A DSS for Water Resources Management under Uncertainty

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Abstract: In this paper we present a scenario analysis approach to perform water system planning and management under climatic and hydrological uncertainty. A DSS with a a graphical interface allows the user a friendly data-input phase and results analysis.

Different generation techniques can be used to set up and analyze a number of scenarios. Uncertainty is modeled by a scenario-tree in a multistage environment, which includes different possible configurations of inflows in a wide time-horizon. The aim is to identify trends and essential features on which to base a robust decision policy.

The DSS prevent obsolescence of optimizer codes exploiting the standard input format MPS. Obtained results show that scenario analysis could be an alternative approach to stochastic optimization when no probabilistic rules can be adopted and deterministic models are inadequate to represent uncertainty. Moreover, experimentation to a real water resources system in Sardinia, Italy, shows that the DSS can be easily used by practitioners and end-users.

Keywords: scenario analysis; multiperiod dynamic network; optimization under uncertainty.

INTRODUCTION

Water Resources (WR) management problems with a multiperiod feature are associated to mathematical optimization models that handle thousands of constraints and variables depending on the level of adherence required to reach a significant representation of the system. See Loucks et al. [1981], Yeh [1985].

Moreover these problems are typically characterized by a level of uncertainty about the value of hydrological exogenous inflows and demand patterns. On the other hand inadequate values assigned to them could invalidate the results of the study. When the statistical information on data estimation is not enough to support a stochastic model or when probabilistic rules are not available, an alternative approach could be in practice that of setting up the scenario analysis technique. Dembo [1991], Rockafellar [1991]. In WR analysis a scenario can represent a possible realization of some sets of uncertain data in the examined time horizon.

In this paper we present a general-purpose scenario-modeling framework to solve water system optimization problems under input data uncertainty, as an alternative to the traditional stochastic approach in order to reach a "robust" decision policy that should minimize the risk of wrong decisions. In the proposed approach, the problem is to be expanded on a set of scenario sub-problems, each of which corresponding to a possible configuration of the data series. By studying the global-scenarios solution, one should discover similarities and trends that should quantify the risk of management operations. Each scenario can be weighted to represent the "importance" assigned to the running configuration. Sometimes the weights can be viewed as the probability of occurrence of the examined scenario. A "robust-barycentric" solution can be then obtained by a postprocessor phase applied to sub-problems solutions.

A WR model is usually defined in a dynamic planning horizon in which management decisions have to be made either sequentially, by adopting a predefined scenario independently.

Extending the analysis to a set of scenarios, an *aggregation condition* will guarantee that the solution referred to a period t is independent of the information that is not yet available. In other words, model evolution is only based on the information available at the moment, a time when the future configuration may diversify.

The availability, in the proposed DSS, of an efficient computer graphical interfaces, designed to facilitate the use of models and database, help end-users to evaluate the best choice in a friendly-to-use way starting from physical system to reach a robust solution.
The proposed tool can perform scenario analysis by generating a scenario-tree structure. It allows the exploitation of the state-of-art efficient computer codes for general purpose mathematical programming supporting up to thousand of variables and constraints.
The tool is a greatly improved version of the DSS WARGI, presented by authors in Sechi and Zuddas[2000].

1. WATER RESOURCES DYNAMIC MODEL

In this section we formulate a water resources management model in a deterministic framework, i.e. having a previous knowledge of the time sequence of inflows and demand. We extend the analysis to a sufficiently wide time horizon and assuming a time step (period), ***t***. The scale and number of time-steps considered must be adequate to reach a significant representation of the variability of hydrological inflows and water demands in the system.
Referring to a "static" or single-period situation, we can represent the physical system by a direct network (basic graph), derived from the physical sketch. Nodes could represent sources, demands, reservoirs, groundwater, diversion canal site, hydropower station site, etc. shows a physical sketch and the basic graph of a simple water system. A dynamic multiperiod network derived by replicating the basic graph for each period supports the dynamic problem. We then connect the corresponding reservoir nodes for different consecutive periods by additional arcs carrying water stored at the end of each period. Figure 1 shows a segment of a dynamic network generated by a simple basic graph.

