Design and Polarimetric Calibration of Dual-Polarized Phased Array Feeds for Radio Astronomy

Taylor D. Webb
Brigham Young University - Provo

Follow this and additional works at: https://scholarsarchive.byu.edu/etd

Part of the Electrical and Computer Engineering Commons

BYU ScholarsArchive Citation

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact scholarsarchive@byu.edu, ellen_amatangelo@byu.edu.
Design and Polarimetric Calibration of Dual-Polarized Phased Array Feeds

for Radio Astronomy

Taylor Webb

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

Karl F. Warnick, Chair
Brian D. Jeffs
Neal K. Bangerter

Department of Electrical and Computer Engineering
Brigham Young University
August 2012

Copyright © 2012 Taylor Webb
All Rights Reserved
ABSTRACT

Design and Polarimetric Calibration of Dual-Polarized Phased Array Feeds for Radio Astronomy

Taylor Webb
Department of Electrical and Computer Engineering
Master of Science

Research institutions around the globe are developing phased array feed (PAF) systems for wide-field L-band radio astronomical observations. PAFs offer faster survey speeds and larger fields of view than standard single-pixel feeds, which enable rapid sky surveys and significantly increased scientific capability.

Because deep space astronomical signals are inherently weak, PAF systems must meet stringent noise and sensitivity requirements. Meeting these requirements requires detailed modeling of the phased array itself as well as the reflector it is mounted on. This thesis details a novel approach to dual-pol PAF design that models the array and reflector as a complete system in order to achieve a more optimal sensitivity and system noise temperature. The design and construction of two arrays designed using this technique is discussed. The implementation of a data acquisition system to receive data from the first of these arrays is also detailed.

Polarization state information plays an important role in understanding cosmological processes for many deep space sources. Because of phase and gain imbalances in the LNAs and receiver chains calibration is required for accurate measurement of polarization by phased array feeds. As a result accurate polarimetric calibration techniques are essential for many observations. Existing polarimetric calibration methods are based on assumptions about the form of the system Mueller matrix that limit the generality of the method or require long observations of a polarized source which is time-consuming for multiple PAF beams. This thesis introduces a more efficient method of calibration that uses only three snapshot observations of bright astronomical calibrator sources, one unpolarized and two partially polarized.

The design of an engineering array for the Green Bank Telescope is discussed. Measured results from a prototype element are presented along with simulated on-reflector results for the full array. Simulations predict that the array will be the highest sensitivity dual pol feed built by the Radio Astronomy Systems group at Brigham Young University to date.

Keywords: radio astronomy, phased array feeds, phased array antennas, polarimetric calibration, active impedances, beamforming, electromagnetics
ACKNOWLEDGMENTS

It is impossible to capture the impact of the people I love in a standard and short acknowledgment. Their efforts, examples, and support are central to my success as a Master’s student and as a human being. My parents taught me to value hard work and education. They instilled in me the drive to do my best and continue to support my ongoing efforts to live up to that ideal. My sisters share my love for learning and provide laughter, relaxation, conversation, support, and friendship. We are friends for life. My baby sister battled cancer for much of my time as a student at BYU. Her zeal for life and her courage in the face of immense difficulty inspire me to work harder and live life more purposefully. She reminds me that it is a privilege to work, to learn, and to live.

My wife came into my life during the first year of my Master’s program. Her quiet and consistent support of my work, as well as her pride in my accomplishments, have brought new meaning and excitement to my education. Our unborn and as yet unnamed son makes me want to provide an example of hard work and excellence. He already motivates me to give my best effort to whatever I am involved in.

I have worked extensively with Professor Jeffs as part of the Radio Astronomy group. He has been a great mentor in engineering and in life in general. I have never questioned that he is intensely interested in my personal success and happiness. My advisor, Professor Warnick, has sacrificed his own time to tutor me, write letters of recommendation, edit proposals, and this thesis. There is no question that I would not have achieved as much as I have without the guidance and extra effort of Professor Warnick. He has continually put my interests ahead of his own and it has made an immense difference to my experience and accomplishments as a student.
# Table of Contents

## List of Tables

<table>
<thead>
<tr>
<th>List of Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>vii</td>
</tr>
</tbody>
</table>

## List of Figures

<table>
<thead>
<tr>
<th>List of Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>viii</td>
</tr>
</tbody>
</table>

## 1 Introduction

1.1 Phased Arrays and Phased Array Feeds ................................. 1

1.1.1 Phased Arrays in Radio Astronomy ................................. 1

1.2 Thesis Contributions .................................................. 4

1.3 Thesis Outline ......................................................... 5

## 2 Background

2.1 Signal Model .......................................................... 6

2.2 Beamforming .......................................................... 8

2.2.1 Active Impedances ................................................ 8

2.3 Figures of Merit ....................................................... 9

2.3.1 Radiation Efficiency .............................................. 9

2.3.2 Aperture Efficiency ............................................... 10

2.3.3 Spillover Efficiency ............................................. 11

2.3.4 Sensitivity ......................................................... 12

2.3.5 Effective Field Of View .......................................... 13

2.3.6 LNA, Receiver, and System Noise Temperatures ................... 13
2.4 Polarimetry ................................................................. 14
  2.4.1 Representing the Polarization State ......................... 15
  2.4.2 Modeling Phased Array Polarimeters ...................... 16
  2.4.3 The Mueller Matrix .............................................. 18
2.5 Polarimetric Figures of Merit ......................................... 19
  2.5.1 Cross-polarization Discrimination .......................... 19
  2.5.2 Cross-polarization Isolation .................................. 20

3 Design and Implementation of Two PAF Systems ................. 22
  3.1 System Overview ..................................................... 22
  3.2 Array Design ....................................................... 24
    3.2.1 The MATLAB Optimization Package ....................... 24
    3.2.2 Modeling Dual-Polarized Arrays ......................... 26
    3.2.3 The Ear Dipole Element .................................... 26
    3.2.4 The Kite Array .............................................. 27
    3.2.5 A NASA Application for the MATLAB Optimization Package ........... 28
  3.3 Data Acquisition .................................................... 29
  3.4 Summary .................................................................. 34

4 Polarimetric Calibration of PAFs ...................................... 35
  4.1 Three Source Calibration ........................................... 36
    4.1.1 The Jones Polarimeter .................................... 37
    4.1.2 The Mueller Polarimeter .................................. 40
  4.2 Numerical Results ..................................................... 41
    4.2.1 Calibration Across The Field of View ................... 44
  4.3 Summary .................................................................. 46
List of Tables

2.1 Table of subscript definitions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14

5.1 Optimization parameters and their final values for the GBT1. . . . . . . . . 52

5.2 Optimization parameters and their final values for the GBT2. . . . . . . . . 55
List of Figures

2.1 Diagram of instrument hardware from antenna elements to calibrated output. 17

3.1 System diagrams for the ‘ear’ and ‘kite’ dipoles. 23
3.2 Block diagram of the MATLAB optimization package. 25
3.3 The ‘ear’ dipole. 27
3.4 Image of the Arecibo telescope. 28
3.5 The ‘kite’ dipole and cryostat. 29
3.6 Images of the completed 40 channel DAQ system. 32
3.7 Block diagram of the DAQ system. 33

4.1 Block diagram of a polarimetric calibration method that uses three sources. 36
4.2 Mueller matrix error and XPD against nonorthogonality in the dipole design. 43
4.3 Stability of sensitivity with respect to mechanical degradations for polarimetric calibration. 43
4.4 Stability of sensitivity across the field of view. 44
4.5 Stability of the Jones matrix and the calibrator error across the field of view. 45
4.6 Error for field of view calibration using interpolation. 46

5.1 The offset fed GBT reflector. 49
5.2 Sensitivity and effective FoV of analytical dipoles as a function of element spacing. 51
5.3 Schematic of the GBT1 element. 52
5.4 Image of the constructed GBT element. ........................................ 54
5.5 Modeled and measured S-parameters for the isolated GBT1 element. .... 54
5.6 Image of the GBT2 element. .......................................................... 55
5.7 Simulated results for the GBT2 array. ............................................. 57
5.8 Modeled XPD for the uncalibrated max SNR beamformer on the GBT2 array. 58
Chapter 1

Introduction

1.1 Phased Arrays and Phased Array Feeds

Phased arrays are arrays of antennas strategically spaced in order to improve signal detection. Signal detection is improved by phased arrays because more signal can be collected and because the signal from each antenna can be strategically combined to favor the signal of interest. This allows phased arrays to detect weaker signals than a single antenna element and to detect signals arriving at wider incidence angles.

The history of phased arrays begins in the early 20th century when they were developed for radar and communication purposes. One of the first ground based phased array radars was built to help land planes in great Britain during WWII [1] and another, The Two Mile Array, was built on the east coast to facilitate communications between London and New Jersey [2]. In modern times phased array technology has become ubiquitous in communications and remote sensing.

In remote sensing, phased arrays are often placed at the focal point of a large reflector. This allows scientists to combine the advantages of phased arrays with the signal collection capacity of reflectors. Phased arrays at the focal point of a reflector are called phased array feeds (PAFs). Reflectors equipped with PAFs can form multiple beams on their target, enabling them to capture multiple pixels at a time. This results in faster data acquisition.

1.1.1 Phased Arrays in Radio Astronomy

In the field of radio astronomy, scientists use large reflector antennas specifically to detect the radio waves emitted by celestial objects. The practice began with the observations of Karl Jansky in the 1930s [3] and quickly grew into its own field because it provided scientists with a new view of the universe that was distinct from the understanding provided by visible
light. There are now many large reflector antennas in observatories world wide dedicated to studying astronomy.

Today, scientists are working to adapt PAF technology for deployment on these telescopes. This research is driven by the potential advantages PAFs could bring to the astronomical community. Telescopes equipped with PAFs would give astronomers the ability to make more rapid discoveries and to study larger sections of the sky in less time. This is because PAFs offer a wider field of view and faster survey speeds, which is a measure of how quickly a telescope can scan the sky, than traditional single-pixel feeds. This gives scientists the ability to gather more information with fewer telescope pointings.

The Radio Astronomy Systems (RAS) group at Brigham Young University (BYU) has been part of this larger international effort to develop design principles and signal processing techniques for radio astronomy PAFs. In the Netherlands the effort is led by the Institute for Radio Astronomy, known as ASTRON. ASTRON is creating a PAF called the APERture Tile In Focus (APERTIF) array to be deployed on their Westerbork Synthesis Telescope. The array boasts an increase in survey speed by a factor of 20 over traditional single element feeds [4]. In China preparations are underway to build the Five hundred meter Aperture Spherical Telescope (FAST). PAFs will play an important role in extending the field of view of this massive telescope [5].

The scientific community is coming together to build the Square Kilometer Array (SKA) to answer important questions about the origins of the universe. The SKA will be an array of coherently connected antennas that will cover 3000 square kilometers and will achieve extremely high sensitivities [6]. Sites for telescope construction have been selected in Australia and South Africa. ASTRON [7] and the Commonwealth Scientific and Industrial Research Organisation (CSIRO) [8] have designed arrays for the SKA Pathfinder telescope, a series of reflectors built in Australia as prototypes in preparation for the final array. A similar telescope, known as the Karoo Array Telescope (meerKAT), is under construction in South Africa [9].

