The Application of Origami to the Design of Lamina Emergent Mechanisms (LEMs) with Extensions to Collapsible, Compliant and Flat-Folding Mechanisms

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The Application of Origami to the Design of Lamina Emergent Mechanisms (LEMs) with Extensions to Collapsible, Compliant and Flat-Folding Mechanisms

Holly C. Greenberg

A thesis submitted to the faculty of Brigham Young University in partial fulfillment of the requirements for the degree of Master of Science

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ABSTRACT

The Application of Origami to the Design of Lamina Emergent Mechanisms (LEMs) with Extensions to Collapsible, Compliant and Flat-Folding Mechanisms

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Master of Science

Lamina emergent mechanisms (LEMs) are a subset of compliant mechanisms which are fabricated from planar materials; use compliance, or flexibility of the material, to transfer energy; and have motion that emerges out of the fabrication plane. LEMs provide potential design advantages by reducing the number of parts, reducing cost, reducing weight, improving recyclability, increasing precision, and eliminating assembly, to name a few. However, there are inherent design and modeling challenges including complexities in large, non-linear deflections, singularities that exist when leaving the planar state, and the coupling of material properties and geometry in predicting mechanism behavior. This thesis examines the planar and spherical LEMs and their relation to origami. Origami, the art of paper folding, is used to better understand spherical LEMs and flat-folding mechanisms in general. All single-layer planar four-bar LEMs are given with their respective layouts. These are all change-point pinned mechanisms (i.e. no slider cranks). Graph representations are used to show the similarities between action origami and mechanisms. Origami principles of flat-folding are shown to be analogous to principles of mechanisms including rules for assembly and motion.

Keywords: origami, compliant mechanism, lamina emergent mechanism
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6.1 Forms of folding can be present along this entire timeline.
NOMENCLATURE

\[ N_e \] Number of edges in a graph
\[ N_n \] Number of nodes in a graph
\[ s \] Refers to the shortest link in a mechanism
\[ l \] Refers to the longest link in a mechanism
CHAPTER 1. INTRODUCTION

Altschuller said there is a law of ideality which states that “any technical system, throughout its lifetime, tends to become more reliable, simple, effective—more ideal. Every time we improve a technical system, we nudge that system closer to ‘ideality.’ It costs less, requires less space, wastes less energy, etc” [2]. Many products become more ideal as the product is viewed holistically and not merely as the sum of its parts. As products are refined their design tends to become more highly integrated. This is seen in the evolution of mechanisms towards the use of compliance.

“A mechanism is a mechanical device used to transfer or transform motion, force, or energy [3]. A compliant mechanism also transfers or transforms motion, force, or energy, [but] unlike rigid-link mechanisms...compliant mechanisms gain at least some of their mobility from the deflection of flexible members rather than from movable joints only [3].” By using the deflection of flexible members to achieve mobility, parts can be reduced, thereby reducing materials, assembly, costs, weight, and even improving recyclability and precision. However, this results in the design of more highly integrated, coupled mechanisms where geometries and material properties affect motion.

1.1 Motivation

The motivation for this thesis is to take the design of compliant mechanisms a step further to make more compact compliant mechanisms, compact meaning flat packing. This thesis draws on origami literature to inspire the design of Lamina Emergent Mechanisms (LEMs). Preliminary work in LEMs had examined pop-up paper mechanisms so it was expected that origami would provide insights as well [4–8]. LEMs are a type of compliant mechanism fabricated from planar materials (lamina) and having motion that emerges from the fabrication plane [5]. As such, LEMs have the potential to perform sophisticated mechanical tasks with simple topology [5]. It is expected that the use of LEMs in design can help products become more ideal than regular com-
pliant mechanisms by reducing the number of parts, reducing cost, reducing weight, improving recyclability, increasing precision, and eliminating assembly [6–8].

The body of origami literature is drawn upon because like LEMs, origami is fabricated from planar materials and because origami is essentially a compliant mechanism since creases can act as joints to allow motion.

1.1.1 Collapsible Mechanisms

A primary utility of LEMs is their property of collapsibility. Collapsibility is especially useful in space exploration technologies, as well as other areas where packing tightly is valued. In terms of applications, ortho-planar springs (a type of LEM) can be used in an industrial valve to demonstrate the advantages in compactness, ease of manufacture, and robustness [9]. Other potential applications for LEMs include novel arrays that give tactile responses, shock absorbing armor, disposable mechanisms, and deployable structures [7]. Some of the primary fields of opportunity include space and bio-implantable devices, e.g. surgical stents [7,10–12].

This thesis examines the collapsibility of planar and spherical mechanisms and their synthesis. The scope of planar mechanisms in single layer LEMs examined in this thesis are change-point mechanisms. Metamorphic, slider and reconfigurable mechanisms are outside the scope of this research.

Spherical mechanisms are examined in origami and it is shown that origami design principles can be especially pertinent to the design of spherical LEMs. Planar LEMs are a simple case and a single lamina constrains the design space to the set of change-point mechanisms and mechanisms with a slider link.

1.1.2 Synthesis Techniques

As mentioned previously, the design of lamina emergent mechanisms is complex and often relies on designer intuition, which results from experience. Since LEMs are a new technology, designers in industry lack familiarity with them and their promise. This thesis provides insights for the synthesis of LEMs and presents a systematic approach for LEM design. This approach
draws upon the origami literature and relates the principles of model design from a laminar sheet to LEM design.

Previous research has outlined different steps towards LEM design which are presented more in depth in the coming chapter. The research in this thesis takes a step back and abstractly considers LEM design through the lens of origami. The principle of abstraction is used to understand LEM design in a way that reduces complexity and emphasizes the essential characteristics of LEMs [13]. Using the principle of abstraction a more generic level of interrelationship between LEMs and origami is examined, or more specifically between linkages/joints and folds [13]. This interrelationship is seen in the use of graph representations and theory.

1.1.3 Applications

As mentioned previously there are many opportunities for the application of LEMs in various technologies. When viewing the primary purpose of LEMs as a way to achieve collapsible, compact and compliant mechanisms more opportunities for application arise.

Origami, beginning from a laminar sheet and using compliant joints to achieve packing in the creation of both forms and motion, is expected to yield insights into the design of LEMs. Likewise the principles of mechanism design are likely to yield insight into kinetic origami design as well as the general origami folding process. As such, origami is deemed to be highly applicable to the designing of collapsible mechanisms.

1.2 Objective

The primary objective of this thesis is to provide a new perspective on LEM design through the lens of origami. This is achieved by relating origami to mechanisms through principles of graph theory and using that relationship to identify a method for synthesizing LEMs.

Secondary objectives include the introduction of origami concepts a means to design for collapsibility and the application of origami to engineering in general.
1.3 Overview

An overview of the material in each chapter of this thesis is provided here. Chapter 2 reviews the relevant literature in mechanisms, origami and graph theory. Chapter 3 provides an overview of the relation between origami and spherical LEM design. The relationship between origami and mechanisms is established through the use of topological graphs. Chapter 4 introduces principles of origami as they relate to the simple case of planar lamina emergent mechanisms. Chapter 5 applies principles from origami to graph representations to generate designs for LEMs. Chapter 6 concludes this thesis and summarizes the findings of this research. It also suggests future areas of research in the design of collapsible mechanisms and the continued development of origami design tools in the design of LEMs.
CHAPTER 2. BACKGROUND AND ANALYSIS

The purpose of this chapter is to provide a review and analysis of the relevant literature to improve understanding of the developments discussed in the later chapters. First an overview of the evolution of mechanisms from rigid-body to compliant and finally to LEMs will be given. This is followed by an introduction of origami as it relates to engineering. This chapter concludes with a brief discussion of the intersection of origami and mechanisms along with some background on graph theory as it pertains to this thesis.

2.1 Evolution of Mechanisms: from Rigid-body to LEMs

To begin, “A mechanism is a mechanical device used to transfer or transform motion, force, or energy [3].” Mechanisms are both man-made and found in nature. From the lifting system of a construction crane to the flapping of insect wings, mechanisms can be identified nearly everywhere. A four-bar linkage is often regarded as the simplest mechanism. Usually when referring to a four-bar linkage a planar mechanism is implied, where each link translates in a plane and rotates around an axis perpendicular to that plane. Planar mechanisms are considered a special case of spherical mechanisms; in a spherical mechanism any point in a moving body is confined to move within a spherical surface, and all spherical surfaces of motion are concentric [14]. In a spherical mechanism all axes perpendicular to the joint intersect at a point and in planar mechanisms that point is understood to be at infinity. In addition, rigid-body mechanisms are mechanisms where rigid links are joined together via joints, such as a revolute or pin joint.

