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Matthew R. Jones
Brigham Young University - Provo, mrjones@byu.edu

Yukio Yamada
Mechanical Engineering Laboratory, AIST-MITI, yyamada@aist.go.jp

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Determination of the Asymmetry Parameter and Scattering Coefficient of Turbid Media from Spatially Resolved Reflectance Measurements

Matthew R. JONES[1] and Yukio YAMADA[2]

[1]*Department of Aerospace & Mechanical Engineering, University of Arizona, Tucson, Arizona 85721 USA,*
[2]*Biomechanics Division, Mechanical Engineering Laboratory, AIST-MITI, Namiki, 1-2, Tsukuba, Ibaraki, 305 Japan*

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We present a technique for determining the asymmetry parameter and scattering coefficient of turbid media from spatially resolved reflectance measurements. This technique will contribute to the development of medical applications in which it is necessary to predict the distribution and propagation of light in tissue. Based on Monte Carlo simulations, we derived correlations which relate the reduced scattering coefficient and the asymmetry parameter to the relative reflectance curve. Initial estimates of the optical properties are obtained from these correlations. Final values are obtained by adjusting the optical parameters and repeating the Monte Carlo simulations until the simulated reflectance pattern matches the measured reflectance pattern. Preliminary experimental results indicate that this technique can be used to determine the asymmetry parameter to within 10% and the reduced scattering coefficient to within 5%.

Key words: turbid media, optical properties, reflectance measurements, Monte Carlo

1. Introduction

Noninvasive determination of the scattering and absorption properties of turbid media has received considerable attention in the recent literature. The parameters of primary interest are the scattering coefficient, μ_s, the absorption coefficient, μ_a, and the asymmetry parameter, g. Interest in techniques for measuring these properties is due to emerging medical applications of light such as photodynamic therapy, laser surgery, and near infra-red imaging and spectroscopy. In each of these applications, it is necessary to know the optical properties of various types of biological tissue in order to predict how light will propagate and be distributed throughout tissues.

It appears that Langerholc[1)] was the first to suggest that reflection and transmission measurements could be used to determine the scattering parameter of biological tissues. More recently, Marquet *et al.*[2)] describe a method of measuring the reduced scattering coefficient, $\mu_s'=(1-g)\mu_s$, and μ_a using spatially resolved transmission measurements. Surveys[3,4)] of methods for measuring the optical properties of tissue indicate that most of the currently employed methods are *in vitro* techniques similar to those proposed by Langerholc[1)] and Marquet *et al.*[2)]

Due to the large uncertainties associated with using *in vitro* measurements *in vivo*, methods have been proposed which allow *in vivo* determination of tissue optical properties. Groenhuis *et al.*[5)] and Farrell *et al.*[6,7)] describe techniques for determining μ_s' and μ_a from spatially resolved reflectance measurements. The method proposed by Groenhuis *et al.*[5)] requires the use of absolute measurements, while the method proposed by Farrell *et al.*[6,7)] uses only the shape of the reflectance curve. Wang and Jacques[8)] recently proposed a simple method of determining μ_s' by measuring the shift in the center of the diffuse reflectance of a narrow laser beam with an oblique angle of incidence. Kienle *et al.*[9)] describe the use of a neural network to extract the optical properties from reflectance measurements obtained using a video reflectometer similar to the system used by Wang and Jacques.[8)] Bays *et al.*[10)] developed prototype probes for endoscopic spatially resolved reflectometry and are using these probes in a clinical setting. However, each of these methods is limited in that it is not possible to use them to determine both μ_s and g. This limitation arises because these methods are based on the diffusion approximation to the radiative transfer equation. Since μ_s and $(1-g)$ always appear as a product in the diffusion approximation, it is only possible to determine μ_s'. Rastegar *et al.*[11)] showed that the asymmetry parameter plays an important role in determining the ablation depth and temperature profile during the laser ablation of tissues, so a method of determining g is desirable.

