Structures and Performances of Five Rainfall-Runoff Models for Continuous River-Flow Simulation

M. Goswami

K. M. O'Connor

A. Y. Shamseldin

Structures and Performances of Five Rainfall-Runoff Models for Continuous River-Flow Simulation

Goswami, M.[a], O'Connor, K.M.[a] and Shamseldin, A.Y.[b]

[a] *Department of Engineering Hydrology, National University of Ireland, Galway, Ireland*

[b] *Department of Civil Engineering, The University of Birmingham, Edgbaston, Birmingham B15 2TT, U.K.*

Abstract: Four black-box-type rainfall-runoff models, namely, the Simple Linear Model, the seasonally-based Linear Perturbation Model, the wetness-index-based Linearly Varying Gain Factor Model, and the Artificial Neural Network Model, along with the conceptual Soil Moisture Accounting and Routing Model, were used for this study. The models exhibit a considerable range of variation in degree of structural complexity and associated parameter parsimony, with corresponding degrees of complication in objective function evaluation. Hence they represent a family of models suitable for application in both developed and developing countries. Operating in continuous river-flow simulation mode, these models and techniques were applied to six test catchments representing wide variability in geographic location, climatic condition, areal extent and physiographical characteristics. A number of performance evaluation criteria were used to comparatively assess model efficiency. The 'Galway Real-Time River Flow Forecasting System' software package, developed at the Department of Engineering Hydrology, of the National University of Ireland, Galway, was used to produce all the numerical results presented in the study.

Keywords: Black-box model, Conceptual model, Simulation, River flow forecasting system, Neural network

1. INTRODUCTION AND THE OBJECTIVE OF THE STUDY

In recent decades, the advent of increasingly efficient computing technology has provided hydrologists with exciting new tools for the mathematical modelling of hydrological systems including, but extending far beyond, the more traditional river-flow forecasting applications. Elaborate physically-based distributed modelling, and elegant mathematical techniques using Artificial Neural Networks, Fuzzy systems, Wavelets, etc. are being used, all with high levels of complexity, but not necessarily with increased levels of efficiency attainment, particularly in the context of flow forecasting. Most such exercises are certainly significant from a research point of view, as they attempt to throw more light on the physical processes involved, but data demands, lack of parsimony in model parameters, and structural complexity can still be a major deterrent when it comes to applying these models in real-life problem solving. In the discharge forecasting context, even simple black-box type system-theoretic models, or physically-inspired lumped conceptual models, can produce better and more reliable discharge forecasting results than complex distributed models.

The Galway River Flow Forecasting System (GFFS) is a software package developed at the Department of Engineering Hydrology, National University of Ireland, Galway [O'Connor et al, 2001]. It comprises a suite of models for simulation, updating and real-time forecasting applications. The degree of structural complexity, associated parameter parsimony, and difficulty in objective function evaluation of these models, varies considerably. The models and techniques used in the present study, all from the GFFS package, are applied to six test catchments representing wide variability in geographic location, climatic conditions, areal extent and various physiographical characteristics. Five performance evaluation criteria are used to assess model efficiency.

2. THE MODELS USED

Three system-theoretic black-box models, an Artificial Neural Network Model and a simple conceptual Soil Moisture Accounting & Routing Model were used. For completeness, brief descriptions of these models are provided in this section.

2.1 The Simple Linear Model (SLM)

The intrinsic hypothesis of the naïve SLM, introduced by Nash and Foley [1982], is the assumption of a linear time-invariant relationship between the total rainfall R_i and the total discharge Q_i. In discrete form, the SLM, is expressed by the convolution summation relation [Kachroo and Liang, 1992],

$$Q_i = \sum_{j=1}^{m} R_{i-j+1} h'_j + e_i \quad = \quad G \sum_{j=1}^{m} R_{i-j+1} B_j \tag{1}$$

where $\sum_{j=1}^{m} B_j = 1$ and Q_i and R_i are the discharge and rainfall respectively at the *i-th* time-step, h'_j is the *j-th* discrete pulse response ordinate or weight, m is the memory length of the system, G is the gain factor, and e_i is the forecast error term.