2.1.1 Compliant Mechanisms

In comparison, “a compliant mechanism also transfers or transforms motion, force, or energy, [but] unlike rigid-link mechanisms...compliant mechanisms gain at least some of their mo-
ility from the deflection of flexible members rather than from movable joints only [3].” The deflection of flexible members stores energy in the form of strain energy.

By using the deflection of flexible members to achieve mobility, parts are reduced. For example compliant mechanisms such as tweezers may be made as a single piece since the compliance [of the material] gives it springiness. Designers can use compliance to their advantage by considering material properties when designing the geometry of a part since this springiness is related to both. Some advantages from incorporating compliance include the reduction of materials, assembly, costs, weight, and even improvement in recyclability and precision [4, 6–8].

The design of compliant mechanisms is not trivial. Historically it has occurred more through trial-and-error methods, dating back to the era of hunting bows and catapults. However in the latter half of the 20th Century the pseudo-rigid-body model (PRBM) was developed which provides a compliant mechanism with a model of its rigid-body counterpart. More specifically, the pseudo-rigid-body model uses traditional rigid-body components, rigid links and springs, that have equivalent force-deflection characteristics to model the deflection of flexible members [3]. This allows the kinematics of a compliant mechanism to be predicted, analyzed and designed to meet the requirements of a particular mechanism.

The field of compliant mechanisms has advanced in applications, modeling techniques, design, materials, manufacturing and analysis, to name a few areas of research. Specifically in the area of design various synthesis techniques have been investigated. Topological optimization is a synthesis technique which models an initial mechanism shape and through topological analysis generates a final design showing where material can be eliminated without compromising the mechanism or the part [3]. Another technique is the concept of rigid-body replacement synthesis which uses a rigid-body mechanism as a basis to develop its compliant analog [15]. Furthermore a classification of compliant mechanism fundamental components has been generated as a catalog to aid in the design of compliant mechanisms [16].

As the design of compliant mechanisms has become increasingly understood, engineers have explored how to make them more ideal. More ideal in terms of reducing the rates of failure, improving precision, manufactured more cheaply, and shipped more efficiently, e.g. flat-packing. Technology in the past decades has followed a trend of making things smaller or collapsible. For example computers have become increasingly portable, as have keyboards, video cameras, thumb
drives, surgical tools, etc. And while compliance can reduce the number of parts in a mechanism, the concept of lamina emergent mechanisms was conceived, to reduce the size of a mechanism and potentially further lower the cost.

2.1.2 Lamina Emergent Mechanisms

LEMs are a relatively new technology developed by Brigham Young University’s Compliant Mechanism Research Laboratory. LEMs use the deflection of flexible members to achieve motion and as such are considered a subset of compliant mechanisms. Moreover, LEMs are manufactured from planar materials (lamina) which have motion that emerges out of the fabrication plane [5]. This allows for the overall volume of the mechanism and manufacturing costs to be reduced, since planar fabrication methods can be used.

LEMs are expected to provide feasible, repeatable solutions to advance mechanism design and manufacture [6]. The incorporation and use of LEMs offer many potential advantages in the design of mechanical products, which include: reducing the number of parts, reducing cost, reducing weight, improving recyclability, increasing precision, and eliminating assembly [4, 5, 7]. As such, these mechanisms can provide opportunities for more cost-effective, compact, easy to assemble, and modular products.

In terms of applications, Parise et al. used an ortho-planar spring, a type of LEM, in an industrial valve to demonstrate the advantages in compactness, ease of manufacture, and robustness [9]. Albrechtsen et al. has identified a variety of potential applications for LEMs, which include novel arrays that give tactile responses, shock absorbing armor, disposable mechanisms, and deployable structures [7]. In addition, multilayer and metamorphic LEMs have been explored [17].

While there are numerous potential advantages to the use of LEM technology, there are also challenges inherent to their modeling and fabrication. The modeling challenges include complexities in large, nonlinear deflections, singularities that exist when leaving the planar state, and coupling of material properties and geometry in predicting mechanism behavior [5].

Some preliminary research has investigated the design of lamina emergent mechanisms. For example Berglund et al. published a paper that established design rules for rigid-body replacement synthesis in the design of compliant mechanisms, some of which is expected to apply to LEMs as well [15]. Winder et al. examined pop-up paper mechanisms and synthesized new ones
which provide a foundation for LEM design [4]. They also proposed the kinematic modeling of pop-up paper mechanisms with the pseudo-rigid-body model (PRBM). In addition in 2008, Winder published a paper identifying joints suitable for LEMs [18]. Jacobsen et al. discussed the Lamina Emergent Torsional joint as a way to achieve high angular rotation [19]. A review of the literature identified a few frameworks for LEM synthesis which are outlined in Table 2.1.

Table 2.1 is not exhaustive; rather it shows that among the different synthesis methods proposed there are similar considerations. Carroll’s proposed synthesis method considers the limitations of manufacturing first, and calls for the design of the rigid-body configuration first before a the planar configuration is designed [20]. Jacobsen’s method is most abstract beginning with the understanding of the flexibility of materials leading to the forming of components and the development of interaction strategies for the design of LEMs [5]. This approach is similar to the approach taken in this thesis since folds in origami are examined as a way to achieve motion. Berglund’s method is the most specific and this research yields a method with similar steps for the design of planar LEMs.

For the design of spherical mechanisms it may be better understood that an orimimetic approach to LEM synthesis is taken, where orimimetic means the ability to imitate folds [8]. (In an engineering design context, it refers to the ability to use the concept of folding to solve problems. Specifically it refers to the use of folding, either figuratively or literally, in mechanism design.) The following sections provide a brief background on origami, its history, evolution and potential for application in mechanism design.

2.2 Origami

Origami is the art of folding paper where ori- means fold and -kami means paper. Initially origami was used to create figures and animals. Traditionally origami is one square sheet of paper folded, with no cuts, often resulting in a static and representational model. Historically origami has its beginning in several countries dating back to the Muromachi period, from 1333-1573 AD [21,22]. Closely related to origami are other forms of paper engineering, such as kirigami, pop-up paper mechanisms, and origami architecture [4].

A more current origami movement began in the early 20th century. Akira Yosihzawa and his origami works have been largely credited for the creative explosion in origami in the last century
Since the 1920’s, origami design has become increasingly complex and varied. Significant amounts of research has been done with the advancement of computational power and as the art has progressed, the models have become increasingly complex, starting with a few to a few dozen steps to a few hundred or even a thousand steps. In the last twenty years new branches of origami have begun to be explored by more than artists; a growing number of mathematicians, educators, engineers and scientists have gathered to discuss the applications of origami in their respective fields [21, 22].

2.2.1 Origami Terms

*Origami*. Origami is defined as the art of folding paper, and in the context of engineering, it is the use of folding to solve mechanical problems [22].

*Kirigami*. Kirigami is defined as the art of folding and cutting paper [23]. An example of kirigami is shown in Figure 2.1 with the side view showing how creases are made such that the house pops out of the paper. Kinetic kirigami models are often LEMs, especially kirigami pop-up models.
Hinge creases. In traditional origami hinge creases define the boundaries between flaps. In the context of mechanisms they are creases along which lies an interface of two planar faces and it is the axis about which both facets rotate [22].

Construction creases. Creases used to create references in the construction of the mechanism, but are not directly used to create motion, are referred to as construction creases. In some cases, they coincide with hinge creases [22].

Structural creases. Structural creases are used to define the shape of flaps; they can be hinge creases as well. These creases are not needed in most mechanisms and they are not feasible for many materials. They may/can be substituted or eliminated in various ways [22].

Crease vs. fold. A fold is an action and a crease is the product of that action. Creases may be folded in one of two ways: mountain or valley.

Mountain crease. A convex crease [24].

Valley crease. A concave crease [24].

2.2.2 Research in Origami

Origami design is governed by mathematical theorems, which if better understood, could be applied in engineering. Some theorems of particular interest to LEMs are those concerning flat-foldability or when the paper model can be folded flat.

Flat-Foldability

In determining flat-foldability it is assumed models can be folded perfectly, ignoring folding error, that paper has zero thickness, and that creases have no width [24]. The following criteria were developed for determining local flat-foldability, where local means about a single vertex.

Theorem 1 (Kawasaki, Justin, Hull) A single-vertex crease pattern defined by angles \( \theta_1 + \theta_2 + \ldots + \theta_n = 360^\circ \) is flat-foldable if and only if \( n \) (the number of creases) is even and the sum of the odd angles \( \theta_{2i+1} \) is equal to the sum of the even angles \( \theta_{2i} \), or equivalently either sum is equal to 180° [22]. This theorem is generally known as Kawasaki’s Theorem, though also discovered independently by Justin in 1989 [24–26].
**Theorem 2 (Maekawa [Kasahara and Takahama], Justin)** In a flat-foldable single-vertex mountain-valley pattern defined by angles $\theta_1 + \theta_2 + \ldots + \theta_n = 360^\circ$, the number of mountains and the number of valleys differ by $\pm 2$ [22]. Theorem 2 is generally know as Maekawa’s Theorem [24–26].

