Lexicographic optimisation for water resources planning: the case of Lake Verbano, Italy

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Abstract: Lake Verbano is a natural lake used as multipurpose reservoir. The lake supplies water for irrigation and hydropower generation to downstream users, while flood controls are applied to protect the lake shores and the downstream populations on the Ticino river, and environmental preservation constraints must be respected. All these objectives are conflicting and they have different priorities, as stated by the Italian regulation on water use. This paper explores a methodology aimed at solving this conflict. The stakeholders involvement in the decision making process is supported by a Multi Objective Decision Support System for the management of water reservoirs. It is designed to be used at the planning level by water agencies to generate management policies and release plans over various time horizons. The DSS also supports the analysis of various scenarios of possible structural modification of the lake outlet and the introduction of a minimum flow rate on the Ticino river. Solving a multi-objective problem, the DSS algorithm generates ”set-valued” policies, providing the decision makers with multiple choices, thus integrating, not substituting, human intelligence. The knowledge of a set of equivalent controls instead of a single value can be advantageous for decision makers since they can choose the control most suitable to a particular situation on the basis of their experience and information. This set-valued policy, obtained solving the primary optimal control problem, can also be exploited for a further lexicographic optimisation. The primary policy, ensuring the performance of a prioritised group of objectives, can be used to define the feasible set of controls for the secondary optimal control problem, defined for a new set of secondary objectives.

Keywords: Water management and planning, lexicographic optimization, set-valued policies, DSS.

1 INTRODUCTION

Italian law (Galli Act, L.N. 36/1994) defines water as a public resource. It must be carefully managed in every possible use made of it. This law is a component of a raising global concern on water related issues: the Dublin Statement on Water and Sustainable Development (ICWE [1992]), states: “*Water development and management should be based on a participatory approach, involving users, planners and policy-makers at all levels*” (Pr.2). Later on, “*Water has an economic value in all its competing uses and should be recognized as an economic good*” (Pr.4).

Therefore, water use should be inspired by three main guidelines: 1) the integration of the whole water cycle with respect to environmental aspects; 2) the economic efficiency in water management; 3) different priority levels in the resource allocation.

Italy empowered water authorities at the basin level in order to enforce the first two guidelines. The ”basin plan” is the main instrument designed by water authorities to assign water resources to users on the basis of the water balance. The implementation of such a plan can be assisted by a Multi Objective Decision Support System such as *TwoLe* (Soncini-Sessa et al. [1999]), which has been created to assist water authorities in the design of management policies and release plans for lakes and reservoirs over various time horizons. The TwoLe DSS also supports the analysis of various scenarios of the possible impacts of structural modifications of the lake outlet and of the imposition of a minimum flow rate on outflowing rivers.

The basin plan also addresses the last guideline, the Galli Act explicitly mentions the scale of priorities in the use of water: first, drinking water demand; then, during droughts, after having satisfied demand for drinking water, water should be supplied for agricultural use; finally, in case of persisting water emergency, the hydro-electrical water storage should be depleted. The Italian approach to water management is similar to other international legislations. There are three different and hierarchical levels: drinking water, environmental and agricultural use, and finally, industrial use.

In this paper we show how a lexicographic optimisation scheme has been incorporated in the TwoLe DSS to implement the hierarchical approach to water management, thus fulfilling the last guideline requiring priority levels in water use. In the remeinder of this paper, first we describe the problem definition and the lexicographic algorithm, then we intrdouce a case study (Lake Verbano), and finally we draw some conclusions.

2 THE LEXICOGRAPHIC APPROACH TO MULTI-OBJECTIVE CONTROL PROBLEM DECOMPOSITION

The approach we propose to efficiently manage water resource in compliance with the different priorities is based on a series of successive multi-objective optimisations. A primary optimal control problem is formulated for a given set of objectives and an optimal policy is computed. Given the system regulated with this policy, a second set of objectives is chosen and a new control problem is formulated and solved: a new optimal policy is obtained. The process can be iterated, taking in consideration new objectives at every step. The optimal policies are computed using Stochastic Dynamic Programming (SDP) techniques (Bellman [1957], Yakowitz [1982]).

In this paper, this procedure has been referred to as *lexicographic* thus extending the meaning of this term, traditionally used in the context of multi-objective problem decomposition (Rentmeesters et al. [1996], Volgenant [2002]).

