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# A web-based tool for Fuzzy Cognitive Map Modeling

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Abstract: A Fuzzy Cognitive Map (FCM) is an efficient and a relatively easy to understand semiquantitative extension of cognitive maps, combining aspects of fuzzy logic and neural networks. It has found a great applicability in diverse scientific domains for modeling and providing decision making support in complex problems with high uncertainty, including environmental management. In essence, the FCM modeling approach attempts to capture the functional and causal interactions about complex systems by relying on experts' domain knowledge, or using stakeholders' views and perceptions, which can reduce conflicts among stakeholders by capturing the different inter-sectorial synergies and tradeoffs, helping hence to reach consensus about wicked environmental problems. Although there is a number of software products that have been used in the literature which allow drawing FCMs by nonexpert users, there is a gap in the literature concerning web-based and open source tools for building, analyzing and visualizing FCMs. In this paper we present a new, intuitive and easy to use web-based software tool, called FCM Wizard, which is fully accessible by experts and lay experts to experiment with, by investigating their real decision making problems. The FCM Wizard is oriented to modeling and inference tasks, considering ways of engaging decision-makers and stakeholders in the modelling process through the different stages of the decision making cycle. The web-based tool can be applied for policy making and environmental management; some computer simulations will be presented to illustrate its feasibility.

Keywords: Fuzzy Cognitive Maps; software tool; modelling; decision making.

#### 1 INTRODUCTION

A Fuzzy Cognitive Map (FCM) is a method for modeling complex systems by relying on existing knowledge and human beliefs and experiences. FCMs were introduced by (Kosko, 1986) as an extension to Cognitive Maps (Axelrod, 1976), providing a powerful tool for modeling dynamical systems. As a knowledge representation and reasoning technique, it depicts a system in a form that corresponds closely to the way humans perceives it. FCMs are particularly useful when it is impractical to build a

model empirically or mathematically, they can be designed to encode experts' domain knowledge and beliefs, and they also have learning capabilities which improve their structure and computational behavior.

FCMs can model any real world system as a collection of concepts and causal relationships among these concepts. They combine Fuzzy Logic and recurrent Neural Networks, inheriting their main advantages. From an Artificial Intelligence perspective, FCMs are dynamic networks with learning capabilities, as more data is available to model the problem at stake, the system becomes better at adapting itself and reaching a solution (Papageorgiou and Salmeron, 2014). They gained momentum due to their dynamic characteristics and learning capabilities. These capabilities make them essential for modeling and decision making tasks as they can be very helpful to managers, they can increase system understanding and reduce uncertainties about how the studied phenomenon may respond to changes in the main system drivers using simulation.

In recent years, FCMs have been applied in many scientific areas because they present numerous advantages. Aside from the capacity to support uncertainty due to ambiguities and ill-defined variables, feedback, and integration of both qualitative and quantitative data, FCMs can be used in a participatory approach to model mental views, and can serve as a useful learning tool that helps improve system understanding and foster system thinking. Maps produced by individuals can also be aggregated to produce a combined FCM, enriched with knowledge from all experts and stakeholders. FCMs can also be used as a quick and simple method to capture where key sensitivities are before applying other models, especially in "wicked" environmental problems which are complex, involve many parties, and have no straightforward answers or solutions (Hobbs et al., 2002; Kontogianni et al., 2012; Samarasinghe and Strickert, 2013; Gray et al., 2014a; Jetter and Kok, 2014; Henly-Shepard et al., 2015; Mourhir et al., 2016). A recent review on FCM research during the last decade can be found in (Papageorgiou and Salmeron, 2013; Jetter and Kok, 2014).

There are also some FCM design tools presented in the literature; however only a few of them are freely available and only one is open source, the R package, called "fcm", available in CRAN. In this research, a new software tool, called FCM Wizard, is presented to provide the researchers and the FCM community, the necessary simulation tool to make decisions and perform policy simulations on their own case studies. The FCM Wizard allows construction of FCMs by either using experts' knowledge or by using data when available. The FCM Wizard is a web-based tool with simulation and learning capabilities, freely available for use by experts and stakeholders in different scientific domains. Through the FCM Wizard, the experts can entirely define FCM-based systems (i.e. the associated directed graphs) by using a very intuitive Graphical User Interface. Moreover, the FCM Wizard incorporates numerous simulation and inference options, as well as Hebbian-based learning algorithms that can significantly improve the system's performance. We provide also a step-by-step instructions on how to use the FCM web-based tool, and how to run the FCM inference and perform simulations about the studied problem starting from different possible scenarios.