2.2 The Linear Perturbation Model (LPM)

In the LPM [Nash and Barsi, 1983], it is assumed that, during a year in which the rainfall is identical to its seasonal expectation, the corresponding discharge hydrograph is also identical to its seasonal expectation. However, in all other years, when the rainfall and the discharge values depart from their respective seasonal expectations, these departures series are assumed to be related by a linear time invariant system. The relation between the departure (i.e. perturbation) series of the LPM has the convolution summation form

$$Q'_i = \sum_{j=1}^{m} R'_{i-j+1} h'_j + e_i \tag{2}$$

where R'_i and Q'_i are the respective departures of rainfall and discharge from their seasonal expectations and e_i is the error output term. Model-estimated departure values are added to the seasonal expectations to give the estimated discharge series.

2.3 The Linearly Varying Gain Factor Model (LVGFM)

The LVGFM, proposed by Ahsan and O'Connor [1994] for the single-input to single-output case, involves only the variation of the gain factor with the selected index of the prevailing catchment wetness, but not the shape (i.e. the weights) of the response function. Using a time-varying gain factor G_i, the model output has the structure

$$Q_i = G_j \sum_{j=1}^{m} R_{i-j+1} B_j, \text{ where } \sum_{j=1}^{m} B_j = 1 \tag{3}$$

In its simplest form, G_i is linearly related to an index of the soil moisture state z_i by the equation $G_i = a + bz_i$, where a and b are constants. The value of z_i is obtained from the outputs of the naïve SLM, operating as an auxiliary model, using

$$z_i = \frac{\hat{G}}{\overline{Q}} \sum_{j=1}^{m} R_{i-j+1} \hat{h}_j \tag{4}$$

where $\hat{G}$ and $\hat{h}_j$ are estimates of the gain factor and the pulse response ordinates respectively of the SLM and $\overline{Q}$ is the mean calibration discharge.

2.4 The Artificial Neural Network Model (ANNM)

The *"multi-layer feed-forward network"* type of artificial neural network, used in this study, consists of an input layer, an output layer and only one *"hidden"* layer located between the input and the output layers [Shamseldin, 1997]. Each neuron of a particular layer has connection pathways to all the neurons in the following adjacent layer, but none to those of its own layer or to those of the previous layer (if any). Likewise, nodes in non-adjacent layers are unconnected. In the output layer, there is only one neuron, for the single output. Because the neural network itself does not incorporate storage effects, storage is implicitly accounted for by the use of the output series of the naïve SLM. For a neuron either in the hidden or in the output layer, each received input y_i is transformed to its output y_{out} by the mathematical transfer function

$$y_{out} = f\left(\sum_{i=1}^{M} w_i y_i + w_o\right) \tag{5}$$

where *f()* denotes the transfer function, w_i are the input connection pathway weights, M is the total number of inputs (which equals the number of neurons in the preceding layer), and w_o is the neuron threshold (or bias). The non-linear transfer function adopted for the neurons of the hidden and output layers is the widely-used logistic/sigmoid function

$$\mathrm{f}\left(\sum_{i=1}^{M} w_i y_i + w_o\right) = \frac{1}{1 + \mathrm{e}^{-\sigma\left(\sum_{i=1}^{M} w_i y_i + w_o\right)}} \tag{6}$$

bounded in the range [0,1]. The neuron weights w_i, the threshold w_o and σ can all be interpreted as parameters of the network configuration.

2.5 The Soil Moisture Accounting And Routing (SMAR) Model

The SMAR Model is a development of the 'Layers' conceptual rainfall-runoff model introduced by O'Connell et al. [1970], its water-balance component having been proposed in 1969 by Nash and Sutcliffe [Clarke, p.307, 1994]. Using a number of empirical and assumed relations, which are considered to be at least physically plausible, the non-linear water balance (i.e. soil moisture accounting) component ensures satisfaction of the continuity equation, over each time-step. The routing component, on the other hand, simulates the attenuation and the diffusive effects of the catchment by routing the various generated runoff components through conservative linear time-invariant storage elements. For each time-step, the combined output of the two routing elements adopted (i.e. one for generated *'surface runoff'* as input and the other for generated *'groundwater runoff'* as input) becomes the simulated discharge forecast. The variant of the SMAR model applied on all catchments, except Fergus, has nine parameters, while that applied on that karstic catchment has ten [Khan, 1986; Kachroo, 1992, a & b; Liang, 1992].