In this paper we propose a method for measuring the asymmetry paper as well as the reduced scattering coefficient. The measurement of g is possible using highly resolved spatial reflectance measurements because most of the light detected near the incident beam has undergone relatively few scattering events. Therefore, these measurements still contain information regarding the direction of scatter. This can be seen by examining the relative reflectance curves in Fig. 1 which were obtained using Monte Carlo simulations. The relative reflectance is defined as the reflected power measured by a detector divided by the sum of the reflected power measured by all the detectors. At points far from the point of incidence, the relative reflectance curve depends only on μ_s'. Similarity relationships which show that the reflectance depends only on μ_s' at points sufficiently removed from the source are discussed by Wyman *et al.*[12)] However, note that near the point of incidence the relative reflectance depends on g as well as μ_s'. We will show that spatially resolved reflectance mea-

E-mail: jones@ame.arizona.edu

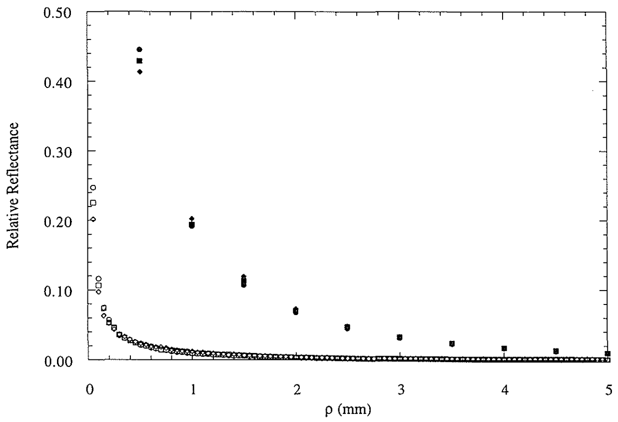

Fig. 1. Relative reflectance curves which illustrate the effect of the asymmetry parameter on the reflectance pattern. Reflectance measurements near the incident point are sensitive to g, and measurement far from the incident point are insensitive to g. Relative reflectance curves are shown for a fiber array composed of 50 μm fibers (open symbols) and for a fiber array composed of 500 μm fibers (filled symbols). ○, g=0.5 d=50 μm; □, g=0.7 d=50 μm; ◇, g=0.9 d=50 μm; ●, g=0.5 d=500 μm; ■, g=0.7 d=500 μm; ◆, g=0.9 d=500 μm.

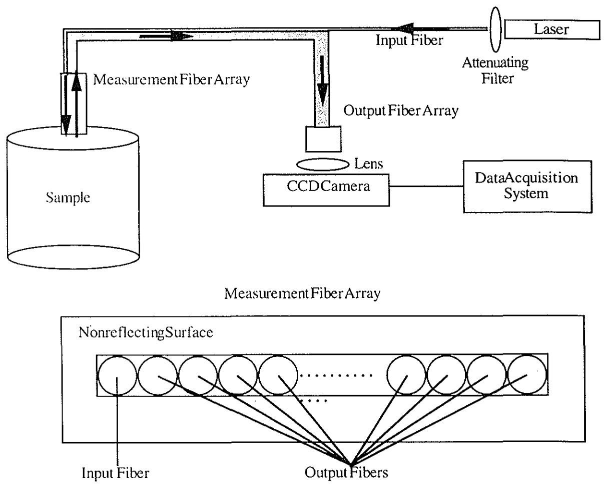

Fig. 2. An array of 25 optical fibers with a diameter of 500 μm is positioned above the sample. Light from a HeNe laser (λ=632.8 nm) is introduced into the sample through the fiber on the end of the array and reflected light is collected by the remaining fibers. After magnification, the intensity of the reflected light collected by each fiber is measured using a CCD camera.

surements near the point of incidence can be used to determine the asymmetry parameter.

2. Experimental Method

Reflectance curves were measured according to the procedure illustrated in Fig. 2. A linear array of 25 fiber optics (d=500 μm, NA=0.5) was positioned slightly above the turbid medium. The beam from a 1 mW Helium-Neon laser was introduced into the medium through the fiber located on the edge of the array, and the reflected light was transmitted to a CCD camera through the remaining 24 fibers. The CCD camera has a 14 bit dynamic range. A 12 cm focal length lens was used to magnify the image of the output fibers before measuring the reflected intensity transmitted by each fiber. As shown in Fig. 2, the surface of the detector array had a non-reflecting coating to prevent reflections back into the sample.

