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Combined Trajectory, Propulsion and Battery Mass Optimization for Solar-Regenerative High-Altitude Long Endurance Unmanned Aircraft

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Combined optimization of propulsion system design, flight trajectory planning and battery mass optimization for solar-regenerative high-altitude long endurance (SRHALE) aircraft through a sequential iterative approach yields an increase of 20.2% in the end-of-day energy available on the winter solstice at 35°N latitude, resulting in an increase in flight time of 2.36 hours. The optimized flight path is obtained by using nonlinear model predictive control to solve flight and energy system dynamics over a 24 hour period with a 15 second time resolution. The optimization objective is to maximize the total energy in the system while flying a station-keeping mission, staying within a 3 km radius and above 60,000 ft. The propulsion system design optimization minimizes the total energy required to fly the optimal path. It uses a combination of blade element momentum theory, blade composite structures, empirical motor and motor controller mass data, as well as a first order motor performance model. The battery optimization seeks to optimally size the battery for a circular orbit. Fixed point iteration between these optimization frameworks yields a flight path and propulsion system that slightly decreases solar capture, but significantly decreases power expended. Fully coupling the trajectory and design optimizations with this level of accuracy is infeasible with current computing resources. These efforts show the benefits of combining design and trajectory optimization to enable the feasibility of SRHALE flight.

Nomenclature

\[ \bar{N} \] Wing surface normal
\[ SN \] Sun direction vector
\[ \gamma \] Flight path angle (rad)
\[ \mu_{\text{solar}} \] Obliquity factor
\[ \phi \] Bank angle (rad)
\[ \phi_s \] Solar azimuth (rad)
\[ \psi \] Heading Angle (rad)
\[ \rho \] Air density (kg/m\(^3\))
\[ \theta \] Pitch angle (rad)
\[ \theta_s \] Solar zenith (rad)
\[ AR \] Aspect ratio
\[ C_D \] Drag coefficient
\[ C_L \] Lift coefficient
\[ D \] Drag (N)
\[ D_{\text{prop}} \] Propeller diameter (m)
\[ E_{\text{battery}} \] Energy stored in battery (kWh)
\[ \epsilon_{\text{inv}} \] Inviscid span efficiency
\[ E_{\text{potential}} \] Potential energy storage (kWh)
\[ g \] Gravity (m/s\(^2\))
\[ h \] Altitude (m)
\[ I_0 \] Motor no load current (amps)
\[ K_v \] Motor rotational constant (rad/s)
\[ L \] Lift (N)
\[ m \] Aircraft mass (kg)
\[ m_{\text{motor}} \] Motor mass (kg)
\[ P_{\text{battery}} \] Power to battery (W)
\[ P_{\text{payload}} \] Power required for payload (W)
\[ P_{\text{total}} \] Total power balance (W)
\[ q \] Dynamic pressure (Pa)
\[ r \] Propeller radial position (m)

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American Institute of Aeronautics and Astronautics
### I. Introduction

Satellites and manned aircraft are currently heavily used resources in areas such as public communication, surveillance, agriculture, weather, and environmental monitoring. Although effective, they come with their share of concerns. One of the primary issues is cost. Both satellites and manned aircraft systems cost millions of dollars to manufacture and maintain. There are also environmental concerns that accompany the fossil-fuel-powered propulsion systems and satellites’ contributions to space debris in orbit. Solar Regenerative High Altitude Long Endurance (SRHALE) aircraft have the potential to provide a more cost effective, energy efficient, and environmentally friendly alternative to space satellites.

Using aircraft as pseudo-satellites has been considered a feasible possibility only recently. However, the technology leading up to this consideration has long been in development. Electric, battery powered aircraft have been experimented with since the 1800s, and the addition of solar panels to electric aircraft has been studied since the 1970s. Experimentation with solar-electric aircraft has significantly progressed since the 1800s. One notable SRHALE aircraft is NASA’s Helios Prototype, a remotely piloted flying wing, which achieved a world record altitude of 96,843 feet in 2001, but experienced catastrophic failure over the Pacific Ocean in 2003. Other recent, notable SRHALE aircraft ventures include Qinetiq’s Zephyr (which holds the current heavier than air aircraft endurance record), Facebook’s Aquila, Aurora’s Odysseus, and Boeing’s SolarEagle.

Topics relevant to SRHALE optimization and design are present in the published literature, and show many unique ways to increase the feasibility of SRHALE flight. These include structures, controls, multidisciplinary design optimization, and path optimization.

Unique structural considerations have been found to be of importance for HALE aircraft. McDonnell, Cesnik, Palacios, and Reichenbach review conventional structural design procedures and their application to highly flexible aircraft generally proposed for HALE applications. Hesse and Palacios also propose reduced-order aeroelastic models for flexible aircraft.

Special considerations in HALE aircraft controls have also been studied. Kubica and Livet developed flight control laws for flexible aircraft. Furthermore, Caverly et. al. as well as Haghighat, Liu, and Martins present methods for gust load alleviation for flexible HALE aircraft. The questions of inherent feasibility and multidisciplinary interactions of SRHALE aircraft have also been considered. Brandt and Gilliam exhibit a basic method for the analysis of solar powered aircraft. Baldock and Mokhtarzadeh-Delghan presented a methodology for determining the feasibility of SRHALE aircraft and suggested the use of a braced-wing concept. Morrisey performed Multidisciplinary Design Optimization (MDO) to investigate a pinned wing SRHALE concept. Ozoroski, Nickol, and Guynn have created an analysis tool for station keeping considerations for both SRHALE airplanes and airships.

