A One-Parameter Groundwater Discharge Model Linked to the IHACRES Rainfall-Runoff Model

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Abstract: A simple groundwater discharge model that uses a modest number of parameters (1-3) has been developed. The model uses three parameters (transmissivity *(T)*, effective porosity *(g)* and hillslope length *(L)*) to model groundwater level as a function of position along the hillslope, and discharge to the stream. If discharge alone is required (groundwater level is not modelled), then the model requires only one parameter. The model has been linked with the IHACRES rainfall-runoff model, with recharge being estimated within the IHACRES model. The discharge formulation within the groundwater model is expressed as a series of exponential terms, and is therefore similar to the commonly used form of the unit hydrograph approach, implemented in streamflow models such as IHACRES. The model is being tested on a variety of catchments in the Lachlan and Macquarie Basins, located in the Murray-Darling Basin in NSW, Australia. The catchments range from 1.6 km^2 to 2000km^2. This allows for the catchments to be represented by single, or multiple hillslopes. Details of the revised groundwater model are presented, as well as modifications made to the IHACRES rainfall-runoff model. Future developments of the model are also discussed.

Keywords: Groundwater; Dupuit-Boussinesq; Rainfall-Runoff Model.

1. INTRODUCTION

Application of rainfall runoff models to systems with varying groundwater levels requires a suitable formulation that appropriately represents the effect of recharge on groundwater discharge, taking into account the change in groundwater storage. This is particularly true when modelling the impact of groundwater salinity on streamflow salinity and salt loads. This paper discusses the integration of a groundwater discharge model developed by Sloan [2000] within the IHACRES rainfall-runoff model [Jakeman *et al.* 1990]. This enables the IHACRES model to explicitly model the groundwater discharge, with parameters of the model based on measurable physical attributes. The model described in this paper applies to closed catchments; that is, catchments with insignificant subsurface inflow or outflow of groundwater. In order for the model to be applied to catchments where the boundary defined by groundwater flow and that defined by topography do not coincide, the subsurface inflow and outflow must be explicitly modelled.

In Section 2, a simplified form of the groundwater discharge model developed by Sloan [2000] is presented. Section 3 discusses the number of terms in the groundwater model that are required to adequately reproduce the full model solution. A baseflow filter based on the groundwater discharge model is described in Section 4, with details of the derivation of the filter presented in Appendix A. Section 5 discusses different possible forms of the model, and in Section 6, the modified form of the IHACRES model is presented.

2. THE SIMPLIFIED GROUNDWATER DISCHARGE MODEL

The groundwater discharge model developed by Sloan [2000] is a parsimonious, lumped, physics-based hillslope model. Sloan [2000] showed that for a homogeneous aquifer (constant aquifer properties), the solution to the Dupuit-Boussinesq equation can be derived analytically, using three parameters to characterise the hillslope response: transmissivity *(T)*, effective porosity *(g)* and hillslope length *(L)*. If only the groundwater discharge is required (groundwater level is not modeled), then the number of parameters is reduced to one *(ω)* which can be either estimated from the three measurable quantities listed above (given by T/gL^2) or optimised.

The model at timestep t can be written as a series of transfer functions of the form:

$$Q_{b,i}(t) = -\alpha_i Q_{b,i}(t-1) + \beta_i R(t) \qquad (1)$$

$$Q_b(t) = \sum_{i=1}^{\infty} Q_{b,i}(t)$$

where $Q_b(t)$ is the ith component of the groundwater discharge $Q_b(t)$, α_i and β_i are the constants for each exponential decay, given by:

$$\alpha_i = \frac{-1}{1+\lambda_i},\ \beta_i = -2\alpha_i\omega \qquad (2)$$

and λ_i is the eigenvalue as defined by Sloan [2000]:

$$\lambda_i = \left[(2i-1)\frac{\pi}{2}\right]^2 \omega \qquad (3)$$

The advantage of expressing the model in this form is that it is similar to the classic unit hydrograph approach used by many surface hydrology models (such as IHACRES), enabling straightforward integration with existing models.

