A Generic Framework for the Identification of Parsimonious Rainfall-Runoff Models

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Abstract: A task which is often central to hydrological modelling is the identification of an appropriate model structure and a suitable parameter set for a specific case, i.e. a given set of modelling objectives, catchment characteristics and data. However, this identification process is difficult and will often result in a range of possible models, i.e. different parameter sets within a certain model structure, or different model structures. Two generic rainfall-runoff modelling and Monte Carlo analysis toolboxes have been developed to allow for the implementation and subsequent comparison of spatially lumped, metric and parametric model components in order to identify the model(s) and model structure(s) most suitable for a given application. These toolboxes include the use of multi-objective and novel dynamic approaches to performance and identifiability analysis. This enables a more objective analysis of the level of model complexity that is supported by the data. It also enables the modeller to test whether a given model structure is consistent with underlying assumptions, reducing model structural uncertainty. An application of these approaches to a catchment located in the South of England demonstrates the advantages of a flexible framework, combined with novel approaches to model identification.

Keywords: Rainfall-runoff modelling; Model identification; Generic frameworks; Uncertainty.

1. INTRODUCTION

Hydrological models are well-established tools that are widely utilized in engineering practice. The majority of model structures currently used can be classified as conceptual when the definition of Wheater et al. [1993] is applied, i.e. their structure is defined prior to any modelling being undertaken, and at least some of the model parameters are identified from calibration using observed system output.

Conceptual model structures suffer from a number of problems despite their frequent use and development over a number of decades. One of the major constraints is the lack of identifiability, i.e. different combinations of parameters [e.g. Johnston and Pilgrim, 1976], or even different model structures [e.g. Uhlenbrock et al., 1999] yield similar results in terms of a defined performance measure, or objective function. This results in difficulties in interpreting past behaviour of the catchment system, and hence in the propagation of uncertainty into future predictions in the form of wide confidence limits, i.e. a wide range of possible system behaviours [Beven and Binley, 1992].

The need for model calibration is a major limitation when ungauged catchments have to be modelled. One approach to deal with this problem is the regionalisation or regional transfer of parameters of a certain model structure. Uncertainty in the model parameters or structure due to a lack of identifiability significantly limits the use of models for this kind of regionalisation because it is difficult to establish sensible statistical relationships [e.g. Wheater et al., 1993]. A model structure with identifiable parameters, i.e. a high *regionalisation potential*, is therefore a prerequisite for successful regionalisation.

Possible directions of improvement with respect to producing better identified models are: (1) the reduction of model complexity to contain only those components, and therefore parameters, that can be identified from the available data, i.e. parsimonious modelling. There are many publications on this topic and many different ways of addressing the problem; three typical publications are Jakeman and Hornberger [1993]; Young et al. [1996] and Wagener et al. [2002a], (2) the improved use of available information, e.g. using different data periods to identify different parameters or groups of parameters [e.g. Wheater et al., 1986; Wagener

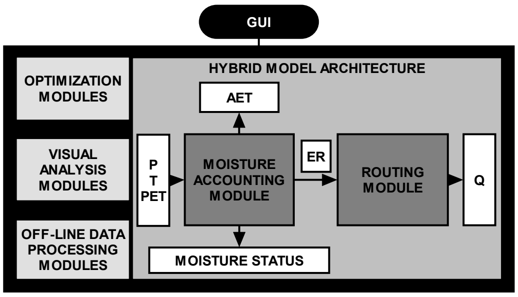

Figure 1. System architecture of the Rainfall-Runoff Modelling Toolbox.