A possible, though comparatively less-explored region of SRHALE technology is in trajectory optimization. Martin showed that, in comparison to cases without trajectory optimization, solar aircraft can increase their energy capture and efficiency of energy use. To make use of this idea, in this paper the aircraft path is optimized dynamically at a fine time resolution, balancing competing objectives to fly efficiently and increase solar energy capture in order to maximizing the total energy in the system. It also stores potential energy by climbing when the battery is full. The net result is to increase the state of charge at the end of the day.

Despite the presence of quality research and development in SRHALE technology, it has been found that there is room for increased detail in a highly multi-disciplinary, system-level design optimization. Published research in the areas of SRHALE aircraft have generally included a small subset of the many models that go into a complete aircraft. Furthermore, most tools developed thus far have been primarily for analysis, and have not been applied to optimization-based design. Some of the published research has been found to be more complete in the inclusion of a large number of important models, as well as optimization considerations, but none capture the design space between dynamics and design as proposed here. In this research, a unique contribution to the already present literature is made by combining trajectory and propulsion design for SRHALE aircraft.

The importance of this path and design optimization is highlighted by the computational expense of
combining the two. A full day of dynamic optimization with a simplified aircraft model at a 15 second time step requires 5,760 successful horizon optimizations. Each horizon requires on average 25 to 100 iterations to converge, and each iteration makes multiple function calls to evaluate the states. If the full aircraft model (including structures) and mid-fidelity aerodynamics (including flexible wing stability constraints) were to be implemented with a 5 second function evaluation time, the total optimization would take 800 hours (33.3 days). This makes the unlikely assumptions that the optimization problem converges in the same amount of iterations and includes analytic derivatives. Parallelization is extremely limited due the inherent sequential progression of a dynamic system, and would only be able to affect the time of each individual iteration. Efforts to parallelize these iterations have failed to meaningfully improve the convergence speeds, with the slight reduction in an already fast iteration time failing to offset the expense of increased overhead.

In this paper, it is shown how an extremely simple point-based iteration method can significantly increase the feasibility of an SRHALE aircraft design at a fraction of the computational cost. By extracting the simplified (or condensed) physics of the propulsion system in the form of continuous splines, the simple dynamic aircraft performance and propulsion models can be evaluated several orders of magnitude faster, which shrinks the total optimization time-frame from months to hours. First, the models will be described, along with the assumptions and limitations that accompany them. These include models of aircraft dynamics, aircraft drag and lift, motor efficiency, propeller efficiency, and solar panel efficiency. Next, the optimization problem and setup for the combined iterative approach will be explained, which is comprised an iterative approach between the three subsystems of propulsion, path, and battery mass optimization. Finally, the results and conclusions will be presented, which show significant increases in HALE mission performance.

II. Model Description

The models used in this work were chosen in order to achieve sufficient accuracy while remaining computationally tractable. The design of the aircraft was kept largely constant, save for changes to the propulsion system and battery mass. Aircraft structural constraints were assumed to be satisfied despite slight changes in the overall aircraft mass. In addition, this study was performed in the absence of wind.

A. Aircraft Dynamic Model

The aircraft dynamic model is used in the path optimization, which solves a nonlinear programming problem with inequality constraints. It describes the fundamental physics of the aircraft flight, and must be solved simultaneously across a large time horizon.

1. Aircraft Parameters and Mission Conditions

The aircraft definition follows the same approach taken in our previous work, where the SRHALE unmanned aircraft is modeled as a large flying wing similar to Facebook’s Aquila. The parameters are reproduced here in Table 1.

Similarly, the flight mission conditions are specified to mimic an SRHALE aircraft acting as a fixed node in a communications network. The flight altitude is constrained to be above U.S. regulated airspace (18,288 m, or 60,000 ft), and the flight radius to be within 3 km from a central point. The time and location are chosen to represent a particularly demanding case within the range of applicable SRHALE aircraft missions: the 2017 winter solstice (December 21), the darkest night of the year, and 35°N latitude, below which a large portion of the world’s population live. Albuquerque, New Mexico lies at this latitude, and is used as the flight location for this work (35.0°N, 106.6°W).

2. Aircraft Dynamics

The flight dynamics model for the SRHALE system is adapted from the six degree-of-freedom model developed by Beard and McLain. The aerodynamic forces are computed as follows:

\[ q = \frac{1}{2} \rho V^2 \]  
\[ L = qSC_L \]  
\[ D = qSC_D \]
Table 1: Parameters for the SRHALE aircraft.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass</td>
<td>$m$</td>
<td>339.6 kg</td>
</tr>
<tr>
<td>Battery mass</td>
<td>$m_{\text{battery}}$</td>
<td>126.6 kg</td>
</tr>
<tr>
<td>Payload mass</td>
<td>$m_{\text{payload}}$</td>
<td>25 kg</td>
</tr>
<tr>
<td>Battery specific energy</td>
<td>$U_{\text{batt}}$</td>
<td>$350 \frac{W\cdot h}{kg}$</td>
</tr>
<tr>
<td>Maximum flight radius</td>
<td>$R_{\text{max}}$</td>
<td>3 km</td>
</tr>
<tr>
<td>Sweep angle</td>
<td>$\Lambda$</td>
<td>$20^\circ$</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>$AR$</td>
<td>30</td>
</tr>
<tr>
<td>Payload power</td>
<td>$P_{\text{payload}}$</td>
<td>250 W</td>
</tr>
<tr>
<td>Wing planform area</td>
<td>$A$</td>
<td>$60 \ m^2$</td>
</tr>
<tr>
<td>Solar panel area</td>
<td>$S$</td>
<td>$60 \ m^2$</td>
</tr>
<tr>
<td>Wingspan</td>
<td>$b$</td>
<td>42 m</td>
</tr>
<tr>
<td>Chord</td>
<td>$c$</td>
<td>1.4 m</td>
</tr>
<tr>
<td>Inviscid span efficiency</td>
<td>$e_s$</td>
<td>0.98</td>
</tr>
<tr>
<td>Maximum battery energy</td>
<td>$E_{\text{batt}}_{\text{max}}$</td>
<td>59.5 kWh</td>
</tr>
<tr>
<td>Minimum altitude</td>
<td>$h_{\text{min}}$</td>
<td>60,000 ft</td>
</tr>
<tr>
<td>Maximum altitude</td>
<td>$h_{\text{max}}$</td>
<td>90,000 ft</td>
</tr>
<tr>
<td>Minimum $C_L$</td>
<td>$C_{L_{\text{min}}}$</td>
<td>0</td>
</tr>
<tr>
<td>Maximum $C_L$</td>
<td>$C_{L_{\text{max}}}$</td>
<td>1.1</td>
</tr>
<tr>
<td>Minimum bank angle</td>
<td>$\phi_{\text{min}}$</td>
<td>$-5^\circ$</td>
</tr>
<tr>
<td>Maximum bank angle</td>
<td>$\phi_{\text{max}}$</td>
<td>$5^\circ$</td>
</tr>
<tr>
<td>Minimum flight path angle</td>
<td>$\gamma_{\text{min}}$</td>
<td>$-5^\circ$</td>
</tr>
<tr>
<td>Maximum flight path angle</td>
<td>$\gamma_{\text{max}}$</td>
<td>$5^\circ$</td>
</tr>
</tbody>
</table>

where $q$ is the dynamic pressure, $\rho$ is the air density, $V$ is the aircraft velocity, and $S$ is the surface area of the wing. The variables $L$ and $D$ represent the lift and drag forces, while $C_L$ and $C_D$ are the wing lift and drag coefficients, respectively.

The aircraft dynamics are modeled using a point-mass model which describes the aircraft behavior in response to the inputs of thrust, angle of attack, and bank angle. The equations of motion for flight in the absence of wind are given as follows:

\begin{align*}
\dot{x} &= V \cos \psi \cos \gamma \quad (4) \\
\dot{y} &= V \sin \psi \cos \gamma \quad (5) \\
\dot{h} &= V \sin \gamma \quad (6) \\
\dot{V} &= \frac{F_{\text{thrust}}}{m} - \frac{D}{m} - g \sin \gamma \quad (7) \\
\dot{\psi} &= \frac{L}{mV \cos \gamma} \quad (8) \\
\dot{\gamma} &= \frac{L}{mV \sin \phi} - \frac{g V}{V \cos \gamma} \quad (9)
\end{align*}

where the states $x$, $y$, and $h$ correspond to the aircraft north, east, and altitude positions in the inertial reference frame. The variable $F_{\text{thrust}}$ represents thrust, $\phi$ is the aircraft bank angle, $\gamma$ is the flight path angle, $\psi$ is the heading angle, $m$ is the aircraft mass, and $g$ is the force of gravity.

The aircraft polars are determined using the approach described in our previous work, however an older version of the aircraft model is used in this paper. The base airfoil used is the SG6043, and XFLR5 and XFOIL are used to obtain both the parasitic drag polar and the lift curve. These data are then fit with nonlinear and linear equations, respectively, and the drag coefficient $C_D$ is computed as follows:
Here, $C_L$ is the lift coefficient, $\alpha$ is the angle of attack, $C_{Dp}$ is the parasitic drag coefficient, $AR$ is the aspect ratio, and $e_{inv}$ is the inviscid span efficiency. The coefficients used in the parasitic drag fit are listed in Table 2.

Table 2: Coefficient values for parasitic drag surface fit.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>9.68780E-02</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-1.19143E-02</td>
</tr>
<tr>
<td>$c_3$</td>
<td>1.46589E-07</td>
</tr>
<tr>
<td>$c_4$</td>
<td>-7.49336E-09</td>
</tr>
<tr>
<td>$c_5$</td>
<td>-1.64442E-01</td>
</tr>
<tr>
<td>$c_6$</td>
<td>3.97917E-05</td>
</tr>
<tr>
<td>$c_7$</td>
<td>-4.18256E-06</td>
</tr>
</tbody>
</table>

3. Solar Flux

Solar flux values are computed using the Simple Model of the Atmospheric Radiative Transfer of Sunshine (SMARTS).\textsuperscript{22} This model is used by researchers in many different fields to predict clear-sky spectral irradiance as a function of latitude, longitude, altitude, time, and environmental factors. To increase computation speed, solar flux values for the entire day were precomputed at 1-minute intervals, and linear interpolation was used to sample the data smoothly at smaller time intervals during the dynamic path optimization.

In order to correct the raw solar flux values for the changing aircraft orientation during flight, the obliquity factor is used as described in our previous work.\textsuperscript{20} The equations are reproduced here for completeness:

$$SN = \begin{bmatrix} \cos(\phi_s) \sin(\theta_s) \\ \sin(\phi_s) \sin(\theta_s) \\ \cos(\theta_s) \end{bmatrix}$$

$$\bar{N} = \begin{bmatrix} \cos(-\phi) \sin(-\theta) \sin(\psi) - \cos(\psi) \sin(-\phi) \\ \cos(-\phi) \cos(\psi) \sin(-\theta) + \sin(-\phi) \sin(\psi) \\ \cos(-\phi) \cos(-\theta) \end{bmatrix}$$

$$\mu_{solar} = \frac{SN}{||SN||} \cdot \frac{\bar{N}}{||\bar{N}||}$$

Here, $SN$ is the sun direction vector, which is calculated from the solar azimuth ($\phi_s$) and zenith ($\theta_s$); $\bar{N}$ is the surface normal of the wing, which is calculated using the aircraft orientation, including the aircraft pitch angle, which is defined as $\theta = \alpha + \gamma$; and $\mu_{solar}$ is the obliquity factor, which is the product of the normalized sun vector and the normalized surface normal.
4. **Air Properties**