The requirements for these radio astronomy instruments are inherently stringent. Many signals of interest in radio astronomy observations have traveled billions of light years and are therefore extremely weak. As a result, radio astronomy applications place strict
requirements on the sensitivity and stability of PAFs. PAFs must be designed to meet these specifications if they are to become useful as radio astronomy instruments.

While the international community has made many technological advances, PAF technology is not yet sufficiently advanced to meet these rigid requirements. PAF designs have not yet been able to show sensitivities comparable to those achieved by traditional single-pixel fed telescopes. This is largely because the closely spaced elements in phased arrays interact with each other. These interactions are called mutual coupling, and they lead to array mismatch and loss. To avoid mutual coupling, engineers must model the entire array and reflector in the design process. Efficient design techniques that account for mutual coupling do not exist.

Phase and gain imbalances in the various input channels of the array also decrease sensitivity and stability if they are not accurately corrected. Correction requires several different calibration measurements, including polarimetric calibration, intensity calibration, and computation of the steering vectors. Polarimetric calibration refers to accurate recovery of the polarization state of a source through correction of phase and gain imbalances between the two measured polarizations. Intensity calibration is the accurate measurement of the flux density of a source which is accomplished through the measurement of a bright source with known intensity. Computation of the steering vectors refers to calculating the appropriate beamformer weights to steer the array on a grid. All of these calibrations must be stable across the field of view and over time in order to resolve weak celestial sources.

To help make science-ready PAFs for radio astronomy a reality, the RAS group at BYU has worked extensively on the development of high sensitivity PAFs. Our PAFs achieve higher sensitivity through better matched element design [10], [11], [12] and signal processing techniques for cancelling noise and interference [13], [14], [15].

The RAS group has also worked toward more efficient and stable calibration techniques. The group has contributed procedures for array calibration [14] and done studies to quantify the stability of that calibration over time [16]. Most recently the RAS group has become involved in polarimetric calibration. Polarimetric calibration of PAFs is still a relatively new field within the radio astronomy community. The BYU RAS group has collaborated with ASTRON and built on the work of Heiles [17] and Hamaker [18], [19].
This work produced two publications which establish theoretical framework and notation for understanding polarimetric calibration and suggest some approximate calibration methods [20], [21]. However, the papers stop short of developing full polarimetric calibration procedures for PAFs.

1.2 Thesis Contributions

This thesis addresses the difficulties presented by high sensitivity PAF design as well as polarimetric calibration of dual-polarized PAFs. The contributions of this thesis fall into three categories: high sensitivity design of dual-polarized PAFs, polarimetric calibration, and systems implementation. Tangible results of my work include a PAF design package, two phased arrays that were constructed and verified, another array that is currently under construction, a 40 channel data acquisition system, and a polarimetric calibration procedure.

My PAF design package allows our group to include the reflector geometry and mutual coupling of the array elements in our design process. The design package combines EM models of an array feed with in-house EM models of the reflector. This allows us to optimize our PAF designs for sensitivity or any other figure of merit. Two phased array feeds that were designed with this algorithm have been constructed and mounted on radio telescopes in Arecibo, Puerto Rico and Green Bank, West Virginia.

A coworker and I developed a 40 channel data acquisition (DAQ) system to support the deployment of the PAF that was deployed on the Arecibo Telescope. The full system includes 5 data acquiring node PCs and a hub PC to synchronize acquisition and store the data. The first data collected by the RAS group with a dual-polarized array was acquired by this system.

A fellow graduate student and I are designing a third PAF that will be deployed on the Green Bank Telescope (GBT). Though the PAF will not be science-ready, it represents an important step toward the goal of designing and deploying a science-ready PAF on this telescope. The array is still under construction, but measured and modeled results of an element prototype, as well as simulated performance for the full array, are given in this thesis.
My method for efficient and accurate calibration of PAF polarimeters is presented in Chapter 4. The method uses snapshot measurements of three celestial sources to determine the transformation caused by a PAF system on the measured polarization state of a celestial object. Simulated results show a high level of accuracy and stability.

1.3 Thesis Outline

Chapter 2 establishes the notation that will be used in the remainder of the document and reviews key results from previous students and the scientific literature.

Chapter 3 outlines my contributions to the designs of the two phased array systems mentioned in the preceding section. This chapter also contains a detailed explanation of the design procedure I developed in order to create lower noise designs. Section 3.3 outlines the development of the dual-polarized DAQ system.

Chapter 4 defines a polarimetric calibration method that can be achieved with snapshot measurements of only three celestial sources. Simulated results are given to show the accuracy and stability of the calibration.

Chapter 5 reviews the design of the GBT array. A single element was constructed to verify our model and modeled and measured results are shown.

Chapter 6 summarizes the most important results and offers suggestions for future work.
Chapter 2

Background

In this chapter we derive a signal model for on reflector observations with PAFs and summarize key results from earlier work such as PAF figures of merit. The signal model has been developed over time by the RAS group at BYU and much of the summary and notation is taken from prior theses. We begin by describing a model for the antenna element response to a signal of interest. We then develop beamforming and several key figures of merit which are used to motivate active impedance matching. Finally, we use the signal model to describe the polarimetric response of the system.

In the following chapters we will use bold faced lower case letters to denote vector quantities and bold faced upper case letters to denote matrix quantities. The conjugate transpose will be denoted by $H$ and $\langle \cdot \rangle$ will denote time averaging.

2.1 Signal Model

In an on reflector observation, signal impinging on the array excites a voltage on the antenna elements. The voltage from each element is then magnified by a low noise amplifier (LNA) and passed into a receiver. The receiver again magnifies the signal and mixes it down to baseband for sampling by an analog to digital converter. We represent the sampled signal from the array as

$$\mathbf{v} = [v_1 \ v_2 \ \ldots \ v_N]^T,$$

where $v_n$ is the sampled signal from the $n$th antenna element and $N$ is the total number of antenna elements in the array.

We can write the vector $\mathbf{v}$ as a combination of noise and the signal of interest,

$$\mathbf{v} = \mathbf{v}_{\text{sig}} + \mathbf{v}_n,$$
where $v_{\text{sig}}$ is the signal of interest and $v_n$ is the contribution from noise. It is often convenient to write $v_n$ as

$$v_n = v_{\text{sp}} + v_{\text{sky}} + v_{\text{loss}} + v_{\text{rec}}, \quad (2.3)$$

where $v_{\text{sp}}$ represents spillover noise, $v_{\text{sky}}$ represents sky noise, $v_{\text{loss}}$ represents the thermal noise introduced by the lossy antenna elements, and $v_{\text{rec}}$ represents the noise added by the receiver chains. Because the dominant contribution to system noise is a result of mismatch between the LNAs and the antenna elements it is often convenient to further break down $v_{\text{rec}}$ as

$$v_{\text{rec}} = v_{\text{LNA}} + v_{\text{rec2}}. \quad (2.4)$$

Here, $v_{\text{LNA}}$ represents the noise introduced at the LNAs and $v_{\text{rec2}}$ represents the noise contributed by the rest of the receiver chains. Because the gain of the LNAs is high, $v_{\text{rec2}}$ is typically small.

In PAF observations, we compute correlations of the voltage signal. These correlations are represented by the correlation matrix which is

$$R = E[vv^H], \quad (2.5)$$

where $E[\cdot]$ is the time expectation. If we assume that the various contributions to $v$ are independent of one another (i.e., $E[v_{\text{sky}}v_{\text{loss}}^H] = 0$) then the correlation matrix can be written as

$$R = E\left[v_{\text{sig}}v_{\text{sig}}^H\right] + E\left[v_{\text{sp}}v_{\text{sp}}^H\right] + E\left[v_{\text{sky}}v_{\text{sky}}^H\right] + E\left[v_{\text{loss}}v_{\text{loss}}^H\right] + E\left[v_{\text{rec}}v_{\text{rec}}^H\right]$$

$$= R_{\text{sig}} + R_{\text{sp}} + R_{\text{sky}} + R_{\text{loss}} + R_{\text{rec}}. \quad (2.6)$$

Another useful formulation is the Thévenin equivalent network that uses open circuited voltages, or the voltage that would appear across each antenna port if the elements were open circuited. The open circuit voltages are related to the receiver outputs by

$$v = Qv_{\text{oc}}, \quad (2.7)$$
where $Q$ is a function of the receiver chains, the LNAs, and the array network parameters [10].

2.2 Beamforming

Perhaps the most powerful benefit offered by phased arrays is the ability to electronically steer the beam pattern. Electronic steering is accomplished through a signal processing technique called beamforming in which weights are applied to the signal from each antenna. These weights can be selected to steer the pattern and to optimize array figures of merit.

Beamforming is achieved by selecting a set of weights and summing the weighted signal from each antenna element. The beamformer weights are selected based on the desired look angle and other optimization parameters. For example, the max SNR beamformer, introduced in section 2.3.4, maximizes the sensitivity achieved by the array for a given look angle. Once the weights have been selected, we collect them into a vector, $w$, which allows us to represent the output power of a formed beam as

$$p = E \left[ (v^H w)^2 \right],$$

$$= w^H E [vv^H] w,$$

$$= w^H R w. \tag{2.8}$$

Multiple sets of beamformer weights can be used to steer the beam in a grid on the sky. We can then generate an image using the output powers of each beam to form the individual pixels. This enables us to observe several pixels with a single telescope pointing.

2.2.1 Active Impedances

An important design requirement for antennas is an impedance match between the antenna and the transmission line carrying the power to or from the antenna element itself. A well matched impedance maximizes the power transfer between the transmission line and the antenna and minimizes the loss due to reflections. In phased arrays, beamforming and mutual coupling between the closely spaced antenna elements cause a change to the effective impedance presented by the antenna elements. This effective impedance is called the active
impedance of the array and it is given by \([10]\)

\[
Z_{act,m} = \frac{1}{w_{oc,m}^*} \sum_{n=1}^{M} w_{oc,n}^* Z_{A,nm},
\]  

where \(\ast\) is the complex conjugate, \(M\) is the number of elements, and \(w_{oc}\) is the open circuit beamformer weights defined such that multiplication with the open circuited voltages gives the same result as \(w^H v\). \(w_{oc}\) can also be defined by multiplication with the \(Q\) matrix defined in equation (2.7).

A good impedance match for a phased array requires matching the active impedances. This is difficult to do because it requires modeling the entire array and reflector in order to obtain the correct beamformer weights. Developing design techniques for obtaining a good match is the focus of Chapter 3.

### 2.3 Figures of Merit

In this section we define and describe the most important figures of merit for radio astronomy PAFs.

#### 2.3.1 Radiation Efficiency

Radiation efficiency measures how much of the power that is delivered to the array is radiated into free space. It is given by

\[
\eta_{rad} = \frac{P_{rad}}{P_{in}},
\]

where \(P_{rad}\) is the total power radiated and \(P_{in}\) is the total power input to the array. For receiving arrays this becomes \([22]\)

\[
\eta_{rad} = \frac{P_{iso}}{P_t} = \frac{P_{iso}}{P_{iso} + P_{loss}},
\]

where \(P_{iso}\) is the noise power due to an external, isotropic noise environment, \(P_t\) is the external and loss noise, and \(P_{loss}\) is the noise power due to losses in the antenna elements. \(P_{loss}\) is measured when the antenna and the external environment are in thermal equilibrium.
We usually assume that $\eta_{\text{rad}} \approx 1$ because small losses are difficult to model accurately. Due to losses created by the antenna element design and the material from which it is fabricated, however, $\eta_{\text{rad}}$ may be less than unity.