3. NUMBER OF TERMS

When using the model, it is necessary to truncate the infinite series in equation (1). Since the time constant $\tau_i = -1/ln(-\alpha_i)$ tends to zero as i tends to infinity, truncating the series impacts only on the initial response to recharge. In addition, conservation of mass requires the volume of the groundwater discharge unit hydrograph to be equal to one (due to the use of recharge in equation 1):

$$\sum_{i=1}^{\infty} V_i = 1, \text{ where } V_i = \frac{\beta_i}{1+\alpha_i} \qquad (4)$$

However the volume of the truncated series will necessarily be less than one, so there is a need to scale the volume components (V_i). Scaling all of the components would incorrectly scale the slowest flow components as well as the faster components. The best solution is to scale the volume of the last (nth, largest eigenvalue) component in the truncated set, so that:

$$V_n' = \frac{\beta_n}{1+\alpha_n} + \sum_{i=n+1}^{\infty} V_i \qquad (5)$$

It is desirable to establish the number of summands that are necessary for a reasonable reproduction of $Q_b(t)$. Table 1 shows the number needed for a 1% error in the first day's flow following a recharge event, assuming negligible contribution from earlier recharge events. The first day of flow will be the day with the largest error due to truncation. Note that the tabulated results give an upper limit in relative error for the modeled flow, since the effect of flows from preceding recharge days are ignored. For example, the e-folding time (time constant) for the first component for ω = 0.01 is 94.5 days, implying that there would be significant contribution to flow from events over the past several months, reducing the error in the total flow below 1%.

Table 1: Number of terms needed for ε <0.01

ω	Time constant	Number of terms
0.1	10.4	7
0.01	94.5	17
0.001	934	33

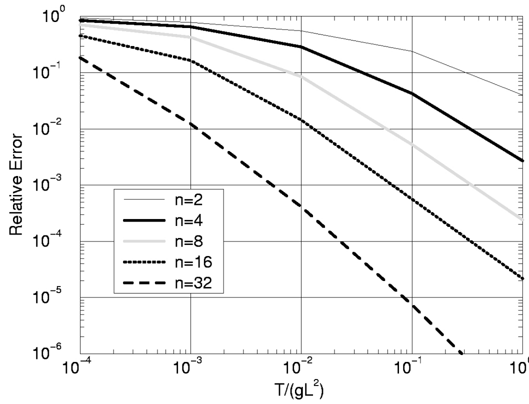

Figure 1: Relative error as a function of ω

Figure 1 gives a plot of the maximum relative error in the truncated series compared with the infinite series for the timestep with recharge, as a function of values of ω for selected values of n. For responsive systems (ω > 0.1), 8 terms are sufficient to accurately reproduce the results from the infinite series (in this case, 100000 terms were used for the infinite sum). For systems with slower responses, more terms are needed to accurately reproduce the increment in groundwater discharge. However, the error in the total flow is considerably smaller if there were recharge events within the preceding e-folding time. Thus, while 17 terms are needed to accurately model the response from a system with ω = 0.01, the large time constant for such a system (95 days) implies that the groundwater discharge could be accurately modeled with fewer terms.

4. BASEFLOW FILTER

This groundwater model can be used as the basis for generating a baseflow filter, using the approach of Chapman [1999]. To derive the baseflow, an estimate of the daily recharge R is needed. This estimate is obtained by assuming that the effective rainfall (U) is partitioned between runoff (Q_{ro}) and recharge (R), using a constant fraction γ. If the surface runoff does not contribute to the following days' flow (duration of event is less than 1 timestep), then:

$$U = R + Q_{ro} = \gamma U + (1-\gamma)U \qquad (6)$$

giving,

$$R = \gamma U = \frac{\gamma}{1-\gamma} Q_{ro} = \kappa Q_{ro} \qquad (7)$$

For a single flow pathway with an exponential decay, the baseflow is then given by:

$$Q_b(t) = \alpha Q_b(t-1) + \beta\kappa Q_{ro}(t) \qquad (8)$$

where α is the slow flow recession rate, and β is the fraction of recharge that appears as slowflow discharge in the first timestep. Assuming that $Q = Q_{ro} + Q_b$ and collecting the Q_b terms on the left gives:

$$Q_b(t) = \frac{1}{1+\beta\kappa}\left[\alpha Q_b(t-1) + \beta\kappa Q(t)\right] \qquad (9)$$

Chapman [1999] used this methodology to generate baseflow filters using the Boughton and IHACRES models. The filter described above is the Boughton filter, since it assumes that the contribution from surface runoff leaves the catchment within one timestep. The IHACRES filter generalises this by assuming an exponential decay in Q_{ro}.