**Theorem 3 (Kawasaki, Justin)** If an angle $\theta_i$ is a strict local minimum (i.e., $\theta_{i-1} > \theta_i < \theta_{i+1}$), then the two creases bounding angle $\theta_i$ have an opposite mountain-valley assignment in any flat-foldable mountain-valley pattern [22, 24–26].

An additional criterion is that a sheet can never penetrate a fold. To determine this it is required to have the geometry specified as well as the angles.

While criteria exist for determining the local flat-foldability (for a single vertex crease pattern), determining global flat-foldability is \textit{np-hard} (non-deterministic polynomial-time hard).

### 2.2.3 Origami Applied in Engineering

The scope and complexity of origami has exploded in the last twenty years creating many different schools of thought [27]. Recently in the fields of science, mathematics and engineering, origami has been used to solve complex problems such as airbag folding, shock absorption (crash box), and deployable telescopic lenses [11, 28, 29].

Previous work has focused on studying the folding process to map the transition between initial and final states. An example is airbag folding designs which unfold smoothly from their flat starting state to their final volume [28]. The Diffractive Optics Group at Lawrence Livermore Laboratory has worked on developing a telescope having with a 100 meter diameter lens that could collapse to fit into a space vehicle having a 4 meter diameter and 10 meter length. Lang used origami to identify a design that would fit and maintain the integrity of the surface when deployed into space [11, 30, 31]. Origami has also been applied to the crash box of a car to improve energy absorption in a low speed collision [29]. Miura explored methods of folding maps that could be unfolded with the simple pull of a corner [32]. In the 1980’s he invented the Miura-ori pattern, which is used as a basis for folding solar arrays [12]. In 2005, Mahadevan published findings that this same pattern exists in leaf folding, wings, and flower petals [33–35].
2.3 Graph Theory Background for Understanding the Proposed Intersection of Origami and Mechanisms

The research in later chapters uses graph theory as a way to help define the relationship between origami and mechanisms. As such this section provides a brief background along with definitions as an introduction. The process of folding origami has been examined using kinematic theory [36–39]. In addition to the process of folding, certain origami models exist which may be considered as “action origami” [40].

While the term “action origami” is a more generally known term it sometimes refers to models that are played with like paper airplanes. Therefore the term kinetic origami is introduced to refer to models that exhibit a mechanical motion and to differentiate from models that are generically referred to as action origami [8]. Figure 2.2 shows two examples, where one is a static origami structure and the other is an action origami mechanism called a flasher hat; it is shown in its fabricated and deployed forms. The way an origami model folds also provides insight and another way to classifying origami. The class of rigidly foldable origami focuses on motion where the creases often act as joints and the faces act as links with bending stress only occurring at the creases [37, 38, 41, 42].

Principles of graph theory have been used in mechanisms research. Specifically Dai’s carton folding research applies graph theory to show an equivalent mechanism [36].

Figure 2.2: Example of an origami structure (left) and flasher hat mechanism in its fabricated (middle) and deployed form (right).
2.3.1 Terms from Graph Theory

*Graph.* A graph consists of points, called vertices, and connections, called edges [43].

*Planar graph.* A graph with no edges crossing is a planar graph [43].

*Facet.* The area enclosed by a planar graph is referred to as a facet.

*Cycle.* A cycle is a non-trivial closed trail in which no vertex is repeated.

*Weighted graph.* A graph having a number assigned to each edge which may represent cost, length, etc.

*Degree.* The degree of a vertex is the number of edges that occur at that vertex [43].

*Degree sequence.* The degree sequence of a graph is the degrees of each vertex listed in decreasing order [43].

*Regular graph, d-regular graph.* A graph is considered regular if all its vertices are of the same degree. It can be referred to as a $d$-regular graph, where $d$ is a non-negative integer where each vertex has degree $d$ [43].

*Generically rigid.* A graph is generically rigid if almost every realization of the graph is rigid. A realization of a graph is an assignment of coordinates to joints but without restrictions on edge lengths. Generic realizations avoid degeneracies such as having three points collinear, four points concyclic, or inducing parallel edges [22].
Table 2.1: Proposed Design Methods from Previous Work

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<tbody>
<tr>
<td>Step 1</td>
<td>Evaluate manufacturing possibilities</td>
<td>Understand the fundamentals governing flexibility</td>
<td>Rigid-body design alternatives are screened to identify the most viable alternative to be converted to a compliant mechanism</td>
</tr>
<tr>
<td>Step 2</td>
<td>Select or design of mechanism</td>
<td>Form components</td>
<td>The conversion process yields alternative compliant designs</td>
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<tr>
<td>Step 3</td>
<td>Possible initial degree of freedom calculation</td>
<td>Create interaction strategies</td>
<td>Compliant alternatives are reviewed using the general design rules to assure good designs</td>
</tr>
<tr>
<td>Step 4</td>
<td>Design of planar configuration</td>
<td>Develop modeling and analysis approaches, synthesize mechanisms</td>
<td>Design alternatives are evaluated to select the best design</td>
</tr>
<tr>
<td>Step 5</td>
<td>Initial and final degree of freedom check</td>
<td>Design, fabricate and test demonstrators</td>
<td>The best design is compared to the best rigid-body alternative</td>
</tr>
<tr>
<td>Step 6</td>
<td>Manufacturing compatibility check</td>
<td></td>
<td>Design rules can again be used to refine the performance of the design, or help select another alternative if it is not superior to the rigid-body design</td>
</tr>
</tbody>
</table>
The previous chapter gave a basic overview for the design of planar single-layer four-bar change-point LEMs. Also presented were the analogs of mechanism design to origami flat-folding rules. Planar mechanisms are the straightforward case for understanding this relationship. The strength and promise of the relationship between origami and mechanisms lies in the design of spherical LEMs. This chapter introduces the relationship between origami and spherical mechanisms and presents it through graph theory.

3.1 Origami and Compliance

Since origami relies on the deflection of flexible materials it is a compliant mechanism. The origami mechanisms examined herein are all flat-folding in their final-folded state, and so they are part of a subgroup of compliant mechanisms called lamina emergent mechanisms (LEMs) [5]. Lamina emergent mechanisms are compliant mechanisms made from planar materials (lamina) with motion that emerges out of the fabrication plane.

A productive connection between origami and compliant mechanisms can be developed by drawing upon principles from graph theory to depict the origami mechanisms via simple, planar connected graphs. These graphs can show how the folds and facets interact with each other in their motion. Graphs allow for improved understanding of the interaction between motion and structure of origami, and can help in understanding how to predict complex motion and develop corresponding mechanisms.

Origami’s rich history and research in design optimization can provide an orimimetic perspective to LEM design that can lead to the generation of novel mechanisms [8].
3.2 Origami Mechanisms

This thesis considers kinetic origami or origami mechanisms as opposed to origami structures. The terms kinetic origami and origami mechanism are used interchangeably.

3.2.1 Kinematic Modeling of Origami Mechanisms

Winder showed that the pseudo-rigid body model can be used to model pop-up paper mechanisms. Likewise, origami mechanisms are modeled with hinge creases as joints and facets as links [4]. Origami that is rigid-foldable is a “piecewise linear origami that is continuously transformable without the deformation of each facet” [44]. Rigid origami is also defined as having regions of the paper between crease lines that do not need to bend or twist in the folding process (i.e. the facet could be replaced with sheet metal and hinge creases replaced by hinges and it would still fold up) [38, 41]. Figure 3.1 shows an example of rigid origami made by modeling the links of a square twist, a common origami unit for tessellations, converted to a polypropylene compliant mechanism.

Figure 3.1: A rigid-foldable version of the square twist origami constructed from polypropylene sheet and paper. The polypropylene shows that it behaves rigidly, where bending is only allowed at the folds.
3.2.2 Origami and Compliant Mechanisms

Origami mechanisms are compliant mechanisms; their motion is a result of the deflection of the material. As a material undergoes deformation, the resulting stored strain energy gives rise to an internal spring force. Balkcom notes that “the configuration of paper is determined by internal spring forces as well as external forces and constraints” [37]. Thus, the origami can be viewed as a compliant mechanism which can be modeled using the pseudo-rigid-body model.

By focusing on origami in a fabricated state we only examine the hinge creases that contribute to the mechanism’s structure and motion, not the folds that were used to construct it.