Estimates for air temperature and pressure are obtained from the NASA 1976 Standard Atmosphere Model. The ideal gas law is used to calculate air density: \( \rho = \frac{p}{R T} \) where \( R \) is the specific gas constant, and \( p \) and \( T \) are, pressure and temperature respectively, at design altitude. Sutherland’s law is used to calculate air viscosity:

\[
\mu_{\text{air}} = \mu_{\text{sl}} \left( \frac{T_{\text{air}}}{T_{\text{sl}}} \right)^{3/2} \frac{T_{\text{sl}} + Sc}{T_{\text{air}} + Sc}
\]

where the subscript “sl” indicates values at sea level and \( Sc \) is the Sutherland temperature (110.4 Kelvin for air).

5. **Energy Balance**

The system energy balance, as in our previous work, is computed as follows:

\[ P_{\text{total}} = P_{\text{solar}} + P_{\text{battery}} - P_{\text{propulsion}} - P_{\text{payload}} \]  

where \( P_{\text{solar}} \) is the power entering the system through the solar panels, \( P_{\text{battery}} \) is the power leaving (+) or entering (−) the battery, and \( P_{\text{propulsion}} \) and \( P_{\text{payload}} \) are the power requirements of the propulsion system and payload, respectively. The balance of power in the system, \( P_{\text{total}} \), must satisfy

\[ P_{\text{total}} \geq 0 \]  

for feasible flight. This constraint is posed as an inequality to give the optimizer more freedom, but the objective function provides incentives to minimize this value, with the end result that there is essentially no un-utilized power, and the constraint is satisfied with equality.

Electrical energy storage in the battery is modeled as follows:

\[ \dot{E}_{\text{battery}} = P_{\text{battery}} \]  

Due to the slow charge and discharge that occurs as the battery cycles over an entire day, the battery is assumed to be perfectly efficient, and no constraints are imposed on the charge and discharge rates.

Potential energy storage is modeled with the following relation:

\[ E_{\text{potential}} = mg(h - h_{0}) \]  

where \( h_{0} \) is the minimum height (60,000 ft). Combining the electrical and potential energy storage gives the total energy:

\[ E_{\text{total}} = E_{\text{battery}} + E_{\text{potential}} \]  

This is used in the objective function for the dynamic path optimization, as will be described later on.

B. **Propeller Performance**

1. **Blade Element Momentum Method**

We use CCBlade, an open source blade element momentum (BEM) code formulated to give guaranteed convergence and in turn allow for a continuously differentiable output. It includes a non-normal inflow correction which allows us to mount the props in line with the wing and include the angle of attack (AOA) of the wing as the inflow angle to the prop. For atmospheric properties we use NASA’s 1976 Standard Atmosphere Model. This BEM formulation uses 2D airfoil data to calculate the induction factors for each annular disk of the propeller including Prandtl hub and tip loss correction factors.
2. Airfoil Pre-computation

Blade element momentum theory is dependent on accurate airfoil data for propeller performance prediction. To calculate the airfoil data, we use XFOIL\textsuperscript{c} to run the Eppler 212 airfoil for 13 Reynolds numbers ranging from $5 \times 10^4$ to $1 \times 10^8$, and fifty-one angles of attack ranging between negative and positive stall (approximately negative 10° to positive 20°, depending on the Reynolds and Mach number).

We use Airfoilpreppy\textsuperscript{d} to model the stall delay experienced by local sections on rotating blades. This code applies the rotational correction on lift by Du et al.\textsuperscript{28} and drag by Eggers et al.\textsuperscript{29} as well as extrapolation to high angles of attack by Viterna et al.\textsuperscript{30}

3. Propeller Performance Comparison

To validate that the BEM code with XFOIL airfoil data calculation was consistent with Epema’s published experimental cases,\textsuperscript{31} we compared his published experimental results with our computational model of his setup, as described in our previous work.\textsuperscript{32} The results for a constant RPM and varying freestream as seen in Fig. 1. Since the 2D airfoil data includes stall, the modeled effects of stall can be seen on the 3D propeller performance in the very low advance ratios. The more linear regions at the lowest advance ratios are comprised of post-stall angles of attack calculated by the Viterna extrapolation.

![Figure 1: Comparison of propeller efficiency with data collected by Epema and the BEM code using XFOIL airfoil data. A maximum error of 5% can be seen in the normal regions of operation which are advance ratios below the region of steep drop-off.](https://example.com/figure1.png)

4. Propeller Structures

We model propeller structures with a simple flap-wise cumulative bending stress calculation and unidirectional carbon fiber failure stress of $1.5 \times 10^9$ Pa with a conservative safety factor of 10. The mass of the propellers is estimated as a solid propeller with a density of 1440 kg/m\textsuperscript{3} for carbon fiber prepreg, similar to propellers manufactured by Warp Drive Propellers\textsuperscript{e}.

C. Power System

1. Solar Panel Model

In order to obtain the actual power output from the solar panels, the solar panel efficiency is modeled following the methods presented by Vika.\textsuperscript{33} This modeling framework relates solar panel current and voltage as a function of solar flux and panel temperature. This model is fit to data from Alta Devices 2017 Specifications for single-junction GaAs cells, which represent a light, flexible, efficient solar array appropriate for high altitude aircraft applications. A constant atmospheric temperature of -56.49 °C is assumed, as the altitudes in this work lie within the tropopause.