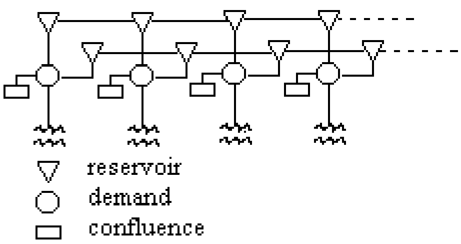

Figure 1. Segment of a dynamic network

Even if the aim of this paper is not to detail the components of the mathematical model, we give a formulation of a reduced model that can be adopted to formalize the uncertainty in water resources management. To illustrate our approach, we adopt a deterministic Linear Programming (LP) described in the next section.

1.1 Definition of Water Resources Optimization Model Components

Even if it is quite impossible to define a general mathematical model for water resources planning and management problem, our DSS allows to take into account the components of a system as general as possible based on the most typical characterization of this type of models. Different components can be considered or ignored updating constraints and objective. In this paper we describe only some of them due to limited space allowed. More detailed description of this approach can be found in Sechi and Zuddas [2000], Onnis et al. [1999]. In the following we refer to the dynamic network ***G = (N, A)*** where ***N*** is the set of nodes and ***A*** is the set of arcs. ***T*** represents the set of time-steps ***t***.

1.1.1 Identification of Hydraulic Network Components and Sets.

Nodes (subsets of ***N***):

res set of reservoir nodes: these represent surface water resources with storage capacity.

dem set of demand nodes: such as for civil and industrial irrigation among others. They can be consumptive or non-consumptive water demand nodes

hyp set of hydroelectric nodes: they are non-consumptive nodes associated with hydroelectric plants

con set of confluence nodes: such as river confluence, withdraw connections for demands satisfaction, etc.

Other sets of nodes can represent groundwater, desalinization, wastewater-treatment plant, among others.

Arcs: (subsets of ***A***)

R set of weighted arcs: these represent arcs whose flow produce a cost or a benefit per unit of flow, such as conveyance work arcs, artificial channels among others;

TRF set of transfer arcs: these represent transfer works in operational or in project state;

Other sets of arcs are present in the tool referred to emergency transfers, spilling arcs, among others.

1.1.2 Required Data

Data marked with (+) refer to operational state (existing works with a known dimension) while data marked with (*) are refer project state (works to be constructed). Unmarked data refer to operational and project state.

Required data for a reservoir j:

Y_{jmax} (+) max storage volume for inter-periods transfer.

ρ^t_{jmax} (+) ratio between max volume usable in each period t and the reservoir capacity.

ρ^t_{jmin} (+) ratio between min stored volume in each period and reservoir capacity.

δ_j gradient of the relationship between the reservoir surfaces and volumes.

l_j evaporation losses per unit of reservoir surface.

inp^t_j hydrological input to the reservoir

c^t_j (+)spilling cost

M_j (*) max allowed capacity

m_j (*) min allowed capacity

γ_j (*) construction costs;

required data for a civil demand j:

P_j (+) population

d^t_j (+) unitary demand

π^t_j (+) request program

c_j deficit cost;

P_{jmax} (*) max population

P_{jmin} (*) min population

β_j (*) net construction benefits;

in the same way can be defined required data for industrial and irrigation demands;

required data for a hydroelectric power station j:

H_j (+) production capacity.

π^t_j (+) production program .

b_j production benefit.

H_{jmax} (*) max production capacity

H_{jmin} (*) min production capacity

γ_j (*) construction cost;

required data for a confluence node j :

I^t_j hydrologic input (if arcs are natural streams);

required data for a transfer arc a:

F_a (+) transfer capacity

ρ^t_{amax} (+) ratio between max transferred volumes and capacity

ρ^t_{amin} (+) ratio between min transferred volumes and capacity

c_a operating cost.

