# Selecting the appropriate time step in an integrated systems model

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**Abstract:** In recent years a variety of integrated system models to support the management of water systems appeared. Two examples are the RaMCo model for coastal-zone management and WadBOS for the Dutch Wadden Sea. A key aspect of the design is that ecological, economic, and physical process models are combined in an integrated systems network. In most cases a uniform mesh size and time step are used for all models, exceptions being made for processes requiring a lower or higher level of detail. The choice for a particular model is primarily based on data availability and modeling experience rather than sound scientific principles, because an objective methodology for the selection of spatial and temporal scales in integrated system models is still lacking. In this paper we examine one aspect of model integration: the problem of how to estimate the appropriate time step for a model in an integrated systems network. The approach followed is based on the comparison of the numerical truncation error for the discrete system equations with the model uncertainties. The example discussed pertains to cockle fisheries in the Dutch Wadden Sea.

***Keywords:*** Decision support systems; Integrated assessment; Appropriate modeling

## 1. INTRODUCTION

A growing interest can be noticed among the scientific community and water managers for tools that can support the decision making process. Such tools can be used to examine the spatial and temporal consequences of different management strategies. Two examples are the RaMCo model for sustainable management of the coastal zone of Southwest Sulawesi, Indonesia [Uljee and Engelen, 1996; De Kok et al., 2001] and WadBOS for the Dutch Wadden Sea [Huizing et al., 1997]. After selecting a combination of scenarios and measures the users of WadBOS can examine the interaction of the social-economic, physical and ecological processes in order to gain understanding of the effectiveness of different strategies. WadBOS [Huizing et al. 1997] has been developed as a case study for a decision-support system for the Dutch Wadden Sea. The model describes the interaction between e.g. recreational navigation, cockle fisheries, and military activities with the natural functions that exist in the Wadden Sea. These include seals, different migratory bird populations, and shell species. The model enables local authorities to study the consequences of zoning plans, fishing quota, and other measures on the development of the state of the Wadden Sea over the course of ten years. Model results are shown in the form of maps and changes of state variables over time. The WadBOS model is based on a large number of coupled difference equations, which are solved with a uniform time step. In the cockle fisheries model the Wadden Sea is divided into twelve different compartments, which allow for the exchange of nutrients and pollutants. For each compartment the changes in the cockle biomass depend on fishing pressure, fluctuations of the water temperature and algae concentration, and grazing by migratory birds (Figure 1). The central aim of model integration is to analyze and rank different management options such as the closure of fishing grounds or fish quota on the basis of the long-term consequences for the cockle population and income of the cockle fisheries sector. A key problem during the design is that models and data at different levels of spatial-temporal detail can be applied. For example, the biomass in each compartment is described for a monthly time step. It is not obvious, however, whether this value is appropriate, given the uncertainties of model parameters and data. The common practice is to let the data availability and process modeling experience guide the selection of models and data