\textsuperscript{c}Xfoil.jl on BYU FLOW Lab GitHub https://github.com/byuflowlab/Xfoil.jl
\textsuperscript{d}AirfoilPreppy on BYU FLOW Lab GitHub https://github.com/byuflowlab/AirfoilPreppy
\textsuperscript{e}Warp Drive Propellers Inc. Commercial Website warpdriveinc.com, accessed 6/7/18
2. Electric Motor

For calculating motor efficiency and power, we use a fundamental first order motor model. Comparing the first order model to the Maxon 305013 Brushless Motor data, we found the efficiency, current, and required voltage to all be within 1.5% for the nominal RPM and torque.

In order to model the motor mass, we created a linear fit to the motor data from the Astroflight line of motors, as described in our previous work. By way of review, Astroflight motors were chosen due to the availability of data and the favorable range of both power and Kv. We found a linear relationship between the mass and the motor peak current divided by the motor Kv parameter. The line fit in Eq. (21) shows the trend of the motor mass, with an $R^2$ value of 0.94. The motors included in this empirically-based model ranged from 1.5 kW to 15 kW and $K_v$ from 32 to 1355.

$$m_{\text{motor}} = 2.464 \frac{I_m}{K_v} + 0.368$$  \hspace{1cm} (21)

To accurately model motor performance in addition to mass, we investigated the relationship between all of the motor parameters. We found that there were no interdependencies other than those first between the mass, motor peak current, and $K_v$, and second between the no-load resistance and no-load current. The trend for the latter is described by the fit in Eq. (22) with an $R^2$ value of 0.93.

$$R_0 = 0.0467(I_0)^{-1.892}$$  \hspace{1cm} (22)

3. Linearized Battery and Motor Controller Masses

Due to the scope and nature of this comparative conceptual design study, we used a simplified approach to model the motor controller and battery masses. The motor controller model for mass was assumed to be linear based on the specific power of 22,059 W/kg taken from the Astroflight high voltage motor controller. Efficiency of the motor controller was assumed to be a constant 97%. The battery was modeled with a specific energy parameter of 300 Wh/kg, representative of a mid-life, currently available Li-S battery.

III. Optimization Setup

A. Propulsion Optimization

The propulsion optimization is a static optimization, as opposed to the trajectory which changes with time. Solving the propulsion optimization repeatedly at every time step of the optimized path presents a challenge in terms of computational power and time. In order to feasibly solve the propulsion optimization in a reasonable amount of time while using the dynamic trajectory for an entire day as an input, the following simplification is made. Similar to translating data into the frequency domain, the thrust and velocity at each time for a given path are discretized into 100 bins, and the weighting (or probability of occurrence) of each bin is computed. This reduces the number of required propulsion system evaluations from around 6,000 to just 100. We then neglect any bin that has a probability of occurrence less than 0.35% (or about 5 minutes), which further reduces the multi-point evaluation to around 20 points. This allows us to successfully optimize the propulsion system at multiple points, considering the climbing, descending, and night-time orbit flight modes, in an average time of under one hour. Fig. 2 shows the thrust-velocity profile for a given dynamic trajectory, with the bins and their probability of occurrence designated. The white bins occur greater than 0.35% of the time, and are the ones included in the optimization. The night-time orbit is the dominant flight mode, occurring for more than 40% of the day, with an average velocity and thrust of 30 m/s and 80 N, respectively.

The propulsion optimization framework is formulated to minimize the total energy expended during the day. The constraints are formulated to meet the required thrust at each bin’s velocity, in addition to a propeller material failure constraint and tip speed constraint. Figure 3 gives a visual representation of the optimization. The total energy is calculated using a weighted average based on the probability of occurrence.
Figure 2: Example thrust velocity input that has been discretized into 100 bins. White boxes have a weighted occurrence, or probability of greater than 5%, teal greater than 10%, and black greater than 40% of each bin and the required motor voltage and current. The mass of the system is chosen at the maximum operating point that satisfies the constraints. The propulsion optimization is performed in Julia using the SNOPT solver.\textsuperscript{36}

Figure 3: Propulsion optimization framework with design variables in red, models in green, and outputs in blue. For the multipoint evaluation, the only duplicated design variable is RPM in order to simulate a fixed pitch propeller.

To make the resulting optimized propulsion model simple enough for the dynamic optimization, the non-dimensional propeller performance curves were tabulated and passed into the dynamic optimization framework. There, a continuous polynomial-spline is made and used during the dynamic optimization (shown later in Fig. 12). The motor performance model with splines satisfied the requirements of the modeling language used for the dynamic path optimization, and simply required the motor parameters to be passed in.
B. Dynamic Path Optimization

1. Problem Formulation

The objective for SRHALE aircraft is to maximize endurance, subject to the constraints of feasible flight operation, enabling the aircraft to fly for long periods of time to accomplish a given mission objective. In pursuit of this goal, dynamic optimization is used to compute the optimal path for an aircraft to fly throughout the whole day on the winter solstice, optimally angling toward the sun to increase solar energy capture while flying as efficiently as possible, and climbing to store potential energy when optimal. In order to produce useful paths, the time-step of the dynamic optimization must be small enough so that the discrete spatial locations are close-enough to discern the aircraft behavior during turns, and to reasonably satisfy the assumption of constant solar flux and energy consumption between time-steps. In this work, the time-step for the path optimization is 15 seconds. This fine time resolution substantially increases the difficulty of solving the optimization problem throughout the day. To overcome this challenge, the problem is formulated using a receding horizon approach, where the optimization problem is solved over a horizon of 75 time-steps (or 18.75 minutes), data for the first 10 points is saved, and then the horizon advances by 10 time-steps (or 2.5 minutes). This process is repeated until the path for the entire day is computed.