Errors due to differences in the sensitivity of the pixels in the CCD array were eliminated using the following equation

$$I_c=\frac{I-D}{U-D}, \tag{1}$$

where I_c is the corrected image, I is the raw image, D is a dark image, and U is the image of an uniformly illuminated field. A uniformly illuminated field was obtained using an integrating sphere. The reflectance curves were calculated by integrating over the face of each fiber in the corrected image. In order to correct for variations in the power input, relative reflectance curves were calculated by normalizing the reflectance curves by the total detected power.

The turbid media examined in this study consist of dilute suspensions of polystyrene spheres in water. The first sample was created using 0.65 μm spheres (g=0.87), and the second sample was created using 1.12 μm spheres (g=0.93). The scattering coefficient of the samples is directly proportional to the volume fraction of polystyrene spheres, so the value of μ_s can be controlled by adjusting the concentration of the spheres in the suspensions.[13] Both samples used in this study were diluted such that μ_s had a value of 9.15 mm^{-1}. We assumed that absorption is negligible and that the refractive index of the samples is equal to 1.33 which is the refractive index of water at 632.8 nm.

3. Monte Carlo Simulations

Monte Carlo simulations have been frequently used to study light propagation in biological tissues.[14–17] Monte Carlo simulations are based on the stochastic nature of the interaction of photons with attenuating media. In our implementation of the Monte Carlo method a photon bundle is tracked for a maximum time of flight of 3 ns or until it leaves the medium. We doubled the maximum time of flight for several sets of optical properties to ensure that this time limit did not affect the results of the simulations. We also assumed that the path lengths of the detected photons were short, so absorption does not affect the results of the simulations. Comparison of simulations in which the absorption properties of water were used with simulations in which absorption was neglected showed that absorption is negligible.

In our model of the incident beam, we assumed that the intensity was uniform over the face of the input fiber, and the initial directions of the photon bundles were uniformly distributed within the acceptance cone of the fiber.

The distance a photon bundle travels between interactions is given by

$$s=-\ln(R)/\mu_s, \tag{2}$$

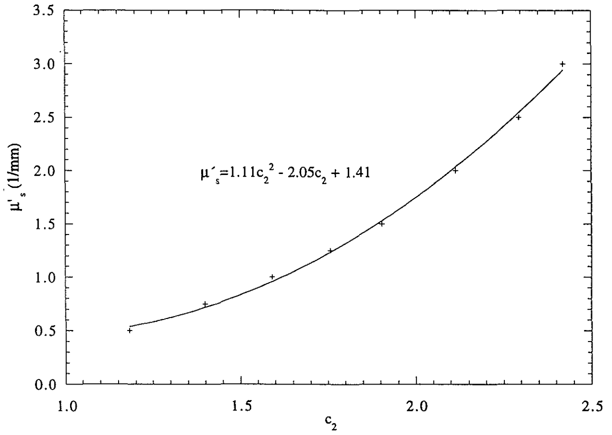

Fig. 3. There exists a quadratic relationship between μ_s' and c_2 for the fiber array composed of 500 μm fibers. A value for c_2 is obtained by fitting Eq. (4) to the measured reflectance pattern, and Eq. (5) is then used to obtain an initial estimate for μ_s'.