3. THE FIVE MODEL EFFICIENCY EVALUATION CRITERIA USED

Five performance evaluation criteria have been used in the study [Kachroo, 1992a; Legates and McCabe, 1999; Beran, 1999]

The coefficient of efficiency [Nash and Sutcliffe, 1970], is defined by the dimensionless expression

$$R^2 = 1 - (MSE/F_o), \qquad (7)$$

with
$$F_o = \frac{1}{N}\sum_1^N\left[(Q_o)_i - \overline{Q}_c\right]^2 \qquad (8)$$

and
$$MSE = \frac{1}{N}\sum_1^N\left[(Q_o)_i - (Q_e)_i\right]^2 \qquad (9)$$

MSE being the mean square error. In expressions (8) and (9), $(Q_o)_i$ is the observed discharge and $(Q_e)_i$ the estimated discharge at the i^{th} time step, N is the total number of discharge values, and $\overline{Q}_c$ the mean of the $(Q_o)_i$ series over the calibration period.

The index of agreement, IoA, is defined as [Willmott, 1981]

$$IoA = 1.0 - \frac{\sum_1^N\left[(Q_o)_i - (Q_e)_i\right]^2}{\sum_{i=1}^N\left(\left|(Q_o)_i - \overline{Q}_c\right| + \left|(Q_e)_i - \overline{Q}_c\right|\right)^2} \qquad (10)$$

in which the numerator is N times the MSE and the denominator is called the potential error. The other symbols have the same meaning as for R^2.

The coefficient of determination, r^2, is given by

$$r^2 = \left[\frac{\sum_1^N\left[(Q_o)_i - \overline{Q}_o\right]\left[(Q_o)_i - \overline{Q}_e\right]}{\left\{\sum_{i=1}^N\left[(Q_o)_i - \overline{Q}_o\right]\right\}^{0.5}\left\{\sum_{i=1}^N\left[(Q_e)_i - \overline{Q}_e\right]\right\}^{0.5}}\right]^2 \qquad (11)$$

where $\overline{Q}_o$ and $\overline{Q}_e$ are the mean of the observed and the estimated discharge data series over the data period considered, and the other symbols have the same meanings as given above.

The index of volumetric fit, IVF, the ratio of the total volume of $(Q_e)_i$ to the total volume of $(Q_e)_i$, is

$$IVF = \sum_{i=1}^N (Q_e)_i \,/\, \sum_{i=1}^N (Q_o)_i \qquad (12)$$

The relative error of the peak (RE) is defined as

$$RE = \frac{\left|(Q_p)_e - (Q_p)_o\right|}{(Q_p)_o} \qquad (13)$$

$(Q_p)_o$ and $(Q_p)_e$ being the observed and estimated peak flows respectively.

4. THE TEST CATCHMENTS

Six test catchments were used in this study. These are Fergus (562 km^2) and Brosna (1,207 km^2) in Ireland, Sagana (2,365 km^2) in Kenya, Sunkosi-1 (18,000 km^2) in Nepal, Halda (779 km^2) in Bangladesh, and Baihe (61,780 km^2) in China. Topographically, Fergus is predominantly flat with karstic features, Brosna is flat, Sagana and Sunkosi-1 are hilly, and Halda and Baihe are mixed. Regarding vegetation, Fergus has farmland, with some scrubland, coniferous plantation, natural woodland and mixed woodland, Brosna has peat bogs with little woodland, Sagana has forest, grassland and tea plantations, Sunkosi-1 has forest and grassland, Halda has trees (20%) and rice fields, and Baihe has mixed vegetations. Climatologically, Fergus and Brosna are temperate, Sagana is tropical, Sunkosi-1 is semi-arid, influenced by monsoons, Halda is tropical, influenced by monsoons, and Baihe is semi-arid.