2.3.2 Aperture Efficiency

Aperture efficiency is a measure of how effectively a beam’s radiation pattern illuminates the physical area of the reflector. In order to define aperture efficiency we start with some IEEE standard definitions.

Antenna efficiency is defined as

$$\eta_{\text{ant}} = \frac{A_e}{A_{\text{phy}}}, \quad (2.12)$$

where $A_{\text{phy}}$ is the physical area of the aperture and $A_e$ is the effective area. The effective area is defined by

$$A_e = \frac{P_{\text{av}}}{S_{\text{sig}}} = \frac{\lambda^2}{4\pi \eta_{\text{rad}} D_0}, \quad (2.13)$$

or the ratio of the power available at the antenna terminals to the power incident on the antenna.

Equation (2.13) can be rewritten as [23]

$$A_e = \frac{\lambda^2}{4\pi} G_0 = \frac{\lambda^2}{4\pi} \eta_{\text{rad}} D_0, \quad (2.14)$$

where $G_0$ is the maximum gain of the antenna and $D_0$ is the maximum directivity.

Aperture efficiency for an aperture antenna is defined as the ratio of the directivity of the antenna, $D_0$ to the standard directivity, $D_{\text{std}}$. The standard directivity for an aperture antenna that is much larger than a wavelength can be written as [23]

$$D_{\text{std}} = \frac{4\pi}{\lambda^2} A_{\text{phy}}. \quad (2.15)$$
We can combine (2.13) and (2.15) with the definition of aperture efficiency to get

\[ \eta_{ap} = \frac{A_e}{\eta_{rad} A_{phy}}. \]  

(2.16)

It is difficult to extend this definition to PAFs. Following [22] we do so by noting that the power available at the terminals of a passive antenna in thermal equilibrium is

\[ P = k_b T_{iso} B, \]  

(2.17)

where \( k_b \) is Boltzmann’s constant, \( T_{iso} \) is the isotropic noise, and \( B \) is the bandwidth. We can scale the output of the beamformer to equal this power by setting the available power for an active array, \( P_a \), to

\[ P_a = \frac{k_b T_{iso} B}{w^H R_{iso} w} w^H R w. \]  

(2.18)

Here, \( R \) is the total measured correlation matrix and \( R_{iso} \) is the correlation matrix due to the isotropic noise. The matrix, \( R_{iso} \) is measured when the array is in thermal equilibrium. The aperture efficiency is then

\[ \eta_{ap} = \left( \frac{k_b T_{iso} B}{S_{sig} A_{phy}} \right) \frac{w^H R_{sig} w}{w^H R_{iso} w}. \]  

(2.19)

This is a convenient representation of aperture efficiency for PAFs since it ties the aperture efficiency to the beamformer weights.

2.3.3 Spillover Efficiency

Spillover efficiency is similar to aperture efficiency in that it characterizes how much of the radiation pattern of the array is on the reflector. Aperture efficiency quantifies how much of the reflector is illuminated by the array while spillover efficiency measures how much of the radiation pattern “spills over” the side of the dish. Because of the hot temperature of the ground, poor spillover efficiency can seriously degrade an array’s performance.
The spillover efficiency for a PAF is derived in [24] and is given by

$$\eta_{sp} = 1 - \frac{T_{iso}}{T_g} \frac{w^H R_{sp} w}{w^H R_{iso} w},$$

(2.20)

where $R_{sp}$ is the correlation matrix corresponding to the voltages produced by the spillover noise and $T_g$ is the ground temperature. If we define $T_{sp}$ to be the equivalent spillover noise temperature we can rewrite this as

$$\eta_{sp} = 1 - \frac{T_{sp}}{T_g}.$$  

(2.21)

### 2.3.4 Sensitivity

Sensitivity is the most important figure of merit for radio astronomy arrays and other receivers. It measures the instrument’s response to the signal of interest compared to noise and determines the weakest signal that could be detected by the radiometer. For a given beamformer with weights $w$ the sensitivity is

$$S = \frac{A_e}{T_{sys}} = \frac{kB}{S_{sig}} \frac{w^H R_{sig} w}{w^H R_n w}.$$  

(2.22)

Because sensitivity is a key figure of merit for receive arrays, the most commonly used beamformer in recent astronomical array development is the set of weights that maximizes the arrays sensitivities. We can find these weights by choosing the weight vector, $w$, that maximizes (2.22). It is given by [25]

$$w_{maxSNR} = R_n^{-1} v_{sig}(\Omega_s),$$  

(2.23)

where $v_{sig}(\Omega_s)$ is the signal resulting from a plane wave arriving at an angle $\Omega_s$. 

12
2.3.5 Effective Field Of View

PAFs are often touted for their ability to increase the speed with which an astronomer can scan the sky. One measure of a PAF’s ability to scan the sky quickly is the effective field of view (FoV). Effective FoV measures the angular size over which high sensitivity beams can be formed. It is given by [26]

\[
FoV_{\text{eff}} = \frac{1}{S_{\text{max}}} \int_{\text{FoV}} S^2(\theta, \phi) \, d\Omega, \tag{2.24}
\]

where \( S \) is the sensitivity of the array at the pointing direction given by \( \Omega \).

2.3.6 LNA, Receiver, and System Noise Temperatures

In order to maintain a high sensitivity it is essential to minimize the amount of noise added by the PAF system. We represent this noise using equivalent noise temperatures (see equation (2.17)). The equivalent noise temperature corresponding to the total noise added by a PAF system is \( T_{\text{sys}} \). This temperature is given by [27]

\[
T_{\text{sys}} = T_{\text{iso}} \frac{P_n}{P_t} = T_{\text{iso}} \frac{w^H R_n w}{w^H R_t w}, \tag{2.25}
\]

where \( R_t \) is the correlation matrix resulting from external and loss noise. As noted in Section 2.1, \( R_n \) can be broken down based on the noise contributions from different system components. Similarly, we can write \( T_{\text{sys}} \) as

\[
T_{\text{sys}} = T_{\text{sp}} + T_{\text{sky}} + T_{\text{loss}} + T_{\text{rec}}. \tag{2.26}
\]

In this equation, the subscripts match the ones found in section 2.1. Table 2.3.6 reviews these subscripts and the system components they represent.

The largest contribution to equation (2.26) is \( T_{\text{rec}} \). To see why it is the largest contribution recall from cascaded systems theory [28] that the total system temperature of
Table 2.1: Table reviewing the meanings of the different subscripts used with correlation matrices, noise temperatures, and powers.

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Component of the system denoted</th>
</tr>
</thead>
<tbody>
<tr>
<td>n or sys</td>
<td>Includes all noise sources both external and internal to the system</td>
</tr>
<tr>
<td>sp</td>
<td>Contributions from spillover</td>
</tr>
<tr>
<td>sky</td>
<td>Contributions from the sky (approximately 5K at L-band)</td>
</tr>
<tr>
<td>rec</td>
<td>Contributions from the receivers</td>
</tr>
<tr>
<td>LNA</td>
<td>The contribution of the LNAs to $T_{rec}$. $T_LNA$ is the largest component of $T_{rec}$ and we often assume $T_LNA \approx T_{rec}$</td>
</tr>
<tr>
<td>loss</td>
<td>Thermal noise resulting from lossy antenna elements</td>
</tr>
<tr>
<td>iso</td>
<td>Power or noise temperature resulting from an ideal isotropic thermal noise environment</td>
</tr>
<tr>
<td>t</td>
<td>Contributions from the environment and the lossy antenna elements</td>
</tr>
</tbody>
</table>

A cascaded system is

$$T_{cascaded} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots,$$  \hspace{1cm} (2.27)

where $T_1$, $T_2$, and $T_3$ are the noise contributions from components one, two, and three respectively, and $G_1$ and $G_2$ are the gains of components one and two. Using this equation we can write the noise temperature contributed by the receiver as

$$T_{rec} = T_{LNA} + \frac{T_{rec2}}{G_{LNA}} + \ldots,$$ \hspace{1cm} (2.28)

where $G_{LNA}$ is the gain of the LNAs. Since the gain of the LNAs is quite large (on the order of 40 dB), other contributions to the receiver noise are small. This allows us to make the approximation

$$T_{rec} \approx T_{LNA},$$ \hspace{1cm} (2.29)

where $T_{LNA}$ is given by

$$T_{LNA} = T_{iso} \left( \frac{w^HR_{LNA}w}{w^HRgw} \right).$$ \hspace{1cm} (2.30)

### 2.4 Polarimetry

In radio astronomy it is often desirable to measure the polarization state of received radiation. Polarization is a characterization of electromagnetic radiation based on the direction of the electric field vector. Most natural sources do not emit radiation in only one
polarization, meaning that the direction of the electric field vector is usually random. However, several natural sources are partially polarized. One example of a celestial source that emits polarized radiation is a pulsar as described in chapter 1. This partial polarization can be measured by a polarimeter.

In this section we review the mathematical representation of polarized and partially polarized radiation and how it can be measured by a PAF polarimeter.

2.4.1 Representing the Polarization State

The polarization state of a propagating wave can be represented by the Stokes vector [29]

\[
\mathbf{S} = \begin{bmatrix}
I \\
Q \\
U \\
V
\end{bmatrix} = \begin{bmatrix}
\langle |E_u|^2 \rangle + \langle |E_v|^2 \rangle \\
\langle |E_u|^2 \rangle - \langle |E_v|^2 \rangle \\
2\text{Re} [\langle E_u E_v^* \rangle] \\
2\text{Im} [\langle E_u E_v^* \rangle]
\end{bmatrix},
\]

(2.31)

where $E_u$ and $E_v$ represent the electric field components in two orthogonal polarizations such that

\[
\mathbf{E} = E_u \hat{u} + E_v \hat{v}
\]

(2.32)

and $\langle \cdot \rangle$ is the time expectation. This representation is convenient because it has a clear connection to the physical characteristics of the polarized radiation. $I$ gives a measure of the total intensity of the signal, $Q$ and $U$ are measures of the linearly polarized component, and $V$ is a measure of the circularly polarized component of the signal.

Another representation of the polarization state of propagating radiation is the coherency or correlation matrix. This representation is often more convenient from a signal processing point of view. The coherency or correlation matrix, $\mathbf{R}$, is given by

\[
\mathbf{R} = \begin{bmatrix}
\langle |E_u|^2 \rangle & \langle E_u E_v^* \rangle \\
\langle E_u^* E_v \rangle & \langle |E_v|^2 \rangle
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
I + Q & U + jV \\
U - jV & I - Q
\end{bmatrix}.
\]

(2.33)

This notation offers a more compact representation of several system properties and facilitates the mathematical treatment of the system polarimetric response.
2.4.2 Modeling Phased Array Polarimeters

Single feed polarimeters measure the intensity of incoming radiation with two orthogonally polarized antenna elements. The measurements from each antenna can then be combined to reconstruct the complete polarization state of the impinging electromagnetic wave. PAFs accomplish this by forming two sets of beamformer weights for each pixel on the sky. The two beamformers are chosen to have nominally orthogonal polarizations and we will refer to them as $w_1$ and $w_2$. Methods for finding beamformers with nominally orthogonal polarizations can be found in [20] and [21]. Accurate recovery of the polarization state of measured radiation requires a calibration of this beamformer pair and is the subject of Chapter 4.

