



Jul 1st, 12:00 AM

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K. Bogner

B. Hingray

A. Musy

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Bogner, K.; Hingray, B.; and Musy, A., "Data-based Mechanistic Modelling of Rainfall-Runoff Processes and Its Application in a Complex Hydrological Context" (2002). *International Congress on Environmental Modelling and Software*. 97.
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Data-based Mechanistic Modelling of Rainfall-Runoff Processes and Its Application in a Complex Hydrological Context

K. Bogner^a, B. Hingray^a and A. Musy^a

^aSwiss Federal Institute of Technology - Lausanne,
ENAC-HYDRAM, CH-1015 Lausanne, Switzerland
(*konrad.bogner@epfl.ch*)

Abstract: Although the inherent uncertainty associated with rainfall-runoff processes is well known, most mathematical models of such systems are completely deterministic in nature. Stochastic modelling requires that the uncertainty, which is associated with both the model parameters and the stochastic inputs, should be quantified in some manner as an inherent part of the modelling analysis. To achieve these objectives, a Data-based mechanistic (DBM) modelling approach will be tested for the Jura lake system (Switzerland). In DBM modelling, the most parsimonious model structure is first inferred statistically from the available time series. State dependent non-linear dependencies can be identified objectively from the rainfall and runoff data and will be used as the bases for the estimation of non-linear transfer function models of the rainfall-runoff processes. After this the model will be accepted if it can be interpreted in a physically meaningful, mechanistic manner. Before this approach will be applied some preprocessing of the data has been done using Wavelet transformations. Furthermore a simplified snow-melt model has been applied in order to calculate equivalent rainfall. First results of this preprocessing and of the DBM modelling will be shown in this paper.

Keywords: Data-based mechanistic modelling; Transfer functions; Snowmelt modelling; Wavelet transformation.

1 INTRODUCTION

Rainfall-runoff models are well-established tools that are widely utilized in engineering practice, e.g. for water resource planning. The majority of model structures currently used can be classified as conceptual. Usually these kind of models have a lot of parameters, which cannot be estimated without imposing prior restrictions. Recently Beven and Freer [2001] have described these problems using the term *equifinality*. This means that many different parameter sets and model structures exist, which are able to explain the observed data equally well, so that no unique solution can be obtained. The data-based modelling methodology of Young and Beven [1994] tries to solve the problems of identification and over-parametrisation of conceptual hydrological models by the use of simple and parsimonious mechanistic model structures. Thus no prior assumptions are made about the form of the model other than a general linear transfer function approach can be used to relate an effective rainfall

input to the total discharge.

This model will be applied to a very large (about 8000 km²) and complex catchment in Switzerland. The overall objective will be to study the impacts of climate variability and change on the sustainable use of water within a project called SWURVE (Sustainable Water : Uncertainty, Risk and Vulnerability in Europe) funded by the EU and the Swiss Federal Office of Education and Sciences. The aim of SWURVE is the elaboration of a probabilistic framework to assess climate change impacts on the sustainable use of water. This methodological framework takes into account the combined uncertainty due to the natural variability and the error due to incomplete knowledge of future conditions. The Jura lake system is a complex hydrological system located in the western part of Swiss plain, which will be analysed in detail. This system of three interconnected lakes (Neuchâtel, Bièvre and Morat) was formed 15,000 year ago, when the huge Rhone glacier had retired. It was significantly modified in

