# Deciding Break Interval in the Discoutinuous Trend Unit Root Test

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**Abstract:** Dickey and Fuller proposed the tests for unit root hypotheses in a uni-variate time series. Perron [1989] extended the t-ratio type unit-root tests so that they allow for a break in the deterministic trend and/ or in the intercept term. In practice, it seems difficult to specify the break point correctly. Zivot and Andrews [1992] proposed a test in which the break point is statistically determined. Morimune and Nakagawa [1999] studied the effect of a miss specified break point on the Perron tests, and the accuracy of the asymptotic expression is examined under various specifications of the error. This paper proposes to set an interval that possibly covers a break point in the Perron tests. This helps to avoid miss specifying the break point, and the unit root test is less susceptible to the choice of a particular break point. Furthermore, an orthogonal decomposition of the F-ratio type test is proposed to find the correlation between the first difference of the series and the trend.

***Key Word:*** unit-root test; discontinuous-trend; break-interval.
**JEL Classification Number:** C22

## 1. INTRODUCTION

The alternative regression in the C test by Perron [1992] for the unit root which allows for a break in the deterministic trend as well as in the intercept term is

$$(1)\quad y_t = \sum_{i=1,2}\{\alpha_i^* + \beta_i^*(t-\bar{t}_i)\}DU_{it} + \gamma D_t + u_t,$$
$$u_t = (1+\phi)u_{t-1} + \varepsilon_t$$

where $\varepsilon_t$ is the white noise with variance $\sigma^2$. The null hypothesis of the test is $\phi = 0$. The sub-interval dummy variables $DU_{1t}$ and $DU_{2t}$ are 1 for $1 \le t \le T_1$ and $T_1 + 2 \le t \le T$, and 0 otherwise, $D_t$ is a shock dummy variable which is 1 when t is at the break point $T_1 + 1$, and 0 otherwise. This shock dummy variable has an effect of jumping the observation at the break point in the estimation. The mean of the trend in each sub-interval is denoted $\bar{t}_1$ or $\bar{t}_2$. The all right-hand side variables are orthogonal to each other. The other tests which are named A and B by Perron include the trend with shifting intercepts or only the shifting intercepts, respectively. An extension of the analyses to A and B tests are straightforward and is omitted from the paper to save space. Estimating (1) by OLS and calculating residuals $\hat{u}_t$, the test statistic is the t-ratio of the $\phi$ coefficient in the regression

$$(2)\quad \Delta\hat{u}_t = \phi\hat{u}_{t-1} + \gamma D_t + \text{error}.$$

This test is similar in the sense that the null size is not affected by nuisance parameters such as $\beta^*_1$ and $\beta^*_2$, and the F-ratio type test does not follow from this formulation. This approach to the unit root test is found in Schmidt and Phillips [1992], Oya and Toda [1995], and Morimune and Nakagawa [2001]. (Once $\phi$ is estimated, $\alpha$ and $\beta$ coefficients can be re-estimated using $\hat{\phi}$. This leads to the nonlinear estimation of $\phi$ and the resulting test statistics may have more complicated properties than that of the t-ratio of the $\phi$ coefficient in (2).) If the Cochrane-Orcutt transformation is applied to (1), it is recast as

$$(3)\quad \Delta y_t = \phi\, y_{t-1} + \sum_{i=1,2}[\alpha_i + \beta_i^*\phi(t-\bar{t}_i)]DU_{it}\,. + \gamma D_t + \varepsilon_t$$

The Dickey-Fuller type t-test $\hat{\tau}_\tau$ is derived as the t-ratio of the $\phi$ coefficient in this regression equation neglecting the nonlinear constraints in the trend coefficients. The difference between the two t-ratios is negligible. See Morimune and Nakagawa [1999, 2001]. In this paper, it is assumed that a break occurs in an interval since it is not easy to specify a break point but easier to set a break interval in empirical studies. The rigorous approach by Zivot and Andrews [1992] does not necessarily lead to a break point that satisfies empirical researchers.

