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Geophysical Institute, Graduate School of Science, Tohoku University

Aoba-Ku, Sendai, 980-8578, Japan (sha@wind.geophys.tohoku.ac.jp)

Abstract: We intend to develop a dynamical core for a new atmospheric meso-scale numerical model, which is expected to suitably treat the steep topography and complex objects on the earth's surface with a finer resolution. In this work, the finite volume method (FVM) in conjunction with the SIMPLER (Semi-Implicit Method for Pressure-Linked Equation Revised) algorithms is used for calculations of the unsteady, three-dimensional, compressible Navier-Stokes equations on a staggered grid. Abandoning the customary terrain-following normalization, we choose the Cartesian coordinate in which the height is used as the vertical one. Blocking-off method is introduced to handle all of the steep topography and complex objects above the earth's sea-mean level. For the spatial and temporal discretizations, higher-order upwind convection scheme is employed, and fully time implicit scheme is utilized. As a preliminary test, the model has been run on flows over a cube mounted on surface. Result of simulations is present, which shows the potential of our proposed approaches for the next-generation atmospheric meso-scale model development.

Keywords: Cartesian coordinate; fully time implicit scheme; higher-order upwind convection scheme; finite volume method; steep topography; complex object; flow modelling

1. INTRODUCTION

Since terrain-following vertical coordinate (sigma) system (Phillips 1957; Gal－Chen and Somerville 1975) has been used extensively to accommodate orography in models for atmospheric flows, most of existing community meso-scale atmospheric numerical model in the world are using the terrain-following coordinate as the vertical coordinate. However, a problem that has received attention rather early in the development of sigma system primitive equation models is that of the noncancellation errors in the two terms of the gradient force in the momentum equation (Smagoringsky, et al. 1967). The two terms on the expression of the pressure gradient force have comparable magnitudes with opposite sign over steep topography and thus their sum may be subject to large errors. Mesinger and Janjic (1985), among others (Sundqvist, 1976), have found that errors in computing the horizontal pressure gradient force in models using a sigma coordinate can be substantial in the vicinity of steep topography. To minimize this error, a step-mountain vertical coordinate, the so-called "eta coordinate", is implemented in the National Centers for Environmental prediction (NCEP) Meso Eta Model (Mesinger et al. 1988) in which the topography is represented as discrete steps (step mountain). However, the step-mountain representation can cause spurious perturbations at step corners and its accuracy may depend strongly on the horizontal scale of the terrain and the resolution of the actual terrain by the vertical grid (Gallus and Klemp, 2000).

Recently, representation of topography, i.e., the "shaved cell" approach, and the related numerical schemes for the equations of geophysical flows in ocean and atmosphere models in which the height is used as vertical coordinate, have been proposed formulated on the finite volume method (Adcroft et al. 1997; Marshall et al. 1997; Bonaventura 2000). With a rapid evolution of computing technology and implementation of massively computers in applications, it is reasonable to expect that a goal of running a regional model at a horizontal resolution of O (100) m may be attainable in the near future, and the topography may then be more accurately represented by the increased resolution. In such a situation, it seems natural to search for an alternative that will be better suited to handle the steep topography and complex objects on the surface for the high-resolution models currently used as well as future next-generation models expected to run with a finer resolution.

In this work, we present several advanced numerical methods based on finite volume discretization (i.e., fully time implicit scheme and higher-order upwind convection scheme, SIMPLER algorithm, blocking-off method for handling complex geometry), and incorporate them into coding of a robust, efficient and accurate dynamical core for the next-generation atmospheric meso-scale numerical model which is expected to suitably treat the steep topography and complex objects with a finer resolution for high-resolution mesoscale flow simulations.

2. DYNAMICAL CORE DESCRIPTIONS

2.1 Governing equations

Three-dimensional, unsteady Navier-Stokes equations with the energy equation, continuity equation and equation of ideal gas state for viscous compressible Newtonian fluid are used.