F_{amax} (*) max transfer capacity

F_{amin} (*) min transfer capacity

γ_a (*) construction cost

1.1.3 Decision Variables and Constraints

Variables considered in the LP model can be divided in flow and project variables. Flow variables can refer to different type of water transfer as: water-transfer in space along arc connecting different nodes at the same time, water transfer in arc connecting homologous nodes at different time and so on. Project variables refer to the project state and they are associated to the dimension of future works: reservoirs capacities, pipes dimensions, irrigation areas, etc. Constraints in the LP model can represent: mass balance equations, demands for the centers of water consumption, evaporation at reservoirs. relations between flows variables and planning works, upper and lower bounds on decision variables. In what follows some variables and corresponding constraints are described in more details.

y^t_j portion of stored water at reservoir k at the end of period t that can be used in next periods. The corresponding constraints, for each time period t is:

$$\rho^t_{kmin} Y_{jmax} \leq y^t_j \leq \rho^t_{jmax} Y_{jmax} \quad \forall j \in \textbf{\textit{res}}$$

this constraints ensures that, in each period, the used volume of the reservoir k be in the prescribed range. In a operational state is a data while in a project state it is a decision variable. In the last case it is bounded by:

$$m_j \leq Y_{jmax} \leq M_j \,, \qquad \forall j \in \textbf{\textit{res}}$$

p^t_j water demand at civil demand center j in period t. The corresponding constraints, for each time period t is:

$$p^t_j = \pi^t_j \, d^t_j P_j \,, \qquad \forall j \in \textbf{\textit{dem}}$$

this constraints ensures the fulfillment of the demand in each period, no matter if coming from the system or from a dummy resources. In a operational state P_j is a data while in a project state it is a decision variable. In the last case it is bounded by:

$$P_{jmin} \leq P_j. \leq P_{jmax} \,, \qquad \forall j \in \textbf{\textit{dem}}$$

Variable and constraints are defined in the same way for other demand sets.

h^t_j water trough hydropower plant j (hydroelectric power station) in period t. The corresponding constraints, for each time period t is:

$h^t_j \leq \pi^t_j \, \alpha_j H_j$, $\forall j \in$ ***hyp***

this constraints expresses the dependence of flow on production capacity. In a operational state H_j is a data while in a project state it is a decision variable. In the last case it is bounded by:

$H_{jmin} \leq H_j. \leq H_{jmax}$, $\forall j \in$ ***hyp***

x^t_a flow on arc a. In case of a transfer arc a, the corresponding constraints, for each time period t is:

$\rho^t_{amin} F_a \leq x^t_a \leq \rho^t_{amax} F_a$, $\forall a \in$ ***TRF***

this constraints ensures that, in each period, the transferred volume in arc a be in the prescribed range. In a operational state F_a is a data while in a project state it is a decision variable. In the last case it is bounded by:

$F_{a\,min} \leq F_a. \leq F_{amax}$, $\forall a \in$ ***TRF***

Variable and constraints are defined in the same way for other arc sets. Referring to the multiperiod dynamic network structure, mass balance constraints are defined in each node $i \in \boldsymbol{N}$. Moreover, lower and upper bounds constraints are defined in some arcs $a \in \boldsymbol{A}$ to represent some particular limits as for transfer arcs ***TRF***.

1.1.4 Objective Function

The objective function considers weights on variables, that is costs and benefits as well as penalties, associated to flow and project variables. Following simplified notation defined in this paper the objective function is the following:

$\Sigma_{j \in \boldsymbol{res}} \gamma_j Y_{jmax}$ - $\Sigma_{j \in \boldsymbol{dem}} \beta_j P_j$ + $\Sigma_{j \in \boldsymbol{dem}} \gamma_j H_j$ + $\Sigma_{a \in \boldsymbol{TRF}} \gamma_a F_a$ + $\Sigma_{a \in \boldsymbol{R}} c_a x^t_a$