At each step, the optimal policies are used to generate the sets of admissible values for the control variables, which are then used to constrain the search space while optimising the new problem, which has been defined with a new set of hierarchically less important objectives. A key point of this computational architecture are *set-valued* policies, that is, the policies are multiple-valued functions of the system's state. In other words, for the same value of the state, alternative and fully equivalent control actions are available. The theory behind set-valued policies is summarised in the next paragraphs.

Set-valued policies. A feedback control law solving an optimal control problem is a function $u_t = m_t(x_t)$ that, given a state value x_t at time t, with $x_t \in X_t$ (e.g. the value of the water storage), returns a single value for the control u_t, with $u_t \in U_t$. A sequence $[m_0(\cdot), m_1(\cdot), ..., m_{T-1}(\cdot)]$ of control laws represents a policy p. By applying SDP, the problem of finding a policy can be reduced to the solution of a cascade of subproblems. Each problem produces the control law for a given time t once given a function $h_{t+1}(x_{t+1})$, which is obtained recursively solving the following problem backwards in time from $t = 0$:

$$h_t(x_t) = \min_{u_t} E_{\varepsilon_t}\left[g_t(x_t, u_t, \varepsilon_t) + h_{t+1}(x_{t+1})\right] \quad (1a)$$

$$x_{t+1} = f_t(x_t, u_t, \varepsilon_t) \quad (1b)$$

$$u_t \in U_t(x_t) \quad (1c)$$

$$\varepsilon_t \sim \phi_t(\varepsilon_t | x_t, u_t) \quad (1d)$$

$$u_t = m_t(x_t) \quad (1e)$$

$$p = [m_0(\cdot), m_1(\cdot), ..., m_{T-1}(\cdot)] \quad (1f)$$

where $\phi_t(\varepsilon_t|\cdot,\cdot)$ is the conditional probability distribution of the disturbance ε_t and $g_t(\cdot,\cdot,\cdot)$ is the step-cost function defined as weighted sum of selected objectives. It can be proven (Bertsekas [1976]) that the function $h_t(\cdot)$ converges asymptotically to a periodic function of time, called *Bellman function*, that is specified by T functions $h^*_t(\cdot)$ with $t = 0, ..., T-1$. When this function has been computed, the optimal policy is given by the following relationship:

$$m^*_t(x_t) = \arg \min_{u_t \in U_t(x_t)} \mathop{E}_{\varepsilon_t \sim \phi_t} \left[g_t(x_t, u_t, \varepsilon_t) + h^*_{t+1 \mod T}(x_{t+1})\right] \quad (2)$$

In the typical applications of Control Theory (usu. the control of electro-mechanical systems) every control law is a single-valued function, since otherwise the controller would not be able to operate automatically if there were multiple control choices. However, in the management of environmental systems only rarely the control action is directly delegated to an automatic machine. More frequently, the machine suggests a value and the final decision is left to an human operator (or decision maker (DM)). When dealing with this kind of systems the term *control action* is generally substituted by *decision*, and the control law is embedded into a Decision Support System (DSS) (Loucks and da Costa [1991]).

When the control law is obtained by solving an optimal control problem, it may happen that for a given state x_t the optimal control value u_t is not unique; i.e. there exists a set $M_t^*(x_t)$ of equivalent optimal controls (i.e. producing same performances), suggested from the DSS. Then the DM would take advantage of knowing the set $M_t(x_t)$, since (s)he may choose the more suitable one for a particular situation. In fact, it is generally impossible to capture the complexity of the real word into the definition of the optimal control problem and thus only a limited set of all the possible goals is actually taken into account.

The primary control problem. The solution of the primary control problem, minimizing the main objectives, is a set-valued policy defined by:

$$P^* = [M_t^*(\cdot),\ t = 0, 1, \ldots] \tag{3}$$

where the set-valued control law is the following:

$$M_t^*(x_t) = \Big\{ u_t \in U_t(x_t) : \mathop{E}_{\varepsilon_t \sim \phi_t} \Big[[g_t(x_t, u_t, \varepsilon_t)]_p + \\ + [h^*_{t \bmod T}(f_t(x_t, u_t, \varepsilon_t))]_p \Big] \le l^* \Big\} \tag{4}$$

with

$$l^* = \min_{x_0 \in X_0} h_0^*(x_0) \tag{5}$$

where $[g_t(\cdot,\cdot,\cdot)]_p$ is the step-cost function valued on the primary objectives and $[h_t(\cdot,\cdot,\cdot)]_p$ is the Bellman function valued on the same objectives (for a more detailed definition see Aufiero et al. [2001a, 2001b]).

