPC normalized by the average rate of the strongest BS-to-MS link for all channels in routes 1 and 2 for an SNR of 10 dB.

Figure 6.5(a)-(b) illustrate the two dimensional pdf of the LGF and the average rate achieved by IBF with the PBPC using all three BSs and using the two best BSs ($C_{P,2BS}$), respectively, normalized by the average rate of cooperative BS communication with uninformed transmitters. These results reinforce our observation that allocating all available power to the two best BSs achieves superior performance when the channel gain from the weakest BS is low. However, when all three BSs have similar channel gains to the MS, allocating power to the third BS provides benefit. Figure 6.5(c) plots the two dimensional pdf of the LGF and the average rate achieved by IBF with PBPC using the two strongest BS-to-MS links normalized by the rate achieved using IBF with PBPC. This result demonstrates that for LGF values larger than $-8$ dB, allocating power to all three BSs achieves a higher rate than that obtained when using only the two strongest BSs. Figure 6.5(d) therefore plots the two dimensional pdf of the LGF and the average rate achieved using IBF with the PBPC ($C_{P,8}$).
Figure 6.8: Average rate achieved using IBF with the PBPC as a function of the low geometry factor threshold level for all channels in routes 1 and 2 for an SNR of 10 dB.

normalized by the rate achieved using uninformed transmitters when the LGFT is set to −8 dB. This result is able to achieve the performance obtained using only two BSs for small values of the LGF while minimizing degradation for larger values of the LGF (compare to Figures 6.5(a)-(b)).

Figures 6.6, 6.7, 6.8, and 6.9 demonstrate similar plots to Figures 6.2, 6.3, 6.4, and 6.5 for SNR of 10 dB instead, respectively. The results basically follow the same behavior as the previous results given at low SNR of 0 dB though the transmit beamforming is less beneficial at this high SNR. Besides, comparing the dashed curves in Figures 6.7(b) and 6.8(b) with corresponding curves in Figures 6.3(b) and 6.4(b), there is a noticeable degradation in the
achieved rate for LGFT values larger than $-12$ dB, which is the result of assigning all the power only to the strongest BS, where the other neglected two links are slightly worse than the best link of average SNR of 10 dB.

6.3 Summary

This work uses fully-coherent measurements from three BS sites to a single MS in a macrocellular environment to study the potential gains achievable with cooperative BS communication. The analysis compares the performance achieved using transmit beamforming with either a sum-power and a per-BS power constraint to that achieved using uninformed
cooperative transmitters as well as the best BS-to-MS link. The results demonstrate that under low SNR conditions, transmit beamforming offers notable performance improvements for either power constraint. However, as the SNR increases, only the sum-power constraint can offer significant improvements. If the system uses simple processing to determine whether or not to use the weakest BS in the cooperative communication, additional performance gains are possible.
Chapter 7

Conclusions

This work has investigated modeling of the full spatial covariance matrix of the MIMO channel as an important quantity in channel modeling, communication system signal processing, and performance analysis. The discussion begins with investigating a generalized framework for computing the full MIMO spatial covariance. Then the presentation continues with development of a general approach for constructing MIMO transmit and receive beamformers that can accommodate either the spatial covariance or the channel state information, which forms a basis for many simulations in the dissertation. The results are summarized as following.

7.1 Summary

Chapter 3 presents an approach for modeling the spatial covariance and formulates a closed-form expression for the full MIMO spatial covariance matrix based on knowledge of the propagation environment PAS and the antenna radiation patterns. A simplified representation for the case where the multipath can be modeled using discrete plane waves is used with ray-tracing data to explore the suitability of different simplifying assumptions. Finally, for the case where the transmitter and receiver have linear arrays of isotropic radiators and the PAS can be described using clusters described using uniform, truncated Gaussian, and truncated Laplacian distributions, the integrations used in the formulation can be computed analytically, leading to a series form for the covariance matrix elements. Exploration of this formulation shows that the number of required series terms to achieve high accuracy depends on the cluster properties.

We continue this work using the insight obtained from this covariance analysis in Chapters 4 and 5 by developing a general approach for constructing MIMO transmit and
receive beamforming vectors based on either the full spatial covariance or the channel state information. Chapter 4 proposes an iterative algorithm that specifies transmit and receive beamformers based on different capabilities of the receiver in the case of available CDI information at the transmitter and CSI information at the receiver. Chapter 5 generalizes the formulation and proposes an iterative algorithm that specifies transmit and receive beamformers based on different power constraints in addition to different types of channel information and different capabilities of the receiver, thereby offering beamforming solutions for specific situations where solutions have not yet been available. Simulations using the method in measured and modeled MIMO channels show that its performance matches that obtained with existing algorithms when used in conjunction with optimal receiver architectures but that it further allows construction of near-optimal transmit precoding for simple MMSE receivers. The simulations further demonstrate the impact of different system parameters on communication performance.