Once the beamformers have been selected the correlated output of the two beams becomes a $2 \times 2$ matrix given by

$$R_{\text{out}} = E \left\langle \begin{bmatrix} w_1 & w_2 \end{bmatrix} H v v^H \begin{bmatrix} w_1 & w_2 \end{bmatrix} \right\rangle$$

$$= E \left\langle \begin{bmatrix} w_1^H v v^H w_1 & w_1^H v v^H w_2 \\ w_2^H v v^H w_1 & w_2^H v v^H w_2 \end{bmatrix} \right\rangle, \tag{2.34}$$

where we have added the subscript “out” to represent that this is the output of the system before polarimetric calibration. This matrix output of the two beamformers is reminiscent of the correlation or coherency matrix shown in equation (2.33). However, phase and gain differences in the LNAs and receiver channels are inherent in the system response, $v$, and need to be calibrated out in order for the measured $R_{\text{out}}$ to equal the actual coherency matrix of the impinging radiation.

Figure 2.0 shows a diagram of the full system, including beamformers and polarimetric calibration. This figure helps to illustrate polarimetric calibration and provides a summary of the background discussed in this section. The signal that impinges on the antenna is amplified by the LNA’s and mixed down to baseband by the receiver boards. A beamformer is then formed for each polarization. The calibration block modifies this beamformer so that the output of the system is a reconstructed version of the original signal.
In order to understand how the system transforms the polarization state of the incoming radiation we define the vectors \( \mathbf{v}_u \) and \( \mathbf{v}_v \) to be the response of the array to the purely polarized components of the electric field of unit intensity. Then \( \mathbf{v} \) becomes

\[
\mathbf{v} = \mathbf{v}_u E_u + \mathbf{v}_v E_v,
\]  

(2.35)

where we have broken down the electric field, \( \mathbf{E} \), at the antenna into orthogonally polarized components, \( E_u \) and \( E_v \) as in (2.32).

The response of the system, \( \mathbf{v}_u \) and \( \mathbf{v}_v \), to orthogonally polarized waves is difficult to measure in practice for a PAF on a large reflector, but the notation provides a compact
expression of (2.34). If we define

$$V = \begin{bmatrix} v_u & v_v \end{bmatrix},$$  \hspace{1cm} (2.36) \\
$$W = \begin{bmatrix} w_1 & w_2 \end{bmatrix},$$  \hspace{1cm} (2.37) \\

then $R_{\text{out}}$ becomes

$$R_{\text{out}} = W^H V R_{\text{in}} V^H W,$$  \hspace{1cm} (2.38) \\
$$= J R_{\text{in}} J^H,$$

where $R_{\text{in}}$ is the actual coherency matrix of the incoming radiation. The matrix

$$J = W^H V$$  \hspace{1cm} (2.39)$$

is called the Jones matrix and is a mathematical representation of the transformation between the measured coherency matrix, $R_{\text{out}}$, and the actual coherency matrix, $R_{\text{in}}$. For an ideal polarimeter $J$ is the identity matrix and $R_{\text{in}} = R_{\text{out}}$.

2.4.3 The Mueller Matrix

A more common representation of the transformation to the polarization state caused by the system is known as the Mueller matrix $M$ [17]. In the Mueller notation the output of the antenna is a Stokes vector, $S_{\text{out}}$, such that

$$S_{\text{out}} = M S_{\text{in}}.$$  \hspace{1cm} (2.40)

The Mueller matrix is convenient because it allows us to work directly with the Stokes vector.
The Mueller and Jones representations are mathematically equivalent. In order to show the relationship between the two we define the coherency vector \( \mathbf{c} \) as

\[
\mathbf{c} = \text{vec}(\mathbf{R}^*) = \begin{bmatrix}
\langle |E_u|^2 \rangle \\
\langle E_u E_v^* \rangle \\
\langle E_v^* E_v \rangle \\
\langle |E_v|^2 \rangle
\end{bmatrix}.
\]

The coherency vector is related to the Stokes vector by \( \mathbf{S} = \mathbf{T} \mathbf{c} \) where

\[
\mathbf{T} = \begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & -1 \\
0 & 1 & 1 & 0 \\
0 & -j & j & 0
\end{bmatrix}.
\]

Using this relationship it can be shown that the Mueller and Jones matrices are related by

\[
\mathbf{M} = \mathbf{T} (\mathbf{J} \otimes \mathbf{J}^*) \mathbf{T}^{-1}.
\]

For a more in depth treatment see [30] and [31].

2.5 Polarimetric Figures of Merit

In this section we define the figures of merit used to evaluate antenna polarimeters and how their definitions extend to PAF polarimeters.

2.5.1 Cross-polarization Discrimination

According to the standard IEEE definition the cross-polarization discrimination (XPD) is “the ratio of the power level at the output of a receiving antenna, nominally co-polarized with the transmitting antenna, to the output of a receiving antenna of the same gain but nominally orthogonally polarized to the transmitting antenna [32].”
For a phased array this means that the XPD is

\[
XPD_u = \frac{|v_{1,u}|^2}{|v_{2,u}|^2},
\]
\[
XPD_v = \frac{|v_{2,v}|^2}{|v_{1,v}|^2},
\]

(2.44)

where we have defined \(v_{1,u}\) and \(v_{2,u}\) to be the outputs of the first and second beamformers to the field \(E_u\) and we have defined \(v_{1,v}\) and \(v_{2,v}\) to be the outputs of the first and second beamformers to the field \(E_v\). These equations are ratios of the output power of the two beamformers when the array is illuminated by a wave of the opposite polarization.

The outputs of the beamformer are given by the elements of \(R_{\text{out}}\) as defined by equation (2.34). Following [20] we substitute the elements of \(R_{\text{out}}\) into equation (2.47) to get

\[
XPD_u = \frac{w_1^H v_u v_u^H w_1}{w_2^H v_u v_u^H w_2},
\]
\[
XPD_v = \frac{w_2^H v_v v_v^H w_2}{w_1^H v_v v_v^H w_1}.
\]

(2.45)

Equation (2.48) is further simplified using the definition of the Jones matrix. With this simplification, XPD becomes

\[
XPD_u = \frac{|J_{11}|^2}{|J_{21}|^2},
\]
\[
XPD_v = \frac{|J_{22}|^2}{|J_{12}|^2}.
\]

(2.46)

This result shows that the XPD is directly related to Jones matrix. As we would expect, the XPD increases as the Jones matrix approaches the identity matrix.

### 2.5.2 Cross-polarization Isolation

Cross-polarization isolation (XPI) is a metric similar to XPD that is used to characterize transmit antennas. The XPI is defined here for completeness even though the arrays described by this thesis are receive antennas. The IEEE standard definition of XPI is “The
ratio of the wanted power to the unwanted power in the same receiver channel when the transmitting antenna is radiating nominally orthogonally polarized signals at the same frequency and power level [32]."

For a phased array this leads to

\[
\begin{align*}
\text{XPI}_1 &= \frac{|v_{1,u}|^2}{|v_{1,v}|^2}, \\
\text{XPI}_2 &= \frac{|v_{2,v}|^2}{|v_{2,u}|^2}.
\end{align*}
\]

Since the beamformers \(w_1\) and \(w_2\) are nominally polarized for the \(u\) and \(v\) polarizations respectively this represents a ratio of wanted to unwanted power at the output for each polarization. We can obtain a relation to the Jones matrix by again following [20] through an analysis similar to the previous section. Doing so leads to

\[
\begin{align*}
\text{XPI}_1 &= w_1^H v_u v_u^H w_1 = \frac{|J_{11}|^2}{|J_{12}|^2}, \\
\text{XPI}_2 &= w_2^H v_v v_v^H w_2 = \frac{|J_{22}|^2}{|J_{21}|^2}.
\end{align*}
\]

This result shows that the XPI is also directly related to the Jones matrix. As we would expect, the XPI also increases as the Jones matrix approaches the identity matrix.
Chapter 3

Design and Implementation of Two PAF Systems

The RAS group at BYU is part of a larger international effort to develop science-ready PAFs. In order to do this the RAS group has designed and implemented a number of PAF systems, complete with the array itself, and all the back-end processing necessary to test it. This chapter details the development and implementation of two of these systems and my contributions to that process. The final systems were deployed on the Arecibo Telescope in Arecibo, Puerto Rico and at the National Radio Astronomy Observatory (NRAO) in West Virginia.

The RAS group has focused a lot of energy on the development of high sensitivity PAFs. Both of the PAF systems summarized in this chapter were designed to demonstrate high sensitivity through well optimized elements. The demonstration of high sensitivity PAFs is essential to the future of PAFs in radio astronomy since insufficient sensitivity is one of the key reasons PAFs are not yet considered science-ready.

3.1 System Overview

The construction of a full PAF system is a large project in which the entire RAS group is involved. A PAF system consists of the antenna array, a chain of amplifiers and receivers for each antenna element, analog to digital conversion, and digital signal processing which includes beamforming and calibration. In addition to the design and construction of the system by the RAS group, the final product has to be mounted on a reflector and integrated with the antenna systems of an observatory.

The first two dual-polarized PAF systems that were constructed by the RAS group were the ‘ear’ and ‘kite’ dipole arrays. The ‘ear’ array was tested on the Arecibo telescope
in Puerto Rico and the ‘kite’ array was verified on the 20 meter reflector at the NRAO. Figure 3.0 shows a high level block diagram of these two PAF systems.

The ‘ear’ PAF was designed by David Carter [11] using an optimization package I developed to combine EM models of the PAF with EM models of the reflectors. I will refer to this package as the MATLAB (MathWorks, Inc., Natick, MA) optimization package. The receivers for this array were designed by Michael Elmer and Vikas Asthana [33]. The 40 channel DAQ system was implemented by Michael Elmer and me and the DSP algorithms were developed by Dr. Brian Jeffs, Dr. Karl Warnick, and Michael Elmer.

The ‘kite’ PAF was also designed by David Carter [11] using the MATLAB optimization package. The LNAs were cooled using a cryostat designed and built by NRAO. The receiver cards and DSP algorithms were the same as those used with the ‘ear’ PAF.
3.2 Array Design

In this section we detail the development of the MATLAB optimization package and the dual-polarized model that supports it. The design of the ‘ear’ and ‘kite’ arrays with the MATLAB optimization package is summarized. The development of a similar optimization package for NASA is also mentioned.

3.2.1 The MATLAB Optimization Package

The MATLAB optimization package was created to address the difficulty of designing PAFs. The design of phased array feeds is more complicated than single element feeds because of mutual coupling and beamforming. Mutual coupling and beamforming change the effective impedance presented by each antenna element to the LNAs (see equation (2.9)). As a result of these effects, achieving a good active impedance match requires a full EM model of the PAF and reflector. The MATLAB optimization package allows a user to combine EM models of the array created in commercial software with models of the reflector. This full system model approach allows us to optimize the array elements for active impedance match, sensitivity, FoV, or any other figure of merit.

Arrays that were designed and implemented before the MATLAB optimizer was created relied primarily on EM models of the array generated in commercial software. The active impedance match was achieved by running an initial design through the MATLAB reflector model to estimate initial beamformer weights. It was then assumed that the weights would remain fairly constant and the initial beamformer weights were used to estimate the active impedances in subsequent optimization iterations. Several single pol arrays were designed using this method [34], [11].