The regression equation under the alternative hypothesis of the test is

$$(4)\quad \Delta y_t = \phi\, y_{t-1} + \sum_{i=1,2}[\alpha_i + \beta_i\ (t - \bar{t}_i)]DU_{it} + \sum_{i=1,m}\ \gamma_i D_{it} + \varepsilon_t. \qquad t = 2,\cdots,T$$

The shock dummy variables are defined over the m continuous periods, such that $D_{it} = 1$ at only one point in the interval and 0 otherwise. This implies these m observations are not used in estimating regression coefficients such as $\phi$, $\alpha$, and $\beta$. This alternative modeling may, at least, reduce the risk of specifying an erroneous break point when the break point is not known. The null hypothesis $\phi$ =0 automatically implies $\beta$ =0 as can be seen by (3).

The regression equation under the null hypothesis of the test is, for $t = 2,\cdots,T$

$$(5)\quad \Delta y_t = \sum_{i=1,2}\alpha_i\, DU_{it} + \sum_{i=1,m'}\ \gamma_i D_{it} + \varepsilon_t.$$

The break interval under the null hypothesis need not be the same as that under the alternative hypothesis. The null regression need not include the intercept terms. There is a possibility of a shorter break interval under the null hypothesis, and m' is assumed smaller than or equal to m.

## 2. ORTHOGONAL DECOMPOSITION OF $\Psi$

There are alternative test statistics. The regression (4) is transformed as

$$(6)\quad \Delta y_t = \phi\, y^*_{t-1} + \sum_{i=1,2}[\alpha_i^{**} + \beta_i^{**}\ (t - \bar{t}_i)]DU_{it} + \sum_{j=1,m}\ \gamma_j D_{jt} + \varepsilon_t,\quad t = 2,\cdots,T$$

where $y^*_{t-1}$ is the residual of regressing $y_{t-1}$ on an intercept, a trend variable in the two sub-intervals, and shock dummy variables. The t-ratio of the $\phi$ coefficient is an extension of the $\hat{\tau}_\tau$ test statistics by Dickey and Fuller [1981]. Jumping observations over the break interval, the test statistics is defined as

$$(7)\quad \hat{\tau}_\tau = \frac{\sum_{t=2,\cdots,T_1,T_1+m+1,\cdots,T}\ \Delta y_t\, y^*_{t-1}}{\hat{\sigma}\sqrt{\sum_{t=2,\cdots,T_1,T_1+m+1,\cdots,T}\ (y^*_{t-1})^2}}$$

$$\Rightarrow \tau_\tau \equiv \frac{\sum_{i=1,2}\lambda_i \int_0^1 \widetilde{B}_i(r)d\widetilde{B}_i(r)}{\sqrt{\sum_{i=1,2}\ \lambda_i^2 \int_0^1 \widetilde{B}_i(r)^2 dr}},$$

the arrow implies the weak convergence of the statistics under the null hypothesis assuming m to be fixed, $\lambda$ s are the break ratio of the two intervals, such that $T_1/T \to \lambda_1, \text{and}\, \lambda_2 = (1-\lambda_1)$, $\hat{\sigma}^2$ is the mean of the squared residuals calculated under the alternative regression, and $\widetilde{B}_i(r)$, i=1,2, are the demeaned and detrended Brownian motions,

$$\widetilde{B}_i(r) = B_i(r) - \int_0^1 B_i(s)ds - 12\left(r - \tfrac{1}{2}\right)\int_0^1 B_i(s)\left(s - \tfrac{1}{2}\right)ds,$$

where $B_i(r)$, i=1,2, are the standard Brownian motions which are mutually independent. It is easy to prove that $\hat{\tau}_\tau$ test is consistent. The F-ratio statistic is also used for testing the unit root. The sum of the squared residuals denoted RSS hereafter is calculated under the null as well as the alternative regression (5) and (4), respectively. Define

$$(8)\quad \tau_{t1} = \frac{\{\sum_{t=2,T_1}\ \Delta y_t (t - \bar{t}_1)\}^2}{\hat{\sigma}^2 \sum_{t=2,T_1}\ (t - \bar{t}_1)^2},$$

and $\tau_{t2}$ in the second sub-interval for $t=T_1+m+1$ to T, the F-ratio test statistic is extended to the discontinuous trend model and expanded as

$$(9)\quad \Psi = \frac{RSS(5) - RSS(4)}{RSS(4)/((T-1) - m - 5)} = (\hat{\tau}_\tau)^2 + \tau_{t1} + \tau_{t2} \Rightarrow (\tau_\tau)^2 + \chi^2(2).$$