Pseudo-Rigid-Body Model

The pseudo-rigid-body model (PRBM) uses rigid-body components, rigid links and springs, that have equivalent force-deflection characteristics to model the deflection of flexible members [3]. Figure 3.2 shows a common kirigami double slit and its corresponding PRBM [18].

![Figure 3.2: A common double slit kirigami representation and its corresponding PRBM both shown placed in the crease of a folded sheet of paper.](image)

3.3 Origami as a Lamina Emergent Mechanism

Since origami mechanisms are made from lamina materials it follows that they are LEMs. Traditionally, origami designs are judged by their efficiency and accuracy in terms of material usage and number of folds [21]. For a model of an animal, efficiency is often defined by the ratio
of the size of the final model to the initial sheet. This measure of efficiency can be especially useful in an orimimetic approach to LEM design. Already orimemtic principles have been applied to solve packing problems in the design of airbags, telescopes and space sails, which are also LEMs [12, 28].

A branch of origami that is of special relevance to LEMs is flat-folding origami, which is origami that folds to a flat state, and tessellations or tilings, which could be useful for LEM applications in arrays. The repetition of basic folds in tessellations leads to the development of an origami mechanism that expands and contracts, thereby capturing the motion that would be required for an array-type structure.

Table 3.1 lists examples of origami corollaries in each of the six categories. The potential for origami insights in LEM design can be seen by mapping existing applications of origami to the six application categories for LEMs suggested by Albrechtsen et al. [7].

<table>
<thead>
<tr>
<th>LEM Application Class</th>
<th>Origami Corollaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposable mechanisms</td>
<td>Packaging [36, 45–47]</td>
</tr>
<tr>
<td>Novel Array mechanisms</td>
<td>Space sails</td>
</tr>
<tr>
<td></td>
<td>Telescopes [11]</td>
</tr>
<tr>
<td>Scalable mechanisms</td>
<td>Origami at nano-level [39]</td>
</tr>
<tr>
<td></td>
<td>Cellular origami [10]</td>
</tr>
<tr>
<td>Surprising Motion mechanisms</td>
<td>Pop-up books [18]</td>
</tr>
<tr>
<td>Shock-absorbing mechanisms</td>
<td>Crash box [29]</td>
</tr>
<tr>
<td>Deployable mechanisms</td>
<td>Airbags [28]</td>
</tr>
<tr>
<td></td>
<td>Stents [10, 48]</td>
</tr>
</tbody>
</table>

### 3.4 Graphs and Mechanisms

Principles from graph theory can be used as a tool to understand how origami functions as a mechanism [49]. Topological graphs can model both origami and mechanisms on an abstract level and show their similarities. The abstraction of origami mechanisms to graphs demonstrates
how the origami functions as a mechanism. Origami mechanisms can be reduced to graphs which show how they behave.

3.5 Linking Origami to Mechanisms with Graphs

In a graph of a kinetic origami model, folds become line segments (edges) and links become nodes (vertexes). From such graphs it can be seen that origami mechanisms may be thought of as interconnected linkages, with each ‘loop’ representing a linkage system. It is important to note that in graph theory the shape of an edge or the position of the vertexes does not matter since graphs depict connections [43]. The crease pattern can be related to the graph of the origami mechanism in some cases where all creases are hinge creases and the origami can be flattened via actuation. In this case the graph and crease pattern are considered dual graphs. Figure 3.3 shows the square twist with its crease pattern and corresponding graph overlaid. However, for the general case, the crease pattern may include structural and construction creases which aid in the folding of the origami but do not contribute to the motion and therefore are not considered in the origami’s corresponding graph.

Figure 3.3: The crease pattern for the square twist shown in Figure 3.1 is shown with its corresponding graph overlaid in black. In the crease pattern the red corresponds to mountain folds and blue to valley folds.

Any simple planar connected graph with four segments connecting four vertexes that is a 2-regular graph represents a four-bar mechanism. The graphs depict the degree of interconnection for each linkage system. This is shown in how each facet relates to its neighboring facets. Each facet in a graph represents a linkage having the same number of links as nodes. For example, a
facet (enclosed region) having four vertexes (links) on the boundaries separating four segments (joints) is representative of a four-bar linkage.

When we reduce an origami model to a simple planar connected graph, the graph only describes how each facet is connected to others. This basically becomes a one-dimensional representation of the model. From it we can infer a little regarding the folding process and which facets are mobile. A crease pattern defines the geometry of different facets and is regarded as a two-dimensional graph. From a crease pattern we can determine whether it will fold flat and how to fold or assemble the model. Essentially a crease pattern contains information that would be useful in understanding the folding, or assembly, of an object manufactured in a flat state. From a crease pattern it can be determined whether it is flat-folding as well as a folding sequence.

It should be noted that in origami, flat foldability refers to when a crease pattern is folded along all the edges and it folds to a flat state. However, in mechanisms the joints do not need to be “folded” in the flat position. So we can have mechanisms which are flat but not necessarily folded along each “edge.” Origami would say that we could not have a flat foldable five-bar linkage with all joints folded, but we know that 5-bar change-point mechanisms do exist by having at least one joint extended in the flat position. In this way, mechanism design can be understood in the frame of origami, but not restricted to it.

### 3.6 Paper Mechanism Examples

Four paper mechanisms are shown in Table 3.2 with their corresponding kinematic and graphical representations. For simplification, all four paper mechanisms are flat-folding mechanisms and are rigid-foldable.

The kinematic representations for each mechanism examine a portion of the larger mechanism and its motion. When the paper mechanisms have a flat initial state and they can be represented with kinematics, by definition they can be realized as a LEM. In addition, the graphs representing each paper mechanism show more abstractly the mechanism and how they are coupled. The graphs also indicate the type of fold, where mountain folds are shown in red and valley folds are in blue.

The four-bar double slit mechanism (Table 3.2, row 1) uses the PRBM to show how each crease can be modeled as a joint with a torsional spring. Its graph is a 2-regular graph. It has three
valley folds, and one mountain fold as indicated by the colors of edges in the graph. Each link has only two joints that define its motion.

For the 45-degree-fold twisting mechanism, also known as the pop-up spinner card (Table 3.2, row 2), each center fold, along the spine of the mechanism, is a 45 degree fold [50]. The kinematic representation shows only four links along the spine. The 45-degree-fold twisting mechanism is a chain of four-bar mechanisms where those links not on the ends share two links between each neighboring linkage system, and each inside link is constrained by three joints with one of those joints shared between each neighboring linkage system.

The square twist mechanism (Table 3.2, row 3) is a series of coupled spherical mechanisms which collapse onto one another. Figure 3.1 shows the basic square twist, and the kinematic representation shows one of the four-bar linkages seen at the corner of the twisting square platform. It may be better to express the square twist as inter-connected four-bar spherical mechanisms, which may be seen in the graph representation. The outer corner links each connect to only two joints. There are two neighboring links on each end that are connected to three joints and are shared between two linkage systems. Of the 7 nodes of degree four, 3 are shared between four linkage systems and 4 are shared between three linkage systems.

Lastly, the kinematic representation for the water bomb base tessellation is shown in Table 3.2, row 4. As a graph the water bomb base tessellation is a 3-regular graph. The water bomb base tessellation contains six-bar linkage systems that are interconnected. Each link is shared between three linkage systems via three joints. This is also the basic tessellation used in the origami stent design which allows for the mechanism to compact radially as well as lengthwise [51].

3.7 Discussion

The examples in Table 3.2 examine a single four-bar mechanism, four-bars connected in series, four bars in series and parallel, and a six-bar spherical mechanism. The use of graph theory helps bridge the fields of origami and mechanisms. Visually the graphs show how the origami models are interconnected spherical mechanisms.
Square Twist

The square twist crease pattern, shown in Figure 3.4, has $2^{12}$ possible assignments and of those only 16 are flat-foldable [25]. Figure 3.5 shows the 16 valid assignments for the square twist. It should be noted that only 6 of these 16 are unique. From a mechanisms standpoint however, only 4 are unique because 2 inversions are present.

![Figure 3.4: Crease pattern for square twist.](image1)

Next by applying Theorem 2 we can determine the remaining crease designations which are shown in Figure 3.6.

It is easier to use the graph (on the right) as opposed to the crease pattern for visualizing how Theorem 2 is applied. Note that each subgraph for one of the four four-bars has either 3 mountain and 1 valley folds or 1 mountain and 3 valley folds. Corresponding paper representations of each are shown in Figure 3.7.
Figure 3.6: The 6 possible unique square twist crease patterns and their corresponding graphs given Theorem 2.

Figure 3.7: Folded origami models corresponding to the 6 possible square twists corresponding to those shown in Figure 3.6.