The MATLAB optimization package improves on this method by running the full reflector model during every iteration. This improves on the prior technique by eliminating inaccuracies that result from the assumption that the beamformer weights do not change with each iteration. More importantly, this allows the user to design a cost function that includes FoV, bandwidth, sensitivity, or any other figure of merit in addition to the active impedance match.
Figure 3.2: Block diagram showing the structure of the MATLAB optimization package. The user selects which parameters should be allowed to change, what the starting parameters are, how to compute the cost function, and which optimization algorithm to use. The optimizer then computes the cost by running the EM models of the array and reflector and computing signal and noise correlation matrices and beamformer weights. The optimization continues until an acceptable cost is reached.

Figure 3.1 shows a block diagram of the MATLAB optimization package. To use the package the user creates an element design in commercial modeling software. The element design must be parameterized. This allows the user to control the overall design by changing desired parameters such as dipole arm length or radius. The user then inputs the desired parameters into the MATLAB optimization package along with upper and lower bounds on their values and their initial conditions, selects an optimization algorithm, and creates a cost function. The user can then choose from any of the built-in optimization algorithms offered by MATLAB or can write their own.

The flow of the optimization package is as follows: the MATLAB optimization package uses visual basic scripts to open the commercial EM software, set the optimization parameters to their current values, execute the simulation, and export the results. The results from the EM model of the array are the far electric fields, $E_{\text{far, array}}$, and the mutual impedance matrix, $Z$. The exported results are imported into MATLAB and used to generate the reflector response, $E_{\text{far}}$, and the system noise response. These are then combined to generate beamformers and compute the cost. Iterations continue until the design meets the desired tolerances. State information is saved after each iteration, enabling the optimization to be restarted if something goes wrong. This is particularly useful with full array simulations because each iteration is quite long and the optimization often runs for several weeks.
The MATLAB optimization package was used to design both the ‘ear’ and the ‘kite’ dipole arrays. Most of the optimizations used either a quasi newton or a genetic optimization algorithm. The first versions of the MATLAB optimizer could only optimize for a single frequency but subsequent versions have added the capacity to design for multiple frequency points. This adds bandwidth as a potential contribution to the cost function.

3.2.2 Modeling Dual-Polarized Arrays

In order to use the MATLAB optimization package to create dual-polarized designs, I expanded the model to be able to handle dual-polarized arrays. This included expanding the beamforming code to be capable of dual polarized beamforming. Polarimetric beamforming is the subject of chapter 4 and the details of polarimetric beamforming are discussed there. Those concepts were used to develop a dual-pol model of the array.

For dual-polarized arrays the figures of merit for each polarization vary slightly. The optimizer is designed to allow the user to optimize for X-pol figures of merit, Y-pol figures of merit, both X and Y polarizations, or to optimize for symmetry between the X and Y polarizations.

The optimizer also includes the ability to optimize for polarimeter figures of merit such as XPD, though we have not done so to date. This is because our focus has been on demonstration of high sensitivity arrays. As a result, we have chosen to optimize our designs for sensitivity and efficiency figures of merit and have not yet used the algorithm’s capacity to optimize for polarimeter figures of merit.

3.2.3 The Ear Dipole Element

An exhaustive review of the dipoles that were designed using the MATLAB optimization package is not given in this thesis since it was presented in [11]. We do, however, give a brief overview of the design and construction of the two dual-polarized arrays. This is given to demonstrate the success of the MATLAB optimization package.

The first PAF design to be created was the ‘ear’ dipole. The name comes from the ear-like structure that connects the dipole arms to the supporting posts. Figure 3.2(a) shows an image of the ‘ear’ dipole.
The design process used six optimization parameters. These included dipole arm length, dipole radius, separation between the dipole arms and the ground plane, the length of the cut on the side of the arms, the length of the cut on top of the arms, and the feed line radius [11].

The first opportunity to verify this design came as part of an experiment we were participating in at the Arecibo Telescope in Puerto Rico in the summer of 2010. Figure 3.2(b) shows the full array mounted on the telescope. The Arecibo telescope, pictured in Figure 3.3, is currently the largest radio telescope in the world.

3.2.4 The Kite Array

As with the ‘ear’ dipole design, the ‘kite’ elements were designed by David Carter using the MATLAB optimization package [11]. The name comes from the kite-like shape of the arms.

The dipoles were designed to be used with cryo-cooled LNAs in order to further reduce system temperature. The design was optimized over seven parameters, including the kite length from the feed to the outside corner, the length from the feed to the center corner, the thickness of the kite, the separation between the arms and the ground plane, the angle
between the feed axis and the outside corner of the kite, the angle between the support posts and the arms, and the feed line radius [11].

The supporting cryostat, shown in Figure 3.4(b), was built by NRAO and the array was mounted on NRAO’s 20 meter telescope during the summer of 2011. Figure 3.4(c) shows the array connected to the cryostat.

While this array did show significant improvement in sensitivity over prior arrays, the beamformed sensitivity improvement was not as high as expected. In order to diagnose this issue a new array is currently being designed and constructed to be mounted on both the 20 meter telescope and the 100 meter GBT. The details of this array can be found in Chapter 5.

3.2.5 A NASA Application for the MATLAB Optimization Package

Using the experience gained while writing the MATLAB optimization package, I developed a similar software package to aid in the design of phased arrays for NASA remote sensing missions. The details of the software are largely the same but different commercial EM software was used and the beamforming algorithm was different.
The package is currently being used to aid in the development of the NASA EcoSAR instrument. EcoSAR is an active Ku-band SAR and interferometer instrument that will be used for remote sensing of biomass.

The new package was designed to be easily applied to other array designs. As a result, it will benefit future NASA missions as well as the EcoSAR instrument.

3.3 Data Acquisition

The ‘kite’ array was the first dual-polarized PAF constructed by the RAS group. Because dual-polarized arrays have twice as many antenna input channels as single-polarized
arrays a new DAQ system was required in order to verify the new array. To fill this need, Michael Elmer and I implemented a 40 channel DAQ system. The system was successfully used to acquire data during our experiments at the Arecibo observatory.

The new DAQ system was designed to accommodate engineering verification and not to be used for actual scientific observations. As a result, the required bandwidth was relatively small (≈ 450KHz). To accommodate this bandwidth the prior 20 channel single-polarized DAQ system operated at 1.25 mega-samples per second. The new DAQ system was designed to meet this same specification.

Multiple PCs were used in order to achieve the high aggregate data rate required of the DAQ system. The aggregate data rate needed to achieve 1.25 mega-samples per second on every channel is 600 megabits per second. To achieve this, data is collected by five node PCs that are connected to a central hub PC via 1 gigabit Ethernet. Each node PC has two Adlink analog to digital cards with four channels each and data is acquired directly to the hard drive of each node. After acquisition the data is aggregated on the hub PC.

The hub PC also manages communications between the DAQ system and the telescope control software (TCS). Coordination between the DAQ system and the TCS is required in order to make sure that the telescope is pointing in the appropriate direction for each acquisition. The new DAQ system handles this coordination using simple handshakes. The hub acts as a slave to the TCS and passes commands from the TCS to the nodes.

When an acquisition is begun, the hub waits for a message from the TCS. When the TCS asserts that the telescope is in position the hub alerts the nodes to begin data acquisition. Once the data is acquired the hub alerts the TCS and the telescope is repositioned for the next acquisition. This process is repeated until all the desired data have been acquired.

Synchronization of the nodes is achieved by using a trigger generated by a function generator. When the telescope is in position the hub PC alerts each node to begin acquisition on the trigger. Each node sends back a message as a handshake to the hub that it is ready for acquisition. After all the nodes have indicated they are ready, the hub PC sends a command to the function generator that causes it to generate a trigger pulse and the cards begin acquiring data. The trigger is distributed to the nodes through a distribution network created by engineers at Arecibo.
The clock is also generated by a function generator distributed to each of the 10 cards. The cards require an input clock rate of at least twice the sample rate, so the function generator supplies a 2.5MHz clock.

An issue arose when we discovered that our data synchronization was imperfect. We noticed that data collected on different cards was often out of sync by 1/2 to 3/2 clock cycles. This resulted in a phase shift on certain channels. Because phase information is critical in PAF observations, this phase shift presented a crucial problem with the DAQ system.

Solving this problem required a considerable amount of time and proved to be one of the most difficult obstacles we had to overcome in implementing the DAQ system. In the end, we could not find a way to achieve perfect synchronization. We circumvented the problem by adding a sinusoidal pulse to the beginning of each acquisition. The sinusoidal pulse allowed us to extract the phase information for each card. We were then able to add a correction to any cards that were out of sync.

The pulse was injected by a third function generator. This function generator turned on with the trigger and injected a sine wave into the first channel of each card for a few milliseconds at the beginning of every acquisition. The signal was injected into the appropriate channel using combiners which can be seen in Figure 3.5(c). A MATLAB script was used to extract the phase information for each acquisition and the appropriate correction was added at correlation.

The DAQ software package can be run from the command line on the hub PC. The hub uses remote desktop commands to launch the appropriate software on each node so that the user only has to execute commands on the hub. When starting the program, the user specifies a file and location for data storage and the integration time or the amount of time that data will be acquired for each telescope pointing.

Figure 3.6 is a block diagram detailing the DAQ process. The diagram shows details of the communication between the TCS, the hub PC, and the nodes. In every instance strict handshaking is required to assure that commands are received and successfully executed.

The whole system, including all six PCs and all three function generators, was assembled in a shielded rack to avoid causing interference in the observations since the system was located in the feed dome above the Arecibo reflector. The signal from the array was brought
in through connectors that were placed in the top of the rack as seen in Figure 3.5(c). The final product can be seen in Figure 3.5(d) which shows an image of a colleague and me standing next to the DAQ system where it was mounted in the Arecibo feed dome. Several other images of the completed system can be seen in Figure 3.5.

The 40 channel system was successfully used to collect dual-polarized data during experiments at the Arecibo telescope, providing the RAS group with its first dual-polarized data set.
Figure 3.7: Block diagram of the DAQ system. The user starts the DAQ system from the command line on the hub PC. The hub then sends a message to each node causing them to launch their DAQ software. Upon receiving tReady from the telescope control software the hub alerts each node to prepare to acquire data. Once all the nodes are ready, the hub activates a function generator which triggers the nodes to begin data acquisition. When the desired amount of data has been acquired, the hub alerts the telescope control software that it is ready for the next pointing. This process continues until the observation is completed.
3.4 Summary

This chapter has reviewed the construction of two PAF systems with details of my contributions to each system. Both arrays were designed using a design technique I implemented. The technique allows a user to optimize for any figure of merit. This is achieved by combining models of the array with in-house models of the reflector. The implementation of a similar technique to aid in the design of phased arrays for NASA missions is mentioned. The chapter also details the construction of a 40 channel DAQ system that was implemented to verify the ‘ear’ array.
Chapter 4

Polarimetric Calibration of PAFs

The ability of PAFs to accurately measure the polarization state of received waves is essential for many astronomical observations. Pulsars, supernova remnants, and other deep space objects radiate partially polarized radio waves. Pulsars are particularly important to the astronomical community because they are expected to enable detection of gravitational waves and provide sensitive tests of the laws of fundamental physics [29].