To derive the baseflow filter for the Sloan model, the technique needs to be modified to allow for multiple exponential terms in the baseflow response. Details of the derivation are given in Appendix A. The baseflow filter is given by:

$$Q_{b,i}(t) = -\alpha_i Q_{b,i}(t-1) + \frac{\gamma\beta_i}{\beta_{ro} + \kappa\sum_{i=1}^{n}\beta_i} \left[Q(t) + \alpha_{ro}Q(t-1) + \sum_{i=1}^{n}(\alpha_i - \alpha_{ro})Q_{b,i}(t-1)\right] \qquad (10)$$

where α_{ro} is the quick flow recession rate. If there is no contribution from the surface runoff of the previous timestep, then this simplifies considerably as $\alpha_{ro} = 0$ and $\beta_{ro} = 1$.

The problem with this methodology is that it assumes that the baseflow filter is adequately representing the baseflow component. If the baseflow component is underestimated, then the surface runoff component will be overestimated, resulting in an overestimation of the recharge, as well as a tendency for continual recharge. Thus the derived baseflow will partly depend on how well the assumed filter matches actual catchment response characteristics.

A way around this problem is to constrain the recharge to only days with rainfall, or even better, to less than the observed rainfall. However, to minimise the measurement error introduced, the observed streamflow can be used by assuming that wet timesteps correspond to timesteps with increasing streamflow; that is, $Q(t) > Q(t-1)$. Thus estimated recharge is given by:

$$R(t) = \frac{\gamma\delta(t)}{(1+\alpha_{ro}) + \kappa\sum_{i=1}^{n}\beta_i} \left[Q(t) + \alpha_{ro}Q(t-1) + \sum_{i=1}^{n}(\alpha_i - \alpha_{ro})Q_{b,i}(t-1)\right] \qquad (11)$$

where

$$\delta(t) = \begin{cases} 0 & Q(t) \le Q(t-1) \\ 1 & Q(t) > Q(t-1) \end{cases} \qquad (12)$$

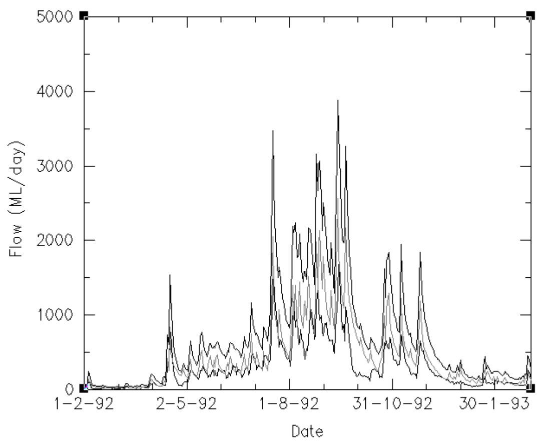

Figure 2: Baseflow filter applied to a gauge in the Goulburn-Broken Basin, Victoria. The grey line is the estimated baseflow, while the lower black line is the estimated surface runoff.

Figure 2 shows the estimated baseflow and runoff derived from applying this filter to observed streamflow at a gauge (405215 – Howqua River, 368km^2) in the Goulburn-Broken basin in Victoria, Australia. The baseflow filter was optimised using a grid search of possible values of ω. For each value of ω, the γ parameter was optimised to the maximum value that gave less than 1% of days with baseflow exceeding observed flow. The value of ϖ which maximised the number of days with near zero quickflow, while maximising the total quickflow volume was then selected; yielding parameter values of $\varpi = 0.003$ and $\gamma = 0.83$.

5. MODEL APPLICATION

In applying this model the spatial distribution of recharge needs to be approximated. This can be done in a variety of ways; the simplest being that proposed by Sloan [2000], where the entire catchment is modeled by a single representative hillslope.