All six paper models in Figure 3.7 are flat-folded versions of the square twist which are kinetic origami. An important distinction to make is that while an origami can be folded flat it does not mean that it is kinetic. Since all folds here are simple folds and they are non-locking folds this allows us to actuate the square twist to have the twisting motion going from a folded state (half the size of the original square) back to the original square.
3.8 Summary

Topological graphs of kinetic origami and kirigami models were used to show how the fields of origami and mechanisms are related. We propose that some of the literature from the origami field can be applied to mechanisms and vice versa. This was shown by comparing the criteria for flat-foldability to some conditions required for change-point mechanisms to be assembled. As such it is anticipated that the abstraction to graph theory will improve the ability to synthesize LEMs.

In addition, by examining the graphs of some paper mechanisms it is understood that their motion is achieved because they are a system of coupled, or interconnected, mechanisms.
Table 3.2: Paper Mechanisms with their Corresponding Kinematic and Graph Representations

<table>
<thead>
<tr>
<th>Paper Mechanism</th>
<th>Kinematic Representation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Four-Bar Double Slit</td>
<td><img src="image1" alt="Four-Bar Double Slit Kinematic Representation" /></td>
<td><img src="image2" alt="Four-Bar Double Slit Graph" /></td>
</tr>
<tr>
<td>2. 45-degree-fold Twisting Mechanism</td>
<td><img src="image3" alt="45-degree-fold Twisting Mechanism Kinematic Representation" /></td>
<td><img src="image4" alt="45-degree-fold Twisting Mechanism Graph" /></td>
</tr>
<tr>
<td>3. Square Twist</td>
<td><img src="image5" alt="Square Twist Kinematic Representation" /></td>
<td><img src="image6" alt="Square Twist Graph" /></td>
</tr>
<tr>
<td>4. Water Bomb Base Tessellation</td>
<td><img src="image7" alt="Water Bomb Base Tessellation Kinematic Representation" /></td>
<td><img src="image8" alt="Water Bomb Base Tessellation Graph" /></td>
</tr>
</tbody>
</table>
CHAPTER 4. PLANAR LAMINA EMERGENT MECHANISMS

This chapter discusses characteristics of planar LEMs and in particular four-bar change-point LEMs. LEMs are a class of compliant mechanisms that are fabricated from planar materials (lamina) and have motion emerging out of the fabrication plane. Origami, the art of paper folding, is believed to provide insights into LEM designs because, like LEMs, origami begins from an initial lamina and can be folded to have motion emerging out of the fabrication plane. This chapter identifies connections between origami and mechanisms. Also introduced in this chapter are the concepts of assembly in four-bar change-point mechanisms and their origami rule counterpart.

4.1 The Set of Single Layer Four-Bar Change-Point LEM Designs

While previous research has been conducted regarding multi-layer LEMs (MLEMs) [17], the scope of this thesis is single-layer LEMs. Single-layer LEMs allow for the application of origami rules because traditional origami is from a single sheet of paper. The term modular origami is used when considering multiple sheets, but that is outside of the scope here. A single-layer LEM four-bar will be a change-point mechanism. This means that

\[ s + l = p + q \]  

(4.1)

where \( s \) is the length of the shortest link and \( l \) is the length of the longest link and \( p \) and \( q \) are the lengths of the other links [1]. There are twenty-four possible layouts for a single-layer LEM four-bar, six of which are primary linkages and eighteen that are inversions of the six. This is excluding the cases where \( s = p \) and \( l = q \). This case does not require a unique layout as is discussed later.

The twenty-four unique linkages can be classified by one distinct characteristic: whether link \( s \) is joined with link \( l \). Where links \( s \) and \( l \) are not joined together, the four-bar layout requires “folding” at each joint. This results in the four folds corresponding to Layout B in Table 4.1.
However when links $s$ and $l$ are joined together the four-bar layout requires the equivalent of two folds. This corresponds to Layout A in Table 4.1. This can be seen when organizing the permutations and identifying which designs are inversions of each other seen in Table 4.1.

The reasoning behind the two layouts can be understood through origami conditions for flat-foldability about a single vertex. This is a simple case since it is a planar mechanism, however this lays the foundation for the case of spherical mechanisms. In the following section the flat-folding theorems are reiterated and related to rules of mechanism design.

Barker provides a complete classification of change-point four-bar mechanisms listed in the following Table 4.2 [1].

4.2 Flat-Foldability and Rules for Mechanism Design

This section provides an overview of the origami rules for local flat-foldability, local meaning about a single vertex, and how this can be applied to mechanism design.

Theorem 1 (Kawasaki, Justin, Hull) A single-vertex crease pattern defined by angles $\theta_1 + \theta_2 + \ldots + \theta_n = 360^\circ$ is flat-foldable if and only if $n$ [the number of creases] is even and the sum of the odd angles $\theta_{2i+1}$ is equal to the sum of the even angles $\theta_{2i}$, or equivalently either sum is equal to $180^\circ$ [22, 24–26].

This theorem, commonly known as Kawasaki’s Theorem, is analogous to the definition of a change-point mechanism, where a change-point four-bar exists when links $s + l = p + q$. Link lengths correspond to angles about a vertex. A second condition for flat foldability, called Maekawa’s Theorem helps us understand how the links are connected to one another.

Theorem 2 (Maekawa [Kasahara and Takahama], Justin) In a flat-foldable single-vertex mountain-valley pattern defined by angles $\theta_1 + \theta_2 + \ldots + \theta_n = 360^\circ$, the number of mountains and the number of valleys differ by $\pm 2$ [22, 24–26].

Origami indicates that there is only one unique way to fold a change-point four-bar mechanism where each joint is folded. This is shown in Figure 4.3. Three of the lines will represent one type of fold while one will represent the opposite type of fold. This corresponds to having either three mountains and one valley or vice versa. The folds attached to the $s$ link will have opposite
Table 4.1: Identifies Linkage and Corresponding Layout. Dashed Green link represents Ground link.

<table>
<thead>
<tr>
<th>Key</th>
<th>Linkage</th>
<th>Inversions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><img src="image1" alt="Linkage Image" /></td>
<td><img src="image2" alt="Inversions Image" /></td>
</tr>
<tr>
<td>s</td>
<td><img src="image3" alt="Linkage Image" /></td>
<td><img src="image4" alt="Inversions Image" /></td>
</tr>
<tr>
<td>l</td>
<td><img src="image5" alt="Linkage Image" /></td>
<td><img src="image6" alt="Inversions Image" /></td>
</tr>
<tr>
<td>p</td>
<td><img src="image7" alt="Linkage Image" /></td>
<td><img src="image8" alt="Inversions Image" /></td>
</tr>
<tr>
<td>q</td>
<td><img src="image9" alt="Linkage Image" /></td>
<td><img src="image10" alt="Inversions Image" /></td>
</tr>
</tbody>
</table>

**Layout A**

| 3   | ![Linkage Image](image11) | ![Inversions Image](image12) |
| s   | ![Linkage Image](image13) | ![Inversions Image](image14) |
| l   | ![Linkage Image](image15) | ![Inversions Image](image16) |
| p   | ![Linkage Image](image17) | ![Inversions Image](image18) |
| q   | ![Linkage Image](image19) | ![Inversions Image](image20) |

**Layout B**

| 4   | ![Linkage Image](image21) | ![Inversions Image](image22) |
| s   | ![Linkage Image](image23) | ![Inversions Image](image24) |
| l   | ![Linkage Image](image25) | ![Inversions Image](image26) |
| p   | ![Linkage Image](image27) | ![Inversions Image](image28) |
| q   | ![Linkage Image](image29) | ![Inversions Image](image30) |
Table 4.2: Barker’s Complete Classification of Change-point Four-bar Mechanisms [1].

<table>
<thead>
<tr>
<th>C.P. Class</th>
<th>Characteristic Length</th>
<th>Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( R_1 = s )</td>
<td>c.p. crank-crank-crank</td>
<td>CPCCC</td>
</tr>
<tr>
<td>2</td>
<td>( R_2 = s )</td>
<td>c.p. crank-rocker-rocker</td>
<td>CPCRR</td>
</tr>
<tr>
<td>3</td>
<td>( R_3 = s )</td>
<td>c.p. rocker-crank-rocker</td>
<td>CPRCR</td>
</tr>
<tr>
<td>4</td>
<td>( R_4 = s )</td>
<td>c.p. rocker-rocker-crank</td>
<td>CPRRC</td>
</tr>
<tr>
<td>5</td>
<td>two equal pairs</td>
<td>double change point</td>
<td>CP2X</td>
</tr>
<tr>
<td>6</td>
<td>( R_1 = R_2 = R_3 = R_4 )</td>
<td>triple change point</td>
<td>CP3X</td>
</tr>
</tbody>
</table>

Figure 4.1: Example showing that the folds on either side of link \( s \) should be opposite to enable flat-foldability.

different type folds which can be understood from Demaine’s discussion on crimp versus edge folds [22]. This may be more readily understood in Figure 4.1.