1.2 Compact Deterministic LP Model

As is well known in LP theory, the described mathematical model can be expressed in a compact standard form as follows:

min $\boldsymbol{c\,x}$
s.t. $\boldsymbol{A\,x} = \boldsymbol{b}$
$\boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u}$

where:
$\boldsymbol{x}$ represents the vector comprehensive of all operating and projects variables;
$\boldsymbol{c}$ represents the vector comprehensive of all weights on operating and projects variables ;
$\boldsymbol{l}$ and $\boldsymbol{u}$ represent vectors comprehensive of all lower and upper bounds on operating and projects variables;
$\boldsymbol{b}$ represent the vector of R.H.S.;
$\boldsymbol{A\,x} = \boldsymbol{b}$ represent the set of all constraints.
In what follows for the sake of simplicity we adopt the compact model to illustrate the scenario approach.

2. WATER RESOURCES CHANCE DYNAMIC MODEL

The presented model is named chance-model to put in evidence that it is not stochastic based but, due to the impossibility to adopt probabilistic rules, try to represent a set of possible performances of the system as uncertain parameters vary. When a set of different and independent scenarios are generated, the structure of the chance-model is based on *scenario aggregation* condition generating a graph structure named "scenario tree".

2.1 Further Components in Chance Model and Scenario-tree Generation

Data defined for deterministic model are required for each scenario in the chance model plus the further data:

G set of synthetic hydrological sequences (parallel scenarios)
w_g weight assigned to a scenario $\mathbf{g} \in \mathbf{G}$

Figure 2.a shows a set of nine parallel scenarios before aggregation. Each dot represents the system in a time-period. Figure 2.b shows an example of the scenario-tree derived from the parallel sequences.
To perform scenario aggregation a number of stages are defined, where stage *0* corresponds to the initial hydrological characterization of the system up to the first branch time-period. In the scenario-tree this represents the root. In stage *1* a number, β_1 (3 in the figure) , of different possible hydrological configurations can occur, in stage *2* a number, $\beta_1 * \beta_2$ (9 in the figure), can occur, and so on and so forth.
The figure represents a tree with two branches: the first branching-time is the 4th time-period, the second is the 8th period. In time periods that precede the first branch, all scenarios are gathered in a single bundle and three bundles are operated at second branch. The zero bundle includes a group of all scenarios; in the 1st stage 3 bundles are generated including 3 scenarios in each group, while in the 2nd stage the 9 scenarios run until they reach the end of the time-horizon.

Finally, the main rules adopted to organize the set of scenarios are:

Branching: to identify branching-times τ as time-periods in which to apply bundles on parallel sequences, while identifying the stages in which to divide the scenario horizon.

Bundling: to identify the number, β_τ, of bundles at each branching-time.

Grouping: to identify groups, Γ_τ of scenarios to include in each bundle.

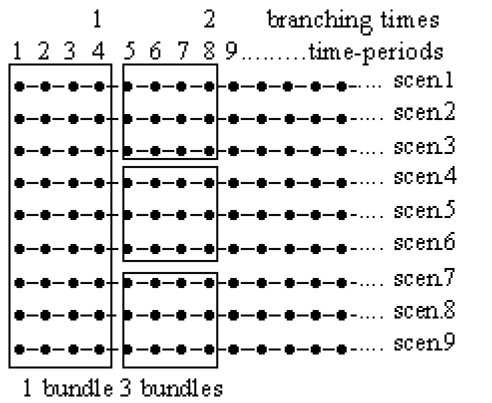

Figure 2.a: Set **G** of nine parallel scenarios

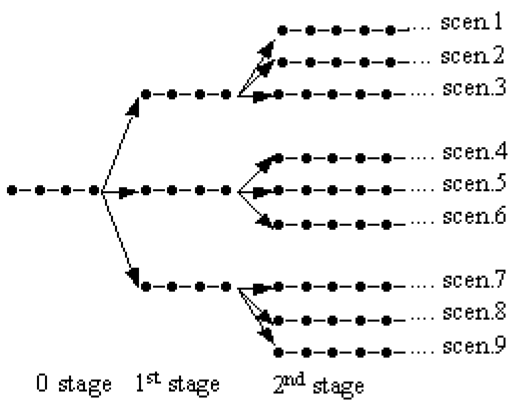

Figure 2.b: Scenario-tree aggregation

2.2 The Chance Mathematical Model

The chance model describe the collection of one deterministic model for each scenario $\mathbf{g} \in \mathbf{G}$ plus a set of congruity constraints representing requirements of equal inter-stage flow transfers in all scenarios between two consecutive stages. The stochastic model will have the following structure:

$$
\begin{aligned}
& \min \Sigma_g\, w_g\, c_g\, x_g \\
& s.t. \\
& \qquad A_g x_g = b_g \qquad\qquad \forall g \in G \\
& \qquad x_g \geq 0 \\
& \qquad x^* \in S
\end{aligned}
$$

where $x^* \in S$ represents the linking constraints on inter-stage flows. An alternative formulation of the objective function can be expressed as

$$\min \quad \Sigma_g\, w_g \,\| c_g (x_g - x_g^*) \|$$

where x_g^* is an optimal policy expected by water manager.

This kind of model can be solved by decomposition methods such as Benders decomposition techniques, which exploit the special structure of constraints. Cai et al. [2001]. When the size of the problem becomes huge, it is possible to resort to parallel computing.

The resolution approach can be described as a three-phase algorithm:

– scenario-tree generation and identification of the chance model;
– resolution of the chance model. At the end of this phase we obtain a solution-set $x_G = U_g\, x_g$;
– obtaining a "robust" solution by a postprocessor on the solution-set .

The postprocessor refers to identify the most performable solution or the most profitable solution or the most "barycentric" solution, and so on, depending on the features of the system and on the end-user point of view.

3. USING DSS FOR WR

The Decision Support System has been developed in order to

– be friendly to use in input phase, in scenarios setting and in processing output results;
– be easy to modify system configuration and related data to perform sensitivity analysis and to process data uncertainty;
– prevent obsolescence of the optimizer exploiting the standard input format in optimization codes.

A graphical interface allows performing scenario analysis starting from physical system following the main steps:

– time period definition and scenario settlement;
– system elements characterization;
– connections topology and transfer constraints.
– links to hydrological data and demand requirements files;
– planning and management rules definition;
– benefits and costs attribution;
– call to optimizers;
– output processing.

The DSS has been developed and tested within an HP-Unix and PC-Linux environment. The various software components have been coded in C++ and TCL-TK graphic language.

4. TEST CASE

Following the three-phase algorithm described in section 2.2, aided by the DSS, scenario analysis was performed on the Flumendosa-Campidano system, Sardinia, Italy. A correct evaluation of the system performances and requirements became increasingly urgent, as the system managers were obliged to face the serious resource deficits caused by the drought events of the past decade. Different hydrological and demand scenarios therefore must be considered to obtain system optimization. A synthetic series has been generated with different techniques, starting from a database of a time-horizon of 75 years, corresponding to 900 monthly time-periods. A set of 30 scenarios was then submitted to statistical validation and selected. Scenario analysis was performed on a scenario-tree of 3 stages up to 30 leaves. Since each scenario involves about 3,000 variables, the chance model supports several thousand variables and constraints.

5. CONCLUSION AND PERSPECTIVES

This paper is aimed to give a contribution to the mathematical optimization of water resources systems, when the role of uncertainty is particularly important. In such a problem, which involves social, economical, political, and physical events, no probabilistic description of the unknown elements is available, either because a substantial statistical base is lacking or because it is impossible to derive a probabilistic law from conceptual considerations. Another not secondary aim planning the presented approach for WR analysis is to create a tool to help water managers in a DSS context, friendly to use but able to take into account the improvements made in the field of computer science and operation research. The state of the art in Mathematical Programming codes evolves continuously producing algorithms that improve computational efficiency thanks to new methodologies and computer science development. See CPLEX [1993]. The standard input format allows to insert the best state-of-the-art codes in the DSS.

Moreover, experimentation, with regional water managers on a real water resources system in Sardinia, Italy, has been performed showing that practitioners and end-users can adopt the DSS as a useful aid in decision making

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