5. RESULTS FROM MODEL TESTS AND DISCUSSION OF THE RESULTS

Each of the five basic models is applied to each of the six test catchments, using split-record evaluation, involving the use of calibration and verification periods (about two-thirds for calibration and one-third for verification). In terms of increasing complexity, the SLM is the simplest, followed by the LPM and the LVGFM. These three are system-theoretic in structure, and ordinary least squares solution is used for estimation of the pulse response function. The SMAR model parameters are estimated using the simplex method of optimisation. As for the ANNM, the number of weights depends on the number of neurons chosen in the input layer and the hidden layer. If *'l'* is the total number of neurons in the input layer and *'m'* is that in the hidden layer, then the total number of weights to be estimated is $(l+1)m + (m+1)$. As such, the ANNM is the least parsimonious amongst the models chosen in the study. The simplex method of automatic optimisation is also used for calibration of the ANNM. The results of performances of the five substantive models are shown in Tables 2a. and 2b.

Table 2a. Calibration and verification results from different rainfall-runoff models

Model	Baihe (61,780 km^2) China						Brosna (1,207 km^2) Ireland						Fergus (562 km^2) Ireland					
	R^2	IoA	r^2	IVF	RE	Rank	R^2	IoA	r^2	IVF	RE	Rank	R^2	IoA	r^2	IVF	RE	Rank
Calibration																		
SLM	.704	.904	.707	1.076	.517	5	.409	.742	.409	1.007	.500	5	.695	.897	.708	1.075	.241	5
LPM	.745	.922	.745	1.000	.480	4	.708	.911	.709	1.015	.447	2	.877	.967	.877	1.011	.168	2
LVGFM	.867	.965	.873	0.870	.005	1	.428	.772	.428	0.989	.297	3	.736	.922	.737	1.002	.047	3
ANNM	.841	.955	.841	1.004	.090	3	.424	.769	.425	1.029	.469	4	.735	.919	.728	1.006	.067	4
SMAR	.842	.957	.843	1.000	.310	2	.844	.958	.846	1.009	.262	1	.977	.994	.978	1.023	.081	1
Verification																		
SLM	.706	.919	.737	1.372	.332	4	.470	.728	.563	.867	.544	5	.771	.921	.798	1.004	.233	5
LPM	.733	.928	.747	1.260	.320	3	.785	.921	.839	.874	.493	2	.911	.974	.917	0.957	.169	2
LVGFM	.635	.937	.871	1.182	.442	5	.502	.760	.582	.858	.466	4	.807	.945	.808	0.970	.102	4
ANNM	.817	.957	.849	1.297	.137	1	.522	.776	.591	.878	.511	3	.810	.944	.813	0.953	.017	3
SMAR	.757	.949	.846	1.335	.069	2	.865	.958	.877	.949	.343	1	.982	.995	.983	1.029	.035	1

Table 2b. Calibration and verification results from different rainfall-runoff models