5.1. One Hillslope Model

When using a single representative hillslope, GIS data are used to determine the hillslope length L. The recharge distribution function $f(x)$, at a distance x down the hillslope, is then a constant, with its value determined by the catchment area divided by L. In this way $f(x)$ is effectively the hillslope width. This gives a catchment represented by a single block, as in Figure 3.

While being simple to implement, this model has serious limitations, as discussed in Croke *et al.* [2001]. Generally the model is unable to reproduce the observed dynamic response of a catchment. This leads to consideration of an extension of the model, with the catchment being represented by not one, but two hillslopes.

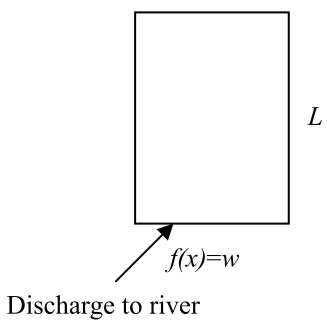

Figure 3: Single hillslope catchment

5.2. Two-Hillslope Model

The simulation of the dynamic response of a catchment is improved by interpreting the catchment as being comprised of two representative hillslopes. This provides the model with both quick and slow flow components for the baseflow, with the shorter hillslope giving a faster response.

The practical implementation of this model is similar to the one hillslope model. However w is now the total hillslope width, so that $w = w_1 + w_2$ and the two hillslope areas add to give the total catchment area, that is $L_1w_1 + L_2w_2 = A$. This results in the number of parameters increasing from 1 (L) to 3 (L_1, L_2 and w_1), since w_2 is uniquely determined by the catchment area (A) together with L_1, L_2 and w_1.

It is possible to consider the two distinct hillslopes as being a single hillslope with a variable width. In this case $f(x)$ is a step function, with

$$f(x) = \begin{cases} w_1 + w_2 & 0 < x \le L_2 \\ w_1 & L_2 < x \le L_1 \end{cases} \qquad (13)$$

With this model the catchment is represented by two adjacent blocks, one long and thin, the other short and wide, as in Figure 4.

The two-hillslope model gives a more accurate representation of the slowflow component of the baseflow discharge due to the longer hillslope. However it does assume that discharge to the river after a recharge event is almost instantaneous. This is because the infiltration from recharge is assumed to converge rapidly onto major flow pathways.

The standard two-hillslope model can be extended to a multiple hillslope model with any number of representative hillslopes. However this comes at a cost of two parameters per additional hillslope (one parameter each for the extra hillslope length and width). To reasonably capture the dynamic response of the catchment without dramatically increasing the number of parameters needed one could consider a variable width hillslope model.

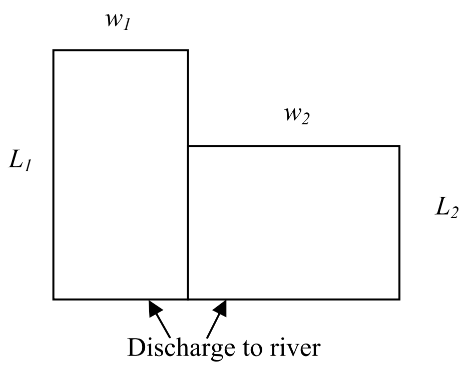

Figure 4: Two-hillslope catchment

5.3. Variable Width Hillslope Model

Note that β_i in Equation (2) is valid only for $f(x)$=1. In general

$$\beta_i = \alpha_i \frac{T}{L} c_i \frac{d}{dx} \phi_i(x) \qquad (14)$$

where

$$c_i = \frac{2}{gL}\int_0^L f(x)\phi_i(x)dx \qquad (15)$$

and

$$\phi_i(x) = \cos\left[(2i-1)\frac{\pi}{2}\frac{x}{L}\right] \qquad (16)$$

$\phi_i(x)$ being the eigenfunction corresponding to λ_i.