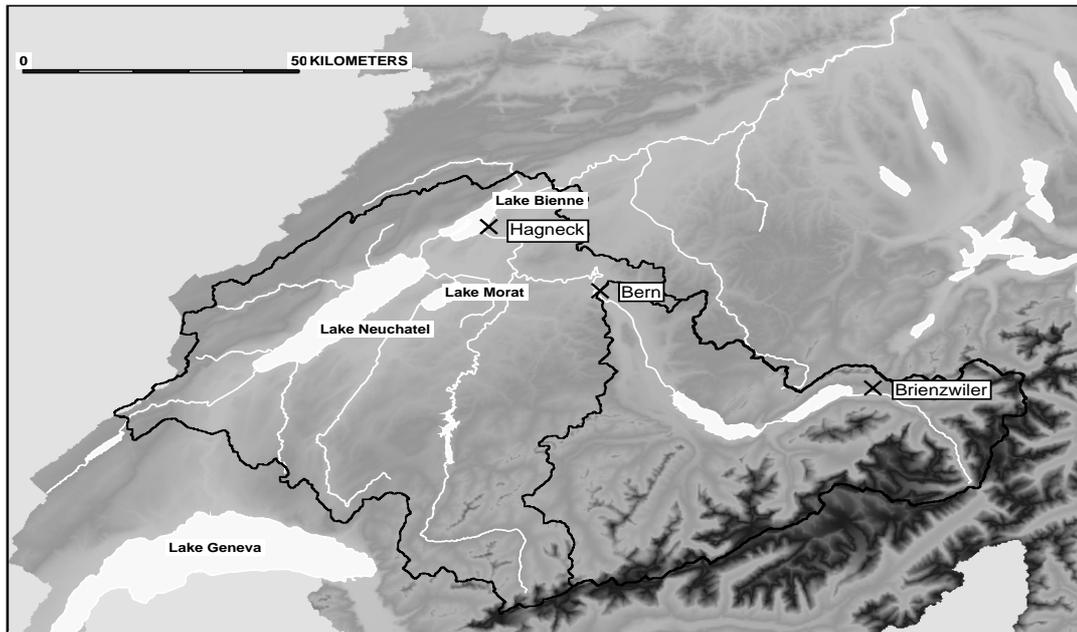


Figure 1: DEM of the Jura lake system (©Swiss federal Office of Topography)

the periods 1833-1884 and 1963-1972 by the construction of channels. Different regulation waterworks have been built, in order to reduce the risks of inundation and to transform the former unproductive areas into fertile arable land.

In the beginning of this project the hydrological system has to be analysed for the present times and a rainfall-runoff model will be applied in order to calculate future scenarios subsequently. The most important inflow to the Jura lake system comes from the Aare River (approximately 70%) and it drains an area of 5128 km² up to the station Hagneck, where the river flows in the lake of Bienné (see Figure 1). The hydrological system is influenced by various components like glaciers, artificial and regulated natural lakes, which make it difficult to apply a more physically based model. Furthermore the essential contribution of the snow-melt have to be taken into consideration. Thus some preprocessing of the data are necessary, which will be explained in the next chapter. Afterwards the DBM modelling approach will be described in more detail and the preliminary results will be shown and discussed.

2 PREPROCESSING THE DATA

The observed discharge series of the Aare river are mostly superposed and altered by various different impacts (e.g. hydro power station management and regulation of lakes), which can be interpreted as noise. For the hydrological modelling it will be

necessary to remove this noise and to select runoff gauges, where the discharge series are more or less noise-free. The WaveShrink methodology developed by Donoho [1995] for estimating an unknown signal from data has been applied to remove the noise from the discharge series. The advantage of this procedure, which is based on Wavelet transformations, is that the peaks are preserved, whereas traditional noise reduction methods, such as splines or kernel smoothers would result in some smoothing of the spikes. In particular, the non-decimated discrete Wavelet transformation has been used in shrinking the Wavelet coefficient towards zero, because it leads to both better prediction and fewer artifacts (e.g. Nason and Sapatinas [2002]). Applying the WaveShrink algorithm to three different discharge series observed at gauges at the Aare river (shown in Figure 2) indicates that the gauge located at Bern will be best suited for further analysis. The residuals (=noise) of this data series are much less in comparison to the series observed at Hagneck (downstream) and Brienzwiler (upstream), which are highly influenced by hydro power production. Thus the catchment of Bern is chosen representing approximately 2/3 of the Aare catchment area (2969 km²).

To improve modelling capabilities, rainfall series are modified here to account for snowfall and snowmelt, producing series of daily equivalent rainfall, which will be the input for rainfall-runoff modelling. The process also accounts for the orographic

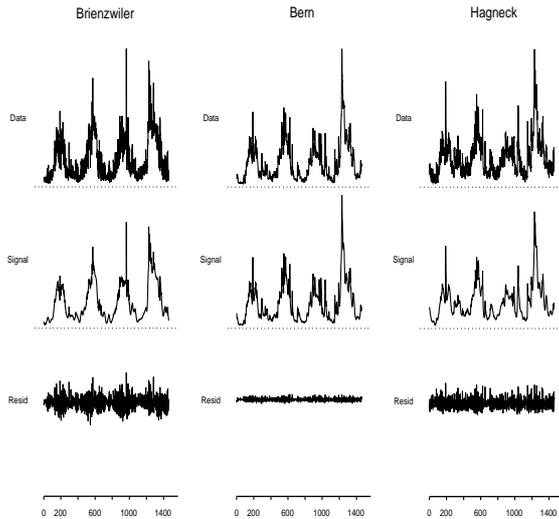


Figure 2: Signal and noise separation of daily discharge series (1996-1999) by the use of Wavelet transformations