The arrow implies the weak convergence of the statistics under the null hypothesis assuming m to be fixed, and $\chi^2(2)$ is the $\chi^2$ random variable with two degrees of freedom. The difference between the two RSS is decomposed into the sum of three terms since all right-hand side variables in (9) are orthogonal. Implication of each term in the decomposition is of interest. The first term is $(\hat{\tau}_\tau)^2$ which is the square of the test statistic (7), and remaining two terms are the correlation between $\Delta y$ and the trend in each sub-interval, in short. Then a large $\Psi$ value does not necessarily imply a large $\hat{\tau}_\tau$ value. A large $\Psi$ value can be resulted from a high correlation between $\Delta y$ and the trend in either or both of the two sub-intervals.

Further, the second term is asymptotically $\chi^2(1)$ under the null hypothesis, but it diverges to infinity under the alternative hypothesis. The same holds for the third term. This means that the second and the third term can also be used as the test statistic for the unit root. Further, the rough idea on the sum of these two terms is obtained by examining the plots of the differenced series. For example, they may be zero if fluctuations in $\Delta y$ do not show any trend, but away from zero if they increase with time. These properties are summarized by the next theorem.

**THEOREM 1**: Under the null hypothesis of the test where the regression is defined by (5), $\tau_{t1}$ and $\tau_{t2}$ are asymptotically distributed as $\chi^2$ with one degree of freedom. Both $\tau_{t1}$ and $\tau_{t2}$ diverge to infinity under the alternative hypothesis of the test where the regression is defined by (4).

The asymptotic distribution under the null hypothesis is proved by replacing $\varepsilon_t$ by $\Delta y_t$. Under the alternative hypothesis, it diverges if $\Delta y_t$ is replaced by $\sum_{i=1,2}\beta_i\ (t-\bar{t}_i)DU_{it}$, in short.

The t-ratios on intercept and trend coefficients are denoted $\tau_{\alpha\tau}$ and $\tau_{\beta\tau}$ in the continuous trend regression. They can be used for testing the unit root. (Dickey and Fuller [1981]) However, $\tau_\tau, \tau_{\alpha\tau}$, and $\tau_{\beta\tau}$ are correlated with each other. The three right-hand side terms in (9) are made up with three orthogonal vectors, and relation among the statistics is simple. The relation between $\tau_t$ and $\tau_{\beta\tau}$ needs to be studied more in detail further. An example will be found later. (The latter is the t-ratio of the trend term and is the unit root test statistic. Dickey and Fuller [1981])

## 3. ORTHOGONAL DECOMPOSITION OF $\Psi$ IN THE AUGMENTED TEST

The decomposition in (9) is generalized to the augmented test. The regression equation includes lagged differenced terms so that

$$(10)\quad \Delta y_t = \phi\, y_{t-1} + \sum_{i=1,2}[\alpha_i + \beta_i\ (t-\bar{t}_i)]DU_{it} + \sum_{i=1,m}\ \gamma_i D_{it} + \sum_{k=1,L}\ \theta_k \Delta y_{t-k} + \varepsilon_t,$$

for $t = L+2,\cdots,T$, and the null regression is

$$(11)\quad \Delta y_t = \sum_{i=1,2}\alpha_i\, DU_{it} + \sum_{i=1,m}\ \gamma_i D_{it} + \sum_{k=1,L}\ \theta_k \Delta y_{t-k} + \varepsilon_t.$$

This regression equation (10) is transformed as

$$(12)\quad \Delta y_t = \phi\, y^*_{t-1} + \sum_{i=1,2}\ \beta_i^{**}\ t_i^* + \sum_{i=1,2}\ \alpha_i^{**} DU_{it} + \sum_{i=1,m}\ \gamma_i D_{it} + \sum_{k=1,L}\ \theta_k^{**} \Delta y_{t-k} + \varepsilon_t\,,$$