The rest of the paper is organized as follows: Section 2 provides a brief overview of the FCM modeling methodology. Section 3, includes a review on existing FCM software tools. Section 4 presents the main aspects of the proposed FCM Wizard, along with an illustration for policy making with simulation results. Finally, Section 5 outlines some conclusions and highlights directions for future perspectives.

#### 2. FUZZY COGNITIVE MAPS (FCMs)

FCMs are an extension of Cognitive Maps (CMs), which were initiated by (Axelrod, 1976) to model political systems. A CM is a system model, that is on the structural level represented in the form of a *graph* whose *nodes* represent the main factors that define the studied system, and *edges* represent perceived causal relationships (caused or not) between concepts. FCMs, introduced by Kosko (1986), allows representing knowledge in the form of a directed graph whose nodes denote the main factors of the analyzed problem, and links represent the causal relationships between concepts. FCMs introduced fuzziness to Cognitive Maps, by using numeric descriptions (fuzzy binaries) of causal influences, instead of positive or negative symbols. Each FCM's edge between two concepts  $C_i$  and  $C_j$  is associated with a weight value  $w_{ij}$  which varies from -1 to 1, that describes the strength of the corresponding relation. There are three different types of possible causalities between every pair of concepts  $C_i$  and  $C_j$ .

W<sub>ij</sub> > 0, which designates a positive causality. That is, if the value of C<sub>i</sub> increases (respectively decreases), it will result in the value of C<sub>j</sub> increasing (respectively decreasing).

- W<sub>ij</sub> < 0, which designates a negative causality. That is, if the value of C<sub>i</sub> increases (respectively decreases), it will result in the value of C<sub>j</sub> decreasing (respectively increasing).
- W<sub>ij</sub> = 0, which designates no causality. That is the concepts C<sub>i</sub> and C<sub>j</sub> do not exert an influence on each other's.

Typically, a FCM of *n* concepts could be represented mathematically by a  $n \times n$  weight matrix (*W*). Figure 1 shows an example of a FCM with its corresponding adjacency weight matrix.



Figure 1. Fuzzy cognitive map (left) and the corresponding weight adjacency matrix (right), showing the influence weights

A FCM is not a static representation of the world; FCM's graph structure facilitates causal reasoning, calculations can be made to perform an assessment of the consequences of a specific system state. <u>Kosko (1987)</u> used *auto-associative neural network* mechanisms to study the system dynamics of FCM models and produce projections for different possible scenarios.

By feeding the fuzzy cognitive map with an initial stimulus state vector  $X^{(t)}$  (state vector at time (t)), it can model the evolution of a scenario over time by evolving forward and letting concepts interact with one another. Each subsequent value of the concept state  $X^{(t+1)}$  can be computed as previous state  $X^{(t)}$  and weight matrix multiplication, according to Eq. (1).

$$X_i^{(\kappa+1)} = f\left(\sum_{j=1, j\neq i}^n w_{ji} \times X_j^\kappa\right)$$
(1)

Based on the literature, two other equations have been proposed for FCM inference, the modified Kosko shown by Eq. (2) and the rescaled Kosko shown by Eq.(3).

$$X_{i}^{(\kappa+1)} = f\left(X_{i}(t) + \sum_{\substack{j=1\\ j \neq i}}^{n} X_{j}(t) \cdot w_{j,i}\right)$$
(2)

$$X_{i}^{(\kappa+1)} = f\left((2 \times X_{i}^{\kappa} - 1) + \sum_{j=1, j \neq i}^{n} w_{ji} \times (2 \times X_{j}^{\kappa} - 1)\right)$$
(3)

where  $X_i^{(\kappa+1)}$  is the value of concept  $C_i$  at simulation step  $\kappa + 1$ ,  $X_j^{(\kappa)}$  is the value of concept  $C_j$  at the simulation step  $\kappa$ ,  $w_{ji}$  is the weight of the interconnection between concept  $C_j$  and concept  $C_i$ , and  $f(\cdot)$  is the threshold transfer function, used to retain the values within the range [0, 1] or [-1, 1]. Generally, the most commonly used transfer function is Sigmoid as shown by Eq.(4) (Tsadiras, 2008).

$$f(x) = \frac{1}{1 + e^{-\lambda x}} \tag{4}$$

where  $\lambda$  is a real positive number ( $\lambda > 0$ ) which determines the steepness of the continuous function *f* and *x* is the value  $X_i^{(\kappa)}$  for a given iteration.