Model	Halda (779 km^2) Bangladesh						Sagana (2,365 km^2) Kenya						Sunkosi – 1 (18,000 km^2) Nepal					
	R^2	IoA	r^2	IVF	RE	Rank	R^2	IoA	r^2	IVF	RE	Rank	R^2	IoA	r^2	IVF	RE	Rank
Calibration																		
SLM	.799	.941	.799	1.014	.129	5	.695	.894	.707	1.063	.311	5	.834	.954	.835	0.977	.390	5
LPM	.824	.950	.824	1.002	.104	2	.744	.922	.744	1.005	.284	4	.920	.979	.920	1.000	.286	1
LVGFM	.814	.947	.815	0.951	.059	3	.756	.929	.758	0.969	.041	2	.858	.963	.864	0.931	.074	3
ANNM	.800	.940	.800	1.042	.218	4	.768	.931	.768	.996	.193	1	.857	.960	.857	1.000	.339	4
SMAR	.840	.954	.841	1.000	.025	1	.744	.927	.748	1.000	.099	3	.900	.975	.905	0.979	.289	2
Verification																		
SLM	.729	.942	.818	1.361	.125	5	.726	.894	.771	0.908	.621	5	.822	.939	.862	.786	.358	5
LPM	.775	.952	.852	1.327	.129	2	.758	.917	.774	0.891	.588	4	.909	.972	.935	.829	.236	1
LVGFM	.763	.949	.829	1.250	.318	3	.826	.947	.832	0.909	.420	1	.823	.941	.876	.732	.281	4
ANNM	.742	.942	.822	1.431	.238	4	.821	.944	.829	0.925	.534	3	.839	.944	.888	.794	.307	3
SMAR	.841	.966	.894	1.229	.088	1	.821	.948	.821	1.120	.472	2	.860	.956	.887	.729	.297	2

From these results, it is clear that the simulation performance of the naïve SLM is, in each case, inferior to that of all other models. As expected, the LVGFM, which is a modification of the SLM, incorporating an element of linear variation of the gain factor G_i with the catchment wetness index z_i at each time-step, performs consistently better than the SLM. In the case of very large catchments, such as Baihe, the LVGFM performs better than both SMAR and ANN models. This is due to the lumped-parameter effect in the SMAR model applied to a large catchment, and to over-parameterisation effects in the case of the ANNM.

For catchments characterised by strong seasonality, such as Sunkosi-1, Halda, Brosna, and Fergus, the LPM, with its inherent component of seasonal variation, outperforms the LVGFM. For Sunkosi-1, having a catchment area of 18,000 km^2, and characterised by physiographical and hydro-meteorological variability, but displaying strong seasonality, the LPM performs better than both the SMAR and the ANN models. For smaller catchments, however, such as Fergus, Brosna and Halda, the SMAR model performs consistently better than the other models. For Sagana, the SMAR model, the LVGFM, and the LPM are all found to perform at nearly the same level, while the ANNM performance is just marginally higher than the others.

These results indicate that simple models, involving fewer parameters or weights to be evaluated, and relying on simple mathematical procedures (e.g. the ordinary least squares solution), are often better in discharge forecasting than models which involve a significantly higher number of parameters or weights to be evaluated and which rely on complex mathematical computations (e.g. automatic optimisation).

6. CONCLUSIONS

The performance of the naïve SLM is clearly inferior to that of all other models. The LVGFM, which is an extension of the SLM, performs better than all other models for very large catchments. For catchments, characterised by strong seasonality, the LPM outperforms the LVGFM. For large catchments with such seasonality, the LPM performs even better than the SMAR model. For smaller catchments, however, the SMAR conceptual model performs consistently better than the LPM. The ANNM, although characterised by a large number of weights (parameters), does not generally perform better than the simpler models. The SMAR model variants, having either nine or ten parameters, fail to adequately simulate the hydrological behaviour of the large catchments.

The values of three performance evaluation criteria namely, the coefficient of efficiency, the Index of agreement and the coefficient of determination, are very similar and consistent. The index of volumetric fit and the relative error of peak are more appropriate for use as auxiliary indices, when the performances of two or more models are indistinguishable on the basis of the first three. The value of the relative error of peak is a useful index in simulating events such as floods.

In conclusion, this study confirms that simpler models for continuous river-flow simulation can surpass their complex counterparts in performance. There is a strong justification, therefore, for the claim that increasing the model complexity, thereby increasing the number of parameters, does not necessarily enhance the model performance. It is suggested that, in practical hydrology, the simpler models, *"based largely on exercises in pattern recognition and curve fitting, through analysis of the available data"* [O'Connor, 1998], can still play a significant role as effective simulation tools, and that performance enhancement is not guaranteed by the adoption of complex model structures.

7. ACKNOWLEDGEMENTS

The first author acknowledges the support from the Millennium Research Fund of NUI, Galway in developing the GFFS package used in this study.