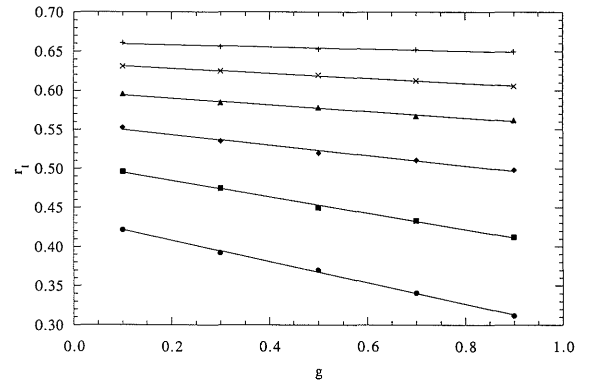

Fig. 4. For a given μ_s', there exists a linear relationship between r_1, which is the reflectance measured by the output fiber nearest the input fiber and the asymmetry parameter, g. ●, $\mu_s'=0.5\,\text{mm}^{-1}$, ◆, $\mu_s'=1.5\,\text{mm}^{-1}$; ×, $\mu_s'=2.5\,\text{mm}^{-1}$; ■, $\mu_s'=1.0\,\text{mm}^{-1}$; ▲, $\mu_s'=2.0\,\text{mm}^{-1}$; +, $\mu_s'=3.0\,\text{mm}^{-1}$.

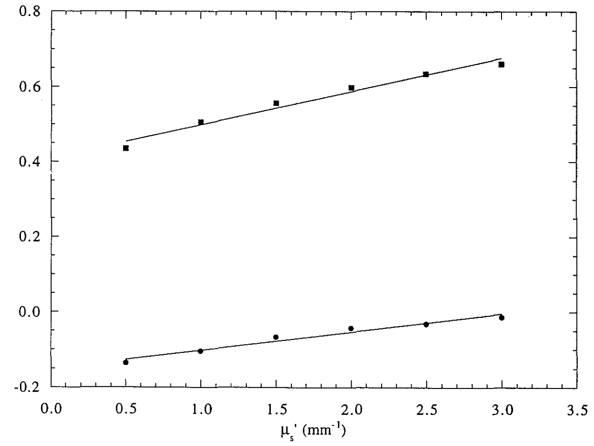

Fig. 5. The slopes, m, and the intercepts, b, of the lines shown in Fig. 4 depended linearly on μ_s'. The estimate of μ_s' obtained from Eq. (5) is used in Eq. (6) to obtain values for m and b. These values are then used in Eq. (7) to obtain an estimate for g. ●, $m=0.049\,\mu_s'-0.15$; ■, $b=0.089\,\mu_s'+0.41$.

where R is a uniformly distributed pseudorandom number between 0 and 1. The scattering direction is determined by assuming that the azimuthal scattering is isotropic and that the Henyey-Greenstein phase function accurately represents the polar scattering. The Henyey-Greenstein phase function is a simple approximation and is widely used to model the propagation of light in tissue.[11,15,17] The direction of scatter is given by

$$\theta=\cos^{-1}\left\{\frac{1}{2g}\left[1+g^2-\left(\frac{1-g^2}{1+g\tilde{R}}\right)^2\right]\right\}, \tag{3}$$

$$\Phi=\pi\tilde{R}$$

where $\tilde{R}$ is a uniformly distributed pseudorandom number between -1 and 1.

When a photon bundle reaches the edge of the sample, Fresnel's relation[18] is used to determine whether or not the bundle is reflected. If the photon bundle is reflected, we continue to track the bundle until the maximum time limit is reached or the bundle emerges from the sample. If the photon bundle is not reflected, we determine whether the line of flight of the photon bundle would lie within the acceptance cone of an output fiber. If this is the case, the number of photon bundles detected by the appropriate output fiber is increased by one. Otherwise, the photon bundle is considered to be lost. Due to symmetry, we assumed that all the photon bundles passing through the annulus that would be swept out by a output fiber if it was rotated about the input fiber are incident on the output fiber. At the completion of the simulation, we multiplied the number of photon bundles detected by each output fiber by the ratio of the fiber area to the area of the corresponding annulus to account for the actual area covered by the output fibers.