Chapter 6 uses fully-coherent measurements from three BS sites to a single MS in a macrocellular environment to study the potential gains achievable with cooperative BS communication. The analysis compares the performance achieved using transmit beamforming already proposed in the previous chapters with either a sum-power and a per-BS power constraint to that achieved using uninformed cooperative transmitters as well as the best BS-to-MS link. The results demonstrate that under low SNR conditions, transmit beamforming offers notable performance improvements for either power constraint. However, as the SNR increases, only the sum-power constraint can offer significant improvements. If the system uses simple processing to determine whether or not to use the weakest BS in the cooperative communication, additional performance gains are possible.

7.2 Suggestions for Future Work

Possible future work on these presented subjects includes the following ideas:

1. For the case where the transmitter and receiver have linear arrays of isotropic radiators and the PAS can be described using clusters satisfying uniform, truncated Gaussian, and truncated Laplacian distributions, a series form for the covariance matrix elements is computed in Chapter 3. This can be further extended to other practical PAS models.
and antenna element arrangements. In addition, the effect of window size on the spatial covariance can be further investigated.

2. Although we have investigated the performance of MIMO transmit and receive beamformers for a single-user scenario using experimental cooperative MIMO channels and have studied situations where the channel gain from one of the basestations is much stronger than that from the others, it would be helpful to modify the beamforming algorithm to accommodate multi-user scenarios and explore the performance for different multi-user cases.

3. Another important aspect of future work could be the physical implementation of the beamforming algorithms proposed in Chapters 4 and 5, which would help to understand the actual behavior and performance of these algorithms in practical applications.
Bibliography


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multipoint trials in the downlink. In *Proc. 5th IEEE Broadband Wireless Access Workshop (BWAWS), 2009 IEEE GLOBECOM Workshops*, pages 1–6, Honolulu, HI, 30 Nov. - 4 Dec. 2009. 76


Appendix A

A.1 IBF with SPC

Our objective is to find the beamforming matrix $B$ that maximizes the rate given in (5.3) or equivalently

$$
C = \sum_{k=1}^{K} \log \left( 1 + \frac{n_k}{d_k} \right), \quad (A.1)
$$

$$
n_k = b_k^\dagger A_k b_k, \quad (A.2)
$$

$$
d_k = \sigma^2_\eta + \sum_{i \in L_k} b_i^\dagger A_k b_i, \quad (A.3)
$$

where $A_k = H^\dagger w_k w_k^\dagger H$. To accomplish this optimization, we form the cost function $\gamma = C - \lambda_f$, where $\lambda_f$ generically represents a Lagrange multiplier term that will in the following be developed for both the SPC and the PAPC.

To find the rate maximizing transmit beamformers, we take the gradient $\nabla_j \gamma$ with respect to $b_j^*$ and set the result equal to zero, where $\{ \cdot \}^*$ represents a conjugate. The gradient of the rate $C$ assumes the form

$$
\nabla_j C = \frac{\partial C}{\partial b_j^*} = \sum_{k=1}^{K} \frac{n'_k}{d_k} - \frac{n_k \left[ n'_k + d'_k \right]}{d_k \left[ n_k + d_k \right]}, \quad (A.4)
$$
where

\[ n'_k = \nabla_j n_k = \begin{cases} 2A_j b_j & j = k \\ 0 & j \neq k, \end{cases} \quad \text{(A.5)} \]

\[ d'_k = \nabla_j d_k = \begin{cases} 2A_k b_j & j \in \mathcal{L}_k \\ 0 & j \notin \mathcal{L}_k. \end{cases} \quad \text{(A.6)} \]