2. Objective Function

Although the ideal objective for SRHALE flight in this case is to maximize endurance, or the total flight time, with this formulation that objective is not possible. Each horizon is independent from each other, and coupling them over the entire day is computationally infeasible. As a proxy, maximizing the total energy in the system across all time serves to accomplish the same goal, while allowing each horizon to be solved independently. Therefore, the objective function for the dynamic optimization problem is posed as follows:

$$\text{maximize} \quad \int_{t_0}^{t_f} E_{\text{total}}(\omega_m, \alpha, \phi, P_{\text{battery}}) \, \text{dt}$$

with respect to $\omega_m, \alpha, \phi, P_{\text{battery}}$

subject to

$$\sqrt{x^2 + y^2} \leq R_{\text{max}}$$

$$h_{\text{min}} \leq h \leq h_{\text{max}}$$

$$C_{L_{\text{min}}} \leq C_L(\alpha) \leq C_{L_{\text{max}}}$$

$$\phi_{\text{min}} \leq \phi \leq \phi_{\text{max}}$$

$$\gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}}$$

$$E_{\text{batt}} \leq E_{\text{batt max}}$$

$$V \geq 0$$

$$P_{\text{total}} \geq 0$$

A key difference between the objective described here and that in our previous work is that the motor rotational speed, $\omega_m$, is controlled instead of the thrust. This allows the removal of the constraints on the maximum thrust, as these are naturally enforced by the increase in energy required to achieve higher thrust (due to the decrease in propulsion system efficiency at high advance ratios, shown in Figure 12c which will be discussed later on in greater detail).

3. Problem Solution

To formulate and solve the dynamic path optimization, the GEKKO optimization suite is used. This is a freely-available Python software package for optimization of systems with differential algebraic equations (DAE). The path optimization problem is written in GEKKO, and the software uses orthogonal collocation on finite elements to transcribe the DAE problem into a purely algebraic system of equations, and automatic differentiation to provide exact first and second derivatives. It then uses a nonlinear solver to solve the system of equations. For this work, the IPOPT solver is used, an open-source software package for large-scale nonlinear optimization. The optimization problem is solved in simultaneous mode, where the equations are evaluated in parallel with the objective function optimization.
4. Initialization

For the first horizon, the nonlinear system is initialized with the results of simulating the aircraft throughout the day in a constant circular orbit, as outlined in our previous work. This process is essential to successfully converging the system, and the circular orbit also serves as a base-case to compare the optimized trajectory against. This process is carried out using a multi-layered approach where (1) a root-finder finds equilibrium conditions for the three differential model terms by manipulating $\phi$, $\alpha$, and in this work $\omega_{\text{min}}$ (instead of $T$), and (2) the root-finder is wrapped in a constrained minimization problem using Sequential Least Squares Programming to determine the power-optimal steady-state trajectory at the minimum altitude. This circular orbit is then (3) integrated forward in time throughout the day using the solar data from SMARTS to compute the total energy performance.

C. Battery Size Optimization

As part of the initialization for the trajectory optimization, the battery size is optimized so that it fills completely during the circular orbit without leaving any excess solar energy uncaptured. This involves taking the same strategy for computing the energy-optimal circular orbit and feeding that into an optimizer that minimizes the squared difference between the maximum energy in the battery and the total solar energy available for capture throughout the flight, which is effectively equivalent to maximizing the total energy in the battery at the end of the day.

In our previous work, the battery-size optimization was carried out to ensure that the base case was sufficiently well-designed to give greater validity to subsequent improvements through trajectory optimization. In this work, the battery size optimization serves as the primary means through which the changes in the propulsion system optimization and the subsequent adjustments by the trajectory optimization translate into improved energy performance.

D. System Optimization

As mentioned above, coupling the path and propulsion optimization at the current level of model fidelity is not possible. A system optimization that includes the effects of propulsion, trajectory, and battery mass optimization therefore must adopt some form of iterative scheme between the three sub-system optimizations.

To bridge the dynamic path and propulsion design optimization, a point based iteration approach is first attempted. This involves simply passing the designs/operating conditions back and forth between the various models and recording the progression as seen in Fig. 4a. The results of this approach are presented in the following sections with surprisingly favorable trends.

![Diagram](image)

(a) Point Based Iteration  (b) System Level Optimization

Figure 4: Comparison of point based iteration and system optimization framework.

IV. Results and Discussion

A. The Effect of Higher Fidelity Propulsion System Models on the Dynamic Trajectory

This work builds upon the trajectory optimization approach in our previous work, wherein we reported an end-of-day energy increase of 8.2%. This increase describes the performance of an optimized trajectory...
compared to a circular state machine strategy with optimal battery mass. These results were obtained using propulsion models derived from momentum-theory, which describe a variable-pitch propeller. A key contribution of the current work is the integration of higher-fidelity propulsion models including a fixed pitch propulsion system into the trajectory optimization framework. This makes possible the combination of design and trajectory optimization through a sequential iterative approach.

With these higher-fidelity propulsion system models, the trajectory optimization now requires propulsion system parameters. For the initial case, these parameters were obtained by optimizing the propulsion system for a circular orbit at the minimum flight altitude. The parameters from this optimization are shown in Tables 3 and 4. The energy performance of the trajectory optimization is shown in Figure 5. The complete cycle of propulsion system optimization, battery mass optimization, and trajectory optimization will be referred to as iteration 1 of the combined sequential optimization.