where $y^*_{t-1}$ is the residual from regressing $y_{t-1}$ on constant dummy variables, dummy trend variables, but also on all the lagged differenced variables, and $t_1^*$ is the residual from regressing $(t-\bar{t}_1)DU_{1t}$ on $(t-\bar{t}_2)DU_{2t}$ and all lagged differenced variables. It is noted that $t_1^*$ is effected by $(t-\bar{t}_2)DU_{2t}$ through the lagged differenced variables even though $(t-\bar{t}_1)DU_{1t}$ and $(t-\bar{t}_2)DU_{2t}$ are orthogonal. $t_2^*$ is similarly defined as $t_1^*$. The regression coefficients are redefined according to these transformations of variables, but the $\phi$ coefficient is unchanged. Observations in the break interval are not used in these calculations of residuals. The t-ratio on $\phi$ may be denoted $\hat{\tau}_\tau$ using the nomenclature for the augmented Dickey-Fuller test statistic. Define T* as a two column matrix which consists with series of $t_1^*$ and $t_2^*$, and a column vector $\Delta y$ with $\Delta y_t$. All of them jump observations in the break interval. The F-ratio statistics is decomposed as

$$(13)\quad \Psi = \frac{RSS(11) - RSS(10)}{RSS(10)/((T-1-L)-5-m-L)} = (\hat{\tau}_\tau)^2 + \tau_t \ \Rightarrow\ (\tau_\tau)^2 + \chi^2(2)$$

where

$$(14)\quad \tau_t = \frac{1}{\hat{\sigma}^2}\Delta y' T^* (T^{*\prime} T^*)^{-1} T^{*\prime} \Delta y\,.$$

This is the F-ratio statistic associated with $\beta_1^{**}$ and $\beta_2^{**}$ coefficients. The arrow implies the weak convergence again under the null hypothesis. Not only the first term $(\hat{\tau}_\tau)^2$ but also the second term diverges to infinity under the alternative hypothesis. This means that $\tau_t$ also serves as a consistent test statistic for the unit root hypothesis. These properties are summarized by the theorem below. Since the sum $(\hat{\tau}_\tau)^2 + \tau_t$ is $\Psi$, $\tau_t$ may be useful only for the interpretation of the $\Psi$ test, in particular, when $\hat{\tau}_\tau$ is insignificant but $\Psi$ is significant. This happens because $\tau_t$ is significant.

**THEOREM 2**: Under the null hypothesis of the test where the regression is defined by (11), $\tau_t$ is asymptotically distributed as $\chi^2$ with two degrees of freedom. Under the alternative hypothesis of the test where the regression is defined by (10), $\tau_t$ diverges to infinity.

## 4. ORTHOGONAL DECOMPOSITION OF $\tau_t$

The test statistic $\tau_t$ has a simple asymptotic distribution under the null hypothesis, and diverges to infinity under the alternative hypothesis. However, it cannot give information whether the deviation from the null hypothesis occurs in the first or/and second sub-interval. It is attempted to decompose $\tau_t$ in the next theorem so that the deviation from the null hypothesis in each sub-interval can be studied.

**THEOREM 3**: Under the null hypothesis of the test where the regression is defined by the equation (11), $\tau_t$ defined by (14) is decomposed as $\tau_t \cong \tau_{t1} + \tau_{t2}$ in the sense that

$$(15) \quad \text{plim}_{t\to\infty}(\tau_t - \tau_{t1} - \tau_{t2})=0,$$

where $\tau_{t1}$ and $\tau_{t2}$ are defined by (8), and are asymptotically distributed as $\chi^2$ with one degree of freedom, respectively, under the null hypothesis of the test.

PROOF: Define $t_1 - \bar{t}_1$, $Q_1$, and $Q_2$ be the ($T_1$-1-L) column vector, ($T_1$-1-L) × L and (T-$T_1$-m) × L matrices consist with the trend in the first sub-interval, and the L lagged dependent variables in the two sub-intervals, respectively. Define further, Q=($Q_1$', $Q_2$')' which is the matrix of the whole observations on the lagged variables, $t_1^*$ is the (T-1-m-L) column vector of residuals regressing the trend in the first sub-interval on the trend in the second sub-interval and the lagged dependent variables that is

$$(16) \quad t_1^* = \{I - S(S'S)^{-1}S'\}\begin{pmatrix} t_1 - \bar{t}_1 \\ 0 \end{pmatrix}$$

where

$$S = \begin{pmatrix} 0 & Q_1 \\ (t_2 - \bar{t}_2) & Q_2 \end{pmatrix}.$$

A normalizing matrix is defined as $N = \text{diag}(T\sqrt{T}, \sqrt{T}I_L)$ . Since $(t_2 - \bar{t}_2)'Q_2 / T\sqrt{T}$ is asymptotically distributed as normal,