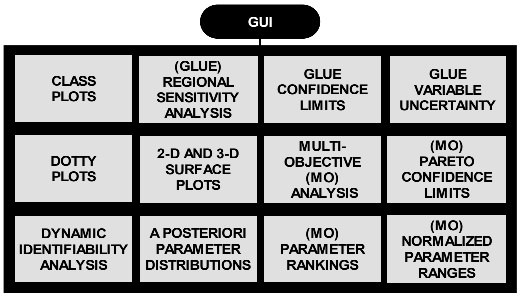

Figure 2. System architecture of the Monte-Carlo Analysis Toolbox.

et al., 2001], and (3) the use of additional information, *i.e.* multi-response data such as water quality data, groundwater levels, or tracer measurements [e.g. Kuczera and Mroczkowski, 1998]. It should be noted that the use of additional output variables is unlikely to be particularly useful with respect to flow regionalisation studies, since relevant multi-response data are not commonly available. Therefore this approach is not investigated further here; instead we focus on methods of reducing model complexity and increasing the information that can be retrieved from streamflow measurements.

To this end, a generic modelling framework has been established to enable the development, analysis and comparison of model structures of different levels of complexity using all available information, i.e. addressing aspects (1) and (2). The aim is to identify the appropriate level of complexity that yields a sufficiently high level of performance, whilst retaining an acceptable level of parameter uncertainty. The framework is described, and a limited number of modelling exercises are presented to illustrate its use.

2. A GENERIC MODELLING FRAMEWORK

The framework consists of two components, a Rainfall-Runoff Modelling Toolbox (RRMT) and a Monte Carlo Analysis Toolbox (MCAT).

2.1 Rainfall-Runoff Modelling Toolbox

As noted above, we seek the development of a model structure of appropriate complexity with respect to model performance and associated uncertainty. The philosophy behind this is the recognition that no model structure is suitable for all modelling tasks, but that the appropriate model structure is a function of: (1) the modelling objectives (e.g. required spatial and temporal discretisation, relevant response modes to be simulated), (2) the characteristics of the hydrological system under investigation (e.g. dominant processes, response times of the system), and (3) the available data (e.g. possible spatial and temporal discretisation).

An increasing number of modelling shells with different levels of complexity can be found in the literature [e.g. Leavesley et al., 2002]. These systems give their user the option to test the suitability of different model components and to combine them in a modular fashion. Components can be modified or added if none of the available components fulfils the problem-specific requirements. The Rainfall-Runoff Modelling Toolbox (RRMT; Wagener et al., 2002a) has been developed in particular to produce parsimonious, lumped model structures with a high level of parameter identifiability (Figure 1).

The RRMT is a generic modelling framework or shell that allows its user to implement different model structures. It can therefore be considered to represent a modelling concept, rather than a specific model structure. The RRMT is implemented in the MATLAB [Mathworks, 1996] programming environment. Each model structure within the RRMT consists of a Soil Moisture Accounting (SMA) and a routing module.

The model structures that can be implemented are spatially lumped with low or medium levels of complexity (in terms of number of parameters). They can be classified as conceptual or hybrid metric-conceptual in type [Wheater et al., 1993]. The latter is related to a systems approach to hydrologic modelling [e.g. Jakeman and Hornberger, 1993]. The aim of this approach is to use observations (the metric paradigm) and other prior knowledge to test hypotheses about the structure of component hydrological stores (the conceptual paradigm).

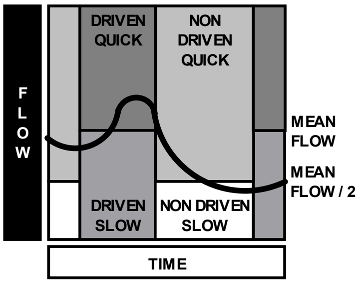

Figure 3. Segmentation scheme used to derive multiple objective functions describing the model performance during different response modes.

2.2 Monte Carlo Analysis Toolbox

The detailed investigation of model performance in terms of parameter sensitivity and identifiability, the suitability of a particular model structure, and prediction uncertainty, are increasingly important parts of any modelling exercise. The understanding of model behaviour and performance increases the transparency of the modelling procedure and helps to assess the reliability of modelling results.