The lexicographic problem. Starting from this solution, a lexicographic problem can be formulated to optimize secondary objectives:

$$\widehat{P}^* = \Big[\widehat{M}_t^*(\cdot),\ t = 0, 1, \ldots\Big] \tag{6}$$

where the set-valued control law is defined by:

$$\widehat{M}_t^*(x_t) = \Big\{ u_t \in M_t^*(x_t) : \mathop{E}_{\varepsilon_t \sim \phi_t} [[g_t(x_t, u_t, \varepsilon_t)]_s + \\ + \Big[\widehat{h}^*_{t \bmod T}(f_t(x_t, u_t, \varepsilon_t))\Big]_s\Big] \le \widehat{l}^* \Big\} \tag{7}$$

with

$$\widehat{l}^* = \min_{x_0 \in X_0} \widehat{h}_0^*(x_0) \tag{8}$$

where $[g_t(\cdot,\cdot,\cdot)]_s$ is the step-cost function valued on secondary objectives and $[h_t(\cdot,\cdot,\cdot)]_s$ is the Bellman function valued on the same objectives. Note that the control u_t is now defined on the control set $M_t^*(\cdot)$ produced by the primary optimisation. This lexicographic optimisation can be further iterated, as represented in Figure 1.

3 THE CASE STUDY: LAKE VERBANO

The proposed optimisation approch was applied to a planning problem in the water system of Lake Verbano. This lake, also known as Lago Maggiore, is a natural lake located south of the Alps between Italy and Switzerland (see Figure 2). Its catchment has a surface of 6,600 square kilometres, its outflow is the Ticino river, a tributary of the Po river, the main water flow in the Padana plain and in Italy. A general description of the system can be found in Soncini-Sessa et al. [2000].

The Release Manager (RM) is the technical director of the Ticino Water Agency (Consorzio del Ticino), an institution representing the interests of the Italian downstream water users: crop irrigation and hydropower generation. While the water demand for hydroelectric power generation does not show distinctive seasonal fluctuations, a seasonal peak located in April-September characterises water demand for agriculture. Therefore, the RM is induced to increase the water storage when the inflow is high, in order to be able to satisfy the water demand during low inflow periods. Thus, satisfying downstream users involves an increase of the flooding risk for both the Italian and Swiss populations living by the lake and river shores. Finally, the environmental patrimony of lake and river must be protected. The design of the reservoir control scheme is therefore a multi-objective problem (Losa [1999]).

The interests of the stakeholders involved in the regulation of Lake Verbano are very articulated and described by conflicting objectives. The Swiss prefer a lake level which is nearly constant, to avoid flooding of the lakeshores during heavy rainfall periods and to reduce navigation problems in drought periods. The Italians share these goals, i.e. to protect their coastline population, but they also need to

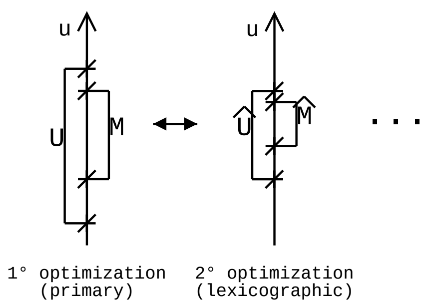

Figure 1: In the lexicographic scheme policy M is used to generate the control set $\hat{U}$ which in turn generates policy $\hat{M}$ and so on.

Figure 2: The Lake Verbano system.

guarantee the satisfaction of downstream water demand. Therefore they need to store water in the lake during periods of high inflows, thus increasing the lake level right when the Swiss would like to keep it constant. Moreover, the Italian need to use Lake Verbano as a lamination reservoir during floods, to protect the Ticino and Po riverine populations.

To guarantee the safety level required by the coastline population, the Swiss authorities devised some proposals for structural enhancements (the most recent can be found in Anastasi [1994]), aimed at increasing the outflow rate at the Miorina dam in anticipation of a flood. These proposals – mainly different degrees of dredging of the lake outlet section in correspondence with the Miorina dam – did not encounter a wide consensus on the Italian side. To make them more attractive to the Italian counterpart, it has been suggested to accompany the Swiss proposals with a review of the regulation range, that should produce a reduction of the supply deficits, and with the imposition of a minimum environmental flow on the outlet. In the next paragraphs we describe how the lexicographic approach has been used to solve the Lake Verbano's management problem under this modified scenario.