We found that two hundred thousand photon bundles were sufficient to calculate the reflectance curve using this method. When run on Digital Alpha Station 250 4/266, many of the simulations required less than 20 min of CPU time. Simulations in which the transport mean free path is relatively small (large μ_s') or the penetration depth is relatively large (large g) require longer CPU times. The median CPU time was 97 min. The Monte Carlo simulations generated using this algorithm agree well with the measured reflectance curves and with the reflectance model based on diffusion theory that was derived by Farrell *et al.*[6,7]

4. Determination of the Reduced Scattering Coefficient and the Asymmetry Parameter

Based on the diffusion approximation it can be shown that the radial dependence of the reflectance curve of a non-absorbing media can be approximated as[6,7,19]

$$r(\rho)=c_1/\rho^{c_2}\,. \tag{4}$$

By fitting the results of Monte Carlo simulations for media with $0.5\,\text{mm}^{-1}\leq\mu_s'\leq3.0\,\text{mm}^{-1}$ to Eq. (4), we determined that $c_1\approx0.2$ for all μ_s' and that there exists an approximately quadratic relationship between μ_s' and c_2 (see Fig.

Table 1. Retrieved optical parameters.

Parameter	μ_s' (mm^{-1})		g	
	Retrieved	Theoretical	Retrieved	Theoretical
0.65 μm polystyrene spheres	1.15	1.20	0.90	0.87
1.12 μm polystyrene spheres	0.64	0.64	0.84	0.93

3). Therefore, we found that an initial approximation for μ_s' could be determined from the value of c_2 obtained by fitting Eq. (4) to the measured reflectance curve. For the fiber array used in this study, we obtained the following correlation for μ_s'.

$$\mu_s'=1.11c_2^2-2.05c_2+1.41\,. \tag{5}$$

Monte Carlo simulations were also conducted for media with $0.1\leq g\leq 0.9$ and $0.5\,\text{mm}^{-1}\leq \mu_s'\leq 3.0\,\text{mm}^{-1}$. As illustrated in Fig. 4, these simulations show that for a given μ_s' the reflectance measured by the output fiber nearest the input fiber, r_1, is a linear function of g. If the slope, m, and intercept, b, of the lines shown in Fig. 4 are plotted as a function of μ_s', we observe that both m and b depend nearly linearly on μ_s' (see Fig. 5). For the fiber array used in this study, we obtained the following correlations.

$$\begin{aligned} m&=0.049\,\mu_s'-0.15 \\ b&=0.089\,\mu_s'+0.41 \end{aligned}\,. \tag{6}$$

Therefore, we can use the estimate of μ_s' obtained from Eq. (5) to calculate the appropriate values of m and b. An estimate for g can then be calculated from

$$g=\frac{r_1-b}{m}\,, \tag{7}$$

where r_1 is reflectance measurement obtained nearest the input fiber.

The proposed procedure for determining the optical parameters of a turbid medium is as follows. We first obtain a value for c_2 by fitting Eq. (4) to the measured relative reflectance curve. We then use Eq. (5) to estimate μ_s'. The initial estimate of μ_s' is used in Eqs. (6) to determine m and b. We then use Eq. (7) to calculate an initial estimate for g. The initial estimates of μ_s' and g are then improved by adjusting these parameters until the rms error between the measured and simulated reflectance curves is a minimum. These adjustments were performed by repeated use of the Monte Carlo algorithm, and the minimum rms error was obtained in less than five iterations. We found that it is best to adjust the value of μ_s' until the rms difference between the measurements other than r_1 and the simulated reflectance curve is minimized. We then adjust the value of g until a minimum in the total rms error is obtained. In general, the magnitude of the slope of the reflectance curve can be increased by increasing the value of μ_s', and the value of r_1 decreases as g is increased and μ_s' is held constant. Table 1 summarizes the results of this procedure for the samples that were described previously. In these preliminary experiments, the