8. REFERENCES

Ahsan, M. and K.M. O'Connor, A simple non-linear rainfall-runoff model with a variable gain factor, *Journal of Hydrology*, 155: 151-183, 1994.

Beran, M., Hydrograph Prediction — How much skill? *Hydrology & Earth System Sciences*, 3(2), 305-307, 1999.

Clarke, R.T., *Statistical Modelling in Hydrology*, John Wiley and Sons, 1994.

Kachroo, R.K., G.C. Liang, and K.M. O'Connor, Application of the linear perturbation model (LPM) to flood routing on the Mekong River, *Hydrological Sciences Journal*, 33, 2-4, 193-214, 1988.

Kachroo, R.K., River flow forecasting, Part 1, A discussion of principles, *Journal of Hydrology*, 133: 1-15, 1992a.

Kachroo, R.K., River Flow forecasting, Part 5, Applications of a conceptual model, *Journal of Hydrology*, 133: 141-178, 1992b.

Kachroo, R.K. and G.C. Liang, River flow forecasting, Part 2, Algebraic development of linear modelling techniques, *Journal of Hydrology*, 133: 17-40, 1992.

Khan, H., Conceptual Modelling of rainfall-runoff systems, *M. Eng. Thesis*, National University of Ireland, Galway, 1986.

Legates D.R. and G.J. McCabe Jr., Evaluating the use of "goodness-of-fit" measures in hydrologic and hydroclimatic model validation, *Water Resources Research*, 35(1): 233-241, 1999.

Liang, G.C., A note on the revised SMAR model, Workshop Memorandum, Department of Engineering Hydrology, National University College of Ireland, Galway (Unpublished), 1992.

Liang, G.C., K.M. O'Connor, and R.K. Kachroo, A multiple-input single-output variable gain factor model, *Journal of Hydrology*, 155, 185-198, 1994.

Nash, J.E. and J.V. Sutcliffe, River flow forecasting through conceptual models, Part 1, A discussion of principles, *Journal of Hydrology*, 10: 282-290, 1970.

Nash, J.E. and J.J. Foley, Linear models of rainfall-runoff systems. In: 'Rainfall-Runoff Relationship', Proceedings of the *International Symposium on Rainfall-Runoff modelling*. Mississippi State University, May 1981, USA. Edited by V.P. Singh, Water Resources Publications pp.: 51-66, 1982.

Nash, J.E. and B.I. Barsi, A hybrid model for flow forecasting on large catchments, *Journal of Hydrology*, 65: 125-137, 1983.

O'Connell, P.E., J.E. Nash, and J.P. Farrell, River Flow forecasting through conceptual models. Part 2. The Brosna catchment at Ferbane, *Journal of Hydrology*., 10: 317-329, 1970.

O'Connor, K.M., 'The Education and Training of Hydrologists', A Discussion Paper submitted to the *Unesco Steering Committee of IHPV Project 8.1* in the context of its Draft Education Policy Document.(unpublished), 1998.

O'Connor, K.M., M. Goswami, G.C. Liang, R.K. Kachroo, and A.Y. Shamseldin, The Development of The 'Galway Real-Time River Flow Forecasting System (GFFS), The proceedings of the *19th European Regional Conference of ICID*, Czech Republic, 2001.

Shamseldin, A. Y., Application of a neural network technique to rainfall-runoff modeling, *Journal of Hydrology*, 199, 272-294, 1997.

Shamseldin, A.Y., K.M. O'Connor, and G.C. Liang, Methods for combining the outputs of different rainfall-runoff models. *Journal of Hydrology*, 197: 203-229, 1997.

Tan, B.Q. and K.M. O'Connor, Application of an empirical infiltration equation in the SMAR conceptual Model. *Journal of Hydrology*, 185: 275-295, 1996.

Willmott, C.J., On the validation of models, *Phys. Geogr.*, 2, 184-194, 1981.

Xiong, L., K.M. O'Connor, and M. Goswami, Application of the artificial neural network (ANN) in flood forecasting on a karstic catchment. The *XXIX IAHR Congress*, Beijing, 2001.