enhancement of rainfall, calculating the equivalent rainfall for different elevation bands before averaging these series for the whole catchment (see Figure 3). The degree-day approach to snow-melt modelling provides a simple methodology for the satisfactory calculation of snow-melt, relying on daily average air temperature to represent the major heat fluxes in operation. As found in an international comparison of snow-melt runoff models WMO [1986], the degree-day method has an accuracy comparable to more complex energy budget formulations (Rango and Martinec [1995]). The most widely used degree-day model was developed by Martinec et al. [1983] and is centered on the formula

$$M = k(T - T_0) \quad (1)$$

where M is daily snow-melt runoff (mm), T is daily mean temperature ($^{\circ}\text{C}$), T_0 is a base temperature (usually 0°C), and k is the degree-day or melt factor ($\text{mm}/^{\circ}\text{C}/\text{day}$). As well as the ability to optimise both T_0 and the degree-day factor k , there is potential for modification of the basic formula to include factors such as albedo (for example based on the age of the snowpack).

With the availability of detailed topographic data from Digital Elevation Models (DEMs), the possibility of considering separate elevation bands within a catchment also provides opportunity for improvement. The main difficulty encountered with the

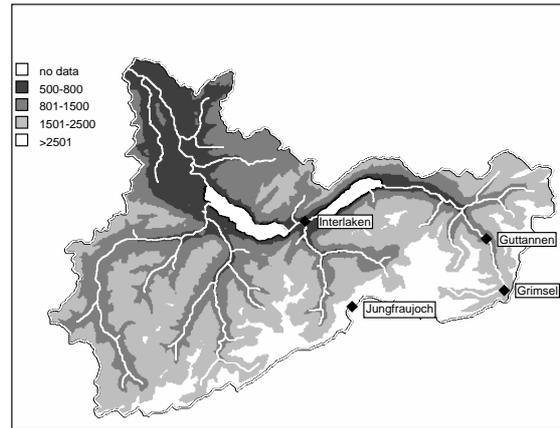


Figure 3: Elevation bands of the Bern catchment and locations of the temperature recording gauges (©Swiss federal Office of Topography)

degree-day method is the variability of the degree-day factor. Work by Schreider et al. [1997] showed the ability of a degree-day type model, used together with IHACRES (Jakeman and Hornberger [1993]), to model snow-affected catchments in the Australian alpine region to the same efficiency as snow-free basins. Although the model of Schreider et al. [1997] was initially developed for use in the southern hemisphere, its success with IHACRES, degree-day methodology basis, and incorporation of topographic data made it a suitable starting point in the snowmelt modelling process. Its reliance on daily temperature and precipitation data make it extremely useful for modelling snow processes in regions with a lack of regular snow observations, or historical periods with limited data.

There are several parts of the model which has been altered by Steel [1999] and have been applied in this study. Some changes were structural such as changing the dates used for the start and end of the snow season (and albedo factor) to be suitable for the northern hemisphere, and changes to the grid cell system. Other changes required parameter value optimisation, notably where values had been based on empirical observations (for example the degree-day factor used and albedo factor). The day degree factor has been estimated by Monte Carlo simulations using different values for k ranging from 0.3 to 10. The result of the calculations of the equivalent rainfall is shown in Figure 4 with $k = 4.4$.

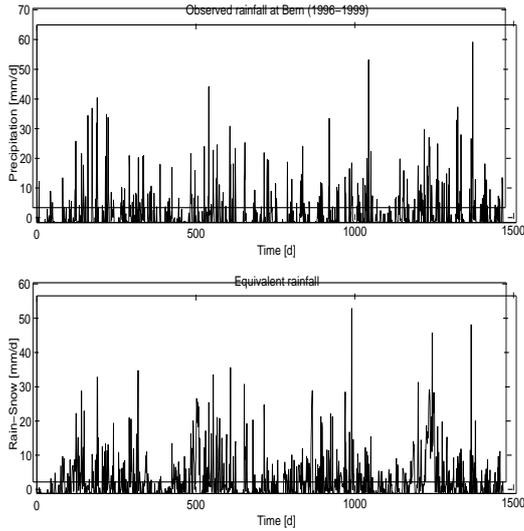


Figure 4: Comparison of observed and calculated equivalent daily rainfall (1996-1999)