The trivial case of flat-folding occurs when not requiring each joint to fold. This exists when only two joints are folded at 0° and 180°, corresponding to Figure 4.2.

Figure 4.2 and Figure 4.3 show the two possible layouts for a change-point four-bar having different link lengths \( l, s, p \) and \( q \). These figures are not meant to be folded, but rather show how the connections need to occur to allow function as a mechanism. Therefore, by definition a

Figure 4.2: Joints are located where dashed lines are in an arbitrary change-point mechanism having links \( s \) and \( l \) joined together.
change-point four-bar follows Equation 4.1 when the linkage is assembled it will fall into one of the two layouts, one requiring every joint to fold and the other requiring only half the joints to fold.

Similarly, origami tells us that a vertex is only flat-foldable when the number of creases about it are even [25]. This however is only when requiring folding along each crease. Since we are considering mechanisms there is no requirement that there be “folding” at each joint. For linkages with odd number of links (e.g. a 5-bar), this means that at least two connected links must not fold in the flat position, resulting in the equivalent of an even number of folds. This is shown in Figure 4.4. Thus flat-folding linkages can have any number of links greater than three and the sum of the half the link lengths is equal to the sum of the remaining link lengths.
4.3 Single-layer LEM Physical Layouts

Previous papers [7, 19] have shown the layout of a LEM four-bar mechanism. The LEM four-bars use compliant [19] “joints” resulting in motion by the deflection of flexible members. This allows for the reduction of parts, however the range of motion for a LEM four-bar is likely limited by material and geometric constraints when compared with the rigid-body counterpart. The arbitrary layouts shown in Figure 4.2 and Figure 4.3 correspond to the physical layouts shown in Figure 4.5 and Figure 4.6, respectively.

In Figure 4.5 and Figure 4.6 some links are doubled so as to reduce parasitic error, and maintain balance such that when a load is applied it could be more evenly distributed across the mechanism. Also joints called inside lamina emergent torsional (LET) joints are shown. However, additional joint types are being developed as alternatives and to meet additional user specification. For Layout A in Figure 4.5, the longest and the shortest links (s and l) run parallel to the other two links (p and q).

4.3.1 Layout types

It is also useful to divide these layouts of unique linkages into types which have specific behaviors. Input links are numbered as link 2 in Table 4.1, while coupler links are numbered as
Figure 4.6: An example lamina emergent four-bar, with LET joints at folds in Layout B

link 3 and output links as link 4. In a four-bar mechanism both input and output links are connected to ground, defined as link 1 in Table 4.1.

**Type I:** any linkage having $s$ and $l$ in either (i) the input and coupler link positions or (ii) in the follower and ground link positions

**Type II:** any linkage having $s$ and $l$ in either (i) the ground and input link positions, or (ii) the coupler and follower link positions
Type III: any linkage having $s$ and $l$ alternating in link positions

These different types of linkages are shown in Table 4.1. Type II linkages by convention require the input angle, $\theta_2$, to start at 180° rather than at 0°. Further behavioral differences are discussed later.

4.4 Mechanism Design Considerations

The design of a single-layer planar LEM will have special considerations including loading requirements, LEM joint requirements, whether it would follow an open or closed circuit, and failure situations. A brief discussion of each of these is provide.

4.4.1 Loading Requirements

The load that a LEM will be required to carry or transmit is a necessary design specification to know at the beginning of the design process. This information will limit the design space significantly by narrowing material choices, thickness of the lamina material and possible fabrication methods. If there is still feasible space the next step is to consider LEM joint requirements.

4.4.2 LEM Joint Requirements

Given the loading conditions, specific joints will have to be selected and optimized to achieve the bending, torsion and stiffness characteristics desired. Some work on joint optimization has been done [].

4.4.3 Circuits: Principle of Virtual Work

It is interesting to consider how mechanisms emerge from the change-point position and whether it will follow the open or crossed circuit. It is hypothesized that there is a preferred circuit based on the concept that the mechanism will go into its least energy state. For the Type III mechanisms it is proposed that they will follow the crossed circuit.
4.4.4 Failure Modes

The pseudo-rigid body model has limitations in modeling large deflections, this should be considered when determining whether a LEM is an appropriate design to replace a rigid-body mechanism.

It is expected that a LEM would fail at the joints, specifically at the joint where the strain energy is sufficiently high to induce failure or after a certain number of cycles. This joint is likely to be the joint that achieves its maximum angular displacement found through the failure analysis for the given joint design. To determine which joint may fail first it is necessary to find the rates of change for the angles with respect to the input angle. The joint that fails first is referred to as the limiting angle, or limiting joint.

4.5 Rigid-body Replacement Synthesis

Rigid-body replacement methods can be used to synthesize the set of planar change-point four-bar LEMs. This set of LEM four-bar mechanisms can be classified according to three types. These types in turn correspond to two different layouts based upon the classic LEM four-bar design. From this exercise new considerations have been identified for planar LEM synthesis.

All of these considerations can be put into a method for rigid-body replacement synthesis. This method takes a rigid-body four-bar and determines how it can be made into a single-layer LEM. The range of motion is dependent on the material and the geometry of the joints. More or less angular deflection can be achieved. However there is usually a maximum of 180° in order to avoid self-intersection.

After the generic layout of the LEM is known the loading conditions and the desired scope of actuation should be used to determine the material and its thickness to determine whether the LEM is a feasible replacement. Then the joint design should be optimized according to the bending, torsion and stiffness requirements. The joint designs will also affect the overall area required by the LEM since their size is not assumed in the PRBM. Finally, an analysis of failure modes should be performed to determine if the LEM meets the functional requirements.
CHAPTER 5. ORIGAMI APPLIED TO LEM SYNTHESIS

The purpose of this chapter is to introduce principles for the synthesis of LEMs. The principles identified pair graph representations and theory with origami for the purpose of synthesizing LEMs. From the principles of graph theory it was shown how origami relates to mechanisms.

Graphical representations help us understand when any simple planar connected graph could be realized as a mechanism. When a graph is represented in two dimensions, whereby geometry is assigned, be it as a crease pattern or other form, we can generally know whether it can be folded flat. It is believed that graphs can serve as a useful tool in synthesis of mechanisms and also in understanding packaging and assembling processes from two to three dimensional space. As such, while the previous chapter only discussed kinetic origami as it exists in its final folded state, the folding itself is a kinetic process and can be modeled mechanically [53].

The kinematic modeling of the folding process allows us to consider the whole timeline of the folding process shown in Figure 5. From the planar manufacturing stage folding may occur while prestressing the raw material, folding may also occur in the assembly and packaging process, and folding or more broadly collapsing and expanding during usage and storage. In addition to the mechanism or product being produced, folding may occur in the production of packaging and the opening of packaging to yield a desired user experience for opening a product. Folding or bending may occur even previous to this depiction in the design and modeling phase as well.

By modeling the folding process, we can model also the unfolding process. This may occur in the packaging, and the using and storing process shown in the loop in Figure 5. The use of origami for modeling of less rigid materials, such as airbags, shows how modeling folding can be useful for predicting its deployment [28]. The ability to model folding and unfolding, collapsing and expanding of laminar and compliant materials would be especially useful in automating processes in manufacturing [47].
Folding occurs along Product Life Cycle

Figure 5.1: This is an example timeline of transforming a raw material to being used and stored as a final product. Forms of folding can be present along this entire timeline.

So far it has been established that the folding processes for all origami models and the final folded state of an action origami can be modeled with kinematics [31,36,44,45,53]. In addition, the mobility of the origami model can be explored and understood using graph theory to identify the joining of links and diagram them. The following section relates kinematic and origami concepts for the purposes of applying the principles to the synthesis of LEMs.

Table 5.1: Similarities between Mechanisms and Origami

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Origami</th>
</tr>
</thead>
<tbody>
<tr>
<td>joints</td>
<td>creases, folds</td>
</tr>
<tr>
<td>links</td>
<td>faces</td>
</tr>
<tr>
<td>motion</td>
<td>motion during folding process and sometimes after</td>
</tr>
<tr>
<td>rigid-body or compliant</td>
<td>rigid foldable or compliant</td>
</tr>
</tbody>
</table>

5.1 Principles for Synthesis

It is proposed that any 1-D graph having a cycle of at least four nodes can be realized as a mechanism by following a few rules. This 1-D graph can be developed into a 2-D graph (think crease pattern), whereby geometry is considered. Then materials can be selected and the design can continue to be adjusted given material properties and subsequent constraints. For example, the thickness of the material will affect mobility, desired deflections, failure predictions, and most design considerations for the mechanism. As the mechanism is adapted to the additional constraints...
from material and thickness, the deflections should be checked to ensure that the desired deflections can still be obtained. The following synthesis principles presuppose the existence of a graph and are used identify whether a graph can be realized as a LEM.