2.2 Temporal-spatial discretizations and SIMPLER algorithm

The integration of the momentum equations over the control volume would give

$$\frac{\rho_P\phi_P - \rho_P^0\phi_P^0}{\Delta t}\Delta x\Delta y\Delta z + J_e - J_w + J_n - J_s + J_t - J_b = \left(S_C + S_P\phi_P\right)\Delta x\Delta y\Delta z$$

In a similar manner, we can integrate the continuity equation over the control volume and obtain

$$\frac{\rho_P - \rho_P^0}{\Delta t}\Delta x\Delta y\Delta z + F_e - F_w + F_n - F_s + F_t - F_b = 0$$

After some arrangement, we obtain the three-dimensional discretization equation as

$$a_P\phi_P = a_E\phi_E + a_W\phi_W + a_N\phi_N + a_S\phi_S + a_T\phi_T + a_B\phi_B + b \qquad (1)$$

where

$$a_P = a_E + a_W + a_N + a_S + a_T + a_B + a_P^0 - S_P\Delta x\Delta y\Delta z$$

$$b = S_C\Delta x\Delta y\Delta z + a_P^0\phi_P^0$$

$$a_P^0 = \frac{\rho_P^0\Delta x\Delta y\Delta z}{\Delta t}$$

$$a_E = D_e A\left(|P_e|\right) + \langle -F_e, 0\rangle$$

$$a_W = D_w A\left(|P_w|\right) + \langle F_w, 0\rangle$$

$$a_N = D_n A\left(|P_n|\right) + \langle -F_n, 0\rangle$$

$$a_S = D_s A\left(|P_s|\right) + \langle F_s, 0\rangle$$

$$a_T = D_t A\left(|P_t|\right) + \langle -F_t, 0\rangle$$

$$a_B = D_b A\left(|P_b|\right) + \langle F_b, 0\rangle$$

The function $A(|P|)$ can be selected from Table 1 for desired scheme, and for more details we would like to make a reference to Pataker(1980), Sha, et al(1991), Ferziger and Peric(1997).

So we see that for the temporal integration of the equation the fully time implicit scheme is utilized. As the fully implicit temporal discretization is used, the time step can be determined only physical criteria and accuracy considerations. The spatial discretization is obtained by the finite volume technique on the staggered grid, and higher-order upwind convection scheme can be chosen to relate the flux at each control volume face.

The coupling system for the velocity and the pressure in the discretized equations is solved by the SIMPLER (Semi-Implicit Method for Pressure-Linked Equation Revised) algorithm(Patanker, 1980).

2.3 Treatment of irregularly shaped objective in calculation domain

We now describe the manner in which we treat arbitrary geometries by the blocking-off method (Patanker, 1980). This is done by blocking off some of the control volumes of the regular grid, so that the remaining inactive control volumes form the desired irregular domain. Example is shown in Fig.1, where the shaded areas denote the inactive control volumes. It is obvious that arbitrary geometries are approximated by a series of the rectangular grids.

Idea of the blocking-off operation consists of establishing known values of the relevant ϕ's in the inactive control volumes. Here is a simple way in which the desired values can be obtained in the inactive control volumes by setting a large source term in the discretization equations. For example, setting S_C and S_P in Eq. (1) for the internal grid points (i.e., in the solid interior) as

$$S_C = 10^{30}\phi_{p,desired}$$

$$S_P = -10^{30}$$

where 10^{30} denotes a number large enough to make the other terms in the discretization equation negligible. The consequence is that

$$S_C + S_P \phi_p \approx 0,$$

$$\phi_P = -\frac{S_C}{S_P} = \phi_{P,desired}$$

Note that this procedure can be easily used to represent irregularly shaped objective in the calculation domain by inserting such the internal boundary conditions

3. PRELIMINARY TEST RESULT

As a preliminary test, the model has been carried on a direct numerical simulation (DNS) of flows over a surface-mounted cube. The number of grids is $111*111*57$ in x,y,z directions, respectively. In Fig. 2, we show the result of a neutral flow (i.e., Fr=∞) at a Reynolds number Re=800 based on the uniform upstream velocity and the cube's height. It illustrates the flow pattern characterized by a horse-shoe vortex, originating upstream of the obstacle and deflected downstream along the lateral sides by the oncoming flow. We found that no spurious flows are generated around the cube, and the simulations show a satisfying result. This inspires our confidence in the present numerical framework. Further work including three-dimensional computations on flow over/around a steep mountain is in progress.

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Scheme	Formula for $A(\|P\|)$
Central difference	$1 - 0.5\|P\|$
First order upwind	1
Hybrid	$\langle 0, 1 - 0.5\|P\| \rangle$
Power law	$\langle 0, (1 - 0.1\|P\|)^5 \rangle$
Exponential(exact)	$\|P\| / [\exp(\|P\|) - 1]$
Higher order upwind	QUICK