Table 3: Initial motor parameters obtained from optimizing the propulsion system for a circular path.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_V$</td>
<td>32.0 rpm/V</td>
</tr>
<tr>
<td>$I_0$</td>
<td>2.0 A</td>
</tr>
<tr>
<td>mass</td>
<td>8.69 kg</td>
</tr>
</tbody>
</table>

Table 4: Initial propeller parameters obtained from optimizing the propulsion system for a circular path.

<table>
<thead>
<tr>
<th>r</th>
<th>Chord</th>
<th>Twist</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20000</td>
<td>0.0390</td>
<td>66.9486</td>
</tr>
<tr>
<td>0.288875</td>
<td>0.2121</td>
<td>56.5429</td>
</tr>
<tr>
<td>0.377750</td>
<td>0.2511</td>
<td>49.0805</td>
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<td>42.9949</td>
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<td>0.555500</td>
<td>0.2617</td>
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</tr>
<tr>
<td>0.733250</td>
<td>0.1862</td>
<td>31.6920</td>
</tr>
<tr>
<td>0.822125</td>
<td>0.0985</td>
<td>29.5858</td>
</tr>
<tr>
<td>0.911000</td>
<td>0.0870</td>
<td>27.2552</td>
</tr>
</tbody>
</table>

Figure 5: Energy performance of initial trajectory optimization, iteration 1.

The end-of-day energy increase obtained by the trajectory optimization in this initial case (iteration 1) is 6.9%. This is slightly lower than the 8.2% we reported previously, and is consistent with the fact that the propulsion system is now modeled with a fixed-pitch propeller. Since the propulsion system was optimized for
a circular orbit, the climbing and descending from the trajectory optimization is less efficient than with the momentum-theory propulsion models that are more like a variable-pitch propeller. An additional reason for this difference is due to slight differences in the drag model used. For a point of reference, earlier trajectory optimization results that were performed with the momentum-theory propulsion model but using the exact same drag model as the current paper showed an energy increase of 9.5%. This suggests that the initial energy increase of 6.9% reported above may be biased upward by perhaps 1.3%. Similarly to our previous work, the trajectory optimization yields four distinct flight modes. These flight modes are shown in Figure 6 for the first iteration of the combined optimization.

Figure 6: Stages of the 24 hour optimized trajectory for the first iteration.

The first flight mode is the charge stage (Figure 6a). This occurs at dawn and continues until the battery is nearly full. The aircraft follows a fabiform path while climbing and descending, changing the angle of attack to increase the solar energy obtained. A net maximum energy is achieved by balancing an increase in angle of attack with the accompanying increase in induced drag losses. This path precesses with the sun, and the maximum solar energy is obtained while the aircraft is flying away from the sun with a high angle of attack and lower airspeed. The path is wavy to maximize the total time spent on this part. After turning around, the aircraft flies towards the sun in a direct path with a low angle of attack and higher airspeed, minimizing the amount of time spent in an orientation with sub-optimal solar capture. This flight mode lasts about 6 hours.

The second flight mode is the climb stage (Figure 6b). This stage begins when the battery is nearly full, and the aircraft begins to store potential energy by climbing in altitude aggressively. The climb continues in
a circular path that angles toward the sun and precesses with it, and includes a climb and descent strategy throughout the climb. The maximum thrust is reached during this time, and with it the greatest requirements for the propulsion system. This flight mode lasts about 2 hours.

The third flight mode is the descent stage (Figure 6c). This stage begins once the solar energy available is not sufficient to keep the battery fully charged. The aircraft begins to descend, optimally balancing electrical and potential energy storage. It follows a modified circular path that angles toward the sun to collect as much solar energy as possible despite the low sun angle. Once the sun has set completely, the aircraft transitions to a perfect circular orbit in a zero-thrust glide downward to the minimum altitude constraint. This flight mode lasts about 2 hours.

The final flight mode is the night time power conservation stage (Figure 6d). This begins once the aircraft reaches the minimum altitude constraint. It flies in a completely circular path, with an optimal velocity and bank angle that minimizes the power needed. This is the longest of the flight modes, and lasts about 14 hours. It continues through the night until the sun comes out in the morning, whereupon the battery begins to charge again.

These flight modes remain similar to those in our previous work, notwithstanding the higher-fidelity propulsion system models. One interesting difference is that tight circular orbits are much more prevalent just before sunset, as seen in the Descent stage (Figure 6c). This corresponds with the previously published observation that near sunset, tight circles are able to point the solar array in a way that gathers more energy than larger turns do.

B. Combined Sequential Optimization

The energy performance of the combined optimization is shown in Figure 7, which details the total energy and the battery state-of-charge throughout the day for each iteration. The total energy includes electrical energy stored in the battery and potential energy stored by climbing above the minimum altitude of 60,000 ft. The energy performance increases with each iteration, with the largest increase occurring between iteration 1 and 2 and the second largest between iterations 6 and 7.

![Graph of total energy and battery state-of-charge for each combined iteration.](image)

Figure 7: Total energy and battery state-of-charge for each combined iteration.

The maximum energy increases as the combined optimization progresses (Figure 7a), primarily due to the increase in battery mass, which is shown in Figure 8b. The battery mass is sized according to the steady-state orbit, and as the propulsion efficiency increases, the energy required for flight decreases, which means that the aircraft can store additional solar energy in the battery. As the aircraft mass increases, it requires more energy to fly, and thus the battery mass addition requires finding the optimum between the two.