The Monte-Carlo Analysis Toolbox [MCAT, Wagener et al., 2002a] includes a number of analysis methods to evaluate the results of Monte Carlo parameter sampling experiments or model optimisation methods based on population evolution techniques (Figure 2). Functions contained in the MCAT include an extension of the Regional Sensitivity Analysis [RSA, Spear and Hornberger, 1980] proposed by Freer et al. [1996], various components of the Generalized Likelihood Uncertainty Estimation (GLUE) method [Beven and Binley, 1992; Freer et al., 1996], options for the use of multiple-objectives for model assessment [Gupta et al., 1998; Boyle et al., 2000], response surface plots, and an empirical measure to evaluate parametric identifiability for a selected objective function (OF) or in a dynamic fashion.

3. APPLICATION EXAMPLE

3.1 Data

To illustrate the use of the framework, an application example is presented based on the River Medway catchment (1256.1km^2), which is located in South East England. Almost seven years of daily-naturalized flows, rainfall, potential evapotranspiration (PE) and temperature data are available for use in this modelling exercise.

The Medway catchment is characterized by a mixture of permeable (chalk) and impermeable (clay) geologies subject to a temperate climate (annual

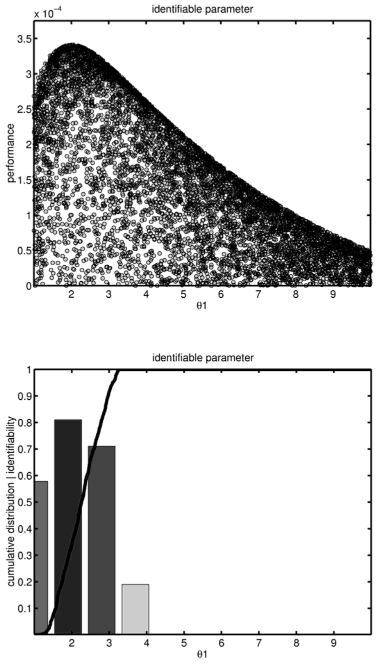

Figure 4. Example of a well identified parameter. The top diagram shows scatter plots of parameter versus measure of performance. The bottom row shows the cumulative distribution of the best performing 10% of parameter sets and the corresponding gradients within each segment of the parameter range.

rainfall of 772 mm and an annual PE of 663 mm over the period 1989-96).

3.2 Methods and Models Structures

Multi-objective (MO) analysis and dynamic identifiability approaches are applied based on the results of Monte Carlo sampling procedures. For the MO analysis, 20000 parameter sets, i.e. models, are randomly sampled from the feasible parameter space for each individual model structure, based on a uniform distribution.

For each of these models, five OFs are calculated. These are the overall RMSE and four OFs derived for different response modes of the catchment. The segmentation applied uses the slope of the hydrograph and an additional threshold as segmentation criteria to split the hydrograph into different response modes (Figure 3). The slope separates periods when the catchment is wetting up or is *driven* [Boyle et al., 2000] by rainfall, i.e. positive slope, and when the catchment is draining, i.e. negative slope. A threshold is used to separate periods of high and low flow, i.e. the mean flow during driven periods and 50% of the mean flow during drainage periods. Four OFs are therefore derived when the residuals during the different periods are aggregated separately using the RMSE criterion: FDH, driven flow during high flow, FDL, driven flow during low flow, FQ, quick drainage (high flows), and FS, slow drainage (low flows). This is a modification of the initial approach by Boyle et al. [2000], which was based on the analysis of flow and rainfall. However, the approach used here has shown to be more suitable for British catchments as modelled in the example presented. These OFs are based on the assumption that different processes are dominant during periods of high and low flow, and during periods of catchment wetting-up and drainage. The residuals, i.e. the differences between observed and simulated flows, are calculated and summarised in the form of the RMSE for each period,

$$RMSE(\theta) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(y_k - \hat{y}_k(\theta)\right)^2} \qquad (1)$$

where $\hat{y}_k(\theta)$ is the calculated flow at time step k using parameter set θ, and y_k is the corresponding observed flow, while N is the total number of time steps considered. Both the performance and identifiability analyses are based on these OFs.