In this analysis, we have used the fact that the receive beamformer weights are unit length \((w_k^\dagger w_k = 1)\). Substitution of (A.5) and (A.6) into (A.4) leads to

\[ \nabla_j C = 2d_j A_j b_j - 2 \sum_{j \in \hat{\mathcal{L}}_k} \frac{n_k}{d_k(n_k + d_k)} A_k b_j, \quad \text{(A.7)} \]

where \( \hat{\mathcal{L}}_k = \mathcal{L}_k \cup k \) and the summation is over all values of \( k \) such that \( j \in \hat{\mathcal{L}}_k \). Furthermore, we will see that for the power constraints considered, we can express the gradient of the Lagrange multiplier term as

\[ \nabla_j \lambda_f = 2A'_f b_j, \quad \text{(A.8)} \]

where \( A'_f \) is a diagonal matrix. Use of (A.4) and (A.8) leads to

\[ \nabla_j \gamma = 2d_j A_j b_j - 2 \left[ A'_f + \sum_{j \in \hat{\mathcal{L}}_k} \frac{n_k}{d_k(n_k + d_k)} A_k \right] b_j. \quad \text{(A.9)} \]

We can now set (A.9) to zero with the goal of solving for the beamforming vector \( b_j \). However, we recognize that the beamformers appear explicitly within \( n_k \) and \( d_k \), and
therefore directly solving for \( b_j \) is difficult. Instead, we can rearrange the resulting equation into the form

\[
\begin{align*}
  b_j &= \frac{1}{d_j} G_j^{-1} A_j b_j, \\
  G_j &= \Lambda_j' + \sum_{j \in \hat{L}_k} \frac{n_k}{d_k(n_k + d_k)} A_k,
\end{align*}
\]  

(A.10)  

(A.11)

which can be solved iteratively to obtain a solution for \( b_j, 1 \leq j \leq K \).

If we assume the SPC, we have

\[
\lambda_f = \lambda \left( \sum_{k=1}^{K} b_k^* b_k - P \right),
\]

(A.12)

where \( \lambda \) is the Lagrange variable. The gradient of \( \lambda_f \) with respect to \( b^*_j \) is \( 2\lambda b_j \), leading to

\[
\Lambda_j' = \lambda I.
\]

(A.13)

The Lagrange multiplier \( \lambda \) can be determined by recognizing that since \( \nabla_j \gamma = 0, b_j^* \nabla_j \gamma = 0 \) and therefore \( \sum_j b_j^* \nabla_j \gamma = 0 \). Since \( P = \sum_j b_j^* b_j \), use of (A.9) leads to

\[
\lambda = \frac{1}{P} \sum_{j=1}^{K} b_j^* F_j b_j,
\]

(A.14)

\[
F_j = \frac{1}{d_j} A_j - \sum_{j \in \hat{L}_k} \frac{n_k}{d_k(n_k + d_k)} A_k,
\]

(A.15)
which is equivalent to
\[
\lambda = \sum_{j=1}^{K} \frac{n_j}{d_j(n_j + d_j)} \frac{\sigma_n^2}{P}.
\]  
(A.16)

This procedure can be applied to determine the receive beamformers \( \mathbf{W} \). Specifically, incorporating the Lagrange multiplier \( \lambda_f = \lambda \left( \mathbf{w}_j^\dagger \mathbf{w}_j - 1 \right) \), taking the gradient of the cost function with respect to \( \mathbf{w}_j^* \), and setting the result equal to zero leads to

\[
\mathbf{w}_j = \tilde{\mathbf{G}}_j^{-1} \tilde{\mathbf{A}}_j \mathbf{w}_j, \tag{A.17}
\]

\[
\tilde{\mathbf{G}}_j = \frac{n_j}{n_j + d_j} \left[ \sigma_n^2 \mathbf{I} + \sum_{k \in \mathcal{L}_j} \tilde{\mathbf{A}}_k \right], \tag{A.18}
\]

\[
\tilde{\mathbf{A}}_k = \mathbf{H} \mathbf{b}_k \mathbf{b}_k^\dagger \mathbf{H}^\dagger. \tag{A.19}
\]

While this iterative form is useful for implementation with CDIR, it is unnecessary for CSIR. Specifically, we recognize that expansion of \( \tilde{\mathbf{A}}_j \) on the right-hand side of (A.17) leaves a right-most product of \( \mathbf{b}_j^\dagger \mathbf{H}^\dagger \mathbf{w}_j \) that is simply a constant, and since \( \mathbf{w}_j \) should be normalized to have unit length (zero gain), this constant is removed through normalization. Therefore, for CSIR, (A.17) reduces to the MMSE beamformer (with or without SIC) [35, 39, 40, 45]

\[
\mathbf{w}_j = \left[ \sigma_n^2 \mathbf{I} + \sum_{k \in \mathcal{L}_j} \mathbf{H} \mathbf{b}_k \mathbf{b}_k^\dagger \mathbf{H}^\dagger \right]^{-1} \mathbf{H} \mathbf{b}_j. \tag{A.20}
\]