The simulation stops when the system reaches equilibrium, i.e., a limit vector is reached as  $X^t = X^{t-1}$  or when  $X^t - X^{t-1} \le e$ ; where *e* is a residual, describing the minimum error difference among the subsequent concepts. Its value depends on the application type and it is typically set to 0.001.

### 3 A REVIEW ON FCM DESIGN TOOLS

This section provides a brief summary about the different FCM design and software tools as presented in the literature. One of the first FCM analysis tools available online is FCMapper<sup>1</sup>, it is based on MS Excel, and it is free for non-commercial use. It was developed to save time during the process of analyzing and visualizing FCMs. Since it supports net-file formats, you will need additional socialnetwork software like Pajek<sup>2</sup> or Visone<sup>3</sup> in order to visualize FCMs. León et al. (2011) proposed the FCM TOOL, which offers a learning algorithm for estimating the causal weights, in addition to having a user friendly graphical user interface. However, the tool was designed for one specific application which is the public transportation problem. Nápoles et al. (2015) proposed a generic computational framework, as an extension of the FCM TOOL, for designing, learning and simulating FCM-based systems, especially in pattern classification problems. A new version of the tool was released recently under a different name which is the FCM Expert<sup>4</sup>. Some other researchers took a different approach by exposing the FCM capabilities as libraries. For example, JFCM is a small and simple open source library that can be used to create a variety of cognitive networks, and it allows also loading networks from XML files (De Franciscis, 2014). An open source FCM library in R language ned "fcm" has been recently proposed under the Comprehensive R Archive Network (CRAN) (Dikopoulou and Papageorgiou, 2017). The fundamental "fcm" package provides to users, a selection of six different inference methods. The main inference methods are: Kosko, modified-Kosko, rescale and the clamping versions are: clamped-Kosko, clamped-modified-Kosko, clamped-rescale. In the latter inference methods, the clamped concepts have a static activation value (does not change dynamically) in the FCM simulation until the output is stabilized. The library also allows the FCM modeler to choose among four threshold functions, namely, bivalent, trivalent, sigmoid and hyperbolic tangent. Another interesting approach to model FCMs is offered by Mental Modeler<sup>5</sup>, which facilitates the aggregation and analysis of group models by combining FCMs created by different individuals or stakeholders, using a web-based interface (Gray et al., 2013). There is also FuzzyDANCES<sup>6</sup> which is a tool to draw, and analyze FCMs by calculating network graph indices. It also contains algorithms for sensitivity analysis (Winding Stairs) and map learning using Differential Evolution. FuzzyDANCES was developed by the Farming Systems Ecology group (FSE) of Wageningen University, as part of a multi-scale agricultural framework (Groot et al., 2012), and it is available for free. FCM software has been also proposed in the form of an applet<sup>7</sup> that is accessible through web browsers. Aguilar and Contreras (2010) developed the FCM designer tool with a Spanish interface, it offers a variety of ways to describe causal links. Relationships can be static, or dynamic based on logic rules, mathematical equations, or Fuzzy Logic, however, there is no indication about the availability of the software for reuse.

#### 4. PRESENTATION OF THE FCM WIZARD TOOL

This section provides an overview of the FCM Wizard<sup>8</sup> tool and illustrates how it can be used to model an ecological problem. The main menu of the FCM Wizard is presented in the screenshot of Figure 2. The tool offers three main modes of FCM construction, namely: (i) expert mode (experts and/or stakeholders can construct manually a FCM of a real problem), (ii) data-based mode (FCM model is constructed automatically using available data), and (iii) through merging of FCMs, where different individual FCMs can be combined to provide an augmented and collective model in order to generate aggregated system complexities.