The variable width hillslope model was considered by Sloan [2000], using GIS data and a distance to stream calculation to obtain the distribution of hillslope width *f(x)*. For a homogeneous aquifer a variable width representative hillslope leaves the eigenvalues λ_i from Equation (3) and their corresponding eigenfunctions $\phi_i(x)$ from Equation (16) unchanged. Since the eigenvalues λ_i are unchanged, the values of α_i in Equation (2) are also unchanged. However a variable width hillslope causes c_i from Equation (15) and hence β_i from Equations (14) and (1) to both change. This means that by implementing a variable width hillslope model the decay of flow with time (α_i) remains unchanged but the fraction of recharge that comes out in the first time step (β_i) is altered.

It is worth considering whether perhaps some function could simply be substituted for *f(x)*, enabling an analytical solution, or whether a data-driven production of *f(x)* via GIS is necessary. The gamma distribution has been used with success in various areas of hydrology and would seem to have the flexibility required to accurately represent *f(x)*. However the gamma distribution requires two parameters (a scale and a shape parameter). These parameters can be accurately estimated once the sample mean and standard deviation is known, but to find these one would need a DEM. If a DEM with sufficiently high resolution is available, then the best approach would be to directly generate *f(x)*. If a suitable DEM is not available then the two parameters for the gamma distribution need to be estimated, either through calibration or estimation from other means (e.g. regionalisation).

6. MODIFIED IHACRES MODEL

The standard form of the IHACRES rainfall-runoff model is a non-linear loss module yielding an effective rainfall, which is passed to a linear routing module, which partitions the effective rainfall between a quickflow and slowflow transfer function (e.g. Jakeman and Hornberger, 1993). Here, the non-linear module has been converted into a form that calculates both the effective rainfall and the recharge. In this case the effective rainfall only relates to the overland, or near surface flow components (the quick component). This formulation of the model allows variable partitioning of rainfall between the quick flow component and groundwater discharge.

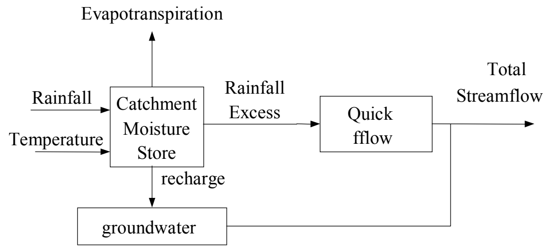

Figure 5: IHACRES_GW model

The form of the non-linear module used in this case is based on the catchment moisture deficit module of Evans and Jakeman [1998], and is further developed in Croke and Jakeman (in prep.). The non-linear module was altered to give an estimate of the recharge per timestep (usually daily), assuming that recharge only occurs during rain events (i.e. the model does not take into consideration the time required for the water to percolate down to the groundwater table). Since the groundwater model developed by Sloan [2000] can be expressed as a series of exponential decay terms, it is easy to incorporate within the IHACRES model. Currently, the two-hillslope version has been tested [Croke *et al.* 2001].

7. CONCLUSIONS

The groundwater model developed by Sloan [2000] can be easily linked to existing hydrological models, and provides a parsimonious representation of groundwater discharge. Generally, accurate estimation of the groundwater discharge can be obtained by using the first 10 terms of the infinite series, with the last term (shortest time constant) adjusted to ensure that the volume of the groundwater discharge unit hydrograph has a volume equal to 1.

The single hillslope model tends to poorly reproduce observed streamflow, due primarily to the range of hillslope lengths that exist within a catchment. The two-hillslope model gives a better representation of the dynamics, at the cost of two extra parameters. The variable hillslope model should be able to represent the range of hillslope lengths without the need for additional parameters providing the function *f(x)* can be derived from the available spatial data.

Possible future developments of the model include adaptation to a sloping aquifer, so that groundwater discharge from upland catchments can be more realistically measured. Work is underway to investigate using perturbation theory to develop a sloping aquifer model. In addition, rising groundwater levels will result in development of new discharge sites. The existing version of the groundwater discharge model does not allow for this possibility.

8. REFERENCES

Chapman, T., A comparison of algorithms for streamflow recession and baseflow separation, *Hydrological Processes*, 13, 701-714, 1999.