Primary optimistaion. The goals of the primary optimisation level are flood protection and minimisation of the deficit of the water supply for irrigation, since the lake is not used as a drinking water reservoir. The control problem solution is the projection of the Pareto boundary shown in Figure 3, in

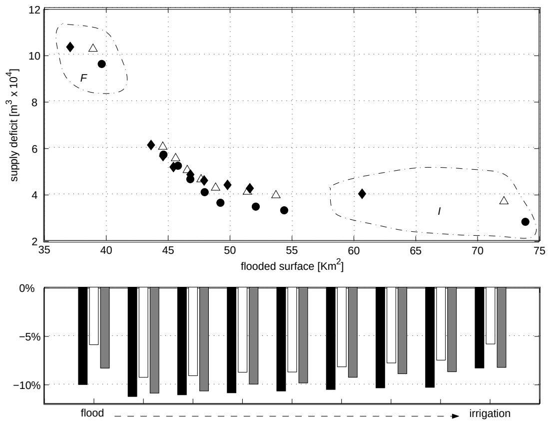

Figure 3: Results of primary optimisation.

the flood and supply deficit space. Since the optimisation problem has been solved using set-valued SDP, we have designed *extractors* which are models of the behaviour of the RM in the choice of the release decision from the optimal control set. The extractors reproduce three different RMs: the first one is concerned with always meeting the irrigation target (ti); the second one is more interested in the sum of the irrigation and the hydropower targets (tih); the last one always chooses to adopt the maximum possible release, choosing the superior value of the control set (sup). For this reason, each point in the Pareto space is identified by a triplet, representing the three different RM's behaviours and each point of a triplet is obtained by simulating the water system over historical inflow scenarii (1974-1988). For instance triplets F and I respectively refer to solely flood protection and supply deficit minimization.

Note that the historical point has been purposedly disregarded since these solutions are obtained under a planned new structural configuration of the system and therefore we cannot compare the current performances with the historical ones. The histogram of Figure 3 shows the hydropower production percentage loss of each triplet with respect to the the optimisation of hydropower generation.

Lexicographic optimisation. We introduce now the secondary objective: the maximisation of hydropower generation. The results of the lexicographic optimisation are represented by the Pareto boundary shown in Figure 4. Since the optimal sets of controls are narrower, being constrained by the results of the primary optimisation, the difference in the performance among points belonging to a same triplet is reduced. Note that the hydropower perfor-

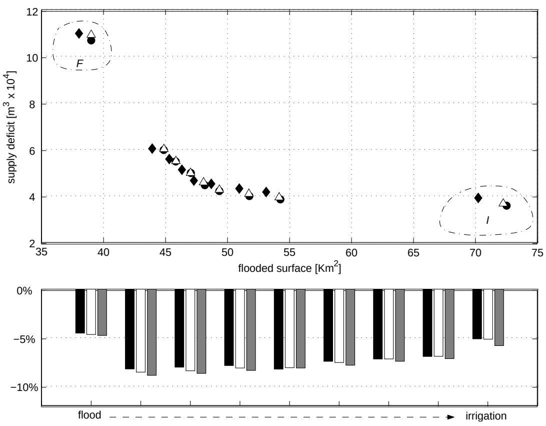

Figure 4: Results of lexicographic optimization.

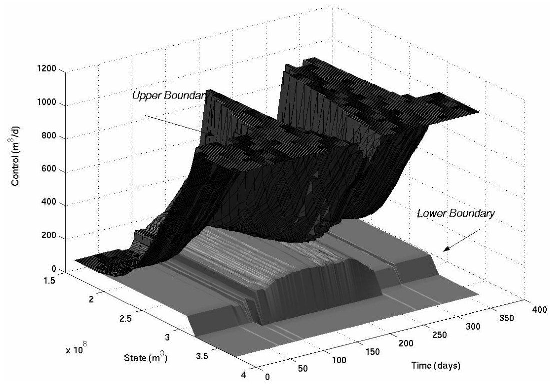

Figure 5: Optimal set controls for primary optimisation.