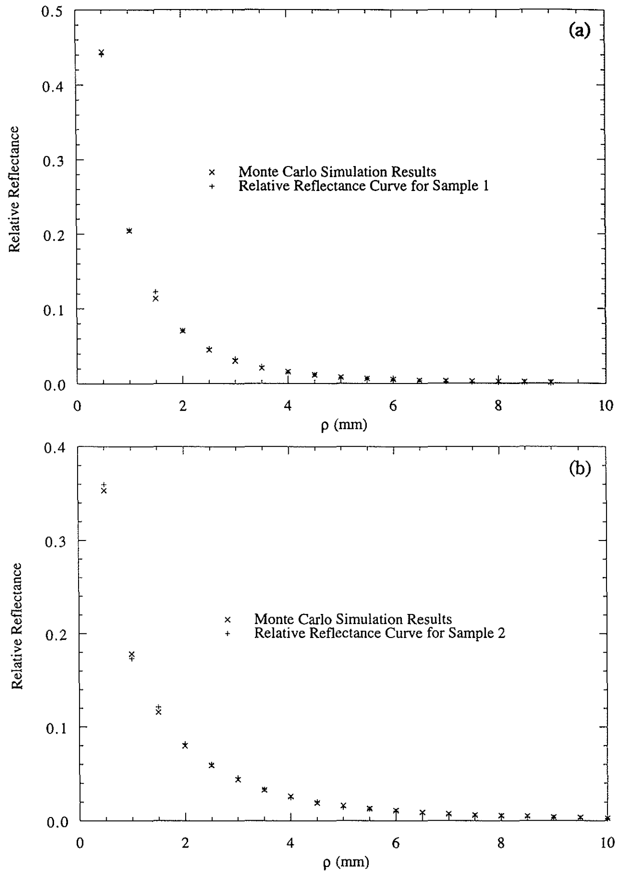

Fig. 6. (a) Comparison of the measured and simulated relative reflectance curves for Sample 1. The optical properties used in the Monte Carlo simulations are $\mu_s'=1.15$ and $g=0.90$. (b) Comparison of the measured and simulated relative reflectance curves for Sample 2. The optical properties used in the Monte Carlo simulations are $\mu_s'=0.64$ and $g=0.84$.

reduced scattering coefficient was measured with an accuracy of 5% and the asymmetry parameter was measured with an accuracy of 10%. The theoretical optical parameters were calculated using the Mie code written by Bohren and Huffman.[20] The measured reflectance curves are compared with the simulated reflectance curves in Figs. 6(a) and 6(b). These figures show that the Monte Carlo simulations are in very good agreement with the measured relative reflectance curves.

5. Discussion of Results

The measured values for the scattering parameter and the estimates based on Mie theory are in excellent agreement. The slight discrepancies between the measured and theoretical values are probably due to random experimental error and to uncertainties in the actual size and optical properties of the polystyrene spheres that were used to perform the Mie calculations.

Since information regarding the relationship between the incident direction and the direction of scatter is lost after each scattering event, we cannot expect to measure g if the photons are scattered too many times before being detected. Therefore, we should not expect that this technique will yield accurate results if the number of transport mean free paths between the incident point and the point at which r_1 is measured is too large. From Eq. (7), it is clear that errors in g will be amplified as the magnitude of m decreases. Figure 4 shows that for the fiber array used in this study, m decreases as μ_s' increases, so it is clear that the accuracy of the proposed method for measuring g decreases as the value of μ_s' increases.

Monte Carlo simulations were also performed for a fiber array consisting of 50 μm fibers, and the results are shown in Fig. 1. These results show that the sensitivity of the measurements to variations in g increases as the separation between the source and the initial detector is decreased. Therefore, the ability to accurately retrieve g would be enhanced by using smaller fibers.

It should also be noted that the accuracy of the value obtained for the asymmetry parameter depends on the validity of the assumed form of the phase function. In an earlier paper,[21] we indicated that there is a region near the source where the reflectance pattern depends only on μ_s' and the first moment of the phase function (g). However, subsequent studies showed that the reflectance pattern near the source depends on μ_s' and on the exact phase function. Therefore, the accuracy of the asymmetry parameter measurements depends on the extent to which the Henyey-Greenstein phase function accurately approximates the actual phase function. It may have been possible to measure g of the samples used in this study with greater accuracy if the Mie phase functions had been used rather than the Henyey-Greenstein phase function, but our intent is to develop a technique useful for determining the optical parameters of biological tissues. The Henyey-Greenstein approximation is widely accepted as an accurate model of the phase function of biological tissues.[15,17]