$$(17) \quad \text{plim}_{T\to\infty} \frac{1}{T^2}(t_2 - \bar{t}_2)'Q_2 = 0,$$

$$(18) \quad \text{plim}_{T\to\infty} N^{-1}(S'S)N^{-1} = \begin{pmatrix} \frac{1}{12}(\lambda_2)^3 & 0 \\ 0 & C \end{pmatrix}$$

where $\lambda_2$ is the limit of the break ratio of the second sub-interval, and C= $\text{plim}_{T\to\infty} Q'Q/T$ is a non-singular fixed matrix since the equation (11) is a stationary autoregression. Using (18), (16) is approximated as

$$(19) \quad t_1^* = \{I - Q(Q'Q)^{-1}Q'\}\begin{pmatrix} t_1 - \bar{t}_1 \\ 0 \end{pmatrix}.$$

It is obvious that the trend is O(T $\sqrt{T}$ ), the second term in the right-hand side is $O_P$(T), and

$$\text{plim}_{T\to\infty} \frac{1}{T\sqrt{T}}\{T^* - \begin{pmatrix} t_1 - \bar{t}_1 & 0 \\ 0 & t_2 - \bar{t}_2 \end{pmatrix}\} = 0.$$

(QED)

This approximation is simple because $\tau_{t1}$ and $\tau_{t2}$ do not depend on the lagged values which vary from regression to regression. However, $\tau_{t1} + \tau_{t2}$ is not close to $\tau_t$ in our studies. The approximation is modified so that the effect of lagged dependent variables is removed from the trend terms. The proof of the corollary is obvious.

**COROLLARY**: Define $t_{(1)}^*$ the ($T_1$-1- L) column vector of residuals regressing the demeaned trend in the first sub-interval on $Q_1$ that is

$$(20) \quad t_{(1)}^* = \{I - Q_1(Q_1'Q_1)^{-1}Q_1'\}(t_1 - \bar{t}_1),$$

and $t_{(2)}^* = \{I - Q_2(Q_2'Q_2)^{-1}Q_2'\}(t_2 - \bar{t}_2)$ . Under the null hypothesis of the test where the regression is defined by (11), $\tau_t \cong \tau_{t1}^* + \tau_{t2}^*$ in the sense of (15) where

$$(21) \quad \tau_{t1}^* = \frac{\{\sum_{i=L+2,T_1} \Delta y_t t_{(1)i}^*\}^2}{\hat{\sigma}^2 \sum_{i=L+2,T_1} t_{(1)i}^{*\,2}},$$

and $\tau_{t2}^*$, t=$T_1$+m+1 to T, for the second interval.

Fig 1.1 Natural Log of Nominal GNP (A model)

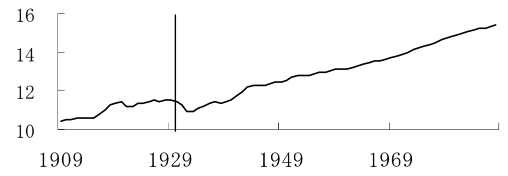



Fig 1.2 Difference and Time

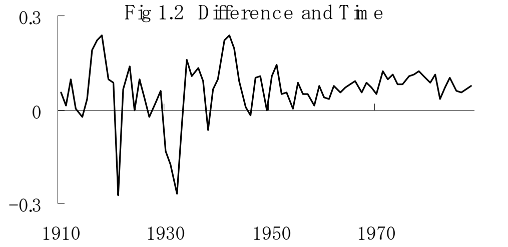



Fig 1.3 Difference and Level(-1)

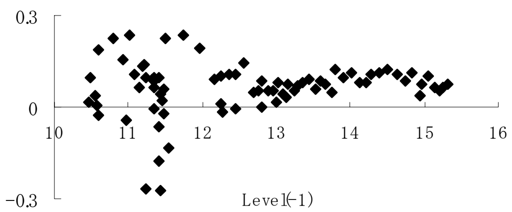
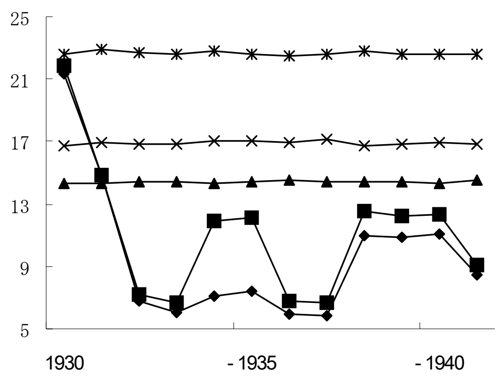