5.1.1 Mobility

Beginning with a 1-D graph it can be determined whether it is generically rigid or flexible and subsequently realizeable as a mechanism or a structure, where a mechanism would be classified as generically flexible and a structure as generically rigid. A generic graph can be realized as a mechanism in two dimensions, having 2 translations and 1 rotation, when

$$N_e < 2N_n - 3 \quad (5.1)$$

or in three dimensions when

$$N_e < 3N_n - 6 \quad (5.2)$$

where $N$ is the number of nodes and $N_e$ is the number of edges in the graph [22]. When

$$N_e = 2N_n - 3 \quad (5.3)$$

the graph is generically rigid.

It should be noted that this is the step where it any existing mechanism can begin to be analyzed to develop a flat-folding counterpart. It is also anticipated that any $n$-bar mechanism could be developed into a flat-folding counterpart where $n$ is greater than or equal to 4.

The graph in Figure 5.2 is used as an example.

The equations regarding generic rigidity are useful for describing the rigidity or flexibility of a mechanism where the Gruebler equation gives misleading results. The Gruebler equation for mobility is given

$$M = 3(L - 1) - 2J \quad (5.4)$$

where $M$ is the degree of freedom or mobility, $L$ is the number of links and $J$ is the number of joints [54]. The Gruebler equation finds that this mechanism has 0 degrees of freedom.
However, to incorporate the interconnectedness of links, the number of degrees of freedom can be written as

\[ F = 3N_e - 2 \sum_r n_r(r - 1) \]  

(5.5)

where \( N_e \) represent the number of edges, \( n_r \) are the number of vertexes having \( r \) edges attached, \( r \) can more easily be thought of as the degree of the vertex [55]. Equation 5.5 finds \( F = 6 \) for the graph in Figure 5.2.

5.1.2 Dual Graph

The previous graph focused on where joints existed between links. However the dual graph allows us to begin working with link and mechanism shape. After it is determined that a 1-D graph is generically flexible its dual graph should be drawn to generate the “crease pattern” for the mechanism to allow the application of the geometric constraints from origami flat-foldability rules. The dual graph is constructed by designating a vertex to correspond to each plane region or facet with an edge joining two neighboring regions.

Figure 5.3 shows an example of a planar graph, in blue, and its dual graph, in red. The top vertex represents the infinite facet. In applying the dual graph to a finite sheet material the edges of the dual graph that connect vertexes representing enclosed regions to vertex representing the infinite region, or facet, should extend to the edge of the material. This is shown in Figure 5.4.
5.1.3 Mechanism Assembly

For a four-bar mechanism to be assembled it is required to meet the criteria of

\[ l < s + p + q \] (5.6)

where \( l \) refers to the length of the longest link, \( s \) to the length of the shortest link and \( p \) and \( q \) the lengths of the other links. Likewise for an origami to be folded flat it must follow certain criteria. In determining flat-foldability it is assumed models can be folded perfectly, ignoring folding error,
that paper has zero thickness, and that creases have no width [24]. The following criteria were
developed for determining local flat-foldability, where local means about a single vertex.

**Theorem 1 (Kawasaki, Justin, Hull)** A single-vertex crease pattern defined by angles
$\theta_1 + \theta_2 + \ldots + \theta_n = 360^\circ$ is flat-foldable if and only if $n$ [the number of creases] is
even and the sum of the odd angles $\theta_{2i+1}$ is equal to the sum of the even angles $\theta_{2i}$, or
equivalently either sum is equal to $180^\circ$ [22].

This theorem is generally known as Kawasaki’s Theorem, though also discovered independ-
ently by Justin in 1989. Its analogy in mechanisms is the condition for a change-point found in
equation 4.1.

Following the same example, Kawasaki’s theorem is used to develop a possible crease
pattern is seen in Figure 5.1.3.

**Theorem 2 (Maekawa [Kasahara and Takahama], Justin)** In a flat-foldable single-
vertex mountain-valley pattern defined by angles $\theta_1 + \theta_2 + \ldots + \theta_n = 360^\circ$, the number
of mountains and the number of valleys differ by $\pm 2$ [22].

Theorem 2 is generally know as Maekawa’s Theorem. It is essentially a condition for
assembling a four-bar closed kinematic chain.
Theorem 3 (Kawasaki, Justin) If an angle $\theta_i$ is a strict local minimum (i.e., $\theta_{i-1} > \theta_i < \theta_{i+1}$), then the two creases bounding angle $\theta_i$ have an opposite mountain-valley assignment in any flat-foldable mountain-valley pattern [22].

This theorem is analogous to the conditions defining the classes for spherical four-bar LEMs where the shortest link has to connect to two links having opposite direction assignment.

In the example, Kawasaki’s theorem was applied to check local flat-foldability and given Maekawa’s theorem make mountain-valley assignments can be made. From the rules for flat-folding where the most acute angle is an opposite mountain-valley assignment must be made [22, 26]. The possible crease pattern assignment for flat-folding is seen in Figure 5.1.3.

5.1.4 Avoid Self-Intersection

An additional criterion is that a sheet can never penetrate a fold. To determine this it is required to have the geometry specified as well as the angles. Therefore, the design is refined to make sure it can be folded flat without folds wanting to intersect one another. This step is what makes identifying global flat-foldability hard; despite the criteria that exists for determining the local flat-foldability (for a single vertex crease pattern), determining global flat-foldability
Figure 5.7: An example mechanism pattern for square platform. There is too much material to fold flat. The smaller dashed square is an example of a viable solution.

is \textit{np-hard} (non-deterministic polynomial-time hard). The order of folding is all that may need to be adjusted or the link facet dimensions may need to change. Once this is all in order the ideal mechanism is designed and then comes the design of the realizable mechanism as seen in Figure 5.1.4. It should be noted that this is only one example of a variety of mechanisms that can be realized from the graph. The links can change shape and angles can be adjusted so that as long as the principles are followed a flat foldable design can be generated.

Through experimentation by folding the mechanism it is recognized that the center square is too small for the amount of material provided, therefore it is not only possible to use less material but necessary to achieve flat folding. Flat folding can be achieved by either increasing the size of the center square or cutting down the material in the flaps as indicated with the inside dotted square. Figure 5.1.4 shows the folded paper models. The model on the left will not fold flat without the sheet penetrating the folds, in other words it is self intersecting. The model on the right is an flat-folding square platform.

Table 5.2 shows a graph and two possible flat folding results from the given graph.
5.1.5 **Nuanced Understanding of Graphs**

The above discussion regarding whether a graph represents a mechanism requires the understanding of the dimension the graph is in. The utility of a graph depends on whether it is a 1D graph or a 2D graph. The difference between the two is whether or not geometry is assigned. This is better understood when thinking about a crease pattern and determining flat-foldability and folding sequence. A 1D graph can lead to a false assumption that it can be realized as a mechanism because it may meet the first three criteria for flat-foldability. However a 2D graph indicates the geometry of the edges, facets and their placement which is required to determine whether or not the sheet would penetrate a fold.

5.2 **Summary**

Graph theory includes theories for identifying whether a generic graph is rigid or flexible in $n$-dimensions [22]. As such it is believed that given any generic graph we can know whether it represents a structure or mechanism by understanding whether the graph is generically rigid, and therefore a structure, or generically flexible, and therefore a mechanism. Showing the relationship between origami and mechanisms will facilitate the application of origami design principles in the design of compliant mechanisms and LEMs.
### Table 5.2: Table of Mechanisms from Graphs

<table>
<thead>
<tr>
<th>Graph</th>
<th>Example Mechanism 1</th>
<th>Example Mechanism 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Four-Bar</td>
<td><img src="image1" alt="Four-Bar Mechanism 1" /></td>
<td><img src="image1" alt="Four-Bar Mechanism 2" /></td>
</tr>
<tr>
<td>2. Two connected four-bars</td>
<td><img src="image2" alt="Two-connected four-bars Mechanism 1" /></td>
<td><img src="image2" alt="Two-connected four-bars Mechanism 2" /></td>
</tr>
<tr>
<td>3. Two connected six-bars</td>
<td><img src="image3" alt="Two-connected six-bars Mechanism 1" /></td>
<td><img src="image3" alt="Two-connected six-bars Mechanism 2" /></td>
</tr>
</tbody>
</table>

#### 5.2.1 Refined Design

The next design decision is the material the mechanism will be realized in. Jacobsen provides a good discussion of developing flexible segments for LEMs relative to geometry, material properties and loading conditions [6]. Once the material and its thickness are known we can move on to assign joint types to each crease, whether it would be a living hinge, a LET joint, a split tube flexure, etc. It is hypothesized that where bending is desired along a line, a LET joint would be amenable, but where bending occurs about a point, slits or an inside LET joint would be more
practical. To determine when a slit would be appropriate, the designer should think about limit of a slit, as its length approaches 0, becomes a hole. The purpose of a hole might be to eliminate material, reduce stress concentrations, or reduce the energy required for bending because of the previous two reasons. The properties of a hole include, but are not limited to the hole size determines the “springiness” of that node, while the hole shape may play a role in the stresses about it, whether self-intersection may occur, and possibly in determining the ease or specified order of folding. These are subjects for further investigation and are outside the scope of this thesis.