Because array element imperfections and phase and gain imbalances in array receiver chains cause a transformation in the measured polarization state of radiation incident on the array, polarimetric calibration is required in order to recover the actual polarization state of the source. Calibration is generally accomplished by measuring the transformation caused by a PAF system on the measured polarization. Once this transformation is known an inverse transformation is applied and the actual polarization state of the source is recovered.

Existing polarimetric calibration methods used for single-pixel feeds suffer from limitations that make them impractical for PAFs. Calibration methods used for optical systems require as many as four partially polarized calibrator sources. Other methods only partially calibrate the array or are based on assumptions about the form of the system Mueller matrix that limit the generality of the method [20], [21]. Astronomical antennas can be calibrated by a multi-hour observation of a single partially polarized source [17], but this approach must be repeated with the source located near the peak of each electronically formed PAF beam and would be time-consuming for a wide-field instrument.

This chapter introduces an efficient polarimetric calibration method that uses only three snapshot observations of celestial sources, one unpolarized and two polarized. The method makes no assumptions about the orientation of the feed elements, has stability in sensitivity with respect to reasonable mechanical degradations such as non-orthogonal
polarization responses of the phased array antenna elements, and is robust with respect to large degradations. Simulation results demonstrate high calibration stability across the field of view.

4.1 Three Source Calibration

We begin by establishing notation for a three source calibration technique. Figure 4.0 shows a block diagram for a calibration method that uses three sources, one polarized and and two unpolarized, and provides a nice summary for the following discussion.

The goal of a three source calibration method is to find two transformations that together recover the correct polarization state of measured radiation. These transformations can be Mueller or Jones matrices that multiply the uncalibrated Stokes vectors or covariance matrices respectively. We will call them $M_1$ and $M_2$ in the Mueller domain and $J_1$ and $J_2$ in the Jones domain. We solve for $J_1$ and $M_1$ with a single measurement of an unpolarized source and we find $J_2$ and $M_2$ with snapshot measurements of two polarized sources.

In the Mueller domain, the measured Stokes vector after calibration is

$$S_{\text{out}} = M_2 M_1 S_{\text{in}},$$

(4.1)

Figure 4.1: Block diagram of a three source calibration method. To make the order of matrix multiplication more intuitive we begin at the right and move left. Radiation from a source characterized by $S_{\text{in}}$ impinges on the array and passes through the LNAs and receiver chains. This causes a transformation of the polarization state of the radiation. The transformation is represented by $M$. The first calibration source is used to generate $M_1$ which calibrates the system to within a rotation and successfully recovers the polarization state for an unpolarized source. With a second and third source that are polarized we generate the matrix $M_2$ which attempts to undue the rotation of the polarization state caused by $M_1 M$. A similar treatment can be applied in the Jones domain.
where $M_1$ and $M_2$ are the Mueller calibration matrices from Figure 4.0, $M$ is the transformation defined by the properties of the system and the raw beam pair before calibration, and $S_{in}$ is the Stokes vector representing the actual polarization state of the radiation.

In the Jones domain the final measured covariance matrix, $R_{out}$, can be written as

$$R_{out} = J_2 J_1 J R_{in} J^H J_1^H J_2^H,$$

(4.2)

where $J_1$ and $J_2$ are Jones calibration matrices, $J$ is the Jones matrix defined by the properties of the system and the raw beam pair before calibration, and $R_{in}$ is the covariance matrix representing the actual polarization state of the impingent radiation. In order to make the notation simpler we define

$$\hat{M} = M_2 M_1 M,$$

(4.3)

$$\hat{J} = J_2 J_1 J,$$

(4.4)

so that we have

$$S_{out} = \hat{M} S_{in} \approx S_{in},$$

(4.5)

$$R_{out} = \hat{J} R_{in} \hat{J}^H \approx R_{in}.$$  

(4.6)

If calibration is perfect these approximations become equalities.

4.1.1 The Jones Polarimeter

Before any calibration, the measured covariance matrix of the first unpolarized calibration source is

$$R_{1} = J R_{in,1} J^H,$$

(4.7)

where we have defined $R_{1}'$ to be the covariance matrix measured by the system without calibration and we use the subscript 1 to denote that we are looking at the first source.
Solving this equation for $J$ gives:

$$J = R_1'^{1/2} U R_{in,1}^{-1/2},$$  \hspace{1cm} (4.8)$$

where $U$ is an arbitrary unitary matrix. We can simplify (4.8) further because the first calibration source is unpolarized and is therefore of the form $\sigma^2 I$, where $I$ is the identity matrix and $\sigma^2$ is the intensity of the source. Using this form for $R_{in,1}$ we can rewrite (4.8) as

$$J = \frac{1}{\sigma_1} R_1'^{1/2} U.$$

Based on this solution for $J$ we choose $J_1$ to be the inverse of the known part of (4.9) or

$$J_1 = \sigma_1 R_1'^{-1/2}$$  \hspace{1cm} (4.10)$$

and the system Jones matrix after the first calibration source becomes

$$J_1 J = \sigma_1 R_1'^{-1/2} \frac{1}{\sigma_1} R_1'^{1/2} U = U.$$  \hspace{1cm} (4.11)$$

which shows that this choice for $J_1$ makes the Jones matrix after the first calibration step unitary.

After multiplication with $J_1$ the system is calibrated for intensity and can accurately measure $I$ and the percent polarization of a partially polarized source. Section 4.1.2 shows that this unitary degree of freedom represents a rotation between the Stokes parameters $Q$, $U$, and $V$. In order to obtain accurate polarimetric measurements we must solve for the rotation. We can then choose $J_2$ to be the inverse rotation, $U^H$, so that $J_2 J_1 J$ becomes the identity matrix.

Solving for $U$ requires measurements of partially polarized sources with known polarization states. We can use these measurements to construct a system of equations and unknowns to solve for the elements of the unitary matrix. The first equation is constructed by noting the form of the measured coherency matrix of the first polarized source. Our first set of equations is given by the measured coherency matrix after calibration with $J_1$. The
measured coherency matrix is given by

\[ R'_2 = J_1 J R_{\text{in},2} J^H J_1^H \]
\[ = U R_{\text{in},2} U^H. \quad (4.12) \]

We can create another set of equations by considering the properties of unitary matrices. The elements of a unitary matrix satisfy the relationships

\[ |u_{1,1}|^2 + |u_{1,2}|^2 = 1, \]
\[ |u_{1,1}|^2 + |u_{2,1}|^2 = 1, \]
\[ |u_{2,2}|^2 + |u_{2,1}|^2 = 1, \]
\[ |u_{2,2}|^2 + |u_{1,2}|^2 = 1, \]
\[ u_{1,1} u_{2,1}^* + u_{1,2} u_{2,2}^* = 0, \]
\[ u_{1,1} u_{1,2}^* + u_{2,1} u_{2,2}^* = 0, \]
\[ u_{1,1} u_{1,2} + u_{2,1} u_{2,2} = 0, \]
\[ u_{2,1} u_{2,2} + u_{1,2} u_{2,2} = 0. \quad (4.14) \]

Because of the structure of equation (4.12), we can also restrict the \( u_{1,1} \) element to be real since the overall phase cancels. Making this assumption and using the properties of (4.13) we can write a unitary matrix as a function of three parameters:

\[ U = \begin{bmatrix} \sqrt{1 - \beta^2} & \beta e^{j\phi} \\ -\beta e^{j(\theta - \phi)} & \sqrt{1 - \beta^2 e^{j\theta}} \end{bmatrix}. \quad (4.15) \]

Using these parameters and equation (4.12) we can write four equations in terms of estimates of the parameters of \( U \), which we will call \( \hat{\beta}, \hat{\theta}, \) and \( \hat{\phi} \), and the elements of \( R'_2 \), and \( R_{\text{in},2} \):

\[ f_1 = \hat{\beta}^2 (r_{\text{in},2}^{2,2} - r_{\text{in},2}^{2,1}) + \hat{\beta} \sqrt{1 - \hat{\beta}^2 (e^{j\hat{\phi}} r_{\text{in},2}^{2,1} + e^{-j\hat{\phi}} r_{\text{in},2}^{2,1}) + r_{\text{in},2}^{2,1} - r_{\text{in},2}^{2,1}}, \quad (4.16) \]
\[ f_2 = \hat{\beta}^2 (r_{\text{in},2}^{2,1} - r_{\text{in},2}^{2,2}) - \hat{\beta} \sqrt{1 - \hat{\beta}^2 (e^{j\hat{\phi}} r_{\text{in},2}^{2,1} + e^{-j\hat{\phi}} r_{\text{in},2}^{2,1}) + r_{\text{in},2}^{2,2} - r_{\text{in},2}^{2,2}}, \quad (4.17) \]
\[ f_3 = \hat{\beta} \sqrt{1 - \hat{\beta}^2 e^{j(\theta - \phi)} (r_{\text{in},2}^{2,2} - r_{\text{in},2}^{2,1}) - \hat{\beta}^2 (e^{j\hat{\theta}} r_{\text{in},2}^{2,1} + e^{j(\hat{\theta} - 2\hat{\phi})} r_{\text{in},2}^{2,1}) + e^{j\hat{\theta}} r_{\text{in},2}^{2,1} - r_{\text{in},2}^{2,1}. \quad (4.18) \]
The fourth equation involving $r_2^{2,1}$ is the complex conjugate of (4.18) and is not independent. In these equations we have used $r_2'$ to represent the elements of $R'_2$ and $r_{in,2}$ to represent the elements of $R_{in,2}$ with superscripts to represent the element indices.

Numerically, it can be shown that there is insufficient information to obtain $\hat{\beta}$, $\hat{\theta}$, and $\hat{\phi}$. In order to constrain the solution, observation of an additional partially polarized source is needed. With a second partially polarized source we can generate another set of equations like (4.16)-(4.18). We similarly label these equations $f_4$, $f_5$, and $f_6$. The solution is obtained when $\{f_1, f_2, ..., f_6\}$ are exactly zero. A numerical optimizer can be used to find the solution with respect to $\hat{\beta}$, $\hat{\theta}$, and $\hat{\phi}$ by minimizing the cost function

$$\text{cost} = \sum_{i=1}^{6} |f_i|.$$  

(4.19)

Once we have $\hat{\beta}$, $\hat{\theta}$, and $\hat{\phi}$, the unitary degree of freedom $U$ can be found and we can set $J_2 = U^H$. We can then use $J_1$ and $J_2$ to calibrate the beamformer pair.

4.1.2 The Mueller Polarimeter

While the actual calibration is performed using the Jones matrix a great deal of insight can be gained by considering the Mueller matrix defined in equation (2.40). We can find the form of the Mueller matrix after the initial calibration step by transforming according to (2.43)

$$M_1M = T(U \otimes U^*)T^{-1}.$$  

(4.20)

We will not show it in detail here but this transformation on a unitary Jones matrix results in a Mueller matrix of the form

$$M_1M = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & l_{11} & l_{12} & l_{13} \\
0 & l_{21} & l_{22} & l_{23} \\
0 & l_{31} & l_{32} & l_{33}
\end{bmatrix},$$  

(4.21)
where we have used \( l_{ij} \) to represent unknown values. This equation shows that after the first calibration the system is calibrated for the Stokes \( I \) and the remaining calibration can be thought of as rotations of the polarization vector \( [Q \ U \ V]^T \). These rotations of the polarization vector are referred to as pol-rotations [35]. Since the remaining inaccuracy is simply a rotation of the polarization vector, \( M_2 \) has the form

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & \left[ R_{\theta,\phi} \right] \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

where \( R_{\theta,\phi} \) is a rotation matrix.