Fig 1.4 Interval $\Psi$ ratio and $\tau^2$




From the top, 1%, 5%, 10% simulated critical values, $\Psi$ and $\tau^2$ are plotted

## 5. US MACRO SERIES

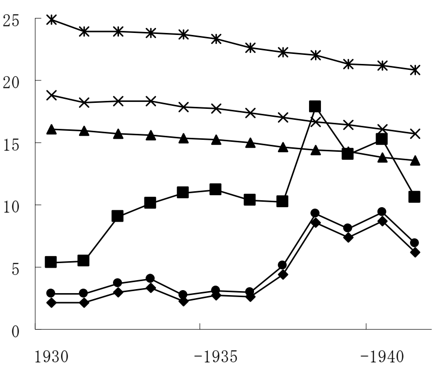
**NOMINAL GNP:** The test with a break interval is applied to the US nominal GNP series from 1909 to 1988. The lag orders of the augmented regressions are chosen by the same rule used by Perron. The starting year of the break interval is fixed at 1930 in our study for simplification. The highest lag order term is kept in the regression when its t-ratio is larger than 1.6, but removed when it is less than 1.6. The A model is used for this series as Perron did where the trend term is common for the both sub-intervals. Then, the decomposition of $\tau_t$ between $\tau_{t1}$ and $\tau_{t2}$ is not applicable. $\tau_t$ has only one degree of freedom in this case.

Figures 1.1 to 1.4 show the results on the natural log of nominal GNP. Figures 1.2 and 1.3 are the scatter diagrams between the first difference and the trend, and between the first difference and the lagged level variable, respectively. These figures give some idea about $\tau_t$ and $\hat{\tau}_\tau$, respectively. Both Figures show some negative relations, but it is more conspicuous in Figure 1.3. Figure 1.3 may imply $\hat{\tau}_\tau$ is significantly different from zero if it is calculated over the whole periods since the first difference decreases as the lagged level increases.

Values of $\Psi$ and the squared values of $\hat{\tau}_\tau$ are plotted in Figure 1.4. We can see these two values move closely except for 1930-1934 and 1930-1935 intervals. The break intervals start from a one-year interval 1930-1930 up to a twelve-year interval 1930-1941. The $\Psi$ ratio is almost significant by the 1% test in the one-year interval test, but is insignificant by the 10% test when the break interval is longer than two years. Since the nominal GNP returned to the 1930 level in 1938 as is seen by Figure 1.1, it may be fair to say that the unit root hypothesis cannot be rejected by the $\Psi$ test once the break interval is taken into account.

The difference between $\Psi$ and the squared $\hat{\tau}_\tau$ is $\tau_t$ defined by (12), and $\tau_t$ value is small in most cases. This is contrary to what is expected from Figure 1.2 since it shows some negative relation. Values of $\tau_t$ imply this relation is weak once the break interval and the lagged differenced variables are taken into account.

Fig 2 Interval $\Psi$ ratio and $\tau^2$




From the top, 1%, 5%, 10% simulated critical values, $\Psi$, $\tau_{t1}^* + \tau^2$, and $\tau^2$ are plotted

**REAL WAGE:** The next example is on the real wage series from 1900 to 1988. The lag order for this series is selected by the same rule as the nominal GDP, but the maximum lag order is six instead of twelve since longer lag orders resulted in positive $\phi$ values. Time series is explosive if $\phi$ is positive.

The $\Psi$ values are plotted in Figure 2 which are insignificant for most break intervals. Then, the null hypothesis of the unit root cannot be rejected by this test either. The $\Psi$ values are decomposed in Figure 2. The second curve from the bottom shows the variation of $\tau_{t1}{}^*$. It is almost negligible. The difference between $\Psi$ and the squared $\hat{\tau}_\tau$ is mostly explained by $\tau_{t2}{}^*$ which is significant. This gives some confusion on the interpretation of this time series. If $\phi$ is zero but $\beta_2^*$ not zero, the level series has quadratic trend in the second sub-interval. Alternatively, this series is stationary but $\phi$ value is almost zero.