3 DATA-BASED MECHANISTIC (DBM) MODELLING

The DBM philosophy (Young and Lees [1994]) and its application are discussed fully in Young [1993], Young and Beven [1994], Young et al. [1997], Young [1998], Beven [2001] and the references therein. The first step of the DBM modelling approach involves the statistical identification and estimation of a model representing the stochastic, dynamic system. The prior assumptions about the structure are kept at a minimum. The general DBM model for a single input (e.g. effective rainfall) takes the form:

$$y_t = \frac{B(z^{-1})}{A(z^{-1})} u_{t-\delta} + \xi_t \quad (2)$$

where the transfer function polynomials are defined as:

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \dots + a_n z^{-n} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + \dots + b_m z^{-m} \end{aligned} \quad (3)$$

and z is a backward shift operator (i.e. $z^{-i} y_t = y_{t-i}$). $y(t)$ is the measured runoff; $u(t)$ is the observed input, which is in our case the equivalent rainfall; δ is a time delay and ξ_t is the stochastic noise. The order of the associated transfer function model is defined by the triad $[n, m, \delta]$. The residuals

ξ_t are defined as follows,

$$\hat{\xi}_t = y_t - \frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})} u_t \quad (4)$$

where $\hat{A}(z^{-1})$ and $\hat{B}(z^{-1})$ are the estimates in (3) and represent measurement noise and unmeasured effects.

3.1 Model structure identification

The following identification and optimisation methods are implemented in the CAPTAIN Matlab[©] Toolbox¹. More information about this toolbox can be found at <http://www.es.lancs.ac.uk/cres/captain>. Appropriate model structures identification, i.e. finding the best values of $[n, m, \delta]$ and the corresponding coefficients, will be done by a process of objective statistical inference. Thus in a first step the input-output data will be analysed and the parameters in a constant parameter transfer function model will be estimated based on optimal Instrumental Variable (IV) algorithm (see Young [1984] and Young and Beven [1994]). This will often result in a first or second order model (see Figure 5). The evaluation of the identified model structure can be done by applying different criteria for the goodness of fit like the coefficient of determination (R^2), YIC and AIC, which are defined in Young and Beven [1994]. Comparing the two models with different input (observed and equivalent rainfall) indicates that the application of the snow-melt model in front of the transfer function model improves the simulation result significantly (see Table 1).

Table 1: Criteria of goodness of fit for 2 different $[1,1,1]$ models

Model-Criteria	YIC	R^2	AIC
observed rainfall	-7.462	0.492	0.998
equivalent rainfall	-8.727	0.740	0.329

3.2 Non-linear model estimation

If standard statistical tests indicate some non-linearity the fixed interval smoothing (FIS) method will be used to obtain non-parametric time variable parameter estimates (Young and Beven [1994]). Considering the case of a single input variable (e.g. rainfall) a nonlinear model can be approximated by

¹ thanks to Prof. Peter C. Young, Department of Environmental Science, Lancaster University, Lancaster, LA1 4YQ, United Kingdom for making the CAPTAIN toolbox available

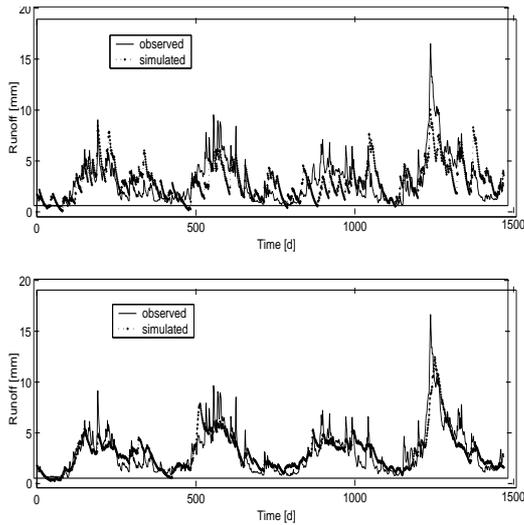


Figure 5: Simulated runoff with observed rainfall as input (top panel) and equivalent rainfall as input (lower panel). The structure of the transfer function model is [1,1,1]

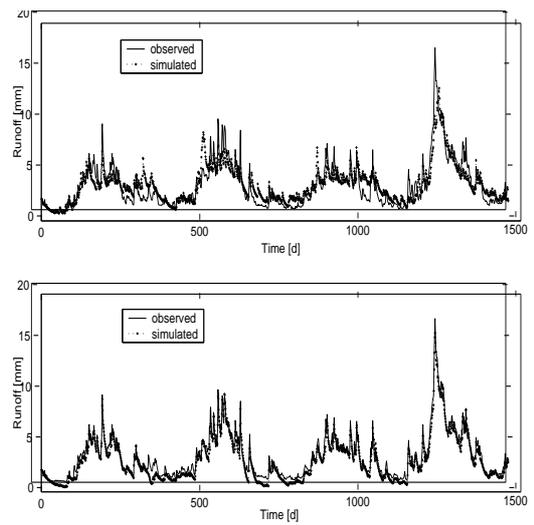


Figure 6: Simulated runoff (transfer function model structure is [3,3,1]) with constant parameters (top panel) and time varying parameters (lower panel)