A.2 IBF with PAPC

IBF with a PAPC uses the same update equation (A.10) for rate maximization but modifies the term \( \mathbf{A}_f \) in (A.11). Specifically, the PAPC requires multiple Lagrange variables
\( \lambda_i \) leading to the constraint equation

\[
\lambda_f = \sum_{i=1}^{N_t} \lambda_i \left( \hat{b}_i \hat{b}_i^\dagger - P_i \right).
\]  

(A.21)

The gradient \( \nabla_j \lambda_f \) is \( 2 \text{diag}(\lambda) b_j \) so that

\[
\Lambda_f' = \text{diag}(\lambda),
\]  

(A.22)

where \( \lambda \) represents \([\lambda_1, \lambda_2, \cdots, \lambda_{N_t}]^T\), and \( \text{diag}(\cdot) \) produces a diagonal matrix from the input vector. The Lagrange variables are constructed as discussed in Appendix A.1, after first prescaling with indicator matrices \( J_i \) that are defined as all-zero matrices with a 1 on the \( i \)th diagonal element. This operation leads to

\[
\lambda_i = \frac{1}{P_i} \sum_{j=1}^{K} b_j^\dagger J_i F_j b_j.
\]  

(A.23)

The receive beamformers for this case are again given by the MMSE (with or without SIC) form in (A.20).

A.3 Approximating the Expected SINR

The transmit precoder for CDIT relies on the approximation \( \mathbb{E}\{\text{SINR}_k\} \approx \mathbb{E}\{n_k\}/\mathbb{E}\{d_k\} \).

While rigorously justifying this assumption is difficult, it can be shown that

\[
\mathbb{E}\left\{ \frac{n_k}{d_k} \right\} \approx \frac{\mathbb{E}\{n_k\}}{\mathbb{E}\{d_k\}} - \frac{\text{cov}(n_k, d_k)}{\mathbb{E}\{d_k\}^2} + \frac{\text{var}(d_k)\mathbb{E}\{n_k\}}{\mathbb{E}\{d_k\}^3},
\]  

(A.24)
where $\text{cov}(x, y)$ is the covariance of $x$ and $y$ and $\text{var}(x)$ is the variance of $x$. Examining (5.4), if the transmit beamformers are nearly orthogonal, then it is reasonable that $\text{cov}(n_k, d_k)$ is small. Furthermore, over a block that might be used for covariance estimation, it is likely that the variance of $d_k$ is also small. These observations support the approximation. Furthermore, even if the approximation is poor, as long as finding the beamformers that maximize the rate under this approximation also maximize the actual expected rate, then use of the approximation is justified. Since our results demonstrate that the performance of the algorithm with this approximation matches that of other algorithms (for an MMSE-SIC receiver), use of this approximation has merit.

### A.4 Expressing $A$ and $\tilde{A}$ in terms of $R$

It is straightforward to express $A$ and $\tilde{A}$ in terms of elements of the full covariance matrix $R$. Specifically, we can write

\[
A_k = \text{reshape}(\rho_t \text{vec}(w_k w_k^H), N_t, N_t), \quad (A.25)
\]

\[
\tilde{A}_k = \text{reshape}(\rho_r \text{vec}(b_k b_k^H), N_r, N_r), \quad (A.26)
\]

where $\text{vec}(\cdot)$ stacks the matrix argument columnwise into a vector, $\text{reshape}(\cdot, m, n)$ reshapes the vector into a $m \times n$ matrix, and

\[
\rho_t = \mathbb{E}\{H^T \otimes H^\dagger\}, \quad (A.27)
\]

\[
\rho_r = \mathbb{E}\{H^* \otimes H\}, \quad (A.28)
\]
where \{\cdot\}^T and \otimes represent a transpose and a Kronecker product, respectively. Clearly, the elements of \( \rho_t \) and \( \rho_t \) correspond to elements of the full covariance matrix \( R \) as

\[
\rho_t((i-1) \ast N_r + 1 : i \ast N_r, (j-1) \ast N_t + 1 : j \ast N_t) = \rho^T,
\]
\[
\rho_t((j-1) \ast N_t + 1 : j \ast N_t, (i-1) \ast N_r + 1 : i \ast N_r) = \rho^*,
\]

where

\[
\rho \triangleq \text{reshape}(R((i-1) \ast N_t + j :), N_t, N_r),
\]
\[
i = 1, \cdots, N_r,
\]
\[
j = 1, \cdots, N_t.
\]