Once the battery is full, the aircraft begins to climb, as shown in Figure 8a. The maximum height between iterations decreases with the first 3 iterations, and remains nearly constant from then on. One reason for this decrease is because the increased battery mass increases the amount of gravitational potential
energy stored per unit height, decreasing the amount of height required to store the same amount of potential energy. Other factors at play include the increase in motor and propeller efficiencies (summarized in Figure 11), which allow more efficient potential energy storage by reducing the energy requirements for the climb and descent (see Appendix, Figure 14c).

Figure 8: Height and battery mass for each combined iteration.

Figure 9 compares the performance of the steady-state and optimized trajectories for each iteration. While the energy performance increases with each iteration, the energy benefit from the trajectory optimization remains nearly constant, decreasing slightly with each iteration (Figure 9b). One explanation for this decrease is that the propulsion system optimization increases the energy efficiency of the night-time circular orbit, and the trajectory optimization does not affect the night-time behavior. Since the steady-state trajectory becomes more energy efficient, there is less room for the optimized trajectory to improve upon it.

Figure 9: Energy performance of each iteration of the combined sequential optimization.

The energy benefit of the combined sequential optimization is summarized in Figure 10. The total energy increase is 20.4%, which is quite significant. This combined effect is more than 3 times the benefit of optimizing the trajectory or the propulsion system alone, and indicates the benefit that combining design and trajectory optimization can have on HALE flight. With the benefits of the combined optimization
established, a closer look will be taken at where they are manifest, and how the trajectory and propulsion system optimizations interact through sequential iterations to produce the benefit.

**Figure 10:** Energy benefits of combined propulsion and trajectory optimization. Each value is the difference in the end-of-day battery state-of-charge between the optimized trajectory and the initial steady-state orbit (iteration 1 in Figure 9a).

Figure 11 shows the progression of the point-based iterations. Three readily apparent main factors should be noted: first, the battery mass significantly increases; second, the flight path height decreases; third, the overall propulsion efficiency increases.

**Figure 11:** Summary of results for each iteration. Percent energy increase and increased flight time are in reference to the optimized trajectory in the first iteration of the sequential optimization scheme.

These three effects are inter-related at a system level: greater battery mass requires the aircraft to fly faster with increased thrust but also provides the capability to store more solar energy. The increased storage will only be useful if the propulsion system is able to increase efficiency to offset the extra power required to fly a heavier aircraft. A main driver in the average efficiency of a fixed pitch propeller is the breadth of the flight requirements, which drives the aircraft to climb much more conservatively.

Figure 12 shows heat maps of the flight requirements on the propeller performance curves at each iteration. A greater thickness or density of the line indicates more operation at that point. The left-most dense region in the heat map is when the aircraft is climbing, the center dense region is during the night time circular orbit, and the right-most dense region is in descent. Increasing the propeller average efficiency (see Fig. 12c) requires both a very high curve as well as a very large weighted integral below the curve. As seen in the final iteration, iteration 7, the propeller peak efficiency is slightly higher than the rest, but the spread of the curve is less than the previous iteration, iteration 6. Since the weighted integral is the objective in order to minimize the total power of the system, iteration 7 outperforms iteration 6 by several percent.

**V. Conclusion and Future Work**

A framework for combined path and propulsion optimization has been described, and the initial results show promise for increasing the feasibility of solar-regenerative high-altitude long endurance (SRHALE) aircraft in station-keeping missions. Compared to a steady-state orbit with a propulsion system optimized for such, after 7 iterations of the point-based combined optimization method the end-of-day energy increased.
by 20.2%, translating to an increase in flight time of 2.36 hours. This is a significant improvement, and is evidence of the powerful role of combining aircraft design and trajectory optimization.

The current work is not without limitations, and efforts are currently underway to address them. Since the changes in battery mass are a key driver of the energy improvements, future work is needed to ensure that the effects of changing mass on the propulsion system optimization are fully accounted for. Changes in the aircraft mass influence the thrust required to stay airborne, and the feasibility of SRHALE flight is most sensitive to the mass. Therefore, more work is needed to more clearly separate out the effects of changes in the battery mass from the effects of changes in the propulsion design and the optimal path. In future work, additional consideration will be given to the treatment of the battery mass generally and the optimization strategy behind it. Additionally, the assumption that structural mass remains constant while battery mass increases is likely optimistic, and a commensurate structural mass increase of at least one-to-one may be warranted.

The base-case design used for comparison deserves additional consideration, as its level of optimality is a strong determiner of the potential gains from combined optimization. Future work will include a more detailed approach to establish the base-case design of the propulsion system and battery size for a circular orbit. This will give increased weight to the comparisons against the base-case performance. Finally, future plans include extending this combined optimization framework to a multidisciplinary system.

Figure 12: Torque coefficient, thrust coefficient, and propeller efficiency by iteration, with heat maps indicating frequency of use.
design architecture using a high-level system optimizer (such as an Analytical Target Cascading algorithm\textsuperscript{41}) in lieu of point-based iteration to further extend the feasibility of SRHALE aircraft through combined path and propulsion design optimization.

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References


American Institute of Aeronautics and Astronautics


Appendix

Figure 13: Power consumption, with the difference ($\Delta$) between the first and last iteration shown.
Figure 14: Solar power generation, with the difference ($\Delta$) between the first and last iteration shown.

Figure 15: Total energy
Figure 16: Thrust-velocity profiles divided into bins, with a cutoff probability of 0.5%: bins less than that are not included in the optimization (and are grayed out). Colored boxes mark the bins with the highest probabilities: black > 50%, red > 10%, cyan > 5%.
Figure 17: Thrust-velocity profiles by stage throughout the day.

(a) Stage 1: Morning

(b) Stage 2: Climb

(c) Stage 3: Descent

(d) Stage 4: Evening