Our tool can be seen as having four main functionality categories. Any type of users can use the first category without a thorough knowledge of the mathematical foundations of the FCM methodology. They can design new FCMs independently from the type of the application by adding new concepts and

<sup>&</sup>lt;sup>1</sup> <u>http://www.fcmappers.net/joomla/</u>

<sup>&</sup>lt;sup>2</sup> <u>http://mrvar.fdv.uni-lj.si/pajek/</u>

<sup>&</sup>lt;sup>3</sup> <u>http://visone.info/</u>

<sup>&</sup>lt;sup>4</sup> <u>http://www.fcmexpert.net/</u>

<sup>&</sup>lt;sup>5</sup> http://www.mentalmodeler.org/

<sup>&</sup>lt;sup>6</sup> <u>https://sites.google.com/site/fuzzydances/</u>

<sup>7 &</sup>lt;u>http://www.ochoadeaspuru.com/fuzcogmap/software.php</u>

<sup>&</sup>lt;sup>8</sup> <u>http://fcmwizard.com</u>

causal relationships between them using influence weights. Moreover, when group modeling is used, the different individuals' cognitive maps can be combined in an easy and meaningful way to produce a collective FCM, which in general might help reaching consensus, promote learning and reduce conflicts (Gray et al., 2014b). Our FCM Wizard offers the possibility to aggregate different FCMs by augmenting their adjacency matrices in order to reflect all proposed concepts from the different participants, then averaging is used to produce the group map.

As FCMs have their theoretical foundation roots in recurrent neural networks, selecting the appropriate transfer function and inference rule is a key factor when designing a FCM model. These aspects will impact the model's capability of producing meaningful and accurate results. Hence, the tool offers a second category of functionalities that is meant for users with more expertise like FCM analysts that are familiar with the method's mechanics. It involves customizing the influence rule, the transfer function and the different parameters that might impact the convergence of the system into a stable state, like the number of iterations and the minimum residual error. This category can be somewhat challenging to other types of users like stakeholders or policy analysts and will necessitate a learning curve.

The construction of FCM maps relies heavily on expert and human input; they express their beliefs using heuristics and their mental models. One way to perform comparisons across stakeholder groups or individuals is by analyzing the map structure using graph theory indices (<u>Özesmi and Özesmi, 2004</u>; <u>Gray et al., 2014b</u>). Our FCM Wizard has a statistical package to compute graph theory indices like total number of concepts, number of connections, connection to concept ratio, outdegree, indegree, number of transmitter variables, the number of receiver variables, the number of ordinary variables, complexity ratio and the hierarchy index.

A major shortcoming of FCMs is the potential convergence to undesired steady states or unacceptable decisions according to problem constraints and a critical dependence on experts. In order to overcome these shortcomings, learning algorithms have been investigated and proposed for FCMs by using historical data, contributing hence to adaptive Fuzzy Cognitive Maps and improving the robustness and accuracy of FCM output, especially in prediction tasks. To that end, we developed a last category of functionalities that allow FCM models to be constructed and calibrated using observed historical data. The tool contains a package of Hebb-based learning algorithms such as non-linear hebbian, differential hebbian, data-driven hebbian and active hebbian learning (Papageorgiou, 2012).

In the rest of this section, we will illustrate how the FCM Wizard can be used to build a FCM model and visualize its results. Figure 2b shows the FCM model constructed by the FCM Wizard tool for the ecological problem described in (Özesmi and Özesmi, 2004) and showed in Figure 2a.



Figure 2. (a) Example of a Fuzzy Cognitive Map. (b) FCM of ecological model example constructed using the FCM Wizard.

The considered ecological problem consists of five concepts, namely:  $C_1$  – Amount of wetland,  $C_2$  – Fish Population,  $C_3$  – Pollution,  $C_4$  – Livelihood,  $C_5$  – Laws, and 11 causal effect interconnections (as shown in Figure 3b). It should be noted that the positive edges are defined with a green color; while, the negative edges are drawn with a red color. Figure 3 depicts the weight matrix produced after construction of the model by the FCM Wizard.

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🛃 Model Design	Matrix 🗠 Scenario Ana	lysis 🖩 Stru	ictural Analysis			
Weight Matrix						
	Wetland	Fish	Law Enforcement	Income	Lake pollution	
Wetland	0	1	0	0.8	-0.1	
Fish	0	0	0	1	0	
Law Enforcement	0.2	0.5	0	-0.2	-0.5	
Income	0	0	0	0	0	
Lake pollution	-0.2	0	0	-0.2	0	

Figure 3. Weight matrix produced by FCM Wizard.