This gives a physical interpretation to the matrices \( J_1 \), \( M_1 \) and \( J_2 \), \( M_2 \). \( J_1 \) and \( M_1 \) correct for pol-conversion, or the mixing of the Stokes \( I \) into \( Q \), \( U \), and \( V \) and vice versa. \( J_2 \) and \( M_2 \) correct for pol-rotation which is the rotation of the polarized portion of the Stokes vector [35].

### 4.2 Numerical Results

In this section we present results generated by using a modeled \( 19 \times 2 \) element PAF and reflector system. The model assumes a 20 meter diameter dish with a focal length to diameter ratio of 0.43. The array consists of crossed dipole elements arranged on a hexagonal grid and spaced \( 0.6\lambda \) wavelengths apart at a center frequency of 1.6 GHz. Analytical dipole radiation patterns are used to compute the array element responses. Reflector scattering is computed using the physical optics approximation. Microwave network theory is used to couple the array elements and low noise amplifiers in order to provide a complete model of the PAF system signal and noise response, including sky noise, spillover noise, electronics noise, and mutual coupling effects. Imperfections are introduced by varying the angle between the dipoles randomly. The angle parameter for each crossed dipole can vary between \( 0^\circ \) (parallel) and \( 90^\circ \).
There are several ways of measuring the accuracy of the calibrated beamformer. The most recognizable measure for astronomical applications is the relative RMS Stokes error

\[ E_S = \frac{||\Delta S||}{I_{in}} = \sqrt{(I_{out} - I_{in})^2 + (Q_{out} - Q_{in})^2 + (U_{out} - U_{in})^2 + (V_{out} - V_{in})^2} \]  

(4.22)

Because \( E_S \) is specific to a single source a more general measure of the error is needed. A useful metric is the Mueller matrix deviation from identity which is given by

\[ E_M = ||M_{cal} - I||_{fro} \]  

(4.23)

where \( M_{cal} \) represents the final Mueller matrix after correction. \( E_M \) provides an upper bound on the RMS stokes error for the calibrated system [20].

The calibration needs to be stable with respect to several different parameters. It must be robust with respect to mechanical errors caused by poor construction of the antenna elements and accurate across the field of view. In addition to requiring stability with respect to calibrator error, the calibration method must maintain high sensitivity. Our first simulations demonstrate the stability of the array with respect to mechanical degradations as well as the impact of polarimetric calibration on the sensitivity.

We simulated fabrication and mechanical imperfections by introducing random perturbations to the orthogonality of the crossed-dipole elements. Figure 4.1(a) shows the Mueller matrix error and the Stokes error for a randomly generated source as the variance of these perturbations increases. The error is stable even for highly non-orthogonal dipoles. The stability of the calibration method with respect to large mechanical degradation gives high confidence in the polarimetric accuracy of the calibrator.

Another important measure of the accuracy of polarimetric calibration is the performance criteria defined by IEEE for single feed polarimeters. These definitions have been extended to PAFs by Warnick et al. [20] and are presented in chapter 2. Figure 4.1(b) shows the XPD for the two polarization channels. As expected, the method shows the same stability with respect to XPD as it did with respect to calibrator error and the XPD goes to infinity for perfectly orthogonal dipoles.
(a) Mueller matrix error and RMS Stokes error for a randomly generated source as the deviation from perfect orthogonality increases. The plot shows that the calibrator is accurate even with poorly aligned antenna elements.

**Figure 4.2:** Mueller matrix error and XPD against nonorthogonality in the dipole design.

(b) XPD as the variance of the angle between the dipoles increases. The XPD is high even for poorly aligned antenna elements. As expected, the XPD goes to infinity for perfectly orthogonal elements.

(a) Sensitivity as the dipole angle perturbation increases. Sensitivity is only stable when the perturbation is less than about 5 degrees.

(b) Inverse sensitivity figure of merit as the dipole angle perturbation increases. It is only stable when the perturbation is less than about 5 degrees.

**Figure 4.3:** Stability of sensitivity with respect to mechanical degradations for polarimetric calibration.

Figures 4.2(a) and 4.2(b) show the variation in sensitivity and $T_{sys}/\eta_{ap}$ with respect to the same set of perturbations. These figures of merit show some degradation with the decreasing orthogonality of the dipoles but are stable as long as the dipoles are orthogonal to within a few degrees. This is well within construction tolerances for most arrays.
In addition to requiring stability with respect to degradations caused by poor construction, the calibration must be stable over the PAF instrument field of view. Figures 4.3(a) and 4.3(b) show that sensitivity and $T_{sys}/\eta_{ap}$ are stable over the field of view and unchanged by the polarimetric calibration. This is a significant result because it shows that we can do polarimetry without loss in sensitivity.

4.2.1 Calibration Across The Field of View

Calibration of the full field of view requires that each calibrator source be placed at the center of every beam. For a large PAF this means several hours of calibration time. In order to save calibration time, this section investigates the possibility of calibrating the full field of view using a few sparse measurements of the calibrator sources.

To determine the feasibility of this method we looked at the stability of the Jones matrix and our calibrator error across the field of view. The stability of the calibrator error, $E_M$, across the field of view is shown in figure 4.4(b). The dotted line shows the error when a measurement of the calibrator sources is acquired at the center of each beam. As expected, this leads to a highly accurate recovery of the polarization state for all beams in the field of view. The solid line shows $E_M$ when the bore sight estimate of the Jones matrix is used.
Figure 4.5: Stability of the Jones matrix and the calibrator error across the field of view.

to calibrate the remainder of the field of view. For analytical dipoles the plot shows that calibration of the field of view out to about 2 HPBWs is possible with only the bore sight measurement of the calibrator sources.

Figure 4.4(a) shows the variation of the Jones matrix across the field of view. The plot demonstrates that the elements of the Jones matrix vary slowly and smoothly with respect to steering angle. The slow variation of the Jones matrix suggests that calibration of the full field of view is feasible using only sparse measurements of the calibration sources and interpolation in between. This would save hours of calibration time.

To test this possibility we used one of our EM models of a full 19 × 2 element PAF. This is more accurate than the analytical dipoles used to generate figures 4.4(b) and 4.4(a) and offers a better idea of what can be accomplished using interpolation to calibrate the field of view. Figure 4.5 shows the results of this experiment. The green line in the figure was generated using re-measurements of the calibrator sources once every HPBW and interpolation in between. The blue and red lines are to provide a baseline. The blue line used the biscalare scaled max SNR method of [21] to perform a partial recalibration at every pointing and the red line used only the bore sight measurement to calibrate each subsequent beam.
Figure 4.6: Error for field of view calibration using interpolation. The blue line gives the error when using the one source biscalar method to provide a baseline. The red line gives the error when the system is calibrated using only the bore sight measurement. The green line shows the error when fresh measurements of the calibrator sources are used to perform full re-calibration at each HPBW. Interpolation is then used to calibrated the beams in between. Since most observations do not operate on a grid finer than points at each HPBW this plot shows that something more sophisticated than simple interpolation is required to calibrate the field of view.

While the figure shows that interpolation achieves reasonable accuracy for angles less than one HPBW, it shows no significant improvement for angles greater than one HPBW. Because most observations do not form beams on a scale finer than one HPBW we concluded that field of view calibration cannot be achieved with simple interpolation. It is still possible that full field of view calibration could be achieved with sparse measurements of the calibration sources but the method must use something more sophisticated than basic interpolation.

4.3 Summary

This chapter presents an efficient polarimetric calibration method for radio astronomy PAFs. Calibration is accomplished with three snap shot measurements of celestial sources. One of these sources is the unpolarized array calibrator source and the other two sources are additional partially polarized calibrators. All three sources must be placed at the center
of each beam in order to achieve full calibration. The method is stable with respect to mechanical degradations and across the field of view.
Chapter 5

A Dual-polarized PAF for the Green Bank Telescope

The BYU RAS group has a long history of collaboration with the National Radio Astronomy Observatory (NRAO). The goal of this collaboration is to design and implement a science ready PAF for the GBT. The GBT is the largest fully steerable telescope in the world. It is located in West Virginia and managed by the NRAO. This chapter details the design of a PAF for that telescope. Though the technology is not yet mature enough to create a science ready PAF this array is an important step in that direction.

For the RAS group at BYU the design is also an important step toward understanding a problem that was discovered with earlier arrays. The cryo ‘kite’ array mentioned in Chapter 3 did not achieve the expected sensitivity improvement in on-reflector measurements. An important design goal for this array is to help diagnose the reason that the cryo ‘kite’ array did not meet expectations.

This chapter details the creation of an EM model for the GBT and its use to design a PAF for the GBT. The issue of array spacing is addressed. Measured and modeled results for a prototype element are given along with simulated results for the latest iteration of the design. The simulated array results are given at two different spacings. These results will aid in finalizing the array design and verifying our EM models.

5.1 An EM Model for The Green Bank Telescope

The first step toward designing a new PAF was the development of an EM model for the GBT. The GBT, pictured in Figure 5.0(b), is an offset reflector. Offset reflectors are designed to minimize the amount of signal blocked by the feed support structures. As shown in Figure 5.0(a), this is accomplished by placing the feed near the edge of the dish rather than at the center. Of course, this means that the focal point of the reflector must also be
To facilitate the design of an array for the GBT, I completed an EM model of the reflector that allows us to account for its offset geometry. The model builds on an existing design that was created by Dr. Karl Warnick, Jonathan Landon, and David Jones [34]. The existing model successfully modeled single polarized arrays but failed to adequately account for the offset reflector geometry when implementing the second polarization. I corrected this error and completed an EM model for the GBT.

The reflector model is based on the physical optics approximation. A separate model is used to generate the far fields resulting from the array. The equivalence principle is then used to convert these fields into equivalent currents on the reflector surface which are then passed into the far field radiation integral in order to compute the secondary far field patterns.

The secondary patterns are passed into existing radio astronomy codes where the signal and noise correlation matrices are computed. These matrices can then be combined with the far fields to compute beamformer coefficients and figures of merit.

With the reflector model updated to handle dual polarized arrays we are now able to follow the same basic design principles that were used to design the ‘kite’ and ‘ear’ dipole.
arrays. This capability was not only essential for the designs described by this chapter but will also be an important asset for the RAS group as it creates more designs for the GBT in the future.

### 5.2 Element Spacing

For the GBT, the ratio of the focal length to the diameter is larger than that of the 20 meter telescope. This means that the illumination pattern of the array must be narrower. Because the current set up of the GBT allows for only 19×2 array elements, this must be achieved by altering the spacing of the elements.

Our first test for determining the appropriate spacing was to use analytical dipoles. The phrase ‘analytical dipoles’ refers to the use of analytical radiation patterns. These patterns are less accurate than the patterns generated by commercial modeling software but they run faster and provide a good first order estimate.

Figures 5.1(a) and 5.1(b) show the modeled sensitivity and effective FoV for various spacings of the analytical dipoles. The plots show agreement that the ideal spacing is around 16 cm. This result will vary based on the final element design and a more thorough analysis will need to be performed after the design is finalized and before construction of the ground plane.