6. Conclusions

A technique for determining the reduced scattering coefficient and the asymmetry parameter from highly resolved spatial reflectance measurements has been described. Preliminary experimental results indicate that the reduced scattering coefficient can be determined to within 5% and the asymmetry parameter can be determined to within 10%. Since it is generally accepted that the asymmetry parameter of various biological tissues is in the range of 0.7 to nearly 1.0,[15] measurements of g with an uncertainty of 10% are not particularly valuable. However, it has been shown that the distance between the incident point and the point at which r_1 is measured is a critical parameter, and it appears that the performance of the proposed technique can be improved by decreasing this distance. In addition, if the lateral extent of the probe and the sample are sufficiently large, it may be possible to obtain information regarding μ_a as well as μ_s and g.[9,10] We are currently investigating the possibility that all three optical parameters can be determined based on the relative reflectance curve. Use of a probe similar to the prototype endoscopic probes described by Bays *et al.*[10] in which a single movable optical fiber is used for detection could provide the flexibility necessary to obtain measurements near enough to the incident point to determine g and far enough from the incident point to determine μ_s' and μ_a.

References

1) J. Langerholc: Appl. Opt. **21** (1982) 1593.
2) P. Marquet, F. Bevilacqua, C. Depeursinge and E.B. de Haller: Opt. Eng. **34** (1995) 2055.
3) W. Cheong: Ph.D. Dissertation, U. Texas at Austin (1990).
4) M. Firbank: Ph.D. Dissertation, University College London (1994).
5) R.A.J. Groenhuis, H.A. Ferwerda and J.J. Ten Bosch: Appl. Opt. **22** (1983) 2456.
6) T.J. Farrell, M.S. Patterson and B. Wilson: Med. Phys. **19** (1992) 879.
7) T.J. Farrell, B.C. Wilson and M.S. Patterson: Phys. Med. Biol. **37** (1992) 2281.
8) L. Wang and S.L. Jacques: Appl. Opt. **34** (1995) 2362.
9) A. Kienle, L. Lilge, M.S. Patterson, R. Hibst, R. Steiner and B.C. Wilson: Appl. Opt. **35**, (1996) 2304.
10) R. Bays, B. Wagnières, D. Robert, D. Braichotte, J. Savary, P. Monnier and H. van den Bergh: Appl. Opt. **35** (1996) 1756.
11) S. Rastegar, M. Motamedi, A.J. Welch and L.J. Hayes: IEEE Trans. Biomed. Eng. **36** (1989) 1180.
12) D.R. Wyman, M.S. Patterson and B.C. Wilson: J. Comput. Phys. **81** (1989) 137.
13) S. Proskurin, Y. Yamada and Y. Takahashi: Opt. Rev. **2** (1995) 292.
14) B.C. Wilson and G. Adam: Med. Phys. **10** (1983) 824.
15) S.T. Flock, M.S. Patterson, B.C. Wilson and D.R. Wyman: IEEE Trans. Biomed. Eng. **36** (1989) 1162.
16) Y. Hasegawa, Y. Yamada, M. Tamura and Y. Nomura: Appl. Opt. **30** (1991) 4515.
17) L.H. Wang, S.L. Jacques and L.Q. Zheng: Comput. Methods Programs Biomed. **47** (1995) 131.
18) Y. Takahashi, Y. Yamada and Y. Hasegawa: Proceedings of the ASME/JSME Thermal Engineering Symposium, Eds. L.S. Fletcher and T. Aihara, Book No. H0933D (1995).
19) M.S. Patterson, B. Chance and B.C. Wilson: Appl. Opt. **28** (1989) 2331.
20) C.F. Bohren and D.R. Huffman: *Absorption and Scattering of Light by Small Particles* (John Wiley & Sons, New York, 1983) p. 477.
21) M.R. Jones and Y. Yamada: *Advances in Optical Imaging and Photon Migration*, eds. N.R. Alfano and J.G. Fujimoto (Optical Society of America, Washington, DC, 1996) Trends in Optics and Photonics Series Vol. II. p. 379