Croke, B.F.W., W.R. Evans, S. Yu. Schreider and C. Buller, Recharge Estimation for Jerrabomberra Creek Catchment, the Australian Capital Territory, *Proc. Of MODSIM 2001*, Canberra December 2001, Modelling and Simulation Society of Australia and New Zealand Inc., eds F. Ghassemi, P. Whetton, R. Little and M. Littleboy, ISBN 0 86740 525 2, vol 2, 555-560, 2001.

Croke, B.F.W. and A.J. Jakeman, Revision of the IHACRES rainfall-evapotranspiration-runoff model, in preparation.

Evans, J.P. and A.J. Jakeman, Development of a simple, catchment-scale rainfall-evapo-transpiration-runoff model, *Environmental Modelling and Software,* 13,385-393.

Jakeman, A.J. and G. M. Hornberger, How Much Complexity is Warranted in a Rainfall-Runoff Model?, *Water Resources Research*, 29, 2637-2649.

Jakeman, A.J., I.G. Littlewood, and P.G. Whitehead, 1990, Computation of the instantaneous unit hydrograph and identifiable component flows with application to two small upland catchments*, Journal of Hydrology*, 117, 275-300.

Sloan, W.T., A physics-based function for modeling transient groundwater discharge at the watershed scale, *Water Resources Research*, 36, 225-241, 2000.

9. APPENDIX A

This appendix outlines the methodology used for deriving the baseflow filter for a baseflow comprising multiple exponential terms. Firstly, the total streamflow is considered to be given by the sum of the surface runoff and the baseflow (i.e. there are no other components):

$$Q(t) = Q_{ro}(t) + Q_b(t) \tag{A-1}$$

Assuming that the surface runoff can be represented by a single exponential decay, driven by a constant fraction (1-γ) of the effective rainfall *U(t)*, gives:

$$Q_{ro}(t) = -\alpha_{ro} Q_{ro}(t-1) + (1+\alpha_{ro})(1-\gamma)U(t) \tag{A-2}$$

For a baseflow comprising of a number (n) of exponential terms, the total baseflow is given by:

$$Q_b(t) = \sum_{i=1}^{n} Q_{b,i}(t) \tag{A-3}$$

where each individual exponential term is given by:

$$Q_{b,i}(t) = -\alpha_i Q_{b,i}(t-1) + \gamma\beta_i U(t) \tag{A-4}$$

Therefore the total streamflow can be expressed as:

$$\begin{aligned} Q(t) = -\alpha_{ro} Q(t-1) + (1+\alpha_{ro})(1-\gamma)U(t) + \\ \sum_{i=1}^{n} -\alpha_i Q_{b,i}(t-1) + \gamma\beta_i U(t) \end{aligned} \tag{A-5}$$

Rearranging this expression gives:

$$U(t) = \frac{Q(t) + \alpha_{ro} Q(t-1) + \sum_{i=1}^{n} (\alpha_i - \alpha_{ro}) Q_{b,i}(t-1)}{(1+\alpha_{ro})(1-\gamma) + \gamma \sum_{i=1}^{n} \beta_i} \tag{A-6}$$

Substituting the above expression for the effective rainfall into (A-4) gives the estimated time-series of flow for each baseflow component.

This methodology can easily be extended to multiple hillslope models by considering the fraction of recharge that goes to each hillslope. For example, a two-hillslope model would yield:

$$Q(t) = Q_{ro}(t) + Q_{b1}(t) + Q_{b2}(t) \tag{A-7}$$

where

$$Q_{b1,i}(t) = -\alpha_{b1,i} Q_{b1,i}(t-1) + \varepsilon\gamma\beta_{b1,i} U(t) \tag{A-8}$$

$$Q_{b2,i}(t) = -\alpha_{b2,i} Q_{b2,i}(t-1) + (1-\varepsilon)\gamma\beta_{b2,i} U(t) \tag{A-9}$$

where ε is the fraction of recharge which goes to the aquifer for the first hillslope, while the remainder (1-ε) goes to the aquifer underlying the second hillslope. Substituting (A-8) and (A-9) into (A-7) gives an expression for the effective rainfall for the two-hillslope model, which can then be used to define the two-hillslope filter.