Table 1: Difference in the performances (%) between primary and lexicographic optimisation (f: flood performances; s: supply performances).

	ti		tih		sup	
	f	s	f	s	f	s
i	1.8	-27.8	-0.2	-0.2	-15.8	2.3
1	0.2	-16.9	-0.9	0.1	-3.0	1.8
2	0.6	-15.7	-0.6	0.0	-2.3	1.6
3	-0.2	-16.2	-1.0	-0.0	-1.6	1.3
4	-0.5	-9.5	-0.9	0.9	-1.0	3.6
5	-0.5	-7.6	-1.0	0.7	-2.0	0.6
6	-0.1	-5.2	-0.5	0.6	-1.6	0.7
7	-0.5	-5.0	-0.7	0.2	-0.7	1.2
f	1.6	-11.4	-0.2	-6.7	-2.5	-6.5

mances of triplet *F* have sensibly increased. This is due to the availability of a wide set of optimal controls before the application of the lexicographic optimisation.

Note that even the performances of the optimal policies computed at the previous level change after the lexicographic optimisation, while, being optimal, they should have not changed. The reason why is that a policy is efficient in the Pareto sense only over an infinite horizon, but not over the finite horizon we have used for the deterministic simulation. Table 1 summarises these differences.

The optimal control set of the primary optimization for the triplet *I* is shown in Figure 5. The lower boundary follows the demand of water supply. The range between boundaries for low values of storage is small over the first part of the year, thus creating water storage for irrigation purpose; from approximately June through August, the range is small and, therefore, the water supply is preserved.

As a result of the lexicografic optimisation, the shrinkage of the control set u_t is observed, as previously anticipated. Figure 6 represents an example of two sections of control set over time, for two different values of the storage, $x_t^1 = 278 \cdot 10^6 m^3$ and $x_t^2 = 225 \cdot 10^6 m^3$. The leftmost graph suggests that, before lexicographic optimization, whenever the storage of the system takes on that particular value x_t^1, the DM can choose the control from a wide set for the whole year with the exception of months with high drought risk. When the state is x_t^2 (rightmost graph), the DM is not given the same discretionality in the choice of controls. After lexicographic optimisation, the boundaries of both states x_t^1 and x_t^2 manifest a periodical pattern due to a smaller energy demand on Sundays.

4 CONCLUSIONS

Italian and international laws assign priorities to water uses. Such regulations add further complexity to the duties of water authorities who are faced with difficult multi-objective management problems. We claim that the use of DSSs in the decision making process can also support water management withn different levels of priority. The paper explored a lexicographic optimisation scheme to achieve this goal using set-valued control policies obtained thanks to stochastic dynamic programming . We explored this procedure applying it to the management of Lake Verbano as a multi-purpose water reservoir: flood protection and supply deficit minimization were the main regulation objectives, and hydropower generation the secondary one. Pareto boundary projections

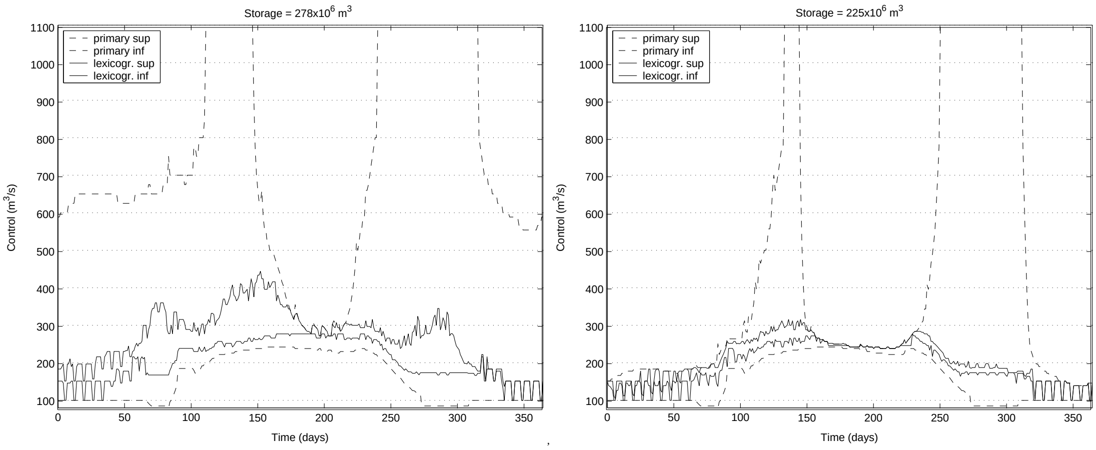

Figure 6: Sections of the optimal control set before and after lexicographic optimization.

were obtained for both primary and secondary optimisation problems showing how the sets of admissible control values shrink during the iterated application of lexicographic optimisations. Nevertheless, the decision maker retains some autonomy in his/her decisions, while respecting the priority constraints in water use and optimising the overall performance of the water system.

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