the following transfer function model with time or state dependent parameters:

$$\begin{aligned}
 y(t) &= \frac{B(t, z^{-1})}{A(t, z^{-1})} u_{t-\delta} + \xi_t \\
 &= \frac{b_{0,t} + b_{1,t}z^{-1} + \dots + b_{m,t}z^{-m}}{1 + a_{1,t}z^{-1} + \dots + a_{n,t}z^{-n}} \\
 &\quad \times u_{t-\delta} + \xi_t \quad (5)
 \end{aligned}$$

The model (5) is a time/state dependent parameter version of the standard transfer function model given in (2), where the polynomial parameters are assumed to be possible functions of the time t . In Figure 6 and 7 the results of this time variable parameter procedure are shown. The gain parameter b_0 is the only parameter, which shows significant variation over the observation period, which is likely to be caused by the soil moisture non-linearity.

In Figure 8 the standard error of this time variable parameter are shown also. The advantage of this methodology is the possibility of evaluating the standard errors, which can be used for the physical interpretation of the model. The final step will be the evaluation of the nature of the parameter variation in relation to other variables in order to identify an appropriate rainfall filter. Furthermore the transfer function model will be decomposed by partial fraction expansion. This decomposition in fast and

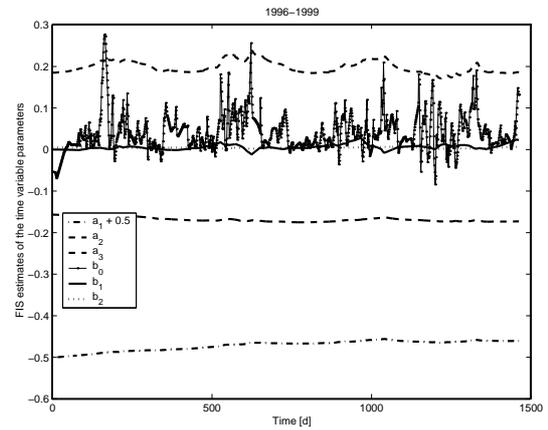


Figure 7: Estimated time variable parameters

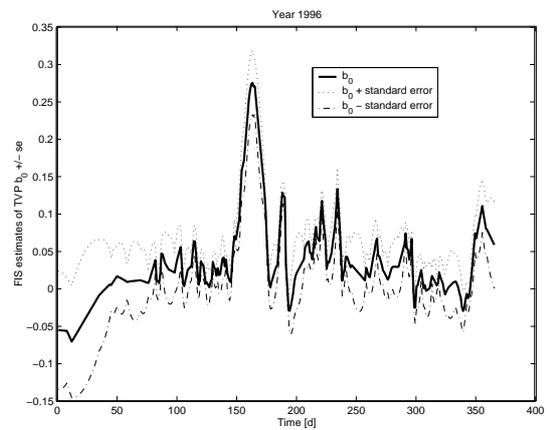


Figure 8: Time variable parameter and standard error bounds of the year 1996

slow pathways has an interesting hydrological interpretation, which could be considered as a representation of a contributing area function for the fast responses in the catchment, whether the fast responses be due to surface or subsurface flow processes.

4 CONCLUSION

In this paper the preliminary results of the application of the DBM approach in a complex hydrological catchment are shown. Preprocessing of the data has been done by the use of Wavelet transformation in order to select noise-free runoff data series. The observed seasonality in the runoff series, influenced by snow and snow-melt processes, have to be taken into account also. Therefore a snow-melt model has been applied to calculate an equivalent rainfall. This preprocessing improved the transfer function modelling significantly. Applying a time variable parameter transfer function model indicates the non-linearity of one parameter, which has to be interpreted in a physical (mechanistic) way at next. In consideration of the complexity of the investigated catchment this preliminary results are quite satisfying demonstrating very well the applicability of a parsimonious transfer function model.

ACKNOWLEDGMENTS

This study is part of the SWURVE project funded by the EU Energy, Environment and Sustainable Development Programme and by the Swiss Federal Office of Education and Sciences. The data has been made available by the Swiss Institute of Meteorology and Swiss Federal Office for Water and Geology. The authors acknowledge these institutes for their support.

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