Table 1 Unit Root Test of Real Wage

| Interval | 1930 | -1932 | -1934 | -1936 | -1938 | -1940 | -1942 |
|---|---|---|---|---|---|---|---|
| $\Psi$ | 5.3 | 9.1 | 10.9 | 10.3 | 17.9 | 15.2 | 6.2 |
| $\tau(\tau)$ | −1.5 | −1.7 | −1.5 | −1.6 | −2.9 | −3.0 | −1.6 |
| $\tau(\tau)^2$ | 2.2 | 3.0 | 2.3 | 2.6 | 8.6 | 8.7 | 2.5 |
| $\tau$(t1*) | 0.6 | 0.7 | 0.4 | 0.4 | 0.8 | 0.7 | 0.4 |
| $\tau$(t2*) | 2.5 | 5.4 | 8.2 | 7.3 | 8.4 | 5.7 | 3.4 |
| Error | 0.04 | 0.06 | 0.00 | 0.02 | 0.14 | 0.12 | 0.01 |
| $\tau(\beta\tau1)$ | 1.5 | 1.7 | 1.4 | 1.5 | 2.6 | 2.7 | 1.6 |
| $\tau(\beta\tau2)$ | 0.9 | 0.8 | 0.3 | 0.4 | 1.5 | 1.7 | 0.7 |

Symbols in the parentheses are given as subscripts in the text.
Error is caused by the approximation in the corollary. They are all very sma
$\tau(\beta\tau1)$ and $\tau(\beta\tau2)$ are the t-ratios of the trend terms which are the Dickey test statisitcs of the unit root. None of the values are significant.
$\tau(\tau2*)$ is significant when it is larger than 4 by the 5% test.

Table 1 tabulates results of various tests. The bottom two rows are not in the figures. They are the t-ratio of trend terms in each sub-interval. It seems to us that the deviation from the null hypothesis affects $\tau_{t1}{}^*$ and $\tau_{t2}{}^*$ more directly than these t-ratios of trend terms. The vectors that form $\hat{\tau}_\tau$, $\tau_{t1}{}^*$ and $\tau_{t2}{}^*$ are orthogonal. This property does not hold in $\hat{\tau}_\tau$ and $\tau_{\beta\tau}$. The row in Table 1 that is titled "error" tabulates the difference ($\Psi - \tau_{t1}{}^* - \tau_{t2}{}^*$). Errors cannot be avoided since the corollary gives only an approximation. However, these errors are small. Further, $\tau_{t2}{}^*$ is larger than 4 once the break interval is taken longer than three years. In Figure 2, $\tau_{t2}{}^*$ is the difference between the $\Psi$ and the ($\tau_{t1}{}^* + \tau_{t2}{}^*$) curves. The $\tau_{t1}{}^*$ values are very small for all break intervals.

## 6 CONCLUSION

It is reasonable to specify a break interval instead of a break point in the test so that the test has a higher probability of including the true break point in the interval. This is also natural since, for example, the exact break point caused by the oil shock in 1974 cannot be exactly specified but an interval of a few years is easy to be set. Setting a break interval does not guarantee avoiding a miss specification but it makes plausible to avoid one. If the true break point is in a break interval, all bias of the test is avoided even though the power of the test is less than that of a correct break point test.

A formal procedure to select a proper interval is not proposed by this paper. We have studied only the effect of break intervals on the test. A typical example where the break interval and the break point tests show opposite results is found in the nominal GNP series. It can be concluded that the tests are susceptible to the choice of a break point, and it is reasonable to set a break-interval.

A decomposition of the F ratio type unit root test statistic was also proposed. This decomposition is useful in studying the effect of the two trend terms in the regression equation under the alternative hypothesis. ((4), (6), (10) and (12).)

The Zivot and Andrews [1992] test avoids specifying the break point prior to the test, and the break point is statistically determined by the first round test. However, there is no guarantee that the chosen break point is correct. It may be appropriate to set an interval that covers a break point chosen by the Zivot and Andrew test.

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