The analog of a joint in paper folding is a crease. Creases are negligible in thickness. As such, once the joint types are assigned we implement the origami principle of grafting to add material to the pattern for the joints. The purpose of this step is to preserve the effective link lengths. This is similar to using the PRBM to redesign a rigid-body mechanism. In origami “most application of grafting serve to create distinct flaps,” [21] these distinct flaps are essentially links. Grafting works in origami to allow features to be added to a base without taking anything away from the features already present [21]. In addition, grafting can be used to somewhat disconnect flaps by reducing the coupling between them.

While Jacobsen has laid the groundwork for LEM design with his research in joints this research has yielded a synthesis method to generate families of mechanisms that come from the same topological graphs. An area for future research is the refinement of those generated designs and realizing them in materials other than paper.
LEMs have been defined as a type of compliant mechanism fabricated from lamina and having motion emerging out of the fabrication plane. They show promise in many areas where products are required to be collapsible or deployable. Similarly, origami has been applied to products where collapsibility and/or deployability are a concern. For example the origami stent [?,51], modeling of an airbag [28] and the crash box [29].

The research focused mainly on the collapsibility principles of folding and creasing by drawing relations between origami and LEMS. Thus it was shown that the principles of origami design and of flat-folding from the origami literature can be applied in mechanism design [8].

The power of the origami principles lies mainly in the relation to spherical mechanisms. Planar mechanisms were shown to be the trivial case when concerned with single layer change-point mechanisms. As planar and spherical mechanisms were compared graphically, it was shown that a spherical four-bar and a planar four-bar reduce to the same graph.

Graphs of origami and kirigami models were used to show how the fields of origami and mechanisms are related. It is proposed that some of the literature from the origami field can be applied to mechanisms and vice versa. This was shown by comparing the criteria for flat-foldability to some conditions required for change-point mechanisms to be assembled. As such it was anticipated that the abstraction to graph theory will improve the ability to synthesize LEMs.

A basic LEM synthesis method was developed using the principles from graph theory and shown in the development of a simple model of a square platform. The method was divided into two parts which separated the theoretical design of a mechanism from its realizable design.

Finally this chapter concludes with ideas for future research. It is proposed that other areas of collapsibility be investigated along with areas of materials and fabrication methods for realizing collapsible compliant mechanisms. In addition, an overview of other origami design tools were discussed as additional venues of opportunity for mechanism design. Of these, tree representation
6.1 Opportunities for Future Research

There is much that engineering can learn from art and vice versa. The field of origami has grown at a fast pace in the past decades as a result of the identification of mathematical principles and scientific investigation in origami [56]. This section serves as an overview of how this work serves as a foundation for future investigations of applying origami principles to the design of mechanisms.

6.1.1 Collapsibility

The work done in this thesis has primarily focused on mechanisms that collapse to and from a flat or planar state. However, there are many opportunities to pursue in the design of mechanisms that collapse to other flat states, such as to a sphere or cylinder or other polyhedra. The work in understanding folding and collapsibility, and its counterpart of deployability or erectability would be useful along the fabrication timeline seen in Figure 6.1 as mentioned previously in Chapter 5 [57].

Collapsibility principles are relevant to the manufacturing to the storage and use stages of a product. The principles of collapsibility also extend to the fabrication processes themselves such that they can be applied to optimize the paths traveled by robots. In addition, collapsibility principles can be applied to enhance the efficiency of material flow.
The following section looks at twelve principles of collapsibility and suggests areas for further research.

**Principles of Collapsibility**

Currently, there are twelve principles of collapsibility which describe actions by which objects may collapse or expand, actions necessary for that, or the structure of particular categories of collapsibles [58]. These principles are not exclusive but can and often overlap. They are: stress, folding, creasing, bellows, assembling, hinging, rolling, sliding, nesting, inflation, fanning and concertina. In addition, collapsibility can refer to both when an object would be used as well as for its storage state.

Stress is considered a dual principle because it includes both the notions of compression and expansion, pressure and tension serve as methods for collapsing. The collapsing principles can also apply for both states of storage and use. For example, when a rubber band is used it acts through tension and is expanded to bundle vegetables together. Also, in its stored state a sleeping bag is compressed and when in use it is relaxed.

Folding provides ways to reduce the size of objects for storage as well as a way to assemble objects to work. Closely tied with folding is creasing which facilitates the act of folding. Pleats, which are parallel creases, have also been used in folding doors and window screens [58]. Bellows is a principle which describes collapsibility from the object of the same name. It is “essentially a contractible and expandable bag with sides folded in pleats [58].” It is considered quasi-collapsible as its collapsed state is both active and passive.

Assembly is another principle of collapsibility where parts are put together into a whole and later dismantled for storage. LEMs may or may not operate on this principle in their deployment.

Hinging is “the *primus inter parus* of collapsibility principles [58].” Traditionally speaking hinging would imply the use of a traditional joint, however in compliant mechanisms we can model the deflections that occur from flexible members as torsional hinges. In addition, ‘living hinges’ are a type of compliant mechanism where creasing or folding makes a material behave like a hinge.

Rolling is another way to collapse materials. Architectural blueprints, scriptural scrolls, black-out window shades are all examples of collapsing by rolling.
The sliding principle of collapsibility is seen in antennae, tripods and extending ladders. The first two examples may also be considered in exemplifying the principle of nesting. Also included are *matrioshka* dolls and luggage sets.

Inflation is seen all around us in balloons, water wings, air mattresses and bounce houses.

### 6.1.2 Future Design of Collapsible Compliant Mechanisms

While the previous sections have provided an overview for the principles of collapsibility and ideas for application, there remains substantial opportunity for investigation in the areas of materials, manufacturing and assembly, and other types of mechanisms.

Further research should be conducted in materials to identify suitable joints specific to materials. The current joint options, including but not limited to the LET joint, have been designed assuming that the materials used would be isotropic, linear, and elastic. It is expected that composite materials could be used to bias bending and deflection.

In addition materials research manufacturing and assembly methods could be developed to allow “folding” to be more precise and to control mechanical properties local to the “fold.” Other methods relating to packaging in manufacturing process could benefit substantially from eliminating waste by incorporating materials into the folds and pleats of packaging.

### 6.1.3 Tessellations

It is advised that tessellations be investigated separately from general origami applications to mechanisms. The concept of repeated patterns maintained by minimal actuators shows promise for array applications. Research has been conducted examining the design and kinematics of basic tessellations as well as cylindrical shaped tessellations [59–62]. In addition basic research in the kinematics of folded sheets has been conducted and this is a possible area to expand [63].

### Future of Design Methods for Synthesis

In the previous chapter the use of grafting has been suggested as a two-dimensional analog for implementing the PRBM. Grafting has its limits in origami in that it only permits paper to be added. Other tools used in origami design to circumvent this limitation. These other methods are
expected to provide insight and possibly methods, for the design of mechanisms. One such tool is the concept of tiling.

A tile is defined as “a portion of a crease pattern, usually consisting of one or more axial polygons and decorated by circles and rivers, that can be assembled into crease patterns by matching circle and river boundaries [21].” In more plain terms a tile represents a basic component of a crease pattern and can be matched with other tiles to form a crease pattern as long as certain rules are followed in the matching process.

Tiles can be represented by stick figures. Sticks will the same length of flaps in the tile, but their orientation is not significant [21]. The lengths and connections matter. The use of tiling culminates in the circle/river/molecule method for designing uniaxial bases [21]. A stick figure is more than just a stick figure, it is essentially a graph, a tree graph to be specific. When working with uniaxial bases the tree graph can be interpreted as “the shadow cast by the base in a plane perpendicular to the layers of the base [21],” however this analogy does not hold when overlapping occurs.

It is expected that the use of tiling, circle/river/molecule methods and tree graphs will each generate new tools and even methods in the design of LEMs. These concepts should be explored and put into context with the use of cutting. Since we have come full circle to graph theory it is anticipated that future mechanism synthesis methods depend on a higher mathematical understanding of graph theory.
REFERENCES


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