Table 1 The function A (|P|) for different schemes

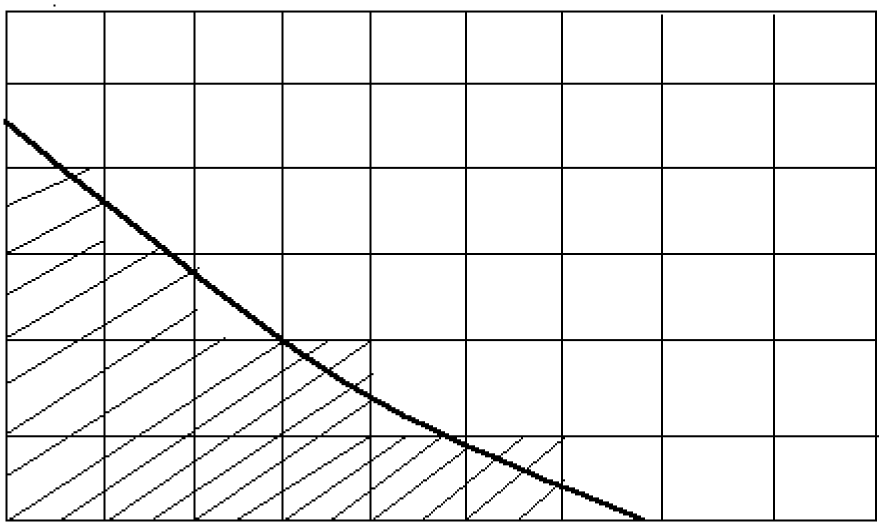

Fig. 1 Blocked-off regions in regular grid

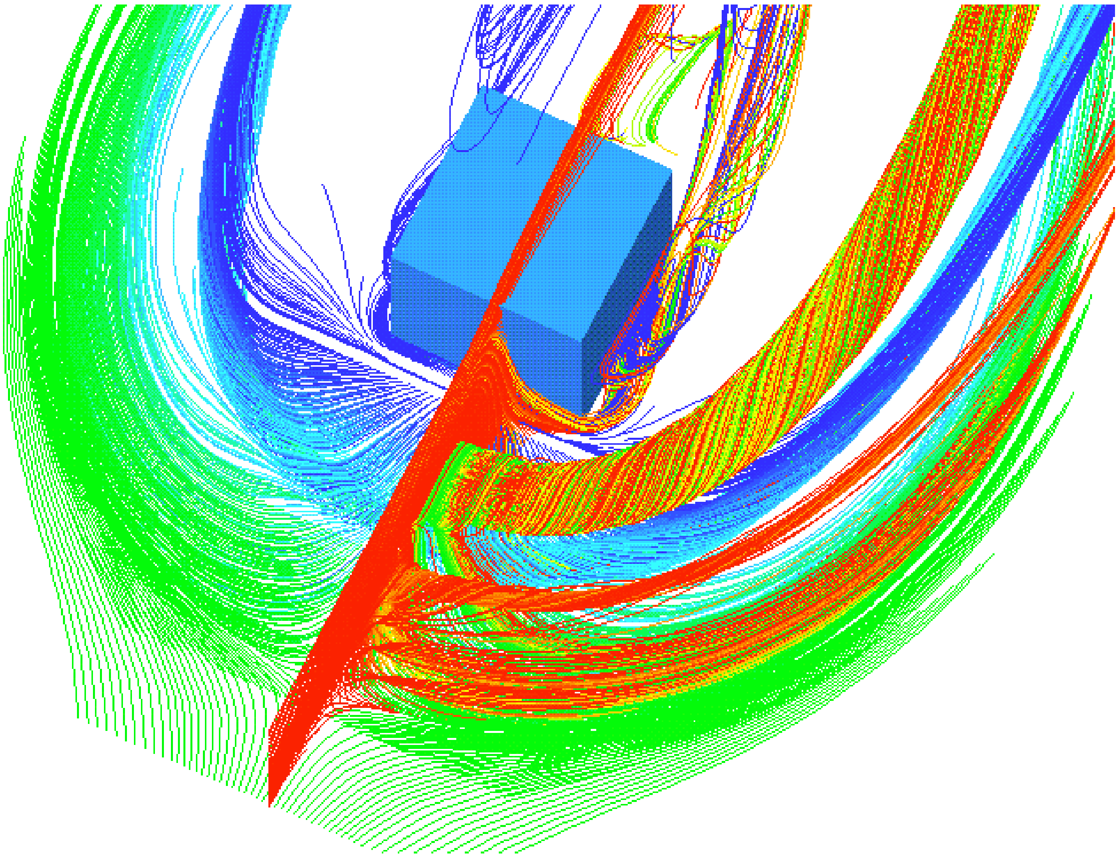

Fig.2 Flow over a cube mounted on surface (Re=800)