Based on these results and the center frequency of our array we have chosen to design our elements for a spacing of 14.99 cm (0.7λ at 1.4 GHz). All of the optimizations used to arrive at our final design assumed this element spacing.

### 5.3 Element Design

Prior arrays were designed with an element spacing of 11.24 centimeters. At the 1.6 GHz center frequency of prior arrays this is a spacing of 0.6λ. The decision to design the GBT array for a spacing of 0.7λ heavily impacted our design process because the mutual coupling at 0.7λ is less than the mutual coupling at 0.6λ.

Smaller mutual coupling means that we can design an isolated element first and then populate this into an array without as much change to the S-parameters as a result of mutual coupling. It also means that active impedances should deviate less from the mutual

50
Figure 5.2: Figures of merit vs element spacing. For the design frequency of 1.4 GHz, the spacing goes from approximately 0.6λ to 0.9λ

impedances than they did in our more closely spaced arrays. This is because smaller mutual coupling means that the off diagonal elements of $Z$ in equation (2.9) will be small.

As a result of these two assumptions we did most of the array design directly in our commercial EM modeling software. We started with an isolated element design and optimized it for a 50 ohm impedance match. When the 50 ohm match was satisfactory we populated an array with the isolated element design. We then optimized the design further by attempting to minimize the reflection coefficients of the center “x-polarized” and “y-polarized” elements.

The element design is based on the ‘kite’ element from the previous chapter. One important addition was a change in the shape of the Teflon bead located at the top of the coax near the feed point (see parameter P6 in figure 5.3). The purpose of this change was to add more mechanical stability to the vertical location of the inner conductor. To accomplish this we introduced a shoulder in the Teflon bead by changing the diameter of the inner conductor as it passes through the bead. This was done to address one of the issues that was thought to be a potential cause of the less than expected sensitivity achieved by the ‘kite’ array. The issue was a lack of stability for vertical mobility of the inner conductor. This resulted in random changes in its vertical position across the array [11].
Table 5.1: Optimization parameters and their final values for the GBT1.

<table>
<thead>
<tr>
<th>Optimization Parameter</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm Length (P1)</td>
<td>24.85mm</td>
</tr>
<tr>
<td>Gap between the two inner conductors (P2)</td>
<td>0.4mm</td>
</tr>
<tr>
<td>Angle made by the side of the dipole arm (P3)</td>
<td>26.84°</td>
</tr>
<tr>
<td>Distance between the LNAs and the ground plane (P4)</td>
<td>30.2mm</td>
</tr>
<tr>
<td>Angle between the arms and the supports (P5)</td>
<td>50.02°</td>
</tr>
<tr>
<td>The percent by which the width of the inner conductor is reduced at the Teflon bead (P6)</td>
<td>20.00%</td>
</tr>
<tr>
<td>The length of the upper Teflon bead (P7)</td>
<td>4.39mm</td>
</tr>
</tbody>
</table>

The design was optimized within our commercial EM software. The optimization parameters were the length from the inner corner to the feed point, the length of the Teflon beads, the distance between the LNA and the ground plane, the gap between the inner conductors, the size of the horizontal portion of the inner conductor compared to the vertical
portion, the angle made by the side edge of the arm with the back edge of the arm, and the angle between the dipole arms and the posts.

The cost function we used varied over the design process but always included the minimization of $S_{11}$, corresponding to the “x-polarized” dipole, and $S_{22}$, corresponding to the “y-polarized” dipole. Most cost functions also included some measure of how symmetric $S_{11}$ and $S_{22}$ were. This is because it turned out to be a difficult problem to get the two polarizations to have the same reflection coefficient over the band of interest even though the design is mostly symmetric. The only asymmetry between the two polarizations is in the length of the inner conductors since one of them is higher than the other in order to avoid an intersection. We found that this slight difference could cause major differences in the response of the individual dipoles. This was overcome by adding a variable to the cost function that measured how different $S_{11}$ was from $S_{22}$ across the band of interest.

Figure 5.3 shows a schematic drawing of the final dipole design which we call the GBT1 element. The optimization parameters are noted as P1 through P7. Table 5.0 shows the final values for these parameters in the GBT1 design.

### 5.3.1 Measured Results

Before constructing the full $19 \times 2$ element array we constructed a single element to verify our model. The element was constructed by the precision machine lab at BYU based on mechanical drawings created by Stephen Carlson, an electrical engineering student at BYU. A picture of the constructed element is shown in figure 5.3.

The constructed element was mounted on a ground plane and measured using a network analyzer. The measurements were taken with the element placed inside a box of absorber. Figure 5.4 shows the measured and modeled reflection coefficients for each polarization. The results show reasonable agreement between measured and modeled results though there is a frequency shift in one of the polarizations. This is an issue since it is important that the two polarizations are balanced. We are working to correct this error in a modified design.
Figure 5.4: Image of the constructed GBT element.

Figure 5.5: Modeled and measured S-parameters for the isolated GBT1 element.
Table 5.2: Optimization parameters and their final values for the GBT2.

<table>
<thead>
<tr>
<th>Optimization Parameter</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm Length (P1)</td>
<td>26.99mm</td>
</tr>
<tr>
<td>Gap between the two inner conductors (P2)</td>
<td>0.498mm</td>
</tr>
<tr>
<td>Angle made by the side of the dipole arm (P3)</td>
<td>41.07°</td>
</tr>
<tr>
<td>Distance between the LNAs and the ground plane (P4)</td>
<td>30.2mm</td>
</tr>
<tr>
<td>Angle between the arms and the supports (P5)</td>
<td>82.0°</td>
</tr>
<tr>
<td>The percent by which the width of the inner conductor is reduced at the Teflon bead (P6)</td>
<td>10.00%</td>
</tr>
<tr>
<td>The length of the upper Teflon bead (P7)</td>
<td>5.31mm</td>
</tr>
</tbody>
</table>

Figure 5.6: Image of the GBT2 element.

5.4 Simulated Array Results

During construction and verification of a prototype for the GBT1 element design, we continued working toward a more optimal array. The latest design resulted from one of the optimizations being run by Manoj Adhikari and is referred to as the GBT2 element. The basic design is the same as the GBT1 and the new parameter values are given in Table 5.1. The parameters P1-P7 are the same as those shown in Figure 5.3. A simulated computer graphic is shown in Figure 5.4. The design has not yet been finalized but it is likely that the GBT2 will be constructed.
The simulated array results are given at two different element spacings in Figure 5.6. The first set of results are given at a spacing of 11.24 cm (0.52λ at 1.4 GHz) which matches the spacing of the ‘kite’ array and the cryostat that was built to support it. Results are also given at 14.99 cm (0.70λ at 1.4 GHz). This is closer to the optimal spacing for the array. A thorough analysis of the optimal array spacing will need to be performed after the design is finalized and before the array is constructed. Figure 5.6(a) shows that at the smaller spacing there is no significant loss of sensitivity in the boresight beam at the center frequency. This means that the peak sensitivity of the array could be verified at the lower spacing with the old cryostat. This would save both the time and money required to build a new one.

Figures 5.6(b) and 5.6(c) show a Smith chart view of the active and self impedances across the band of interest at boresight. These figures are included to show the robustness of the GBT2 design. Mechanical imperfections in the construction of an array inevitably lead to perturbations in the array impedances which will cause shifts to the locations of these curves on the Smith chart. Our goal is to design the array to be as robust to these shifts as possible. The figures show that the combined active and self impedance curves of both designs are well centered and robust to mechanical perturbations.

Finally, figures 5.7(a) and 5.7(b) show the modeled XPD for the GBT2 array. The XPD is modeled using an uncalibrated version of the max SNR beamformer defined in (2.23). Polarization is achieved by using only the signal from the co-polarized elements to form the beam. The plot shows excellent polarimetric performance for the new array. Polarimetric calibration using the method detailed in Chapter 4 increases the XPD into the hundreds of decibels.

5.5 Summary

This chapter has outlined the ongoing process of designing a PAF for the GBT. The construction and verification of a prototype element has been discussed. Simulated results for the latest iteration of the design have been presented. While the design is still not finalized, it represents significant progress toward the design of the first RAS PAF to be deployed on the GBT. The final array will also help the RAS group to understand why the measured sensitivity for the ‘kite’ array did not meet expectations.
(a) Beamformed $T_{\text{sys}}/\eta_{\text{ap}}$ for the “x-polarized” beamformer across the field of view. Curves are shown at various frequencies for the ‘kite’ array and the GBT2 array at two different spacings.

(b) Active and self impedances of the GBT2 array plotted on the smith chart for the 11.24 cm spacing of the ‘kite’ array (0.6$\lambda$ at 1.6 GHz). The frequency range is 1.2 to 1.5 GHz.

(c) Active and self impedances of the GBT2 array plotted on the smith chart for 14.99 cm spacing (0.7$\lambda$ at 1.4 GHz). The frequency range is 1.2 to 1.5 GHz.

Figure 5.7: Simulated results for the GBT2 array.
Figure 5.8: Modeled XPD for the uncalibrated max SNR beamformer on the GBT2 array.
Chapter 6

Conclusion and Future Work

The BYU RAS group has a long history of significant contributions to the radio astronomy community through PAF development. My contributions to this effort include a new design method for PAFs that was successfully used to design two arrays. Other contributions include the development of a dual-polarized DAQ system, the development of a polarimetric calibration algorithm, and the design of a dual pol PAF for the GBT.

I designed the MATLAB optimization package to allow the user to optimize an array design for any figure of merit. It accomplishes this by combining EM models of the array with in-house models of a reflector. The package was successfully used to develop two low noise PAFs. These PAFs were deployed and verified on the Arecibo telescope in Arecibo, Puerto Rico, and on the 20 meter reflector at the NRAO in Green Bank, West Virginia. The package will remain with the RAS group and will benefit the design of future arrays.

In order to verify one of the arrays designed by the MATLAB optimization package, a coworker and I developed a 40 channel DAQ system. The 40 channel system consists of five acquisition nodes controlled by one central hub machine. Three function generators were also used to achieve synchronous acquisition.

I developed an efficient calibration procedure for PAF polarimeters. The method is an improvement over older methods that required tracking a source for several hours because it only requires snapshot observations of two sources in addition to the source required to find the array steering vectors. The method provides a high level of accuracy and is stable across the FoV. It is also robust with respect to mechanical degradations in the array elements.

A PAF design for the GBT is well underway. This new array is an important step toward the RAS group’s goal of deploying a science ready PAF on the GBT. It will also help to diagnose the lower than expected sensitivity achieved by the ‘kite’ array. A prototype
element has been constructed and measured and modeled results are given in this thesis. Simulated results for the latest iteration of the full array design are also presented.

6.1 Future Work

While great strides have been made toward the design of a sufficiently sensitive PAF work still needs to be done to obtain the sensitivity and stability of single element feeds. There is significant design work that can still be done to achieve these goals.

The polarimetric calibration method presented in this thesis is an important first step toward efficient calibration of phased array polarimeters. A large amount of time could be saved if it is possible to calibrate the entire field of view without re-measuring the calibrator sources at the center of every individual beam. Future work could look for ways to calibrate the field of view using only sparse measurements of the polarized sources combined with the measurement of the unpolarized source that is required for array calibration.
Bibliography