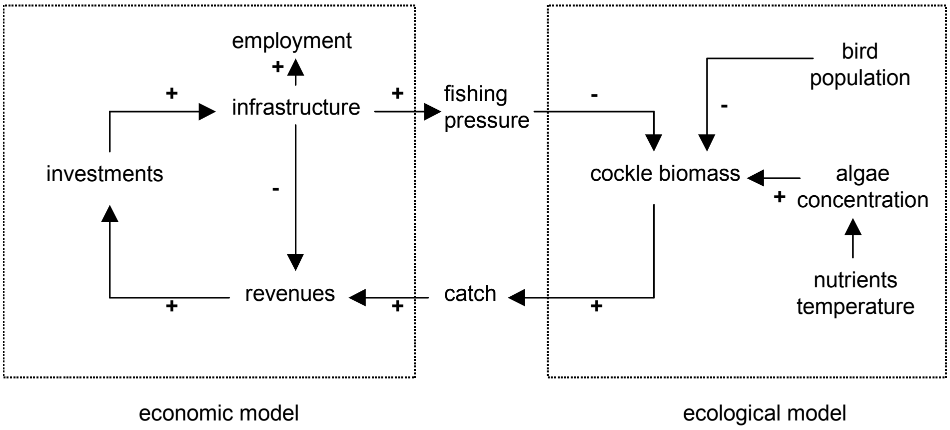


**Figure 1.** The cockle fisheries model.

during the design. This has major disadvantages for the design of integrated system models. In the first place, the design process becomes an art rather than a science, with a strongly case-dependent approach. Secondly, data and models collected can be either too coarse hence introducing unacceptable uncertainty in the system model, or too detailed, which will raise the costs of the design. This can lead to an unnecessary system complexity, which may hinder interpretation of the system behavior by decision-makers. The key aspect here is that the purpose of the integrated system should be kept in mind, while the accuracy of the different process models used in the system should not be more complex than necessary to distinguish the consequences of different management alternatives (Figure 2).

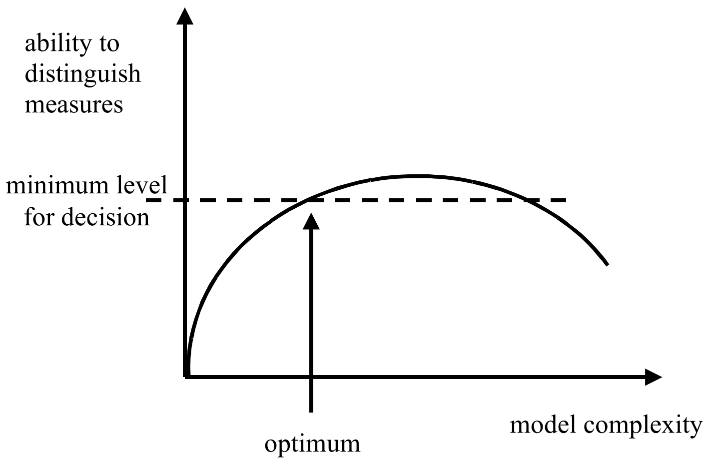


**Figure 2.** Influence of model complexity on ability to distinguish measures.

The purpose of appropriate modeling is to identify the model of optimal complexity. For integrated model networks such as the cockle fisheries subsystem of Figure 1 the problem is more difficult due to the propagation of uncertainties between interacting models. One of the key aspects of appropriate modelling is the identification of order-of-magnitude estimates for the time step and spatial mesh size for each model in the system. The ideal situation would be to have order-of-magnitude information on the appropriate mesh size and time step for the different process models prior to the definitive design of the system. A possible approach to validate the time step is to compare the numerical truncation error for the discrete system equations with the intrinsic model uncertainties. A previous study for a flood damage model [De Kok, 2001] showed how the appropriate spatial resolution can be estimated in a similar way. Here we address the problem of verifying the time step for the cockle biomass model. The approach is based on a comparison of the intrinsic model uncertainties resulting from parameter distributions and the input noise from interacting models with the error induced by the numerical procedure used to solve the difference equation. The example discussed pertains to a single model for biomass growth, but can be generalized to integrated model networks. A limitation of the method is that it requires calculation of the functional derivatives for each system equation. This means that the required computations become more intensive for chains of interacting models.

## 2. ERROR PROPAGATION

The general form for a (non)linear dynamic state equation is:

$$\frac{\partial B}{\partial t} = f(B(t),t) \tag{1}$$

where the functional *f(t)* is a characteristic of the differential equation and *B(t)*, for example, is the cockle biomass. In WadBOS the solution to (1) is obtained with the simple Euler method:

$$b_{i+1} = b_i + hf\left(B(t_i),t_i\right) + \varepsilon_i \tag{2}$$

Here *h* is the selected time step for the numerical procedure, and $\varepsilon_i$ is the truncation or *local* error, given by

$$\varepsilon_i = \frac{h^2}{2}B''(t_i) \quad \sim O\left(h^2\right) \tag{3}$$

where the accent refers to differentiation with respect to time. Although the exact solution of (1) with an infinitely small time step is not known, the second derivative in (3) can be approximated with the biomass time series of WadBOS:

$$B''\left(t_i\right) \approx \frac{b_{i+2} - 2b_{i+1} + b_i}{{h_0}^2} \tag{4}$$

where $h_0$ is the time step used in WadBOS. The accumulated or *global* error at time $t_i$ is given by the difference between the exact and numerical solution:

$$E_i = B\left(t_i\right) - b_i \tag{5}$$

Provided $f(B(t),t)$ is sufficiently differentiable [Henrici, 1963, Atkinson, 1978] the global truncation error can be written in the form:

$$E_i = hD_i + O\left(h^2\right) \tag{6}$$

where $D_i \equiv D\left(t_i\right)$ is the approximate solution to

$$D'(t) = \frac{\partial f}{\partial B}D(t) + \frac{B''(t)}{2} \tag{7}$$

which can be solved for by the Euler method for the selected time step *h*:

$$D_{i+1} = D_i\left(1 + h\left[\frac{\partial f}{\partial B}\right]_i\right) + h\frac{B''\left(t_i\right)}{2} \tag{8}$$

Combining (3), (6) and (8) we obtain a difference equation for the global truncation error:

$$E_{i+1} = E_i\left(1 + h\left[\frac{\partial f}{\partial B}\right]_i\right) + \varepsilon_i \tag{9}$$

The functional derivative of *f(B)* with respect to *B* can be obtained through chain differentiation:

$$B'' = f' = \frac{\partial f}{\partial B}B' \tag{10}$$

This gives:

$$\left[\frac{\partial f}{\partial B}\right]_i \approx \frac{b_{i+2} - 2b_{i+1} + b_i}{h_0\left(b_{i+1} - b_i\right)} \quad \left(b_i \neq b_{i+1}\right) \tag{11}$$

which is a functional property of *f(B)* and does not depend on the time step *h*. Finally, we compare the global error with the intrinsic model uncertainties $\sigma_i$ by imposing the condition that the numerical error is an order of magnitude smaller than the model uncertainty:

$$\left|E_i\left(h\right)\right| << \sigma_i \tag{12}$$

The reason is that the numerical error can be controlled. In contrast the model accuracy is limited and depends on the degree of process knowledge. The dependence of the global error on the time step can be examined for the example

$$\frac{\partial f}{\partial B} = a \qquad \varepsilon_i = \varepsilon_0 \tag{13}$$

This shows that the solution of (9) is of first order in the time step $h$:

$$E_i = \varepsilon_0 \sum_{j=0}^{i-1} (1+ah)^j$$
$$\approx \varepsilon_0 \left[ \frac{(1+ah)^{i-1} - 1}{ah} \right] \quad (14)$$
$$\rightarrow \varepsilon_0 \left[ \frac{e^{aih} - 1}{ah} \right] \quad \sim O(h) \quad (h \rightarrow 0)$$

The last step in the analysis is to select the value of $h$ for which the error $E_i$ is significantly smaller than the model uncertainties $\sigma_i$. For systems with only a few state variables these can be obtained by analytical error propagation techniques. For more complex systems the model uncertainties are usually determined by means of a Monte Carlo approach.

## 3. RESULTS

Figure 3 shows the variation of the total cockle biomass for the Wadden Sea over the simulation period of 120 months. Two scenarios are shown: a fixed yearly quota of 67 metric tons and a zero yearly quota scenario.

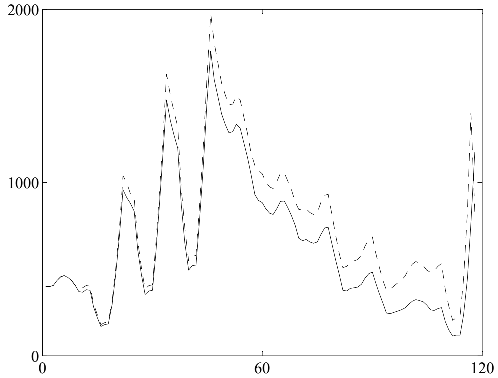


**Figure 3.** Cockle biomass in the Wadden Sea as calculated by WadBOS (ton fresh weight) with fixed quota (solid) and zero quota (dashed).