The identifiability measure used is based on the (initially uniform) univariate parameter distributions conditioned on the different OFs (Figure 4). One can select the top 10% of values for each parameter, calculate their cumulative distribution and subsequently split each parameter range individually into 10 bins. The gradient of the cumulative distribution in each bin serves as an empirical measure of identifiability. The use of system identification techniques might also allow answering the question of how well some parameters are defined in a statistical manner without referring to a more general Monte Carlo approach. This option is not yet (completely) implemented in the framework described here.

The resulting parameter populations are used to rank all models or model structures, with respect to their performance and identifiability, using the identifiability measure introduced above. The best model structures are retained and a more thorough analysis using a dynamic approach (termed DYNIA) is performed (Wagener et al., 2002b). DYNIA is based on a random sampling procedures using 2500 parameter sets collected from a uniform distribution. The smaller sample size is due to computational limitations of the current DYNIA application in the MATLAB [Mathworks, 1996] environment. Within the DYNIA approach, the OF

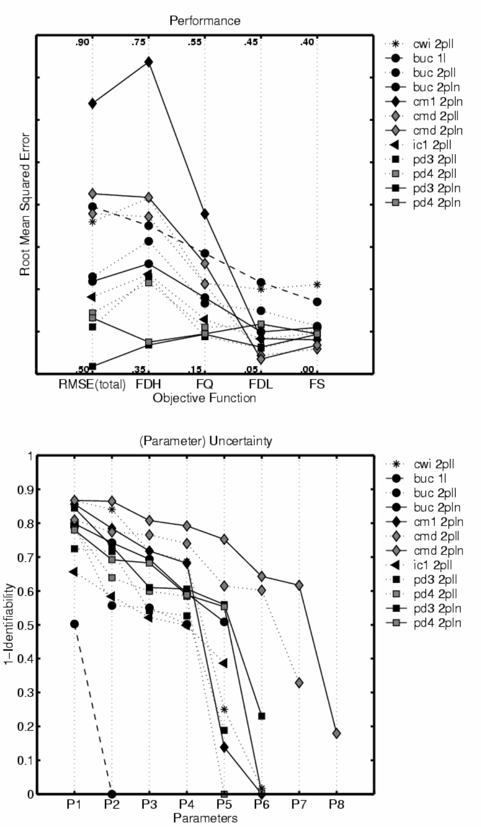

Figure 5. Results of the model structure comparison.

(and therefore the parameter conditioning) is calculated as a moving average value. An identifiability value is thus derived for every time-step and periods of information and noise can be separated.

A large variety of lumped parsimonious model structures can be found in the literature [e.g. Singh and Frevert, 2002]. However, the range of components on which these structures are based is relatively small. Some of the most commonly found components are selected here.

The SMA components used are:

- The Catchment Moisture Deficit [cmd, Evans and Jakeman, 1998]. A conceptual bucket with a bottom outlet to sustain drainage into the summer periods.
- The Catchment Wetness Index [cwi, Jakeman and Hornberger, 1993]. A metric approach based on the Antecedent Precipitation Index (API).
- The Probability Distributed soil moisture stores [pd3 and pd4, Moore, 1999]. A probability distribution of conceptual buckets based on a Pareto distribution. Evapotranspiration is either at the potential rate (pd3), as long as soil moisture is available, or at a rate declining linearly with soil moisture content (pd4).