The tool also offers users the possibility to perform "what-if" scenario analysis by introducing the initial stimulus state vector, selecting the inference rule's type, the transfer function, its learning parameter and the number of iterations or the convergence step (see Figure 4). Furthermore, the open/closed lock option in the screenshot of Figure 4 allows the user to set the value of a concept as being "clamped".

GENERAL ACTIONS		🛔 Model Design	🇰 Weight Matrix	Scenario Analysis	E Structural Analysis				
Construct FCM	v								
Expert Mode		Concept Initial Values	5						
Data-based mode		Wetland	Fish		Law Enforcement	Incor	me	Lake pollution	
Merge FCMs		1	1		1	1		1	
FCM Learning									
FCM Classification			•	<b>A</b>		•	•	<b>•</b>	
S FCM History									)
2 Coming Soon		Inference Rules		Transfo	Transformation Functions		Lambda Parameter (0	.0~10.0)	
<ul> <li>Conning Soon</li> </ul>		Modified Kosko's Ac	tivation Rule •	Sign	hoid	*	1		
		Iterations and Converge	nce step						
		0	0.001	\$					
		Simulate 💼 0	lear Simulation	Hold Current Simulation					
57 ()									

Figure 4. Scenario analysis performed by FCM Wizard.

We tested the constructed ecological FCM model using two main scenarios (see Table 1), using the three main inference rules supported by the tool and the sigmoid threshold function as a nonlinear transformation function. Simulations were conducted until the system reached convergence. The initial state vectors and the equilibrium state for each scenario are shown in Table 1.

It can be concluded from the results that increasing concept  $C_1$  (Amount of wetland), increases significantly the other concepts, especially,  $C_4$  (Livelihood) and  $C_2$  (Fish population). Furthermore, the plot pane in Figure 5 demonstrates graphically the evolution of concepts over time to equilibrium.

Scenario I	<b>C</b> 1	<b>C</b> <sub>2</sub>	C₃	<b>C</b> <sub>4</sub>	C₅	Iterations
Kosko	0.5037	0.5813	0.5	0.6898	0.4255	3
Modified Kosko	0.6662	0.8654	0.6891	0.8855	0.5345	7
Rescale	0.5001	0.5004	0.5	0.5019	0.4999	14
Scenario II	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C₅	Iterations
Kosko	1	0.6977	0.5	0.7884	0.4134	4
Modified Kosko	1	0.8378	0.6589	0.9097	0.5238	11
Rescale	1	0.8598	0.4998	0.9142	0.4513	10

Table	1.	Experimental	Results
IUNIC			resound



Figure 5. Graphical plot of convergence performed by FCM Wizard.

It should be noted that the software was developed using web-based programming technologies, namely, php, CSS and JavaScript. The software is freely available to users upon a password request from the principal investigator and supervisor of the FCM Wizard<sup>1</sup>. The tool keeps separate accounts for the different users after registration and allows them to keep track of their FCM modeling history. No API packages are provided for the moment as the tool is still under continuous development.

#### 5 CONCUSIONS

In this paper, we introduced a new web-based software tool for fuzzy cognitive modeling, that we named FCM Wizard. The tool offers a variety of simulation options and different learning parameters. Particularly, the tool supports three FCM inference rules, four threshold functions, besides the possibility of customizing a number of parameters. The FCM Wizard provides also other learning functionalities not presented in this paper.

The software was designed as a flexible and interactive tool that allows users to customize their simulations. It can be used intuitively by any type of users (stakeholders, policy analysts or others) to create FCM models and build simulations for different scenarios. However, a minimum understanding of the method's mechanics is assumed in order to be able to customize the different inference or learning parameters.

The method was also designed to be generic and reusable, and has the potential to facilitate consideration of different types of applications with simple instantiation and without implementation efforts.

The FCM Wizard is under continuous development and we seek to bridge the gap between theoretical advances in the FCM field and the development of sound practical applications. Future work is directed towards including other evolutionary learning algorithms, support of dynamic and nonlinear relationships, time delays, among other extensions that have been proposed in FCM literature. Another major development axis is the generic characterization of the potentially reducible uncertainties emanating from incomplete understanding or disagreement between experts which can be supported by the use of Fuzzy Logic.

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