The seasonal fluctuations in the biomass are due to winter mortality and the temporary opening of the Wadden Sea for cockle fisheries. It must be noted that the results shown in Figure 3 have not yet been validated. The model parameters have been estimated by the ecologists participating in the design of WadBOS. However, the fisheries statistics office collects samples on a yearly basis to estimate the total biomass of cockles in the Wadden Sea. These can be used for validation of Figure 3. Condition (12) must be tested with the uncertainty bounds for the biomass, which can be determined from uncertainty analyses with the integrated system. Unfortunately, these have not yet been conducted with WadBOS. Therefore, we use the absolute difference between the two fishing scenarios of Figure 3 instead of the model uncertainty. This is also consistent with our definition of appropriate modeling. Figure 4 shows the variation in the absolute value of the accumulated error $E_i$ against this difference.

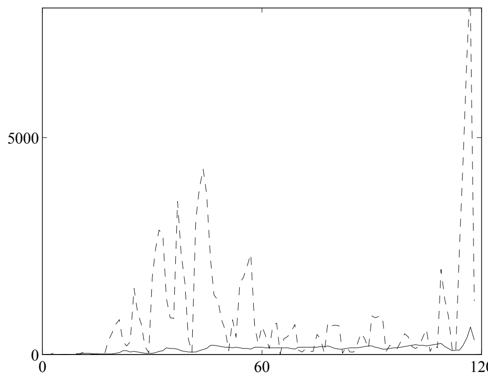


**Figure 4.** Difference between the two fishing scenarios (solid) and absolute value of the global error $E_i$ (dashed) during the simulation period.

The area under the curve for numerical error is roughly seven times larger. This means that the error introduced by the numerical method is significant, mainly due to the contribution during large fluctuations in the biomass. From (14) we know that the global error depends linearly on the time step. This means that condition (12) can be met if the time step is reduced by a factor 70, which suggests a time step of twelve hours instead of a month. Of course, this is only an order-of-magnitude estimate.

## 4. DISCUSSION

A model-independent procedure to test the time step in discrete models has been presented. A case example showed how the numerical discretization error can be compared with the intrinsic model uncertainties. Although the example pertains to a single process involving only one state variable, the method is suitable for application to an integrated systems network. In that case the truncation errors have to be generated with an automatic procedure, while the model uncertainties that appearing in (11) can be obtained by means of Monte Carlo analyses. The proper value for the time step could also be estimated directly from the graph of Figure 3, but this becomes more difficult for complex systems with a large number of interacting models, where uncertainties propagate from one model to another. In this case equation (1) becomes a matrix equation with the biomass now replaced by a vector of state variables and the functional derivatives have to be calculated for all the state variables. Using a test system of simple models as the starting point one can determine order-of-magnitude estimates for the appropriate time step, pointing to the type of models to use in the definitive design. Planned research comprises solving (1) with a different numerical procedure, the incorporation of model uncertainty in the global error, and the application to systems of coupled differential equations.

## 5. REFERENCES


Atkinson, K.E., *An introduction to numerical analysis*, Wiley, New York, 1978.

Henrici, P., *Error propagation for difference methods*, John Wiley & Sons, New York, 1963.

Huizing, J.J., Van de Ven, K., and Pothof, I., WadBOS: A prototype of a knowledge-based system for policy analysis of the Wadden Sea, Final Report, Directorate-General of Public Works and Water Management (RWS), National Institute for Coastal and Marine Management, RIKS bv, INFRAM bv, The Netherlands, 1998. (In Dutch).

Kok, J.L. de, Appropriate spatial resolution in flood damage modelling. In Falconer, R.A. & Blain, W.R. (Ed.*), River basin management.* (pp. 161-165). Cardiff, Great Britain: WIT Press, 2001.

Kok, J.L. de, and Wind, H. G., Integration of social and ecological processes in quantitative models for the management of water systems. In: Matthies, M., Malchow, H. & Kriz., J. (Ed.), *Integrative Systems Approaches to Natural and Social Dynamics*, Springer Verlag, Berlin, Heidelberg, New York, 2001.

Kok, J.L. de, and Wind, H.G., Rapid Assessment of Water Systems based on Internal Consistency, *Journal of Water Resources Planning and Management* (ASCE), accepted for publication, July 2002.

Uljee I., G. Engelen and R. White, RamCo Demo Guide, Workdocument CZM-C 96.08, Coastal Zone Management Centre, National Institute for Coastal and Marine Management, The Hague, 1996.