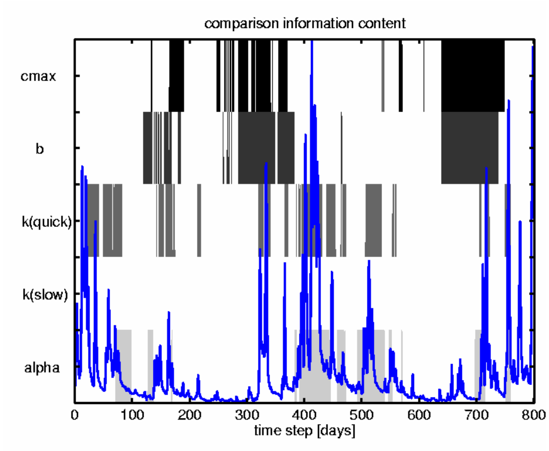

Figure 6. Periods of high information content for individual parameters. Plotted are the 100 most informative time-steps, i.e. with the smallest width of confidence limits (cfls) width. Values for k(slow) are not plotted in this example due to consistently wide cfls.

- A simple bucket type structure (buc), evaporating at the potential rate as long as soil moisture is available.
- The Penman storage model [ic1, Penman, 1949]. A two layered structure connected by an overflow mechanism. Evapotranspiration occurs at potential rate from the upper layer, similar to the root zone, and at a reduced rate, 1/12 of PE, from the bottom layer. An additional bypass mechanism diverts a fraction of the rainfall from the SMA component to contribute to the effective rainfall at time-steps where rainfall exceeds PE.

The routing components used are:

- Conceptual reservoirs in various combinations and in linear and non-linear form. In detail (Figure 5): a single linear reservoir (1l), two linear reservoirs in parallel (2pll), and a linear and a non-linear reservoir in parallel (2pln).

3.3 Results and Discussion

The main results of the MO analysis as shown in Figure 5 are as follows:

- At a general level for the SMA modules (Figure 5, top): the probability distributions of storage elements (pd3 and pd4) seem to perform best, followed by the simple bucket (buc), and the cmd and cwi modules.
- The cm1, i.e. a cmd that always evaporates at the potential rate, performs much more poorly than the rest with respect to those objective functions which mainly describe periods of high flow, RMSE(total), FDH and FQ. This is also the case for the cmd module, but not as pronounced. However, the cmd and cm1 modules do very well during low flow periods. This is caused by the bottom outlet of the bucket, which sustains the production of effective rainfall even during periods of severe moisture deficits in the SMA module.
- The overall result of the performance analysis is that the pd3 and pd4 SMA modules in combination with 2pll or 2pln routing modules are superior. The cmd is a useful component when the modelling purpose demands the accurate prediction of low flow periods and periods of high flows are of minor importance.
- A detailed analysis of the routing components shows that the use of a non-linear conceptual reservoir in parallel with a linear one (2pln), performs better at the peaks (RMSE(total) and FDH), see Figure 5(top).
- The uncertainty analysis (Figure 5, bottom) however reveals that the identifiability of the cmd parameters is very low and this module can be rejected on this basis. For some applications, this aspect might be of minor importance, however.

The pd3 and the pd4 SMA components are retained for further analysis with the DYNIA approach. Assuming that our interest is in low flows, e.g. for water resources purposes, only a linear parallel routing structure (2pll) is considered. A non-linear component would be advisable for high flow periods.

The DYNIA results for pd4-2pll in Figure 6 show the periods of identifiability (information) for the different model parameters. This information can for example be used to create tailor-made OFs for individual parameters or parameter groups. In general, DYNIA enables a more detailed assessment of model structures, e.g. based on an investigation of the temporal variability in optimum parameter estimates.

4. CONCLUSIONS

The toolkit presented here facilitates the development and analysis of lumped and parsimonious model structures using state-of-the-art modelling techniques.

- The RRMT allows the implementation of conceptual, or hybrid metric-conceptual model structures. Its major advantage is a high degree of structural flexibility, which allows the quick implementation, and evaluation of different model structures to identify the most suitable one(s) for the circumstances at hand.

- The MCAT enables the detailed investigation of model performance, parameter sensitivity and identifiability, model structure suitability, and prediction uncertainty.
- The application example shows how MO and dynamic approaches can be used to derive a more objective model analysis.

Both toolboxes can be freely downloaded for non-commercial use from http://ewre.cv.ic.ac.uk.

5. ACKNOWLEDGEMENTS

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